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vcj239

20 a) $S = \begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix} \xrightarrow{R_2 \to R_2 - 3R_1} \sim \begin{bmatrix} 1 & 3 \\ 0 & -7 \end{bmatrix}$

$D = \begin{bmatrix} 1 & 0 \\ 0 & -7 \end{bmatrix}$ $\quad L = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ $\quad L^T = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

$S = \begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -7 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

b) $S = \begin{bmatrix} 1 & b \\ b & c \end{bmatrix} \xrightarrow{R_2 \to R_2 - bR_1} \begin{bmatrix} 1 & b \\ 0 & c-b^2 \end{bmatrix}$ $\quad D = \begin{bmatrix} 1 & 0 \\ 0 & c-b^2 \end{bmatrix}$

$L = \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix}$ $\quad L^T = \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix}$ $\quad S = \begin{bmatrix} 1 & b \\ b & c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix} \cdots$

$\cdots \begin{bmatrix} 1 & 0 \\ 0 & c-b^2 \end{bmatrix} \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix}$

c) $S = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \sim \begin{bmatrix} 2 & -1 & 0 \\ 0 & 3/2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \sim \begin{bmatrix} 2 & -1 & 0 \\ 0 & 3/2 & -1 \\ 0 & 0 & 4/3 \end{bmatrix}$

$D: \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3/2 & 0 \\ 0 & 0 & 4/3 \end{bmatrix}$ $\quad L = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 0 & -2/3 & 1 \end{bmatrix}$

$S = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 0 & -2/3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3/2 & 0 \\ 0 & 0 & 4/3 \end{bmatrix} \begin{bmatrix} 1 & -1/2 & 0 \\ 0 & 1 & -2/3 \\ 0 & 0 & 1 \end{bmatrix}$

22 a) $PA = LU$, $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

$P: \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 3 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix} = U$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix} \qquad PA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

b) $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 2 & 4 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \sim U$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \qquad PA = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

19) a) $(A^TSA)^T = A^{TT}S^TA^T = AS^TA^T = A^TSA$
(since $S = S^T$ for $n \times n$ matrix)

b) for A^TA, $A^T_{ii} = A_{ii}$, so on diagonal, $(A^TA)_{ii} = (A)_{ii}^2 = (A^T)_{ii}^2 \geq 0$, so there will be no negative values on the diagonal.

3.1 10) a) Yes, $b_1 = b_2$ in $(b_1\ b_2\ b_3)$ is in $\mathbb{R}^3$

b) No, $b_1 = 1$ in (b_1, b_2, b_3) is not in $\mathbb{R}^3$ (addition gets $b_1 = 2$)

c) No, $b_1 b_2 b_3 = 0$ in $(b_1\ b_2\ b_3)$ is not $\mathbb{R}^3$ (addition won't result in $b_1, b_2, b_3 = 0$)

d) Yes, $v = (1\ 4\ 0)$ and $w = (2, 2, 2)$ in $\mathbb{R}^3_2$

e) $b_1 + b_2 + b_3 = 0$ is in $\mathbb{R}^3$

f) No $b_1 \leq b_2 \leq b_3$ is not in $\mathbb{R}^3$ (addition won't result in rule)

18 a) True, forms subspace

b) True, forms subspace

c) False, doesn't form subspace.

19. A: span $\left\{\begin{pmatrix}1\\0\\0\end{pmatrix}\right\}$ or x-axis $\begin{pmatrix}x\\0\\0\end{pmatrix}^T$.

B: xy-plane, $\begin{pmatrix}x\\y\\0\end{pmatrix}$; C is line of vectors on ~~span $\left\{\begin{pmatrix}1\\2\\0\end{pmatrix}\right\}$~~ or on $\begin{pmatrix}x\\2x\\0\end{pmatrix}$

B.2 1) a)

$$\begin{bmatrix}1&2&2&4&6\\1&2&3&6&9\\0&0&1&2&3\end{bmatrix} \sim \begin{bmatrix}1&2&2&4&6\\0&0&1&2&3\\0&0&1&2&3\end{bmatrix} \sim \begin{bmatrix}1&2&2&4&6\\0&0&1&2&3\\0&0&0&0&0\end{bmatrix}$$

x_2, x_4, x_5 are free

b)

$$\begin{bmatrix}2&4&2\\0&4&4\\0&8&8\end{bmatrix} \sim \begin{bmatrix}2&4&2\\0&4&4\\0&0&0\end{bmatrix} \sim \begin{bmatrix}1&2&1\\0&1&1\\0&0&0\end{bmatrix}$$

x_3 is free

2) a) For $x_2 = 1$: we get $(-2, 1, 0, 0, 0)^T$

For $x_4 = 1$: we get $(0, 0, -2, 1, 0)^T$

For $x_5 = 1$: we get $(0, 0, -3, 0, 1)^T$

b) Only sol: $(1, -1, 1)^T$

4) a) $Ax = 0$, $A = \begin{bmatrix} -1 & 3 & 5 \\ -2 & 6 & 10 \end{bmatrix} \sim \begin{bmatrix} -1 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$

x_2 : x_3 free

$x_2 = 1$: we get $(3, 1, 0)$, $x_3 = 1$: $(5\ 0\ 1)$

b) $x = 0$, $B = \begin{bmatrix} -1 & 3 & 5 \\ -2 & 6 & 7 \end{bmatrix} \sim \begin{bmatrix} -1 & 3 & 5 \\ 0 & 0 & -3 \end{bmatrix}$

x_2 is free

$x_2 = 1$: weget $(3\ 1\ 0)$

For $m \times n$, number of pivot vars.s + number of free var.s $= n$