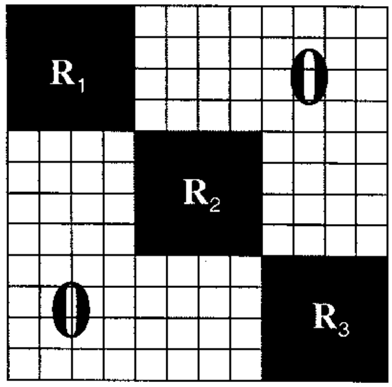
Block Diagonal Covariance Matrix



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This slide illustrates a simple example of a block diagonal covariance matrix. There are four repeated measures on each of the three subjects. So you have a total of 12 observations and therefore a 12×12 covariance matrix. Each block corresponds to the covariance structure for each subject. In PROC MIXED, V_1, V_2, and V_3 must have the same structure but can have different parameter estimates (use the GROUP= option). In this matrix, the observations within each block (subject) can take on a variety of covariance structures while the observations outside of the blocks are assumed to be independent. In other words, the subjects are still assumed to be independent of each other and the measurements within each subject can be interdependent.

Covariance Matrix in Each Block — Variance Component (VC) (Default)

σ_1^2			**0**
	σ_1^2		
		σ_1^2	
0			σ_1^2

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The simplest covariance structure in each block is the default variance component (VC) or simple structure. This is an independent and equal variance structure, where the within-subject error correlation is zero. This is usually not a reasonable structure for repeated measures data because the repeated measures within a subject are often correlated.

Covariance Matrix in Each Block — Compound Symmetry

$$\sigma^2 \begin{bmatrix} 1.0 & \rho & \rho & \rho \\ & 1.0 & \rho & \rho \\ & & 1.0 & \rho \\ & & & 1.0 \end{bmatrix} = \begin{bmatrix} \sigma_b^2+\sigma_e^2 & \sigma_b^2 & \sigma_b^2 & \sigma_b^2 \\ & \sigma_b^2+\sigma_e^2 & \sigma_b^2 & \sigma_b^2 \\ & & \sigma_b^2+\sigma_e^2 & \sigma_b^2 \\ & & & \sigma_b^2+\sigma_e^2 \end{bmatrix}$$

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The simplest correlation model is compound symmetry (CS), also referred to as exchangeable correlation structure. It assumes that correlation is constant regardless of the distance between time points. It may not be a valid assumption for observations collected over time, but may be reasonable in situations where the repeated measurements are not obtained over time. The univariate ANOVA analysis performed previously is equivalent to assuming a CS structure in error terms for the ij^{th} subject. Recall you had $\text{var}(y_{ijk}) = \sigma_b^2 + \sigma_e^2$ and $\text{cov}(y_{ijk}, y_{ijl}) = \sigma_b^2$ for $k \neq l$, which is identical to the CS model with $\sigma^2 = \sigma_b^2 + \sigma_e^2$ and $\rho = \dfrac{\sigma_b^2}{\sigma_b^2 + \sigma_e^2}$.

The MIXED procedure provides the estimate of the covariance parameter σ_b^2 rather than the correlation ρ.

Covariance Matrix in Each Block — First-order Autoregressive AR(1)

$$\sigma^2 \begin{bmatrix} 1.0 & \rho & \rho^2 & \rho^3 \\ & 1.0 & \rho & \rho^2 \\ & & 1.0 & \rho \\ & & & 1.0 \end{bmatrix}$$

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The AR(1) model assumes the correlation between adjacent observations is ρ, regardless of whether the pair of the observations is the first and second, second and third, third and fourth, and so on. It also assumes that the correlation for any pairs of observations two units apart (for example, first and third, second and fourth) is ρ^2. In general, observations d units apart have correlation ρ^d. So, the correlation between observations is a function of their distance in time. This appears to be a reasonable assumption in repeated measures.

AR(1) structure does not model unequally spaced time points, in which case, you probably want to consider one of the spatial covariance structures.

Covariance Matrix in Each Block — Spatial Power

$$
\sigma^2 \begin{bmatrix} 1.0 & \rho^{|t_1-t_2|} & \rho^{|t_1-t_3|} & \rho^{|t_1-t_4|} \\ & 1.0 & \rho^{|t_2-t_3|} & \rho^{|t_2-t_4|} \\ & & 1.0 & \rho^{|t_3-t_4|} \\ & & & 1.0 \end{bmatrix}
$$

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Spatial covariance structures allow for unequal spacing of the repeated measures. These structures are mainly used in geostatistical models, and will be discussed in detail later in the course. Because repeated measures data can be viewed as a spatial process in one dimension, the spatial covariance structures are very useful for modeling repeated measures data, especially when the time points are unequally spaced and the correlations decline as a function of time difference.

The spatial power structure provides a direct generalization of the AR(1) structure for equally spaced data.

Covariance Matrix in Each Block — Unstructured Covariance

σ_1^2	σ_{12}	σ_{13}	σ_{14}
	σ_2^2	σ_{23}	σ_{24}
		σ_3^2	σ_{34}
			σ_4^2

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The most complex covariance structure is unstructured, where observations for each pair of times have their own unique correlation. This is essentially what multivariate ANOVA assumes, and many times it is more complicated than is necessary.