

Functions Gone Wild 2023

This problem will be worth 16 points. Each successfully implemented function will earn you a point. There is the student tester and one test for each of the 10 functions for a total of 11 possible points. Additional bonus points will be awarded for correctly implementing functions 1-3, functions 4 - 6, functions 7 and 8, and functions 9 and 10. A final bonus point will be awarded for correctly implementing all 10 functions.

This problem will ask you to implement the following functions. One function is defined recursively. On some problems I will give you functions from the `java.lang.Math` class with a brief description, and sometimes you just need to implement the function. In all the following functions you do **NOT** need worry about domain issues. That is, you may assume that all test data will not cause any exceptions to be thrown. In total, there are ten different functions in this problem. All methods which return a `double` must return a value 'close enough' (less than some $\delta > 0$) to the correct answer to be considered correct.

You should use:

• <code>Math.max(a,b)</code> for $\max(a, b)$.	• <code>Math.min(a, b)</code> for $\min(a, b)$
• <code>Math.abs(x)</code> for $ x $.	• <code>Math.cos(a)</code> for $\cos(a)$
• <code>Math.sqrt(x)</code> for \sqrt{x}	• <code>Math.sin(a)</code> for $\sin(a)$
• <code>Math.pow(x, 1.0/n)</code> for $\sqrt[n]{x}$	• <code>Math.tan(a)</code> for $\tan(a)$
• <code>Math.log(a)</code> for $\ln(a)$	• <code>Math.log10(a)</code> for $\log_{10}(a)$ or $\log(a)$
• <code>Math.ceil(a)</code> for $\lceil a \rceil$.	• <code>Math.floor(a)</code> for $\lfloor a \rfloor$.

note:

- All trig functions are in radians.
- `Math.abs(int a)` returns an `int` and `Math.abs(double a)` returns a `double`
- `Math.max(int a, int b)` and `Math.min(int a, int b)` returns an `int`.
- `Math.max(double a, double b)` and `Math.min(double a, double b)` returns a `double`.
- Return type of all other methods is `double`.
- $\lfloor x \rfloor$ is the largest (Closes to positive infinity) double value smaller than or equal to the argument, x , and is equal to a mathematical integer.. For example, $\lfloor 2.9 \rfloor = 2.0$ and $\lfloor -14.3 \rfloor = -15.0$.
- $\lceil x \rceil$ is the smallest (Closes to negative infinity) double value that is greater than or equal to the argument, x , and is equal to a mathematical integer. For example, $\lceil 1.1 \rceil = 2.0$ and $\lceil -24.9 \rceil = -24$.
- Use the following constant for pi (π)

<code>Math.PI</code>	The <code>double</code> value that is closer than any other to π , the ratio of the circumference of a circle to its diameter.
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- Use the following constant for e

<code>Math.E</code>	The <code>double</code> value that is closer than any other to e , the base of the natural logarithms.
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1. FizzBuzz Problem: Given an `int`, return a `String` according to the following rules:

First break the number in to a front half and a back half. For example:

1234 has a front half of 12 and a back half of 34

1212 has a front half of 12 and a back half of 12

When there are an odd number of digits, ignore the middle digit. For example:

12345 has a front half of 12 and a back half of 45

23456 has a front half of 23 and a back half of 56

return a `String` according to the following rules

- If the front half of the number is greater than the back half return the `String` "FRONT".
- If the front half of the number is less than the back half return the `String` "BACK".
- If the two halves are equal return the `String` "FRONTBACK".

Use following function heading:

```
public static String f1(int n)
```

Test data: `f1(1234)` returns "BACK"
`f1(27826)` returns "FRONT"
`f1(527527)` returns "FRONTBACK"

2. The motivation for this question comes from the following problem:

Given a `String str` ("d3739d0" for example) and an `int div` (60 for example), return the smallest digit that replaces `d` so that `d3739d0` is divisible by 60? You may assume $0 \leq d < 9$.

Use following function heading:

```
public static int f2(String str, int div)
```

Test data: `f2("d3739d0", 60)` returns 4
373900 % 60 = 40
1373910 % 60 = 30
2373920 % 60 = 20
3373930 % 60 = 10
4373940 % 60 = 0

`f2("16d2d", 60)` returns 0
16020 % 60 = 0

`f2("16d", 7)` returns 1
160 % 7 = 6
161 % 7 = 0
168 % 7 = 0

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3. Implement the following integer recursive function. All calculations shall be computed using Integer math.

$$f3(n) = \begin{cases} f3\left(\frac{n^2 + 21}{5(n-5)}\right) + \frac{n}{3} & n \geq 200, n \text{ is even} \\ f3\left(\frac{7n-13}{3(n-4)} + \frac{n}{5}\right) + 2 & n \geq 200, n \text{ is odd} \\ f3\left(2n \% \left(1 + \frac{n}{4}\right)\right) + \frac{2n+1}{3} & 50 < n < 200 \\ |n-2|^{(n\%5)} & \text{otherwise} \end{cases}$$

Use following function heading:

```
public static int f3(int n)
```

```
Test data:  f3(22) returns 400 = |22 - 2|22%5
            f3(135) returns 990 = f3(32) + 90
                                     = 900 + 90
            f3(501) returns 470 = f3(102) + 2
                                     = f3(22) + 68 + 2
                                     = 400+68+2
            f3(500) returns 234 = f3(101) + 166
                                     = f3(20) + 67 + 166
                                     = 1 + 67 + 166
```

4. Implement the following function

Consider the mathematical notation $\sum_{i=m}^n (someFunction)$ used to represent the summation of many similar terms.

The notation $\sum_{i=m}^n (h(i))$ is defined as: $h(m)+h(m+1)+h(m+2)+...+h(n)$

The subscript gives the symbol for an index variable, i . Here, i represents the index of summation; m is the lower bound of summation, and n is the upper bound of summation. In this case, $i = m$ under the summation symbol means that the index i starts equal to m . Successive values of i are found by adding 1 to the previous value of i , continuing up to and including when i equals n . An example: $\sum_{k=2}^6 k^2 = 2^2 + 3^2 + 4^2 + 5^2 + 6^2 = 90$.

Your task in this problem is to implement the following function.

Note – all calculations are to be completed using integer math

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$$f4(a, b, c) = \sum_{i=\max(ab-c, ac-b)}^{\min(2ab+c, 2bc+a)} \left(\left\lceil \frac{2c^2(a+b)}{a+|b-c|} \right\rceil i^2 + \left\lceil \frac{\max(b, c) * \max(ab, |a-c|)}{\max(\min(ac, bc), \min(2b, 3c))} \right\rceil i \right)$$

This is equivalent to the following:

$$\begin{aligned} & \left(\left\lceil \frac{2c^2(a+b)}{a+|b-c|} \right\rceil (\max(ab-c, ac-b))^2 + \left\lceil \frac{\max(b, c) * \max(ab, |a-c|)}{\max(\min(ac, bc), \min(2b, 3c))} \right\rceil (\max(ab-c, ac-b)) \right) \\ & + \left(\left\lceil \frac{2c^2(a+b)}{a+|b-c|} \right\rceil (1 + \max(ab-c, ac-b))^2 \right. \\ & \quad \left. + \left\lceil \frac{\max(b, c) * \max(ab, |a-c|)}{\max(\min(ac, bc), \min(2b, 3c))} \right\rceil (1 + \max(ab-c, ac-b)) \right) \\ & + \left(\left\lceil \frac{2c^2(a+b)}{a+|b-c|} \right\rceil (2 + \max(ab-c, ac-b))^2 \right. \\ & \quad \left. + \left\lceil \frac{\max(b, c) * \max(ab, |a-c|)}{\max(\min(ac, bc), \min(2b, 3c))} \right\rceil (2 + \max(ab-c, ac-b)) \right) \\ & ++ \\ & + \left(\left\lceil \frac{2c^2(a+b)}{a+|b-c|} \right\rceil (\min(2ab+c, 2bc+a))^2 \right. \\ & \quad \left. + \left\lceil \frac{\max(b, c) * \max(ab, |a-c|)}{\max(\min(ac, bc), \min(2b, 3c))} \right\rceil (\min(2ab+c, 2bc+a)) \right) \end{aligned}$$

Special Note: if $\min(ab+c, bc+a) < \max(a-b, 1+|b-c|)$, return 0.

Use following function heading:

```
public static int f4(int a, int b, int c)
```

Test data:

```
f4(1, 1, 2)
returns = f(1) + f(2) + f(3) + f(4) = 9 + 34 + 75 + 132 = 250
```

```
f4(3, 2, 6)
returns = f(16) + f(17) + f(18) = 7712 + 8704 + 9756 = 44472
```

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5. Phone math: The distance your fingers walk when dialing the number:

$$\text{"385-5395"} = 3 + 2\sqrt{2} + \sqrt{5}$$

$$\text{"1-372-485-6709"} = 4 + 6\sqrt{2} + 2\sqrt{5}$$

The distance from 1 to 2 and 1 to 4 is 1.

The distance from 1 to 5 is $\sqrt{2}$.

The distance from 1 to 8 is $\sqrt{5}$.

The distance from 1 to 9 is $2\sqrt{2}$.

The distance from 1 to 0 is $\sqrt{10}$.

All phone numbers will be either a 7 digit numbers with a single hyphen or an 11 digit number with three hyphens as shown in the examples.



Use following function heading:

-

```
public static double f5(String num)
```

Test data: `f5("385-5395")` returns $3 + 2\sqrt{2} + \sqrt{5} = 8.064495102$

Test data: `f5("1-372-485-6709")` returns $4 + 6\sqrt{2} + 2\sqrt{5} = 16.95741733$

6. Implement the following function

$$f6(x, y, z) = \begin{cases} \log(e^{2x-y}) & \tan(x) > \cos\left(\frac{z+\pi}{3y}\right) \\ \pi * \ln(y^{e+z}) & \tan(x) \leq \cos\left(\frac{z+\pi}{3y}\right) \end{cases}$$

Use following function heading:

```
public static double f6(double x, double y, double z)
```

Test data: `f6(Math.PI/4., 5*Math.PI/6., Math.PI/8)` returns -0.4547921179

`f6(7*Math.PI/13., 7*Math.PI/12., Math.PI/9)` returns 5.83706310

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7. The motivation for this problem comes from the following:

This problem was asked by Microsoft.

Given a string, generate all possible subsequences of the string.

For example, given the string `xyz`, return an array or set with the following strings:

```
x
y
z
xy
xz
yz
xyz
```

Note that `zx` is not a valid subsequence since it is not in the order of the given string.

In this problem you will be given a Master String and a potential subsequence. You will implement a function that returns true if the potential subsequence is a subsequence of the Master String.

Use following function heading:

```
public static boolean f7(String ms, String sq)
```

Test data: `f7("xyz", "xz")` returns true

Test data: `f7("xyz", "zx")` returns false

8 Implement the following function

This problem was asked by Slack.

You are given a string formed by concatenating several words corresponding to the integers zero through nine and then anagramming.

For example, the input could be `"NIESEVEHRTFEEV"`, which is an anagram of `"THREEFIVESEVEN"`. Note that there can be multiple instances of each integer.

Given a String, return A String containing the original integers in sorted order. In the example above, this would be `"357"`.

Use following function heading:

```
public static String f8(String phrase)
```

Test data: `f8("NIESEVEHRTFEEV")` returns `"357"`

`f8("WZOZEOERTROTWO")` returns `"0022"`

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9. Implement the following function

$$f_9(x, y, z) = \begin{cases} \text{false} & x = \text{false} & y = \text{false} & z = \text{false} \\ \text{false} & x = \text{false} & y = \text{false} & z = \text{true} \\ \text{false} & x = \text{false} & y = \text{true} & z = \text{false} \\ \text{true} & x = \text{false} & y = \text{true} & z = \text{true} \\ \text{true} & x = \text{true} & y = \text{false} & z = \text{false} \\ \text{true} & x = \text{true} & y = \text{false} & z = \text{true} \\ \text{false} & x = \text{true} & y = \text{true} & z = \text{false} \\ \text{false} & x = \text{true} & y = \text{true} & z = \text{true} \end{cases}$$

Use following function heading:

```
public static boolean f9(boolean x, boolean y, boolean z)
```

Test data: `f9(false, false, false)` returns false

10. Implement the following function

$$f_9(x, y, z) = \begin{cases} \text{true} & j = \text{false} & k = \text{false} & m = \text{false} & n = \text{false} \\ \text{true} & j = \text{false} & k = \text{false} & m = \text{false} & n = \text{true} \\ \text{true} & j = \text{false} & k = \text{false} & m = \text{true} & n = \text{false} \\ \text{false} & j = \text{false} & k = \text{false} & m = \text{true} & n = \text{true} \\ \text{false} & j = \text{false} & k = \text{true} & m = \text{false} & n = \text{false} \\ \text{true} & j = \text{false} & k = \text{true} & m = \text{false} & n = \text{true} \\ \text{true} & j = \text{false} & k = \text{true} & m = \text{true} & n = \text{false} \\ \text{false} & j = \text{false} & k = \text{true} & m = \text{true} & n = \text{true} \\ \text{false} & j = \text{true} & k = \text{false} & m = \text{false} & n = \text{false} \\ \text{true} & j = \text{true} & k = \text{false} & m = \text{false} & n = \text{true} \\ \text{false} & j = \text{true} & k = \text{false} & m = \text{true} & n = \text{false} \\ \text{false} & j = \text{true} & k = \text{false} & m = \text{true} & n = \text{true} \\ \text{true} & j = \text{true} & k = \text{true} & m = \text{false} & n = \text{false} \\ \text{true} & j = \text{true} & k = \text{true} & m = \text{false} & n = \text{true} \\ \text{true} & j = \text{true} & k = \text{true} & m = \text{true} & n = \text{false} \\ \text{true} & j = \text{true} & k = \text{true} & m = \text{true} & n = \text{true} \end{cases}$$

Use following function heading:

```
public static boolean f10(boolean j, boolean k, boolean m, boolean n)
```

Test data: `f10(false, false, false, false)` returns true