
Chapter 7: Inferences about Population Variances

- 7.1 a. 0.01
 b. 0.90
 c. $1 - 0.99 = 0.01$
 d. $1 - 0.01 - 0.01 = 0.98$
- 7.2 Let χ_a^2 be the upper $100a$ -percentile from a Chi-square distribution.
 a. 21.92
 b. 3.186
- 7.3 a. Let y be the quantity in a randomly selected jar:
 Proportion $= P(y < 32) = P(z < \frac{32-32.3}{.15}) = 0.0228 \Rightarrow 2.28\%$
 b. The plot indicates that the distribution is approximately normal because the data values are reasonably close to the straight-line.
 c. 95% C.I. on σ : $\left(\sqrt{\frac{(50-1)(.135)^2}{70.22}}, \sqrt{\frac{(50-1)(.135)^2}{31.55}} \right) \Rightarrow (0.113, 0.168)$
 d. $H_o : \sigma \leq 0.15$ versus $H_a : \sigma > 0.15$
 Reject H_o if $\frac{(n-1)(s)^2}{(.15)^2} \geq 66.34$
 $\frac{(50-1)(.135)^2}{(.15)^2} = 39.69 < 66.34 \Rightarrow$
 Fail to reject H_o and conclude the data does not support σ greater than 0.15.
 e. p-value $= P(\frac{(n-1)(s)^2}{(.15)^2} \geq 39.69)$
 Using the Chi-square tables with $df = 49$, $0.10 < \text{p-value} < 0.90$
 (Using a computer program, p-value = 0.8262).
- 7.4 a. The box plot is symmetric with whiskers of approximately the same length. The only possible deviation from a normal distribution would be that there is an outlier, however, with a large sample size, a single outlier is not completely inconsistent with samples from a normal population.
 b. 95% C.I. on σ : $\left(\sqrt{\frac{(100-1)(11.35)^2}{128.42}}, \sqrt{\frac{(100-1)(11.35)^2}{73.36}} \right) \Rightarrow (9.97, 13.19)$
 c. $H_o : \sigma \leq 10$ versus $H_a : \sigma > 10$
 Reject H_o if $\frac{(n-1)(s)^2}{(.15)^2} \geq 123.23$
 $\frac{(100-1)(11.35)^2}{(10)^2} = 127.53 > 123.23 \Rightarrow$
 Reject H_o and conclude the data supports the contention that σ is greater than 10.
- 7.5 a. The box plot is symmetric but there are four outliers. Since the sample size is 150, a few outliers would be expected. However, four out of 150 may indicate the population distribution may have heavier tails than a normal distribution. This may cause the values of s to be inflated.

b. 99% C.I. on σ : $\left(\sqrt{\frac{(150-1)(9.537)^2}{197.21}}, \sqrt{\frac{(150-1)(9.537)^2}{108.29}} \right) \Rightarrow (8.290, 11.187)$

c. $H_o : \sigma^2 \leq 90$ versus $H_a : \sigma^2 > 90$

With $\alpha = 0.05$, reject H_o if $\frac{(n-1)(s)^2}{90} \geq 178.49$

$\frac{(150-1)(9.537)^2}{90} = 150.58 < 178.49 \Rightarrow$

Fail to reject H_o and conclude the data fails to support the statement that σ^2 is greater than 90.

7.6 p-value = $P(\chi^2 \geq 150.58) = 0.4484$

7.7 a. The box plot is symmetric with a single outlier. Since the sample size is 81, a few outliers would be expected. Thus, the normality of the population distribution appears to be satisfied.

b. $H_o : \sigma \geq 2$ versus $H_a : \sigma < 2$

With $\alpha = 0.05$, reject H_o if $\frac{(n-1)(s)^2}{(2)^2} \leq 60.39$

$\frac{(81-1)(1.771)^2}{(2)^2} = 62.73 > 60.39 \Rightarrow$

Fail to reject H_o and conclude the data fails to support the contention that σ is less than 2. p-value = 0.0772

c. 95% C.I. on σ : $\left(\sqrt{\frac{(81-1)(1.771)^2}{106.63}}, \sqrt{\frac{(81-1)(1.771)^2}{57.15}} \right) \Rightarrow (1.534, 2.095)$

7.8 We need to assume that the two samples were independently selected from normally distributed populations.

$H_o : \sigma_1^2 = \sigma_2^2$ versus $H_a : \sigma_1^2 \neq \sigma_2^2$

With $\alpha = 0.10$, reject H_o if $\frac{s_1^2}{s_2^2} \leq \frac{1}{3.68} = 0.272$ or $\frac{s_1^2}{s_2^2} \geq 3.29$

$s_1^2/s_2^2 = 0.583 \Rightarrow 0.272 < 0.583 < 3.29 \Rightarrow$

Fail to reject H_o and conclude the data does not support the contention that the population variances are different.

7.9 $H_o : \sigma_A^2 \leq \sigma_B^2$ versus $H_a : \sigma_A^2 > \sigma_B^2$

With $\alpha = 0.05$, reject H_o if $\frac{s_A^2}{s_B^2} \geq 3.79$

$s_A^2/s_B^2 = 3.15 < 3.79 \Rightarrow$

Fail to reject H_o and conclude the data does not support σ_A^2 being greater than σ_B^2 .

7.10 a. 95% C.I. on σ_{Old} : $\left(\sqrt{\frac{(61-1)(0.231)^2}{83.30}}, \sqrt{\frac{(61-1)(0.231)^2}{40.48}} \right) \Rightarrow (0.196, 0.281)$

95% C.I. on σ_{New} : $\left(\sqrt{\frac{(61-1)(0.162)^2}{83.30}}, \sqrt{\frac{(61-1)(0.162)^2}{40.48}} \right) \Rightarrow (0.137, 0.197)$

b. $H_o : \sigma_{Old}^2 \leq \sigma_{New}^2$ versus $H_a : \sigma_{Old}^2 > \sigma_{New}^2$

With $\alpha = 0.05$, reject H_o if $\frac{s_{Old}^2}{s_{New}^2} \geq 1.53$

$s_{Old}^2/s_{New}^2 = 2.033 > 1.53 \Rightarrow$

Reject H_o and conclude the data supports the statement that σ_{New}^2 is less than σ_{Old}^2 .

- c. The box plots indicate the both population distributions are normally distributed. From the problem description, the two samples appear to be independently selected random samples.

$$7.11 \text{ 95\% C.I. on } \sigma_{Comp.} : \left(\sqrt{\frac{(91-1)(53.77)^2}{118.14}}, \sqrt{\frac{(61-1)(53.77)^2}{65.65}} \right) \Rightarrow (46.93, 62.96)$$

$$\text{95\% C.I. on } \sigma_{Conv.} : \left(\sqrt{\frac{(91-1)(36.94)^2}{118.14}}, \sqrt{\frac{(91-1)(36.94)^2}{65.65}} \right) \Rightarrow (32.24, 43.25)$$

$$\text{95\% C.I. on } \mu_{Comp.} : 484.45 \pm (1.987)(53.77)/\sqrt{91} \Rightarrow (473.60, 495.30)$$

$$\text{95\% C.I. on } \mu_{Conv.} : 487.38 \pm (1.987)(36.94)/\sqrt{91} \Rightarrow (479.69, 495.07)$$

$$H_o : \sigma_{Comp.}^2 = \sigma_{Conv.}^2 \text{ versus } H_a : \sigma_{Comp.}^2 \neq \sigma_{Conv.}^2$$

$$\text{With } \alpha = 0.05, \text{ reject } H_o \text{ if } \frac{s_{Comp.}^2}{s_{Conv.}^2} \leq \frac{1}{1.52} = 0.660 \text{ or } \frac{s_{Comp.}^2}{s_{Conv.}^2} \geq 1.52$$

$$s_{Old}^2/s_{New}^2 = 2.12 > 1.52 \Rightarrow$$

Reject H_o and conclude there is significant evidence that $\sigma_{Comp.}^2$ and $\sigma_{Conv.}^2$ are different.

$$H_o : \mu_{Comp.} = \mu_{Conv.} \text{ versus } H_a : \mu_{Comp.} \neq \mu_{Conv.}$$

$$t = \frac{484.45 - 487.38}{\sqrt{\frac{(53.77)^2}{91} + \frac{(36.94)^2}{91}}} = -0.06 \Rightarrow \text{p-value} = 2P(t \geq |-0.06|) = 0.95$$

Fail to reject H_o and conclude there is not significant evidence that the mean SAT math exam scores are different.

The two methods yield similar mean scores but the computer testing method has a higher degree of variability than the conventional method.

- 7.12 a. From the box plots, the distribution for Additive 3 appears to be normal. The plots for Additive 1 and Additive 2 are skewed to the right with outliers. These conclusions are confirmed by examining the normal probability plots which show deviations from a straight-line for Additives 1 and 2 but the points for Additive 3 are relatively close to the straight-line.
- b. With $\alpha = 0.05$, reject H_o if $F_{max} > 5.34$
 $F_{max} = \frac{(15.33)^2}{(2.69)^2} = 32.48 > 5.34 \Rightarrow$
 Reject H_o and conclude there is a significant difference in the population variances.
- c. No, Levine's test determined there was insignificant evidence of a difference in the population variances, whereas, the Hartley's test found a difference.
- d. The Levine test since two of the three populations appears to be nonnormally distributed.
- e. A set of 95% C.I.'s for the Additive mean increases in mpg are given here:
 95% C.I. on μ_{Add_1} : $7.05 \pm (2.262)(7.11)/\sqrt{10} \Rightarrow (1.96, 12.14)$
 95% C.I. on μ_{Add_2} : $10.60 \pm (2.262)(15.33)/\sqrt{10} \Rightarrow (-0.37, 21.57)$
 95% C.I. on μ_{Add_3} : $9.15 \pm (2.262)(2.69)/\sqrt{10} \Rightarrow (7.23, 11.08)$
 Because there is a large overlap between the three C.I.'s, the study does not provide significant evidence of a difference in the mean increase in mpg for the three additives.

7.13 The data is summarized in the following table:

Method	n	Mean	95% C.I. on μ	St.Dev.	95% C.I. on σ
L	10	5.90	(2.34, 9.46)	4.9766	(3.42, 9.09)
L/R	7	7.29	(2.31, 12.26)	5.3763	(3.46, 11.84)
L/C	9	16.00	(9.57, 22.43)	8.3666	(5.65, 16.03)
C	9	17.67	(5.45, 29.88)	15.8902	(10.73, 30.44)

From the box plots, it appears that the L/R distribution is right skewed but the other 3 distributions appear to be random samples from normal distributions.

Test $H_o : \sigma_L = \sigma_{L/R} = \sigma_{L/C} = \sigma_C$ versus $H_a : \sigma's$ are different

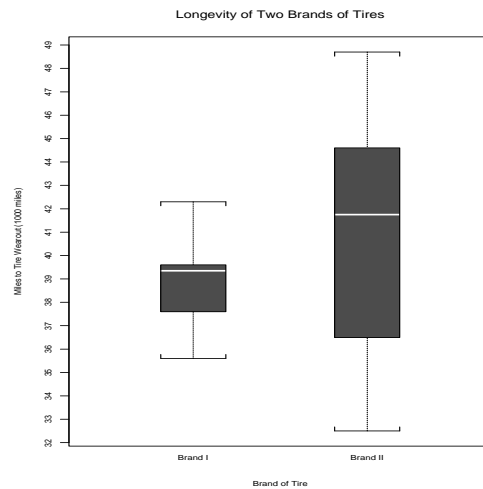
Reject H_o at level $\alpha = 0.05$ if $L \geq F_{.05,3,31} = 2.91$

From the data, $L = 2.345 < 2.91 \Rightarrow$

Fail to reject H_o and conclude there is not significant evidence of difference in variability of the increase in test scores.

Based on the C.I.'s for the $\mu's$, we can conclude that there is very little difference in the average change in test scores for the four methods of instruction. However, lecture only method yielded somewhat smaller mean change in test score than the computer instruction only procedure. These confidence intervals have an overall level of confidence of $(.95)^4 = 0.81$ since the data from the four procedures are independent. Thus, our conclusion would have a relatively large chance of committing a Type I error in attempting to determine if any pair of instructional methods have different means. An improved procedure for comparing the four instructional methods will be covered in Chapter 8. This procedure would determine that there is a significant difference in the instructional means (p-value = 0.032).

7.14 a. The box plots are given here:



The box plots and normal probability plots indicate that both samples are from normally distributed populations.

b. The C.I.'s are given here:

Method	n	Mean	95% C.I. on μ	St.Dev.	95% C.I. on σ
I	10	38.79	(37.39, 40.19)	1.9542	(1.34, 3.57)
II	10	40.67	(36.68, 44.66)	5.5791	(3.84, 10.19)

c. A comparison of the population variances yields:

$$H_o : \sigma_I^2 = \sigma_{II}^2 \text{ versus } H_a : \sigma_I^2 \neq \sigma_{II}^2$$

With $\alpha = 0.01$, reject H_o if $\frac{s_I^2}{s_{II}^2} \leq \frac{1}{6.54} = 0.15$ or $\frac{s_1^2}{s_2^2} \geq 6.54$

$$s_1^2/s_2^2 = (5.5791)^2/(1.9542)^2 = 8.15 > 6.54 \Rightarrow$$

Reject H_o and conclude there is significant evidence that the population variances are different.

A comparison of the population means using the separate variance t-test yields:

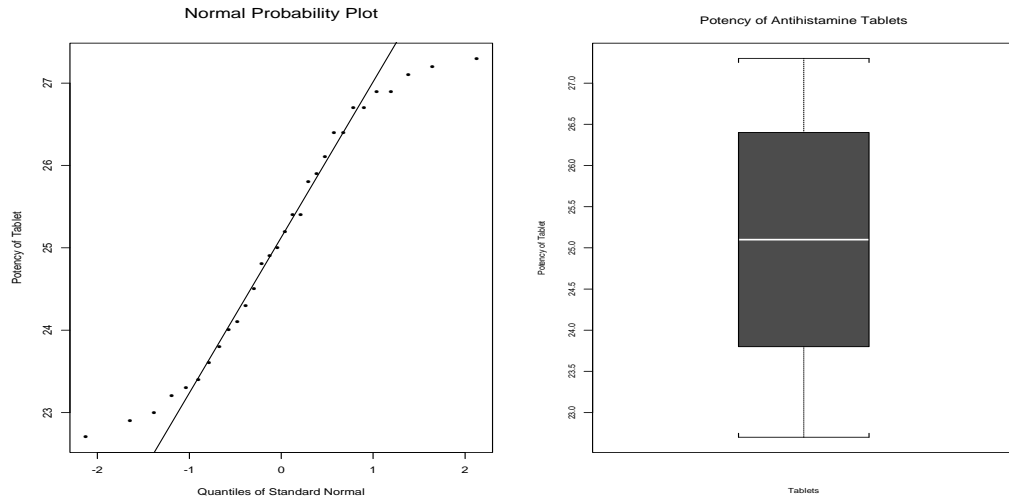
$$H_o : \mu_I = \mu_{II} \text{ versus } H_a : \mu_I \neq \mu_{II}$$

$$t = \frac{38.79 - 40.67}{\sqrt{\frac{(1.9542)^2}{10} + \frac{(5.5791)^2}{10}}} = -1.01 \text{ with df}=11 \Rightarrow \text{p-value} = 0.336$$

Fail to reject H_o and conclude that the data does not support a difference in the mean tread wear for the two brands of tires. However, Brand I has a more uniform tread wear as reflected by its significantly lower standard deviation.

7.15 a. $25 \times 90\% = 22.5$ and $25 \times 110\% = 27.5$ implies the limits are 22.5 to 27.5

b. The box plot and normal probability plot are given here:



The box plot indicates a symmetric distribution with no outliers. The normal probability plot shows the data values reasonably close to a straight line, although there is some deviation at both ends which indicates that the data may be a random sample from a distribution which has shorter tails than a normally distributed population.

- c. Range = $27.5 - 22.5 = 5 \Rightarrow \hat{\sigma} = 5/4 = 1.25$

$H_o : \sigma = 1.25$ versus $H_a : \sigma \neq 1.25$

With $\alpha = 0.05$, reject H_o if $\frac{(n-1)s^2}{(1.25)^2} \leq 16.05$ or $\frac{(n-1)s^2}{(1.25)^2} \geq 45.72$

$$\frac{(30-1)(1.4691)^2}{(1.25)^2} = 40.06 \Rightarrow 16.06 < 40.06 < 45.72$$

Fail to reject H_o and conclude there is insufficient evidence that the product standard deviation is greater than 1.25. Thus, it appears that the potencies are within the required bounds.

- 7.16 a. $H_o : \sigma_1^2 \geq \sigma_2^2$ versus $H_a : \sigma_1^2 < \sigma_2^2$

With $\alpha = 0.05$, reject H_o if $\frac{s_2^2}{s_1^2} \geq 3.18$

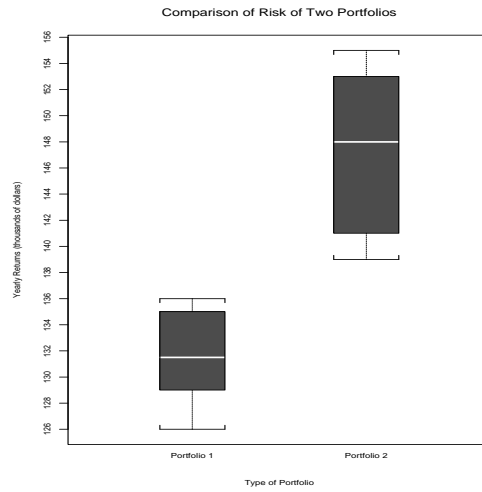
$$s_2^2/s_1^2 = (5.9591)^2/(3.5963)^2 = 2.75 < 3.18 \Rightarrow$$

Fail to reject H_o and conclude there is not significant evidence that portfolio 2 has a larger variance than portfolio 1.

$$95\% \text{ C.I. on } \frac{\sigma_2^2}{\sigma_1^2} : \left(\frac{(5.9591)^2}{(3.5963)^2} (0.248), \frac{(5.9591)^2}{(3.5963)^2} (4.03) \right) \Rightarrow (0.68, 11.07)$$

- b. p-value = $P(F_{(9,9)} \geq 2.75) \Rightarrow 0.05 < \text{p-value} < 0.10$

- c. The box plots are given here:



From the box plots, the condition of normality appears to be satisfied for both portfolios.

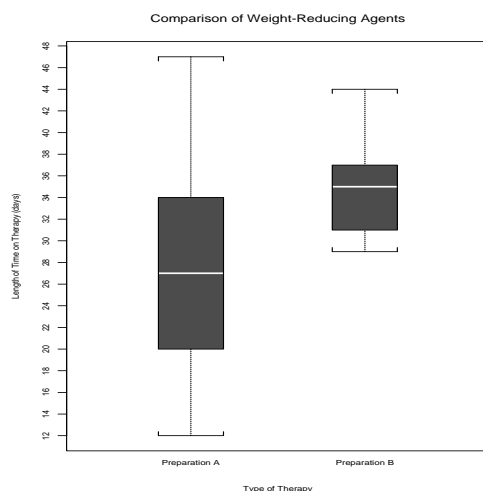
- 7.17 Since the box plots indicate that data from both portfolios has a normal distribution. Also, the C.I. on the ratio of the variances contained 1 which indicates equal variances. Thus, a pooled variance t-test will be used as the test statistic.

$$H_o : \mu_1 = \mu_2 \text{ versus } H_a : \mu_1 \neq \mu_2$$

$$t = \frac{131.60 - 147.20}{4.92 \sqrt{\frac{1}{10} + \frac{1}{10}}} = -7.09 \text{ with df}=18 \Rightarrow \text{p-value} < 0.0005$$

Reject H_o and conclude that the data strongly supports a difference in the mean returns of the two portfolios.

7.18 Box plots are given here:



The box plots indicate that both samples are from normally distributed populations but with different levels of variability.

The C.I.'s are given here:

Method	n	Mean	95% C.I. on μ	St.Dev.
A	13	27.62	(21.68, 33.55)	9.83
B	13	34.69	(32.26, 37.13)	4.03

A comparison of the population variances yields:

$$H_o : \sigma_A^2 = \sigma_B^2 \text{ versus } H_a : \sigma_A^2 \neq \sigma_B^2$$

$$s_A^2/s_B^2 = (9.83)^2/(4.03)^2 = 5.955 \Rightarrow \text{p-value} = 0.0021 \Rightarrow$$

Reject H_o and conclude there is significant evidence that the population variances are different.

A comparison of the population means using the separate variance t-test yields:

$$H_o : \mu_A = \mu_B \text{ versus } H_a : \mu_A \neq \mu_B$$

$$t = \frac{27.62 - 34.69}{\sqrt{\frac{(9.83)^2}{13} + \frac{(4.03)^2}{13}}} = -2.40 \text{ with df}=15 \Rightarrow \text{p-value} = 0.030$$

Reject H_o and conclude that the data indicates a difference in the mean length of time people remain on the two therapies.

7.19 We would now run a 1-tail test:

$$H_o : \mu_A \geq \mu_B \text{ versus } H_a : \mu_A < \mu_B$$

$$t = \frac{27.62 - 34.69}{\sqrt{\frac{(9.83)^2}{13} + \frac{(4.03)^2}{13}}} = -2.40 \text{ with df}=15 \Rightarrow \text{p-value} = 0.0149$$

Reject H_o and conclude that the data indicates the mean length of time people remain on therapy B is longer than the mean for therapy A.