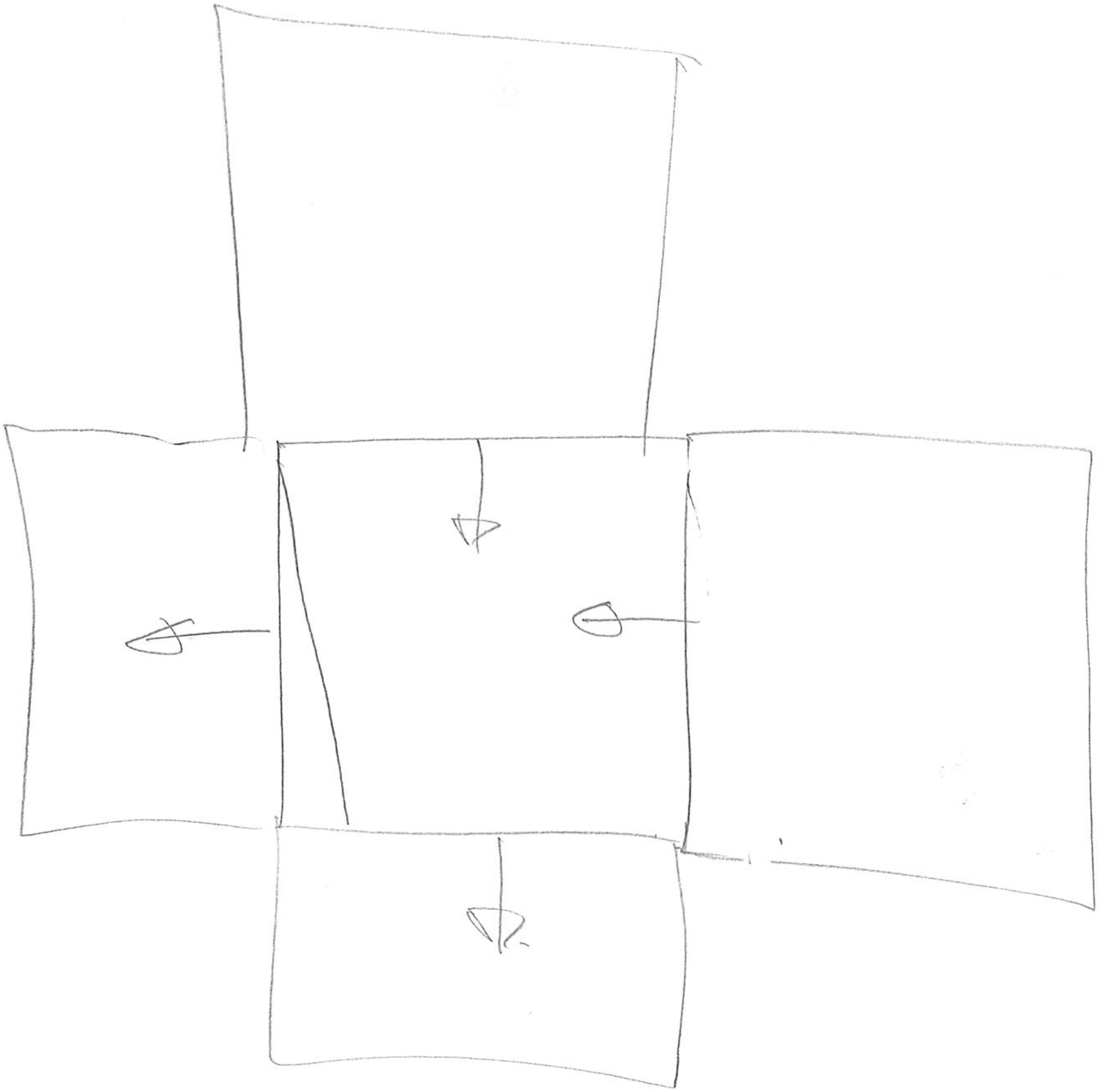
phiAdjust

$\boxed{\text{cellVofphiadjust}}$



idea - limit fluxes leaving cell to less than contained in cell, give CFL < 1 based on outflow.

ie $\frac{\sum u_{f,out} A \Delta t}{V} < 1$

$$\underset{\sim}{u}_l = \begin{cases} \underset{\sim}{u}_n & \text{if } \underset{\sim}{u} \text{ is directed out of cell} \\ 0 & \text{if } \underset{\sim}{u} \text{ is directed into cell.} \end{cases}$$

$$\phi_{check,d} = \phi^0 - \Delta t\, \underset{\sim}{\nabla} \cdot \phi \underset{\sim}{u}_l.$$

$$(1-\phi_{check}) = 1 - \phi^0 - \Delta t\, \underset{\sim}{\nabla} \cdot (1-\phi) \underset{\sim}{u}_l.$$

$$\phi_{check,c} = \phi^0 + \Delta t \left(\underset{\sim}{\nabla} \cdot \phi \underset{\sim}{u}_l - \underset{\sim}{\nabla} \cdot \underset{\sim}{u}_l \right).$$

$$0 \leq \phi_{check,d}, \phi_{check,c} \leq 1$$

Now, with CFL ≤ 1, $0 \leq \Delta t\, \underset{\sim}{\nabla} \cdot \underset{\sim}{u}_l \leq 1$ &

$$0 \leq \Delta t\, \underset{\sim}{\nabla} \cdot \phi \underset{\sim}{u}_l \leq 1.$$

Also, as $\phi < 1$, $\underset{\sim}{\nabla} \cdot \phi \underset{\sim}{u}_l < \underset{\sim}{\nabla} \cdot \underset{\sim}{u}_l$

$\therefore$ $\phi_{check,d} \leq \phi^0$, so only goes negative. 1

$$\therefore \Delta t \left(\underset{\sim}{\nabla} \cdot \phi \underset{\sim}{u}_l - \underset{\sim}{\nabla} \cdot \underset{\sim}{u}_l \right) \leq 0.$$

$$-\Delta t \left(\underset{\sim}{\nabla} \cdot \phi \underset{\sim}{u}_l - \underset{\sim}{\nabla} \cdot \underset{\sim}{u}_l \right) \geq 0.$$

$\therefore$ $\phi_{check,c} \geq \phi^0$, so only goes > 1.

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So from 1, ensure $\phi^0 - \Delta t\, \underset{\sim}{\nabla} \cdot \phi_a \underset{\sim}{u}_l \geq 0$ or

$$\Delta t\, \underset{\sim}{\nabla} \cdot \phi_a^{\text{adjusted}} \underset{\sim}{u}_l \leq \phi^0$$

from 2 $\phi^0 - \Delta t \left(\underset{\sim}{\nabla} \cdot \phi_a \underset{\sim}{u}_l - \underset{\sim}{\nabla} \cdot \underset{\sim}{u}_l \right) \leq 1.$

$\therefore \quad \Delta t \, \nabla \cdot \phi_a \underline{u}_\ell \geq \phi_0 + \underbrace{(\Delta t \, \nabla \cdot \underline{u}_\ell - 1)}_{\text{negative}}$

ie $\quad \phi_0 + \Delta t \, \nabla \cdot \underline{u}_\ell - 1 \leq \Delta t \, \nabla \cdot \phi_a \underline{u}_\ell \leq \phi_0 \quad \text{--- (3)}$

where $\phi_a = \min(\max(\phi_f + \Delta\phi, 0), 1)$.

$\Delta\phi$ is a cell centred adjustment to all outwardly directed fluxes to ensure (3)

$\boxed{\text{cellvofphiadjust}}$

needs flux, phif, phi[t=1], dt
3) 2) 1) 4)

check $\Delta t \, \nabla \cdot \underline{u}_\ell < 1$, otherwise adjust flux velocities by constant factor.

interp = 0.

$\Delta\phi = 0 \Rightarrow \Delta t \, \nabla \cdot \phi_a \underline{u}_\ell = A(\Delta\phi)$.

if $(A > \phi_0)$ { increase ϕ_a.

$\Delta\phi = \min(1-\phi, \text{faces})$ & $> \Delta\phi$.

calc new $A(\Delta\phi)$. interp = 1.

if $(A < \phi_0 - 1 + A_0)$

$\Delta\phi = -\max(\phi, \text{faces})$ & $< \Delta\phi^0$ interp = 1.

else bounded;

if interp

else $\Delta\phi = 0$ exit

- $\phi_o + \Delta t \underset{\sim}{\nabla} \cdot \underset{\sim}{u}_l - 1 \leq \Delta t \underset{\sim}{\nabla} \cdot [\phi_f + \Delta\phi]_l \underset{\sim}{u}_l \leq \phi_o$.

where $\phi_o, \Delta t, \phi_f$ are explicit (& hence don't depend on unknowns, here written as ω_m)

- when constraint is in effect,

$$\Delta t \underset{\sim}{\nabla} \cdot [\phi_f + \Delta\phi]_l \underset{\sim}{u}_l = A \Delta t \underset{\sim}{\nabla} \cdot \underset{\sim}{u}_l + B.$$

where $A = \{0, 1\}$ & $B = B(\phi_o)$.

Now $\underset{\sim}{\nabla} \cdot = \sum_j d_{ij} \underset{\sim}{N}_{ij} \cdot$ where here $d_{ij} = \frac{A_j}{V_i}$

ie - $|\mathrm{divop}(i,j)|$.

$$\therefore \Delta t \sum_j d_{ij} \underset{\sim}{N}_{ij} \cdot ([\phi_{f_{ij}} + \Delta\phi_{iup}]_l - A) \underset{\sim}{u}_{l,j} = B.$$

$$u_{l,j} = \underset{\sim}{N}_{ij} \cdot \underset{\sim}{u}_{l,j} = \begin{cases} \underset{\sim}{N}_{ij} \cdot \underset{\sim}{u}_j & \text{if } > 0 \\ 0 & \text{otherwise}. \end{cases}$$

$$\therefore \Delta t \sum_j d_{ij} ([\phi_{f_{ij}} + \Delta\phi_{iup}]_l - A) u_{l,j} = B$$

$$\therefore \frac{\partial}{\partial \omega_m} \Rightarrow \cancel{\Delta t} \sum_j d_{ij} \left[\theta_{\phi_{ij}} \left(\frac{\partial \Delta\phi_{iup}}{\partial \omega_m} \right) u_{l,j} + ([\phi_{f_{ij}} + \Delta\phi_{iup}]_l - A) \frac{\partial u_{l,j}}{\partial \omega_m} \right] = 0.$$

$$\therefore \quad \frac{\partial \Delta\phi_{inp}}{\partial w_m} \sum_j d_{ij}\theta_{\phi,j}\, u_{\ell,j} + \sum_j d_{ij}\left([\phi_{f,j} + \Delta\phi_{inp}]_\ell - A\right)\frac{\partial u_{\ell,j}}{\partial w_m} = 0$$

$$\therefore \quad \frac{\partial \Delta\phi_{inp}}{\partial w_m} = \frac{-\sum_j d_{ij}\left([\phi_{f,j} + \Delta\phi_{inp}]_\ell - A\right)\frac{\partial u_{\ell,j}}{\partial w_m}}{\sum_j d_{ij}\, u_{\ell,j}\,\theta_{\phi,j}}$$

thetaphi_list

$$\therefore \quad \frac{\partial \Delta\phi_{inp}}{\partial w_m} = \frac{\sum_j c_j \frac{\partial u_{\ell,j}}{\partial w_m}}{D} \quad \text{where } D = \sum_j d_{ij}\, u_{\ell,j}\, \theta_{\phi,j}.$$

$$\& \quad c_j = -d_{ij}\left([\phi_{f,j} + \Delta\phi_{inp}]_\ell - A\right)\theta_{u,j}$$

$$(\text{ie, } c_j = 0 \text{ if } \theta_{u,j} = 0).$$

Notation;

$$[\phi_{f,j} + \Delta\phi_{inp}]_\ell = \max(\min(\phi_{f,j} + \Delta\phi_{inp}, 1), 0)$$

thetaphi_list

if $\phi_{f,j} + \Delta\phi_{inp} > 1$ or < 0, $\theta_{\phi,j} = 0$, else $\theta_{\phi,j} = 1$

$$\theta_{u,j} = \underbrace{\underset{\sim}{N}_{i,j} \cdot \underset{\sim}{n}_j}_{(\pm 1)} \text{ if } \underset{\sim}{N}_{i,j} \cdot \underset{\sim}{u}_j > 0, \text{ or } 0 \text{ ~~otherwise~~}.$$

thetau_list

$$\therefore \quad u_{\ell,j} = u_{f,j}\,\theta_{u,j}$$

Change:

Now make $d'_{ij} = d_{ij} \Theta_{u,j}$

$$\therefore d_{ij} u_{e,j} = d'_{ij} u_{f,j}$$

areadvol_list

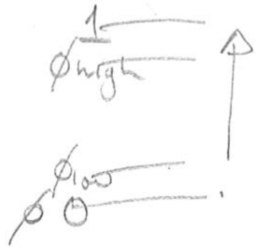
phif_list

$$\therefore C_j = -d'_{ij} \left([\phi_{f,j} + \Delta\phi_{iup}]_{d} - A \right)$$

$$D = \sum_j d'_{ij} u_{f,j} \Theta_{\phi,j}$$

thetaphi_list (init)

flux_list



$$\phi_f = 0 \ldots \phi_n \ldots 1 .$$

$$(0 \le)\,\phi_{low} \ldots \phi_n \ldots \phi_{high}\ (\le 1)$$

$$\Delta\phi = -\phi_{high} \to 1 - \phi_{low} .$$

$$\underbrace{-\phi_{high} \to}_{\ominus\text{'ve}} 0 \underbrace{\to 1 - \phi_{low}}_{\oplus\text{'ve}}$$

$$-1,\ \underbrace{-\phi_{high} \ldots -\phi_{low}}_{\text{to decrease } A},\ 0,\ \underbrace{1-\phi_{high} \ldots 1-\phi_{low}}_{\text{to increase } A},\ 1$$

$$\cancel{A = \Delta t}$$

$$A(\Delta\phi) = \Delta t\, \underset{\sim}{\nabla} \cdot \max(\min(\phi_f + \Delta\phi, 1), 0)\, \underset{\sim}{u}_l .$$

$$A(0) = \Delta t\, \underset{\sim}{\nabla} \cdot \phi_f \underset{\sim}{u}_l . \qquad \text{A\_0}$$

$$A(1) = \Delta t\, \underset{\sim}{\nabla} \cdot \underset{\sim}{u}_l \qquad \text{A\_1}$$

cellvofphiadjust $\rightarrow$ cylindrial. 2/6/14

conservative.

$$\frac{\partial \phi}{\partial t} + \underbrace{\frac{\underline{\nabla} \cdot \Gamma_f \phi \underline{u}}{\Gamma_c}} = 0.$$

as $\underline{\nabla} \cdot \Gamma_f \underline{u} = 0$

$$\frac{\underline{\nabla} \cdot \Gamma_f \phi \underline{u}}{\Gamma_c} = \frac{\cancel{\phi \underline{\nabla} \cdot \Gamma_f \underline{u}}^{\,0} + \Gamma_f \underline{u} \cdot \underline{\nabla} \phi}{\Gamma_c}.$$

$$= \underline{u} \cdot \underline{\nabla} \phi \quad ✓.$$

(=0 when converged)

$$\frac{\partial \phi}{\partial t} + \underline{u} \cdot \underline{\nabla} \phi = 0 \Rightarrow \underbrace{\frac{\partial \phi}{\partial t} + \frac{\underline{\nabla} \cdot \Gamma_f \phi \underline{u} - \phi \underline{\nabla} \cdot \Gamma_f \underline{u}}{\Gamma_c}}_{\text{equation solved.}} = 0.$$

Now, from previous analysis.

$$\phi_{check} = \phi^0 - \frac{\Delta t}{\Gamma_c} \underline{\nabla} \cdot \Gamma_f \phi \underline{u}_l$$

& $\phi_{check} = \phi^0 - \frac{\Delta t}{\Gamma_c} \left(\underline{\nabla} \cdot \Gamma_f \phi \underline{u}_l - \underline{\nabla} \cdot \Gamma_f \underline{u}_l \right).$

$\hookrightarrow$ redefing flux as $\frac{\Gamma_f \underline{u}_l}{\Gamma_c}$ $\leftarrow$ lastcell ov. should work consistently.