## Lesson 9 - Homework

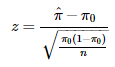
**1.** (15 pts) In 1995, 10.8% of all U.S. families had incomes below the poverty level, as reported by the Census Bureau in *Current Population Reports*. During that same year, of 378 randomly selected families whose householder had a Bachelor's degree or more, 11 had incomes below the poverty level. At the 1% significance level, do the data provide sufficient evidence to conclude that in 1995, families whose householder had a Bachelor's degree or more had a lower percentage earning incomes below the poverty level than the national percentage of 10.8%? (Use rejection region approach.)

n π0 = 378 \* 0.108 = 40.824 > 5

n(1 - πo) = 378 \* (1-0.108) = 337.176 > 5

Null and alternate hypothesis: Ho: π >= 0.108, Ha: < 0.108, According to the question, α = 0.01

π^ = 11/378 = 0.029

Test Statistic Z\* :  = (0.029-0.108) / sqrt(0.108\*(1-0.108) / 378) = -4.95

The rejection region is Z < -2.326. Therefore since the test statistic lies in the rejection region we reject the null hypothesis. We can conclude that that in 1995, families whose householder had a Bachelor's degree or more had a lower percentage earning incomes below the poverty level than the national percentage of 10.8%

**2.** (15 pts) The Chicago Title Insurance Company publishes statistics on recent home buyers in the *The Guarantor*. According to that publication, 83.1% of home buyers in 1995 purchased single-family houses. Out of 200 randomly selected home buyers for this year, 151 purchased single-family houses. Do the data provide sufficient evidence to conclude that this year's percentage of home buyers purchasing single-family houses is different from the 1995 figure of 83.1%? Use α = 0.021. (Use p-value approach.)

n π0 = 200 \* 0.831 = 166.2 > 5

n(1 - πo) = 200 \* (1-0.831) = 33.8 > 5

H0 :π = 0.831 and Ha :π ≠ 0.831 This is a two-sided test and α = 0.021

πˆ = 151/200 = 0.755

Test Statistic: z\* = (0.755 – 0.831) / sqrt(0.831 \* (1 – 0.831)/200) = -2.868

P-value = 2\*P(z > 2.868) = 2 \* (1 – P(Z<=2.868)) = 2 \* (1 - 0.9979) = 0.0042

P-value of 0.0042 < α of 0.021 therby we can reject the null hypothesis.

**3.** (15 pts) The owner of an exercise club believes the mean value of the amount of weight lost in a program during six months should be at least 30 pounds. The number of pounds lost by a sample of 29 people over six months are given below:

3, 4, 5, 5, 6, 7, 7, 10, 10, 10, 11, 12, 15, 15, 18, 18, 20, 23, 24, 25, 25, 28, 29, 30, 35, 40, 41, 44, 45

Can you reject the owner’s claim at α = 0.01? Check conditions before you perform the test. Use Minitab>stat>basic stat> 1-sample t to get the p-value and then use p-value approach.

Ho: µ <= 30; Ha: µ> 30

**One-Sample T: pounds**

Test of μ = 30 vs > 30

Variable N Mean StDev SE Mean 95% Lower Bound T P

pounds 29 19.48 12.83 2.38 15.43 -4.41 1.000

Now, we obtain that the observed level of significance (p-valu) is 1.0 For this problem, we see that p-value = 1.0 > 0.01. Thus, we fail to reject the null hypothesis. That is, we fail to conclude the research hypothesis.

**4**. (10 pts) Test the claim (research hypothesis) that for the population of U.S. commercial jets, the mean age is less than 18 years. A sample of 36 jets has a mean of 16.2 years and a sample standard deviation of 8.1 years. (use α=0.05 and the rejection region approach).

Null and alternate hypothesis: Ho: mu >= 18, Ha: mu < 18, According to the question, α = 0.05

Test Statistic t\* :  = (16.2 – 18) / (8.1 / sqrt(36)) = -1.33

Since n = 36, degrees of freedom = 35, and the critical values are t0.05. The value from Minitab is

t0.025 = -1.69

Since -1.33 does not fall in the rejection region, we cannot reject Ho at α = 0.05. Therefore at α = 0.05, the data does not provide sufficient evidence to conclude that the mean age is less than 18 years.

**5.** (15 pts) The city of Providence supervisor claims that the mean weight of all garbage discarded by households each week is 29 lb. A random sample of 30 households shows that the sample mean is 27.02 with a standard deviation of 12.16 lb. Is there evidence to doubt the supervisor’s claim? (use α=0.01 and the rejection region approach).

Null and alternate hypothesis: Ho: mu = 29, Ha: mu ≠ 29, According to the question, α = 0.01

Test Statistic t\* :  = (27.02-29) / (12.16 / sqrt(30)) = -0.892

Since n = 30, degrees of freedom = 29, and the critical values are +- t0.005. The value from Minitab is

t0.005 = -2.756 so The critical values are ± 2.756

Since -0.892 does not fall in the rejection region, we cannot reject Ho at α = 0.01. There is not enough evidence to doubt the claim.

**6.** (15 pts) A random sample of 30 standard metropolitan statistical areas (SMSAs) was selected and ratio (per 1000) of registered voters to the total number of persons 18 years and over was recorded in each area. Use the data given to test the research hypothesis that μ, the average ratio, is different from 675, last year’s average ratio. Give the level of significance for your test. In addition, use the p-value approach to perform the test at a default level of significance.

802 497 653 600 729 812 751 730 635 605 760 681 807 747 728

561 696 710 641 848 672 740 818 725 694 854 674 683 695 803

Null and alternate hypothesis: Ho: mu = 675, Ha: mu ≠ 675

**One-Sample T: voters**

Test of μ = 675 vs ≠ 675

Variable N Mean StDev SE Mean 95% CI T P

voters 30 711.7 84.2 15.4 (680.3, 743.1) 2.39 0.024

Now, we obtain that the observed level of significance (p-valu) is 0.024. For this problem, since alpha is not given, we use the default value of 0.05. We see that p-value = 0.024 < 0.05. Thus, we reject the null hypothesis.

**7.** (15 pts) The stated weight of detergent is supposed to be 48 oz. The manufacturer wants to detect a difference of 1 oz from the stated weight. It is observed from past data that the weight of a bottle of detergent ranges from 49 to 55 oz. How many bottles of detergent should one sample so that one can test the claim that "the mean weight is different from 48 oz" with 80% power at 1% level of significance? (Please use minitab: “Power and sample size”, you can provide an estimate for the value of sigma using the formula Range/4, also, remember to adjust the alpha level in minitab).

H0: mu = 48 and Ha: mu ≠ 48, Delta: 1oz, Power=0.8 so B=0.2, alpha = 0.01, sigma = (55-49)/4 = 1.5

Z0.005 = 2.576 and Z0.2 = 0.842

Use  = (1.5^2 \* (2.576 + 0.842)^2) / 1^2 = 26.28 = 27 bottles

**Using Minitab: Power and Sample Size**

1-Sample t Test

Testing mean = null (versus ≠ null)

Calculating power for mean = null + difference

α = 0.01 Assumed standard deviation = 1.5

Sample Target

Difference Size Power Actual Power

1 30 0.8 0.806362

Therefore sample size = 30 bottles.

**https://courses.worldcampus.psu.edu/stat500sp03/images/index.gif Reading:** *An Introduction to Statistical Methods and Data Analysis*, chapters 5.6, 5.7。

**Practice Homework:** (please do not submit these problems): problems 5.43, 5.46, 5.47 in 6th edition (which are problems 5.57, 5.61, 5.62 in 5th edition) (use minitab instead of the formula to compute type II error).

**Practice problems and its solutions:**

**Practice problem 1**. Problem 5.44 in 6th edition (5.58 in 5th edition) b and c (skip a since a is contained in c)

**Solutions to practice problem 1:**

b. N=18<30, use normal probability plot to see if the data is normal distribution.

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**Practice problem 2**. The undergraduate GPA of 18 students from a large MBA class of 800 students are selected. The data are given below:

3.76 3.10 3.15 3.45 4.00 3.80 3.00 3.20 3.30

3.76 3.60 3.01 3.74 3.90 3.70 3.83 4.00 3.40.

Use the data given to test the research hypothesis that the average undergraduate GPA of the MBA class is more than 3.5. Give the level of significance of your test. Use the p-value approach to perform the test at a default level of significance.

**Solution to practice problem 2**:

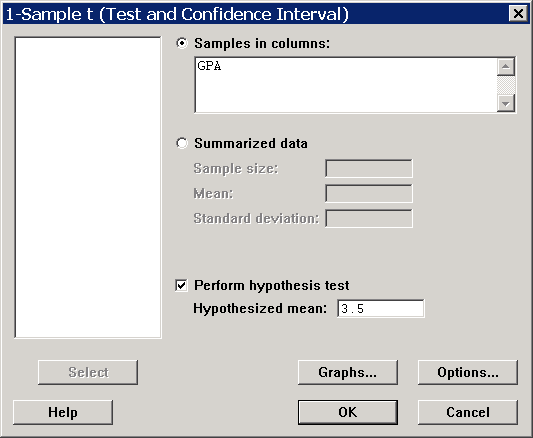
Since the sample size is 18, less than 30, we use normal probability plot to plot the data. Since all the points fall within the confidence bands, we conclude that the data may come from a normal distribution and proceed to use the t-test.

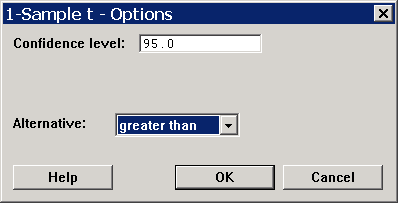


Since research hypothesis is mu greater than 3.5, we set up the hypotheses as:

Ho: µ≤ 3.5; Ha: µ> 3.5

Using **Minitab > Stat > Basic Statistics > 1-sample t :**





We need to check the box : Perform hypothesis test and then get into the option to select the alternative. The output of minitab is:

**One-Sample T: GPA**

Test of mu = 3.5 vs > 3.5

95%

Lower

Variable N Mean StDev SE Mean Bound T P

GPA 18 3.53889 0.34210 0.08063 3.39862 0.48 0.318

Now, we obtain that the observed level of significance (p-valu) is 0.318. For this problem, since alpha is not given, we use the default value of 0.05. We see that p-value = 0.318 > 0.05. Thus, we fail to reject the null hypothesis. That is, we fail to conclude the research hypothesis.