Stat 502 Exam 1 - Solutions

1. A study was conducted to evaluate the durability of 4 types of paint used for highway lanes. Each of the 4 paints (p1, p2, p3, p4) were applied at random locations along a stretch of highway to complete 4 replications. The response is an index of durability. The data were as follows:

|  |  |  |  |
| --- | --- | --- | --- |
| Paint | | | |
| P1 | P2 | P3 | P4 |
| 28 | 21 | 26 | 16 |
| 35 | 36 | 38 | 25 |
| 27 | 25 | 27 | 22 |
| 21 | 18 | 17 | 18 |
|  |  |  |  |

The overall mean was 25.0 and the total sums of squares was 692.0

a) (5 pts) Show the computations to find the Sums of Squares for the Treatment effect

|  |  |
| --- | --- |
| Treatment | Mean |
| P1 | 27.75 |
| P2 | 25 |
| P3 | 27 |
| P4 | 20.25 |

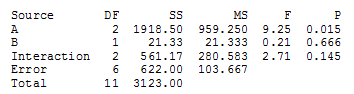
b) (15 pts) Construct the ANOVA table

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | df | SS | MS | F |
| Trt | 3 | 136.5 | 45.5000 | 0.98 |
| Error | 12 | 555.5 | 46.2917 |  |
| Total | 15 | 692.0 |  |  |

c) (5pts) Find the Fcritical value to test at a 95% confidence level.

d) (5pts) Conclude: \_\_***Do Not Reject***\_\_ .

2) (10 pts) The output below shows a two-factor ANOVA with no significant interaction and no significant main effect for Treatment B. Given that the researcher has expressed no interest in reporting the lack of significant effect of Treatment B or Interaction, complete the one-way ANOVA for treatment A.



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | DF | SS | MS | F |
| A | 2 | 1918.50 | 959.25 | 7.17 |
| Error | 9 | 1204.50 | 133.83 |
| Total | 11 | 3123.00 |  |

The DF for B and interaction get added to Error DF, similarly the SS.

3) (5X2=10 points) The interaction plots for the ANOVA1 and ANOVA2 outputs shown below are Plot1 and Plot2 respectively. State whether each of the following statements is TRUE or FALSE, and correct the statements that are false.

x

B1

Plot 1

A2

A1

B2

**ANOVA 1 : Yield versus Ingred1, ingred2**

Source DF SS MS F-Value P-Value

Ingred1 1 18.06 18.06 2.12 0.171

Ingred2 1 90.25 90.25 10.59 0.007

Ingred1\*ingred2 1 3.42 3.42 0.40 0.538

Error 12 102.30 8.52

Total 15 214.03

X

B1

B2

A1

A2

Plot 2

**ANOVA 2**

**General Linear Model: weight versus school, gender**

Source DF SS MS F-Value P-Value

school 1 5.00 5.00 29.41 0.000

gender 1 0.24 0.24 1.42 0.250

school\*gender 1 5.61 5.61 33.05 0.000

Error 16 2.72 0.17

Total 19 13.57

1. The factors A and B in Plot 1 are ingred1 and ingred2, respectively. **TRUE**
2. In ANOVA2, the factor school has a statistically significant main effect, and corresponds to factor A in Plot2. **FALSE. In ANOVA2, the factor school has a statistically significant main effect, and corresponds to factor B in Plot2**
3. The test statistic value for testing the main effect of factor Ingred2 is F=10.59, with numerator and denominator degrees of freedom equal to 1 and 12, respectively**. *Note: Multiple answers possible – no points taken off. The interaction may be retained, as would be the usual situation for a designed experiment. However, the other option is to remove the interaction term and ‘reduce’ the model to an additive effect only model. Model reduction is typically employed in observational studies where the objective is to identify significant predictors (as in MlR), whereas all the terms in the original model are retained in a designed experiment.***
4. The appropriate model for the effects of school and gender on weight is:

weight = mean + school + gender + error.

**FALSE. The correct model is :**

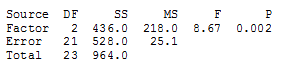
**weight = mean + school + gender + school\*gender + error.**

1. The appropriate model for the effects of Ingred1 and Ingred2 on Yield is:

Yield = mean + Ingred1 + Ingred2 + error. ***Note: Multiple answers possible – no points taken off. Again, the interaction may be retained, as would be the usual situation for a designed experiment, but the other option is to remove the interaction term and ‘reduce’ the model to an additive effect only model. Model reduction is typically employed in observational studies where the objective is to identify significant predictors (as in MlR) as is not a necessary step to perform for a designed experiment..***

4) (10 pts) Using a balanced, one-factor ANOVA, the following are scores on a fifty-question vocabulary test administered to twenty-four (24) students after one year of studying a foreign language under one of three teaching methods (B1, B2, and B3). The ANOVA output is also provided. Perform a Tukey multiple comparison for the teaching method effect at a 0.05 alpha level of significance. Include a letter-labeling comparison of means in your conclusion.

|  |  |  |
| --- | --- | --- |
| Aural-Oral  Method (A)  B1 | Translation  Method (T)  B2 | Combined  Method (C)  B3 |
| 37  30  (etc)  37  28 | 27  24  (etc)    14  15 | 20  31  (etc)    23  18 |



The ranked sample means are:

Since *v=21* is not on table can use 20 or 24:

*v=*20: w *=* q0.05(3,21) = 3.58 = 3.58\*1.77 = 6.34

*v=*24: w *=* q0.05(3,21) = 3.53 = 3.53\*1.77 = 6.25

The difference between sample means for B1 and B3 is 30 – 21.5 = 8.5 which exceeds either *w* value. Conclude means differ.

The difference between sample means for B1 and B2 is 30 – 20.5 = 9.5 which exceeds either *w* value. Conclude means differ.

The difference between sample means for B3 and B2 is 21.5 – 20.5 = 1.0 which is less than either *w* value. Conclude means do not differ.

Lettering:

|  |  |  |
| --- | --- | --- |
| Treatment | Mean | Grouping |
| B1 | 30.0 | A |
| B3 | 21.5 | B |
| B2 | 20.5 | B |

5) A study is designed to compare three Companies A, B, and C, that produce heat sinks (a heat sink is a device for cooling electronic components). Each company makes two models of heat sink and they are interested in comparing these as well. To measure the effectiveness of the six heat sinks, each one is randomly applied to five identical central processing units (CPUs) and run under identical conditions for 10 minutes. The temperature of each CPU is then recorded.

a) (5pts) Either write the statistical model for this experiment, or alternatively, you may write the model statement in SAS that would be used for this situation.

Model: temperature = company model(company);

b) (10 pts) Construct the first two columns (SS and df) of the ANOVA table for this experiment

|  |  |
| --- | --- |
| Source | df |
| Company | 3-1=2 |
| Model(Company) | 3\*(2-1)=3 |
| Error | 29-5=24 |
| Total | (2\*3\*5)-1=29 |

6) Suppose a study was made of attendance of elementary, junior high, and senior high school students for four ethnic groups (I, II, III, and IV). The study was balanced and included n=24 students at each school level.

a) (10 pts) Portions of the ANOVA table from the analysis are given below. Complete the table.

With n=24 in each school and three schools, N = 3\*24 = 72

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | DF | SS | MS | F |
| School Level (S) | (3-1) = 2 | 900 | (900/2) = 450 | (450/100) = 4.5 |
| Ethnicity (E) | (4-1) = 3 | (250\*3) = 750 | 250 | (250/100) = 2.5 |
| S x E | (3-1)\*(4-1) = 6 | 1200 | (1200/6) = 200 | (200/100) = 2.0 |
| Error | (72-11-1) = 60 | 6000 | (6000/60) = 100 |  |

b) (5 pts) Provide the correct hypotheses for testing each main effect and interaction.

c) (5 pts) What are critical F-values for each hypothesis test above in part b at alpha = 0.10 or 10%?

School Level: F0.10, 2,60 = 2.39

Ethnicity: F0.10, 3,60 = 2.18

Interaction: F0.10, 6,60 = 1.87

d) (5 pts) Does the F-test for the S x E interaction indicate that the attendance trend did not follow the same pattern for the four ethnic groups at a 10% level of significance?

Yes. The F-test statistic for the interaction is 2.0 which exceeds the critical F-value of 1.87 at a 0.10 level of significance. ***Note: There were problems with interpreting the term “attendence trend” and no points were deducted accordingly.***