

The sample means are:

x1bar = 12/3= 4 and x2bar = 3/3 = 1

Therefore the sample mean vector x¯=

We have the following variances and covariance:

|  |  |  |  |
| --- | --- | --- | --- |
|  | ***x*1** | ***x*2** | XijXik |
|  | 3 | 4 | 12 |
|  | 6 | -2 | -12 |
|  | 3 | 1 | 3 |
|  |  |  | 0 |
|  |  |  |  |
| Avg | 4 | 1 |  |
| Sum | 12 | 3 | 3 |
| Covariance | |  | -4.5 |
| Variance | 3 | 9 |  |

Therefore sample covariance matrix S =

The correlation between X1 and X2 is = -4.5/(sqrt(3) \* sqrt(9)) = -0.86603

Therefore sample correlation matrix R =

To get to the generalized sample variance we have to get the determinant of S i.e. |S| = 3\*9 – (-4.5\*-4.5) = 6.75



Here b = [1,1,1] and c = [1,2,-3]

The sample mean vector x¯=

The estimated mean of b`x = 5+3+4 = 12

The estimated mean of c`x = 5+2\*3-3\*4 = -1

We have the following variances and covariance

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | ***x*1** | ***x*2** | ***X3*** | ***X1X2*** | ***X2X3*** | ***X1X3*** |
|  | 1 | 4 | 3 | 4 | 12 | 3 |
|  | 6 | 2 | 6 | 12 | 12 | 36 |
|  | 8 | 3 | 3 | 24 | 9 | 24 |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| Avg | 5 | 3 | 4 |  |  |  |
| Sum | 15 | 9 | 12 | 40 | 33 | 63 |
| Covariance | |  |  | -2.5 | -1.5 | 1.5 |
| Variance | 13 | 1 | 3 |  |  |  |

Therefore sample covariance matrix S =

s2b`x = 13+1+3+(2\*-2.5)+ (2\*-1.5)+ (2\*1.5) = 12

s2c`x = 13+(2^2\*1)+(-3^2\*3)+(2\*1\*2\*-2.5)+ (2\*2\*-3\*-1.5)+ (2\*1\*-3\*1.5) = 43

sb`x, c`x = 1\*1\* s21 + 1\*2\* s22 + 1\*-3\* s23 + 1\*2\*s12 + 1\*-3\*s13 + 1\*1\*s21 + 1\*-3\*s23 + 1\*1\*s31 + 1\*2\*s32

= 1\*1\*13 + 1\*2\* 1 + 1\*-3\* 3 + 1\*2\*-2.5 + 1\*-3\*1.5 + 1\*1\*-2.5 + 1\*-3\*-1.5 + 1\*1\*1.5 + 1\*2\*-1.5

= -3

[above can be verified by performing b`Sb (see text)



]



We have the following variances and covariance

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Sales | Profits | Assets | S\*P | P\*A | S\*A |
|  | 126974 | 4224 | 173297 | 536338176 | 732006528 | 22004213278 |
|  | 96933 | 3835 | 160893 | 371738055 | 617024655 | 15595841169 |
|  | 86656 | 3510 | 83219 | 304162560 | 292098690 | 7211425664 |
|  | 63438 | 3758 | 77734 | 238400004 | 292124372 | 4931289492 |
|  | 55264 | 3939 | 128344 | 217684896 | 505547016 | 7092802816 |
|  | 50976 | 1809 | 39080 | 92215584 | 70695720 | 1992142080 |
|  | 39069 | 2946 | 38528 | 115097274 | 113503488 | 1505250432 |
|  | 36156 | 359 | 51038 | 12980004 | 18322642 | 1845329928 |
|  | 35209 | 2480 | 34715 | 87318320 | 86093200 | 1222280435 |
|  | 32416 | 2413 | 25636 | 78219808 | 61859668 | 831016576 |
|  |  |  |  |  |  |  |
| Avg | 62309.1 | 2927.3 | 81248.4 |  |  |  |
| Sum | 623091 | 29273 | 812484 | 2054154681 | 2.789E+09 | 64231591870 |
| Covariance |  |  |  | 25575599.6 | 45654618 | 1511827230 |
| Variance | 1000509114 | 1430020 | 2980489782 |  |  |  |
| SD | 31630.8254 | 1195.834 | 54593.86212 |  |  |  |

Therefore sample covariance matrix S =

We also have Overhead b = [1, -1, 0] and Assets after Sales c = [1,0,1]

1. Sample means:

The sample mean vector x¯=

Sample mean of Overhead = 62309.1 – 2927.3 = 59381.8

Sample mean of Assets after Sales = 62309.1 + 81248.4 = 143557.5

1. s2overhead = 1000509114 + (-1)^2 \* 1430020 + (2 \* 1 \* -1 \* 25575599.6) = 950787934.8

s2assets = 1000509114 + 2980489782 + (2 \* 1 \* 1 \* 1511827230) = 7004653356

1. Sample covariance

Soverhead, assets = 1\*1\* s21 + 1\*1\*s13 + (-1)\*1\*s21 + (-1)\*1\*s23

= 1\*1\* 1000509114 + 1\*1\*1511827230 + (-1)\*1\*25575599.6 + (-1)\*1\*45654618 = 2441106126

roverhead, assets = Soverhead, assets / (Soverhead \* Sassets) = 2441106126 / sqrt(950787934.8 \* 7004653356) = 0.946

We find that both covariance and correlation provide us the understanding that overhead and assets after sales are positively related:

* Covariance > 0 implies that the two variables are positively correlated; i.e., values of variable overhead tend to increase with increasing values of assets. The larger the covariance, the stronger the positive association between the two variables.
* The close to +1 value of correlation indicates a strong positive dependence between the two.