



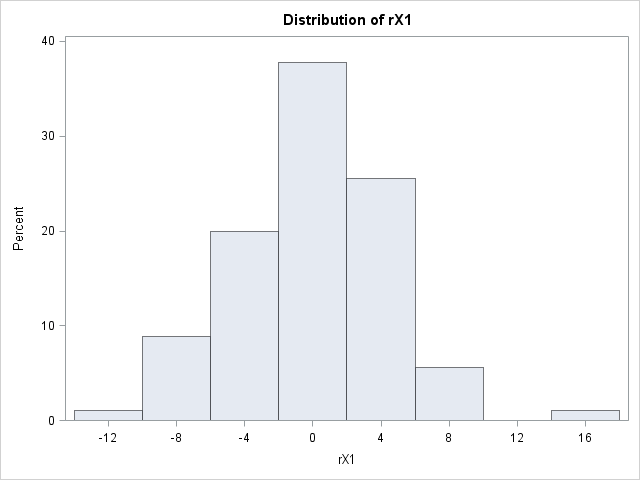
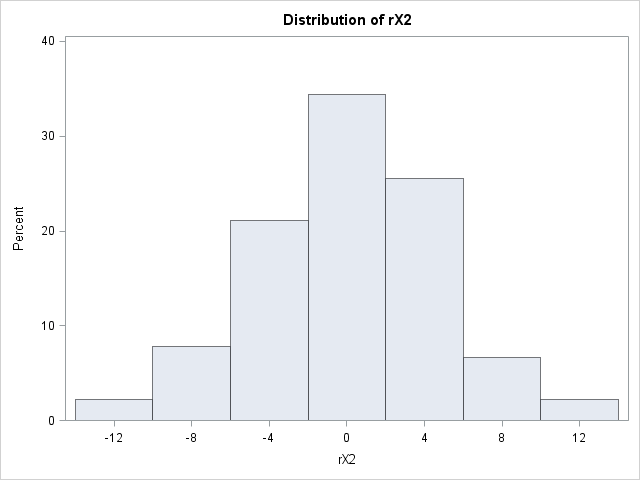
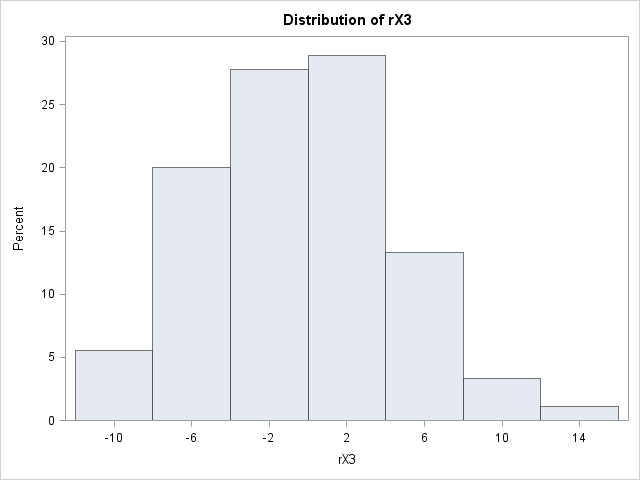
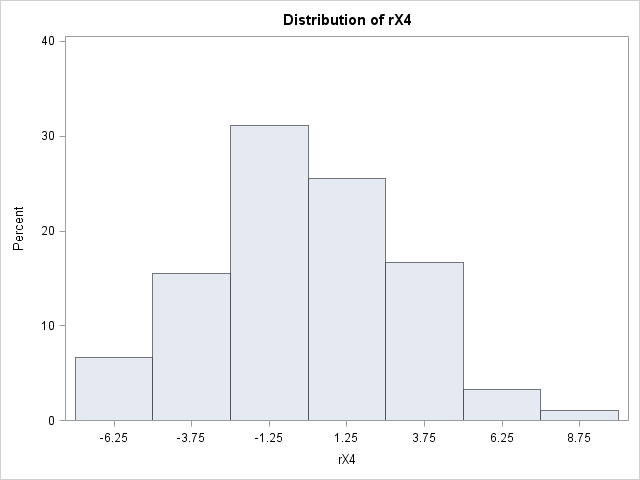
Assumption 1: The data from group i has common mean vector μi

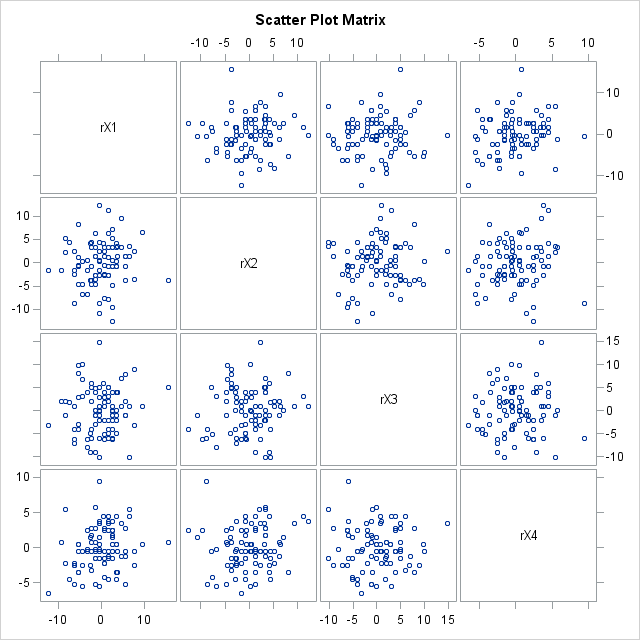
This assumption says that there are no subpopulations with different mean vectors. Here, this assumption might be violated if skulls collected from a given time period has inconsistency due to some reason.

Assumption 3: Independence: The subjects are independently sampled. Even though we see that the samples were selected over a period of time, time is a part of the hypothesis.

Assumption 4: Normality: The data are multivariate normally distributed.

We have from SAS:



Histograms suggest that the distributions are relatively symmetric.

Assumption 2: The data from all groups have common variance-covariance matrix Σ.

|  |
| --- |
| Bartlett's Test |

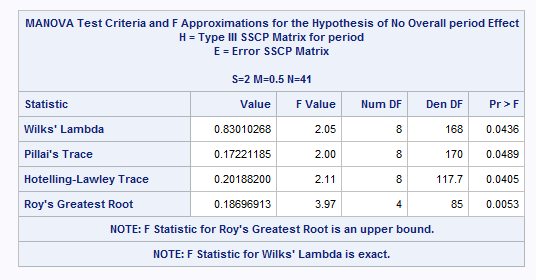
The DISCRIM Procedure

Test of Homogeneity of Within Covariance Matrices

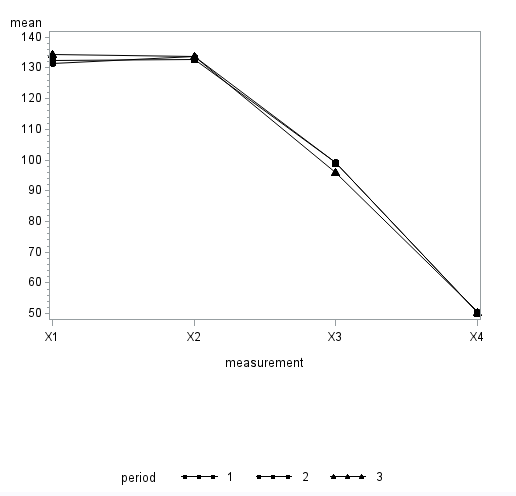
| **Chi-Square** | **DF** | **Pr > ChiSq** |
| --- | --- | --- |
| 21.048436 | 20 | 0.3943 |

We find no statistically significant evidence against the null hypothesis that the variance-covariance matrices are homogeneous (L' = 21.05; d.f. = 20; p = 0.39).

Manova output:



The measurements of the skull depends on the period from which the sample was obtained ( Λ\* = 0.8301; F = 2.05; d.f. = 8, 168; p = 0.0436 < 0.05). It was found therefore, that there are differences in the measurements of the skull of at least one dimension between at least one pair of periods.



|  |  |  |  |
| --- | --- | --- | --- |
| **Skull Dimension** | **F** | **df** | **SAS *p*-value** |
| **X1** | **3.66** | **2, 87** | **0.0298** |
| **X2** | **0.47** | **2, 87** | **0.6293** |
| **X3** | **3.84** | **2, 87** | **0.0251** |
| **X4** | **0.1** | **2, 87** | **0.9007** |

**Analysis of Individual Chemical Elements – Naïve approach**

F(2, 87, 0.05) = 3.1013

No we see that F-value is > Fcritical for X1 and X3 dimensions. Also for these variables p < 0.05 and therefore we see that for X1 and X3 we have significant results

**Analysis of Individual Chemical Elements – Bonferroni correction**

Here, p = 4 variables, g = 3 groups, and a total of N = 90 observations. So, for an α = 0.05 level test, we reject

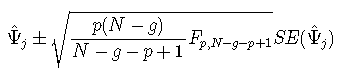
H0:μ1k=μ2k=⋯=μgk if F > F(g−1,N−g,α/p) = F(2, 87, 0.05/4) = 4.61

Since none of the F-statistics exceed the critical value of 4.61, or equivalently, since the SAS p-values all fall above 0.0125 (= 0.05/4), we can see that none of the dimensions are significant at the 0.05 level under the Bonferroni correction.

**Conclusion**: Means for all dimensions don’t differ significantly among the periods.

We set up contrasts as follows:

|  |  |  |  |
| --- | --- | --- | --- |
| **Contrast** | **Period 1** | **Period 2** | **Period 3** |
| 1 – 1 vs 2 | 1 | -1 | 0 |
| 2 – 1 vs 3 | 1 | 0 | -1 |
| 3 – 2 vs 3 | 0 | 1 | -1 |

We have 

Recall that we have p = 4 dimensions, g = 3 periods, and a total of N = 90 observations. From the F-table, we have F[4,90-3-4+1,0.05] = F[4,84,0.05] = 2.48. Then our multiplier is:

M = sqrt( ((p \* (N-g)) / (N-g-p+1)) \* F[p, N-g-p+1]) = sqrt( ((4 \* (90-3)) / (90-3-4+1)) \* 2.48) = 3.205

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | M | SQRT( ((4 \* (90-3)) / (90-3-4+1)) \* 2.48) | 3.205353 |  |
| **Contrast 1** | 1 vs 2 |  |  |  |  |
|  |  |  |  |  |  |
|  | **Ψ^** | **SE** | **M X SE** | **Lower CI** | **Upper CI** |
| **X1** | -1 | 1.17 | 3.750262619 | -4.75026 | 2.75026262 |
| **X2** | 0.9 | 1.214 | 3.891298137 | -2.9913 | 4.79129814 |
| **X3** | 0.1 | 1.284 | 4.115672823 | -4.31567 | 3.91567282 |
| **X4** | 0.3 | 0.802 | 2.570692838 | -2.27069 | 2.87069284 |
| **Contrast 2** | 1 vs 3 |  |  |  |  |
|  |  |  |  |  |  |
|  | **Ψ^** | **SE** | **M X SE** | **Lower CI** | **Upper CI** |
| **X1** | -3.1 | 1.17 | 3.750262619 | -6.85026 | 0.65026262 |
| **X2** | -0.2 | 1.214 | 3.891298137 | -4.0913 | 3.69129814 |
| **X3** | 3.133 | 1.284 | 4.115672823 | -0.98267 | 7.24867282 |
| **X4** | -0.033 | 0.802 | 2.570692838 | -2.60369 | 2.53769284 |
| **Contrast 3** | 2 vs 3 |  |  |  |  |
|  |  |  |  |  |  |
|  | **Ψ^** | **SE** | **M X SE** | **Lower CI** | **Upper CI** |
| **X1** | -2.1 | 1.17 | 3.750262619 | -5.85026 | 1.65026262 |
| **X2** | -1.1 | 1.214 | 3.891298137 | -4.9913 | 2.79129814 |
| **X3** | 3.033 | 1.284 | 4.115672823 | -1.08267 | 7.14867282 |
| **X4** | -0.333 | 0.802 | 2.570692838 | -2.90369 | 2.23769284 |



1. At alpha = 0.05 (no Bonferrorni correction) - The dependant variables depends on the location of the seeds ( Λ\* = 0.10651620; F = 11.18; d.f. = 3, 4; p = 0.0205 < 0.05). We reject the null hypothesis.

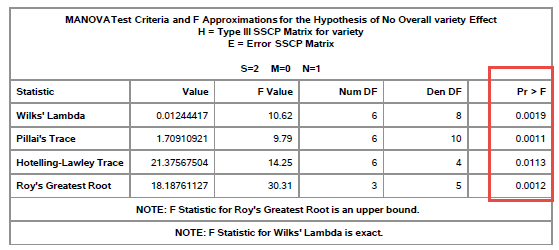
At alpha = 0.05 after Bonferrorni correction = 0.05/3 = 0.0167 - The dependant variables don’t depends on the location of the seeds ( Λ\* = 0.10651620; F = 11.18; d.f. = 3, 4; p = 0.0205 > 0.0167). We fail to reject the null hypothesis.

>>

The *p*-value is .0205, which is signi\_cant evidence that the location effects (when aver-

aging over varieties) are different for at least one *X* response.

1. The p-values are:



Making conclusions based on Wilks Lambda:

At alpha = 0.05 (no Bonferrorni correction) - The dependant variable means depends on the variety of the seeds ( Λ\* = 0.01244417; F = 10.62; d.f. = 6, 8; p = 0.0019 < 0.05). We reject the null hypothesis.

At alpha = 0.05 after Bonferrorni correction = 0.05/3 = 0.0167 - The dependant variable means depends on the variety of the seeds ( Λ\* = 0.01244417; F = 10.62; d.f. = 6, 8; p = 0.0019 < 0.0167). We reject the null hypothesis.

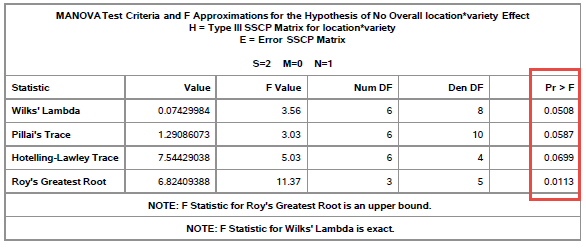
>>

The *p*-value for Wilk's Lambda test is .0019 (the *p*-values for the other three test statistics

are also well below 0.05). So, there is signi\_cant evidence that the variety effects (when

averaging over locations) are different for at least one *X* response.

1. The p-values are:



Making conclusions based on Wilks Lambda:

Here we have H0: there is no interaction and HA: an interaction exists

At alpha = 0.05, we see that the interaction term is not significant. Since the interaction term is not significant we can interpret the individual factors as we did in parts a and b

>>

The p-values for Wilk's Lambda, Pillai's Trace, Hotelling-Lawley Trace, and Roy's Great-

est Root test statistics are .0508, .0587, .0699, and .0113, respectively. It seems the last

test is nding more evidence of interaction than the other three. Ideally, these tests

would all agree, but given that three of the four show insignicant evidence of interac-

tion, we will side with them. As always, however, failing to reject a null hypothesis|in

this case the assumption of no interaction|does not mean it's true. It means only that

the amount of evidence to reject it fell short of the signicant level threshold.

1. ANOVA for the 3 variables gives us:

|  |  |  |  |
| --- | --- | --- | --- |
| **~~Dimension~~** | **~~F~~** | **~~df~~** | **~~SAS~~*~~p~~*~~-value~~** |
| **~~X1~~** | **~~4.63~~** | **~~5, 6~~** | **~~0.0446~~** |
| **~~X2~~** | **~~6.92~~** | **~~5, 6~~** | **~~0.0177~~** |
| **~~X3~~** | **~~5.6~~** | **~~5, 6~~** | **~~0.0292~~** |

**~~Analysis of individual variables – Naïve approach~~**

~~F(5, 6, 0.05) = 4.387~~

~~Now we see that F-value is > Fcritical for all dimensions. Also for these variables p < 0.05 and therefore we see that all X1, X2 and X3 we have significant results~~

**~~Analysis of individual variables – Bonferroni correction~~**

~~Here, p = 3 variables. So, for an α = 0.05 level test, we reject~~

~~H0:μ1k=μ2k=⋯=μgk if F > F(5, 6, 0.05/3) = 7.10~~

~~Now we see that F-value is < Fcritical for all dimensions. Therefore we see that all X1, X2 and X3 we don’t have significant results~~

>>

Considering the three univariate ANOVA tests simultaneously, the signi\_cance level is

adjusted with a Bonferroni correction to *:*05*=*3 = *:*017. At this level, both main effects

and their interaction are insignicant for *X*1. For *X*2 and *X*3, only the variety main

effect is signicant; the other terms are insignicant.

1. For contrast “diff56”

Lets make the table for all the contrasts

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Contrast | https://onlinecourses.science.psu.edu/stat505/sites/onlinecourses.science.psu.edu.stat505/files/lesson13/formula_80.gif | F | Num d.f. | Denom d.f. | p |
| diff56 | 0.069559 | 17.83 | 3 | 4 | 0.0089 |
| diff58 | 0.23363 | 4.37 | 3 | 4 | 0.094 |
| diff68 | 0.06095 | 20.54 | 3 | 4 | 0.0068 |

The mean dimension of seeds from variety 5 differs in at least one variable from that of variety 6 (https://onlinecourses.science.psu.edu/stat505/sites/onlinecourses.science.psu.edu.stat505/files/lesson13/formula_80.gif = 0.069559; F = 17.83; d.f. = 3, 4; p = 0.0089 < 0.05). We reject the null.

>>

The *p*-value for this contrast is .0089, which is less than .05 and also less than *:*05*=*3 =

*:*017, which would be the Bonferroni-corrected signi\_cance level if this contrast and the

two other MANOVA contrasts are considered simultaneously. In either case, this is

signi\_cant evidence that varieties 5 and 6 differ for at least one *X* response.

1. Refer table above - There is no significant difference in the mean dimension of seeds from variety 5 and 8 (https://onlinecourses.science.psu.edu/stat505/sites/onlinecourses.science.psu.edu.stat505/files/lesson13/formula_80.gif = 0.23363; F = 4.37; d.f. = 3, 4; p = 0.094 > 0.05). We fail to reject the null.

>>

The *p*-value for this contrast is .094, which is greater than .05. This is insigni\_cant

evidence that varieties 5 and 8 differ for at least one *X* response.

1. Refer table above - The mean dimension of seeds from variety 6 differs in at least one variable from that of variety 8 (https://onlinecourses.science.psu.edu/stat505/sites/onlinecourses.science.psu.edu.stat505/files/lesson13/formula_80.gif = 0.06095; F = 20.54; d.f. = 3, 4; p = 0.0068 < 0.05). We reject the null.

>>

The *p*-value for this contrast is .0068, which is less than .05 and also less than *:*05*=*3 =

*:*017, which would be the Bonferroni-corrected signi\_cance level if this contrast and the

two other MANOVA contrasts are considered simultaneously. In either case, this is

signi\_cant evidence that varieties 6 and 8 differ for at least one *X* response.

1. Here we will use the following variables:

Variable: X1



Variable: X2



Variable: X3



Based on the above tables we see that for contrast diff56, we have significant differences for variable X1 (p=0.0154) and X2 (p=0.0186)

>>

First note that there would need to be a Bonferroni correction of *:*05*=*3 = *:*017 if these

contrasts are viewed together as a follow-up to the MANOVA `diff56' contrast only

(separate from the other MANOVA contrasts). If all univariate follow-ups for all three

MANOVA contrasts are viewed together, then the Bonferroni correction for multiplicity

would be *:*05*=*9 = *:*0056. This is the approach taken here.

For the response variables *X*1, *X*2, and *X*3, the *p*-values are .0154, .0186, and .6661,

respectively. None are individually signicant at the .0056 level.

1. Based on the above tables we see that for contrast diff58, we have significant differences for variable X2 (p=0.0064) and X3 (p=0.0082)

>>

For the response variables *X*1, *X*2, and *X*3, the *p*-values are .1074, .0064, and .0082,

respectively. None are individually signi\_cant at the .0056 level.

1. Based on the above tables we see that for contrast diff68, we have significant differences for variable X3 (p=0.0141)

>>

For the response variables *X*1, *X*2, and *X*3, the *p*-values are .1948, .4028, .0141, respec-

tively. None are individually signi\_cant at the .0056 level.

1. The null-hypothesis of this test is that the population is normally distributed. Thus if the p-value is less than the chosen alpha level, then the null hypothesis is rejected and there is evidence that the data tested are not from a normally distributed population. In other words, the data are not normal.

Residuals: res1



We have p-value > alpha = 0.05, we fail to reject the null hypothesis. We can conclude that the data came from a normal distribution.

Residuals: res2



We have p-value > alpha = 0.05, we fail to reject the null hypothesis. We can conclude that the data came from a normal distribution.

Residuals: res3



We have p-value > alpha = 0.05, we fail to reject the null hypothesis. We can conclude that the data came from a normal distribution.

>>

For the response variables *X*1, *X*2, and *X*3, the *p*-values are .6552, .9949, and (approxi-

mately) 1.0, respectively. Thus, there is little evidence that univariate normality fails to

hold for any of the three response *X* variables.