**Stat 500 Solution- HW5**

1. *(25 pts; each 5 pts)*
2. z = (y –μ)/σ = (600 – 500)/100 = 1. From Table 1, P (y > 600) = P (z >1) = 1 – P (z ≤ 1) = 1 – 0.8413 = 0.1587. Therefore, 15.87% of scores can be expected to be greater than 600.
3. z = (y – μ)/σ = (700 – 500)/100 = 2. From Table 1, P (y > 700) = P (z > 2) = 1 – P (z ≤ 2) = 1 - .9772 = .0228. So 2.28% would be expected to be greater than 700.
4. z = (y –μ)/σ = (450 – 500)/100 = - 0.5. P (y < 450) = P (z < -0.5) = 0.3085. Thus 30.85% can be expected to be less than 450.
5. From part a, P (y < 600) = 0.8413; from part c, P (y < 450) = 0.3085. Then P (450 < y < 600) = P (y < 600) – P (y < 450) = 0.8413 – 0.3085 = 0.5328. So 53.28% of scores can be expected to be between 450 and 600.
6. **,** = 0.9999683- 0.0082=0.9918 Therefore, 99.18% of the class average SAT score is between 440 and 600.
7. *(10 pts)*

Randomly generate 28 integers from 1 to 1725 *without replacement* from Minitab:

|  |
| --- |
| ID 713 731 899 1818 458 370 855 171 1822 1343 1028  1410 1578 1078 148 1204 577 1726 1713 1235 980 292  1759 668 1084 1480 565 83 |

1. *(15 pts)*

Individual baggage weight has ** and = 35, so  ⇒  or**

(Alternative method:

Total weight has mean = 200 × 95 = 19000 with the standard deviation == = 494.97. So P (> 20,000) = P {z > (20,000 – 19,000)/494.97} = P (z > 2.02) = 1 – 0.9783 = 0.0217. The probability that the total weight of the passengers’ baggage will exceed the 20000-pound limit is 2.17%.)

1. *(20 pts; (a), (b) 10 pts; (c) extra 3 credits)*
2. μ = 160 and σ = 20 ⇒ P (y<150) = P {z < (150 -160)/20} = P (z < - 0.5) = 0.3085. If a single measurement is taken, the probability that this measurement will fail detect that the patient has high blood pressure is 30.85%.
3. ⇒ P (< 150) = P {z < (150 – 160)/8.94} = P (z < - 1.12) = 0.1314. If five measurements (n=5) are taken, there is 13.14% probability that the average of 5 blood pressure readings will be less than 150.
4. From Table 1, closest value to 0.01 is 0.0099 with corresponding z value = -2.33 ⇒ z = -2.33 = (150-160)/(20/) ⇒ n = 21.72 ≈ 22. So at least 22 measurements are needed.
5. (*20 pts; (a), (b) 10 pts*)

**a)** (x-µ) / (σ)=(2265-2250) / 10.2=1.47

P(x>2265) = P (z>1.47)= 1- P(z<1.47)=1-0.9292=0.0708

**b)** µy = 2250, σy = 10.2/ = 2.63 According to the central limit theorem,the sampling distribution for y-bar based on random samples of 15 1-foot sections is approximately a normal distribution.

**6)** *(10 pts(a), (b) 4pts each (c)2pts )*

**a)** population mean = \_\_16\_\_, population st. dev. = \_\_5

|  |  |  |  |
| --- | --- | --- | --- |
| Sample size | mean of sample mean | st. dev. Of sample mean | Skew |
| N=2 | 16.01 | 3.54 | -0.02 |
| N=5 | 16 | 2.25 | -0.01 |
| N=25 | 16.00 | 0.99 | 0.02 |

  b) population mean = 8.08 , population st. dev. = \_6.22

|  |  |  |  |
| --- | --- | --- | --- |
| Sample size | mean of sample mean | st. dev. of sample mean | skew |
| N=2 | 8.06 | 4.36 | 0.56 |
| N=5 | 8.13 | 2.79 | 0.32 |
| N=25 | 8.08 | 1.28 | 0.20 |

c) The central limit theorem that for a large sample size (rule of thumb: n 30), is approximately normally distributed, regardless of the distribution of the population one samples from. If the population has mean and standard deviation, then  has mean  and standard deviation.

The simulation results indicate the central limit theorem seem correct. In (a), the population distribution is a normal. The sample mean is close to the population mean for different sample size . The standard deviation decreases in proportion of. The skew is close to 0. Hence, the sample distribution seems to be close to a normal distribution. In (b), the population distribution is a skewed one. The sample is getting close to the population mean. The standard deviation also decreases in proportion of. The skew is high (0.56) for small, but it is getting close to 0 for large. In this case, its sample mean seems approximately normally distributed. Overall, the central limit theory holds true regardless of the population distribution as long as the sample size gets large.