

1. We have the following table:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Factor1 | Factor2 | Factor3 | Li1^2 | Li2^2 | Li3^2 | h^2 (Communality) | Specific Variance 1- Communality |
| symptoms | 0.763 | 0.239 | 0.133 | 0.582169 | 0.057121 | 0.017689 | 0.656979 | 0.343021 |
| activity | 0.899 | 0.036 | -0.057 | 0.808201 | 0.001296 | 0.003249 | 0.812746 | 0.187254 |
| sleep | 0.061 | 0.898 | 0.146 | 0.003721 | 0.806404 | 0.021316 | 0.831441 | 0.168559 |
| eat | 0.562 | 0.556 | -0.034 | 0.315844 | 0.309136 | 0.001156 | 0.626136 | 0.373864 |
| appetite | 0.618 | 0.65 | -0.109 | 0.381924 | 0.4225 | 0.011881 | 0.816305 | 0.183695 |
| skinreact | 0.016 | 0.067 | 0.985 | 0.000256 | 0.004489 | 0.970225 | 0.97497 | 0.02503 |
|  |  |  | Variance explained by factor | 2.092115 | 1.600946 | 1.025516 |  |  |
|  |  |  | Proportion of total variance | 0.348686 | 0.266824 | 0.170919 |  |  |

Here we see communality for symptoms = 0.656979

Interpretation: if we perform multiple regression of symptoms against the three common factors, we obtain an R2 = 0.656979, indicating that about 65.7% of the variation in symptoms is explained by the factor model.

1. The specific variances can be computed by subtracting the communality from the variance as expressed below:

Ψ^i=1−h^2i

(Since the data are standardized in this case, the variance for standardized data is going to be equal to one.)

Ψ^I = 1 – 0.657 = 0.343

1. The amount of variance explained by the first factor: 

We can get this from the table = 2.092115

1. The proportion is given by: 2.092115 / 6 = 0.348685833
2. In order to interpret these factors we reproduce the table:

|  |  |  |  |
| --- | --- | --- | --- |
|  | Factor1 | Factor2 | Factor3 |
| symptoms | **0.763** | 0.239 | 0.133 |
| activity | **0.899** | 0.036 | -0.057 |
| sleep | 0.061 | **0.898** | 0.146 |
| eat | **0.562** | **0.556** | -0.034 |
| appetite | **0.618** | **0.65** | -0.109 |
| skinreact | 0.016 | 0.067 | **0.985** |

We have marked the entries using a benchmark of 0.5

Factor 1 is correlated most strongly with activity and symptoms but also to a lesser extent with eat and appetite. So, in this case you can say that the first factor is primarily a measure of these variables

Factor 2 is correlated most strongly with sleep but also to a lesser extent with eat and appetite. Indicates that as sleep increases so does eat and appetite.

Factor 3 seems like a measure of the skinreact variable.



1. We get the following results:

|  |  |
| --- | --- |
| Before Rotation: | After Rotation: |
| | **Factor Pattern** | | | | --- | --- | --- | |  | **Factor1** | **Factor2** | | **wind** | -0.36202 | 0.32781 | | **solar** | 0.31424 | -0.61997 | | **CO** | 0.84242 | -0.00803 | | **NO** | 0.57724 | 0.51174 | | **NO2** | 0.76129 | 0.23518 | | **O3** | 0.49613 | -0.66749 | | **HC** | 0.48826 | 0.36247 | | | **Rotated Factor Pattern** | | | | --- | --- | --- | |  | **Factor1** | **Factor2** | | **wind** | -0.12397 | -0.47238 | | **solar** | -0.07574 | 0.69093 | | **CO** | 0.70104 | 0.46719 | | **NO** | 0.76309 | -0.11300 | | **NO2** | 0.76605 | 0.21919 | | **O3** | 0.05060 | 0.83013 | | **HC** | 0.60699 | -0.03664 | |

1. We have highlighted the loadings with the cutoff at .45. Based on this we can say the following:

**Before Rotation**

Factor 1 is correlated most strongly with CO (0.842) but also with NO2 and to a lesser extent NO, O3 and HC. So, in this case you can say that the first factor is primarily a measure of these variables

Factor 2 is primarily related to solar and O3 and to a lesser extent with NO. Here we see as solar and O3 decrease, NO increases

**After Rotation**

Factor 1 is correlated most strongly with NO2 but also with NO, CO and HC. So, in this case you can say that the first factor is primarily a measure of these variables

Factor 2 is primarily related to solar and O3 and to a lesser extent with wind and CO. Here we see as wind decreases, solar, O3 and NO increases.

**Summary**: We see that rotation has given us clearer loadings since there are less number of overlaps and more coverage (wind is included in factor 2)

1. Table including commanalities after rotation is:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Rotated Factor Pattern** | | |  |  |  |  |
|  | **Factor1** | **Factor2** | Li1^2 | Li2^2 | h^2  (Communality) | Specific Variance |
| **wind** | -0.12397 | -0.47238 | 0.015369 | 0.223143 | 0.238511425 | 0.761489 |
| **solar** | -0.07574 | 0.69093 | 0.005737 | 0.477384 | 0.483120813 | 0.516879 |
| **CO** | 0.70104 | 0.46719 | 0.491457 | 0.218266 | 0.709723578 | 0.290276 |
| **NO** | 0.76309 | -0.113 | 0.582306 | 0.012769 | 0.595075348 | 0.404925 |
| **NO2** | 0.76605 | 0.21919 | 0.586833 | 0.048044 | 0.634876859 | 0.365123 |
| **O3** | 0.0506 | 0.83013 | 0.00256 | 0.689116 | 0.691676177 | 0.308324 |
| **HC** | 0.60699 | -0.03664 | 0.368437 | 0.001342 | 0.36977935 | 0.630221 |
|  |  | Variance explained by factor | 2.052698 | 1.670065 |  |  |
|  |  | Proportion of total variance | 0.293243 | 0.238581 |  |  |

1. You can think of these values as multiple R2 values for regression models predicting the variables of interest from the 2 factors. The communality for a given variable can be interpreted as the proportion of variation in that variable explained by the two factors. In other words, if we perform multiple regression of wind against the two common factors, we obtain an R2 = 0.2385, indicating that about 23.85% of the variation in wind is explained by the factor model. The results suggest that the factor analysis does the best job of explaining variation in CO, NO2 and O3
2. The specific variance is highlighted in the below table:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Rotated Factor Pattern** | | |  |  |  |  |
|  | **Factor1** | **Factor2** | Li1^2 | Li2^2 | h^2  (Communality) | Specific Variance |
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|  |  | Variance explained by factor | 2.052698 | 1.670065 |  |  |
|  |  | Proportion of total variance | 0.293243 | 0.238581 |  |  |

1. Assuming that we are using the rotated factors the variance explained and proportion of total variance for the two factors is:

|  |  |  |
| --- | --- | --- |
|  | **Factor1** | **Factor2** |
| Variance explained by factor | 2.052698 | 1.670065 |
| Proportion of total variance | 0.293243 | 0.238581 |

Highlighted value shows proportion of the total variance of the seven variables is explained by the first

Factor.

1. We get the following results:

|  |  |
| --- | --- |
| Before Rotation: | After Rotation: |
| | **Factor Pattern** | | | | | --- | --- | --- | --- | |  | **Factor1** | **Factor2** | **Factor3** | | **wind** | -0.36202 | 0.32781 | 0.70608 | | **solar** | 0.31424 | -0.61997 | 0.24631 | | **CO** | 0.84242 | -0.00803 | -0.12466 | | **NO** | 0.57724 | 0.51174 | -0.44671 | | **NO2** | 0.76129 | 0.23518 | 0.21568 | | **O3** | 0.49613 | -0.66749 | 0.17540 | | **HC** | 0.48826 | 0.36247 | 0.59369 | | | **Rotated Factor Pattern** | | | | | --- | --- | --- | --- | |  | **Factor1** | **Factor2** | **Factor3** | | **wind** | -0.02870 | -0.17361 | 0.84030 | | **solar** | 0.04294 | 0.73598 | -0.01674 | | **CO** | 0.70551 | 0.27469 | -0.38996 | | **NO** | 0.64527 | -0.38271 | -0.48145 | | **NO2** | 0.81137 | 0.15187 | 0.00409 | | **O3** | 0.16599 | 0.81968 | -0.15173 | | **HC** | 0.70529 | 0.07134 | 0.46874 | |

1. We have highlighted the loadings with the cutoff at .45. Based on this we can say the following:

**Before Rotation**

Factor 1 is correlated most strongly with CO (0.842) but also with NO2 and to a lesser extent NO, O3 and HC. So, in this case you can say that the first factor is primarily a measure of these variables

Factor 2 is primarily related to solar and O3 and to a lesser extent with NO. Here we see as solar and O3 decrease, NO increases

Factor 3 is primarily related to wind and to a lesser extent to HC

**After Rotation**

Factor 1 is correlated most strongly with NO2 but also with NO, CO and HC. So, in this case you can say that the first factor is primarily a measure of these variables

Factor 2 is primarily related to solar and O3.

Factor 3 is primarily related to wind and to a lesser extent to NO and HC.

**Summary**: We see that rotation has given us slightly clearer loadings since there are less number of overlaps. The coverage benefit is not relevant since the 3 factors covered all variables even before rotation.

1. Table including commanalities after rotation is:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Rotated Factor Pattern** | | | |  |  |  |  |  |
|  | **Factor1** | **Factor2** | **Factor3** | Li1^2 | Li2^2 | Li3^2 | h^2  (Communality) | Specific Variance |
| **wind** | -0.0287 | -0.17361 | 0.8403 | 0.000824 | 0.030140432 | 0.706104 | 0.737068212 | 0.262931788 |
| **solar** | 0.04294 | 0.73598 | -0.01674 | 0.001844 | 0.54166656 | 0.00028 | 0.543790632 | 0.456209368 |
| **CO** | 0.70551 | 0.27469 | -0.38996 | 0.497744 | 0.075454596 | 0.152069 | 0.725267758 | 0.274732242 |
| **NO** | 0.64527 | -0.38271 | -0.48145 | 0.416373 | 0.146466944 | 0.231794 | 0.79463442 | 0.205365581 |
| **NO2** | 0.81137 | 0.15187 | 0.00409 | 0.658321 | 0.023064497 | 1.67E-05 | 0.681402502 | 0.318597498 |
| **O3** | 0.16599 | 0.81968 | -0.15173 | 0.027553 | 0.671875302 | 0.023022 | 0.722449975 | 0.277550025 |
| **HC** | 0.70529 | 0.07134 | 0.46874 | 0.497434 | 0.005089396 | 0.219717 | 0.722240567 | 0.277759433 |
|  |  |  | Variance explained by factor | 2.100093 | 1.493757728 | 1.333003 |  |  |
|  |  |  | Proportion of total variance | 0.300013 | 0.213393961 | 0.190429 |  |  |

1. You can think of these values as multiple R2 values for regression models predicting the variables of interest from the 3 factors. The communality for a given variable can be interpreted as the proportion of variation in that variable explained by the three factors. In other words, if we perform multiple regression of wind against the three common factors, we obtain an R2 = 0.737, indicating that about 73.7% of the variation in wind is explained by the factor model. The results suggest that the factor analysis does the best job of explaining variation in wind, CO, NO, NO2, O3 and HC
2. The specific variance is highlighted in the below table:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Rotated Factor Pattern** | | | |  |  |  |  |  |
|  | **Factor1** | **Factor2** | **Factor3** | Li1^2 | Li2^2 | Li3^2 | h^2  (Communality) | Specific Variance |
| **wind** | -0.0287 | -0.17361 | 0.8403 | 0.000824 | 0.030140432 | 0.706104 | 0.737068212 | 0.262931788 |
| **solar** | 0.04294 | 0.73598 | -0.01674 | 0.001844 | 0.54166656 | 0.00028 | 0.543790632 | 0.456209368 |
| **CO** | 0.70551 | 0.27469 | -0.38996 | 0.497744 | 0.075454596 | 0.152069 | 0.725267758 | 0.274732242 |
| **NO** | 0.64527 | -0.38271 | -0.48145 | 0.416373 | 0.146466944 | 0.231794 | 0.79463442 | 0.205365581 |
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| **HC** | 0.70529 | 0.07134 | 0.46874 | 0.497434 | 0.005089396 | 0.219717 | 0.722240567 | 0.277759433 |
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1. Assuming that we are using the rotated factors the variance explained and proportion of total variance for the two factors is:

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Factor1** | **Factor2** | **Factor3** |
| Variance explained by factor | 2.100093 | 1.493757728 | 1.333003 |
| Proportion of total variance | 0.300013 | 0.213393961 | 0.190429 |

Highlighted value shows proportion of the total variance of the seven variables is explained by the first

Factor.