## Lesson 3 - Homework

**Notes:**

* The answers to the problems will be posted on Sunday after the due date. To view the answers, click on the **Homework Solutions** link in the menu bar to the left.

**1.** Problem 4.11 of the 6th edition (which is Problem 4.6 of the 5th edition) hint: list out the events in A,B,C respectively, it is then easy to compute their probabilities.

The events are

|  |  |  |  |
| --- | --- | --- | --- |
|  | Toss 1 | Toss 2 | Toss 3 |
| 1 | H | H | H |
| 2 | H | H | T |
| 3 | H | T | H |
| 4 | H | T | T |
| 5 | T | H | H |
| 6 | T | H | T |
| 7 | T | T | H |
| 8 | T | T | T |

Total possibilities: 2^3 = 8

1. Exactly 1 head: Events 4, 6, 7. Probability = 3/8
2. 1 or more heads: all events except 8. Probability = 7/8
3. No heads: only event 8. Probability = 1/8

**2.** For the events given in the above problem, compute:

1. P (A | B)
2. P (A | C)
3. P (B | C)
4. Are A, B independent? Are A, B mutually exclusive?
5. Are B, C independent? Are B,C mutually exclusive?
6. P(A|B) = P(A ∩ B)/P(B) = 3 / 7
7. P (A | C) = P(A ∩ C)/P(C) = 0 (since A and C have an empty intersection)
8. P (B | C) = P(B ∩ C)/P(C) = 0 (since B and C have an empty intersection)
9. A and B are not independent as we see above that P(A|B) ≠ P(A). Also, P(A∩B) = 3/8 ≠ P(A) \* P(B) = 3/8\*7/8=21/64

They are also not mutually exclusive as event B is a superset of event A. Therefore when event A occurs, B definitely occurs but not vice versa.

1. B and C are not independent as we see above that P(B|C) ≠ P(B). Also, P(B ∩ C) = 0 ≠ P(B) \* P(C) = 1/8\*7/8=7/64

They are however mutually exclusive. When we know either event B or event C occurs that completely excludes the possibility of occurrence of the other event.

**3.** Consider the following outcomes for an experiment:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Outcome | 1 | 2 | 3 | 4 | 5 |
| Probability | .20 | .25 | .15 | .10 | .30 |

Let A consists of outcomes 1, 2, 4 and B consist of outcomes 4 and 5

a. Find P(A) and P(B)

b. Find P( both A and B occur)

c. Find P( either A or B occur)

d. Are A, B independent?

e. Are A, B mutually exclusive.

1. P(A) = .55 and P(B) = .4
2. P(A ∩ B) = P(4) = .1
3. P(A U B) = P(A) + P(B) - P(A ∩ B) = .55 + .4 - .1 = .85
4. A and B are not independent because:
   1. Also P(A ∩ B) = P(4) = .1 ≠ P(A) P(B) = .55 \* .4 = .22
   2. The knowledge of one of the events has an effect on the conditional probability of the other. So if we know that event B occurred that changes the probability of A i.e. P(A|B) = .1 / .4 = .25 ≠ P(A)
5. A and B are not mutually exclusive as P(A ∩ B) = .1 ≠ 0

**4.** Problem 4.27 of the 6th edition which is Problem 4.21 of the 5th edition

1. Full payment by both customers: 0.70 \* 0.70 = 0.49 or 49%
2. Atleast one in full = P(1 pays full) + P(both pay full) = 2\*0.7\*0.3 + 0.49 = 0.91 or 91 %

**5.** Problem 4.28 of the 6th edition whichisProblem 4.22 of the 5th edition

Use Notation

A – 1st bill in full

A^ - 1st bill not in full

B – 2nd bill in full

B^ - 2nd bill not in full

We know from the question P(B | A) = .95 and P(B | A^) = .1

We can deduce: P(B^ | A^) = 1-.1 = .9, P(B^ | A) = 1-.95 = .05

1. P(A ∩ B) = P(A) \* P(B|A) = 0.7 \* 0.95 = 0.665
2. P(A^ ∩ B^) = P(A^) \* P(B^|A^) = 0.3 \* 0.9 = 0.27
3. We have to find P(A^ ∩ B) + P(A ∩ B^) = P(A^) \* P(B|A^) + P(A) \* P(B^|A) = 0.3 \* 0.1 + 0.7 \* 0.05 = .03 + .035 = .065

**6.** Problem 4.41 of the 6th edition which is Problem 4.34 of the 5th edition

1. P(4)+P(5)+P(6)+ P(7)+P(8)+P(9)+P(10) = .12+.1+.08+.07+.06+.04+.03 = 0.5
2. P(4)+P(5)+P(6) = .12+.1+.08 = 0.3
3. P(9)+P(10) = .04+.03 = 0.07

**7.** If the table in sample problem 2 is given as:

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Test** | **Result** |  |
| **True Status** | **(+)** | **(-)** | **Total** |
| **Pregnant** | 70 | 2 | 72 |
| **Not Pregnant** | 18 | 910 | 928 |
| **Total** | 88 | 912 | 1000 |

Give the probabilities asked in a, b, c, d of the sample question 2.

a. What percentage of the women in the sample of 1,000 women were pregnant?

P( pregnant) = 72/1000 = 0.072

1. What percentage of the women in the sample of 1,000 women tested positive?

P( tested positive) = 88/1000 = 0.088

1. Given a woman is pregnant, what is the probability that she gets a positive pregnancy test?

P( test positive given pregnant)

= P(test positive and pregnant)/P(pregnant)

= (70/1000)/(72/1000) = 70/72 = 0.97

1. Given a women receives a positive pregnancy test, what is the probability that she is truly pregnant?   
       
   P(pregnant given test positive)

= P(pregnant and test positive)/P(test positive)

= (70/1000)/( 88/1000) = 70 / 88 = 0.795

**Reading:** *An Introduction to Statistical Methods and Data Analysis*, chapters 4.1 - 4.4, 4.6, 4.7.

**Sample problem 1:**

**1. A die is to be rolled and we are to observe the number of that falls face up. Find the probabilities of these events:**

a. A : observe a 6

b. B : observe an even number

c. C : observe a number greater than 2

d. D: observe an even number and a number greater than 2

e. find P( A given B).

f. Are C, D independent?

g. Are C, D mutually exclusive?

**Solution to sample problem 1.**

**For this problem, the possible outcomes are: {1, 2, 3, 4, 5, 6}, points shown on the face of a die. If this is a fair die, then:**

1. first list out A: A = { 6 } , P(A) = 1/6
2. first list out B: B = { 2,4, 6 }, P(B) = 3/6= ½
3. since C = { 3,4,5,6 }, P(C) = 4/6 = 2/3
4. since D = { 4,6 }, P(D) = 2/6 = 1/3
5. P(A given B) = P( A intersect B)/P(B) = P( {6})/P(B) = (1/6)/(3/6) = 1/3
6. To check whether C, D are independent, we need to check whether

P(C intersects D) = P( C) \* P(D)?

C intersects D = {4, 6}, P(C intersects D) = 2/6

P( C) \* P(D) = (2/3) \* (1/3) = 2/9,

Since P(C intersects D) ≠ P( C) \* P(D) ,

C, D are not independent.

1. Mutually exclusive means there is nothing in common to C and D, since

C intersects D = {4, 6} , these two outcomes are in common to C and D, thus C, D are not mutually exclusive.

**Sample problem 2. Example on Conditional probability:** A female student wants to determine whether to PANIC or not about the positive result she received when performing a home pregnancy test.  To answer her question, she finds the following data on the accuracy of the pregnancy test she used when performed on 1,000 college-aged women.  

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Test** | **Result** |  |
| **True Status** | **(+)** | **(-)** | **Total** |
| **Pregnant** | 48 | 2 | 50 |
| **Not Pregnant** | 38 | 912 | 950 |
| **Total** | 86 | 914 | 1000 |

1. What percentage of the women in the sample of 1,000 women were pregnant?
2. What percentage of the women in the sample of 1,000 women tested positive?
3. Given a woman is pregnant, what is the probability that she gets a positive pregnancy test?
4. Given a woman receives a positive pregnancy test, what is the probability that she is truly pregnant?   
       
   **Solution to Sample problem 2**

a. What percentage of the women in the sample of 1,000 women were pregnant?

P( pregnant) = 50/1000 = 0.05

1. What percentage of the women in the sample of 1,000 women tested positive?

P( tested positive) = 86/1000 = 0.086

1. Given a woman is pregnant, what is the probability that she gets a positive pregnancy test?

P( test positive given pregnant)

= P(test positive and pregnant)/P(pregnant)

= (48/1000)/(50/1000) = 48/50 = 0.96

* 1. Given a women receives a positive pregnancy test, what is the probability that she is truly pregnant?   
         
     P(pregnant given test positive)

= P(pregnant and test positive)/P(test positive)

= (48/1000)/( 86/1000) = 0.558