Chapter 5: Inferences about Population Central Values

5.1 a. All registered voters in the state.

b. Simple random sample from a list of registered voters.

5.2 We might think that the actual average lifetime is less than the proposed 1500 hours.

a. The population is the lifetime of all fuses produced by the manufacturer during a selected period of time.

b. Testing a hypothesis.

5.3 It would depend on many factors relating to the manner and locations in which the fuses are manufactured and stored.

5.4 a. $12.3 \pm (1.96)\frac{0.2}{\sqrt{25}} = (12.22, 12.38)$

b. We are 95% confident that the average weight of a box of corn flakes is between 12.22 and 12.38 oz.

5.5 a. The width of the interval will be decreased.

b. The width of the interval will be increased.

5.6 a. $5.2 \pm (2.58)(\frac{7.5}{\sqrt{10}}) = 5.2 \pm 6.12 = (-.92, 11.32)$

b. Since the sample size is small, the condition that the distribution of profit margins needs to be normal is crucial. Similarly, with n=10, replacing σ with s is questionable. Section 5.7 will provide more details about this type of situation.

5.7 a. $10.4 \pm (2.58)(\frac{4.2}{\sqrt{400}}) = 10.4 \pm 0.54 = (9.86, 10.94)$

b. No, we are 99% confident that the average tire pressure is between 9.86 and 10.94 psi. underflated. Since a tire is considered to be seriously underinflated only if its tire pressure is more than 10 psi underinflated, this is a good chance that tires may not be seriously underinflated.

c. The 90% C.I. is $10.4 \pm (1.645)(\frac{4.2}{\sqrt{400}}) = 10.4 \pm 0.35 = (10.05, 10.75)$. Yes, since this interval is completely above 10 psi.

5.8 $3.2 \pm (1.96)(\frac{1.1}{\sqrt{150}}) = 3.2 \pm 0.18 = (3.02, 3.38)$

5.9 $850 \pm (1.96)(\frac{100}{\sqrt{60}}) = 850 \pm 25.3 = (824.7, 875.3)$

5.10 $3.27 \pm (1.96)(\frac{0.23}{\sqrt{100}}) = 3.27 \pm 0.045 = (3.225, 3.315)$

We are 95% confident that the mean scanner reading for the population is between 3.225 and 3.315.

5.11 a. $n = \frac{(1.96)^2(2.5)^2}{(1/2)^2} = 97$

b. $n = \frac{(2.58)^2(2.5)^2}{(1.2/2)^2} = 116$

c. $n = \frac{(1.645)^2(2.5)^2}{(1.2/2)^2} = 47$

5.12 Increase sample size by four times.

5.13 $\hat{\sigma} = 13, E = 3, \alpha = .01 \Rightarrow n = \frac{(2.58)^2(13)^2}{(3)^2} = 125$

5.14 $\hat{\sigma} = (1500 - 200)/4 = 325, E = 50, \alpha = .05 \Rightarrow n = \frac{(1.96)^2(325)^2}{(50)^2} = 163$

5.15 a. $\hat{\sigma} = \frac{1500-200}{4} = 325 \Rightarrow n = \frac{(2.58)^2(325)^2}{(25)^2} = 1125$

b. The 95% level of confidence implies that there will be a 1 in 20 chance, over a large number of samples, that the confidence interval will not contain the population average rent. The 99% level of confidence implies there is only a 1 in 100 chance of not containing the average. Thus, we would increase the odds of not containing the true average five-fold.

5.16 a. $z = \frac{40.1-38}{5.6/\sqrt{50}} = 2.65$; Because the observed value of $\bar{y}$ lies more than 1.645 ($\alpha = .05$) standard deviations above the hypothesized mean 38, we reject H_o and conclude that there is significant evidence that the mean is greater than 38.

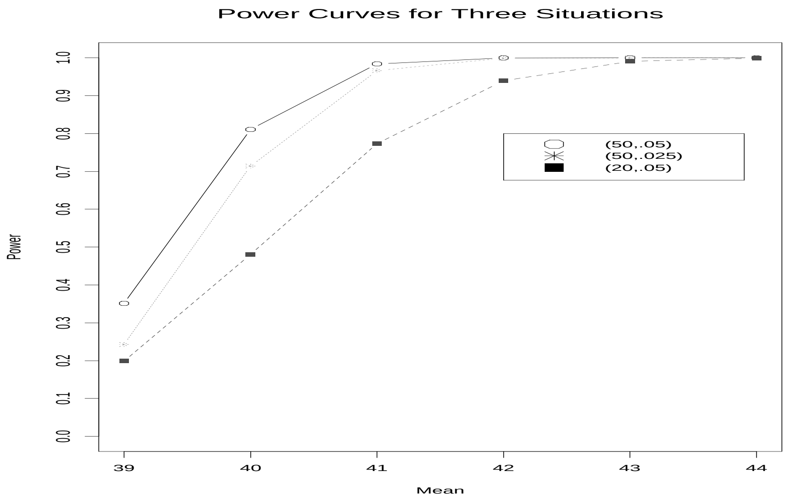
b. No. A Type II error can only be committed if we fail to reject H_o.

c. $\beta(39) = P(z \leq 1.645 - \frac{|38-39|}{5.6/\sqrt{50}}) = P(z \leq 0.38) = 0.6489$

5.17 a.-c. The power values,$PWR(\mu_a)$, are given here:

		μ_a					
n	α	39	40	41	42	43	44
50	0.05	0.3511	0.8107	0.9840	0.9997	1.0000	1.0000
50	0.025	0.2428	0.7141	0.9662	0.9990	0.9999	1.0000
20	0.05	0.1987	0.4809	0.7736	0.9394	0.9906	0.9992

The power curves are plotted here:



5.18 $H_o : \mu \geq 16$ vs $H_a : \mu < 16$

$\alpha = 0.05, \beta = 0.10$, whenever $\mu \leq 12, \sigma = 7.64$

$z_{0.05} = 1.645, z_{0.10} = 1.28, n = \frac{(7.64)^2(1.645+1.28)^2}{(12-16)^2} = 31.2 \Rightarrow n = 32$

5.19 $H_o : \mu \leq 2$ vs $H_a : \mu > 2, \bar{y} = 2.17, s = 1.05, n = 90$

a. $z = \frac{2.17-2}{1.05/\sqrt{90}} = 1.54 < 1.645 = z_{0.05} \Rightarrow$
Fail to reject H_o. The data does not support the hypothesis that the mean has been decreased from 2.

b. $\beta(2.1) = P\left(z \leq 1.645 - \frac{|2-2.1|}{1.05/\sqrt{90}}\right) = P(z \leq 0.74) = 0.7704$

5.20 The power values are computed using the following formulas:

$n = 90, \alpha = 0.05, PWR(\mu_a) = P(z > 1.645 - \frac{|2-\mu_a|}{1.05/\sqrt{90}})$

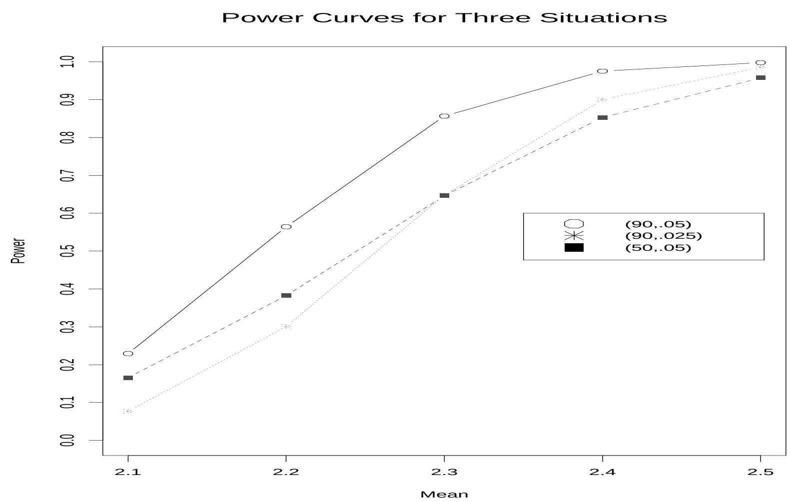
$n = 90, \alpha = 0.01, PWR(\mu_a) = P(z > 2.33 - \frac{|2-\mu_a|}{1.05/\sqrt{90}})$

$n = 50, \alpha = 0.05, PWR(\mu_a) = P(z > 1.645 - \frac{|2-\mu_a|}{1.05/\sqrt{50}})$

The power values are given here:

		μ_a				
n	α	2.1	2.2	2.3	2.4	2.5
90	0.05	0.2292	0.5644	0.8567	0.9755	0.9980
90	0.01	0.0769	0.3005	0.6482	0.9004	0.9857
50	0.05	0.1656	0.3828	0.6463	0.8529	0.9575

The power curves are plotted here:



a. Reducing α from 0.05 to 0.01 reduces the power of the test.

b. Reducing the sample size from 90 to 50 reduces the power of the test.

5.21 $n = \frac{(80)^2(1.645+1.96)^2}{(525-550)^2} = 133.1 \Rightarrow n = 134$

5.22 $z = \frac{542-525}{76/\sqrt{100}} = 2.24 > 1.645 = z_{0.05} \Rightarrow$ Reject H_o.

There is sufficient evidence to conclude that the mean has been increased above 525.

5.23 $H_o : \mu \leq 30$ vs $H_a : \mu > 30$

$\alpha = 0.05, n = 37, \bar{y} = 37.24, s = 37.12$

a. $z = \frac{37.24-30}{37.12/\sqrt{37}} = 1.19 < 1.645 = z_{0.05} \Rightarrow$ Fail to reject H_o.
There is not sufficient evidence to conclude that the mean lead concentration exceeds 30 mg kg^{-1} dry weight.

b. $\beta(50) = P(z \leq 1.645 - \frac{|30-50|}{37.12/\sqrt{37}}) = P(z \leq -1.63) = 0.0513$.

c. No, the data values are not very close to the straight-line in the normal probability plot.

d. No, since there is a substantial deviation from a normal distribution, the sample size should be somewhat larger to use the $z-$ test. Section 5.8 provides an alternative test statistic for handling this situation.

5.24 p-value $= P(z \geq \frac{48.2-45}{12.57/\sqrt{50}}) = P(z \geq 1.80) = 0.0359 < 0.05 = \alpha \Rightarrow$

Yes, there is significant evidence that the mean is greater than 45.

5.25 p-value $= 0.0359 > 0.025 = \alpha \Rightarrow$

No, there is not significant evidence that the mean is greater than 45. With $\alpha = 0.025$, the researcher is demanding greater evidence in the data to support the research hypothesis.

5.26 $H_o: \mu \leq 14$ versus $H_a: \mu > 14, n = 300, \bar{y} = 14.6, s = 3.8, \alpha = 0.01$.

p-value $= P(z \geq \frac{14.6-14}{3.8/\sqrt{300}}) = P(z \geq 2.73) = 0.0032 < 0.01 = \alpha \Rightarrow$

Yes, there is significant evidence that the mean nicotine content is greater than 14.

5.27 $H_o: \mu = 1.6$ versus $H_a: \mu \neq 1.6$,

$n = 36, \bar{y} = 2.2, s = 0.57, \alpha = 0.05$.

p-value $= 2P(z \geq \frac{|2.2-1.6|}{.57/\sqrt{36}}) = 2P(z \geq 6.32) < 0.0001 < 0.05 = \alpha \Rightarrow$

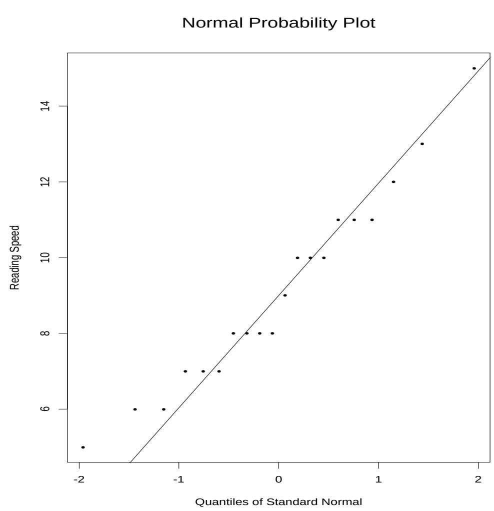
Yes, there is significant evidence that the mean time delay differs from 1.6 seconds.

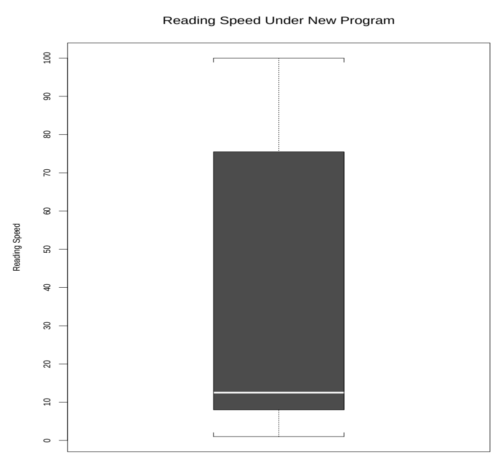
5.28
a. Reject H_o if $t \leq -1.761$
b. Reject H_o if $|t| \geq 2.074$
c. Reject H_o if $t \geq 2.015$

5.29
a. Reject H_o if $t \leq -2.624$
b. Reject H_o if $|t| \geq 2.819$
c. Reject H_o if $t \geq 3.365$

5.30 $n = 20, \bar{y} = 9.1, s = 2.573, t_{.025,19} = 2.093$

a. $9.1 \pm (2.093)(2.573)/\sqrt{20} \Rightarrow 9.1 \pm 1.2 \Rightarrow (7.9, 10.3)$ is a 95% C.I. on μ
b. A normal probability plot and boxplot are given here:





The data set appears to be a sample from a normal distribution.

c. We are 95% confident that the mean reading speed for the population is between 7.9 and 10.3.

d. $9.1 \pm (2.539)(2.573)/\sqrt{20} \Rightarrow 9.1 \pm 1.46 \Rightarrow (7.64, 10.56)$ is a 98% C.I. on $\mu \Rightarrow$ The inference is less precise.

5.31 $H_o : \mu \leq 80$ versus $H_a : \mu > 80, n = 20, \bar{y} = 82.05, s = 10.88$

$t = \frac{82.05-80}{10.88/\sqrt{20}} = 0.84 \Rightarrow$ Reject H_o if $t \geq 1.729$.

Fail to reject H_o and conclude data does not support the hypothesis that the mean reading comprehension is greater than 80.

The level of significance is given by p-value $= P(t \geq 0.84) \approx 0.20$.

5.32 $n = 15, \bar{y} = 31.47, s = 5.04$

a. $31.47 \pm (2.977)(5.04)/\sqrt{15} \Rightarrow 31.47 \pm 3.87 \Rightarrow (27600, 35340)$ is a 99% C.I. on the mean miles driven.

b. $H_o : \mu \geq 35$ versus $H_a : \mu < 35$ Reject H_o if $t \leq -2.624$.

$t = \frac{31.47-35}{5.04/\sqrt{15}} = -2.71 \Rightarrow$

Reject H_o and conclude data supports the hypothesis that the mean miles driven is less than 35,000 miles.

Level of significance is given by p-value $= P(t \leq -2.71) \Rightarrow 0.005 <$ p-value < 0.01.

5.33 a. Yes, the plotted points are all close to the straight line.

b. Fairly close.

c. Yes, since the confidence interval contains values which are greater than 35. However, the C.I. is a two-sided procedure which yields a one-sided α of 0.005 which would require greater evidence from the data to support the research hypothesis.

5.34 a. $4.95 \pm (2.365)(0.45)/\sqrt{8} \Rightarrow 4.95 \pm 0.38 \Rightarrow (4.57, 5.33)$ is a 95% C.I. on the mean dissolved oxygen level.

b. There is inconclusive evidence that the mean is less than 5 since the C.I. contains values both less and greater than 5.

c. $H_o : \mu \geq 5$ versus $H_a : \mu < 5$, p-value $= P(t \leq -0.31) \Rightarrow 0.25 <$ p-value < 0.40 (Using a computer program p-value $= 0.3828$). Fail to reject H_o and conclude the data does not support that the mean is less than 5.

5.35 a. Untreated: $43.6 \pm (1.833)(5.7)/\sqrt{10} \Rightarrow (40.3, 46.9)$

Treated: $36.1 \pm (1.833)(4.9)/\sqrt{10} \Rightarrow (33.3, 38.9)$

We are 90% confident that the average height of untreated shrups is between 40.3 cm and 46.9 cm. We are 90% confident that the average height of treated shrups is between 33.3 cm and 38.9 cm.

b. The two intervals do not overlap. This would indicate that the average heights of the treated and untreated shrups are significantly different.

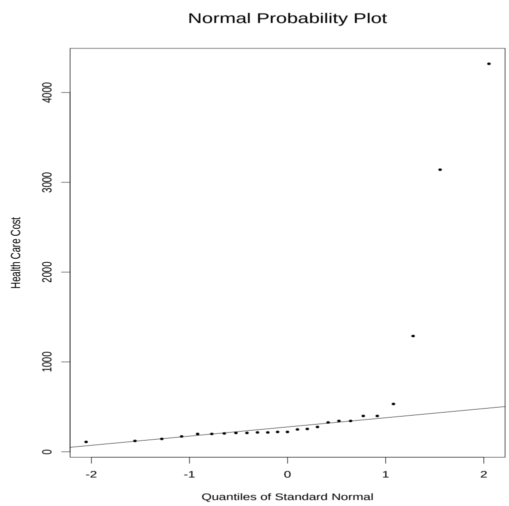
5.36 a. $L_{.05} = 5, U_{.05} = 16$

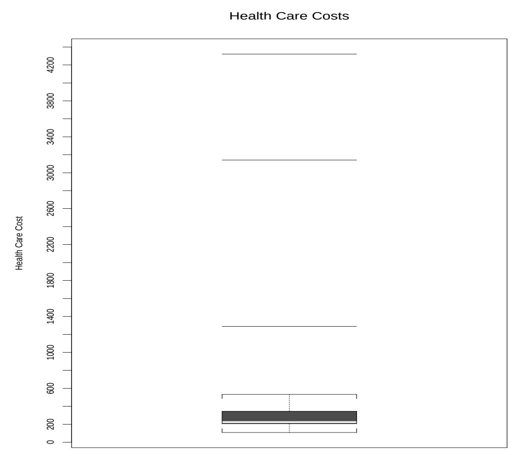
b. $L_{.05} = 6, U_{.05} = 15$
The intervals are somewhat narrower.

5.37 Reject H_o if $B \geq 20$

5.38 Reject H_o if $B_{ST} \geq 1.645$

5.39 a. The normal probability plot is given here:





The data set does not appear to be a sample from a normal distribution, since a large proportion of the values are outliers as depicted in the box plot and several points are a considerable distance from the line in the normal probability plot. The data appears to be from an extremely right skewed distribution.

b. Because of the skewness, the median would be a better choice than the mean.

c. (208, 342) We are 95% confident that the median amount spent on healthcare by the population of hourly workers is between \$208 and \$342 per year.

d. Reject $H_o : M \leq 400$ if $B \geq 25 - 7 = 18$.
We obtain B = 4; Since $4 < 18$, do not reject $H_o : M \leq 400$. The data fails to demonstrate that the median amount spend on health care is greater than \$400.

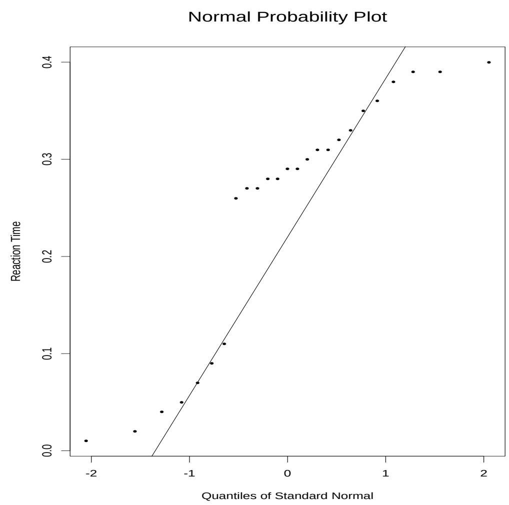
5.40 a. 99% C.I. on Mean: $0.247 \pm (2.797)(0.129)/\sqrt{25} \Rightarrow (0.175, 0.319)$
99% C.I. on Median: $(y_{(6)}, y_{(20)}) \Rightarrow (0.09, 0.35)$

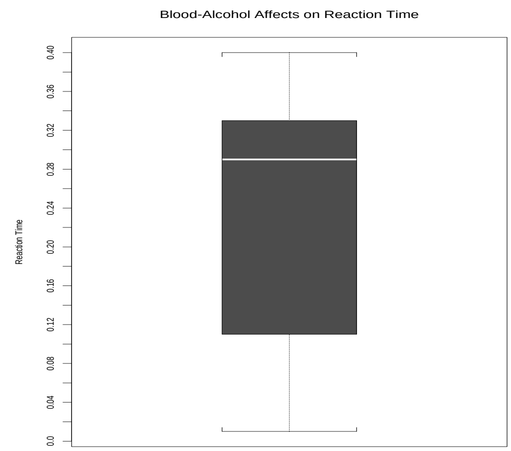
b. Yes, $t = \frac{0.247-0}{0.129/\sqrt{25}} = 9.57 \Rightarrow$ p-value $= P(t \geq 9.57) < 0.0001$. Thus, there is significant evidence of an increase in mean reaction time.

c. Yes, $B = 25 > 21 \Rightarrow$ reject H_o at the $\alpha = .001$ level. Thus, there is significant evidence of an increase in median reaction time.

d. From the normal probability plot or box plot it is observed that the data appears to be from distribution which is bimodal, skewed to the left. Thus, the median is a more appropriate representative of reaction time differences.

Normal probability and box plots are given here:





5.41 a. Reject $H_o: M \leq 0.25$ in favor of $H_a: M > 0.25$ at level $\alpha = 0.01$ if $B \geq 25 - 6 = 19$. From the differences, $y_i - 0.25$, we obtain $B = 18$ positive values. Thus, we fail to reject H_o and conclude that the data does not support a median increase in reaction time of at least 0.25 seconds.

b. Weight of driver, experience(age) of driver, amount of sleep in previous 24 hours, etc.

5.42 a. $\bar{y} = 28.7$

b. $s/\sqrt{n} = 3.8/\sqrt{38} = 0.62$

c. 95% C.I.: $28.7 \pm (1.96)(3.8)/\sqrt{38} \Rightarrow (27.49, 29.91)$

d. Test $H_a: \mu < 30$, p-value $= P(z \leq \frac{28.7-30}{38/\sqrt{3.8}}) = P(z \leq -2.11) = 0.0174 < 0.05 = \alpha \Rightarrow$ There is sufficient evidence that the mean time has been reduced.

5.43 a. $\bar{y} = 1.466$

b. 95% C.I.: $1.466 \pm (2.145)(.3765)/\sqrt{15} \Rightarrow (1.26, 1.67)$

We are 95% confident that the average mercury content after the accident is between 1.28 and $1.66 mg/m^3$

c. $H_a: \mu > 1.20$ Reject H_o if $t \geq 1.761$.

$t = \frac{1.466-1.2}{.3765/\sqrt{15}} = 2.74 \Rightarrow$

There is sufficient evidence that the mean mercury concentration has increased.

d. Using Table 4, we obtain the following with $d = \frac{|\mu_a - 1.2|}{.32}$

μ_a	d	$PWR(\mu_a)$
1.28	0.250	0.235
1.32	0.375	0.396
1.36	0.500	0.578
1.40	0.625	0.744

5.44 a. Number the mothers from 1 to n. Use a random number generator to choose 100 numbers from the numbers 1 to n. Those mothers whose "numbers" are chosen shall be in the study.

b. 95% C.I.: $9.2 \pm (1.96)(12.4)/\sqrt{100} \Rightarrow (6.77, 11.63)$

We are 95% confident that the average number of days to birth beyond the due date is between 6.77 and 11.63 days.

c. $H_a : \mu < 13$ p-value $= P(z \leq \frac{9.2-13}{12.4/\sqrt{100}}) = P(z \leq -3.06) = 0.0011$.

Since the p-value is very small,there is substantial evidence that the average number of days to birth beyond the due data has been reduced. The level of significance is 0.0011.

d. Various answers

5.45 $H_0 : \mu \geq 300$ versus $H_a : \mu < 300$,

$n = 20, \bar{y} = 160, s = 90, \alpha = 0.05$

p-value $= P(t \leq \frac{160-300}{90/\sqrt{20}}) = P(t \leq -6.95) < 0.0001 < 0.05 = \alpha$.

Yes, there is sufficient evidence to conclude that the average is less than \$300.

5.46 $H_0 : \mu \leq 25$ versus $H_a : \mu > 25$,

$n = 15, \bar{y} = 28.20, s = 11.44, \alpha = 0.05$

p-value $= P(t \geq \frac{28.20-25}{11.44/\sqrt{15}}) = P(t \geq 1.08) = 0.1492 > 0.05 = \alpha$.

No , there is not sufficient evidence to conclude that the average time to fill an order is greater than 25 minutes.

5.47 a. $\bar{y} = 74.2$ 95% C.I.: $74.2 \pm (2.145)(44.2)/\sqrt{15} \Rightarrow (49.72, 98.68)$

b. $H_0 : \mu \leq 50$ versus $H_a : \mu > 50$,

$n = 15, \alpha = 0.05$

p-value $= P(t \geq \frac{74.2-50}{44.2/\sqrt{15}}) = P(t \geq 2.12) = 0.0262 < 0.05 = \alpha$.

Yes, there is sufficient evidence to conclude that the average daily output is greater than 50 tons of ore.

5.48 a. $H_o : \mu \geq 5.2; H_a : \mu < 5.2$

$z = \frac{5.0-5.2}{0.7/\sqrt{50}} = -2.02$

Reject Ho if $z \leq -1.645$

b. Reject H_o and conclude that the mean dissolved oxygen count is less than 5.2 ppm.

5.49 a. The summary statistics are given here:

Time	Mean	Std.Dev	n	95% C.I.
6 A.M.	0.128	0.0355	15	(0.108, 0.148)
2 P.M.	0.116	0.0406	15	(0.094, 0.138)
10 P.M.	0.142	0.0428	15	(0.118, 0.166)
All Day	0.129	0.0403	45	(0.117, 0.141)

b. No, the three C.I.'s have a considerable overlap.

c. $H_o : \mu \geq 0.145$ versus $H_a : \mu < 0.145$

p-value $= P(t \leq \frac{.129-.145}{.0403/\sqrt{45}}) = P(t \leq -2.66) = 0.0054$

There is significant evidence (very small p-value) that the average SO_2 level using the new scrubber is less than 0.145.

5.50 a. $\bar{y} = 30.514, s = 12.358, n = 35$

95% C.I.: $30.514 \pm (2.032)(12.358)/\sqrt{35} \Rightarrow (26.27, 34.76)$

We are 95% confident that this interval captures the population mean exercise capacity.

b. 99% C.I.: $30.514 \pm (2.728)(12.358)/\sqrt{35} \Rightarrow (24.81, 36.21)$

The 99% C.I. is somewhat wider that the 95% C.I.

5.51 $n = \frac{(12.36)^2(1.96)^2}{(1)^2} = 586.9 \Rightarrow n = 587$

5.52 $n = 50, \bar{y} = 75, s = 15$

95% C.I. on $\mu : 75 \pm (2.010)(15)/\sqrt{50} \Rightarrow (70.7, 79.3)$

5.53 $n = 40, \bar{y} = 58, s = 10$

99% C.I. on $\mu : 58 \pm (2.708)(10)/\sqrt{40} \Rightarrow (53.7, 62.3)$

5.54 $\hat{\sigma} = \frac{400-40}{4} = 90 \Rightarrow n = \frac{(1.96)^2(90)^2}{(10)^2} = 311.2 \Rightarrow n = 312$

5.55 From the material in the Encounters with Real Data section, we obtain the following answers.

a.
1. The population from which the data was randomly selected is a population of female nurses.
2. Some possible dietary variables are total calories consumed per day; amount of salt, amount of protein, amount of fiber in the diet; amount of liquids consumed; and many other factors.
3. Percent body fat, amount of daily exercise, blood pressure, family health history, age, and many other factors.
4. Obtain a list of names of all nurses in the population from which the sample is to be selected. Assign a number to each name. Randomly select 168 numbers from 1 to N, the total number of nurses in the population, using a computer program or the random number table in the Appendix. The nurses corrsponding to each of these 168 numbers will be in the study.

5. Does the population of nurses have a higher proportion with PCF above 50% than populations of other professionals, blue collar workers, etc.? Is there greater variability in the PCF values for nurses in comparison to populations of other groups of people?