## Axiom Quick Reference (January 2008)

#### **Command Line**

)cd (pathname)

)clear all – clear workspace

)display op \( \)function \( \) – function arguments

)set message autoload off – quietly load algebra

)set message bottom on – show selection process

)set stream calculate 20 – number of terms to calculate

)show (domain) – list all functions

)spool (filename) – start save session

)spool – close spool file

)trace (domain) )math – trace execution

)quit – exit Axiom

)read  $\langle \text{filename} \rangle [\text{.input}] - \text{evaluate a file}$ 

)sys (command line) – execute command

 $_{-}$  continues input lines or escapes chars  $a_{-}$  b = "a b"

% is last value

%%(n) is *n*th value

- and ++ start comment lines

# Programming

assignment: var := value

x := 3

conditional: if  $\langle pred \rangle$  then  $\langle truecase \rangle$  else  $\langle falsecase \rangle$ 

if (2 > 4) then 4 else 5

loop: for  $\langle pred \rangle$  repeat  $\langle block \rangle$ 

for i in 1..5 repeat print i

while i < 3 repeat (print i ; i:=i+1)</pre>

function:  $f(x) = x^2$ 

 $f(x)==x^2$ 

anon. function: g:=x +-> x+1  $g(3) \rightarrow 4$ 

Indentation is significant:

f(x) == (x > 3 => x ; 0)

f(x)==

 $x > 3 \Rightarrow x$ 

Basic constants and functions

 $\pi = \text{\%pi} \quad e = \text{\%e} \quad i = \text{\%i} \quad \infty = \text{\%infinity} \\ + \infty = \text{\%plusInfinity} \quad - \infty = \text{\%minusInfinity}$ 

numeric(%pi) = 3.1415926535 897932385

Functions: sin cos tan sec csc cot sinh cosh tanh

 $\mathtt{sech}\ \mathtt{csch}\ \mathtt{coth}\ \mathtt{log}\ \mathtt{ln}\ \mathtt{exp}$ 

ab = a\*b  $\frac{a}{b} = a/b$   $a^b = a^b$   $\sqrt{x} = \operatorname{sqrt}(x)$   $\sqrt[n]{x} = x^{(1/n)} |x| = abs(x) \log_b(x) = \log(x)/\log(b)$ 

Operations on expressions

factor(...) expand(...) simplify(...)

Symbolic equations: f(x)=g(x)

Solve f(x) = g(x): solve(f(x)=g(x),x)

 $solve([x^2*y-1,x*y^2-2],.01)$ 

 $\rightarrow [[y = 1.5859375, x = 0.79296875]]$ 

complexSolve( $[x^2*y-1,x*y^2-2],1/1000$ )

 $radicalSolve([x^2/a+a+y^3-1,a*y+a+1],[x,y])$ 

 $\sum_{i=1}^{n} f(i) = \texttt{reduce(+,[f(i) for i in k..n])}$ 

 $\prod_{i=k} f(i) = \texttt{reduce(*,[f(i) for i in k..n])}$ 

## Pattern Matching

 $\begin{array}{c} \text{logrule:=rule log(x)+log(y) == log(x*y)} \rightarrow \\ \text{log(y)+log(x)+\%B==log(x y)+\%B} \end{array}$ 

 $f{:=}log \ sin \ x \ + \ log \ x \ \rightarrow \ log(sin(x)){+}log(x)$ 

 $logrule f \rightarrow log(x sin(x))$ 

#### Calculus

 $\lim_{x \to a} f(x) = \text{limit(f(x), x=a)}$ 

 $\lim_{x \to a} f(x) = \operatorname{limit}(f(x), x=a, "left")$ 

 $\lim_{x \to 0} f(x) = \text{limit(f(x), x=a, "right")}$ 

 $\lim f(x) = \lim f(x), x=%plusInfinity)$ 

 $limit(sin(x)/x, x=\%plusInfinity) \rightarrow 0$ 

 $complexLimit(sin(x)/x,x=%infinity) \rightarrow "failed"$ 

 $\frac{d}{dx}(f(x)) = D(f(x),x)$ 

 $\frac{\partial}{\partial x}(f(x,y)) = \mathrm{D}(\mathrm{f}(\mathrm{x},\mathrm{y}),\mathrm{x})$ 

 $\int f(x)dx = integrate(f(x),x)$ 

 $\int_a^b f(x)dx = integrate(f(x), x=a..b)$ 

#### Series

x:=series 'x

y:=sin(x)  $\rightarrow x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + O(x^9)$ coefficient(y,3)  $\rightarrow -\frac{1}{6}$ 

 $_{\rm taylor(f(x),x=a)}$ 

laurent(x/log(x),x=1)
puiseux(sqrt(sec(x)),x=3\*%pi/2)

## 2D graphics

draw(cos(5\*t/8),t=0..16\*%pi,coordinates==polar)

f(t:SF):SF == sin(3\*t/5)

g(t:SF):SF == sin(t)

draw(curve(f,g),0..%pi)

 $draw(x^2+y^3-1=0,x,y,range==[-1..1,-1..1])$ 

v1:=draw(Gamma(i),i=-4.2..4,adaptive==true)

v2:=draw(1/Gamma(i),i=-4.2..4,adaptive==true)

putGraph(v2,getGraph(v1,1),2)

makeViewport2D(v2)

options: adaptive clip toScale curveColor pointColor unit range coordinates

## 3D graphics

m(u:SF,v:SF):SF == 1

draw(m,0..2\*%pi,0..%pi,coordinates==spherical)

options: title style color Function coordinates tubeRadius tubePoints var1Steps var2Steps space

#### Discrete math

|x| = floor(x) [x] = ceiling(x)

Remainder of n divided by k = rem(n,k) , k|n iff n%k==0

n! = factorial(n)  $\binom{x}{m} = binomial(x,m)$   $\phi(n) = eulerPhi(n)$  Tuples: (1,'Hello,x)

## **Type Conversions**

r:=(2/3)\*x^2-y+4/5  $\rightarrow -y + \frac{2}{3}x^2 + \frac{4}{5}$ 

Type: Polynomial Fraction Integer

r::FRAC POLY INT  $\rightarrow \frac{-15y+10x^2+12}{15}$ 

Type: Fraction Polynomial Integer

 $s := (3+4*\%i)/(7+3*\%i) \rightarrow \frac{33}{58} + \frac{19}{58}\%i$ 

s::FRAC COMPLEX INT  $\rightarrow \frac{3+4\%i}{7+3\%i}$ 

# Equation

eq1:= $3*x+4*y=5 \rightarrow 4y + 3x = 5$ 

eq2:= $2*x+2*v=3 \rightarrow 2y+2x=3$ 

lhs eq $1 \to 4y + 3x$ 

rhs eq $1 \to 5$ 

 $eq1 + eq2 \to 6y + 5x = 8$ 

#### Factored

 $g := factor(4312) \rightarrow 2^37^211$ unit  $g \to 1$ numberOfFactors  $g \rightarrow 3$  $nthFactor(g,2) \rightarrow 7$  $nthExponent(g,2) \rightarrow 2$  $nthFlag(g,2) \rightarrow "prime"$  $map(factor, 55739/2520) \rightarrow \frac{139 \ 401}{2^{3} \ 3^{2} \ 5 \ 7}$ 

#### List

 $a := [1,2,3,4] \rightarrow [1,2,3,4]$  $b := [3,4,5,6] \rightarrow [3,4,5,6]$ append(a,b)  $\rightarrow$  [1, 2, 3, 4, 3, 4, 5, 6]  $cons(10,a) \rightarrow [10,1,2,3,4]$ empty?  $a \rightarrow false$  $a.2 \rightarrow 2$  $a.2 := 99 \rightarrow [1, 99, 3, 4]$ reverse  $b \rightarrow [6, 5, 4, 3]$ 

#### MakeFunction

expr:=(x+a)^3  $\rightarrow x^3 + 3ax^2 + 3a^2x + a^3$ function(expr,f,x)  $\rightarrow$  f  $f(2) \rightarrow a^3 + 6a^2 + 12a + 8$ function(expr,g,a)  $\rightarrow$  g  $g(2) \rightarrow x^3 + 6x^2 + 12x + 8$ 

#### Matrix

A:=matrix([[1,2],[3,4]])  $\rightarrow \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ determinant  $A \rightarrow -2$  $v:=vector([1,2]) \rightarrow [1,2]$  $A*v \to [5, 11]$  $A^{-1} \rightarrow \left[\begin{array}{cc} 2 & 1 \\ \frac{3}{2} & \frac{1}{2} \end{array}\right]$  $transpose(A) \rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ 

nrows  $A \rightarrow 2$ ncols  $A \rightarrow 2$ 

nullity  $A \to 0$ 

rank  $A \rightarrow 2$ trace  $A \rightarrow 5$ 

## Polynomial

x+1 yields Type Polynomial Integer z-2.3 yields Type Polynomial Float y^2-z+3/4 yields Type Polynomial Fraction Integer  $p:=(y-1)^2*x*z \to (xy^2 - 2xy + x)z$  $q:=(y-1)*x*(z+5) \rightarrow (xy-x)z + 5xy - 5x$  $\gcd(p,q) \to xy - x$ mainVariable p  $\rightarrow z$ variables  $p \to [z, y, x]$  $degree(p,y) \rightarrow 2$ totaldegree p  $\rightarrow 4$  $eval(p,x,w) \rightarrow (wy^2 - 2wy + w)z$  $D(p,x) \to (y^2 - 2y + 1)z$ integrate(p,x)  $\rightarrow (\frac{1}{2}x^2y^2 - x^2y + \frac{1}{2}x^2)z$ 

## PrimeField

x:PrimeField(7):= $5 \rightarrow 5$  $x^3 \rightarrow 6$  $1/x \rightarrow 3$ 

#### Set

 $s:=brace([1,2,3,4,5]) \rightarrow \{1,2,3,4,5\}$  $t:=brace([2,3,5,7]) \rightarrow \{2,3,5,7\}$  $intersect(s,t) \rightarrow \{2,3,5\}$ union(s,t)  $\rightarrow \{1, 2, 3, 4, 5, 7\}$  $difference(s,t) \rightarrow \{1,4\}$ insert! $(7,s) \rightarrow \{1,2,3,4,5,7\}$ remove! $(7,s) \rightarrow \{1, 2, 3, 4, 5\}$  $\{1,2,1,a\} = brace([1,2,1,'a]) (= \{1,2,a\})$  $\{f(x): x \in X, x > 0\} \approx \operatorname{brace}([f(x) \text{ for x in X | x>0}])_{\operatorname{resultant}(p,q) \to 0}$ 

# **Special Functions**

[fibonacci(k) for k in 0..]  $\rightarrow$  [0,1,1,2,3,5,...] [legendre(i,11) for i in 0..5]  $\rightarrow$  [0,1,-1,1,1,1]  $[jacobi(i,15) \text{ for i in } 0..5] \rightarrow [0,1,1,0,1,0]$ [eulerPhi i for i in 1..]  $\rightarrow$  [1,1,2,2,4,2,...] [moebius Mu i for i in 1..]  $\rightarrow$  [1,- 1,- 1,0,- 1,1,...]  $E1(0.01) \rightarrow 4.0379295765381134$  $Gamma(0.01) \rightarrow 99.432585119150588$ 

#### Stream

)set streams calculate 6

ints := [i for i in 1..]  $\rightarrow$  [1,2,3,4,5,6,...] ints.20  $\rightarrow$  20 [i for i in ints | odd? i]  $\rightarrow$  [1,3,5,7,9,11,...] String

creation: s:= "Hello" concatenate "He" "llo"  $\rightarrow$  "Hello" s(1)='H' s.1='H' s(2..3)='e1's(4..)='lo'split("hi there",char " ") → ["hi", "there"] prefix?("He","Hello")  $\rightarrow$  true substring?("ll","Hello",3)  $\rightarrow$  true

## **TwoDimensionalArray**

creation: arr:ARRAY2 INT:=new(2,3,0)  $\rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ nrows arr  $\rightarrow 2$  $ncols arr \rightarrow 3$  $setelt(arr,1,1,17) \rightarrow \begin{bmatrix} 17 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$  $arr(1,1) \rightarrow 17$ 

# Univariate Polynomial

 $q:UP(x,INT):=(1-6*x+9*x^2)^2$ leadingCoefficient p  $\rightarrow$  18 degree  $p \rightarrow 3$ reductum p  $\rightarrow 60x^2 - 46x + 8$  $gcd(p,q) \to 9x^2 - 6x + 1$  $lcm(p.q) \rightarrow 162x^5 + 432x^4 - 756x^3 + 408x^2 - 94x + 8$  $p(2) \rightarrow 300$  (used as function)

creation:  $p:UP(x,INT):=(3*x-1)^2*(2*x+8)$ 

 $D(p) \rightarrow 54x^2 + 120x - 46$  (derivative)

#### Vector

creation:  $v := vector([1,2,3,4,5]) \rightarrow [1,2,3,4,5]$ length:  $\#v \rightarrow 5$ access:  $v.2 \rightarrow 2$ add:  $v+v \rightarrow [2, 4, 6, 8, 10]$ multiply:  $5*v \rightarrow [5, 10, 15, 20, 25]$ assign: v.2 :=  $7 \rightarrow [1, 7, 3, 4, 5]$