**STAT 501 – Homework 4 (covering Lesson 4) – due date September 20th**

**Instructions**: Use Word to type your answers within this document. Then, submit your answers in the appropriate dropbox in ANGEL by the due date. The point distribution is located next to each question. If there are multiple parts, then the points are divided equally over the subparts.

1. **(3x8 = 24 points)** Say whether the following statements about residual analysis for simple linear regression are true or false and briefly explain your answers.
   1. If the sample mean of the residuals is zero, then this supports the linearity condition for the model.
   2. An ideal residual plot for a valid model will display residuals with a strong positive or negative linear trend.

False.

* 1. An ideal residual plot for a valid model will display residuals with similar variation no matter the value on the horizontal axis.

False.

* 1. We should only assess the linearity and equal variance conditions after first confirming the normality condition using a normal probability plot.
  2. A residual plot that has a predictor variable excluded from the model on the horizontal axis provides no useful information about the adequacy of the model.
  3. We can accept the validity of our model if at least one of the four LINE conditions is supported.

False.

* 1. We should question the validity of our model only if all four LINE conditions seem in doubt.

True.

* 1. Residual analysis is entirely objective so that every question has a right or wrong answer.

False.

1. **(2x4 = 8 points)** The dataset “Auto” gives information on 392 different cars including their mpg, weight, horsepower, displacement, year of make, country of origin etc. A car buyer is interested in regressing mpg (Y) on horsepower (X). Given below is the residual plot of the regression.



1. What departure, if any, from the simple linear regression model assumptions can be assessed from this plot?

The following departures are evident from the plot:

First, the pattern seems curved (slight U) that indicates that the wrong type of equation was used.

This plot of residuals versus fits shows that the residual variance increases as the fitted values increase. This violates the assumption of constant error variance.



1. The residuals are now regressed on another predictor weight. Residual plot of this regression is given below. Would you say that inclusion of weight as an additional predictor has improved the residual plot? Briefly justify your answer.

Yes. The inclusion of weight predictor has reduced both of the effects highlighted in part a.

1. First, the pattern still seems curved but is less markedly less curved than the previous.
2. This plot of residuals versus weight still shows variation in the variance of the residuals. However even in this case the variation is markedly reduced.
3. **(8 points)** “Auto” data includes cars from three different origins: 1 (USA), 2 (Europe) and 3 (Japan). The variable *acceleration* (time to accelerate) is regressed on the variable *displacement*, separately for the cars from different origins. Following are the residual plots from three separate regression equations.

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Compare the residual plots for the cars of three different origins in terms of (i) whether linearity holds (ii) whether the response follows normal distribution and (iii) whether the errors have equal variances.

1. (**6x5 = 30 points**) Consider the “Desired Height” dataset. The data are from females in two introductory statistics classes at Penn State. The variables are *x* = Height (student's height in centimeters) and *y* = DesiredHt (how tall the student said they would like to be, in centimeters, if they could be any height at all). (Note: Students gave responses in inches, but we converted them to centimeters.)
   1. Draw a scatterplot of *y* = DesiredHt versus *x* = Height. Include the graph and write a brief description of the graph's main features in the context of simle linear regression. [In Minitab, the menu sequence is Graph > Scatterplot. To copy a Minitab graph, right click on it and use “Copy Graph” in the resulting menu.]



The scatterplot shows the following features

1. The plot indicates that the mean desired height increases as height increases.
2. The variance of desired height at different height level is not completely uniform
   1. Determine the estimated sample regression equation for a straight-line relationship between desired height and actual height. [In Minitab, use the menu sequence Stat > Regression > Regression > Fit Regression Model (v17, exclude last part in v16).]

The model from minitab gives:

**Regression Analysis: DesiredHt versus Height**

Analysis of Variance

Source DF Adj SS Adj MS F-Value P-Value

Regression 1 3716.4 3716.39 197.72 0.000

Height 1 3716.4 3716.39 197.72 0.000

Error 249 4680.3 18.80

Lack-of-Fit 16 634.5 39.66 2.28 0.004

Pure Error 233 4045.8 17.36

Total 250 8396.7

Model Summary

S R-sq R-sq(adj) R-sq(pred)

4.33549 44.26% 44.04% 43.25%

Coefficients

Term Coef SE Coef T-Value P-Value VIF

Constant 77.84 6.45 12.06 0.000

Height 0.5518 0.0392 14.06 0.000 1.00

Regression Equation

DesiredHt = 77.84 + 0.5518 Height

* 1. The output that you created for the previous part will include a value of *R*2. What is that value? Write a sentence that interprets the value in the context of this situation.

The value of R2 is 44.26%

We can say that 44.26% of the variation in the DesiredHt is reduced by taking into account Height. Or, we can say that 44.26% of the variation in DesiredHt is 'explained by' Height.

* 1. In the output that you created, find evidence that the observed relationship is statistically significant. Describe the evidence that you found (for instance, the test for . . . had a *p*-value of . . ., which means . . . .)

The evidence lies in the following section of the output:

Coefficients

Term Coef SE Coef T-Value P-Value VIF

Constant 77.84 6.45 12.06 0.000

Height 0.5518 0.0392 14.06 0.000 1.00

Here we can see that the P-Value for the height co-efficient has value < 0.001

For the testing of the null hypothesis that the population slope β1 equals 0, the t-statistic = 14.06 and the p-value = 0.000 (< 0.001)

Since the p-value < alpha = 0.01 and therefore we reject the null hypothesis that the population slope equals 0. Therefore, the conclusion we draw is that there is a relationship between the variables InfctRsk and Stay

* 1. Create a plot of residuals versus predicted values (fits) for this situation. Include the plot and briefly discuss what the plot indicates about the validity of the model and assumptions about the errors. [In Minitab, click the Graphs button in the Regression dialog and then select “Residuals versus Fits.”]



The residual vs fit plot indicates:

1. There is no obvious pattern in the residuals indicating that the model is valid.
2. The variance of the residuals is varying for different values of the fitted values.
   1. Create a histogram of the residuals and a normal probability plot of the residuals. Include the plots and briefly discuss what they indicate about the validity of the model and assumptions about the errors. [In Minitab, click the Graphs button in the Regression dialog and then select “Histogram of residuals” and also “Normal probability plot of residuals.”]





The plots above indicate that the residuals are not perfectly normal. The histogram and the normal probability plot suggests that the residuals are slightly skewed (to the right).

1. (**6x5 = 30 points**) Consider the “Compare” dataset. There is a single response variable, *Y*, and four potential predictor variables, *X1*, *X2*, *X3*, and *X4*. Fit four simple linear regression models, with each of the predictor variables, *X1*, *X2*, *X3*, and *X4* in turn. For each model create a residual plot with residuals on the vertical axis and fitted values on the horizontal axis.
   1. Compare the values of *S* (root mean square error), *R2*, and the slope *t*-statistic for each model (the clearest way to present this information is in a table). *Based solely on these values*, order the four models from “best” to “worst” (in terms of predicting or explaining *Y*). If two or more models appear to be essentially equivalent based on these criteria, make a note of this.

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| --- | --- | --- | --- | --- | --- |
| Model | S | R2 | T-Statistic (slope) | P-Value | Model Rank |
| X1 | 1.06996 | 80.65% | 14.14 | 0.000 | 4 |
| X2 | 1.01855 | 82.46% | 15.02 | 0.000 | 1 |
| X3 | 1.01873 | 82.46% | 15.02 | 0.000 | 2 |
| X4 | 1.01878 | 82.45% | 15.02 | 0.000 | 3 |

Note: While model 2 has slightly less root mean square error, they are essentially equivalent

* 1. What does the residual plot for the model with *X1* as the predictor indicate about the validity of this regression model and assumptions made about the errors?



* 1. What does the residual plot for the model with *X2* as the predictor indicate about the validity of this regression model and assumptions made about the errors?



* 1. What does the residual plot for the model with *X3* as the predictor indicate about the validity of this regression model and assumptions made about the errors?



* 1. What does the residual plot for the model with *X4* as the predictor indicate about the validity of this regression model and assumptions made about the errors?



* 1. Given your answers for parts (b) to (e), which of the four models now appears to be “best?” Explain your answer.

The best model now appears to be where the predictor is X1. This was ranked the 4th in the initial answer.