ACSL
American Computer Science League

2014 - 2015 **Contest #4**

Quine-McCluskyAlgorithm
Senior Division

PROBLEM: Use a part of the Quine-McClusky Algorithm method to simplify Boolean functions.

As an example $\mathbf{ABC\overline{D} + ABCD}$ is simplified by using DeMorgan's Theorem as follows
$\mathbf{ABC(\overline{D} + D)\ = ABC(1)\ = ABC.}$

If instead we are given which of the 16 possible ordered binary quadruples make the function true (1110 and 1111 which are 14 and 15 in decimal) and we also note that they only differ in one place value, the two quadruples can be combined and one digit can be eliminated.

$$\begin{array}{c} 1\ 1\ 1\ 0 \\ \underline{1\ 1\ 1\ 1\ } \\ 1\ 1\ 1\ x \end{array}$$

Converting 111x to its Boolean function representation gives **ABC** as above.

The above can be expressed mathematically as f(A,B,C,D) = $\sum m$(14, 15) = $\sum m$(1110, 1111) = ABC.

f(A,B,C,D) = $\sum m$ (8, 9, 10, 11, 12, 14, 15) shows where the terms evaluate to 1 (true). That is shown in the f column in the chart on the left. The chart on the right groups those binary representations by the number of 1's (index) in the binary representation. Combining takes place with values that have an index of n and n+1 and only differ in one place value.

#	A	B	C	D	*f*
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	1
11	1	0	1	1	1
12	1	1	0	0	1
13	1	1	0	1	0
14	1	1	1	0	1
15	1	1	1	1	1

index	Term number	Binary	Simplified pairs
1	8	1000	m(8, 9) = 100x m(8,10) = 10x0 m(8,12) = 1x00
2	9 10 12	1001 1010 1100	m(9,11) = 10x1 m(10,11) = 101x m(10,14) = 1x10 m(12, 14) = 11x0
3	11 14	1011 1110	m(11,15) = 1x11 m(14,15) = 111x
4	15	1111	

The process of combining continues by trying to combine 2 of the simplified pairs values. Combining takes place with values that have an index of n and n+1, only differ in one place value and the x must be at the same place value.

In translating to a Boolean function lower case characters will be used to show negation. It is possible for two or more simplified pairs and extended simplifications to be the same. Extended simplification *m*(8,12,10,14) gives 1xx0 which is the same as *m*(8,10, 12,14). In that case only one is used to write the Boolean function.

$$
\begin{array}{rl}
m(10,11) = & 101x \\
m(8,9) \;=\; & 100x \\
\hline
m(8,9,10,11) = & 10xx = \text{ Ab}
\end{array}
$$

Index	Term number	Binary	Simplified pairs	Extended simplification
1	8	1000	*m*(8, 9) = 100x *m*(8,10) = 10x0 *m*(8,12) = 1x00	*m*(8,10,12,14) = 1xx0 *m*(8,9,10,11) = 10xx
2	9 10 12	1001 1001 1100	*m*(9,11) = 10x1 *m*(10,11) = 101x *m*(10,14) = 1x10 *m*(12, 14) = 11x0	*m*(10,11,14, 15) = 1x1x
3	11 14	1011 1110	*m*(11,15) = 1x11 *m*(14,15) = 111x	
4	15	1111		

10xx + 1xx0 + 1x1x = Ab + Ad + AC

INPUT: There will be 5 lines of input. Each line will contain a listing of the term numbers of the function. Each line will end with a -1. The first 3 input lines will give the term numbers of a function with 3 variables (A, B and C) so the term numbers will be in the range 0 - 7. The last 2 input lines will give the term numbers of a function with 4 variables (A, B, C and D) so the term numbers will be in the range 0 - 15. We guarantee there will always be a simplification.

OUTPUT: Print the simplified Boolean function for each input. The terms can be printed in any order.

SAMPLE INPUT	SAMPLE OUTPUT
1. 2, 6, -1	1. Bc
2. 2, 3, 5, 7, -1	2. AC + aB +BC
3. 1, 3, 4, 5, 6, 7, -1	3. A + C
4. 8, 9, 10, 11, 12, 14, 15, -1	4. Ab + Ad + AC
5. 9, 10, 11, 12, 13, 14, 15, -1	5. AB + AC + AD