STAT505 Assessment #6

1. J&W Exercise 6.24. The data can be found in "skulls.dat".

   The MANOVA null hypothesis of interest is $H_0 : \boldsymbol{\mu_1} = \boldsymbol{\mu_2} = \boldsymbol{\mu_3}$, where $\boldsymbol{\mu_i}$ is the skull measurement vector for the ith period. The sum of squares and cross products (SSCP) matrices for this hypothesis and error, respectively, are

$$\mathbf{H} = \begin{bmatrix} 150.2 & 20.3 & -161.83 & 5.03 \\ 20.3 & 20.6 & -38.73 & 6.43 \\ -161.8 & -38.73 & 190.29 & -10.86 \\ 5.0 & 6.4 & -10.86 & 2.02 \end{bmatrix} \quad \text{and} \quad \mathbf{E} = \begin{bmatrix} 1785.40 & 172.5 & 128.97 & 289.6 \\ 172.50 & 1924.3 & 178.80 & 171.9 \\ 128.97 & 178.8 & 2153.00 & -1.7 \\ 289.63 & 171.9 & -1.70 & 840.2 \end{bmatrix}$$

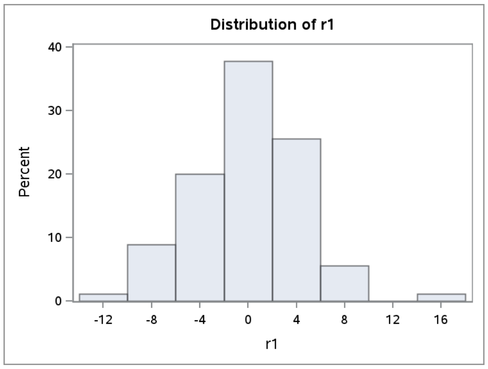
   With $\mathbf{T} = \mathbf{H} + \mathbf{E}$, we can collect these in a MANOVA table:

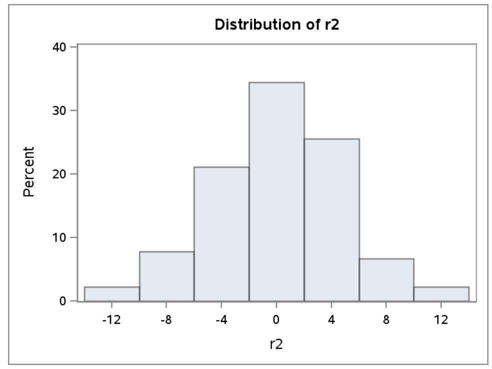
Source	SSCP	df
Treatments	$\mathbf{H}$	2
Error	$\mathbf{E}$	87
Total	$\mathbf{T}$	89

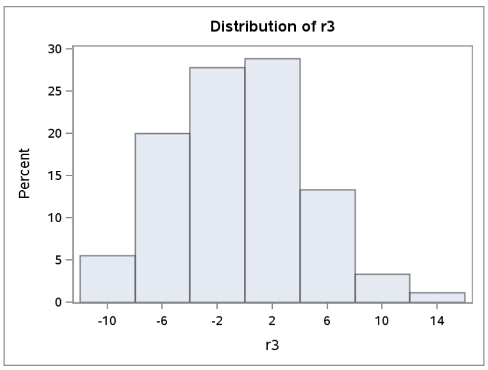
   Wilk's Lambda value is .83 and corresponds to an F statistic of 2.05 with 8 and 168 degrees of freedom. This is significant evidence (p-value 0.0436) that at least two of the periods differ in at least one mean measurement. Simultaneous 95% confidence limits are given below for each pair of periods and measurement. A Bonferroni adjustment of .05/12 is made to account for the multiplicity (individual confidence level is 99.58%).

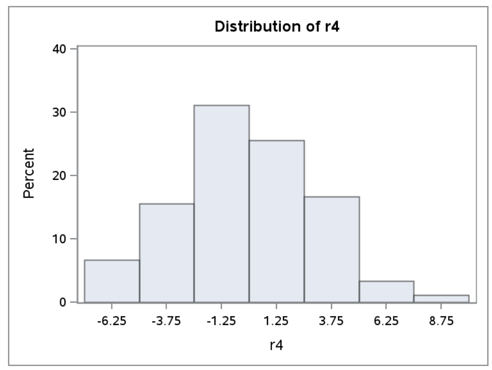
Obs	Dependent	i	j	LowerCL	UpperCL
1	breadth	1	2	-4.442312	2.442312
2	breadth	1	3	-6.542312	0.342312
3	breadth	2	3	-5.542312	1.342312
4	b_height	1	2	-2.673706	4.473706
5	b_height	1	3	-3.773706	3.373706
6	b_height	2	3	-4.673706	2.473706
7	length	1	2	-3.680111	3.880111
8	length	1	3	-0.646777	6.913444
9	length	2	3	-0.746777	6.813444
10	n_height	1	2	-2.061423	2.661423
11	n_height	1	3	-2.394757	2.328090
12	n_height	2	3	-2.694757	2.028090

   To assess univariate normality, histograms of the residuals for each skull measurement are given below. They all appear to be reasonably normally distributed.









Also, the sample covariance matrices seem to be reasonably similar, so the assumption of equal population covariance matrices is not rejected. Bartlett's test stat and p-value (for testing equal covariance matrices) are 21.05 and 0.3943, respectively. SAS code for this problem is

```
proc glm data=skulls;
  class type;
  model breadth b_height length n_height = type / clparm alpha=.00416666;
  output out=resids r=r1 r2 r3 r4;
  estimate '1 vs 2' type 1 -1 0;
  estimate '1 vs 3' type 1 0 -1;
  estimate '2 vs 3' type 0 1 -1;
  lsmeans type / cl pdiff alpha=.00416666;
  manova h=type / printe printh;
  run; quit;
proc print data=out_lsm;
  var dependent i j lowercl uppercl;
  run;
proc univariate noprint data=resids;
  histogram r4;
  run;
proc discrim method=normal pool=test data=skulls;
  class type;
  var breadth b_height length n_height;
  run;
```

2. Three peanut varieties (coded 5, 6, 8) are grown in two geographical locations (coded 1,2). There were two replications (growing plots) per combination of variety and location. The variables are X_1 = yield (weight), X_2 = sound mature kernels (weight), and X_3 = seed size.

   The data for this problem is in "peanut.dat". The following program reads the data into SAS, computes a two-factor MANOVA (and univariate ANOVAs), gives results for pairwise contrasts among varieties, gives means for the factors in the model, and does tests of normality for the residuals (output also included separately on ANGEL).

```
data peanut;
infile "v:\peanut.dat";
input location variety x1 x2 x3;
run;
proc glm;
class location variety;
model x1 x2 x3 =location variety location*variety;
contrast 'diff56' variety 1 -1 0;
contrast 'diff58' variety 1 0 -1;
contrast 'diff68' variety 0 1 -1;
lsmeans location variety location*variety;
manova h = _all_;
output out = resids r = res1-res3;
run;
proc univariate normal;
var res1 res2 res3 ;
run;
```

   (a) In the output, locate the MANOVA tests for the location factor. What is the p-value and what conclusion can we make?

   The p-value is .0205, which is significant evidence that the location effects (when averaging over varieties) are different for at least one X response.

   (b) In the output, locate the MANOVA tests for the variety factor. What are the p-values for the four different tests and what conclusion can we make?

   The p-value for Wilk's Lambda test is .0019 (the p-values for the other three test statistics are also well below 0.05). So, there is significant evidence that the variety effects (when averaging over locations) are different for at least one X response.

   (c) In the output, locate the MANOVA tests for the location*variety interaction factor. What are the p-values for the four different tests and what conclusion can we make?

   The p-values for Wilk's Lambda, Pillai's Trace, Hotelling-Lawley Trace, and Roy's Greatest Root test statistics are .0508, .0587, .0699, and .0113, respectively. It seems the last test is finding more evidence of interaction than the other three. Ideally, these tests would all agree, but given that three of the four show insignificant evidence of interaction, we will side with them. As always, however, failing to reject a null hypothesis—in this case the assumption of no interaction—does not mean it's true. It means only that the amount of evidence to reject it fell short of the significant level threshold.

   (d) Locate the univariate ANOVAs for the three variables. Briefly summarize the results; that is, for each variable, what are the significant factors?

   Considering the three univariate ANOVA tests simultaneously, the significance level is adjusted with a Bonferroni correction to $.05/3 = .017$. At this level, both main effects and their interaction are insignificant for X_1. For X_2 and X_3, only the variety main effect is significant; the other terms are insignificant.

(e) Locate the MANOVA test for the 'diff56' contrast, the difference between varieties 5 and 6. What is the p-value, and what conclusion can we make?

The p-value for this contrast is .0089, which is less than .05 and also less than $.05/3 = .017$, which would be the Bonferroni-corrected significance level if this contrast and the two other MANOVA contrasts are considered simultaneously. In either case, this is significant evidence that varieties 5 and 6 differ for at least one X response.

(f) Repeat part e) for the 'diff58' contrast.

The p-value for this contrast is .094, which is greater than .05. This is insignificant evidence that varieties 5 and 8 differ for at least one X response.

(g) Repeat part e) for the 'diff68' contrast.

The p-value for this contrast is .0068, which is less than .05 and also less than $.05/3 = .017$, which would be the Bonferroni-corrected significance level if this contrast and the two other MANOVA contrasts are considered simultaneously. In either case, this is significant evidence that varieties 6 and 8 differ for at least one X response.

(h) Locate the univariate test results for the 'diff56' contrast. For which variables is there a significant difference between varieties 5 and 6?

First note that there would need to be a Bonferroni correction of $.05/3 = .017$ if these contrasts are viewed together as a follow-up to the MANOVA 'diff56' contrast only (separate from the other MANOVA contrasts). If all univariate follow-ups for all three MANOVA contrasts are viewed together, then the Bonferroni correction for multiplicity would be $.05/9 = .0056$. This is the approach taken here.

For the response variables X_1, X_2, and X_3, the p-values are .0154, .0186, and .6661, respectively. None are individually significant at the .0056 level.

(i) Repeat part h) for the 'diff58' contrast.

For the response variables X_1, X_2, and X_3, the p-values are .1074, .0064, and .0082, respectively. None are individually significant at the .0056 level.

(j) Repeat part h) for the 'diff68' contrast.

For the response variables X_1, X_2, and X_3, the p-values are .1948, .4028, .0141, respectively. None are individually significant at the .0056 level.

(k) Locate the tests of normality for the three residual variables (res1, res2, res3). Four tests are given, but we'll use just the Wilk-Shapiro test. The null hypothesis is that the residuals have a normal distribution. What can we conclude about the normality of the residuals? Briefly explain.

For the response variables X_1, X_2, and X_3, the p-values are .6552, .9949, and (approximately) 1.0, respectively. Thus, there is little evidence that univariate normality fails to hold for any of the three response X variables.