San Diego State University
CS 370 Computer Architecture

Homework Assignment 1 [100 points, Weight 8%]

- You are required to write by hand legibly to solve each problem. When you plot a circuit, please make sure that you use a ruler. You are not allowed to use any software to draw a circuit or answer any question. An answer that is not written by your hand will receive zero credit for the corresponding problem.
- Please write down your full name and SDSU Red ID on the first scratch paper. After you finish writing all your answers on multiple scratch papers (print papers preferred), please take a photo for each of them. Next, please insert these image files into a Word file. Finally, convert the Word file into a single PDF file, and then, submit the PDF file from within Canvas.
- Please note that we only accept your PDF file submissions. A submission in any other format will not be graded and will receive zero credit. For example, submitting multiple image files will receive zero credit.
- Please pay special attention to the due date – a late submission will receive zero.
- For your convenience, the text of each question is appended to this document.
- Please do the following problems from the textbook, and submit solutions.

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1. [6 points] Problem 1-9 {please ignore the "Octal" part} (each sub-question is 1 point)

1-9. *Convert the following numbers from the given base to the other three bases listed in the table:

Decimal	Binary	Hexadecimal
369.3125	? $\boxed{101110001.0101}$	? $\boxed{171.5}$
? $\boxed{189.625}$	10111101.101	? $\boxed{BD.A}$
? $\boxed{62407.625}$	? $\boxed{1111\ 0011\ 1100\ 0111.101}$	F3C7.A

Decimal

369.3125 →

Binary

$\frac{369.3125}{2^4} \rightarrow 369.3125 \cdot 2^4 = 5909$

0) 5909/2 = 2954.5 1
1) 2954/2 = 1477 0
2) 1477/2 = 738.5 1
3) 738/2 = 369 0
decimal point
4) 369/2 = 184.5 1
5) 184/2 = 92 0
6) 92/2 = 46 0
7) 46/2 = 23 0
8) 23/2 = 11.5 1
9) 11/2 = 5.5 1
10) 5/2 = 2.5 1
11) 2/2 = 1 0
12) 1/2 = 0.5 1

$\boxed{101110001.0101}$

→

Hexadecimal

$\frac{369.3125}{16^1} \rightarrow 369.3125 \cdot 16^1 = 5909$

0) 5909/16 = 369.125 5
decimal point
1) 369/16 = 23.0625 1
2) 23/16 = 1.4375 7
3) 1/16 = 0.0625 1

$\boxed{171.5}$

10111101.101

$(1\times2^7)+(0\times2^6)+(1\times2^5)+(1\times2^4)+$
$(1\times2^3)+(1\times2^2)+(0\times2^1)+(1\times2^0)+$
$(1\times2^{-1})+(0\times2^{-2})+(1\times2^{-3})$
$= \boxed{189.625}$

← 10111101.101 →

$\underbrace{1011}_{B}\,\underbrace{1101}_{D}\,.\,\underbrace{1010}_{A}$ → add 0 for 3 binary digits

$\boxed{BD.A}$

F3C7.A

$(15\times16^3)+(3\times16^2)+$
$(12\times16^1)+(7\times16^0)+$
(10×16^{-1})
$= \boxed{62407.625}$

← F3C7.A ← F3C7.A

F: 1111
3: 0011
C: 1100
7: 0111
A: 1010

$\boxed{1111\ 0011\ 1100\ 0111.101}$

2. [6 points] Problem 1-12 (each sub-question is 2 points)

1-12. Perform the following binary multiplications:

(a) 1010 × 1100 $\boxed{1111000}$ **(b)** 0110 × 1001 $\boxed{110110}$ **(c)** 1111001 × 011101 $\boxed{110110110101}$

(a) 1010 × 1100

$$\begin{array}{r} 1010 \\ \times\ 1100 \\ \hline 101000 \\ 1010000 \\ \hline 1111000 \end{array}$$

$\boxed{1111000}$

(b) 0110 × 1001

$$\begin{array}{r} 0110 \\ \times\ 1001 \\ \hline 110 \\ 110000 \\ \hline 110110 \end{array}$$

$\boxed{110110}$

(c) 1111001 × 011101

$$\begin{array}{r} 1111001 \\ \times\ 11101 \\ \hline 1111001 \\ 111100100 \\ 1111001000 \\ 11110010000 \\ \hline 110110110101 \end{array}$$

$\boxed{110110110101}$

3. [6 points] Problem 2-3 (a)

2-3. +Prove the identity of each of the following Boolean equations, using algebraic manipulation:

(a) $AB\overline{C} + B\overline{C}\,\overline{D} + BC + \overline{C}D = B + \overline{C}D$

$$AB\overline{C} + B\overline{C}\,\overline{D} + BC + \overline{C}D$$
$$= AB\overline{C} + ABC + BC + B\overline{C}\,\overline{D} + B\overline{C}D + \overline{C}D$$
$$= AB(\overline{C}+C) + B\overline{C}(\overline{D}+D) + BC + \overline{C}D$$
$$= AB + B\overline{C} + BC + \overline{C}D$$
$$= B + AB + CD$$
$$= B + \overline{C}D$$

$$\therefore AB\overline{C} + B\overline{C}\,\overline{D} + BC + \overline{C}D = B + \overline{C}D \quad ✓$$

4. [10 points] Problem 2-6 (each sub-question is 2 points)

2-6. Simplify the following Boolean expressions to expressions containing a minimum number of literals:

(a) $\overline{A}\,\overline{C} + \overline{A}BC + \overline{B}C$

(b) $(\overline{A + B + C}) \cdot \overline{ABC}$

(c) $AB\overline{C} + AC$

(d) $\overline{A}\,\overline{B}D + \overline{A}\,\overline{C}D + BD$

(e) $(A + B)(A + C)(A\overline{B}C)$

(a) $\overline{A}\,\overline{C} + \overline{A}BC + \overline{B}C$

$= \overline{A}\,\overline{C} + \overline{A}BC + (\overline{A}\,\overline{B}C + \overline{B}C)$

$= \overline{A}\,\overline{C} + \overline{A}BC + \overline{A}\,\overline{B}C + \overline{B}C$

$= \boxed{(\overline{A}\,\overline{C} + \overline{A}C) + \overline{B}C = \overline{A} + \overline{B}C}$

(b) $(\overline{A + B + C}) \cdot \overline{ABC}$

$= \overline{A}\,\overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C}\,\overline{C} \longrightarrow (\overline{A}\,\overline{A})\overline{B}\,\overline{C} + \overline{A}(\overline{B}\,\overline{B})\overline{C} + \overline{A}\,\overline{B}(\overline{C}\,\overline{C})$

$= \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C}$

$= \boxed{\overline{A}\,\overline{B}\,\overline{C}}$

(c) $AB\overline{C} + AC$

$= A(B\overline{C} + C)$

$= \boxed{A(B + C)}$

(d) $\overline{A}\,\overline{B}D + \overline{A}\,\overline{C}D + BD$

$= D(AB + B + AC)$ // Factor out D

$= D(\overline{A} + \overline{A}\,\overline{C} + B)$

$= \boxed{D(\overline{A} + B)}$

(e) $(A + B)(A + C)(A\overline{B}C)$

$= A\underline{AA}\overline{B}C + A\underline{C}A\overline{B}C + B\underline{AA}\overline{B}C + B\underline{CA}\overline{B}C$

$= \boxed{A\overline{B}C}$

5. [10 points] Problem 2-11 {please ignore "maxterms of each function" in (a)} (each sub-question is 2 points)

2-11. For the Boolean functions E and F, as given in the following truth table:

X	Y	Z	E	F
0	0	0	0	1
0	0	1	1	0
0	1	0	1	1
0	1	1	0	0
1	0	0	1	1
1	0	1	0	0
1	1	0	1	0
1	1	1	0	1

(a) List the minterms of each function.

(b) List the minterms of $\overline{E}$ and $\overline{F}$

(c) List the minterms of $E + F$ and $E \cdot F$.

(d) Express E and F in sum-of-minterms algebraic form.

(e) Simplify E and F to expressions with a minimum of literals.

(a) List the minterms of each function.

E and F Functions

$$E = \Sigma m(\underbrace{1,2,4,6}_{\text{binary 1's}}) = \Pi M(\underbrace{0,3,5,7}_{\text{binary 0's}})$$

$$F = \Sigma m(\underbrace{0,2,4,7}_{\text{binary 1's}}) = \Pi M(\underbrace{1,3,5,6}_{\text{binary 0's}})$$

(b) List the minterms of $\overline{E}$ and $\overline{F}$

$$\overline{E} = \Sigma m(\underbrace{0,3,5,7}_{\text{binary 0's}})$$

$$\overline{F} = \Sigma m(\underbrace{1,3,5,6}_{\text{binary 0's}})$$

(c) List the minterms of $E + F$ and $E \cdot F$.

$$E + F = \Sigma m(0,1,2,4,6,7)$$

$$E \cdot F = \Sigma m(2,4)$$

(d) Express E and F in sum-of-minterms algebraic form.

$$E = \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xy\bar{z}$$

$$F = \bar{x}\bar{y}\bar{z} + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xyz$$

(e) Simplify E and F to expressions with a minimum of literals.

$$E = \bar{z}(x+y) + \bar{x}\bar{y}z$$

$$F = \bar{z}(\bar{x}+\bar{y}) + xyz$$

6. [6 points] Problem 2-17 (each sub-question is 3 points; in each sub-question, a correct K-map is 1 point and a correct equation is 2 points)

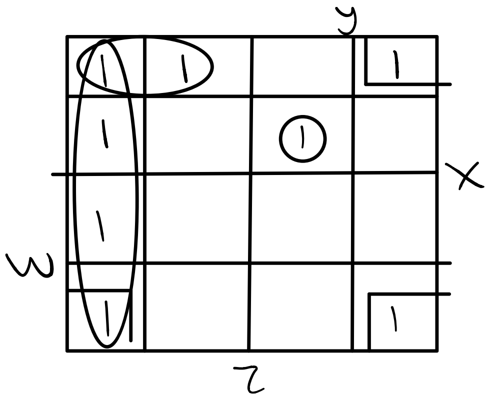
2-17. Optimize the following Boolean functions, using a map:

(a) $F(W, X, Y, Z) = \Sigma m(0, 1, 2, 4, 7, 8, 10, 12)$

(b) $F(A, B, C, D) = \Sigma m(1, 4, 5, 6, 10, 11, 12, 13, 15)$

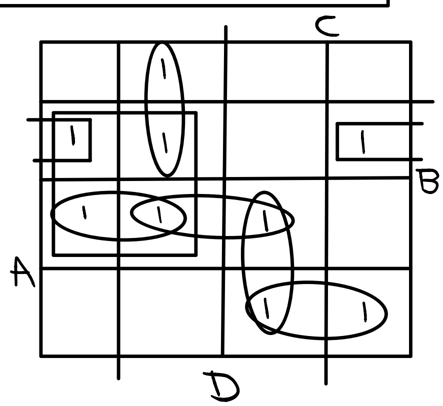
(a) $F(W, X, Y, Z) = \Sigma m(0, 1, 2, 4, 7, 8, 10, 12)$

$$\boxed{F = \bar{X}\bar{Z} + \bar{Y}\bar{Z} + \bar{W}\bar{X}\bar{Y} + \bar{W}XYZ}$$



(b) $F(A, B, C, D) = \Sigma m(1, 4, 5, 6, 10, 11, 12, 13, 15)$

$$\boxed{F = B\bar{C} + \bar{A}\bar{C}D + \bar{A}B\bar{D} + A\bar{B}C + ABD}$$



7. [9 points] Problem 2-20 (each sub-question is 3 points; in each sub-question, finding all prime implicants deserves 1 point, finding all essential prime implicants deserves 1 point, final function after applying the selection rule deserves 1 point)

2-20. Optimize the following Boolean functions by finding all prime implicants and essential prime implicants and applying the selection rule:

(a) $F(A, B, C, D) = \Sigma m\,(1, 5, 6, 7, 11, 12, 13, 15)$

(b) $F(W, X, Y, Z) = \Sigma m\,(0, 1, 2, 3, 4, 5, 10, 11, 13, 15)$

(c) $F(W, X, Y, Z) = \Sigma m\,(0, 1, 2, 5, 7, 8, 10, 12, 14, 15)$

(a) $F(A, B, C, D) = \Sigma m\,(1, 5, 6, 7, 11, 12, 13, 15)$

Prime Implicant:

$= BD, \bar{A}\bar{C}D, \bar{A}BC, AB\bar{C}, ACD$

Essential Prime Implicant:

$= \bar{A}\bar{C}D, \bar{A}BC, AB\bar{C}, ACD$

Final Function:

$\rightarrow F = \bar{A}\bar{C}D + \bar{A}BC + AB\bar{C} + AC\bar{D}$

(b) $F(W, X, Y, Z) = \Sigma m\,(0, 1, 2, 3, 4, 5, 10, 11, 13, 15)$

Prime Implicant:

$= \bar{W}\bar{Y}, \bar{X}Y, WXZ, \bar{W}\bar{X}, X\bar{Y}Z, WYZ$

Essential Prime Implicant:

$= \bar{W}\bar{Y}, \bar{X}\bar{Y}$

Final Function:

$\rightarrow F = \bar{W}\bar{Y} + \bar{X}Y + WXZ$

(c) $F(W, X, Y, Z) = \Sigma m\,(0, 1, 2, 5, 7, 8, 10, 12, 14, 15)$

Prime Implicant:

$= W\bar{Z}, \bar{X}\bar{Z}, \bar{W}\bar{Y}Z, XYZ, \bar{W}\bar{X}\bar{Y}, \bar{W}XZ, WXY$

Essential Prime Implicant:

$= W\bar{Z}, \bar{X}\bar{Z}$

Final Function:

$\rightarrow F = W\bar{Z} + \bar{X}\bar{Z} + \bar{W}\bar{Y}Z + XYZ$

8. [11 points] Problem 3-9 (the truth table is 2 points, each **optimized** equation deserves 2 points, a K-map for each equation deserves 1 point). For each output, you need to draw a K-map and then write down an optimized equation based on your K-map. Providing a non-optimized equation will receive zero for that equation part.

3-9. +Design a combinational circuit that accepts a 4-bit number and generates a 3-bit binary number output that approximates the square root of the number. For example, if the square root is 3.5 or larger, give a result of 4. If the square root is < 3.5 and ≥ 2.5, give a result of 3.

A	B	C	D	S2	S1	S0
0	0	0	0	0	0	0
0	0	0	1	0	0	1
0	0	1	0	0	0	1
0	0	1	1	0	1	0
0	1	0	0	0	1	0
0	1	0	1	0	1	0
0	1	1	0	0	1	0
0	1	1	1	0	1	1
1	0	0	0	0	1	1
1	0	0	1	0	1	1
1	0	1	0	0	1	1
1	0	1	1	0	1	1
1	1	0	0	0	1	1
1	1	0	1	1	0	0
1	1	1	0	1	0	0
1	1	1	1	1	0	0

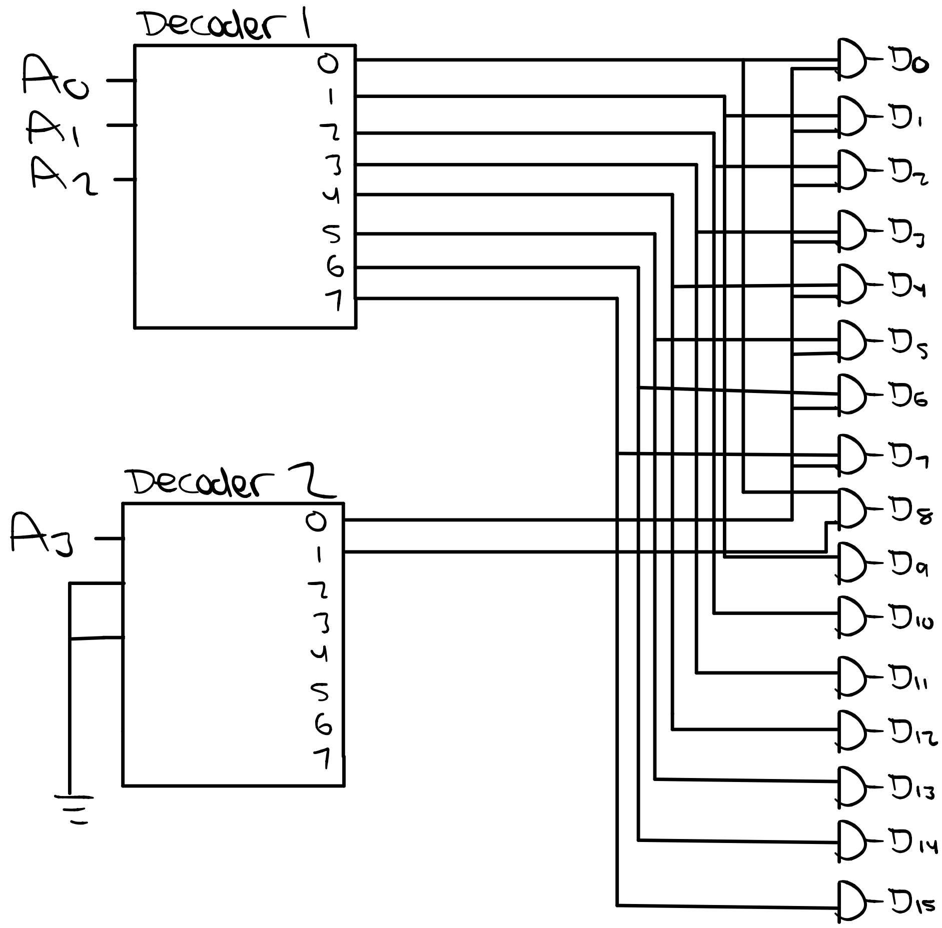
$S_0 = \bar{B}\bar{C}D + \bar{B}C\bar{D} + A\bar{B} + A\bar{C}\bar{D} + \bar{A}BCD$

$S_1 = \bar{A}B + A\bar{B} + \bar{A}CD + B\bar{C}\bar{D}$

$S_2 = ABC + ABD$

9. [6 points] Problem 3-28 (**hint:** You can fully use one 3-8 decoder so that you can have 3 inputs and 8 outputs. Meanwhile, you can partially use the second 3-8 decoder so that you have 1 input and 2 outputs. For this second 3-8 decoder, you only need to use one of its inputs [you can connect the other two inputs to the ground so that these two inputs are always zero] and two of its 8 outputs [the other six outputs are simply not used]. Note that each of the two outputs of the second 3-8 decoder controls 8 2-input AND gates).

3-28. Design a 4–to–16-line decoder using two 3–to–8-line decoders and 16 2-input AND gates.



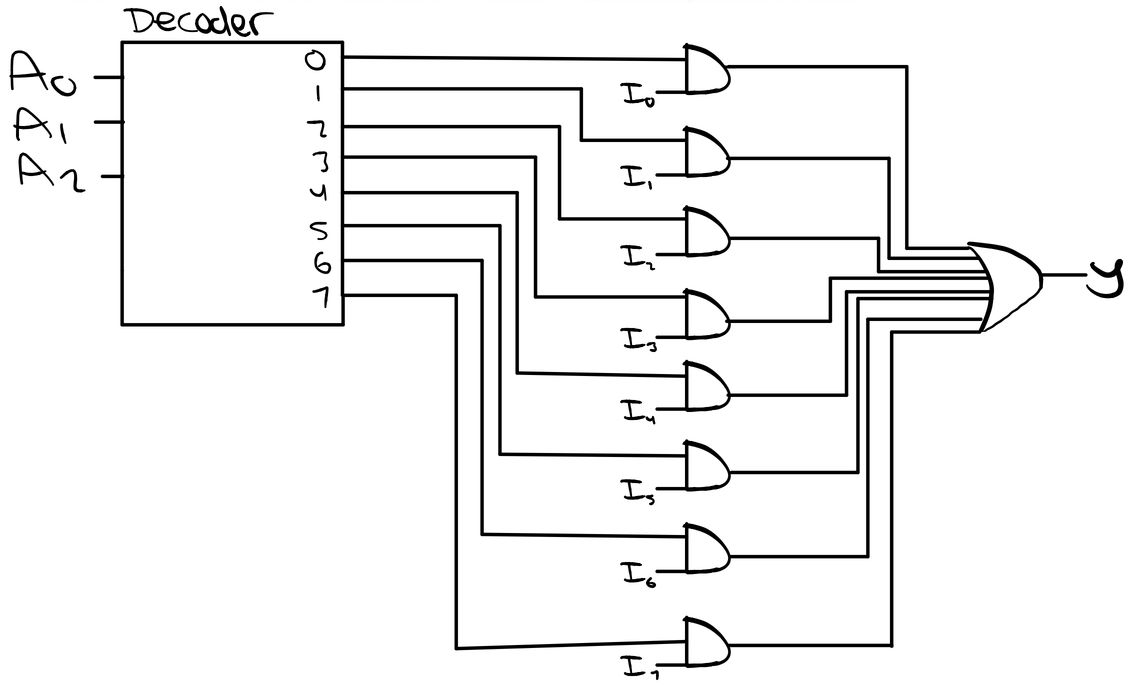
10. [6 points] Problem 3-36.

3-36. Derive the truth table of a decimal-to-binary priority encoder. There are 10 inputs I_0 through I_9 and outputs A_3 through A_0 and V. Input I_9 has the highest priority.

9	8	7	6	5	4	3	2	1	0	A_3	A_2	A_1	A_0	V
0	0	0	0	0	0	0	0	0	0	X	X	X	X	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
0	0	0	0	0	0	0	0	1	X	0	0	0	1	1
0	0	0	0	0	0	0	1	X	X	0	0	1	0	1
0	0	0	0	0	0	1	X	X	X	0	0	1	1	1
0	0	0	0	0	1	X	X	X	X	0	1	0	0	1
0	0	0	0	1	X	X	X	X	X	0	1	0	1	1
0	0	0	1	X	X	X	X	X	X	0	1	1	0	1
0	0	1	X	X	X	X	X	X	X	0	1	1	1	1
0	1	X	X	X	X	X	X	X	X	1	0	0	0	1
1	X	X	X	X	X	X	X	X	X	1	0	0	1	1

11. [6 points] Problem 3-37 (a) (**hint:** An 8x2 AND-OR means 8 AND gates [each of these 8 AND gates has two inputs] and one OR gate with 8 inputs. The 8 outputs of the 8 AND gates are the inputs of the OR gate).

3-37. **(a)** Design an 8–to–1-line multiplexer using a 3–to–8-line decoder and an 8 × 2 AND-OR.



12. [4 points] Problem 3-52 (each sub-question is 1 point)

3-52. Perform the indicated subtraction with the following unsigned binary numbers by taking the 2s complement of the subtrahend:

(a) 11010 − 10001 $\boxed{01001}$

(b) 11110 − 1110 $\boxed{10000}$

(c) 1111110 − 1111110 $\boxed{0000000}$

(d) 101001 − 101 $\boxed{100100}$

(a) 11010 − 10001

$$\begin{array}{r} 11010 \\ +\,01111 \\ \hline 01001 \end{array}$$

$\boxed{01001}$

(b) 11110 − 1110

$$\begin{array}{r} 11110 \\ +\,10010 \\ \hline 10000 \end{array}$$

$\boxed{10000}$

(c) 1111110 − 1111110

$$\begin{array}{r} 1111110 \\ +\,0000010 \\ \hline 0000000 \end{array}$$

$\boxed{0000000}$

(d) 101001 − 101

$$\begin{array}{r} 101001 \\ +\,111011 \\ \hline 100100 \end{array}$$

$\boxed{100100}$

13. [4 points] Problem 3-53 (each sub-question is 1 point)

3-53. Repeat Problem 3-52, assuming the numbers are 2s complement signed numbers. Use extension to equalize the length of the operands. Indicate whether overflow occurs during the complement operations for any of the given subtrahends. Indicate whether overflow occurs overall for any of the given subtractions. When an overflow does occur, repeat the operation with the minimum number of bits required to perform the operation without overflow.

(a) 11010 − 10001

$$\begin{array}{r} 11010 \\ +\,01111 \\ \hline 01001 \end{array}$$

$\boxed{01001}$

(b) 11110 − 1110

$$\begin{array}{r} 11110 \\ +\,00010 \\ \hline 00000 \end{array}$$

$\boxed{00000}$

(c) 1111110 − 1111110

$$\begin{array}{r} 1111110 \\ +\,0000010 \\ \hline 0000000 \end{array}$$

$\boxed{0000000}$

(d) 101001 − 101

$$\begin{array}{r} 101001 \\ +\,000011 \\ \hline 101100 \end{array}$$

$\boxed{101100}$

14. [4 points] Problem 3-55 (each sub-question is 1 point)

3-55. The following binary numbers have a sign in the leftmost position and, if negative, are in 2s complement form. Perform the indicated arithmetic operations and verify the answers.

(a) 100111 + 111001 $\boxed{100000}$

(b) 001011 + 100110 $\boxed{110001}$

(c) 110001 − 010010 $\boxed{011111}$

(a) 100111 + 111001

$$\begin{array}{r} 100111 \\ +111001 \\ \hline 100000 \end{array} \qquad \boxed{100000} \qquad \begin{array}{r} -25 \\ -7 \\ \hline -32 \end{array} \checkmark$$

(b) 001011 + 100110

$$\begin{array}{r} 00101111 \\ +100110 \\ \hline 110001 \end{array} \qquad \boxed{110001} \qquad \begin{array}{r} 11 \\ -26 \\ \hline -15 \end{array} \checkmark$$

(c) 110001 − 010010

$$\begin{array}{r} 110001 \\ +101110 \\ \hline 011111 \end{array} \qquad \boxed{011111} \qquad \begin{array}{r} -15 \\ -18 \\ \hline -23 \end{array} \checkmark$$

15. [6 points] Problem 3-57

3-57. Use contraction beginning with a 4-bit adder with carry out to design a 4-bit increment-by-2 circuit with carry out that adds the binary value 0010 to its 4-bit input. The function to be implemented is $S = A + 0010$.