UNIVERSITY OF NEW SOUTH WALES

GSOE9712 - 20T1 - Assignment 1

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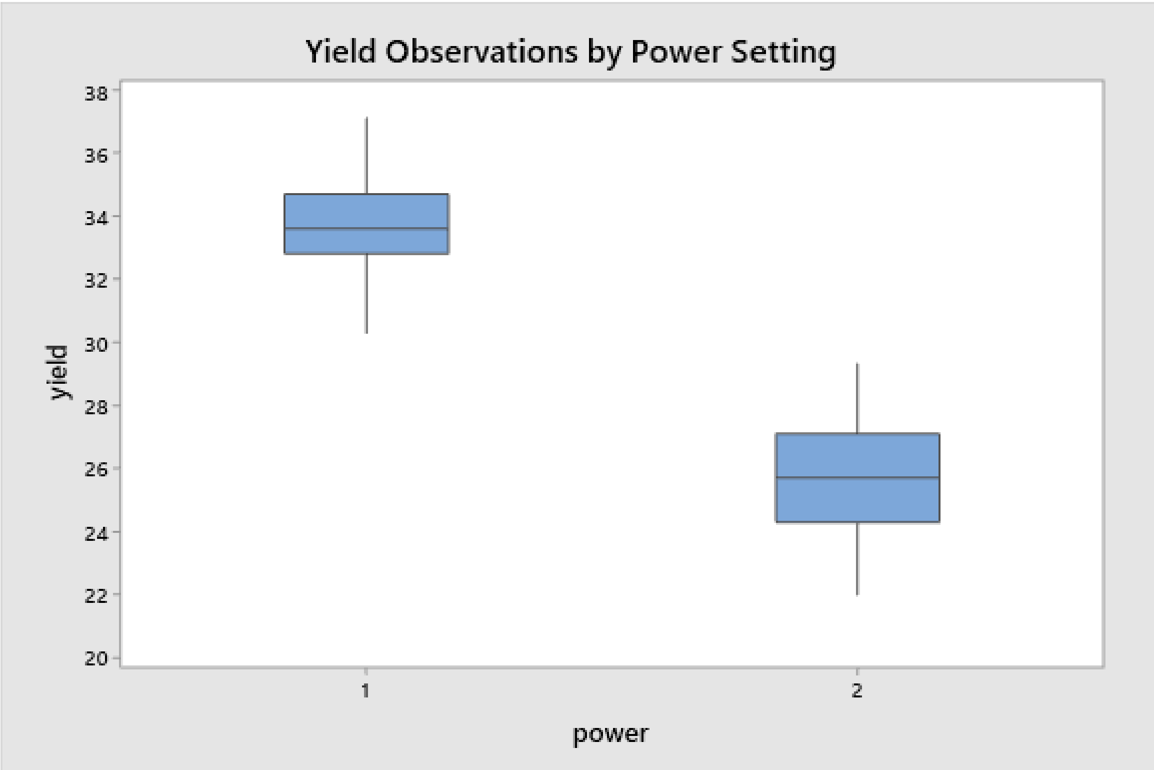
September 8, 2020

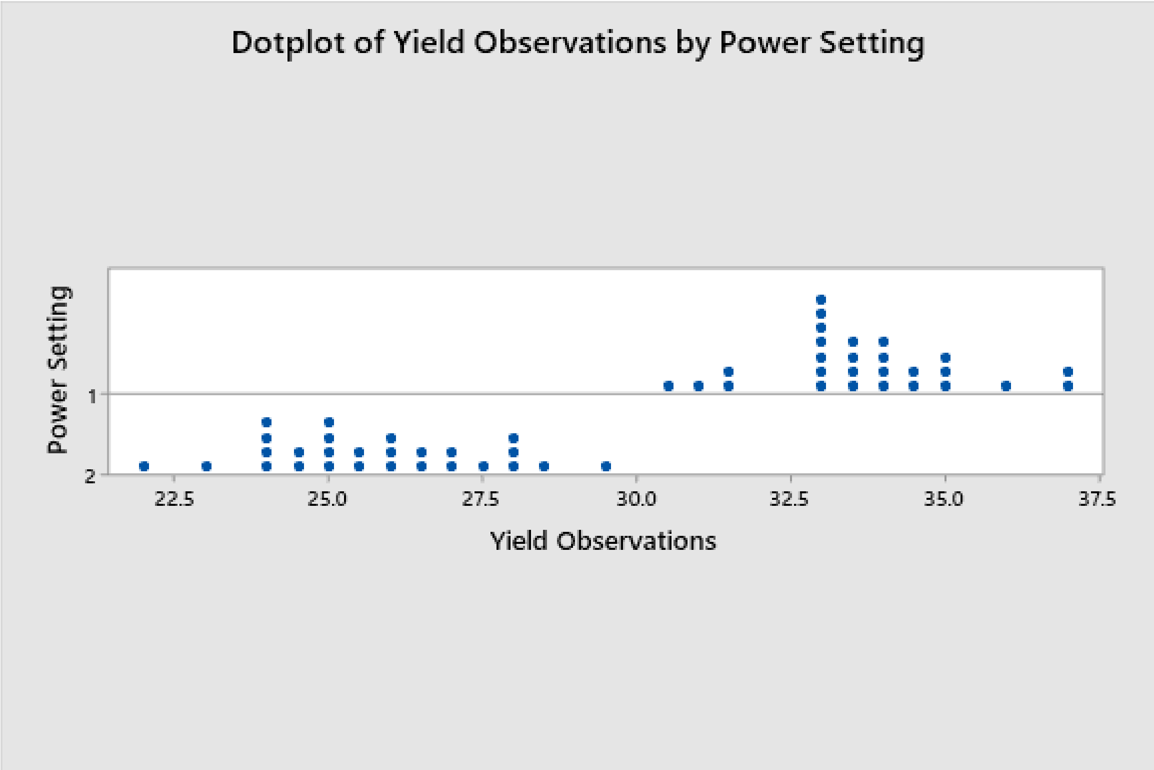
1 Scenario A

1.1 1)

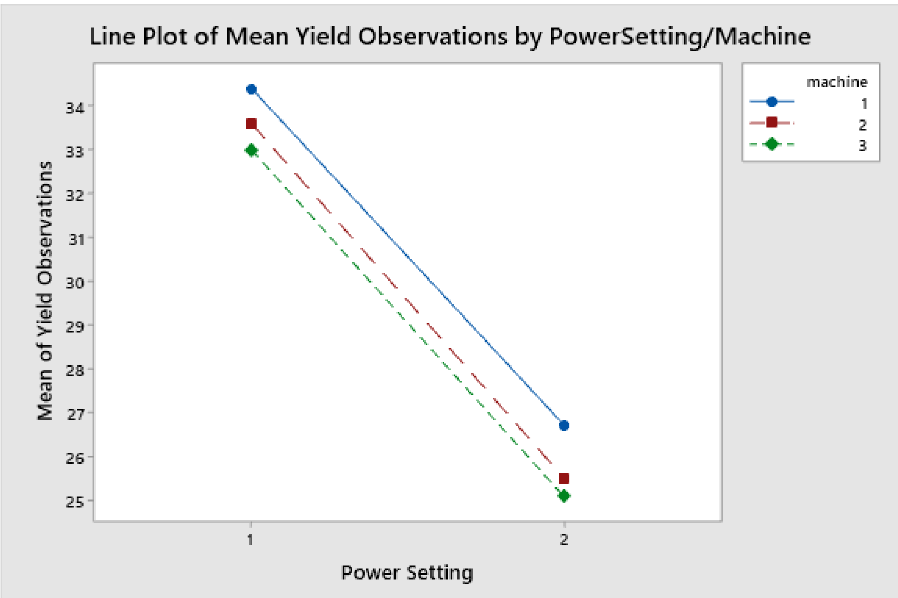
- Due to the sample size being relatively low (n = 58), we cannot establish the distribution shape from a histogram. We can, however, observe that the power setting factor has a considerable effect on the yield observations. The first figure demonstrates that the two different power settings have segregable observations with their medians and 1st/3rd quartiles separable from each other. We can derive from this figure the fact that power setting 2 tends to average around a higher yield value of 33 with low variance, compared to power setting 1 with a median close to 26 and low variance. This low variance suggests that the 2 are segregable and might "potentially" be significantly different.

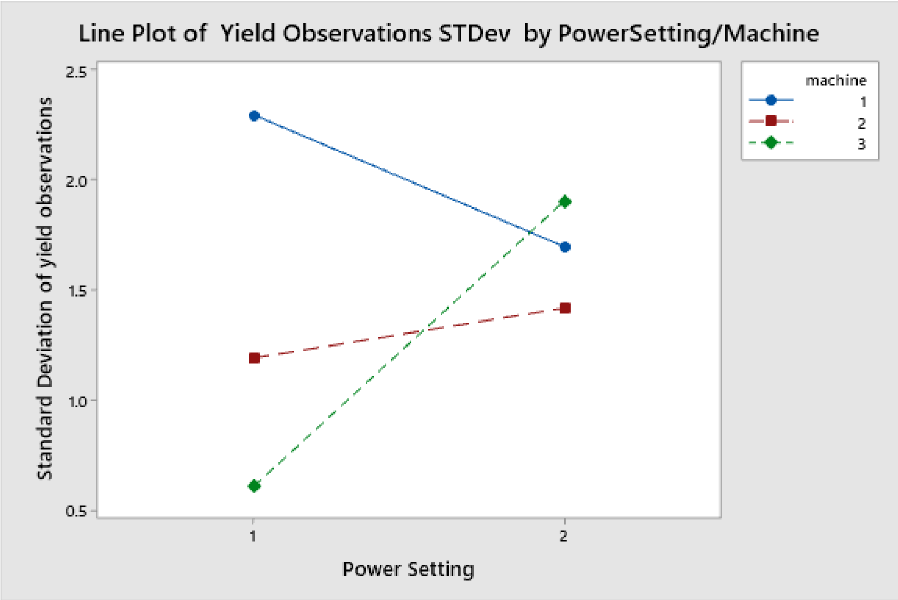


- To further support this, the following dot plot shows that the yield observations for both power settings are also separable, more clearly showing the actual values they take. Again, power setting 2 tends to have lower yield observations than power setting 1, which supports the above boxplot.



- Another interesting observation can be seen in the following figure. While we know from the above 2 plots that the power setting has an effect on the yield observations, it can be extended to machines per power setting as well. Machine 1 shows a higher mean than 2, and 2 shows a higher mean than 3 at both power setting values. This parallelism shows that there is no interaction between the power setting and machine when considering the mean of the yield observations, however if we switch to standard deviation of the yield observations and then consider the 2 different factors, we see that there is strong interaction between them. This suggests that a particular mix of the 2 factors somehow effects the variability of the yield observations, unlike the mean yields.





1.2 2)

1. **Null and Alternative Hypothesis:**

$$H_0 : \mu_{ps1} = \mu_{ps2}$$
$$H_1 : \mu_{ps1} \neq \mu_{ps2}$$

where:

μ: mean
$ps1$: Yield at power setting 1
$ps2$: Yield at power setting 2.

2. **Assumptions:**

- Yield observations are normally distributed
- Yield observations are identically-independently-distributed (IID)
- Yield standard deviations by power settings are equal since:

$$\frac{\sigma_{ps2}}{\sigma_{ps1}} = \frac{1.766}{1.585} = 1.114 < 2$$

Suggesting a pooled t-test is to be used.

3. **Level of significance:** $\alpha = 0.05$

4. **Test statistics:**

$$S_p^2 = \frac{(n_{ps1}-1)S_{ps1}^2+(n_{ps2}-1)S_{ps2}^2}{n_{ps1}+n_{ps2}-2}$$

where:

$nps1$: number of data points for $ps1 = 27$
$nps2$: number of data points for $ps2 = 27$
S_1^2: power setting 1 sample variance $= 2.51$
S_2^2: power setting 2 sample variance $= 3.12$

so, $S_p^2 = \frac{(27-1)2.51+(27-1)3.12}{27+27-2} = 2.815$

$$t_0 = \frac{\bar{y}_{ps1} - \bar{y}_{ps2}}{S_p\sqrt{\frac{1}{n_{ps1}} + \frac{1}{n_{ps2}}}}$$

where:

$\bar{y}_{ps1}$: yield sample mean for power setting 1 $= 33.68$
$\bar{y}_{ps2}$: yield sample mean for power setting 2 $= 25.76$

so, $t_0 = \frac{33.68-25.76}{1.68\sqrt{\frac{1}{27}+\frac{1}{27}}} = 17.32$

Therefore, p-value « 0.001 (referencing the T-distribution table)[1].

5. Since p-value $< \sigma$, there is significant evidence to suggest that H_0 is rejected.
6. Thus, the yields obtained from Power setting 1 are significantly different from those obtained from power setting 2.

1.3 3)

1. **Null and Alternative Hypothesis:**

$H_0 : \mu_{m1} = \mu_{m2} = \mu_{m3}$
H_1: At least one of the means (μ_{mi}) is different from the others.

where:

μ: mean
$m1$: Yield when using machine 1
$m2$: Yield when using machine 2
$m3$: Yield when using machine 3.

2. **Assumptions:**

- Yield observations at each machine level are normally distributed
- Yield observations Yield observations at each machine are identically-independently-distributed (IID)
- Yield variance Yield observations at each machine by power settings is equal

$$\sigma_{m1} = 19.53, \sigma_{m2} = 19.03, \sigma_{m3} = 18.54$$

Suggesting that, with a high level of confidence, they are in fact equal.

[1] To back this up, a MiniTab pooled 2-sample t test was conducted and produced a t-value of 17.33, p-value of 0.

3. We calculate the Sum of squares for the error, machines and the total:

$$\bar{y}_{..} = 34.1 + 30.3 + 31.6 + ... + 22.0 + 24.8 = 29.72 \text{ (Grand Mean)}$$

$$SS_{total} = \Sigma\Sigma(y_{ij} - \bar{y}_{..})^2 =$$
$$(34.1 - 29.72)^2 + (30.3 - 29.72)^2 + ... + (22.0 - 29.72)^2 + (24.8 - 29.72)^2 = 992.07$$

$$SS_{machine} = n\Sigma(\bar{y}_{i.} - \bar{y}_{..})^2 = 18((30.56 - 29.72)^2 + (29.56 - 29.72)^2 + (29.04 - 29.72)^2) =$$
$$18(1.1936) = 21.4848$$

$$\text{Thus, } SS_{error} = SS_{total} = SS_{machine} = 970.5852$$

Now, we calculate the Mean squares for machine and error:

$$MS_{machine} = \frac{SS_{machine}}{a-1} \text{ where a is the number of machines (a = 3)}$$

$$= \frac{21.4848}{2} = 10.7424$$

$$MS_{error} = \frac{SS_{error}}{N-a} \text{ where N is the total number of observations (N = 54)}$$

$$= \frac{970.5852}{51} = 19.031$$

So, the F statistic (F_0) is:

$$F_0 = \frac{MS_{machine}}{MS_{error}} = \frac{10.7424}{19.031} = 0.565$$

This F statistic corresponds to a p-value of 0.573 (which is larger than $\alpha = 0.05$), suggesting that there is not enough evidence to reject the null hypothesis. Hence, there is not enough evidence to suggest that the yield observations for all 3 machines are different.

1.4 4)

The randomisation of the experiments is important to ensure that the samples obtained are identically and independently distributed. If randomisation is not conducted, there may be nuisance factors that could potentially cause unnecessary bias. For example, conducting all the experiments that are to be done on an arbitrary machine in a short period of time (ie. 5 minute intervals), may lead to skewed results. The machine would perform differently if it has been used frequently in a short period of time as opposed to being used after longer periods of time. This dependence on the machine's activity status is not desirable and is considered a nuisance factor, resulting in skewed results and errors for the experiments conducted.

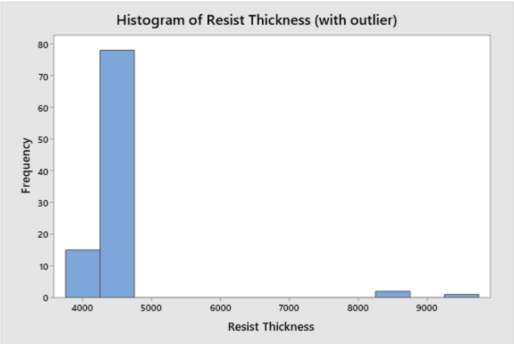
1.5 5)

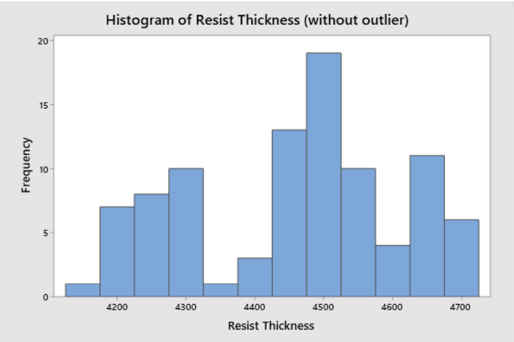
The results and evidence obtained from this experiment suggest that different power settings lead to different yields produced from the experiment, whereas using different machines would not suggest that different yield results will be produced.

2 Scenario B

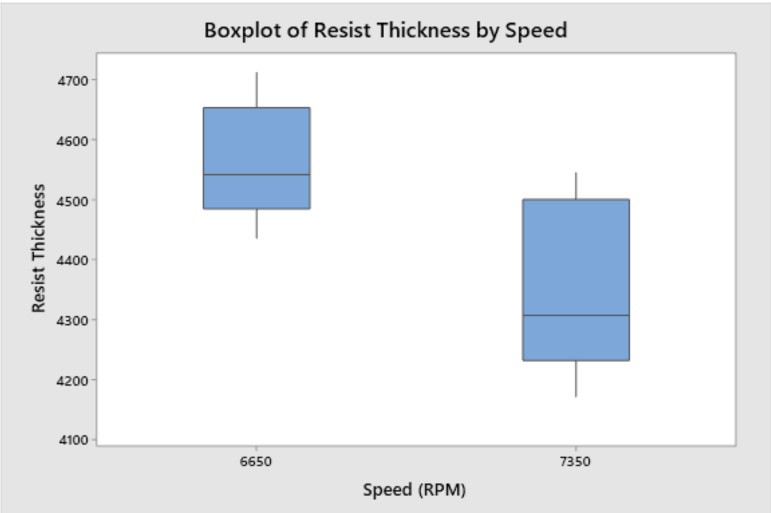
2.1 1)

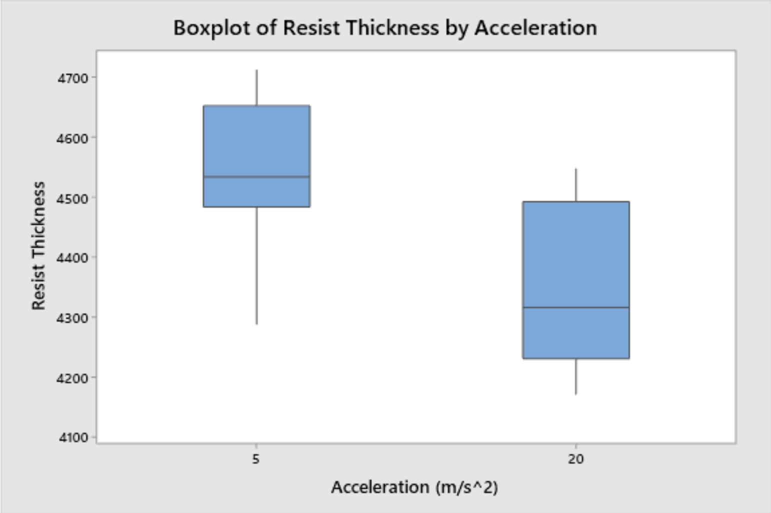
- The first significant observation that can be made is that the dataset contains very effectual outliers that result in skewing the results quite a lot. This can be shown in the following 2 histograms where the outliers are blatantly skewing the results:

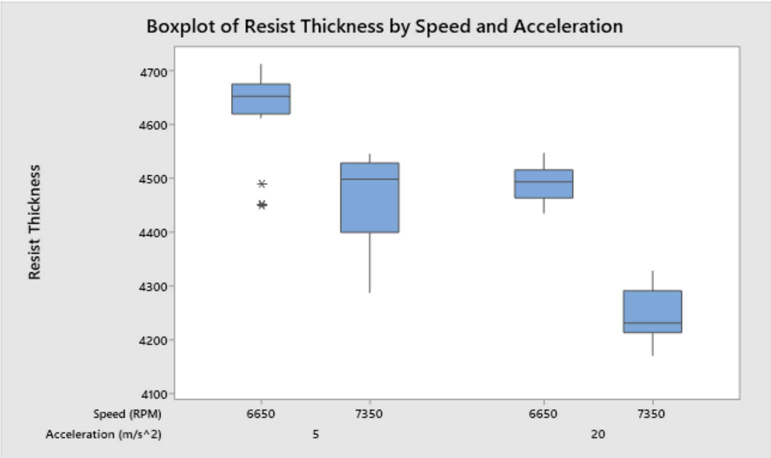




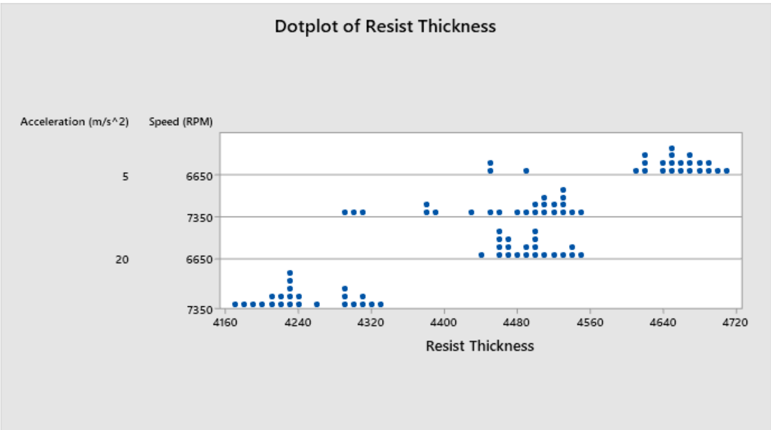
- Once the outliers are removed, we can observe that the spin speed and acceleration may potentially have a significant effect on the resist thicknesses obtained from the experiment. The 2 different levels for each factor have substantially different means and when we consider the cross combination of the 2, we can also observe that the 2 extremities - when combined together- (the 2 lowest levels and 2 highest levels) further fine grade the potential differences between the 2 factors:



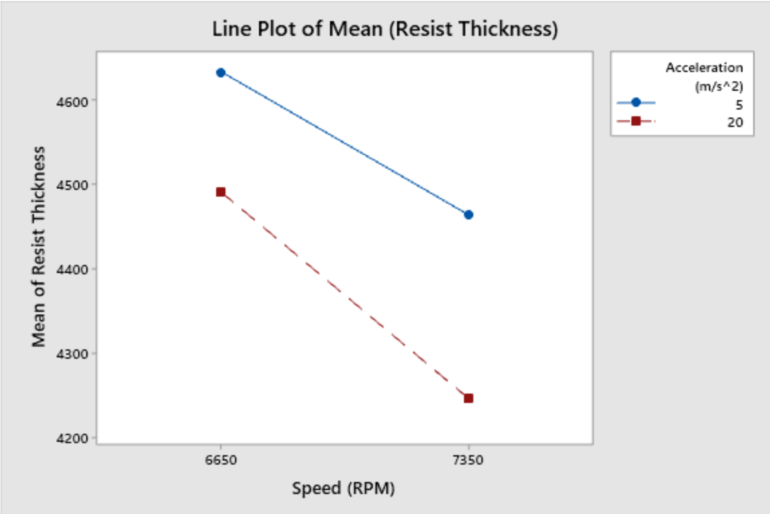


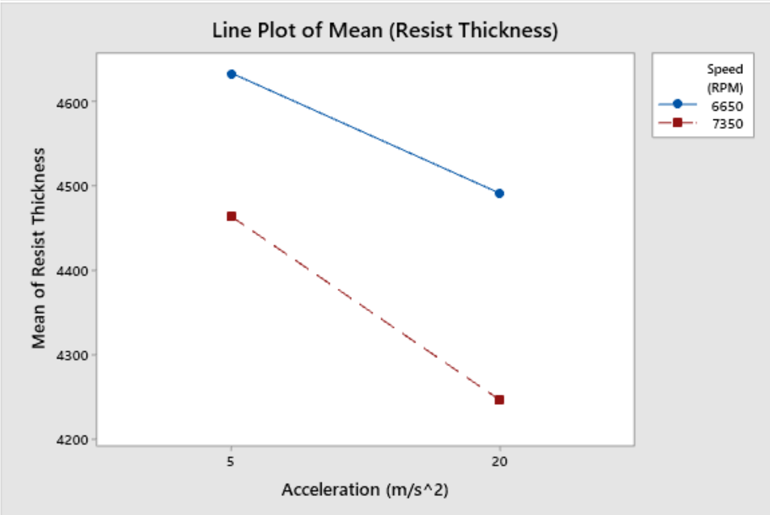


This can be shown as well using a dot plot split between the 2 factors' levels:

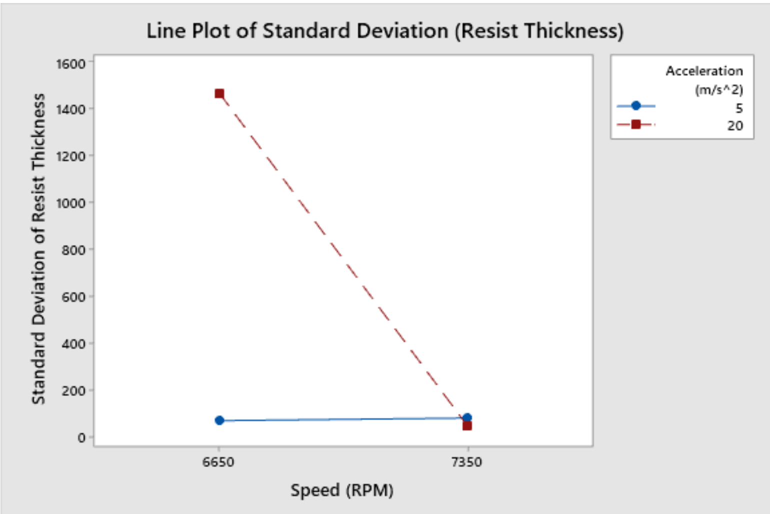


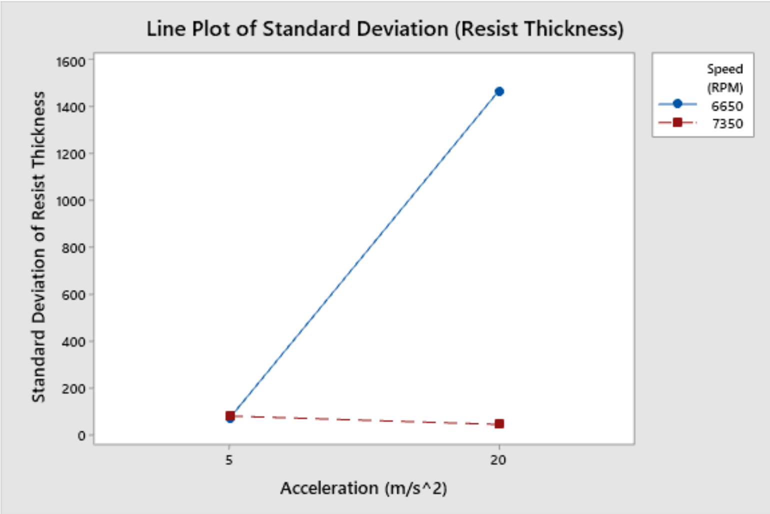
- Finally, if we consider the line plots of the resist thickness means, we observe a potentially weak interaction between spin speed and acceleration:





Whereas, when we consider standard deviation line plots of the same factors, we observe a much stronger interaction, suggesting that the variances of the 2 may also be related strongly:





2.2 2)

1. **Null and Alternative Hypothesis:**

$$H_0 : \mu_{v1} = \mu_{v2}$$
$$H_1 : \mu_{v1} \neq \mu_{v2}$$

where:

μ: mean
$v1$: Resist Average at Volume 1 (3cc)
$v2$: Resist Average at Volume 2 (5cc).

2. **Assumptions:**

- A data-point was removed as it proved to be an effectual outlier to the dataset.
- The 3 readings (left, centre and right) are considered to be 3 distinct readings of the test wafer under setting condition, and thus can be considered 3 replicates for a single setting option.
- Resist averages are normally distributed
- Resist averages are identically-independently-distributed (IID)
- Resist averages standard deviations by volume are equal since:

$$\frac{\sigma_{v2}}{\sigma_{v1}} = \frac{127.8}{90.6} = 1.42 < 2$$

Suggesting a pooled t-test is to be used.

3. **Level of significance:** $\alpha = 0.1$

4. **Test statistics:**

$$S_p^2 = \frac{(n_{v1}-1)S_{v1}^2+(n_{v2}-1)S_{v2}^2}{n_{v1}+n_{v2}-2}$$

where:

$$nv1\text{: number of data points for } v1 = 45$$
$$nv2\text{: number of data points for } v2 = 48$$
$$S_1^2\text{: volume 1 resist averages sample variance} = 25281$$
$$S_2^2\text{: volume 2 resist averages sample variance} = 22500$$

so, $S_p^2 = \frac{(45-1)25281+(48-1)22500}{45+48-2} = 23844.66$

$$t_0 = \frac{\bar{y}_{v1} - \bar{y}_{v2}}{S_p\sqrt{\frac{1}{n_{v1}} + \frac{1}{n_{v2}}}}$$

where:

$$\bar{y}_{ps1}\text{: Resist average sample mean for volume 1} = 4452.3$$
$$\bar{y}_{ps2}\text{: Resist average sample mean for volume 2} = 4463.3$$

so, $t_0 = \frac{4452.3-4463.3}{154.42\sqrt{\frac{1}{45}+\frac{1}{48}}} = -0.343$

Therefore, p-value $= 0.733$ [2].

5. Since p-value $> \alpha$, there is not enough evidence to suggest that H_0 is rejected.
6. Thus, there is insufficient evidence to say that resist averages obtained from Volume 1 (3cc) are different to from those obtained from volume 2 (5cc).

2.3 3)

1. **Null and Alternative Hypothesis:**

$$H_0 : \sigma_{ss1}^2 = \sigma_{ss2}^2$$
$$H_1 : \sigma_{ss1}^2 \neq \sigma_{ss2}^2$$

where:

$$\sigma^2\text{: variance}$$
$$ss1\text{: Resist Average at Spin Speed 1 (7350 rpm)}$$
$$ss2\text{: Resist Average at Spin Speed 2 (6650 rpm).}$$

2. **Assumptions:**
   - A data-point was removed as it proved to be an effectual outlier to the dataset.
   - The 3 readings (left, centre and right) are considered to be 3 distinct readings of the test wafer under setting condition, and thus can be considered 3 replicates for a single setting option.
   - Sample size of Spin Speed 1 (n_1)= 45, Sample size of Spin Speed 1 (n_2)= 48.
3. **Level of significance:** $\alpha = 0.1$

[2] a MiniTab pooled 2-sample t test was conducted and produced a t-value of -0.34, p-value of 0.733 as confirmation

4. **Test statistics:**

$$F_0 = \frac{S_1^2}{S_2^2}$$

where:

S_1^2: Spin Speed 1 resist averages sample variance $= 8208.36$
S_2^2: Spin Speed 2 resist averages sample variance $= 16332.84$

so, $F_0 = \frac{8208.36}{16332.84} = 0.5026$
H_0 is rejected if $F_0 > F_{0.05,44,47}$ or $F_0 < F_{0.95,44,47}$
so, $F_{0.05,44,47} = [1.53,\ 1.69]$, $F_0 < F_{0.95,44,47} = \frac{1}{F_{0.05,47,48}} = [0.59,\ 0.65]$.
Therefore, F_0 is less than $F_{0.95,44,47}$

5. Since $F_0 < F_{0.95,44,47}$, there is significant evidence to suggest that H_0 is rejected.

6. Thus, resist averages variance obtained from Speed Spin 1 (7350 rpm) is significantly different from those obtained from spin speed 2 (6650 rpm).

2.4 4)

From question 2, the p-value, for checking whether the difference between the resist averages using different volumes is significant, is 0.733 for a 2-tailed test. Thus, the level of significance α such that the volume of the resist applied with make a significant different to the average resist thickness is 0.37.
$\alpha = 0.37$

2.5 5)

One way ANOVA can be performed on this data to analysis the differences in variance between the 6 factors in the experiment. This, in turn, will inference when considering X number of factors whether or not at least one of them differ from the rest. This is enough to suggest that all the factors do not have the same means. Another method is regression which models relationships between dependent and independent variables from the data. One factor at a time may be used to understand the resist thicknesses where as linear regression can model a relationship between the 2, or multiple factors can be modelled against the resist thicknesses using multiple linear regression.

2.6 6)

A potential issue is the ambiguity surrounding the choice of certain factor levels for the experiment runs and is not exhaustive. The provided table shows a collection of settings chosen for each run, but the method in which which setting is chosen is not clear. The design factor choices are not exhaustive, as the number of runs is 32, whereas the maximum number of distinct runs that can be run is 64. Perhaps the other 32 possibilities may show interesting outcomes that cannot be observed in the 32 conducted in the experiment. Another potential problem is the usage of different test wafers for each run. An assumption has been made that a different test wafer is used for each run, which indicates that the resist thickness *intra*-variability of the wafers should only be considered and NOT the resist thickness *inter*-variability or mean across the different wafers. A better approach is to use the same wafer or wafers with the same measurements for different runs to obtain more consistent results.

2.7 7)

From the results and evidence obtained, we can conclude that The volume setting has no effect on the resist thickness and any can be used where no significant difference is observable. On the contrary, using different spin speed settings will result in different resist thicknesses. Further analysis on acceleration is required to conclude if it affects the resist thickness, as shown in the data visualisation. Finally, the experiment can be improved on by exploring more meaningful settings using an established DOE method.