**Stat 510 Week 2 Homework Solutions**

**1.** Consider the following short time series data set:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *t* | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| *xt* | 13 | 2 | 5 | 20 | 2 | 4 | 12 | 8 | 7 | 19 | 7 | 1 | 20 | 5 | 12 | 18 | 12 | 20 | 8 | 5 |

Evaluate each of the following expressions (i.e., give a numerical answer):

a. 

**== 20 – 1 = 19**

b. 

**= = 7**

c. 

**== 20 – 7 = 13**

d. 

**=() − () = (20 − 1) – (1 – 7) = 19 – (-6) = 25**

e. 

**= = 20 – 13 = 7**

f. 

**= =** **= =20 – 1.7(1) + 1.2(7)-0.5(19) = 17.2**

**2.** Write out the autoregressive and moving average polynomials for the following model:



Hint: The key structure for this answer is (AR polynomial)xt = (MA polynomial)wt .

**, giving**

** ; **

**3**. Consider the MA(1) model  with the wt assumed to be iid N(0,).

A. Give a numerical value for the first lag autocorrelation.



B. Give a numerical value for the second lag autocorrelation.

**0**

C. Describe the appearance of the ACF for this model.

**There will be a single spike at the first lag. All other autocorrelations will be 0.**

D. Use R to sketch the ACF for this model. The commands are:

acfprob3=ARMAacf(ma=c(.6), lag.max=10)

plot(seq(0,10), acfprob3, xlim=c(1,10), xlab="lags", type="h")

(In the plot command, the type="h" causes projections from the value to the axis as we usually do in an ACF.)



E. To get the PACF of this model, modify the command by adding the pacf option:

pacfprob3 = ARMAacf(ma=c(.6), lag.max=10, pacf=TRUE)

plot(pacfprob3, type="h")



**4**. Consider the AR(1) model 

A. Give a numerical value for the first lag autocorrelation. (You might want to look back at the week 1 notes.)

**ρ1 = -.6**

B. Give a numerical value for the second lag autocorrelation.

**ρ2 = (-.6)2 = 0.36**

C. Give a numerical value for the third lag autocorrelation.

**ρ3 = (-.6)3 = -0.216**

D. Describe the appearance of the PACF for this model.

**There will be a single spike at the first lag. All other partial autocorrelations will be 0.**

E. Use R to sketch the ACF for this model. The commands are:

acfprob4=ARMAacf(ar=c(-.6), lag.max=10)

lags=0:10 #creates a variable named lags that ranges from 0 to 10.

plot(lags, acfprob4, xlim=c(1,10), type="h")



F. To get the PACF of this model, modify the command by adding the pacf option:

pacfprob4 = ARMAacf(ar=c(-.6), lag.max=10, pacf=TRUE)

plot(pacfprob4, type="h")

****

**5.** The time series for this problem is stride length measured every 30 seconds for a runner on a treadmill moving at pace of 7 minutes per mile.

Following are the ACF and PACF for the series. Briefly describe what model(s) may be suggested by these plots (and explain why).

**Either an AR(1) or an AR(2), probably an AR(2). The ACF tapers to 0, but possible with a sinusoidal pattern. The PACF has one clearly significant spike at the first lag and a barely significant spike at the second lag.**



