axiom™

The 30 Year Horizon

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Volume 10.2: Axiom Algebra: Categories

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Craig Lucas	Michael Lucks	Richard Luczak
Camm Maguire	Dave Mainey	Francois Maltey
William Martin	Ursula Martin	Osni Marques
Alasdair McAndrew	Bob McElrath	Michael McGettrick
Bob McNeill	Edi Meier	Ian Meikle
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Contents

New Foreword

On October 1, 2001 Axiom was withdrawn from the market and ended life as a commercial product. On September 3, 2002 Axiom was released under the Modified BSD license, including this document. On August 27, 2003 Axiom was released as free and open source software available for download from the Free Software Foundation's website, Savannah.

Work on Axiom has had the generous support of the Center for Algorithms and Interactive Scientific Computation (CAISS) at City College of New York. Special thanks go to Dr. Gilbert Baumslag for his support of the long term goal.

The online version of this documentation is roughly 1000 pages. In order to make printed versions we've broken it up into three volumes. The first volume is tutorial in nature. The second volume is for programmers. The third volume is reference material. We've also added a fourth volume for developers. All of these changes represent an experiment in print-on-demand delivery of documentation. Time will tell whether the experiment succeeded.

Axiom has been in existence for over thirty years. It is estimated to contain about three hundred man-years of research and has, as of September 3, 2003, 143 people listed in the credits. All of these people have contributed directly or indirectly to making Axiom available. Axiom is being passed to the next generation. I'm looking forward to future milestones.

With that in mind I've introduced the theme of the "30 year horizon". We must invent the tools that support the Computational Mathematician working 30 years from now. How will research be done when every bit of mathematical knowledge is online and instantly available? What happens when we scale Axiom by a factor of 100, giving us 1.1 million domains? How can we integrate theory with code? How will we integrate theorems and proofs of the mathematics with space-time complexity proofs and running code? What visualization tools are needed? How do we support the conceptual structures and semantics of mathematics in effective ways? How do we support results from the sciences? How do we teach the next generation to be effective Computational Mathematicians?

The "30 year horizon" is much nearer than it appears.

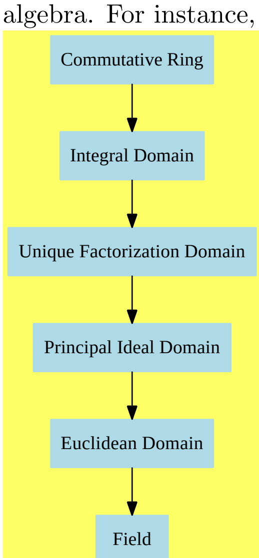
Tim Daly
CAISS, City College of New York
November 10, 2003 ((iHy))

Chapter 1

Categories

Axiom has 3 main algebra components, Categories, Domains, and Packages. If we make an analogy to dressmaking, you can consider the Categories to be hierarchies of properties of things, like patterns, colors, or fabrics. Domains are instances of things based on category choices, such as a dress with a particular style, fabric, color, etc. Packages are tools that work with dresses such as irons, sewing machines, etc.

Axiom is based on abstract algebra and uses it as a scaffolding for constructing well-formed algebra. For instance, in abstract algebra there is a strict subset hierarchy, like:



— **algebrahierarchy.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"Commutative Ring"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=COMRING"];
```

```
"Integral Domain"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=INTDOM"];
"Unique Factorization Domain"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=UFD"];
"Principal Ideal Domain"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=PID"];
"Euclidean Domain"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=EUCDOM"];
"Field"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=FIELD"];

"Commutative Ring" -> "Integral Domain"
"Integral Domain" -> "Unique Factorization Domain"
"Unique Factorization Domain" -> "Principal Ideal Domain"
"Principal Ideal Domain" -> "Euclidean Domain"
"Euclidean Domain" -> "Field"

}
```

Chapter 2

Category Layer 1

In general, we use several colors in the graph images. The “lightblue” color indicates a category that is in the direct inheritance path. The “green” (#00EE00) color indicates a category or domain used in the exports. The “seagreen” (a dark green, indicates a category or domain which is used but does not correspond to the signature of an existing category. The system can infer that this “subsumption node” matches the category. A “yellow” color indicates a domain.

2.0.1 Category (CATEGORY)

Category Laws:

- Left identity: id . f = f
- Right identity: f . id = f
- Associativity: f . (g . h) = (f . g) . h

This is the root of the category hierarchy and is not represented by code.

See:

⇒ “ArcHyperbolicFunctionCategory” (AHYP) 2.0.6 on page 13
⇒ “ArcTrigonometricFunctionCategory” (ATRIG) 2.0.7 on page 16
⇒ “BasicType” (BASTYPE) 2.0.9 on page 27
⇒ “CoercibleTo” (KOERCE) 2.0.14 on page 39
⇒ “CombinatorialFunctionCategory” (CFCAT) 2.0.15 on page 42
⇒ “ConvertibleTo” (KONVERT) 2.0.17 on page 47
⇒ “ElementaryFunctionCategory” (ELEMFUN) 2.0.18 on page 51
⇒ “Eltable” (ELTAB) 2.0.19 on page 54
⇒ “FullyEvalableOver” (FEVALAB) 4.0.67 on page 246

⇒ "HyperbolicFunctionCategory" (HYPCAT) 2.0.21 on page 59
⇒ "InnerEvalable" (IEVALAB) 2.0.22 on page 64
⇒ "Logic" (LOGIC) 3.0.52 on page 168
⇒ "ModularAlgebraicGcdOperations" (MAGCDOC) 2.0.26 on page 74
⇒ "OpenMath" (OM) 2.0.31 on page 88
⇒ "PartialTranscendentalFunctions" (PTRANFN) 2.0.33 on page 94
⇒ "Patternable" (PATAB) 2.0.34 on page 100
⇒ "PrimitiveFunctionCategory" (PRIMCAT) 2.0.35 on page 104
⇒ "RadicalCategory" (RADCAT) 2.0.36 on page 106
⇒ "RetractableTo" (RETRACT) 2.0.37 on page 110
⇒ "SpecialFunctionCategory" (SPFCAT) 2.0.40 on page 119
⇒ "TrigonometricFunctionCategory" (TRIGCAT) 2.0.41 on page 123
⇒ "Type" (TYPE) 2.0.42 on page 127

— **CATEGORY.dotabb** —

```
"CATEGORY"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CATEGORY"];
```

———-

— **CATEGORY.dotfull** —

```
"Category"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CATEGORY"];
```

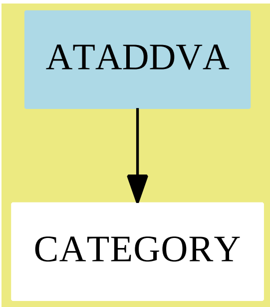
———-

— **CATEGORY.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"Category" [color=lightblue];
}
```

———-

2.0.2 AdditiveValuationAttribute (ATADDVA)



— **AdditiveValuationAttribute.input** —

```
)set break resume
)sys rm -f AdditiveValuationAttribute.output
)spool AdditiveValuationAttribute.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AdditiveValuationAttribute
--R
--R AdditiveValuationAttribute is a category constructor
--R Abbreviation for AdditiveValuationAttribute is ATADDVA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ATADDVA
--R
--E 1
)spool
)lisp (bye)
```

———-

— **AdditiveValuationAttribute.help** —

```
====================================================================
AdditiveValuationAttribute
====================================================================

The class of all euclidean domains such that
euclideanSize(a*b) = euclideanSize(a)*euclideanSize(b)

See Also:
o )show AdditiveValuationAttribute
```

———-

— **AdditiveValuationAttribute.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ATADDVA">
AdditiveValuationAttribute (ATADDVA)</a>
</h2>

The class of all euclidean domains such that<br/>
euclideanSize(a*b)=euclideanSize(a)+euclideanSize(b)
</body>
```

— category ATADDVA AdditiveValuationAttribute —

```
)abbrev category ATADDVA AdditiveValuationAttribute
++ Description:
++ The class of all euclidean domains such that
++ \spad{euclideanSize(a*b) = euclideanSize(a)+euclideanSize(b)}

AdditiveValuationAttribute() : Category == SIG where

  SIG ==> with nil
```

— ATADDVA.dotabb —

```
"ATADDVA"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATADDVA"];
"ATADDVA" -> "CATEGORY"
```

— ATADDVA.dotfull —

```
"AdditiveValuationAttribute()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATADDVA"];
"AdditiveValuationAttribute()" -> "Category"
```

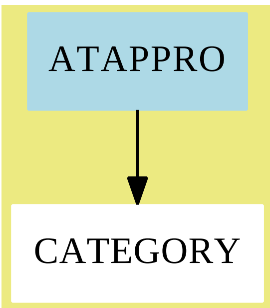
— ATADDVA.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"AdditiveValuationAttribute()" [color=lightblue];
"AdditiveValuationAttribute()" -> "Category"

"Category" [color=lightblue];
```

```
}
```

———-

2.0.3 ApproximateAttribute (ATAPPRO)



— ApproximateAttribute.input —

```
)set break resume
)sys rm -f ApproximateAttribute.output
)spool ApproximateAttribute.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ApproximateAttribute
--R
--R ApproximateAttribute is a category constructor
--R Abbreviation for ApproximateAttribute is ATAPPRO
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ATAPPRO
--R
--E 1
)spool
)lisp (bye)
```

———-

— ApproximateAttribute.help —

```
====================================================================
ApproximateAttribute
====================================================================

An approximation to the real numbers.

See Also:
```

```
o )show ApproximateAttribute
```

———-

— ApproximateAttribute.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ATAPPRO">
ApproximateAttribute (ATAPPRO)</a>
</h2>

An approximation of the real numbers.
</body>
```

———-

— category ATAPPRO ApproximateAttribute —

```
)abbrev category ATAPPRO ApproximateAttribute
++ Description:
++ An approximation to the real numbers.

ApproximateAttribute() : Category == SIG where

  SIG ==> with nil
```

———-

— ATAPPRO.dotabb —

```
"ATAPPRO"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATAPPRO"];
"ATAPPRO" -> "CATEGORY"
```

———-

— ATAPPRO.dotfull —

```
"ApproximateAttribute()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATAPPRO"];
"ApproximateAttribute()" -> "Category"
```

———-

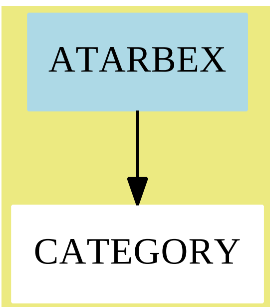
— ATAPPRO.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];
```

```
"ApproximateAttribute()" [color=lightblue];
"ApproximateAttribute()" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.4 ArbitraryExponentAttribute (ATARBEX)



— **ArbitraryExponentAttribute.input** —

```
)set break resume
)sys rm -f ArbitraryExponentAttribute.output
)spool ArbitraryExponentAttribute.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ArbitraryExponentAttribute
--R
--R ArbitraryExponentAttribute is a category constructor
--R Abbreviation for ArbitraryExponentAttribute is ATARBEX
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ATARBEX
--R
--E 1
)spool
)lisp (bye)
```

———-

— **ArbitraryExponentAttribute.help** —

```
====================================================================
ArbitraryExponentAttribute
```

```
====================================================================

Approximate numbers with arbitrarily large exponents

See Also:
o )show ArbitraryExponentAttribute
```

———-

— **ArbitraryExponentAttribute.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ATARBEX">
ArbitraryExponentAttribute (ATARBEX)</a>
</h2>

Approximate numbers with arbitrarily large exponents.
</body>
```

———-

— **category ATARBEX ArbitraryExponentAttribute** —

```
)abbrev category ATARBEX ArbitraryExponentAttribute
++ Description:
++ Approximate numbers with arbitrarily large exponents

ArbitraryExponentAttribute() : Category == SIG where

  SIG ==> with nil
```

———-

— **ATARBEX.dotabb** —

```
"ATARBEX"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATARBEX"];
"ATARBEX" -> "CATEGORY"
```

———-

— **ATARBEX.dotfull** —

```
"ArbitraryExponentAttribute()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATARBEX"];
"ArbitraryExponentAttribute()" -> "Category"
```

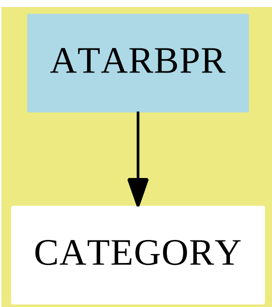
———-

— ATARBEX.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"ArbitraryExponentAttribute()" [color=lightblue];
"ArbitraryExponentAttribute()" -> "Category"

"Category" [color=lightblue];
}
```

———

2.0.5 ArbitraryPrecisionAttribute (ATARBPR)



— ArbitraryPrecisionAttribute.input —

```
)set break resume
)sys rm -f ArbitraryPrecisionAttribute.output
)spool ArbitraryPrecisionAttribute.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ArbitraryPrecisionAttribute
--R
--R ArbitraryPrecisionAttribute is a category constructor
--R Abbreviation for ArbitraryPrecisionAttribute is ATARBPR
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ATARBPR
--R
--E 1
)spool
)lisp (bye)
```

———

— ArbitraryPrecisionAttribute.help —

```
====================================================================
ArbitraryPrecisionAttribute
====================================================================

Approximate numbers for which the user can set the precision
for subsequent calculations.

See Also:
o )show ArbitraryPrecisionAttribute
```

— ArbitraryPrecisionAttribute.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ATARBPR">
ArbitraryPrecisionAttribute (ATARBPR)</a>
</h2>

Approximate numbers for which the user can set the precision
for subsequent calculations.
</body>
```

— category ATARBPR ArbitraryPrecisionAttribute —

```
)abbrev category ATARBPR ArbitraryPrecisionAttribute
++ Description:
++ Approximate numbers for which the user can set the precision
++ for subsequent calculations.

ArbitraryPrecisionAttribute() : Category == SIG where

  SIG ==> with nil
```

— ATARBPR.dotabb —

```
"ATARBPR"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATARBPR"];
"ATARBPR" -> "CATEGORY"
```

— ATARBPR.dotfull —

```
"ArbitraryPrecisionAttribute()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATARBPR"];
"ArbitraryPrecisionAttribute()" -> "Category"
```

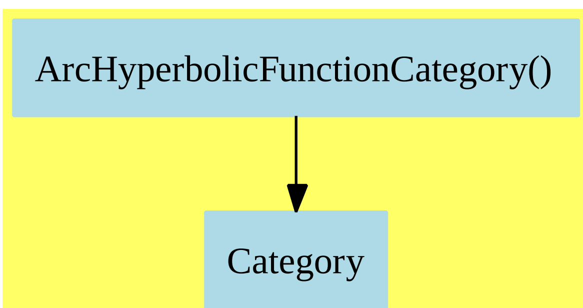
———-

— **ATARBPR.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"ArbitraryPrecisionAttribute()" [color=lightblue];
"ArbitraryPrecisionAttribute()" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.6 ArcHyperbolicFunctionCategory (AHYP)



— **ArcHyperbolicFunctionCategory.input** —

```
)set break resume
)sys rm -f ArcHyperbolicFunctionCategory.output
)spool ArcHyperbolicFunctionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ArcHyperbolicFunctionCategory
--R
--R ArcHyperbolicFunctionCategory is a category constructor
--R Abbreviation for ArcHyperbolicFunctionCategory is AHYP
--R This constructor is exposed in this frame.
```

```
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for AHYP
--R
--R------------------------------- Operations --------------------------------
--R acosh : % -> %                         acoth : % -> %
--R acsch : % -> %                         asech : % -> %
--R asinh : % -> %                         atanh : % -> %
--R
--E 1

)spool
)lisp (bye)
```

— **ArcHyperbolicFunctionCategory.help** —

```
====================================================================
ArcHyperbolicFunctionCategory examples
====================================================================

This is the Category for the inverse hyperbolic trigonometric functions

See Also:
o )show ArcHyperbolicFunctionCategory
```

See:

⇒ "TranscendentalFunctionCategory" (TRANFUN) 3.0.58 on page 192
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

acosh acoth acsch asech asinh atanh

These are directly exported but not implemented:

```
acosh : % -> %
acoth : % -> %
acsch : % -> %
asech : % -> %
asinh : % -> %
atanh : % -> %
```

— **ArcHyperbolicFunctionCategory.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#AHYP">
ArcHyperbolicFunctionCategory (AHYP)</a>
</h2>

Category for the inverse hyperbolic trigonometric functions.
```

```
<br/><br/><br/>

acosh : % -&gt; %<br/>
  acosh(x) returns the hyperbolic arc-cosine of x.<br/><br/>

acoth : % -&gt; %<br/>
  acoth(x) returns the hyperbolic arc-cotangent of x.<br/><br/>

acsch : % -&gt; %<br/>
  acsch(x) returns the hyperbolic arc-cosecant of x.<br/><br/>

asech : % -&gt; %<br/>
  asech(x) returns the hyperbolic arc-secant of x.<br/><br/>

asinh : % -&gt; %<br/>
  asinh(x) returns the hyperbolic arc-sine of x.<br/><br/>

atanh : % -&gt; %<br/>
  atanh(x) returns the hyperbolic arc-tangent of x.<br/><br/>

</body>
```

— category AHYP ArcHyperbolicFunctionCategory —

```
)abbrev category AHYP ArcHyperbolicFunctionCategory
++ Date Last Updated: 14 May 1991
++ Description:
++ Category for the inverse hyperbolic trigonometric functions;

ArcHyperbolicFunctionCategory() : Category == SIG where

  SIG ==> with

    acosh : $ -> $
      ++ acosh(x) returns the hyperbolic arc-cosine of x.

    acoth : $ -> $
      ++ acoth(x) returns the hyperbolic arc-cotangent of x.

    acsch : $ -> $
      ++ acsch(x) returns the hyperbolic arc-cosecant of x.

    asech : $ -> $
      ++ asech(x) returns the hyperbolic arc-secant of x.

    asinh : $ -> $
      ++ asinh(x) returns the hyperbolic arc-sine of x.

    atanh : $ -> $
      ++ atanh(x) returns the hyperbolic arc-tangent of x.
```

— **AHYP.dotabb** —

```
"AHYP"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=AHYP"];
"AHYP" -> "CATEGORY"
```

————

— **AHYP.dotfull** —

```
"ArcHyperbolicFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=AHYP"];
"ArcHyperbolicFunctionCategory()" -> "Category"
```

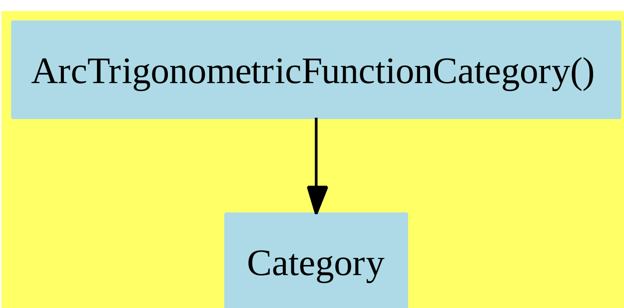
————

— **AHYP.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"ArcHyperbolicFunctionCategory()" [color=lightblue];
"ArcHyperbolicFunctionCategory()" -> "Category"

"Category" [color=lightblue];
}
```

————

2.0.7 ArcTrigonometricFunctionCategory (ATRIG)



The `asec` and `acsc` functions were modified to include an intermediate test to check that the argument has a reciprocal values.

— **ArcTrigonometricFunctionCategory.input** —

```
)set break resume
)sys rm -f ArcTrigonometricFunctionCategory.output
)spool ArcTrigonometricFunctionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ArcTrigonometricFunctionCategory
--R
--R ArcTrigonometricFunctionCategory is a category constructor
--R Abbreviation for ArcTrigonometricFunctionCategory is ATRIG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ATRIG
--R
--R------------------------------- Operations --------------------------------
--R acos : % -> %                         acot : % -> %
--R acsc : % -> %                         asec : % -> %
--R asin : % -> %                         atan : % -> %
--R
--E 1

)spool
)lisp (bye)
```

— **ArcTrigonometricFunctionCategory.help** —

```
====================================================================
ArcTrigonometricFunctionCategory examples
====================================================================

This is the Category for the inverse trigonometric functions

See Also:
o )show ArcTrigonometricFunctionCategory
```

See:

⇒ "TranscendentalFunctionCategory" (TRANFUN) 3.0.58 on page 192
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

acos acot acsc asec asin atan

These are directly exported but not implemented:

```
acos : % -> %
acot : % -> %
asin : % -> %
atan : % -> %
```

These are implemented by this category:

```
acsc : % -> %
asec : % -> %
```

— ArcTrigonometricFunctionCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ATRIG">
ArcTrigonometricFunctionCategory (ATRIG)</a>
</h2>

Category for the inverse trigonometric functions;
<br/><br/><br/>

acos : % -> %<br/>
  acos(x) returns the arc-cosine of x. When evaluated<br/>
  into some subset of the complex numbers, one<br/>
  branch cut for acos lies along the negative real axis<br/>
  to the left of -1 (inclusive), continuous with the<br/>
  upper half plane, the other along the positive real axis to<br/>
  the right of 1 (inclusive), continuous with the lower half<br/>
  plane.<br/><br/>

acot : % -> %<br/>
  acot(x) returns the arc-cotangent of x.<br/><br/>

acsc : % -> %<br/>
  acsc(x) returns the arc-cosecant of x.<br/><br/>

asec : % -> %<br/>
  asec(x) returns the arc-secant of x.<br/><br/>

asin : % -> %<br/>
  asin(x) returns the arc-sine of x. When evaluated into some<br/>
  subset of the complex numbers, one branch cut for asin lies<br/>
  along the negative real axis to the left of -1 (inclusive),<br/>
  continuous with the upper half plane, the other along the<br/>
  positive real axis to the right of 1 (inclusive), continuous<br/>
  with the lower half plane.<br/><br/>

atan : % -> %<br/>
  atan(x) returns the arc-tangent of x. When evaluated into some<br/>
  subset of the complex numbers, one branch cut for atan lies<br/>
  along the positive imaginary axis above %i (exclusive),<br/>
  continuous with the left half plane, the other along the<br/>
  negative imaginary axis below -%i (exclusive) continuous<br/>
  with the right half plane. The domain does not contain %i<br/>
  and -%i<br/><br/>

</body>
```

———

— category ATRIG ArcTrigonometricFunctionCategory —

```
)abbrev category ATRIG ArcTrigonometricFunctionCategory
++ Date Last Updated: 14 May 1991
++ Description:
++ Category for the inverse trigonometric functions;

ArcTrigonometricFunctionCategory() : Category == SIG where

  SIG ==> with

    acos : $ -> $
      ++ acos(x) returns the arc-cosine of x. When evaluated
      ++ into some subset of the complex numbers, one
      ++ branch cut for acos lies along the negative real axis
      ++ to the left of -1 (inclusive), continuous with the
      ++ upper half plane, the other along the positive real axis to
      ++ the right of 1 (inclusive), continuous with the lower half
      ++ plane.

    acot : $ -> $
      ++ acot(x) returns the arc-cotangent of x.

    acsc : $ -> $
      ++ acsc(x) returns the arc-cosecant of x.

    asec : $ -> $
      ++ asec(x) returns the arc-secant of x.

    asin : $ -> $
      ++ asin(x) returns the arc-sine of x. When evaluated into some
      ++ subset of the complex numbers, one branch cut for asin lies
      ++ along the negative real axis to the left of -1 (inclusive),
      ++ continuous with the upper half plane, the other along the
      ++ positive real axis to the right of 1 (inclusive), continuous
      ++ with the lower half plane.

    atan : $ -> $
      ++ atan(x) returns the arc-tangent of x. When evaluated into some
      ++ subset of the complex numbers, one branch cut for atan lies
      ++ along the positive imaginary axis above %i (exclusive),
      ++ continuous with the left half plane, the other along the
      ++ negative imaginary axis below -%i (exclusive) continuous
      ++ with the right half plane. The domain does not contain %i and -%i

   add

     if $ has Ring then

       asec(x) ==
         (a := recip x) case "failed" => error "asec: no reciprocal"
         acos(a::$)
```

```
    acsc(x) ==
      (a := recip x) case "failed" => error "acsc: no reciprocal"
      asin(a::$)
```

———-

— **ATRIG.dotabb** —

```
"ATRIG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATRIG"];
"ATRIG" -> "CATEGORY"
```

———-

— **ATRIG.dotfull** —

```
"ArcTrigonometricFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATRIG"];
"ArcTrigonometricFunctionCategory()" -> "Category"
```

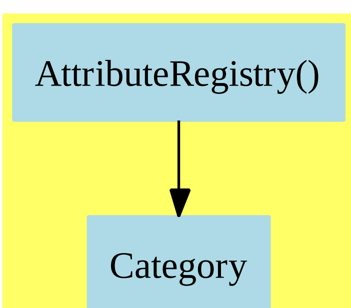
———-

— **ATRIG.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"ArcTrigonometricFunctionCategory()" [color=lightblue];
"ArcTrigonometricFunctionCategory()" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.8 AttributeRegistry (ATTREG)



— AttributeRegistry.input —

```
)set break resume
)sys rm -f AttributeRegistry.output
)spool AttributeRegistry.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AttributeRegistry
--R
--R AttributeRegistry is a category constructor
--R Abbreviation for AttributeRegistry is ATTREG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ATTREG
--R
--E 1

)spool
)lisp (bye)
```

— AttributeRegistry.help —

```
====================================================================
AttributeRegistry examples
====================================================================

This category exports the attributes in the AXIOM Library.

See Also:
o )show BasicType
```

See:

⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports: Nothing

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **arbitraryPrecision** means the user can set the precision for subsequent calculations.
- **partiallyOrderedSet** is true if a set with $<$ which is transitive, but not($a < b$ or $a = b$) does not necessarily imply $b < a$.
- **central** is true if, given an algebra over a ring R, the image of R is the center of the algebra, that is, the set of members of the algebra which commute with all others is precisely the image of R in the algebra.
- **noetherian** is true if all of its ideals are finitely generated.
- **additiveValuation** implies `euclideanSize(a*b)=euclideanSize(a)+euclideanSize(b)`.
- **multiplicativeValuation** implies `euclideanSize(a*b)=euclideanSize(a)*euclideanSize(b)`.
- **NullSquare** means that $[x, x] = 0$ holds. See `LieAlgebra`.
- **JacobiIdentity** means that $[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0$ holds. See `LieAlgebra`.
- **canonical** is true if and only if distinct elements have distinct data structures. For example, a domain of mathematical objects which has the `canonical` attribute means that two objects are mathematically equal if and only if their data structures are equal.
- **approximate** means "is an approximation to the real numbers".
- **complex** means that this domain has $\sqrt{-1}$

— AttributeRegistry.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ATTREG">
AttributeRegistry (ATTREG)</a>
</h2>

This category exports the attributes in the AXIOM Library.
<br/><br/><br/>

finiteAggregate<br/>
  finiteAggregate is true if it is an aggregate with a <br/>
  finite number of elements.<br/><br/>

commutative("*")<br/>
```

```
  commutative("*") is true if it has an operation<br/>
  "*": (D,D) -> D which is commutative.<br/><br/>

shallowlyMutable<br/>
  shallowlyMutable is true if its values<br/>
  have immediate components that are updateable (mutable).<br/>
  Note that the properties of any component domain are<br/>
  irrelevant to the \spad{shallowlyMutable} proper.<br/><br/>

unitsKnown<br/>
  unitsKnown is true if a monoid (a multiplicative semigroup <br/>
  with a 1) has unitsKnown means that<br/>
  the operation recip can only return "failed" <br/>
  if its argument is not a unit.<br/><br/>

leftUnitary<br/>
  leftUnitary is true if 1 * x = x for all x.<br/><br/>

rightUnitary<br/>
  rightUnitary is true if x * 1 = x for all x.<br/><br/>

noZeroDivisors<br/>
  noZeroDivisors is true if x * y ~= 0 implies <br/>
  both x and y are non-zero.<br/><br/>

canonicalUnitNormal<br/>
  canonicalUnitNormal is true if we can choose a canonical<br/>
  representative for each class of associate elements, that is<br/>
  associates?(a,b) returns true if and only if <br/>
  unitCanonical(a) = unitCanonical(b).<br/><br/>

canonicalsClosed<br/>
  canonicalsClosed is true if <br/>
  unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b).<br/><br/>

arbitraryPrecision<br/>
  arbitraryPrecision means the user can set the <br/>
  precision for subsequent calculations.<br/><br/>

partiallyOrderedSet<br/>
  partiallyOrderedSet is true if<br/>
  a set with &lt; which is transitive, <br/>
  but not(a &lt; b or a = b)<br/>
  does not necessarily imply b &lt; a.<br/><br/>

central<br/>
  central is true if, given an algebra over a ring R,<br/>
  the image of R is the center of the algebra, For example,<br/>
  the set of members of the algebra which commute with all<br/>
  others is precisely the image of R in the algebra.<br/><br/>

noetherian<br/>
  noetherian is true if all of its ideals are<br/>
  finitely generated.<br/><br/>
```

```
additiveValuation<br/>
  additiveValuation implies<br/>
  euclideanSize(a*b)=euclideanSize(a)+euclideanSize(b).<br/><br/>

multiplicativeValuation<br/>
  multiplicativeValuation implies<br/>
  euclideanSize(a*b)=euclideanSize(a)*euclideanSize(b).<br/><br/>

NullSquare<br/>
  NullSquare means that [x,x] = 0 holds.<br/>
  See LieAlgebra.<br/><br/>

JacobiIdentity<br/>
  JacobiIdentity means that <br/>
  [x,[y,z]]+[y,[z,x]]+[z,[x,y]] = 0 holds.<br/>
  See LieAlgebra.<br/><br/>

canonical<br/>
  canonical is true if and only if distinct elements have <br/>
  distinct data structures. For example, a domain of mathematical <br/>
  objects which has the canonical attribute means that two <br/>
  objects are mathematically equal if and only if their data <br/>
  structures are equal.<br/><br/>

approximate<br/>
  approximate means "is an approximation to the <br/>
  real numbers".<br/><br/>

</body>
```

— category ATTREG AttributeRegistry —

```
)abbrev category ATTREG AttributeRegistry
++ Description:
++ This category exports the attributes in the AXIOM Library

AttributeRegistry() : Category == SIG where

  SIG ==> with

    finiteAggregate
      ++ \spad{finiteAggregate} is true if it is an aggregate with a
      ++ finite number of elements.

    commutative("*")
      ++ \spad{commutative("*")} is true if it has an operation
      ++ \spad{"*": (D,D) -> D} which is commutative.

    shallowlyMutable
      ++ \spad{shallowlyMutable} is true if its values
      ++ have immediate components that are updateable (mutable).
```

```
  ++ Note that the properties of any component domain are
  ++ irrevelant to the \spad{shallowlyMutable} proper.

unitsKnown
  ++ \spad{unitsKnown} is true if a monoid (a multiplicative semigroup
  ++ with a 1) has \spad{unitsKnown} means that
  ++ the operation \spadfun{recip} can only return "failed"
  ++ if its argument is not a unit.

leftUnitary
  ++ \spad{leftUnitary} is true if \spad{1 * x = x} for all x.

rightUnitary
  ++ \spad{rightUnitary} is true if \spad{x * 1 = x} for all x.

noZeroDivisors
  ++ \spad{noZeroDivisors} is true if \spad{x * y \~~= 0} implies
  ++ both x and y are non-zero.

canonicalUnitNormal
  ++ \spad{canonicalUnitNormal} is true if we can choose a canonical
  ++ representative for each class of associate elements, that is
  ++ \spad{associates?(a,b)} returns true if and only if
  ++ \spad{unitCanonical(a) = unitCanonical(b)}.

canonicalsClosed
  ++ \spad{canonicalsClosed} is true if
  ++ \spad{unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)}.

arbitraryPrecision
  ++ \spad{arbitraryPrecision} means the user can set the
  ++ precision for subsequent calculations.

partiallyOrderedSet
  ++ \spad{partiallyOrderedSet} is true if
  ++ a set with \spadop{<} which is transitive,
  ++ but \spad{not(a < b or a = b)}
  ++ does not necessarily imply \spad{b<a}.

central
  ++ \spad{central} is true if, given an algebra over a ring R,
  ++ the image of R is the center of the algebra, For example,
  ++ the set of members of the algebra which commute with all
  ++ others is precisely the image of R in the algebra.

noetherian
  ++ \spad{noetherian} is true if all of its ideals are
  ++ finitely generated.

additiveValuation
  ++ \spad{additiveValuation} implies
  ++ \spad{euclideanSize(a*b)=euclideanSize(a)+euclideanSize(b)}.

multiplicativeValuation
```

```
      ++ \spad{multiplicativeValuation} implies
      ++ \spad{euclideanSize(a*b)=euclideanSize(a)*euclideanSize(b)}.

    NullSquare
      ++ \axiom{NullSquare} means that \axiom{[x,x] = 0} holds.
      ++ See \axiomType{LieAlgebra}.

    JacobiIdentity
      ++ \axiom{JacobiIdentity} means that
      ++ \axiom{[x,[y,z]]+[y,[z,x]]+[z,[x,y]] = 0} holds.
      ++ See \axiomType{LieAlgebra}.

    canonical
      ++ \spad{canonical} is true if and only if distinct elements have
      ++ distinct data structures. For example, a domain of mathematical
      ++ objects  which has the \spad{canonical} attribute means that two
      ++ objects are mathematically equal if and only if their data
      ++ structures are equal.

    approximate
      ++ \spad{approximate} means "is an approximation to the
      ++ real numbers".
```

———-

— ATTREG.dotabb —

```
"ATTREG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATTREG"];
"ATTREG" -> "CATEGORY"
```

———-

— ATTREG.dotfull —

```
"AttributeRegistry()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATTREG"];
"AttributeRegistry()" -> "Category"
```

———-

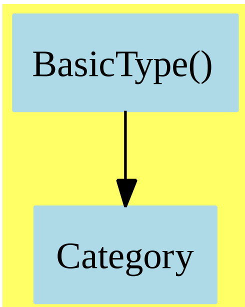
— ATTREG.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"AttributeRegistry()" [color=lightblue];
"AttributeRegistry()" -> "Category"
```

```
"Category" [color=lightblue];
}
```

———-

2.0.9 BasicType (BASTYPE)



— **BasicType.input** —

```
)set break resume
)sys rm -f BasicType.output
)spool BasicType.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BasicType
--R
--R BasicType is a category constructor
--R Abbreviation for BasicType is BASTYPE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for BASTYPE
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

———-

— **BasicType.help** —

```
====================================================================
BasicType examples
====================================================================
```

```
BasicType is the basic category for describing a collection
of elements with = (equality).

See Also:
o )show BasicType
```

———-

See:

⇒ "BlowUpMethodCategory" (BLMETCT) 4.0.60 on page 206
⇒ "SetCategory" (SETCAT) 3.0.57 on page 187
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

?=? ?~=?

These are directly exported but not implemented:

```
?=? : (%,%) -> Boolean
```

These are implemented by this category:

```
?~=? : (%,%) -> Boolean
```

— **BasicType.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#BASTYPE">
BasicType (BASTYPE)</a>
</h2>

BasicType is the basic category for describing a collection
of elements with = (equality).
<br/><br/><br/>

"=" : (%,%) -> Boolean<br/>
  x=y tests if x and y are equal.<br/><br/>

"~=" : (%,%) -> Boolean<br/>
  x~=y tests if x and y are not equal.<br/><br/>

</body>
```

———-

— **category BASTYPE BasicType** —

```
)abbrev category BASTYPE BasicType
--% BasicType
++ Description:
++ BasicType is the basic category for describing a collection
++ of elements with = (equality).
```

```
BasicType() : Category == SIG where

  SIG ==> with

    "=" : (%,%) -> Boolean
      ++ x=y tests if x and y are equal.

    "~=" : (%,%) -> Boolean
      ++ x~=y tests if x and y are not equal.

   add

     _~_=(x:%,y:%) : Boolean == not(x=y)
```

———-

— **COQ BASTYPE** —

```
(* category BASTYPE *)
(* From the Coq.Init.Logic library we know that
     Definition not (A:Prop) := A -> False
   and
     Notation "~ x" := (not x) : type_scope. *)

(*
      ~=: (%,%) -> Boolean
      ~=(x:%,y:%) : Boolean == not(x=y)
*)
```

———-

— **BASTYPE.dotabb** —

```
"BASTYPE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BASTYPE"];
"BASTYPE" -> "CATEGORY"
```

———-

— **BASTYPE.dotfull** —

```
"BasicType()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BASTYPE"];
"BasicType()" -> "Category"
```

———-

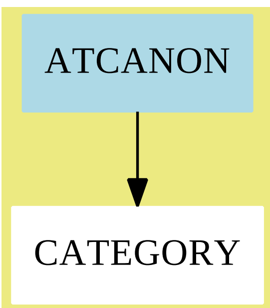
— **BASTYPE.dotpic** —

```
digraph pic {
```

```
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.10 CanonicalAttribute (ATCANON)



— CanonicalAttribute.input —

```
)set break resume
)sys rm -f CanonicalAttribute.output
)spool CanonicalAttribute.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show CanonicalAttribute
--R
--R CanonicalAttribute is a category constructor
--R Abbreviation for CanonicalAttribute is ATCANON
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ATCANON
--R
--E 1
)spool
)lisp (bye)
```

———-

— CanonicalAttribute.help —

```
====================================================================
CanonicalAttribute
====================================================================

The class of all domains which have canonical represenntation,
that is, mathematically equal elements ahve the same data structure.

See Also:
o )show CanonicalAttribute
```

———-

— CanonicalAttribute.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ATCANON">
CanonicalAttribute (ATCANON)</a></h2>

The class of all domains which have canonical represenntation,
that is, mathematically equal elements have the same data structure.
</body>
```

———-

— category ATCANON CanonicalAttribute —

```
)abbrev category ATCANON CanonicalAttribute
++ Description:
++ The class of all domains which have canonical represenntation,
++ that is, mathematically equal elements have the same data structure.

CanonicalAttribute() : Category == SIG where

  SIG ==> with nil
```

———-

— ATCANON.dotabb —

```
"ATCANON"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATCANON"];
"ATCANON" -> "CATEGORY"
```

———-

— ATCANON.dotfull —

```
"CanonicalAttribute()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATCANON"];
"CanonicalAttribute()" -> "Category"
```

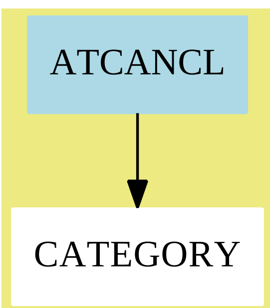
———-

— **ATCANON.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"CanonicalAttribute()" [color=lightblue];
"CanonicalAttribute()" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.11 CanonicalClosedAttribute (ATCANCL)



— **CanonicalClosedAttribute.input** —

```
)set break resume
)sys rm -f CanonicalClosedAttribute.output
)spool CanonicalClosedAttribute.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show CanonicalClosedAttribute
--R
--R CanonicalClosedAttribute is a category constructor
--R Abbreviation for CanonicalClosedAttribute is ATCANCL
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ATCANCL
--R
--E 1
)spool
)lisp (bye)
```

— **CanonicalClosedAttribute.help** —

```
====================================================================
CanonicalClosedAttribute
====================================================================

The class of all integral domains such that
unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)

See Also:
o )show CanonicalClosedAttribute
```

— **CanonicalClosedAttribute.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ATCANCL">
CanonicalClosedAttribute (ATCANCL)</a></h2>

The class of all integral domains such that <br/>
unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)

</body>
```

— **category ATCANCL CanonicalClosedAttribute** —

```
)abbrev category ATCANCL CanonicalClosedAttribute
++ Description:
++ The class of all integral domains such that
++ \spad{unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)}

CanonicalClosedAttribute() : Category == SIG where

  SIG ==> with nil
```

— **ATCANCL.dotabb** —

```
"ATCANCL"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATCANCL"];
"ATCANCL" -> "CATEGORY"
```

— ATCANCL.dotfull —

```
"CanonicalClosedAttribute()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATCANCL"];
"CanonicalClosedAttribute()" -> "Category"
```

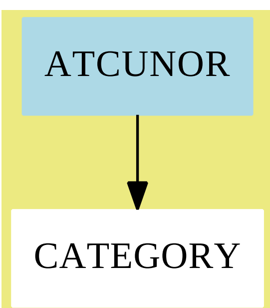
————

— ATCANCL.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"CanonicalClosedAttribute()" [color=lightblue];
"CanonicalClosedAttribute()" -> "Category"

"Category" [color=lightblue];
}
```

————

2.0.12 CanonicalUnitNormalAttribute (ATCUNOR)



— CanonicalUnitNormalAttribute.input —

```
)set break resume
)sys rm -f CanonicalUnitNormalAttribute.output
)spool CanonicalUnitNormalAttribute.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show CanonicalUnitNormalAttribute
--R
--R CanonicalUnitNormalAttribute is a category constructor
--R Abbreviation for CanonicalUnitNormalAttribute is ATCUNOR
```

```
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ATCUNOR
--R
--E 1
)spool
)lisp (bye)
```

— CanonicalUnitNormalAttribute.help —

```
====================================================================
CanonicalUnitNormalAttribute
====================================================================

The class of all integral domains such that we can choose a canonical
representative for each class of associate elements. That is,
associates?(a,b) returns true if and only if
unitCanonical(a) = unitCanonical(b)

See Also:
o )show CanonicalUnitNormalAttribute
```

— CanonicalUnitNormalAttribute.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ATCUNOR">
CanonicalUnitNormalAttribute (ATCUNOR)</a></h2>

The class of all integral domains such that we can choose a canonical
representative for each class of associate elements. That is,
associates?(a,b) returns true if and only if<br/>
unitCanonical(a) = unitCanonical(b)

</body>
```

— category ATCUNOR CanonicalUnitNormalAttribute —

```
)abbrev category ATCUNOR CanonicalUnitNormalAttribute
++ Description:
++ The class of all integral domains such that we can choose a canonical
++ representative for each class of associate elements. That is,
++ \spad{associates?(a,b)} returns true if and only if
++ \spad{unitCanonical(a)} = \spad{unitCanonical(b)}

CanonicalUnitNormalAttribute() : Category == SIG where
```

```
  SIG ==> with nil
```

———-

— ATCUNOR.dotabb —

```
"ATCUNOR"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATCUNOR"];
"ATCUNOR" -> "CATEGORY"
```

———-

— ATCUNOR.dotfull —

```
"CanonicalUnitNormalAttribute()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATCUNOR"];
"CanonicalUnitNormalAttribute()" -> "Category"
```

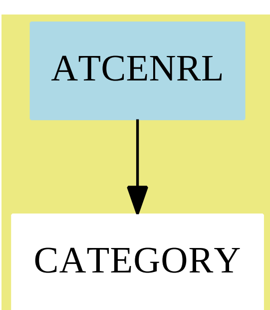
———-

— ATCUNOR.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"CanonicalUnitNormalAttribute()" [color=lightblue];
"CanonicalUnitNormalAttribute()" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.13 CentralAttribute (ATCENRL)



— **CentralAttribute.input** —

```
)set break resume
)sys rm -f CentralAttribute.output
)spool CentralAttribute.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show CentralAttribute
--R
--R CentralAttribute is a category constructor
--R Abbreviation for CentralAttribute is ATCENRL
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ATCENRL
--R
--E 1
)spool
)lisp (bye)
```

———

— **CentralAttribute.help** —

```
====================================================================
CentralAttribute
====================================================================

Central is true if, given an algebra over a ring R, the image of R
is the center of the algebra. For example, the set of members of the
algebra which commute with all others is precisely the image of R
in the algebra.

See Also:
o )show CentralAttribute
```

———

— **CentralAttribute.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ATCENRL">
CentralAttribute (ATCENRL)</a></h2>

Central is true if, given an algebra over a ring R, the image of R
is the center of the algebra. For example, the set of members of the
algebra which commute with all others is precisely the image of R
in the algebra.

</body>
```

———-

— category ATCENRL CentralAttribute —

```
)abbrev category ATCENRL CentralAttribute
++ Description:
++ Central is true if, given an algebra over a ring R, the image of R
++ is the center of the algebra. For example, the set of members of the
++ algebra which commute with all others is precisely the image of R
++ in the algebra.

CentralAttribute() : Category == SIG where

  SIG ==> with nil
```

———-

— ATCENRL.dotabb —

```
"ATCENRL"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATCENRL"];
"ATCENRL" -> "CATEGORY"
```

———-

— ATCENRL.dotfull —

```
"CentralAttribute()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATCENRL"];
"CentralAttribute()" -> "Category"
```

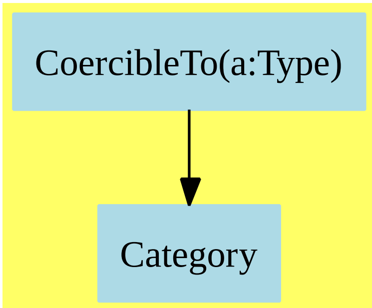
———-

— ATCENRL.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"CentralAttribute()" [color=lightblue];
"CentralAttribute()" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.14 CoercibleTo (KOERCE)



— **CoercibleTo.input** —

```
)set break resume
)sys rm -f CoercibleTo.output
)spool CoercibleTo.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show CoercibleTo
--R
--R CoercibleTo(S: Type) is a category constructor
--R Abbreviation for CoercibleTo is KOERCE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for KOERCE
--R
--R------------------------------- Operations --------------------------------
--R coerce : % -> S
--R
--E 1

)spool
)lisp (bye)
```

———-

— **CoercibleTo.help** —

```
====================================================================
CoercibleTo examples
====================================================================

A is coercible to B means any element of A can automatically be
converted into an element of B by the interpreter.

See Also:
o )show CoercibleTo
```

———-

See:

⇒ "BlowUpMethodCategory" (BLMETCT) 4.0.60 on page 206
⇒ "DirectProductCategory" (DIRPCAT) 12.0.184 on page 1173
⇒ "FortranProgramCategory" (FORTCAT) 3.0.49 on page 155
⇒ "PlottablePlaneCurveCategory" (PPCURVE) 3.0.53 on page 172
⇒ "PlottableSpaceCurveCategory" (PSCURVE) 3.0.54 on page 176
⇒ "PolynomialSetCategory" (PSETCAT) 6.0.115 on page 598
⇒ "SetCategory" (SETCAT) 3.0.57 on page 187
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

coerce

This is directly exported but not implemented:

```
coerce : % -> S
```

— **CoercibleTo.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#KOERCE">
CoercibleTo (KOERCE)</a></h2>

A is coercible to B means any element of A can automatically be
converted into an element of B by the interpreter.
<br/><br/><br/>

coerce : % -> S<br/>
  coerce(a) transforms a into an element of S.

</body>
```

———-

— **category KOERCE CoercibleTo** —

```
)abbrev category KOERCE CoercibleTo
++ Author: Manuel Bronstein
++ Date Last Updated: 14 May 1991
++ Description:
++ A is coercible to B means any element of A can automatically be
++ converted into an element of B by the interpreter.

CoercibleTo(S) : Category == SIG where
  S : Type

  SIG ==> with

    coerce : % -> S
```

```
    ++ coerce(a) transforms a into an element of S.
```

———-

— **KOERCE.dotabb** —

```
"KOERCE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=KOERCE"];
"KOERCE" -> "CATEGORY"
```

———-

— **KOERCE.dotfull** —

```
"CoercibleTo(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=KOERCE"];
"CoercibleTo(a:Type)" -> "Category"

"CoercibleTo(OutputForm)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KOERCE"];
"CoercibleTo(OutputForm)" ->
    "CoercibleTo(a:Type)"

"CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet()))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KOERCE"];
"CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet()))"
  -> "CoercibleTo(a:Type)"
```

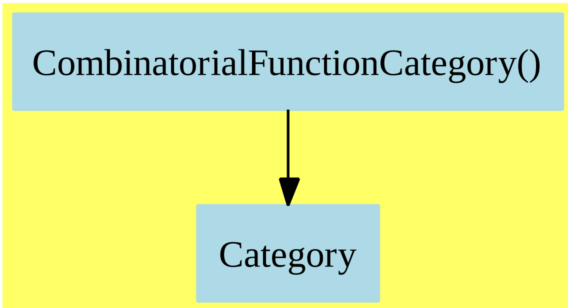
———-

— **KOERCE.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.15 CombinatorialFunctionCategory (CFCAT)



— **CombinatorialFunctionCategory.input** —

```
)set break resume
)sys rm -f CombinatorialFunctionCategory.output
)spool CombinatorialFunctionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show CombinatorialFunctionCategory
--R
--R CombinatorialFunctionCategory is a category constructor
--R Abbreviation for CombinatorialFunctionCategory is CFCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for CFCAT
--R
--R------------------------------- Operations --------------------------------
--R binomial : (%,%) -> %                 factorial : % -> %
--R permutation : (%,%) -> %
--R
--E 1

)spool
)lisp (bye)
```

— **CombinatorialFunctionCategory.help** —

```
====================================================================
CombinatorialFunctionCategory examples
====================================================================

This is the Category for the usual combinatorial functions

See Also:
o )show CombinatorialFunctionCategory
```

See:

⇒ "CombinatorialOpsCategory" (COMBOPC) 3.0.45 on page 138
⇒ "IntegerNumberSystem" (INS) 16.0.210 on page 1420
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

binomial factorial permutation

These are directly exported but not implemented:

```
binomial : (%,%) -> %
factorial : % -> %
permutation : (%,%) -> %
```

— CombinatorialFunctionCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#CFCAT">
CombinatorialFunctionCategory (CFCAT)</a></h2>

Category for the usual combinatorial functions.
<br/><br/><br/>

binomial : (%, %) -> %<br/>
  binomial(n,r) returns the (n,r) binomial coefficient<br/>
  (often denoted in the literature by C(n,r)).<br/>
  Note that C(n,r) = n!/(r!(n-r)!) where n &gt;= r &gt;= 0}.<br/>
 <br/>
  [binomial(5,i) for i in 0..5]<br/><br/>

factorial : % -> %<br/>
  factorial(n) computes the factorial of n<br/>
  (denoted in the literature by n!)<br/>
  Note that n! = n (n-1)! when n &gt; 0; also, 0! = 1.<br/><br/>

permutation : (%, %) -> %<br/>
  permutation(n, m) returns the number of<br/>
  permutations of n objects taken m at a time.<br/>
  Note that permutation(n,m) = n!/(n-m)!.<br/>

</body>
```

— category CFCAT CombinatorialFunctionCategory —

```
)abbrev category CFCAT CombinatorialFunctionCategory
++ Author: Manuel Bronstein
++ Date Last Updated: 14 May 1991
++ Description:
```

```
++ Category for the usual combinatorial functions;

CombinatorialFunctionCategory() : Category == SIG where

  SIG ==> with

    binomial : ($, $) -> $
      ++ binomial(n,r) returns the \spad{(n,r)} binomial coefficient
      ++ (often denoted in the literature by \spad{C(n,r)}).
      ++ Note that \spad{C(n,r) = n!/(r!(n-r)!)} where \spad{n >= r >= 0}.
      ++
      ++X [binomial(5,i) for i in 0..5]

    factorial : $ -> $
      ++ factorial(n) computes the factorial of n
      ++ (denoted in the literature by \spad{n!})
      ++ Note that \spad{n! = n (n-1)! when n > 0}; also, \spad{0! = 1}.

    permutation : ($, $) -> $
      ++ permutation(n, m) returns the number of
      ++ permutations of n objects taken m at a time.
      ++ Note that \spad{permutation(n,m) = n!/(n-m)!}.
```

———

— CFCAT.dotabb —

```
"CFCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CFCAT"];
"CFCAT" -> "CATEGORY"
```

———

— CFCAT.dotfull —

```
"CombinatorialFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CFCAT"];
"CombinatorialFunctionCategory()" -> "Category"
```

———

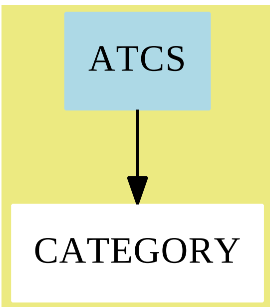
— CFCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"CombinatorialFunctionCategory()" [color=lightblue];
"CombinatorialFunctionCategory()" -> "Category"
```

```
"Category" [color=lightblue];
}
```

———-

2.0.16 CommutativeStarAttribute (ATCS)



— **CommutativeStarAttribute.input** —

```
)set break resume
)sys rm -f CommutativeStarAttribute.output
)spool CommutativeStarAttribute.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show CommutativeStarAttribute
--R
--R CommutativeStarAttribute is a category constructor
--R Abbreviation for CommutativeStarAttribute is ATCS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ATCS
--R
--E 1
)spool
)lisp (bye)
```

———-

— **CommutativeStarAttribute.help** —

```
====================================================================
CommutativeStarAttribute
====================================================================

The class of all commutative semigroups in multiplicative notation.
In other words domain D with "*": (D,D) -> D which is commutative.
```

```
Typially applied to rings.

See Also:
o )show CommutativeStarAttribute
```

— CommutativeStarAttribute.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ATCS">
CommutativeStarAttribute (ATCS)</a></h2>

The class of all commutative semigroups in multiplicative notation.
In other words domain D with "*": (D,D) -> D which is
commutative. Typically applied to rings.

</body>
```

— category ATCS CommutativeStarAttribute —

```
)abbrev category ATCS CommutativeStarAttribute
++ Description:
++ The class of all commutative semigroups in multiplicative notation.
++ In other words domain \spad{D} with \spad{"*": (D,D) -> D} which is
++ commutative. Typically applied to rings.

CommutativeStarAttribute() : Category == SIG where

  SIG ==> with nil
```

— ATCS.dotabb —

```
"ATCS"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATCS"];
"ATCS" -> "CATEGORY"
```

— ATCS.dotfull —

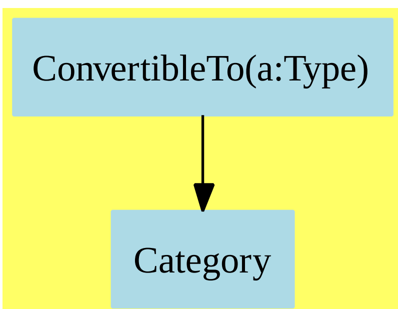
```
"CommutativeStarAttribute()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATCS"];
"CommutativeStarAttribute()" -> "Category"
```

— **ATCS.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"CommutativeStarAttribute()" [color=lightblue];
"CommutativeStarAttribute()" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.17 ConvertibleTo (KONVERT)



— **ConvertibleTo.input** —

```
)set break resume
)sys rm -f ConvertibleTo.output
)spool ConvertibleTo.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ConvertibleTo
--R
--R ConvertibleTo(S: Type) is a category constructor
--R Abbreviation for ConvertibleTo is KONVERT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for KONVERT
--R
--R------------------------------- Operations --------------------------------
--R convert : % -> S
--R
--E 1
```

```
)spool
)lisp (bye)
```

———-

— ConvertibleTo.help —

```
====================================================================
ConvertibleTo examples
====================================================================

A is convertible to B means any element of A can be converted into
an element of B, but not automatically by the interpreter.

See Also:
o )show ConvertibleTo
```

———-

See:

⇒ "Collection" (CLAGG) 5.0.94 on page 403
⇒ "FunctionSpace" (FS) 17.0.219 on page 1544
⇒ "IntegerNumberSystem" (INS) 16.0.210 on page 1420
⇒ "MonogenicAlgebra" (MONOGEN) 19.0.238 on page 1847
⇒ "QuotientFieldCategory" (QFCAT) 17.0.222 on page 1595
⇒ "RealConstant" (REAL) 3.0.55 on page 180
⇒ "RealNumberSystem" (RNS) 17.0.224 on page 1622
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

convert

This is directly exported but not implemented:

```
convert : % -> S
```

— ConvertibleTo.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#KONVERT">
ConvertibleTo (KONVERT)</a></h2>

A is convertible to B means any element of A
can be converted into an element of B,
but not automatically by the interpreter.
<br/><br/><br/>

convert : % -> S<br/>
  convert(a) transforms a into an element of S.

</body>
```

— category KONVERT ConvertibleTo —

```
)abbrev category KONVERT ConvertibleTo
++ Author: Manuel Bronstein
++ Date Last Updated: 14 May 1991
++ Description:
++ A is convertible to B means any element of A
++ can be converted into an element of B,
++ but not automatically by the interpreter.

ConvertibleTo(S) : Category == SIG where
  S : Type

  SIG ==> with

    convert : % -> S
      ++ convert(a) transforms a into an element of S.
```

— KONVERT.dotabb —

```
"KONVERT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=KONVERT"];
"KONVERT" -> "CATEGORY"
```

— KONVERT.dotfull —

```
"ConvertibleTo(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(a:Type)" -> "Category"

"ConvertibleTo(DoubleFloat)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(DoubleFloat)" -> "ConvertibleTo(a:Type)"

"ConvertibleTo(Float)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(Float)" -> "ConvertibleTo(a:Type)"

"ConvertibleTo(InputForm)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(InputForm)" -> "ConvertibleTo(a:Type)"

"ConvertibleTo(Integer)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(Integer)" -> "ConvertibleTo(a:Type)"
```

```
"ConvertibleTo(Pattern(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(Pattern(Integer))" -> "ConvertibleTo(a:Type)"

"ConvertibleTo(Pattern(Float))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(Pattern(Float))" -> "ConvertibleTo(a:Type)"

"ConvertibleTo(Complex(Float))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(Complex(Float))" ->  "ConvertibleTo(a:Type)"

"ConvertibleTo(Complex(DoubleFloat))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(Complex(DoubleFloat))" ->  "ConvertibleTo(a:Type)"

"ConvertibleTo(String)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(String)" ->  "ConvertibleTo(a:Type)"

"ConvertibleTo(Symbol)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(Symbol)" ->  "ConvertibleTo(a:Type)"

"ConvertibleTo(SExpression)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(SExpression)" ->  "ConvertibleTo(a:Type)"

"ConvertibleTo(Pattern(Base))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(Pattern(Base))" ->  "ConvertibleTo(a:Type)"

"ConvertibleTo(List(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(List(Integer))" ->  "ConvertibleTo(a:Type)"

"ConvertibleTo(List(Character))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(List(Character))" ->  "ConvertibleTo(a:Type)"

"ConvertibleTo(UnivariatePolynomialCategory(CommutativeRing))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
"ConvertibleTo(UnivariatePolynomialCategory(CommutativeRing))" ->
    "ConvertibleTo(a:Type)"
```

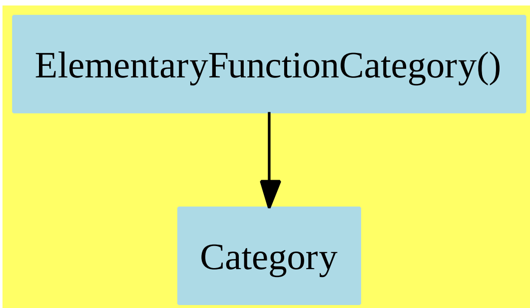
— KONVERT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];
```

```
"ConvertibleTo(a:Type)" [color=lightblue];
"ConvertibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

————-

2.0.18 ElementaryFunctionCategory (ELEMFUN)



— **ElementaryFunctionCategory.input** —

```
)set break resume
)sys rm -f ElementaryFunctionCategory.output
)spool ElementaryFunctionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ElementaryFunctionCategory
--R
--R ElementaryFunctionCategory is a category constructor
--R Abbreviation for ElementaryFunctionCategory is ELEMFUN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ELEMFUN
--R
--R------------------------------- Operations --------------------------------
--R ?**? : (%,%) -> %                     exp : % -> %
--R log : % -> %
--R
--E 1

)spool
)lisp (bye)
```

————-

— ElementaryFunctionCategory.help —

```
====================================================================
ElementaryFunctionCategory examples
====================================================================

This is the Category for the elementary functions.

See Also:
o )show ElementaryFunctionCategory
```

——————

See:

⇒ "TranscendentalFunctionCategory" (TRANFUN) 3.0.58 on page 192
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

?**? exp log

These are directly exported but not implemented:

```
exp : % -> %
log : % -> %
```

These are implemented by this category:

```
?**? : (%,%) -> %
```

— ElementaryFunctionCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ELEMFUN">
ElementaryFunctionCategory (ELEMFUN)</a></h2>

Category for the elementary functions.
<br/><br/><br/>

log : % -> %<br/>
  log(x) returns the natural logarithm of x. When evaluated<br/>
  into some subset of the complex numbers, the branch cut lies<br/>
  along the negative real axis, continuous with quadrant II. The<br/>
  domain does not contain the origin.<br/><br/>

exp : % -> %<br/>
  exp(x) returns %e to the power x.<br/><br/>

"**": (%, %) -> %<br/>
  x**y returns x to the power y.

</body>
```

——————

— **category ELEMFUN ElementaryFunctionCategory** —

```
)abbrev category ELEMFUN ElementaryFunctionCategory
++ Category for the elementary functions
++ Author: Manuel Bronstein
++ Date Last Updated: 14 May 1991
++ Description:
++ Category for the elementary functions;

ElementaryFunctionCategory() : Category == SIG where

  SIG ==> with

    log : $ -> $
      ++ log(x) returns the natural logarithm of x. When evaluated
      ++ into some subset of the complex numbers, the branch cut lies
      ++ along the negative real axis, continuous with quadrant II. The
      ++ domain does not contain the origin.

    exp : $ -> $
      ++ exp(x) returns %e to the power x.

    "**": ($, $) -> $
      ++ x**y returns x to the power y.

   add

     if $ has Monoid then

       x ** y == exp(y * log x)
```

————

— **COQ ELEMFUN** —

```
(* category ELEMFUN *)
(*
   if $ has Monoid then

     **: ($, $) -> $  ++ x**y returns x to the power y.
     x ** y == exp(y * log x)
*)
```

————

— **ELEMFUN.dotabb** —

```
"ELEMFUN"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ELEMFUN"];
"ELEMFUN" -> "CATEGORY"
```

———-

— ELEMFUN.dotfull —

```
"ElementaryFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ELEMFUN"];
"ElementaryFunctionCategory()" -> "Category"
```

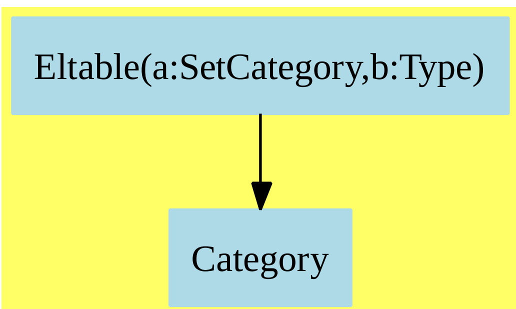
———-

— ELEMFUN.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"ElementaryFunctionCategory()" [color=lightblue];
"ElementaryFunctionCategory()" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.19 Eltable (ELTAB)



— Eltable.input —

```
)set break resume
)sys rm -f Eltable.output
)spool Eltable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Eltable
```

```
--R
--R Eltable(S: SetCategory,Index: Type) is a category constructor
--R Abbreviation for Eltable is ELTAB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ELTAB
--R
--R------------------------------- Operations --------------------------------
--R ?.? : (%,S) -> Index
--R
--E 1

)spool
)lisp (bye)
```

— **Eltable.help** —

```
====================================================================
Eltable examples
====================================================================

An eltable over domains D and I is a structure which can be viewed
as a function from D to I. Examples of eltable structures range from
data structures, for example, those of type List, to algebraic structures
like Polynomial.

See Also:
o )show Eltable
```

See:

⇒ "EltableAggregate" (ELTAGG) 3.0.47 on page 146
⇒ "LinearOrdinaryDifferentialOperatorCategory" (LODOCAT) 11.0.180 on page 1145
⇒ "UnivariatePolynomialCategory" (UPOLYC) 17.0.228 on page 1708
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

?.?

This syntax for elt is supported by the interpreter and compiler.

This is directly exported but not implemented:

```
 ?.? : (%,S) -> Index
```

— **Eltable.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ELTAB">
Eltable(S,Index (ELTAB)</a></h2>
```

```
&bull; S : SetCategory<br/>
&bull; Index : Type<br/><br/>

An eltable over domains D and I is a structure which can be viewed<br/>
as a function from D to I. Examples of eltable structures range from <br/>
data structures, For example, those of type List, to algebraic <br/>
structures like Polynomial.
<br/><br/><br/>

elt : (%, S) -> Index<br/>
  elt(u,i) (also written: u.i) returns the element of u indexed by i.<br/>
  Error: if i is not an index of u.

</body>
```

— **category ELTAB Eltable** —

```
)abbrev category ELTAB Eltable
++ Author: Michael Monagan; revised by Manuel Bronstein and Manuel Bronstein
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ An eltable over domains D and I is a structure which can be viewed
++ as a function from D to I. Examples of eltable structures range from
++ data structures, For example, those of type List, to algebraic
++ structures like Polynomial.

Eltable(S,Index) : Category == SIG where
  S : SetCategory
  Index : Type

  SIG ==> with

    elt : (%, S) -> Index
      ++ elt(u,i) (also written: u . i) returns the element of u indexed by i.
      ++ Error: if i is not an index of u.
```

— **ELTAB.dotabb** —

```
"ELTAB" [color=lightblue,href="bookvol10.2.pdf#nameddest=ELTAB"];
"ELTAB" -> "CATEGORY"
```

— **ELTAB.dotfull** —

```
"Eltable(a:SetCategory,b:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ELTAB"];
```

```
"Eltable(a:SetCategory,b:Type)" -> "Category"

"Eltable(a:UnivariatePolynomialCategory(a:Ring),b:UnivariatePolynomialCategory(a:Ring))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ELTAB"];
"Eltable(a:UnivariatePolynomialCategory(a:Ring),b:UnivariatePolynomialCategory(a:Ring))" ->
   "Eltable(a:SetCategory,b:Type)"

"Eltable(a:Ring,b:Ring)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ELTAB"];
"Eltable(a:Ring,b:Ring)" ->
   "Eltable(a:SetCategory,b:Type)"

"Eltable(a:SetCategory,b:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ELTAB"];
"Eltable(a:SetCategory,b:SetCategory)" ->
   "Eltable(a:SetCategory,b:Type)"
```

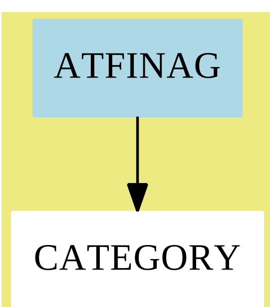
————

— **ELTAB.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"Eltable(a:SetCategory,b:Type)" [color=lightblue];
"Eltable(a:SetCategory,b:Type)" -> "Category"

"Category" [color=lightblue];
}
```

————

2.0.20 FiniteAggregateAttribute (ATFINAG)



— **FiniteAggregateAttribute.input** —

```
)set break resume
)sys rm -f FiniteAggregateAttribute.output
)spool FiniteAggregateAttribute.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteAggregateAttribute
--R
--R FiniteAggregateAttribute is a category constructor
--R Abbreviation for FiniteAggregateAttribute is ATFINAG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ATFINAG
--R
--E 1
)spool
)lisp (bye)
```

— FiniteAggregateAttribute.help —

```
====================================================================
FiniteAggregateAttribute
====================================================================

The class of all aggregates with a finite number of arguments

See Also:
o )show FiniteAggregateAttribute
```

— FiniteAggregateAttribute.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ATFINAG">
FiniteAggregateAttribute (ATFINAG)</a></h2>

The class of all aggregates with a finite number of arguments.
</body>
```

— category ATFINAG FiniteAggregateAttribute —

```
)abbrev category ATFINAG FiniteAggregateAttribute
++ Description:
++ The class of all aggregates with a finite number of arguments
```

```
FiniteAggregateAttribute() : Category == SIG where

  SIG ==> with nil
```

———-

— ATFINAG.dotabb —

```
"ATFINAG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATFINAG"];
"ATFINAG" -> "CATEGORY"
```

———-

— ATFINAG.dotfull —

```
"FiniteAggregateAttribute()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATFINAG"];
"FiniteAggregateAttribute()" -> "Category"
```

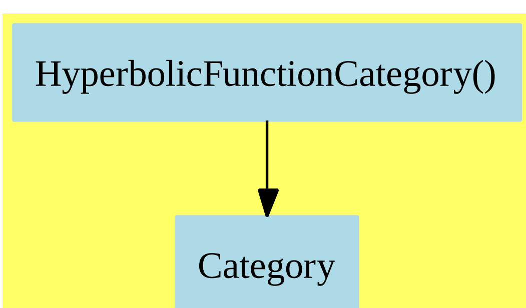
———-

— ATFINAG.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FiniteAggregateAttribute()" [color=lightblue];
"FiniteAggregateAttribute()" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.21 HyperbolicFunctionCategory (HYPCAT)



The `csch` and `sech` functions were modified to include an intermediate test to check that the argument has a reciprocal values.

— HyperbolicFunctionCategory.input —

```
)set break resume
)sys rm -f HyperbolicFunctionCategory.output
)spool HyperbolicFunctionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show HyperbolicFunctionCategory
--R
--R HyperbolicFunctionCategory is a category constructor
--R Abbreviation for HyperbolicFunctionCategory is HYPCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for HYPCAT
--R
--R------------------------------- Operations --------------------------------
--R cosh : % -> %                         coth : % -> %
--R csch : % -> %                         sech : % -> %
--R sinh : % -> %                         tanh : % -> %
--R
--R
--E 1

)spool
)lisp (bye)
```

— HyperbolicFunctionCategory.help —

```
====================================================================
HyperbolicFunctionCategory examples
====================================================================

This is the Category for the hyperbolic trigonometric functions.

See Also:
o )show HyperbolicFunctionCategory
```

See:

⇒ "TranscendentalFunctionCategory" (TRANFUN) 3.0.58 on page 192
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

cosh coth csch sech sinh tanh

These are implemented by this category:

```
cosh : % -> %
coth : % -> %
csch : % -> %
sech : % -> %
sinh : % -> %
tanh : % -> %
```

— HyperbolicFunctionCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#HYPCAT">
HyperbolicFunctionCategory (HYPCAT)</a></h2>

Category for the hyperbolic trigonometric functions.
<br/><br/><br/>

cosh : % -> %<br/>
  cosh(x) returns the hyperbolic cosine of x.<br/><br/>

coth : % -> %<br/>
  coth(x) returns the hyperbolic cotangent of x.<br/><br/>

csch : % -> %<br/>
  csch(x) returns the hyperbolic cosecant of x.<br/><br/>

sech : % -> %<br/>
  sech(x) returns the hyperbolic secant of x.<br/><br/>

sinh : % -> %<br/>
  sinh(x) returns the hyperbolic sine of x.<br/><br/>

tanh : % -> %<br/>
  tanh(x) returns the hyperbolic tangent of x.<br/><br/>

</body>
```

— category HYPCAT HyperbolicFunctionCategory —

```
)abbrev category HYPCAT HyperbolicFunctionCategory
++ Date Last Updated: 14 May 1991
++ Category for the hyperbolic trigonometric functions;

HyperbolicFunctionCategory() : Category == SIG where

  SIG ==> with

    cosh : $ -> $
      ++ cosh(x) returns the hyperbolic cosine of x.
```

```
    coth : $ -> $
      ++ coth(x) returns the hyperbolic cotangent of x.

    csch : $ -> $
      ++ csch(x) returns the hyperbolic cosecant of x.

    sech : $ -> $
      ++ sech(x) returns the hyperbolic secant of x.

    sinh : $ -> $
      ++ sinh(x) returns the hyperbolic sine of x.

    tanh : $ -> $
      ++ tanh(x) returns the hyperbolic tangent of x.

   add

     if $ has Ring then

       csch x ==
         (a := recip(sinh x)) case "failed" => error "csch: no reciprocal"
         a::$

       sech x ==
         (a := recip(cosh x)) case "failed" => error "sech: no reciprocal"
         a::$

       tanh x == sinh x * sech x

       coth x == cosh x * csch x

       if $ has ElementaryFunctionCategory then

         cosh x ==
           e := exp x
           (e + recip(e)::$) * recip(2::$)::$

         sinh(x):$ ==
           e := exp x
           (e - recip(e)::$) * recip(2::$)::$
```

———-

— COQ HYPCAT —

```
(* category HYPCAT *)
(*
    if $ has Ring then

       csch: $ -> $
       csch x ==
         (a := recip(sinh x)) case "failed" => error "csch: no reciprocal"
```

```
        a::$

      sech: $ -> $
      sech x ==
        (a := recip(cosh x)) case "failed" => error "sech: no reciprocal"
        a::$

      tanh: $ -> $
      tanh x == sinh x * sech x

      coth: $ -> $
      coth x == cosh x * csch x

      if $ has ElementaryFunctionCategory then

        cosh: $ -> $
        cosh x ==
          e := exp x
          (e + recip(e)::$) * recip(2::$)::$

        sinh: $ -> $
        sinh(x):$ ==
          e := exp x
          (e - recip(e)::$) * recip(2::$)::$
*)
```

— HYPCAT.dotabb —

```
"HYPCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=HYPCAT"];
"HYPCAT" -> "CATEGORY"
```

— HYPCAT.dotfull —

```
"HyperbolicFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=HYPCAT"];
"HyperbolicFunctionCategory()" -> "Category"
```

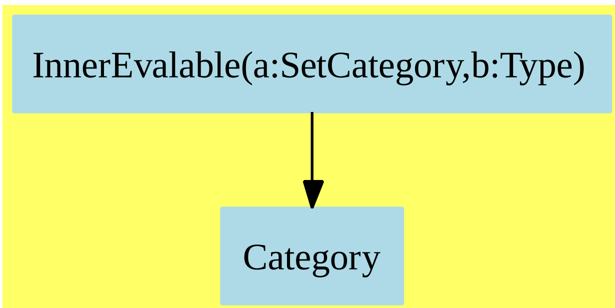
— HYPCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];
```

```
"HyperbolicFunctionCategory()" [color=lightblue];
"HyperbolicFunctionCategory()" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.22 InnerEvalable (IEVALAB)



— InnerEvalable.input —

```
)set break resume
)sys rm -f InnerEvalable.output
)spool InnerEvalable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InnerEvalable
--R
--R InnerEvalable(A: SetCategory,B: Type) is a category constructor
--R Abbreviation for InnerEvalable is IEVALAB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for IEVALAB
--R
--R------------------------------- Operations --------------------------------
--R eval : (%,A,B) -> %                  eval : (%,List(A),List(B)) -> %
--R
--E 1

)spool
)lisp (bye)
```

———-

— InnerEvalable.help —

```
====================================================================
InnerEvalable examples
====================================================================

This category provides eval operations. A domain may belong to this
category if it is possible to make "evaluation" substitutions.  The
difference between this and Evalable is that the operations in this
category specify the substitution as a pair of arguments rather than
as an equation.

See Also:
o )show InnerEvalable
```

———-

See:

⇒ "DesingTreeCategory" (DSTRCAT) 4.0.61 on page 210
⇒ "Evalable" (EVALAB) 3.0.48 on page 151
⇒ "ExpressionSpace" (ES) 5.0.96 on page 418
⇒ "MultivariateTaylorSeriesCategory" (MTSCAT) 15.0.206 on page 1384
⇒ "PolynomialCategory" (POLYCAT) 16.0.213 on page 1449
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

eval

These are directly exported but not implemented:

```
eval : (%,A,B) -> %
```

These are implemented by this category:

```
eval : (%,List A,List B) -> %
```

— **InnerEvalable.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#IEVALAB">
InnerEvalable (IEVALAB)</a></h2>

This category provides eval operations.
A domain may belong to this category if it is possible to make
"evaluation" substitutions.  The difference between this
and Evalable is that the operations in this category
specify the substitution as a pair of arguments rather than as
an equation.
<br/><br/><br/>
eval : (%, A, B) -> %<br/>
  eval(f, x, v) replaces x by v in f.<br/><br/>

eval : (%, List A, List B) -> %<br/>
  eval(f, [x1,...,xn], [v1,...,vn]) replaces xi by vi in f.<br/><br/>
```

```
</body>
```

———-

— category IEVALAB InnerEvalable —

```
)abbrev category IEVALAB InnerEvalable
-- FOR THE BENEFIT OF LIBAXO GENERATION
++ Date Last Updated: June 3, 1991
++ Description:
++ This category provides \spadfun{eval} operations.
++ A domain may belong to this category if it is possible to make
++ "evaluation" substitutions.  The difference between this
++ and \spadtype{Evalable} is that the operations in this category
++ specify the substitution as a pair of arguments rather than as
++ an equation.

InnerEvalable(A,B) : Category == SIG where
  A : SetCategory
  B : Type

  SIG ==> with

    eval : ($, A, B) -> $
      ++ eval(f, x, v) replaces x by v in f.

    eval : ($, List A, List B) -> $
      ++ eval(f, [x1,...,xn], [v1,...,vn]) replaces xi by vi in f.

   add

     eval(f:$, x:A, v:B) == eval(f, [x], [v])
```

———-

— COQ IEVALAB —

```
(* category IEVALAB *)
(*
    eval: ($, A, B) -> $
    eval(f:$, x:A, v:B) == eval(f, [x], [v])
*)
```

———-

— IEVALAB.dotabb —

```
"IEVALAB"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=IEVALAB"];
"IEVALAB" -> "CATEGORY"
```

———-

— **IEVALAB.dotfull** —

```
"InnerEvalable(a:SetCategory,b:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=IEVALAB"];
"InnerEvalable(a:SetCategory,b:Type)" -> "Category"

"InnerEvalable(a:SetCategory,b:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=IEVALAB"];
"InnerEvalable(a:SetCategory,b:SetCategory)" ->
    "InnerEvalable(a:SetCategory,b:Type)"

"InnerEvalable(a:OrderedSet,b:Ring)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=IEVALAB"];
"InnerEvalable(a:OrderedSet,b:Ring)" ->
    "InnerEvalable(a:SetCategory,b:Type)"

"InnerEvalable(a:OrderedSet,b:PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=IEVALAB"];
"InnerEvalable(a:OrderedSet,b:PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)" ->
    "InnerEvalable(a:SetCategory,b:Type)"

"InnerEvalable(a:Ring,MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=IEVALAB"];
"InnerEvalable(a:Ring,MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet))"
     -> "InnerEvalable(a:SetCategory,b:Type)"

"InnerEvalable(Kernal(ExpressionSpace),ExpressionSpace)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=IEVALAB"];
"InnerEvalable(Kernal(ExpressionSpace),ExpressionSpace)" ->
    "InnerEvalable(a:SetCategory,b:Type)"
```

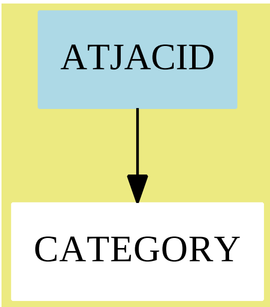
———-

— **IEVALAB.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"InnerEvalable(a:SetCategory,b:Type)" [color=lightblue];
"InnerEvalable(a:SetCategory,b:Type)" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.23 JacobiIdentityAttribute (ATJACID)



— **JacobiIdentityAttribute.input** —

```
)set break resume
)sys rm -f JacobiIdentityAttribute.output
)spool JacobiIdentityAttribute.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show JacobiIdentityAttribute
--R
--R JacobiIdentityAttribute is a category constructor
--R Abbreviation for JacobiIdentityAttribute is ATJACID
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ATJACID
--R
--E 1
)spool
)lisp (bye)
```

————

— **JacobiIdentityAttribute.help** —

```
====================================================================
JacobiIdentityAttribute
====================================================================

JacobiIdentity means that [x,[y,z]]+[y,[z,x]]+[z,[x,y]] = 0 holds.
See LieAlgebra.

See Also:
o )show JacobiIdentityAttribute
```

————

— JacobiIdentityAttribute.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ATJACID">
JacobiIdentityAttribute (ATJACID)</a></h2>

JacobiIdentity means that [x,[y,z]]+[y,[z,x]]+[z,[x,y]] = 0 holds.
<br/><br/>
&bull; LieAlgebra
</body>
```

— category ATJACID JacobiIdentityAttribute —

```
)abbrev category ATJACID JacobiIdentityAttribute
++ Description:
++ JacobiIdentity means that \spad{[x,[y,z]]+[y,[z,x]]+[z,[x,y]] = 0} holds.
++ See LieAlgebra.

JacobiIdentityAttribute() : Category == SIG where

  SIG ==> with nil
```

— COQ ATJACID —

```
(* category ATJACID *}
(*
Axiom
  [x,[y,z]]+[y,[z,x]]+[z,[x,y]] = 0

*)
```

— ATJACID.dotabb —

```
"ATJACID"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATJACID"];
"ATJACID" -> "CATEGORY"
```

— ATJACID.dotfull —

```
"JacobiIdentityAttribute()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATJACID"];
"JacobiIdentityAttribute()" -> "Category"
```

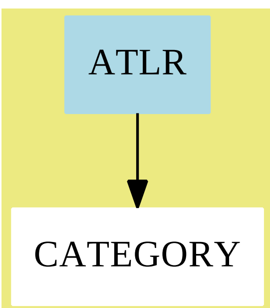
———-

— ATJACID.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"JacobiIdentityAttribute()" [color=lightblue];
"JacobiIdentityAttribute()" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.24 LazyRepresentationAttribute (ATLR)



— LazyRepresentationAttribute.input —

```
)set break resume
)sys rm -f LazyRepresentationAttribute.output
)spool LazyRepresentationAttribute.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show LazyRepresentationAttribute
--R
--R LazyRepresentationAttribute is a category constructor
--R Abbreviation for LazyRepresentationAttribute is ATLR
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ATLR
--R
--E 1
)spool
)lisp (bye)
```

———-

— LazyRepresentationAttribute.help —

```
====================================================================
LazyRepresentationAttribute
====================================================================

The class of all domains which have a lazy representation

See Also:
o )show LazyRepresentationAttribute
```

———-

— LazyRepresentationAttribute.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ATLR">
LazyRepresentationAttribute (ATLR)</a></h2>

The class of all domains which have a lazy representation.
</body>
```

———-

— category ATLR LazyRepresentationAttribute —

```
)abbrev category ATLR LazyRepresentationAttribute
++ Description:
++ The class of all domains which have a lazy representation

LazyRepresentationAttribute() : Category == SIG where

  SIG ==> with nil
```

———-

— ATLR.dotabb —

```
"ATLR"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATLR"];
"ATLR" -> "CATEGORY"
```

———-

— ATLR.dotfull —

```
"LazyRepresentationAttribute()"
```

```
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATLR"];
"LazyRepresentationAttribute()" -> "Category"
```

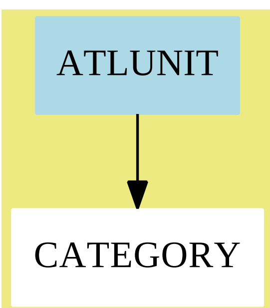
———-

— **ATLR.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"LazyRepresentationAttribute()" [color=lightblue];
"LazyRepresentationAttribute()" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.25 LeftUnitaryAttribute (ATLUNIT)



— **LeftUnitaryAttribute.input** —

```
)set break resume
)sys rm -f LeftUnitaryAttribute.output
)spool LeftUnitaryAttribute.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show LeftUnitaryAttribute
--R
--R LeftUnitaryAttribute is a category constructor
--R Abbreviation for LeftUnitaryAttribute is ATLUNIT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ATLUNIT
```

```
--R
--E 1
)spool
)lisp (bye)
```

———-

— **LeftUnitaryAttribute.help** —

```
====================================================================
LeftUnitaryAttribute
====================================================================

LeftUnitary is true if 1 * x = x for all x.

See Also:
o )show LeftUnitaryAttribute
```

———-

— **LeftUnitaryAttribute.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ATLUNIT">
LeftUnitaryAttribute (ATLUNIT)</a></h2>

LeftUnitary is true if 1 * x = x for all x.
</body>
```

———-

— **category ATLUNIT LeftUnitaryAttribute** —

```
)abbrev category ATLUNIT LeftUnitaryAttribute
++ Description:
++ LeftUnitary is true if \spad{1 * x = x} for all x.

LeftUnitaryAttribute() : Category == SIG where

  SIG ==> with nil
```

———-

— **COQ ATLUNIT** —

```
(* category ATLUNIT *)
(*
   LeftUnitary is true if 1 * x = x for all x.

*)
```

———-

— ATLUNIT.dotabb —

```
"ATLUNIT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATLUNIT"];
"ATLUNIT" -> "CATEGORY"
```

———-

— ATLUNIT.dotfull —

```
"LeftUnitaryAttribute()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATLUNIT"];
"LeftUnitaryAttribute()" -> "Category"
```

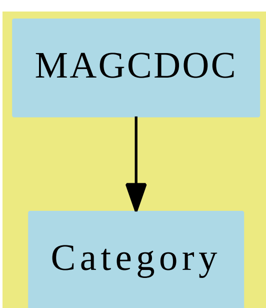
———-

— ATLUNIT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"LeftUnitaryAttribute()" [color=lightblue];
"LeftUnitaryAttribute()" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.26 ModularAlgebraicGcdOperations (MAGCDOC)



— ModularAlgebraicGcdOperations.input —

```
)set break resume
)sys rm -f ModularAlgebraicGcdOperations.output
)spool ModularAlgebraicGcdOperations.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ModularAlgebraicGcdOperations
--R
--R ModularAlgebraicGcdOperations(MPT: Type,MD: Type) is a category constructor
--R Abbreviation for ModularAlgebraicGcdOperations is MAGCDOC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for MAGCDOC
--R
--R------------------------------- Operations --------------------------------
--R degree : MPT -> Integer               pseudoRem : (MPT,MPT,MD) -> MPT
--R zero? : MPT -> Boolean
--R MPtoMPT : (Polynomial(Integer),Symbol,List(Symbol),MD) -> MPT
--R canonicalIfCan : (MPT,MD) -> Union(MPT,"failed")
--R packExps : (Integer,Integer,MD) -> SortedExponentVector
--R packModulus : (List(Polynomial(Integer)),List(Symbol),Integer) -> Union(MD,"failed")
--R repack1 : (MPT,U32Vector,Integer,MD) -> Void
--R
--E 1

)spool
)lisp (bye)
```

— **ModularAlgebraicGcdOperations.help** —

```
====================================================================
ModularAlgebraicGcdOperations
====================================================================

This category specifies operations needed by ModularAlgebraicGcd
package.  Since we have multiple implementations we specify interface
here and put implementations in separate packages.  Most operations
are done using special purpose abstract representation.  Apropriate
types are passesd as parametes: MPT is type of modular polynomials in
one variable with coefficients in some algebraic extension.  MD is
type of modulus.  Final results are converted to packed representation,
with coefficients (from prime field) stored in one array and exponents
(in main variable and in auxilary variables representing generators
of algebrac extension) stored in parallel array.
```

Exports:

canonicalIfCan	degree	MPtoMPT	packExps	packModulus
pseudoRem	repack1	zero?		

These are directly exported but not implemented:

```
canonicalIfCan : (MPT,MD) -> Union(MPT,"failed")
degree : MPT -> Integer
MPtoMPT : (Polynomial(Integer),Symbol,List(Symbol),MD) -> MPT
packExps : (Integer,Integer,MD) -> SortedExponentVector
packModulus : (List(Polynomial(Integer)),List(Symbol),Integer) ->
                Union(MD,"failed")
pseudoRem : (MPT,MPT,MD) -> MPT
repack1 : (MPT,U32Vector,Integer,MD) -> Void
zero? : MPT -> Boolean
```

— ModularAlgebraicGcdOperations.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#MAGCDOC">
ModularAlgebraicGcdOperations (MAGCDOC)</a></h2>

&bull; MPT : Type<br/>
&bull; MD : TYpe<br/>
<br/>
This category specifies operations needed by
ModularAlgebraicGcd package.  Since we have multiple
implementations we specify interface here and put
implementations in separate packages.  Most operations
are done using special purpose abstract representation.
Apropriate types are passesd as parametes: MPT is type
of modular polynomials in one variable with coefficients
in some algebraic extension.  MD is type of modulus.
Final results are converted to packed representation,
with coefficients (from prime field) stored in one
array and exponents (in main variable and in auxilary
variables representing generators of algebrac extension)
stored in parallel array.
<br/><br/><br/>

pseudoRem : (MPT, MPT, MD) -> MPT<br/>
  pseudoRem(x, y, m) computes pseudoremainder of x by y<br/>
  modulo m.<br/><br/>

canonicalIfCan : (MPT, MD) -> Union(MPT, "failed")<br/>
  canonicalIfCan(x, m) tries to divide x by its leading<br/>
  coefficient modulo m.<br/><br/>

packModulus : (List MP, List(Symbol), Integer) -> Union(MD, "failed")<br/>
  packModulus(lp, ls, p) converts lp, ls and prime p which<br/><br/>
  together describe algebraic extension to packed<br/>
  representation.<br/><br/>

MPtoMPT : (MP, Symbol, List(Symbol), MD) -> MPT<br/>
  MPtoMPT(p, s, ls, m) coverts p to packed represntation.<br/><br/>
```

```
zero? : MPT -> Boolean<br/>
  zero?(x) checks if x is zero.<br/><br/>

degree : MPT -> Integer<br/>
  degree(x) gives degree of x.<br/><br/>

packExps : (Integer, Integer, MD) -> SortedExponentVector<br/>
  packExps(d, s, m) produces vector of exponents up<br/><br/>
  to degree d.  s is size (degree) of algebraic extension.<br/>
  Use together with repack1.<br/><br/>

repack1 : (MPT, PA, Integer, MD) -> Void<br/>
  repack1(x, a, d, m) stores coefficients of x in a.<br/>
  d is degree of x.  Corresponding exponents are given<br/>
&nsbp; by packExps.
</body>
```

— category MAGCDOC ModularAlgebraicGcdOperations —

```
)abbrev category MAGCDOC ModularAlgebraicGcdOperations
++ Description: This category specifies operations needed by
++ ModularAlgebraicGcd package.  Since we have multiple
++ implementations we specify interface here and put
++ implementations in separate packages.  Most operations
++ are done using special purpose abstract representation.
++ Apropriate types are passesd as parametes: MPT is type
++ of modular polynomials in one variable with coefficients
++ in some algebraic extension.  MD is type of modulus.
++ Final results are converted to packed representation,
++ with coefficients (from prime field) stored in one
++ array and exponents (in main variable and in auxilary
++ variables representing generators of algebrac extension)
++ stored in parallel array.

ModularAlgebraicGcdOperations(MPT,MD) : Category == SIG where
  MPT : Type
  MD : Type

  MP ==> Polynomial Integer
  PA ==> U32Vector

  SIG ==> with

    pseudoRem : (MPT, MPT, MD) -> MPT
      ++ pseudoRem(x, y, m) computes pseudoremainder of x by y
      ++ modulo m.

    canonicalIfCan : (MPT, MD) -> Union(MPT, "failed")
      ++ canonicalIfCan(x, m) tries to divide x by its leading
      ++ coefficient modulo m.
```

```
    packModulus : (List MP, List(Symbol), Integer) -> Union(MD, "failed")
      ++ packModulus(lp, ls, p) converts lp, ls and prime p which
      ++ together describe algebraic extension to packed
      ++ representation.

    MPtoMPT : (MP, Symbol, List(Symbol), MD) -> MPT
      ++ MPtoMPT(p, s, ls, m) coverts p to packed represntation.

    zero? : MPT -> Boolean
      ++ zero?(x) checks if x is zero.

    degree : MPT -> Integer
      ++ degree(x) gives degree of x.

    packExps : (Integer, Integer, MD) -> SortedExponentVector
      ++ packExps(d, s, m) produces vector of exponents up
      ++ to degree d.  s is size (degree) of algebraic extension.
      ++ Use together with repack1.

    repack1 : (MPT, PA, Integer, MD) -> Void
      ++ repack1(x, a, d, m) stores coefficients of x in a.
      ++ d is degree of x.  Corresponding exponents are given
      ++ by packExps.
```

— MAGCDOC.dotabb —

```
"MAGCDOC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MAGCDOC"];
"MAGCDOC" -> "CATEGORY"
```

— MAGCDOC.dotfull —

```
"ModularAlgebraicGcdOperations()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MAGCDOC"];
"ModularAlgebraicGcdOperations(a:Type,b:Type)" -> "Category"
```

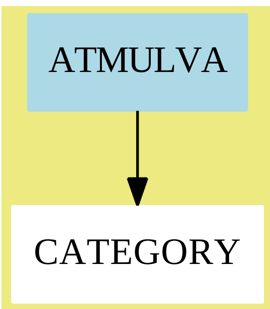
— MAGCDOC.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"MAGCDOC" [color=lightblue];
"MAGCDOC" -> "Category"
```

```
"Category" [color=lightblue];
}
```

———-

2.0.27 MultiplicativeValuationAttribute (ATMULVA)



— **MultiplicativeValuationAttribute.input** —

```
)set break resume
)sys rm -f MultiplicativeValuationAttribute.output
)spool MultiplicativeValuationAttribute.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MultiplicativeValuationAttribute
--R
--R MultiplicativeValuationAttribute is a category constructor
--R Abbreviation for MultiplicativeValuationAttribute is ATMULVA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ATMULVA
--R
--E 1
)spool
)lisp (bye)
```

———-

— **MultiplicativeValuationAttribute.help** —

```
====================================================================
MultiplicativeValuationAttribute
====================================================================

The class of all euclidean domains such that
```

```
euclideanSize(a*b)=euclideanSize(a)*euclideanSize(b)

See Also:
o )show MultiplicativeValuationAttribute
```

—— MultiplicativeValuationAttribute.html ——

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ATMULVA">
MultiplicativeValuationAttribute (ATMULVA)</a></h2>

The class of all euclidean domains such that<br/>
euclideanSize(a*b)=euclideanSize(a)*euclideanSize(b)

</body>
```

—— category ATMULVA MultiplicativeValuationAttribute ——

```
)abbrev category ATMULVA MultiplicativeValuationAttribute
++ Description:
++ The class of all euclidean domains such that
++ \spad{euclideanSize(a*b)=euclideanSize(a)*euclideanSize(b)}

MultiplicativeValuationAttribute() : Category == SIG where

  SIG ==> with nil
```

—— COQ ATMULVA ——

```
(* category ATMULVA *)
(*
Axiom
   euclideanSize(a*b)=euclideanSize(a)*euclideanSize(b)

*)
```

—— ATMULVA.dotabb ——

```
"ATMULVA"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATMULVA"];
"ATMULVA" -> "CATEGORY"
```

— **ATMULVA.dotfull** —

```
"MultiplicativeValuationAttribute()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATMULVA"];
"MultiplicativeValuationAttribute()" -> "Category"
```

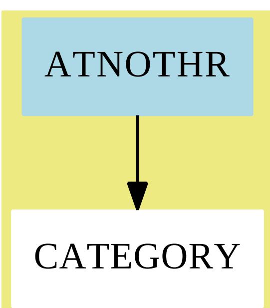
————

— **ATMULVA.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"MultiplicativeValuationAttribute()" [color=lightblue];
"MultiplicativeValuationAttribute()" -> "Category"

"Category" [color=lightblue];
}
```

————

2.0.28 NotherianAttribute (ATNOTHR)



— **NotherianAttribute.input** —

```
)set break resume
)sys rm -f NotherianAttribute.output
)spool NotherianAttribute.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NotherianAttribute
--R
--R NotherianAttribute is a category constructor
```

```
--R Abbreviation for NotherianAttribute is ATNOTHR
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ATNOTHR
--R
--E 1
)spool
)lisp (bye)
```

———-

— NotherianAttribute.help —

```
====================================================================
NotherianAttribute
====================================================================

Notherian is true if all of its ideals are finitely generated.

See Also:
o )show NotherianAttribute
```

———-

— NotherianAttribute.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ATNOTHR">
NotherianAttribute (ATNOTHR)</a></h2>

Notherian is true if all of its ideals are finitely generated.
</body>
```

———-

— category ATNOTHR NotherianAttribute —

```
)abbrev category ATNOTHR NotherianAttribute
++ Description:
++ Notherian is true if all of its ideals are finitely generated.

NotherianAttribute() : Category == SIG where

  SIG ==> with nil
```

———-

— ATNOTHR.dotabb —

```
"ATNOTHR"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATNOTHR"];
"ATNOTHR" -> "CATEGORY"
```

———-

— **ATNOTHR.dotfull** —

```
"NotherianAttribute()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATNOTHR"];
"NotherianAttribute()" -> "Category"
```

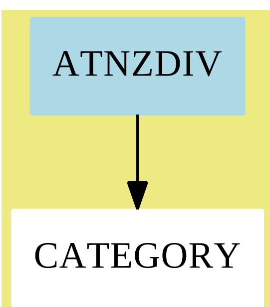
———-

— **ATNOTHR.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"NotherianAttribute()" [color=lightblue];
"NotherianAttribute()" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.29 NoZeroDivisorsAttribute (ATNZDIV)



— **NoZeroDivisorsAttribute.input** —

```
)set break resume
)sys rm -f NoZeroDivisorsAttribute.output
)spool NoZeroDivisorsAttribute.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
```

```
)show NoZeroDivisorsAttribute
--R
--R NoZeroDivisorsAttribute is a category constructor
--R Abbreviation for NoZeroDivisorsAttribute is ATNZDIV
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ATNZDIV
--R
--E 1
)spool
)lisp (bye)
```

———

— NoZeroDivisorsAttribute.help —

```
====================================================================
NoZeroDivisorsAttribute
====================================================================

The class of all semirings such that x * y ~= 0 implies
both x and y are non-zero.

See Also:
o )show NoZeroDivisorsAttribute
```

———

— NoZeroDivisorsAttribute.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ATNZDIV">
NoZeroDivisorsAttribute (ATNZDIV)</a></h2>

The class of all semirings such that x * y ~= 0 implies
both x and y are non-zero.
</body>
```

———

— category ATNZDIV NoZeroDivisorsAttribute —

```
)abbrev category ATNZDIV NoZeroDivisorsAttribute
++ Description:
++ The class of all semirings such that \spad{x * y ~= 0} implies
++ both x and y are non-zero.

NoZeroDivisorsAttribute() : Category == SIG where

  SIG ==> with nil
```

———

— COQ ATNZDIV —

```
(* category ATNZDIV *)
(*
Axiom
  The class of all semirings such that x * y ~= 0 implies
  both x and y are non-zero.

*)
```

———-

— ATNZDIV.dotabb —

```
"ATNZDIV"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATNZDIV"];
"ATNZDIV" -> "CATEGORY"
```

———-

— ATNZDIV.dotfull —

```
"NoZeroDivisorsAttribute()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATNZDIV"];
"NoZeroDivisorsAttribute()" -> "Category"
```

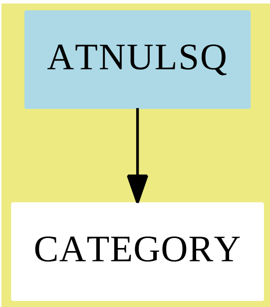
———-

— ATNZDIV.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"NoZeroDivisorsAttribute()" [color=lightblue];
"NoZeroDivisorsAttribute()" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.30 NullSquareAttribute (ATNULSQ)



— **NullSquareAttribute.input** —

```
)set break resume
)sys rm -f NullSquareAttribute.output
)spool NullSquareAttribute.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NullSquareAttribute
--R
--R NullSquareAttribute is a category constructor
--R Abbreviation for NullSquareAttribute is ATNULSQ
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ATNULSQ
--R
--E 1
)spool
)lisp (bye)
```

———-

— **NullSquareAttribute.help** —

```
====================================================================
NullSquareAttribute
====================================================================

NullSquare means that [x,x] = 0 holds. See LieAlgebra.

See Also:
o )show NullSquareAttribute
```

———-

— **NullSquareAttribute.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ATNULSQ">
NullSquareAttribute (ATNULSQ)</a></h2>

NullSquare means that [x,x] = 0 holds<br/><br/>

&bull; LieAlgebra
</body>
```

— category ATNULSQ NullSquareAttribute —

```
)abbrev category ATNULSQ NullSquareAttribute
++ Description:
++ NullSquare means that \spad{[x,x] = 0} holds. See LieAlgebra.

NullSquareAttribute() : Category == SIG where

  SIG ==> with nil
```

— COQ ATNULSQ —

```
(* category ATNULSQ *)
(*
Axiom
  NullSquare means that [x,x] = 0 holds. See LieAlgebra.

*)
```

— ATNULSQ.dotabb —

```
"ATNULSQ"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATNULSQ"];
"ATNULSQ" -> "CATEGORY"
```

— ATNULSQ.dotfull —

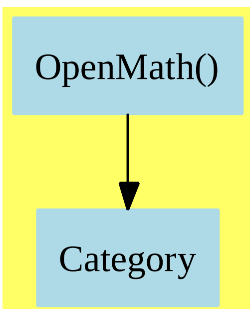
```
"NullSquareAttribute()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATNULSQ"];
"NullSquareAttribute()" -> "Category"
```

— **ATNULSQ.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"NullSquareAttribute()" [color=lightblue];
"NullSquareAttribute()" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.31 OpenMath (OM)



— **OpenMath.input** —

```
)set break resume
)sys rm -f OpenMath.output
)spool OpenMath.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OpenMath
--R
--R OpenMath is a category constructor
--R Abbreviation for OpenMath is OM
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for OM
--R
--R------------------------------- Operations --------------------------------
--R OMwrite : % -> String                 OMwrite : (%,Boolean) -> String
--R OMwrite : (OpenMathDevice,%) -> Void
--R OMwrite : (OpenMathDevice,%,Boolean) -> Void
--R
--E 1
```

```
)spool
)lisp (bye)
```

— OpenMath.help —

```
====================================================================
OpenMath examples
====================================================================

OpenMath provides operations for exporting an object in OpenMath format.

See Also:
o )show OpenMath
```

See:

⇒ "StringCategory" (STRICAT) 10.0.173 on page 1093
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

OMwrite

These are directly exported but not implemented:

```
OMwrite : % -> String
OMwrite : (%,Boolean) -> String
OMwrite : (OpenMathDevice,%) -> Void
OMwrite : (OpenMathDevice,%,Boolean) -> Void
```

See: Corless[Corl00], Fateman[Fate01a]

— OpenMath.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#OM">
OpenMath (OM)</a></h2>

OpenMath provides operations for exporting an object in OpenMath format.
<br/><br/><br/>
OMwrite : % -> String<br/>
  OMwrite(u) returns the OpenMath XML encoding of u as a<br/>
  complete OpenMath object.<br/><br/>

OMwrite : (%, Boolean) -> String<br/>
  OMwrite(u, true) returns the OpenMath XML encoding of u<br/>
  as a complete OpenMath object; OMwrite(u, false) returns the<br/>
  OpenMath XML encoding of \axiom{u} as an OpenMath fragment.<br/><br/>

OMwrite : (OpenMathDevice, %) -> Void<br/>
  OMwrite(dev, u) writes the OpenMath form of u to the<br/>
```

```
  OpenMath device \axiom{dev} as a complete OpenMath object.<br/><br/>

OMwrite : (OpenMathDevice, %, Boolean) -> Void<br/>
  OMwrite(dev, u, true) writes the OpenMath form of \axiom{u} to<br/>
  the OpenMath device \axiom{dev} as a complete OpenMath object;<br/>
  OMwrite(dev, u, false) writes object as an OpenMath fragment.<br/><br/>
<br/><br/>
<b>REFERENCES:</b><br/>
&bull; Corl00 According to Abramowitz and Stegun or arccoth
needn't be Uncouth<br/>
&bull; Fate01a A Critique of OpenMath and Thoughts on Encoding Mathematics

</body>
```

— category OM OpenMath —

```
)abbrev category OM OpenMath
++ Author: Mike Dewar & Vilya Harvey
++ Basic Functions: OMwrite
++ References:
++ Corl00 According to Abramowitz and Stegun or arccoth needn't be Uncouth
++ Fate01a A Critique of OpenMath and Thoughts on Encoding Mathematics
++ Description:
++ \spadtype{OpenMath} provides operations for exporting an object
++ in OpenMath format.

OpenMath() : Category == SIG where

  SIG ==> with

    OMwrite : % -> String
      ++ OMwrite(u) returns the OpenMath XML encoding of \axiom{u} as a
      ++ complete OpenMath object.

    OMwrite : (%, Boolean) -> String
      ++ OMwrite(u, true) returns the OpenMath XML encoding of \axiom{u}
      ++ as a complete OpenMath object; OMwrite(u, false) returns the
      ++ OpenMath XML encoding of \axiom{u} as an OpenMath fragment.

    OMwrite : (OpenMathDevice, %) -> Void
      ++ OMwrite(dev, u) writes the OpenMath form of \axiom{u} to the
      ++ OpenMath device \axiom{dev} as a complete OpenMath object.

    OMwrite : (OpenMathDevice, %, Boolean) -> Void
      ++ OMwrite(dev, u, true) writes the OpenMath form of \axiom{u} to
      ++ the OpenMath device \axiom{dev} as a complete OpenMath object;
      ++ OMwrite(dev, u, false) writes the object as an OpenMath fragment.
```

— OM.dotabb —

```
"OM"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OM"];
"OM" -> "CATEGORY"
```

———-

— **OM.dotfull** —

```
"OpenMath()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OM"];
"OpenMath()" -> "Category"
```

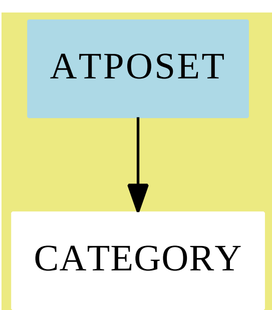
———-

— **OM.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"OpenMath()" [color=lightblue];
"OpenMath()" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.32 PartiallyOrderedSetAttribute (ATPOSET)



— **PartiallyOrderedSetAttribute.input** —

```
)set break resume
)sys rm -f PartiallyOrderedSetAttribute.output
)spool PartiallyOrderedSetAttribute.output
)set message test on
)set message auto off
```

```
)clear all

--S 1 of 1
)show PartiallyOrderedSetAttribute
--R
--R PartiallyOrderedSetAttribute is a category constructor
--R Abbreviation for PartiallyOrderedSetAttribute is ATPOSET
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ATPOSET
--R
--E 1
)spool
)lisp (bye)
```

———-

— PartiallyOrderedSetAttribute.help —

```
====================================================================
PartiallyOrderedSetAttribute
====================================================================

PartiallyOrderedSet is true if a set with < is transitive,
but not(a <b or a = b). It does not imply b < a.

See Also:
o )show PartiallyOrderedSetAttribute
```

———-

— PartiallyOrderedSetAttribute.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ATPOSET">
PartiallyOrderedSetAttribute (ATPOSET)</a></h2>

PartiallyOrderedSet is true if a set with &lt; is transitive,
but not(a &lt; b or a = b). It does not imply b &lt; a
</body>
```

———-

— category ATPOSET PartiallyOrderedSetAttribute —

```
)abbrev category ATPOSET PartiallyOrderedSetAttribute
++ Description:
++ PartiallyOrderedSet is true if a set with \spad{<} is transitive,
++ but \spad{not(a <b or a = b)}. It does not imply \spad{b < a}

PartiallyOrderedSetAttribute() : Category == SIG where
```

```
  SIG ==> with nil
```

— **COQ ATPOSET** —

```
(* category ATPOSET *)
(*
Axiom
  PartiallyOrderedSet is true if a set with < is transitive,
  but not(a <b or a = b). It does not imply b < a

*)
```

— **ATPOSET.dotabb** —

```
"ATPOSET"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATPOSET"];
"ATPOSET" -> "CATEGORY"
```

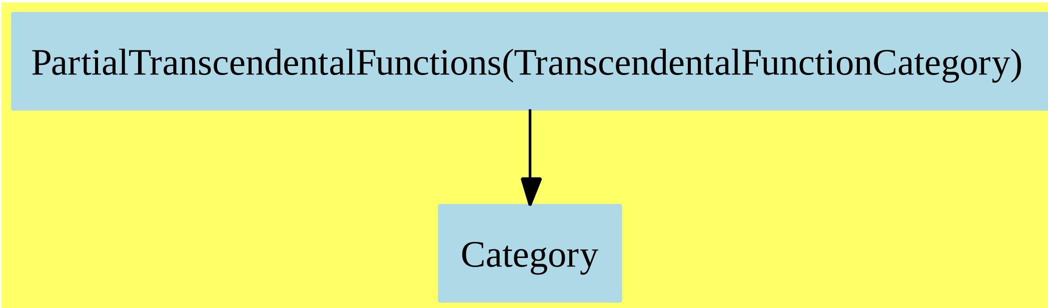
— **ATPOSET.dotfull** —

```
"PartiallyOrderedSetAttribute()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATPOSET"];
"PartiallyOrderedSetAttribute()" -> "Category"
```

— **ATPOSET.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"PartiallyOrderedSetAttribute()" [color=lightblue];
"PartiallyOrderedSetAttribute()" -> "Category"

"Category" [color=lightblue];
}
```

2.0.33 PartialTranscendentalFunctions (PTRANFN)



— **PartialTranscendentalFunctions.input** —

```
)set break resume
)sys rm -f PartialTranscendentalFunctions.output
)spool PartialTranscendentalFunctions.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PartialTranscendentalFunctions
--R
--R PartialTranscendentalFunctions(K: TranscendentalFunctionCategory) is a category constructor
--R Abbreviation for PartialTranscendentalFunctions is PTRANFN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PTRANFN
--R
--R------------------------------- Operations --------------------------------
--R acosIfCan : K -> Union(K,"failed")    acoshIfCan : K -> Union(K,"failed")
--R acotIfCan : K -> Union(K,"failed")    acothIfCan : K -> Union(K,"failed")
--R acscIfCan : K -> Union(K,"failed")    acschIfCan : K -> Union(K,"failed")
--R asecIfCan : K -> Union(K,"failed")    asechIfCan : K -> Union(K,"failed")
--R asinIfCan : K -> Union(K,"failed")    asinhIfCan : K -> Union(K,"failed")
--R atanIfCan : K -> Union(K,"failed")    atanhIfCan : K -> Union(K,"failed")
--R cosIfCan : K -> Union(K,"failed")     coshIfCan : K -> Union(K,"failed")
--R cotIfCan : K -> Union(K,"failed")     cothIfCan : K -> Union(K,"failed")
--R cscIfCan : K -> Union(K,"failed")     cschIfCan : K -> Union(K,"failed")
--R expIfCan : K -> Union(K,"failed")     logIfCan : K -> Union(K,"failed")
--R secIfCan : K -> Union(K,"failed")     sechIfCan : K -> Union(K,"failed")
--R sinIfCan : K -> Union(K,"failed")     sinhIfCan : K -> Union(K,"failed")
--R tanIfCan : K -> Union(K,"failed")     tanhIfCan : K -> Union(K,"failed")
--R nthRootIfCan : (K,NonNegativeInteger) -> Union(K,"failed")
--R
--E 1

)spool
)lisp (bye)
```

————

— PartialTranscendentalFunctions.help —

```
====================================================================
PartialTranscendentalFunctions examples
====================================================================

This is the description of any package which provides partial
functions on a domain belonging to TranscendentalFunctionCategory.

See Also:
o )show PartialTranscendentalFunctions
```

———-

See:

⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

acosIfCan	acoshIfCan	acotIfCan	acothIfCan	acscIfCan
acschIfCan	asecIfCan	asechIfCan	asinIfCan	asinhIfCan
atanIfCan	atanhIfCan	cosIfCan	coshIfCan	cotIfCan
cothIfCan	cscIfCan	cschIfCan	expIfCan	logIfCan
nthRootIfCan	secIfCan	sechIfCan	sinIfCan	sinhIfCan
tanIfCan	tanhIfCan			

These are directly exported but not implemented:

```
acosIfCan : K -> Union(K,"failed")
acoshIfCan : K -> Union(K,"failed")
acotIfCan : K -> Union(K,"failed")
acothIfCan : K -> Union(K,"failed")
acscIfCan : K -> Union(K,"failed")
acschIfCan : K -> Union(K,"failed")
asecIfCan : K -> Union(K,"failed")
asechIfCan : K -> Union(K,"failed")
asinIfCan : K -> Union(K,"failed")
asinhIfCan : K -> Union(K,"failed")
atanIfCan : K -> Union(K,"failed")
atanhIfCan : K -> Union(K,"failed")
cosIfCan : K -> Union(K,"failed")
coshIfCan : K -> Union(K,"failed")
cotIfCan : K -> Union(K,"failed")
cothIfCan : K -> Union(K,"failed")
cscIfCan : K -> Union(K,"failed")
cschIfCan : K -> Union(K,"failed")
expIfCan : K -> Union(K,"failed")
logIfCan : K -> Union(K,"failed")
nthRootIfCan : (K,NonNegativeInteger) -> Union(K,"failed")
secIfCan : K -> Union(K,"failed")
sechIfCan : K -> Union(K,"failed")
sinIfCan : K -> Union(K,"failed")
sinhIfCan : K -> Union(K,"failed")
```

```
 tanIfCan : K -> Union(K,"failed")
 tanhIfCan : K -> Union(K,"failed")
```

— PartialTranscendentalFunctions.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#PTRANFN">
PartialTranscendentalFunctions (PTRANFN)</a></h2>
&bull; K : TranscendentalFunctionCategory
<br/><br/>
A package which provides partial transcendental
functions, for example, functions which return an answer or "failed"
This is the description of any package which provides partial
functions on a domain belonging to TranscendentalFunctionCategory.
<br/><br/><br/>

nthRootIfCan : (K,NNI) -> Union(K,"failed")<br/>
  nthRootIfCan(z,n) returns the nth root of z if possible,<br/>
  and "failed" otherwise.<br/><br/>

expIfCan : K -> Union(K,"failed")<br/>
  expIfCan(z) returns exp(z) if possible, and "failed" otherwise.<br/><br/>

logIfCan : K -> Union(K,"failed")<br/>
  logIfCan(z) returns log(z) if possible, and "failed" otherwise.<br/><br/>

sinIfCan : K -> Union(K,"failed")<br/>
  sinIfCan(z) returns sin(z) if possible, and "failed" otherwise.<br/><br/>

cosIfCan : K -> Union(K,"failed")<br/>
  cosIfCan(z) returns cos(z) if possible, and "failed" otherwise.<br/><br/>

tanIfCan : K -> Union(K,"failed")<br/>
  tanIfCan(z) returns tan(z) if possible, and "failed" otherwise.<br/><br/>

cotIfCan : K -> Union(K,"failed")<br/>
  cotIfCan(z) returns cot(z) if possible, and "failed" otherwise.<br/><br/>

secIfCan : K -> Union(K,"failed")<br/>
  secIfCan(z) returns sec(z) if possible, and "failed" otherwise.<br/><br/>

cscIfCan : K -> Union(K,"failed")<br/>
  cscIfCan(z) returns csc(z) if possible, and "failed" otherwise.<br/><br/>

asinIfCan : K -> Union(K,"failed")<br/>
  asinIfCan(z) returns asin(z) if possible, and "failed" otherwise.<br/><br/>

acosIfCan : K -> Union(K,"failed")<br/>
  acosIfCan(z) returns acos(z) if possible, and "failed" otherwise.<br/><br/>

atanIfCan : K -> Union(K,"failed")<br/>
  atanIfCan(z) returns atan(z) if possible, and "failed" otherwise.<br/><br/>
```

```
acotIfCan : K -> Union(K,"failed")<br/>
  acotIfCan(z) returns acot(z) if possible, and "failed" otherwise.<br/><br/>

asecIfCan : K -> Union(K,"failed")<br/>
  asecIfCan(z) returns asec(z) if possible, and "failed" otherwise.<br/><br/>

acscIfCan : K -> Union(K,"failed")<br/>
  acscIfCan(z) returns acsc(z) if possible, and "failed" otherwise.<br/><br/>

sinhIfCan : K -> Union(K,"failed")<br/>
  sinhIfCan(z) returns sinh(z) if possible, and "failed" otherwise.<br/><br/>

coshIfCan : K -> Union(K,"failed")<br/>
  coshIfCan(z) returns cosh(z) if possible, and "failed" otherwise.<br/><br/>

tanhIfCan : K -> Union(K,"failed")<br/>
  tanhIfCan(z) returns tanh(z) if possible, and "failed" otherwise.<br/><br/>

cothIfCan : K -> Union(K,"failed")<br/>
  cothIfCan(z) returns coth(z) if possible, and "failed" otherwise.<br/><br/>

sechIfCan : K -> Union(K,"failed")<br/>
  sechIfCan(z) returns sech(z) if possible, and "failed" otherwise.<br/><br/>

cschIfCan : K -> Union(K,"failed")<br/>
  cschIfCan(z) returns csch(z) if possible, and "failed" otherwise.<br/><br/>

asinhIfCan : K -> Union(K,"failed")<br/>
  asinhIfCan(z) returns asinh(z) if possible, and "failed" otherwise.<br/><br/>

acoshIfCan : K -> Union(K,"failed")<br/>
  acoshIfCan(z) returns acosh(z) if possible, and "failed" otherwise.<br/><br/>

atanhIfCan : K -> Union(K,"failed")<br/>
  atanhIfCan(z) returns atanh(z) if possible, and "failed" otherwise.<br/><br/>

acothIfCan : K -> Union(K,"failed")<br/>
  acothIfCan(z) returns acoth(z) if possible, and "failed" otherwise.<br/><br/>

asechIfCan : K -> Union(K,"failed")<br/>
  asechIfCan(z) returns asech(z) if possible, and "failed" otherwise.<br/><br/>

acschIfCan : K -> Union(K,"failed")<br/>
  acschIfCan(z) returns acsch(z) if possible, and "failed" otherwise.<br/><br/>

</body>
```

— category PTRANFN PartialTranscendentalFunctions —

```
)abbrev category PTRANFN PartialTranscendentalFunctions
++ Author: Clifton J. Williamson
++ Date Created: 12 February 1990
```

```
++ Date Last Updated: 14 February 1990
++ Description:
++ A package which provides partial transcendental
++ functions, for example, functions which return an answer or "failed"
++ This is the description of any package which provides partial
++ functions on a domain belonging to TranscendentalFunctionCategory.

PartialTranscendentalFunctions(K) : Category == SIG where
  K : TranscendentalFunctionCategory

  NNI ==> NonNegativeInteger

  SIG ==> with

    --% Exponentials and Logarithms

    nthRootIfCan : (K,NNI) -> Union(K,"failed")
      ++ nthRootIfCan(z,n) returns the nth root of z if possible,
      ++ and "failed" otherwise.

    expIfCan : K -> Union(K,"failed")
      ++ expIfCan(z) returns exp(z) if possible, and "failed" otherwise.

    logIfCan : K -> Union(K,"failed")
      ++ logIfCan(z) returns log(z) if possible, and "failed" otherwise.

    --% TrigonometricFunctionCategory

    sinIfCan : K -> Union(K,"failed")
      ++ sinIfCan(z) returns sin(z) if possible, and "failed" otherwise.

    cosIfCan : K -> Union(K,"failed")
      ++ cosIfCan(z) returns cos(z) if possible, and "failed" otherwise.

    tanIfCan : K -> Union(K,"failed")
      ++ tanIfCan(z) returns tan(z) if possible, and "failed" otherwise.

    cotIfCan : K -> Union(K,"failed")
      ++ cotIfCan(z) returns cot(z) if possible, and "failed" otherwise.

    secIfCan : K -> Union(K,"failed")
      ++ secIfCan(z) returns sec(z) if possible, and "failed" otherwise.

    cscIfCan : K -> Union(K,"failed")
      ++ cscIfCan(z) returns csc(z) if possible, and "failed" otherwise.

    --% ArcTrigonometricFunctionCategory

    asinIfCan : K -> Union(K,"failed")
      ++ asinIfCan(z) returns asin(z) if possible, and "failed" otherwise.

    acosIfCan : K -> Union(K,"failed")
      ++ acosIfCan(z) returns acos(z) if possible, and "failed" otherwise.
```

```
atanIfCan : K -> Union(K,"failed")
  ++ atanIfCan(z) returns atan(z) if possible, and "failed" otherwise.

acotIfCan : K -> Union(K,"failed")
  ++ acotIfCan(z) returns acot(z) if possible, and "failed" otherwise.

asecIfCan : K -> Union(K,"failed")
  ++ asecIfCan(z) returns asec(z) if possible, and "failed" otherwise.

acscIfCan : K -> Union(K,"failed")
  ++ acscIfCan(z) returns acsc(z) if possible, and "failed" otherwise.

--% HyperbolicFunctionCategory

sinhIfCan : K -> Union(K,"failed")
  ++ sinhIfCan(z) returns sinh(z) if possible, and "failed" otherwise.

coshIfCan : K -> Union(K,"failed")
  ++ coshIfCan(z) returns cosh(z) if possible, and "failed" otherwise.

tanhIfCan : K -> Union(K,"failed")
  ++ tanhIfCan(z) returns tanh(z) if possible, and "failed" otherwise.

cothIfCan : K -> Union(K,"failed")
  ++ cothIfCan(z) returns coth(z) if possible, and "failed" otherwise.

sechIfCan : K -> Union(K,"failed")
  ++ sechIfCan(z) returns sech(z) if possible, and "failed" otherwise.

cschIfCan : K -> Union(K,"failed")
  ++ cschIfCan(z) returns csch(z) if possible, and "failed" otherwise.

--% ArcHyperbolicFunctionCategory

asinhIfCan : K -> Union(K,"failed")
  ++ asinhIfCan(z) returns asinh(z) if possible, and "failed" otherwise.

acoshIfCan : K -> Union(K,"failed")
  ++ acoshIfCan(z) returns acosh(z) if possible, and "failed" otherwise.

atanhIfCan : K -> Union(K,"failed")
  ++ atanhIfCan(z) returns atanh(z) if possible, and "failed" otherwise.

acothIfCan : K -> Union(K,"failed")
  ++ acothIfCan(z) returns acoth(z) if possible, and "failed" otherwise.

asechIfCan : K -> Union(K,"failed")
  ++ asechIfCan(z) returns asech(z) if possible, and "failed" otherwise.

acschIfCan : K -> Union(K,"failed")
  ++ acschIfCan(z) returns acsch(z) if possible, and "failed" otherwise.
```

— **PTRANFN.dotabb** —

```
"PTRANFN"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PTRANFN"];
"PTRANFN" -> "CATEGORY"
```

————

— **PTRANFN.dotfull** —

```
"PartialTranscendentalFunctions(TranscendentalFunctionCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PTRANFN"];
"PartialTranscendentalFunctions(TranscendentalFunctionCategory)" ->
   "Category()"
```

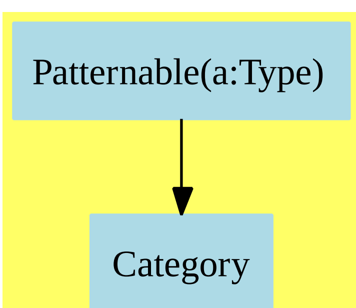
————

— **PTRANFN.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"PartialTranscendentalFunctions(TranscendentalFunctionCategory)"
 [color=lightblue];
"PartialTranscendentalFunctions(TranscendentalFunctionCategory)" ->
   "Category"

"Category" [color=lightblue];

}
```

————

2.0.34 Patternable (PATAB)



— Patternable.input —

```
)set break resume
)sys rm -f Patternable.output
)spool Patternable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Patternable
--R
--R Patternable(R: Type) is a category constructor
--R Abbreviation for Patternable is PATAB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PATAB
--R
--R------------------------------- Operations --------------------------------
--R convert : % -> Pattern(Integer) if R has KONVERT(PATTERN(INT))
--R convert : % -> Pattern(Float) if R has KONVERT(PATTERN(FLOAT))
--R
--E 1

)spool
)lisp (bye)
```

— Patternable.help —

```
====================================================================
Patternable examples
====================================================================

Category of sets that can be converted to useful patterns. An object
S is Patternable over an object R if S can lift the conversions from
R into Pattern(Integer) and Pattern(Float) to itself.

See Also:
o )show Patternable
```

See:

⇒ "ComplexCategory" (COMPCAT) 20.0.240 on page 1869
⇒ "FunctionSpace" (FS) 17.0.219 on page 1544
⇒ "IntegerNumberSystem" (INS) 16.0.210 on page 1420
⇒ "QuotientFieldCategory" (QFCAT) 17.0.222 on page 1595
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

convert

These exports come from (p47) ConvertibleTo(Pattern(Integer)):

```
convert : % -> Pattern Integer if R has KONVERT PATTERN INT
```

These exports come from (p47) ConvertibleTo(Pattern(Float)):

```
convert : % -> Pattern Float if R has KONVERT PATTERN FLOAT
```

— **Patternable.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#PATAB">
Patternable (PATAB)</a></h2>

Category of sets that can be converted to useful patterns.
An object S is Patternable over an object R if S can
lift the conversions from R into Pattern(Integer) and
Pattern(Float) to itself.
<br/><br/><br/>

convert : % -> Pattern Float if R has ConvertibleTo Pattern Float<br/>
  from ConvertibleTo Pattern Float<br/><br/>

convert : % -> Pattern Integer if R has ConvertibleTo Pattern Integer<br/>
  from ConvertibleTo Pattern Integer
</body>
```

————

— **category PATAB Patternable** —

```
)abbrev category PATAB Patternable
++ Author: Manuel Bronstein
++ Date Created: 29 Nov 1989
++ Date Last Updated: 29 Nov 1989
++ Description:
++ Category of sets that can be converted to useful patterns
++ An object S is Patternable over an object R if S can
++ lift the conversions from R into \spadtype{Pattern(Integer)} and
++ \spadtype{Pattern(Float)} to itself;

Patternable(R) : Category == SIG where
  R : Type

  SIG ==> with

    if R has ConvertibleTo Pattern Integer then
             ConvertibleTo Pattern Integer

    if R has ConvertibleTo Pattern Float then
             ConvertibleTo Pattern Float
```

————

— **PATAB.dotabb** —

```
"PATAB"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PATAB"];
"PATAB" -> "CATEGORY"
```

————

— **PATAB.dotfull** —

```
"Patternable(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PATAB"];
"Patternable(a:Type)" -> "Category"

"Patternable(IntegralDomain)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PATAB"];
"Patternable(IntegralDomain)" -> "Patternable(a:Type)"

"Patternable(OrderedSet)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PATAB"];
"Patternable(OrderedSet)" -> "Patternable(a:Type)"

"Patternable(CommutativeRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PATAB"];
"Patternable(CommutativeRing)" -> "Patternable(a:Type)"
```

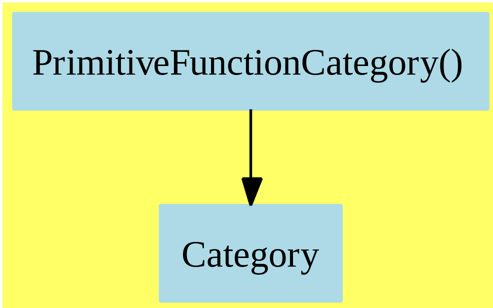
————

— **PATAB.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"Patternable(a:Type)" [color=lightblue];
"Patternable(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

————

2.0.35 PrimitiveFunctionCategory (PRIMCAT)



— **PrimitiveFunctionCategory.input** —

```
)set break resume
)sys rm -f PrimitiveFunctionCategory.output
)spool PrimitiveFunctionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PrimitiveFunctionCategory
--R
--R PrimitiveFunctionCategory is a category constructor
--R Abbreviation for PrimitiveFunctionCategory is PRIMCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PRIMCAT
--R
--R------------------------------- Operations --------------------------------
--R integral : (%,Symbol) -> %
--R integral : (%,SegmentBinding(%)) -> %
--R
--E 1

)spool
)lisp (bye)
```

— **PrimitiveFunctionCategory.help** —

```
====================================================================
PrimitiveFunctionCategory examples
====================================================================

This is the  Category for the functions defined by integrals.

See Also:
o )show PrimitiveFunctionCategory
```

See:

⇒ "LiouvillianFunctionCategory" (LFCAT) 4.0.75 on page 295
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

integral

These are directly exported but not implemented:

```
integral : (%,Symbol) -> %
integral : (%,SegmentBinding %) -> %
```

— PrimitiveFunctionCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#PRIMCAT">
PrimitiveFunctionCategory (PRIMCAT)</a></h2>

Category for the functions defined by integrals.
<br/><br/><br/>

integral : (%, Symbol) -> %<br/>
  integral(f, x) returns the formal integral of f dx.<br/><br/>

integral : (%, SegmentBinding %) -> %<br/>
  integral(f, x = a..b) returns the formal definite integral<br/>
  of f dx for x between a and b.

</body>
```

— category PRIMCAT PrimitiveFunctionCategory —

```
)abbrev category PRIMCAT PrimitiveFunctionCategory
++ Author: Manuel Bronstein
++ Date Last Updated: 14 May 1991
++ Description:
++ Category for the functions defined by integrals;

PrimitiveFunctionCategory() : Category == SIG where

  SIG ==> with

    integral : ($, Symbol) -> $
      ++ integral(f, x) returns the formal integral of f dx.

    integral : ($, SegmentBinding $) -> $
      ++ integral(f, x = a..b) returns the formal definite integral
      ++ of f dx for x between \spad{a} and b.
```

———-

— PRIMCAT.dotabb —

```
"PRIMCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PRIMCAT"];
"PRIMCAT" -> "CATEGORY"
```

———-

— PRIMCAT.dotfull —

```
"PrimitiveFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PRIMCAT"];
"PrimitiveFunctionCategory()" -> "Category"
```

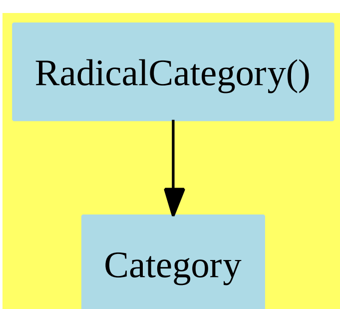
———-

— PRIMCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"PrimitiveFunctionCategory()" [color=lightblue];
"PrimitiveFunctionCategory()" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.36 RadicalCategory (RADCAT)



— **RadicalCategory.input** —

```
)set break resume
)sys rm -f RadicalCategory.output
)spool RadicalCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RadicalCategory
--R
--R RadicalCategory is a category constructor
--R Abbreviation for RadicalCategory is RADCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for RADCAT
--R
--R------------------------------ Operations -------------------------------
--R ?**? : (%,Fraction(Integer)) -> %     nthRoot : (%,Integer) -> %
--R sqrt : % -> %
--R
--E 1

)spool
)lisp (bye)
```

————

— **RadicalCategory.help** —

```
====================================================================
RadicalCategory examples
====================================================================

The RadicalCategory is a model for the rational numbers.

See Also:
o )show RadicalCategory
```

————

See:

⇒ "AlgebraicallyClosedField" (ACF) 17.0.215 on page 1497
⇒ "IntervalCategory" (INTCAT) 14.0.200 on page 1341
⇒ "RealClosedField" (RCFIELD) 17.0.223 on page 1611
⇒ "RealNumberSystem" (RNS) 17.0.224 on page 1622
⇒ "UnivariateLaurentSeriesCategory" (ULSCAT) 17.0.226 on page 1686
⇒ "UnivariatePuiseuxSeriesCategory" (UPXSCAT) 17.0.227 on page 1697
⇒ "UnivariateTaylorSeriesCategory" (UTSCAT) 16.0.214 on page 1476
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

nthRoot sqrt ?**?

These are directly exported but not implemented:

```
?**? : (%,Fraction Integer) -> %
```

These are implemented by this category:

```
nthRoot : (%,Integer) -> %
sqrt : % -> %
```

— RadicalCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#RADCAT">
RadicalCategory (RADCAT)</a></h2>

The RadicalCategory is a model for the rational numbers.
<br/><br/><br/>

sqrt : % -> %<br/>
  sqrt(x) returns the square root of x. The branch cut lies along<br/>
  the negative real axis, continuous with quadrant II.<br/><br/>

nthRoot : (%, Integer) -> %<br/>
  nthRoot(x,n) returns the nth root of x.<br/><br/>

** : (%, Fraction Integer) -> %<br/>
  x ** y is the rational exponentiation of x by the power y.<br/><br/>
</body>
```

— category RADCAT RadicalCategory —

```
)abbrev category RADCAT RadicalCategory
++ Description: The RadicalCategory is a model for the rational numbers.

RadicalCategory() : Category == SIG where

  SIG ==> with

    sqrt : % -> %
      ++ sqrt(x) returns the square root of x. The branch cut lies along
      ++ the negative real axis, continuous with quadrant II.

    nthRoot : (%, Integer) -> %
      ++ nthRoot(x,n) returns the nth root of x.

    _*_* : (%, Fraction Integer) -> %
      ++ x ** y is the rational exponentiation of x by the power y.

   add

     sqrt x == x ** inv(2::Fraction(Integer))
```

```
nthRoot(x, n) == x ** inv(n::Fraction(Integer))
```

———-

— **COQ RADCAT** —

```
(* category RADCAT *)
(*
  sqrt : % -> %
  sqrt x == x ** inv(2::Fraction(Integer))

  nthRoot: (%, Integer) -> %
  nthRoot(x, n) == x ** inv(n::Fraction(Integer))

*)
```

———-

— **RADCAT.dotabb** —

```
"RADCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RADCAT"];
"RADCAT" -> "CATEGORY"
```

———-

— **RADCAT.dotfull** —

```
"RadicalCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RADCAT"];
"RadicalCategory()" -> "Category"
```

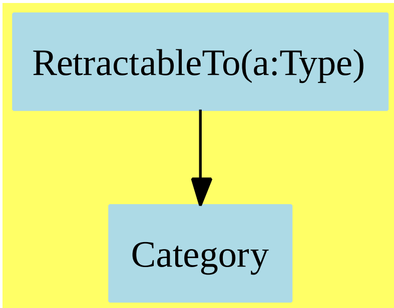
———-

— **RADCAT.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"RadicalCategory()" [color=lightblue];
"RadicalCategory()" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.37 RetractableTo (RETRACT)



— **RetractableTo.input** —

```
)set break resume
)sys rm -f RetractableTo.output
)spool RetractableTo.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RetractableTo
--R
--R RetractableTo(S: Type) is a category constructor
--R Abbreviation for RetractableTo is RETRACT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for RETRACT
--R
--R------------------------------- Operations --------------------------------
--R coerce : S -> %                       retract : % -> S
--R retractIfCan : % -> Union(S,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

— **RetractableTo.help** —

```
====================================================================
RetractableTo examples
====================================================================

A is retractable to B means that some elementsif A can be converted
into elements of B and any element of B can be converted into an
element of A.

See Also:
```

```
o )show RetractableTo
```

———-

See:

⇒ "DifferentialPolynomialCategory" (DPOLCAT) 17.0.216 on page 1512
⇒ "DifferentialVariableCategory" (DVARCAT) 5.0.95 on page 411
⇒ "ExtensionField" (XF) 18.0.230 on page 1754
⇒ "ExpressionSpace" (ES) 5.0.96 on page 418
⇒ "FiniteAlgebraicExtensionField" (FAXF) 19.0.237 on page 1829
⇒ "FortranMachineTypeCategory" (FMTC) 13.0.195 on page 1295
⇒ "FreeAbelianMonoidCategory" (FAMONC) 7.0.127 on page 719
⇒ "FreeModuleCat" (FMCAT) 10.0.164 on page 1034
⇒ "FullyRetractableTo" (FRETRCT) 3.0.50 on page 158
⇒ "FunctionSpace" (FS) 17.0.219 on page 1544
⇒ "GradedAlgebra" (GRALG) 5.0.97 on page 436
⇒ "IntegerNumberSystem" (INS) 16.0.210 on page 1420
⇒ "IntervalCategory" (INTCAT) 14.0.200 on page 1341
⇒ "PolynomialCategory" (POLYCAT) 16.0.213 on page 1449
⇒ "QuotientFieldCategory" (QFCAT) 17.0.222 on page 1595
⇒ "RealNumberSystem" (RNS) 17.0.224 on page 1622
⇒ "UnivariateLaurentSeriesConstructorCategory" (ULSCCAT) 18.0.235 on page 1801
⇒ "UnivariatePuiseuxSeriesConstructorCategory" (UPXSCCA) 18.0.236 on page 1817
⇒ "XFreeAlgebra" (XFALG) 11.0.183 on page 1164
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

coerce retract retractIfCan

These are directly exported but not implemented:

```
coerce : S -> %
retractIfCan : % -> Union(S,"failed")
```

These are implemented by this category:

```
retract : % -> S
```

— RetractableTo.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#RETRACT">
RetractableTo(S) (RETRACT)</a></h2>
&bull; S : Type
<br/><br/>
A is retractable to B means that some elements if A can be converted
into elements of B and any element of B can be converted into an
element of A.
<br/><br/><br/>
coerce : S -> %<br/>
  coerce(a) transforms a into an element of %.<br/><br/>
```

```
retractIfCan : % -> Union(S,"failed")<br/>
  retractIfCan(a) transforms a into an element of S if possible.<br/>
  Returns "failed" if a cannot be made into an element of S.<br/><br/>

retract : % -> S<br/>
  retract(a) transforms a into an element of S if possible.<br/>
  Error: if a cannot be made into an element of S.
</body>
```

—— category RETRACT RetractableTo ——

```
)abbrev category RETRACT RetractableTo
++ Date Last Updated: 14 May 1991
++ Description:
++ A is retractable to B means that some elements if A can be converted
++ into elements of B and any element of B can be converted into an
++ element of A.

RetractableTo(S) : Category == SIG where
  S : Type

  SIG ==> with

    coerce : S -> %
      ++ coerce(a) transforms a into an element of %.

    retractIfCan : % -> Union(S,"failed")
      ++ retractIfCan(a) transforms a into an element of S if possible.
      ++ Returns "failed" if a cannot be made into an element of S.

    retract : % -> S
      ++ retract(a) transforms a into an element of S if possible.
      ++ Error: if a cannot be made into an element of S.

   add

     retract(s) ==
       (u:=retractIfCan s) case "failed" => error "not retractable"
       u
```

—— COQ RETRACT ——

```
(* category RETRACT *)
(*
    retract: % -> S
    retract(s) ==
      (u:=retractIfCan s) case "failed" => error "not retractable"
      u
```

```
*)
```

— **RETRACT.dotabb** —

```
"RETRACT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RETRACT" -> "CATEGORY"
```

— **RETRACT.dotfull** —

```
"RetractableTo(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(a:Type)" -> "Category"

"RetractableTo(SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(SetCategory)" -> "RetractableTo(a:Type)"

"RetractableTo(OrderedSet)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(OrderedSet)" -> "RetractableTo(a:Type)"

"RetractableTo(Symbol)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(Symbol)" -> "RetractableTo(a:Type)"

"RetractableTo(Integer)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(Integer)" -> "RetractableTo(a:Type)"

"RetractableTo(NonNegativeInteger)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(NonNegativeInteger)" -> "RetractableTo(a:Type)"

"RetractableTo(Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(Fraction(Integer))" -> "RetractableTo(a:Type)"

"RetractableTo(Float)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(Float)" -> "RetractableTo(a:Type)"

"RetractableTo(Kernel(ExpressionSpace))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(Kernel(ExpressionSpace))" -> "RetractableTo(a:Type)"

"RetractableTo(CommutativeRing)"
```

```
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(CommutativeRing)" -> "RetractableTo(a:Type)"

"RetractableTo(UnivariatePuiseuxSeriesCategory(Ring))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(UnivariatePuiseuxSeriesCategory(Ring))"
  -> "RetractableTo(a:Type)"

"RetractableTo(Field)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(Field)" -> "RetractableTo(a:Type)"

"RetractableTo(IntegralDomain)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(IntegralDomain)" -> "RetractableTo(a:Type)"

"RetractableTo(OrderedFreeMonoid(OrderedSet))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(OrderedFreeMonoid(OrderedSet))" -> "RetractableTo(a:Type)"
```

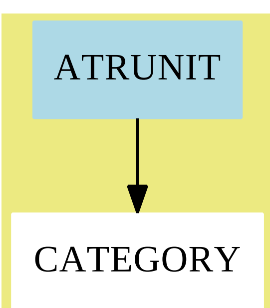
———-

— **RETRACT.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.38 RightUnitaryAttribute (ATRUNIT)



— RightUnitaryAttribute.input —

```
)set break resume
)sys rm -f RightUnitaryAttribute.output
)spool RightUnitaryAttribute.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
--R
--R RightUnitaryAttribute is a category constructor
--R Abbreviation for RightUnitaryAttribute is ATRUNIT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ATRUNIT
--R
--E 1
)spool
)lisp (bye)
```

— RightUnitaryAttribute.help —

```
====================================================================
RightUnitaryAttribute
====================================================================

RightUnitary is true if x * 1 = x for all x.

See Also:
o )show RightUnitaryAttribute
```

— RightUnitaryAttribute.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ATRUNIT">
RightUnitaryAttribute (ATRUNIT)</a></h2>

RightUnitary is true if x * 1 = x for all x.
</body>
```

— category ATRUNIT RightUnitaryAttribute —

```
)abbrev category ATRUNIT RightUnitaryAttribute
++ Description:
++ RightUnitary is true if \spad{x * 1 = x} for all x.
```

```
RightUnitaryAttribute() : Category == SIG where

  SIG ==> with nil
```

———-

— COQ ATRUNIT —

```
(* category ATRUNIT *)
(*
Axiom
   RightUnitary is true if x * 1 = x for all x.

*)
```

———-

— ATRUNIT.dotabb —

```
"ATRUNIT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATRUNIT"];
"ATRUNIT" -> "CATEGORY"
```

———-

— ATRUNIT.dotfull —

```
"RightUnitaryAttribute()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATRUNIT"];
"RightUnitaryAttribute()" -> "Category"
```

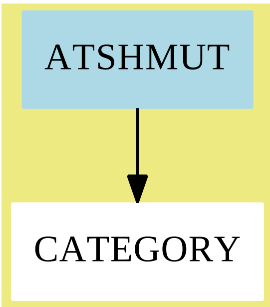
———-

— ATRUNIT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"RightUnitaryAttribute()" [color=lightblue];
"RightUnitaryAttribute()" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.39 ShallowlyMutableAttribute (ATSHMUT)



— ShallowlyMutableAttribute.input —

```
)set break resume
)sys rm -f ShallowlyMutableAttribute.output
)spool ShallowlyMutableAttribute.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ShallowlyMutableAttribute
--R
--R ShallowlyMutableAttribute is a category constructor
--R Abbreviation for ShallowlyMutableAttribute is ATSHMUT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ATSHMUT
--R
--E 1
)spool
)lisp (bye)
```

———-

— ShallowlyMutableAttribute.help —

```
====================================================================
ShallowlyMutableAttribute
====================================================================

The class of all domains which have immediate components that
are updateable in place (mutable). The properties of any component
domain are irrevelant to the ShallowlyMutableAttribute.

See Also:
o )show ShallowlyMutableAttribute
```

———-

— ShallowlyMutableAttribute.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ATSHMUT">
ShallowlyMutableAttribute (ATSHMUT)</a></h2>

The class of all domains which have immediate components that
are updateable in place (mutable). The properties of any component
domain are irrevelant to the ShallowlyMutableAttribute.

</body>
```

————

— category ATSHMUT ShallowlyMutableAttribute —

```
)abbrev category ATSHMUT ShallowlyMutableAttribute
++ Description:
++ The class of all domains which have immediate components that
++ are updateable in place (mutable). The properties of any component
++ domain are irrevelant to the ShallowlyMutableAttribute.

ShallowlyMutableAttribute() : Category == SIG where

  SIG ==> with nil
```

————

— ATSHMUT.dotabb —

```
"ATSHMUT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATSHMUT"];
"ATSHMUT" -> "CATEGORY"
```

————

— ATSHMUT.dotfull —

```
"ShallowlyMutableAttribute()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATSHMUT"];
"ShallowlyMutableAttribute()" -> "Category"
```

————

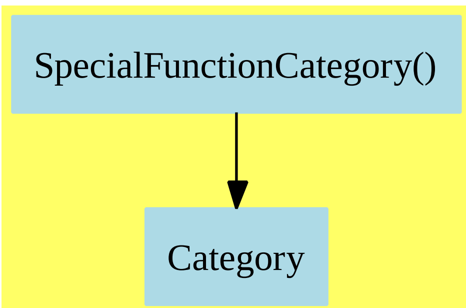
— ATSHMUT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];
```

```
"ShallowlyMutableAttribute()" [color=lightblue];
"ShallowlyMutableAttribute()" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.40 SpecialFunctionCategory (SPFCAT)



— **SpecialFunctionCategory.input** —

```
)set break resume
)sys rm -f SpecialFunctionCategory.output
)spool SpecialFunctionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SpecialFunctionCategory
--R
--R SpecialFunctionCategory is a category constructor
--R Abbreviation for SpecialFunctionCategory is SPFCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SPFCAT
--R
--R------------------------------- Operations --------------------------------
--R Beta : (%,%) -> %                      Gamma : % -> %
--R Gamma : (%,%) -> %                     abs : % -> %
--R airyAi : % -> %                        airyBi : % -> %
--R besselI : (%,%) -> %                   besselJ : (%,%) -> %
--R besselK : (%,%) -> %                   besselY : (%,%) -> %
--R digamma : % -> %                       polygamma : (%,%) -> %
--R
--E 1

)spool
```

```
)lisp (bye)
```

———-

— SpecialFunctionCategory.help —

```
====================================================================
SpecialFunctionCategory examples
====================================================================

This is the  Category for the other special functions.

See Also:
o )show SpecialFunctionCategory
```

———-

See:

⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

abs	airyAi	airyBi	besselI	besselJ
besselK	besselY	Beta	digamma	Gamma
polygamma				

These are directly exported but not implemented:

```
abs : % -> %
airyAi : % -> %
airyBi : % -> %
besselI : (%,%) -> %
besselJ : (%,%) -> %
besselK : (%,%) -> %
besselY : (%,%) -> %
Beta : (%,%) -> %
digamma : % -> %
Gamma : % -> %
Gamma : (%,%) -> %
polygamma : (%,%) -> %
```

— SpecialFunctionCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#SPFCAT">
SpecialFunctionCategory (SPFCAT)</a></h2>

Category for the other special functions.
<br/><br/><br/>
abs : $ -> $<br/>
  abs(x) returns the absolute value of x.<br/><br/>

Gamma : $ -> $<br/>
```

```
  Gamma(x) is the Euler Gamma function.<br/><br/>

Beta : ($,$)->$<br/>
  Beta(x,y) is Gamma(x) * Gamma(y)/Gamma(x+y).<br/><br/>

digamma : $ -> $<br/>
  digamma(x) is the logarithmic derivative of Gamma(x)<br/>
  (often written psi(x) in the literature).<br/><br/>

polygamma : ($, $) -> $<br/>
  polygamma(k,x) is the k-th derivative of digamma(x),<br/>
  (often written psi(k,x) in the literature).<br/><br/>

Gamma : ($, $) -> $<br/>
  Gamma(a,x) is the incomplete Gamma function.<br/><br/>

besselJ : ($,$) -> $<br/>
  besselJ(v,z) is the Bessel function of the first kind.<br/><br/>

besselY : ($,$) -> $<br/>
  besselY(v,z) is the Bessel function of the second kind.<br/><br/>

besselI : ($,$) -> $<br/>
 
besselI(v,z) is the modified Bessel function of the first kind.<br/><br/>

besselK : ($,$) -> $<br/>
 
besselK(v,z) is the modified Bessel function of the second kind.<br/><br/>

airyAi : $ -> $<br/>
  airyAi(x) is the Airy function Ai(x).<br/><br/>

airyBi : $ -> $<br/>
  airyBi(x) is the Airy function Bi(x).<br/><br/>
</body>
```

— category SPFCAT SpecialFunctionCategory —

```
)abbrev category SPFCAT SpecialFunctionCategory
++ Author: Manuel Bronstein
++ Date Last Updated: 11 May 1993
++ Description:
++ Category for the other special functions;

SpecialFunctionCategory() : Category == SIG where

  SIG ==> with

    abs : $ -> $
      ++ abs(x) returns the absolute value of x.
```

```
    Gamma : $ -> $
      ++ Gamma(x) is the Euler Gamma function.

    Beta : ($,$)->$
      ++ Beta(x,y) is \spad{Gamma(x) * Gamma(y)/Gamma(x+y)}.

    digamma : $ -> $
      ++ digamma(x) is the logarithmic derivative of \spad{Gamma(x)}
      ++ (often written \spad{psi(x)} in the literature).

    polygamma : ($, $) -> $
      ++ polygamma(k,x) is the \spad{k-th} derivative of \spad{digamma(x)},
      ++ (often written \spad{psi(k,x)} in the literature).

    Gamma : ($, $) -> $
      ++ Gamma(a,x) is the incomplete Gamma function.

    besselJ : ($,$) -> $
      ++ besselJ(v,z) is the Bessel function of the first kind.

    besselY : ($,$) -> $
      ++ besselY(v,z) is the Bessel function of the second kind.

    besselI : ($,$) -> $
      ++ besselI(v,z) is the modified Bessel function of the first kind.

    besselK : ($,$) -> $
      ++ besselK(v,z) is the modified Bessel function of the second kind.

    airyAi : $ -> $
      ++ airyAi(x) is the Airy function \spad{Ai(x)}.

    airyBi : $ -> $
      ++ airyBi(x) is the Airy function \spad{Bi(x)}.
```

— **SPFCAT.dotabb** —

```
"SPFCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SPFCAT"];
"SPFCAT" -> "CATEGORY"
```

— **SPFCAT.dotfull** —

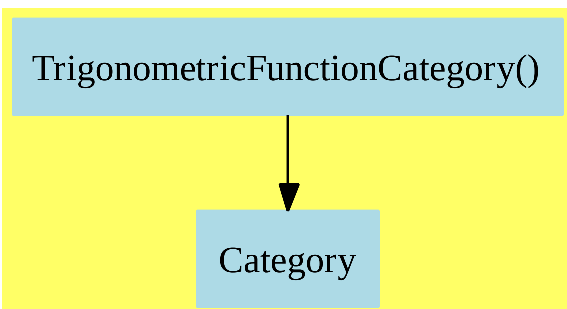
```
"SpecialFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SPFCAT"];
"SpecialFunctionCategory()" -> "Category"
```

— **SPFCAT.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"SpecialFunctionCategory()" [color=lightblue];
"SpecialFunctionCategory()" -> "Category"

"Category" [color=lightblue];
}
```

————-

2.0.41 TrigonometricFunctionCategory (TRIGCAT)



The `csc` and `sec` functions were modified to include an intermediate test to check that the argument has a reciprocal values.

— **TrigonometricFunctionCategory.input** —

```
)set break resume
)sys rm -f TrigonometricFunctionCategory.output
)spool TrigonometricFunctionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show TrigonometricFunctionCategory
--R
--R TrigonometricFunctionCategory is a category constructor
--R Abbreviation for TrigonometricFunctionCategory is TRIGCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for TRIGCAT
--R
--R------------------------------- Operations --------------------------------
--R cos : % -> %                          cot : % -> %
```

```
--R csc : % -> %                    sec : % -> %
--R sin : % -> %                    tan : % -> %
--R
--E 1

)spool
)lisp (bye)
```

— TrigonometricFunctionCategory.help —

```
====================================================================
TrigonometricFunctionCategory examples
====================================================================

This is the Category for the trigonometric functions.

See Also:
o )show TrigonometricFunctionCategory
```

See:

⇒ "TranscendentalFunctionCategory" (TRANFUN) 3.0.58 on page 192
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

cos cot csc sec sin tan

These are directly exported but not implemented:

```
cos : % -> %
sin : % -> %
```

These are implemented by this category:

```
cot : % -> %
csc : % -> %
sec : % -> %
tan : % -> %
```

— TrigonometricFunctionCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#TRIGCAT">
TrigonometricFunctionCategory (TRIGCAT)</a></h2>
Category for the trigonometric functions;
<br/><br/><br/>
cos : $ -> $<br/>
  cos(x) returns the cosine of x.<br/><br/>

cot : $ -> $<br/>
```

```
  cot(x) returns the cotangent of x.<br/><br/>

csc : $ -> $<br/>
  csc(x) returns the cosecant of x.<br/><br/>

sec : $ -> $<br/>
  sec(x) returns the secant of x.<br/><br/>

sin : $ -> $<br/>
  sin(x) returns the sine of x.<br/><br/>

tan : $ -> $<br/>
  tan(x) returns the tangent of x.<br/><br/>

</body>
```

— category TRIGCAT TrigonometricFunctionCategory —

```
)abbrev category TRIGCAT TrigonometricFunctionCategory
++ Date Last Updated: 14 May 1991
++ Description:
++ Category for the trigonometric functions;

TrigonometricFunctionCategory() : Category == SIG where

  SIG ==> with

    cos : $ -> $
      ++ cos(x) returns the cosine of x.

    cot : $ -> $
      ++ cot(x) returns the cotangent of x.

    csc : $ -> $
      ++ csc(x) returns the cosecant of x.

    sec : $ -> $
      ++ sec(x) returns the secant of x.

    sin : $ -> $
      ++ sin(x) returns the sine of x.

    tan : $ -> $
      ++ tan(x) returns the tangent of x.

   add

     if $ has Ring then

       csc x ==
         (a := recip(sin x)) case "failed" => error "csc: no reciprocal"
         a::$
```

```
      sec x ==
        (a := recip(cos x)) case "failed" => error "sec: no reciprocal"
        a::$

      tan x == sin x * sec x

      cot x == cos x * csc x
```

———-

— COQ TRIGCAT —

```
(* category TRIGCAT *)
(*
    if $ has Ring then

      csc: $ -> $        ++ csc(x) returns the cosecant of x.
      csc x ==
        (a := recip(sin x)) case "failed" => error "csc: no reciprocal"
        a::$

      sec: $ -> $        ++ sec(x) returns the secant of x.
      sec x ==
        (a := recip(cos x)) case "failed" => error "sec: no reciprocal"
        a::$

      tan: $ -> $        ++ tan(x) returns the tangent of x.
      tan x == sin x * sec x

      cot: $ -> $        ++ cot(x) returns the cotangent of x.
      cot x == cos x * csc x

*)
```

———-

— TRIGCAT.dotabb —

```
"TRIGCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=TRIGCAT"];
"TRIGCAT" -> "CATEGORY"
```

———-

— TRIGCAT.dotfull —

```
"TrigonometricFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=TRIGCAT"];
"TrigonometricFunctionCategory()" -> "Category"
```

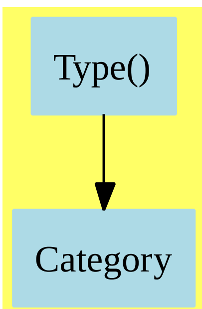
———-

— **TRIGCAT.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"TrigonometricFunctionCategory()" [color=lightblue];
"TrigonometricFunctionCategory()" -> "Category"

"Category" [color=lightblue];
}
```

————

2.0.42 Type (TYPE)



— **Type.input** —

```
)set break resume
)sys rm -f Type.output
)spool Type.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Type
--R
--R Type is a category constructor
--R Abbreviation for Type is TYPE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for TYPE
--R
--R------------------------------- Operations --------------------------------
--R Type is a category constructor.
--R Abbreviation for Type is TYPE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for TYPE
```

```
--R
--R------------------------------ Operations --------------------------------
--R
--R
--R
--E 1

)spool
)lisp (bye)
```

— Type.help —

```
====================================================================
Type examples
====================================================================

The fundamental Type.

See Also:
o )show Type
```

See:

⇒ “Aggregate” (AGG) 3.0.44 on page 133
⇒ “FortranProgramCategory” (FORTCAT) 3.0.49 on page 155
⇒ “FullyPatternMatchable” (FPATMAB) 3.0.51 on page 164
⇒ “SegmentCategory” (SEGCAT) 3.0.56 on page 183
⇐ “Category” (CATEGORY) 2.0.1 on page 3

Attributes exported:

- **nil**

— Type.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#TYPE">
Type (TYPE)</a></h2>
The fundamental Type.
</body>
```

— category TYPE Type —

```
)abbrev category TYPE Type
++ Author: Richard Jenks
++ Date Created: 14 May 1992
++ Date Last Updated: 14 May 1992
```

```
++ Description:
++ The fundamental Type.

Type() : Category == SIG where

  SIG ==> with nil
```

———-

— TYPE.dotabb —

```
"TYPE" [color=lightblue,href="bookvol10.2.pdf#nameddest=TYPE"];
"TYPE" -> "CATEGORY"
```

———-

— TYPE.dotfull —

```
"Type()" [color=lightblue,href="bookvol10.2.pdf#nameddest=TYPE"];
"Type()" -> "Category"
```

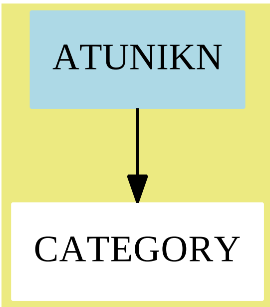
———-

— TYPE.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

———-

2.0.43 UnitsKnownAttribute (ATUNIKN)



— **UnitsKnownAttribute.input** —

```
)set break resume
)sys rm -f UnitsKnownAttribute.output
)spool UnitsKnownAttribute.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnitsKnownAttribute
--R
--R UnitsKnownAttribute is a category constructor
--R Abbreviation for UnitsKnownAttribute is ATUNIKN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ATUNIKN
--R
--E 1
)spool
)lisp (bye)
```

———-

— **UnitsKnownAttribute.help** —

```
====================================================================
UnitsKnownAttribute
====================================================================

The class of all monoids (multiplicative semigroups with a 1)
such that the operation recop can only return "failed"
if its argument is not a unit.

See Also:
o )show UnitsKnownAttribute
```

———-

— UnitsKnownAttribute.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ATUNIKN">
UnitsKnownAttribute (ATUNIKN)</a></h2>

The class of all monoids (multiplicative semigroups with a 1)
such that the operation recop can only return "failed"
if its argument is not a unit.

</body>
```

— category ATUNIKN UnitsKnownAttribute —

```
)abbrev category ATUNIKN UnitsKnownAttribute
++ Description:
++ The class of all monoids (multiplicative semigroups with a 1)
++ such that the operation recop can only return "failed"
++ if its argument is not a unit.

UnitsKnownAttribute() : Category == SIG where

  SIG ==> with nil
```

— ATUNIKN.dotabb —

```
"ATUNIKN"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATUNIKN"];
"ATUNIKN" -> "CATEGORY"
```

— ATUNIKN.dotfull —

```
"UnitsKnownAttribute()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ATUNIKN"];
"UnitsKnownAttribute()" -> "Category"
```

— ATUNIKN.dotpic —

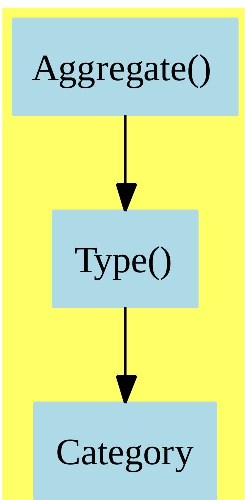
```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];
```

```
"UnitsKnownAttribute()" [color=lightblue];
"UnitsKnownAttribute()" -> "Category"

"Category" [color=lightblue];
}
```

Chapter 3

Category Layer 2

3.0.44 Aggregate (AGG)



— **Aggregate.input** —

```
)set break resume
)sys rm -f Aggregate.output
)spool Aggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Aggregate
--R
--R Aggregate is a category constructor
--R Abbreviation for Aggregate is AGG
--R This constructor is exposed in this frame.
```

```
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for AGG
--R
--R------------------------------- Operations --------------------------------
--R copy : % -> %                          empty : () -> %
--R empty? : % -> Boolean                  eq? : (%,%) -> Boolean
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R less? : (%,NonNegativeInteger) -> Boolean
--R more? : (%,NonNegativeInteger) -> Boolean
--R size? : (%,NonNegativeInteger) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

— **Aggregate.help** —

```
====================================================================
Aggregate examples
====================================================================

The notion of aggregate serves to model any data structure aggregate,
designating any collection of objects, with heterogenous or homogeneous
members, with a finite or infinite number of members, explicitly or
implicitly represented. An aggregate can in principle represent
everything from a string of characters to abstract sets such
as "the set of x satisfying relation r(x)"

An attribute "finiteAggregate" is used to assert that a domain
has a finite number of elements.

See Also:
o )show Aggregate
```

See:

⇒ "HomogeneousAggregate" (HOAGG) 4.0.73 on page 280
⇐ "Type" (TYPE) 2.0.42 on page 127

Attributes exported:

- **nil**

Exports:

empty? eq? less? more? sample size?

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.

These are implemented by this category:

```
empty? : % -> Boolean
eq? : (%,%) -> Boolean
less? : (%,NonNegativeInteger) -> Boolean
more? : (%,NonNegativeInteger) -> Boolean
sample : () -> %
size? : (%,NonNegativeInteger) -> Boolean
```

— Aggregate.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#AGG">
Aggregate (AGG)</a></h2>

The notion of aggregate serves to model any data structure aggregate,
designating any collection of objects, with heterogenous or homogeneous
members, with a finite or infinite number of members, explicitly or
implicitly represented. An aggregate can in principle represent
everything from a string of characters to abstract sets such
as "the set of x satisfying relation r(x)"
An attribute "finiteAggregate" is used to assert that a domain
has a finite number of elements.
<br/><br/><br/>
eq? : (%,%) -> Boolean<br/>
  eq?(u,v) tests if u and v are same objects.<br/><br/>

copy : % -> %<br/>
  copy(u) returns a top-level (non-recursive) copy of u.<br/>
  Note that for collections, copy(u) == [x for x in u].<br/><br/>

empty : () -> %<br/>
  empty()$D creates an aggregate of type D with 0 elements.<br/>
  Note that The $D can be dropped if understood by context,<br/>
  for example u: D := empty().<br/><br/>

empty? : % -> Boolean<br/>
  empty?(u) tests if u has 0 elements.<br/><br/>

less? : (%,NonNegativeInteger) -> Boolean<br/>
  less?(u,n) tests if u has less than n elements.<br/><br/>

more? : (%,NonNegativeInteger) -> Boolean<br/>
  more?(u,n) tests if u has greater than n elements.<br/><br/>

size? : (%,NonNegativeInteger) -> Boolean<br/>
  size?(u,n) tests if u has exactly n elements.<br/><br/>

sample : constant -> %<br/>
  sample yields a value of type %<br/><br/>

 "#" : % -> NonNegativeInteger if % has finiteAggregate <br/>
  #u returns the number of items in u.<br/><br/>
</body>
```

———-

— category AGG Aggregate —

```
)abbrev category AGG Aggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ The notion of aggregate serves to model any data structure aggregate,
++ designating any collection of objects, with heterogenous or homogeneous
++ members, with a finite or infinite number of members, explicitly or
++ implicitly represented. An aggregate can in principle represent
++ everything from a string of characters to abstract sets such
++ as "the set of x satisfying relation r(x)"
++ An attribute "finiteAggregate" is used to assert that a domain
++ has a finite number of elements.

Aggregate() : Category == SIG where

  SIG ==> Type with

    eq? : (%,%) -> Boolean
      ++ eq?(u,v) tests if u and v are same objects.

    copy : % -> %
      ++ copy(u) returns a top-level (non-recursive) copy of u.
      ++ Note that for collections, \axiom{copy(u) == [x for x in u]}.

    empty : () -> %
      ++ empty()$D creates an aggregate of type D with 0 elements.
      ++ Note that The $D can be dropped if understood by context,
      ++ for example \axiom{u: D := empty()}.

    empty? : % -> Boolean
      ++ empty?(u) tests if u has 0 elements.

    less? : (%,NonNegativeInteger) -> Boolean
      ++ less?(u,n) tests if u has less than n elements.

    more? : (%,NonNegativeInteger) -> Boolean
      ++ more?(u,n) tests if u has greater than n elements.

    size? : (%,NonNegativeInteger) -> Boolean
      ++ size?(u,n) tests if u has exactly n elements.

    sample : constant -> %
      ++ sample yields a value of type %

    if % has finiteAggregate then

      "#" : % -> NonNegativeInteger
        ++ # u returns the number of items in u.
```

```
add

  eq?(a,b) == EQ(a,b)$Lisp

  sample() == empty()

  if % has finiteAggregate then

    empty? a  == #a = 0

    less?(a,n) == #a < n

    more?(a,n) == #a > n

    size?(a,n) == #a = n
```

— COQ AGG —

```
(* category AGG *)
(*

  eq?: (%,%) -> Boolean
  eq?(a,b) == EQ(a,b)$Lisp

  sample: constant -> %
  sample() == empty()

  if % has finiteAggregate then

    empty?: % -> Boolean
    empty? a   == #a = 0

    less?: (%,NonNegativeInteger) -> Boolean
    less?(a,n) == #a < n

    more?: (%,NonNegativeInteger) -> Boolean
    more?(a,n) == #a > n

    size?: (%,NonNegativeInteger) -> Boolean
    size?(a,n) == #a = n

*)
```

— AGG.dotabb —

```
"AGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=AGG"];
"AGG" -> "TYPE"
```

———-

— **AGG.dotfull** —

```
"Aggregate()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=AGG"];
"Aggregate()" -> "Type()"
```

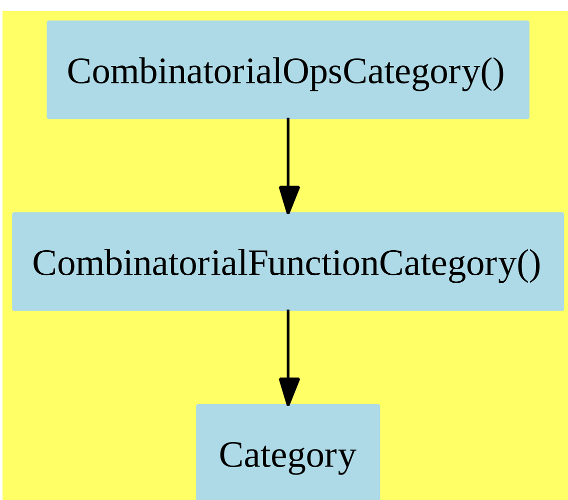
———-

— **AGG.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

———-

3.0.45 CombinatorialOpsCategory (COMBOPC)



— **CombinatorialOpsCategory.input** —

```
)set break resume
)sys rm -f CombinatorialOpsCategory.output
)spool CombinatorialOpsCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show CombinatorialOpsCategory
--R
--R CombinatorialOpsCategory is a category constructor
--R Abbreviation for CombinatorialOpsCategory is COMBOPC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for COMBOPC
--R
--R------------------------------- Operations --------------------------------
--R binomial : (%,%) -> %                factorial : % -> %
--R factorials : (%,Symbol) -> %         factorials : % -> %
--R permutation : (%,%) -> %             product : (%,Symbol) -> %
--R summation : (%,Symbol) -> %
--R product : (%,SegmentBinding(%)) -> %
--R summation : (%,SegmentBinding(%)) -> %
--R
--E 1

)spool
)lisp (bye)
```

— **CombinatorialOpsCategory.help** —

```
====================================================================
CombinatorialOpsCategory examples
====================================================================

CombinatorialOpsCategory is the category obtaining by adjoining
summations and products to the usual combinatorial operations;

See Also:
o )show CombinatorialOpsCategory
```

See:

⇐ "CombinatorialFunctionCategory" (CFCAT) 2.0.15 on page 42

Exports:

binomial	factorial	factorials	permutation	product
summation				

These are directly exported but not implemented:

```
factorials : % -> %
factorials : (%,Symbol) -> %
product : (%,Symbol) -> %
product : (%,SegmentBinding %) -> %
summation : (%,Symbol) -> %
summation : (%,SegmentBinding %) -> %
```

These exports come from (p42) CombinatorialFunctionCategory():

```
binomial : (%,%) -> %
factorial : % -> %
permutation : (%,%) -> %
```

— CombinatorialOpsCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#COMBOPC">
CombinatorialOpsCategory (COMBOPC)</a></h2>

CombinatorialOpsCategory is the category obtaining by adjoining
summations and products to the usual combinatorial operations.
<br/><br/><br/>

binomial: ($,$) -> $<br/>
  binomial(n,r) returns the (n,r) binomial coefficient (often<br/>
  denoted in the literature as C(n,r)). Note:<br/>
  C(n,r) = n!/(r!(n-r)!) where n >= r >= 0<br/>
  from CombinatorialFunctionCategory<br/><br/>

factorial: $ -> $
  factorial(n) computes the factorial of n (denoted in the literature<br/>
  by n!). Note: n! = n * (n-1)! when n > 0; also, 0! = 1<br/>
  from CombinatorialFunctionCategory<br/><br/>

permutation: ($,$) -> $
  permutation(n,m) returns the number of permutations of n objects<br/>
  taken m at a time. Note: permutations(n,m) = n!/(n-m)!<br/>
  from CombinatorialFunctionCategory<br/><br/>

factorials : $ -> $<br/>
  factorials(f) rewrites the permutations and binomials in f<br/>
  in terms of factorials.<br/><br/>

factorials : ($, Symbol) -> $<br/>
  factorials(f, x) rewrites the permutations and binomials in f<br/>
  involving x in terms of factorials.<br/><br/>

summation : ($, Symbol) -> $<br/>
  summation(f(n), n) returns the formal sum S(n) which verifies<br/>
  S(n+1) - S(n) = f(n)<br/><br/>

summation : ($, SegmentBinding $) -> $<br/>
  summation(f(n), n = a..b) returns f(a) + ... + f(b) as a<br/>
  formal sum.<br/><br/>
```

```
product : ($, Symbol) -> $<br/>
  product(f(n), n) returns the formal product P(n) which verifies<br/>
  P(n+1)/P(n) = f(n)<br/><br/>

product : ($, SegmentBinding  $) -> $<br/>
  product(f(n), n = a..b) returns f(a) * ... * f(b) as a<br/>
  formal product.<br/><br/>

&bull; CombinatorialFunctionCategory<br/><br/>

</body>
```

— category COMBOPC CombinatorialOpsCategory —

```
)abbrev category COMBOPC CombinatorialOpsCategory
++ Author: Manuel Bronstein
++ Date Last Updated: 22 February 1993 (JHD/BMT)
++ Description:
++ CombinatorialOpsCategory is the category obtaining by adjoining
++ summations and products to the usual combinatorial operations.

CombinatorialOpsCategory() : Category == SIG where

  SIG ==> CombinatorialFunctionCategory with

    factorials : $ -> $
      ++ factorials(f) rewrites the permutations and binomials in f
      ++ in terms of factorials;

    factorials : ($, Symbol) -> $
      ++ factorials(f, x) rewrites the permutations and binomials in f
      ++ involving x in terms of factorials;

    summation : ($, Symbol) -> $
      ++ summation(f(n), n) returns the formal sum S(n) which verifies
      ++ S(n+1) - S(n) = f(n);

    summation : ($, SegmentBinding $) -> $
      ++ summation(f(n), n = a..b) returns f(a) + ... + f(b) as a
      ++ formal sum;

    product : ($, Symbol) -> $
      ++ product(f(n), n) returns the formal product P(n) which verifies
      ++ P(n+1)/P(n) = f(n);

    product : ($, SegmentBinding  $) -> $
      ++ product(f(n), n = a..b) returns f(a) * ... * f(b) as a
      ++ formal product;
```

— **COMBOPC.dotabb** —

```
"COMBOPC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=COMBOPC"];
"COMBOPC" -> "CFCAT"
```

————

— **COMBOPC.dotfull** —

```
"CombinatorialOpsCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=COMBOPC"];
"CombinatorialOpsCategory()" -> "CombinatorialFunctionCategory()"
```

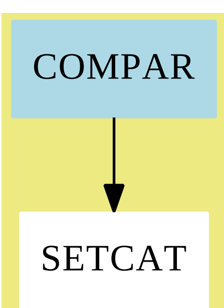
————

— **COMBOPC.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"CombinatorialOpsCategory()" [color=lightblue];
"CombinatorialOpsCategory()" -> "CombinatorialFunctionCategory()"

"CombinatorialFunctionCategory()" [color=lightblue];
"CombinatorialFunctionCategory()" -> "Category"

"Category" [color=lightblue];
}
```

————

3.0.46 Comparable (COMPAR)



— Comparable.input —

```
)set break resume
)sys rm -f Comparable.output
)spool Comparable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Comparable
--R
--R Comparable is a category constructor
--R Abbreviation for Comparable is COMPAR
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for COMPAR
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger            latex : % -> String
--R smaller? : (%,%) -> Boolean          ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

— Comparable.help —

```
====================================================================
Comparable examples
====================================================================

See Also:
o )show Comparable
```

See:

?=? coerce hash latex smaller? ?~=?

— Comparable.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#COMPAR">
Comparable (COMPAR)</a></h2>

The class of set equipped with possibly unnatural linear order
(needed for technical reasons).
<br/><br/><br/>
```

```
"=" : (%,%) -> Boolean<br/>
  x=y tests if x and y are equal.<br/>
  from BasicType<br/><br/>

"~=" : (%,%) -> Boolean<br/>
  x~=y tests if x and y are not equal.<br/>
  from BasicType<br/><br/>

coerce : % -> OutputForm<br/>
  coerce(a) transforms a into an element of OutputForm<br/>
  from CoercibleTo OutputForm<br/><br/>

hash : % -> SingleInteger<br/>
  hash(s) calculates a hash code for s.<br/>
  from SetCategory<br/><br/>

latex : % -> String<br/>
  latex(s) returns a LaTeX-printable output<br/>
  representation of s.<br/>
  from SetCategory<br/><br/>

smaller? : (%,%) -> Boolean<br/>
 
smaller?(x, y) is a strict total ordering on the elements of the set.<br/><br/>

&bull; BasicType<br/>
&bull; CoercibleTo OutputForm<br/>
&bull; SetCategory
</body>
```

— category COMPAR Comparable —

```
)abbrev category COMPAR Comparable
++ Description:
++ The class of set equipped with possibly unnatural linear order
++ (needed for technical reasons).

Comparable() : Category == SIG where

  SIG ==> SetCategory with

    smaller? : (%,%) -> Boolean
      ++ smaller?(x, y) is a strict total ordering on the elements of the set.
```

— COMPAR.dotabb —

```
"COMPAR" [color=lightblue,href="bookvol10.2.pdf#nameddest=COMPAR"];
"COMPAR" -> "SETCAT"
```

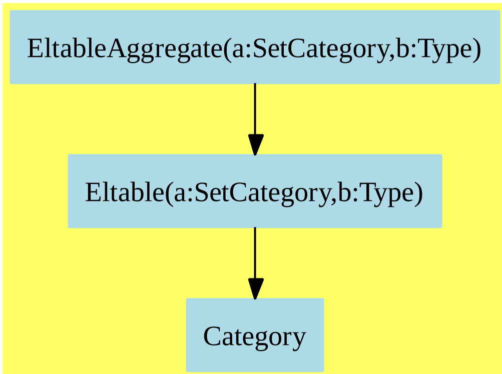
— **COMPAR.dotfull** —

```
"Comparable()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=COMPAR"];
"Comparable()" -> "SetCategory()"
```

— **COMPAR.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"Comparable()" [color=lightblue];
"Comparable()" -> "SetCatgory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

3.0.47 EltableAggregate (ELTAGG)



— **EltableAggregate.input** —

```
)set break resume
)sys rm -f EltableAggregate.output
)spool EltableAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show EltableAggregate
--R
--R EltableAggregate(Dom: SetCategory,Im: Type) is a category constructor
--R Abbreviation for EltableAggregate is ELTAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ELTAGG
--R
--R------------------------------- Operations --------------------------------
--R elt : (%,Dom,Im) -> Im                ?.? : (%,Dom) -> Im
--R qelt : (%,Dom) -> Im
--R qsetelt! : (%,Dom,Im) -> Im if $ has shallowlyMutable
--R setelt : (%,Dom,Im) -> Im if $ has shallowlyMutable
--R
--E 1

)spool
)lisp (bye)
```

————

— **EltableAggregate.help** —

```
====================================================================
EltableAggregate examples
====================================================================

An eltable aggregate is one which can be viewed as a function.
```

```
For example, the list [1,7,4] can applied to 0,1, and 2 respectively
will return the integers 1, 7, and 4; thus this list may be viewed
as mapping 0 to 1, 1 to 7 and 2 to 4. In general, an aggregate
can map members of a domain Dom to an image domain Im.

See Also:
o )show EltableAggregate
```

———-

See:

⇒ "IndexedAggregate" (IXAGG) 5.0.98 on page 441
⇐ "Eltable" (ELTAB) 2.0.19 on page 54

Exports:

elt qelt qsetelt! setelt ?.?

Attributes Used:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are part of this category's direct exports:

```
elt : (%,Dom,Im) -> Im
setelt : (%,Dom,Im) -> Im if $ has shallowlyMutable
```

These are implemented by this category:

```
qelt : (%,Dom) -> Im
qsetelt! : (%,Dom,Im) -> Im if $ has shallowlyMutable
```

These exports come from (p54) Eltable():

```
?.? : (%,Dom) -> Im
```

— EltableAggregate.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ELTAGG">
EltableAggregate(Dom,Im) (ELTAGG)</a></h2>
&bull; Dom: SetCategory<br/>
&bull; Im: Type
<br/><br/>
An eltable aggregate is one which can be viewed as a function.
For example, the list [1,7,4] can applied to 0,1, and 2 respectively
will return the integers 1, 7, and 4; thus this list may be viewed as
mapping 0 to 1, 1 to 7 and 2 to 4. In general, an aggregate
can map members of a domain Dom to an image domain Im.
<br/><br/><br/>

elt : (%, Dom, Im) -> Im<br/>
  elt(u, x, y) applies u to x if x is in the domain of u,<br/>
```

```
  and returns y otherwise.<br/>
  For example, if u is a polynomial in x over the rationals,<br/>
  elt(u,n,0) may define the coefficient of x<br/>
  to the power n, returning 0 when n is out of range.<br/><br/>

?.? : (%,Dom) -> Im<br/>
  x.u applies u to x  if x is in the domain of u<br/>
  It will give an index out of range error otherwise<br/><br/>

qelt: (%, Dom) -> Im<br/>
  qelt(u, x) applies u to x without checking whether<br/>
  x is in the domain of u.  If x is not in the domain of u<br/>
  a memory-access violation may occur. <br/>
  If a check on whether x is in the domain of u <br/>
  is required, use the function elt.<br/><br/>

setelt : (%, Dom, Im) -> Im if % has shallowlyMutable<br/>
  setelt(u,x,y) sets the image of x to be y under u,<br/>
  assuming x is in the domain of u.<br/>
  Error: if x is not in the domain of u.<br/><br/>

qsetelt! : (%, Dom, Im) -> Im if % has shallowlyMutable<br/>
  qsetelt!(u,x,y) sets the image of x to be y under u, without
  checking that x is in the domain of u.
  If such a check is required use the function setelt.<br/><br/>

</body>
```

— category ELTAGG EltableAggregate —

```
)abbrev category ELTAGG EltableAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ An eltable aggregate is one which can be viewed as a function.
++ For example, the list [1,7,4] can applied to 0,1, and 2 respectively
++ will return the integers 1, 7, and 4; thus this list may be viewed as
++ mapping 0 to 1, 1 to 7 and 2 to 4. In general, an aggregate
++ can map members of a domain Dom to an image domain Im.

EltableAggregate(Dom,Im) : Category == SIG where
  Dom : SetCategory
  Im : Type

  SIG ==> Eltable(Dom, Im) with

    elt : (%, Dom, Im) -> Im
       ++ elt(u, x, y) applies u to x if x is in the domain of u,
       ++ and returns y otherwise.
       ++ For example, if u is a polynomial in \axiom{x} over the rationals,
       ++ \axiom{elt(u,n,0)} may define the coefficient of \axiom{x}
```

```
      ++ to the power n, returning 0 when n is out of range.

   qelt: (%, Dom) -> Im
      ++ qelt(u, x) applies \axiom{u} to \axiom{x} without checking whether
      ++ \axiom{x} is in the domain of \axiom{u}.  If \axiom{x} is not
      ++ in the domain of \axiom{u} a memory-access violation may occur.
      ++ If a check on whether \axiom{x} is in the domain of \axiom{u}
      ++ is required, use the function \axiom{elt}.

   if % has shallowlyMutable then

      setelt : (%, Dom, Im) -> Im
          ++ setelt(u,x,y) sets the image of x to be y under u,
          ++ assuming x is in the domain of u.
          ++ Error: if x is not in the domain of u.
          -- this function will soon be renamed as setelt!.

      qsetelt_! : (%, Dom, Im) -> Im
          ++ qsetelt!(u,x,y) sets the image of \axiom{x} to be \axiom{y}
          ++ under \axiom{u}, without checking that \axiom{x} is in
          ++ the domain of \axiom{u}.
          ++ If such a check is required use the function \axiom{setelt}.

  add

    qelt(a, x) == elt(a, x)

    if % has shallowlyMutable then

      qsetelt_!(a, x, y) == (a.x := y)
```

———-

— COQ ELTAGG —

```
(* category ELTAGG *)
(*

  qelt: (%, Dom) -> Im
  qelt(a, x) == elt(a, x)

  if % has shallowlyMutable then

    qsetelt_!: (%, Dom, Im) -> Im
    qsetelt_!(a, x, y) == (a.x := y)

*)
```

———-

— ELTAGG.dotabb —

```
"ELTAGG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ELTAGG"];
"ELTAGG" -> "ELTAB"
```

———-

— **ELTAGG.dotfull** —

```
"EltableAggregate(a:SetCategory,b:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ELTAGG"];
"EltableAggregate(a:SetCategory,b:Type)" -> "Eltable(a:SetCategory,b:Type)"
```

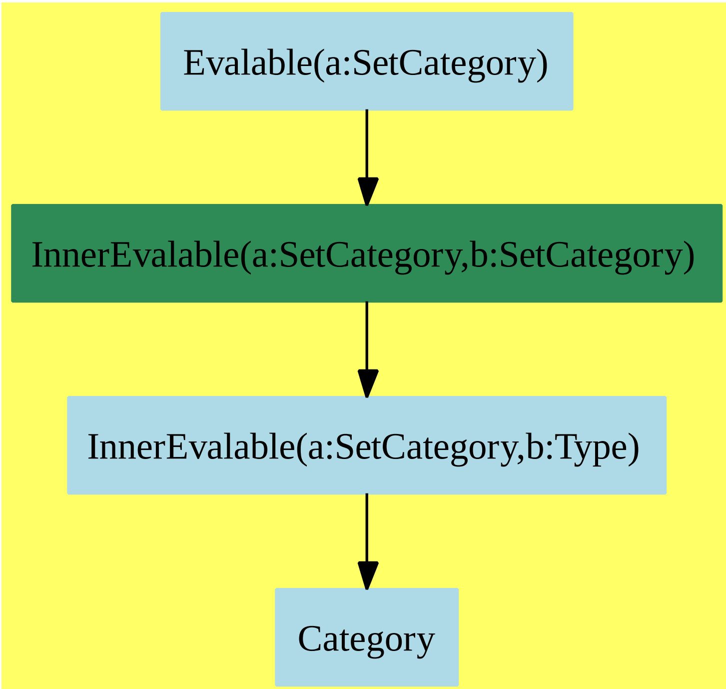
———-

— **ELTAGG.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"EltableAggregate(a:SetCategory,b:Type)" [color=lightblue];
"EltableAggregate(a:SetCategory,b:Type)" -> "Eltable(a:SetCategory,b:Type)"

"Eltable(a:SetCategory,b:Type)" [color=lightblue];
"Eltable(a:SetCategory,b:Type)" -> "Category"

"Category" [color=lightblue];
}
```

———-

3.0.48 Evalable (EVALAB)



— **Evalable.input** —

```
)set break resume
)sys rm -f Evalable.output
)spool Evalable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Evalable
--R
--R Evalable(R: SetCategory) is a category constructor
--R Abbreviation for Evalable is EVALAB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for EVALAB
--R
--R------------------------------- Operations --------------------------------
--R eval : (%,List(Equation(R))) -> %     eval : (%,Equation(R)) -> %
--R eval : (%,R,R) -> %                   eval : (%,List(R),List(R)) -> %
--R
--E 1

)spool
)lisp (bye)
```

———-

— **Evalable.help** —

```
====================================================================
Evalable examples
====================================================================

This category provides eval operations. A domain may belong to this
category if it is possible to make "evaluation" substitutions.

See Also:
o )show Evalable
```

———-

See:

⇒ "ExpressionSpace" (ES) 5.0.96 on page 418
⇒ "MultivariateTaylorSeriesCategory" (MTSCAT) 15.0.206 on page 1384
⇒ "PolynomialCategory" (POLYCAT) 16.0.213 on page 1449
⇐ "InnerEvalable" (IEVALAB) 2.0.22 on page 64

Exports:

eval

These are directly exported but not implemented:

```
eval : (%,List Equation R) -> %
```

These are implemented by this category:

```
eval : (%,Equation R) -> %
eval : (%,List R,List R) -> %
```

These exports come from (p64) InnerEvalable(R:SetCategory,R:SetCategory):

```
eval : (%,R,R) -> %
```

— **Evalable.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#EVALAB">
Evalable(R) (EVALAB)</a></h2>
&bull; R : SetCategory<br/><br/>

A domain may belong to this category if it is possible to make
"evaluation" substitutions.
<br/><br/><br/>

eval : ($, Equation R) -> $<br/>
  eval(f,x = v) replaces x by v in f.<br/><br/>

eval : ($, List Equation R) -> $<br/>
```

```
  eval(f, [x1 = v1,...,xn = vn]) replaces xi by vi in f.<br/><br/>

eval : ($, List R, List R) -> $<br/>
  eval(f, [x1,...,xn], [v1,...,vn]) replaces xi by vi in f.<br/>
  from InnerEvalable(R,R)<br/><br/>

eval : ($, R, R) -> $<br/>
  eval(f, x, v) replaces x by v in f.<br/>
  from InnerEvalable(R,R)<br/><br/>

&bull; InnerEvalable(R,R)
</body>
```

— category EVALAB Evalable —

```
)abbrev category EVALAB Evalable
++ Date Last Updated: June 3, 1991
++ Description:
++ This category provides \spadfun{eval} operations.
++ A domain may belong to this category if it is possible to make
++ "evaluation" substitutions.

Evalable(R) : Category == SIG where
  R : SetCategory

  SIG ==> InnerEvalable(R,R) with

    eval : ($, Equation R) -> $
      ++ eval(f,x = v) replaces x by v in f.

    eval : ($, List Equation R) -> $
      ++ eval(f, [x1 = v1,...,xn = vn]) replaces xi by vi in f.

   add

     eval(f:$, eq:Equation R) == eval(f, [eq])

     eval(f:$, xs:List R,vs:List R) == eval(f,[x=v for x in xs for v in vs])
```

— COQ EVALAB —

```
(* category EVALAB *)
(*
    eval: ($, Equation R) -> $
    eval(f:$, eq:Equation R) == eval(f, [eq])

    eval: ($, List Equation R) -> $
    eval(f:$, xs:List R,vs:List R) == eval(f,[x=v for x in xs for v in vs])
```

```
*)
```

— **EVALAB.dotabb** —

```
"EVALAB"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=EVALAB"];
"EVALAB" -> "IEVALAB"
```

— **EVALAB.dotfull** —

```
"Evalable(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=EVALAB"];
"Evalable(a:SetCategory)" -> "InnerEvalable(a:SetCategory,b:SetCategory)"

"Evalable(MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=EVALAB"];
"Evalable(MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet))" ->
    "Evalable(a:SetCategory)"

"Evalable(ExpressionSpace)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=EVALAB"];
"Evalable(ExpressionSpace)" -> "Evalable(a:SetCategory)"

"Evalable(PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=EVALAB"];
"Evalable(PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet))"
  -> "Evalable(a:SetCategory)"
```

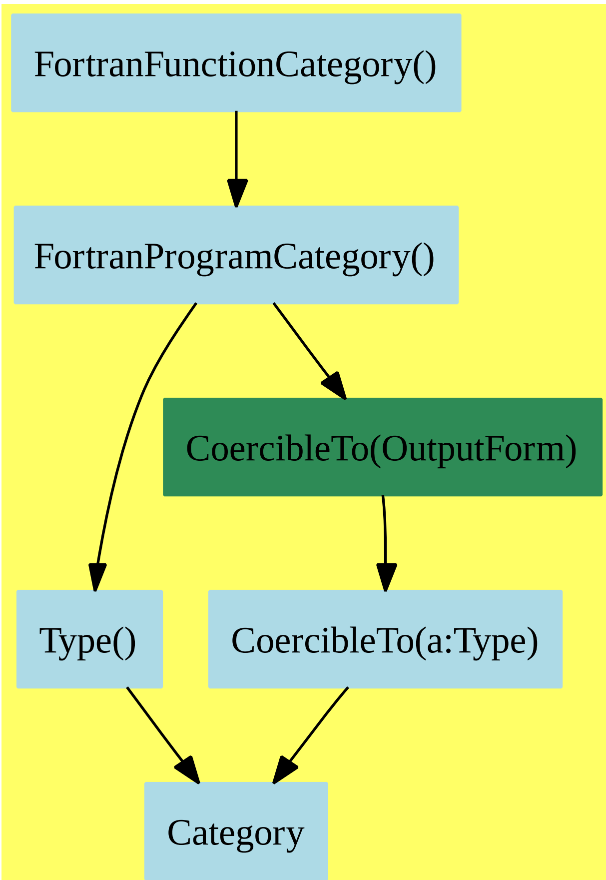
— **EVALAB.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"Evalable(a:SetCategory)" [color=lightblue];
"Evalable(a:SetCategory)" -> "InnerEvalable(a:SetCategory,b:SetCategory)"

"InnerEvalable(a:SetCategory,b:SetCategory)" [color=seagreen];
"InnerEvalable(a:SetCategory,b:SetCategory)" ->
    "InnerEvalable(a:SetCategory,b:Type)"

"InnerEvalable(a:SetCategory,b:Type)" [color=lightblue];
"InnerEvalable(a:SetCategory,b:Type)" -> "Category"
```

```
"Category" [color=lightblue];
}
```

————-

3.0.49 FortranProgramCategory (FORTCAT)



— **FortranProgramCategory.input** —

```
)set break resume
)sys rm -f FortranProgramCategory.output
)spool FortranProgramCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FortranProgramCategory
--R
```

```
--R FortranProgramCategory is a category constructor
--R Abbreviation for FortranProgramCategory is FORTCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FORTCAT
--R
--R------------------------------- Operations --------------------------------
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R
--E 1

)spool
)lisp (bye)
```

— FortranProgramCategory.help —

```
====================================================================
FortranProgramCategory examples
====================================================================

FortranProgramCategory provides various models of FORTRAN subprograms.
These can be transformed into actual FORTRAN code.

See Also:
o )show FortranProgramCategory
```

See:

⇒ "FortranFunctionCategory" (FORTFN) 4.0.62 on page 219
⇒ "FortranMatrixCategory" (FMC) 4.0.63 on page 225
⇒ "FortranMatrixFunctionCategory" (FMFUN) 4.0.64 on page 229
⇒ "FortranVectorCategory" (FVC) 4.0.65 on page 236
⇒ "FortranVectorFunctionCategory" (FVFUN) 4.0.66 on page 240
⇐ "CoercibleTo" (KOERCE) 2.0.14 on page 39
⇐ "Type" (TYPE) 2.0.42 on page 127

Exports:

coerce outputAsFortran

Attributes:

- **nil**

These are directly exported but not implemented:

```
outputAsFortran : % -> Void
```

These exports come from (p39) CoercibleTo(OutputForm):

```
coerce : % -> OutputForm
```

— FortranProgramCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FORTCAT">
FortranProgramCategory (FORTCAT)</a></h2>

FortranProgramCategory provides various models of FORTRAN subprograms.
These can be transformed into actual FORTRAN code.
<br/><br/><br/>

coerce : % -> OutputForm<br/>
  coerce(a) transforms a into an element of OutputForm.<br/>
  from CoercibleTo OutputForm<br/><br/>

outputAsFortran : $ -> Void<br/>
 
outputAsFortran(u) translates u into a legal FORTRAN subprogram<br/><br/>

&bull; CoercibleTo OutputForm
</body>
```

— category FORTCAT FortranProgramCategory —

```
)abbrev category FORTCAT FortranProgramCategory
++ Author: Mike Dewar
++ Date Created: November 1992
++ Description:
++ FortranProgramCategory provides various models of FORTRAN subprograms.
++ These can be transformed into actual FORTRAN code.

FortranProgramCategory() : Category == SIG where

  SIG ==> Join(Type,CoercibleTo OutputForm) with

    outputAsFortran : $ -> Void
      ++ \axiom{outputAsFortran(u)} translates \axiom{u} into a legal FORTRAN
      ++ subprogram.
```

— FORTCAT.dotabb —

```
"FORTCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FORTCAT"];
"FORTCAT" -> "KOERCE"
"FORTCAT" -> "TYPE"
```

— FORTCAT.dotfull —

```
"FortranProgramCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FORTCAT"];
"FortranProgramCategory()" -> "Type()"
"FortranProgramCategory()" -> "CoercibleTo(OutputForm)"
```

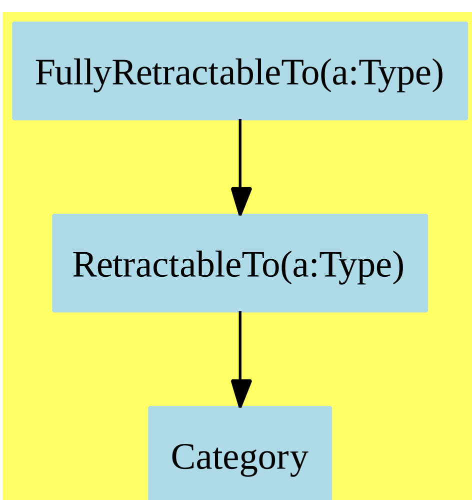
———-

— **FORTCAT.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FortranProgramCategory()" [color=lightblue];
"FortranProgramCategory()" -> "Type()"
"FortranProgramCategory()" -> "CoercibleTo(OutputForm)"

"Type()" [color=lightblue];
"Type()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

———-

3.0.50 FullyRetractableTo (FRETRCT)



— FullyRetractableTo.input —

```
)set break resume
)sys rm -f FullyRetractableTo.output
)spool FullyRetractableTo.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FullyRetractableTo
--R
--R FullyRetractableTo(S: Type) is a category constructor
--R Abbreviation for FullyRetractableTo is FRETRCT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FRETRCT
--R
--R------------------------------- Operations --------------------------------
--R coerce : S -> %                       retract : % -> S
--R coerce : Integer -> % if S has RETRACT(INT)
--R coerce : Fraction(Integer) -> % if S has RETRACT(FRAC(INT))
--R retract : % -> Integer if S has RETRACT(INT)
--R retract : % -> Fraction(Integer) if S has RETRACT(FRAC(INT))
--R retractIfCan : % -> Union(Integer,"failed") if S has RETRACT(INT)
--R retractIfCan : % -> Union(Fraction(Integer),"failed") if S has RETRACT(FRAC(INT))
--R retractIfCan : % -> Union(S,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— FullyRetractableTo.help —

```
====================================================================
FullyRetractableTo examples
====================================================================

A is fully retractable to B means that A is retractable to B and
if B is retractable to the integers or rational numbers then so is A.
In particular, what we are asserting is that there are no integers
(rationals) in A which don't retract into B.

See Also:
o )show FullyRetractableTo
```

See:

⇒ “ComplexCategory” (COMPCAT) 20.0.240 on page 1869
⇒ “DirectProductCategory” (DIRPCAT) 12.0.184 on page 1173
⇒ “FiniteAbelianMonoidRing” (FAMR) 14.0.199 on page 1331
⇒ “FunctionSpace” (FS) 17.0.219 on page 1544
⇒ “MonogenicAlgebra” (MONOGEN) 19.0.238 on page 1847
⇒ “OctonionCategory” (OC) 12.0.190 on page 1238
⇒ “QuaternionCategory” (QUATCAT) 12.0.191 on page 1252
⇒ “RealClosedField” (RCFIELD) 17.0.223 on page 1611
⇒ “SquareMatrixCategory” (SMATCAT) 12.0.192 on page 1266
⇒ “UnivariateSkewPolynomialCategory” (OREPCAT) 10.0.174 on page 1101
⇐ “RetractableTo” (RETRACT) 2.0.37 on page 110

Exports:

coerce retract retractIfCan

These are implemented by this category:

```
coerce : Integer -> %
    if S has RETRACT INT
coerce : Fraction Integer -> %
    if S has RETRACT FRAC INT
retract : % -> Integer
    if S has RETRACT INT
retract : % -> Fraction Integer
    if S has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed")
    if S has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
    if S has RETRACT FRAC INT
```

These exports come from (p110) RetractableTo(S:Type):

```
coerce : S -> %
retract : % -> S
retractIfCan : % -> Union(S,"failed")
```

— FullyRetractableTo.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FRETRCT">
FullyRetractableTo (FRETRCT)</a></h2>
&bull; S : Type<br/><br/>
A is fully retractable to B means that A is retractable to B and
if B is retractable to the integers or rational numbers then so is A.
In particular, what we are asserting is that there are no integers
(rationals) in A which don't retract into B.
<br/><br/><br/>

coerce: S -> %<br/>
  from RetractableTo S<br/><br/>

retract : % -> S<br/>
  from RetractableTo S<br/><br/>
```

```
coerce : Integer -> % if S has RETRACT(INT)<br/>
  from RetractableTo Integer<br/><br/>

coerce : Fraction(Integer) -> % if S has RETRACT(FRAC(INT))<br/>
  from RetractableTo Fraction Integer<br/><br/>

retract : % -> Integer if S has RETRACT(INT)<br/>
  from RetractableTo Integer<br/><br/>

retract : % -> Fraction(Integer) if S has RETRACT(FRAC(INT))<br/>
  from RetractableTo Fraction Integer<br/><br/>

retractIfCan : % -> Union(Integer,"failed") if S has RETRACT(INT)<br/>
  from RetractableTo Integer<br/><br/>

retractIfCan : % ->
Union(Fraction(Integer),"failed") if S has RETRACT(FRAC(INT))<br/>
  from RetractableTo Fraction Integer<br/><br/>

retractIfCan : % -> Union(S, "failed")<br/>
  from RetractableTo S<br/><br/>

&bull; RetractableTo Fraction Integer<br/>
&bull; RetractableTo Integer<br/>
</body>
```

— category FRETRCT FullyRetractableTo —

```
)abbrev category FRETRCT FullyRetractableTo
++ Author: Manuel Bronstein
++ Date Created: March 1990
++ Date Last Updated: 9 April 1991
++ Description:
++ A is fully retractable to B means that A is retractable to B and
++ if B is retractable to the integers or rational numbers then so is A.
++ In particular, what we are asserting is that there are no integers
++ (rationals) in A which don't retract into B.

FullyRetractableTo(S) : Category == SIG where
  S : Type

  SIG ==> RetractableTo(S) with

    if (S has RetractableTo Integer) then RetractableTo Integer

    if (S has RetractableTo Fraction Integer) then
              RetractableTo Fraction Integer

   add

    if not(S is Integer) then
```

```
      if (S has RetractableTo Integer) then    -- induction

        coerce(n:Integer):%  == n::S::%

        retract(r:%):Integer == retract(retract(r)@S)

        retractIfCan(r:%):Union(Integer, "failed") ==
          (u:= retractIfCan(r)@Union(S,"failed")) case "failed"=> "failed"
          retractIfCan(u::S)

    if not(S is Fraction Integer) then

      if (S has RetractableTo Fraction Integer) then   -- induction

        coerce(n:Fraction Integer):%  == n::S::%

        retract(r:%):Fraction(Integer) == retract(retract(r)@S)

        retractIfCan(r:%):Union(Fraction Integer, "failed") ==
          (u:=retractIfCan(r)@Union(S,"failed")) case "failed"=>"failed"
          retractIfCan(u::S)
```

— **COQ FRETRCT** —

```
(* category FRETRCT *)
(*
    if not(S is Integer) then

      if (S has RetractableTo Integer) then    -- induction

        coerce : Integer -> %
        coerce(n:Integer):%  == n::S::%

        retract : % -> Integer
        retract(r:%):Integer == retract(retract(r)@S)

        retractIfCan : % -> Union(Integer,"failed")
        retractIfCan(r:%):Union(Integer, "failed") ==
          (u:= retractIfCan(r)@Union(S,"failed")) case "failed"=> "failed"
          retractIfCan(u::S)

    if not(S is Fraction Integer) then

      if (S has RetractableTo Fraction Integer) then   -- induction

        coerce : Fraction Integer -> %
        coerce(n:Fraction Integer):%  == n::S::%

        retract : % -> Fraction Integer
        retract(r:%):Fraction(Integer) == retract(retract(r)@S)
```

```
        retractIfCan : % -> Union(Fraction Integer,"failed")
        retractIfCan(r:%):Union(Fraction Integer, "failed") ==
          (u:=retractIfCan(r)@Union(S,"failed")) case "failed"=>"failed"
          retractIfCan(u::S)

*)
```

———-

— FRETRCT.dotabb —

```
"FRETRCT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FRETRCT"];
"FRETRCT" -> "RETRACT"
```

———-

— FRETRCT.dotfull —

```
"FullyRetractableTo(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FRETRCT"];
"FullyRetractableTo(a:Type)" -> "RetractableTo(a:Type)"

"FullyRetractableTo(a:Ring)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FRETRCT"];
"FullyRetractableTo(a:Ring)" -> "FullyRetractableTo(a:Type)"

"FullyRetractableTo(a:CommutativeRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FRETRCT"];
"FullyRetractableTo(a:CommutativeRing)" -> "FullyRetractableTo(a:Type)"

"FullyRetractableTo(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FRETRCT"];
"FullyRetractableTo(a:SetCategory)" -> "FullyRetractableTo(a:Type)"

"FullyRetractableTo(Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FRETRCT"];
"FullyRetractableTo(Fraction(Integer))" -> "FullyRetractableTo(a:Type)"
```

———-

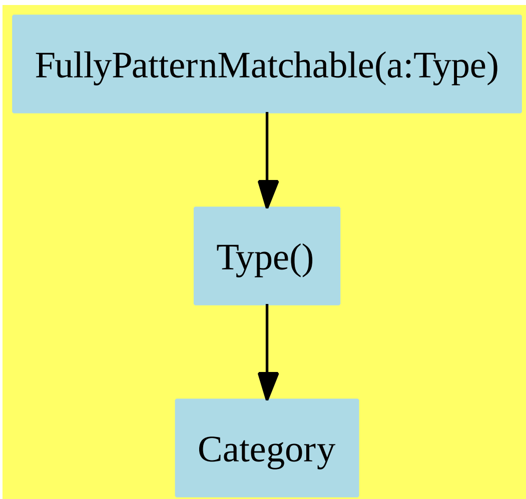
— FRETRCT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FullyRetractableTo(a:Type)" [color=lightblue];
"FullyRetractableTo(a:Type)" -> "RetractableTo(a:Type)"
```

```
"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

———-

3.0.51 FullyPatternMatchable (FPATMAB)



— **FullyPatternMatchable.input** —

```
)set break resume
)sys rm -f FullyPatternMatchable.output
)spool FullyPatternMatchable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FullyPatternMatchable
--R
--R FullyPatternMatchable(R: Type) is a category constructor
--R Abbreviation for FullyPatternMatchable is FPATMAB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FPATMAB
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean if R has PATMAB(INT) or R has PATMAB(FLOAT)
--R coerce : % -> OutputForm if R has PATMAB(INT) or R has PATMAB(FLOAT)
--R hash : % -> SingleInteger if R has PATMAB(INT) or R has PATMAB(FLOAT)
--R latex : % -> String if R has PATMAB(INT) or R has PATMAB(FLOAT)
--R patternMatch : (%,Pattern(Integer),PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%) if R has
```

```
--R patternMatch : (%,Pattern(Float),PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%) if R has PATMAB
--R ?~=? : (%,%) -> Boolean if R has PATMAB(INT) or R has PATMAB(FLOAT)
--R
--E 1

)spool
)lisp (bye)
```

———

— **FullyPatternMatchable.help** —

```
====================================================================
FullyPatternMatchable examples
====================================================================

A set S is PatternMatchable over R if S can lift the pattern-matching
functions of S over the integers and float to itself (necessary for
matching in towers).

See Also:
o )show FullyPatternMatchable
```

———

See:

⇒ "ComplexCategory" (COMPCAT) 20.0.240 on page 1869
⇒ "FunctionSpace" (FS) 17.0.219 on page 1544
⇒ "QuotientFieldCategory" (QFCAT) 17.0.222 on page 1595
⇐ "Type" (TYPE) 2.0.42 on page 127

Attributes exported:

- **nil**

Exports:

coerce hash latex patternMatch ?=?
?~=?

These exports come from (p337) PatternMatchable(Integer):

```
coerce : % -> OutputForm
    if R has PATMAB INT
    or R has PATMAB FLOAT
hash : % -> SingleInteger
    if R has PATMAB INT
    or R has PATMAB FLOAT
latex : % -> String
    if R has PATMAB INT
    or R has PATMAB FLOAT
patternMatch :
  (%,Pattern Integer,PatternMatchResult(Integer,%))
     -> PatternMatchResult(Integer,%) if R has PATMAB INT
?=? : (%,%) -> Boolean
```

```
     if R has PATMAB INT
     or R has PATMAB FLOAT
 ?~=? : (%,%) -> Boolean
     if R has PATMAB INT
     or R has PATMAB FLOAT
```

These exports come from (p337) PatternMatchable(Float):

```
 patternMatch :
   (%,Pattern Float,PatternMatchResult(Float,%))
      -> PatternMatchResult(Float,%) if R has PATMAB FLOAT
```

These exports come from (p127) Type():

— FullyPatternMatchable.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FPATMAB">
FullyPatternMatchable(R) (FPATMAB)</a></h2>
&bull; R : Type<br/><br/>

A set S is PatternMatchable over R if S can lift the
pattern-matching functions of S over the integers and float
to itself (necessary for matching in towers).
<br/><br/><br/>

?=? : (%,%) -> Boolean if R has PATMAB(INT) or R has PATMAB(FLOAT)<br/><br/>

coerce : % -> OutputForm if R has PATMAB(INT) or R has PATMAB(FLOAT)<br/><br/>

hash : % -> SingleInteger if R has PATMAB(INT) or R has PATMAB(FLOAT)<br/><br/>

latex : % -> String if R has PATMAB(INT) or R has PATMAB(FLOAT)<br/><br/>

patternMatch : (%,Pattern(Integer),PatternMatchResult(Integer,%))
 -> PatternMatch(Result(Integer,%) if R hs PATMAB(INT)<br/><br/>

patternMatch : (%,Pattern(Float),PatternMatchResult(Float,%))
 -> PatternMatchResult(Float,%) if R has PATMAB(FLOAT)<br/><br/>

?~=? : (%,%) -> Boolean if R has PATMAB(INT) or R has PATMAB(FLOAT)<br/><br/>

&bull; PatternMatchable Float<br/>
&bull; PatternMatchable Integer<br/>
&bull; Pattern Float<br/>
&bull; Pattern Integer<br/>

</body>
```

— category FPATMAB FullyPatternMatchable —

```
)abbrev category FPATMAB FullyPatternMatchable
++ Author: Manuel Bronstein
++ Date Created: 28 Nov 1989
++ Date Last Updated: 29 Nov 1989
++ Description:
++ A set S is PatternMatchable over R if S can lift the
++ pattern-matching functions of S over the integers and float
++ to itself (necessary for matching in towers).

FullyPatternMatchable(R) : Category == SIG where
  R : Type

  SIG ==> Type with

    if R has PatternMatchable Integer then PatternMatchable Integer

    if R has PatternMatchable Float   then PatternMatchable Float
```

———-

— **FPATMAB.dotabb** —

```
"FPATMAB"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FPATMAB"];
"FPATMAB" -> "TYPE"
```

———-

— **FPATMAB.dotfull** —

```
"FullyPatternMatchable(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FPATMAB"];
"FullyPatternMatchable(a:Type)" -> "Type()"

"FullyPatternMatchable(IntegralDomain)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FPATMAB"];
"FullyPatternMatchable(IntegralDomain)" ->
  "FullyPatternMatchable(a:Type)"

"FullyPatternMatchable(OrderedSet)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FPATMAB"];
"FullyPatternMatchable(OrderedSet)" ->
  "FullyPatternMatchable(a:Type)"

"FullyPatternMatchable(CommutativeRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FPATMAB"];
"FullyPatternMatchable(CommutativeRing)" ->
  "FullyPatternMatchable(a:Type)"
```

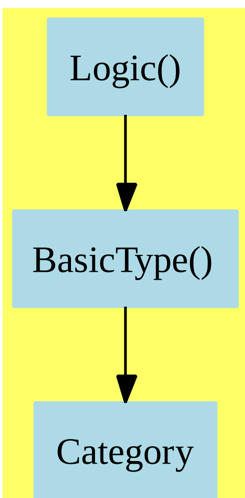
———-

— **FPATMAB.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FullyPatternMatchable(a:Type)" [color=lightblue];
"FullyPatternMatchable(a:Type)" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

———-

3.0.52 Logic (LOGIC)



— **Logic.input** —

```
)set break resume
)sys rm -f Logic.output
)spool Logic.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Logic
--R
--R Logic is a category constructor
```

```
--R Abbreviation for Logic is LOGIC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for LOGIC
--R
--R------------------------------- Operations --------------------------------
--R ?/\? : (%,%) -> %                     ?=? : (%,%) -> Boolean
--R ?\/? : (%,%) -> %                     ~? : % -> %
--R ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

———-

— **Logic.help** —

```
====================================================================
Logic examples
====================================================================

Logic provides the basic operations for lattices, for example,
boolean algebra.

See Also:
o )show Logic
```

———-

See:

⇒ "BitAggregate" (BTAGG) 9.0.147 on page 907
⇐ "BasicType" (BASTYPE) 2.0.9 on page 27

Exports:

?/\? ?=? ?\/? ~? ?~=?

These are directly exported but not implemented:

```
~? : % -> %
?/\? : (%,%) -> %
```

These are implemented by this category:

```
?\/? : (%,%) -> %
```

These exports come from (p27) BasicType():

```
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

— **Logic.html** —

```
<body>
<h2>
```

```
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#LOGIC">
Logic (LOGIC)</a></h2>
Logic provides the basic operations for lattices,
for example, boolean algebra.
<br/><br/><br/>
?/\? : (%,%) -> %<br/>
  /\ returns the logical ‘meet’, for example, ‘and’.<br/><br/>

?\/? : (%,%) -> %<br/>
  \/ returns the logical ‘join’, for example, ‘or’.<br/><br/>

~? : % -> %<br/>
  ~x returns the logical complement of x.<br/><br/>

?=? : (%,%) -> Boolean<br/>
  x=y tests if x and y are equal.<br/>
  from BasicType<br/><br/>

?~= : (%,%) -> Boolean<br/>
  x~=y tests if x and y are not equal.<br/>
  from BasicType<br/><br/>

&bull; BasicType

</body>
```

———

— category LOGIC Logic —

```
)abbrev category LOGIC Logic
++ Description:
++ Logic provides the basic operations for lattices,
++ for example, boolean algebra.

Logic() : Category == SIG where

  SIG ==> BasicType with

    _~ : % -> %
      ++ ~(x) returns the logical complement of x.

    _/_\ : (%, %) -> %
      ++ \spadignore{/\} returns the logical ‘meet’, for example, ‘and’.

    _\_/ : (%, %) -> %
      ++ \spadignore{\/} returns the logical ‘join’, for example, ‘or’.

   add

     _\_/(x: %,y: %) == _~( _/_\(_~(x), _~(y)))
```

———

— COQ LOGIC —

```
(* category LOGIC *)
(*
    _\_/: (%, %) -> %
    _\_/(x: %,y: %) == _~( _/_\(_~(x), _~(y)))

*)
```

— LOGIC.dotabb —

```
"LOGIC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LOGIC"];
"LOGIC" -> "BASTYPE"
```

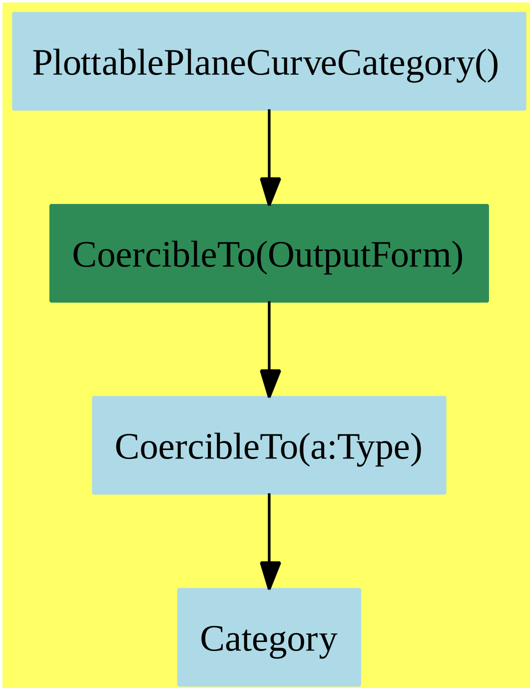
— LOGIC.dotfull —

```
"Logic()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LOGIC"];
"Logic()" -> "BasicType()"
```

— LOGIC.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"Logic()" [color=lightblue];
"Logic()" -> "BasicType()"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"Category" [color=lightblue];
}
```

3.0.53 PlottablePlaneCurveCategory (PPCURVE)



— **PlottablePlaneCurveCategory.input** —

```
)set break resume
)sys rm -f PlottablePlaneCurveCategory.output
)spool PlottablePlaneCurveCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PlottablePlaneCurveCategory
--R
--R PlottablePlaneCurveCategory is a category constructor
--R Abbreviation for PlottablePlaneCurveCategory is PPCURVE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PPCURVE
--R
--R------------------------------- Operations --------------------------------
--R coerce : % -> OutputForm              xRange : % -> Segment(DoubleFloat)
--R yRange : % -> Segment(DoubleFloat)
--R listBranches : % -> List(List(Point(DoubleFloat)))
--R
--E 1

)spool
)lisp (bye)
```

———-

— PlottablePlaneCurveCategory.help —

```
====================================================================
PlottablePlaneCurveCategory examples
====================================================================

PlottablePlaneCurveCategory is the category of curves in the plane
which may be plotted via the graphics facilities.  Functions are
provided for obtaining lists of lists of points, representing the
branches of the curve, and for determining the ranges of the
x-coordinates and y-coordinates of the points on the curve.

See Also:
o )show PlottablePlaneCurveCategory
```

———-

See:

⇐ "CoercibleTo" (KOERCE) 2.0.14 on page 39

Exports:

coerce listBranches xRange yRange

These are directly exported but not implemented:

```
listBranches : % -> List List Point DoubleFloat
xRange : % -> Segment DoubleFloat
yRange : % -> Segment DoubleFloat
```

These exports come from (p39) CoercibleTo(OutputForm):

```
coerce : % -> OutputForm
```

— PlottablePlaneCurveCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#PPCURVE">
PlottablePlaneCurveCategory (PPCURVE)</a></h2>
PlotablePlaneCurveCategory is the category of curves in the plane
which may be plotted via the graphics facilities. Functions are
provided for obtaining lists of lists of points, representing the
branches of the curve, and for determining the ranges of the
x-coordinates and y-coordinates of the points on the curve.
<br/><br/><br/>

listBranches : % -> List List Point DoubleFloat<br/>
  listBranches(c) returns a list of lists of points, representing the<br/>
  branches of the curve c.<br/><br/>

xRange : % -> Segment DoubleFloat<br/>
  xRange(c) returns the range of the x-coordinates of the points<br/>
```

```
  on the curve c.<br/><br/>

yRange : % -> Segment DoubleFloat<br/>
  yRange(c) returns the range of the y-coordinates of the points<br/>
  on the curve c.<br/><br/>

coerce : % -> OutputForm<br/>
  from CoercibleTo OutputForm<br/><br/>

&bull; CoercibleTo OutputForm

</body>
```

— category PPCURVE PlottablePlaneCurveCategory —

```
)abbrev category PPCURVE PlottablePlaneCurveCategory
++ Author: Clifton J. Williamson
++ Date Created: 11 January 1990
++ Date Last Updated: 15 June 1990
++ Description:
++ PlotablePlaneCurveCategory is the category of curves in the plane
++ which may be plotted via the graphics facilities. Functions are
++ provided for obtaining lists of lists of points, representing the
++ branches of the curve, and for determining the ranges of the
++ x-coordinates and y-coordinates of the points on the curve.

PlottablePlaneCurveCategory() : Category == SIG where

  L     ==> List
  SEG   ==> Segment
  SF    ==> DoubleFloat
  POINT ==> Point DoubleFloat

  SIG ==> CoercibleTo OutputForm with

    listBranches : % -> L L POINT
      ++ listBranches(c) returns a list of lists of points, representing the
      ++ branches of the curve c.

    xRange : % -> SEG SF
      ++ xRange(c) returns the range of the x-coordinates of the points
      ++ on the curve c.

    yRange : % -> SEG SF
      ++ yRange(c) returns the range of the y-coordinates of the points
      ++ on the curve c.
```

— PPCURVE.dotabb —

```
"PPCURVE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PPCURVE"];
"PPCURVE" -> "KOERCE"
```

———-

— PPCURVE.dotfull —

```
"PlottablePlaneCurveCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PPCURVE"];
"PlottablePlaneCurveCategory()" -> "CoercibleTo(OutputForm)"
```

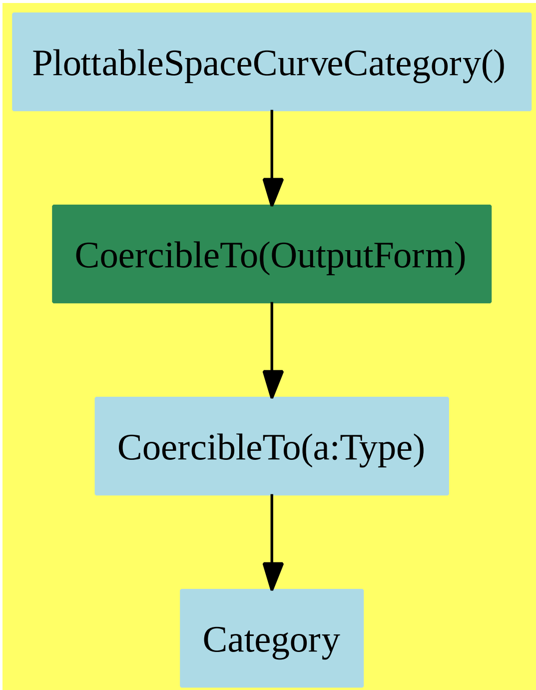
———-

— PPCURVE.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"PlottablePlaneCurveCategory()" [color=lightblue];
"PlottablePlaneCurveCategory()" -> "CoercibleTo(OutputForm)"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

———-

3.0.54 PlottableSpaceCurveCategory (PSCURVE)



— **PlottableSpaceCurveCategory.input** —

```
)set break resume
)sys rm -f PlottableSpaceCurveCategory.output
)spool PlottableSpaceCurveCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PlottableSpaceCurveCategory
--R
--R PlottableSpaceCurveCategory is a category constructor
--R Abbreviation for PlottableSpaceCurveCategory is PSCURVE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PSCURVE
--R
--R------------------------------- Operations --------------------------------
--R coerce : % -> OutputForm              xRange : % -> Segment(DoubleFloat)
--R yRange : % -> Segment(DoubleFloat)    zRange : % -> Segment(DoubleFloat)
--R listBranches : % -> List(List(Point(DoubleFloat)))
--R
--E 1

)spool
)lisp (bye)
```

———-

— **PlottableSpaceCurveCategory.help** —

```
====================================================================
PlottableSpaceCurveCategory examples
====================================================================

PlottableSpaceCurveCategory is the category of curves in 3-space which
may be plotted via the graphics facilities.  Functions are provided for
obtaining lists of lists of points, representing the branches of the
curve, and for determining the ranges of the x-, y-, and z-coordinates
of the points on the curve.

See Also:
o )show PlottableSpaceCurveCategory
```

———-

See:

⇐ "CoercibleTo" (KOERCE) 2.0.14 on page 39

Exports:

coerce listBranches xRange yRange zRange

These are directly exported but not implemented:

```
listBranches : % -> List List Point DoubleFloat
xRange : % -> Segment DoubleFloat
yRange : % -> Segment DoubleFloat
zRange : % -> Segment DoubleFloat
```

These exports come from (p39) CoercibleTo(OutputForm):

```
coerce : % -> OutputForm
```

— **PlottableSpaceCurveCategory.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#PSCURVE">
PlottableSpaceCurveCategory (PSCURVE)</a></h2>
PlottableSpaceCurveCategory is the category of curves in
3-space which may be plotted via the graphics facilities.  Functions are
provided for obtaining lists of lists of points, representing the
branches of the curve, and for determining the ranges of the
x-, y-, and z-coordinates of the points on the curve.
<br/><br/><br/>

listBranches : % -> List List Point DoubleFloat<br/>
  listBranches(c) returns a list of lists of points, representing the<br/>
  branches of the curve c.<br/><br/>

xRange : % -> Segment DoubleFloat<br/>
```

```
  xRange(c) returns the range of the x-coordinates of the points<br/>
  on the curve c.<br/><br/>

yRange : % -> Segment DoubleFloat<br/>
  yRange(c) returns the range of the y-coordinates of the points<br/>
  on the curve c.<br/><br/>

zRange : % -> Segment DoubleFloat<br/>
  zRange(c) returns the range of the z-coordinates of the points<br/>
  on the curve c.<br/><br/>

coerce : % -> OutputForm<br/>
  from CoercibleTo OutputForm<br/><br/>

&bull; CoercibleTo OutputForm

</body>
```

— category PSCURVE PlottableSpaceCurveCategory —

```
)abbrev category PSCURVE PlottableSpaceCurveCategory
++ Author: Clifton J. Williamson
++ Date Created: 11 January 1990
++ Date Last Updated: 15 June 1990
++ Description:
++ PlottableSpaceCurveCategory is the category of curves in
++ 3-space which may be plotted via the graphics facilities.  Functions are
++ provided for obtaining lists of lists of points, representing the
++ branches of the curve, and for determining the ranges of the
++ x-, y-, and z-coordinates of the points on the curve.

PlottableSpaceCurveCategory() : Category == SIG where

  L     ==> List
  SEG   ==> Segment
  SF    ==> DoubleFloat
  POINT ==> Point DoubleFloat

  SIG ==> CoercibleTo OutputForm with

    listBranches : % -> L L POINT
      ++ listBranches(c) returns a list of lists of points, representing the
      ++ branches of the curve c.

    xRange : % -> SEG SF
      ++ xRange(c) returns the range of the x-coordinates of the points
      ++ on the curve c.

    yRange : % -> SEG SF
      ++ yRange(c) returns the range of the y-coordinates of the points
      ++ on the curve c.
```

```
    zRange : % -> SEG SF
      ++ zRange(c) returns the range of the z-coordinates of the points
      ++ on the curve c.
```

———-

— **PSCURVE.dotabb** —

```
"PSCURVE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PSCURVE"];
"PSCURVE" -> "KOERCE"
```

———-

— **PSCURVE.dotfull** —

```
"PlottableSpaceCurveCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PSCURVE"];
"PlottableSpaceCurveCategory()" -> "CoercibleTo(OutputForm)"
```

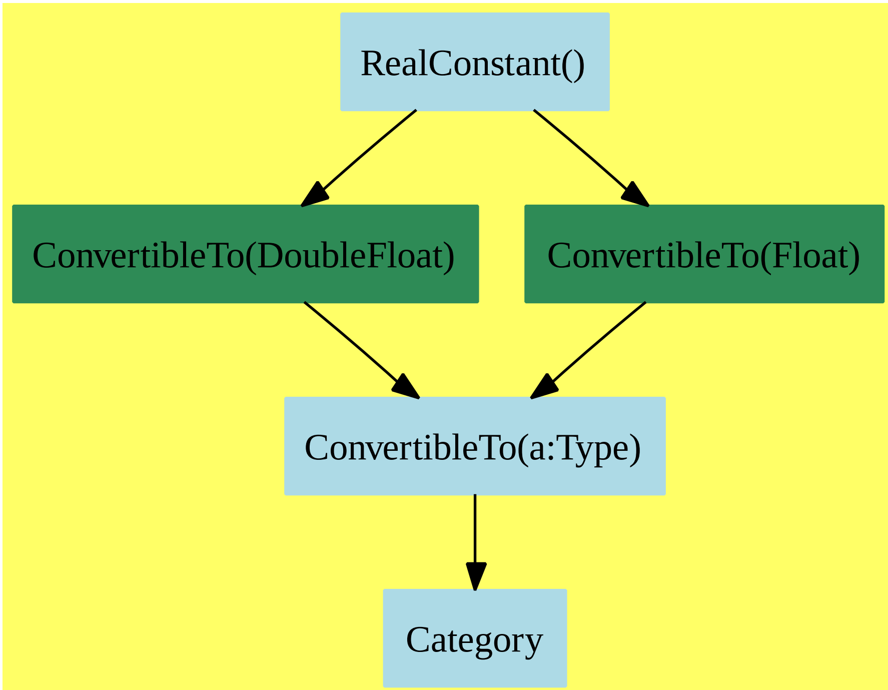
———-

— **PSCURVE.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"PlottableSpaceCurveCategory()" [color=lightblue];
"PlottableSpaceCurveCategory()" -> "CoercibleTo(OutputForm)"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

———-

3.0.55 RealConstant (REAL)



— **RealConstant.input** —

```
)set break resume
)sys rm -f RealConstant.output
)spool RealConstant.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RealConstant
--R
--R RealConstant is a category constructor
--R Abbreviation for RealConstant is REAL
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for REAL
--R
--R------------------------------- Operations --------------------------------
--R convert : % -> Float                  convert : % -> DoubleFloat
--R
--E 1

)spool
)lisp (bye)
```

— RealConstant.help —

```
====================================================================
RealConstant examples
====================================================================

The category of real numeric domains, that is, convertible to floats.

See Also:
o )show RealConstant
```

See:

⇒ "IntegerNumberSystem" (INS) 16.0.210 on page 1420
⇒ "QuotientFieldCategory" (QFCAT) 17.0.222 on page 1595
⇒ "RealNumberSystem" (RNS) 17.0.224 on page 1622
⇐ "ConvertibleTo" (KONVERT) 2.0.17 on page 47

Exports:

convert

These exports come from (p47) ConvertibleTo(DoubleFloat):

```
convert : % -> DoubleFloat
```

These exports come from (p47) ConvertibleTo(Float):

```
convert : % -> Float
```

— RealConstant.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#REAL">
RealConstant (REAL)</a></h2>
The category of real numeric domains, that is, convertible to floats.
<br/><br/><br/>

convert : % -> Float<br/>
  from ConvertibleTo Float<br/><br/>

convert : % -> DoubleFloat<br/>
  from ConvertibleTo DoubleFloat<br/><br/>

&bull; ConvertibleTo Float<br/>
&bull; ConvertibleTo DoubleFloat
</body>
```

— category REAL RealConstant —

```
)abbrev category REAL RealConstant
++ Description:
++ The category of real numeric domains, that is, convertible to floats.

RealConstant() : Category == SIG where

  SIG ==> Join(ConvertibleTo DoubleFloat, ConvertibleTo Float)
```

———-

— **REAL.dotabb** —

```
"REAL"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=REAL"];
"REAL" -> "KONVERT"
```

———-

— **REAL.dotfull** —

```
"RealConstant()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=REAL"];
"RealConstant()" -> "ConvertibleTo(DoubleFloat)"
"RealConstant()" -> "ConvertibleTo(Float)"
```

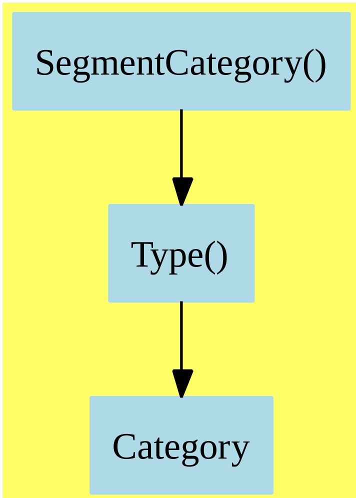
———-

— **REAL.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"RealConstant()" [color=lightblue];
"RealConstant()" -> "ConvertibleTo(DoubleFloat)"
"RealConstant()" -> "ConvertibleTo(Float)"

"ConvertibleTo(DoubleFloat)" [color=seagreen];
"ConvertibleTo(DoubleFloat)" -> "ConvertibleTo(a:Type)"

"ConvertibleTo(Float)" [color=seagreen];
"ConvertibleTo(Float)" -> "ConvertibleTo(a:Type)"

"ConvertibleTo(a:Type)" [color=lightblue];
"ConvertibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

———-

3.0.56 SegmentCategory (SEGCAT)



— **SegmentCategory.input** —

```
)set break resume
)sys rm -f SegmentCategory.output
)spool SegmentCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SegmentCategory
--R
--R SegmentCategory(S: Type) is a category constructor
--R Abbreviation for SegmentCategory is SEGCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SEGCAT
--R
--R------------------------------- Operations --------------------------------
--R BY : (%,Integer) -> %                ?..? : (S,S) -> %
--R convert : S -> %                     hi : % -> S
--R high : % -> S                        incr : % -> Integer
--R lo : % -> S                          low : % -> S
--R segment : (S,S) -> %
--R
--E 1

)spool
)lisp (bye)
```

———

— **SegmentCategory.help** —

```
====================================================================
SegmentCategory examples
====================================================================

This category provides operations on ranges, or segments as they are called.

See Also:
o )show SegmentCategory
```

See:

⇒ "SegmentExpansionCategory" (SEGXCAT) 4.0.84 on page 348
⇐ "Type" (TYPE) 2.0.42 on page 127

Exports:

BY	convert	hi	high	incr
lo	low	segment	?..?	

Attributes Exported:

- **nil**

These are directly exported but not implemented:

```
BY : (%,Integer) -> %
convert : S -> %
hi : % -> S
high : % -> S
incr : % -> Integer
lo : % -> S
low : % -> S
segment : (S,S) -> %
?..? : (S,S) -> %
```

— SegmentCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#SEGCAT">
SegmentCategory(S) (SEGCAT)</a></h2>
&bull; S : Type<br/><br/>
This category provides operations on ranges, or segments
as they are called.
<br/><br/><br/>

SEGMENT : (S, S) -> %<br/>
  l..h creates a segment with l and h as the endpoints.<br/><br/>

BY : (%, Integer) -> %<br/>
  s by n creates a new segment in which only every <br/>
  n-th element is used.<br/><br/>

lo : % -> S<br/>
```

```
  lo(s) returns the first endpoint of s.<br/>
  Note that lo(l..h) = l.<br/><br/>

hi : % -> S<br/>
  hi(s) returns the second endpoint of s.<br/>
  Note that hi(l..h) = h.<br/><br/>

low : % -> S<br/>
  low(s) returns the first endpoint of s.<br/>
  Note that low(l..h) = l.<br/><br/>

high : % -> S<br/>
  high(s) returns the second endpoint of s.<br/>
  Note that high(l..h) = h.<br/><br/>

incr : % -> Integer<br/>
  incr(s) returns n, where s is a segment in which every<br/>
  n-th element is used.<br/>
  Note that incr(l..h by n) = n.<br/><br/>

segment : (S, S) -> %<br/>
  segment(i,j) is an alternate way to create the segment i..j<br/><br/>

convert : S -> %<br/>
  convert(i) creates the segment i..i.<br/><br/>

</body>
```

— **category SEGCAT SegmentCategory** —

```
)abbrev category SEGCAT SegmentCategory
++ Author:  Stephen M. Watt
++ Date Created:  December 1986
++ Date Last Updated: June 3, 1991
++ Description:
++ This category provides operations on ranges, or segments
++ as they are called.

SegmentCategory(S) : Category == SIG where
  S : Type

  SIG ==> Type with

    SEGMENT : (S, S) -> %
      ++ \spad{l..h} creates a segment with l and h as the endpoints.

    BY : (%, Integer) -> %
      ++ \spad{s by n} creates a new segment in which only every
      ++ \spad{n}-th element is used.

    lo : % -> S
      ++ lo(s) returns the first endpoint of s.
```

```
      ++ Note that \spad{lo(l..h) = l}.

    hi : % -> S
      ++ hi(s) returns the second endpoint of s.
      ++ Note that \spad{hi(l..h) = h}.

    low : % -> S
      ++ low(s) returns the first endpoint of s.
      ++ Note that \spad{low(l..h) = l}.

    high : % -> S
      ++ high(s) returns the second endpoint of s.
      ++ Note that \spad{high(l..h) = h}.

    incr : % -> Integer
      ++ incr(s) returns \spad{n}, where s is a segment in which every
      ++ \spad{n}-th element is used.
      ++ Note that \spad{incr(l..h by n) = n}.

    segment : (S, S) -> %
      ++ segment(i,j) is an alternate way to create the segment
      ++ \spad{i..j}.

    convert : S -> %
      ++ convert(i) creates the segment \spad{i..i}.
```

———-

— SEGCAT.dotabb —

```
"SEGCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SEGCAT"];
"SEGCAT" -> "TYPE"
```

———-

— SEGCAT.dotfull —

```
"SegmentCategory(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SEGCAT"];
"SegmentCategory(a:Type)" -> "Type()"

"SegmentCategory(OrderedRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=SEGCAT"];
"SegmentCategory(OrderedRing)" -> "SegmentCategory(a:Type)"
```

———-

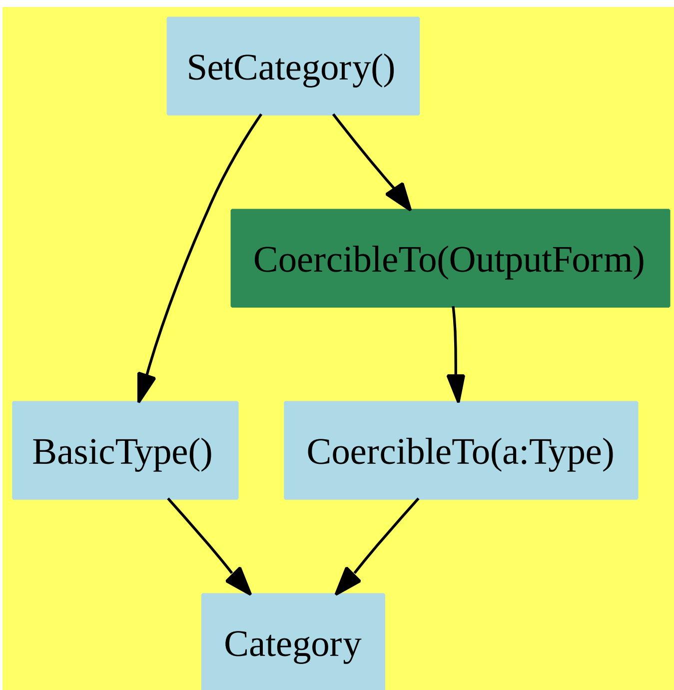
— SEGCAT.dotpic —

```
digraph pic {
```

```
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"SegmentCategory(a:Type)" [color=lightblue];
"SegmentCategory(a:Type)" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

————-

3.0.57 SetCategory (SETCAT)



— **SetCategory.input** —

```
)set break resume
)sys rm -f SetCategory.output
)spool SetCategory.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show SetCategory
--R
--R SetCategory is a category constructor
--R Abbreviation for SetCategory is SETCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SETCAT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

— **SetCategory.help** —

```
====================================================================
SetCategory examples
====================================================================

SetCategory is the basic category for describing a collection
of elements with = (equality) and coerce to output form.

Conditional Attributes canonical data structure equality is the same as =

See Also:
o )show SetCategory
```

See:

⇒ "AbelianSemiGroup" (ABELSG) 4.0.59 on page 201
⇒ "BlowUpMethodCategory" (BLMETCT) 4.0.60 on page 206
⇒ "FileCategory" (FILECAT) 4.0.68 on page 252
⇒ "FileNameCategory" (FNCAT) 4.0.70 on page 263
⇒ "Finite" (FINITE) 4.0.69 on page 258
⇒ "GradedModule" (GRMOD) 4.0.71 on page 269
⇒ "HomogeneousAggregate" (HOAGG) 4.0.73 on page 280
⇒ "IndexedDirectProductCategory" (IDPC) 4.0.74 on page 290
⇒ "NumericalIntegrationCategory" (NUMINT) 4.0.77 on page 309
⇒ "NumericalOptimizationCategory" (OPTCAT) 4.0.78 on page 315
⇒ "OrderedSet" (ORDSET) 4.0.80 on page 326
⇒ "OrdinaryDifferentialEquationsSolverCategory" (ODECAT) 4.0.79 on page 321
⇒ "PartialDifferentialEquationsSolverCategory" (PDECAT) 4.0.81 on page 331
⇒ "PatternMatchable" (PATMAB) 4.0.82 on page 337
⇒ "PolynomialSetCategory" (PSETCAT) 6.0.115 on page 598

⇒ "RealRootCharacterizationCategory" (RRCC) 4.0.83 on page 341
⇒ "SemiGroup" (SGROUP) 4.0.85 on page 353
⇒ "SetAggregate" (SETAGG) 6.0.118 on page 628
⇒ "SExpressionCategory" (SEXCAT) 4.0.87 on page 361
⇒ "StepThrough" (STEP) 4.0.88 on page 367
⇒ "StringCategory" (STRICAT) 10.0.173 on page 1093
⇒ "ThreeSpaceCategory" (SPACEC) 4.0.89 on page 371
⇐ "BasicType" (BASTYPE) 2.0.9 on page 27
⇐ "CoercibleTo" (KOERCE) 2.0.14 on page 39

Exports:

coerce hash latex ?=? ?~=?

These are implemented by this category:

```
hash : % -> SingleInteger
latex : % -> String
```

These exports come from (p27) BasicType():

```
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

These exports come from (p39) CoercibleTo(OutputForm):

```
coerce : % -> OutputForm
```

— **SetCategory.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#SETCAT">
SetCategory (SETCAT)</a></h2>

?=? : (%,%) -> Boolean<br/>
  x=y tests if x and y are equal.<br/>
  from BasicType<br/><br/>

?~=? : (%,%) -> Boolean<br/>
  x~=y tests if x and y are not equal.<br/>
  from BasicType<br/><br/>

coerce : % -> OutputForm<br/>
  coerce(a) transforms a into an element of OutputForm<br/>
  from CoercibleTo OutputForm<br/><br/>

hash : % -> SingleInteger<br/>
  hash(s) calculates a hash code for s.<br/><br/>

latex : % -> String<br/>
  latex(s) returns a LaTeX-printable output<br/>
  representation of s.<br/><br/>

&bull; CoercibleTo OutputForm<br/>
&bull; BasicType<br/>
```

```
</body>
```

— category SETCAT SetCategory —

```
)abbrev category SETCAT SetCategory
++ Date Last Updated: November 10, 2009 tpd happy birthday
++ Description:
++ \spadtype{SetCategory} is the basic category for describing a collection
++ of elements with \spadop{=} (equality) and \spadfun{coerce} to
++ output form.
++
++ Conditional Attributes\br
++ \tab{5}canonical\tab{5}data structure equality is the same as \spadop{=}

SetCategory() : Category == SIG where

  SIG ==> Join(BasicType,CoercibleTo OutputForm) with

    hash : % -> SingleInteger
      ++ hash(s) calculates a hash code for s.

    latex : % -> String
      ++ latex(s) returns a LaTeX-printable output representation of s.

   add

      hash(s : %):  SingleInteger == SXHASH(s)$Lisp

      latex(s : %): String == "\mbox{\bf Unimplemented}"
```

— COQ SETCAT —

```
(* category SETCAT *)
(*

    hash: % -> SingleInteger
    hash(s : %): SingleInteger == SXHASH(s)$Lisp

    latex: % -> String
    latex(s : %): String == "\mbox{\bf Unimplemented}"

*)
```

— SETCAT.dotabb —

```
"SETCAT"
```

```
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SETCAT"];
"SETCAT" -> "BASTYPE"
"SETCAT" -> "KOERCE"
```

———-

— SETCAT.dotfull —

```
"SetCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SETCAT"];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"
```

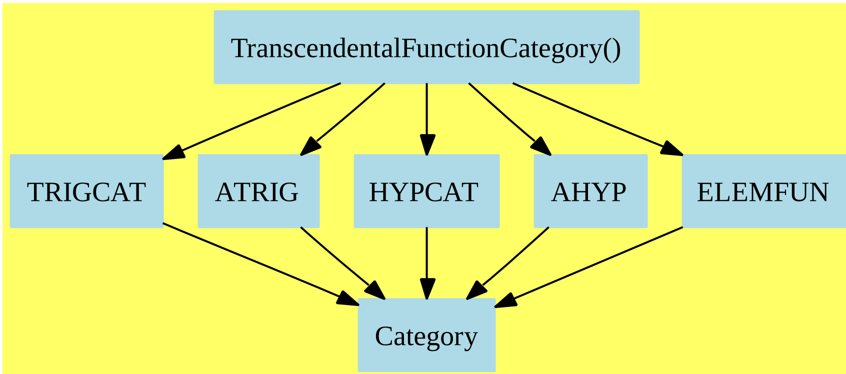
———-

— SETCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

———-

3.0.58 TranscendentalFunctionCategory (TRANFUN)



— **TranscendentalFunctionCategory.input** —

```
)set break resume
)sys rm -f TranscendentalFunctionCategory.output
)spool TranscendentalFunctionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show TranscendentalFunctionCategory
--R
--R TranscendentalFunctionCategory is a category constructor
--R Abbreviation for TranscendentalFunctionCategory is TRANFUN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for TRANFUN
--R
--R------------------------------- Operations --------------------------------
--R ?**? : (%,%) -> %                    acos : % -> %
--R acosh : % -> %                       acot : % -> %
--R acoth : % -> %                       acsc : % -> %
--R acsch : % -> %                       asec : % -> %
--R asech : % -> %                       asin : % -> %
--R asinh : % -> %                       atan : % -> %
--R atanh : % -> %                       cos : % -> %
--R cosh : % -> %                        cot : % -> %
--R coth : % -> %                        csc : % -> %
--R csch : % -> %                        exp : % -> %
--R log : % -> %                         pi : () -> %
--R sec : % -> %                         sech : % -> %
--R sin : % -> %                         sinh : % -> %
--R tan : % -> %                         tanh : % -> %
--R
--E 1

)spool
)lisp (bye)
```

— **TranscendentalFunctionCategory.help** —

```
====================================================================
TranscendentalFunctionCategory examples
====================================================================

This is the Category for the transcendental elementary functions.

See Also:
o )show TranscendentalFunctionCategory
```

The `acsch`, `asech`, and `acoth` functions were modified to include an intermediate test to check that the argument has a reciprocal values.

See:

⇒ “IntervalCategory” (INTCAT) 14.0.200 on page 1341
⇒ “LiouvillianFunctionCategory” (LFCAT) 4.0.75 on page 295
⇒ “UnivariateLaurentSeriesCategory” (ULSCAT) 17.0.226 on page 1686
⇒ “UnivariatePuiseuxSeriesCategory” (UPXSCAT) 17.0.227 on page 1697
⇒ “UnivariateTaylorSeriesCategory” (UTSCAT) 16.0.214 on page 1476
⇐ “ArcHyperbolicFunctionCategory” (AHYP) 2.0.6 on page 13
⇐ “ArcTrigonometricFunctionCategory” (ATRIG) 2.0.7 on page 16
⇐ “ElementaryFunctionCategory” (ELEMFUN) 2.0.18 on page 51
⇐ “HyperbolicFunctionCategory” (HYPCAT) 2.0.21 on page 59
⇐ “TrigonometricFunctionCategory” (TRIGCAT) 2.0.41 on page 123

Exports:

?**?	acos	acosh	acot	acoth
acsc	acsch	asec	asech	asin
asinh	atan	atanh	cos	cosh
cot	coth	csc	csch	exp
log	pi	sec	sech	sin
sinh	tan	tanh		

These are implemented by this category:

```
pi : () -> %
```

These exports come from (p123) TrigonometricFunctionCategory():

```
cos : % -> %
cot : % -> %
csc : % -> %
sec : % -> %
sin : % -> %
tan : % -> %
```

These exports come from (p16) ArcTrigonometricFunctionCategory():

```
acos : % -> %
```

```
acot : % -> %
acsc : % -> %
asec : % -> %
asin : % -> %
atan : % -> %
```

These exports come from (p59) HyperbolicFunctionCategory():

```
cosh : % -> %
coth : % -> %
csch : % -> %
sech : % -> %
sinh : % -> %
tanh : % -> %
```

These exports come from (p13) ArcHyperbolicFunctionCategory():

```
acosh : % -> %
acoth : % -> %
acsch : % -> %
asech : % -> %
asinh : % -> %
atanh : % -> %
```

These exports come from (p51) ElementaryFunctionCategory():

```
?**? : (%,%) -> %
exp : % -> %
log : % -> %
```

— TranscendentalFunctionCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#TRANFUN">
TranscendentalFunctionCategory (TRANFUN)</a></h2>
Category for the transcendental elementary functions.
<br/><br/><br/>

?**? : (%,%) -> %<br/>
  x**y returns x to the power y.<br/>
  from ElementaryFunctionCategory<br/><br/>

acos : % -> %<br/>
  acos(x) returns the arc-cosine of x. When evaluated<br/>
  into some subset of the complex numbers, one<br/>
  branch cut for acos lies along the negative real axis<br/>
  to the left of -1 (inclusive), continuous with the<br/>
  upper half plane, the other along the positive real axis to<br/>
  the right of 1 (inclusive), continuous with the lower half plane.<br/>
  from ArcTrigonometricFunctionCategory<br/><br/>

acosh : % -> %<br/>
  acosh(x) returns the hyperbolic arc-cosine of x.<br/>
  from ArcHyperbolicFunctionCategory<br/><br/>

acot : % -> %<br/>
```

```
  acot(x) returns the arc-cotangent of x.<br/>
  from ArcTrigonometricFunctionCategory<br/><br/>

acoth : % -> %<br/>
  acoth(x) returns the hyperbolic arc-cotangent of x.<br/>
  from ArcHyperbolicFunctionCategory<br/><br/>

acsc : % -> %<br/>
  acsc(x) returns the arc-cosecant of x.<br/>
  from ArcTrigonometricFunctionCategory<br/><br/>

acsch : % -> %<br/>
  acsch(x) returns the hyperbolic arc-cosecant of x.<br/>
  from ArcHyperbolicFunctionCategory<br/><br/>

asec : % -> %<br/>
  asec(x) returns the arc-secant of x.<br/>
  from ArcTrigonometricFunctionCategory<br/><br/>

asech : % -> %<br/>
  asech(x) returns the hyperbolic arc-secant of x.<br/>
  from ArcHyperbolicFunctionCategory<br/><br/>

asin : % -> %<br/>
  asin(x) returns the arc-sine of x. When evaluated into some<br/>
  subset of the complex numbers, one branch cut for asin lies<br/>
  along the negative real axis to the left of -1 (inclusive),<br/>
  continuous with the upper half plane, the other along the<br/>
  positive real axis to the right of 1 (inclusive), continuous<br/>
  with the lower half plane.<br/>
  from ArcTrigonometricFunctionCategory<br/><br/>

asinh : % -> %<br/>
  asinh(x) returns the hyperbolic arc-sine of x.<br/>
  from ArcHyperbolicFunctionCategory<br/><br/>

atan : % -> %<br/>
  atan(x) returns the arc-tangent of x. When evaluated into some<br/>
  subset of the complex numbers, one branch cut for atan lies<br/>
  along the positive imaginary axis above %i (exclusive),<br/>
  continuous with the left half plane, the other along the<br/>
  negative imaginary axis below -%i (exclusive) continuous<br/>
  with the right half plane. The domain does not contain %i and -%i<br/>
  from ArcTrigonometricFunctionCategory<br/><br/>

atanh : % -> %<br/>
  atanh(x) returns the hyperbolic arc-tangent of x.<br/>
  from ArcHyperbolicFunctionCategory<br/><br/>

cos : % -> %<br/>
  cos(x) returns the cosine of x.<br/>
  from TrigonometricFunctionCategory<br/><br/>

cosh : % -> %<br/>
```

```
  cosh(x) returns the hyperbolic cosine of x.<br/>
  from HyperbolicFunctionCategory<br/><br/>

cot : % -> %<br/>
  cot(x) returns the cotangent of x.<br/>
  from TrigonometricFunctionCategory<br/><br/>

coth : % -> %<br/>
  coth(x) returns the hyperbolic cotangent of x.<br/>
  from HyperbolicFunctionCategory<br/><br/>

csc : % -> %<br/>
  csc(x) returns the cosecant of x.<br/>
  from TrigonometricFunctionCategory<br/><br/>

csch : % -> %<br/>
  csch(x) returns the hyperbolic cosecant of x.<br/>
  from HyperbolicFunctionCategory<br/><br/>

exp : % -> %<br/>
  exp(x) returns %e to the power x.<br/>
  from ElementaryFunctionCategory<br/><br/>

log : % -> %<br/>
  log(x) returns the natural logarithm of x. When evaluated<br/>
  into some subset of the complex numbers, the branch cut lies<br/>
  along the negative real axis, continuous with quadrant II. The<br/>
  domain does not contain the origin.<br/>
  from ElementaryFunctionCategory<br/><br/>

pi : () -> %<br/>
  pi() returns the constant pi.<br/><br/>

sec : % -> %<br/>
  sec(x) returns the secant of x.<br/>
  from TrigonometricFunctionCategory<br/><br/>

sech : % -> %<br/>
  sech(x) returns the hyperbolic secant of x.<br/>
  from HyperbolicFunctionCategory<br/><br/>

sin : % -> %<br/>
  sin(x) returns the sine of x.<br/>
  from TrigonometricFunctionCategory<br/><br/>

sinh : % -> %<br/>
  sinh(x) returns the hyperbolic sine of x.<br/>
  from HyperbolicFunctionCategory<br/><br/>

tan : % -> %<br/>
  tan(x) returns the tangent of x.<br/>
  from TrigonometricFunctionCategory<br/><br/>

tanh : % -> %<br/>
```

```
  tanh(x) returns the hyperbolic tangent of x.<br/>
  from HyperbolicFunctionCategory<br/><br/>

&bull; ElementaryFunctionCategory<br/>
&bull; ArcTrigonometricFunctionCategory<br/>
&bull; ArcHyperbolicFunctionCategory<br/>
&bull; TrigonometricFunctionCategory<br/>
&bull; HyperbolicFunctionCategory<br/>
&bull; ElementaryFunctionCategory<br/>

</body>
```

— category TRANFUN TranscendentalFunctionCategory —

```
)abbrev category TRANFUN TranscendentalFunctionCategory
++ Author: Manuel Bronstein
++ Date Last Updated: 14 May 1991
++ Description:
++ Category for the transcendental elementary functions;

TranscendentalFunctionCategory() : Category == SIG where

  TFC  ==> TrigonometricFunctionCategory
  ATFC ==> ArcTrigonometricFunctionCategory
  HFC  ==> HyperbolicFunctionCategory
  AHFC ==> ArcHyperbolicFunctionCategory
  EFC  ==> ElementaryFunctionCategory

  SIG ==> Join(TFC,ATFC,HFC,AHFC,EFC) with

    pi : () -> $
      ++ pi() returns the constant pi.

   add

     if $ has Ring then

       pi() == 2*asin(1)

       acsch x ==
         (a := recip x) case "failed" => error "acsch: no reciprocal"
         asinh(a::$)

       asech x ==
         (a := recip x) case "failed" => error "asech: no reciprocal"
         acosh(a::$)

       acoth x ==
         (a := recip x) case "failed" => error "acoth: no reciprocal"
         atanh(a::$)

     if $ has Field and $ has sqrt: $ -> $ then
```

```
  asin x == atan(x/sqrt(1-x**2))

  acos x == pi()/2::$ - asin x

  acot x == pi()/2::$ - atan x

  asinh x == log(x + sqrt(x**2 + 1))

  acosh x == 2*log(sqrt((x+1)/2::$) + sqrt((x-1)/2::$))

  atanh x == (log(1+x)-log(1-x))/2::$
```

———-

— COQ TRANFUN —

```
(* category TRANFUN *)
(*
     if $ has Ring then

       pi : () -> $        ++ pi() returns the constant pi.
       pi() == 2*asin(1)

       acsch : % -> %
       acsch x ==
         (a := recip x) case "failed" => error "acsch: no reciprocal"
         asinh(a::$)

       asech : % -> %
       asech x ==
         (a := recip x) case "failed" => error "asech: no reciprocal"
         acosh(a::$)

       acoth : % -> %
       acoth x ==
         (a := recip x) case "failed" => error "acoth: no reciprocal"
         atanh(a::$)

     if $ has Field and $ has sqrt: $ -> $ then

       asin : % -> %
       asin x == atan(x/sqrt(1-x**2))

       acos : % -> %
       acos x == pi()/2::$ - asin x

       acot : % -> %
       acot x == pi()/2::$ - atan x

       asinh : % -> %
       asinh x == log(x + sqrt(x**2 + 1))
```

```
       acosh : % -> %
       acosh x == 2*log(sqrt((x+1)/2::$) + sqrt((x-1)/2::$))

       atanh : % -> %
       atanh x == (log(1+x)-log(1-x))/2::$

*)
```

———-

— TRANFUN.dotabb —

```
"TRANFUN"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=TRANFUN"];
"TRANFUN" -> "TRIGCAT"
"TRANFUN" -> "ATRIG"
"TRANFUN" -> "HYPCAT"
"TRANFUN" -> "AHYP"
"TRANFUN" -> "ELEMFUN"
```

———-

— TRANFUN.dotfull —

```
"TranscendentalFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=TRANFUN"];
"TranscendentalFunctionCategory()" ->
    "TrigonometricFunctionCategory()"
"TranscendentalFunctionCategory()" ->
    "ArcTrigonometricFunctionCategory()"
"TranscendentalFunctionCategory()" ->
    "HyperbolicFunctionCategory()"
"TranscendentalFunctionCategory()" ->
    "ArcHyperbolicFunctionCategory()"
"TranscendentalFunctionCategory()" ->
    "ElementaryFunctionCategory()"
```

———-

— TRANFUN.dotpic —

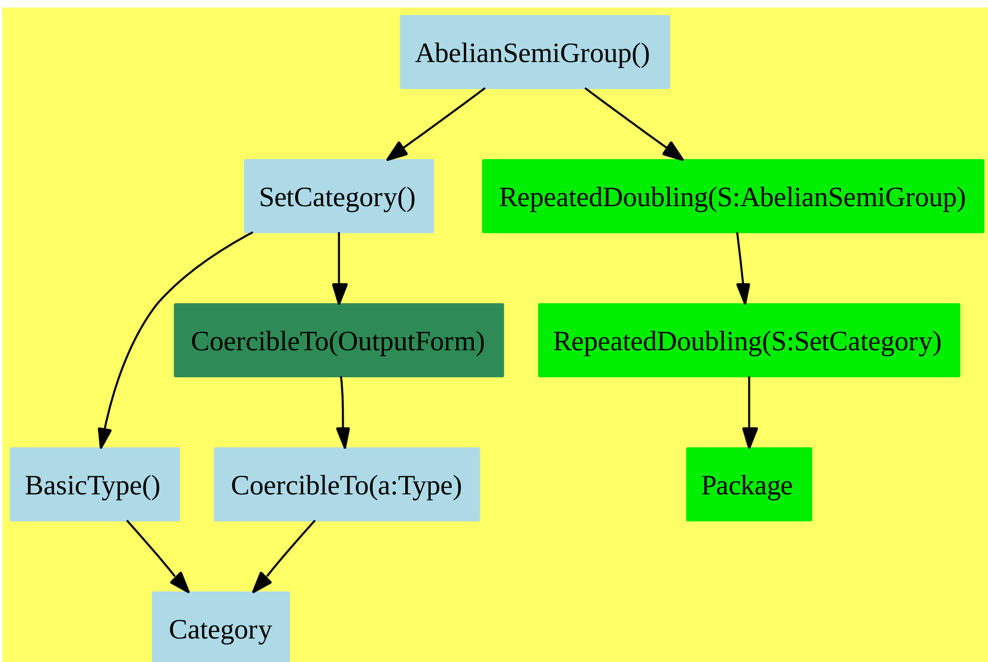
```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"TranscendentalFunctionCategory()" [color=lightblue];
"TranscendentalFunctionCategory()" ->
    "TRIGCAT"
"TranscendentalFunctionCategory()" ->
    "ATRIG"
"TranscendentalFunctionCategory()" ->
```

```
    "HYPCAT"
"TranscendentalFunctionCategory()" ->
    "AHYP"
"TranscendentalFunctionCategory()" ->
    "ELEMFUN"

"TRIGCAT" [color=lightblue];
"TRIGCAT" -> "Category"

"ATRIG" [color=lightblue];
"ATRIG" -> "Category"

"HYPCAT" [color=lightblue];
"HYPCAT" -> "Category"

"AHYP" [color=lightblue];
"AHYP" -> "Category"

"ELEMFUN" [color=lightblue];
"ELEMFUN" -> "Category"

"Category" [color=lightblue];
}
```

Chapter 4

Category Layer 3

4.0.59 AbelianSemiGroup (ABELSG)



— **AbelianSemiGroup.input** —

```
)set break resume
)sys rm -f AbelianSemiGroup.output
)spool AbelianSemiGroup.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show AbelianSemiGroup
--R
--R AbelianSemiGroup is a category constructor
--R Abbreviation for AbelianSemiGroup is ABELSG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ABELSG
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (PositiveInteger,%) -> %        ?+? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

— **AbelianSemiGroup.help** —

```
====================================================================
AbelianSemiGroup examples
====================================================================

This is the class of all additive (commutative) semigroups, that is,
a set with a commutative and associative operation +.

Axioms:
  associative("+":(%,%)->%)   (x+y)+z = x+(y+z)
  commutative("+":(%,%)->%)   x+y = y+x

See Also:
o )show AbelianSemiGroup
```

See:

⇒ "AbelianMonoid" (ABELMON) 5.0.90 on page 383
⇒ "FunctionSpace" (FS) 17.0.219 on page 1544
⇐ "SetCategory" (SETCAT) 3.0.57 on page 187

Exports:

coerce	hash	latex	?*?	?+?
?=?	?~=?			

These are directly exported but not implemented:

```
?+? : (%,%) -> %
```

These are implemented by this category:

```
?*? : (PositiveInteger,%) -> %
```

These exports come from (p187) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

— **AbelianSemiGroup.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ABELSG">
AbelianSemiGroup (ABELSG)</a></h2>
The class of all additive (commutative) semigroups, that is,
a set with a commutative and associative operation +.
<br/><br/><br/>

?*? : (PositiveInteger,%) -> %<br/>
  n*x computes the left-multiplication of x by the positive <br/>
  integer n. This is equivalent to adding x to itself n times.<br/><br/>

?+? : (%,%) -> %<br/>
  x+y computes the sum of x and y.<br/><br/>

?=? : (%,%) -> Boolean<br/>
  x=y tests if x and y are equal.<br/>
  from BasicType<br/><br/>

coerce : % -> OutputForm<br/>
  coerce(a) transforms a into an element of OutputForm<br/>
  from CoercibleTo OutputForm<br/><br/>

hash : % -> SingleInteger<br/>
  hash(s) calculates a hash code for s.<br/>
  from SetCategory<br/><br/>

latex : % -> String<br/>
  latex(s) returns a LaTeX-printable output representation of s.<br/>
  from SetCategory<br/><br/>

?~=? : (%,%) -> Boolean<br/>
  x~=y tests if x and y are not equal.<br/>
  from BasicType<br/><br/>

&bull; BasicType<br/>
&bull; SetCategory<br/>
&bull; CoercibleTo OutputForm<br/>

</body>
```

———

— category ABELSG AbelianSemiGroup —

```
)abbrev category ABELSG AbelianSemiGroup
++ Description:
++ The class of all additive (commutative) semigroups, that is,
++ a set with a commutative and associative operation \spadop{+}.
++
++ Axioms\br
++ \tab{5}\spad{associative("+":(%,%)->%)}\tab{5}\spad{(x+y)+z = x+(y+z)}\br
++ \tab{6}\spad{commutative("+":(%,%)->%)}\tab{5}\spad{x+y = y+x}

AbelianSemiGroup() : Category == SIG where

  SIG ==> SetCategory with

    "+" : (%,%) -> %
      ++ x+y computes the sum of x and y.

    "*" : (PositiveInteger,%) -> %
      ++ n*x computes the left-multiplication of x by the positive
      ++ integer n. This is equivalent to adding x to itself n times.

   add

     import RepeatedDoubling(%)

     if not (% has Ring) then

        n:PositiveInteger * x:% == double(n,x)
```

———-

— COQ ABELSG —

```
(* category ABELSG *)
(*
      import RepeatedDoubling(%)

      if not (% has Ring) then

        "*": (PositiveInteger,%) -> %
        n:PositiveInteger * x:% == double(n,x)

*)
```

———-

— ABELSG.dotabb —

```
"ABELSG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ABELSG"];
"ABELSG" -> "SETCAT"
"ABELSG" -> "REPDB"
```

———-

— ABELSG.dotfull —

```
"AbelianSemiGroup()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ABELSG"];
"AbelianSemiGroup()" -> "SetCategory()"
"AbelianSemiGroup()" -> "RepeatedDoubling(a:SetCategory)"
```

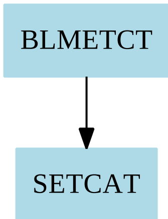
———-

— ABELSG.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SetCategory()"
"AbelianSemiGroup()" -> "RepeatedDoubling(AbelianSemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" ->
    "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"RepeatedDoubling(AbelianSemiGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianSemiGroup)" -> "RepeatedDoubling(a:SetCategory)"

"RepeatedDoubling(a:SetCategory)" [color="#00EE00"];
"RepeatedDoubling(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];
}
```

———-

4.0.60 BlowUpMethodCategory (BLMETCT)



— **BlowUpMethodCategory.input** —

```
)set break resume
)sys rm -f BlowUpMethodCategory.output
)spool BlowUpMethodCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BlowUpMethodCategory
--R
--R BlowUpMethodCategory is a category constructor
--R Abbreviation for BlowUpMethodCategory is BLMETCT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for BLMETCT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                chartCoord : % -> Integer
--R coerce : List(Integer) -> %           coerce : % -> OutputForm
--R excepCoord : % -> Integer             hash : % -> SingleInteger
--R infClsPt? : % -> Boolean              latex : % -> String
--R quotValuation : % -> Integer          ramifMult : % -> Integer
--R transCoord : % -> Integer             ?~=? : (%,%) -> Boolean
--R createHN : (Integer,Integer,Integer,Integer,Integer,Boolean,Union(left,center,right,vertical,horizontal)) ->
--R type : % -> Union(left,center,right,vertical,horizontal)
--R
--E 1

)spool
)lisp (bye)
```

— **BlowUpMethodCategory.help** —

```
====================================================================
BlowUpMethodCategory examples
====================================================================

See Also:
o )show BlowUpMethodCategory
```

———-

See:

⇐ "BasicType" (BASTYPE) 2.0.9 on page 27
⇐ "CoercibleTo" (KOERCE) 2.0.14 on page 39
⇐ "SetCategory" (SETCAT) 3.0.57 on page 187

Exports:

?=?	?~=?	chartCoord	coerce	createHN
excepCoord	hash	infClsPt?	latex	quotValuation
ramifMult	transCoord	type		

These are directly exported but not implemented:

```
coerce: List Integer -> %
excepCoord: % -> Integer
chartCoord: % -> Integer
transCoord: % -> Integer
createHN:  ( Integer, Integer, Integer , Integer, Integer, Boolean,_
             Union("left","center","right","vertical","horizontal")) -> %
ramifMult: % -> Integer
infClsPt_? : % -> Boolean
quotValuation : % -> Integer
type: % -> Union("left","center","right","vertical","horizontal")
```

These exports come from (p187) SetCategory():

```
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
```

— **BlowUpMethodCategory.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#BLMETCT">
BlowUpMethodCategory (BLMETCT)</a></h2>

?=? : (%,%) -> Boolean<br/>
  x=y tests if x and y are equal.<br/>
  from BasicType<br/><br/>

chartCoord : % -> Integer<br/><br/>

coerce : List(Integer) -> %<br/>
  coerce(a) transforms a into an element of S.<br/>
  from CoercibleTo List Integer<br/><br/>

coerce : % -> OutputForm<br/>
  coerce(a) transforms a into an element of S.<br/>
  from CoercibleTo OutputForm<br/><br/>
```

```
excepCoord : % -> Integer<br/><br/>

hash : % -> SingleInteger<br/>
  hash(s) calculates a hash code for s.<br/>
  from SetCategory<br/><br/>

infClsPt? : % -> Boolean<br/><br/>

latex : % -> String<br/>
  latex(s) returns a LaTeX-printable output representation of s.<br/>
  from SetCategory<br/><br/>

quotValuation : % -> Integer<br/><br/>

ramifMult : % -> Integer<br/><br/>

transCoord : % -> Integer<br/><br/>

?~=? : (%,%) -> Boolean<br/>
  x~=y tests if x and y are not equal.<br/>
  from BasicType<br/><br/>

createHN : (Integer,Integer,Integer,Integer,Integer,Boolean,<br/>
   Union(left,center,right,vertical,horizontal)) -> %<br/><br/>

type : % -> Union(left,center,right,vertical,horizontal)<br/><br/>

&bull; SetCategory<br/>
&bull; BasicType<br/>
&bull; CoercibleTo List Integer<br/>
&bull; CoercibleTo OutputForm<br/>

</body>
```

— category BLMETCT BlowUpMethodCategory —

```
)abbrev category BLMETCT BlowUpMethodCategory

BlowUpMethodCategory() : Category == SIG where

  SIG ==> SetCategory with

    coerce : List Integer -> %

    excepCoord : % -> Integer

    chartCoord : % -> Integer

    transCoord : % -> Integer

    createHN : ( Integer, Integer, Integer , Integer, Integer, Boolean,_
                 Union("left","center","right","vertical","horizontal")) -> %
```

```
    ramifMult : % -> Integer

    infClsPt_? : % -> Boolean

    quotValuation : % -> Integer

    type : % -> Union("left","center","right","vertical","horizontal")
```

———-

— BLMETCT.dotabb —

```
"BLMETCT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BLMETCT"];
"BLMETCT" -> "SETCAT"
```

———-

— BLMETCT.dotfull —

```
"BlowUpMethodCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BLMETCT"];
"BlowUpMethodCategory()" -> "SetCategory()"
```

———-

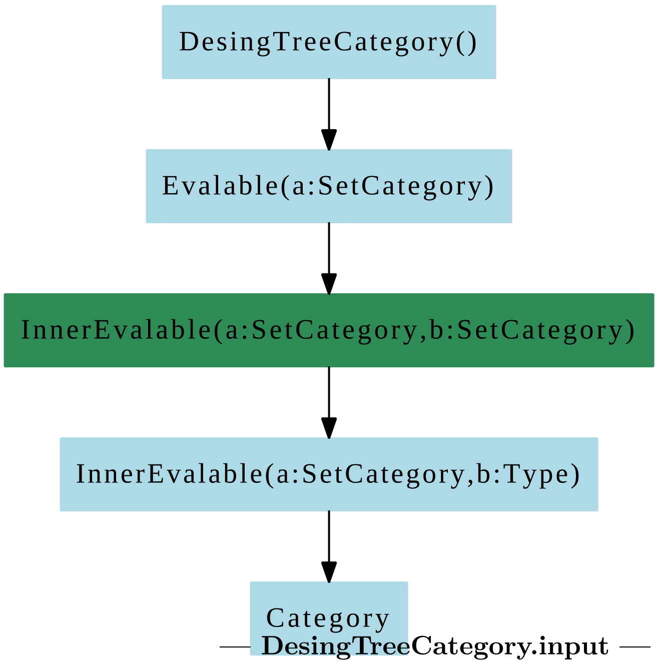
— BLMETCT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"BlowUpMethodCategory()" [color=lightblue];
"BlowUpMethodCategory()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" ->
    "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"
```

```
"Category" [color=lightblue];
}
```

———-

4.0.61 DesingTreeCategory (DSTRCAT)



— **DesingTreeCategory.input** —

```
)set break resume
)sys rm -f DesingTreeCategory.output
)spool DesingTreeCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DesingTreeCategory
--R
--R DesingTreeCategory(S: SetCategory) is a category constructor
--R Abbreviation for DesingTreeCategory is DSTRCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for DSTRCAT
--R
--R------------------------------- Operations --------------------------------
--R children : % -> List(%)              copy : % -> %
--R cyclic? : % -> Boolean               distance : (%,%) -> Integer
--R ?.value : (%,value) -> S             empty : () -> %
--R empty? : % -> Boolean                eq? : (%,%) -> Boolean
```

```
--R latex : % -> String if S has SETCAT   leaf? : % -> Boolean
--R leaves : % -> List(S)                 map : ((S -> S),%) -> %
--R nodes : % -> List(%)                  sample : () -> %
--R tree : List(S) -> %                   tree : S -> %
--R tree : (S,List(%)) -> %               value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List(S) if $ has finiteAggregate
--R setchildren! : (%,List(%)) -> % if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

— **DesingTreeCategory.help** —

```
====================================================================
DesingTreeCategory examples
====================================================================

This category is part of the PAFF package.

See Also:
o )show DesingTreeCategory
```

See:

⇒ “Evalable” (EVALAB) 3.0.48 on page 151

Exports:

#?	?.value	?=?	?~=?	any?
child?	children	coerce	copy	count
count	cyclic?	distance	empty	empty?
eq?	eval	every?	hash	latex
leaf?	leaves	less?	map	map!
member?	members	more?	node?	nodes
parts	sample	setchildren!	setelt	setvalue!
size?	tree	value		

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
tree : (S,List %) -> %
tree : List S -> %
tree : S -> %
```

These exports come from (p475) RecursiveAggregate(Type)

```
#? : % -> NonNegativeInteger if $ has finiteAggregate
?.value : (%,value) -> S
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
child? : (%,%) -> Boolean if S has SETCAT
children : % -> List %
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
cyclic? : % -> Boolean
distance : (%,%) -> Integer
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
leaf? : % -> Boolean
leaves : % -> List S
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
```

```
 more? : (%,NonNegativeInteger) -> Boolean
 node? : (%,%) -> Boolean if S has SETCAT
 nodes : % -> List %
 parts : % -> List S if $ has finiteAggregate
 sample : () -> %
 setchildren! : (%,List %) -> % if $ has shallowlyMutable
 setelt : (%,value,S) -> S if $ has shallowlyMutable
 setvalue! : (%,S) -> S if $ has shallowlyMutable
 size? : (%,NonNegativeInteger) -> Boolean
 value : % -> S
```

— DesingTreeCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#DSTRCAT">
DesingTreeCategory(S) (DSTRCAT)</a></h2>
&bull; S : SetCategory<br/><br/>
This category is part of the PAFF package.
<br/><br/><br/>

children : % -> List(%)<br/>
  children(u) returns a list of the children of aggregate u.<br/>
  from RecursiveAggregate(S)<br/><br/>

copy : % -> %<br/>
  copy(u) returns a top-level (non-recursive) copy of u.<br/>
  Note that for collections, \axiom{copy(u) == [x for x in u]}.<br/>
  from Aggregate<br/><br/>

cyclic? : % -> Boolean<br/>
  cyclic?(u) tests if u has a cycle.<br/>
  from RecursiveAggregate(S)<br/><br/>

distance : (%,%) -> Integer<br/>
 
distance(u,v) returns the path length (an integer) from node u to v.<br/>
  from RecursiveAggregate(S)<br/><br/>

?.value : (%,value) -> S<br/>
  elt(u,"value") (also written: \axiom{a. value}) is<br/>
  equivalent to \axiom{value(a)}.<br/>
  from RecursiveAggregate(S)<br/><br/>

empty : () -> %<br/>
  empty()$D creates an aggregate of type D with 0 elements.<br/>
  Note that The $D can be dropped if understood by context,<br/>
  for example \axiom{u: D := empty()}.<br/>
  from Aggregate<br/><br/>

empty? : % -> Boolean<br/>
  empty?(u) tests if u has 0 elements.<br/>
  from Aggregate<br/><br/>
```

```
eq? : (%,%) -> Boolean<br/>
  eq?(u,v) tests if u and v are same objects.<br/>
  from Aggregate<br/><br/>

latex : % -> String if S has SETCAT<br/>
  latex(s) returns a LaTeX-printable output representation of s.<br/>
  from SetCategory<br/><br/>

leaf? : % -> Boolean<br/>
  leaf?(u) tests if u is a terminal node.<br/>
  from RecursiveAggregate(S)<br/><br/>

leaves : % -> List(S)<br/>
  leaves(t) returns the list of values in obtained by visiting the<br/>
  nodes of tree \axiom{t} in left-to-right order.<br/>
  from RecursiveAggregate(S)<br/><br/>

map : ((S -> S),%) -> %<br/>
  map(f,u) returns a copy of u with each element x replaced by f(x).<br/>
  For collections, \axiom{map(f,u) = [f(x) for x in u]}.<br/>
  from HomogeneousAggregate(S)<br/><br/>

nodes : % -> List(%)<br/>
  nodes(u) returns a list of all of the nodes of aggregate u.<br/>
  from RecursiveAggregate(S)<br/><br/>

sample : () -> %<br/>
  sample yields a value of type %<br/>
  from Aggregate<br/><br/>

tree : List(S) -> %<br/>
  tree(l) creates a chain tree from the list l<br/><br/>

tree : S -> %<br/>
  tree(nd) creates a tree with value nd, and no children<br/><br/>

tree : (S,List(%)) -> %<br/>
  tree(nd,ls) creates a tree with value nd, and children ls.<br/><br/>

value : % -> S<br/>
  value(u) returns the value of the node u.<br/>
  from RecursiveAggregate(S)<br/><br/>

#? : % -> NonNegativeInteger if $ has finiteAggregate<br/>
  # u returns the number of items in u.<br/>
  from Aggregate<br/><br/>

?=? : (%,%) -> Boolean if S has SETCAT<br/>
  x=y tests if x and y are equal.<br/>
  from BasicType<br/><br/>

any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate<br/>
  any?(p,u) tests if \axiom{p(x)} is true for any element x of u.<br/>
  For collections, any?(p,u) = reduce(or,map(f,u),false,true).<br/>
```

```
  from HomogeneousAggregate(S)<br/><br/>

child? : (%,%) -> Boolean if S has SETCAT<br/>
  child?(u,v) tests if node u is a child of node v.<br/>
  from RecursiveAggregate(S)<br/><br/>

coerce : % -> OutputForm if S has SETCAT<br/>
  coerce(a) transforms a into an element of S.<br/>
  from CoercibleTo OutputForm<br/><br/>

count : (S,%) -> NonNegativeInteger
if S has SETCAT and $ has finiteAggregate<br/>
  count(x,u) returns the number of occurrences of x in u. For<br/>
  collections, \axiom{count(x,u) = reduce(+,[x=y for y in u],0)}.
  from HomogeneousAggregate(S)<br/><br/>

count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate<br/>
  count(p,u) returns the number of elements x in u<br/>
  such that \axiom{p(x)} is true. For collections,<br/>
  count(p,u) = reduce(+,[1 for x in u | p(x)],0).<br/>
  from HomogeneousAggregate(S)<br/><br/>

eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT<br/>
  eval(f, [x1,...,xn], [v1,...,vn]) replaces xi by vi in f.<br/>
  from InnerEvalable<br/><br/>

eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT<br/>
  eval(f, x, v) replaces x by v in f.<br/>
  from InnerEvalable<br/><br/>

eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT<br/>
  eval(f,x = v) replaces x by v in f.<br/>
  from Evalable<br/><br/>

eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT<br/>
  eval(f, [x1 = v1,...,xn = vn]) replaces xi by vi in f.<br/>
  from Evalable<br/><br/>

every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate<br/>
  every?(f,u) tests if p(x) is true for all elements x of u.<br/>
  For collections, every?(p,u) = reduce(and,map(f,u),true,false).<br/>
  from HomogeneousAggregate(S)<br/><br/>

hash : % -> SingleInteger if S has SETCAT<br/>
  hash(s) calculates a hash code for s.<br/>
  from SetCategory<br/><br/>

less? : (%,NonNegativeInteger) -> Boolean<br/>
  less?(u,n) tests if u has less than n elements.<br/>
  from Aggregate<br/><br/>

map! : ((S -> S),%) -> % if $ has shallowlyMutable<br/>
  map!(f,u) destructively replaces each element x of u by f(x).<br/>
  from HomogeneousAggregate(S)<br/><br/>
```

```
member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate<br/>
  member?(x,u) tests if x is a member of u.<br/>
  For collections, member?(x,u) = reduce(or,[x=y for y in u],false)<br/>
  from HomogeneousAggregate(S)<br/><br/>

members : % -> List(S) if $ has finiteAggregate<br/>
  members(u) returns a list of the consecutive elements of u.<br/>
  For collections, \axiom{parts([x,y,...,z]) = (x,y,...,z)}.<br/>
  from HomogeneousAggregate(S)<br/><br/>

more? : (%,NonNegativeInteger) -> Boolean<br/>
  more?(u,n) tests if u has greater than n elements.<br/>
  from Aggregate<br/><br/>

node? : (%,%) -> Boolean if S has SETCAT<br/>
  node?(u,v) tests if node u is contained in node v<br/>
  (either as a child, a child of a child, etc.).<br/>
  from RecursiveAggregate(S)<br/><br/>

parts : % -> List(S) if $ has finiteAggregate<br/>
  parts(u) returns a list of the consecutive elements of u.<br/>
  For collections, \axiom{parts([x,y,...,z]) = (x,y,...,z)}.<br/>
  from HomogeneousAggregate(S)<br/><br/>

setchildren! : (%,List(%)) -> % if $ has shallowlyMutable<br/>
  setchildren!(u,v) replaces the current children of node u<br/>
  with the members of v in left-to-right order.<br/>
  from RecursiveAggregate(S)<br/><br/>

setelt : (%,value,S) -> S if $ has shallowlyMutable<br/>
  setelt(a,"value",x) (also written \axiom{a . value := x})<br/>
  is equivalent to \axiom{setvalue!(a,x)}<br/>
  from RecursiveAggregate(S)<br/><br/>

setvalue! : (%,S) -> S if $ has shallowlyMutable<br/>
  setvalue!(u,x) sets the value of node u to x.<br/>
  from RecursiveAggregate(S)<br/><br/>

size? : (%,NonNegativeInteger) -> Boolean<br/>
  size?(u,n) tests if u has exactly n elements.<br/>
  from Aggregate<br/><br/>

?~=? : (%,%) -> Boolean if S has SETCAT<br/>
  x~=y tests if x and y are not equal.<br/>
  from BasicType<br/><br/>

&bull; RecursiveAggregate(S)<br/>
&bull; HomogeneousAggregate(S)<br/>
&bull; Aggregate<br/>
&bull; SetCategory<br/>
&bull; BasicType<br/>
&bull; Evalable<br/>
&bull; InnerEvalable<br/>
```

```
&bull; CoercibleTo OutputForm<br/>

</body>
```

— category DSTRCAT DesingTreeCategory —

```
)abbrev category DSTRCAT DesingTreeCategory
++ Authors: Gaetan Hache
++ Date Created: jan 1998
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This category is part of the PAFF package

DesingTreeCategory(S) : Category == SIG where
  S : SetCategory

  SIG ==> RecursiveAggregate(S) with

    shallowlyMutable

    finiteAggregate

    tree : (S,List %) -> %
      ++ tree(nd,ls) creates a tree with value nd, and children ls.

    tree : S -> %
      ++ tree(nd) creates a tree with value nd, and no children

    tree : List(S) -> %
      ++ tree(l) creates a chain tree from the list l
```

— DSTRCAT.dotabb —

```
"DSTRCAT" [color=lightblue,href="bookvol10.2.pdf#nameddest=DSTRCAT"];
"EVALAB" [color="#4488FF",href="bookvol10.2.pdf#nameddest=EVALAB"]
"DSTRCAT" -> "EVALAB"
```

— DSTRCAT.dotfull —

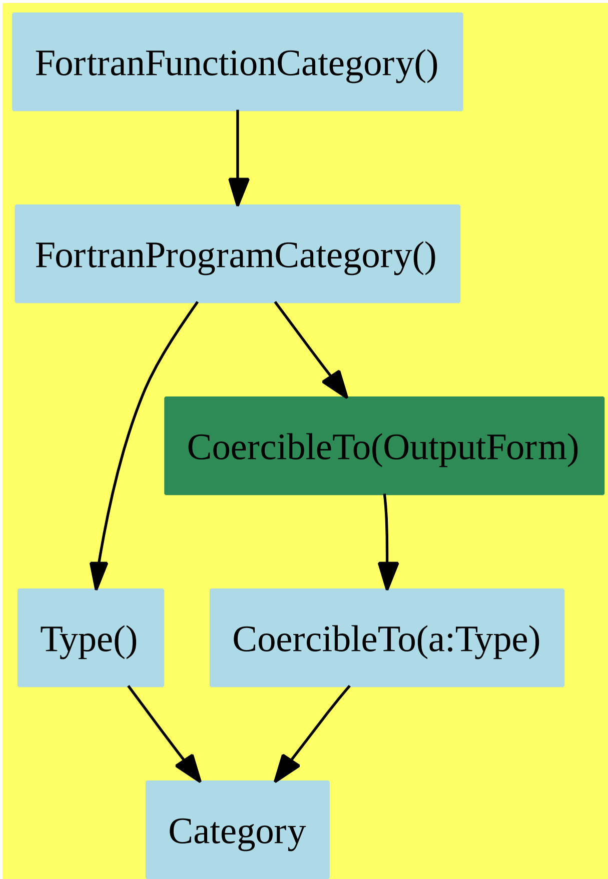
```
"DesingTreeCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DSTRCAT"];
"DesingTreeCategory()" -> "Evalable()"
```

— **DSTRCAT.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"DesingTreeCategory()" [color=lightblue];
"DesingTreeCategory()" -> "Evalable(a:SetCategory)"

"Evalable(a:SetCategory)" [color=lightblue];
"Evalable(a:SetCategory)" -> "InnerEvalable(a:SetCategory,b:SetCategory)"

"InnerEvalable(a:SetCategory,b:SetCategory)" [color=seagreen];
"InnerEvalable(a:SetCategory,b:SetCategory)" ->
    "InnerEvalable(a:SetCategory,b:Type)"

"InnerEvalable(a:SetCategory,b:Type)" [color=lightblue];
"InnerEvalable(a:SetCategory,b:Type)" -> "Category"

"Category" [color=lightblue];

}
```

————

4.0.62 FortranFunctionCategory (FORTFN)



— **FortranFunctionCategory.input** —

```
)set break resume
)sys rm -f FortranFunctionCategory.output
)spool FortranFunctionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FortranFunctionCategory
--R
--R FortranFunctionCategory is a category constructor
--R Abbreviation for FortranFunctionCategory is FORTFN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FORTFN
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List(FortranCode) -> %
```

```
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R retract : Polynomial(Integer) -> %    retract : Polynomial(Float) -> %
--R retract : Expression(Integer) -> %    retract : Expression(Float) -> %
--R coerce : Record(localSymbols: SymbolTable,code: List(FortranCode)) -> %
--R retract : Fraction(Polynomial(Integer)) -> %
--R retract : Fraction(Polynomial(Float)) -> %
--R retractIfCan : Fraction(Polynomial(Integer)) -> Union(%,"failed")
--R retractIfCan : Fraction(Polynomial(Float)) -> Union(%,"failed")
--R retractIfCan : Polynomial(Integer) -> Union(%,"failed")
--R retractIfCan : Polynomial(Float) -> Union(%,"failed")
--R retractIfCan : Expression(Integer) -> Union(%,"failed")
--R retractIfCan : Expression(Float) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— **FortranFunctionCategory.help** —

```
====================================================================
FortranFunctionCategory examples
====================================================================

FortranFunctionCategory is the category of arguments to NAG Library
routines which return (sets of) function values.

See Also:
o )show FortranFunctionCategory
```

See:

⇐ "FortranProgramCategory" (FORTCAT) 3.0.49 on page 155

Exports:

coerce outputAsFortran retract retractIfCan

Attributes:

- **nil**

These are directly exported but not implemented:

```
coerce : FortranCode -> %
coerce : List FortranCode -> %
coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
retract : Expression Float -> %
retract : Expression Integer -> %
retract : Polynomial Float -> %
retract : Polynomial Integer -> %
retract : Fraction Polynomial Integer -> %
```

```
retract : Fraction Polynomial Float -> %
retractIfCan : Fraction Polynomial Integer -> Union(%,"failed")
retractIfCan : Fraction Polynomial Float -> Union(%,"failed")
retractIfCan : Polynomial Integer -> Union(%,"failed")
retractIfCan : Polynomial Float -> Union(%,"failed")
retractIfCan : Expression Integer -> Union(%,"failed")
retractIfCan : Expression Float -> Union(%,"failed")
```

These exports come from (p155) FortranProgramCategory():

```
coerce : % -> OutputForm
outputAsFortran : % -> Void
```

— FortranFunctionCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FORTFN">
FortranFunctionCategory (FORTFN)</a></h2>
FortranFunctionCategory is the category of arguments to
NAG Library routines which return (sets of) function values.
<br/><br/><br/>
coerce : List FortranCode -> $<br/>
  coerce(e) takes an object from List FortranCode and<br/>
  uses it as the body of an ASP.<br/><br/>

coerce : FortranCode -> $<br/>
  coerce(e) takes an object from FortranCode and<br/>
  uses it as the body of an ASP.<br/><br/>

coerce : Record(localSymbols:SymbolTable,code:List(FortranCode)) -> $<br/>
  coerce(e) takes the component of e from<br/>
  List FortranCode and uses it as the body of the ASP,<br/>
  making the declarations in the SymbolTable component.<br/><br/>

retract : Expression Float -> $<br/>
  retract(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retractIfCan : Expression Float -> Union($,"failed")<br/>
  retractIfCan(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retract : Expression Integer -> $<br/>
  retract(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retractIfCan : Expression Integer -> Union($,"failed")<br/>
  retractIfCan(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retract : Polynomial Float -> $<br/>
  retract(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>
```

```
retractIfCan : Polynomial Float -> Union($,"failed")<br/>
  retractIfCan(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retract : Polynomial Integer -> $<br/>
  retract(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retractIfCan : Polynomial Integer -> Union($,"failed")<br/>
  retractIfCan(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retract : Fraction Polynomial Float -> $<br/>
  retract(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retractIfCan : Fraction Polynomial Float -> Union($,"failed")<br/>
  retractIfCan(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retract : Fraction Polynomial Integer -> $<br/>
  retract(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retractIfCan : Fraction Polynomial Integer -> Union($,"failed")<br/>
  retractIfCan(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

coerce : % -> OutputForm<br/>
  coerce(a) transforms a into an element of OutputForm<br/>
  from CoercibleTo OutputForm<br/><br/>

outputAsFortran : % -> Void<br/>
  outputAsFortran(u) translates u into a legal FORTRAN subprogram.<br/>
  from FortranProgramCategory<br/><br/>

&bull; CoercibleTo OutputForm<br/>
&bull; FortranProgramCategory<br/>


</body>
```

— category FORTFN FortranFunctionCategory —

```
)abbrev category FORTFN FortranFunctionCategory
++ Author: Mike Dewar
++ Date Created: 13 January 1994
++ Date Last Updated: 18 March 1994
++ Description:
++ \axiomType{FortranFunctionCategory} is the category of arguments to
++ NAG Library routines which return (sets of) function values.
```

```
FortranFunctionCategory() : Category == SIG where

  SIG ==> FortranProgramCategory with

    coerce : List FortranCode -> $
      ++ coerce(e) takes an object from \spadtype{List FortranCode} and
      ++  uses it as the body of an ASP.

    coerce : FortranCode -> $
      ++ coerce(e) takes an object from \spadtype{FortranCode} and
      ++  uses it as the body of an ASP.

    coerce : Record(localSymbols:SymbolTable,code:List(FortranCode)) -> $
      ++ coerce(e) takes the component of \spad{e} from
      ++ \spadtype{List FortranCode} and uses it as the body of the ASP,
      ++ making the declarations in the \spadtype{SymbolTable} component.

    retract : Expression Float -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retractIfCan : Expression Float -> Union($,"failed")
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retract : Expression Integer -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retractIfCan : Expression Integer -> Union($,"failed")
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retract : Polynomial Float -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retractIfCan : Polynomial Float -> Union($,"failed")
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retract : Polynomial Integer -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retractIfCan : Polynomial Integer -> Union($,"failed")
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retract : Fraction Polynomial Float -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retractIfCan : Fraction Polynomial Float -> Union($,"failed")
```

```
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retract : Fraction Polynomial Integer -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retractIfCan : Fraction Polynomial Integer -> Union($,"failed")
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    -- NB: These ASPs also have a coerce from an appropriate instantiation
    --     of FortranExpression.
```

— **FORTFN.dotabb** —

```
"FORTFN"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FORTFN"];
"FORTFN" -> "FORTCAT"
```

— **FORTFN.dotfull** —

```
"FortranFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FORTFN"];
"FortranFunctionCategory()" -> "FortranProgramCategory()"
```

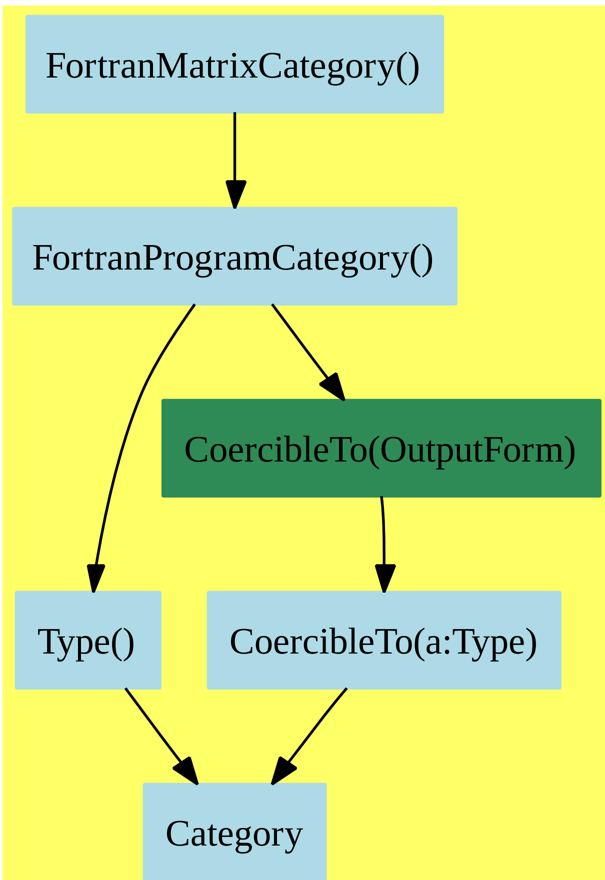
— **FORTFN.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FortranFunctionCategory()" [color=lightblue];
"FortranFunctionCategory()" -> "FortranProgramCategory()"

"FortranProgramCategory()" [color=lightblue];
"FortranProgramCategory()" -> "Type()"
"FortranProgramCategory()" -> "CoercibleTo(OutputForm)"

"Type()" [color=lightblue];
"Type()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"
```

```
"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

———–

4.0.63 FortranMatrixCategory (FMC)



— **FortranMatrixCategory.input** —

```
)set break resume
)sys rm -f FortranMatrixCategory.output
)spool FortranMatrixCategory.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show FortranMatrixCategory
--R
--R FortranMatrixCategory is a category constructor
--R Abbreviation for FortranMatrixCategory is FMC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FMC
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List(FortranCode) -> %
--R coerce : Matrix(MachineFloat) -> %    coerce : % -> OutputForm
--R outputAsFortran : % -> Void
--R coerce : Record(localSymbols: SymbolTable,code: List(FortranCode)) -> %
--R
--E 1

)spool
)lisp (bye)
```

— **FortranMatrixCategory.help** —

```
====================================================================
FortranMatrixCategory examples
====================================================================

FortranMatrixCategory provides support for producing Functions and
Subroutines when the input to these is an AXIOM object of type Matrix
or in domains involving FortranCode.

See Also:
o )show FortranMatrixCategory
```

See:

⇐ "FortranProgramCategory" (FORTCAT) 3.0.49 on page 155

Exports:

coerce outputAsFortran

Attributes:

- **nil**

These are directly exported but not implemented:

```
coerce : Matrix MachineFloat -> %
coerce : List FortranCode -> %
coerce : FortranCode -> %
coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
```

These exports come from (p155) FortranProgramCategory():

```
coerce : % -> OutputForm
outputAsFortran : % -> Void
```

— FortranMatrixCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FMC">
FortranMatrixCategory (FMC)</a></h2>

FortranMatrixCategory provides support for producing Functions and
Subroutines when the input to these is an AXIOM object of type Matrix
or in domains involving FortranCode.
<br/><br/><br/>

coerce : Matrix MachineFloat -> $<br/>
  coerce(v) produces an ASP which returns the value of v.<br/><br/>

coerce : List FortranCode -> $<br/>
  coerce(e) takes an object from List FortranCode and<br/>
  uses it as the body of an ASP.<br/><br/>

coerce : FortranCode -> $<br/>
  coerce(e) takes an object from FortranCode and<br/>
  uses it as the body of an ASP.<br/><br/>

coerce : Record(localSymbols:SymbolTable,code:List(FortranCode)) -> $
  coerce(e) takes the component of \spad{e} from<br/>
  List FortranCode and uses it as the body of the ASP,<br/>
  making the declarations in the SymbolTable component.<br/><br/>

coerce : % -> OutputForm<br/>
  coerce(a) transforms a into an element of OutputForm<br/>
  from CoercibleTo OutputForm<br/><br/>

outputAsFortran : % -> Void<br/>
  outputAsFortran(u) translates u into a legal FORTRAN subprogram.<br/>
  from FortranProgramCategory<br/><br/>

&bull; CoercibleTo OutputForm<br/>
&bull; FortranProgramCategory<br/>

</body>
```

— category FMC FortranMatrixCategory —

```
)abbrev category FMC FortranMatrixCategory
++ Author: Mike Dewar
++ Date Created: 21 March 1994
++ Description:
++ \axiomType{FortranMatrixCategory} provides support for
```

```
++ producing Functions and Subroutines when the input to these
++ is an AXIOM object of type \axiomType{Matrix} or in domains
++ involving \axiomType{FortranCode}.

FortranMatrixCategory() : Category == SIG where

  SIG ==> FortranProgramCategory with

    coerce : Matrix MachineFloat -> $
      ++ coerce(v) produces an ASP which returns the value of \spad{v}.

    coerce : List FortranCode -> $
      ++ coerce(e) takes an object from \spadtype{List FortranCode} and
      ++  uses it as the body of an ASP.

    coerce : FortranCode -> $
      ++ coerce(e) takes an object from \spadtype{FortranCode} and
      ++  uses it as the body of an ASP.

    coerce : Record(localSymbols:SymbolTable,code:List(FortranCode)) -> $
      ++ coerce(e) takes the component of \spad{e} from
      ++ \spadtype{List FortranCode} and uses it as the body of the ASP,
      ++ making the declarations in the \spadtype{SymbolTable} component.
```

— FMC.dotabb —

```
"FMC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FMC"];
"FMC" -> "FORTCAT"
```

— FMC.dotfull —

```
"FortranMatrixCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FMC"];
"FortranMatrixCategory()" -> "FortranProgramCategory()"
```

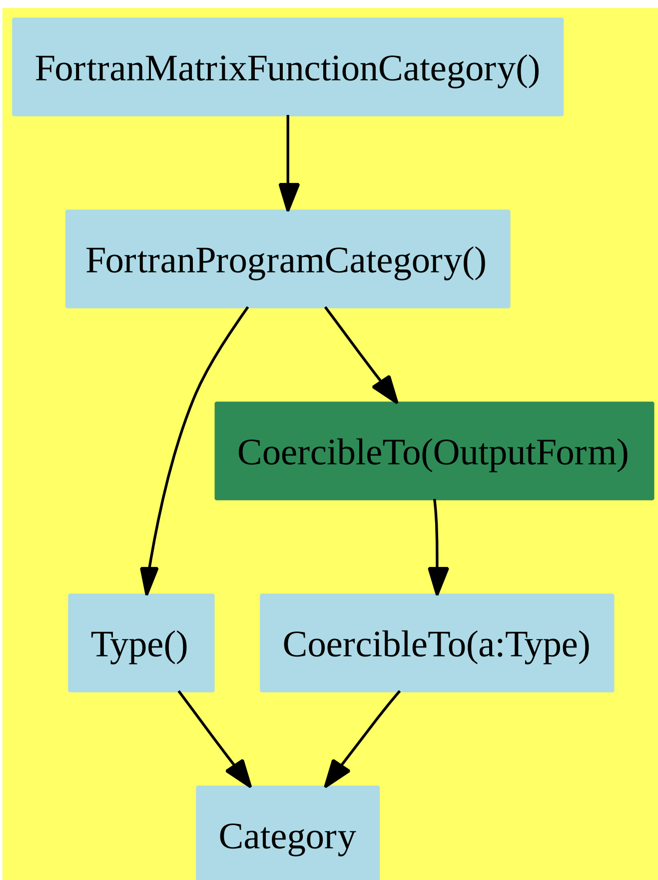
— FMC.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FortranMatrixCategory()" [color=lightblue];
"FortranMatrixCategory()" -> "FortranProgramCategory()"
```

```
"FortranProgramCategory()" [color=lightblue];
"FortranProgramCategory()" -> "Type()"
"FortranProgramCategory()" -> "CoercibleTo(OutputForm)"

"Type()" [color=lightblue];
"Type()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

———-

4.0.64 FortranMatrixFunctionCategory (FMFUN)



— FortranMatrixFunctionCategory.input —

```
)set break resume
)sys rm -f FortranMatrixFunctionCategory.output
)spool FortranMatrixFunctionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FortranMatrixFunctionCategory
--R
--R FortranMatrixFunctionCategory is a category constructor
--R Abbreviation for FortranMatrixFunctionCategory is FMFUN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FMFUN
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %            coerce : List(FortranCode) -> %
--R coerce : % -> OutputForm             outputAsFortran : % -> Void
--R coerce : Record(localSymbols: SymbolTable,code: List(FortranCode)) -> %
--R retract : Matrix(Fraction(Polynomial(Integer))) -> %
--R retract : Matrix(Fraction(Polynomial(Float))) -> %
--R retract : Matrix(Polynomial(Integer)) -> %
--R retract : Matrix(Polynomial(Float)) -> %
--R retract : Matrix(Expression(Integer)) -> %
--R retract : Matrix(Expression(Float)) -> %
--R retractIfCan : Matrix(Fraction(Polynomial(Integer))) -> Union(%,"failed")
--R retractIfCan : Matrix(Fraction(Polynomial(Float))) -> Union(%,"failed")
--R retractIfCan : Matrix(Polynomial(Integer)) -> Union(%,"failed")
--R retractIfCan : Matrix(Polynomial(Float)) -> Union(%,"failed")
--R retractIfCan : Matrix(Expression(Integer)) -> Union(%,"failed")
--R retractIfCan : Matrix(Expression(Float)) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— FortranMatrixFunctionCategory.help —

```
====================================================================
FortranMatrixFunctionCategory examples
====================================================================

FortranMatrixFunctionCategory provides support for producing Functions
and Subroutines representing matrices of expressions.

See Also:
o )show FortranMatrixFunctionCategory
```

———-

See:

⇐ "FortranProgramCategory" (FORTCAT) 3.0.49 on page 155

Exports:

coerce outputAsFortran retract retractIfCan

Attributes:

- **nil**

These are directly exported but not implemented:

```
coerce : List FortranCode -> %
coerce : FortranCode -> %
coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
retract : Matrix Expression Float -> %
retract : Matrix Expression Integer -> %
retract : Matrix Polynomial Float -> %
retract : Matrix Polynomial Integer -> %
retract : Matrix Fraction Polynomial Float -> %
retract : Matrix Fraction Polynomial Integer -> %
retractIfCan : Matrix Fraction Polynomial Integer -> Union(%,"failed")
retractIfCan : Matrix Fraction Polynomial Float -> Union(%,"failed")
retractIfCan : Matrix Polynomial Integer -> Union(%,"failed")
retractIfCan : Matrix Polynomial Float -> Union(%,"failed")
retractIfCan : Matrix Expression Integer -> Union(%,"failed")
retractIfCan : Matrix Expression Float -> Union(%,"failed")
```

These exports come from (p155) FortranProgramCategory():

```
coerce : % -> OutputForm
outputAsFortran : % -> Void
```

— FortranMatrixFunctionCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FMFUN">
FortranMatrixFunctionCategory (FMFUN)</a></h2>
FortranMatrixFunctionCategory provides support for
producing Functions and Subroutines representing matrices of expressions.
<br/><br/><br/>

coerce : % -> OutputForm<br/>
  coerce(a) transforms a into an element of OutputForm<br/>
  from CoercibleTo OutputForm<br/><br/>

coerce : List FortranCode -> $<br/>
  coerce(e) takes an object from List FortranCode and<br/>
  uses it as the body of an ASP.<br/><br/>

coerce : FortranCode -> $<br/>
```

```
  coerce(e) takes an object from FortranCode and<br/>
  uses it as the body of an ASP.<br/><br/>

coerce : Record(localSymbols:SymbolTable,code:List(FortranCode)) -> $<br/>
  coerce(e) takes the component of e from<br/>
  List FortranCode and uses it as the body of the ASP,<br/>
  making the declarations in the SymbolTable component.<br/><br/>

retract : Matrix Expression Float -> $<br/>
  retract(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retractIfCan : Matrix Expression Float -> Union($,"failed")<br/>
  retractIfCan(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retract : Matrix Expression Integer -> $<br/>
  retract(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retractIfCan : Matrix Expression Integer -> Union($,"failed")<br/>
  retractIfCan(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retract : Matrix Polynomial Float -> $<br/>
  retract(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retractIfCan : Matrix Polynomial Float -> Union($,"failed")<br/>
  retractIfCan(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retract : Matrix Polynomial Integer -> $<br/>
  retract(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retractIfCan : Matrix Polynomial Integer -> Union($,"failed")<br/>
  retractIfCan(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retract : Matrix Fraction Polynomial Float -> $<br/>
  retract(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retractIfCan : Matrix Fraction Polynomial Float -> Union($,"failed")<br/>
  retractIfCan(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retract : Matrix Fraction Polynomial Integer -> $<br/>
  retract(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retractIfCan : Matrix Fraction Polynomial Integer -> Union($,"failed")<br/>
  retractIfCan(e) tries to convert e into an ASP, checking that<br/>
```

```
  legal Fortran-77 is produced.<br/><br/>

outputAsFortran : % -> Void<br/>
  outputAsFortran(u) translates u into a legal FORTRAN subprogram.<br/>
  from FortranProgramCategory<br/><br/>

&bull; CoercibleTo OutputForm<br/>
&bull; FortranProgramCategory<br/>

</body>
```

— category FMFUN FortranMatrixFunctionCategory —

```
)abbrev category FMFUN FortranMatrixFunctionCategory
++ Author: Mike Dewar
++ Date Created: March 18 1994
++ Description:
++ \axiomType{FortranMatrixFunctionCategory} provides support for
++ producing Functions and Subroutines representing matrices of
++ expressions.

FortranMatrixFunctionCategory() : Category == SIG where

  SIG ==> FortranProgramCategory with

    coerce : List FortranCode -> $
      ++ coerce(e) takes an object from \spadtype{List FortranCode} and
      ++  uses it as the body of an ASP.

    coerce : FortranCode -> $
      ++ coerce(e) takes an object from \spadtype{FortranCode} and
      ++  uses it as the body of an ASP.

    coerce : Record(localSymbols:SymbolTable,code:List(FortranCode)) -> $
      ++ coerce(e) takes the component of \spad{e} from
      ++ \spadtype{List FortranCode} and uses it as the body of the ASP,
      ++ making the declarations in the \spadtype{SymbolTable} component.

    retract : Matrix Expression Float -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retractIfCan : Matrix Expression Float -> Union($,"failed")
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retract : Matrix Expression Integer -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retractIfCan : Matrix Expression Integer -> Union($,"failed")
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
```

```
      ++  legal Fortran-77 is produced.

    retract : Matrix Polynomial Float -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retractIfCan : Matrix Polynomial Float -> Union($,"failed")
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retract : Matrix Polynomial Integer -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retractIfCan : Matrix Polynomial Integer -> Union($,"failed")
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retract : Matrix Fraction Polynomial Float -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retractIfCan : Matrix Fraction Polynomial Float -> Union($,"failed")
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retract : Matrix Fraction Polynomial Integer -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retractIfCan : Matrix Fraction Polynomial Integer -> Union($,"failed")
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    -- NB: These ASPs also have a coerce from an appropriate instantiation
    --     of Matrix FortranExpression.
```

— FMFUN.dotabb —

```
"FMFUN"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FMFUN"];
"FMFUN" -> "FORTCAT"
```

— FMFUN.dotfull —

```
"FortranMatrixFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FMFUN"];
"FortranMatrixFunctionCategory()" -> "FortranProgramCategory()"
```

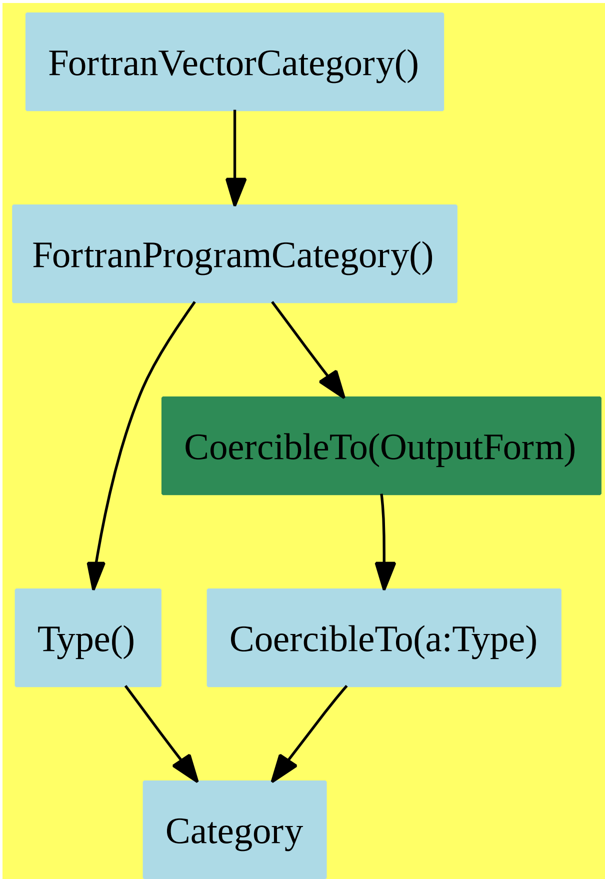
———-

— **FMFUN.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FortranMatrixFunctionCategory()" [color=lightblue];
"FortranMatrixFunctionCategory()" -> "FortranProgramCategory()"

"FortranProgramCategory()" [color=lightblue];
"FortranProgramCategory()" -> "Type()"
"FortranProgramCategory()" -> "CoercibleTo(OutputForm)"

"Type()" [color=lightblue];
"Type()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

———-

4.0.65 FortranVectorCategory (FVC)



— **FortranVectorCategory.input** —

```
)set break resume
)sys rm -f FortranVectorCategory.output
)spool FortranVectorCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FortranVectorCategory
--R
--R FortranVectorCategory is a category constructor
--R Abbreviation for FortranVectorCategory is FVC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FVC
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List(FortranCode) -> %
```

```
--R coerce : Vector(MachineFloat) -> %    coerce : % -> OutputForm
--R outputAsFortran : % -> Void
--R coerce : Record(localSymbols: SymbolTable,code: List(FortranCode)) -> %
--R
--E 1

)spool
)lisp (bye)
```

———–

— **FortranVectorCategory.help** —

```
====================================================================
FortranVectorCategory examples
====================================================================

FortranVectorCategory provides support for producing Functions and
Subroutines when the input to these is an AXIOM object of type
Vector or in domains involving FortranCode.

See Also:
o )show FortranVectorCategory
```

———–

See:

⇐ "FortranProgramCategory" (FORTCAT) 3.0.49 on page 155

Exports:

Attributes:

- **nil**

These are directly exported but not implemented:

```
coerce : FortranCode -> %
coerce : List FortranCode -> %
coerce : Vector MachineFloat -> %
coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
```

These exports come from (p155) FortranProgramCategory():

```
coerce : % -> OutputForm
outputAsFortran : % -> Void
```

— **FortranVectorCategory.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FVC">
FortranVectorCategory (FVC)</a></h2>
```

```
FortranVectorCategory provides support for producing Functions and
Subroutines when the input to these is an Axiom object of type Vector
or in domains involving FortranCode.
<br/><br/><br/>

coerce : Vector MachineFloat -> $<br/>
  coerce(v) produces an ASP which returns the value of v.<br/><br/>

coerce : List FortranCode -> $<br/>
  coerce(e) takes an object from List FortranCode and<br/>
  uses it as the body of an ASP.<br/><br/>

coerce : FortranCode -> $<br/>
  coerce(e) takes an object from FortranCode and<br/>
  uses it as the body of an ASP.<br/><br/>

coerce : Record(localSymbols:SymbolTable,code:List(FortranCode)) -> $<br/>
  coerce(e) takes the component of e from<br/>
  List FortranCode and uses it as the body of the ASP,<br/>
  making the declarations in the SymbolTable component.<br/><br/>

coerce : % -> OutputForm<br/>
  coerce(a) transforms a into an element of OutputForm<br/>
  from CoercibleTo OutputForm<br/><br/>

outputAsFortran : % -> Void<br/>
  outputAsFortran(u) translates u into a legal FORTRAN subprogram.<br/>
  from FortranProgramCategory<br/><br/>

&bull; CoercibleTo OutputForm<br/>
&bull; FortranProgramCategory<br/>
</body>
```

— category FVC FortranVectorCategory —

```
)abbrev category FVC FortranVectorCategory
++ Author: Mike Dewar
++ Date Created: October 1993
++ Date Last Updated: 18 March 1994
++ Description:
++ \axiomType{FortranVectorCategory} provides support for
++ producing Functions and Subroutines when the input to these
++ is an AXIOM object of type \axiomType{Vector} or in domains
++ involving \axiomType{FortranCode}.

FortranVectorCategory() : Category == SIG where

  SIG ==> FortranProgramCategory with

    coerce : Vector MachineFloat -> $
      ++ coerce(v) produces an ASP which returns the value of \spad{v}.
```

```
    coerce : List FortranCode -> $
      ++ coerce(e) takes an object from \spadtype{List FortranCode} and
      ++  uses it as the body of an ASP.

    coerce : FortranCode -> $
      ++ coerce(e) takes an object from \spadtype{FortranCode} and
      ++  uses it as the body of an ASP.

    coerce : Record(localSymbols:SymbolTable,code:List(FortranCode)) -> $
      ++ coerce(e) takes the component of \spad{e} from
      ++ \spadtype{List FortranCode} and uses it as the body of the ASP,
      ++ making the declarations in the \spadtype{SymbolTable} component.
```

———-

— FVC.dotabb —

```
"FVC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FVC"];
"FVC" -> "FORTCAT"
```

———-

— FVC.dotfull —

```
"FortranVectorCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FVC"];
"FortranVectorCategory()" -> "FortranProgramCategory()"
```

———-

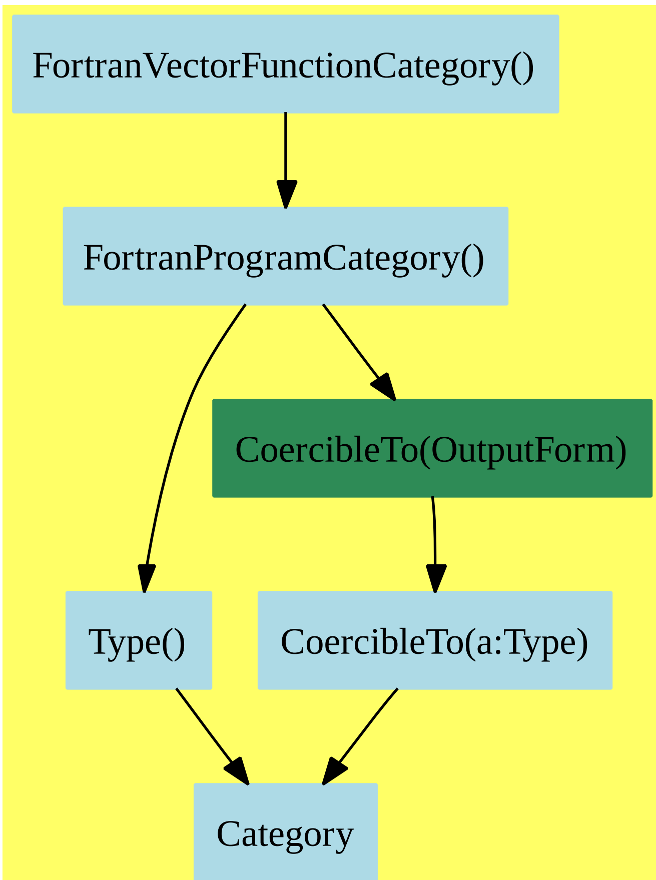
— FVC.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FortranVectorCategory()" [color=lightblue];
"FortranVectorCategory()" -> "FortranProgramCategory()"

"FortranProgramCategory()" [color=lightblue];
"FortranProgramCategory()" -> "Type()"
"FortranProgramCategory()" -> "CoercibleTo(OutputForm)"

"Type()" [color=lightblue];
"Type()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"
```

```
"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

———-

4.0.66 FortranVectorFunctionCategory (FVFUN)



— **FortranVectorFunctionCategory.input** —

```
)set break resume
)sys rm -f FortranVectorFunctionCategory.output
)spool FortranVectorFunctionCategory.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show FortranVectorFunctionCategory
--R
--R FortranVectorFunctionCategory is a category constructor
--R Abbreviation for FortranVectorFunctionCategory is FVFUN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FVFUN
--R
--R------------------------------- Operations --------------------------------
--R coerce : FortranCode -> %             coerce : List(FortranCode) -> %
--R coerce : % -> OutputForm              outputAsFortran : % -> Void
--R coerce : Record(localSymbols: SymbolTable,code: List(FortranCode)) -> %
--R retract : Vector(Fraction(Polynomial(Integer))) -> %
--R retract : Vector(Fraction(Polynomial(Float))) -> %
--R retract : Vector(Polynomial(Integer)) -> %
--R retract : Vector(Polynomial(Float)) -> %
--R retract : Vector(Expression(Integer)) -> %
--R retract : Vector(Expression(Float)) -> %
--R retractIfCan : Vector(Fraction(Polynomial(Integer))) -> Union(%,"failed")
--R retractIfCan : Vector(Fraction(Polynomial(Float))) -> Union(%,"failed")
--R retractIfCan : Vector(Polynomial(Integer)) -> Union(%,"failed")
--R retractIfCan : Vector(Polynomial(Float)) -> Union(%,"failed")
--R retractIfCan : Vector(Expression(Integer)) -> Union(%,"failed")
--R retractIfCan : Vector(Expression(Float)) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— **FortranVectorFunctionCategory.help** —

```
====================================================================
FortranVectorFunctionCategory examples
====================================================================

FortranVectorFunctionCategory is the catagory of arguments
to NAG Library routines which return the values of vectors of functions.

See Also:
o )show FortranVectorFunctionCategory
```

See:

⇐ "FortranProgramCategory" (FORTCAT) 3.0.49 on page 155

Exports:

coerce outputAsFortran retract retractIfCan

Attributes:

- **nil**

These are directly exported but not implemented:

```
coerce : FortranCode -> %
coerce : List FortranCode -> %
coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
retract : Vector Fraction Polynomial Integer -> %
retract : Vector Fraction Polynomial Float -> %
retract : Vector Polynomial Integer -> %
retract : Vector Polynomial Float -> %
retract : Vector Expression Integer -> %
retract : Vector Expression Float -> %
retractIfCan : Vector Fraction Polynomial Integer -> Union(%,"failed")
retractIfCan : Vector Fraction Polynomial Float -> Union(%,"failed")
retractIfCan : Vector Polynomial Integer -> Union(%,"failed")
retractIfCan : Vector Polynomial Float -> Union(%,"failed")
retractIfCan : Vector Expression Integer -> Union(%,"failed")
retractIfCan : Vector Expression Float -> Union(%,"failed")
```

These exports come from (p155) FortranProgramCategory():

```
coerce : % -> OutputForm
outputAsFortran : % -> Void
```

— FortranVectorFunctionCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FVFUN">
FortranVectorFunctionCategory (FVFUN)</a></h2>

FortranVectorFunctionCategory is the catagory of arguments
to NAG Library routines which return the values of vectors of functions.
<br/><br/><br/>

coerce : List FortranCode -> $<br/>
  coerce(e) takes an object from List FortranCode and<br/>
  uses it as the body of an ASP.<br/><br/>

coerce : FortranCode -> $<br/>
  coerce(e) takes an object from FortranCode and<br/>
  uses it as the body of an ASP.<br/><br/>

coerce : Record(localSymbols:SymbolTable,code:List(FortranCode)) -> $<br/>
  coerce(e) takes the component of e from<br/>
  List FortranCode and uses it as the body of the ASP,<br/>
  making the declarations in the SymbolTable component.<br/><br/>

retract : Vector Expression Float -> $<br/>
  retract(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retractIfCan : Vector Expression Float -> Union($,"failed")<br/>
  retractIfCan(e) tries to convert e into an ASP, checking that<br/>
```

```
  legal Fortran-77 is produced.<br/><br/>

retract : Vector Expression Integer -> $<br/>
  retract(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retractIfCan : Vector Expression Integer -> Union($,"failed")<br/>
  retractIfCan(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retract : Vector Polynomial Float -> $<br/>
  retract(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retractIfCan : Vector Polynomial Float -> Union($,"failed")<br/>
  retractIfCan(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retract : Vector Polynomial Integer -> $<br/>
  retract(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retractIfCan : Vector Polynomial Integer -> Union($,"failed")<br/>
  retractIfCan(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retract : Vector Fraction Polynomial Float -> $<br/>
  retract(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retractIfCan : Vector Fraction Polynomial Float -> Union($,"failed")<br/>
  retractIfCan(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retract : Vector Fraction Polynomial Integer -> $<br/>
  retract(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

retractIfCan : Vector Fraction Polynomial Integer -> Union($,"failed")<br/>
  retractIfCan(e) tries to convert e into an ASP, checking that<br/>
  legal Fortran-77 is produced.<br/><br/>

coerce : % -> OutputForm<br/>
  coerce(a) transforms a into an element of OutputForm<br/>
  from CoercibleTo OutputForm<br/><br/>

outputAsFortran : % -> Void<br/>
  outputAsFortran(u) translates u into a legal FORTRAN subprogram.<br/>
  from FortranProgramCategory<br/><br/>

&bull; CoercibleTo OutputForm<br/>
&bull; FortranProgramCategory<br/>
</body>
```

———-

— category FVFUN FortranVectorFunctionCategory —

```
)abbrev category FVFUN FortranVectorFunctionCategory
++ Author: Mike Dewar
++ Date Created: 11 March 1994
++ Date Last Updated: 18 March 1994
++ Description:
++ \axiomType{FortranVectorFunctionCategory} is the catagory of arguments
++ to NAG Library routines which return the values of vectors of functions.

FortranVectorFunctionCategory() : Category == SIG where

  SIG ==> FortranProgramCategory with

    coerce : List FortranCode -> $
      ++ coerce(e) takes an object from \spadtype{List FortranCode} and
      ++  uses it as the body of an ASP.

    coerce : FortranCode -> $
      ++ coerce(e) takes an object from \spadtype{FortranCode} and
      ++  uses it as the body of an ASP.

    coerce : Record(localSymbols:SymbolTable,code:List(FortranCode)) -> $
      ++ coerce(e) takes the component of \spad{e} from
      ++ \spadtype{List FortranCode} and uses it as the body of the ASP,
      ++ making the declarations in the \spadtype{SymbolTable} component.

    retract : Vector Expression Float -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retractIfCan : Vector Expression Float -> Union($,"failed")
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retract : Vector Expression Integer -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retractIfCan : Vector Expression Integer -> Union($,"failed")
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retract : Vector Polynomial Float -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retractIfCan : Vector Polynomial Float -> Union($,"failed")
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    retract : Vector Polynomial Integer -> $
```

```
  ++ retract(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.

retractIfCan : Vector Polynomial Integer -> Union($,"failed")
  ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.

retract : Vector Fraction Polynomial Float -> $
  ++ retract(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.

retractIfCan : Vector Fraction Polynomial Float -> Union($,"failed")
  ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.

retract : Vector Fraction Polynomial Integer -> $
  ++ retract(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.

retractIfCan : Vector Fraction Polynomial Integer -> Union($,"failed")
  ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
  ++  legal Fortran-77 is produced.

-- NB: These ASPs also have a coerce from an appropriate instantiation
--     of Vector FortranExpression.
```

— **FVFUN.dotabb** —

```
"FVFUN"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FVFUN"];
"FVFUN" -> "FORTCAT"
```

— **FVFUN.dotfull** —

```
"FortranVectorFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FVFUN"];
"FortranVectorFunctionCategory()" -> "FortranProgramCategory()"
```

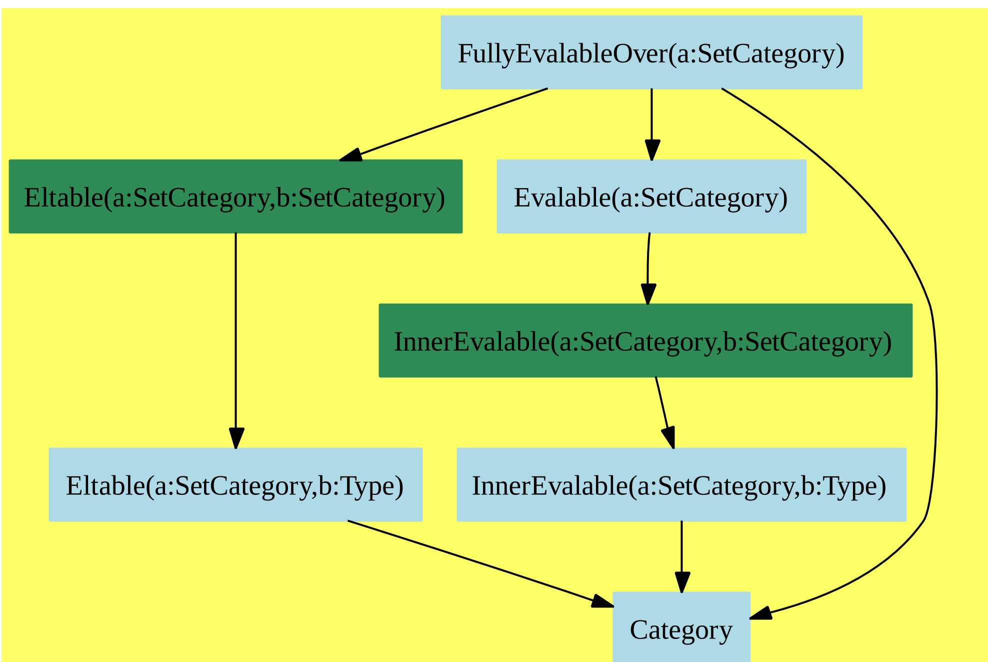
— **FVFUN.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];
```

```
"FortranVectorFunctionCategory()" [color=lightblue];
"FortranVectorFunctionCategory()" -> "FortranProgramCategory()"

"FortranProgramCategory()" [color=lightblue];
"FortranProgramCategory()" -> "Type()"
"FortranProgramCategory()" -> "CoercibleTo(OutputForm)"

"Type()" [color=lightblue];
"Type()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

———-

4.0.67 FullyEvalableOver (FEVALAB)



— **FullyEvalableOver.input** —

```
)set break resume
)sys rm -f FullyEvalableOver.output
```

```
)spool FullyEvalableOver.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FullyEvalableOver
--R
--R FullyEvalableOver(R: SetCategory) is a category constructor
--R Abbreviation for FullyEvalableOver is FEVALAB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FEVALAB
--R
--R------------------------------- Operations --------------------------------
--R map : ((R -> R),%) -> %
--R ?.? : (%,R) -> % if R has ELTAB(R,R)
--R eval : (%,List(R),List(R)) -> % if R has EVALAB(R)
--R eval : (%,R,R) -> % if R has EVALAB(R)
--R eval : (%,Equation(R)) -> % if R has EVALAB(R)
--R eval : (%,List(Equation(R))) -> % if R has EVALAB(R)
--R eval : (%,List(Symbol),List(R)) -> % if R has IEVALAB(SYMBOL,R)
--R eval : (%,Symbol,R) -> % if R has IEVALAB(SYMBOL,R)
--R
--E 1

)spool
)lisp (bye)
```

———-

— **FullyEvalableOver.help** —

```
====================================================================
FullyEvalableOver examples
====================================================================

This category provides a selection of evaluation operations depending
on what the argument type R provides.

See Also:
o )show FullyEvalableOver
```

———-

See:

⇒ "ComplexCategory" (COMPCAT) 20.0.240 on page 1869
⇒ "OctonionCategory" (OC) 12.0.190 on page 1238
⇒ "QuaternionCategory" (QUATCAT) 12.0.191 on page 1252
⇒ "QuotientFieldCategory" (QFCAT) 17.0.222 on page 1595
⇐ "Category" (CATEGORY) 2.0.1 on page 3

Exports:

eval map ?.?

These are directly exported but not implemented:

```
map : ((R -> R),%) -> %
```

These are implemented by this category:

```
?.? : (%,R) -> % if R has ELTAB(R,R)
eval : (%,Equation R) -> % if R has EVALAB R
eval : (%,List Symbol,List R) -> % if R has IEVALAB(SYMBOL,R)
```

These exports come from (p151) Evalable(a:Type):

```
eval : (%,List Equation R) -> % if R has EVALAB R
eval : (%,R,R) -> % if R has EVALAB R
eval : (%,List R,List R) -> % if R has EVALAB R
```

These exports come from (p64) InnerEvalable(a:Symbol,b:SetCategory):

```
eval : (%,Symbol,R) -> % if R has IEVALAB(SYMBOL,R)
```

— FullyEvalableOver.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FEVALAB">
FullyEvalableOver (FEVALAB)</a></h2>
&bull; R : SetCategory<br/><br/>

This category provides a selection of evaluation operations
depending on what the argument type R provides.
<br/><br/><br/>

map : (R -> R, $) -> $<br/>
  map(f, ex) evaluates ex, applying f to values of type R in ex.<br/><br/>

?.? : (%,R) -> % if R has ELTAB(R,R)<br/>
  elt(u,i) (also written: u . i) returns the element of u
indexed by i.<br/>
  Error: if i is not an index of u.<br/>
  from Eltable(R,$)<br/><br/>

eval : (%,List(R),List(R)) -> % if R has EVALAB(R)<br/>
  eval(f, [x1,...,xn], [v1,...,vn]) replaces xi by vi in f.<br/>
  from InnerEval(R,R)<br/><br/>

eval : (%,R,R) -> % if R has EVALAB(R)<br/>
  eval(f, x, v) replaces x by v in f.<br/>
  from InnerEval(R,R)<br/><br/>

eval : (%,Equation(R)) -> % if R has EVALAB(R)<br/>
  eval(f,x = v) replaces x by v in f.<br/>
  from Evalable(R)<br/><br/>

eval : (%,List(Equation(R))) -> % if R has EVALAB(R)<br/>
  eval(f, [x1 = v1,...,xn = vn]) replaces xi by vi in f.<br/>
  from Evalable(R)<br/><br/>
```

```
eval : (%,List(Symbol),List(R)) -> % if R has IEVALAB(SYMBOL,R)<br/>
  eval(f, [x1,...,xn], [v1,...,vn]) replaces xi by vi in f.<br/>
  from InnerEval(Symbol,R)<br/><br/>

eval : (%,Symbol,R) -> % if R has IEVALAB(SYMBOL,R)<br/>
  eval(f, x, v) replaces x by v in f.<br/>
  from InnerEval(Symbol,R)<br/><br/>

&bull; Eltable(R, $)<br/>
&bull; Evalable(R)<br/>
&bull; InnerEvalable(Symbol, R)<br/>
&bull; Eltable(R, R)<br/>

</body>
```

— category FEVALAB FullyEvalableOver —

```
)abbrev category FEVALAB FullyEvalableOver
++ Date Last Updated: June 3, 1991
++ Description:
++ This category provides a selection of evaluation operations
++ depending on what the argument type R provides.

FullyEvalableOver(R) : Category == SIG where
  R : SetCategory

  SIG ==> with

    map : (R -> R, $) -> $
      ++ map(f, ex) evaluates ex, applying f to values of type R in ex.

    if R has Eltable(R, R) then Eltable(R, $)

    if R has Evalable(R) then Evalable(R)

    if R has InnerEvalable(Symbol, R) then InnerEvalable(Symbol, R)

   add

     if R has Eltable(R, R) then

       elt(x:$, r:R) == map(y +-> y(r), x)

     if R has Evalable(R) then

       eval(x:$, l:List Equation R) == map(y +-> eval(y, l), x)

     if R has InnerEvalable(Symbol, R) then

       eval(x:$, ls:List Symbol, lv:List R) == map(y +-> eval(y, ls, lv), x)
```

———-

— COQ FEVALAB —

```
(* category FEVALAB *)
(*
    if R has Eltable(R, R) then

      elt(x:$, r:R) == map(y +-> y(r), x)

    if R has Evalable(R) then

      eval : (%,List(Equation(R))) -> %
      eval(x:$, l:List Equation R) == map(y +-> eval(y, l), x)

    if R has InnerEvalable(Symbol, R) then

      eval : (%,List(Symbol),List(R)) -> %
      eval(x:$, ls:List Symbol, lv:List R) == map(y +-> eval(y, ls, lv), x)

*)
```

———-

— FEVALAB.dotabb —

```
"FEVALAB"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FEVALAB"];
"FEVALAB" -> "ELTAB"
"FEVALAB" -> "EVALAB"
"FEVALAB" -> "IEVALAB"
"FEVALAB" -> "CATEGORY"
```

———-

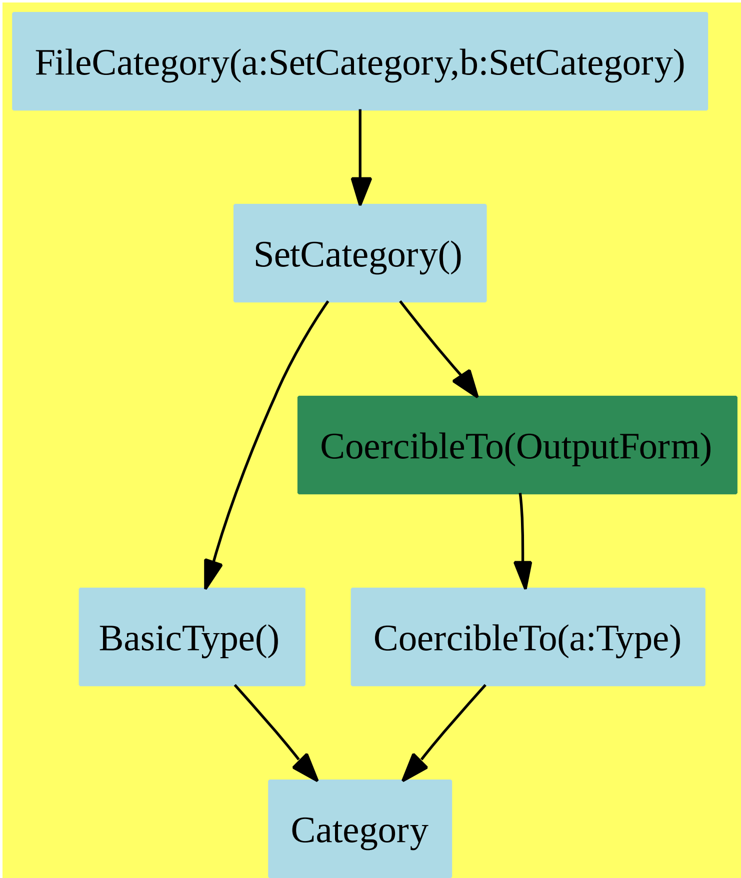
— FEVALAB.dotfull —

```
"FullyEvalableOver(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FEVALAB"];
"FullyEvalableOver(a:SetCategory)" -> "Eltable(a:SetCategory,b:Type)"
"FullyEvalableOver(a:SetCategory)" -> "Evalable(a:SetCategory)"
"FullyEvalableOver(a:SetCategory)" -> "Category"

"FullyEvalableOver(IntegralDomain)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FEVALAB"];
"FullyEvalableOver(IntegralDomain)" ->
  "FullyEvalableOver(a:SetCategory)"

"FullyEvalableOver(CommutativeRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FEVALAB"];
"FullyEvalableOver(CommutativeRing)" ->
  "FullyEvalableOver(a:SetCategory)"
```

— **FEVALAB.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FullyEvalableOver(a:SetCategory)" [color=lightblue];
"FullyEvalableOver(a:SetCategory)" -> "Eltable(a:SetCategory,b:SetCategory)"
"FullyEvalableOver(a:SetCategory)" -> "Evalable(a:SetCategory)"
"FullyEvalableOver(a:SetCategory)" -> "Category"

"Eltable(a:SetCategory,b:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ELTAB"];
"Eltable(a:SetCategory,b:SetCategory)" ->
   "Eltable(a:SetCategory,b:Type)"

"Eltable(a:SetCategory,b:Type)" [color=lightblue];
"Eltable(a:SetCategory,b:Type)" -> "Category"

"Evalable(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=EVALAB"];
"Evalable(a:SetCategory)" -> "InnerEvalable(a:SetCategory,b:SetCategory)"

"InnerEvalable(a:SetCategory,b:SetCategory)" [color=seagreen];
"InnerEvalable(a:SetCategory,b:SetCategory)" ->
    "InnerEvalable(a:SetCategory,b:Type)"

"InnerEvalable(a:SetCategory,b:Type)" [color=lightblue];
"InnerEvalable(a:SetCategory,b:Type)" -> "Category"

"Category" [color=lightblue];
}
```

4.0.68 FileCategory (FILECAT)



— **FileCategory.input** —

```
)set break resume
)sys rm -f FileCategory.output
)spool FileCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FileCategory
--R
--R FileCategory(Name: SetCategory,S: SetCategory) is a category constructor
--R Abbreviation for FileCategory is FILECAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FILECAT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                close! : % -> %
```

```
--R coerce : % -> OutputForm             flush : % -> Void
--R hash : % -> SingleInteger            iomode : % -> String
--R latex : % -> String                  name : % -> Name
--R open : (Name,String) -> %            open : Name -> %
--R read! : % -> S                       reopen! : (%,String) -> %
--R write! : (%,S) -> S                  ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

— **FileCategory.help** —

```
====================================================================
FileCategory examples
====================================================================

This category provides an interface to operate on files in the
computer's file system.  The precise method of naming files
is determined by the Name parameter.  The type of the contents
of the file is determined by S.

See Also:
o )show FileCategory
```

See:

⇐ "SetCategory" (SETCAT) 3.0.57 on page 187

Exports:

close!	coerce	hash	iomode	latex
name	open	read!	reopen!	write!
?=?	?~=?			

These are directly exported but not implemented:

```
close! : % -> %
iomode : % -> String
name : % -> Name
open : Name -> %
open : (Name,String) -> %
read! : % -> S
reopen! : (%,String) -> %
write! : (%,S) -> S
```

These exports come from SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
```

```
 ?=? : (%,%) -> Boolean
 ?~=? : (%,%) -> Boolean
```

— **FileCategory.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FILECAT">
FileCategory(Name,S) (FILECAT)</a></h2>
&bull; Name : SetCategory<br/>
&bull; S : SetCategory<br/><br/>

This category provides an interface to operate on files in the
computer's file system.  The precise method of naming files
is determined by the Name parameter.  The type of the contents
of the file is determined by S.
<br/><br/><br/>

open : Name -> %<br/>
  open(s) returns the file s open for input.<br/><br/>

open : (Name, IOMode) -> %<br/>
  open(s,mode) returns a file s open for operation in the <br/>
  indicated mode: "input" or "output".<br/><br/>

reopen! : (%, IOMode) -> %<br/>
  reopen!(f,mode) returns a file f reopened for operation in the<br/>
  indicated mode: "input" or "output".<br/>
  reopen!(f,"input") will reopen the file f for input.<br/><br/>

close! : % -> %<br/>
  close!(f) returns the file f closed to input and output.<br/><br/>

name : % -> Name<br/>
  name(f) returns the external name of the file f.<br/><br/>

iomode : % -> IOMode<br/>
  iomode(f) returns the status of the file f. The input/output <br/>
  status of f may be "input", "output" or "closed" mode.<br/><br/>

read! : % -> S<br/>
  read!(f) extracts a value from file f.  The state of f is<br/>
  modified so a subsequent call to read! will return<br/>
  the next element.<br/><br/>

write! : (%,S) -> S<br/>
  write!(f,s) puts the value s into the file f. <br/>
  The state of f is modified so subsequents call to write!<br/>
  will append one after another.<br/><br/>

flush : % -> Void<br/>
  flush(f) makes sure that buffered data is written out<br/><br/>

?=? : (%,%) -> Boolean<br/>
```

```
  x=y tests if x and y are equal.<br/>
  from BasicType<br/><br/>

coerce : % -> OutputForm<br/>
  coerce(a) transforms a into an element of OutputForm<br/>
  from CoercibleTo OutputForm<br/><br/>

hash : % -> SingleInteger<br/>
  hash(s) calculates a hash code for SingleInteger<br/>
  from SetCategory<br/><br/>

latex : % -> String<br/>
  latex(s) returns a LaTeX-printable output representation of s.<br/>
  from SetCategory<br/><br/>

?~=? : (%,%) -> Boolean<br/>
  x~=y tests if x and y are not equal.<br/>
  from BasicType<br/><br/>

&bull; SetCategory<br/>
&bull; BasicType<br/>
&bull; CoercibleTo OutputForm<br/>

</body>
```

— category FILECAT FileCategory —

```
)abbrev category FILECAT FileCategory
++ Author: Stephen M. Watt, Victor Miller
++ Date Last Updated: June 4, 1991
++ Description:
++ This category provides an interface to operate on files in the
++ computer's file system.  The precise method of naming files
++ is determined by the Name parameter.  The type of the contents
++ of the file is determined by S.

FileCategory(Name, S) : Category == SIG where
  Name : SetCategory
  S : SetCategory

  IOMode ==> String  -- Union("input", "output", "closed")

  SIG ==> SetCategory with

    open : Name -> %
      ++ open(s) returns the file s open for input.

    open : (Name, IOMode) -> %
      ++ open(s,mode) returns a file s open for operation in the
      ++ indicated mode: "input" or "output".

    reopen_! : (%, IOMode) -> %
```

```
      ++ reopen!(f,mode) returns a file f reopened for operation in the
      ++ indicated mode: "input" or "output".
      ++ \spad{reopen!(f,"input")} will reopen the file f for input.

    close_! : % -> %
      ++ close!(f) returns the file f closed to input and output.

    name : % -> Name
      ++ name(f) returns the external name of the file f.

    iomode : % -> IOMode
      ++ iomode(f) returns the status of the file f. The input/output
      ++ status of f may be "input", "output" or "closed" mode.

    read_! : % -> S
      ++ read!(f) extracts a value from file f.  The state of f is
      ++ modified so a subsequent call to \spadfun{read!} will return
      ++ the next element.

    write_! : (%,S) -> S
      ++ write!(f,s) puts the value s into the file f.
      ++ The state of f is modified so subsequents call to \spad{write!}
      ++ will append one after another.

    flush : % -> Void
      ++ flush(f) makes sure that buffered data is written out
```

— FILECAT.dotabb —

```
"FILECAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FILECAT"];
"FILECAT" -> "SETCAT"
```

— FILECAT.dotfull —

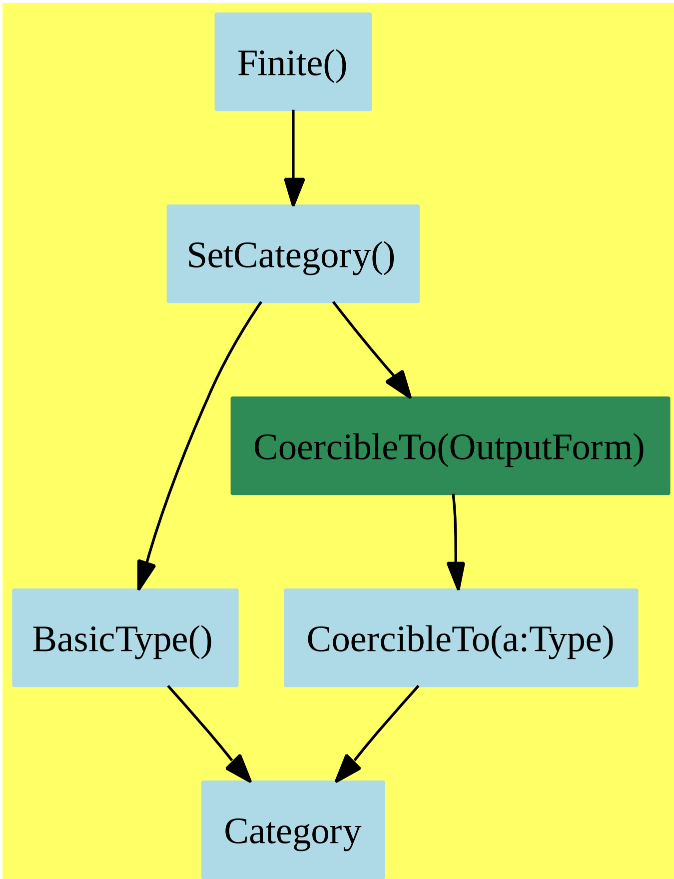
```
"FileCategory(a:SetCategory,b:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FILECAT"];
"FileCategory(a:SetCategory,b:SetCategory)" -> "SetCategory()"
```

— FILECAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];
```

```
"FileCategory(a:SetCategory,b:SetCategory)" [color=lightblue];
"FileCategory(a:SetCategory,b:SetCategory)" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.0.69 Finite (FINITE)



— **Finite.input** —

```
)set break resume
)sys rm -f Finite.output
)spool Finite.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Finite
--R
--R Finite is a category constructor
--R Abbreviation for Finite is FINITE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FINITE
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
```

```
--R enumerate : () -> List(%)           hash : % -> SingleInteger
--R index : PositiveInteger -> %        latex : % -> String
--R lookup : % -> PositiveInteger       random : () -> %
--R size : () -> NonNegativeInteger     ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

————

— **Finite.help** —

```
====================================================================
Finite examples
====================================================================

The category of domains composed of a finite set of elements. We include
the functions lookup and index to give a bijection between the finite set
and an initial segment of positive integers.

Axioms:
         lookup(index(n)) = n
         index(lookup(s)) = s

See Also:
o )show Finite
```

————

See:

⇒ "DirectProductCategory" (DIRPCAT) 12.0.184 on page 1173
⇒ "FiniteFieldCategory" (FFIELDC) 18.0.231 on page 1762
⇒ "OrderedFinite" (ORDFIN) 5.0.101 on page 461
⇐ "SetCategory" (SETCAT) 3.0.57 on page 187

Exports:

coerce hash	index	latex	lookup	random
size	?=?	?~=?		

These are directly exported but not implemented:

```
index : PositiveInteger -> %
lookup : % -> PositiveInteger
random : () -> %
size : () -> NonNegativeInteger
```

These exports come from (p187) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
```

```
 ?~=? : (%,%) -> Boolean
```

— **Finite.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FINITE">
Finite (FINITE)</a></h2>

The category of domains composed of a finite set of elements.
We include the functions lookup and index
to give a bijection between the finite set and an initial
segment of positive integers.
<br/><br/><br/>

size : () ->  NonNegativeInteger<br/>
  size() returns the number of elements in the set.<br/><br/>

index : PositiveInteger -> %<br/>
  index(i) takes a positive integer i less than or equal<br/>
  to size() and returns the i-th element of the set. <br/>
  This operation establishs a bijection<br/>
  between the elements of the finite set and 1..size().<br/><br/>

lookup : % -> PositiveInteger<br/>
  lookup(x) returns a positive integer such that<br/>
  x = index lookup x.<br/><br/>

random : () -> %<br/>
  random() returns a random element from the set.<br/><br/>

 enumerate : () -> List %<br/>
  enumerate() returns a list of elements of the set<br/><br/>

  <b>Example</b> enumerate()$OrderedVariableList([p,q])<br/><br/>

hash : % -> SingleInteger<br/>
  hash(s) calculates a hash code for s.<br/>
  from SetCategory<br/><br/>

latex : % -> String<br/>
  latex(s) returns a LaTeX-printable output representation of s.<br/>
  from SetCategory<br/><br/>

"=" : (%,%) -> Boolean<br/>
  x=y tests if x and y are equal.<br/>
  from BasicType<br/><br/>

"~=" : (%,%) -> Boolean<br/>
  x~=y tests if x and y are not equal.<br/>
  from BasicType<br/><br/>

coerce : % -> S<br/>
  coerce(a) transforms a into an element of OutputForm<br/>
```

```
  from CoercibleTo OutputForm<br/><br/>

&bull; CoercibleTo OutputForm<br/>
&bull; SetCategory<br/>
&bull; BasicType<br/>

</body>
```

———

— category FINITE Finite —

```
)abbrev category FINITE Finite
++ Description:
++ The category of domains composed of a finite set of elements.
++ We include the functions \spadfun{lookup} and \spadfun{index}
++ to give a bijection between the finite set and an initial
++ segment of positive integers.
++
++ Axioms:\br
++ \tab{5}\spad{lookup(index(n)) = n}\br
++ \tab{5}\spad{index(lookup(s)) = s}

Finite() : Category == SIG where

  SIG ==> SetCategory with

    size : () ->  NonNegativeInteger
      ++ size() returns the number of elements in the set.

    index : PositiveInteger -> %
      ++ index(i) takes a positive integer i less than or equal
      ++ to \spad{size()} and
      ++ returns the \spad{i}-th element of the set.
      ++ This operation establishs a bijection
      ++ between the elements of the finite set and \spad{1..size()}.

    lookup : % -> PositiveInteger
      ++ lookup(x) returns a positive integer such that
      ++ \spad{x = index lookup x}.

    random : () -> %
      ++ random() returns a random element from the set.

    enumerate : () -> List %
      ++ enumerate() returns a list of elements of the set
      ++
      ++X enumerate()$OrderedVariableList([p,q])

   add

     random() == index((1+random(size()$%))::PositiveInteger)

     enumerate() == [index(i::PositiveInteger) for i in 1..size()]
```

———-

— COQ FINITE —

```
(* category FINITE *)
(*

Axioms:
  lookup(index(n)) = n
  index(lookup(s)) = s

  random: () -> %
  random() == index((1+random(size()$%))::PositiveInteger)

  enumerate: () -> List %
  enumerate() == [index(i::PositiveInteger) for i in 1..size()]

*)
```

———-

— FINITE.dotabb —

```
"FINITE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FINITE"];
"FINITE" -> "SETCAT"
```

———-

— FINITE.dotfull —

```
"Finite()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FINITE"];
"Finite()" -> "SetCategory()"
```

———-

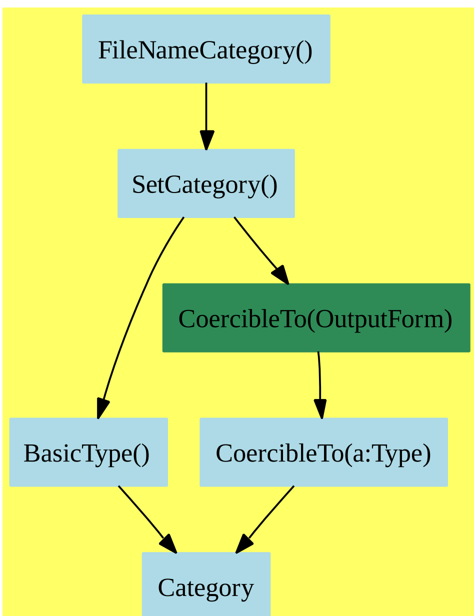
— FINITE.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"Finite()" [color=lightblue];
"Finite()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"
```

```
"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

———-

4.0.70 FileNameCategory (FNCAT)



— **FileNameCategory.input** —

```
)set break resume
)sys rm -f FileNameCategory.output
)spool FileNameCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FileNameCategory
```

```
--R
--R FileNameCategory is a category constructor
--R Abbreviation for FileNameCategory is FNCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FNCAT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> String
--R coerce : String -> %                  coerce : % -> OutputForm
--R directory : % -> String               exists? : % -> Boolean
--R extension : % -> String               hash : % -> SingleInteger
--R latex : % -> String                   name : % -> String
--R new : (String,String,String) -> %     readable? : % -> Boolean
--R writable? : % -> Boolean              ?~=? : (%,%) -> Boolean
--R filename : (String,String,String) -> %
--R
--E 1

)spool
)lisp (bye)
```

— **FileNameCategory.help** —

```
====================================================================
FileNameCategory examples
====================================================================

This category provides an interface to names in the file system.

See Also:
o )show FileNameCategory
```

See:

⇐ "SetCategory" (SETCAT) 3.0.57 on page 187

Exports:

coerce	directory	exists?	extension	filename
hash	latex	name	new	readable?
writable?	?=?	?~=?		

These are directly exported but not implemented:

```
coerce : String -> %
coerce : % -> String
directory : % -> String
exists? : % -> Boolean
extension : % -> String
filename : (String,String,String) -> %
name : % -> String
```

```
new : (String,String,String) -> %
readable? : % -> Boolean
writable? : % -> Boolean
```

These exports come from (p187) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

— FileNameCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FNCAT">
FileNameCategory (FNCAT)</a></h2>

This category provides an interface to names in the file system.
<br/><br/><br/>

coerce : String -> %<br/>
  coerce(s) converts a string to a file name<br/>
  according to operating system-dependent conventions.<br/><br/>

coerce : % -> String<br/>
  coerce(fn) produces a string for a file name<br/>
  according to operating system-dependent conventions.<br/><br/>

filename : (String, String, String) -> %<br/>
  filename(d,n,e) creates a file name with<br/>
  d as its directory, n as its name and e as its extension.<br/>
  This is a portable way to create file names.<br/>
  When d or t is the empty string, a default is used.<br/><br/>

directory : % -> String<br/>
  directory(f) returns the directory part of the file name.<br/><br/>

name : % -> String<br/>
  name(f) returns the name part of the file name.<br/><br/>

extension : % -> String<br/>
  extension(f) returns the type part of the file name.<br/><br/>

exists? : % -> Boolean<br/>
  exists?(f) tests if the file exists in the file system.<br/><br/>

readable? : % -> Boolean<br/>
  readable?(f) tests if the named file exist and can it be opened<br/>
  for reading.<br/><br/>

writable? : % -> Boolean<br/>
  writable?(f) tests if the named file be opened for writing.<br/>
  The named file need not already exist.<br/><br/>
```

```
new : (String, String, String) -> %<br/>
  new(d,pref,e) constructs the name of a new writable file with<br/>
  d as its directory, pref as a prefix of its name and<br/>
  e as its extension.<br/>
  When d or t is the empty string, a default is used.<br/>
  An error occurs if a new file cannot be written in the given<br/>
  directory.<br/><br/>

hash : % -> SingleInteger<br/>
  hash(s) calculates a hash code for s.<br/>
  from SetCategory<br/><br/>

latex : % -> String<br/>
  latex(s) returns a LaTeX-printable output representation of s.<br/>
  from SetCategory<br/><br/>

"=" : (%,%) -> Boolean<br/>
  x=y tests if x and y are equal.<br/>
  from BasicType<br/><br/>

"~=" : (%,%) -> Boolean<br/>
  x~=y tests if x and y are not equal.<br/>
  from BasicType<br/><br/>

coerce : % -> S<br/>
  coerce(a) transforms a into an element of OutputForm<br/>
  from CoercibleTo OutputForm<br/><br/>

&bull; CoercibleTo OutputForm<br/>
&bull; SetCategory<br/>
&bull; BasicType<br/>

</body>
```

— category FNCAT FileNameCategory —

```
)abbrev category FNCAT FileNameCategory
++ Author: Stephen M. Watt
++ Date Created: 1985
++ Date Last Updated: June 20, 1991
++ Description:
++ This category provides an interface to names in the file system.

FileNameCategory() : Category == SIG where

  SIG ==> SetCategory with

    coerce : String -> %
      ++ coerce(s) converts a string to a file name
      ++ according to operating system-dependent conventions.
```

```
    coerce : % -> String
      ++ coerce(fn) produces a string for a file name
      ++ according to operating system-dependent conventions.

    filename : (String, String, String) -> %
      ++ filename(d,n,e) creates a file name with
      ++ d as its directory, n as its name and e as its extension.
      ++ This is a portable way to create file names.
      ++ When d or t is the empty string, a default is used.

    directory : % -> String
      ++ directory(f) returns the directory part of the file name.

    name : % -> String
      ++ name(f) returns the name part of the file name.

    extension : % -> String
      ++ extension(f) returns the type part of the file name.

    exists? : % -> Boolean
      ++ exists?(f) tests if the file exists in the file system.

    readable? : % -> Boolean
      ++ readable?(f) tests if the named file exist and can it be opened
      ++ for reading.

    writable? : % -> Boolean
      ++ writable?(f) tests if the named file be opened for writing.
      ++ The named file need not already exist.

    new : (String, String, String) -> %
      ++ new(d,pref,e) constructs the name of a new writable file with
      ++ d as its directory, pref as a prefix of its name and
      ++ e as its extension.
      ++ When d or t is the empty string, a default is used.
      ++ An error occurs if a new file cannot be written in the given
      ++ directory.
```

———-

— FNCAT.dotabb —

```
"FNCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FNCAT"];
"FNCAT" -> "SETCAT"
```

———-

— FNCAT.dotfull —

```
"FileNameCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FNCAT"];
```

```
"FileNameCategory()" -> "SetCategory()"
```

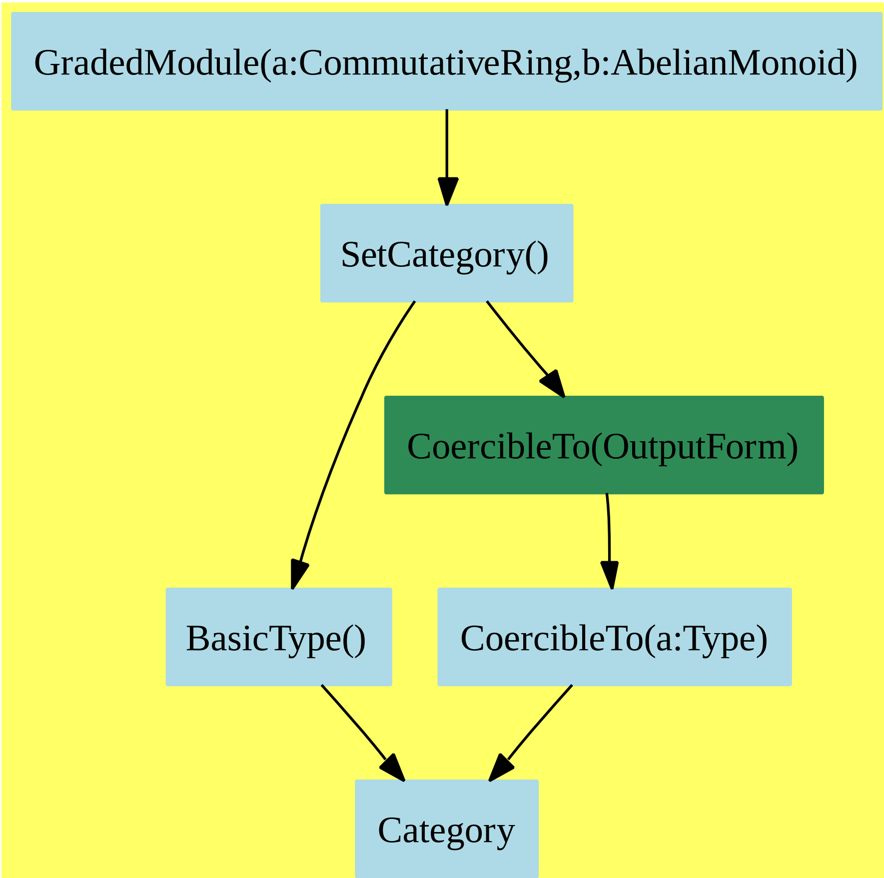
———-

— **FNCAT.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FileNameCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FNCAT"];
"FileNameCategory()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

———-

4.0.71 GradedModule (GRMOD)



— **GradedModule.input** —

```
)set break resume
)sys rm -f GradedModule.output
)spool GradedModule.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show GradedModule
--R
--R GradedModule(R: CommutativeRing,E: AbelianMonoid) is a category constructor
--R Abbreviation for GradedModule is GRMOD
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for GRMOD
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
```

```
--R ?+? : (%,%) -> %                    ?-? : (%,%) -> %
--R -? : % -> %                         ?=? : (%,%) -> Boolean
--R 0 : () -> %                         coerce : % -> OutputForm
--R degree : % -> E                     hash : % -> SingleInteger
--R latex : % -> String                 ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

— **GradedModule.help** —

```
====================================================================
GradedModule examples
====================================================================

GradedModule(R,E) denotes "E-graded R-module", that is, collection of
R-modules indexed by an abelian monoid E. An element g of G[s] for
some specific s in E is said to be an element of G with degree s.
Sums are defined in each module G[s] so two elements of G have a
sum if they have the same degree.

Morphisms can be defined and composed by degree to give the mathematical
category of graded modules.

See Also:
o )show GradedModule
```

See:

⇒ "GradedAlgebra" (GRALG) 5.0.97 on page 436
⇐ "SetCategory" (SETCAT) 3.0.57 on page 187

Exports:

0	coerce	degree	hash	latex
?~=?	?*?	?+?	?-?	-?
?=?				

These are directly exported but not implemented:

```
0 : () -> %
degree : % -> E
?*? : (%,R) -> %
?*? : (R,%) -> %
-? : % -> %
?+? : (%,%) -> %
```

These are implemented by this category:

```
?-? : (%,%) -> %
```

These exports come from (p187) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?~=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
```

See: Algebra [Saun79]

— GradedModule.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#GRMOD">
GradedModule(R,E) (GRMOD)</a></h2>

&bull; R : CommutativeRing<br/>
&bull; E : AbelianMonoid<br/><br/>

GradedModule(R,E) denotes "E-graded R-module", that is, collection of
R-modules indexed by an abelian monoid E.<br/>
An element \spad{g} of G[s] for some specific s in E
is said to be an element of G with degree s.<br/>
Sums are defined in each module G[s] so two elements of G
have a sum if they have the same degree.<br/>
Morphisms can be defined and composed by degree to give the
mathematical category of graded modules.
<br/><br/><br/>

degree : % -> E<br/>
  degree(g) names the degree of g.  The set of all elements<br/>
  of a given degree form an R-module.<br/><br/>

0 : constant -> %<br/>
  \spad{0} denotes the zero of degree 0.<br/><br/>

?*? : (R,%) -> %<br/>
  r*g is left module multiplication.<br/><br/>

?*? : (%,R) -> %<br/>
  g*r is right module multiplication.<br/><br/>

-? : % -> %<br/>
  -g is the additive inverse of g in the module of elements<br/>
  of the same grade as g.<br/><br/>

?+? : (%,%) -> %<br/>
  g+h is the sum of g and h in the module of elements of<br/>
  the same degree as g and h.  Error: if g and h<br/>
  have different degrees.<br/><br/>

?-? : (%,%) -> %<br/>
  g-h is the difference of g and h in the module of elements of<br/>
  the same degree as g and h.  Error: if g and h<br/>
  have different degrees.<br/><br/>
```

```
hash : % -> SingleInteger<br/>
  hash(s) calculates a hash code for s.<br/>
  from SetCategory<br/><br/>

latex : % -> String<br/>
  latex(s) returns a LaTeX-printable output representation of s.<br/>
  from SetCategory<br/><br/>

"=" : (%,%) -> Boolean<br/>
  x=y tests if x and y are equal.<br/>
  from BasicType<br/><br/>

"~=" : (%,%) -> Boolean<br/>
  x~=y tests if x and y are not equal.<br/>
  from BasicType<br/><br/>

coerce : % -> S<br/>
  coerce(a) transforms a into an element of OutputForm<br/>
  from CoercibleTo OutputForm<br/><br/>

&bull; CoercibleTo OutputForm<br/>
&bull; SetCategory<br/>
&bull; BasicType<br/>

</body>
```

— category GRMOD GradedModule —

```
)abbrev category GRMOD GradedModule
++ Author: Stephen M. Watt
++ Date Created: May 20, 1991
++ Date Last Updated: May 20, 1991
++ References:
++ Saun79 Algebra 2d Edition, MacLane and Birkhoff, MacMillan 1979
++ Description:
++ GradedModule(R,E) denotes ``E-graded R-module'', that is, collection of
++ R-modules indexed by an abelian monoid E.
++ An element \spad{g} of \spad{G[s]} for some specific \spad{s} in \spad{E}
++ is said to be an element of \spad{G} with degree \spad{s}.
++ Sums are defined in each module \spad{G[s]} so two elements of \spad{G}
++ have a sum if they have the same degree.
++
++ Morphisms can be defined and composed by degree to give the
++ mathematical category of graded modules.

GradedModule(R,E): Category == SIG where
  R : CommutativeRing
  E : AbelianMonoid

  SIG ==> SetCategory with

    degree : % -> E
```

```
      ++ degree(g) names the degree of g.  The set of all elements
      ++ of a given degree form an R-module.

    0 : constant -> %
      ++ \spad{0} denotes the zero of degree 0.

    _* : (R, %) -> %
      ++ r*g is left module multiplication.

    _* : (%, R) -> %
      ++ g*r is right module multiplication.

    _- : % -> %
      ++ -g is the additive inverse of g in the module of elements
      ++ of the same grade as g.

    _+ : (%, %) -> %
      ++ g+h is the sum of g and h in the module of elements of
      ++ the same degree as g and h.  Error: if g and h
      ++ have different degrees.

    _- : (%, %) -> %
      ++ g-h is the difference of g and h in the module of elements of
      ++ the same degree as g and h.  Error: if g and h
      ++ have different degrees.

   add

      (x: %) - (y: %) == x+(-y)
```

— COQ GRMOD —

```
(* category GRMOD *)
(*

    -: (%, %) -> %
    (x: %) - (y: %) == x+(-y)

*)
```

— GRMOD.dotabb —

```
"GRMOD"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=GRMOD"];
"GRMOD" -> "SETCAT"
```

— GRMOD.dotfull —

```
"GradedModule(a:CommutativeRing,b:AbelianMonoid)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=GRMOD"];
"GradedModule(a:CommutativeRing,b:AbelianMonoid)" -> "SetCategory()"
```

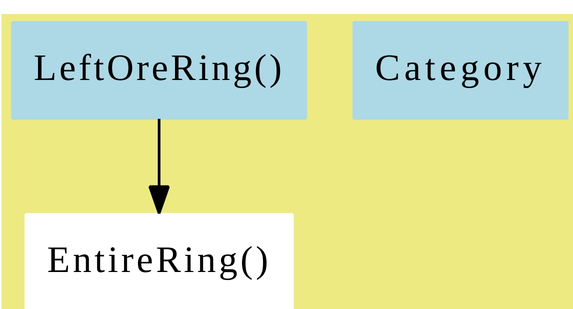
————

— GRMOD.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"GradedModule(a:CommutativeRing,b:AbelianMonoid)" [color=lightblue];
"GradedModule(a:CommutativeRing,b:AbelianMonoid)" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

————

4.0.72 LeftOreRing (LORER)



— **LeftOreRing.input** —

```
)set break resume
)sys rm -f LeftOreRing.output
)spool LeftOreRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show LeftOreRing
--R
--R LeftOreRing is a category constructor
--R Abbreviation for LeftOreRing is LORER
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for LORER
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R one? : % -> Boolean                   recip : % -> Union(%,"failed")
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— **LeftOreRing.help** —

```
====================================================================
LeftOreRing
====================================================================

This is the category of left ore rings, that is, the commutative
rings without zero divisors where we can compute least left
common multiples.

See [Dele06], [Bron96a]

See Also:
o )show LeftOreRing
```

———-

See Delenclos [Dele06], Bronstein [Bron96a]

— LeftOreRing.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#LORER">
LeftOreRing (LORER)</a></h2>

This is the category of left ore rings, that is noncommutative
rings without zero divisors where we can compute the least left
common multiple.
<br/><br/><br/>

0 : constant -> % <br/>
  0 is the additive identity element.<br/>
  from AbelianMonoid<br/><br/>

1 : constant ->  %<br/>
  1 is the multiplicative identity. <br/>
  from Monoid<br/><br/>

?*? : (%,%) -> %<br/>
  x*y returns the product of x and y.<br/>
&nbps; from SemiGroup<br/><br/>

?*? : (Integer,%) -> %            <br/>
  n*x is the product of x by the integer n.<br/>
  from AbelianGroup<br/><br/>

?*? : (PositiveInteger,%) -> %<br/>
  n*x computes the left-multiplication of x by the positive <br/>
  integer n. This is equivalent to adding x to itself n times.<br/>
  from AbelianSemiGroup<br/><br/>

?+? : (%,%) -> %<br/>
  x+y computes the sum of x and y.<br/>
  from AbelianSemiGroup<br/><br/>

-? : % -> %<br/>
  -x is the additive inverse of x.<br/>
  from AbelianGroup<br/><br/>

?-? : (%,%) -> %<br/>
  x-y is the difference of x and y x + (-y).<br/>
  from AbelianGroup<br/><br/>

?^? : (%,NonNegativeInteger) -> %<br/>
  x**n returns the repeated product of x n times, exponentiation.<br/>
  from Monoid<br/><br/>

?^? : (%,PositiveInteger) -> %<br/>
```

```
  x^n returns the repeated product of x n times, exponentiation.<br/>
  from SemiGroup<br/><br/>

?**? : (%,PositiveInteger) -> %<br/>
  x^n returns the repeated product of x n times, exponentiation.<br/>
  from SemiGroup<br/><br/>

characteristic : () -> NonNegativeInteger<br/>
  characteristic() returns the characteristic of the ring<br/>
  this is the smallest positive integer n such that<br/>
  n*x=0 for all x in the ring, or zero if no such n exists.<br/>
  from Ring<br/><br/>

coerce : Integer -> %<br/>
  coerce(i) converts the integer i to a member of the given domain.<br/>
  from Ring<br/><br/>

lcmCoef : (%, %) -> Record(llcm_res : %, coeff1 : %, coeff2 : %)<br/>
  lcmCoef(c1, c2) computes (llcm_res, coeff1, coeff2) such that<br/>
  llcm_res is least left common multiple of c1 and c2<br/>
  and llcm_res = coeff1*c1 = coeff2*c2<br/><br/>

one? : % -> Boolean<br/>
  one?(x) tests if x is equal to 1.<br/>
  from Monoid<br/><br/>

recip : % -> Union(%,"failed")<br/>
  recip(x) tries to compute the multiplicative inverse for x<br/>
  or "failed" if it cannot find the inverse (see unitsKnown).<br/>
  from Monoid<br/><br/>

sample : constant -> %<br/>
  sample yields a value of type %<br/>
  from AbelianMonoid<br/><br/>

subtractIfCan : (%,%) -> Union(%,"failed")<br/>
  subtractIfCan(x, y) returns an element z such that z+y=x<br/>
  or "failed" if no such element exists.<br/>
  from CancellationAbelianMonoid<br/><br/>

zero? : % -> Boolean<br/>
  zero?(x) tests if x is equal to 0.<br/>
  from AbelianMonoid<br/><br/>

noZeroDivisors<br/>
  if a product is zero then one of the factors<br/>
  must be zero.<br/>
  from EntireRing<br/><br/>

leftUnitary <br/>
  1 * x = x<br/>
  from BiModule(%,%)<br/><br/>

rightUnitary <br/>
```

```
  x * 1 = x<br/>
  from BiModule(%,%)<br/><br/>

unitsKnown<br/>
  recip truly yields<br/>
  reciprocal or "failed" if not a unit.<br/>
  Note that recip(0) = "failed".<br/>
  from Ring<br/><br/>

hash : % -> SingleInteger<br/>
  hash(s) calculates a hash code for s.<br/>
  from SetCategory<br/><br/>

latex : % -> String<br/>
  latex(s) returns a LaTeX-printable output representation of s.<br/>
  from SetCategory<br/><br/>

"=" : (%,%) -> Boolean<br/>
  x=y tests if x and y are equal.<br/>
  from BasicType<br/><br/>

"~=" : (%,%) -> Boolean<br/>
  x~=y tests if x and y are not equal.<br/>
  from BasicType<br/><br/>

coerce : % -> OutputForm<br/>
  coerce(a) transforms a into an element of OutputForm<br/>
  from CoercibleTo OutputForm<br/><br/>

&bull; AbelianSemiGroup<br/>
&bull; AbelianMonoid<br/>
&bull; CancellationAbelianMonoid<br/>
&bull; CoercibleTo OutputForm<br/>
&bull; SetCategory<br/>
&bull; BasicType<br/>
&bull; Ring<br/>
&bull; BiModule(%,%)<br/>
&bull; EntireRing<br/>
&bull; Rng<br/>
&bull; Monoid<br/>
&bull; LeftModule(%)<br/>
&bull; AbelianGroup<br/>
&bull; SemiGroup<br/>

</body>
```

— category LORER LeftOreRing —

```
)abbrev category LORER LeftOreRing
++ References:
++ Dele06 Noncommutative Symmetric functions and W-polynomials,
++ Bron96a An introduction to pseudo-linear algebra
```

```
++ Description:
++ This is the category of left ore rings, that is noncommutative
++ rings without zero divisors where we can compute the least left
++ common multiple.

LeftOreRing() : Category == SIG where

  SIG ==> EntireRing with

    lcmCoef : (%, %) -> Record(llcm_res : %, coeff1 : %, coeff2 : %)
      ++ lcmCoef(c1, c2) computes (llcm_res, coeff1, coeff2) such that
      ++ llcm_res is least left common multiple of c1 and c2
      ++ and llcm_res = coeff1*c1 = coeff2*c2
```

— LORER.dotabb —

```
"LORER"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LORER"];
"LORER" -> "BMODULE"
```

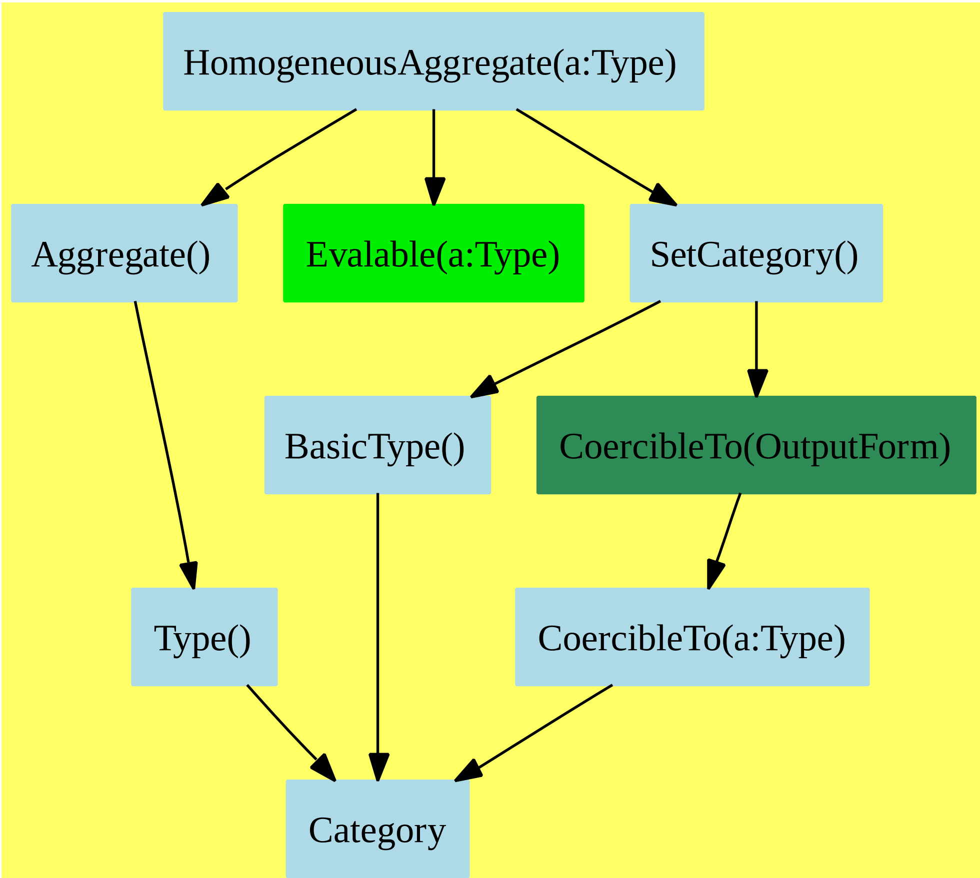
— LORER.dotfull —

```
"LeftOreRing()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LORER"];
"LeftOreRing()" -> "EntireRing()"
```

— LORER.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"LeftOreRing()" [color=lightblue];
"LeftOreRing()" -> "EntireRing()"

"Category" [color=lightblue];
}
```

4.0.73 HomogeneousAggregate (HOAGG)



— **HomogeneousAggregate.input** —

```
)set break resume
)sys rm -f HomogeneousAggregate.output
)spool HomogeneousAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show HomogeneousAggregate
--R
--R HomogeneousAggregate(S: Type) is a category constructor
--R Abbreviation for HomogeneousAggregate is HOAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for HOAGG
--R
--R------------------------------- Operations --------------------------------
--R copy : % -> %                         empty : () -> %
```

```
--R empty? : % -> Boolean                eq? : (%,%) -> Boolean
--R latex : % -> String if S has SETCAT   map : ((S -> S),%) -> %
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(S) if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———-

— **HomogeneousAggregate.help** —

```
====================================================================
HomogeneousAggregate examples
====================================================================

A homogeneous aggregate is an aggregate of elements all of the same type.

In the current system, all aggregates are homogeneous. Two attributes
characterize classes of aggregates. Aggregates from domains with
attribute "finiteAggregate" have a finite number of members. Those
with attribute "shallowlyMutable" allow an element to be modified
or updated without changing its overall value.

See Also:
o )show HomogeneousAggregate
```

———-

See:

⇒ "BagAggregate" (BGAGG) 5.0.92 on page 393

⇒ "Collection" (CLAGG) 5.0.94 on page 403
⇒ "IndexedAggregate" (IXAGG) 5.0.98 on page 441
⇒ "RectangularMatrixCategory" (RMATCAT) 10.0.171 on page 1075
⇒ "RecursiveAggregate" (RCAGG) 5.0.104 on page 475
⇒ "TwoDimensionalArrayCategory" (ARR2CAT) 5.0.105 on page 481
⇐ "Aggregate" (AGG) 3.0.44 on page 133
⇐ "Evalable" (EVALAB) 3.0.48 on page 151
⇐ "SetCategory" (SETCAT) 3.0.57 on page 187

Attributes exported:

- **nil**

Exports:

any?	coerce	copy	count	empty
empty?	eq?	eval	every?	hash
latex	less?	map	map!	members
member?	more?	parts	sample	size?
#?	?=?	?~=?		

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
parts : % -> List S if $ has finiteAggregate
```

These are implemented by this category:

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if S has SETCAT
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
```

These exports come from (p133) Aggregate:

```
copy : % -> %
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
```

```
less? : (%,NonNegativeInteger) -> Boolean
more? : (%,NonNegativeInteger) -> Boolean
sample : () -> %
size? : (%,NonNegativeInteger) -> Boolean
```

These exports come from (p151) Evalable(a:Type):

```
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
```

These exports come from (p187) SetCategory():

```
hash : % -> SingleInteger if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
latex : % -> String if S has SETCAT
```

— HomogeneousAggregate.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#HOAGG">
HomogeneousAggregate(S) (HOAGG)</a></h2>
&bull; S : Type<br/><br/>

A homogeneous aggregate is an aggregate of elements all of the same
type.  In the current system, all aggregates are homogeneous.  Two
attributes characterize classes of aggregates.  Aggregates from
domains with attribute finiteAggregate have a finite number of
members.  Those with attribute shallowlyMutable allow an element to be
modified or updated without changing its overall value.
<br/><br/><br/>

eval : (%,List(R),List(R)) -> % if R has EVALAB(R)<br/>
  eval(f, [x1,...,xn], [v1,...,vn]) replaces xi by vi in f.<br/>
  from InnerEval(R,R)<br/><br/>

eval : (%,R,R) -> % if R has EVALAB(R)<br/>
  eval(f, x, v) replaces x by v in f.<br/>
  from InnerEval(R,R)<br/><br/>

eval : (%,Equation(R)) -> % if R has EVALAB(R)<br/>
  eval(f,x = v) replaces x by v in f.<br/>
  from Evalable(R)<br/><br/>

eval : (%,List(Equation(R))) -> % if R has EVALAB(R)<br/>
  eval(f, [x1 = v1,...,xn = vn]) replaces xi by vi in f.<br/>
  from Evalable(R)<br/><br/>

eq? : (%,%) -> Boolean<br/>
  eq?(u,v) tests if u and v are same objects.<br/>
  from Aggregate<br/><br/>
```

```
copy : % -> %<br/>
  copy(u) returns a top-level (non-recursive) copy of u.<br/>
  Note that for collections, copy(u) == [x for x in u].<br/>
  from Aggregate<br/><br/>

empty : () -> %<br/>
  empty()$D creates an aggregate of type D with 0 elements.<br/>
  Note that The $D can be dropped if understood by context,<br/>
  for example u: D := empty().<br/>
  from Aggregate<br/><br/>

empty? : % -> Boolean<br/>
  empty?(u) tests if u has 0 elements.<br/>
  from Aggregate<br/><br/>

less? : (%,NonNegativeInteger) -> Boolean<br/>
  less?(u,n) tests if u has less than n elements.<br/>
  from Aggregate<br/><br/>

more? : (%,NonNegativeInteger) -> Boolean<br/>
  more?(u,n) tests if u has greater than n elements.<br/>
  from Aggregate<br/><br/>

size? : (%,NonNegativeInteger) -> Boolean<br/>
  size?(u,n) tests if u has exactly n elements.<br/>
  from Aggregate<br/><br/>

sample : constant -> %<br/>
  sample yields a value of type %<br/>
  from Aggregate<br/><br/>

# : % -> NonNegativeInteger<br/>
  # u returns the number of items in u.<br/>
  from Aggregate<br/><br/>

map : (S->S,%) -> %<br/>
  map(f,u) returns a copy of u with each element x replaced by f(x).<br/>
  For collections, map(f,u) = [f(x) for x in u].<br/><br/>

map! : (S->S,%) -> %<br/>
  map!(f,u) destructively replaces each element x of u <br/>
  by f(x).<br/><br/>

any? : (S->Boolean,%) -> Boolean<br/>
  any?(p,u) tests if p(x) is true for any element x of u.<br/>
  Note that for collections,<br/>
  any?(p,u) = reduce(or,map(f,u),false,true).<br/><br/>

every? : (S->Boolean,%) -> Boolean<br/>
  every?(f,u) tests if p(x) is true for all elements x of u.<br/>
  Note that for collections,<br/>
  every?(p,u) = reduce(and,map(f,u),true,false).<br/><br/>

count : (S->Boolean,%) -> NonNegativeInteger<br/>
```

```
  count(p,u) returns the number of elements x in u<br/>
  such that p(x) is true. For collections,<br/>
  count(p,u) = reduce(+,[1 for x in u | p(x)],0).<br/><br/>

parts : % -> List S<br/>
  parts(u) returns a list of the consecutive elements of u.<br/>
  For collections, parts([x,y,...,z]) = (x,y,...,z).<br/><br/>

members : % -> List S<br/>
  members(u) returns a list of the consecutive elements of u.<br/>
  For collections, parts([x,y,...,z]) = (x,y,...,z).<br/><br/>

count : (S,%) -> NonNegativeInteger<br/>
  count(x,u) returns the number of occurrences of x in u. For<br/>
  collections, count(x,u) = reduce(+,[x=y for y in u],0).<br/><br/>

member? : (S,%) -> Boolean<br/>
  member?(x,u) tests if x is a member of u.<br/>
  For collections,<br/>
  member?(x,u) = reduce(or,[x=y for y in u],false).<br/><br/>

hash : % -> SingleInteger<br/>
  hash(s) calculates a hash code for s.<br/>
  from SetCategory<br/><br/>

latex : % -> String<br/>
  latex(s) returns a LaTeX-printable output representation of s.<br/>
  from SetCategory<br/><br/>

"=" : (%,%) -> Boolean<br/>
  x=y tests if x and y are equal.<br/>
  from BasicType<br/><br/>

"~=" : (%,%) -> Boolean<br/>
  x~=y tests if x and y are not equal.<br/>
  from BasicType<br/><br/>

coerce : % -> OutputForm<br/>
  coerce(a) transforms a into an element of OutputForm<br/>
  from CoercibleTo OutputForm<br/><br/>

&bull; CoercibleTo OutputForm<br/>
&bull; SetCategory<br/>
&bull; BasicType<br/>
&bull; Aggregate
&bull; Evalable S
&bull; shallowlyMutable
&bull; finiteAggregate

</body>
```

— category HOAGG HomogeneousAggregate —

```
)abbrev category HOAGG HomogeneousAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991, May 1995
++ Description:
++ A homogeneous aggregate is an aggregate of elements all of the
++ same type.
++ In the current system, all aggregates are homogeneous.
++ Two attributes characterize classes of aggregates.
++ Aggregates from domains with attribute \spadatt{finiteAggregate}
++ have a finite number of members.
++ Those with attribute \spadatt{shallowlyMutable} allow an element
++ to be modified or updated without changing its overall value.

HomogeneousAggregate(S) : Category == SIG where
  S : Type

  SIG ==> Aggregate with

    if S has SetCategory then SetCategory

    if S has SetCategory then
       if S has Evalable S then Evalable S

    map : (S->S,%) -> %
      ++ map(f,u) returns a copy of u with each element x replaced by f(x).
      ++ For collections, \axiom{map(f,u) = [f(x) for x in u]}.

    if % has shallowlyMutable then

      map_! : (S->S,%) -> %
        ++ map!(f,u) destructively replaces each element x of u
        ++ by \axiom{f(x)}.

    if % has finiteAggregate then

       any? : (S->Boolean,%) -> Boolean
         ++ any?(p,u) tests if \axiom{p(x)} is true for any element x of u.
         ++ Note that for collections,
         ++ \axiom{any?(p,u) = reduce(or,map(f,u),false,true)}.

       every? : (S->Boolean,%) -> Boolean
         ++ every?(f,u) tests if p(x) is true for all elements x of u.
         ++ Note that for collections,
         ++ \axiom{every?(p,u) = reduce(and,map(f,u),true,false)}.

       count : (S->Boolean,%) -> NonNegativeInteger
         ++ count(p,u) returns the number of elements x in u
         ++ such that \axiom{p(x)} is true. For collections,
         ++ \axiom{count(p,u) = reduce(+,[1 for x in u | p(x)],0)}.

       parts : % -> List S
         ++ parts(u) returns a list of the consecutive elements of u.
```

```
      ++ For collections, \axiom{parts([x,y,...,z]) = (x,y,...,z)}.

    members : % -> List S
      ++ members(u) returns a list of the consecutive elements of u.
      ++ For collections, \axiom{parts([x,y,...,z]) = (x,y,...,z)}.

    if S has SetCategory then

      count : (S,%) -> NonNegativeInteger
        ++ count(x,u) returns the number of occurrences of x in u. For
        ++ collections, \axiom{count(x,u) = reduce(+,[x=y for y in u],0)}.

      member? : (S,%) -> Boolean
        ++ member?(x,u) tests if x is a member of u.
        ++ For collections,
        ++ \axiom{member?(x,u) = reduce(or,[x=y for y in u],false)}.

add

  if S has Evalable S then

    eval(u:%,l:List Equation S):% == map(x +-> eval(x,l),u)

  if % has finiteAggregate then

    #c == # parts c

    any?(f, c)  == _or/[f x for x in parts c]

    every?(f, c) == _and/[f x for x in parts c]

    count(f:S -> Boolean, c:%) == _+/[1 for x in parts c | f x]

    members x == parts x

    if S has SetCategory then

      count(s:S, x:%) == count(y +-> s = y, x)

      member?(e, c) == any?(x +-> e = x,c)

      x = y ==
         size?(x, #y) and _and/[a = b for a in parts x for b in parts y]

      coerce(x:%):OutputForm ==
        bracket
           commaSeparate [a::OutputForm for a in parts x]$List(OutputForm)
```

— COQ HOAGG —

```
(* category HOAGG *)
```

```
(*
   if S has Evalable S then

     eval : (%,List(Equation(S))) -> %
     eval(u:%,l:List Equation S):% == map(x +-> eval(x,l),u)

   if % has finiteAggregate then

     #? : % -> NonNegativeInteger
     #c == # parts c

     any?: (S->Boolean,%) -> Boolean
     any?(f, c)  == _or/[f x for x in parts c]

     every?: (S->Boolean,%) -> Boolean
     every?(f, c) == _and/[f x for x in parts c]

     count: (S->Boolean,%) -> NonNegativeInteger
     count(f:S -> Boolean, c:%) == _+/[1 for x in parts c | f x]

     members: % -> List S
     members x == parts x

     if S has SetCategory then

       count: (S,%) -> NonNegativeInteger
       count(s:S, x:%) == count(y +-> s = y, x)

       member?: (S,%) -> Boolean
       member?(e, c)   == any?(x +-> e = x,c)

       ?=? : (%,%) -> Boolean
       x = y ==
          size?(x, #y) and _and/[a = b for a in parts x for b in parts y]

       coerce : % -> OutputForm
       coerce(x:%):OutputForm ==
         bracket
            commaSeparate [a::OutputForm for a in parts x]$List(OutputForm)

*)
```

————

— HOAGG.dotabb —

```
"HOAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=HOAGG"];
"HOAGG" -> "AGG"
```

————

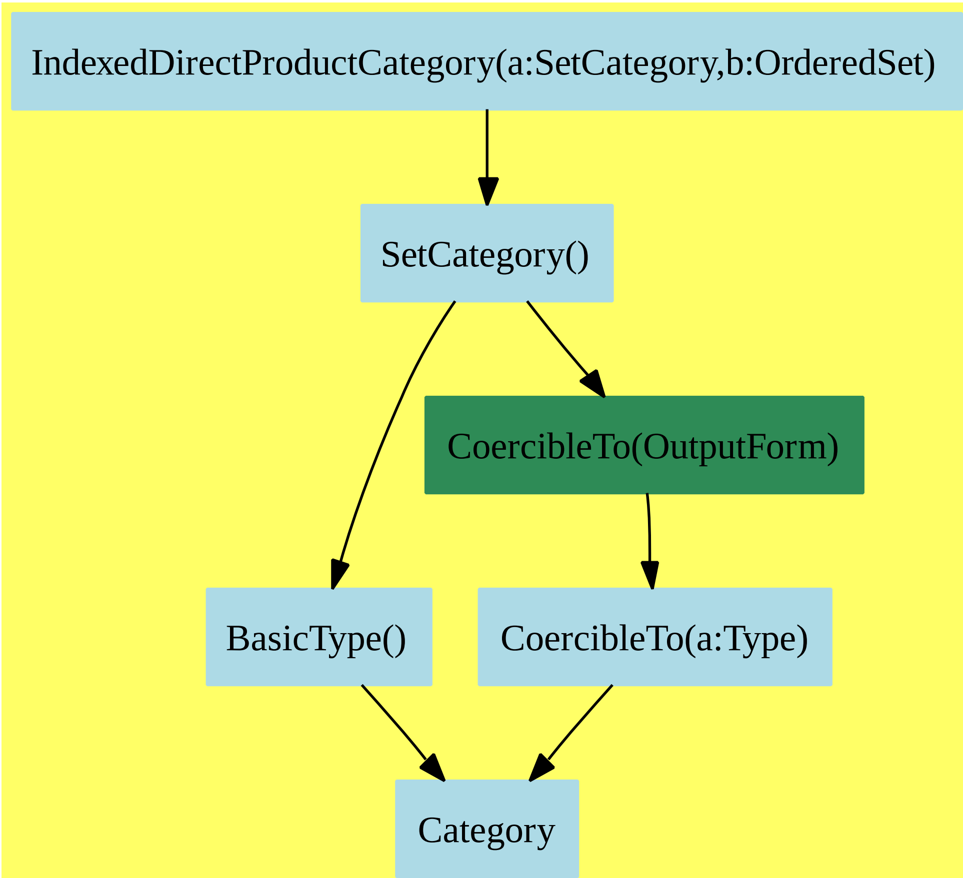
— HOAGG.dotfull —

```
"HomogeneousAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=HOAGG"];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"HomogeneousAggregate(Ring)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=HOAGG"];
"HomogeneousAggregate(Ring)"
  -> "HomogeneousAggregate(a:Type)"
```

— **HOAGG.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"
"HomogeneousAggregate(a:Type)" -> "Evalable(a:Type)"
"HomogeneousAggregate(a:Type)" -> "SetCategory()"

"Evalable(a:Type)" [color="#00EE00"];

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];

}
```

4.0.74 IndexedDirectProductCategory (IDPC)



— **IndexedDirectProductCategory.input** —

```
)set break resume
)sys rm -f IndexedDirectProductCategory.output
)spool IndexedDirectProductCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedDirectProductCategory
--R
--R IndexedDirectProductCategory(A: SetCategory,S: OrderedSet)
--R is a category constructor
--R Abbreviation for IndexedDirectProductCategory is IDPC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for IDPC
--R
--R------------------------------- Operations --------------------------------
```

```
--R ?=? : (%,%) -> Boolean              coerce : % -> OutputForm
--R hash : % -> SingleInteger           latex : % -> String
--R leadingCoefficient : % -> A         leadingSupport : % -> S
--R map : ((A -> A),%) -> %             monomial : (A,S) -> %
--R reductum : % -> %                   ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

— **IndexedDirectProductCategory.help** —

```
====================================================================
IndexedDirectProductCategory examples
====================================================================

This category represents the direct product of some set with respect
to an ordered indexing set.

See Also:
o )show IndexedDirectProductCategory
```

See:

⇐ "SetCategory" (SETCAT) 3.0.57 on page 187

Exports:

coerce	hash	latex	leadingCoefficient	leadingSupport
map	monomial	reductum	?=?	?~=?

These are directly exported but not implemented:

```
leadingCoefficient : % -> A
leadingSupport : % -> S
map : ((A -> A),%) -> %
monomial : (A,S) -> %
reductum : % -> %
```

These exports come from (p187) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

— **IndexedDirectProductCategory.html** —

```
<body>
<h2>
```

```
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#IDPC">
IndexedDirectProductCategory (IDPC)</a></h2>
&bull; A : SetCategory<br/>
&bull; S : OrderedSet<br/><br/>

This category represents the direct product of some set with
respect to an ordered indexing set.
<br/><br/><br/>

map : (A -> A, %) -> %<br/>
  map(f,z) returns the new element created by applying the<br/>
  function f to each component of the direct product element z.<br/><br/>

monomial : (A, S) -> %<br/>
  monomial(a,s) constructs a direct product element with the s<br/>
  component set to \spad{a}<br/><br/>

leadingCoefficient : % -> A<br/>
  leadingCoefficient(z) returns the coefficient of the leading<br/>
  (with respect to the ordering on the indexing set)<br/>
  monomial of z.<br/>
  Error: if z has no support.<br/><br/>

leadingSupport : % -> S<br/>
  leadingSupport(z) returns the index of leading<br/>
  (with respect to the ordering on the indexing set) monomial of z.<br/>
  Error: if z has no support.<br/><br/>

reductum : % -> %<br/>
  reductum(z) returns a new element created by removing the<br/>
  leading coefficient/support pair from the element z.<br/>
  Error: if z has no support.<br/><br/>

hash : % -> SingleInteger<br/>
  hash(s) calculates a hash code for s.<br/>
  from SetCategory<br/><br/>

latex : % -> String<br/>
  latex(s) returns a LaTeX-printable output representation of s.<br/>
  from SetCategory<br/><br/>

"=" : (%,%) -> Boolean<br/>
  x=y tests if x and y are equal.<br/>
  from BasicType<br/><br/>

"~=" : (%,%) -> Boolean<br/>
  x~=y tests if x and y are not equal.<br/>
  from BasicType<br/><br/>

coerce : % -> OutputForm<br/>
  coerce(a) transforms a into an element of OutputForm<br/>
  from CoercibleTo OutputForm<br/><br/>

&bull; CoercibleTo OutputForm<br/>
```

```
&bull; SetCategory<br/>
&bull; BasicType<br/>

</body>
```

— category IDPC IndexedDirectProductCategory —

```
)abbrev category IDPC IndexedDirectProductCategory
++ Author: James Davenport
++ Description:
++ This category represents the direct product of some set with
++ respect to an ordered indexing set.

IndexedDirectProductCategory(A,S) : Category == SIG where
  A : SetCategory
  S : OrderedSet

  SIG ==> SetCategory with

    map : (A -> A, %) -> %
      ++ map(f,z) returns the new element created by applying the
      ++ function f to each component of the direct product element z.

    monomial : (A, S) -> %
      ++ monomial(a,s) constructs a direct product element with the s
      ++ component set to \spad{a}

    leadingCoefficient : % -> A
      ++ leadingCoefficient(z) returns the coefficient of the leading
      ++ (with respect to the ordering on the indexing set)
      ++ monomial of z.
      ++ Error: if z has no support.

    leadingSupport : % -> S
      ++ leadingSupport(z) returns the index of leading
      ++ (with respect to the ordering on the indexing set) monomial of z.
      ++ Error: if z has no support.

    reductum : % -> %
      ++ reductum(z) returns a new element created by removing the
      ++ leading coefficient/support pair from the element z.
      ++ Error: if z has no support.
```

— IDPC.dotabb —

```
"IDPC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=IDPC"];
"IDPC" -> "SETCAT"
```

———-

— **IDPC.dotfull** —

```
"IndexedDirectProductCategory(a:SetCategory,b:OrderedSet)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=IDPC"];
"IndexedDirectProductCategory(a:SetCategory,b:OrderedSet)" ->
   "SetCategory()"
```

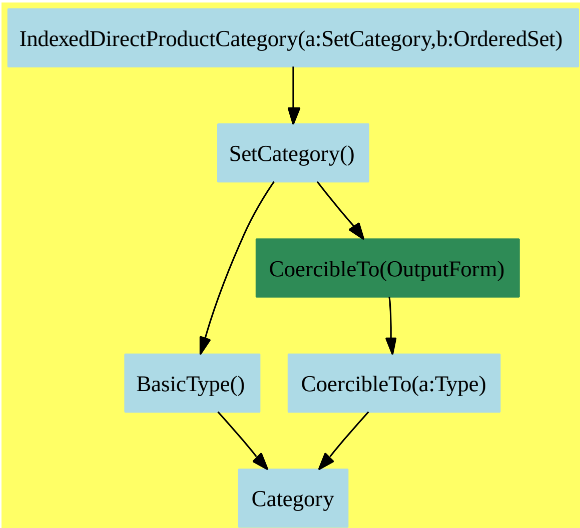
———-

— **IDPC.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"IndexedDirectProductCategory(a:SetCategory,b:OrderedSet)" [color=lightblue];
"IndexedDirectProductCategory(a:SetCategory,b:OrderedSet)" ->
   "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

———-

4.0.75 LiouvillianFunctionCategory (LFCAT)



— **LiouvillianFunctionCategory.input** —

```
)set break resume
)sys rm -f LiouvillianFunctionCategory.output
)spool LiouvillianFunctionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show LiouvillianFunctionCategory
--R
--R LiouvillianFunctionCategory is a category constructor
--R Abbreviation for LiouvillianFunctionCategory is LFCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for LFCAT
--R
--R------------------------------- Operations --------------------------------
--R ?**? : (%,%) -> %                     Ci : % -> %
--R Ei : % -> %                           Si : % -> %
--R acos : % -> %                         acosh : % -> %
--R acot : % -> %                         acoth : % -> %
--R acsc : % -> %                         acsch : % -> %
--R asec : % -> %                         asech : % -> %
--R asin : % -> %                         asinh : % -> %
--R atan : % -> %                         atanh : % -> %
--R cos : % -> %                          cosh : % -> %
--R cot : % -> %                          coth : % -> %
--R csc : % -> %                          csch : % -> %
--R dilog : % -> %                        erf : % -> %
--R exp : % -> %                          fresnelC : % -> %
```

```
--R fresnelS : % -> %                    integral : (%,Symbol) -> %
--R li : % -> %                          log : % -> %
--R pi : () -> %                         sec : % -> %
--R sech : % -> %                        sin : % -> %
--R sinh : % -> %                        tan : % -> %
--R tanh : % -> %
--R integral : (%,SegmentBinding(%)) -> %
--R
--E 1

)spool
)lisp (bye)
```

— **LiouvillianFunctionCategory.help** —

```
====================================================================
LiouvillianFunctionCategory examples
====================================================================

This is the Category for the transcendental Liouvillian functions.

See Also:
o )show LiouvillianFunctionCategory
```

See:

⇐ "PrimitiveFunctionCategory" (PRIMCAT) 2.0.35 on page 104
⇐ "TranscendentalFunctionCategory" (TRANFUN) 3.0.58 on page 192

Exports:

Ci	Ei	Si	acos	acosh
acot	acoth	acsc	acsch	asec
asech	asin	asinh	atan	atanh
cos	cosh	cot	coth	csc
csch	dilog	erf	exp	frsenelC
frsenelS	integral	li	log	pi
sec	sech	sin	sinh	tan
tanh	?**?			

These are directly exported but not implemented:

```
Ci : % -> %
dilog : % -> %
Ei : % -> %
erf : % -> %
li : % -> %
Si : % -> %
fresnelC : % -> %
fresnelS : % -> %
```

These exports come from (p104) PrimitiveFunctionCategory()

```
integral : (%,Symbol) -> %
integral : (%,SegmentBinding %) -> %
```

These exports come from (p192) TranscendentalFunctionCategory():

```
?**? : (%,%) -> %
acos : % -> %
acosh : % -> %
acot : % -> %
acoth : % -> %
acsc : % -> %
acsch : % -> %
asec : % -> %
asech : % -> %
asin : % -> %
asinh : % -> %
atan : % -> %
atanh : % -> %
cos : % -> %
cosh : % -> %
cot : % -> %
coth : % -> %
csc : % -> %
csch : % -> %
exp : % -> %
log : % -> %
pi : () -> %
sec : % -> %
sech : % -> %
sin : % -> %
sinh : % -> %
tan : % -> %
tanh : % -> %
```

— LiouvillianFunctionCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#LFCAT">
LiouvillianFunctionCategory (LFCAT)</a></h2>

Category for the transcendental Liouvillian functions.
<br/><br/><br/>

log : $ -> $<br/>
  log(x) returns the natural logarithm of x. When evaluated<br/>
  into some subset of the complex numbers, the branch cut lies<br/>
  along the negative real axis, continuous with quadrant II. The<br/>
  domain does not contain the origin.<br/>
  from ElementaryFunctionCategory<br/><br/>

exp : $ -> $<br/>
  exp(x) returns %e to the power x.<br/>
  from ElementaryFunctionCategory<br/><br/>
```

```
?**? : (%,%) -> %
  x**y returns x to the power y.<br/>
  from ElementaryFunctionCategory<br/><br/>

acosh : $ -> $<br/>
  acosh(x) returns the hyperbolic arc-cosine of x.<br/>
  from ArcHyperbolicFunctionCategory<br/><br/>

acoth : $ -> $<br/>
  acoth(x) returns the hyperbolic arc-cotangent of x.<br/>
  from ArcHyperbolicFunctionCategory<br/><br/>

acsch : $ -> $<br/>
  acsch(x) returns the hyperbolic arc-cosecant of x.<br/>
  from ArcHyperbolicFunctionCategory<br/><br/>

asech : $ -> $<br/>
  asech(x) returns the hyperbolic arc-secant of x.<br/>
  from ArcHyperbolicFunctionCategory<br/><br/>

asinh : $ -> $<br/>
  asinh(x) returns the hyperbolic arc-sine of x.<br/>
  from ArcHyperbolicFunctionCategory<br/><br/>

atanh : $ -> $<br/>
  atanh(x) returns the hyperbolic arc-tangent of x.<br/>
  from ArcHyperbolicFunctionCategory<br/><br/>

cosh : $ -> $<br/>
  cosh(x) returns the hyperbolic cosine of x.<br/>
  from HyperbolicFunctionCategory<br/><br/>

coth : $ -> $<br/>
  coth(x) returns the hyperbolic cotangent of x.<br/>
  from HyperbolicFunctionCategory<br/><br/>

csch : $ -> $<br/>
  csch(x) returns the hyperbolic cosecant of x.<br/>
  from HyperbolicFunctionCategory<br/><br/>

sech : $ -> $<br/>
  sech(x) returns the hyperbolic secant of x.<br/>
  from HyperbolicFunctionCategory<br/><br/>

sinh : $ -> $<br/>
  sinh(x) returns the hyperbolic sine of x.<br/>
  from HyperbolicFunctionCategory<br/><br/>

tanh : $ -> $<br/>
  tanh(x) returns the hyperbolic tangent of x.<br/>
  from HyperbolicFunctionCategory<br/><br/>

acos : $ -> $<br/>
```

```
  acos(x) returns the arc-cosine of x. When evaluated<br/>
  into some subset of the complex numbers, one<br/>
  branch cut for acos lies along the negative real axis<br/>
  to the left of -1 (inclusive), continuous with the<br/>
  upper half plane, the other along the positive real axis to<br/>
  the right of 1 (inclusive), continuous with the lower half plane.<br/>
  from ArcTrigonometricFunctionCategory<br/><br/>

acot : $ -> $<br/>
  acot(x) returns the arc-cotangent of x.<br/>
  from ArcTrigonometricFunctionCategory<br/><br/>

acsc : $ -> $<br/>
  acsc(x) returns the arc-cosecant of x.<br/>
  from ArcTrigonometricFunctionCategory<br/><br/>

asec : $ -> $<br/>
  asec(x) returns the arc-secant of x.<br/>
  from ArcTrigonometricFunctionCategory<br/><br/>

asin : $ -> $<br/>
  asin(x) returns the arc-sine of x. When evaluated into some<br/>
  subset of the complex numbers, one branch cut for asin lies<br/>
  along the negative real axis to the left of -1 (inclusive),<br/>
  continuous with the upper half plane, the other along the<br/>
  positive real axis to the right of 1 (inclusive), continuous<br/>
  with the lower half plane.<br/>
  from ArcTrigonometricFunctionCategory<br/><br/>

atan : $ -> $<br/>
  atan(x) returns the arc-tangent of x. When evaluated into some<br/>
  subset of the complex numbers, one branch cut for atan lies<br/>
  along the positive imaginary axis above %i (exclusive),<br/>
  continuous with the left half plane, the other along the<br/>
  negative imaginary axis below -%i (exclusive) continuous<br/>
  with the right half plane. The domain does not contain %i and -%i<br/>
  from ArcTrigonometricFunctionCategory<br/><br/>

cos : $ -> $<br/>
  cos(x) returns the cosine of x.<br/>
  from TrigonometricFunctionCategory<br/><br/>

cot : $ -> $<br/>
  cot(x) returns the cotangent of x.<br/>
  from TrigonometricFunctionCategory<br/><br/>

csc : $ -> $<br/>
  csc(x) returns the cosecant of x.<br/>
  from TrigonometricFunctionCategory<br/><br/>

sec : $ -> $<br/>
  sec(x) returns the secant of x.<br/>
  from TrigonometricFunctionCategory<br/><br/>
```

```
sin : $ -> $<br/>
  sin(x) returns the sine of x.<br/>
  from TrigonometricFunctionCategory<br/><br/>

tan : $ -> $<br/>
  tan(x) returns the tangent of x.<br/>
  from TrigonometricFunctionCategory<br/><br/>

pi : () -> $<br/>
  pi() returns the constant pi.<br/>
  from TranscendentalFunctionCategory<br/><br/>

integral : ($, Symbol) -> $<br/>
  integral(f, x) returns the formal integral of f dx.<br/>
  from PrimitiveFunctionCategory<br/><br/>

integral : ($, SegmentBinding $) -> $<br/>
  integral(f, x = a..b) returns the formal definite integral<br/>
  of f dx for x between a and b.<br/>
  from PrimitiveFunctionCategory<br/><br/>

Ei : $  -> $<br/>
  Ei(x) returns the exponential integral of x, that is,<br/>
  the integral of exp(x)/x dx.<br/><br/>

Si : $  -> $<br/>
  Si(x) returns the sine integral of x, that is,<br/>
  the integral of sin(x) / x dx.<br/><br/>

Ci : $  -> $<br/>
  Ci(x) returns the cosine integral of x, that is,<br/>
  the integral of cos(x) / x dx.<br/><br/>

li : $  -> $<br/>
  li(x) returns the logarithmic integral of x, that is,<br/>
  the integral of dx / log(x).<br/><br/>

dilog : $  -> $<br/>
  dilog(x) returns the dilogarithm of x, that is,<br/>
  the integral of log(x) / (1 - x) dx.<br/><br/>

erf : $  -> $<br/>
  erf(x) returns the error function of x, that is,<br/>
  2 / sqrt(%pi) times the integral of exp(-x**2) dx.<br/><br/>

fresnelS : $ -> $<br/>
  fresnelS(x) is the Fresnel integral S, defined by<br/>
  S(x) = integrate(sin(t^2),t=0..x)<br/><br/>

fresnelC : $ -> $<br/>
  fresnelC(x) is the Fresnel integral C, defined by<br/>
  C(x) = integrate(cos(t^2),t=0..x)<br/><br/>

&bull; PrimitiveFunctionCategory<br/>
```

```
&bull; TranscendentalFunctionCategory<br/>
&bull; TrigonometricFunctionCategory
&bull; ArcTrigonometricFunctionCategory
&bull; HyperbolicFunctionCategory
&bull; ArcHyperbolicFunctionCategory
&bull; ElementaryFunctionCategory

</body>
```

———-

— category LFCAT LiouvillianFunctionCategory —

```
)abbrev category LFCAT LiouvillianFunctionCategory
++ Category for the transcendental Liouvillian functions
++ Author: Manuel Bronstein
++ Date Last Updated: 14 May 1991
++ Description:
++ Category for the transcendental Liouvillian functions;

LiouvillianFunctionCategory() : Category == SIG where

  SIG ==> Join(PrimitiveFunctionCategory, TranscendentalFunctionCategory) with

    Ei : $  -> $
      ++ Ei(x) returns the exponential integral of x, that is,
      ++ the integral of \spad{exp(x)/x dx}.

    Si : $  -> $
      ++ Si(x) returns the sine integral of x, that is,
      ++ the integral of \spad{sin(x) / x dx}.

    Ci : $  -> $
      ++ Ci(x) returns the cosine integral of x, that is,
      ++ the integral of \spad{cos(x) / x dx}.

    li : $  -> $
      ++ li(x) returns the logarithmic integral of x, that is,
      ++ the integral of \spad{dx / log(x)}.

    dilog : $  -> $
      ++ dilog(x) returns the dilogarithm of x, that is,
      ++ the integral of \spad{log(x) / (1 - x) dx}.

    erf : $  -> $
      ++ erf(x) returns the error function of x, that is,
      ++ \spad{2 / sqrt(%pi)} times the integral of \spad{exp(-x**2) dx}.

    fresnelS : $ -> $
      ++ fresnelS(x) is the Fresnel integral S, defined by
      ++ S(x) = integrate(sin(t^2),t=0..x)

    fresnelC : $ -> $
      ++ fresnelC(x) is the Fresnel integral C, defined by
```

```
++ C(x) = integrate(cos(t^2),t=0..x)
```

———-

— **LFCAT.dotabb** —

```
"LFCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LFCAT"];
"LFCAT" -> "PRIMCAT"
"LFCAT" -> "TRANFUN"
```

———-

— **LFCAT.dotfull** —

```
"LiouvillianFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LFCAT"];
"LiouvillianFunctionCategory()" -> "PrimitiveFunctionCategory()"
"LiouvillianFunctionCategory()" -> "TranscendentalFunctionCategory()"
```

———-

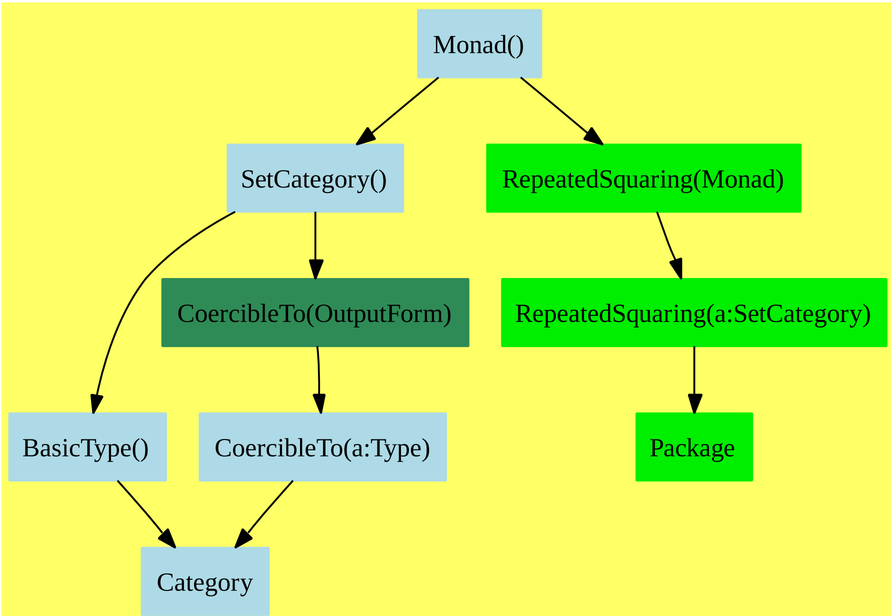
— **LFCAT.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"LiouvillianFunctionCategory()" [color=lightblue];
"LiouvillianFunctionCategory()" -> "PrimitiveFunctionCategory()"
"LiouvillianFunctionCategory()" -> "TranscendentalFunctionCategory()"

"PrimitiveFunctionCategory()" [color=lightblue];
"PrimitiveFunctionCategory()" -> "Category"

"TranscendentalFunctionCategory()" [color=lightblue];
"TranscendentalFunctionCategory()" ->
    "TRIGCAT"
"TranscendentalFunctionCategory()" ->
    "ATRIG"
"TranscendentalFunctionCategory()" ->
    "HYPCAT"
"TranscendentalFunctionCategory()" ->
    "AHYP"
"TranscendentalFunctionCategory()" ->
    "ELEMFUN"

"TRIGCAT" [color=lightblue];
"TRIGCAT" -> "Category"
```

```
"ATRIG" [color=lightblue];
"ATRIG" -> "Category"

"HYPCAT" [color=lightblue];
"HYPCAT" -> "Category"

"AHYP" [color=lightblue];
"AHYP" -> "Category"

"ELEMFUN" [color=lightblue];
"ELEMFUN" -> "Category"

"Category" [color=lightblue];
}
```

——–

4.0.76 Monad (MONAD)



— **Monad.input** —

```
)set break resume
)sys rm -f Monad.output
)spool Monad.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
```

```
)show Monad
--R
--R Monad is a category constructor
--R Abbreviation for Monad is MONAD
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for MONAD
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?**? : (%,PositiveInteger) -> %
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R leftPower : (%,PositiveInteger) -> %
--R rightPower : (%,PositiveInteger) -> %
--R
--E 1

)spool
)lisp (bye)
```

— **Monad.help** —

```
====================================================================
Monad examples
====================================================================

Monad is the class of all multiplicative monads, that is, sets
with a binary operation.

See Also:
o )show Monad
```

See:

⇒ "MonadWithUnit" (MONADWU) 5.0.99 on page 449
⇒ "NonAssociativeRng" (NARNG) 8.0.140 on page 849

Exports:

coerce	hash	latex	leftPower	rightPower
?**?	?*?	?=?	?~=?	

These are directly exported but not implemented:

```
?*? : (%,%) -> %
```

These are implemented by this category:

```
leftPower : (%,PositiveInteger) -> %
rightPower : (%,PositiveInteger) -> %
?**? : (%,PositiveInteger) -> %
```

These exports come from (p187) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

See: Jacobson [Jaco68], Jacobson [Jaco51]

— Monad.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#MONAD">
Monad (MONAD)</a></h2>

Monad is the class of all multiplicative monads, that is sets
with a binary operation.
<br/><br/><br/>

?*? : (%,%) -> %
  a*b is the product of a and b in a set with<br/>
  a binary operation.<br/><br/>

rightPower : (%,PositiveInteger) -> %<br/>
  rightPower(a,n) returns the n-th right power of a,<br/>
  that is, rightPower(a,n) := rightPower(a,n-1) * a and<br/>
  rightPower(a,1) := a.<br/><br/>

leftPower : (%,PositiveInteger) -> %<br/>
  leftPower(a,n) returns the n-th left power of a,<br/>
  that is, leftPower(a,n) := a * leftPower(a,n-1) and<br/>
  leftPower(a,1) := a.<br/><br/>

?**? : (%,PositiveInteger) -> %<br/>
  a**n returns the n-th power of a,<br/>
  defined by repeated squaring.<br/><br/>

hash : % -> SingleInteger<br/>
  hash(s) calculates a hash code for s.<br/>
  from SetCategory<br/><br/>

latex : % -> String<br/>
  latex(s) returns a LaTeX-printable output representation of s.<br/>
  from SetCategory<br/><br/>

?=? : (%,%) -> Boolean<br/>
  x=y tests if x and y are equal.<br/>
  from BasicType<br/><br/>

?~=? : (%,%) -> Boolean<br/>
  x~=y tests if x and y are not equal.<br/>
  from BasicType<br/><br/>

coerce : % -> OutputForm<br/>
  coerce(a) transforms a into an element of OutputForm<br/>
```

```
  from CoercibleTo OutputForm<br/><br/>

&bull; CoercibleTo OutputForm<br/>
&bull; SetCategory<br/>
&bull; BasicType<br/>

</body>
```

— category MONAD Monad —

```
)abbrev category MONAD Monad
++ Authors: J. Grabmeier, R. Wisbauer
++ Date Created: 01 March 1991
++ Date Last Updated: 11 June 1991
++ Reference:
++ Jaco68 Structure and Representations of Jordan Algebras
++ Jaco51 General Representation Theory of Jordan Algebras
++ Description:
++ Monad is the class of all multiplicative monads, that is sets
++ with a binary operation.

Monad() : Category == SIG where

  SIG ==> SetCategory with

    "*" : (%,%) -> %
      ++ a*b is the product of \spad{a} and b in a set with
      ++ a binary operation.

    rightPower : (%,PositiveInteger) -> %
      ++ rightPower(a,n) returns the \spad{n}-th right power of \spad{a},
      ++ that is, \spad{rightPower(a,n) := rightPower(a,n-1) * a} and
      ++ \spad{rightPower(a,1) := a}.

    leftPower : (%,PositiveInteger) -> %
      ++ leftPower(a,n) returns the \spad{n}-th left power of \spad{a},
      ++ that is, \spad{leftPower(a,n) := a * leftPower(a,n-1)} and
      ++ \spad{leftPower(a,1) := a}.

    "**" : (%,PositiveInteger) -> %
      ++ a**n returns the \spad{n}-th power of \spad{a},
      ++ defined by repeated squaring.

   add

      import RepeatedSquaring(%)

      x:% ** n:PositiveInteger == expt(x,n)

      rightPower(a,n) ==
        (n = 1) => a
        res := a
```

```
        for i in 1..(n-1) repeat res := res * a
        res

      leftPower(a,n) ==
        (n = 1) => a
        res := a
        for i in 1..(n-1) repeat res := a * res
        res
```

———-

— COQ MONAD —

```
(* category MONAD *)
(*

      import RepeatedSquaring(%)

      "**": (%,PositiveInteger) -> %
      x:% ** n:PositiveInteger == expt(x,n)

      rightPower: (%,PositiveInteger) -> %
      rightPower(a,n) ==
        (n = 1) => a
        res := a
        for i in 1..(n-1) repeat res := res * a
        res

      leftPower: (%,PositiveInteger) -> %
      leftPower(a,n) ==
        (n = 1) => a
        res := a
        for i in 1..(n-1) repeat res := a * res
        res

*)
```

———-

— MONAD.dotabb —

```
"MONAD"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MONAD"];
"MONAD" -> "SETCAT"
```

———-

— MONAD.dotfull —

```
"Monad()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MONAD"];
```

```
"Monad()" -> "SetCategory()"
"Monad()" -> "RepeatedSquaring(Monad)"
```

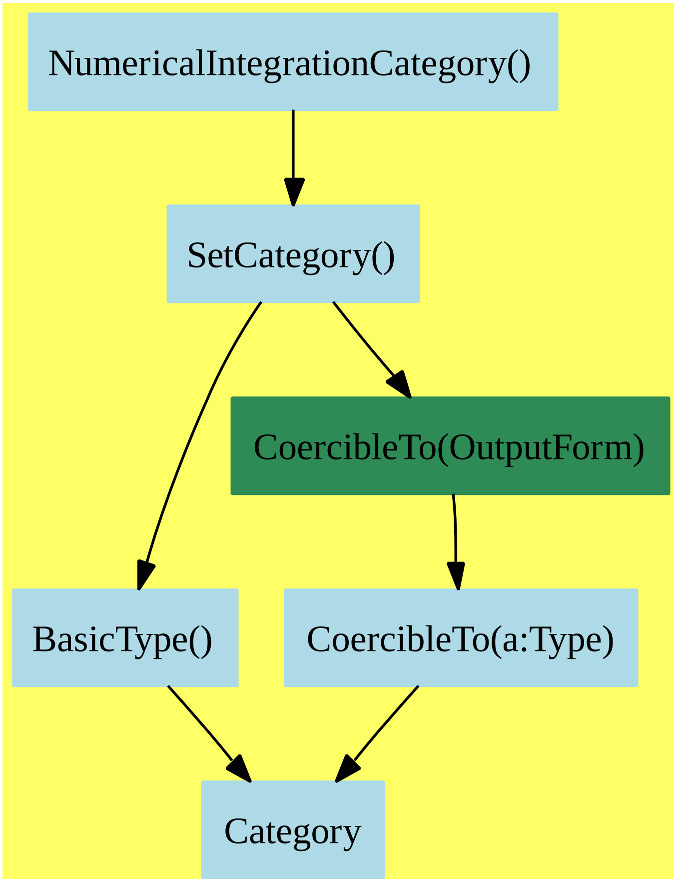
———-

— **MONAD.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"Monad()" [color=lightblue];
"Monad()" -> "SetCategory()"
"Monad()" -> "RepeatedSquaring(Monad)"

"RepeatedSquaring(Monad)" [color="#00EE00"];
"RepeatedSquaring(Monad)" -> "RepeatedSquaring(a:SetCategory)"

"RepeatedSquaring(a:SetCategory)" [color="#00EE00"];
"RepeatedSquaring(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

———-

4.0.77 NumericalIntegrationCategory (NUMINT)



— **NumericalIntegrationCategory.input** —

```
)set break resume
)sys rm -f NumericalIntegrationCategory.output
)spool NumericalIntegrationCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NumericalIntegrationCategory
--R
--R NumericalIntegrationCategory is a category constructor
--R Abbreviation for NumericalIntegrationCategory is NUMINT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for NUMINT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
```

```
--R hash : % -> SingleInteger              latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R measure : (RoutinesTable,Record(fn: Expression(DoubleFloat),range: List(Segment(OrderedCompletion(DoubleFloa
--R measure : (RoutinesTable,Record(var: Symbol,fn: Expression(DoubleFloat),range: Segment(OrderedCompletion(Dou
--R numericalIntegration : (Record(fn: Expression(DoubleFloat),range: List(Segment(OrderedCompletion(DoubleFloat
--R numericalIntegration : (Record(var: Symbol,fn: Expression(DoubleFloat),range: Segment(OrderedCompletion(Doub
--R
--E 1

)spool
)lisp (bye)
```

— **NumericalIntegrationCategory.help** —

```
====================================================================
NumericalIntegrationCategory examples
====================================================================

NumericalIntegrationCategory is the category for describing the set of
Numerical Integration domains with measure and numericalIntegration.

See Also:
o )show NumericalIntegrationCategory
```

See:

⇐ "SetCategory" (SETCAT) 3.0.57 on page 187

Exports:

coerce	hash	latex	measure
numericalIntegration	?=?	?~=?	

These are directly exported but not implemented:

```
measure :
 (RoutinesTable,
  Record(fn: Expression DoubleFloat,
         range: List Segment OrderedCompletion DoubleFloat,
         abserr: DoubleFloat,
         relerr: DoubleFloat)) ->
    Record(measure: Float,explanations: String,extra: Result)
measure :
 (RoutinesTable,
  Record(var: Symbol,
         fn: Expression DoubleFloat,
         range: Segment OrderedCompletion DoubleFloat,
         abserr: DoubleFloat,
         relerr: DoubleFloat)) ->
   Record(measure: Float,explanations: String,extra: Result)
numericalIntegration :
```

```
  (Record(fn: Expression DoubleFloat,
          range: List Segment OrderedCompletion DoubleFloat,
          abserr: DoubleFloat,relerr: DoubleFloat),
   Result) ->
  Result
 numericalIntegration :
  (Record(var: Symbol,
          fn: Expression DoubleFloat,
          range: Segment OrderedCompletion DoubleFloat,
          abserr: DoubleFloat,
          relerr: DoubleFloat),
   Result) ->
  Result
```

These exports come from (p187) SetCategory():

```
 coerce : % -> OutputForm
 hash : % -> SingleInteger
 latex : % -> String
 ?=? : (%,%) -> Boolean
 ?~=? : (%,%) -> Boolean
```

— NumericalIntegrationCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#NUMINT">
NumericalIntegrationCategory (NUMINT)</a></h2>

NumericalIntegrationCategory is the category for
describing the set of Numerical Integration domains with
measure and numericalIntegration.
<br/><br/><br/>

measure : (RoutinesTable,NIAE) ->
 Record(measure:Float,explanations:String,extra:Result)<br/>
  measure(R,args) calculates an estimate of the ability of a<br/>
  particular method to solve a problem.  <br/>
 <br/>
  This method may be either a specific NAG routine or a strategy<br/>
  (such as transforming the function from one which is difficult<br/>
  to one which is easier to solve).<br/>
 <br/>
  It will call whichever agents are needed to perform analysis on the<br/>
  problem in order to calculate the measure. There is a parameter,<br/>
  labelled sofar, which would contain the best compatibility<br/>
  found so far.<br/><br/>

numericalIntegration : (NIAE, Result) -> Result<br/>
  numericalIntegration(args,hints) performs the integration of the<br/>
  function given the strategy or method returned by measure.<br/><br/>

measure : (RoutinesTable,MDNIAE) ->
 Record(measure:Float,explanations:String,extra:Result)<br/>
  measure(R,args) calculates an estimate of the ability of a<br/>
```

```
  particular method to solve a problem.  <br/>
 <br/>
  This method may be either a specific NAG routine or a strategy<br/>
  (such as transforming the function from one which is difficult<br/>
  to one which is easier to solve).<br/>
 <br/>
  It will call whichever agents are needed to perform analysis on the<br/>
  problem in order to calculate the measure. There is a parameter,<br/>
  labelled sofar, which would contain the best compatibility<br/>
  found so far.<br/><br/>

numericalIntegration : (MDNIAE, Result) -> Result<br/>
  numericalIntegration(args,hints) performs the integration of the<br/>
  function given the strategy or method returned by measure.<br/><br/>

hash : % -> SingleInteger<br/>
  hash(s) calculates a hash code for s.<br/>
  from SetCategory<br/><br/>

latex : % -> String<br/>
  latex(s) returns a LaTeX-printable output representation of s.<br/>
  from SetCategory<br/><br/>

?=? : (%,%) -> Boolean<br/>
  x=y tests if x and y are equal.<br/>
  from BasicType<br/><br/>

?~=? : (%,%) -> Boolean<br/>
  x~=y tests if x and y are not equal.<br/>
  from BasicType<br/><br/>

coerce : % -> OutputForm<br/>
  coerce(a) transforms a into an element of OutputForm<br/>
  from CoercibleTo OutputForm<br/><br/>

&bull; CoercibleTo OutputForm<br/>
&bull; SetCategory<br/>
&bull; BasicType<br/>

</body>
```

— category NUMINT NumericalIntegrationCategory —

```
)abbrev category NUMINT NumericalIntegrationCategory
++ Author: Brian Dupee
++ Date Created: February 1994
++ Date Last Updated: March 1996
++ Description:
++ \axiomType{NumericalIntegrationCategory} is the \axiom{category} for
++ describing the set of Numerical Integration \axiom{domains} with
++ \axiomFun{measure} and \axiomFun{numericalIntegration}.
```

```
NumericalIntegrationCategory() : Category == SIG where

  EDFE   ==> Expression DoubleFloat
  SOCDFE ==> Segment OrderedCompletion DoubleFloat
  DFE    ==> DoubleFloat
  NIAE   ==> Record(var:Symbol,fn:EDFE,range:SOCDFE,abserr:DFE,relerr:DFE)
  MDNIAE ==> Record(fn:EDFE,range:List SOCDFE,abserr:DFE,relerr:DFE)

  SIG ==> SetCategory with

    measure : (RoutinesTable,NIAE) -> _
     Record(measure:Float,explanations:String,extra:Result)
      ++ measure(R,args) calculates an estimate of the ability of a particular
      ++ method to solve a problem.
      ++
      ++ This method may be either a specific NAG routine or a strategy (such
      ++ as transforming the function from one which is difficult to one which
      ++ is easier to solve).
      ++
      ++ It will call whichever agents are needed to perform analysis on the
      ++ problem in order to calculate the measure. There is a parameter,
      ++ labelled \axiom{sofar}, which would contain the best compatibility
      ++ found so far.

    numericalIntegration : (NIAE, Result) -> Result
      ++ numericalIntegration(args,hints) performs the integration of the
      ++ function given the strategy or method returned by \axiomFun{measure}.

    measure : (RoutinesTable,MDNIAE) -> _
     Record(measure:Float,explanations:String,extra:Result)
      ++ measure(R,args) calculates an estimate of the ability of a particular
      ++ method to solve a problem.
      ++
      ++ This method may be either a specific NAG routine or a strategy (such
      ++ as transforming the function from one which is difficult to one which
      ++ is easier to solve).
      ++
      ++ It will call whichever agents are needed to perform analysis on the
      ++ problem in order to calculate the measure. There is a parameter,
      ++ labelled \axiom{sofar}, which would contain the best compatibility
      ++ found so far.

    numericalIntegration : (MDNIAE, Result) -> Result
      ++ numericalIntegration(args,hints) performs the integration of the
      ++ function given the strategy or method returned by \axiomFun{measure}.
```

— **NUMINT.dotabb** —

```
"NUMINT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NUMINT"];
```

```
"NUMINT" -> "SETCAT"
```

———-

— NUMINT.dotfull —

```
"NumericalIntegrationCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NUMINT"];
"NumericalIntegrationCategory()" -> "SetCategory()"
```

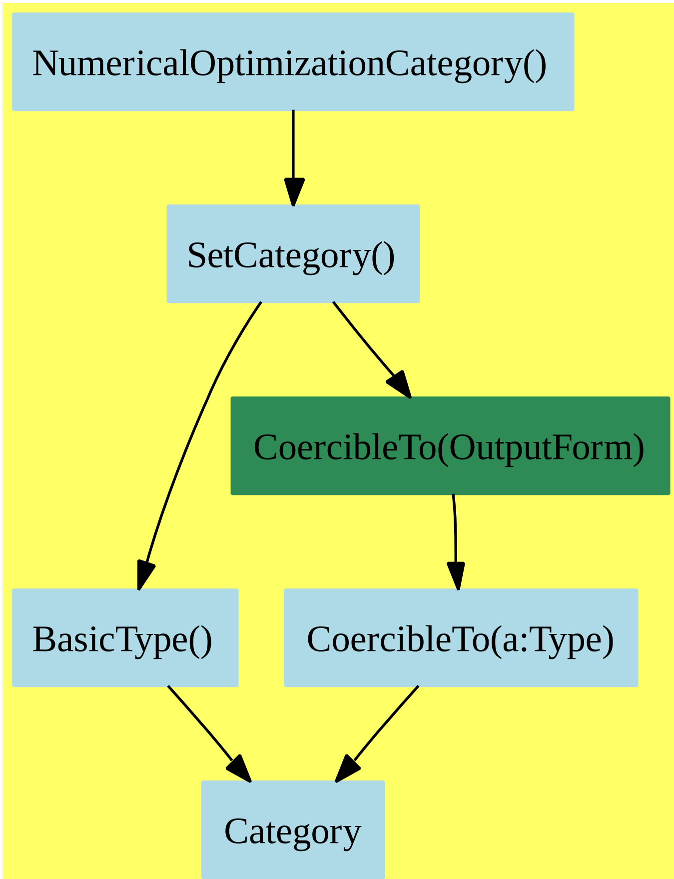
———-

— NUMINT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"NumericalIntegrationCategory()" [color=lightblue];
"NumericalIntegrationCategory()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

———-

4.0.78 NumericalOptimizationCategory (OPTCAT)



— **NumericalOptimizationCategory.input** —

```
)set break resume
)sys rm -f NumericalOptimizationCategory.output
)spool NumericalOptimizationCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NumericalOptimizationCategory
--R
--R NumericalOptimizationCategory is a category constructor
--R Abbreviation for NumericalOptimizationCategory is OPTCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for OPTCAT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
```

```
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R measure : (RoutinesTable,Record(lfn: List(Expression(DoubleFloat)),init: List(DoubleFloat))) -> Record(measu
--R measure : (RoutinesTable,Record(fn: Expression(DoubleFloat),init: List(DoubleFloat),lb: List(OrderedCompleti
--R numericalOptimization : Record(fn: Expression(DoubleFloat),init: List(DoubleFloat),lb: List(OrderedCompletio
--R numericalOptimization : Record(lfn: List(Expression(DoubleFloat)),init: List(DoubleFloat)) -> Result
--R
--E 1

)spool
)lisp (bye)
```

— **NumericalOptimizationCategory.help** —

```
====================================================================
NumericalOptimizationCategory examples
====================================================================

NumericalOptimizationCategory is the category for describing the set of
Numerical Optimization domains with measure and optimize.

See Also:
o )show NumericalOptimizationCategory
```

See:

⇐ "SetCategory" (SETCAT) 3.0.57 on page 187

Exports:

coerce	hash	latex	measure
numericalOptimization	?=?	?~=?	

These are directly exported but not implemented:

```
measure :
 (RoutinesTable,
  Record(lfn: List Expression DoubleFloat,
         init: List DoubleFloat)) ->
   Record(measure: Float,explanations: String)
measure :
 (RoutinesTable,
  Record(fn: Expression DoubleFloat,
         init: List DoubleFloat,
         lb: List OrderedCompletion DoubleFloat,
         cf: List Expression DoubleFloat,
         ub: List OrderedCompletion DoubleFloat)) ->
   Record(measure: Float,explanations: String)
numericalOptimization :
 Record(fn: Expression DoubleFloat,
        init: List DoubleFloat,
```

```
        lb: List OrderedCompletion DoubleFloat,
        cf: List Expression DoubleFloat,
        ub: List OrderedCompletion DoubleFloat) ->
  Result
 numericalOptimization :
  Record(lfn: List Expression DoubleFloat,
        init: List DoubleFloat) ->
  Result
```

These exports come from (p187) SetCategory():

```
 coerce : % -> OutputForm
 hash : % -> SingleInteger
 latex : % -> String
 ?=? : (%,%) -> Boolean
 ?~=? : (%,%) -> Boolean
```

— NumericalOptimizationCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#OPTCAT">
NumericalOptimizationCategory (OPTCAT)</a></h2>

NumericalOptimizationCategory is the category for
describing the set of Numerical Optimization domains with
measure and optimize.
<br/><br/><br/>

measure : (RoutinesTable,NOAH) ->
 Record(measure:Float,explanations:String)<br/>
  measure(R,args) calculates an estimate of the ability of a<br/>
  particular method to solve an optimization problem.  <br/>
 <br/>
  This method may be either a specific NAG routine or a strategy<br/>
  (such as transforming the function from one which is difficult<br/>
  to one which is easier to solve).<br/>
 <br/>
  It will call whichever agents are needed to perform analysis on the<br/>
  problem in order to calculate the measure. There is a parameter,<br/>
  labelled sofar, which would contain the best compatibility<br/>
  found so far.<br/><br/>

measure : (RoutinesTable,LSAH) ->
 Record(measure:Float,explanations:String)<br/>
  measure(R,args) calculates an estimate of the ability of a<br/>
  particular method to solve an optimization problem.  <br/>
 <br/>
  This method may be either a specific NAG routine or a strategy<br/>
  (such as transforming the function from one which is difficult<br/>
  to one which is easier to solve).<br/>
 <br/>
  It will call whichever agents are needed to perform analysis on the<br/>
  problem in order to calculate the measure. There is a parameter,<br/>
  labelled sofar, which would contain the best compatibility<br/>
```

```
  found so far.<br/><br/>

numericalOptimization : LSAH -> Result<br/>
  numericalOptimization(args) performs the optimization of the<br/>
  function given the strategy or method returned by measure.<br/><br/>

numericalOptimization : NOAH -> Result<br/>
  numericalOptimization(args) performs the optimization of the<br/>
  function given the strategy or method returned by measure.<br/><br/>

hash : % -> SingleInteger<br/>
  hash(s) calculates a hash code for s.<br/>
  from SetCategory<br/><br/>

latex : % -> String<br/>
  latex(s) returns a LaTeX-printable output representation of s.<br/>
  from SetCategory<br/><br/>

?=? : (%,%) -> Boolean<br/>
  x=y tests if x and y are equal.<br/>
  from BasicType<br/><br/>

?~=? : (%,%) -> Boolean<br/>
  x~=y tests if x and y are not equal.<br/>
  from BasicType<br/><br/>

coerce : % -> OutputForm<br/>
  coerce(a) transforms a into an element of OutputForm<br/>
  from CoercibleTo OutputForm<br/><br/>

&bull; CoercibleTo OutputForm<br/>
&bull; SetCategory<br/>
&bull; BasicType<br/>

</body>
```

— category OPTCAT NumericalOptimizationCategory —

```
)abbrev category OPTCAT NumericalOptimizationCategory
++ Author: Brian Dupee
++ Date Created: January 1996
++ Date Last Updated: March 1996
++ Description:
++ \axiomType{NumericalOptimizationCategory} is the \axiom{category} for
++ describing the set of Numerical Optimization \axiom{domains} with
++ \axiomFun{measure} and \axiomFun{optimize}.

NumericalOptimizationCategory() : Category == SIG where

  LDFH   ==> List DoubleFloat
  LEDFH  ==> List Expression DoubleFloat
  LSAH   ==> Record(lfn:LEDFH, init:LDFH)
```

```
  EDFH   ==> Expression DoubleFloat
  LOCDFH ==> List OrderedCompletion DoubleFloat
  NOAH   ==> Record(fn:EDFH, init:LDFH, lb:LOCDFH, cf:LEDFH, ub:LOCDFH)

  SIG ==> SetCategory with

    measure : (RoutinesTable,NOAH) -> Record(measure:Float,explanations:String)
      ++ measure(R,args) calculates an estimate of the ability of a particular
      ++ method to solve an optimization problem.
      ++
      ++ This method may be either a specific NAG routine or a strategy (such
      ++ as transforming the function from one which is difficult to one which
      ++ is easier to solve).
      ++
      ++ It will call whichever agents are needed to perform analysis on the
      ++ problem in order to calculate the measure. There is a parameter,
      ++ labelled \axiom{sofar}, which would contain the best compatibility
      ++ found so far.

    measure : (RoutinesTable,LSAH) -> Record(measure:Float,explanations:String)
      ++ measure(R,args) calculates an estimate of the ability of a particular
      ++ method to solve an optimization problem.
      ++
      ++ This method may be either a specific NAG routine or a strategy (such
      ++ as transforming the function from one which is difficult to one which
      ++ is easier to solve).
      ++
      ++ It will call whichever agents are needed to perform analysis on the
      ++ problem in order to calculate the measure. There is a parameter,
      ++ labelled \axiom{sofar}, which would contain the best compatibility
      ++ found so far.

    numericalOptimization : LSAH -> Result
      ++ numericalOptimization(args) performs the optimization of the
      ++ function given the strategy or method returned by \axiomFun{measure}.

    numericalOptimization : NOAH -> Result
      ++ numericalOptimization(args) performs the optimization of the
      ++ function given the strategy or method returned by \axiomFun{measure}.
```

——–

— **OPTCAT.dotabb** —

```
"OPTCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OPTCAT"];
"OPTCAT" -> "SETCAT"
```

——–

— **OPTCAT.dotfull** —

```
"NumericalOptimizationCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OPTCAT"];
"NumericalOptimizationCategory()" -> "SetCategory()"
```

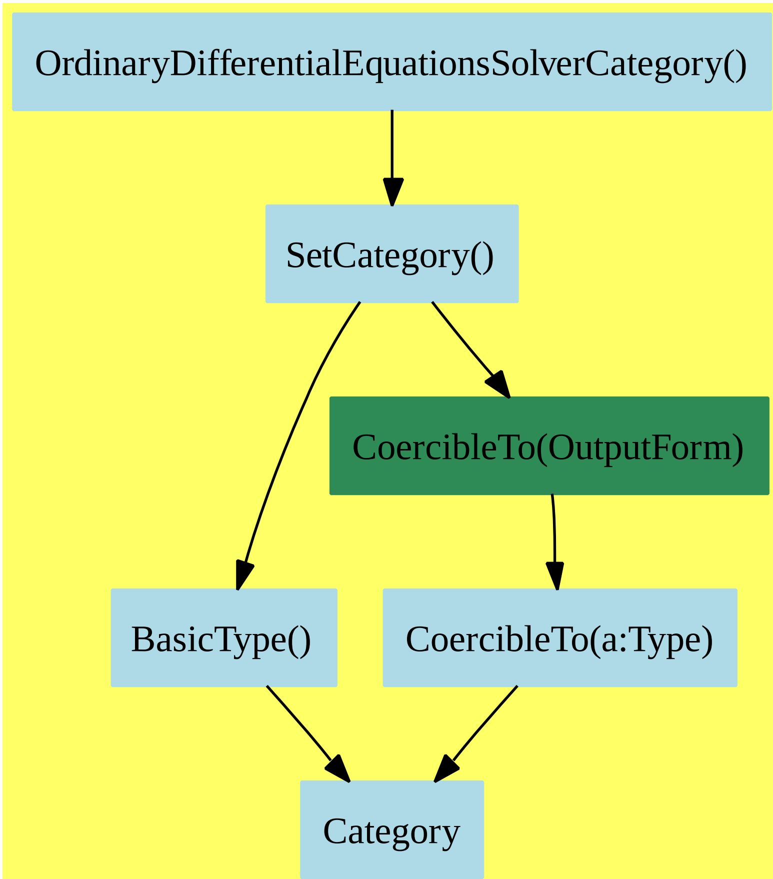
———-

— **OPTCAT.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"NumericalOptimizationCategory()" [color=lightblue];
"NumericalOptimizationCategory()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

———-

4.0.79 OrdinaryDifferentialEquationsSolverCategory (ODECAT)



— **OrdinaryDifferentialEquationsSolverCategory.input** —

```
)set break resume
)sys rm -f OrdinaryDifferentialEquationsSolverCategory.output
)spool OrdinaryDifferentialEquationsSolverCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrdinaryDifferentialEquationsSolverCategory
--R
--R OrdinaryDifferentialEquationsSolverCategory is a category constructor
--R Abbreviation for OrdinaryDifferentialEquationsSolverCategory is ODECAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ODECAT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
```

```
--R hash : % -> SingleInteger              latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R ODESolve : Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector(Expression(DoubleFloat)),yinit: List(Doubl
--R measure : (RoutinesTable,Record(xinit: DoubleFloat,xend: DoubleFloat,fn: Vector(Expression(DoubleFloat)),yin
--R
--E 1

)spool
)lisp (bye)
```

———

— OrdinaryDifferentialEquationsSolverCategory.help —

```
====================================================================
OrdinaryDifferentialEquationsSolverCategory examples
====================================================================

OrdinaryDifferentialEquationsSolverCategory is the category for describing
the set of ODE solver domains with measure and ODEsolve.

See Also:
o )show OrdinaryDifferentialEquationsSolverCategory
```

———

See:

⇐ "SetCategory" (SETCAT) 3.0.57 on page 187

Exports:

coerce	hash	latex	measure	ODESolve
?=?	?~=?			

These are directly exported but not implemented:

```
 ODESolve :
  Record(xinit: DoubleFloat,
         xend: DoubleFloat,
         fn: Vector Expression DoubleFloat,
         yinit: List DoubleFloat,
         intvals: List DoubleFloat,
         g: Expression DoubleFloat,
         abserr: DoubleFloat,
         relerr: DoubleFloat) ->
   Result
 measure :
  (RoutinesTable,
   Record(xinit: DoubleFloat,
          xend: DoubleFloat,
          fn: Vector Expression DoubleFloat,
          yinit: List DoubleFloat,
          intvals: List DoubleFloat,
          g: Expression DoubleFloat,
```

```
          abserr: DoubleFloat,
          relerr: DoubleFloat)) ->
    Record(measure: Float,explanations: String)
```

These exports come from (p187) SetCategory():

```
 coerce : % -> OutputForm
 hash : % -> SingleInteger
 latex : % -> String
 ?=? : (%,%) -> Boolean
 ?~=? : (%,%) -> Boolean
```

— OrdinaryDifferentialEquationsSolverCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ODECAT">
OrdinaryDifferentialEquationsSolverCategory (ODECAT)</a></h2>

OrdinaryDifferentialEquationsSolverCategory is the
category for describing the set of ODE solver domains
with measure and ODEsolve.
<br/><br/><br/>

measure : (RoutinesTable,ODEAF) ->
 Record(measure:Float,explanations:String)<br/>
  measure(R,args) calculates an estimate of the ability of a<br/>
  particular method to solve a problem.  <br/>
 <br/>
  This method may be either a specific NAG routine or a strategy<br/>
  (such as transforming the function from one which is difficult<br/>
  to one which is easier to solve).<br/>
 <br/>
  It will call whichever agents are needed to perform analysis on the<br/>
  problem in order to calculate the measure. There is a parameter,<br/>
  labelled sofar}, which would contain the best compatibility<br/>
  found so far.<br/><br/>

ODESolve : ODEAF -> Result<br/>
  ODESolve(args) performs the integration of the<br/>
  function given the strategy or method returned by measure.<br/><br/>

hash : % -> SingleInteger<br/>
  hash(s) calculates a hash code for s.<br/>
  from SetCategory<br/><br/>

latex : % -> String<br/>
  latex(s) returns a LaTeX-printable output representation of s.<br/>
  from SetCategory<br/><br/>

?=? : (%,%) -> Boolean<br/>
  x=y tests if x and y are equal.<br/>
  from BasicType<br/><br/>

?~=? : (%,%) -> Boolean<br/>
```

```
  x~=y tests if x and y are not equal.<br/>
  from BasicType<br/><br/>

coerce : % -> OutputForm<br/>
  coerce(a) transforms a into an element of OutputForm<br/>
  from CoercibleTo OutputForm<br/><br/>

&bull; CoercibleTo OutputForm<br/>
&bull; SetCategory<br/>
&bull; BasicType<br/>

</body>
```

— category ODECAT OrdinaryDifferentialEquationsSolverCategory —

```
)abbrev category ODECAT OrdinaryDifferentialEquationsSolverCategory
++ Author: Brian Dupee
++ Date Created: February 1995
++ Date Last Updated: June 1995
++ Description:
++ \axiomType{OrdinaryDifferentialEquationsSolverCategory} is the
++ \axiom{category} for describing the set of ODE solver \axiom{domains}
++ with \axiomFun{measure} and \axiomFun{ODEsolve}.

OrdinaryDifferentialEquationsSolverCategory() : Category == SIG where

  DFF   ==> DoubleFloat
  VEDFF ==> Vector Expression DoubleFloat
  LDFF  ==> List DoubleFloat
  EDFF  ==> Expression DoubleFloat
  ODEAF ==> Record(xinit:DFF,xend:DFF,fn:VEDFF,yinit:LDFF,intvals:LDFF,
                 g:EDFF,abserr:DFF,relerr:DFF)

  SIG ==> SetCategory with

    measure : (RoutinesTable,ODEAF) ->
     Record(measure:Float,explanations:String)
      ++ measure(R,args) calculates an estimate of the ability of a particular
      ++ method to solve a problem.
      ++
      ++ This method may be either a specific NAG routine or a strategy (such
      ++ as transforming the function from one which is difficult to one which
      ++ is easier to solve).
      ++
      ++ It will call whichever agents are needed to perform analysis on the
      ++ problem in order to calculate the measure. There is a parameter,
      ++ labelled \axiom{sofar}, which would contain the best compatibility
      ++ found so far.

    ODESolve : ODEAF -> Result
      ++ ODESolve(args) performs the integration of the
```

```
++ function given the strategy or method returned by \axiomFun{measure}.
```

———-

— **ODECAT.dotabb** —

```
"ODECAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ODECAT"];
"ODECAT" -> "SETCAT"
```

———-

— **ODECAT.dotfull** —

```
"OrdinaryDifferentialEquationsSolverCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ODECAT"];
"OrdinaryDifferentialEquationsSolverCategory()" -> "SetCategory()"
```

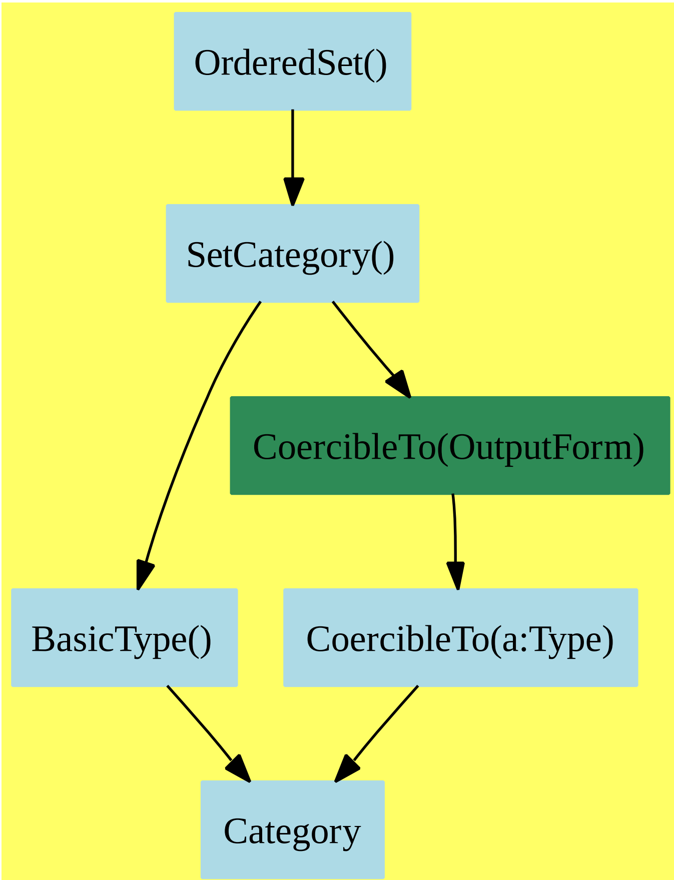
———-

— **ODECAT.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"OrdinaryDifferentialEquationsSolverCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ODECAT"];
"OrdinaryDifferentialEquationsSolverCategory()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

———-

4.0.80 OrderedSet (ORDSET)



— **OrderedSet.input** —

```
)set break resume
)sys rm -f OrderedSet.output
)spool OrderedSet.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderedSet
--R
--R OrderedSet is a category constructor
--R Abbreviation for OrderedSet is ORDSET
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ORDSET
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
```

```
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R max : (%,%) -> %                      min : (%,%) -> %
--R ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

— **OrderedSet.help** —

```
====================================================================
OrderedSet examples
====================================================================

The class of totally ordered sets, that is, sets such that for each
pair of elements (a,b) exactly one of the following relations holds
a<b or a=b or b<a and the relation is transitive, that is,
a<b and b<c => a<c.

See Also:
o )show OrderedSet
```

See:

⇒ "BitAggregate" (BTAGG) 9.0.147 on page 907
⇒ "CachableSet" (CACHSET) 5.0.93 on page 399
⇒ "DifferentialVariableCategory" (DVARCAT) 5.0.95 on page 411
⇒ "ExpressionSpace" (ES) 5.0.96 on page 418
⇒ "FortranMachineTypeCategory" (FMTC) 13.0.195 on page 1295
⇒ "IntervalCategory" (INTCAT) 14.0.200 on page 1341
⇒ "OrderedAbelianSemiGroup" (OASGP) 6.0.113 on page 590
⇒ "OrderedFinite" (ORDFIN) 5.0.101 on page 461
⇒ "OrderedMonoid" (ORDMON) 6.0.114 on page 594
⇒ "PolynomialCategory" (POLYCAT) 16.0.213 on page 1449
⇒ "QuotientFieldCategory" (QFCAT) 17.0.222 on page 1595
⇐ "SetCategory" (SETCAT) 3.0.57 on page 187

Exports:

coerce	hash	latex	max	min
?<?	?<=?	?=?	?>?	?>=?
?~=?				

These are directly exported but not implemented:

```
?<? : (%,%) -> Boolean
```

These are implemented by this category:

```
max : (%,%) -> %
min : (%,%) -> %
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
```

These exports come from (p187) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

— OrderedSet.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ORDSET">
OrderedSet (ORDSET)</a></h2>
</body>
```

— category ORDSET OrderedSet —

```
)abbrev category ORDSET OrderedSet
++ Description:
++ The class of totally ordered sets, that is, sets such that for each
++ pair of elements \spad{(a,b)}
++ exactly one of the following relations holds \spad{a<b or a=b or b<a}
++ and the relation is transitive, that is, \spad{a<b and b<c => a<c}.

OrderedSet() : Category == SIG where

  SIG ==> SetCategory with

    "<" : (%,%) -> Boolean
      ++ x < y is a strict total ordering on the elements of the set.

    ">" : (%, %) -> Boolean
      ++ x > y is a greater than test.

    ">=" : (%, %) -> Boolean
      ++ x >= y is a greater than or equal test.

    "<=" : (%, %) -> Boolean
      ++ x <= y is a less than or equal test.

    max : (%,%) -> %
      ++ max(x,y) returns the maximum of x and y relative to "<".

    min : (%,%) -> %
      ++ min(x,y) returns the minimum of x and y relative to "<".
```

```
add

   x,y: %

   -- These really ought to become some sort of macro

   max(x,y) ==
     x > y => x
     y

   min(x,y) ==
     x > y => y
     x

   ((x: %) >  (y: %)) : Boolean == y < x

   ((x: %) >= (y: %)) : Boolean == not (x < y)

   ((x: %) <= (y: %)) : Boolean == not (y < x)
```

———-

— **COQ ORDSET** —

```
(* category ORDSET *)
(*
    x,y: %

  -- These really ought to become some sort of macro

    max: (%,%) -> %
    max(x,y) ==
      x > y => x
      y

    min: (%,%) -> %
    min(x,y) ==
      x > y => y
      x

    ">": (%, %) -> Boolean
    ((x: %) >  (y: %)) : Boolean == y < x

    ">=": (%, %) -> Boolean
    ((x: %) >= (y: %)) : Boolean == not (x < y)

    "<=": (%, %) -> Boolean
    ((x: %) <= (y: %)) : Boolean == not (y < x)

*)
```

———-

— **ORDSET.dotabb** —

```
"ORDSET"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDSET"];
"ORDSET" -> "SETCAT"
```

———-

— **ORDSET.dotfull** —

```
"OrderedSet()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDSET"];
"OrderedSet()" -> "SetCategory()"
```

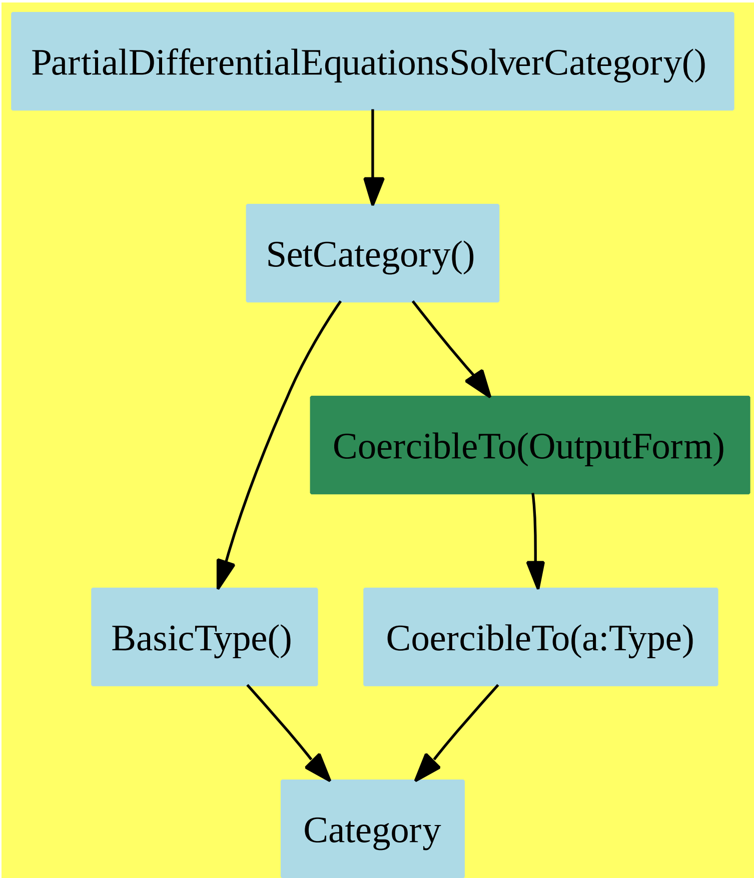
———-

— **ORDSET.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

———-

4.0.81 PartialDifferentialEquationsSolverCategory (PDECAT)



— **PartialDifferentialEquationsSolverCategory.input** —

```
)set break resume
)sys rm -f PartialDifferentialEquationsSolverCategory.output
)spool PartialDifferentialEquationsSolverCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PartialDifferentialEquationsSolverCategory
--R
--R PartialDifferentialEquationsSolverCategory is a category constructor
--R Abbreviation for PartialDifferentialEquationsSolverCategory is PDECAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PDECAT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
```

```
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R PDESolve : Record(pde: List(Expression(DoubleFloat)),constraints: List(Record(start: DoubleFloat,finish: Dou
--R measure : (RoutinesTable,Record(pde: List(Expression(DoubleFloat)),constraints: List(Record(start: DoubleFlo
--R
--E 1

)spool
)lisp (bye)
```

———-

— PartialDifferentialEquationsSolverCategory.help —

```
====================================================================
PartialDifferentialEquationsSolverCategory examples
====================================================================

PartialDifferentialEquationsSolverCategory is the category for describing
the set of PDE solver domains with measure and PDEsolve.

See Also:
o )show PartialDifferentialEquationsSolverCategory
```

———-

See:

⇐ "SetCategory" (SETCAT) 3.0.57 on page 187

Exports:

coerce	hash	latex	measure	PDESolve
?=?	?~=?			

These are directly exported but not implemented:

```
measure :
(RoutinesTable,
 Record(pde: List Expression DoubleFloat,
        constraints:
         List Record(start: DoubleFloat,
                     finish: DoubleFloat,
                     grid: NonNegativeInteger,
                     boundaryType: Integer,
                     dStart: Matrix DoubleFloat,
                     dFinish: Matrix DoubleFloat),
        f: List List Expression DoubleFloat,
        st: String,
        tol: DoubleFloat)) ->
   Record(measure: Float,explanations: String)
PDESolve :
 Record(pde: List Expression DoubleFloat,
        constraints:
         List Record(start: DoubleFloat,
```

```
                    finish: DoubleFloat,
                    grid: NonNegativeInteger,
                    boundaryType: Integer,
                    dStart: Matrix DoubleFloat,
                    dFinish: Matrix DoubleFloat),
         f: List List Expression DoubleFloat,
         st: String,
         tol: DoubleFloat) ->
    Result
```

These exports come from (p687) Dictionary(S:SetCategory):

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

— PartialDifferentialEquationsSolverCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#PDECAT">
PartialDifferentialEquationsSolverCategory (PDECAT)</a></h2>

PartialDifferentialEquationsSolverCategory is the
category for describing the set of PDE solver domains
with measure and PDEsolve.
<br/><br/><br/>

measure : (RoutinesTable,PDEBG) ->
 Record(measure:Float,explanations:String)<br/>
  measure(R,args) calculates an estimate of the ability of a<br/>
  particular method to solve a problem.  <br/>
 <br/>
  This method may be either a specific NAG routine or a strategy<br/>
  (such as transforming the function from one which is difficult<br/>
  to one which is easier to solve).<br/>
 <br/>
  It will call whichever agents are needed to perform analysis on the<br/>
  problem in order to calculate the measure. There is a parameter,<br/>
  labelled sofar, which would contain the best compatibility<br/>
  found so far.<br/><br/>

PDESolve : PDEBG -> Result<br/>
  PDESolve(args) performs the integration of the function<br/>
  given the strategy or method returned by measure.<br/><br/>

hash : % -> SingleInteger<br/>
  hash(s) calculates a hash code for s.<br/>
  from SetCategory<br/><br/>

latex : % -> String<br/>
  latex(s) returns a LaTeX-printable output representation of s.<br/>
  from SetCategory<br/><br/>
```

```
?=? : (%,%) -> Boolean<br/>
  x=y tests if x and y are equal.<br/>
  from BasicType<br/><br/>

?~=? : (%,%) -> Boolean<br/>
  x~=y tests if x and y are not equal.<br/>
  from BasicType<br/><br/>

coerce : % -> OutputForm<br/>
  coerce(a) transforms a into an element of OutputForm<br/>
  from CoercibleTo OutputForm<br/><br/>

&bull; CoercibleTo OutputForm<br/>
&bull; SetCategory<br/>
&bull; BasicType<br/>

</body>
```

— category PDECAT PartialDifferentialEquationsSolverCategory —

```
)abbrev category PDECAT PartialDifferentialEquationsSolverCategory
++ Author: Brian Dupee
++ Date Created: February 1995
++ Date Last Updated: June 1995
++ Description:
++ \axiomType{PartialDifferentialEquationsSolverCategory} is the
++ \axiom{category} for describing the set of PDE solver \axiom{domains}
++ with \axiomFun{measure} and \axiomFun{PDEsolve}.

-- PDEA        ==> Record(xmin:F,xmax:F,ymin:F,ymax:F,ngx:NNI,ngy:NNI,_
--             pde:List Expression Float, bounds:List List Expression Float,_
--             st:String, tol:DF)

--  measure:(RoutinesTable,PDEA) -> Record(measure:F,explanations:String)
--    ++ measure(R,args) calculates an estimate of the ability of a particular
--    ++ method to solve a problem.
--    ++
--    ++ This method may be either a specific NAG routine or a strategy (such
--    ++ as transforming the function from one which is difficult to one which
--    ++ is easier to solve).
--    ++
--    ++ It will call whichever agents are needed to perform analysis on the
--    ++ problem in order to calculate the measure. There is a parameter,
--    ++ labelled \axiom{sofar}, which would contain the best compatibility
--    ++ found so far.

--  PDESolve: PDEA -> Result
--    ++ PDESolve(args) performs the integration of the
--    ++ function given the strategy or method returned by \axiomFun{measure}.

PartialDifferentialEquationsSolverCategory() : Category == SIG where
```

```
  DFG   ==> DoubleFloat
  NNIG  ==> NonNegativeInteger
  INTG  ==> Integer
  MDFG  ==> Matrix DoubleFloat
  PDECG ==> Record(start:DFG, finish:DFG, grid:NNIG, boundaryType:INTG,
                   dStart:MDFG, dFinish:MDFG)
  LEDFG ==> List Expression DoubleFloat
  PDEBG ==> Record(pde:LEDFG, constraints:List PDECG, f:List LEDFG,
                   st:String, tol:DFG)
  SIG ==> SetCategory with

    measure : (RoutinesTable,PDEBG) ->
     Record(measure:Float,explanations:String)
      ++ measure(R,args) calculates an estimate of the ability of a particular
      ++ method to solve a problem.
      ++
      ++ This method may be either a specific NAG routine or a strategy (such
      ++ as transforming the function from one which is difficult to one which
      ++ is easier to solve).
      ++
      ++ It will call whichever agents are needed to perform analysis on the
      ++ problem in order to calculate the measure. There is a parameter,
      ++ labelled \axiom{sofar}, which would contain the best compatibility
      ++ found so far.

    PDESolve : PDEBG -> Result
      ++ PDESolve(args) performs the integration of the
      ++ function given the strategy or method returned by \axiomFun{measure}.
```

———-

— PDECAT.dotabb —

```
"PDECAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PDECAT"];
"PDECAT" -> "SETCAT"
```

———-

— PDECAT.dotfull —

```
"PartialDifferentialEquationsSolverCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PDECAT"];
"PartialDifferentialEquationsSolverCategory()" -> "SetCategory()"
```

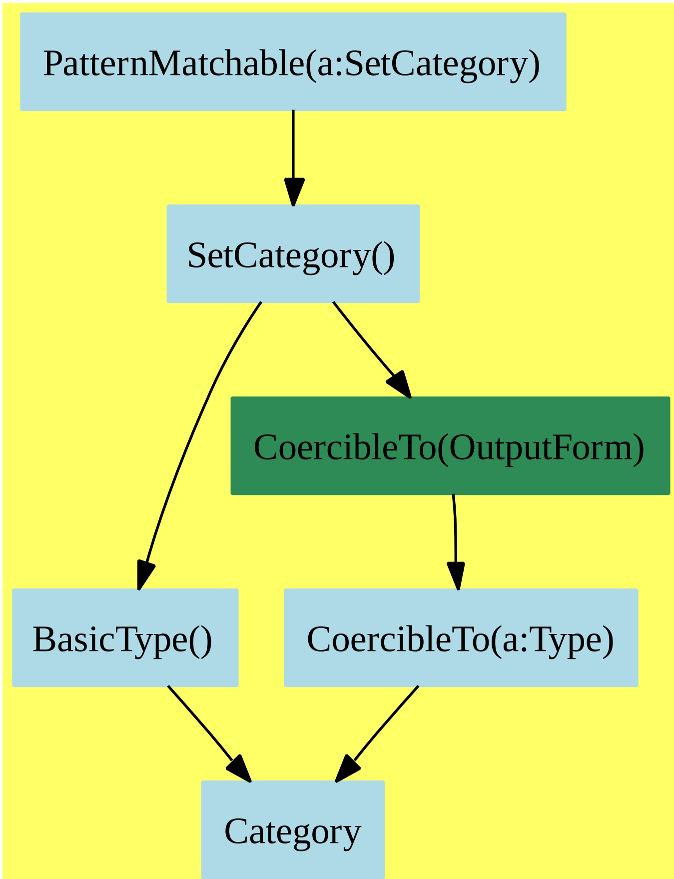
———-

— PDECAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
```

```
 node [shape=box, color=white, style=filled];

"PartialDifferentialEquationsSolverCategory()" [color=lightblue];
"PartialDifferentialEquationsSolverCategory()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.0.82 PatternMatchable (PATMAB)



— **PatternMatchable.input** —

```
)set break resume
)sys rm -f PatternMatchable.output
)spool PatternMatchable.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PatternMatchable
--R
--R PatternMatchable(S: SetCategory) is a category constructor
--R Abbreviation for PatternMatchable is PATMAB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PATMAB
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
```

```
--R hash : % -> SingleInteger             latex : % -> String
--R ?~=? : (%,%) -> Boolean
--R patternMatch : (%,Pattern(S),PatternMatchResult(S,%)) -> PatternMatchResult(S,%)
--R
--E 1

)spool
)lisp (bye)
```

———-

— PatternMatchable.help —

```
====================================================================
PatternMatchable examples
====================================================================

A set R is PatternMatchable over S if elements of R can be matched to
patterns over S.

See Also:
o )show PatternMatchable
```

———-

See:

⇒ "RealNumberSystem" (RNS) 17.0.224 on page 1622
⇐ "SetCategory" (SETCAT) 3.0.57 on page 187

Exports:

coerce hash latex patternMatch ?=?
?~=?

These are directly exported but not implemented:

```
patternMatch :
  (%,Pattern S,PatternMatchResult(S,%))
      -> PatternMatchResult(S,%)
```

These exports come from (p187) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

— PatternMatchable.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#PATMAB">
PatternMatchable (PATMAB)</a></h2>
```

```
&bull; S : SetCategory<br/><br/>

A set R is PatternMatchable over S if elements of R can
be matched to patterns over S.
<br/><br/><br/>

patternMatch : (%, Pattern S, PatternMatchResult(S, %)) ->
 PatternMatchResult(S, %)<br/>
  patternMatch(expr, pat, res) matches the pattern pat to the<br/>
  expression expr. res contains the variables of pat which<br/>
  are already matched and their matches (necessary for recursion).<br/>
  Initially, res is just the result of new<br/>
  which is an empty list of matches.<br/><br/>

hash : % -> SingleInteger<br/>
  hash(s) calculates a hash code for s.<br/>
  from SetCategory<br/><br/>

latex : % -> String<br/>
  latex(s) returns a LaTeX-printable output representation of s.<br/>
  from SetCategory<br/><br/>

?=? : (%,%) -> Boolean<br/>
  x=y tests if x and y are equal.<br/>
  from BasicType<br/><br/>

?~=? : (%,%) -> Boolean<br/>
  x~=y tests if x and y are not equal.<br/>
  from BasicType<br/><br/>

coerce : % -> OutputForm<br/>
  coerce(a) transforms a into an element of OutputForm<br/>
  from CoercibleTo OutputForm<br/><br/>

&bull; CoercibleTo OutputForm<br/>
&bull; SetCategory<br/>
&bull; BasicType<br/>

</body>
```

— category PATMAB PatternMatchable —

```
)abbrev category PATMAB PatternMatchable
++ Category of sets that can be pattern-matched on
++ Author: Manuel Bronstein
++ Date Created: 28 Nov 1989
++ Date Last Updated: 15 Mar 1990
++ Description:
++ A set R is PatternMatchable over S if elements of R can
++ be matched to patterns over S.

PatternMatchable(S) : Category == SIG where
```

```
  S : SetCategory

  SIG ==> SetCategory with

    patternMatch : (%, Pattern S, PatternMatchResult(S, %)) ->
                                                PatternMatchResult(S, %)
      ++ patternMatch(expr, pat, res) matches the pattern pat to the
      ++ expression expr. res contains the variables of pat which
      ++ are already matched and their matches (necessary for recursion).
      ++ Initially, res is just the result of \spadfun{new}
      ++ which is an empty list of matches.
```

———-

— **PATMAB.dotabb** —

```
"PATMAB"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PATMAB"];
"PATMAB" -> "SETCAT"
```

———-

— **PATMAB.dotfull** —

```
"PatternMatchable(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PATMAB"];
"PatternMatchable(a:SetCategory)" -> "SetCategory()"

"PatternMatchable(Integer)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PATMAB"];
"PatternMatchable(Integer)" -> "PatternMatchable(a:SetCategory)"

"PatternMatchable(Float)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PATMAB"];
"PatternMatchable(Float)" -> "PatternMatchable(a:SetCategory)"
```

———-

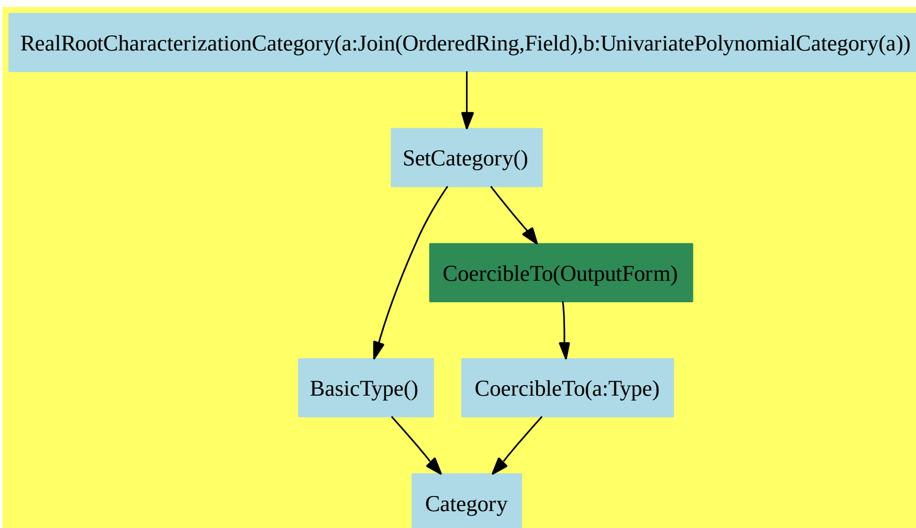
— **PATMAB.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"PatternMatchable(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PATMAB"];
"PatternMatchable(a:SetCategory)" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
```

```
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

——————-

4.0.83 RealRootCharacterizationCategory (RRCC)



— **RealRootCharacterizationCategory.input** —

```
)set break resume
)sys rm -f RealRootCharacterizationCategory.output
)spool RealRootCharacterizationCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RealRootCharacterizationCategory
--R
--R RealRootCharacterizationCategory(TheField: Join(OrderedRing,Field),ThePols: UnivariatePolynomialCategory(t#1
```

```
--R Abbreviation for RealRootCharacterizationCategory is RRCC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for RRCC
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean               allRootsOf : ThePols -> List(%)
--R coerce : % -> OutputForm             definingPolynomial : % -> ThePols
--R hash : % -> SingleInteger            latex : % -> String
--R negative? : (ThePols,%) -> Boolean   positive? : (ThePols,%) -> Boolean
--R sign : (ThePols,%) -> Integer        zero? : (ThePols,%) -> Boolean
--R ?~=? : (%,%) -> Boolean
--R approximate : (ThePols,%,TheField) -> TheField
--R recip : (ThePols,%) -> Union(ThePols,"failed")
--R relativeApprox : (ThePols,%,TheField) -> TheField
--R rootOf : (ThePols,PositiveInteger) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

——

— RealRootCharacterizationCategory.help —

```
====================================================================
RealRootCharacterizationCategory examples
====================================================================

RealRootCharacterizationCategory provides common access functions for
all real root codings.

See Also:
o )show RealRootCharacterizationCategory
```

——

See:

⇐ "SetCategory" (SETCAT) 3.0.57 on page 187

Exports:

allRootsOf	approximate	coerce	definingPolynomial
hash	latex	negative?	positive?
recip	relativeApprox	rootOf	sign
zero?	?=?	?~=?	

These are directly exported but not implemented:

```
approximate : (ThePols,%,TheField) -> TheField
allRootsOf : ThePols -> List %
definingPolynomial : % -> ThePols
relativeApprox : (ThePols,%,TheField) -> TheField
sign : (ThePols,%) -> Integer
```

These are implemented by this category:

```
negative? : (ThePols,%) -> Boolean
positive? : (ThePols,%) -> Boolean
recip : (ThePols,%) -> Union(ThePols,"failed")
rootOf : (ThePols,PositiveInteger) -> Union(%,"failed")
zero? : (ThePols,%) -> Boolean
```

These exports come from (p187) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

— RealRootCharacterizationCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#RRCC">
RealRootCharacterizationCategory(TheField,ThePols) (RRCC)</a></h2>

&bull; TheField : Join(OrderedRing, Field)<br/>
&bull; ThePols : UnivariatePolynomialCategory(TheField)<br/><br/>

RealRootCharacterizationCategory provides common access
functions for all real roots of polynomials
<br/><br/><br/>

sign : ( ThePols, $ ) -> Z<br/>
  sign(pol,aRoot) gives the sign of pol interpreted as aRoot<br/><br/>

zero? : ( ThePols, $ ) -> Boolean<br/>
  zero?(pol,aRoot) answers if pol interpreted as aRoot is 0<br/><br/>

negative? : ( ThePols, $ ) -> Boolean<br/>
  negative?(pol,aRoot) answers if pol<br/>
  interpreted as aRoot is negative<br/><br/>

positive? : ( ThePols, $ ) -> Boolean<br/>
  positive?(pol,aRoot) answers if pol<br/>
  interpreted as aRoot is positive<br/><br/>

recip : ( ThePols, $ ) -> Union(ThePols,"failed") <br/>
  recip(pol,aRoot) tries to inverse pol<br/>
  interpreted as aRoot<br/><br/>

definingPolynomial : $ -> ThePols<br/>
  definingPolynomial(aRoot) gives a polynomial<br/>
  such that definingPolynomial(aRoot).aRoot = 0 <br/><br/>

allRootsOf : ThePols -> List $<br/>
  allRootsOf(pol) creates all the roots of pol <br/>
  in the Real Closure, assumed in order.<br/><br/>
```

```
rootOf : ( ThePols, N ) -> Union($,"failed")<br/>
  rootOf(pol,n) gives the nth root for the order of the<br/>
  Real Closure<br/><br/>

approximate : (ThePols,$,TheField) -> TheField<br/>
  approximate(term,root,prec) gives an approximation <br/>
  of term over root with precision prec<br/><br/>

relativeApprox : (ThePols,$,TheField) -> TheField<br/>
  approximate(term,root,prec) gives an approximation <br/>
  of term over root with precision prec<br/><br/>

hash : % -> SingleInteger<br/>
  hash(s) calculates a hash code for s.<br/>
  from SetCategory<br/><br/>

latex : % -> String<br/>
  latex(s) returns a LaTeX-printable output representation of s.<br/>
  from SetCategory<br/><br/>

?=? : (%,%) -> Boolean<br/>
  x=y tests if x and y are equal.<br/>
  from BasicType<br/><br/>

?~=? : (%,%) -> Boolean<br/>
  x~=y tests if x and y are not equal.<br/>
  from BasicType<br/><br/>

coerce : % -> OutputForm<br/>
  coerce(a) transforms a into an element of OutputForm<br/>
  from CoercibleTo OutputForm<br/><br/>

&bull; CoercibleTo OutputForm<br/>
&bull; SetCategory<br/>
&bull; BasicType<br/>
</body>
```

— **category RRCC RealRootCharacterizationCategory** —

```
)abbrev category RRCC RealRootCharacterizationCategory
++ Author: Renaud Rioboo
++ Date Created: summer 1992
++ Date Last Updated: January 2004
++ Description:
++ \axiomType{RealRootCharacterizationCategory} provides common access
++ functions for all real roots of polynomials

RealRootCharacterizationCategory(TheField,ThePols) : Category == SIG where
  TheField : Join(OrderedRing, Field)
  ThePols : UnivariatePolynomialCategory(TheField)

  Z ==> Integer
```

```
N ==> PositiveInteger

SIG ==> SetCategory with

  sign : ( ThePols, $ ) -> Z
    ++ \axiom{sign(pol,aRoot)} gives the sign of \axiom{pol}
    ++ interpreted as \axiom{aRoot}

  zero? : ( ThePols, $ ) -> Boolean
    ++ \axiom{zero?(pol,aRoot)} answers if \axiom{pol}
    ++ interpreted as \axiom{aRoot} is \axiom{0}

  negative? : ( ThePols, $ ) -> Boolean
    ++ \axiom{negative?(pol,aRoot)} answers if \axiom{pol}
    ++ interpreted as \axiom{aRoot} is negative

  positive? : ( ThePols, $ ) -> Boolean
    ++ \axiom{positive?(pol,aRoot)} answers if \axiom{pol}
    ++ interpreted as \axiom{aRoot} is positive

  recip : ( ThePols, $ ) -> Union(ThePols,"failed")
    ++ \axiom{recip(pol,aRoot)} tries to inverse \axiom{pol}
    ++ interpreted as \axiom{aRoot}

  definingPolynomial : $ -> ThePols
    ++ \axiom{definingPolynomial(aRoot)} gives a polynomial
    ++ such that \axiom{definingPolynomial(aRoot).aRoot = 0}

  allRootsOf : ThePols -> List $
    ++ \axiom{allRootsOf(pol)} creates all the roots of \axiom{pol}
    ++ in the Real Closure, assumed in order.

  rootOf : ( ThePols, N ) -> Union($,"failed")
    ++ \axiom{rootOf(pol,n)} gives the nth root for the order of the
    ++ Real Closure

  approximate : (ThePols,$,TheField) -> TheField
    ++ \axiom{approximate(term,root,prec)} gives an approximation
    ++ of \axiom{term} over \axiom{root} with precision \axiom{prec}

  relativeApprox : (ThePols,$,TheField) -> TheField
    ++ \axiom{approximate(term,root,prec)} gives an approximation
    ++ of \axiom{term} over \axiom{root} with precision \axiom{prec}

 add

   zero?(toTest, rootChar) ==
     sign(toTest, rootChar) = 0

   negative?(toTest, rootChar) ==
     sign(toTest, rootChar) < 0

   positive?(toTest, rootChar) ==
     sign(toTest, rootChar) > 0
```

```
    rootOf(pol,n) ==
      liste:List($):= allRootsOf(pol)
      # liste > n => "failed"
      liste.n

    recip(toInv,rootChar) ==
      degree(toInv) = 0 =>
        res := recip(leadingCoefficient(toInv))
        if (res case "failed") then "failed" else (res::TheField::ThePols)
      defPol := definingPolynomial(rootChar)
      d := principalIdeal([defPol,toInv])
      zero?(d.generator,rootChar) => "failed"
      if (degree(d.generator) ^= 0 )
      then
        defPol := (defPol exquo (d.generator))::ThePols
        d := principalIdeal([defPol,toInv])
      d.coef.2
```

———-

— COQ RRCC —

```
(* category RRCC *)
(*

        zero? : ( ThePols, $ ) -> Boolean
        zero?(toTest, rootChar) ==
          sign(toTest, rootChar) = 0

        negative?: ( ThePols, $ ) -> Boolean
        negative?(toTest, rootChar) ==
          sign(toTest, rootChar) < 0

        positive?: ( ThePols, $ ) -> Boolean
        positive?(toTest, rootChar) ==
          sign(toTest, rootChar) > 0

        rootOf: ( ThePols, N ) -> Union($,"failed")
        rootOf(pol,n) ==
          liste:List($):= allRootsOf(pol)
          # liste > n => "failed"
          liste.n

        recip: ( ThePols, $ ) -> Union(ThePols,"failed")
        recip(toInv,rootChar) ==
          degree(toInv) = 0 =>
            res := recip(leadingCoefficient(toInv))
            if (res case "failed") then "failed" else (res::TheField::ThePols)
          defPol := definingPolynomial(rootChar)
          d := principalIdeal([defPol,toInv])
          zero?(d.generator,rootChar) => "failed"
          if (degree(d.generator) ^= 0 )
```

```
          then
            defPol := (defPol exquo (d.generator))::ThePols
            d := principalIdeal([defPol,toInv])
          d.coef.2

*)
```

———-

— RRCC.dotabb —

```
"RRCC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RRCC"];
"RRCC" -> "SETCAT"
```

———-

— RRCC.dotfull —

```
"RealRootCharacterizationCategory(a:Join(OrderedRing,Field),b:UnivariatePolynomialCategory(a))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RRCC"];
"RealRootCharacterizationCategory(a:Join(OrderedRing,Field),b:UnivariatePolynomialCategory(a))"
  -> "SetCategory()"
```

———-

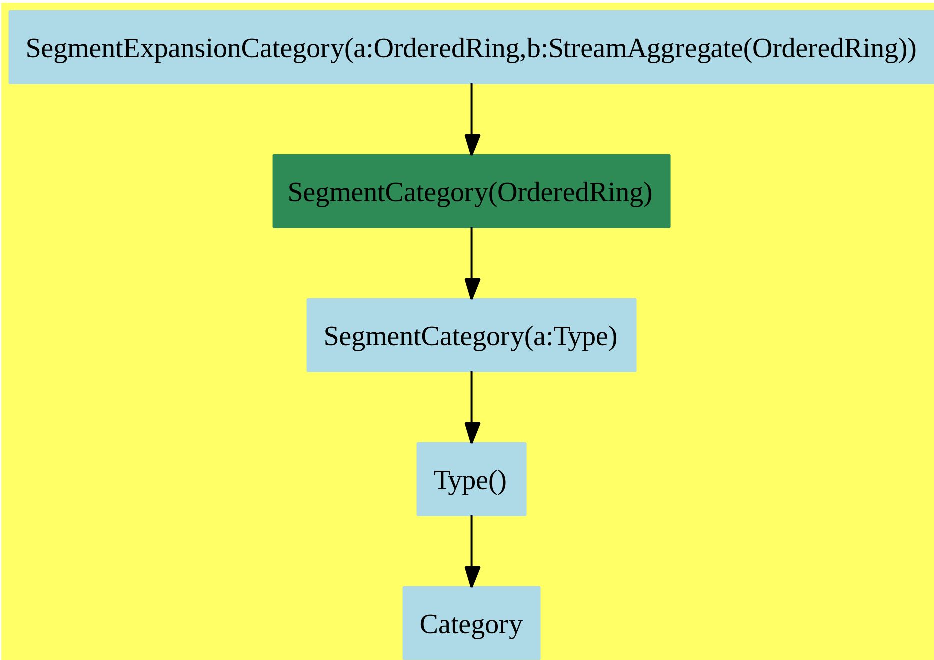
— RRCC.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"RealRootCharacterizationCategory(a:Join(OrderedRing,Field),b:UnivariatePolynomialCategory(a))"
 [color=lightblue];
"RealRootCharacterizationCategory(a:Join(OrderedRing,Field),b:UnivariatePolynomialCategory(a))"
  -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"
```

```
"Category" [color=lightblue];

}
```

———-

4.0.84 SegmentExpansionCategory (SEGXCAT)



— **SegmentExpansionCategory.input** —

```
)set break resume
)sys rm -f SegmentExpansionCategory.output
)spool SegmentExpansionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SegmentExpansionCategory
--R
--R SegmentExpansionCategory(S: OrderedRing,L: StreamAggregate(t#1)) is a category constructor
--R Abbreviation for SegmentExpansionCategory is SEGXCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SEGXCAT
--R
--R------------------------------- Operations --------------------------------
--R BY : (%,Integer) -> %                 ?..? : (S,S) -> %
```

```
--R convert : S -> %                    expand : % -> L
--R expand : List(%) -> L               hi : % -> S
--R high : % -> S                       incr : % -> Integer
--R lo : % -> S                         low : % -> S
--R map : ((S -> S),%) -> L             segment : (S,S) -> %
--R
--E 1

)spool
)lisp (bye)
```

— **SegmentExpansionCategory.help** —

```
====================================================================
SegmentExpansionCategory examples
====================================================================

This category provides an interface for expanding segments to a
stream of elements.

See Also:
o )show SegmentExpansionCategory
```

See:

⇐ "SegmentCategory" (SEGCAT) 3.0.56 on page 183

Exports:

BY	convert	expand	hi	high
incr	lo	low	map	segment
?..?				

Attributes exported:

- **nil**

These are directly exported but not implemented:

```
expand : % -> L
expand : List % -> L
map : ((S -> S),%) -> L
```

These exports come from (p183) SegmentCategory(OrderedRing):

```
BY : (%,Integer) -> %
convert : S -> %
hi : % -> S
high : % -> S
incr : % -> Integer
lo : % -> S
low : % -> S
segment : (S,S) -> %
```

```
 ?..? : (S,S) -> %
```

— SegmentExpansionCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#SEGXCAT">
SegmentExpansionCategory (SEGXCAT)</a></h2>

&bull; S : OrderedRing<br/>
&bull; L : StreamAggregate(S)<br/><br/>

This category provides an interface for expanding segments to
a stream of elements.
<br/><br/><br/>

expand : List % -> L<br/>
  expand(l) creates a new value of type L in which each segment<br/>
  l..h by k is replaced with l, l+k, ... lN,<br/>
  where lN <= h < lN+k.<br/>
  For example, expand [1..4, 7..9] = [1,2,3,4,7,8,9].<br/><br/>

expand : % -> L<br/>
  expand(l..h by k) creates value of type L with elements<br/>
  l, l+k, ... lN where lN <= h < lN+k.<br/>
  For example, expand(1..5 by 2) = [1,3,5].<br/><br/>

map : (S -> S, %) -> L<br/>
  map(f,l..h by k) produces a value of type L by applying f<br/>
  to each of the succesive elements of the segment, that is,<br/>
  [f(l), f(l+k), ..., f(lN)], where lN <= h < lN+k.<br/><br/>

?..? : (S,S) -> %<br/>
  l..h creates a segment with l and h as the endpoints.<br/>
  from  SegmentCategory(S)<br/><br/>

BY : (%, Integer) -> %<br/>
  s by n creates a new segment in which only every <br/>
  n-th element is used.<br/>
  from  SegmentCategory(S)<br/><br/>

lo : % -> S<br/>
  lo(s) returns the first endpoint of s.<br/>
  Note that lo(l..h) = l.<br/>
  from  SegmentCategory(S)<br/><br/>

hi : % -> S<br/>
  hi(s) returns the second endpoint of s.<br/>
  Note that hi(l..h) = h.<br/>
  from  SegmentCategory(S)<br/><br/>

low : % -> S<br/>
  low(s) returns the first endpoint of s.<br/>
  Note that low(l..h) = l.<br/>
```

```
  from  SegmentCategory(S)<br/><br/>

high : % -> S<br/>
  high(s) returns the second endpoint of s.<br/>
  Note that high(l..h) = h.<br/>
  from  SegmentCategory(S)<br/><br/>

incr : % -> Integer<br/>
  incr(s) returns n, where s is a segment in which every<br/>
  n-th element is used.<br/>
  Note that incr(l..h by n) = n.<br/>
  from  SegmentCategory(S)<br/><br/>

segment : (S, S) -> %<br/>
  segment(i,j) is an alternate way to create the segment i..j.<br/>
  from  SegmentCategory(S)<br/><br/>

convert : S -> %<br/>
  convert(i) creates the segment i..i.<br/>
  from  SegmentCategory(S)<br/><br/>

&bull; SegmentCategory(S)<br/>

</body>
```

— category SEGXCAT SegmentExpansionCategory —

```
)abbrev category SEGXCAT SegmentExpansionCategory
++ Author:  Stephen M. Watt
++ Date Created: June 5, 1991
++ Description:
++ This category provides an interface for expanding segments to
++ a stream of elements.

SegmentExpansionCategory(S,L) : Category == SIG where
  S : OrderedRing
  L : StreamAggregate(S)

  SIG ==> SegmentCategory(S) with

    expand : List % -> L
      ++ expand(l) creates a new value of type L in which each segment
      ++ \spad{l..h by k} is replaced with \spad{l, l+k, ... lN},
      ++ where \spad{lN <= h < lN+k}.
      ++ For example, \spad{expand [1..4, 7..9] = [1,2,3,4,7,8,9]}.

    expand : % -> L
      ++ expand(l..h by k) creates value of type L with elements
      ++ \spad{l, l+k, ... lN} where \spad{lN <= h < lN+k}.
      ++ For example, \spad{expand(1..5 by 2) = [1,3,5]}.

    map : (S -> S, %) -> L
```

```
++ map(f,l..h by k) produces a value of type L by applying f
++ to each of the succesive elements of the segment, that is,
++ \spad{[f(l), f(l+k), ..., f(lN)]}, where \spad{lN <= h < lN+k}.
```

———-

— SEGXCAT.dotabb —

```
"SEGXCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SEGXCAT"];
"SEGXCAT" -> "SEGCAT"
```

———-

— SEGXCAT.dotfull —

```
"SegmentExpansionCategory(a:OrderedRing,b:StreamAggregate(OrderedRing))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SEGXCAT"];
"SegmentExpansionCategory(a:OrderedRing,b:StreamAggregate(OrderedRing))"
   -> "SegmentCategory(OrderedRing)"
```

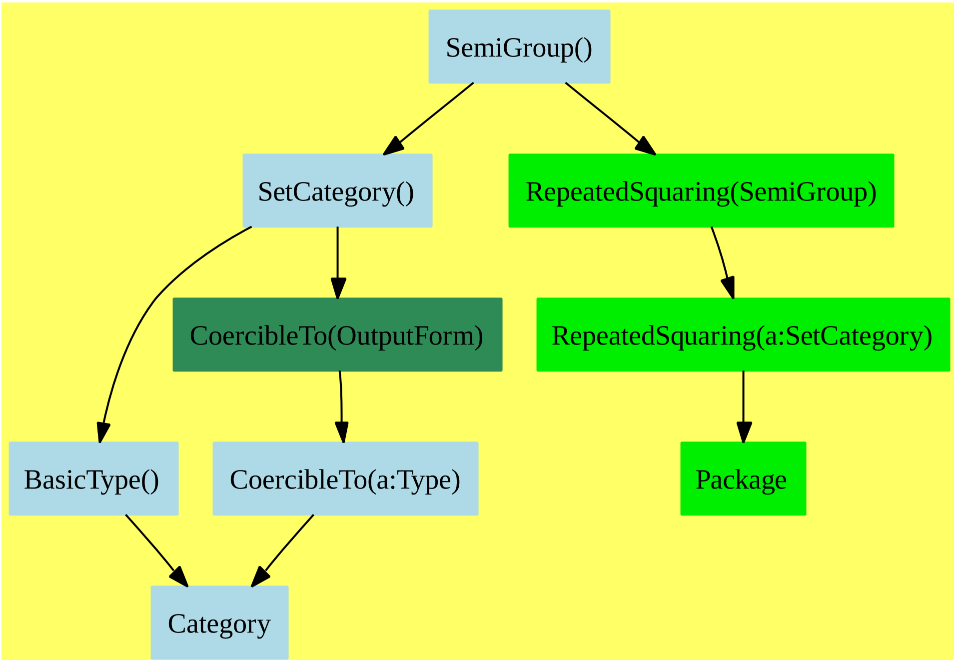
———-

— SEGXCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"SegmentExpansionCategory(a:OrderedRing,b:StreamAggregate(OrderedRing))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SEGXCAT"];
"SegmentExpansionCategory(a:OrderedRing,b:StreamAggregate(OrderedRing))"
   -> "SegmentCategory(OrderedRing)"

"SegmentCategory(OrderedRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=SEGCAT"];
"SegmentCategory(OrderedRing)" -> "SegmentCategory(a:Type)"

"SegmentCategory(a:Type)" [color=lightblue];
"SegmentCategory(a:Type)" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

———-

4.0.85 SemiGroup (SGROUP)



A Semigroup is defined as a set S with a binary multiplicative operator "*". A Semigroup $G(S,*)$ is:

- a set S which can be null
- a binary multiplicative operator "*"
- associative. $\forall a,b,c \in S, a*(b*c) = (a*b)*c$

— **SemiGroup.input** —

```
)set break resume
)sys rm -f SemiGroup.output
)spool SemiGroup.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SemiGroup
--R
--R SemiGroup is a category constructor
--R Abbreviation for SemiGroup is SGROUP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SGROUP
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                       ?**? : (%,PositiveInteger) -> %
--R ?=? : (%,%) -> Boolean                 ?^? : (%,PositiveInteger) -> %
--R coerce : % -> OutputForm               hash : % -> SingleInteger
```

```
--R latex : % -> String                    ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

— SemiGroup.help —

```
====================================================================
SemiGroup examples
====================================================================

The class of all multiplicative semigroups, that is, a set with an
associative operation *.

Axioms:
        associative("*":(%,%)->%)    (x*y)*z = x*(y*z)

Conditional attributes:
        commutative("*":(%,%)->%)    x*y = y*x

See Also:
o )show SemiGroup
```

See:

⇒ "FunctionSpace" (FS) 17.0.219 on page 1544
⇒ "Monoid" (MONOID) 5.0.100 on page 455
⇒ "Rng" (RNG) 8.0.145 on page 896
⇐ "SetCategory" (SETCAT) 3.0.57 on page 187

Exports:

coerce	hash	latex	?*?	?**?
?=?	?^?	?~=?		

These are directly exported but not implemented:

```
?*? : (%,%) -> %
```

These are implemented by this category:

```
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p187) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
```

```
?~=? : (%,%) -> Boolean
```

— SemiGroup.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#SGROUP">
SemiGroup (SGROUP)</a></h2>

The class of all multiplicative semigroups, that is, a set
with an associative operation *.
<br/><br/><br/>

?*? : (%,%) -> %<br/>
  x*y returns the product of x and y.<br/><br/>

?**? : (%,PositiveInteger) -> %<br/>
 
x**n returns the repeated product of x n times, exponentiation.<br/><br/>

?^? : (%,PositiveInteger) -> %<br/>
  x^n returns the repeated product of x n times, exponentiation.<br/><br/>

hash : % -> SingleInteger<br/>
  hash(s) calculates a hash code for s.<br/>
  from SetCategory<br/><br/>

latex : % -> String<br/>
  latex(s) returns a LaTeX-printable output representation of s.<br/>
  from SetCategory<br/><br/>

?=? : (%,%) -> Boolean<br/>
  x=y tests if x and y are equal.<br/>
  from BasicType<br/><br/>

?~=? : (%,%) -> Boolean<br/>
  x~=y tests if x and y are not equal.<br/>
  from BasicType<br/><br/>

coerce : % -> OutputForm<br/>
  coerce(a) transforms a into an element of OutputForm<br/>
  from CoercibleTo OutputForm<br/><br/>

&bull; CoercibleTo OutputForm<br/>
&bull; SetCategory<br/>
&bull; BasicType<br/>

</body>
```

— category SGROUP SemiGroup —

```
)abbrev category SGROUP SemiGroup
```

```
++ Description:
++ the class of all multiplicative semigroups, that is, a set
++ with an associative operation \spadop{*}.
++
++ Axioms\br
++ \tab{5}\spad{associative("*":(%,%)->%)}\tab{5}\spad{(x*y)*z = x*(y*z)}
++
++ Conditional attributes\br
++ \tab{5}\spad{commutative("*":(%,%)->%)}\tab{5}\spad{x*y = y*x}

SemiGroup() : Category == SIG where

  SIG ==> SetCategory with

    "*" : (%,%) -> %
      ++ x*y returns the product of x and y.

    "**" : (%,PositiveInteger) -> %
      ++ x**n returns the repeated product of x n times, exponentiation.

    "^" : (%,PositiveInteger) -> %
      ++ x^n returns the repeated product of x n times, exponentiation.

   add

     import RepeatedSquaring(%)

     x:% ** n:PositiveInteger == expt(x,n)

     _^(x:%, n:PositiveInteger):% == x ** n
```

— COQ SGROUP —

```
(* category SGROUP *)
(*
    import RepeatedSquaring(%)

    "**": (%,PositiveInteger) -> %
    x:% ** n:PositiveInteger == expt(x,n)

    "^": (%,PositiveInteger) -> %
    _^(x:%, n:PositiveInteger):% == x ** n

*)
```

— SGROUP.dotabb —

```
"SGROUP"
```

```
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SGROUP"];
"SGROUP" -> "SETCAT"
```

———–

— SGROUP.dotfull —

```
"SemiGroup()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SGROUP"];
"SemiGroup()" -> "SetCategory()"
"SemiGroup()" -> "RepeatedSquaring(a:SemiGroup)"
```

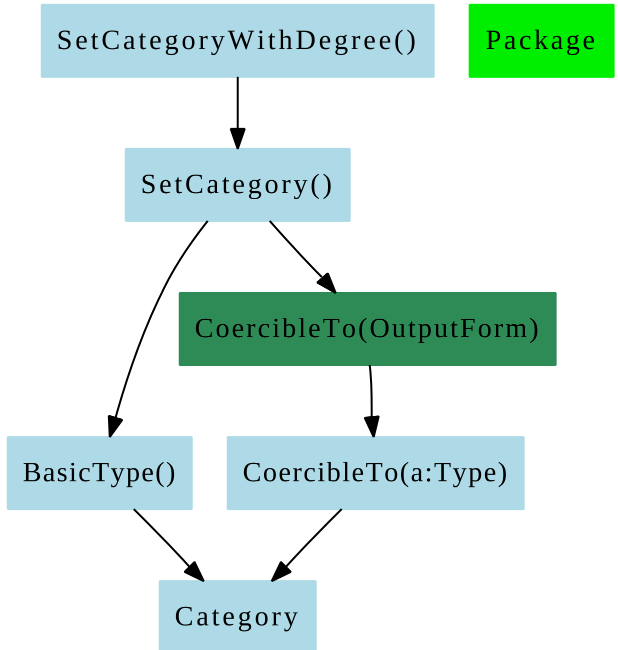
———–

— SGROUP.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SetCategory()"
"SemiGroup()" -> "RepeatedSquaring(a:SemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"RepeatedSquaring(a:SemiGroup)" [color="#00EE00"];
"RepeatedSquaring(a:SemiGroup)" -> "RepeatedSquaring(a:SetCategory)"

"RepeatedSquaring(a:SetCategory)" [color="#00EE00"];
"RepeatedSquaring(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];

}
```

———–

4.0.86 SetCategoryWithDegree (SETCATD)



— **SetCategoryWithDegree.input** —

```
)set break resume
)sys rm -f SetCategoryWithDegree.output
)spool SetCategoryWithDegree.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SetCategoryWithDegree
--R
--R SetCategoryWithDegree is a category constructor
--R Abbreviation for SetCategoryWithDegree is SETCATD
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SETCATD
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R degree : % -> PositiveInteger         hash : % -> SingleInteger
--R latex : % -> String                   ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

— **SetCategoryWithDegree.help** —

```
====================================================================
SetCategoryWithDegree examples
====================================================================

This is part of the PAFF package, related to projective space.

See Also:
o )show SetCategoryWithDegree
```

See:

⇒ “ProjectiveSpaceCategory” (PRSPCAT) 5.0.103 on page 469
⇐ “BasicType” (BASTYPE) 2.0.9 on page 27
⇐ “CoercibleTo” (KOERCE) 2.0.14 on page 39
⇐ “SetCategory” (SETCAT) 3.0.57 on page 187

Exports:

coerce degree hash latex ?=? ?~=?

These are directly exported but not implemented:

```
degree : % -> PositiveInteger
```

These exports come from (p187) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

— **SetCategoryWithDegree.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#SETCATD">
SetCategoryWithDegree (SETCATD)</a></h2>
</body>
```

— **category SETCATD SetCategoryWithDegree** —

```
)abbrev category SETCATD SetCategoryWithDegree
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
```

```
++ This is part of the PAFF package, related to projective space.

SetCategoryWithDegree() : Category == SIG where

  SIG ==> SetCategory with

    degree : % -> PositiveInteger
```

———-

— SETCATD.dotabb —

```
"SETCATD"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SETCATD"];
"SETCATD" -> "SETCAT"
```

———-

— SETCATD.dotfull —

```
"SetCategoryWithDegree()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SETCATD"];
"SetCategoryWithDegree()" -> "SetCategory()"
```

———-

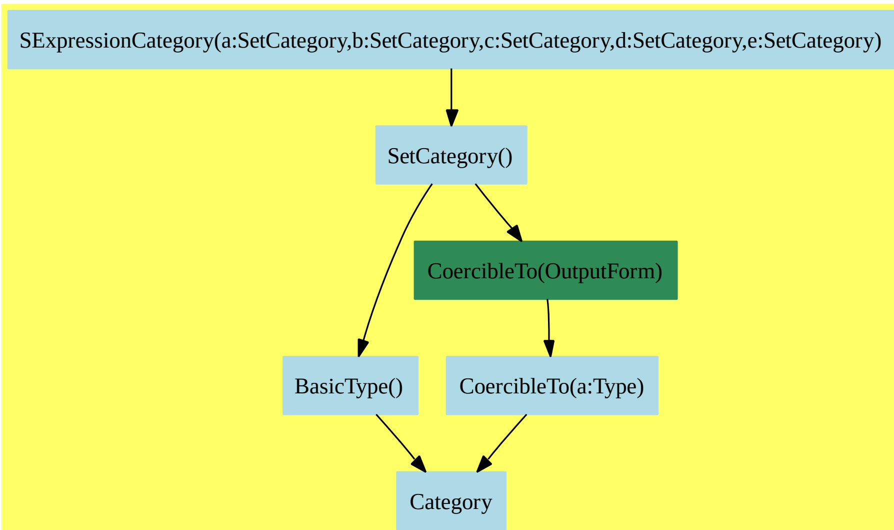
— SETCATD.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"SetCategoryWithDegree()" [color=lightblue];
"SetCategoryWithDegree()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Package" [color="#00EE00"];
```

```
"Category" [color=lightblue];

}
```

———-

4.0.87 SExpressionCategory (SEXCAT)



— **SExpressionCategory.input** —

```
)set break resume
)sys rm -f SExpressionCategory.output
)spool SExpressionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SExpressionCategory
--R
--R SExpressionCategory(Str: SetCategory,Sym: SetCategory,Int: SetCategory,Flt: SetCategory,Expr: SetCategory) i
--R Abbreviation for SExpressionCategory is SEXCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SEXCAT
--R
--R------------------------------- Operations --------------------------------
--R #? : % -> Integer                     ?=? : (%,%) -> Boolean
--R atom? : % -> Boolean                  car : % -> %
--R cdr : % -> %                          coerce : % -> OutputForm
--R convert : Expr -> %                   convert : Flt -> %
--R convert : Int -> %                    convert : Sym -> %
--R convert : Str -> %                    convert : List(%) -> %
```

```
--R destruct : % -> List(%)
--R ?.? : (%,Integer) -> %
--R expr : % -> Expr
--R float? : % -> Boolean
--R integer : % -> Int
--R latex : % -> String
--R null? : % -> Boolean
--R string : % -> Str
--R symbol : % -> Sym
--R ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

```
?.? : (%,List(Integer)) -> %
eq : (%,%) -> Boolean
float : % -> Flt
hash : % -> SingleInteger
integer? : % -> Boolean
list? : % -> Boolean
pair? : % -> Boolean
string? : % -> Boolean
symbol? : % -> Boolean
```

———

— SExpressionCategory.help —

```
====================================================================
SExpressionCategory examples
====================================================================

This category allows the manipulation of Lisp values while keeping
the grunge fairly localized.

See Also:
o )show SExpressionCategory
```

———

See:

⇐ "SetCategory" (SETCAT) 3.0.57 on page 187

Exports:

atom?	car	cdr	coerce	convert
destruct	eq	expr	float	float?
hash	integer	integer?	latex	list?
null?	pair?	string	string?	symbol
symbol?	#?	?=?	?~=?	?.?

These are directly exported but not implemented:

```
atom? : % -> Boolean
car : % -> %
cdr : % -> %
convert : Expr -> %
convert : Flt -> %
convert : Int -> %
convert : Sym -> %
convert : Str -> %
convert : List % -> %
```

```
destruct : % -> List %
eq : (%,%) -> Boolean
expr : % -> Expr
float : % -> Flt
float? : % -> Boolean
integer : % -> Int
integer? : % -> Boolean
list? : % -> Boolean
null? : % -> Boolean
pair? : % -> Boolean
string : % -> Str
string? : % -> Boolean
symbol : % -> Sym
symbol? : % -> Boolean
#? : % -> Integer
?.? : (%,List Integer) -> %
?.? : (%,Integer) -> %
```

These exports come from (p187) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

— SExpressionCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#SEXCAT">
SExpressionCategory (SEXCAT)</a></h2>
</body>
```

———-

— category SEXCAT SExpressionCategory —

```
)abbrev category SEXCAT SExpressionCategory
++ Author: S.M.Watt
++ Date Created: July 1987
++ Date Last Modified: 23 May 1991
++ Description:
++ This category allows the manipulation of Lisp values while keeping
++ the grunge fairly localized.
--  The coerce to expression lets the
--  values be displayed in the usual parenthesized way (displaying
--  them as type Expression can cause the formatter to die, since
--  certain magic cookies are in unexpected places).
--  SMW July 87

SExpressionCategory(Str, Sym, Int, Flt, Expr) : Category == SIG where
  Str : SetCategory
  Sym : SetCategory
  Int : SetCategory
```

```
Flt : SetCategory
Expr : SetCategory

SIG ==> SetCategory with

  eq : (%,%) -> Boolean
    ++ eq(s, t) is true if EQ(s,t) is true in Lisp.

  null? : % -> Boolean
    ++ null?(s) is true if s is the S-expression ().

  atom? : % -> Boolean
    ++ atom?(s) is true if s is a Lisp atom.

  pair? : % -> Boolean
    ++ pair?(s) is true if s has is a non-null Lisp list.

  list? : % -> Boolean
    ++ list?(s) is true if s is a Lisp list, possibly ().

  string? : % -> Boolean
    ++ string?(s) is true if s is an atom and belong to Str.

  symbol? : % -> Boolean
    ++ symbol?(s) is true if s is an atom and belong to Sym.

  integer? : % -> Boolean
    ++ integer?(s) is true if s is an atom and belong to Int.

  float? : % -> Boolean
    ++ float?(s) is true if s is an atom and belong to Flt.

  destruct : % -> List %
    ++ destruct((a1,...,an)) returns the list [a1,...,an].

  string : % -> Str
    ++ string(s) returns s as an element of Str.
    ++ Error: if s is not an atom that also belongs to Str.

  symbol : % -> Sym
    ++ symbol(s) returns s as an element of Sym.
    ++ Error: if s is not an atom that also belongs to Sym.

  integer : % -> Int
    ++ integer(s) returns s as an element of Int.
    ++ Error: if s is not an atom that also belongs to Int.

  float : % -> Flt
    ++ float(s) returns s as an element of Flt;
    ++ Error: if s is not an atom that also belongs to Flt.

  expr : % -> Expr
    ++ expr(s) returns s as an element of Expr;
    ++ Error: if s is not an atom that also belongs to Expr.
```

```
convert : List % -> %
  ++ convert([a1,...,an]) returns an S-expression \spad{(a1,...,an)}.

convert : Str -> %
  ++ convert(x) returns the Lisp atom x;

convert : Sym -> %
  ++ convert(x) returns the Lisp atom x.

convert : Int -> %
  ++ convert(x) returns the Lisp atom x.

convert : Flt -> %
  ++ convert(x) returns the Lisp atom x.

convert : Expr -> %
  ++ convert(x) returns the Lisp atom x.

car : % -> %
  ++ car((a1,...,an)) returns a1.

cdr : % -> %
  ++ cdr((a1,...,an)) returns \spad{(a2,...,an)}.

"#" : % -> Integer
  ++ #((a1,...,an)) returns n.

elt : (%, Integer) -> %
  ++ elt((a1,...,an), i) returns \spad{ai}.

elt : (%, List Integer) -> %
  ++ elt((a1,...,an), [i1,...,im]) returns \spad{(a_i1,...,a_im)}.
```

— SEXCAT.dotabb —

```
"SEXCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SEXCAT"];
"SEXCAT" -> "SETCAT"
```

— SEXCAT.dotfull —

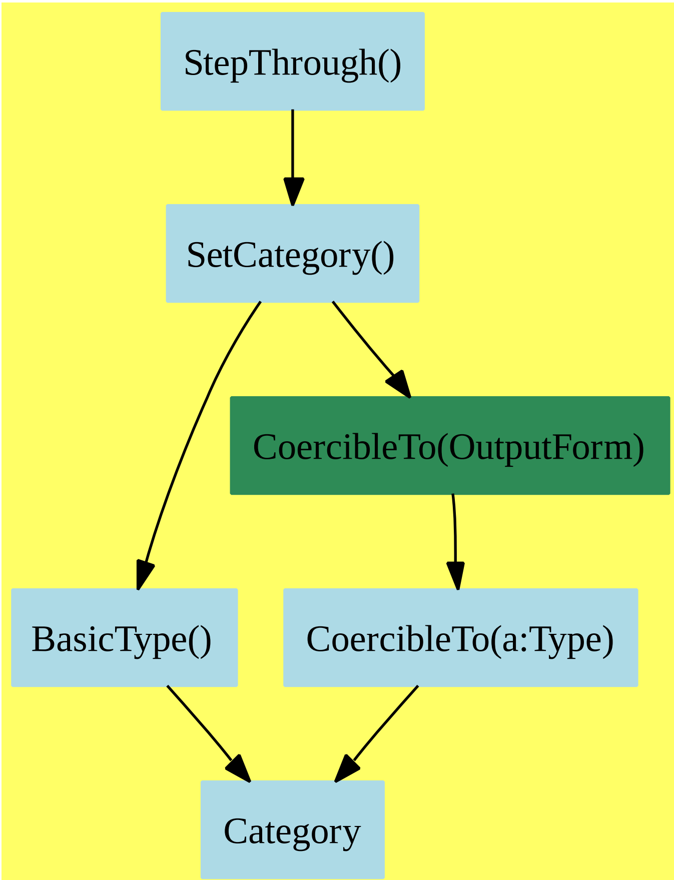
```
"SExpressionCategory(a:SetCategory,b:SetCategory,c:SetCategory,d:SetCategory,e:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SEXCAT"];
"SExpressionCategory(a:SetCategory,b:SetCategory,c:SetCategory,d:SetCategory,e:SetCategory)" ->
   "SetCategory()"
```

— SEXCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"SExpressionCategory(a:SetCategory,b:SetCategory,c:SetCategory,d:SetCategory,e:SetCategory)"
 [color=lightblue];
"SExpressionCategory(a:SetCategory,b:SetCategory,c:SetCategory,d:SetCategory,e:SetCategory)" ->
   "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

———

4.0.88 StepThrough (STEP)



— **StepThrough.input** —

```
)set break resume
)sys rm -f StepThrough.output
)spool StepThrough.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show StepThrough
--R
--R StepThrough is a category constructor
--R Abbreviation for StepThrough is STEP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for STEP
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
```

```
--R hash : % -> SingleInteger            init : () -> %
--R latex : % -> String                  nextItem : % -> Union(%,"failed")
--R ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

— StepThrough.help —

```
====================================================================
StepThrough examples
====================================================================

A class of objects which can be 'stepped through'.

Repeated applications of nextItem is guaranteed never to return
duplicate items and only return "failed" after exhausting all
elements of the domain. This assumes that the sequence starts
with init(). For infinite domains, repeated application of nextItem
is not required to reach all possible domain elements starting from
any initial element.

Conditional attributes:
    infinite -- repeated nextItem's are never "failed".

See Also:
o )show StepThrough
```

See:

⇒ "FiniteFieldCategory" (FFIELDC) 18.0.231 on page 1762
⇒ "IntegerNumberSystem" (INS) 16.0.210 on page 1420
⇒ "QuotientFieldCategory" (QFCAT) 17.0.222 on page 1595
⇒ "UnivariatePolynomialCategory" (UPOLYC) 17.0.228 on page 1708
⇐ "SetCategory" (SETCAT) 3.0.57 on page 187

Exports:

coerce	hash	init	latex	?=?
?~=?	nextItem			

These are directly exported but not implemented:

```
init : () -> %
nextItem : % -> Union(%,"failed")
```

These exports come from (p187) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
```

```
 latex : % -> String
 ?=? : (%,%) -> Boolean
 ?~=? : (%,%) -> Boolean
```

— **StepThrough.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#STEP">
StepThrough (STEP)</a></h2>
</body>
```

— **category STEP StepThrough** —

```
)abbrev category STEP StepThrough
++ Description:
++ A class of objects which can be 'stepped through'.
++ Repeated applications of \spadfun{nextItem} is guaranteed never to
++ return duplicate items and only return "failed" after exhausting
++ all elements of the domain.
++ This assumes that the sequence starts with \spad{init()}.
++ For infinite domains, repeated application
++ of \spadfun{nextItem} is not required to reach all possible domain elements
++ starting from any initial element.
++
++ Conditional attributes\br
++ \tab{5}infinite\tab{5}repeated nextItem's are never "failed".

StepThrough() : Category == SIG where

  SIG ==> SetCategory with

    init : constant -> %
      ++ init() chooses an initial object for stepping.

    nextItem : % -> Union(%,"failed")
      ++ nextItem(x) returns the next item, or "failed"
      ++ if domain is exhausted.
```

— **STEP.dotabb** —

```
"STEP" [color=lightblue,href="bookvol10.2.pdf#nameddest=STEP"];
"STEP" -> "SETCAT"
```

— **STEP.dotfull** —

```
"StepThrough()"
```

```
 [color=lightblue,href="bookvol10.2.pdf#nameddest=STEP"];
"StepThrough()" -> "SetCategory()"
```

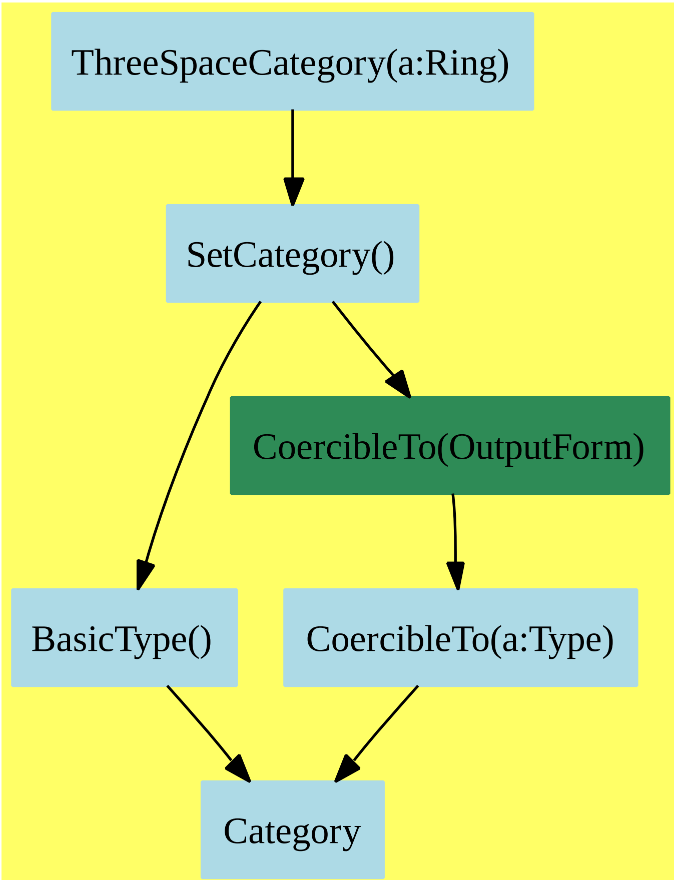
———

— **STEP.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"StepThrough()" [color=lightblue];
"StepThrough()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

———

4.0.89 ThreeSpaceCategory (SPACEC)



— **ThreeSpaceCategory.input** —

```
)set break resume
)sys rm -f ThreeSpaceCategory.output
)spool ThreeSpaceCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ThreeSpaceCategory
--R
--R ThreeSpaceCategory(R: Ring) is a category constructor
--R Abbreviation for ThreeSpaceCategory is SPACEC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SPACEC
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                check : % -> %
```

```
--R closedCurve : % -> List(Point(R))     closedCurve : List(Point(R)) -> %
--R closedCurve? : % -> Boolean           coerce : % -> OutputForm
--R components : % -> List(%)             composite : List(%) -> %
--R composites : % -> List(%)             copy : % -> %
--R create3Space : SubSpace(3,R) -> %     create3Space : () -> %
--R curve : % -> List(Point(R))           curve : List(Point(R)) -> %
--R curve : (%,List(List(R))) -> %        curve : (%,List(Point(R))) -> %
--R curve? : % -> Boolean                 hash : % -> SingleInteger
--R latex : % -> String                   lp : % -> List(Point(R))
--R merge : (%,%) -> %                    merge : List(%) -> %
--R mesh : % -> List(List(Point(R)))      mesh : List(List(Point(R))) -> %
--R mesh? : % -> Boolean                  point : % -> Point(R)
--R point : Point(R) -> %                 point : (%,List(R)) -> %
--R point : (%,Point(R)) -> %             point? : % -> Boolean
--R polygon : % -> List(Point(R))         polygon : List(Point(R)) -> %
--R polygon : (%,List(List(R))) -> %      polygon : (%,List(Point(R))) -> %
--R polygon? : % -> Boolean               subspace : % -> SubSpace(3,R)
--R ?~=? : (%,%) -> Boolean
--R closedCurve : (%,List(List(R))) -> %
--R closedCurve : (%,List(Point(R))) -> %
--R enterPointData : (%,List(Point(R))) -> NonNegativeInteger
--R lllip : % -> List(List(List(NonNegativeInteger)))
--R lllp : % -> List(List(List(Point(R))))
--R llprop : % -> List(List(SubSpaceComponentProperty))
--R lprop : % -> List(SubSpaceComponentProperty)
--R mesh : (List(List(Point(R))),Boolean,Boolean) -> %
--R mesh : (%,List(List(List(R))),Boolean,Boolean) -> %
--R mesh : (%,List(List(Point(R))),Boolean,Boolean) -> %
--R mesh : (%,List(List(List(R))),List(SubSpaceComponentProperty),SubSpaceComponentProperty) -> %
--R mesh : (%,List(List(Point(R))),List(SubSpaceComponentProperty),SubSpaceComponentProperty) -> %
--R modifyPointData : (%,NonNegativeInteger,Point(R)) -> %
--R numberOfComponents : % -> NonNegativeInteger
--R numberOfComposites : % -> NonNegativeInteger
--R objects : % -> Record(points: NonNegativeInteger,curves: NonNegativeInteger,polygons: NonNegativeInteger,con
--R point : (%,NonNegativeInteger) -> %
--R
--E 1

)spool
)lisp (bye)
```

— ThreeSpaceCategory.help —

```
====================================================================
ThreeSpaceCategory examples
====================================================================

The category ThreeSpaceCategory is used for creating three dimensional
objects using functions for defining points, curves, polygons,
constructs and the subspaces containing them.

See Also:
```

```
o )show ThreeSpaceCategory
```

———-

See:

⇐ "SetCategory" (SETCAT) 3.0.57 on page 187

Exports:

check	closedCurve	closedCurve?	coerce
components	composite	composites	copy
create3Space	curve	curve?	enterPointData
hash	latex	lllip	lllp
llprop	lp	lprop	merge
mesh	mesh?	modifyPointData	numberOfComponents
numberOfComposites	objects	point	point?
polygon	polygon?	subspace	?=?
?~=?			

These are directly exported but not implemented:

```
check : % -> %
closedCurve : (%,List List R) -> %
closedCurve : (%,List Point R) -> %
closedCurve : List Point R -> %
closedCurve : % -> List Point R
closedCurve? : % -> Boolean
coerce : % -> OutputForm
components : % -> List %
composite : List % -> %
composites : % -> List %
copy : % -> %
create3Space : () -> %
create3Space : SubSpace(3,R) -> %
curve : (%,List List R) -> %
curve : (%,List Point R) -> %
curve : List Point R -> %
curve : % -> List Point R
curve? : % -> Boolean
enterPointData : (%,List Point R) -> NonNegativeInteger
lllip : % -> List List List NonNegativeInteger
lllp : % -> List List List Point R
llprop : % -> List List SubSpaceComponentProperty
lp : % -> List Point R
lprop : % -> List SubSpaceComponentProperty
merge : List % -> %
merge : (%,%) -> %
mesh : % -> List List Point R
mesh : List List Point R -> %
mesh : (List List Point R,Boolean,Boolean) -> %
mesh : (%,List List List R,Boolean,Boolean) -> %
mesh : (%,List List Point R,Boolean,Boolean) -> %
mesh : (%,List List List R,
```

```
        List SubSpaceComponentProperty,
        SubSpaceComponentProperty) -> %
mesh : (%,List List Point R,
        List SubSpaceComponentProperty,
        SubSpaceComponentProperty) -> %
mesh? : % -> Boolean
modifyPointData : (%,NonNegativeInteger,Point R) -> %
numberOfComponents : % -> NonNegativeInteger
numberOfComposites : % -> NonNegativeInteger
objects : % ->
  Record(points: NonNegativeInteger,
         curves: NonNegativeInteger,
         polygons: NonNegativeInteger,
         constructs: NonNegativeInteger)
point : (%,Point R) -> %
point : (%,List R) -> %
point : (%,NonNegativeInteger) -> %
point : Point R -> %
point : % -> Point R
point? : % -> Boolean
polygon : (%,List Point R) -> %
polygon : (%,List List R) -> %
polygon : List Point R -> %
polygon : % -> List Point R
polygon? : % -> Boolean
subspace : % -> SubSpace(3,R)
```

These exports come from (p187) SetCategory():

```
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

— ThreeSpaceCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#SPACEC">
ThreeSpaceCategory (SPACEC)</a></h2>
</body>
```

————

— category SPACEC ThreeSpaceCategory —

```
)abbrev category SPACEC ThreeSpaceCategory
++ Description:
++ The category ThreeSpaceCategory is used for creating
++ three dimensional objects using functions for defining points, curves,
++ polygons, constructs and the subspaces containing them.

ThreeSpaceCategory(R) : Category == SIG where
  R : Ring
```

```
I    ==> Integer
PI   ==> PositiveInteger
NNI  ==> NonNegativeInteger
L    ==> List
B    ==> Boolean
O    ==> OutputForm
SUBSPACE ==> SubSpace(3,R)
POINT   ==> Point(R)
PROP    ==> SubSpaceComponentProperty()
REP3D ==> Record(lp:L POINT,llliPt:L L L NNI, llProp:L L PROP, lProp:L PROP)
OBJ3D   ==> Record(points:NNI, curves:NNI, polygons:NNI, constructs:NNI)

SIG ==> SetCategory with

  create3Space : () -> %
    ++ create3Space() creates a \spadtype{ThreeSpace} object capable of
    ++ holding point, curve, mesh components and any combination.

  create3Space : SUBSPACE -> %
    ++ create3Space(s) creates a \spadtype{ThreeSpace} object containing
    ++ objects pre-defined within some \spadtype{SubSpace} s.

  numberOfComponents : % -> NNI
    ++ numberOfComponents(s) returns the number of distinct
    ++ object components in the indicated \spadtype{ThreeSpace}, s, such
    ++ as points, curves, polygons, and constructs.

  numberOfComposites : % -> NNI
    ++ numberOfComposites(s) returns the number of supercomponents,
    ++ or composites, in the \spadtype{ThreeSpace}, s; Composites are
    ++ arbitrary groupings of otherwise distinct and unrelated components;
    ++ A \spadtype{ThreeSpace} need not have any composites defined at all
    ++ and, outside of the requirement that no component can belong
    ++ to more than one composite at a time, the definition and
    ++ interpretation of composites are unrestricted.

  merge : L % -> %
    ++ merge([s1,s2,...,sn]) will create a new \spadtype{ThreeSpace} that
    ++ has the components of all the ones in the list; Groupings of
    ++ components into composites are maintained.

  merge : (%,%) -> %
    ++ merge(s1,s2) will create a new \spadtype{ThreeSpace} that has the
    ++ components of \spad{s1} and \spad{s2}; Groupings of components
    ++ into composites are maintained.

  composite : L % -> %
    ++ composite([s1,s2,...,sn]) will create a new \spadtype{ThreeSpace}
    ++ that is a union of all the components from each
    ++ \spadtype{ThreeSpace} in the parameter list, grouped as a composite.

  components : % -> L %
    ++ components(s) takes the \spadtype{ThreeSpace} s, and creates a list
    ++ containing a unique \spadtype{ThreeSpace} for each single component
```

```
    ++ of s. If s has no components defined, the list returned is empty.

  composites : % -> L %
    ++ composites(s) takes the \spadtype{ThreeSpace} s, and creates a list
    ++ containing a unique \spadtype{ThreeSpace} for each single composite
    ++ of s. If s has no composites defined (composites need to be
    ++ explicitly created), the list returned is empty. Note that not all
    ++ the components need to be part of a composite.

  copy : % -> %
    ++ copy(s) returns a new \spadtype{ThreeSpace} that is an exact copy
    ++ of s.

  enterPointData : (%,L POINT) -> NNI
    ++ enterPointData(s,[p0,p1,...,pn]) adds a list of points from p0
    ++ through pn to the \spadtype{ThreeSpace}, s, and returns the index,
    ++ to the starting point of the list;

  modifyPointData : (%,NNI,POINT) -> %
    ++ modifyPointData(s,i,p) changes the point at the indexed
    ++ location i in the \spadtype{ThreeSpace}, s, to that of point p.
    ++ This is useful for making changes to a point which has been
    ++ transformed.

  -- 3D primitives

  point : (%,POINT) -> %
    ++ point(s,p) adds a point component defined by the point, p,
    ++ specified as a list from \spad{List(R)}, to the
    ++ \spadtype{ThreeSpace}, s, where R is the \spadtype{Ring} over
    ++ which the point is defined.

  point : (%,L R) -> %
    ++ point(s,[x,y,z]) adds a point component defined by a list of
    ++ elements which are from the \spad{PointDomain(R)} to the
    ++ \spadtype{ThreeSpace}, s, where R is the \spadtype{Ring} over
    ++ which the point elements are defined.

  point : (%,NNI) -> %
    ++ point(s,i) adds a point component which is placed into a component
    ++ list of the \spadtype{ThreeSpace}, s, at the index given by i.

  point : POINT -> %
    ++ point(p) returns a \spadtype{ThreeSpace} object which is composed
    ++ of one component, the point p.

  point : % -> POINT
    ++ point(s) checks to see if the \spadtype{ThreeSpace}, s, is
    ++ composed of only a single point and if so, returns the point.
    ++ An error is signaled otherwise.

  point? : % -> B
    ++ point?(s) queries whether the \spadtype{ThreeSpace}, s, is
    ++ composed of a single component which is a point and returns the
```

```
  ++ boolean result.

curve : (%,L POINT) -> %
  ++ curve(s,[p0,p1,...,pn]) adds a space curve component defined by a
  ++ list of points \spad{p0} through \spad{pn}, to the
  ++ \spadtype{ThreeSpace} s.

curve : (%,L L R) -> %
  ++ curve(s,[[p0],[p1],...,[pn]]) adds a space curve which is a list of
  ++ points p0 through pn defined by lists of elements from the domain
  ++ \spad{PointDomain(m,R)}, where R is the \spadtype{Ring} over which
  ++ the point elements are defined and m is the dimension of the
  ++ points, to the \spadtype{ThreeSpace} s.

curve : L POINT -> %
  ++ curve([p0,p1,p2,...,pn]) creates a space curve defined
  ++ by the list of points \spad{p0} through \spad{pn}, and returns the
  ++ \spadtype{ThreeSpace} whose component is the curve.

curve : % -> L POINT
  ++ curve(s) checks to see if the \spadtype{ThreeSpace}, s, is
  ++ composed of a single curve defined by a list of points and if so,
  ++ returns the curve, that is, list of points. An error is signaled
  ++ otherwise.

curve? : % -> B
  ++ curve?(s) queries whether the \spadtype{ThreeSpace}, s, is a curve,
  ++ that is, has one component, a list of list of points, and returns
  ++ true if it is, or false otherwise.

closedCurve : (%,L POINT) -> %
  ++ closedCurve(s,[p0,p1,...,pn,p0]) adds a closed curve component
  ++ which is a list of points defined by the first element p0 through
  ++ the last element pn and back to the first element p0 again, to the
  ++ \spadtype{ThreeSpace} s.

closedCurve : (%,L L R) -> %
  ++ closedCurve(s,[[lr0],[lr1],...,[lrn],[lr0]]) adds a closed curve
  ++ component defined by a list of points \spad{lr0} through
  ++ \spad{lrn}, which are lists of elements from the domain
  ++ \spad{PointDomain(m,R)}, where R is the \spadtype{Ring} over which
  ++ the point elements are defined and m is the dimension of the
  ++ points, in which the last element of the list of points contains
  ++ a copy of the first element list, lr0.
  ++ The closed curve is added to the \spadtype{ThreeSpace}, s.

closedCurve : L POINT -> %
  ++ closedCurve(lp) sets a list of points defined by the first element
  ++ of lp through the last element of lp and back to the first elelment
  ++ again and returns a \spadtype{ThreeSpace} whose component is the
  ++ closed curve defined by lp.

closedCurve : % -> L POINT
  ++ closedCurve(s) checks to see if the \spadtype{ThreeSpace}, s, is
```

```
    ++ composed of a single closed curve component defined by a list of
    ++ points in which the first point is also the last point, all of
    ++ which are from the domain \spad{PointDomain(m,R)} and if so,
    ++ returns the list of points.  An error is signaled otherwise.

  closedCurve? : % -> B
    ++ closedCurve?(s) returns true if the \spadtype{ThreeSpace} s
    ++ contains a single closed curve component, that is, the first element
    ++ of the curve is also the last element, or false otherwise.

  polygon : (%,L POINT) -> %
    ++ polygon(s,[p0,p1,...,pn]) adds a polygon component defined by a
    ++ list of points, p0 throught pn, to the \spadtype{ThreeSpace} s.

  polygon : (%,L L R) -> %
    ++ polygon(s,[[r0],[r1],...,[rn]]) adds a polygon component defined
    ++ by a list of points \spad{r0} through \spad{rn}, which are lists of
    ++ elements from the domain \spad{PointDomain(m,R)} to the
    ++ \spadtype{ThreeSpace} s, where m is the dimension of the points
    ++ and R is the \spadtype{Ring} over which the points are defined.

  polygon : L POINT -> %
    ++ polygon([p0,p1,...,pn]) creates a polygon defined by a list of
    ++ points, p0 through pn, and returns a \spadtype{ThreeSpace} whose
    ++ component is the polygon.

  polygon : % -> L POINT
    ++ polygon(s) checks to see if the \spadtype{ThreeSpace}, s, is
    ++ composed of a single polygon component defined by a list of
    ++ points, and if so, returns the list of points;  An error is
    ++ signaled otherwise.

  polygon? : % -> B
    ++ polygon?(s) returns true if the \spadtype{ThreeSpace} s contains
    ++ a single polygon component, or false otherwise.

  mesh : (%,L L POINT,L PROP,PROP) -> %
    ++ mesh(s,[[p0],[p1],...,[pn]],[props],prop) adds a surface component,
    ++ defined over a list curves which contains lists of points, to the
    ++ \spadtype{ThreeSpace} s; props is a list which contains the
    ++ subspace component properties for each surface parameter, and
    ++ prop is the subspace component property by which the points are
    ++ defined.

  mesh : (%,L L L R,L PROP,PROP) -> %
    ++ mesh(s, LLLR, [props], prop)
    ++ where LLLR is of the form:
    ++ [[[r10]...,[r1m]],[[r20]...,[r2m]],...,[[rn0]...,[rnm]]],
    ++ adds a surface component to the \spadtype{ThreeSpace} s, which is
    ++ defined over a rectangular domain of size WxH where W is the number
    ++ of lists of points from the domain \spad{PointDomain(R)} and H is
    ++ the number of elements in each of those lists; lprops is the list
    ++ of the subspace component properties for each curve list, and
    ++ prop is the subspace component property by which the points are
```

```
  ++ defined.

mesh : (%,L L POINT,B,B) -> %
  ++ mesh(s, LLP, close1, close2)
  ++ where LLP is of the form [[p0],[p1],...,[pn]] adds a surface
  ++ component to the \spadtype{ThreeSpace}, which is defined over a
  ++ list of curves, in which each of these curves is a list of points.
  ++ The boolean arguments close1 and close2 indicate how the surface
  ++ is to be closed. Argument close1 equal true
  ++ means that each individual list (a curve) is to be closed, that is,
  ++ the last point of the list is to be connected to the first point.
  ++ Argument close2 equal true
  ++ means that the boundary at one end of the surface is to be
  ++ connected to the boundary at the other end, that is, the boundaries
  ++ are defined as the first list of points (curve) and
  ++ the last list of points (curve).

mesh : (%,L L L R,B,B) -> %
  ++ mesh(s, LLLR, close1, close2)
  ++ where LLLR is of the form
  ++ [[[r10]...,[r1m]],[[r20]...,[r2m]],...,[[rn0]...,[rnm]]],
  ++ adds a surface component to the \spadtype{ThreeSpace} s, which is
  ++ defined over a rectangular domain of size WxH where W is the number
  ++ of lists of points from the domain \spad{PointDomain(R)} and H is
  ++ the number of elements in each of those lists; the booleans close1
  ++ and close2 indicate how the surface is to be closed: if close1 is
  ++ true this means that each individual list (a curve) is to be
  ++ closed (that is,
  ++ the last point of the list is to be connected to the first point);
  ++ if close2 is true, this means that the boundary at one end of the
  ++ surface is to be connected to the boundary at the other end
  ++ (the boundaries are defined as the first list of points (curve)
  ++ and the last list of points (curve)).

mesh : L L POINT -> %
  ++ mesh([[p0],[p1],...,[pn]]) creates a surface defined by a list of
  ++ curves which are lists, p0 through pn, of points, and returns a
  ++ \spadtype{ThreeSpace} whose component is the surface.

mesh : (L L POINT,B,B) -> %
  ++ mesh([[p0],[p1],...,[pn]], close1, close2) creates a surface
  ++ defined over a list of curves, p0 through pn, which are lists of
  ++ points; the booleans close1 and close2 indicate how the surface is
  ++ to be closed: close1 set to true means that each individual list
  ++ (a curve) is to be closed (that is, the last point of the list is
  ++ to be connected to the first point); close2 set to true means
  ++ that the boundary at one end of the surface is to be connected to
  ++ the boundary at the other end (the boundaries are defined as the
  ++ first list of points (curve) and the last list of points (curve));
  ++ the \spadtype{ThreeSpace} containing this surface is returned.

mesh : % -> L L POINT
  ++ mesh(s) checks to see if the \spadtype{ThreeSpace}, s, is
  ++ composed of a single surface component defined by a list curves
```

```
  ++ which contain lists of points, and if so, returns the list of
  ++ lists of points;  An error is signaled otherwise.

mesh? : % -> B
  ++ mesh?(s) returns true if the \spadtype{ThreeSpace} s is composed
  ++ of one component, a mesh comprising a list of curves which are lists
  ++ of points, or returns false if otherwise

lp : % -> L POINT
  ++ lp(s) returns the list of points component which the
  ++ \spadtype{ThreeSpace}, s, contains; these points are used by
  ++ reference, that is, the component holds indices referring to the
  ++ points rather than the points themselves. This allows for sharing
  ++ of the points.

lllip : % -> L L L NNI
  ++ lllip(s) checks to see if the \spadtype{ThreeSpace}, s, is
  ++ composed of a list of components, which are lists of curves,
  ++ which are lists of indices to points, and if so, returns the list
  ++ of lists of lists;  An error is signaled otherwise.

lllp : % -> L L L POINT   -- used by view3D
  ++ lllp(s) checks to see if the \spadtype{ThreeSpace}, s, is
  ++ composed of a list of components, which are lists of curves,
  ++ which are lists of points, and if so, returns the list of
  ++ lists of lists;  An error is signaled otherwise.

llprop : % -> L L PROP      -- used by view3D
  ++ llprop(s) checks to see if the \spadtype{ThreeSpace}, s, is
  ++ composed of a list of curves which are lists of the
  ++ subspace component properties of the curves, and if so, returns the
  ++ list of lists;  An error is signaled otherwise.

lprop : % -> L PROP        -- used by view3D
  ++ lprop(s) checks to see if the \spadtype{ThreeSpace}, s, is
  ++ composed of a list of subspace component properties, and if so,
  ++ returns the list;  An error is signaled otherwise.

objects : % -> OBJ3D
  ++ objects(s) returns the \spadtype{ThreeSpace}, s, in the form of a
  ++ 3D object record containing information on the number of points,
  ++ curves, polygons and constructs comprising the
  ++ \spadtype{ThreeSpace}..

check : % -> %             -- used by mesh
  ++ check(s) returns lllpt, list of lists of lists of point information
  ++ about the \spadtype{ThreeSpace} s.

subspace : % -> SUBSPACE
  ++ subspace(s) returns the \spadtype{SubSpace} which holds all the
  ++ point information in the \spadtype{ThreeSpace}, s.

coerce : % -> O
  ++ coerce(s) returns the \spadtype{ThreeSpace} s to Output format.
```

— **SPACEC.dotabb** —

```
"SPACEC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SPACEC"];
"SPACEC" -> "SETCAT"
```

— **SPACEC.dotfull** —

```
"ThreeSpaceCategory(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SPACEC"];
"ThreeSpaceCategory(a:Ring)" -> "SetCategory()"
```

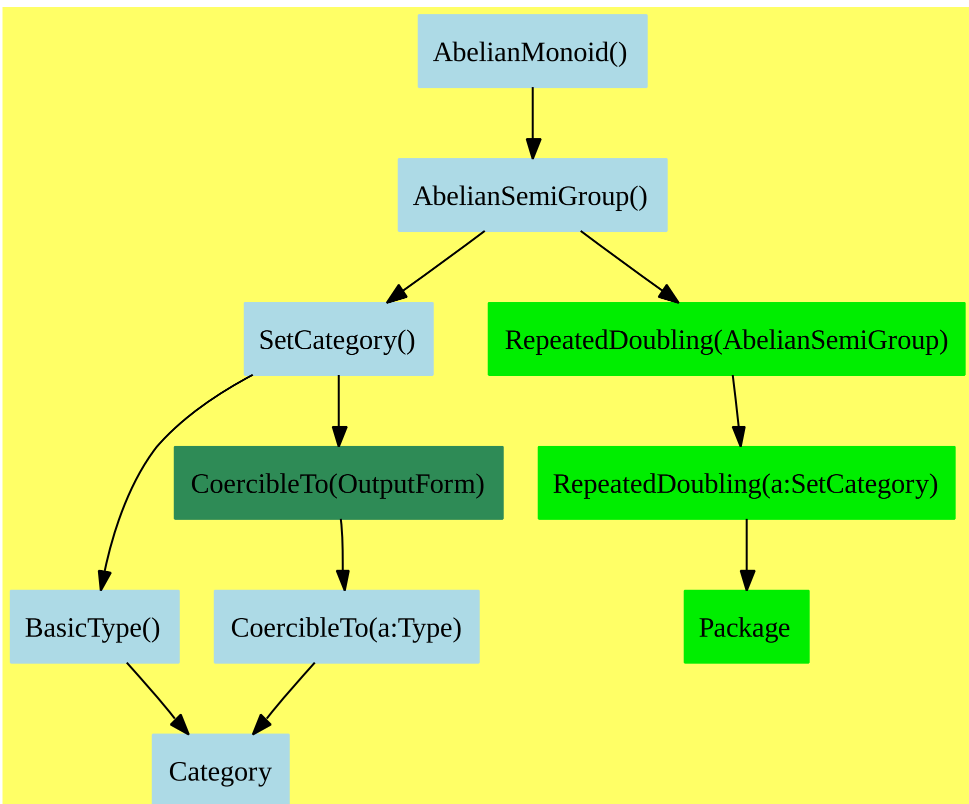
— **SPACEC.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"ThreeSpaceCategory(a:Ring)" [color=lightblue];
"ThreeSpaceCategory(a:Ring)" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

Chapter 5

Category Layer 4

5.0.90 AbelianMonoid (ABELMON)



— **AbelianMonoid.input** —

```
)set break resume
)sys rm -f AbelianMonoid.output
```

```
)spool AbelianMonoid.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AbelianMonoid
--R
--R AbelianMonoid is a category constructor
--R Abbreviation for AbelianMonoid is ABELMON
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ABELMON
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                      ?=? : (%,%) -> Boolean
--R 0 : () -> %                           coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

— **AbelianMonoid.help** —

```
====================================================================
AbelianMonoid examples
====================================================================

The class of multiplicative monoids, that is, semigroups with an
additive identity element.

Axioms:
        leftIdentity("+":(%,%)->%,0)      0+x=x
        rightIdentity("+":(%,%)->%,0)     x+0=x

See Also:
o )show AbelianMonoid
```

See:

⇒ “CancellationAbelianMonoid” (CABMON) 6.0.107 on page 508
⇒ “FunctionSpace” (FS) 17.0.219 on page 1544
⇒ “OrderedAbelianMonoid” (OAMON) 7.0.129 on page 730
⇒ “OrderedAbelianSemiGroup” (OASGP) 6.0.113 on page 590
⇐ “AbelianSemiGroup” (ABELSG) 4.0.59 on page 201

Exports:

0	coerce	hash	latex	sample
zero?	?*?	?+?	?=?	?~=?

These are directly exported but not implemented:

```
0 : () -> %
```

These are implemented by this category:

```
sample : () -> %
zero? : % -> Boolean
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
```

These exports come from (p201) AbelianSemiGroup():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?+? : (%,%) -> %
```

— **AbelianMonoid.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ABELMON">
AbelianMonoid (ABELMON)</a></h2>
</body>
```

———-

— **category ABELMON AbelianMonoid** —

```
)abbrev category ABELMON AbelianMonoid
++ Description:
++ The class of multiplicative monoids, that is, semigroups with an
++ additive identity element.
++
++ Axioms\br
++ \tab{5}\spad{leftIdentity("+":(%,%)->%,0)}\tab{5}\spad{0+x=x}\br
++ \tab{5}\spad{rightIdentity("+":(%,%)->%,0)}\tab{4}\spad{x+0=x}
-- following domain must be compiled with subsumption disabled
-- define SourceLevelSubset to be EQUAL

AbelianMonoid() : Category == SIG where

  SIG ==> AbelianSemiGroup with

    0 : constant -> %
      ++ \spad{0} is the additive identity element.

    sample : constant -> %
      ++ sample yields a value of type %
```

```
    zero? : % -> Boolean
      ++ zero?(x) tests if x is equal to 0.

    "*" : (NonNegativeInteger,%) -> %
      ++ n * x is left-multiplication by a non negative integer

   add

      import RepeatedDoubling(%)

      zero? x == x = 0

      n:PositiveInteger * x:% == (n::NonNegativeInteger) * x

      sample() == 0

      if not (% has Ring) then

        n:NonNegativeInteger * x:% ==
          zero? n => 0
          double(n pretend PositiveInteger,x)
```

———-

— COQ ABELMON —

```
(* category ABELMON *)
(*
      import RepeatedDoubling(%)

      zero?: % -> Boolean
      zero? x == x = 0

      ?*? : (PositiveInteger,%) -> %
      n:PositiveInteger * x:% == (n::NonNegativeInteger) * x

      sample: constant -> %
      sample() == 0

      if not (% has Ring) then

        "*": (NonNegativeInteger,%) -> %
        n:NonNegativeInteger * x:% ==
          zero? n => 0
          double(n pretend PositiveInteger,x)

*)
```

———-

— ABELMON.dotabb —

```
"ABELMON"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ABELMON"];
"ABELMON" -> "ABELSG"
```

———-

— ABELMON.dotfull —

```
"AbelianMonoid()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ABELMON"];
"AbelianMonoid()" -> "AbelianSemiGroup()"
```

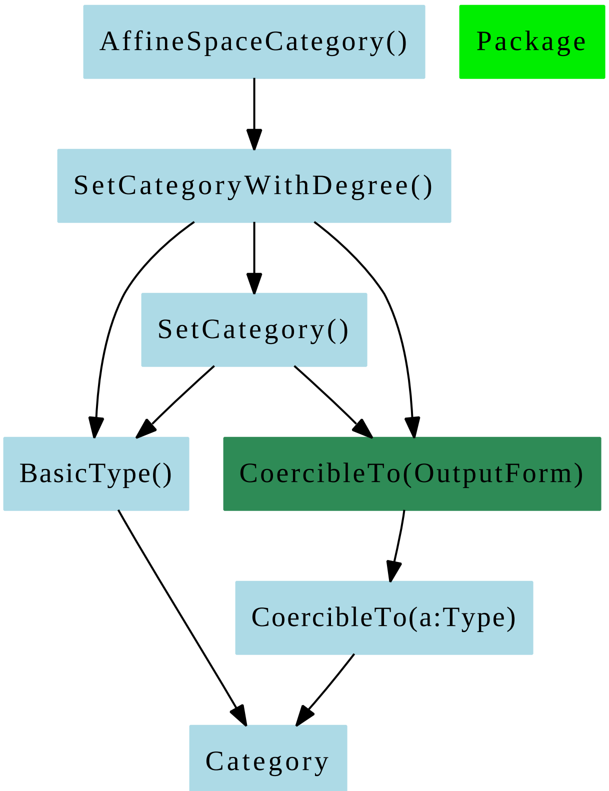
———-

— ABELMON.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SetCategory()"
"AbelianSemiGroup()" -> "RepeatedDoubling(AbelianSemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" ->
    "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"RepeatedDoubling(AbelianSemiGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianSemiGroup)" -> "RepeatedDoubling(a:SetCategory)"

"RepeatedDoubling(a:SetCategory)" [color="#00EE00"];
"RepeatedDoubling(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];
}
```

———-

5.0.91 AffineSpaceCategory (AFSPCAT)



— **AffineSpaceCategory.input** —

```
)set break resume
)sys rm -f AffineSpaceCategory.output
)spool AffineSpaceCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AffineSpaceCategory
--R
--R AffineSpaceCategory(K: Field) is a category constructor
--R Abbreviation for AffineSpaceCategory is AFSPCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for AFSPCAT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                affinePoint : List(K) -> %
```

```
--R coerce : List(K) -> %                coerce : % -> List(K)
--R coerce : % -> OutputForm             conjugate : % -> %
--R definingField : % -> K               degree : % -> PositiveInteger
--R ?.? : (%,Integer) -> K               hash : % -> SingleInteger
--R latex : % -> String                  list : % -> List(K)
--R orbit : % -> List(%)                 origin : () -> %
--R pointValue : % -> List(K)            rational? : % -> Boolean
--R setelt : (%,Integer,K) -> K          ?~=? : (%,%) -> Boolean
--R conjugate : (%,NonNegativeInteger) -> %
--R orbit : (%,NonNegativeInteger) -> List(%)
--R rational? : (%,NonNegativeInteger) -> Boolean
--R removeConjugate : List(%) -> List(%)
--R removeConjugate : (List(%),NonNegativeInteger) -> List(%)
--R
--E 1

)spool
)lisp (bye)
```

— **AffineSpaceCategory.help** —

```
====================================================================
AffineSpaceCategory examples
====================================================================

The following is all the categories and domains related to projective
space and part of the PAFF package

See Also:
o )show AffineSpaceCategory
```

See:

⇐ "SetCategoryWithDegree" (SETCATD) 4.0.86 on page 358

Exports:

?.?	?=?	?~=?	affinePoint	coerce
conjugate	definingField	degree	hash	latex
list	orbit	origin	pointValue	rational?
removeConjugate	setelt			

These are directly exported but not implemented

```
?.? : (%,Integer) -> K
affinePoint : List K -> %
coerce : % -> List K
coerce : List K -> %
conjugate : % -> %
conjugate : (%,NNI) -> %
```

```
definingField : % -> K
list : % -> List K
orbit : % -> List %
orbit : (%,NNI) -> List %
origin : () -> %
pointValue : % -> List K
rational? : % -> Boolean
rational? : (%,NNI) -> Boolean
removeConjugate : (List %,NNI) -> List %
removeConjugate : List % -> List %
setelt : (%,Integer,K) -> K
```

These exports come from (p358) SetCategoryWithDegree

```
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
coerce : % -> OutputForm
degree : % -> PositiveInteger
hash : % -> SingleInteger
latex : % -> String
```

— **AffineSpaceCategory.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#AFSPCAT">
AffineSpaceCategory (AFSPCAT)</a></h2>
</body>
```

———-

— **category AFSPCAT AffineSpaceCategory** —

```
)abbrev category AFSPCAT AffineSpaceCategory
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ The following is all the categories and domains related to projective
++ space and part of the PAFF package

AffineSpaceCategory(K) : Category == SIG where
  K : Field

  LIST ==> List
  INT  ==> Integer
  NNI  ==> NonNegativeInteger

  SIG ==> SetCategoryWithDegree with

    origin : () -> %

    affinePoint : LIST(K) -> %
      ++ affinePoint creates a affine point from a list
```

```
definingField : % -> K

coerce : % -> List(K)

coerce : LIST(K) -> %
  ++ coerce a list of K to a affine point.

orbit : % -> LIST %
  ++ orbit(p) returns the orbit of the point p according to the
  ++ characteristic of K, that is, for q= char K,
  ++ orbit(p) = \{ p, p**q, p**(q**2), p**(q**3), ..... \}

orbit : (%,NNI) -> LIST %
  ++ orbit(p,n) returns the orbit of the point p according to n, that is
  ++ orbit(p,n) = \{ p, p**n, p**(n**2), p**(n**3), ..... \}

conjugate : (%,NNI) -> %
  ++ conjugate(p,n) returns p**n, that is all the coordinates of p
  ++ to the power of n

conjugate : % -> %
  ++ conjugate(p) returns conjugate(p,n) where n is the characteristic
  ++ of the ground field.

removeConjugate : (LIST(%),NNI) -> LIST(%)
  ++ removeConjugate(lp,n) returns a list of points such that no points
  ++ in the list is the conjugate (according to n) of another point.

removeConjugate : LIST(%)  -> LIST(%)
  ++ removeConjugate(lp) returns removeConjugate(lp,n)
  ++ where n is the characteristic of the ground field.

rational? : (%,NNI) -> Boolean
  ++ rational?(p,n) test if the point is rational according to n.

rational? : % -> Boolean
  ++ rational?(p) test if the point is rational according to the
  ++ characteristic of the ground field.

list  : % -> LIST(K)
  ++ list returns the list of the coordinates

elt : (%,INT) -> K
  ++ elt returns the value of a specified coordinates

setelt : (%,INT,K) -> K
  ++ setelt sets the value of a specified coordinates

pointValue : % -> LIST(K)
  ++ pointValue returns the coordinates of the point or of the point
  ++ of origin that represent an infinitly close point
```

———

— AFSPCAT.dotabb —

```
"AFSPCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=AFSPCAT"];
"AFSPCAT" -> "SETCATD"
```

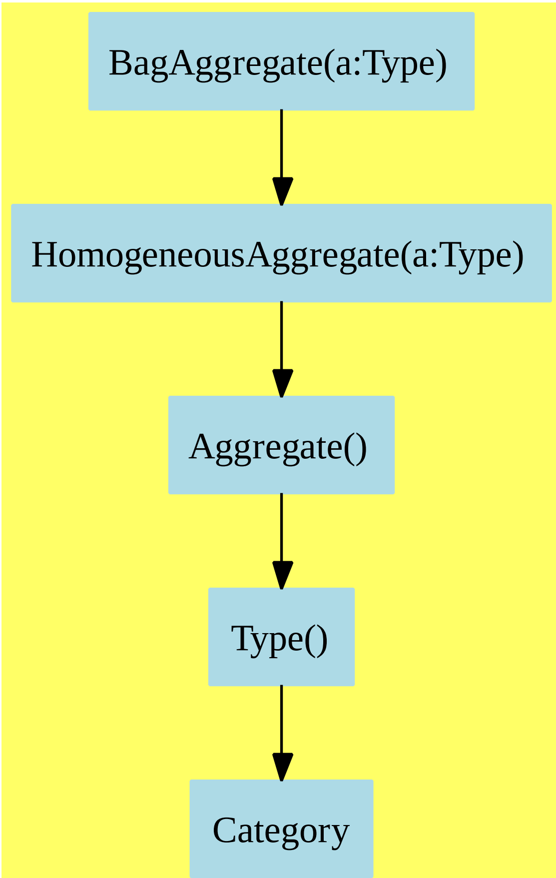
— AFSPCAT.dotfull —

```
"AffineSpaceCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=AFSPCAT"];
"AffineSpaceCategory()" -> "SetCategoryWithDegree()"
```

— AFSPCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"AffineSpaceCategory()" [color=lightblue];
"AffineSpaceCategory()" -> "SetCategoryWithDegree()"

"SetCategoryWithDegree()" [color=lightblue];
"SetCategoryWithDegree()" -> "BasicType()"
"SetCategoryWithDegree()" -> "CoercibleTo(OutputForm)"
"SetCategoryWithDegree()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Package" [color="#00EE00"];

"Category" [color=lightblue];

}
```

5.0.92 BagAggregate (BGAGG)



— **BagAggregate.input** —

```
)set break resume
)sys rm -f BagAggregate.output
)spool BagAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BagAggregate
--R
--R BagAggregate(S: Type) is a category constructor
--R Abbreviation for BagAggregate is BGAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for BGAGG
--R
--R------------------------------- Operations --------------------------------
--R bag : List(S) -> %                    copy : % -> %
```

```
--R empty : () -> %                        empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                 extract! : % -> S
--R insert! : (S,%) -> %                   inspect : % -> S
--R latex : % -> String if S has SETCAT   map : ((S -> S),%) -> %
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(S) if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

— **BagAggregate.help** —

```
====================================================================
BagAggregate examples
====================================================================

A bag aggregate is an aggregate for which one can insert and extract
objects, and where the order in which objects are inserted determines the
order of extraction. Examples of bags are stacks, queues, and dequeues.

See Also:
o )show BagAggregate
```

See:

⇒ "DictionaryOperations" (DIOPS) 6.0.108 on page 512
⇒ "PriorityQueueAggregate" (PRQAGG) 6.0.116 on page 617
⇒ "QueueAggregate" (QUAGG) 6.0.117 on page 622

⇒ "StackAggregate" (SKAGG) 6.0.119 on page 636
⇐ "HomogeneousAggregate" (HOAGG) 4.0.73 on page 280

Exports:

any?	bag	coerce	copy	count
empty	empty?	eq?	eval	every?
extract!	hash	insert!	inspect	latex
less?	map	map!	member?	members
more?	parts	sample	size?	#?
?=?	?~=?			

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are directly exported but not implemented:

```
extract! : % -> S
insert! : (S,%) -> %
inspect : % -> S
```

These are implemented by this category:

```
bag : List S -> %
```

These exports come from (p280) HomogeneousAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
members : % -> List S if $ has finiteAggregate
```

```
 member? : (S,%) -> Boolean
          if S has SETCAT and $ has finiteAggregate
 more? : (%,NonNegativeInteger) -> Boolean
 parts : % -> List S if $ has finiteAggregate
 sample : () -> %
 size? : (%,NonNegativeInteger) -> Boolean
 \#? : % -> NonNegativeInteger if $ has finiteAggregate
 ?=? : (%,%) -> Boolean if S has SETCAT
 ?~=? : (%,%) -> Boolean if S has SETCAT
```

— BagAggregate.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#BGAGG">
BagAggregate (BGAGG)</a></h2>
</body>
```

— category BGAGG BagAggregate —

```
)abbrev category BGAGG BagAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ A bag aggregate is an aggregate for which one can insert and extract
++ objects, and where the order in which objects are inserted determines
++ the order of extraction.
++ Examples of bags are stacks, queues, and dequeues.

BagAggregate(S) : Category == SIG where
  S : Type

  SIG ==> HomogeneousAggregate S with

    shallowlyMutable
      ++ shallowlyMutable means that elements of bags may be
      ++ destructively changed.

    bag : List S -> %
      ++ bag([x,y,...,z]) creates a bag with elements x,y,...,z.

    extract_! : % -> S
      ++ extract!(u) destructively removes a (random) item from bag u.

    insert_! : (S,%) -> %
      ++ insert!(x,u) inserts item x into bag u.

    inspect : % -> S
      ++ inspect(u) returns an (random) element from a bag.

   add
```

```
  bag(l) ==
    x:=empty()
    for s in l repeat x:=insert_!(s,x)
    x
```

— COQ BGAGG —

```
(* category BGAGG *)
(*

   bag: List S -> %
   bag(l) ==
     x:=empty()
     for s in l repeat x:=insert_!(s,x)
     x
*)
```

— BGAGG.dotabb —

```
"BGAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=BGAGG"];
"BGAGG" -> "HOAGG"
```

— BGAGG.dotfull —

```
"BagAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BGAGG"];
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

"BagAggregate(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=BGAGG"];
"BagAggregate(a:SetCategory)" -> "BagAggregate(a:Type)"
```

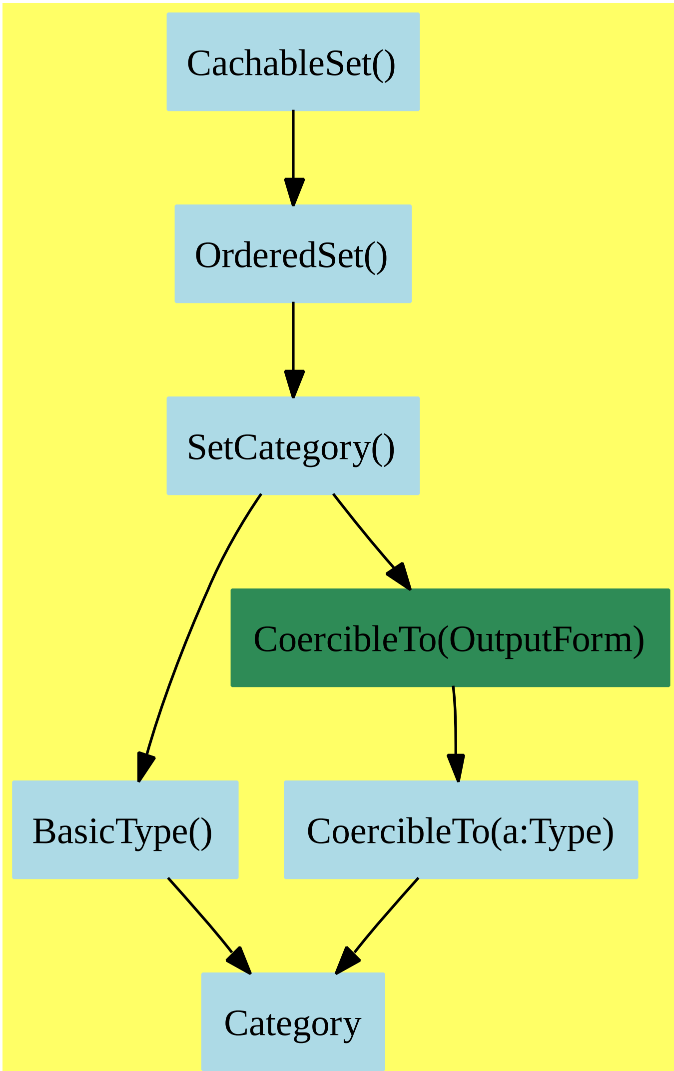
— BGAGG.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"
```

```
"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

———-

5.0.93 CachableSet (CACHSET)



— **CachableSet.input** —

```
)set break resume
)sys rm -f CachableSet.output
)spool CachableSet.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show CachableSet
--R
--R CachableSet is a category constructor
```

```
--R Abbreviation for CachableSet is CACHSET
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for CACHSET
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean              ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean              ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean             coerce : % -> OutputForm
--R hash : % -> SingleInteger           latex : % -> String
--R max : (%,%) -> %                    min : (%,%) -> %
--R position : % -> NonNegativeInteger    ?~=? : (%,%) -> Boolean
--R setPosition : (%,NonNegativeInteger) -> Void
--R
--E 1

)spool
)lisp (bye)
```

———-

— CachableSet.help —

```
====================================================================
CachableSet examples
====================================================================

A cachable set is a set whose elements keep an integer as part
of their structure.

See Also:
o )show CachableSet
```

———-

See:

⇐ "OrderedSet" (ORDSET) 4.0.80 on page 326

Exports:

coerce	hash	latex	max	min
position	setPosition	?~=?	?<?	?<=?
?=?	?>?	?>=?		

These are directly exported but not implemented:

```
position : % -> NonNegativeInteger
setPosition : (%,NonNegativeInteger) -> Void
```

These exports come from (p326) OrderedSet():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
```

```
 ?~=? : (%,%) -> Boolean
 ?<? : (%,%) -> Boolean
 ?<=? : (%,%) -> Boolean
 ?=? : (%,%) -> Boolean
 ?>? : (%,%) -> Boolean
 ?>=? : (%,%) -> Boolean
```

— CachableSet.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#CACHSET">
CachableSet (CACHSET)</a></h2>
</body>
```

— category CACHSET CachableSet —

```
)abbrev category CACHSET CachableSet
++ Author: Manuel Bronstein
++ Date Created: 31 Oct 1988
++ Date Last Updated: 14 May 1991
++ Description:
++ A cachable set is a set whose elements keep an integer as part
++ of their structure.

CachableSet() : Category == SIG where

  SIG ==> OrderedSet with

    position : % -> NonNegativeInteger
      ++ position(x) returns the integer n associated to x.

    setPosition : (%, NonNegativeInteger) -> Void
      ++ setPosition(x, n) associates the integer n to x.
```

— CACHSET.dotabb —

```
"CACHSET"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CACHSET"];
"CACHSET" -> "ORDSET"
```

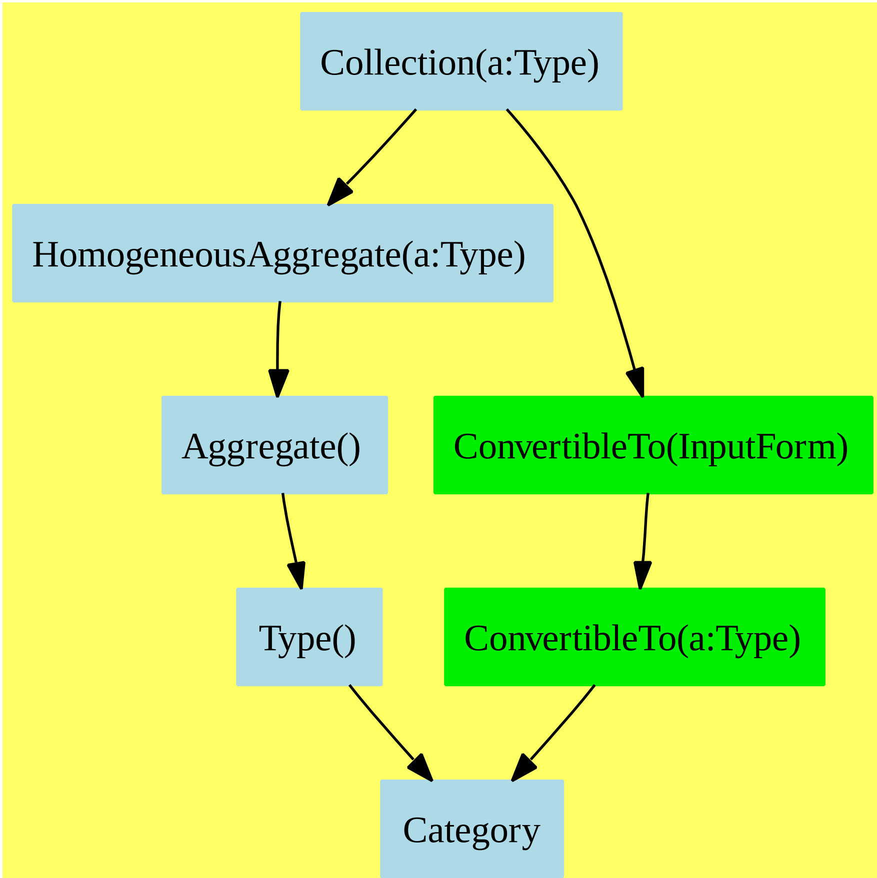
— CACHSET.dotfull —

```
"CachableSet()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CACHSET"];
"CachableSet()" -> "OrderedSet()"
```

— **CACHSET.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"CachableSet()" [color=lightblue];
"CachableSet()" -> "OrderedSet()"

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

5.0.94 Collection (CLAGG)



— **Collection.input** —

```
)set break resume
)sys rm -f Collection.output
)spool Collection.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Collection
--R
--R Collection(S: Type) is a category constructor
--R Abbreviation for Collection is CLAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for CLAGG
--R
--R------------------------------- Operations --------------------------------
--R construct : List(S) -> %              copy : % -> %
```

```
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                latex : % -> String if S has SETCAT
--R map : ((S -> S),%) -> %               sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT(INFORM)
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R hash : % -> SingleInteger if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(S) if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

— Collection.help —

```
====================================================================
Collection examples
====================================================================

A collection is a homogeneous aggregate which can built from
list of members. The operation used to build the aggregate is
generically named construct. However, each collection provides
its own special function with the same name as the data type,
except with an initial lower case letter, for example,
list for List, flexibleArray for FlexibleArray, and so on.

See Also:
```

```
o )show Collection
```

———-

See:

⇒ "DictionaryOperations" (DIOPS) 6.0.108 on page 512
⇒ "LinearAggregate" (LNAGG) 6.0.111 on page 531
⇒ "PolynomialSetCategory" (PSETCAT) 6.0.115 on page 598
⇒ "SetAggregate" (SETAGG) 6.0.118 on page 628
⇐ "ConvertibleTo" (KONVERT) 2.0.17 on page 47
⇐ "HomogeneousAggregate" (HOAGG) 4.0.73 on page 280

Exports:

any?	coerce	construct	copy	convert
count	empty	empty?	eq?	eval
every?	find	hash	latex	less?
map	map!	member?	members	more?
parts	reduce	remove	removeDuplicates	sample
select	size?	#?	?=?	?~=?

Attributes exported:

- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.

These are directly exported but not implemented:

```
construct: List S -> %
```

These are implemented by this category:

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
find : ((S -> Boolean),%) -> Union(S,"failed")
reduce : (((S,S) -> S),%,S,S) -> S
         if S has SETCAT and $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
removeDuplicates : % -> %
         if S has SETCAT and $ has finiteAggregate
select : ((S -> Boolean),%) -> % if $ has finiteAggregate
#? : % -> NonNegativeInteger if $ has finiteAggregate
```

These exports come from (p280) HomogeneousAggregate(S:Type):

```
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
```

```
            if S has SETCAT and $ has finiteAggregate
 empty : () -> %
 empty? : % -> Boolean
 eq? : (%,%) -> Boolean
 eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
 eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
 eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
 eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
 every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
 hash : % -> SingleInteger if S has SETCAT
 latex : % -> String if S has SETCAT
 less? : (%,NonNegativeInteger) -> Boolean
 map : ((S -> S),%) -> %
 map! : ((S -> S),%) -> % if $ has shallowlyMutable
 member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
 members : % -> List S if $ has finiteAggregate
 more? : (%,NonNegativeInteger) -> Boolean
 parts : % -> List S if $ has finiteAggregate
 sample : () -> %
 size? : (%,NonNegativeInteger) -> Boolean
 ?=? : (%,%) -> Boolean if S has SETCAT
 ?~=? : (%,%) -> Boolean if S has SETCAT
```

These exports come from (p47) ConvertibleTo(S:Type):

```
 convert : % -> InputForm if S has KONVERT INFORM
```

— Collection.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#CLAGG">
Collection (CLAGG)</a></h2>
</body>
```

— category CLAGG Collection —

```
)abbrev category CLAGG Collection
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ A collection is a homogeneous aggregate which can built from
++ list of members. The operation used to build the aggregate is
++ generically named construct. However, each collection
++ provides its own special function with the same name as the
++ data type, except with an initial lower case letter, For example,
++ list for List, flexibleArray for FlexibleArray, and so on.
```

```
Collection(S) : Category == SIG where
  S : Type

  SIG ==> HomogeneousAggregate(S) with

    construct : List S -> %
      ++ \axiom{construct(x,y,...,z)} returns the collection of elements
      ++ \axiom{x,y,...,z} ordered as given. Equivalently written as
      ++ \axiom{[x,y,...,z]$D}, where
      ++ D is the domain. D may be omitted for those of type List.

    find : (S->Boolean, %) -> Union(S, "failed")
      ++ \axiom{find(p,u)} returns the first x in u such that
      ++ \axiom{p(x)} is true, and "failed" otherwise.

    if % has finiteAggregate then

      reduce : ((S,S)->S,%) -> S
        ++reduce(f,u) reduces the binary operation f across u.
        ++For example, if u is \axiom{[x,y,...,z]} then \axiom{reduce(f,u)}
        ++returns \axiom{f(..f(f(x,y),...),z)}.
        ++Note that if u has one element x, \axiom{reduce(f,u)} returns x.
        ++Error: if u is empty.
        ++
        ++X )clear all
        ++X reduce(+,[C[i]*x**i for i in 1..5])

      reduce : ((S,S)->S,%,S) -> S
        ++ reduce(f,u,x) reduces the binary operation f across u, where x is
        ++ the identity operation of f.
        ++ Same as \axiom{reduce(f,u)} if u has 2 or more elements.
        ++ Returns \axiom{f(x,y)} if u has one element y,
        ++ x if u is empty.
        ++ For example, \axiom{reduce(+,u,0)} returns the
        ++ sum of the elements of u.

      remove : (S->Boolean,%) -> %
        ++ remove(p,u) returns a copy of u removing all elements x such that
        ++ \axiom{p(x)} is true.
        ++ Note that \axiom{remove(p,u) == [x for x in u | not p(x)]}.

      select : (S->Boolean,%) -> %
        ++ select(p,u) returns a copy of u containing only those elements
        ++ such \axiom{p(x)} is true.
        ++ Note that \axiom{select(p,u) == [x for x in u | p(x)]}.

      if S has SetCategory then

        reduce : ((S,S)->S,%,S,S) -> S
          ++ reduce(f,u,x,z) reduces the binary operation f across u,
          ++ stopping when an "absorbing element" z is encountered.
          ++ As for \axiom{reduce(f,u,x)}, x is the identity operation of f.
          ++ Same as \axiom{reduce(f,u,x)} when u contains no element z.
```

```
        ++ Thus the third argument x is returned when u is empty.

      remove : (S,%) -> %
        ++ remove(x,u) returns a copy of u with all
        ++ elements \axiom{y = x} removed.
        ++ Note that \axiom{remove(y,c) == [x for x in c | x ^= y]}.

      removeDuplicates : % -> %
        ++ removeDuplicates(u) returns a copy of u with all duplicates
        ++ removed.
        ++
        ++X removeDuplicates [1,4,2,-6,0,3,5,4,2,3]

    if S has ConvertibleTo InputForm then ConvertibleTo InputForm

   add

     if % has finiteAggregate then

       #c == # parts c

       count(f:S -> Boolean, c:%) == _+/[1 for x in parts c | f x]

       any?(f, c) == _or/[f x for x in parts c]

       every?(f, c) == _and/[f x for x in parts c]

       find(f:S -> Boolean, c:%) == find(f, parts c)

       reduce(f:(S,S)->S, x:%) == reduce(f, parts x)

       reduce(f:(S,S)->S, x:%, s:S) == reduce(f, parts x, s)

       remove(f:S->Boolean, x:%) ==
         construct remove(f, parts x)

       select(f:S->Boolean, x:%) ==
         construct select(f, parts x)

       if S has SetCategory then

         remove(s:S, x:%) == remove(y +-> y = s, x)

         reduce(f:(S,S)->S, x:%, s1:S, s2:S) == reduce(f, parts x, s1, s2)

         removeDuplicates(x) == construct removeDuplicates parts x
```

— COQ CLAGG —

```
(* category CLAGG *)
(*
```

```
   if % has finiteAggregate then

     #? : % -> NonNegativeInteger
     #c == # parts c

     count : ((S -> Boolean),%) -> NonNegativeInteger
     count(f:S -> Boolean, c:%) == _+/[1 for x in parts c | f x]

     any? : ((S -> Boolean),%) -> Boolean
     any?(f, c) == _or/[f x for x in parts c]

     every? : ((S -> Boolean),%) -> Boolean
     every?(f, c) == _and/[f x for x in parts c]

     find: (S->Boolean, %) -> Union(S, "failed")
     find(f:S -> Boolean, c:%) == find(f, parts c)

     reduce: ((S,S)->S,%) -> S
     reduce(f:(S,S)->S, x:%) == reduce(f, parts x)

     reduce: ((S,S)->S,%,S) -> S
     reduce(f:(S,S)->S, x:%, s:S) == reduce(f, parts x, s)

     remove: (S->Boolean,%) -> %
     remove(f:S->Boolean, x:%) ==
       construct remove(f, parts x)

     select: (S->Boolean,%) -> %
     select(f:S->Boolean, x:%) ==
       construct select(f, parts x)

     if S has SetCategory then

       remove: (S,%) -> %
       remove(s:S, x:%) == remove(y +-> y = s, x)

       reduce: ((S,S)->S,%,S,S) -> S
       reduce(f:(S,S)->S, x:%, s1:S, s2:S) == reduce(f, parts x, s1, s2)

       removeDuplicates: % -> %
       removeDuplicates(x) == construct removeDuplicates parts x
*)
```

— CLAGG.dotabb —

```
"CLAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=CLAGG"];
"CLAGG" -> "HOAGG"
```

— **CLAGG.dotfull** —

```
"Collection(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CLAGG"];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

"Collection(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=CLAGG"];
"Collection(a:SetCategory)" -> "Collection(a:Type)"

"Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet())"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=CLAGG"];
"Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet())"
  -> "Collection(a:Type)"
```

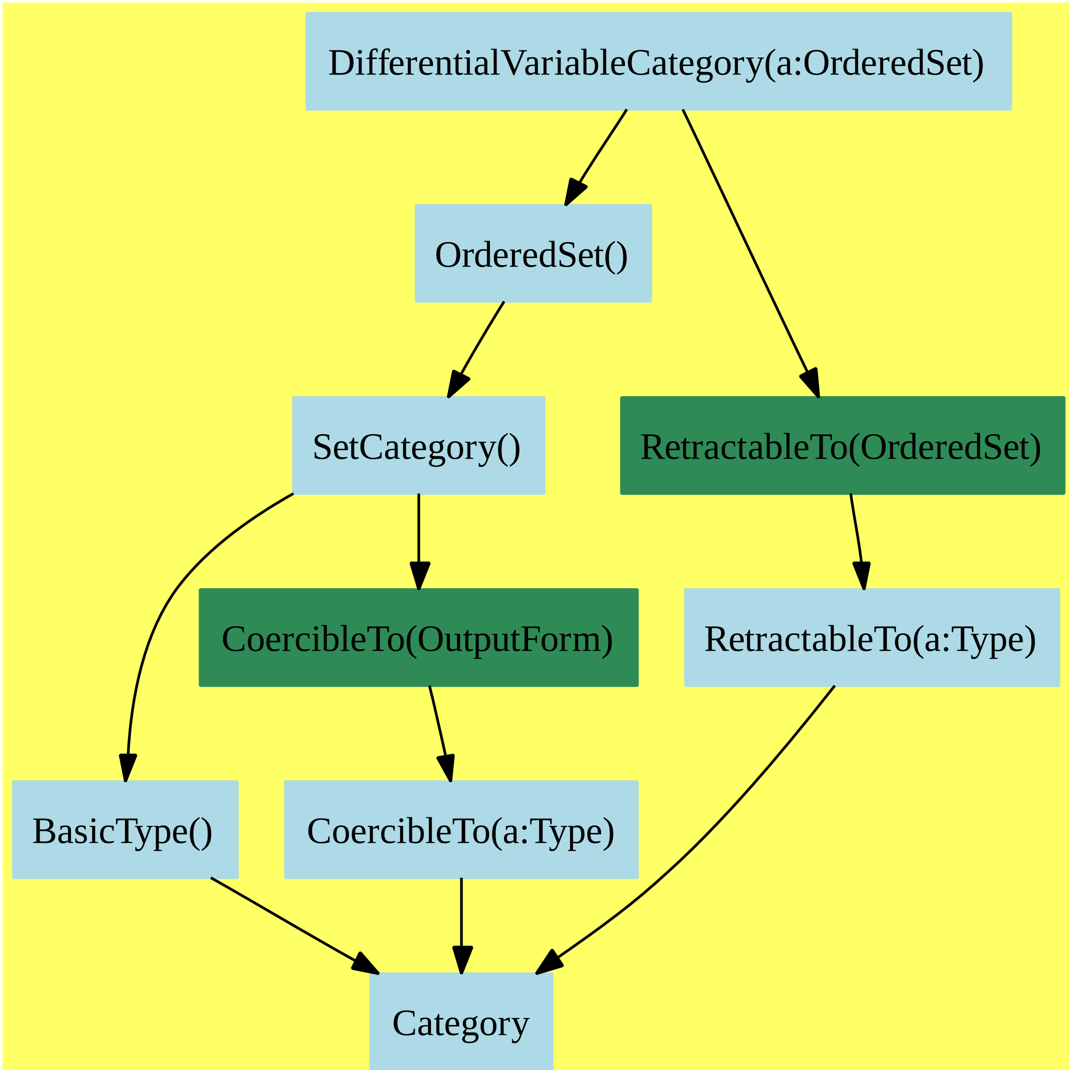
———

— **CLAGG.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"
"Collection(a:Type)" -> "ConvertibleTo(InputForm)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"ConvertibleTo(InputForm)" [color="#00EE00"];
"ConvertibleTo(InputForm)" -> "ConvertibleTo(a:Type)"

"ConvertibleTo(a:Type)" [color="#00EE00"];
"ConvertibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

———

5.0.95 DifferentialVariableCategory (DVARCAT)



— **DifferentialVariableCategory.input** —

```
)set break resume
)sys rm -f DifferentialVariableCategory.output
)spool DifferentialVariableCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DifferentialVariableCategory
--R
--R DifferentialVariableCategory(S: OrderedSet) is a category constructor
```

```
--R Abbreviation for DifferentialVariableCategory is DVARCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for DVARCAT
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               coerce : S -> %
--R coerce : % -> OutputForm              differentiate : % -> %
--R hash : % -> SingleInteger             latex : % -> String
--R max : (%,%) -> %                      min : (%,%) -> %
--R order : % -> NonNegativeInteger       retract : % -> S
--R variable : % -> S                     weight : % -> NonNegativeInteger
--R ?~=? : (%,%) -> Boolean
--R differentiate : (%,NonNegativeInteger) -> %
--R makeVariable : (S,NonNegativeInteger) -> %
--R retractIfCan : % -> Union(S,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— DifferentialVariableCategory.help —

```
====================================================================
DifferentialVariableCategory examples
====================================================================

DifferentialVariableCategory constructs the set of derivatives of a
given set of (ordinary) differential indeterminates. If x,...,y is
an ordered set of differential indeterminates, and the prime notation
is used for differentiation, then the set of derivatives (including
zero-th order) of the differential indeterminates is
    x, x', x'',..., y, y', y'',...
(Note that in the interpreter, the n-th derivative of y is displayed as
y with a subscript n.)  This set is viewed as a set of algebraic
indeterminates, totally ordered in a way compatible with differentiation
and the given order on the differential indeterminates.  Such a total
order is called a ranking of the differential indeterminates.

A domain in this category is needed to construct a differential
polynomial domain.  Differential polynomials are ordered by a ranking
on the derivatives, and by an order (extending the ranking) on the set
of differential monomials.  One may thus associate a domain in this
category with a ranking of the differential indeterminates, just as
one associates a domain in the category OrderedAbelianMonoidSup with
an ordering of the set of monomials in a set of algebraic indeterminates.
The ranking is specified through the binary relation <. For example, one
may define one derivative to be less than another by lexicographically
comparing first the order, then the given order of the differential
indeterminates appearing in the derivatives.  This is the default
```

```
implementation.

The notion of weight generalizes that of degree.  A polynomial domain
may be made into a graded ring if a weight function is given on the set
of indeterminates. Very often, a grading is the first step in ordering
the set of monomials.  For differential polynomial domains, this
constructor provides a function \spadfun{weight}, which allows the
assignment of a non-negative number to each derivative of a differential
indeterminate.  For example, one may define the weight of a derivative
to be simply its order (this is the default assignment). This weight
function can then be extended to the set of all differential polynomials,
providing a graded ring structure.

See Also:
o )show DifferentialVariableCategory
```

See:

⇐ "OrderedSet" (ORDSET) 4.0.80 on page 326
⇐ "RetractableTo" (RETRACT) 2.0.37 on page 110

Exports:

coerce	differentiate	hash	latex	makeVariable
max	min	order	retract	retractIfCan
variable	weight	?~=?	?<?	?<=?
?=?	?>?	?>=?		

These are directly exported but not implemented:

```
makeVariable : (S,NonNegativeInteger) -> %
order : % -> NonNegativeInteger
variable : % -> S
```

These are implemented by this category:

```
coerce : S -> %
coerce : % -> OutputForm
differentiate : % -> %
differentiate : (%,NonNegativeInteger) -> %
retract : % -> S
retractIfCan : % -> Union(S,"failed")
weight : % -> NonNegativeInteger
?<? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
```

These exports come from (p326) OrderedSet():

```
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

```
?~=? : (%,%) -> Boolean
```

These exports come from (p110) RetractableTo(S:OrderedSet):

See: Ritt [Ritt50], Hubbard and Lundell [Hubb]

— DifferentialVariableCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#DVARCAT">
DifferentialVariableCategory (DVARCAT)</a></h2>
</body>
```

— category DVARCAT DifferentialVariableCategory —

```
)abbrev category DVARCAT DifferentialVariableCategory
++ Author:  William Sit
++ Date Created: 19 July 1990
++ Date Last Updated: 13 September 1991
++ References:
++ Ritt50 "Differential Algebra"
++ Hubb "A First Look at Differential Algebra"
++ Description:
++ \spadtype{DifferentialVariableCategory} constructs the
++ set of derivatives of a given set of
++ (ordinary) differential indeterminates.
++ If x,...,y is an ordered set of differential indeterminates,
++ and the prime notation is used for differentiation, then
++ the set of derivatives (including
++ zero-th order) of the differential indeterminates is
++ x,\spad{x'},\spad{x''},..., y,\spad{y'},\spad{y''},...
++ (Note that in the interpreter, the n-th derivative of y is displayed as
++ y with a subscript n.)  This set is
++ viewed as a set of algebraic indeterminates, totally ordered in a
++ way compatible with differentiation and the given order on the
++ differential indeterminates.  Such a total order is called a
++ ranking of the differential indeterminates.
++
++ A domain in this category is needed to construct a differential
++ polynomial domain.  Differential polynomials are ordered
++ by a ranking on the derivatives,  and by an order (extending the
++ ranking) on
++ on the set of differential monomials.  One may thus associate
++ a domain in this category with a ranking of the differential
++ indeterminates, just as one associates a domain in the category
++ \spadtype{OrderedAbelianMonoidSup} with an ordering of the set of
++ monomials in a set of algebraic indeterminates.  The ranking
++ is specified through the binary relation \spadfun{<}.
++ For example, one may define
++ one derivative to be less than another by lexicographically comparing
++ first the \spadfun{order}, then the given order of the differential
++ indeterminates appearing in the derivatives.  This is the default
```

```
++ implementation.
++
++ The notion of weight generalizes that of degree.  A
++ polynomial domain may be made into a graded ring
++ if a weight function is given on the set of indeterminates,
++ Very often, a grading is the first step in ordering the set of
++ monomials.  For differential polynomial domains, this
++ constructor provides a function \spadfun{weight}, which
++ allows the assignment of a non-negative number to each derivative of a
++ differential indeterminate.  For example, one may define
++ the weight of a derivative to be simply its \spadfun{order}
++ (this is the default assignment).
++ This weight function can then be extended to the set of
++ all differential polynomials, providing a graded ring structure.

DifferentialVariableCategory(S) : Category == SIG where
  S : OrderedSet

  SIG ==> Join(OrderedSet, RetractableTo S) with

    makeVariable : (S, NonNegativeInteger) -> $
      ++ makeVariable(s, n) returns the n-th derivative of a
      ++ differential indeterminate s as an algebraic indeterminate.
      -- v:=makeVariable('s, 5)

    order : $ -> NonNegativeInteger
      ++ order(v) returns n if v is the n-th derivative of any
      ++ differential indeterminate.
      -- Example: order(v)

    variable : $ -> S
      ++ variable(v) returns s if v is any derivative of the differential
      ++ indeterminate s.
      -- Example: variable(v)

    weight : $ -> NonNegativeInteger
      ++ weight(v) returns the weight of the derivative v.
      -- Example: weight(v)

    differentiate : $ -> $
      ++ differentiate(v) returns the derivative of v.
      -- Example: differentiate(v)

    differentiate : ($, NonNegativeInteger) -> $
      ++ differentiate(v, n) returns the n-th derivative of v.
      -- Example: differentiate(v,2)

    coerce : S  -> $
      ++ coerce(s) returns s, viewed as the zero-th order derivative of s.
      -- Example: coerce('s); differentiate(%,5)

   add

    import NumberFormats
```

```
    coerce (s:S):$ == makeVariable(s, 0)

    differentiate v == differentiate(v, 1)

    differentiate(v, n) == makeVariable(variable v, n + order v)

    retractIfCan v == (zero?(order v) => variable v; "failed")

    v = u == (variable v = variable u) and (order v = order u)

    coerce(v:$):OutputForm ==
      a := variable(v)::OutputForm
      zero?(nn := order v) => a
      sub(a, outputForm nn)

    retract v ==
      zero?(order v) => variable v
      error "Not retractable"

    v < u ==
      -- the ranking below is orderly, and is the default --
      order v = order u => variable v < variable u
      order v < order u

    weight v == order v
      --  the default weight is just the order
```

———-

— COQ DVARCAT —

```
(* category DVARCAT *)
(*
    import NumberFormats

    coerce : S -> %
    coerce (s:S):$ == makeVariable(s, 0)

    differentiate : $ -> $
    differentiate v     == differentiate(v, 1)

    differentiate : ($, NonNegativeInteger) -> $
    differentiate(v, n) == makeVariable(variable v, n + order v)

    retractIfCan : % -> Union(S,"failed")
    retractIfCan v == (zero?(order v) => variable v; "failed")

    ?=? : (%,%) -> Boolean
    v = u == (variable v = variable u) and (order v = order u)

    coerce : % -> OutputForm
    coerce(v:$):OutputForm ==
```

```
      a := variable(v)::OutputForm
      zero?(nn := order v) => a
      sub(a, outputForm nn)

    retract : % -> S
    retract v ==
      zero?(order v) => variable v
      error "Not retractable"

    ?<? : (%,%) -> Boolean
    v < u ==
      -- the ranking below is orderly, and is the default --
      order v = order u => variable v < variable u
      order v < order u

    weight : $ -> NonNegativeInteger
    weight v == order v
      --  the default weight is just the order

*)
```

——

— DVARCAT.dotabb —

```
"DVARCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DVARCAT"];
"DVARCAT" -> "ORDSET"
"DVARCAT" -> "RETRACT"
```

——

— DVARCAT.dotfull —

```
"DifferentialVariableCategory(a:OrderedSet)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DVARCAT"];
"DifferentialVariableCategory(a:OrderedSet)" -> "OrderedSet()"
"DifferentialVariableCategory(a:OrderedSet)" -> "RetractableTo(OrderedSet)"
```

——

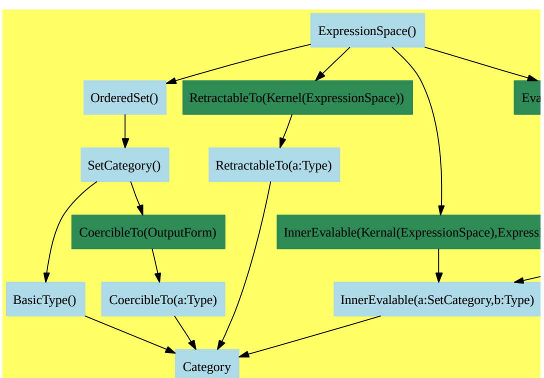
— DVARCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"DifferentialVariableCategory(a:OrderedSet)" [color=lightblue];
"DifferentialVariableCategory(a:OrderedSet)" -> "OrderedSet()"
"DifferentialVariableCategory(a:OrderedSet)" -> "RetractableTo(OrderedSet)"
```

```
"RetractableTo(OrderedSet)" [color=seagreen];
"RetractableTo(OrderedSet)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

———-

5.0.96 ExpressionSpace (ES)



— ExpressionSpace.input —

```
)set break resume
)sys rm -f ExpressionSpace.output
)spool ExpressionSpace.output
)set message test on
)set message auto off
```

```
)clear all

--S 1 of 1
)show ExpressionSpace
--R
--R ExpressionSpace is a category constructor
--R Abbreviation for ExpressionSpace is ES
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ES
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               belong? : BasicOperator -> Boolean
--R box : List(%) -> %                    box : % -> %
--R coerce : Kernel(%) -> %               coerce : % -> OutputForm
--R distribute : (%,%) -> %               distribute : % -> %
--R elt : (BasicOperator,List(%)) -> %    elt : (BasicOperator,%,%,%) -> %
--R elt : (BasicOperator,%,%) -> %        elt : (BasicOperator,%) -> %
--R eval : (%,Symbol,(% -> %)) -> %       eval : (%,List(%),List(%)) -> %
--R eval : (%,%,%) -> %                   eval : (%,Equation(%)) -> %
--R eval : (%,List(Equation(%))) -> %     eval : (%,Kernel(%),%) -> %
--R freeOf? : (%,Symbol) -> Boolean       freeOf? : (%,%) -> Boolean
--R hash : % -> SingleInteger             height : % -> NonNegativeInteger
--R is? : (%,Symbol) -> Boolean           is? : (%,BasicOperator) -> Boolean
--R kernel : (BasicOperator,%) -> %       kernels : % -> List(Kernel(%))
--R latex : % -> String                   map : ((% -> %),Kernel(%)) -> %
--R max : (%,%) -> %                      min : (%,%) -> %
--R paren : List(%) -> %                  paren : % -> %
--R retract : % -> Kernel(%)              subst : (%,Equation(%)) -> %
--R tower : % -> List(Kernel(%))          ?~=? : (%,%) -> Boolean
--R definingPolynomial : % -> % if $ has RING
--R elt : (BasicOperator,%,%,%,%) -> %
--R eval : (%,BasicOperator,(% -> %)) -> %
--R eval : (%,BasicOperator,(List(%) -> %)) -> %
--R eval : (%,List(BasicOperator),List((List(%) -> %))) -> %
--R eval : (%,List(BasicOperator),List((% -> %))) -> %
--R eval : (%,Symbol,(List(%) -> %)) -> %
--R eval : (%,List(Symbol),List((List(%) -> %))) -> %
--R eval : (%,List(Symbol),List((% -> %))) -> %
--R eval : (%,List(Kernel(%)),List(%)) -> %
--R even? : % -> Boolean if $ has RETRACT(INT)
--R kernel : (BasicOperator,List(%)) -> %
--R mainKernel : % -> Union(Kernel(%),"failed")
--R minPoly : Kernel(%) -> SparseUnivariatePolynomial(%) if $ has RING
--R odd? : % -> Boolean if $ has RETRACT(INT)
--R operator : BasicOperator -> BasicOperator
--R operators : % -> List(BasicOperator)
--R retractIfCan : % -> Union(Kernel(%),"failed")
--R subst : (%,List(Kernel(%)),List(%)) -> %
--R subst : (%,List(Equation(%))) -> %
--R
--E 1
```

```
)spool
)lisp (bye)
```

———-

— ExpressionSpace.help —

```
====================================================================
ExpressionSpace examples
====================================================================

An expression space is a set which is closed under certain operators.

See Also:
o )show ExpressionSpace
```

———-

See:

⇒ "FunctionSpace" (FS) 17.0.219 on page 1544
⇐ "Evalable" (EVALAB) 3.0.48 on page 151
⇐ "InnerEvalable" (IEVALAB) 2.0.22 on page 64
⇐ "OrderedSet" (ORDSET) 4.0.80 on page 326
⇐ "RetractableTo" (RETRACT) 2.0.37 on page 110

Exports:

belong?	box	coerce	definingPolynomial
distribute	elt	eval	even?
freeOf?	hash	height	is?
kernel	kernels	latex	mainKernel
map	max	min	minPoly
odd?	operator	operators	paren
retract	retractIfCan	subst	tower
?<?	?<=?	?=?	?>?
?>=?	?~=?		

These are directly exported but not implemented:

```
definingPolynomial : % -> % if $ has RING
eval : (%,List Symbol,List (List % -> %)) -> %
eval : (%,List Kernel %,List %) -> %
eval : (%,List %,List %) -> %
eval : (%,%,%) -> %
eval : (%,Equation %) -> %
eval : (%,Kernel %,%) -> %
kernels : % -> List Kernel %
minPoly : Kernel % -> SparseUnivariatePolynomial % if $ has RING
subst : (%,List Kernel %,List %) -> %
```

These are implemented by this category:

```
belong? : BasicOperator -> Boolean
box : List % -> %
```

```
box : % -> %
distribute : % -> %
distribute : (%,%) -> %
elt : (BasicOperator,%,%,%,%) -> %
elt : (BasicOperator,%,%,%) -> %
elt : (BasicOperator,%,%) -> %
elt : (BasicOperator,%) -> %
elt : (BasicOperator,List %) -> %
eval : (%,List Equation %) -> %
eval : (%,Symbol,(% -> %)) -> %
eval : (%,Symbol,(List % -> %)) -> %
eval : (%,BasicOperator,(% -> %)) -> %
eval : (%,BasicOperator,(List % -> %)) -> %
eval : (%,List BasicOperator,List (% -> %)) -> %
eval : (%,List Symbol,List (% -> %)) -> %
eval : (%,List BasicOperator,List (List % -> %)) -> %
even? : % -> Boolean if $ has RETRACT INT
freeOf? : (%,Symbol) -> Boolean
freeOf? : (%,%) -> Boolean
height : % -> NonNegativeInteger
is? : (%,BasicOperator) -> Boolean
is? : (%,Symbol) -> Boolean
kernel : (BasicOperator,%) -> %
kernel : (BasicOperator,List %) -> %
mainKernel : % -> Union(Kernel %,"failed")
map : ((% -> %),Kernel %) -> %
odd? : % -> Boolean if $ has RETRACT INT
operator : BasicOperator -> BasicOperator
operators : % -> List BasicOperator
paren : % -> %
paren : List % -> %
retract : % -> Kernel %
retractIfCan : % -> Union(Kernel %,"failed")
subst : (%,Equation %) -> %
subst : (%,List Equation %) -> %
tower : % -> List Kernel %
```

These exports come from (p326) OrderedSet():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?<? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
```

These exports come from (p110) RetractableTo(a:Type):

```
coerce : Kernel % -> %
```

These exports come from (p64) InnerEvalable(a:SetCategory,b:Type):

These exports come from (p151) Evalable(a:SetCategory):

— ExpressionSpace.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ES">
ExpressionSpace (ES)</a></h2>
</body>
```

— category ES ExpressionSpace —

```
)abbrev category ES ExpressionSpace
++ Category for domains on which operators can be applied
++ Author: Manuel Bronstein
++ Date Created: 22 March 1988
++ Date Last Updated: 27 May 1994
++ Description:
++ An expression space is a set which is closed under certain operators;

ExpressionSpace() : Category == SIG where

  N   ==> NonNegativeInteger
  K   ==> Kernel %
  OP  ==> BasicOperator
  SY  ==> Symbol
  PAREN  ==> "%paren"::SY
  BOX    ==> "%box"::SY
  DUMMYVAR ==> "%dummyVar"

  OS ==> OrderedSet
  RT ==> RetractableTo(K)
  IE ==> InnerEvalable(K, %)
  EV ==> Evalable(%)

  SIG ==> Join(OS,RT,IE,EV) with

    elt : (OP, %) -> %
      ++ elt(op,x) or op(x) applies the unary operator op to x.

    elt : (OP, %, %) -> %
      ++ elt(op,x,y) or op(x, y) applies the binary operator op to x and y.

    elt : (OP, %, %, %) -> %
      ++ elt(op,x,y,z) or op(x, y, z) applies the ternary operator op
      ++ to x, y and z.

    elt : (OP, %, %, %, %) -> %
      ++ elt(op,x,y,z,t) or op(x, y, z, t) applies the 4-ary operator op
```

```
  ++ to x, y, z and t.

elt : (OP, List %) -> %
  ++ elt(op,[x1,...,xn]) or op([x1,...,xn]) applies the n-ary operator
  ++ op to x1,...,xn.

subst : (%, Equation %) -> %
  ++ subst(f, k = g) replaces the kernel k by g formally in f.

subst : (%, List Equation %) -> %
  ++ subst(f, [k1 = g1,...,kn = gn]) replaces the kernels k1,...,kn
  ++ by g1,...,gn formally in f.

subst : (%, List K, List %) -> %
  ++ subst(f, [k1...,kn], [g1,...,gn]) replaces the kernels k1,...,kn
  ++ by g1,...,gn formally in f.

box : % -> %
  ++ box(f) returns f with a 'box' around it that prevents f from
  ++ being evaluated when operators are applied to it. For example,
  ++ \spad{log(1)} returns 0, but \spad{log(box 1)}
  ++ returns the formal kernel log(1).

box : List % -> %
  ++ box([f1,...,fn]) returns \spad{(f1,...,fn)} with a 'box'
  ++ around them that
  ++ prevents the fi from being evaluated when operators are applied to
  ++ them, and makes them applicable to a unary operator. For example,
  ++ \spad{atan(box [x, 2])} returns the formal kernel \spad{atan(x, 2)}.

paren : % -> %
  ++ paren(f) returns (f). This prevents f from
  ++ being evaluated when operators are applied to it. For example,
  ++ \spad{log(1)} returns 0, but \spad{log(paren 1)} returns the
  ++ formal kernel log((1)).

paren : List % -> %
  ++ paren([f1,...,fn]) returns \spad{(f1,...,fn)}. This
  ++ prevents the fi from being evaluated when operators are applied to
  ++ them, and makes them applicable to a unary operator. For example,
  ++ \spad{atan(paren [x, 2])} returns the formal
  ++ kernel \spad{atan((x, 2))}.

distribute : % -> %
  ++ distribute(f) expands all the kernels in f that are
  ++ formally enclosed by a \spadfunFrom{box}{ExpressionSpace}
  ++ or \spadfunFrom{paren}{ExpressionSpace} expression.

distribute : (%, %) -> %
  ++ distribute(f, g) expands all the kernels in f that contain g in their
  ++ arguments and that are formally
  ++ enclosed by a \spadfunFrom{box}{ExpressionSpace}
  ++ or a \spadfunFrom{paren}{ExpressionSpace} expression.
```

```
height : %  -> N
  ++ height(f) returns the highest nesting level appearing in f.
  ++ Constants have height 0. Symbols have height 1. For any
  ++ operator op and expressions f1,...,fn, \spad{op(f1,...,fn)} has
  ++ height equal to \spad{1 + max(height(f1),...,height(fn))}.

mainKernel : %  -> Union(K, "failed")
  ++ mainKernel(f) returns a kernel of f with maximum nesting level, or
  ++ if f has no kernels (that is, f is a constant).

kernels : %  -> List K
  ++ kernels(f) returns the list of all the top-level kernels
  ++ appearing in f, but not the ones appearing in the arguments
  ++ of the top-level kernels.

tower : %  -> List K
  ++ tower(f) returns all the kernels appearing in f, no matter
  ++ what their levels are.

operators : %  -> List OP
  ++ operators(f) returns all the basic operators appearing in f,
  ++ no matter what their levels are.

operator : OP -> OP
  ++ operator(op) returns a copy of op with the domain-dependent
  ++ properties appropriate for %.

belong? : OP -> Boolean
  ++ belong?(op) tests if % accepts op as applicable to its
  ++ elements.

is? : (%, OP)     -> Boolean
  ++ is?(x, op) tests if x is a kernel and is its operator is op.

is? : (%, SY) -> Boolean
  ++ is?(x, s) tests if x is a kernel and is the name of its
  ++ operator is s.

kernel : (OP, %) -> %
  ++ kernel(op, x) constructs op(x) without evaluating it.

kernel : (OP, List %) -> %
  ++ kernel(op, [f1,...,fn]) constructs \spad{op(f1,...,fn)} without
  ++ evaluating it.

map : (% -> %, K) -> %
  ++ map(f, k) returns \spad{op(f(x1),...,f(xn))} where
  ++ \spad{k = op(x1,...,xn)}.

freeOf? : (%, %)  -> Boolean
  ++ freeOf?(x, y) tests if x does not contain any occurrence of y,
  ++ where y is a single kernel.

freeOf? : (%, SY) -> Boolean
```

```
  ++ freeOf?(x, s) tests if x does not contain any operator
  ++ whose name is s.

eval : (%, List SY, List(% -> %)) -> %
  ++ eval(x, [s1,...,sm], [f1,...,fm]) replaces
  ++ every \spad{si(a)} in x by \spad{fi(a)} for any \spad{a}.

eval : (%, List SY, List(List % -> %)) -> %
  ++ eval(x, [s1,...,sm], [f1,...,fm]) replaces
  ++ every \spad{si(a1,...,an)} in x by
  ++ \spad{fi(a1,...,an)} for any \spad{a1},...,\spad{an}.

eval : (%, SY, List % -> %) -> %
  ++ eval(x, s, f) replaces every \spad{s(a1,..,am)} in x
  ++ by \spad{f(a1,..,am)} for any \spad{a1},...,\spad{am}.

eval : (%, SY, % -> %) -> %
  ++ eval(x, s, f) replaces every \spad{s(a)} in x by \spad{f(a)}
  ++ for any \spad{a}.

eval : (%, List OP, List(% -> %)) -> %
  ++ eval(x, [s1,...,sm], [f1,...,fm]) replaces
  ++ every \spad{si(a)} in x by \spad{fi(a)} for any \spad{a}.

eval : (%, List OP, List(List % -> %)) -> %
  ++ eval(x, [s1,...,sm], [f1,...,fm]) replaces
  ++ every \spad{si(a1,...,an)} in x by
  ++ \spad{fi(a1,...,an)} for any \spad{a1},...,\spad{an}.

eval : (%, OP, List % -> %) -> %
  ++ eval(x, s, f) replaces every \spad{s(a1,..,am)} in x
  ++ by \spad{f(a1,..,am)} for any \spad{a1},...,\spad{am}.

eval : (%, OP, % -> %) -> %
  ++ eval(x, s, f) replaces every \spad{s(a)} in x by \spad{f(a)}
  ++ for any \spad{a}.

if % has Ring then

  minPoly : K -> SparseUnivariatePolynomial %
    ++ minPoly(k) returns p such that \spad{p(k) = 0}.

  definingPolynomial : % -> %
    ++ definingPolynomial(x) returns an expression p such that
    ++ \spad{p(x) = 0}.

if % has RetractableTo Integer then

  even? : % -> Boolean
    ++ even? x is true if x is an even integer.

  odd? : % -> Boolean
    ++ odd? x is true if x is an odd integer.
```

```
add

  -- the 7 functions not provided are:
  --        kernels   minPoly   definingPolynomial
  --        coerce:K -> %  eval:(%, List K, List %) -> %
  --        subst:(%, List K, List %) -> %
  --        eval:(%, List Symbol, List(List % -> %)) -> %

  allKernels : % -> Set K

  listk : % -> List K

  allk : List % -> Set K

  unwrap : (List K, %) -> %

  okkernel : (OP, List %) -> %

  mkKerLists: List Equation % -> Record(lstk: List K, lstv:List %)

  oppren := operator(PAREN)$CommonOperators()

  opbox  := operator(BOX)$CommonOperators()

  box(x:%) == box [x]

  paren(x:%) == paren [x]

  belong? op == op = oppren or op = opbox

  listk f == parts allKernels f

  tower f == sort_! listk f

  allk l == reduce("union", [allKernels f for f in l], {})

  operators f == [operator k for k in listk f]

  height f == reduce("max", [height k for k in kernels f], 0)

  freeOf?(x:%, s:SY) == not member?(s, [name k for k in listk x])

  distribute x == unwrap([k for k in listk x | is?(k, oppren)], x)

  box(l:List %) == opbox l

  paren(l:List %) == oppren l

  freeOf?(x:%, k:%) == not member?(retract k, listk x)

  kernel(op:OP, arg:%) == kernel(op, [arg])

  elt(op:OP, x:%) == op [x]
```

```
elt(op:OP, x:%, y:%) == op [x, y]

elt(op:OP, x:%, y:%, z:%) == op [x, y, z]

elt(op:OP, x:%, y:%, z:%, t:%) == op [x, y, z, t]

eval(x:%, s:SY, f:List % -> %) == eval(x, [s], [f])

eval(x:%, s:OP, f:List % -> %) == eval(x, [name s], [f])

eval(x:%, s:SY, f:% -> %) ==
  eval(x, [s], [(y:List %):% +-> f(first y)])

eval(x:%, s:OP, f:% -> %) ==
  eval(x, [s], [(y:List %):% +-> f(first y)])

subst(x:%, e:Equation %) == subst(x, [e])

eval(x:%, ls:List OP, lf:List(% -> %)) ==
  eval(x, ls, [y +-> f(first y) for f in lf]$List(List % -> %))

eval(x:%, ls:List SY, lf:List(% -> %)) ==
  eval(x, ls, [y +-> f(first y) for f in lf]$List(List % -> %))

eval(x:%, ls:List OP, lf:List(List % -> %)) ==
  eval(x, [name s for s in ls]$List(SY), lf)

map(fn, k) ==
  (l := [fn x for x in argument k]$List(%)) = argument k => k::%
  (operator k) l

operator op ==
  is?(op, PAREN) => oppren
  is?(op, BOX) => opbox
  error "Unknown operator"

mainKernel x ==
  empty?(l := kernels x) => "failed"
  n := height(k := first l)
  for kk in rest l repeat
    if height(kk) > n then
      n := height kk
      k := kk
  k

-- takes all the kernels except for the dummy variables, which are second
-- arguments of rootOf's, integrals, sums and products which appear only
-- in their first arguments

allKernels f ==
  s := brace(l := kernels f)
  for k in l repeat
      t :=
          (u := property(operator k, DUMMYVAR)) case None =>
```

```
              arg := argument k
              s0  := remove_!(retract(second arg)@K, allKernels first arg)
              arg := rest rest arg
              n   := (u::None) pretend N
              if n > 1 then arg := rest arg
              union(s0, allk arg)
          allk argument k
      s := union(s, t)
  s

kernel(op:OP, args:List %) ==
  not belong? op => error "Unknown operator"
  okkernel(op, args)

okkernel(op, l) ==
  kernel(op, l, 1 + reduce("max", [height f for f in l], 0))$K :: %

elt(op:OP, args:List %) ==
  not belong? op => error "Unknown operator"
  ((u := arity op) case N) and (#args ^= u::N)
                              => error "Wrong number of arguments"
  (v := evaluate(op,args)$BasicOperatorFunctions1(%)) case % => v::%
  okkernel(op, args)

retract f ==
  (k := mainKernel f) case "failed" => error "not a kernel"
  k::K::% ^= f => error "not a kernel"
  k::K

retractIfCan f ==
  (k := mainKernel f) case "failed" => "failed"
  k::K::% ^= f => "failed"
  k

is?(f:%, s:SY) ==
  (k := retractIfCan f) case "failed" => false
  is?(k::K, s)

is?(f:%, op:OP) ==
  (k := retractIfCan f) case "failed" => false
  is?(k::K, op)

unwrap(l, x) ==
  for k in reverse_! l repeat
    x := eval(x, k, first argument k)
  x

distribute(x, y) ==
  ky := retract y
  unwrap([k for k in listk x |
          is?(k, "%paren"::SY) and member?(ky, listk(k::%))], x)

-- in case of conflicting substitutions, for example, [x = a, x = b],
-- the first one prevails.
```

```
    -- this is not part of the semantics of the function, but just
    -- a feature of this implementation.
    eval(f:%, leq:List Equation %) ==
      rec := mkKerLists leq
      eval(f, rec.lstk, rec.lstv)

    subst(f:%, leq:List Equation %) ==
      rec := mkKerLists leq
      subst(f, rec.lstk, rec.lstv)

    mkKerLists leq ==
      lk := empty()$List(K)
      lv := empty()$List(%)
      for eq in leq repeat
        (k := retractIfCan(lhs eq)@Union(K, "failed")) case "failed" =>
                          error "left hand side must be a single kernel"
        if not member?(k::K, lk) then
          lk := concat(k::K, lk)
          lv := concat(rhs eq, lv)
      [lk, lv]

    if % has RetractableTo Integer then

      intpred?: (%, Integer -> Boolean) -> Boolean

      even? x == intpred?(x, even?)

      odd? x  == intpred?(x, odd?)

      intpred?(x, pred?) ==
          (u := retractIfCan(x)@Union(Integer, "failed")) case Integer
                 and pred?(u::Integer)
```

———-

— COQ ES —

```
(* category ES *)
(*

-- the 7 functions not provided are:
--        kernels   minPoly   definingPolynomial
--        coerce:K -> %  eval:(%, List K, List %) -> %
--        subst:(%, List K, List %) -> %
--        eval:(%, List Symbol, List(List % -> %)) -> %

    oppren := operator(PAREN)$CommonOperators()
    opbox  := operator(BOX)$CommonOperators()

    box : % -> %
    box(x:%) == box [x]

    paren : % -> %
```

```
paren(x:%) == paren [x]

belong? : OP -> Boolean
belong? op == op = oppren or op = opbox

listk : % -> List K
listk f == parts allKernels f

tower : %  -> List K
tower f == sort_! listk f

allk : List % -> Set K
allk l == reduce("union", [allKernels f for f in l], {})

operators : % -> List OP
operators f  == [operator k for k in listk f]

height : %  -> N
height f == reduce("max", [height k for k in kernels f], 0)

freeOf? : (%, SY) -> Boolean
freeOf?(x:%, s:SY) == not member?(s, [name k for k in listk x])

distribute : % -> %
distribute x == unwrap([k for k in listk x | is?(k, oppren)], x)

box : List % -> %
box(l:List %) == opbox l

paren : List % -> %
paren(l:List %) == oppren l

freeOf? : (%, %)  -> Boolean
freeOf?(x:%, k:%) == not member?(retract k, listk x)

kernel : (OP, %) -> %
kernel(op:OP, arg:%) == kernel(op, [arg])

elt : (OP, %) -> %
elt(op:OP, x:%) == op [x]

elt : (OP, %, %) -> %
elt(op:OP, x:%, y:%) == op [x, y]

elt : (OP, %, %, %) -> %
elt(op:OP, x:%, y:%, z:%) == op [x, y, z]

elt : (OP, %, %, %, %) -> %
elt(op:OP, x:%, y:%, z:%, t:%) == op [x, y, z, t]

eval : (%, SY, List % -> %) -> %
eval(x:%, s:SY, f:List % -> %) == eval(x, [s], [f])

eval : (%, OP, List % -> %) -> %
```

```
    eval(x:%, s:OP, f:List % -> %) == eval(x, [name s], [f])

    eval : (%, SY, % -> %) -> %
    eval(x:%, s:SY, f:% -> %) ==
      eval(x, [s], [(y:List %):% +-> f(first y)])

    eval : (%, OP, % -> %) -> %
    eval(x:%, s:OP, f:% -> %) ==
      eval(x, [s], [(y:List %):% +-> f(first y)])

    subst : (%, Equation %) -> %
    subst(x:%, e:Equation %) == subst(x, [e])

    eval : (%, List OP, List(% -> %)) -> %
    eval(x:%, ls:List OP, lf:List(% -> %)) ==
      eval(x, ls, [y +-> f(first y) for f in lf]$List(List % -> %))

    eval : (%,List(Symbol),List((% -> %))) -> %
    eval(x:%, ls:List SY, lf:List(% -> %)) ==
      eval(x, ls, [y +-> f(first y) for f in lf]$List(List % -> %))

    eval : (%, List SY, List(% -> %)) -> %
    eval(x:%, ls:List OP, lf:List(List % -> %)) ==
      eval(x, [name s for s in ls]$List(SY), lf)

    map : (% -> %, K) -> %
    map(fn, k) ==
      (l := [fn x for x in argument k]$List(%)) = argument k => k::%
      (operator k) l

    operator : BasicOperator -> BasicOperator
    operator op ==
      is?(op, PAREN) => oppren
      is?(op, BOX) => opbox
      error "Unknown operator"

    mainKernel : % -> Union(K, "failed")
    mainKernel x ==
      empty?(l := kernels x) => "failed"
      n := height(k := first l)
      for kk in rest l repeat
        if height(kk) > n then
          n := height kk
          k := kk
      k

-- takes all the kernels except for the dummy variables, which are second
-- arguments of rootOf's, integrals, sums and products which appear only in
-- their first arguments

    allKernels: % -> Set K
    allKernels f ==
      s := brace(l := kernels f)
      for k in l repeat
```

```
      t :=
          (u := property(operator k, DUMMYVAR)) case None =>
              arg := argument k
              s0  := remove_!(retract(second arg)@K, allKernels first arg)
              arg := rest rest arg
              n   := (u::None) pretend N
              if n > 1 then arg := rest arg
              union(s0, allk arg)
          allk argument k
      s := union(s, t)
  s

kernel : (BasicOperator,List(%)) -> %
kernel(op:OP, args:List %) ==
  not belong? op => error "Unknown operator"
  okkernel(op, args)

okkernel : (BasicOperator, List %) -> %
okkernel(op, l) ==
  kernel(op, l, 1 + reduce("max", [height f for f in l], 0))$K :: %

elt : (BasicOperator, List %) -> %
elt(op:OP, args:List %) ==
  not belong? op => error "Unknown operator"
  ((u := arity op) case N) and (#args ^= u::N)
                              => error "Wrong number of arguments"
  (v := evaluate(op,args)$BasicOperatorFunctions1(%)) case % => v::%
  okkernel(op, args)

retract : % -> Kernel(%)
retract f ==
  (k := mainKernel f) case "failed" => error "not a kernel"
  k::K::% ^= f => error "not a kernel"
  k::K

retractIfCan : % -> Union(Kernel(%),"failed")
retractIfCan f ==
  (k := mainKernel f) case "failed" => "failed"
  k::K::% ^= f => "failed"
  k

is? : (%, Symbol) -> Boolean
is?(f:%, s:SY) ==
  (k := retractIfCan f) case "failed" => false
  is?(k::K, s)

is? : (%, BasicOperator) -> Boolean
is?(f:%, op:OP) ==
  (k := retractIfCan f) case "failed" => false
  is?(k::K, op)

unwrap : (List K, %) -> %
unwrap(l, x) ==
  for k in reverse_! l repeat
```

```
        x := eval(x, k, first argument k)
      x

    distribute : (%, %) -> %
    distribute(x, y) ==
      ky := retract y
      unwrap([k for k in listk x |
              is?(k, "%paren"::SY) and member?(ky, listk(k::%))], x)

    -- in case of conflicting substitutions, for example [x = a, x = b],
    -- the first one prevails.
    -- this is not part of the semantics of the function, but just
    -- a feature of this implementation.

    eval : (%,List(Equation(%))) -> %
    eval(f:%, leq:List Equation %) ==
      rec := mkKerLists leq
      eval(f, rec.lstk, rec.lstv)

    subst : (%, List Equation %) -> %
    subst(f:%, leq:List Equation %) ==
      rec := mkKerLists leq
      subst(f, rec.lstk, rec.lstv)

    mkKerLists: List Equation % -> Record(lstk: List K, lstv:List %)
    mkKerLists leq ==
      lk := empty()$List(K)
      lv := empty()$List(%)
      for eq in leq repeat
        (k := retractIfCan(lhs eq)@Union(K, "failed")) case "failed" =>
                          error "left hand side must be a single kernel"
        if not member?(k::K, lk) then
          lk := concat(k::K, lk)
          lv := concat(rhs eq, lv)
      [lk, lv]

    if % has RetractableTo Integer then

      even?: % -> Boolean
      even? x == intpred?(x, even?)

      odd? : % -> Boolean
      odd? x  == intpred?(x, odd?)

      intpred?: (%, Integer -> Boolean) -> Boolean
      intpred?(x, pred?) ==
           (u := retractIfCan(x)@Union(Integer, "failed")) case Integer
                  and pred?(u::Integer)

*)
```

———

— ES.dotabb —

```
"ES"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ES"];
"ES" -> "ORDSET"
"ES" -> "RETRACT"
"ES" -> "IEVALAB"
"ES" -> "EVALAB"
```

————-

— ES.dotfull —

```
"ExpressionSpace()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ES"];
"ExpressionSpace()" -> "OrderedSet()"
"ExpressionSpace()" -> "RetractableTo(Kernel(ExpressionSpace))"
"ExpressionSpace()" ->
    "InnerEvalable(Kernal(ExpressionSpace),ExpressionSpace)"
"ExpressionSpace()" -> "Evalable(ExpressionSpace)"
```

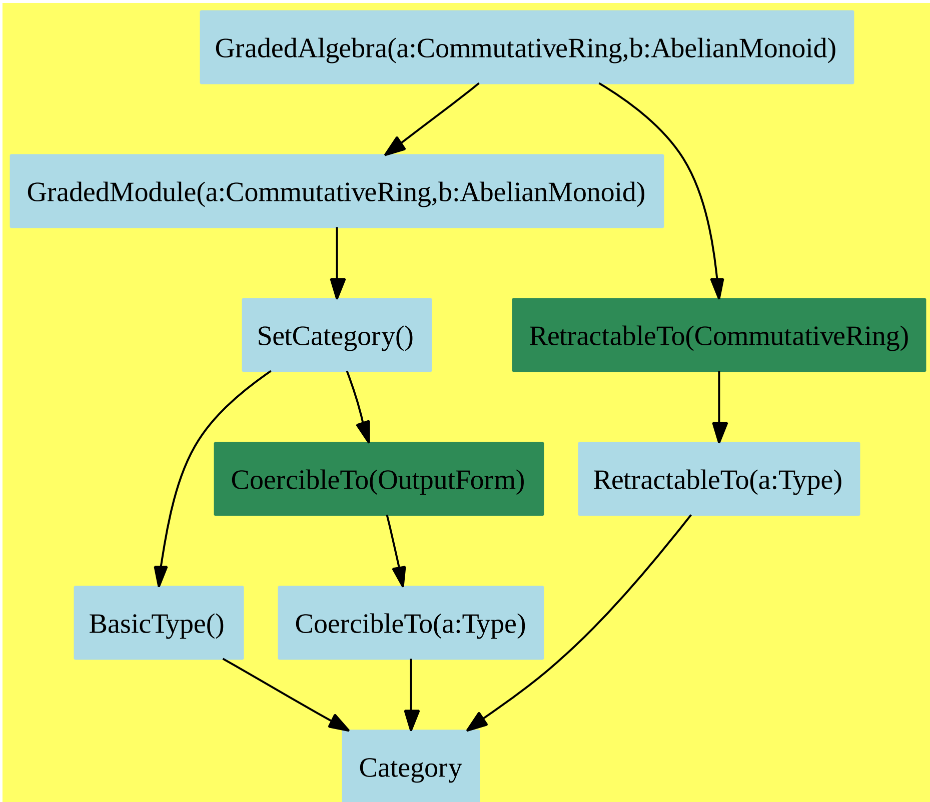
————-

— ES.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"ExpressionSpace()" [color=lightblue];
"ExpressionSpace()" -> "OrderedSet()"
"ExpressionSpace()" -> "RetractableTo(Kernel(ExpressionSpace))"
"ExpressionSpace()" ->
    "InnerEvalable(Kernal(ExpressionSpace),ExpressionSpace)"
"ExpressionSpace()" -> "Evalable(ExpressionSpace)"

"Evalable(ExpressionSpace)" [color=seagreen];
"Evalable(ExpressionSpace)" -> "Evalable(a:SetCategory)"

"Evalable(a:SetCategory)" [color=lightblue];
"Evalable(a:SetCategory)" -> "InnerEvalable(a:SetCategory,b:SetCategory)"

"InnerEvalable(Kernal(ExpressionSpace),ExpressionSpace)" [color=seagreen];
"InnerEvalable(Kernal(ExpressionSpace),ExpressionSpace)" ->
    "InnerEvalable(a:SetCategory,b:Type)"

"InnerEvalable(a:SetCategory,b:SetCategory)" [color=seagreen];
"InnerEvalable(a:SetCategory,b:SetCategory)" ->
    "InnerEvalable(a:SetCategory,b:Type)"

"InnerEvalable(a:SetCategory,b:Type)" [color=lightblue];
"InnerEvalable(a:SetCategory,b:Type)" -> "Category"
```

```
"RetractableTo(Kernel(ExpressionSpace))" [color=seagreen];
"RetractableTo(Kernel(ExpressionSpace))" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

5.0.97 GradedAlgebra (GRALG)



— **GradedAlgebra.input** —

```
)set break resume
)sys rm -f GradedAlgebra.output
)spool GradedAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show GradedAlgebra
--R
--R GradedAlgebra(R: CommutativeRing,E: AbelianMonoid) is a category constructor
--R Abbreviation for GradedAlgebra is GRALG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for GRALG
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
```

```
--R coerce : R -> %                      coerce : % -> OutputForm
--R degree : % -> E                      hash : % -> SingleInteger
--R latex : % -> String                  product : (%,%) -> %
--R retract : % -> R                     ?~=? : (%,%) -> Boolean
--R retractIfCan : % -> Union(R,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

— **GradedAlgebra.help** —

```
====================================================================
GradedAlgebra examples
====================================================================

GradedAlgebra(R,E) denotes "E-graded R-algebra". A graded algebra is a
graded module together with a degree preserving R-linear map, called
the product.

The name "product" is written out in full so inner and outer products
with the same mapping type can be distinguished by name.

See Also:
o )show GradedAlgebra
```

———-

See:

⇐ "GradedModule" (GRMOD) 4.0.71 on page 269
⇐ "RetractableTo" (RETRACT) 2.0.37 on page 110

Exports:

0	1	coerce	degree	hash
latex	product	retract	retractIfCan	?~=?
?*?	?+?	?-?	-?	?=?

These are directly exported but not implemented:

```
product : (%,%) -> %
```

These are implemented by this category:

```
0 : () -> %
1 : () -> %
?*? : (%,R) -> %
?*? : (R,%) -> %
```

These exports come from (p269) GradedModule(R, E)
where R:CommutativeRing and E:AbelianMonoid:

```
coerce : % -> OutputForm
```

```
degree : % -> E
hash : % -> SingleInteger
latex : % -> String
?~=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?-? : (%,%) -> %
-? : % -> %
?+? : (%,%) -> %
```

These exports come from (p110) RetractableTo(R:CommutativeRing):

```
coerce : R -> %
retract : % -> R
retractIfCan : % -> Union(R,"failed")
```

See: Encyclopedic Dictionary of Mathematics [Iyan60]

— **GradedAlgebra.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#GRALG">
GradedAlgebra (GRALG)</a></h2>
</body>
```

————

— **category GRALG GradedAlgebra** —

```
)abbrev category GRALG GradedAlgebra
++ Author: Stephen M. Watt
++ Date Created: May 20, 1991
++ Date Last Updated: May 20, 1991
++ References:
++ Iyan60 Encyclopedic Dictionary of Mathematics, MIT Press, 1977
++ Description:
++ GradedAlgebra(R,E) denotes ``E-graded R-algebra''.
++ A graded algebra is a graded module together with a degree preserving
++ R-linear map, called the product.
++
++ The name ``product'' is written out in full so inner and outer products
++ with the same mapping type can be distinguished by name.

GradedAlgebra(R,E) : Category == SIG where
  R : CommutativeRing
  E : AbelianMonoid

  SIG ==> Join(GradedModule(R, E),RetractableTo(R)) with

    1 : constant -> %
      ++ \spad{1} is the identity for \spad{product}.

    product : (%, %) -> %
      ++ product(a,b) is the degree-preserving R-linear product:
      ++
      ++ \spad{degree product(a,b) = degree a + degree b}
      ++ \spad{product(a1+a2,b) = product(a1,b) + product(a2,b)}
```

```
      ++ \spad{product(a,b1+b2) = product(a,b1) + product(a,b2)}
      ++ \spad{product(r*a,b) = product(a,r*b) = r*product(a,b)}
      ++ \spad{product(a,product(b,c)) = product(product(a,b),c)}

   add

     if not (R is %) then

       0: % == (0$R)::%

       1: % == 1$R::%

       (r: R)*(x: %) == product(r::%, x)

       (x: %)*(r: R) == product(x, r::%)
```

— COQ GRALG —

```
(* category GRALG *)
(*
   if not (R is %) then

      0 : () -> %
      0: % == (0$R)::%

      1 : () -> %
      1: % == 1$R::%

      ?*? : (R,%) -> %
      (r: R)*(x: %) == product(r::%, x)

      ?*? : (%,R) -> %
      (x: %)*(r: R) == product(x, r::%)

*)
```

— GRALG.dotabb —

```
"GRALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=GRALG"];
"GRALG" -> "GRMOD"
"GRALG" -> "RETRACT"
```

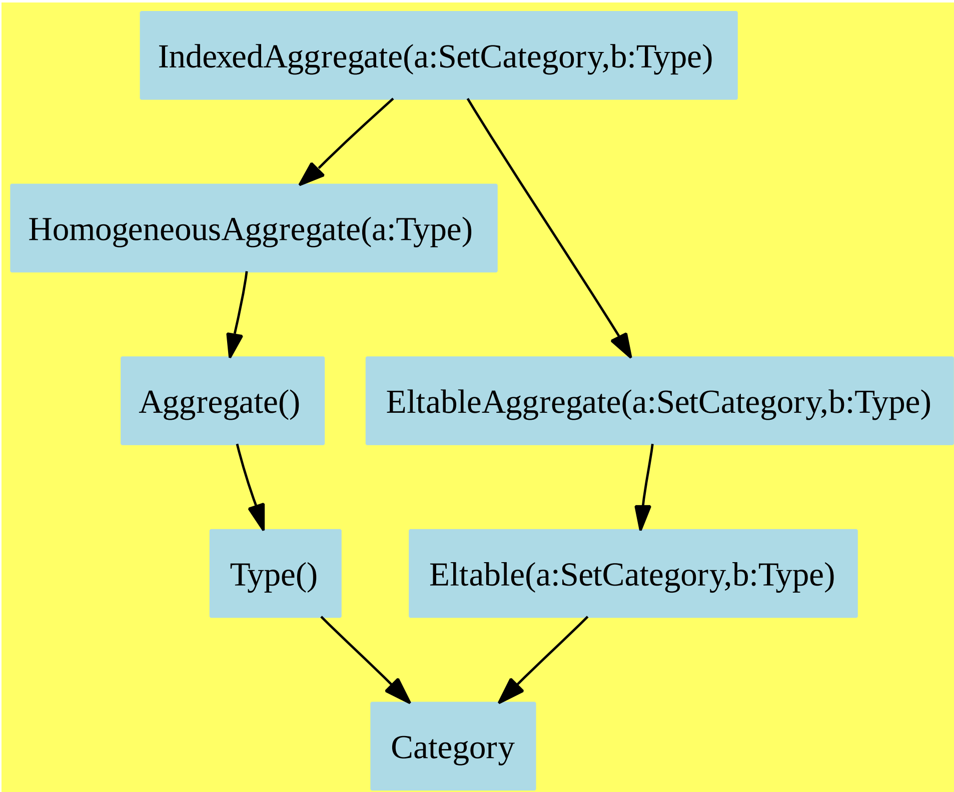
— GRALG.dotfull —

```
"GradedAlgebra(a:CommutativeRing,b:AbelianMonoid)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=GRALG"];
"GradedAlgebra(a:CommutativeRing,b:AbelianMonoid)" ->
    "GradedModule(a:CommutativeRing,b:AbelianMonoid)"
"GradedAlgebra(a:CommutativeRing,b:AbelianMonoid)" ->
    "RetractableTo(CommutativeRing)"
```

— **GRALG.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"GradedAlgebra(a:CommutativeRing,b:AbelianMonoid)" [color=lightblue];
"GradedAlgebra(a:CommutativeRing,b:AbelianMonoid)" ->
    "GradedModule(a:CommutativeRing,b:AbelianMonoid)"
"GradedAlgebra(a:CommutativeRing,b:AbelianMonoid)" ->
    "RetractableTo(CommutativeRing)"

"RetractableTo(CommutativeRing)" [color=seagreen];
"RetractableTo(CommutativeRing)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"GradedModule(a:CommutativeRing,b:AbelianMonoid)" [color=lightblue];
"GradedModule(a:CommutativeRing,b:AbelianMonoid)" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

5.0.98 IndexedAggregate (IXAGG)



— **IndexedAggregate.input** —

```
)set break resume
)sys rm -f IndexedAggregate.output
)spool IndexedAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IndexedAggregate
--R
--R IndexedAggregate(Index: SetCategory,Entry: Type) is a category constructor
--R Abbreviation for IndexedAggregate is IXAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for IXAGG
--R
--R------------------------------- Operations --------------------------------
--R copy : % -> %                         ?.? : (%,Index) -> Entry
--R elt : (%,Index,Entry) -> Entry        empty : () -> %
--R empty? : % -> Boolean                 entries : % -> List(Entry)
--R eq? : (%,%) -> Boolean                index? : (Index,%) -> Boolean
```

```
--R indices : % -> List(Index)           map : ((Entry -> Entry),%) -> %
--R qelt : (%,Index) -> Entry            sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if Entry has SETCAT
--R any? : ((Entry -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if Entry has SETCAT
--R count : (Entry,%) -> NonNegativeInteger if Entry has SETCAT and $ has finiteAggregate
--R count : ((Entry -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R entry? : (Entry,%) -> Boolean if $ has finiteAggregate and Entry has SETCAT
--R eval : (%,List(Entry),List(Entry)) -> % if Entry has EVALAB(Entry) and Entry has SETCAT
--R eval : (%,Entry,Entry) -> % if Entry has EVALAB(Entry) and Entry has SETCAT
--R eval : (%,Equation(Entry)) -> % if Entry has EVALAB(Entry) and Entry has SETCAT
--R eval : (%,List(Equation(Entry))) -> % if Entry has EVALAB(Entry) and Entry has SETCAT
--R every? : ((Entry -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,Entry) -> % if $ has shallowlyMutable
--R first : % -> Entry if Index has ORDSET
--R hash : % -> SingleInteger if Entry has SETCAT
--R latex : % -> String if Entry has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((Entry -> Entry),%) -> % if $ has shallowlyMutable
--R maxIndex : % -> Index if Index has ORDSET
--R member? : (Entry,%) -> Boolean if Entry has SETCAT and $ has finiteAggregate
--R members : % -> List(Entry) if $ has finiteAggregate
--R minIndex : % -> Index if Index has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(Entry) if $ has finiteAggregate
--R qsetelt! : (%,Index,Entry) -> Entry if $ has shallowlyMutable
--R setelt : (%,Index,Entry) -> Entry if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R swap! : (%,Index,Index) -> Void if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if Entry has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

— **IndexedAggregate.help** —

```
====================================================================
IndexedAggregate examples
====================================================================

An indexed aggregate is a many-to-one mapping of indices to entries.
For example, a one-dimensional-array is an indexed aggregate where
the index is an integer.  Also, a table is an indexed aggregate
where the indices and entries may have any type.

See Also:
o )show IndexedAggregate
```

See:

⇒ "DirectProductCategory" (DIRPCAT) 12.0.184 on page 1173
⇒ "LinearAggregate" (LNAGG) 6.0.111 on page 531
⇒ "TableAggregate" (TBAGG) 9.0.156 on page 972
⇐ "EltableAggregate" (ELTAGG) 3.0.47 on page 146
⇐ "HomogeneousAggregate" (HOAGG) 4.0.73 on page 280

Exports:

any?	coerce	copy	count	elt
empty	empty?	entries	entry?	eq?
eval	every?	fill!	first	hash
index?	indices	latex	less?	map
map!	maxIndex	member?	members	minIndex
more?	parts	qelt	qsetelt!	sample
setelt	size?	swap!	?.?	?~=?
#?	?=?			

Attributes exported:

- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
index? : (Index,%) -> Boolean
indices : % -> List Index
```

These are implemented by this category:

```
elt : (%,Index,Entry) -> Entry
entries : % -> List Entry
entry? : (Entry,%) -> Boolean
           if $ has finiteAggregate and Entry has SETCAT
fill! : (%,Entry) -> % if $ has shallowlyMutable
first : % -> Entry if Index has ORDSET
map : ((Entry -> Entry),%) -> %
map! : ((Entry -> Entry),%) -> % if $ has shallowlyMutable
maxIndex : % -> Index if Index has ORDSET
minIndex : % -> Index if Index has ORDSET
swap! : (%,Index,Index) -> Void if $ has shallowlyMutable
```

These exports come from (p280) HomogeneousAggregate(Entry:Type):

```
any? : ((Entry -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if Entry has SETCAT
copy : % -> %
count : (Entry,%) -> NonNegativeInteger
         if Entry has SETCAT and $ has finiteAggregate
```

```
count : ((Entry -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List Entry,List Entry) -> %
      if Entry has EVALAB Entry and Entry has SETCAT
eval : (%,Entry,Entry) -> %
      if Entry has EVALAB Entry and Entry has SETCAT
eval : (%,Equation Entry) -> %
      if Entry has EVALAB Entry and Entry has SETCAT
eval : (%,List Equation Entry) -> %
      if Entry has EVALAB Entry and Entry has SETCAT
every? : ((Entry -> Boolean),%) -> Boolean
      if $ has finiteAggregate
hash : % -> SingleInteger if Entry has SETCAT
latex : % -> String if Entry has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
member? : (Entry,%) -> Boolean
          if Entry has SETCAT and $ has finiteAggregate
members : % -> List Entry if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List Entry if $ has finiteAggregate
sample : () -> %
size? : (%,NonNegativeInteger) -> Boolean
?~=? : (%,%) -> Boolean if Entry has SETCAT
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if Entry has SETCAT
```

These exports come from (p146) EltableAggregate(Index:SetCategory,Entry:Type):

```
qelt : (%,Index) -> Entry
qsetelt! : (%,Index,Entry) -> Entry if $ has shallowlyMutable
setelt : (%,Index,Entry) -> Entry if $ has shallowlyMutable
?.? : (%,Index) -> Entry
```

— IndexedAggregate.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#IXAGG">
IndexedAggregate (IXAGG)</a></h2>
</body>
```

— category IXAGG IndexedAggregate —

```
)abbrev category IXAGG IndexedAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ An indexed aggregate is a many-to-one mapping of indices to entries.
++ For example, a one-dimensional-array is an indexed aggregate where
```

```
++ the index is an integer.  Also, a table is an indexed aggregate
++ where the indices and entries may have any type.

IndexedAggregate(Index, Entry) : Category == SIG where
  Index : SetCategory
  Entry : Type

  HE ==> HomogeneousAggregate(Entry)
  EA ==> EltableAggregate(Index,Entry)

  SIG ==> Join(HE, EA) with

    entries : % -> List Entry
      ++ entries(u) returns a list of all the entries of aggregate u
      ++ in no assumed order.
      -- to become entries: % -> Entry* and
      -- entries: % -> Iterator(Entry,Entry)

    index? : (Index,%) -> Boolean
      ++ index?(i,u) tests if i is an index of aggregate u.

    indices : % -> List Index
      ++ indices(u) returns a list of indices of aggregate u in no
      ++ particular order. to become indices:
      --  % -> Index* and indices: % -> Iterator(Index,Index).

    -- map : ((Entry,Entry)->Entry,%,%,Entry) -> %
    --   ++ exists c = map(f,a,b,x), i:Index where
    --   ++    c.i = f(a(i,x),b(i,x)) | index?(i,a) or index?(i,b)

    if Entry has SetCategory and % has finiteAggregate then

      entry? : (Entry,%) -> Boolean
        ++ entry?(x,u) tests if x equals \axiom{u . i} for some index i.

    if Index has OrderedSet then

      maxIndex : % -> Index
        ++ maxIndex(u) returns the maximum index i of aggregate u.
        ++ Note that in general,
        ++ \axiom{maxIndex(u) = reduce(max,[i for i in indices u])};
        ++ if u is a list, \axiom{maxIndex(u) = #u}.

      minIndex : % -> Index
        ++ minIndex(u) returns the minimum index i of aggregate u.
        ++ Note that in general,
        ++ \axiom{minIndex(a) = reduce(min,[i for i in indices a])};
        ++ for lists, \axiom{minIndex(a) = 1}.

      first : % -> Entry
        ++ first(u) returns the first element x of u.
        ++ Note that for collections, \axiom{first([x,y,...,z]) = x}.
        ++ Error: if u is empty.
```

```
    if % has shallowlyMutable then

      fill_! : (%,Entry) -> %
        ++ fill!(u,x) replaces each entry in aggregate u by x.
        ++ The modified u is returned as value.

      swap_! : (%,Index,Index) -> Void
        ++ swap!(u,i,j) interchanges elements i and j of aggregate u.
        ++ No meaningful value is returned.

   add

     elt(a, i, x) == (index?(i, a) => qelt(a, i); x)

     if % has finiteAggregate then

       entries x == parts x

       if Entry has SetCategory then

         entry?(x, a) == member?(x, a)

     if Index has OrderedSet then

       maxIndex a == "max"/indices(a)

       minIndex a == "min"/indices(a)

       first a == a minIndex a

     if % has shallowlyMutable then

       map(f, a) == map_!(f, copy a)

       map_!(f, a) ==
         for i in indices a repeat qsetelt_!(a, i, f qelt(a, i))
         a

       fill_!(a, x) ==
         for i in indices a repeat qsetelt_!(a, i, x)
         a

       swap_!(a, i, j) ==
         t := a.i
         qsetelt_!(a, i, a.j)
         qsetelt_!(a, j, t)
         void
```

— COQ IXAGG —

```
(* category IXAGG *)
```

```
(*

  elt : (%,Index,Entry) -> Entry
  elt(a, i, x) == (index?(i, a) => qelt(a, i); x)

  if % has finiteAggregate then

    entries: % -> List Entry
    entries x == parts x

    if Entry has SetCategory then

      entry?: (Entry,%) -> Boolean
      entry?(x, a) == member?(x, a)

  if Index has OrderedSet then

    maxIndex: % -> Index
    maxIndex a == "max"/indices(a)

    minIndex: % -> Index
    minIndex a == "min"/indices(a)

    first : % -> Entry
    first a  == a minIndex a

  if % has shallowlyMutable then

    map : ((Entry -> Entry),%) -> %
    map(f, a) == map_!(f, copy a)

    map! : ((Entry -> Entry),%) -> %
    map_!(f, a) ==
      for i in indices a repeat qsetelt_!(a, i, f qelt(a, i))
      a

    fill_!: (%,Entry) -> %
    fill_!(a, x) ==
      for i in indices a repeat qsetelt_!(a, i, x)
      a

    swap_!: (%,Index,Index) -> Void
    swap_!(a, i, j) ==
      t := a.i
      qsetelt_!(a, i, a.j)
      qsetelt_!(a, j, t)
      void

*)
```

— IXAGG.dotabb —

```
"IXAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=IXAGG"];
"IXAGG" -> "HOAGG"
"IXAGG" -> "ELTAGG"
```

———-

— IXAGG.dotfull —

```
"IndexedAggregate(a:SetCategory,b:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=IXAGG"];
"IndexedAggregate(a:SetCategory,b:Type)" ->
    "HomogeneousAggregate(a:Type)"
"IndexedAggregate(a:SetCategory,b:Type)" ->
    "EltableAggregate(a:SetCategory,b:Type)"

"IndexedAggregate(a:SetCategory,b:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=IXAGG"];
"IndexedAggregate(a:SetCategory,b:SetCategory)" ->
    "IndexedAggregate(a:SetCategory,b:Type)"

"IndexedAggregate(b:Integer,a:Type)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=IXAGG"];
"IndexedAggregate(b:Integer,a:Type)" ->
    "IndexedAggregate(a:SetCategory,b:Type)"
```

———-

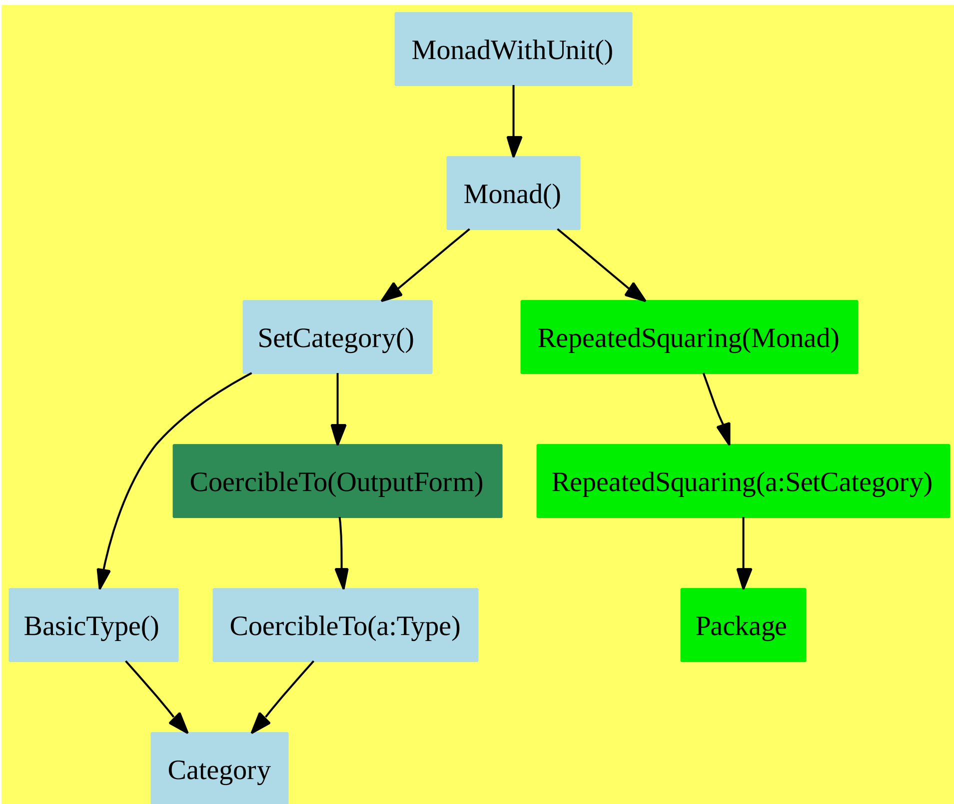
— IXAGG.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"IndexedAggregate(a:SetCategory,b:Type)" [color=lightblue];
"IndexedAggregate(a:SetCategory,b:Type)" ->
    "HomogeneousAggregate(a:Type)"
"IndexedAggregate(a:SetCategory,b:Type)" ->
    "EltableAggregate(a:SetCategory,b:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"EltableAggregate(a:SetCategory,b:Type)" [color=lightblue];
"EltableAggregate(a:SetCategory,b:Type)" -> "Eltable(a:SetCategory,b:Type)"

"Eltable(a:SetCategory,b:Type)" [color=lightblue];
"Eltable(a:SetCategory,b:Type)" -> "Category"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"
```

```
"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

———-

5.0.99 MonadWithUnit (MONADWU)



— **MonadWithUnit.input** —

```
)set break resume
)sys rm -f MonadWithUnit.output
)spool MonadWithUnit.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MonadWithUnit
```

```
--R
--R MonadWithUnit is a category constructor
--R Abbreviation for MonadWithUnit is MONADWU
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for MONADWU
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?**? : (%,NonNegativeInteger) -> %
--R ?**? : (%,PositiveInteger) -> %       ?=? : (%,%) -> Boolean
--R 1 : () -> %                           coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R leftRecip : % -> Union(%,"failed")    one? : % -> Boolean
--R recip : % -> Union(%,"failed")        ?~=? : (%,%) -> Boolean
--R leftPower : (%,NonNegativeInteger) -> %
--R leftPower : (%,PositiveInteger) -> %
--R rightPower : (%,NonNegativeInteger) -> %
--R rightPower : (%,PositiveInteger) -> %
--R rightRecip : % -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— **MonadWithUnit.help** —

```
====================================================================
MonadWithUnit examples
====================================================================

MonadWithUnit is the class of multiplicative monads with unit,
that is, sets with a binary operation and a unit element.

Axioms:
       leftIdentity("*":(%,%)->%,1)      1*x=x
       rightIdentity("*":(%,%)->%,1)     x*1=x

Common Additional Axioms:
       unitsKnown - if "recip" says "failed", it PROVES input wasn't a unit

See Also:
o )show MonadWithUnit
```

See:

⇒ "NonAssociativeRing" (NASRING) 9.0.148 on page 917
⇐ "Monad" (MONAD) 4.0.76 on page 303

Exports:

1	coerce	hash	latex	one?
recip	leftPower	leftRecip	rightPower	rightRecip
?*?	?~=?	?**?	?=?	

These are directly exported but not implemented:

```
1 : () -> %
leftRecip : % -> Union(%,"failed")
recip : % -> Union(%,"failed")
rightRecip : % -> Union(%,"failed")
```

These are implemented by this category:

```
leftPower : (%,NonNegativeInteger) -> %
one? : % -> Boolean
rightPower : (%,NonNegativeInteger) -> %
?**? : (%,NonNegativeInteger) -> %
```

These exports come from (p303) Monad():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
leftPower : (%,PositiveInteger) -> %
rightPower : (%,PositiveInteger) -> %
?**? : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?~=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
```

See: Jacobson Structure and Representations of Jordan Algebras [Jaco68]

— **MonadWithUnit.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#MONADWU">
MonadWithUnit (MONADWU)</a></h2>
</body>
```

— **category MONADWU MonadWithUnit** —

```
)abbrev category MONADWU MonadWithUnit
++ Authors: J. Grabmeier, R. Wisbauer
++ Date Created: 01 March 1991
++ Date Last Updated: 11 June 1991
++ Reference:
++ Jaco68 Structure and Representations of Jordan Algebras
++ Description:
++ MonadWithUnit is the class of multiplicative monads with unit,
++ that is, sets with a binary operation and a unit element.
++
++ Axioms\br
++ \tab{5}leftIdentity("*":(%,%)->%,1) for example, 1*x=x\br
++ \tab{5}rightIdentity("*":(%,%)->%,1) for example, x*1=x
++
++ Common Additional Axioms\br
```

```
++ \tab{5}unitsKnown - if "recip" says "failed", it PROVES input wasn't a unit

MonadWithUnit() : Category == SIG where

  SIG ==> Monad with

    1 : constant ->  %
      ++ \spad{1} returns the unit element, denoted by 1.

    one? : % -> Boolean
      ++ one?(a) tests whether \spad{a} is the unit 1.

    rightPower : (%,NonNegativeInteger) -> %
      ++ rightPower(a,n) returns the \spad{n}-th right power of \spad{a},
      ++ that is, \spad{rightPower(a,n) := rightPower(a,n-1) * a} and
      ++ \spad{rightPower(a,0) := 1}.

    leftPower : (%,NonNegativeInteger) -> %
      ++ leftPower(a,n) returns the \spad{n}-th left power of \spad{a},
      ++ that is, \spad{leftPower(a,n) := a * leftPower(a,n-1)} and
      ++ \spad{leftPower(a,0) := 1}.

    "**" : (%,NonNegativeInteger) -> %
      ++ \spad{a**n} returns the \spad{n}-th power of \spad{a},
      ++ defined by repeated squaring.

    recip : % -> Union(%,"failed")
      ++ recip(a) returns an element, which is both a left and a right
      ++ inverse of \spad{a},
      ++ or \spad{"failed"} if such an element doesn't exist or cannot
      ++ be determined (see unitsKnown).

    leftRecip : % -> Union(%,"failed")
      ++ leftRecip(a) returns an element, which is a left inverse of
      ++ \spad{a}, or \spad{"failed"} if such an element doesn't exist
      ++ or cannot be determined (see unitsKnown).

    rightRecip : % -> Union(%,"failed")
      ++ rightRecip(a) returns an element, which is a right inverse of
      ++ \spad{a}, or \spad{"failed"} if such an element doesn't exist
      ++ or cannot be determined (see unitsKnown).

   add

      import RepeatedSquaring(%)

      one? x == x = 1

      x:% ** n:NonNegativeInteger ==
         zero? n => 1
         expt(x,n pretend PositiveInteger)

      rightPower(a,n) ==
        zero? n => 1
```

```
        res := 1
        for i in 1..n repeat res := res * a
        res

      leftPower(a,n) ==
        zero? n => 1
        res := 1
        for i in 1..n repeat res := a * res
        res
```

———-

— COQ MONADWU —

```
(* category MONADWU *)
(*

Axioms:
 leftIdentity("*":(%,%)->%,1)      1*x=x
 rightIdentity("*":(%,%)->%,1)     x*1=x
 unitsKnown - if "recip" says "failed", it PROVES input wasn't a unit

      import RepeatedSquaring(%)

      one?: % -> Boolean
      one? x == x = 1

      "**": (%,NonNegativeInteger) -> %
      x:% ** n:NonNegativeInteger ==
         zero? n => 1
         expt(x,n pretend PositiveInteger)

      rightPower: (%,NonNegativeInteger) -> %
      rightPower(a,n) ==
        zero? n => 1
        res := 1
        for i in 1..n repeat res := res * a
        res

      leftPower: (%,NonNegativeInteger) -> %
      leftPower(a,n) ==
        zero? n => 1
        res := 1
        for i in 1..n repeat res := a * res
        res

*)
```

———-

— MONADWU.dotabb —

```
"MONADWU"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MONADWU"];
"MONADWU" -> "MONAD"
```

————

— MONADWU.dotfull —

```
"MonadWithUnit()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MONADWU"];
"MonadWithUnit()" -> "Monad()"
```

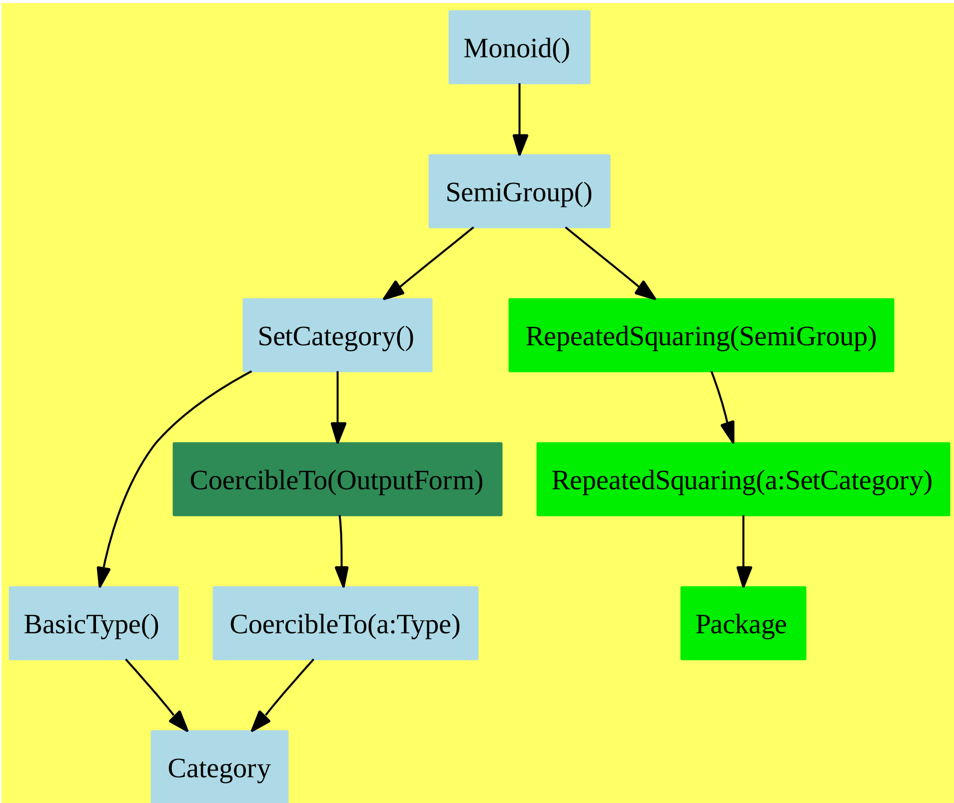
————

— MONADWU.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"MonadWithUnit()" [color=lightblue];
"MonadWithUnit()" -> "Monad()"

"Monad()" [color=lightblue];
"Monad()" -> "SetCategory()"
"Monad()" -> "RepeatedSquaring(Monad)"

"RepeatedSquaring(Monad)" [color="#00EE00"];
"RepeatedSquaring(Monad)" -> "RepeatedSquaring(a:SetCategory)"

"RepeatedSquaring(a:SetCategory)" [color="#00EE00"];
"RepeatedSquaring(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

———-

5.0.100 Monoid (MONOID)



— **Monoid.input** —

```
)set break resume
)sys rm -f Monoid.output
)spool Monoid.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Monoid
--R
--R Monoid is a category constructor
--R Abbreviation for Monoid is MONOID
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for MONOID
--R
```

```
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                   ?**? : (%,NonNegativeInteger) -> %
--R ?**? : (%,PositiveInteger) -> %     ?=? : (%,%) -> Boolean
--R 1 : () -> %                         ?^? : (%,NonNegativeInteger) -> %
--R ?^? : (%,PositiveInteger) -> %      coerce : % -> OutputForm
--R hash : % -> SingleInteger           latex : % -> String
--R one? : % -> Boolean                 recip : % -> Union(%,"failed")
--R sample : () -> %                    ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

———-

— Monoid.help —

```
====================================================================
Monoid examples
====================================================================

The class of multiplicative monoids, that is, semigroups with a
multiplicative identity element.

Axioms:
        leftIdentity("*":(%,%)->%,1)     1*x=x
        rightIdentity("*":(%,%)->%,1)    x*1=x

Conditional attributes:
        unitsKnown - \spadfun{recip} only returns "failed" on non-units

See Also:
o )show Monoid
```

———-

See:

⇒ "FunctionSpace" (FS) 17.0.219 on page 1544
⇒ "Group" (GROUP) 6.0.110 on page 525
⇒ "OrderedMonoid" (ORDMON) 6.0.114 on page 594
⇒ "OrderedRing" (ORDRING) 10.0.168 on page 1055
⇒ "Ring" (RING) 9.0.153 on page 946
⇐ "SemiGroup" (SGROUP) 4.0.85 on page 353

Exports:

1	coerce	hash	latex	one?
recip	sample	?*?	?=?	?~=?
?**?	?^?			

These are directly exported but not implemented:

```
1 : () -> %
```

These are implemented by this category:

```
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,NonNegativeInteger) -> %
```

These exports come from (p353) SemiGroup():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?*? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

— Monoid.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#MONOID">
Monoid (MONOID)</a></h2>
</body>
```

— category MONOID Monoid —

```
)abbrev category MONOID Monoid
++ Description:
++ The class of multiplicative monoids, that is, semigroups with a
++ multiplicative identity element.
++
++ Axioms\br
++ \tab{5}\spad{leftIdentity("*":(%,%)->%,1)}\tab{5}\spad{1*x=x}\br
++ \tab{5}\spad{rightIdentity("*":(%,%)->%,1)}\tab{4}\spad{x*1=x}
++
++ Conditional attributes\br
++ \tab{5}unitsKnown - \spadfun{recip} only returns "failed" on non-units

Monoid() : Category == SIG where

  SIG ==> SemiGroup with

    1 : constant ->  %
      ++ \axiom{1} is the multiplicative identity.

    sample : constant -> %
      ++ sample yields a value of type %

    one? : % -> Boolean
```

```
      ++ one?(x) tests if x is equal to 1.

    "**" : (%,NonNegativeInteger) -> %
      ++ x**n returns the repeated product
      ++ of x n times, that is, exponentiation.

    "^" : (%,NonNegativeInteger) -> %
      ++ x^n returns the repeated product
      ++ of x n times, that is, exponentiation.

    recip : % -> Union(%,"failed")
      ++ recip(x) tries to compute the multiplicative inverse for x
      ++ or "failed" if it cannot find the inverse (see unitsKnown).

   add

      import RepeatedSquaring(%)

      _^(x:%, n:NonNegativeInteger):% == x ** n

      one? x == x = 1

      sample() == 1

      recip x ==
        (x = 1) => x
        "failed"

      x:% ** n:NonNegativeInteger ==
         zero? n => 1
         expt(x,n pretend PositiveInteger)
```

———-

— COQ MONOID —

```
(* category MONOID *)
(*
Axioms:
  leftIdentity("*":(%,%)->%,1)     1*x=x
  rightIdentity("*":(%,%)->%,1)    x*1=x

      import RepeatedSquaring(%)

      "^" : (%,NonNegativeInteger) -> %
      _^(x:%, n:NonNegativeInteger):% == x ** n

      one?: % -> Boolean
      one? x == x = 1

      sample: constant -> %
      sample() == 1
```

```
      recip: % -> Union(%,"failed")
      recip x ==
        (x = 1) => x
        "failed"

      "**": (%,NonNegativeInteger) -> %
      x:% ** n:NonNegativeInteger ==
         zero? n => 1
         expt(x,n pretend PositiveInteger)

*)
```

————

— MONOID.dotabb —

```
"MONOID"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MONOID"];
"MONOID" -> "SGROUP"
```

————

— MONOID.dotfull —

```
"Monoid()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MONOID"];
"Monoid()" -> "SemiGroup()"
```

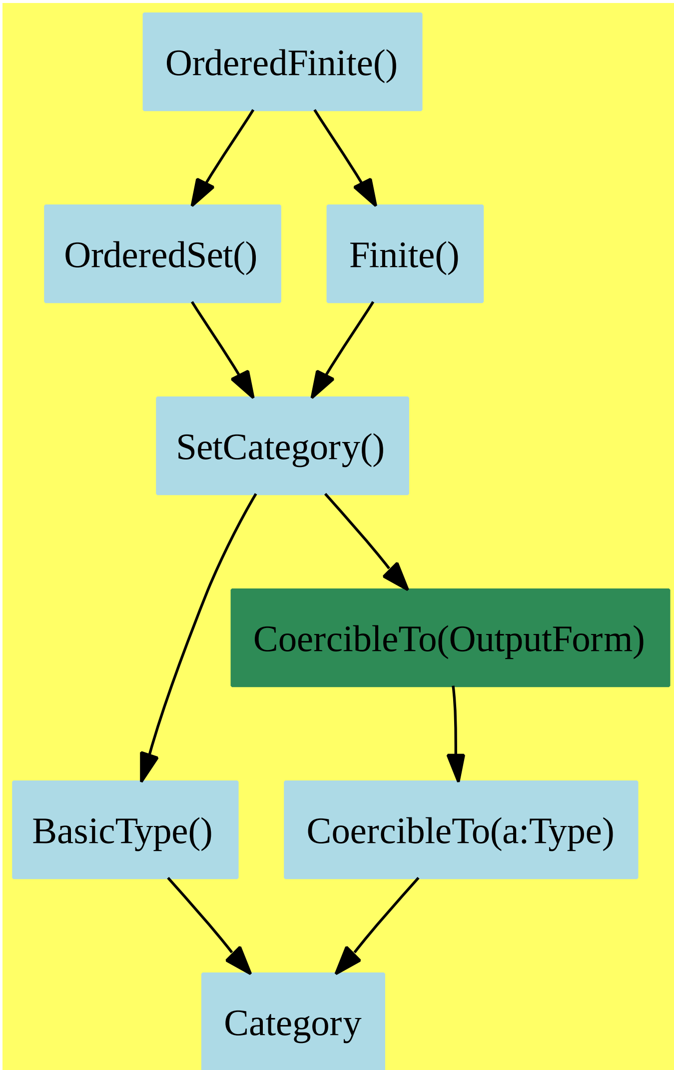
————

— MONOID.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SetCategory()"
"SemiGroup()" -> "RepeatedSquaring(a:SemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"
```

```
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"RepeatedSquaring(a:SemiGroup)" [color="#00EE00"];
"RepeatedSquaring(a:SemiGroup)" -> "RepeatedSquaring(a:SetCategory)"

"RepeatedSquaring(a:SetCategory)" [color="#00EE00"];
"RepeatedSquaring(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];

}
```

5.0.101 OrderedFinite (ORDFIN)



— **OrderedFinite.input** —

```
)set break resume
)sys rm -f OrderedFinite.output
)spool OrderedFinite.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderedFinite
--R
--R OrderedFinite is a category constructor
```

```
--R Abbreviation for OrderedFinite is ORDFIN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ORDFIN
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               coerce : % -> OutputForm
--R enumerate : () -> List(%)             hash : % -> SingleInteger
--R index : PositiveInteger -> %          latex : % -> String
--R lookup : % -> PositiveInteger         max : (%,%) -> %
--R min : (%,%) -> %                      random : () -> %
--R size : () -> NonNegativeInteger       ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

———-

— **OrderedFinite.help** —

```
====================================================================
OrderedFinite examples
====================================================================

This is the category of Ordered finite sets.

See Also:
o )show OrderedFinite
```

———-

See:

⇐ "Finite" (FINITE) 4.0.69 on page 258
⇐ "OrderedSet" (ORDSET) 4.0.80 on page 326

Exports:

coerce	hash	index	latex	lookup
max	min	random	size	?~=?
?<?	?<=?	?=?	?>?	?>=?

These exports come from (p326) OrderedSet():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?<? : (%,%) -> Boolean
```

```
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

These exports come from (p258) Finite():

```
index : PositiveInteger -> %
lookup : % -> PositiveInteger
random : () -> %
size : () -> NonNegativeInteger
```

— OrderedFinite.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ORDFIN">
OrderedFinite (ORDFIN)</a></h2>
</body>
```

— category ORDFIN OrderedFinite —

```
)abbrev category ORDFIN OrderedFinite
++ Description:
++ Ordered finite sets.

OrderedFinite() : Category == SIG where

  SIG ==> Join(OrderedSet, Finite)
```

— ORDFIN.dotabb —

```
"ORDFIN"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDFIN"];
"ORDFIN" -> "ORDSET"
"ORDFIN" -> "FINITE"
```

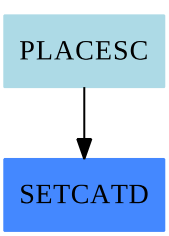
— ORDFIN.dotfull —

```
"OrderedFinite()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDFIN"];
"OrderedFinite()" -> "OrderedSet()"
"OrderedFinite()" -> "Finite()"
```

— ORDFIN.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"OrderedFinite()" [color=lightblue];
"OrderedFinite()" -> "OrderedSet()"
"OrderedFinite()" -> "Finite()"

"Finite()" [color=lightblue];
"Finite()" -> "SetCategory()"

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

5.0.102 PlacesCategory (PLACESC)



— **PlacesCategory.input** —

```
)set break resume
)sys rm -f PlacesCategory.output
)spool PlacesCategory.output
)set message test on
)set message auto off
```

```
)clear all

--S 1 of 1
)show PlacesCategory
--R
--I PlacesCategory(K: Field,PCS: LocalPowerSeriesCategory(t#1))
--R Abbreviation for PlacesCategory is PLACESC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PLACESC
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Integer,%) -> Divisor(%)       ?+? : (%,%) -> Divisor(%)
--R -? : % -> Divisor(%)                  ?-? : (%,%) -> Divisor(%)
--R ?=? : (%,%) -> Boolean                coerce : % -> OutputForm
--R create : Symbol -> %                  create : List(K) -> %
--R degree : % -> PositiveInteger         ?.? : (%,Integer) -> K
--R foundPlaces : () -> List(%)           hash : % -> SingleInteger
--R itsALeaf! : % -> Void                 latex : % -> String
--R leaf? : % -> Boolean                  localParam : % -> List(PCS)
--R reduce : List(%) -> Divisor(%)        setParam! : (%,List(PCS)) -> Void
--R ?~=? : (%,%) -> Boolean
--R ?+? : (%,Divisor(%)) -> Divisor(%)
--R ?+? : (Divisor(%),%) -> Divisor(%)
--R ?-? : (%,Divisor(%)) -> Divisor(%)
--R ?-? : (Divisor(%),%) -> Divisor(%)
--R setDegree! : (%,PositiveInteger) -> Void
--R setFoundPlacesToEmpty : () -> List(%)
--R
--E 1

)spool
)lisp (bye)
```

— **PlacesCategory.help** —

```
====================================================================
PlacesCategory examples
====================================================================

This is part of the PAFF package, related to projective space.

See Also:
o )show PlacesCategory
```

See:

⇐ “SetCategoryWithDegree” (SETCATD) 4.0.86 on page 358

Exports:

-?	?*?	?+?	?-?
?.?	?=?	?~=?	coerce
create	degree	foundPlaces	hash
itsALeaf!	latex	leaf?	localParam
reduce	setDegree!	setFoundPlacesToEmpty	setParam!

These are directly exported but not implemented:

```
-? : % -> Divisor %
?*? : (Integer,%) -> Divisor %
?+? : (%,%) -> Divisor %
?+? : (%,Divisor %) -> Divisor %
?+? : (Divisor %,%) -> Divisor %
?-? : (%,%) -> Divisor %
?-? : (%,Divisor %) -> Divisor %
?-? : (Divisor %,%) -> Divisor %
?.? : (%,Integer) -> K
create : List K -> %
create : Symbol -> %
foundPlaces : () -> List %
itsALeaf! : % -> Void
leaf? : % -> Boolean
localParam : % -> List PCS
reduce : List % -> Divisor %
setDegree! : (%,PositiveInteger) -> Void
setFoundPlacesToEmpty : () -> List %
setParam! : (%,List PCS) -> Void
```

These exports come from (p358) SetCategoryWithDegree

```
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
coerce : % -> OutputForm
degree : % -> PositiveInteger
hash : % -> SingleInteger
latex : % -> String
```

— PlacesCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#PLACESC">
PlacesCategory (PLACESC)</a></h2>
</body>
```

— category PLACESC PlacesCategory —

```
)abbrev category PLACESC PlacesCategory
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This is part of the PAFF package, related to projective space.
```

```
PlacesCategory(K,PCS) : Category == SIG where
  K : Field
  PCS : LocalPowerSeriesCategory(K)

  SIG ==> SetCategoryWithDegree with

    "+" : (%,%) -> Divisor(%)

    "+" : (Divisor(%) , % ) -> Divisor(%)

    "+" : (%, Divisor(%)) -> Divisor(%)

    "-" : (%,%) -> Divisor(%)

    "-" : (Divisor(%) , % ) -> Divisor(%)

    "-" : (%, Divisor(%)) -> Divisor(%)

    "-" : % -> Divisor(%)

    "*" : (Integer,%) -> Divisor(%)

    reduce : List %   -> Divisor(%)

    create : List K -> %

    create : Symbol -> %

    localParam : % -> List PCS
      ++ localParam(pl) returns the local parametrization associated
      ++ to the place pl.

    setParam_! : (%,List PCS)  -> Void
      ++ setParam!(pl,ls) set the local parametrization of pl to ls.

    setDegree_! : (%,PositiveInteger)  -> Void
      ++ setDegree!(pl,ls) set the degree.

    leaf? : % -> Boolean
      ++ leaf?(pl) test if the place pl correspond to a leaf of a
      ++ desingularisation tree.

    itsALeaf_! : % -> Void

    foundPlaces : () -> List %
      ++ foundPlaces() returns the list of all "created" places up to now.

    setFoundPlacesToEmpty : () -> List %
      ++ setFoundPlacesToEmpty() does what it says.
      ++ (this should not be used)!!!

    elt : (%,Integer) -> K
      ++ elt returns the value of a specified coordinates if the places
      ++ correspnd to a simple point
```

———-

— **PLACESC.dotabb** —

```
"PLACESC" [color=lightblue,href="bookvol10.2.pdf#nameddest=PLACESC"];
"SETCATD" [color="#4488FF",href="bookvol10.2.pdf#nameddest=SETCATD"]
"PLACESC" -> "SETCATD"
```

———-

— **PLACESC.dotfull** —

```
"PlacesCategory()" [color=lightblue,href="bookvol10.2.pdf#nameddest=PLACESC"];
"PlacesCategory()" -> "SetCategoryWithDegree()"
```

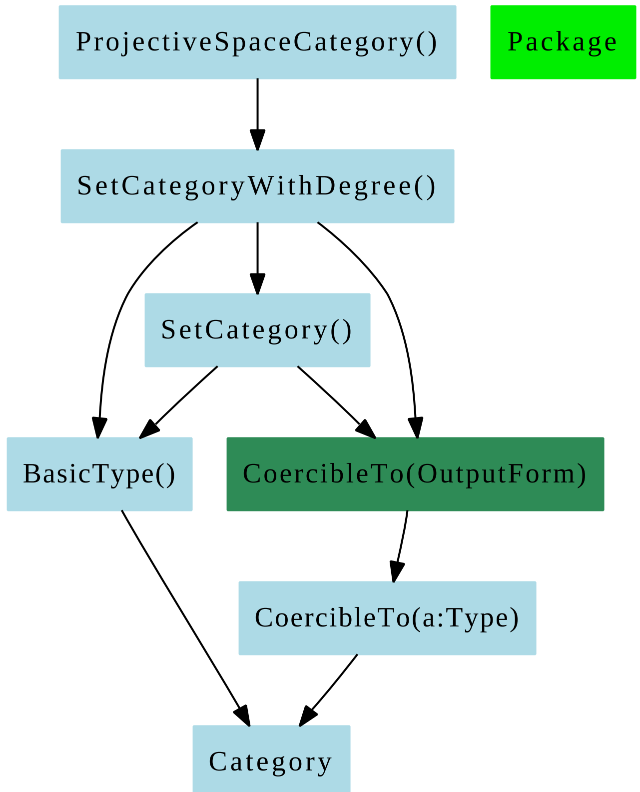
———-

— **PLACESC.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"PlacesCategory()" [color=lightblue];
"PlacesCategory()" -> "SetCategoryWithDegree()"

"SetCategoryWithDegree()" [color=lightblue];
"SetCategoryWithDegree()" -> "BasicType()"
"SetCategoryWithDegree()" -> "CoercibleTo(OutputForm)"
"SetCategoryWithDegree()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Package" [color="#00EE00"];

"Category" [color=lightblue];

}
```

———-

5.0.103 ProjectiveSpaceCategory (PRSPCAT)



— **ProjectiveSpaceCategory.input** —

```
)set break resume
)sys rm -f ProjectiveSpaceCategory.output
)spool ProjectiveSpaceCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ProjectiveSpaceCategory
--R
--R ProjectiveSpaceCategory(K: Field) is a category constructor
--R Abbreviation for ProjectiveSpaceCategory is PRSPCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PRSPCAT
--R
```

```
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean              coerce : List(K) -> %
--R coerce : % -> List(K)               coerce : % -> OutputForm
--R conjugate : % -> %                  definingField : % -> K
--R degree : % -> PositiveInteger       ?.? : (%,Integer) -> K
--R hash : % -> SingleInteger           homogenize : % -> %
--R homogenize : (%,Integer) -> %       lastNonNul : % -> Integer
--R lastNonNull : % -> Integer          latex : % -> String
--R list : % -> List(K)                 orbit : % -> List(%)
--R pointValue : % -> List(K)           projectivePoint : List(K) -> %
--R rational? : % -> Boolean            setelt : (%,Integer,K) -> K
--R ?~=? : (%,%) -> Boolean
--R conjugate : (%,NonNegativeInteger) -> %
--R orbit : (%,NonNegativeInteger) -> List(%)
--R rational? : (%,NonNegativeInteger) -> Boolean
--R removeConjugate : List(%) -> List(%)
--R removeConjugate : (List(%),NonNegativeInteger) -> List(%)
--R
--E 1

)spool
)lisp (bye)
```

———-

— **ProjectiveSpaceCategory.help** —

```
====================================================================
ProjectiveSpaceCategory examples
====================================================================

This is part of the PAFF package, related to projective space.

See Also:
o )show ProjectiveSpaceCategory
```

———-

See:

⇐ "SetCategoryWithDegree" (SETCATD) 4.0.86 on page 358

Exports:

?.?	?=?	? =?	coerce
conjugate	definingField	degree	hash
homogenize	lastNonNul	lastNonNull	latex
list	orbit	orbit	pointValue
projectivePoint	rational?	removeConjugate	setelt

These are directly exported but not implemented:

```
?.? : (%,Integer) -> K
coerce : % -> List K
```

```
coerce : List K -> %
conjugate : % -> %
conjugate : (%,NonNegativeInteger) -> %
definingField : % -> K
homogenize : % -> %
homogenize : (%,Integer) -> %
lastNonNul : % -> Integer
lastNonNull : % -> Integer
list : % -> List K
orbit : % -> List %
orbit : (%,NonNegativeInteger) -> List %
pointValue : % -> List K
projectivePoint : List K -> %
rational? : % -> Boolean
rational? : (%,NonNegativeInteger) -> Boolean
removeConjugate : (List %,NonNegativeInteger) -> List %
removeConjugate : List % -> List %
setelt : (%,Integer,K) -> K
```

These exports come from (p358) SetCategoryWithDegree

```
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
coerce : % -> OutputForm
degree : % -> PositiveInteger
hash : % -> SingleInteger
latex : % -> String
```

See: Hoholdt, van Lint, and Pellikaan [Hold11]

— ProjectiveSpaceCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#PRSPCAT">
ProjectiveSpaceCategory (PRSPCAT)</a></h2>
</body>
```

— category PRSPCAT ProjectiveSpaceCategory —

```
)abbrev category PRSPCAT ProjectiveSpaceCategory
++ Author: Gaetan Hache
++ Date Created: 17 nov 1992
++ Date Last Updated: May 2010 by Tim Daly
++ References:
++ Hold11 Algebraic Geometry Codes
++ Description:
++ This is part of the PAFF package, related to projective space.

ProjectiveSpaceCategory(K) : Category == SIG where
  K : Field

  LIST ==> List
  INT  ==> Integer
  NNI  ==> NonNegativeInteger
```

```
SIG ==> SetCategoryWithDegree with

  lastNonNul : % -> INT

  homogenize : (%,INT) -> %
    ++ homogenize the point according to the coordinate specified
    ++ by the integer

  projectivePoint : LIST(K) -> %
    ++ projectivePoint creates a projective point from a list

  definingField : % -> K

  coerce : % -> List(K)
    ++ coerce a a projective point list of K

  coerce : LIST(K) -> %
    ++ coerce a list of K to a projective point.

  orbit : % -> LIST %
    ++ orbit(p) returns the orbit of the point p according to the
    ++ characteristic of K, that is, for q= char K,
    ++ orbit(p) = \{ p, p**q, p**(q**2), p**(q**3), ..... \}

  orbit : (%,NNI) -> LIST %
    ++ orbit(p,n) returns the orbit of the point p according to n, that is
    ++ orbit(p,n) = \{ p, p**n, p**(n**2), p**(n**3), ..... \}

  conjugate : (%,NNI) -> %
    ++ conjugate(p,n) returns p**n, that is all the coordinates of p
    ++ to the power of n

  conjugate : % -> %
    ++ conjugate(p) returns conjugate(p,n) where n is the
    ++ characteristic of the ground field.

  removeConjugate : (LIST(%),NNI) -> LIST(%)
    ++ removeConjugate(lp,n) returns a list of points such that no points
    ++ in the list is the conjugate (according to n) of another point.

  removeConjugate : LIST(%)  -> LIST(%)
    ++ removeConjugate(lp) returns removeConjugate(lp,n)
    ++ where n is the characteristic of the ground field.

  rational? : (%,NNI) -> Boolean
    ++ rational?(p,n) test if the point is rational according to n.

  rational? : % -> Boolean
    ++ rational?(p) test if the point is rational according to the
    ++ characteristic of the ground field.

  homogenize : % -> %
    ++ homogenize(pt) the point according to the coordinate
```

```
      ++ which is the last non null.

    lastNonNull : % -> INT
      ++ lastNonNull returns the integer corresponding to the last
      ++ non null coordinates.

    list : % -> LIST(K)
      ++ list returns the list of the coordinates

    elt : (%,INT) -> K
      ++ elt returns the value of a specified coordinates

    setelt : (%,INT,K) -> K
      ++ setelt sets the value of a specified coordinates

    pointValue : % -> LIST(K)
      ++ pointValue returns the coordinates of the point or of the point
      ++ of origin that represent an infinitly close point
```

————-

— PRSPCAT.dotabb —

```
"PRSPCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PRSPCAT"];
"PRSPCAT" -> "SETCATD"
```

————-

— PRSPCAT.dotfull —

```
"ProjectiveSpaceCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PRSPCAT"];
"ProjectiveSpaceCategory()" -> "SetCategoryWithDegree()"
```

————-

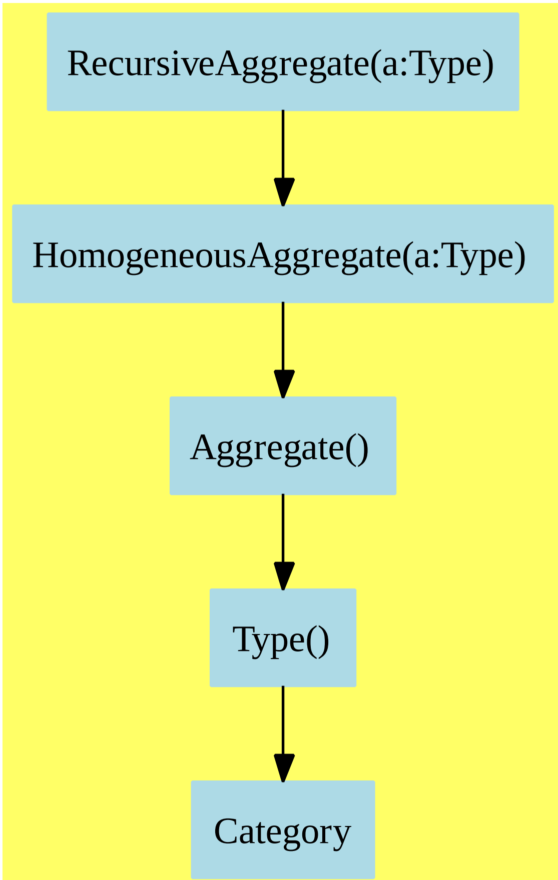
— PRSPCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"ProjectiveSpaceCategory()" [color=lightblue];
"ProjectiveSpaceCategory()" -> "SetCategoryWithDegree()"

"SetCategoryWithDegree()" [color=lightblue];
"SetCategoryWithDegree()" -> "BasicType()"
"SetCategoryWithDegree()" -> "CoercibleTo(OutputForm)"
"SetCategoryWithDegree()" -> "SetCategory()"
```

```
"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Package" [color="#00EE00"];

"Category" [color=lightblue];

}
```

———-

5.0.104 RecursiveAggregate (RCAGG)



— **RecursiveAggregate.input** —

```
)set break resume
)sys rm -f RecursiveAggregate.output
)spool RecursiveAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RecursiveAggregate
--R
--R RecursiveAggregate(S: Type) is a category constructor
--R Abbreviation for RecursiveAggregate is RCAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for RCAGG
--R
--R------------------------------- Operations --------------------------------
--R children : % -> List(%)              copy : % -> %
```

```
--R cyclic? : % -> Boolean               distance : (%,%) -> Integer
--R ?.value : (%,value) -> S             empty : () -> %
--R empty? : % -> Boolean                eq? : (%,%) -> Boolean
--R latex : % -> String if S has SETCAT  leaf? : % -> Boolean
--R leaves : % -> List(S)                map : ((S -> S),%) -> %
--R nodes : % -> List(%)                 sample : () -> %
--R value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List(S) if $ has finiteAggregate
--R setchildren! : (%,List(%)) -> % if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

——————

— RecursiveAggregate.help —

```
====================================================================
RecursiveAggregate examples
====================================================================

A recursive aggregate over a type S is a model for a a directed graph
containing values of type S. Recursively, a recursive aggregate is a node
consisting of a value from S and 0 or more children which are recursive
aggregates. A node with no children is called a leaf node. A recursive
aggregate may be cyclic for which some operations as noted may go into
an infinite loop.

See Also:
```

```
o )show RecursiveAggregate
```

See:

⇒ "BinaryRecursiveAggregate" (BRAGG) 6.0.106 on page 498
⇒ "DoublyLinkedAggregate" (DLAGG) 6.0.109 on page 519
⇒ "UnaryRecursiveAggregate" (URAGG) 6.0.120 on page 642
⇐ "HomogeneousAggregate" (HOAGG) 4.0.73 on page 280

Exports:

any?	child?	children	coerce	copy
count	cyclic?	distance	empty	empty?
eq?	eval	every?	hash	latex
leaf?	leaves	less?	map	map!
member?	members	more?	nodes	node?
parts	sample	setchildren!	setelt	setvalue!
size?	value	?.value	?~=?	#?
?=?				

Attributes exported:

- **nil**

Attributes Used:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
children : % -> List %
cyclic? : % -> Boolean
distance : (%,%) -> Integer
nodes : % -> List %
leaf? : % -> Boolean
leaves : % -> List S
node? : (%,%) -> Boolean if S has SETCAT
setchildren! : (%,List %) -> % if $ has shallowlyMutable
setvalue! : (%,S) -> S if $ has shallowlyMutable
value : % -> S
```

These are implemented by this category:

```
child? : (%,%) -> Boolean if S has SETCAT
setelt : (%,value,S) -> S if $ has shallowlyMutable
?.value : (%,value) -> S
```

These exports come from (p280) HomogeneousAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
```

```
          if S has SETCAT and $ has finiteAggregate
 count : ((S -> Boolean),%) -> NonNegativeInteger
          if $ has finiteAggregate
 empty : () -> %
 empty? : % -> Boolean
 eq? : (%,%) -> Boolean
 eval : (%,List S,List S) -> %
       if S has EVALAB S and S has SETCAT
 eval : (%,S,S) -> %
       if S has EVALAB S and S has SETCAT
 eval : (%,Equation S) -> %
       if S has EVALAB S and S has SETCAT
 eval : (%,List Equation S) -> %
       if S has EVALAB S and S has SETCAT
 every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
 hash : % -> SingleInteger if S has SETCAT
 latex : % -> String if S has SETCAT
 less? : (%,NonNegativeInteger) -> Boolean
 map : ((S -> S),%) -> %
 map! : ((S -> S),%) -> % if $ has shallowlyMutable
 member? : (S,%) -> Boolean
           if S has SETCAT and $ has finiteAggregate
 members : % -> List S if $ has finiteAggregate
 more? : (%,NonNegativeInteger) -> Boolean
 parts : % -> List S if $ has finiteAggregate
 sample : () -> %
 size? : (%,NonNegativeInteger) -> Boolean
 ?~=? : (%,%) -> Boolean if S has SETCAT
 #? : % -> NonNegativeInteger if $ has finiteAggregate
 ?=? : (%,%) -> Boolean if S has SETCAT
```

— RecursiveAggregate.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#RCAGG">
RecursiveAggregate (RCAGG)</a></h2>
</body>
```

— category RCAGG RecursiveAggregate —

```
)abbrev category RCAGG RecursiveAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ A recursive aggregate over a type S is a model for a
++ a directed graph containing values of type S.
++ Recursively, a recursive aggregate is a node
++ consisting of a \spadfun{value} from S and 0 or more \spadfun{children}
++ which are recursive aggregates.
++ A node with no children is called a \spadfun{leaf} node.
```

```
++ A recursive aggregate may be cyclic for which some operations as noted
++ may go into an infinite loop.

RecursiveAggregate(S) : Category == SIG where
  S : Type

  SIG ==> HomogeneousAggregate(S) with

    children : % -> List %
      ++ children(u) returns a list of the children of aggregate u.
      -- should be % -> %* and also needs children: % -> Iterator(S,S)

    nodes : % -> List %
      ++ nodes(u) returns a list of all of the nodes of aggregate u.
      -- to become % -> %* and also nodes: % -> Iterator(S,S)

    leaf? : % -> Boolean
      ++ leaf?(u) tests if u is a terminal node.

    value : % -> S
      ++ value(u) returns the value of the node u.

    elt : (%,"value") -> S
      ++ elt(u,"value") (also written: \axiom{a. value}) is
      ++ equivalent to \axiom{value(a)}.

    cyclic? : % -> Boolean
      ++ cyclic?(u) tests if u has a cycle.

    leaves : % -> List S
      ++ leaves(t) returns the list of values in obtained by visiting the
      ++ nodes of tree \axiom{t} in left-to-right order.

    distance : (%,%) -> Integer
      ++ distance(u,v) returns the path length (an integer) from node u to v.

    if S has SetCategory then

       child? : (%,%) -> Boolean
         ++ child?(u,v) tests if node u is a child of node v.

       node? : (%,%) -> Boolean
         ++ node?(u,v) tests if node u is contained in node v
         ++ (either as a child, a child of a child, etc.).

    if % has shallowlyMutable then

       setchildren_! : (%,List %)->%
         ++ setchildren!(u,v) replaces the current children of node u
         ++ with the members of v in left-to-right order.

       setelt : (%,"value",S) -> S
         ++ setelt(a,"value",x) (also written \axiom{a . value := x})
         ++ is equivalent to \axiom{setvalue!(a,x)}
```

```
      setvalue_! : (%,S) -> S
        ++ setvalue!(u,x) sets the value of node u to x.

  add

    elt(x,"value") == value x

    if % has shallowlyMutable then
      setelt(x,"value",y) == setvalue_!(x,y)

    if S has SetCategory then
      child?(x,l) == member?(x,children(l))
```

———-

— **COQ RCAGG** —

```
(* category RCAGG *)
(*

  elt: (%,"value") -> S
  elt(x,"value") == value x

  if % has shallowlyMutable then

    setelt: (%,"value",S) -> S
    setelt(x,"value",y) == setvalue_!(x,y)

  if S has SetCategory then

    child?: (%,%) -> Boolean
    child?(x,l) == member?(x,children(l))

*)
```

———-

— **RCAGG.dotabb** —

```
"RCAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=RCAGG"];
"RCAGG" -> "HOAGG"
```

———-

— **RCAGG.dotfull** —

```
"RecursiveAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RCAGG"];
"RecursiveAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"
```

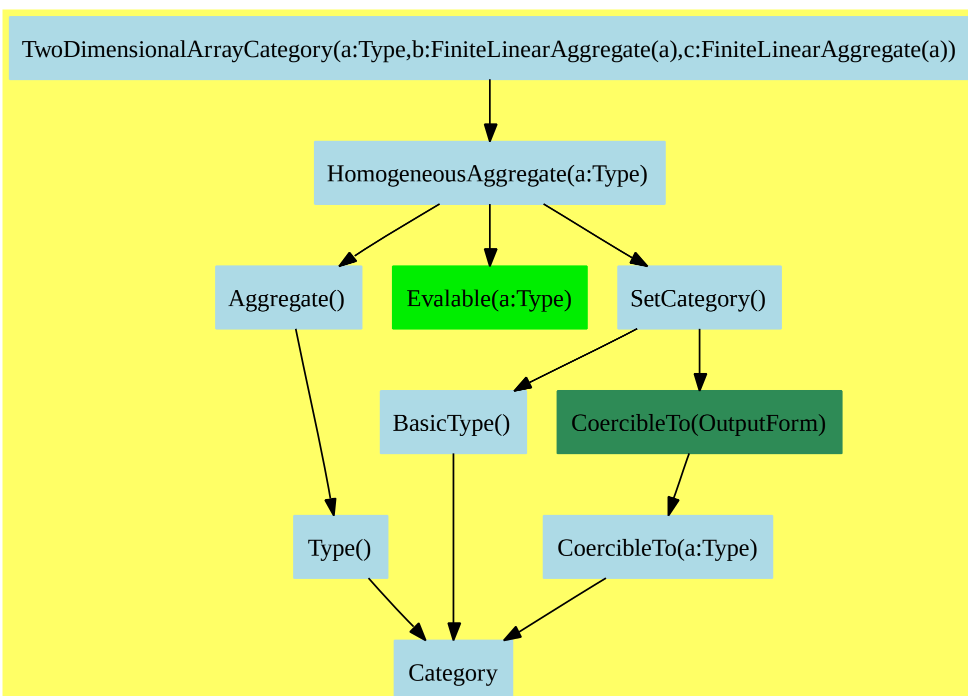
———-

— **RCAGG.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"RecursiveAggregate(a:Type)" [color=lightblue];
"RecursiveAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

———-

5.0.105 TwoDimensionalArrayCategory (ARR2CAT)



TwoDimensionalArrayCategory is a general array category which allows different representations and indexing schemes. Rows and columns may be extracted with rows returned as objects of type Row and columns returned as objects of type Col. The index of the 'first' row may be obtained by calling the function 'minRowIndex'. The index of the 'first' column may be obtained by calling the function 'minColIndex'. The index of the first element of a 'Row' is the same as the index of the first column in an array and vice versa.

— TwoDimensionalArrayCategory.input —

```
)set break resume
)sys rm -f TwoDimensionalArrayCategory.output
)spool TwoDimensionalArrayCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show TwoDimensionalArrayCategory
--R
--R TwoDimensionalArrayCategory(R: Type,Row: FiniteLinearAggregate(t#1),Col: FiniteLinearAggregate(t#1)) is a ca
--R Abbreviation for TwoDimensionalArrayCategory is ARR2CAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ARR2CAT
--R
--R------------------------------- Operations --------------------------------
--R column : (%,Integer) -> Col           copy : % -> %
--R elt : (%,Integer,Integer,R) -> R      elt : (%,Integer,Integer) -> R
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                fill! : (%,R) -> %
--R latex : % -> String if R has SETCAT   map : (((R,R) -> R),%,%,R) -> %
--R map : (((R,R) -> R),%,%) -> %         map : ((R -> R),%) -> %
--R map! : ((R -> R),%) -> %              maxColIndex : % -> Integer
--R maxRowIndex : % -> Integer            minColIndex : % -> Integer
--R minRowIndex : % -> Integer            ncols : % -> NonNegativeInteger
--R nrows : % -> NonNegativeInteger       parts : % -> List(R)
--R qelt : (%,Integer,Integer) -> R       row : (%,Integer) -> Row
--R sample : () -> %                      setColumn! : (%,Integer,Col) -> %
--R setRow! : (%,Integer,Row) -> %        setelt : (%,Integer,Integer,R) -> R
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if R has SETCAT
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if R has SETCAT
--R count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List(R),List(R)) -> % if R has EVALAB(R) and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB(R) and R has SETCAT
--R eval : (%,Equation(R)) -> % if R has EVALAB(R) and R has SETCAT
--R eval : (%,List(Equation(R))) -> % if R has EVALAB(R) and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if R has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
```

```
--R members : % -> List(R) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,NonNegativeInteger,R) -> %
--R qsetelt! : (%,Integer,Integer,R) -> R
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if R has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———-

— TwoDimensionalArrayCategory.help —

```
====================================================================
TwoDimensionalArrayCategory examples
====================================================================

This is the category of two dimensional array categories and domains.

See Also:
o )show TwoDimensionalArrayCategory
```

———-

See:

⇒ "MatrixCategory" (MATCAT) 6.0.112 on page 540
⇐ "HomogeneousAggregate" (HOAGG) 4.0.73 on page 280

Exports:

any?	column	coerce	copy	count
elt	empty	empty?	eq?	eval
every?	fill!	hash	latex	less?
map	map!	maxColIndex	maxRowIndex	member?
members	minColIndex	minRowIndex	more?	ncols
new	nrows	parts	qelt	qsetelt!
row	sample	setColumn!	setRow!	setelt
size?	#?	?=?	?~=?	

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

Attributes Used:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
elt : (%,Integer,Integer) -> R
maxColIndex : % -> Integer
maxRowIndex : % -> Integer
minColIndex : % -> Integer
minRowIndex : % -> Integer
new : (NonNegativeInteger,NonNegativeInteger,R) -> %
ncols : % -> NonNegativeInteger
nrows : % -> NonNegativeInteger
qelt : (%,Integer,Integer) -> R
qsetelt! : (%,Integer,Integer,R) -> R
setelt : (%,Integer,Integer,R) -> R
```

These are implemented by this category:

```
any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
coerce : % -> OutputForm if R has SETCAT
column : (%,Integer) -> Col
copy : % -> %
count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
elt : (%,Integer,Integer,R) -> R
every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
fill! : (%,R) -> %
less? : (%,NonNegativeInteger) -> Boolean
map : ((R -> R),%) -> %
map : (((R,R) -> R),%,%) -> %
map : (((R,R) -> R),%,%,R) -> %
map! : ((R -> R),%) -> %
member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List R
row : (%,Integer) -> Row
setColumn! : (%,Integer,Col) -> %
setRow! : (%,Integer,Row) -> %
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if R has SETCAT
```

These exports come from (p280) HomogeneousAggregate(R:Type)

```
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
hash : % -> SingleInteger if R has SETCAT
latex : % -> String if R has SETCAT
members : % -> List R if $ has finiteAggregate
sample : () -> %
?~=? : (%,%) -> Boolean if R has SETCAT
```

— **TwoDimensionalArrayCategory.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ARR2CAT">
TwoDimensionalArrayCategory (ARR2CAT)</a></h2>
</body>
```

— **category ARR2CAT TwoDimensionalArrayCategory** —

```
)abbrev category ARR2CAT TwoDimensionalArrayCategory
++ Date Created: 27 October 1989
++ Date Last Updated: 27 June 1990
++ Keywords: array, data structure
++ Description:
++ Two dimensional array categories and domains

TwoDimensionalArrayCategory(R,Row,Col) : Category == SIG where
  R   : Type
  Row : FiniteLinearAggregate R
  Col : FiniteLinearAggregate R

  SIG ==> HomogeneousAggregate(R) with

    shallowlyMutable
      ++ one may destructively alter arrays

    finiteAggregate
      ++ two-dimensional arrays are finite

    --% Array creation

    new : (NonNegativeInteger,NonNegativeInteger,R) -> %
      ++ new(m,n,r) is an m-by-n array all of whose entries are r
      ++
      ++X arr : ARRAY2 INT := new(5,4,0)

    fill_! : (%,R) -> %
      ++ fill!(m,r) fills m with r's
      ++
      ++X arr : ARRAY2 INT := new(5,4,0)
      ++X fill!(arr,10)

    --% Size inquiries

    minRowIndex : % -> Integer
      ++ minRowIndex(m) returns the index of the 'first' row of the array m
      ++
      ++X arr : ARRAY2 INT := new(5,4,10)
      ++X minRowIndex(arr)

    maxRowIndex : % -> Integer
```

```
    ++ maxRowIndex(m) returns the index of the 'last' row of the array m
    ++
    ++X arr : ARRAY2 INT := new(5,4,10)
    ++X maxRowIndex(arr)

  minColIndex : % -> Integer
    ++ minColIndex(m) returns the index of the 'first' column of the array m
    ++
    ++X arr : ARRAY2 INT := new(5,4,10)
    ++X minColIndex(arr)

  maxColIndex : % -> Integer
    ++ maxColIndex(m) returns the index of the 'last' column of the array m
    ++
    ++X arr : ARRAY2 INT := new(5,4,10)
    ++X maxColIndex(arr)

  nrows : % -> NonNegativeInteger
    ++ nrows(m) returns the number of rows in the array m
    ++
    ++X arr : ARRAY2 INT := new(5,4,10)
    ++X nrows(arr)

  ncols : % -> NonNegativeInteger
    ++ ncols(m) returns the number of columns in the array m
    ++
    ++X arr : ARRAY2 INT := new(5,4,10)
    ++X ncols(arr)

  --% Part extractions

  elt : (%,Integer,Integer) -> R
    ++ elt(m,i,j) returns the element in the ith row and jth
    ++ column of the array m
    ++ error check to determine if indices are in proper ranges
    ++
    ++X arr : ARRAY2 INT := new(5,4,10)
    ++X elt(arr,1,1)

  qelt : (%,Integer,Integer) -> R
    ++ qelt(m,i,j) returns the element in the ith row and jth
    ++ column of the array m
    ++ NO error check to determine if indices are in proper ranges
    ++
    ++X arr : ARRAY2 INT := new(5,4,10)
    ++X qelt(arr,1,1)

  elt : (%,Integer,Integer,R) -> R
    ++ elt(m,i,j,r) returns the element in the ith row and jth
    ++ column of the array m, if m has an ith row and a jth column,
    ++ and returns r otherwise
    ++
    ++X arr : ARRAY2 INT := new(5,4,10)
    ++X elt(arr,1,1,6)
```

```
  ++X elt(arr,1,10,6)

row : (%,Integer) -> Row
  ++ row(m,i) returns the ith row of m
  ++ error check to determine if index is in proper ranges
  ++
  ++X arr : ARRAY2 INT := new(5,4,10)
  ++X row(arr,1)

column : (%,Integer) -> Col
  ++ column(m,j) returns the jth column of m
  ++ error check to determine if index is in proper ranges
  ++
  ++X arr : ARRAY2 INT := new(5,4,10)
  ++X column(arr,1)

parts : % -> List R
  ++ parts(m) returns a list of the elements of m in row major order
  ++
  ++X arr : ARRAY2 INT := new(5,4,10)
  ++X parts(arr)

--% Part assignments

setelt : (%,Integer,Integer,R) -> R
  ++ setelt(m,i,j,r) sets the element in the ith row and jth
  ++ column of m to r
  ++ error check to determine if indices are in proper ranges
  ++
  ++X arr : ARRAY2 INT := new(5,4,0)
  ++X setelt(arr,1,1,17)

qsetelt_! : (%,Integer,Integer,R) -> R
  ++ qsetelt!(m,i,j,r) sets the element in the ith row and jth
  ++ column of m to r
  ++ NO error check to determine if indices are in proper ranges
  ++
  ++X arr : ARRAY2 INT := new(5,4,0)
  ++X qsetelt!(arr,1,1,17)

setRow_! : (%,Integer,Row) -> %
  ++ setRow!(m,i,v) sets to ith row of m to v
  ++
  ++X T1:=TwoDimensionalArray Integer
  ++X arr:T1:= new(5,4,0)
  ++X T2:=OneDimensionalArray Integer
  ++X arow:=construct([1,2,3,4]::List(INT))$T2
  ++X setRow!(arr,1,arow)$T1

setColumn_! : (%,Integer,Col) -> %
  ++ setColumn!(m,j,v) sets to jth column of m to v
  ++
  ++X T1:=TwoDimensionalArray Integer
  ++X arr:T1:= new(5,4,0)
```

```
    ++X T2:=OneDimensionalArray Integer
    ++X acol:=construct([1,2,3,4,5]::List(INT))$T2
    ++X setColumn!(arr,1,acol)$T1

  --% Map and Zip

  map : (R -> R,%) -> %
    ++ map(f,a) returns \spad{b}, where \spad{b(i,j) = f(a(i,j))}
    ++ for all \spad{i, j}
    ++
    ++X arr : ARRAY2 INT := new(5,4,10)
    ++X map(-,arr)
    ++X map((x +-> x + x),arr)

  map_! : (R -> R,%) -> %
    ++ map!(f,a)  assign \spad{a(i,j)} to \spad{f(a(i,j))}
    ++ for all \spad{i, j}
    ++
    ++X arr : ARRAY2 INT := new(5,4,10)
    ++X map!(-,arr)

  map : ((R,R) -> R,%,%) -> %
    ++ map(f,a,b) returns \spad{c}, where \spad{c(i,j) = f(a(i,j),b(i,j))}
    ++ for all \spad{i, j}
    ++
    ++X adder(a:Integer,b:Integer):Integer == a+b
    ++X arr : ARRAY2 INT := new(5,4,10)
    ++X map(adder,arr,arr)

  map : ((R,R) -> R,%,%,R) -> %
    ++ map(f,a,b,r) returns \spad{c}, where \spad{c(i,j) = f(a(i,j),b(i,j))}
    ++ when both \spad{a(i,j)} and \spad{b(i,j)} exist;
    ++ else \spad{c(i,j) = f(r, b(i,j))} when \spad{a(i,j)} does not exist;
    ++ else \spad{c(i,j) = f(a(i,j),r)} when \spad{b(i,j)} does not exist;
    ++ otherwise \spad{c(i,j) = f(r,r)}.
    ++
    ++X adder(a:Integer,b:Integer):Integer == a+b
    ++X arr1 : ARRAY2 INT := new(5,4,10)
    ++X arr2 : ARRAY2 INT := new(3,3,10)
    ++X map(adder,arr1,arr2,17)

 add

   --% Predicates

   any?(f,m) ==
     for i in minRowIndex(m)..maxRowIndex(m) repeat
       for j in minColIndex(m)..maxColIndex(m) repeat
         f(qelt(m,i,j)) => return true
     false

   every?(f,m) ==
     for i in minRowIndex(m)..maxRowIndex(m) repeat
       for j in minColIndex(m)..maxColIndex(m) repeat
```

```
      not f(qelt(m,i,j)) => return false
  true

size?(m,n) == nrows(m) * ncols(m) = n

less?(m,n) == nrows(m) * ncols(m) < n

more?(m,n) == nrows(m) * ncols(m) > n

--% Size inquiries

# m == nrows(m) * ncols(m)

--% Part extractions

elt(m,i,j,r) ==
  i < minRowIndex(m) or i > maxRowIndex(m) => r
  j < minColIndex(m) or j > maxColIndex(m) => r
  qelt(m,i,j)

count(f:R -> Boolean,m:%) ==
  num : NonNegativeInteger := 0
  for i in minRowIndex(m)..maxRowIndex(m) repeat
    for j in minColIndex(m)..maxColIndex(m) repeat
      if f(qelt(m,i,j)) then num := num + 1
  num

parts m ==
  entryList : List R := nil()
  for i in maxRowIndex(m)..minRowIndex(m) by -1 repeat
    for j in maxColIndex(m)..minColIndex(m) by -1 repeat
      entryList := concat(qelt(m,i,j),entryList)
  entryList

--% Creation

copy m ==
  ans := new(nrows m,ncols m,NIL$Lisp)
  for i in minRowIndex(m)..maxRowIndex(m) repeat
    for j in minColIndex(m)..maxColIndex(m) repeat
      qsetelt_!(ans,i,j,qelt(m,i,j))
  ans

fill_!(m,r) ==
  for i in minRowIndex(m)..maxRowIndex(m) repeat
    for j in minColIndex(m)..maxColIndex(m) repeat
      qsetelt_!(m,i,j,r)
  m

map(f,m) ==
  ans := new(nrows m,ncols m,NIL$Lisp)
  for i in minRowIndex(m)..maxRowIndex(m) repeat
    for j in minColIndex(m)..maxColIndex(m) repeat
      qsetelt_!(ans,i,j,f(qelt(m,i,j)))
```

```
    ans

  map_!(f,m) ==
    for i in minRowIndex(m)..maxRowIndex(m) repeat
      for j in minColIndex(m)..maxColIndex(m) repeat
        qsetelt_!(m,i,j,f(qelt(m,i,j)))
    m

  map(f,m,n) ==
    (nrows(m) ^= nrows(n)) or (ncols(m) ^= ncols(n)) =>
      error "map: arguments must have same dimensions"
    ans := new(nrows m,ncols m,NIL$Lisp)
    for i in minRowIndex(m)..maxRowIndex(m) repeat
      for j in minColIndex(m)..maxColIndex(m) repeat
        qsetelt_!(ans,i,j,f(qelt(m,i,j),qelt(n,i,j)))
    ans

  map(f,m,n,r) ==
    maxRow := max(maxRowIndex m,maxRowIndex n)
    maxCol := max(maxColIndex m,maxColIndex n)
    ans := new(max(nrows m,nrows n),max(ncols m,ncols n),NIL$Lisp)
    for i in minRowIndex(m)..maxRow repeat
      for j in minColIndex(m)..maxCol repeat
        qsetelt_!(ans,i,j,f(elt(m,i,j,r),elt(n,i,j,r)))
    ans

  setRow_!(m,i,v) ==
    i < minRowIndex(m) or i > maxRowIndex(m) =>
      error "setRow!: index out of range"
    for j in minColIndex(m)..maxColIndex(m) _
      for k in minIndex(v)..maxIndex(v) repeat
        qsetelt_!(m,i,j,v.k)
    m

  setColumn_!(m,j,v) ==
    j < minColIndex(m) or j > maxColIndex(m) =>
      error "setColumn!: index out of range"
    for i in minRowIndex(m)..maxRowIndex(m) _
      for k in minIndex(v)..maxIndex(v) repeat
        qsetelt_!(m,i,j,v.k)
    m

  if R has _= : (R,R) -> Boolean then

    m = n ==
      eq?(m,n) => true
      (nrows(m) ^= nrows(n)) or (ncols(m) ^= ncols(n)) => false
      for i in minRowIndex(m)..maxRowIndex(m) repeat
        for j in minColIndex(m)..maxColIndex(m) repeat
          not (qelt(m,i,j) = qelt(n,i,j)) => return false
      true

    member?(r,m) ==
      for i in minRowIndex(m)..maxRowIndex(m) repeat
```

```
          for j in minColIndex(m)..maxColIndex(m) repeat
            qelt(m,i,j) = r => return true
        false

      count(r:R,m:%) == count(x +-> x = r,m)

    if Row has shallowlyMutable then

      row(m,i) ==
        i < minRowIndex(m) or i > maxRowIndex(m) =>
          error "row: index out of range"
        v : Row := new(ncols m,NIL$Lisp)
        for j in minColIndex(m)..maxColIndex(m) _
          for k in minIndex(v)..maxIndex(v) repeat
            qsetelt_!(v,k,qelt(m,i,j))
        v

    if Col has shallowlyMutable then

      column(m,j) ==
        j < minColIndex(m) or j > maxColIndex(m) =>
          error "column: index out of range"
        v : Col := new(nrows m,NIL$Lisp)
        for i in minRowIndex(m)..maxRowIndex(m) _
          for k in minIndex(v)..maxIndex(v) repeat
            qsetelt_!(v,k,qelt(m,i,j))
        v

    if R has CoercibleTo(OutputForm) then

      coerce(m:%) ==
        l : List List OutputForm
        l := [[qelt(m,i,j) :: OutputForm _
                 for j in minColIndex(m)..maxColIndex(m)] _
                 for i in minRowIndex(m)..maxRowIndex(m)]
        matrix l
```

— **COQ ARR2CAT** —

```
(* category ARR2CAT *)
(*

--% Predicates

    any? : ((R -> Boolean),%) -> Boolean
    any?(f,m) ==
      for i in minRowIndex(m)..maxRowIndex(m) repeat
        for j in minColIndex(m)..maxColIndex(m) repeat
          f(qelt(m,i,j)) => return true
      false
```

```
    every? : ((R -> Boolean),%) -> Boolean
    every?(f,m) ==
      for i in minRowIndex(m)..maxRowIndex(m) repeat
        for j in minColIndex(m)..maxColIndex(m) repeat
          not f(qelt(m,i,j)) => return false
      true

    size? : (%,NonNegativeInteger) -> Boolean
    size?(m,n) == nrows(m) * ncols(m) = n

    less? : (%,NonNegativeInteger) -> Boolean
    less?(m,n) == nrows(m) * ncols(m) < n

    more? : (%,NonNegativeInteger) -> Boolean
    more?(m,n) == nrows(m) * ncols(m) > n

--% Size inquiries

    #? : % -> NonNegativeInteger
    # m == nrows(m) * ncols(m)

--% Part extractions

    elt: (%,Integer,Integer,R) -> R
    elt(m,i,j,r) ==
      i < minRowIndex(m) or i > maxRowIndex(m) => r
      j < minColIndex(m) or j > maxColIndex(m) => r
      qelt(m,i,j)

    count : ((R -> Boolean),%) -> NonNegativeInteger
    count(f:R -> Boolean,m:%) ==
      num : NonNegativeInteger := 0
      for i in minRowIndex(m)..maxRowIndex(m) repeat
        for j in minColIndex(m)..maxColIndex(m) repeat
          if f(qelt(m,i,j)) then num := num + 1
      num

    parts: % -> List R
    parts m ==
      entryList : List R := nil()
      for i in maxRowIndex(m)..minRowIndex(m) by -1 repeat
        for j in maxColIndex(m)..minColIndex(m) by -1 repeat
          entryList := concat(qelt(m,i,j),entryList)
      entryList

--% Creation

    copy : % -> %
    copy m ==
      ans := new(nrows m,ncols m,NIL$Lisp)
      for i in minRowIndex(m)..maxRowIndex(m) repeat
        for j in minColIndex(m)..maxColIndex(m) repeat
          qsetelt_!(ans,i,j,qelt(m,i,j))
      ans
```

```
 fill_!: (%,R) -> %
 fill_!(m,r) ==
   for i in minRowIndex(m)..maxRowIndex(m) repeat
     for j in minColIndex(m)..maxColIndex(m) repeat
       qsetelt_!(m,i,j,r)
   m

map: (R -> R,%) -> %
map(f,m) ==
   ans := new(nrows m,ncols m,NIL$Lisp)
   for i in minRowIndex(m)..maxRowIndex(m) repeat
     for j in minColIndex(m)..maxColIndex(m) repeat
       qsetelt_!(ans,i,j,f(qelt(m,i,j)))
   ans

 map_!: (R -> R,%) -> %
 map_!(f,m) ==
   for i in minRowIndex(m)..maxRowIndex(m) repeat
     for j in minColIndex(m)..maxColIndex(m) repeat
       qsetelt_!(m,i,j,f(qelt(m,i,j)))
   m

 map:((R,R) -> R,%,%) -> %
 map(f,m,n) ==
   (nrows(m) ^= nrows(n)) or (ncols(m) ^= ncols(n)) =>
     error "map: arguments must have same dimensions"
   ans := new(nrows m,ncols m,NIL$Lisp)
   for i in minRowIndex(m)..maxRowIndex(m) repeat
     for j in minColIndex(m)..maxColIndex(m) repeat
       qsetelt_!(ans,i,j,f(qelt(m,i,j),qelt(n,i,j)))
   ans

 map:((R,R) -> R,%,%,R) -> %
 map(f,m,n,r) ==
   maxRow := max(maxRowIndex m,maxRowIndex n)
   maxCol := max(maxColIndex m,maxColIndex n)
   ans := new(max(nrows m,nrows n),max(ncols m,ncols n),NIL$Lisp)
   for i in minRowIndex(m)..maxRow repeat
     for j in minColIndex(m)..maxCol repeat
       qsetelt_!(ans,i,j,f(elt(m,i,j,r),elt(n,i,j,r)))
   ans

 setRow_!: (%,Integer,Row) -> %
 setRow_!(m,i,v) ==
   i < minRowIndex(m) or i > maxRowIndex(m) =>
     error "setRow!: index out of range"
   for j in minColIndex(m)..maxColIndex(m) _
     for k in minIndex(v)..maxIndex(v) repeat
       qsetelt_!(m,i,j,v.k)
   m

 setColumn_!: (%,Integer,Col) -> %
 setColumn_!(m,j,v) ==
```

```
    j < minColIndex(m) or j > maxColIndex(m) =>
      error "setColumn!: index out of range"
    for i in minRowIndex(m)..maxRowIndex(m) _
      for k in minIndex(v)..maxIndex(v) repeat
        qsetelt_!(m,i,j,v.k)
    m

  if R has _= : (R,R) -> Boolean then

    ?=? : (%,%) -> Boolean
    m = n ==
      eq?(m,n) => true
      (nrows(m) ^= nrows(n)) or (ncols(m) ^= ncols(n)) => false
      for i in minRowIndex(m)..maxRowIndex(m) repeat
        for j in minColIndex(m)..maxColIndex(m) repeat
          not (qelt(m,i,j) = qelt(n,i,j)) => return false
      true

    member? : (R,%) -> Boolean
    member?(r,m) ==
      for i in minRowIndex(m)..maxRowIndex(m) repeat
        for j in minColIndex(m)..maxColIndex(m) repeat
          qelt(m,i,j) = r => return true
      false

    count : (R,%) -> NonNegativeInteger
    count(r:R,m:%) == count(x +-> x = r,m)

  if Row has shallowlyMutable then

    row: (%,Integer) -> Row
    row(m,i) ==
      i < minRowIndex(m) or i > maxRowIndex(m) =>
        error "row: index out of range"
      v : Row := new(ncols m,NIL$Lisp)
      for j in minColIndex(m)..maxColIndex(m) _
        for k in minIndex(v)..maxIndex(v) repeat
          qsetelt_!(v,k,qelt(m,i,j))
      v

  if Col has shallowlyMutable then

    column: (%,Integer) -> Col
    column(m,j) ==
      j < minColIndex(m) or j > maxColIndex(m) =>
        error "column: index out of range"
      v : Col := new(nrows m,NIL$Lisp)
      for i in minRowIndex(m)..maxRowIndex(m) _
        for k in minIndex(v)..maxIndex(v) repeat
          qsetelt_!(v,k,qelt(m,i,j))
      v

  if R has CoercibleTo(OutputForm) then
```

```
      coerce : % -> OutputForm
      coerce(m:%) ==
        l : List List OutputForm
        l := [[qelt(m,i,j) :: OutputForm _
                for j in minColIndex(m)..maxColIndex(m)] _
                for i in minRowIndex(m)..maxRowIndex(m)]
        matrix l

*)
```

———-

— ARR2CAT.dotabb —

```
"ARR2CAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ARR2CAT"];
"ARR2CAT" -> "HOAGG"
```

———-

— ARR2CAT.dotfull —

```
"TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ARR2CAT"];
"TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a))"
    -> "HomogeneousAggregate(a:Type)"

"TwoDimensionalArrayCategory(a:Type,d:IndexedOneDimensionalArray(a,b),e:IndexedOneDimensionalArray(a,c))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ARR2CAT"];
"TwoDimensionalArrayCategory(a:Type,d:IndexedOneDimensionalArray(a,b),e:IndexedOneDimensionalArray(a,c))"
-> "TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a))"
```

———-

— ARR2CAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a))"
 [color=lightblue];
"TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a))"
    -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"
"HomogeneousAggregate(a:Type)" -> "Evalable(a:Type)"
"HomogeneousAggregate(a:Type)" -> "SetCategory()"
```

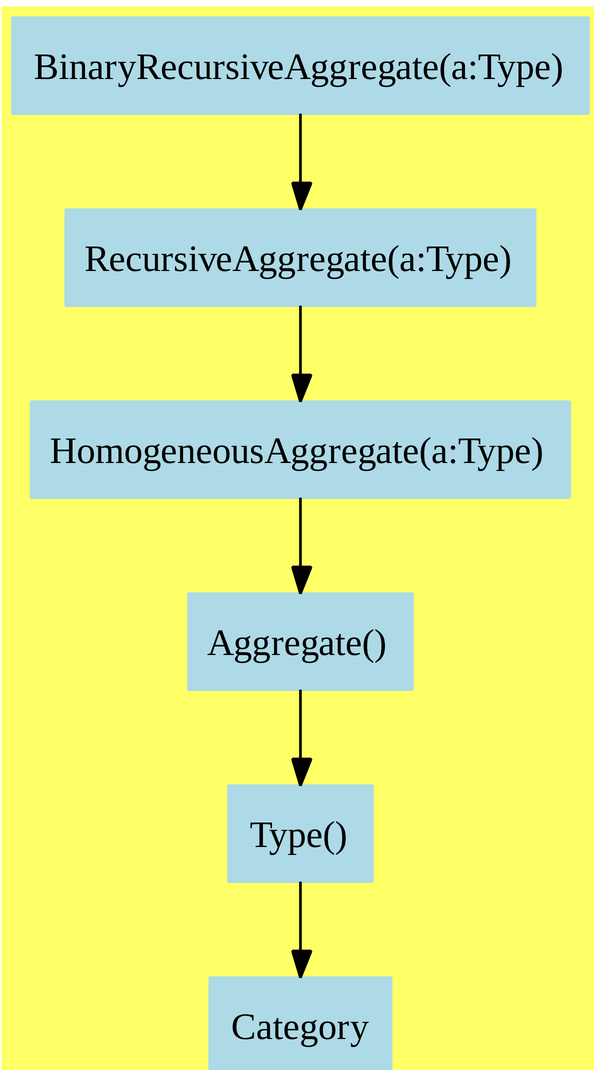
```
"Evalable(a:Type)" [color="#00EE00"];

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];

}
```

———

Chapter 6

Category Layer 5

6.0.106 BinaryRecursiveAggregate (BRAGG)



— BinaryRecursiveAggregate.input —

```
)set break resume
)sys rm -f BinaryRecursiveAggregate.output
)spool BinaryRecursiveAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BinaryRecursiveAggregate
--R
--R BinaryRecursiveAggregate(S: Type) is a category constructor
--R Abbreviation for BinaryRecursiveAggregate is BRAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for BRAGG
--R
--R------------------------------- Operations --------------------------------
--R children : % -> List(%)             copy : % -> %
--R cyclic? : % -> Boolean               distance : (%,%) -> Integer
--R ?.right : (%,right) -> %             ?.left : (%,left) -> %
--R ?.value : (%,value) -> S             empty : () -> %
--R empty? : % -> Boolean                eq? : (%,%) -> Boolean
--R latex : % -> String if S has SETCAT  leaf? : % -> Boolean
--R leaves : % -> List(S)                left : % -> %
--R map : ((S -> S),%) -> %              nodes : % -> List(%)
--R right : % -> %                       sample : () -> %
--R value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List(S) if $ has finiteAggregate
--R setchildren! : (%,List(%)) -> % if $ has shallowlyMutable
--R setelt : (%,right,%) -> % if $ has shallowlyMutable
--R setelt : (%,left,%) -> % if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
```

```
--R setleft! : (%,%) -> % if $ has shallowlyMutable
--R setright! : (%,%) -> % if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

— **BinaryRecursiveAggregate.help** —

```
====================================================================
BinaryRecursiveAggregate examples
====================================================================

A binary-recursive aggregate has 0, 1 or 2 children and serves
as a model for a binary tree or a doubly-linked aggregate structure

See Also:
o )show BinaryRecursiveAggregate
```

See:

⇒ "BinaryTreeCategory" (BTCAT) 7.0.122 on page 680
⇐ "RecursiveAggregate" (RCAGG) 5.0.104 on page 475

Exports:

any?	children	child?	coerce	copy
count	cyclic?	distance	empty	empty?
eq?	eval	every?	hash	latex
leaf?	leaves	left	less?	map
map!	member?	members	more?	nodes
node?	parts	right	sample	setchildren!
setelt	setleft!	setright!	setvalue!	size?
value	#?	?=?	?~=?	?.right
?.left	?.value			

Attributes exported:

- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
left : % -> %
right : % -> %
setelt : (%,right,%) -> % if $ has shallowlyMutable
setelt : (%,left,%) -> % if $ has shallowlyMutable
setleft! : (%,%) -> % if $ has shallowlyMutable
setright! : (%,%) -> % if $ has shallowlyMutable
```

These are implemented by this category:

```
children : % -> List %
coerce : % -> OutputForm if S has SETCAT
cyclic? : % -> Boolean
leaf? : % -> Boolean
leaves : % -> List S
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
nodes : % -> List %
node? : (%,%) -> Boolean if S has SETCAT
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?.right : (%,right) -> %
?.left : (%,left) -> %
```

These exports come from (p475) RecursiveAggregate(S:Type)

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
child? : (%,%) -> Boolean if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
distance : (%,%) -> Integer
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
sample : () -> %
```

```
 setchildren! : (%,List %) -> % if $ has shallowlyMutable
 setelt : (%,value,S) -> S if $ has shallowlyMutable
 setvalue! : (%,S) -> S if $ has shallowlyMutable
 size? : (%,NonNegativeInteger) -> Boolean
 value : % -> S
 ?~=? : (%,%) -> Boolean if S has SETCAT
 ?.value : (%,value) -> S
```

— BinaryRecursiveAggregate.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#BRAGG">
BinaryRecursiveAggregate (BRAGG)</a></h2>
</body>
```

— category BRAGG BinaryRecursiveAggregate —

```
)abbrev category BRAGG BinaryRecursiveAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ A binary-recursive aggregate has 0, 1 or 2 children and serves
++ as a model for a binary tree or a doubly-linked aggregate structure

BinaryRecursiveAggregate(S) : Category == SIG where
  S : Type

  SIG ==> RecursiveAggregate S with

    -- needs preorder, inorder and postorder iterators

    left : % -> %
      ++ left(u) returns the left child.

    elt : (%,"left") -> %
      ++ elt(u,"left") (also written: \axiom{a . left}) is
      ++ equivalent to \axiom{left(a)}.

    right : % -> %
      ++ right(a) returns the right child.

    elt : (%,"right") -> %
      ++ elt(a,"right") (also written: \axiom{a . right})
      ++ is equivalent to \axiom{right(a)}.

    if % has shallowlyMutable then

      setelt : (%,"left",%) -> %
        ++ setelt(a,"left",b) (also written \axiom{a . left := b}) is
        ++ equivalent to \axiom{setleft!(a,b)}.
```

```
   setleft_! : (%,%) -> %
     ++ setleft!(a,b) sets the left child of \axiom{a} to be b.

   setelt : (%,"right",%) -> %
     ++ setelt(a,"right",b) (also written \axiom{b . right := b})
     ++ is equivalent to \axiom{setright!(a,b)}.

   setright_! : (%,%) -> %
     ++ setright!(a,x) sets the right child of t to be x.

add

  cycleMax ==> 1000

  elt(x,"left")  == left x

  elt(x,"right") == right x

  leaf? x == empty? x or empty? left x and empty? right x

  leaves t ==
    empty? t => empty()$List(S)
    leaf? t => [value t]
    concat(leaves left t,leaves right t)

  nodes x ==
    l := empty()$List(%)
    empty? x => l
    concat(nodes left x,concat([x],nodes right x))

  children x ==
    l := empty()$List(%)
    empty? x => l
    empty? left x  => [right x]
    empty? right x => [left x]
    [left x, right x]

  if % has SetAggregate(S) and S has SetCategory then

    node?(u,v) ==
      empty? v => false
      u = v => true
      for y in children v repeat node?(u,y) => return true
      false

    x = y ==
      empty?(x) => empty?(y)
      empty?(y) => false
      value x = value y and left x = left y and right x = right y

    if % has finiteAggregate then

      member?(x,u) ==
```

```
        empty? u => false
        x = value u => true
        member?(x,left u) or member?(x,right u)

    if S has SetCategory then

      coerce(t:%): OutputForm ==
        empty? t =>  "[]"::OutputForm
        v := value(t):: OutputForm
        empty? left t =>
          empty? right t => v
          r := coerce(right t)@OutputForm
          bracket ["."::OutputForm, v, r]
        l := coerce(left t)@OutputForm
        r :=
          empty? right t => "."::OutputForm
          coerce(right t)@OutputForm
        bracket [l, v, r]

    if % has finiteAggregate then

      aggCount: (%,NonNegativeInteger) -> NonNegativeInteger

      #x == aggCount(x,0)

      aggCount(x,k) ==
        empty? x => 0
        k := k + 1
        k = cycleMax and cyclic? x => error "cyclic tree"
        for y in children x repeat k := aggCount(y,k)
        k

    isCycle?:  (%, List %) -> Boolean

    eqMember?: (%, List %) -> Boolean

    cyclic? x == not empty? x and isCycle?(x,empty()$(List %))

    isCycle?(x,acc) ==
      empty? x => false
      eqMember?(x,acc) => true
      for y in children x | not empty? y repeat
        isCycle?(y,acc) => return true
      false

    eqMember?(y,l) ==
      for x in l repeat eq?(x,y) => return true
      false

    if % has shallowlyMutable then

      setelt(x,"left",b)  == setleft_!(x,b)

      setelt(x,"right",b) == setright_!(x,b)
```

———-

— **COQ BRAGG** —

```
(* category BRAGG *)
(*
   cycleMax ==> 1000

   elt: (%,"left") -> %
   elt(x,"left")  == left x

   elt: (%,"right") -> %
   elt(x,"right") == right x

   leaf? : % -> Boolean
   leaf? x == empty? x or empty? left x and empty? right x

   leaves : % -> List(S)
   leaves t ==
     empty? t => empty()$List(S)
     leaf? t => [value t]
     concat(leaves left t,leaves right t)

   nodes : % -> List(%)
   nodes x ==
     l := empty()$List(%)
     empty? x => l
     concat(nodes left x,concat([x],nodes right x))

   children : % -> List(%)
   children x ==
     l := empty()$List(%)
     empty? x => l
     empty? left x  => [right x]
     empty? right x => [left x]
     [left x, right x]

   if % has SetAggregate(S) and S has SetCategory then

     node? : (%,%) -> Boolean
     node?(u,v) ==
       empty? v => false
       u = v => true
       for y in children v repeat node?(u,y) => return true
       false

     ?=? : (%,%) -> Boolean
     x = y ==
       empty?(x) => empty?(y)
       empty?(y) => false
       value x = value y and left x = left y and right x = right y
```

```
    if % has finiteAggregate then

      member? : (S,%) -> Boolean
      member?(x,u) ==
        empty? u => false
        x = value u => true
        member?(x,left u) or member?(x,right u)

  if S has SetCategory then

    coerce : % -> OutputForm
    coerce(t:%): OutputForm ==
      empty? t =>  "[]"::OutputForm
      v := value(t):: OutputForm
      empty? left t =>
        empty? right t => v
        r := coerce(right t)@OutputForm
        bracket ["."::OutputForm, v, r]
      l := coerce(left t)@OutputForm
      r :=
        empty? right t => "."::OutputForm
        coerce(right t)@OutputForm
      bracket [l, v, r]

  if % has finiteAggregate then

    #? : % -> NonNegativeInteger
    #x == aggCount(x,0)

    aggCount: (%,NonNegativeInteger) -> NonNegativeInteger
    aggCount(x,k) ==
      empty? x => 0
      k := k + 1
      k = cycleMax and cyclic? x => error "cyclic tree"
      for y in children x repeat k := aggCount(y,k)
      k

  cyclic? : % -> Boolean
  cyclic? x == not empty? x and isCycle?(x,empty()$(List %))

  isCycle?: (%, List %) -> Boolean
  isCycle?(x,acc) ==
    empty? x => false
    eqMember?(x,acc) => true
    for y in children x | not empty? y repeat
      isCycle?(y,acc) => return true
    false

  eqMember?: (%, List %) -> Boolean
  eqMember?(y,l) ==
    for x in l repeat eq?(x,y) => return true
    false

  if % has shallowlyMutable then
```

```
     setelt: (%,"left",%) -> %
     setelt(x,"left",b)  == setleft_!(x,b)

     setelt: (%,"right",%) -> %
     setelt(x,"right",b) == setright_!(x,b)

*)
```

———-

— **BRAGG.dotabb** —

```
"BRAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=BRAGG"];
"BRAGG" -> "RCAGG"
```

———-

— **BRAGG.dotfull** —

```
"BinaryRecursiveAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BRAGG"];
"BinaryRecursiveAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"

"BinaryRecursiveAggregate(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=BRAGG"];
"BinaryRecursiveAggregate(a:SetCategory)" ->
   "BinaryRecursiveAggregate(a:Type)"
```

———-

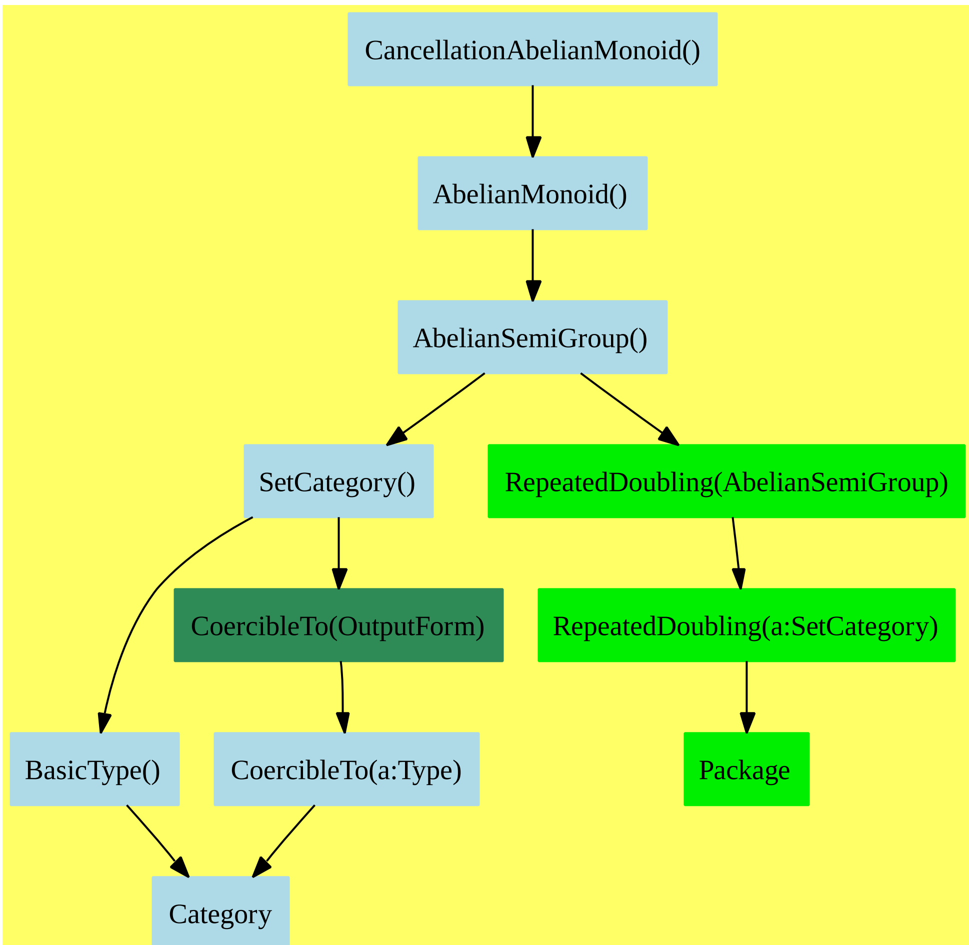
— **BRAGG.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"BinaryRecursiveAggregate(a:Type)" [color=lightblue];
"BinaryRecursiveAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"

"RecursiveAggregate(a:Type)" [color=lightblue];
"RecursiveAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
```

```
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

———-

6.0.107 CancellationAbelianMonoid (CABMON)



— **CancellationAbelianMonoid.input** —

```
)set break resume
)sys rm -f CancellationAbelianMonoid.output
)spool CancellationAbelianMonoid.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show CancellationAbelianMonoid
--R
--R CancellationAbelianMonoid is a category constructor
--R Abbreviation for CancellationAbelianMonoid is CABMON
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for CABMON
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                      ?=? : (%,%) -> Boolean
--R 0 : () -> %                           coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— **CancellationAbelianMonoid.help** —

```
====================================================================
CancellationAbelianMonoid examples
====================================================================

This is an AbelianMonoid with the cancellation property, that is,
     a+b = a+c => b=c
This is formalised by the partial subtraction operator, which satisfies
the Axiom
     c = a+b <=> c-b = a

See Also:
o )show CancellationAbelianMonoid
```

See:

⇒ "AbelianGroup" (ABELGRP) 7.0.121 on page 673
⇒ "FreeAbelianMonoidCategory" (FAMONC) 7.0.127 on page 719
⇒ "OrderedCancellationAbelianMonoid" (OCAMON) 8.0.142 on page 870
⇐ "AbelianMonoid" (ABELMON) 5.0.90 on page 383

Exports:

0	coerce	hash	latex	sample
subtractIfCan	zero?	?~=?	?*?	?+?
?=?				

These are directly exported but not implemented:

```
subtractIfCan : (%,%) -> Union(%,"failed")
```

These exports come from (p383) AbelianMonoid():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
zero? : % -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?~=? : (%,%) -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
```

— CancellationAbelianMonoid.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#CABMON">
CancellationAbelianMonoid (CABMON)</a></h2>
</body>
```

— category CABMON CancellationAbelianMonoid —

```
)abbrev category CABMON CancellationAbelianMonoid
++ Description:
++ This is an \spadtype{AbelianMonoid} with the cancellation property, \br
++ \tab{5}\spad{ a+b = a+c => b=c }.\br
++ This is formalised by the partial subtraction operator,
++ which satisfies the  Axioms\br
++ \tab{5}\spad{c = a+b <=> c-b = a}

CancellationAbelianMonoid() : Category == SIG where

  SIG ==> AbelianMonoid with

    subtractIfCan : (%,%) -> Union(%,"failed")
      ++ subtractIfCan(x, y) returns an element z such that \spad{z+y=x}
      ++ or "failed" if no such element exists.
```

— CABMON.dotabb —

```
"CABMON"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CABMON"];
```

```
"CABMON" -> "ABELMON"
```

———-

— **CABMON.dotfull** —

```
"CancellationAbelianMonoid()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CABMON"];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"
```

———-

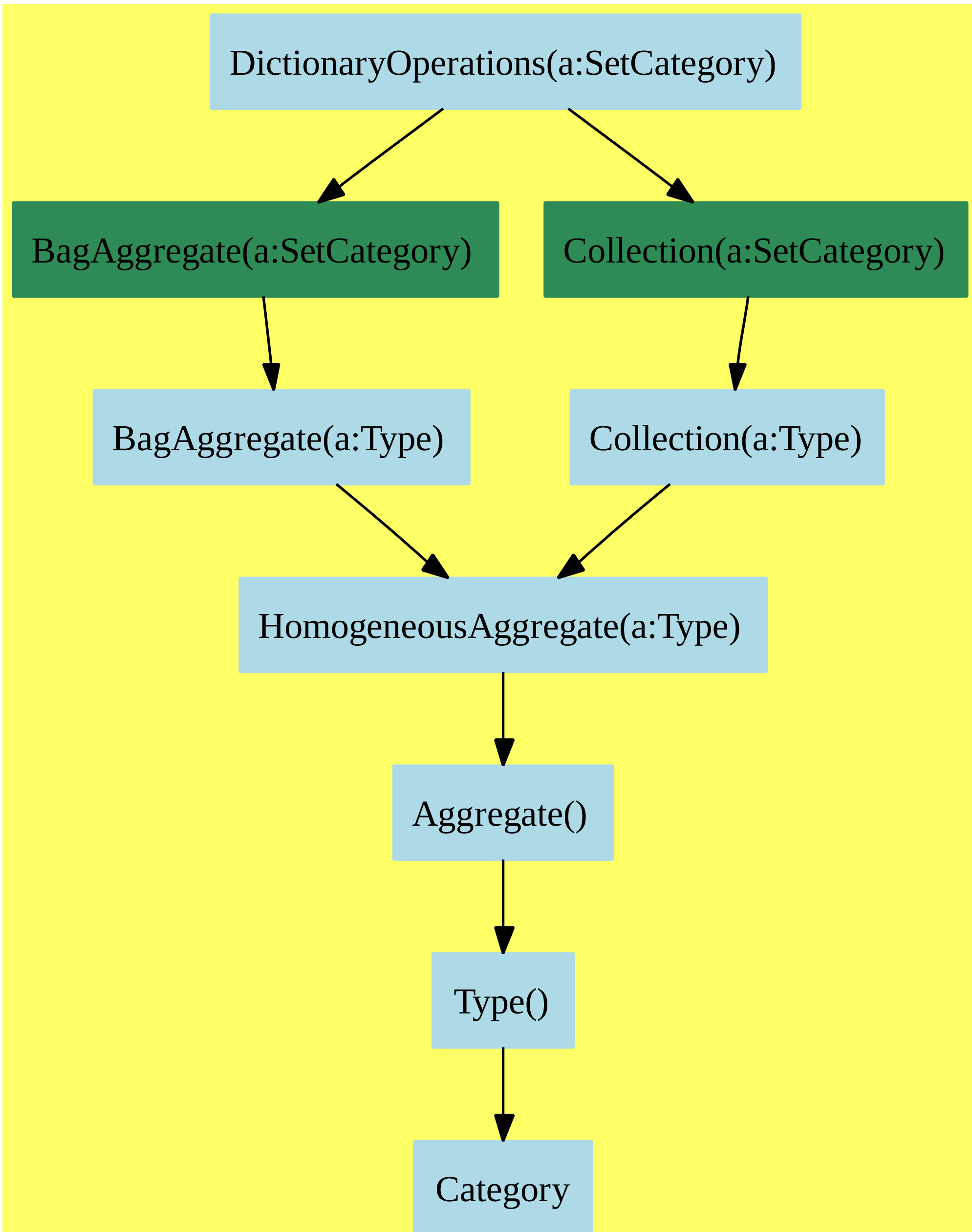
— **CABMON.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SetCategory()"
"AbelianSemiGroup()" -> "RepeatedDoubling(AbelianSemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" ->
    "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"RepeatedDoubling(AbelianSemiGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianSemiGroup)" -> "RepeatedDoubling(a:SetCategory)"

"RepeatedDoubling(a:SetCategory)" [color="#00EE00"];
"RepeatedDoubling(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];
```

```
}
```

———-

6.0.108 DictionaryOperations (DIOPS)



— DictionaryOperations.input —

```
)set break resume
)sys rm -f DictionaryOperations.output
)spool DictionaryOperations.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DictionaryOperations
--R
--R DictionaryOperations(S: SetCategory) is a category constructor
--R Abbreviation for DictionaryOperations is DIOPS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for DIOPS
--R
--R------------------------------- Operations --------------------------------
--R bag : List(S) -> %                    construct : List(S) -> %
--R copy : % -> %                         dictionary : List(S) -> %
--R dictionary : () -> %                  empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R extract! : % -> S                     insert! : (S,%) -> %
--R inspect : % -> S                      latex : % -> String if S has SETCAT
--R map : ((S -> S),%) -> %               sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT(INFORM)
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R hash : % -> SingleInteger if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(S) if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R remove! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (S,%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
```

```
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

— DictionaryOperations.help —

```
====================================================================
DictionaryOperations examples
====================================================================

This category is a collection of operations common to both
categories Dictionary and MultiDictionary.

See Also:
o )show DictionaryOperations
```

See:

⇒ "Dictionary" (DIAGG) 7.0.123 on page 687
⇒ "MultiDictionary" (MDAGG) 7.0.128 on page 724
⇐ "BagAggregate" (BGAGG) 5.0.92 on page 393
⇐ "Collection" (CLAGG) 5.0.94 on page 403

Exports:

any?	bag	coerce	construct	convert
copy	count	dictionary	empty	empty?
eq?	eval	every?	extract!	find
hash	insert!	inspect	latex	less?
map	map!	member?	members	more?
parts	reduce	remove	remove!	removeDuplicates
sample	select	select!	size?	#?
?=?	?~=?			

Attributes exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.

These are directly exported but not implemented:

```
dictionary : List S -> %
remove! : ((S -> Boolean),%) -> % if $ has finiteAggregate
remove! : (S,%) -> % if $ has finiteAggregate
select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
```

These are implemented by this category:

```
coerce : % -> OutputForm if S has SETCAT
construct : List S -> %
copy : % -> %
dictionary : () -> %
```

These exports come from (p393) BagAggregate(S:SetCategory):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
bag : List S -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
extract! : % -> S
hash : % -> SingleInteger if S has SETCAT
insert! : (S,%) -> %
inspect : % -> S
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
sample : () -> %
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
```

These exports come from (p403) Collection(S:SetCategory)

```
convert : % -> InputForm if S has KONVERT INFORM
find : ((S -> Boolean),%) -> Union(S,"failed")
reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
```

```
 reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
 reduce : (((S,S) -> S),%,S,S) -> S
                 if S has SETCAT and $ has finiteAggregate
 remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
 remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
 removeDuplicates : % -> %
          if S has SETCAT and $ has finiteAggregate
 select : ((S -> Boolean),%) -> % if $ has finiteAggregate
```

— **DictionaryOperations.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#DIOPS">
DictionaryOperations (DIOPS)</a></h2>
</body>
```

— **category DIOPS DictionaryOperations** —

```
)abbrev category DIOPS DictionaryOperations
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ This category is a collection of operations common to both
++ categories \spadtype{Dictionary} and \spadtype{MultiDictionary}

DictionaryOperations(S) : Category == SIG where
  S : SetCategory

  SIG ==> Join(BagAggregate S, Collection(S)) with

    dictionary : () -> %
      ++ dictionary()$D creates an empty dictionary of type D.

    dictionary : List S -> %
      ++ dictionary([x,y,...,z]) creates a dictionary consisting of
      ++ entries \axiom{x,y,...,z}.

    if % has finiteAggregate then

      remove_! : (S,%) -> %
        ++ remove!(x,d) destructively changes dictionary d by removing
        ++ all entries y such that \axiom{y = x}.

      remove_! : (S->Boolean,%) -> %
        ++ remove!(p,d) destructively changes dictionary d by removeing
        ++ all entries x such that \axiom{p(x)} is true.

      select_! : (S->Boolean,%) -> %
        ++ select!(p,d) destructively changes dictionary d by removing
        ++ all entries x such that \axiom{p(x)} is not true.
```

```
   add

     construct l == dictionary l

     dictionary() == empty()

     if % has finiteAggregate then

       copy d == dictionary parts d

       coerce(s:%):OutputForm ==
         prefix("dictionary"@String :: OutputForm,
                  [x::OutputForm for x in parts s])
```

———-

— COQ DIOPS —

```
(* category DIOPS *)
(*

   construct : List(S) -> %
   construct l == dictionary l

   dictionary: () -> %
   dictionary() == empty()

   if % has finiteAggregate then

     copy : % -> %
     copy d == dictionary parts d

     coerce : % -> OutputForm
     coerce(s:%):OutputForm ==
       prefix("dictionary"@String :: OutputForm,
                                     [x::OutputForm for x in parts s])

*)
```

———-

— DIOPS.dotabb —

```
"DIOPS" [color=lightblue,href="bookvol10.2.pdf#nameddest=DIOPS"];
"DIOPS" -> "BGAGG"
"DIOPS" -> "CLAGG"
```

———-

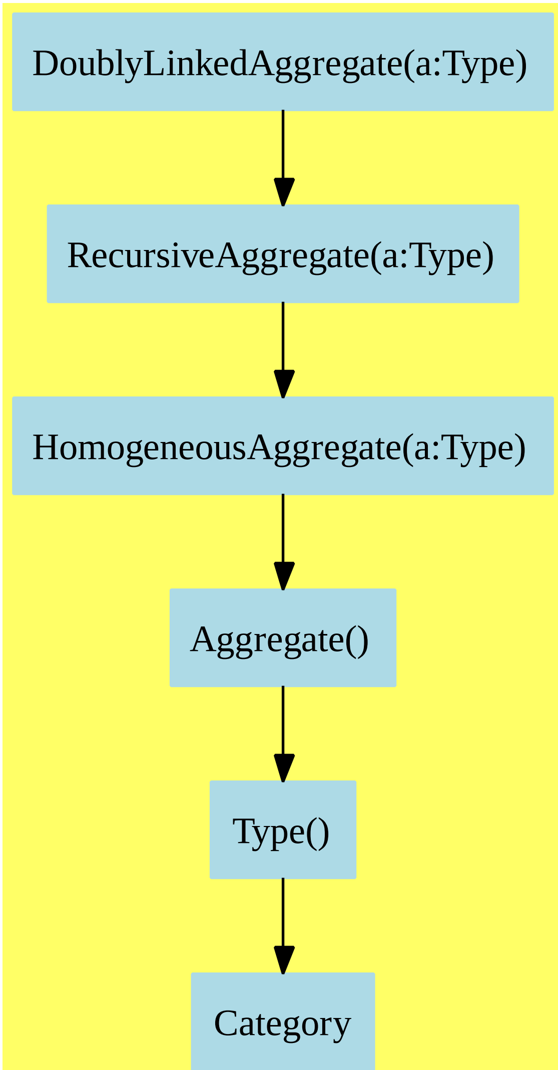
— DIOPS.dotfull —

```
"DictionaryOperations(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIOPS"];
"DictionaryOperations(a:SetCategory)" -> "BagAggregate(a:SetCategory)"
"DictionaryOperations(a:SetCategory)" -> "Collection(a:SetCategory)"
```

— **DIOPS.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"DictionaryOperations(a:SetCategory)" [color=lightblue];
"DictionaryOperations(a:SetCategory)" -> "BagAggregate(a:SetCategory)"
"DictionaryOperations(a:SetCategory)" -> "Collection(a:SetCategory)"

"BagAggregate(a:SetCategory)" [color=seagreen];
"BagAggregate(a:SetCategory)" -> "BagAggregate(a:Type)"

"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

"Collection(a:SetCategory)" [color=seagreen];
"Collection(a:SetCategory)" -> "Collection(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

6.0.109 DoublyLinkedAggregate (DLAGG)



— **DoublyLinkedAggregate.input** —

```
)set break resume
)sys rm -f DoublyLinkedAggregate.output
)spool DoublyLinkedAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DoublyLinkedAggregate
--R
--R DoublyLinkedAggregate(S: Type) is a category constructor
```

```
--R Abbreviation for DoublyLinkedAggregate is DLAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for DLAGG
--R
--R------------------------------- Operations --------------------------------
--R children : % -> List(%)              copy : % -> %
--R cyclic? : % -> Boolean               distance : (%,%) -> Integer
--R ?.value : (%,value) -> S             empty : () -> %
--R empty? : % -> Boolean                eq? : (%,%) -> Boolean
--R head : % -> %                        last : % -> S
--R latex : % -> String if S has SETCAT  leaf? : % -> Boolean
--R leaves : % -> List(S)                map : ((S -> S),%) -> %
--R next : % -> %                        nodes : % -> List(%)
--R previous : % -> %                    sample : () -> %
--R tail : % -> %                        value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R concat! : (%,%) -> % if $ has shallowlyMutable
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List(S) if $ has finiteAggregate
--R setchildren! : (%,List(%)) -> % if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setnext! : (%,%) -> % if $ has shallowlyMutable
--R setprevious! : (%,%) -> % if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

— DoublyLinkedAggregate.help —

```
====================================================================
```

```
DoublyLinkedAggregate examples
====================================================================

A doubly-linked aggregate serves as a model for a doubly-linked
list, that is, a list which can has links to both next and previous
nodes and thus can be efficiently traversed in both directions.

See Also:
o )show DoublyLinkedAggregate
```

———-

See:

⇐ "RecursiveAggregate" (RCAGG) 5.0.104 on page 475

Exports:

any?	children	child?	coerce	concat!
copy	count	cyclic?	distance	empty
empty?	eq?	eval	every?	hash
head	last	latex	leaf?	leaves
less?	map	map!	member?	members
more?	next	nodes	node?	parts
previous	sample	setchildren!	setelt	setnext!
setprevious!	setvalue!	size?	tail	value
#?	?=?	?~=?	?.value	

Attributes exported:

- **nil**

Attributes Used:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
concat! : (%,%) -> % if $ has shallowlyMutable
head : % -> %
last : % -> S
next : % -> %
previous : % -> %
setnext! : (%,%) -> % if $ has shallowlyMutable
setprevious! : (%,%) -> % if $ has shallowlyMutable
tail : % -> %
```

These exports come from (p475) RecursiveAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
children : % -> List %
child? : (%,%) -> Boolean if S has SETCAT
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
```

```
 count : (S,%) -> NonNegativeInteger
          if S has SETCAT and $ has finiteAggregate
 count : ((S -> Boolean),%) -> NonNegativeInteger
          if $ has finiteAggregate
 cyclic? : % -> Boolean
 distance : (%,%) -> Integer
 empty : () -> %
 empty? : % -> Boolean
 eq? : (%,%) -> Boolean
 eval : (%,List S,List S) -> %
          if S has EVALAB S and S has SETCAT
 eval : (%,S,S) -> %
          if S has EVALAB S and S has SETCAT
 eval : (%,Equation S) -> %
          if S has EVALAB S and S has SETCAT
 eval : (%,List Equation S) -> %
          if S has EVALAB S and S has SETCAT
 every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
 hash : % -> SingleInteger if S has SETCAT
 latex : % -> String if S has SETCAT
 leaf? : % -> Boolean
 leaves : % -> List S
 less? : (%,NonNegativeInteger) -> Boolean
 map : ((S -> S),%) -> %
 map! : ((S -> S),%) -> % if $ has shallowlyMutable
 member? : (S,%) -> Boolean
          if S has SETCAT and $ has finiteAggregate
 members : % -> List S if $ has finiteAggregate
 more? : (%,NonNegativeInteger) -> Boolean
 nodes : % -> List %
 node? : (%,%) -> Boolean if S has SETCAT
 parts : % -> List S if $ has finiteAggregate
 sample : () -> %
 setchildren! : (%,List %) -> % if $ has shallowlyMutable
 setelt : (%,value,S) -> S if $ has shallowlyMutable
 setvalue! : (%,S) -> S if $ has shallowlyMutable
 size? : (%,NonNegativeInteger) -> Boolean
 value : % -> S
 #? : % -> NonNegativeInteger if $ has finiteAggregate
 ?=? : (%,%) -> Boolean if S has SETCAT
 ?~=? : (%,%) -> Boolean if S has SETCAT
 ?.value : (%,value) -> S
```

— DoublyLinkedAggregate.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#DLAGG">
DoublyLinkedAggregate (DLAGG)</a></h2>
</body>
```

———-

— category DLAGG DoublyLinkedAggregate —

```
)abbrev category DLAGG DoublyLinkedAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ A doubly-linked aggregate serves as a model for a doubly-linked
++ list, that is, a list which can has links to both next and previous
++ nodes and thus can be efficiently traversed in both directions.

DoublyLinkedAggregate(S) : Category == SIG where
  S : Type

  SIG ==> RecursiveAggregate S with

    last : % -> S
      ++ last(l) returns the last element of a doubly-linked aggregate l.
      ++ Error: if l is empty.

    head : % -> %
      ++ head(l) returns the first element of a doubly-linked aggregate l.
      ++ Error: if l is empty.

    tail : % -> %
      ++ tail(l) returns the doubly-linked aggregate l starting at
      ++ its second element.
      ++ Error: if l is empty.

    previous : % -> %
      ++ previous(l) returns the doubly-link list beginning with its previous
      ++ element.
      ++ Error: if l has no previous element.
      ++ Note that \axiom{next(previous(l)) = l}.

    next : % -> %
      ++ next(l) returns the doubly-linked aggregate beginning with its next
      ++ element.
      ++ Error: if l has no next element.
      ++ Note that \axiom{next(l) = rest(l)} and \axiom{previous(next(l)) = l}.

    if % has shallowlyMutable then

      concat_! : (%,%) -> %
        ++ concat!(u,v) destructively concatenates doubly-linked aggregate v
        ++ to the end of doubly-linked aggregate u.

      setprevious_! : (%,%) -> %
        ++ setprevious!(u,v) destructively sets the previous node of
        ++ doubly-linked aggregate u to v, returning v.

      setnext_! : (%,%) -> %
        ++ setnext!(u,v) destructively sets the next node of doubly-linked
        ++ aggregate u to v, returning v.
```

— DLAGG.dotabb —

```
"DLAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=DLAGG"];
"DLAGG" -> "RCAGG"
```

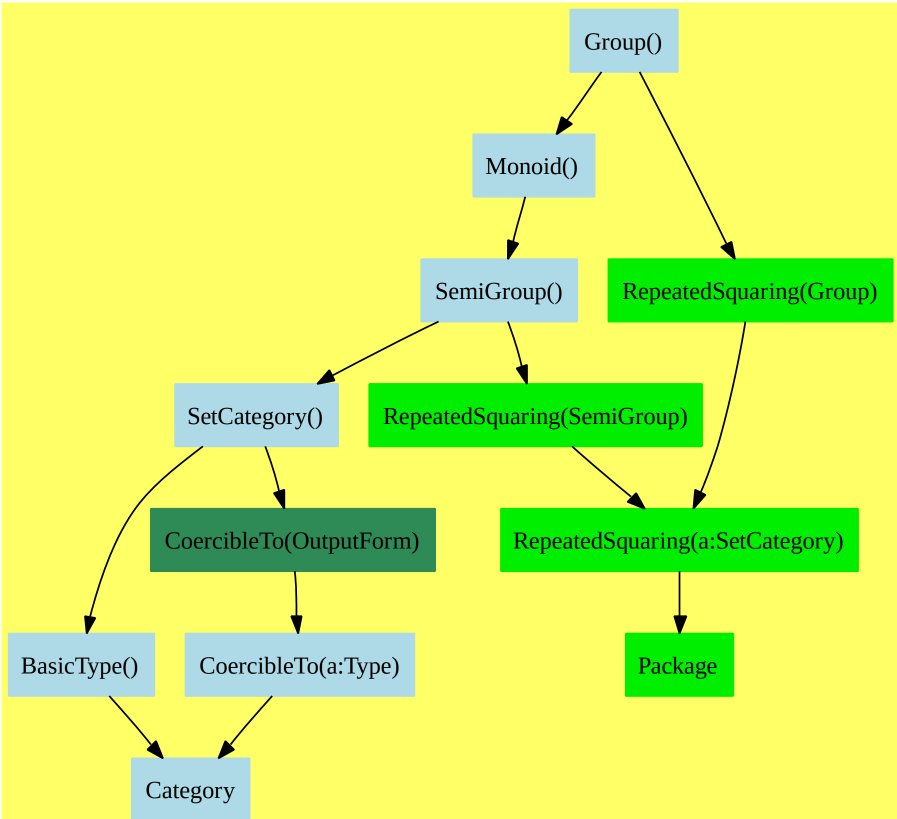
— DLAGG.dotfull —

```
"DoublyLinkedAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DLAGG"];
"DoublyLinkedAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"
```

— DLAGG.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"DoublyLinkedAggregate(a:Type)" [color=lightblue];
"DoublyLinkedAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"

"RecursiveAggregate(a:Type)" [color=lightblue];
"RecursiveAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

6.0.110 Group (GROUP)



— **Group.input** —

```
)set break resume
)sys rm -f Group.output
)spool Group.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Group
--R
--R Group is a category constructor
--R Abbreviation for Group is GROUP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for GROUP
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?**? : (%,Integer) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?/? : (%,%) -> %                      ?=? : (%,%) -> Boolean
```

```
--R 1 : () -> %                           ?^? : (%,Integer) -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R coerce : % -> OutputForm              commutator : (%,%) -> %
--R conjugate : (%,%) -> %                hash : % -> SingleInteger
--R inv : % -> %                          latex : % -> String
--R one? : % -> Boolean                   recip : % -> Union(%,"failed")
--R sample : () -> %                      ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

— Group.help —

```
====================================================================
Group examples
====================================================================

The class of multiplicative groups, monoids with multiplicative inverses.

Axioms:
        leftInverse("*":(%,%)->%,inv)   inv(x)*x = 1
        rightInverse("*":(%,%)->%,inv)  x*inv(x) = 1

See Also:
o )show Group
```

See:

⇒ "FunctionSpace" (FS) 17.0.219 on page 1544
⇒ "PermutationCategory" (PERMCAT) 7.0.130 on page 734
⇐ "Monoid" (MONOID) 5.0.100 on page 455

Exports:

1	coerce	commutator	conjugate	hash
inv	latex	one?	recip	sample
?~=?	?*?	?**?	?/?	?=?
?^?				

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

These are directly exported but not implemented:

```
inv : % -> %
```

These are implemented by this category:

```
commutator : (%,%) -> %
conjugate : (%,%) -> %
recip : % -> Union(%,"failed")
?/? : (%,%) -> %
?^? : (%,Integer) -> %
?**? : (%,Integer) -> %
```

These exports come from (p455) Monoid():

```
1 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
sample : () -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

— Group.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#GROUP">
Group (GROUP)</a></h2>
</body>
```

———-

— category GROUP Group —

```
)abbrev category GROUP Group
++ Description:
++ The class of multiplicative groups, that is, monoids with
++ multiplicative inverses.
++
++ Axioms\br
++ \tab{5}\spad{leftInverse("*":(%,%)->%,inv)}\tab{5}\spad{inv(x)*x = 1}\br
++ \tab{5}\spad{rightInverse("*":(%,%)->%,inv)}\tab{4}\spad{x*inv(x) = 1}

Group() : Category == SIG where

  SIG ==> Monoid with

    inv : % -> %
      ++ inv(x) returns the inverse of x.

    "/" : (%,%) -> %
      ++ x/y is the same as x times the inverse of y.

    "**" : (%,Integer) -> %
```

```
      ++ x**n returns x raised to the integer power n.

    "^" : (%,Integer) -> %
      ++ x^n returns x raised to the integer power n.

    unitsKnown
      ++ unitsKnown asserts that recip only returns
      ++ "failed" for non-units.

    conjugate : (%,%) -> %
      ++ conjugate(p,q) computes \spad{inv(q) * p * q}; this is
      ++ 'right action by conjugation'.

    commutator : (%,%) -> %
      ++ commutator(p,q) computes \spad{inv(p) * inv(q) * p * q}.

   add

     import RepeatedSquaring(%)

     x:% / y:% == x*inv(y)

     recip(x:%) == inv(x)

     _^(x:%, n:Integer):% == x ** n

     x:% ** n:Integer ==
       zero? n => 1
       n<0 => expt(inv(x),(-n) pretend PositiveInteger)
       expt(x,n pretend PositiveInteger)

     conjugate(p,q) == inv(q) * p * q

     commutator(p,q) == inv(p) * inv(q) * p * q
```

——————

— **COQ GROUP** —

```
(* category GROUP *)
(*
Axioms:
  leftInverse("*":(%,%)->%,inv)   inv(x)*x = 1
  rightInverse("*":(%,%)->%,inv)  x*inv(x) = 1

      import RepeatedSquaring(%)

      "/": (%,%) -> %
      x:% / y:% == x*inv(y)

      recip : % -> Union(%,"failed")
      recip(x:%) == inv(x)
```

```
      "^": (%,Integer) -> %
      _^(x:%, n:Integer):% == x ** n

      "**": (%,Integer) -> %
      x:% ** n:Integer ==
         zero? n => 1
         n<0 => expt(inv(x),(-n) pretend PositiveInteger)
         expt(x,n pretend PositiveInteger)

      conjugate: (%,%) -> %
      conjugate(p,q) == inv(q) * p * q

      commutator: (%,%) -> %
      commutator(p,q) == inv(p) * inv(q) * p * q

*)
```

———-

— **GROUP.dotabb** —

```
"GROUP"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=GROUP"];
"GROUP" -> "MONOID"
```

———-

— **GROUP.dotfull** —

```
"Group()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=GROUP"];
"Group()" -> "Monoid()"
```

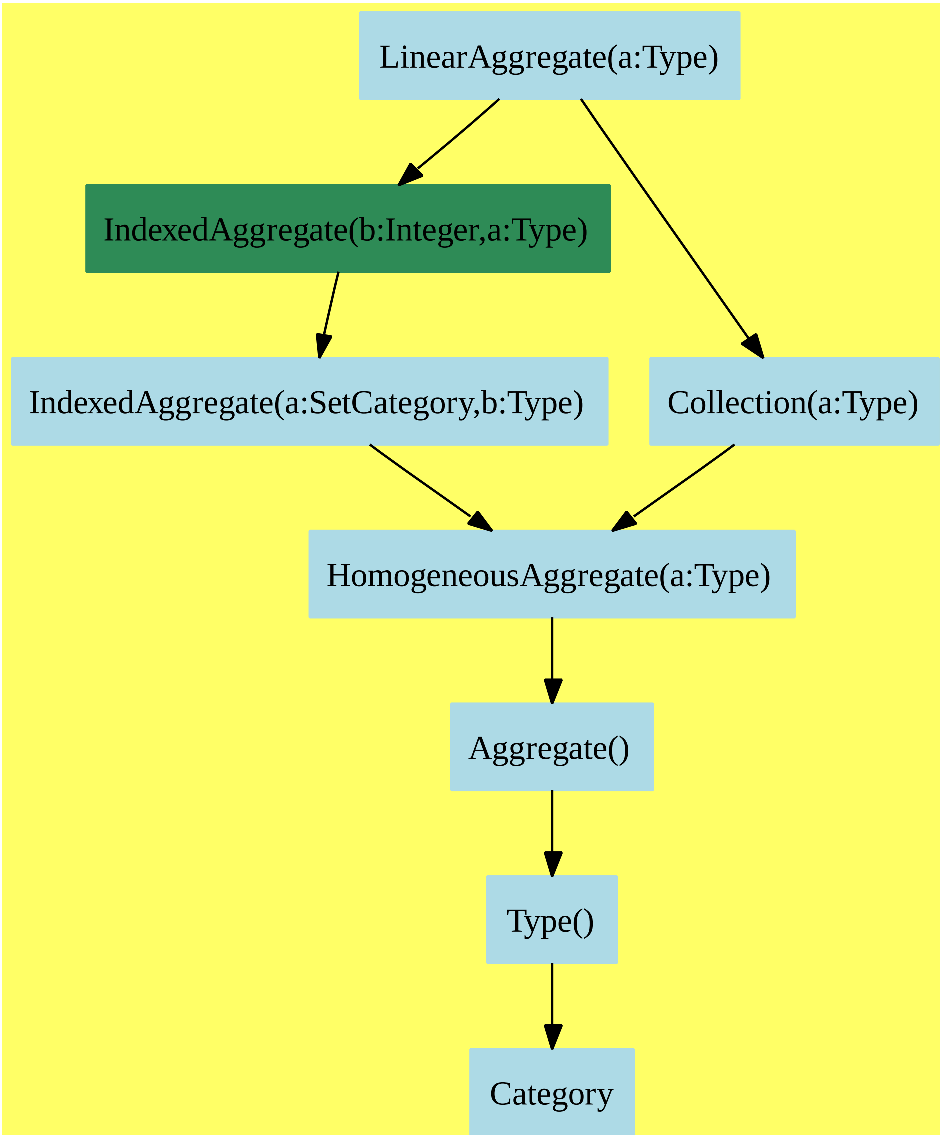
———-

— **GROUP.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"Group()" [color=lightblue];
"Group()" -> "Monoid()"
"Group()" -> "RepeatedSquaring(Group)"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SetCategory()"
```

```
"SemiGroup()" -> "RepeatedSquaring(SemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"RepeatedSquaring(Group)" [color="#00EE00"];
"RepeatedSquaring(Group)" -> "RepeatedSquaring(a:SetCategory)"

"RepeatedSquaring(SemiGroup)" [color="#00EE00"];
"RepeatedSquaring(SemiGroup)" -> "RepeatedSquaring(a:SetCategory)"

"RepeatedSquaring(a:SetCategory)" [color="#00EE00"];
"RepeatedSquaring(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];

}
```

6.0.111 LinearAggregate (LNAGG)



— **LinearAggregate.input** —

```
)set break resume
)sys rm -f LinearAggregate.output
)spool LinearAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
```

```
)show LinearAggregate
--R
--R LinearAggregate(S: Type) is a category constructor
--R Abbreviation for LinearAggregate is LNAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for LNAGG
--R
--R------------------------------- Operations --------------------------------
--R concat : List(%) -> %                concat : (%,%) -> %
--R concat : (S,%) -> %                  concat : (%,S) -> %
--R construct : List(S) -> %             copy : % -> %
--R delete : (%,Integer) -> %            ?.? : (%,Integer) -> S
--R elt : (%,Integer,S) -> S             empty : () -> %
--R empty? : % -> Boolean                entries : % -> List(S)
--R eq? : (%,%) -> Boolean               index? : (Integer,%) -> Boolean
--R indices : % -> List(Integer)         insert : (%,%,Integer) -> %
--R insert : (S,%,Integer) -> %          latex : % -> String if S has SETCAT
--R map : (((S,S) -> S),%,%) -> %        map : ((S -> S),%) -> %
--R new : (NonNegativeInteger,S) -> %    qelt : (%,Integer) -> S
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT(INFORM)
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (%,UniversalSegment(Integer)) -> %
--R ?.? : (%,UniversalSegment(Integer)) -> %
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R first : % -> S if Integer has ORDSET
--R hash : % -> SingleInteger if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(S) if $ has finiteAggregate
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
```

```
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment(Integer),S) -> S if $ has shallowlyMutable
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

— **LinearAggregate.help** —

```
====================================================================
LinearAggregate examples
====================================================================

A linear aggregate is an aggregate whose elements are indexed by integers.
Examples of linear aggregates are strings, lists, and arrays.

Most of the exported operations for linear aggregates are non-destructive
but are not always efficient for a particular aggregate.

For example, concat of two lists needs only to copy its first argument,
whereas concat of two arrays needs to copy both arguments. Most of the
operations exported here apply to infinite objects (for example, streams)
as well to finite ones. For finite linear aggregates, see
FiniteLinearAggregate.

See Also:
o )show LinearAggregate
```

See:

⇒ “ExtensibleLinearAggregate” (ELAGG) 7.0.125 on page 700
⇒ “FiniteLinearAggregate” (FLAGG) 7.0.126 on page 709
⇒ “StreamAggregate” (STAGG) 7.0.131 on page 739
⇐ “Collection” (CLAGG) 5.0.94 on page 403
⇐ “IndexedAggregate” (IXAGG) 5.0.98 on page 441

Exports:

any?	coerce	concat	construct	convert
copy	count	delete	elt	empty
empty?	entries	entry?	eq?	eval
every?	fill!	find	first	hash
index?	indices	insert	latex	less?
map	map!	maxIndex	member?	members
minIndex	more?	new	parts	qelt
qsetelt!	reduce	remove	removeDuplicates	sample
setelt	size?	swap!	?˜=?	#?
?=?	?.?			

Attributes exported:

- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
concat : (%,%) -> %
concat : List % -> %
delete : (%,Integer) -> %
delete : (%,UniversalSegment Integer) -> %
insert : (%,%,Integer) -> %
map : (((S,S) -> S),%,%) -> %
new : (NonNegativeInteger,S) -> %
setelt : (%,UniversalSegment Integer,S) -> S
         if $ has shallowlyMutable
?.? : (%,UniversalSegment Integer) -> %
```

These are implemented by this category:

```
concat : (%,S) -> %
concat : (S,%) -> %
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (S,%,Integer) -> %
maxIndex : % -> Integer if Integer has ORDSET
```

These exports come from (p441) IndexedAggregate(Integer,S:Type)

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
elt : (%,Integer,S) -> S
empty : () -> %
empty? : % -> Boolean
entries : % -> List S
```

```
entry? : (S,%) -> Boolean
         if $ has finiteAggregate and S has SETCAT
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
fill! : (%,S) -> % if $ has shallowlyMutable
first : % -> S if Integer has ORDSET
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
qelt : (%,Integer) -> S
qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
sample : () -> %
setelt : (%,Integer,S) -> S if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
#? : % -> NonNegativeInteger if $ has finiteAggregate
?.? : (%,Integer) -> S
?~=? : (%,%) -> Boolean if S has SETCAT
?=? : (%,%) -> Boolean if S has SETCAT
```

These exports come from (p403) Collection(S:Type):

```
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
find : ((S -> Boolean),%) -> Union(S,"failed")
reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
                if S has SETCAT and $ has finiteAggregate
remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
removeDuplicates : % -> %
         if S has SETCAT and $ has finiteAggregate
select : ((S -> Boolean),%) -> % if $ has finiteAggregate
```

— LinearAggregate.html —

```
<body>
<h2>
```

```
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#LNAGG">
LinearAggregate (LNAGG)</a></h2>
</body>
```

———-

— category LNAGG LinearAggregate —

```
)abbrev category LNAGG LinearAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ A linear aggregate is an aggregate whose elements are indexed by integers.
++ Examples of linear aggregates are strings, lists, and
++ arrays.
++ Most of the exported operations for linear aggregates are non-destructive
++ but are not always efficient for a particular aggregate.
++ For example, \spadfun{concat} of two lists needs only to copy its first
++ argument, whereas \spadfun{concat} of two arrays needs to copy both
++ arguments. Most of the operations exported here apply to infinite
++ objects (for example, streams) as well to finite ones.
++ For finite linear aggregates, see \spadtype{FiniteLinearAggregate}.

LinearAggregate(S) : Category == SIG where
  S : Type

  IA ==> IndexedAggregate(Integer,S)
  CO ==> Collection(S)

  SIG ==> Join(IA,CO) with

    new : (NonNegativeInteger,S) -> %
      ++ new(n,x) returns \axiom{fill!(new n,x)}.

    concat : (%,S) -> %
      ++ concat(u,x) returns aggregate u with additional element x at the end.
      ++ Note that for lists, \axiom{concat(u,x) == concat(u,[x])}

    concat : (S,%) -> %
      ++ concat(x,u) returns aggregate u with additional element at the front.
      ++ Note that for lists: \axiom{concat(x,u) == concat([x],u)}.

    concat : (%,%) -> %
      ++ concat(u,v) returns an aggregate consisting of the elements of u
      ++ followed by the elements of v.
      ++ Note that if \axiom{w = concat(u,v)} then
      ++ \axiom{w.i = u.i for i in indices u}
      ++ and \axiom{w.(j + maxIndex u) = v.j for j in indices v}.

    concat : List % -> %
      ++ concat(u), where u is a lists of aggregates \axiom{[a,b,...,c]},
      ++ returns a single aggregate consisting of the elements of \axiom{a}
      ++ followed by those
```

```
  ++ of b followed ... by the elements of c.
  ++ Note that \axiom{concat(a,b,...,c) = concat(a,concat(b,...,c))}.

map : ((S,S)->S,%,%) -> %
  ++ map(f,u,v) returns a new collection w with elements
  ++ \axiom{z = f(x,y)} for corresponding elements x and y from u and v.
  ++ Note that for linear aggregates, \axiom{w.i = f(u.i,v.i)}.

elt : (%,UniversalSegment(Integer)) -> %
  ++ elt(u,i..j) (also written: \axiom{a(i..j)}) returns the aggregate of
  ++ elements \axiom{u} for k from i to j in that order.
  ++ Note that in general, \axiom{a.s = [a.k for i in s]}.

delete : (%,Integer) -> %
  ++ delete(u,i) returns a copy of u with the \axiom{i}th
  ++ element deleted. Note that for lists,
  ++ \axiom{delete(a,i) == concat(a(0..i - 1),a(i + 1,..))}.

delete : (%,UniversalSegment(Integer)) -> %
  ++ delete(u,i..j) returns a copy of u with the \axiom{i}th through
  ++ \axiom{j}th element deleted.
  ++ Note that \axiom{delete(a,i..j) = concat(a(0..i-1),a(j+1..))}.

insert : (S,%,Integer) -> %
  ++ insert(x,u,i) returns a copy of u having x as its
  ++ \axiom{i}th element.
  ++ Note that \axiom{insert(x,a,k) = concat(concat(a(0..k-1),x),a(k..))}.

insert : (%,%,Integer) -> %
  ++ insert(v,u,k) returns a copy of u having v inserted beginning at the
  ++ \axiom{i}th element.
  ++ Note that \axiom{insert(v,u,k) = concat( u(0..k-1), v, u(k..) )}.

if % has shallowlyMutable then

  setelt : (%,UniversalSegment(Integer),S) -> S
    ++ setelt(u,i..j,x) (also written: \axiom{u(i..j) := x}) destructively
    ++ replaces each element in the segment \axiom{u(i..j)} by x.
    ++ The value x is returned.
    ++ Note that u is destructively change so
    ++ that \axiom{u.k := x for k in i..j};
    ++ its length remains unchanged.

add

  indices a == [i for i in minIndex a .. maxIndex a]

  index?(i, a) == i >= minIndex a and i <= maxIndex a

  concat(a:%, x:S) == concat(a, new(1, x))

  concat(x:S, y:%) == concat(new(1, x), y)

  insert(x:S, a:%, i:Integer) == insert(new(1, x), a, i)
```

```
    if % has finiteAggregate then

      maxIndex l == #l - 1 + minIndex l
```

———-

— COQ LNAGG —

```
(* category LNAGG *)
(*

  indices : % -> List(Integer)
  indices a == [i for i in minIndex a .. maxIndex a]

  index? : (Integer,%) -> Boolean
  index?(i, a) == i >= minIndex a and i <= maxIndex a

  concat: (%,S) -> %
  concat(a:%, x:S) == concat(a, new(1, x))

  concat: (S,%) -> %
  concat(x:S, y:%) == concat(new(1, x), y)

  insert: (S,%,Integer) -> %
  insert(x:S, a:%, i:Integer) == insert(new(1, x), a, i)

  if % has finiteAggregate then

    maxIndex : % -> Integer
    maxIndex l == #l - 1 + minIndex l

--if % has shallowlyMutable then new(n, s)  == fill_!(new n, s)

*)
```

———-

— LNAGG.dotabb —

```
"LNAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=LNAGG"];
"LNAGG" -> "IXAGG"
"LNAGG" -> "CLAGG"
```

———-

— LNAGG.dotfull —

```
"LinearAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LNAGG"];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
```

```
"LinearAggregate(a:Type)" -> "Collection(a:Type)"
```

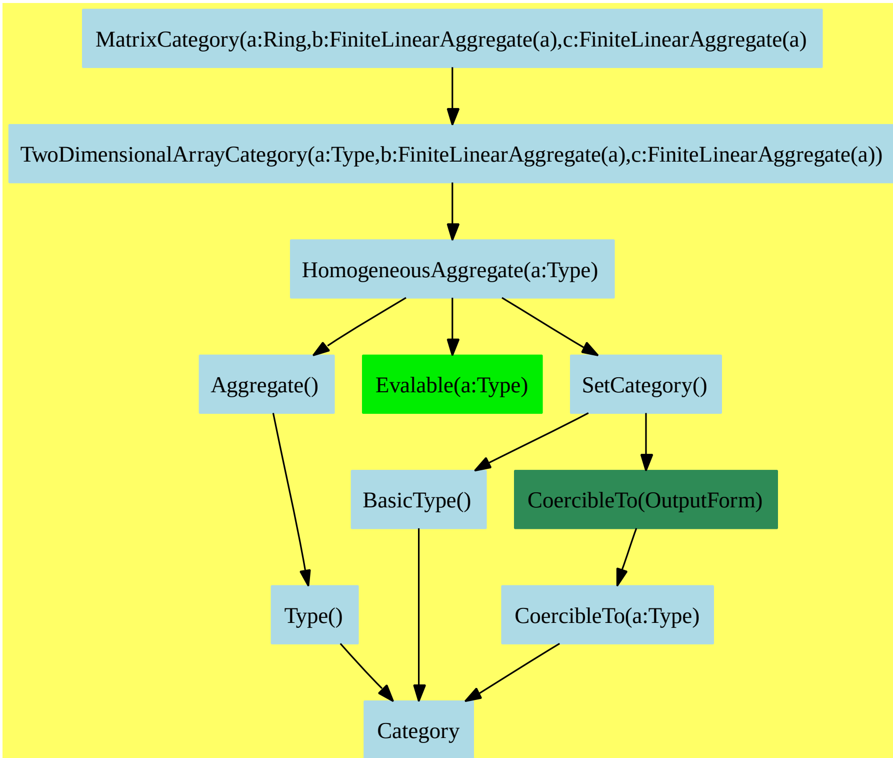
———-

— **LNAGG.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "Collection(a:Type)"

"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" ->
    "IndexedAggregate(a:SetCategory,b:Type)"

"IndexedAggregate(a:SetCategory,b:Type)" [color=lightblue];
"IndexedAggregate(a:SetCategory,b:Type)" ->
    "HomogeneousAggregate(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];

}
```

———-

6.0.112 MatrixCategory (MATCAT)



— **MatrixCategory.input** —

```
)set break resume
)sys rm -f MatrixCategory.output
)spool MatrixCategory.output
)set message test on
)set message auto off
)clear all


--S 1 of 59
square? matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R   (1)  true
--R                                                                 Type: Boolean
--E 1

--S 2 of 59
diagonal? matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R   (2)  false
```

```
--R                                                                 Type: Boolean
--E 2

--S 3 of 59
symmetric? matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R   (3)  false
--R                                                                 Type: Boolean
--E 3

--S 4 of 59
antisymmetric? matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R   (4)  false
--R                                                                 Type: Boolean
--E 4

--S 5 of 59
z:Matrix(INT):=zero(3,3)
--R
--R
--R        +0  0  0+
--R        |       |
--R   (5)  |0  0  0|
--R        |       |
--R        +0  0  0+
--R                                                         Type: Matrix(Integer)
--E 5

--S 6 of 59
matrix [[1,2,3],[4,5,6],[7,8,9],[1,1,1]]
--R
--R
--R        +1  2  3+
--R        |       |
--R        |4  5  6|
--R   (6)  |       |
--R        |7  8  9|
--R        |       |
--R        +1  1  1+
--R                                                         Type: Matrix(Integer)
--E 6

--S 7 of 59
z:Matrix(INT):=scalarMatrix(3,5)
--R
--R
--R        +5  0  0+
--R        |       |
--R   (7)  |0  5  0|
--R        |       |
--R        +0  0  5+
```

```
--R                                                     Type: Matrix(Integer)
--E 7

--S 8 of 59
diagonalMatrix [1,2,3]
--R
--R
--R        +1  0  0+
--R        |       |
--R   (8)  |0  2  0|
--R        |       |
--R        +0  0  3+
--R                                                     Type: Matrix(Integer)
--E 8

--S 9 of 59
diagonalMatrix [matrix [[1,2],[3,4]], matrix [[4,5],[6,7]]]
--R
--R
--R        +1  2  0  0+
--R        |          |
--R        |3  4  0  0|
--R   (9)  |          |
--R        |0  0  4  5|
--R        |          |
--R        +0  0  6  7+
--R                                                     Type: Matrix(Integer)
--E 9

--S 10 of 59
coerce([1,2,3])@Matrix(INT)
--R
--R
--R         +1+
--R         | |
--R   (10)  |2|
--R         | |
--R         +3+
--R                                                     Type: Matrix(Integer)
--E 10

--S 11 of 59
transpose([1,2,3])@Matrix(INT)
--R
--R
--R   (11)  [1  2  3]
--R                                                     Type: Matrix(Integer)
--E 11

--S 12 of 59
transpose matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1   1    1    1 +
```

```
--R          |                  |
--R          |1  2   3    4    5 |
--R          |                  |
--R   (12)   |1  4   9   16   25 |
--R          |                  |
--R          |1  8   27  64   125|
--R          |                  |
--R          +1  16  81  256  625+
--R                                                    Type: Matrix(Integer)
--E 12

--S 13 of 59
squareTop matrix [[j**i for i in 0..2] for j in 1..5]
--R
--R
--R          +1  1  1+
--R          |       |
--R   (13)   |1  2  4|
--R          |       |
--R          +1  3  9+
--R                                                    Type: Matrix(Integer)
--E 13

--S 14 of 59
t1:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R          +1  1  1    1    1 +
--R          |                  |
--R          |1  2  4    8   16 |
--R          |                  |
--R   (14)   |1  3  9   27   81 |
--R          |                  |
--R          |1  4  16  64   256|
--R          |                  |
--R          +1  5  25  125  625+
--R                                                    Type: Matrix(Integer)
--E 14

--S 15 of 59
horizConcat(t1,t1)
--R
--R
--R          +1  1  1    1    1   1  1  1    1    1 +
--R          |                                      |
--R          |1  2  4    8   16   1  2  4    8   16 |
--R          |                                      |
--R   (15)   |1  3  9   27   81   1  3  9   27   81 |
--R          |                                      |
--R          |1  4  16  64   256  1  4  16  64   256|
--R          |                                      |
--R          +1  5  25  125  625  1  5  25  125  625+
--R                                                    Type: Matrix(Integer)
--E 15
```

```
--S 16 of 59
t2:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R        +1  1  1    1    1 +
--R        |                  |
--R        |1  2  4    8   16 |
--R        |                  |
--R   (16) |1  3  9   27   81 |
--R        |                  |
--R        |1  4  16  64   256|
--R        |                  |
--R        +1  5  25  125  625+
--R                                                  Type: Matrix(Integer)
--E 16

--S 17 of 59
vertConcat(t2,t2)
--R
--R
--R        +1  1  1    1    1 +
--R        |                  |
--R        |1  2  4    8   16 |
--R        |                  |
--R        |1  3  9   27   81 |
--R        |                  |
--R        |1  4  16  64   256|
--R        |                  |
--R        |1  5  25  125  625|
--R   (17) |                  |
--R        |1  1  1    1    1 |
--R        |                  |
--R        |1  2  4    8   16 |
--R        |                  |
--R        |1  3  9   27   81 |
--R        |                  |
--R        |1  4  16  64   256|
--R        |                  |
--R        +1  5  25  125  625+
--R                                                  Type: Matrix(Integer)
--E 17

--S 18 of 59
t3:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R        +1  1  1    1    1 +
--R        |                  |
--R        |1  2  4    8   16 |
--R        |                  |
--R   (18) |1  3  9   27   81 |
--R        |                  |
--R        |1  4  16  64   256|
```

```
--R         |                 |
--R         +1  5  25  125  625+
--R                                                     Type: Matrix(Integer)
--E 18

--S 19 of 59
listOfLists t3
--R
--R
--R   (19)
--R   [[1,1,1,1,1],[1,2,4,8,16],[1,3,9,27,81],[1,4,16,64,256],[1,5,25,125,625]]
--R                                                 Type: List(List(Integer))
--E 19

--S 20 of 59
t4:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                 |
--R         |1  2  4    8   16 |
--R         |                 |
--R   (20)  |1  3  9   27   81 |
--R         |                 |
--R         |1  4  16  64   256|
--R         |                 |
--R         +1  5  25  125  625+
--R                                                     Type: Matrix(Integer)
--E 20

--S 21 of 59
elt(t4,3,3)
--R
--R
--R   (21)  9
--R                                                     Type: PositiveInteger
--E 21

--S 22 of 59
t5:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                 |
--R         |1  2  4    8   16 |
--R         |                 |
--R   (22)  |1  3  9   27   81 |
--R         |                 |
--R         |1  4  16  64   256|
--R         |                 |
--R         +1  5  25  125  625+
--R                                                     Type: Matrix(Integer)
--E 22
```

```
--S 23 of 59
setelt(t5,3,3,10)
--R
--R
--R   (23)  10
--R                                                    Type: PositiveInteger
--E 23

--S 24 of 59
t6:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R   (24)  |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                    Type: Matrix(Integer)
--E 24

--S 25 of 59
swapRows!(t6,2,4)
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R   (25)  |1  3  9   27   81 |
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R         +1  5  25  125  625+
--R                                                    Type: Matrix(Integer)
--E 25

--S 26 of 59
t7:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R   (26)  |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                    Type: Matrix(Integer)
```

```
--E 26

--S 27 of 59
swapColumns!(t7,2,4)
--R
--R
--R         +1   1   1   1   1 +
--R         |                  |
--R         |1   8   4   2  16 |
--R         |                  |
--R   (27)  |1  27   9   3  81 |
--R         |                  |
--R         |1  64   16  4  256|
--R         |                  |
--R         +1  125  25  5  625+
--R                                                     Type: Matrix(Integer)
--E 27

--S 28 of 59
t8:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R   (28)  |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                     Type: Matrix(Integer)
--E 28

--S 29 of 59
subMatrix(t8,1,3,2,4)
--R
--R
--R         +1  1  1 +
--R         |        |
--R   (29)  |2  4  8 |
--R         |        |
--R         +3  9  27+
--R                                                     Type: Matrix(Integer)
--E 29

--S 30 of 59
t9:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
```

```
--R   (30)  |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                    Type: Matrix(Integer)
--E 30

--S 31 of 59
setsubMatrix!(t9,2,2,matrix [[3,3],[3,3]])
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  3  3    8   16 |
--R         |                  |
--R   (31)  |1  3  3   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                    Type: Matrix(Integer)
--E 31

--S 32 of 59
t0:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R   (32)  |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                    Type: Matrix(Integer)
--E 32

--S 33 of 59
t0+t0
--R
--R
--R         +2  2   2    2    2  +
--R         |                    |
--R         |2  4   8   16    32 |
--R         |                    |
--R   (33)  |2  6   18  54   162 |
--R         |                    |
--R         |2  8   32  128  512 |
--R         |                    |
--R         +2  10  50  250  1250+
--R                                                    Type: Matrix(Integer)
```

```
--E 33

--S 34 of 59
t0-t0
--R
--R
--R        +0  0  0  0  0+
--R        |             |
--R        |0  0  0  0  0|
--R        |             |
--R   (34) |0  0  0  0  0|
--R        |             |
--R        |0  0  0  0  0|
--R        |             |
--R        +0  0  0  0  0+
--R                                                  Type: Matrix(Integer)
--E 34

--S 35 of 59
-t0
--R
--R
--R        +- 1  - 1  - 1    - 1    - 1 +
--R        |                            |
--R        |- 1  - 2  - 4    - 8   - 16 |
--R        |                            |
--R   (35) |- 1  - 3  - 9   - 27   - 81 |
--R        |                            |
--R        |- 1  - 4  - 16  - 64   - 256|
--R        |                            |
--R        +- 1  - 5  - 25  - 125  - 625+
--R                                                  Type: Matrix(Integer)
--E 35

--S 36 of 59
t0*t0
--R
--R
--R        + 5    15    55     225    979  +
--R        |                               |
--R        |31   129    573   2637   12405 |
--R        |                               |
--R   (36) |121  547   2551   12121  58315 |
--R        |                               |
--R        |341  1593  7585   36561  177745|
--R        |                               |
--R        +781  3711  17871  86841  424731+
--R                                                  Type: Matrix(Integer)
--E 36

--S 37 of 59
1/3*t0
--R
--R
```

```
--R         +1  1  1    1    1 +
--R         |-  -  -    -    - |
--R         |3  3  3    3    3 |
--R         |                  |
--R         |1  2  4    8   16 |
--R         |-  -  -    -   -- |
--R         |3  3  3    3    3 |
--R         |                  |
--R         |1                 |
--R   (37)  |-  1  3    9   27 |
--R         |3                 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |-  -  --  --   ---|
--R         |3  3   3   3    3 |
--R         |                  |
--R         |1  5  25  125  625|
--R         |-  -  --  ---  ---|
--R         +3  3   3   3    3 +
--R                                                  Type: Matrix(Fraction(Integer))
--E 37

--S 38 of 59
m:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R   (38)  |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                            Type: Matrix(Integer)
--E 38

--S 39 of 59
t0*1/3
--R
--R
--R         +1  1  1    1    1 +
--R         |-  -  -    -    - |
--R         |3  3  3    3    3 |
--R         |                  |
--R         |1  2  4    8   16 |
--R         |-  -  -    -   -- |
--R         |3  3  3    3    3 |
--R         |                  |
--R         |1                 |
--R   (39)  |-  1  3    9   27 |
--R         |3                 |
--R         |                  |
```

```
--R         |1  4  16  64   256|
--R         |-  -  --  --   ---|
--R         |3  3   3   3    3 |
--R         |                  |
--R         |1  5  25  125  625|
--R         |-  -  --  ---  ---|
--R         +3  3   3   3    3 +
--R                                          Type: Matrix(Fraction(Integer))
--E 39

--S 40 of 59
3*t0
--R
--R
--R         +3  3   3    3    3  +
--R         |                    |
--R         |3  6   12  24    48 |
--R         |                    |
--R   (40)  |3  9   27  81   243 |
--R         |                    |
--R         |3  12  48  192  768 |
--R         |                    |
--R         +3  15  75  375  1875+
--R                                                    Type: Matrix(Integer)
--E 40

--S 41 of 59
c:=coerce([1,2,3,4,5])@Matrix(INT)
--R
--R
--R         +1+
--R         | |
--R         |2|
--R         | |
--R   (41)  |3|
--R         | |
--R         |4|
--R         | |
--R         +5+
--R                                                    Type: Matrix(Integer)
--E 41

--S 42 of 59
t0*c
--R
--R
--R         + 15 +
--R         |    |
--R         |129 |
--R         |    |
--R   (42)  |547 |
--R         |    |
--R         |1593|
--R         |    |
```

```
--R         +3711+
--R                                                      Type: Matrix(Integer)
--E 42

--S 43 of 59
r:=transpose([1,2,3,4,5])@Matrix(INT)
--R
--R
--R   (43)  [1  2  3  4  5]
--R                                                      Type: Matrix(Integer)
--E 43

--S 44 of 59
r*t0
--R
--R
--R   (44)  [15  55  225  979  4425]
--R                                                      Type: Matrix(Integer)
--E 44

--S 45 of 59
t0**3
--R
--R
--R         + 1279    5995     28635     138385    674175  +
--R         |                                              |
--R         |15775    74581    358021   1735927    8476705 |
--R         |                                              |
--R   (45)  |73655   348927   1677079   8138493   39765355 |
--R         |                                              |
--R         |223825  1061251  5103579   24775909  121090455|
--R         |                                              |
--R         +533935  2532835  12184195  59162185  289195879+
--R                                                      Type: Matrix(Integer)
--E 45

--S 46 of 59
t10:=matrix [[2**i for i in 2..4] for j in 1..5]
--R
--R
--R         +4  8  16+
--R         |        |
--R         |4  8  16|
--R         |        |
--R   (46)  |4  8  16|
--R         |        |
--R         |4  8  16|
--R         |        |
--R         +4  8  16+
--R                                                      Type: Matrix(Integer)
--E 46

--S 47 of 59
exquo(t10,2)
```

```
--R
--R
--R        +2  4  8+
--R        |       |
--R        |2  4  8|
--R        |       |
--R   (47) |2  4  8|
--R        |       |
--R        |2  4  8|
--R        |       |
--R        +2  4  8+
--R                                          Type: Union(Matrix(Integer),...)
--E 47

--S 48 of 59
t10/4
--R
--R
--R        +1  2  4+
--R        |       |
--R        |1  2  4|
--R        |       |
--R   (48) |1  2  4|
--R        |       |
--R        |1  2  4|
--R        |       |
--R        +1  2  4+
--R                                          Type: Matrix(Fraction(Integer))
--E 48

--S 49 of 59
rowEchelon matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R        +1  0  0  0  0 +
--R        |              |
--R        |0  1  1  1  1 |
--R        |              |
--R   (49) |0  0  2  0  2 |
--R        |              |
--R        |0  0  0  6  12|
--R        |              |
--R        +0  0  0  0  24+
--R                                                    Type: Matrix(Integer)
--E 49

--S 50 of 59
columnSpace matrix [[1,2,3],[4,5,6],[7,8,9],[1,1,1]]
--R
--R
--R   (50)  [[1,4,7,1],[2,5,8,1]]
--R                                              Type: List(Vector(Integer))
--E 50
```

```
--S 51 of 59
rank matrix [[1,2,3],[4,5,6],[7,8,9]]
--R
--R
--R   (51)  2
--R                                                     Type: PositiveInteger
--E 51

--S 52 of 59
nullity matrix [[1,2,3],[4,5,6],[7,8,9]]
--R
--R
--R   (52)  1
--R                                                     Type: PositiveInteger
--E 52

--S 53 of 59
nullSpace matrix [[1,2,3],[4,5,6],[7,8,9]]
--R
--R
--R   (53)  [[1,- 2,1]]
--R                                                Type: List(Vector(Integer))
--E 53

--S 54 of 59
determinant matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R   (54)  288
--R                                                     Type: PositiveInteger
--E 54

--S 55 of 59
minordet matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R   (55)  288
--R                                                     Type: PositiveInteger
--E 55

--S 56 of 59
pfaffian [[0,1,0,0],[-1,0,0,0],[0,0,0,1],[0,0,-1,0]]
--R
--R
--R   (56)  1
--R                                                     Type: PositiveInteger
--E 56

--S 57 of 59
inverse matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         + 5    - 10   10   - 5    1  +
--R         |                            |
```

```
--R         |  77  107     39   61     25|
--R         |- --  ---   - --   --   - --|
--R         |  12   6       2    6     12|
--R         |                            |
--R         | 71     59   49     41   35 |
--R         | --   - --   --   - --   -- |
--R   (57)  | 24      6    4      6   24 |
--R         |                            |
--R         |   7   13          11      5|
--R         |- --   --   - 3    --   - --|
--R         |  12    6           6     12|
--R         |                            |
--R         |  1     1    1      1     1 |
--R         | --   - -    -    - -    -- |
--R         + 24     6    4      6    24 +
--R                                   Type: Union(Matrix(Fraction(Integer)),...)
--E 57

--S 58 of 59
(matrix [[j**i for i in 0..4] for j in 1..5]) ** 2
--R
--R
--R         + 5    15    55     225    979  +
--R         |                               |
--R         |31   129    573   2637   12405 |
--R         |                               |
--R   (58)  |121  547   2551   12121  58315 |
--R         |                               |
--R         |341  1593  7585   36561  177745|
--R         |                               |
--R         +781  3711  17871  86841  424731+
--R                                                    Type: Matrix(Integer)
--E 58

--S 59 of 59
)show MatrixCategory
--R
--R MatrixCategory(R: Ring,Row: FiniteLinearAggregate(t#1),Col: FiniteLinearAggregate(t#1)) is a category constr
--R Abbreviation for MatrixCategory is MATCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for MATCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Row,%) -> Row                  ?*? : (%,Col) -> Col
--R ?*? : (Integer,%) -> %                ?*? : (%,R) -> %
--R ?*? : (R,%) -> %                      ?*? : (%,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?+? : (%,%) -> %
--R -? : % -> %                           ?-? : (%,%) -> %
--R ?/? : (%,R) -> % if R has FIELD       antisymmetric? : % -> Boolean
--R coerce : Col -> %                     column : (%,Integer) -> Col
--R copy : % -> %                         diagonal? : % -> Boolean
--R diagonalMatrix : List(%) -> %         diagonalMatrix : List(R) -> %
--R elt : (%,Integer,Integer,R) -> R      elt : (%,Integer,Integer) -> R
--R empty : () -> %                       empty? : % -> Boolean
```

```
--R eq? : (%,%) -> Boolean                fill! : (%,R) -> %
--R horizConcat : (%,%) -> %              latex : % -> String if R has SETCAT
--R listOfLists : % -> List(List(R))      map : (((R,R) -> R),%,%,R) -> %
--R map : (((R,R) -> R),%,%) -> %         map : ((R -> R),%) -> %
--R map! : ((R -> R),%) -> %              matrix : List(List(R)) -> %
--R maxColIndex : % -> Integer            maxRowIndex : % -> Integer
--R minColIndex : % -> Integer            minRowIndex : % -> Integer
--R ncols : % -> NonNegativeInteger       nrows : % -> NonNegativeInteger
--R parts : % -> List(R)                  pfaffian : % -> R if R has COMRING
--R qelt : (%,Integer,Integer) -> R       row : (%,Integer) -> Row
--R sample : () -> %                      setColumn! : (%,Integer,Col) -> %
--R setRow! : (%,Integer,Row) -> %        setelt : (%,Integer,Integer,R) -> R
--R square? : % -> Boolean                squareTop : % -> %
--R symmetric? : % -> Boolean             transpose : % -> %
--R transpose : Row -> %                  vertConcat : (%,%) -> %
--R zero? : % -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?**? : (%,Integer) -> % if R has FIELD
--R ?=? : (%,%) -> Boolean if R has SETCAT
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if R has SETCAT
--R columnSpace : % -> List(Col) if R has EUCDOM
--R count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R determinant : % -> R if R has commutative(*)
--R elt : (%,List(Integer),List(Integer)) -> %
--R eval : (%,List(R),List(R)) -> % if R has EVALAB(R) and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB(R) and R has SETCAT
--R eval : (%,Equation(R)) -> % if R has EVALAB(R) and R has SETCAT
--R eval : (%,List(Equation(R))) -> % if R has EVALAB(R) and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R hash : % -> SingleInteger if R has SETCAT
--R inverse : % -> Union(%,"failed") if R has FIELD
--R less? : (%,NonNegativeInteger) -> Boolean
--R matrix : (NonNegativeInteger,NonNegativeInteger,((Integer,Integer) -> R)) -> %
--R member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
--R members : % -> List(R) if $ has finiteAggregate
--R minordet : % -> R if R has commutative(*)
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,NonNegativeInteger,R) -> %
--R nullSpace : % -> List(Col) if R has INTDOM
--R nullity : % -> NonNegativeInteger if R has INTDOM
--R qsetelt! : (%,Integer,Integer,R) -> R
--R rank : % -> NonNegativeInteger if R has INTDOM
--R rowEchelon : % -> % if R has EUCDOM
--R scalarMatrix : (NonNegativeInteger,R) -> %
--R setelt : (%,List(Integer),List(Integer),%) -> %
--R setsubMatrix! : (%,Integer,Integer,%) -> %
--R size? : (%,NonNegativeInteger) -> Boolean
--R subMatrix : (%,Integer,Integer,Integer,Integer) -> %
--R swapColumns! : (%,Integer,Integer) -> %
--R swapRows! : (%,Integer,Integer) -> %
--R zero : (NonNegativeInteger,NonNegativeInteger) -> %
```

```
--R ?~=? : (%,%) -> Boolean if R has SETCAT
--R
--E 59

)spool
)lisp (bye)
```

———

— MatrixCategory.help —

```
====================================================================
MatrixCategory examples
====================================================================

MatrixCategory is a general matrix category which allows different
representations and indexing schemes.  Rows and columns may be
extracted with rows returned as objects of type Row and colums
returned as objects of type Col. A domain belonging to this category
will be shallowly mutable. The index of the 'first' row may be
obtained by calling the function minRowIndex.  The index of the
'first' column may be obtained by calling the function minColIndex.
The index of the first element of a Row is the same as the index of the
first column in a matrix and vice versa.

Predicates:

square?(m) returns true if m is a square matrix
(if m has the same number of rows as columns) and false otherwise.

  square matrix [[j**i for i in 0..4] for j in 1..5]

diagonal?(m) returns true if the matrix m is square and
diagonal (all entries of m not on the diagonal are zero) and
false otherwise.

  diagonal? matrix [[j**i for i in 0..4] for j in 1..5]

symmetric?(m) returns true if the matrix m is square and
symmetric (\spad{m[i,j] = m[j,i]} for all i and j) and false
otherwise.

  symmetric? matrix [[j**i for i in 0..4] for j in 1..5]

antisymmetric?(m) returns true if the matrix m is square and
antisymmetric (\spad{m[i,j] = -m[j,i]} for all i and j)
and false otherwise.

  antisymmetric? matrix [[j**i for i in 0..4] for j in 1..5]


Creation
```

```
zero(m,n) returns an m-by-n zero matrix.

  z:Matrix(INT):=zero(3,3)

matrix(l) converts the list of lists l to a matrix, where the
list of lists is viewed as a list of the rows of the matrix.

  matrix [[1,2,3],[4,5,6],[7,8,9],[1,1,1]]

scalarMatrix(n,r) returns an n-by-n matrix with r's on the
diagonal and zeroes elsewhere.

  z:Matrix(INT):=scalarMatrix(3,5)

diagonalMatrix(l) returns a diagonal matrix with the elements
of l on the diagonal.

  diagonalMatrix [1,2,3]

diagonalMatrix([m1,...,mk]) creates a block diagonal matrix
M with block matrices m1,...,mk down the diagonal,
with 0 block matrices elsewhere.

More precisly: if ri := nrows mi, ci := ncols mi,
then m is an (r1+..+rk) by (c1+..+ck) - matrix  with entries
m.i.j = ml.(i-r1-..-r(l-1)).(j-n1-..-n(l-1)),
if (r1+..+r(l-1)) < i <= r1+..+rl and
   (c1+..+c(l-1)) < i <= c1+..+cl,
    m.i.j = 0  otherwise.

  diagonalMatrix [matrix [[1,2],[3,4]], matrix [[4,5],[6,7]]]

coerce(col) converts the column col to a column matrix.

  coerce([1,2,3])@Matrix(INT)

transpose(r) converts the row r to a row matrix.

  transpose([1,2,3])@Matrix(INT)

Creation of new matrices from old

transpose(m) returns the transpose of the matrix m.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  transpose m

squareTop(m) returns an n-by-n matrix consisting of the first
n rows of the m-by-n matrix m. Error: if m < n.

  m:=matrix [[j**i for i in 0..2] for j in 1..5]
  squareTop m

horizConcat(x,y) horizontally concatenates two matrices with
```

```
an equal number of rows. The entries of y appear to the right
of the entries of x.  Error: if the matrices
do not have the same number of rows.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  horizConcat(m,m)

vertConcat(x,y) vertically concatenates two matrices with an
equal number of columns. The entries of y appear below
of the entries of x.  Error: if the matrices
do not have the same number of columns.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  vertConcat(m,m)

Part extractions/assignments

listOfLists(m) returns the rows of the matrix m as a list of lists

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  listOfLists m

elt(x,rowList,colList) returns an m-by-n matrix consisting
of elements of x, where m = # rowList and n = # colList
If rowList = [i<1>,i<2>,...,i<m>] and
   colList = [j<1>,j<2>,...,j<n>],
 then the (k,l)-th entry of elt(x,rowList,colList) is x(i<k>,j<l>).

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  elt(m,3,3)

setelt(x,rowList,colList,y) destructively alters the matrix x.
If y is m-by-n,
   rowList = [i<1>,i<2>,...,i<m>] and
   colList = [j<1>,j<2>,...,j<n>],
 then x(i<k>,j<l>)
 is set to y(k,l) for k = 1,...,m and l = 1,...,n

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  setelt(m,3,3,10)

swapRows!(m,i,j) interchanges the i-th and j-th
rows of m. This destructively alters the matrix.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  swapRows!(m,2,4)

swapColumns!(m,i,j) interchanges the i-th and j-th
columns of m. This destructively alters the matrix.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  swapColumns!(m,2,4)

subMatrix(x,i1,i2,j1,j2) extracts the submatrix [x(i,j)]
```

```
where the index i ranges from i1 to i2
and   the index j ranges from j1 to j2.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  subMatrix(m,1,3,2,4)

setsubMatrix(x,i1,j1,y) destructively alters the matrix x.
Here x(i,j) is set to y(i-i1+1,j-j1+1) for
i = i1,...,i1-1+nrows y and j = j1,...,j1-1+ncols y.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  setsubMatrix!(m,2,2,matrix [[3,3],[3,3]])


Arithmetic

x + y is the sum of the matrices x and y.
It is an error if the dimensions are incompatible.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  m+m

x - y is the difference of the matrices x and y.
It is an error if the dimensions are incompatible.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  m-m

-x returns the negative of the matrix x.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  -m

x * y is the product of the matrices x and y.
It is an error if the dimensions are incompatible.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  m*m

r*x is the left scalar multiple of the scalar r and the matrix x.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  1/3*m

x * r is the right scalar multiple of the scalar r and the matrix x.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  m*1/3

n * x is an integer multiple.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  3*m
```

```
x * c is the product of the matrix x and the column vector c.
It is an error if the dimensions are incompatible.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  c:=coerce([1,2,3,4,5])@Matrix(INT)
  m *c

r * x is the product of the row vector r and the matrix x.
It is an error if the dimensions are incompatible.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  r:=transpose([1,2,3,4,5])@Matrix(INT)
  r*m

x ** n computes a non-negative integral power of the matrix x.
It is an error if the matrix is not square.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  m**3

exquo(m,r) computes the exact quotient of the elements
of m by r, returning "failed" if this is not possible.

  m:=matrix [[2**i for i in 2..4] for j in 1..5]
  exquo(m,2)

m/r divides the elements of m by r, r must be non-zero.

  m:=matrix [[2**i for i in 2..4] for j in 1..5]
  m/4

Linear algebra

rowEchelon(m) returns the row echelon form of the matrix m.

  rowEchelon matrix [[j**i for i in 0..4] for j in 1..5]

columnSpace(m) returns a sublist of columns of the matrix m

  columnSpace matrix [[1,2,3],[4,5,6],[7,8,9],[1,1,1]]

rank(m) returns the rank of the matrix m.

  rank matrix [[1,2,3],[4,5,6],[7,8,9]]

nullity(m) returns the nullity of the matrix m. This is
the dimension of the null space of the matrix m.

  nullity matrix [[1,2,3],[4,5,6],[7,8,9]]

nullSpace(m) returns a basis for the null space of the matrix m.

  nullSpace matrix [[1,2,3],[4,5,6],[7,8,9]]
```

```
determinant(m) returns the determinant of the matrix m.
It is an error if the matrix is not square.

  determinant matrix [[j**i for i in 0..4] for j in 1..5]

minordet(m) computes the determinant of the matrix m using minors.
It is an error if the matrix is not square.

  minordet matrix [[j**i for i in 0..4] for j in 1..5]

pfaffian(m) returns the Pfaffian of the matrix m.
It is an error if the matrix is not antisymmetric

  pfaffian [[0,1,0,0],[-1,0,0,0],[0,0,0,1],[0,0,-1,0]]

inverse(m) returns the inverse of the matrix m.
If the matrix is not invertible, "failed" is returned.
It is an error if the matrix is not square.

  inverse matrix [[j**i for i in 0..4] for j in 1..5]

m**n computes an integral power of the matrix m.
It is an error if matrix is not square or
if the matrix is square but not invertible.

  (matrix [[j**i for i in 0..4] for j in 1..5]) ** 2
```

We define three categories for matrices

- MatrixCategory is the category of all matrices
- RectangularMatrixCategory is the category of all matrices of a given dimension
- SquareMatrixCategory inherits from RectangularMatrixCategory

The Matrix domain is the domain of all matrices.

All three domains share the same representation, inherited from Matrix. Most algorithms are only implemented for Matrix but implemented in separate packages.

- MatrixLinearAlgebraFunctions is the top-level package that calls the other packages
- InnerMatrixLinearAlgebraFunctions contains implementations that work over a Field
- InnerMatrixQuotientFieldFunctions contain implementations that work over a quotient field

Implementations that rely on the representation of matrices used in Matrix should be put into these packages.

See:

⇐ "TwoDimensionalArrayCategory" (ARR2CAT) 5.0.105 on page 481
⇒ "RectangularMatrixCategory" (RMATCAT) 10.0.171 on page 1075
⇒ "SquareMatrixCategory" (SMATCAT) 12.0.192 on page 1266

Exports:

antisymmetric?	any?	coerce	column	columnSpace
copy	count	determinant	diagonal?	diagonalMatrix
elt	empty	empty?	eq?	eval
every?	exquo	fill!	hash	horizConcat
inverse	latex	less?	listOfLists	map
map!	matrix	maxColIndex	maxRowIndex	member?
members	minColIndex	minordet	minRowIndex	more?
ncols	new	nrows	nullSpace	nullity
parts	pfaffian	qelt	qsetelt!	rank
row	rowEchelon	sample	scalarMatrix	setColumn!
setelt	setRow!	setsubMatrix!	size?	square?
squareTop	subMatrix	swapColumns!	swapRows!	symmetric?
transpose	vertConcat	zero	#?	?**?
?/?	?=?	?~=?	?*?	?+?
-?	?-?	zero?		

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are directly exported but not implemented:

```
determinant : % -> R if R has commutative *
inverse : % -> Union(%,"failed") if R has FIELD
minordet : % -> R if R has commutative *
nullity : % -> NonNegativeInteger if R has INTDOM
nullSpace : % -> List Col if R has INTDOM
rowEchelon : % -> % if R has EUCDOM
rank : % -> NonNegativeInteger if R has INTDOM
```

These are implemented by this category:

```
antisymmetric? : % -> Boolean
coerce : Col -> %
columnSpace: % -> List Col if R has EUCDOM
diagonal? : % -> Boolean
diagonalMatrix : List % -> %
diagonalMatrix : List R -> %
elt : (%,List Integer,List Integer) -> %
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
horizConcat : (%,%) -> %
listOfLists : % -> List List R
pfaffian: % -> R if R has COMRING
matrix : List List R -> %
matrix : (NonNegativeInteger, NonNegativeInteger, (Integer,Integer) -> R) -> %
scalarMatrix : (NonNegativeInteger,R) -> %
setelt : (%,List Integer,List Integer,%) -> %
setsubMatrix! : (%,Integer,Integer,%) -> %
```

```
square? : % -> Boolean
squareTop : % -> %
subMatrix : (%,Integer,Integer,Integer,Integer) -> %
swapColumns! : (%,Integer,Integer) -> %
swapRows! : (%,Integer,Integer) -> %
symmetric? : % -> Boolean
transpose : Row -> %
transpose : % -> %
vertConcat : (%,%) -> %
zero : (NonNegativeInteger,NonNegativeInteger) -> %
zero? :  % -> Boolean
?/? : (%,R) -> % if R has FIELD
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?*? : (%,R) -> %
?*? : (R,%) -> %
?*? : (Integer,%) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,Integer) -> % if R has FIELD
?*? : (%,Col) -> Col
?*? : (Row,%) -> Row
?*? : (%,%) -> %
```

These exports come from (p481) TwoDimensionalArrayCategory(R,Row,Col)
where R:Ring, Row:FiniteLinearAggregate(R),
Col:FiniteLinearAggregate(R):

```
any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
coerce : % -> OutputForm if R has SETCAT
column : (%,Integer) -> Col
copy : % -> %
count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
elt : (%,Integer,Integer,R) -> R
elt : (%,Integer,Integer) -> R
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
fill! : (%,R) -> %
hash : % -> SingleInteger if R has SETCAT
latex : % -> String if R has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : (((R,R) -> R),%,%,R) -> %
map : (((R,R) -> R),%,%) -> %
map : ((R -> R),%) -> %
map! : ((R -> R),%) -> %
maxColIndex : % -> Integer
maxRowIndex : % -> Integer
member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
```

```
 members : % -> List R if $ has finiteAggregate
 minColIndex : % -> Integer
 minRowIndex : % -> Integer
 more? : (%,NonNegativeInteger) -> Boolean
 new : (NonNegativeInteger,NonNegativeInteger,R) -> %
 ncols : % -> NonNegativeInteger
 nrows : % -> NonNegativeInteger
 parts : % -> List R
 qelt : (%,Integer,Integer) -> R
 qsetelt! : (%,Integer,Integer,R) -> R
 row : (%,Integer) -> Row
 sample : () -> %
 setColumn! : (%,Integer,Col) -> %
 setelt : (%,Integer,Integer,R) -> R
 setRow! : (%,Integer,Row) -> %
 size? : (%,NonNegativeInteger) -> Boolean
 #? : % -> NonNegativeInteger if $ has finiteAggregate
 ?~=? : (%,%) -> Boolean if R has SETCAT
 ?=? : (%,%) -> Boolean if R has SETCAT
```

— MatrixCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#MATCAT">
MatrixCategory (MATCAT)</a></h2>
</body>
```

— category MATCAT MatrixCategory —

```
)abbrev category MATCAT MatrixCategory
++ Authors: Grabmeier, Gschnitzer, Williamson, Gabriel Dos Reis
++ Date Created: 1987
++ Date Last Updated: July 1990
++ Description:
++ \spadtype{MatrixCategory} is a general matrix category which allows
++ different representations and indexing schemes.  Rows and
++ columns may be extracted with rows returned as objects of
++ type Row and colums returned as objects of type Col.
++ A domain belonging to this category will be shallowly mutable.
++ The index of the 'first' row may be obtained by calling the
++ function \spadfun{minRowIndex}.  The index of the 'first' column may
++ be obtained by calling the function \spadfun{minColIndex}.  The index of
++ the first element of a Row is the same as the index of the
++ first column in a matrix and vice versa.

MatrixCategory(R,Row,Col) : Category == SIG where
  R   : Ring
  Row : FiniteLinearAggregate R
  Col : FiniteLinearAggregate R

  SIG ==> TwoDimensionalArrayCategory(R,Row,Col) with
```

```
    shallowlyMutable
      ++ One may destructively alter matrices

    finiteAggregate
      ++ matrices are finite

--% Predicates

    square? : % -> Boolean
      ++square?(m) returns true if m is a square matrix
      ++ (if m has the same number of rows as columns) and false otherwise.
      ++
      ++X square? matrix [[j**i for i in 0..4] for j in 1..5]

    diagonal? : % -> Boolean
      ++diagonal?(m) returns true if the matrix m is square and
      ++ diagonal (that is, all entries of m not on the diagonal are zero) and
      ++ false otherwise.
      ++
      ++X diagonal? matrix [[j**i for i in 0..4] for j in 1..5]

    symmetric? : % -> Boolean
      ++symmetric?(m) returns true if the matrix m is square and
      ++ symmetric (that is, \spad{m[i,j] = m[j,i]} for all i and j) and false
      ++ otherwise.
      ++
      ++X symmetric? matrix [[j**i for i in 0..4] for j in 1..5]

    antisymmetric? : % -> Boolean
      ++antisymmetric?(m) returns true if the matrix m is square and
      ++ antisymmetric (that is, \spad{m[i,j] = -m[j,i]} for all i and j)
      ++ and false otherwise.
      ++
      ++X antisymmetric? matrix [[j**i for i in 0..4] for j in 1..5]

    zero? : % -> Boolean
      ++ \spad{zero?(m)} returns true if m is a zero matrix
      ++
      ++ zero? matrix [[0 for i in 0..4] for j in 1..5]

--% Creation

    zero : (NonNegativeInteger,NonNegativeInteger) -> %
      ++zero(m,n) returns an m-by-n zero matrix.
      ++
      ++X z:Matrix(INT):=zero(3,3)

    matrix : List List R -> %
      ++matrix(l) converts the list of lists l to a matrix, where the
      ++ list of lists is viewed as a list of the rows of the matrix.
      ++
      ++X matrix [[1,2,3],[4,5,6],[7,8,9],[1,1,1]]
```

```
    matrix : (NonNegativeInteger,NonNegativeInteger,(Integer,Integer)->R) -> %
      ++matrix(n,m,f) constructs an \spad{n * m} matrix with
      ++ the \spad{(i,j)} entry equal to \spad{f(i,j)}
      ++
      ++X f(i:INT,j:INT):INT == i+j
      ++X matrix(3,4,f)

    scalarMatrix : (NonNegativeInteger,R) -> %
      ++scalarMatrix(n,r) returns an n-by-n matrix with r's on the
      ++ diagonal and zeroes elsewhere.
      ++
      ++X z:Matrix(INT):=scalarMatrix(3,5)

    diagonalMatrix : List R -> %
      ++diagonalMatrix(l) returns a diagonal matrix with the elements
      ++ of l on the diagonal.
      ++
      ++X diagonalMatrix [1,2,3]

    diagonalMatrix : List % -> %
      ++diagonalMatrix([m1,...,mk]) creates a block diagonal matrix
      ++ M with block matrices m1,...,mk down the diagonal,
      ++ with 0 block matrices elsewhere.
      ++ More precisly: if \spad{ri := nrows mi}, \spad{ci := ncols mi},
      ++ then m is an (r1+..+rk) by (c1+..+ck) - matrix  with entries
      ++ \spad{m.i.j = ml.(i-r1-..-r(l-1)).(j-n1-..-n(l-1))}, if
      ++ \spad{(r1+..+r(l-1)) < i <= r1+..+rl} and
      ++ \spad{(c1+..+c(l-1)) < i <= c1+..+cl},
      ++ \spad{m.i.j} = 0  otherwise.
      ++
      ++X diagonalMatrix [matrix [[1,2],[3,4]], matrix [[4,5],[6,7]]]

    coerce : Col -> %
      ++coerce(col) converts the column col to a column matrix.
      ++
      ++X coerce([1,2,3])@Matrix(INT)

    transpose : Row -> %
      ++transpose(r) converts the row r to a row matrix.
      ++
      ++X transpose([1,2,3])@Matrix(INT)

--% Creation of new matrices from old

    transpose : % -> %
      ++transpose(m) returns the transpose of the matrix m.
      ++
      ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
      ++X transpose m

    squareTop : % -> %
      ++squareTop(m) returns an n-by-n matrix consisting of the first
      ++ n rows of the m-by-n matrix m. Error: if
      ++ \spad{m < n}.
```

```
      ++
      ++X m:=matrix [[j**i for i in 0..2] for j in 1..5]
      ++X squareTop m

    horizConcat : (%,%) -> %
      ++horizConcat(x,y) horizontally concatenates two matrices with
      ++ an equal number of rows. The entries of y appear to the right
      ++ of the entries of x.  Error: if the matrices
      ++ do not have the same number of rows.
      ++
      ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
      ++X horizConcat(m,m)

    vertConcat : (%,%) -> %
      ++vertConcat(x,y) vertically concatenates two matrices with an
      ++ equal number of columns. The entries of y appear below
      ++ of the entries of x.  Error: if the matrices
      ++ do not have the same number of columns.
      ++
      ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
      ++X vertConcat(m,m)

--% Part extractions/assignments

    listOfLists : % -> List List R
      ++listOfLists(m) returns the rows of the matrix m as a list
      ++ of lists.
      ++
      ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
      ++X listOfLists m

    elt : (%,List Integer,List Integer) -> %
      ++elt(x,rowList,colList) returns an m-by-n matrix consisting
      ++ of elements of x, where \spad{m = # rowList} and \spad{n = # colList}
      ++ If \spad{rowList = [i<1>,i<2>,...,i<m>]} and \spad{colList =
      ++ [j<1>,j<2>,...,j<n>]}, then the \spad{(k,l)}th entry of
      ++ \spad{elt(x,rowList,colList)} is \spad{x(i<k>,j<l>)}.
      ++
      ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
      ++X elt(m,3,3)

    setelt : (%,List Integer,List Integer, %) -> %
      ++setelt(x,rowList,colList,y) destructively alters the matrix x.
      ++ If y is \spad{m}-by-\spad{n}, \spad{rowList = [i<1>,i<2>,...,i<m>]}
      ++ and \spad{colList = [j<1>,j<2>,...,j<n>]}, then \spad{x(i<k>,j<l>)}
      ++ is set to \spad{y(k,l)} for \spad{k = 1,...,m} and \spad{l = 1,...,n}
      ++
      ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
      ++X setelt(m,3,3,10)

    swapRows_! : (%,Integer,Integer) -> %
      ++swapRows!(m,i,j) interchanges the \spad{i}th and \spad{j}th
      ++ rows of m. This destructively alters the matrix.
      ++
```

```
      ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
      ++X swapRows!(m,2,4)

    swapColumns_! : (%,Integer,Integer) -> %
      ++swapColumns!(m,i,j) interchanges the \spad{i}th and \spad{j}th
      ++ columns of m. This destructively alters the matrix.
      ++
      ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
      ++X swapColumns!(m,2,4)

    subMatrix : (%,Integer,Integer,Integer,Integer) -> %
      ++subMatrix(x,i1,i2,j1,j2) extracts the submatrix
      ++ \spad{[x(i,j)]} where the index i ranges from \spad{i1} to \spad{i2}
      ++ and the index j ranges from \spad{j1} to \spad{j2}.
      ++
      ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
      ++X subMatrix(m,1,3,2,4)

    setsubMatrix_! : (%,Integer,Integer,%) -> %
      ++setsubMatrix!(x,i1,j1,y) destructively alters the
      ++ matrix x. Here \spad{x(i,j)} is set to \spad{y(i-i1+1,j-j1+1)} for
      ++ \spad{i = i1,...,i1-1+nrows y} and \spad{j = j1,...,j1-1+ncols y}.
      ++
      ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
      ++X setsubMatrix!(m,2,2,matrix [[3,3],[3,3]])

--% Arithmetic

    "+" : (%,%) -> %
      ++\spad{x + y} is the sum of the matrices x and y.
      ++ Error: if the dimensions are incompatible.
      ++
      ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
      ++X m+m

    "-" : (%,%) -> %
      ++\spad{x - y} is the difference of the matrices x and y.
      ++ Error: if the dimensions are incompatible.
      ++
      ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
      ++X m-m

    "-" : % -> %
      ++\spad{-x} returns the negative of the matrix x.
      ++
      ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
      ++X -m

    "*" : (%,%) -> %
      ++\spad{x * y} is the product of the matrices x and y.
      ++ Error: if the dimensions are incompatible.
      ++
      ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
      ++X m*m
```

```
"*" : (R,%) -> %
  ++\spad{r*x} is the left scalar multiple of the scalar r and the
  ++ matrix x.
  ++
  ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
  ++X 1/3*m

"*" : (%,R) -> %
  ++\spad{x * r} is the right scalar multiple of the scalar r and the
  ++ matrix x.
  ++
  ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
  ++X m*1/3

"*" : (Integer,%) -> %
  ++\spad{n * x} is an integer multiple.
  ++
  ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
  ++X 3*m

"*" : (%,Col) -> Col
  ++\spad{x * c} is the product of the matrix x and the column vector c.
  ++ Error: if the dimensions are incompatible.
  ++
  ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
  ++X c:=coerce([1,2,3,4,5])@Matrix(INT)
  ++X m*c

"*" : (Row,%) -> Row
  ++\spad{r * x} is the product of the row vector r and the matrix x.
  ++ Error: if the dimensions are incompatible.
  ++
  ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
  ++X r:=transpose([1,2,3,4,5])@Matrix(INT)
  ++X r*m

"**" : (%,NonNegativeInteger) -> %
  ++\spad{x ** n} computes a non-negative integral power of the matrix x.
  ++ Error: if the matrix is not square.
  ++
  ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
  ++X m**3

if R has IntegralDomain then

  "exquo" : (%,R) -> Union(%,"failed")
    ++\spad{exquo(m,r)} computes the exact quotient of the elements
    ++ of m by r, returning \axiom{"failed"} if this is not possible.
    ++
    ++X m:=matrix [[2**i for i in 2..4] for j in 1..5]
    ++X exquo(m,2)

if R has Field then
```

```
      "/" : (%,R) -> %
        ++\spad{m/r} divides the elements of m by r. Error: if \spad{r = 0}.
        ++
        ++X m:=matrix [[2**i for i in 2..4] for j in 1..5]
        ++X m/4

--% Linear algebra

    if R has EuclideanDomain then

      rowEchelon : % -> %
        ++\spad{rowEchelon(m)} returns the row echelon form of the matrix m.
        ++
        ++X rowEchelon matrix [[j**i for i in 0..4] for j in 1..5]

      columnSpace : % -> List Col
        ++\spad{columnSpace(m)} returns a sublist of columns of the matrix m
        ++ forming a basis of its column space
        ++
        ++X columnSpace matrix [[1,2,3],[4,5,6],[7,8,9],[1,1,1]]

    if R has IntegralDomain then

      rank : % -> NonNegativeInteger
        ++\spad{rank(m)} returns the rank of the matrix m.
        ++
        ++X rank matrix [[1,2,3],[4,5,6],[7,8,9]]

      nullity : % -> NonNegativeInteger
        ++\spad{nullity(m)} returns the nullity of the matrix m. This is
        ++ the dimension of the null space of the matrix m.
        ++
        ++X nullity matrix [[1,2,3],[4,5,6],[7,8,9]]

      nullSpace : % -> List Col
        ++\spad{nullSpace(m)} returns a basis for the null space of
        ++ the matrix m.
        ++
        ++X nullSpace matrix [[1,2,3],[4,5,6],[7,8,9]]

    if R has commutative("*") then

      determinant : % -> R
        ++\spad{determinant(m)} returns the determinant of the matrix m.
        ++ Error: if the matrix is not square.
        ++
        ++X determinant matrix [[j**i for i in 0..4] for j in 1..5]

      minordet : % -> R
        ++\spad{minordet(m)} computes the determinant of the matrix m using
        ++ minors. Error: if the matrix is not square.
        ++
        ++X minordet matrix [[j**i for i in 0..4] for j in 1..5]
```

```
    if R has CommutativeRing then

      pfaffian : % -> R
        ++\spad{pfaffian(m)} returns the Pfaffian of the matrix m.
        ++ Error if the matrix is not antisymmetric
        ++
        ++X pfaffian [[0,1,0,0],[-1,0,0,0],[0,0,0,1],[0,0,-1,0]]

    if R has Field then

      inverse : % -> Union(%,"failed")
        ++\spad{inverse(m)} returns the inverse of the matrix m.
        ++ If the matrix is not invertible, "failed" is returned.
        ++ Error: if the matrix is not square.
        ++
        ++X inverse matrix [[j**i for i in 0..4] for j in 1..5]

      "**" : (%,Integer) -> %
        ++\spad{m**n} computes an integral power of the matrix m.
        ++ Error: if matrix is not square or if the matrix
        ++ is square but not invertible.
        ++
        ++X (matrix [[j**i for i in 0..4] for j in 1..5]) ** 2

   add

     minr ==> minRowIndex
     maxr ==> maxRowIndex
     minc ==> minColIndex
     maxc ==> maxColIndex
     mini ==> minIndex
     maxi ==> maxIndex

--% Predicates

     square? x == nrows x = ncols x

     diagonal? x ==
       not square? x => false
       for i in minr x .. maxr x repeat
         for j in minc x .. maxc x | (j - minc x) ^= (i - minr x) repeat
           not zero? qelt(x, i, j) => return false
       true

     symmetric? x ==
       (nRows := nrows x) ^= ncols x => false
       mr := minRowIndex x; mc := minColIndex x
       for i in 0..(nRows - 1) repeat
         for j in (i + 1)..(nRows - 1) repeat
           qelt(x,mr + i,mc + j) ^= qelt(x,mr + j,mc + i) => return false
       true

     antisymmetric? x ==
       (nRows := nrows x) ^= ncols x => false
```

```
        mr := minRowIndex x; mc := minColIndex x
        for i in 0..(nRows - 1) repeat
          for j in i..(nRows - 1) repeat
            qelt(x,mr + i,mc + j) ^= -qelt(x,mr + j,mc + i) =>
              return false
        true

      zero?(x) ==
        for i in minr(x)..maxr(x) repeat
          for j in minc(x)..maxc(x) repeat
            if qelt(x,i,j) ^= 0 then return false
        true

--% Creation of matrices

      zero(rows,cols) == new(rows,cols,0)

      matrix(l: List List R) ==
        null l => new(0,0,0)
        -- error check: this is a top level function
        rows : NonNegativeInteger := 1; cols := # first l
        cols = 0 => error "matrices with zero columns are not supported"
        for ll in rest l repeat
          cols ^= # ll => error "matrix: rows of different lengths"
          rows := rows + 1
        ans := new(rows,cols,0)
        for i in minr(ans)..maxr(ans) for ll in l repeat
          for j in minc(ans)..maxc(ans) for r in ll repeat
            qsetelt_!(ans,i,j,r)
        ans

      matrix(n,m,f) ==
        mat := new(n,m,0)
        for i in minr mat..maxr mat repeat
          for j in minc mat..maxc mat repeat
            qsetelt!(mat,i,j,f(i,j))
        mat

      scalarMatrix(n,r) ==
        ans := zero(n,n)
        for i in minr(ans)..maxr(ans) for j in minc(ans)..maxc(ans) repeat
          qsetelt_!(ans,i,j,r)
        ans

      diagonalMatrix(l: List R) ==
        n := #l; ans := zero(n,n)
        for i in minr(ans)..maxr(ans) for j in minc(ans)..maxc(ans) _
            for r in l repeat qsetelt_!(ans,i,j,r)
        ans

      diagonalMatrix(list: List %) ==
        rows : NonNegativeInteger := 0
        cols : NonNegativeInteger := 0
        for mat in list repeat
```

```
    rows := rows + nrows mat
    cols := cols + ncols mat
  ans := zero(rows,cols)
  loR := minr ans; loC := minc ans
  for mat in list repeat
    hiR := loR + nrows(mat) - 1; hiC := loC + nrows(mat) - 1
    for i in loR..hiR for k in minr(mat)..maxr(mat) repeat
      for j in loC..hiC for l in minc(mat)..maxc(mat) repeat
        qsetelt_!(ans,i,j,qelt(mat,k,l))
    loR := hiR + 1; loC := hiC + 1
  ans

coerce(v:Col) ==
  x := new(#v,1,0)
  one := minc(x)
  for i in minr(x)..maxr(x) for k in mini(v)..maxi(v) repeat
    qsetelt_!(x,i,one,qelt(v,k))
  x

transpose(v:Row) ==
  x := new(1,#v,0)
  one := minr(x)
  for j in minc(x)..maxc(x) for k in mini(v)..maxi(v) repeat
    qsetelt_!(x,one,j,qelt(v,k))
  x

transpose(x:%) ==
  ans := new(ncols x,nrows x,0)
  for i in minr(ans)..maxr(ans) repeat
    for j in minc(ans)..maxc(ans) repeat
      qsetelt_!(ans,i,j,qelt(x,j,i))
  ans

squareTop x ==
  nrows x < (cols := ncols x) =>
    error "squareTop: number of columns exceeds number of rows"
  ans := new(cols,cols,0)
  for i in minr(x)..(minr(x) + cols - 1) repeat
    for j in minc(x)..maxc(x) repeat
      qsetelt_!(ans,i,j,qelt(x,i,j))
  ans

horizConcat(x,y) ==
  (rows := nrows x) ^= nrows y =>
    error "HConcat: matrices must have same number of rows"
  ans := new(rows,(cols := ncols x) + ncols y,0)
  for i in minr(x)..maxr(x) repeat
    for j in minc(x)..maxc(x) repeat
      qsetelt_!(ans,i,j,qelt(x,i,j))
  for i in minr(y)..maxr(y) repeat
    for j in minc(y)..maxc(y) repeat
      qsetelt_!(ans,i,j + cols,qelt(y,i,j))
  ans
```

```
      vertConcat(x,y) ==
        (cols := ncols x) ^= ncols y =>
          error "HConcat: matrices must have same number of columns"
        ans := new((rows := nrows x) + nrows y,cols,0)
        for i in minr(x)..maxr(x) repeat
          for j in minc(x)..maxc(x) repeat
            qsetelt_!(ans,i,j,qelt(x,i,j))
        for i in minr(y)..maxr(y) repeat
          for j in minc(y)..maxc(y) repeat
            qsetelt_!(ans,i + rows,j,qelt(y,i,j))
        ans

--% Part extraction/assignment

      listOfLists x ==
        ll : List List R := nil()
        for i in maxr(x)..minr(x) by -1 repeat
          l : List R := nil()
          for j in maxc(x)..minc(x) by -1 repeat
            l := cons(qelt(x,i,j),l)
          ll := cons(l,ll)
        ll

      swapRows_!(x,i1,i2) ==
        (i1 < minr(x)) or (i1 > maxr(x)) or (i2 < minr(x)) or _
            (i2 > maxr(x)) => error "swapRows!: index out of range"
        i1 = i2 => x
        for j in minc(x)..maxc(x) repeat
          r := qelt(x,i1,j)
          qsetelt_!(x,i1,j,qelt(x,i2,j))
          qsetelt_!(x,i2,j,r)
        x

      swapColumns_!(x,j1,j2) ==
        (j1 < minc(x)) or (j1 > maxc(x)) or (j2 < minc(x)) or _
            (j2 > maxc(x)) => error "swapColumns!: index out of range"
        j1 = j2 => x
        for i in minr(x)..maxr(x) repeat
          r := qelt(x,i,j1)
          qsetelt_!(x,i,j1,qelt(x,i,j2))
          qsetelt_!(x,i,j2,r)
        x

      elt(x:%,rowList:List Integer,colList:List Integer) ==
        for ei in rowList repeat
          (ei < minr(x)) or (ei > maxr(x)) =>
            error "elt: index out of range"
        for ej in colList repeat
          (ej < minc(x)) or (ej > maxc(x)) =>
            error "elt: index out of range"
        y := new(# rowList,# colList,0)
        for ei in rowList for i in minr(y)..maxr(y) repeat
          for ej in colList for j in minc(y)..maxc(y) repeat
            qsetelt_!(y,i,j,qelt(x,ei,ej))
```

```
    y

  setelt(x:%,rowList:List Integer,colList:List Integer,y:%) ==
    for ei in rowList repeat
      (ei < minr(x)) or (ei > maxr(x)) =>
        error "setelt: index out of range"
    for ej in colList repeat
      (ej < minc(x)) or (ej > maxc(x)) =>
        error "setelt: index out of range"
    ((# rowList) ^= (nrows y)) or ((# colList) ^= (ncols y)) =>
      error "setelt: matrix has bad dimensions"
    for ei in rowList for i in minr(y)..maxr(y) repeat
      for ej in colList for j in minc(y)..maxc(y) repeat
        qsetelt_!(x,ei,ej,qelt(y,i,j))
    y

  subMatrix(x,i1,i2,j1,j2) ==
    (i2 < i1) => error "subMatrix: bad row indices"
    (j2 < j1) => error "subMatrix: bad column indices"
    (i1 < minr(x)) or (i2 > maxr(x)) =>
      error "subMatrix: index out of range"
    (j1 < minc(x)) or (j2 > maxc(x)) =>
      error "subMatrix: index out of range"
    rows := (i2 - i1 + 1) pretend NonNegativeInteger
    cols := (j2 - j1 + 1) pretend NonNegativeInteger
    y := new(rows,cols,0)
    for i in minr(y)..maxr(y) for k in i1..i2 repeat
      for j in minc(y)..maxc(y) for l in j1..j2 repeat
        qsetelt_!(y,i,j,qelt(x,k,l))
    y

  setsubMatrix_!(x,i1,j1,y) ==
    i2 := i1 + nrows(y) -1
    j2 := j1 + ncols(y) -1
    (i1 < minr(x)) or (i2 > maxr(x)) =>
     error _
      "setsubMatrix!: inserted matrix too big, use subMatrix to restrict it"
    (j1 < minc(x)) or (j2 > maxc(x)) =>
     error _
      "setsubMatrix!: inserted matrix too big, use subMatrix to restrict it"
    for i in minr(y)..maxr(y) for k in i1..i2 repeat
      for j in minc(y)..maxc(y) for l in j1..j2 repeat
        qsetelt_!(x,k,l,qelt(y,i,j))
    x

--% Arithmetic

  x + y ==
    ((r := nrows x) ^= nrows y) or ((c := ncols x) ^= ncols y) =>
      error "can't add matrices of different dimensions"
    ans := new(r,c,0)
    for i in minr(x)..maxr(x) repeat
      for j in minc(x)..maxc(x) repeat
        qsetelt_!(ans,i,j,qelt(x,i,j) + qelt(y,i,j))
```

```
      ans

    x - y ==
      ((r := nrows x) ^= nrows y) or ((c := ncols x) ^= ncols y) =>
        error "can't subtract matrices of different dimensions"
      ans := new(r,c,0)
      for i in minr(x)..maxr(x) repeat
        for j in minc(x)..maxc(x) repeat
          qsetelt_!(ans,i,j,qelt(x,i,j) - qelt(y,i,j))
      ans

    - x == map((r1:R):R +-> - r1,x)

    a:R * x:% == map((r1:R):R +-> a * r1,x)

    x:% * a:R == map((r1:R):R +-> r1 * a,x)

    m:Integer * x:% == map((r1:R):R +-> m * r1,x)

    x:% * y:% ==
      (ncols x ^= nrows y) =>
        error "can't multiply matrices of incompatible dimensions"
      ans := new(nrows x,ncols y,0)
      for i in minr(x)..maxr(x) repeat
        for j in minc(y)..maxc(y) repeat
          entry :=
            sum : R := 0
            for k in minr(y)..maxr(y) for l in minc(x)..maxc(x) repeat
              sum := sum + qelt(x,i,l) * qelt(y,k,j)
            sum
          qsetelt_!(ans,i,j,entry)
      ans

    positivePower:(%,Integer) -> %
    positivePower(x,n) ==
--      one? n => x
      (n = 1) => x
      odd? n => x * positivePower(x,n - 1)
      y := positivePower(x,n quo 2)
      y * y

    x:% ** n:NonNegativeInteger ==
      not((nn:= nrows x) = ncols x) => error "**: matrix must be square"
      zero? n => scalarMatrix(nn,1)
      positivePower(x,n)

    --if R has ConvertibleTo InputForm then
      --convert(x:%):InputForm ==
        --convert [convert("matrix"::Symbol)@InputForm,
                 --convert listOfLists x]$List(InputForm)

    if Col has shallowlyMutable then

      x:% * v:Col ==
```

```
    ncols(x) ^= #v =>
      error "can't multiply matrix A and vector v if #cols A ^= #v"
    w : Col := new(nrows x,0)
    for i in minr(x)..maxr(x) for k in mini(w)..maxi(w) repeat
      w.k :=
        sum : R := 0
        for j in minc(x)..maxc(x) for l in mini(v)..maxi(v) repeat
          sum := sum + qelt(x,i,j) * v(l)
        sum
    w

if Row has shallowlyMutable then

  v:Row * x:% ==
    nrows(x) ^= #v =>
      error "can't multiply vector v and matrix A if #rows A ^= #v"
    w : Row := new(ncols x,0)
    for j in minc(x)..maxc(x) for k in mini(w)..maxi(w) repeat
      w.k :=
        sum : R := 0
        for i in minr(x)..maxr(x) for l in mini(v)..maxi(v) repeat
          sum := sum + qelt(x,i,j) * v(l)
        sum
    w

if R has EuclideanDomain then
  columnSpace M ==
    M2 := rowEchelon M
    basis: List Col := []
    n: Integer := ncols M
    m: Integer := nrows M
    indRow: Integer := 1
    for k in 1..n while indRow <= m repeat
      if not zero?(M2.(indRow,k)) then
        basis := cons(column(M,k),basis)
        indRow := indRow + 1
    reverse! basis

if R has CommutativeRing then
  skewSymmetricUnitMatrix(n:PositiveInteger):% ==
    matrix [[(if i=j+1 and odd? j
               then -1
               else if i=j-1 and odd? i
                      then 1
                      else 0) for j in 1..n] for i in 1..n]

  SUPR ==> SparseUnivariatePolynomial R

  PfChar(A:%):SUPR ==
    n := nrows A
    (n = 2) => monomial(1$R,2)$SUPR + qelt(A,1,2)::SUPR
    M:=subMatrix(A,3,n,3,n)
    r:=subMatrix(A,1,1,3,n)
    s:=subMatrix(A,3,n,2,2)
```

```
    p:=PfChar(M)
    d:=degree(p)$SUPR
    B:=skewSymmetricUnitMatrix((n-2)::PositiveInteger)
    C:=r*B
    g:List R := [qelt(C*s,1,1), qelt(A,1,2), 1]
    if d >= 4 then
      B:=M*B
      for i in 4..d by 2 repeat
        C:=C*B
        g:=cons(qelt(C*s,1,1),g)
    g:=reverse! g
    res:SUPR := 0
    for i in 0..d by 2 for j in 2..d+2 repeat
      c:=coefficient(p,i)
      for e in first(g,j) for k in 2..-d by -2 repeat
        res:=res+monomial(c*e,(k+i)::NonNegativeInteger)$SUPR
    res

  pfaffian a ==
    if antisymmetric? a
      then if odd? nrows a
             then 0
             else PfChar(a).0
      else
        error "pfaffian: only defined for antisymmetric square matrices"

if R has IntegralDomain then
  x exquo a ==
    ans := new(nrows x,ncols x,0)
    for i in minr(x)..maxr(x) repeat
      for j in minc(x)..maxc(x) repeat
        entry :=
          (r := (qelt(x,i,j) exquo a)) case "failed" =>
            return "failed"
          r :: R
        qsetelt_!(ans,i,j,entry)
    ans

if R has Field then
  x / r == map((r1:R):R +-> r1 / r,x)

  x:% ** n:Integer ==
    not((nn:= nrows x) = ncols x) => error "**: matrix must be square"
    zero? n => scalarMatrix(nn,1)
    positive? n => positivePower(x,n)
    (xInv := inverse x) case "failed" =>
      error "**: matrix must be invertible"
    positivePower(xInv :: %,-n)
```

— COQ MATCAT —

```
(* category MATCAT *)
(*
     minr ==> minRowIndex
     maxr ==> maxRowIndex
     minc ==> minColIndex
     maxc ==> maxColIndex
     mini ==> minIndex
     maxi ==> maxIndex

--% Predicates

     square?  : % -> Boolean
     square? x == nrows x = ncols x

     diagonal?: % -> Boolean
     diagonal? x ==
       not square? x => false
       for i in minr x .. maxr x repeat
         for j in minc x .. maxc x | (j - minc x) ^= (i - minr x) repeat
           not zero? qelt(x, i, j) => return false
       true

     symmetric?: % -> Boolean
     symmetric? x ==
       (nRows := nrows x) ^= ncols x => false
       mr := minRowIndex x; mc := minColIndex x
       for i in 0..(nRows - 1) repeat
         for j in (i + 1)..(nRows - 1) repeat
           qelt(x,mr + i,mc + j) ^= qelt(x,mr + j,mc + i) => return false
       true

     antisymmetric?: % -> Boolean
     antisymmetric? x ==
       (nRows := nrows x) ^= ncols x => false
       mr := minRowIndex x; mc := minColIndex x
       for i in 0..(nRows - 1) repeat
         for j in i..(nRows - 1) repeat
           qelt(x,mr + i,mc + j) ^= -qelt(x,mr + j,mc + i) =>
             return false
       true

--% Creation of matrices

     zero: (NonNegativeInteger,NonNegativeInteger) -> %
     zero(rows,cols) == new(rows,cols,0)

     matrix: List List R -> %
     matrix(l: List List R) ==
       null l => new(0,0,0)
       -- error check: this is a top level function
       rows : NonNegativeInteger := 1; cols := # first l
       cols = 0 => error "matrices with zero columns are not supported"
       for ll in rest l repeat
         cols ^= # ll => error "matrix: rows of different lengths"
```

```
    rows := rows + 1
  ans := new(rows,cols,0)
  for i in minr(ans)..maxr(ans) for ll in l repeat
    for j in minc(ans)..maxc(ans) for r in ll repeat
      qsetelt_!(ans,i,j,r)
  ans

matrix: (NonNegativeInteger,NonNegativeInteger,(Integer,Integer)->R) -> %
matrix(n,m,f) ==
  mat := new(n,m,0)
  for i in minr mat..maxr mat repeat
    for j in minc mat..maxc mat repeat
      qsetelt!(mat,i,j,f(i,j))
  mat

scalarMatrix: (NonNegativeInteger,R) -> %
scalarMatrix(n,r) ==
  ans := zero(n,n)
  for i in minr(ans)..maxr(ans) for j in minc(ans)..maxc(ans) repeat
    qsetelt_!(ans,i,j,r)
  ans

diagonalMatrix: List R -> %
diagonalMatrix(l: List R) ==
  n := #l; ans := zero(n,n)
  for i in minr(ans)..maxr(ans) for j in minc(ans)..maxc(ans) _
      for r in l repeat qsetelt_!(ans,i,j,r)
  ans

diagonalMatrix: List % -> %
diagonalMatrix(list: List %) ==
  rows : NonNegativeInteger := 0
  cols : NonNegativeInteger := 0
  for mat in list repeat
    rows := rows + nrows mat
    cols := cols + ncols mat
  ans := zero(rows,cols)
  loR := minr ans; loC := minc ans
  for mat in list repeat
    hiR := loR + nrows(mat) - 1; hiC := loC + nrows(mat) - 1
    for i in loR..hiR for k in minr(mat)..maxr(mat) repeat
      for j in loC..hiC for l in minc(mat)..maxc(mat) repeat
        qsetelt_!(ans,i,j,qelt(mat,k,l))
    loR := hiR + 1; loC := hiC + 1
  ans

coerce: Col -> %
coerce(v:Col) ==
  x := new(#v,1,0)
  one := minc(x)
  for i in minr(x)..maxr(x) for k in mini(v)..maxi(v) repeat
    qsetelt_!(x,i,one,qelt(v,k))
  x
```

```
    transpose: Row -> %
    transpose(v:Row) ==
      x := new(1,#v,0)
      one := minr(x)
      for j in minc(x)..maxc(x) for k in mini(v)..maxi(v) repeat
        qsetelt_!(x,one,j,qelt(v,k))
      x

    transpose: % -> %
    transpose(x:%) ==
      ans := new(ncols x,nrows x,0)
      for i in minr(ans)..maxr(ans) repeat
        for j in minc(ans)..maxc(ans) repeat
          qsetelt_!(ans,i,j,qelt(x,j,i))
      ans

    squareTop: % -> %
    squareTop x ==
      nrows x < (cols := ncols x) =>
        error "squareTop: number of columns exceeds number of rows"
      ans := new(cols,cols,0)
      for i in minr(x)..(minr(x) + cols - 1) repeat
        for j in minc(x)..maxc(x) repeat
          qsetelt_!(ans,i,j,qelt(x,i,j))
      ans

    horizConcat: (%,%) -> %
    horizConcat(x,y) ==
      (rows := nrows x) ^= nrows y =>
        error "HConcat: matrices must have same number of rows"
      ans := new(rows,(cols := ncols x) + ncols y,0)
      for i in minr(x)..maxr(x) repeat
        for j in minc(x)..maxc(x) repeat
          qsetelt_!(ans,i,j,qelt(x,i,j))
      for i in minr(y)..maxr(y) repeat
        for j in minc(y)..maxc(y) repeat
          qsetelt_!(ans,i,j + cols,qelt(y,i,j))
      ans

    vertConcat: (%,%) -> %
    vertConcat(x,y) ==
      (cols := ncols x) ^= ncols y =>
        error "HConcat: matrices must have same number of columns"
      ans := new((rows := nrows x) + nrows y,cols,0)
      for i in minr(x)..maxr(x) repeat
        for j in minc(x)..maxc(x) repeat
          qsetelt_!(ans,i,j,qelt(x,i,j))
      for i in minr(y)..maxr(y) repeat
        for j in minc(y)..maxc(y) repeat
          qsetelt_!(ans,i + rows,j,qelt(y,i,j))
      ans

--% Part extraction/assignment
```

```
listOfLists: % -> List List R
listOfLists x ==
  ll : List List R := nil()
  for i in maxr(x)..minr(x) by -1 repeat
    l : List R := nil()
    for j in maxc(x)..minc(x) by -1 repeat
      l := cons(qelt(x,i,j),l)
    ll := cons(l,ll)
  ll

swapRows_!: (%,Integer,Integer) -> %
swapRows_!(x,i1,i2) ==
  (i1 < minr(x)) or (i1 > maxr(x)) or (i2 < minr(x)) or _
      (i2 > maxr(x)) => error "swapRows!: index out of range"
  i1 = i2 => x
  for j in minc(x)..maxc(x) repeat
    r := qelt(x,i1,j)
    qsetelt_!(x,i1,j,qelt(x,i2,j))
    qsetelt_!(x,i2,j,r)
  x

swapColumns_!: (%,Integer,Integer) -> %
swapColumns_!(x,j1,j2) ==
  (j1 < minc(x)) or (j1 > maxc(x)) or (j2 < minc(x)) or _
      (j2 > maxc(x)) => error "swapColumns!: index out of range"
  j1 = j2 => x
  for i in minr(x)..maxr(x) repeat
    r := qelt(x,i,j1)
    qsetelt_!(x,i,j1,qelt(x,i,j2))
    qsetelt_!(x,i,j2,r)
  x

elt : (%,List(Integer),List(Integer)) -> %
elt(x:%,rowList:List Integer,colList:List Integer) ==
  for ei in rowList repeat
    (ei < minr(x)) or (ei > maxr(x)) =>
      error "elt: index out of range"
  for ej in colList repeat
    (ej < minc(x)) or (ej > maxc(x)) =>
      error "elt: index out of range"
  y := new(# rowList,# colList,0)
  for ei in rowList for i in minr(y)..maxr(y) repeat
    for ej in colList for j in minc(y)..maxc(y) repeat
      qsetelt_!(y,i,j,qelt(x,ei,ej))
  y

setelt : (%,List(Integer),List(Integer),%) -> %
setelt(x:%,rowList:List Integer,colList:List Integer,y:%) ==
  for ei in rowList repeat
    (ei < minr(x)) or (ei > maxr(x)) =>
      error "setelt: index out of range"
  for ej in colList repeat
    (ej < minc(x)) or (ej > maxc(x)) =>
      error "setelt: index out of range"
```

```
    ((# rowList) ^= (nrows y)) or ((# colList) ^= (ncols y)) =>
      error "setelt: matrix has bad dimensions"
    for ei in rowList for i in minr(y)..maxr(y) repeat
      for ej in colList for j in minc(y)..maxc(y) repeat
        qsetelt_!(x,ei,ej,qelt(y,i,j))
    y

  subMatrix: (%,Integer,Integer,Integer,Integer) -> %
  subMatrix(x,i1,i2,j1,j2) ==
    (i2 < i1) => error "subMatrix: bad row indices"
    (j2 < j1) => error "subMatrix: bad column indices"
    (i1 < minr(x)) or (i2 > maxr(x)) =>
      error "subMatrix: index out of range"
    (j1 < minc(x)) or (j2 > maxc(x)) =>
      error "subMatrix: index out of range"
    rows := (i2 - i1 + 1) pretend NonNegativeInteger
    cols := (j2 - j1 + 1) pretend NonNegativeInteger
    y := new(rows,cols,0)
    for i in minr(y)..maxr(y) for k in i1..i2 repeat
      for j in minc(y)..maxc(y) for l in j1..j2 repeat
        qsetelt_!(y,i,j,qelt(x,k,l))
    y

  setsubMatrix_!: (%,Integer,Integer,%) -> %
  setsubMatrix_!(x,i1,j1,y) ==
    i2 := i1 + nrows(y) -1
    j2 := j1 + ncols(y) -1
    (i1 < minr(x)) or (i2 > maxr(x)) =>
     error _
      "setsubMatrix!: inserted matrix too big, use subMatrix to restrict it"
    (j1 < minc(x)) or (j2 > maxc(x)) =>
     error _
      "setsubMatrix!: inserted matrix too big, use subMatrix to restrict it"
    for i in minr(y)..maxr(y) for k in i1..i2 repeat
      for j in minc(y)..maxc(y) for l in j1..j2 repeat
        qsetelt_!(x,k,l,qelt(y,i,j))
    x

--% Arithmetic

  "+": (%,%) -> %
  x + y ==
    ((r := nrows x) ^= nrows y) or ((c := ncols x) ^= ncols y) =>
      error "can't add matrices of different dimensions"
    ans := new(r,c,0)
    for i in minr(x)..maxr(x) repeat
      for j in minc(x)..maxc(x) repeat
        qsetelt_!(ans,i,j,qelt(x,i,j) + qelt(y,i,j))
    ans

  "-": (%,%) -> %
  x - y ==
    ((r := nrows x) ^= nrows y) or ((c := ncols x) ^= ncols y) =>
      error "can't subtract matrices of different dimensions"
```

```
  ans := new(r,c,0)
  for i in minr(x)..maxr(x) repeat
    for j in minc(x)..maxc(x) repeat
      qsetelt_!(ans,i,j,qelt(x,i,j) - qelt(y,i,j))
  ans

"-":  %    -> %
- x == map((r1:R):R +-> - r1,x)

"*": (%,R) -> %
a:R * x:% == map((r1:R):R +-> a * r1,x)

"*": (R,%) -> %
x:% * a:R == map((r1:R):R +-> r1 * a,x)

"*": (Integer,%) -> %
m:Integer * x:% == map((r1:R):R +-> m * r1,x)

?*? : (%,%) -> %
x:% * y:% ==
  (ncols x ^= nrows y) =>
    error "can't multiply matrices of incompatible dimensions"
  ans := new(nrows x,ncols y,0)
  for i in minr(x)..maxr(x) repeat
    for j in minc(y)..maxc(y) repeat
      entry :=
        sum : R := 0
        for k in minr(y)..maxr(y) for l in minc(x)..maxc(x) repeat
          sum := sum + qelt(x,i,l) * qelt(y,k,j)
        sum
      qsetelt_!(ans,i,j,entry)
  ans

positivePower:(%,Integer) -> %
positivePower(x,n) ==
  (n = 1) => x
  odd? n => x * positivePower(x,n - 1)
  y := positivePower(x,n quo 2)
  y * y

?**? : (%,NonNegativeInteger) -> %
x:% ** n:NonNegativeInteger ==
  not((nn:= nrows x) = ncols x) => error "**: matrix must be square"
  zero? n => scalarMatrix(nn,1)
  positivePower(x,n)

--if R has ConvertibleTo InputForm then
  --convert(x:%):InputForm ==
    --convert [convert("matrix"::Symbol)@InputForm,
             --convert listOfLists x]$List(InputForm)

if Col has shallowlyMutable then

  "*": (%,Col) -> Col
```

```
    x:% * v:Col ==
      ncols(x) ^= #v =>
        error "can't multiply matrix A and vector v if #cols A ^= #v"
      w : Col := new(nrows x,0)
      for i in minr(x)..maxr(x) for k in mini(w)..maxi(w) repeat
        w.k :=
          sum : R := 0
          for j in minc(x)..maxc(x) for l in mini(v)..maxi(v) repeat
            sum := sum + qelt(x,i,j) * v(l)
          sum
      w

  if Row has shallowlyMutable then

    "*": (Row,%) -> Row
    v:Row * x:% ==
      nrows(x) ^= #v =>
        error "can't multiply vector v and matrix A if #rows A ^= #v"
      w : Row := new(ncols x,0)
      for j in minc(x)..maxc(x) for k in mini(w)..maxi(w) repeat
        w.k :=
          sum : R := 0
          for i in minr(x)..maxr(x) for l in mini(v)..maxi(v) repeat
            sum := sum + qelt(x,i,j) * v(l)
          sum
      w

  if R has EuclideanDomain then

    columnSpace: % -> List Col
    columnSpace M ==
      M2 := rowEchelon M
      basis: List Col := []
      n: Integer := ncols M
      m: Integer := nrows M
      indRow: Integer := 1
      for k in 1..n while indRow <= m repeat
        if not zero?(M2.(indRow,k)) then
          basis := cons(column(M,k),basis)
          indRow := indRow + 1
      reverse! basis

  if R has CommutativeRing then

    skewSymmetricUnitMatrix(n:PositiveInteger):% ==
      matrix [[(if i=j+1 and odd? j
                  then -1
                  else if i=j-1 and odd? i
                         then 1
                         else 0) for j in 1..n] for i in 1..n]

    SUPR ==> SparseUnivariatePolynomial R

    PfChar(A:%):SUPR ==
```

```
    n := nrows A
    (n = 2) => monomial(1$R,2)$SUPR + qelt(A,1,2)::SUPR
    M:=subMatrix(A,3,n,3,n)
    r:=subMatrix(A,1,1,3,n)
    s:=subMatrix(A,3,n,2,2)
    p:=PfChar(M)
    d:=degree(p)$SUPR
    B:=skewSymmetricUnitMatrix((n-2)::PositiveInteger)
    C:=r*B
    g:List R := [qelt(C*s,1,1), qelt(A,1,2), 1]
    if d >= 4 then
      B:=M*B
      for i in 4..d by 2 repeat
        C:=C*B
        g:=cons(qelt(C*s,1,1),g)
    g:=reverse! g
    res:SUPR := 0
    for i in 0..d by 2 for j in 2..d+2 repeat
      c:=coefficient(p,i)
      for e in first(g,j) for k in 2..-d by -2 repeat
        res:=res+monomial(c*e,(k+i)::NonNegativeInteger)$SUPR
    res

  pfaffian: % -> R
  pfaffian a ==
    if antisymmetric? a
      then if odd? nrows a
             then 0
             else PfChar(a).0
      else
        error "pfaffian: only defined for antisymmetric square matrices"

if R has IntegralDomain then

  "exquo": (%,R) -> Union(%,"failed")
  x exquo a ==
    ans := new(nrows x,ncols x,0)
    for i in minr(x)..maxr(x) repeat
      for j in minc(x)..maxc(x) repeat
        entry :=
          (r := (qelt(x,i,j) exquo a)) case "failed" =>
            return "failed"
          r :: R
        qsetelt_!(ans,i,j,entry)
    ans

if R has Field then

  "/": (%,R) -> %
  x / r == map((r1:R):R +-> r1 / r,x)

  "**": (%,Integer) -> %
  x:% ** n:Integer ==
    not((nn:= nrows x) = ncols x) => error "**: matrix must be square"
```

```
        zero? n => scalarMatrix(nn,1)
        positive? n => positivePower(x,n)
        (xInv := inverse x) case "failed" =>
          error "**: matrix must be invertible"
        positivePower(xInv :: %,-n)

*)
```

———-

— MATCAT.dotabb —

```
"MATCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MATCAT"];
"MATCAT" -> "ARR2CAT"
```

———-

— MATCAT.dotfull —

```
"MatrixCategory(a:Ring,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MATCAT"];
"MatrixCategory(a:Ring,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a)"
 ->
"TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a))"
```

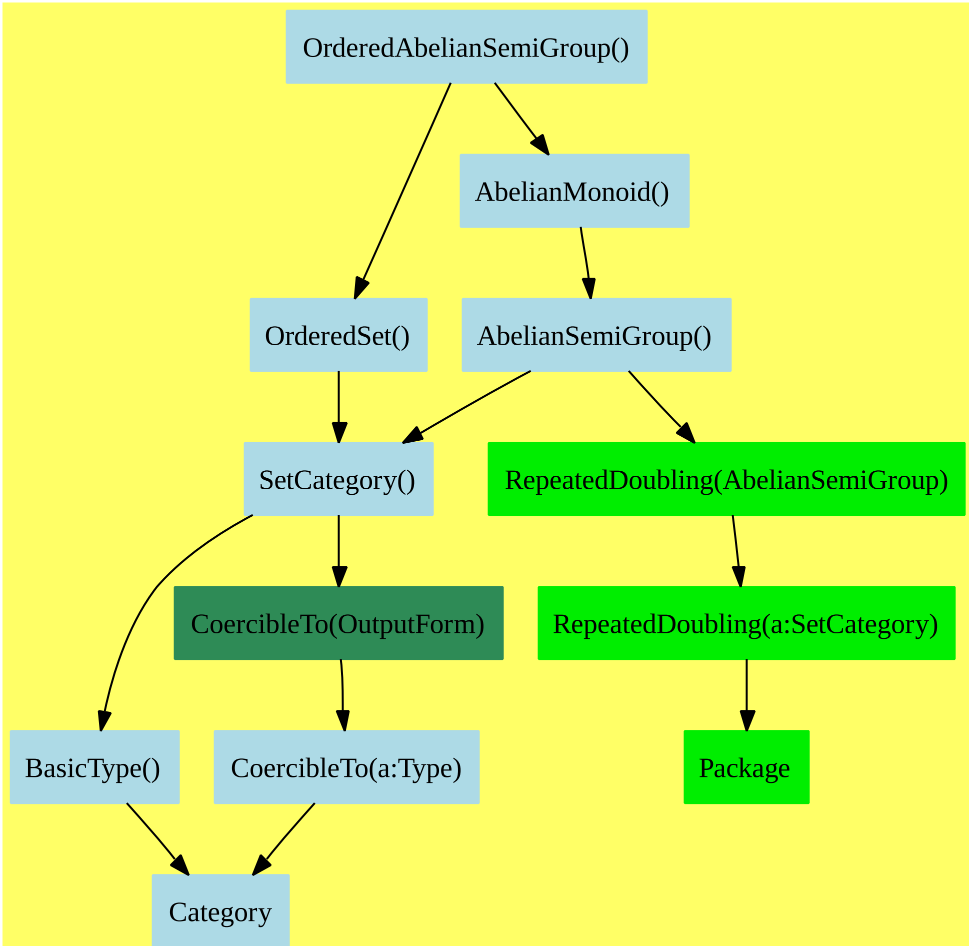
———-

— MATCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"MatrixCategory(a:Ring,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a)"
 [color=lightblue];
"MatrixCategory(a:Ring,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a)"
 ->
"TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a))"

"TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a))"
 [color=lightblue];
"TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a))"
    -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"
"HomogeneousAggregate(a:Type)" -> "Evalable(a:Type)"
"HomogeneousAggregate(a:Type)" -> "SetCategory()"
```

```
"Evalable(a:Type)" [color="#00EE00"];

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];

}
```

6.0.113 OrderedAbelianSemiGroup (OASGP)



— OrderedAbelianSemiGroup.input —

```
)set break resume
)sys rm -f OrderedAbelianSemiGroup.output
)spool OrderedAbelianSemiGroup.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderedAbelianSemiGroup
--R
--R OrderedAbelianSemiGroup is a category constructor
--R Abbreviation for OrderedAbelianSemiGroup is OASGP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for OASGP
--R
```

```
--R------------------------------- Operations --------------------------------
--R ?*? : (PositiveInteger,%) -> %       ?+? : (%,%) -> %
--R ?<? : (%,%) -> Boolean               ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean               ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean              coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R max : (%,%) -> %                     min : (%,%) -> %
--R ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

— **OrderedAbelianSemiGroup.help** —

```
====================================================================
OrderedAbelianSemiGroup examples
====================================================================

Ordered sets which are also abelian semigroups, such that the addition
preserves the ordering.

Axiom:
        x < y => x+z < y+z

See Also:
o )show OrderedAbelianSemiGroup
```

See:

⇒ "OrderedAbelianMonoid" (OAMON) 7.0.129 on page 730
⇐ "AbelianMonoid" (ABELMON) 5.0.90 on page 383
⇐ "OrderedSet" (ORDSET) 4.0.80 on page 326

Exports:

0	coerce	hash	latex	max
min	sample	zero?	?~=?	?*?
?+?	?<?	?<=?	?=?	?>?
?>=?				

These exports come from (p326) OrderedSet():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
?<? : (%,%) -> Boolean
```

```
?>? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

These exports come from (p383) AbelianMonoid():

```
0 : () -> %
sample : () -> %
zero? : % -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
```

— OrderedAbelianSemiGroup.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#OASGP">
OrderedAbelianSemiGroup (OASGP)</a></h2>
</body>
```

— category OASGP OrderedAbelianSemiGroup —

```
)abbrev category OASGP OrderedAbelianSemiGroup
++ Description:
++ Ordered sets which are also abelian semigroups, such that the addition
++ preserves the ordering.\br
++
++ Axiom\br
++ \tab{5} x < y => x+z < y+z

OrderedAbelianSemiGroup() : Category == SIG where

  SIG ==> Join(OrderedSet, AbelianSemiGroup)
```

— COQ OASGP —

```
(* category OASGP *)
(*
Axiom
   x < y => x+z < y+z
*)
```

— OASGP.dotabb —

```
"OASGP"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OASGP"];
```

```
"OASGP" -> "ORDSET"
"OASGP" -> "ABELMON"
```

———-

— **OASGP.dotfull** —

```
"OrderedAbelianSemiGroup()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OASGP"];
"OrderedAbelianSemiGroup()" -> "OrderedSet()"
"OrderedAbelianSemiGroup()" -> "AbelianMonoid()"
```

———-

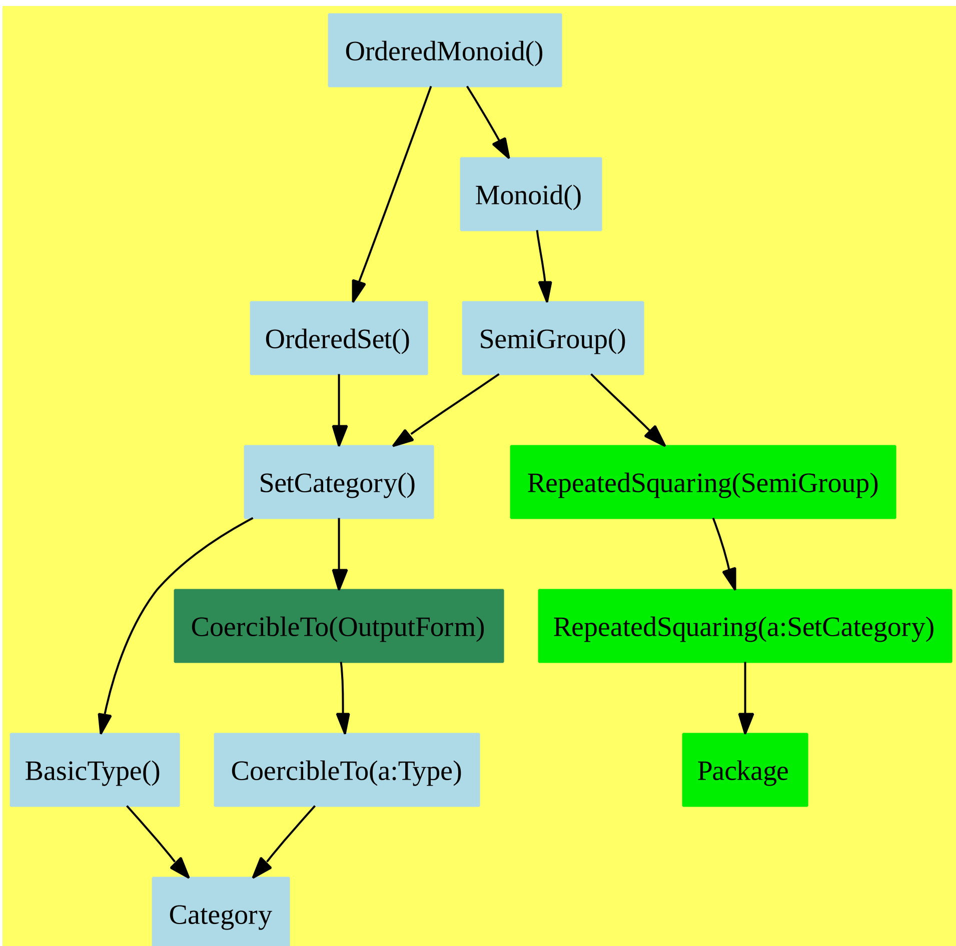
— **OASGP.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"OrderedAbelianSemiGroup()" [color=lightblue];
"OrderedAbelianSemiGroup()" -> "OrderedSet()"
"OrderedAbelianSemiGroup()" -> "AbelianMonoid()"

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SetCategory()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SetCategory()"
"AbelianSemiGroup()" -> "RepeatedDoubling(AbelianSemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" ->
    "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"RepeatedDoubling(AbelianSemiGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianSemiGroup)" -> "RepeatedDoubling(a:SetCategory)"
```

```
"RepeatedDoubling(a:SetCategory)" [color="#00EE00"];
"RepeatedDoubling(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];
}
```

———-

6.0.114 OrderedMonoid (ORDMON)



— **OrderedMonoid.input** —

```
)set break resume
)sys rm -f OrderedMonoid.output
)spool OrderedMonoid.output
```

```
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderedMonoid
--R
--R OrderedMonoid is a category constructor
--R Abbreviation for OrderedMonoid is ORDMON
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ORDMON
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?**? : (%,PositiveInteger) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean               ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean                ?>=? : (%,%) -> Boolean
--R 1 : () -> %                           ?^? : (%,PositiveInteger) -> %
--R ?^? : (%,NonNegativeInteger) -> %     coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R max : (%,%) -> %                      min : (%,%) -> %
--R one? : % -> Boolean                   recip : % -> Union(%,"failed")
--R sample : () -> %                      ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

— **OrderedMonoid.help** —

```
====================================================================
OrderedMonoid examples
====================================================================

Ordered sets which are also monoids, such that multiplication
preserves the ordering.

Axioms:
         x < y => x*z < y*z
         x < y => z*x < z*y

See Also:
o )show OrderedMonoid
```

See:

⇐ "Monoid" (MONOID) 5.0.100 on page 455
⇐ "OrderedSet" (ORDSET) 4.0.80 on page 326

Exports:

1	coerce	hash	latex	max
min	one?	recip	sample	?*?
?**?	?<?	?<=?	?=?	?>?
?>=?	?~=?	?^?		

These exports come from (p455) Monoid():

```
1 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
?*? : (%,%) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

These exports come from (p326) OrderedSet():

```
max : (%,%) -> %
min : (%,%) -> %
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

— OrderedMonoid.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ORDMON">
OrderedMonoid (ORDMON)</a></h2>
</body>
```

———-

— category ORDMON OrderedMonoid —

```
)abbrev category ORDMON OrderedMonoid
++ Description:
++ Ordered sets which are also monoids, such that multiplication
++ preserves the ordering.
++
++ Axioms\br
++ \tab{5}\spad{x < y => x*z < y*z}\br
++ \tab{5}\spad{x < y => z*x < z*y}

OrderedMonoid() : Category == SIG where
```

```
  SIG ==> Join(OrderedSet, Monoid)
```

———-

— COQ ORDMON —

```
(* category ORDMON *)
(*
Axioms:
  x < y => x*z < y*z
  x < y => z*x < z*y
*)
```

———-

— ORDMON.dotabb —

```
"ORDMON"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDMON"];
"ORDMON" -> "ORDSET"
"ORDMON" -> "MONOID"
```

———-

— ORDMON.dotfull —

```
"OrderedMonoid()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDMON"];
"OrderedMonoid()" -> "OrderedSet()"
"OrderedMonoid()" -> "Monoid()"
```

———-

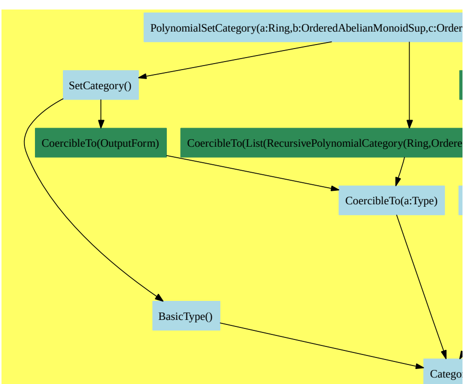
— ORDMON.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"OrderedMonoid()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDMON"];
"OrderedMonoid()" -> "OrderedSet()"
"OrderedMonoid()" -> "Monoid()"

"OrderedSet()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDSET"];
"OrderedSet()" -> "SetCategory()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"
```

```
"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SetCategory()"
"SemiGroup()" -> "RepeatedSquaring(SemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"RepeatedSquaring(SemiGroup)" [color="#00EE00"];
"RepeatedSquaring(SemiGroup)" -> "RepeatedSquaring(a:SetCategory)"

"RepeatedSquaring(a:SetCategory)" [color="#00EE00"];
"RepeatedSquaring(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];

}
```

———-

6.0.115 PolynomialSetCategory (PSETCAT)



— PolynomialSetCategory.input —

```
)set break resume
```

```
)sys rm -f PolynomialSetCategory.output
)spool PolynomialSetCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PolynomialSetCategory
--R
--R PolynomialSetCategory(R: Ring,E: OrderedAbelianMonoidSup,VarSet: OrderedSet,P: RecursivePolynomialCategory(t
--R Abbreviation for PolynomialSetCategory is PSETCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PSETCAT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                coerce : % -> List(P)
--R coerce : % -> OutputForm              collect : (%,VarSet) -> %
--R collectUnder : (%,VarSet) -> %        collectUpper : (%,VarSet) -> %
--R construct : List(P) -> %              copy : % -> %
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                hash : % -> SingleInteger
--R latex : % -> String                   mainVariables : % -> List(VarSet)
--R map : ((P -> P),%) -> %               mvar : % -> VarSet
--R retract : List(P) -> %                sample : () -> %
--R trivialIdeal? : % -> Boolean          variables : % -> List(VarSet)
--R ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R convert : % -> InputForm if P has KONVERT(INFORM)
--R count : ((P -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (P,%) -> NonNegativeInteger if P has SETCAT and $ has finiteAggregate
--R eval : (%,List(Equation(P))) -> % if P has EVALAB(P) and P has SETCAT
--R eval : (%,Equation(P)) -> % if P has EVALAB(P) and P has SETCAT
--R eval : (%,P,P) -> % if P has EVALAB(P) and P has SETCAT
--R eval : (%,List(P),List(P)) -> % if P has EVALAB(P) and P has SETCAT
--R every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R find : ((P -> Boolean),%) -> Union(P,"failed")
--R headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
--R less? : (%,NonNegativeInteger) -> Boolean
--R mainVariable? : (VarSet,%) -> Boolean
--R map! : ((P -> P),%) -> % if $ has shallowlyMutable
--R member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
--R members : % -> List(P) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(P) if $ has finiteAggregate
--R reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P,P) -> P if P has SETCAT and $ has finiteAggregate
--R remainder : (P,%) -> Record(rnum: R,polnum: P,den: R) if R has INTDOM
--R remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
--R retractIfCan : List(P) -> Union(%,"failed")
--R rewriteIdealWithHeadRemainder : (List(P),%) -> List(P) if R has INTDOM
```

```
--R rewriteIdealWithRemainder : (List(P),%) -> List(P) if R has INTDOM
--R roughBase? : % -> Boolean if R has INTDOM
--R roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
--R roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
--R roughUnitIdeal? : % -> Boolean if R has INTDOM
--R select : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : (%,VarSet) -> Record(under: %,floor: %,upper: %)
--R triangular? : % -> Boolean if R has INTDOM
--R
--E 1

)spool
)lisp (bye)
```

— **PolynomialSetCategory.help** —

```
====================================================================
PolynomialSetCategory examples
====================================================================

A category for finite subsets of a polynomial ring.  Such a set is
only regarded as a set of polynomials and not identified to the ideal
it generates. So two distinct sets may generate the same the ideal.
Furthermore, for R being an integral domain, a set of polynomials may
be viewed as a representation of the ideal it generates in the polynomial
ring (R)^(-1) P, or the set of its zeros (described for instance by the
radical of the previous ideal, or a split of the associated affine
variety) and so on.  So this category provides operations about
those different notions.

See Also:
o )show PolynomialSetCategory
```

See:

⇒ "TriangularSetCategory" (TSETCAT) 7.0.132 on page 750
⇐ "CoercibleTo" (KOERCE) 2.0.14 on page 39
⇐ "Collection" (CLAGG) 5.0.94 on page 403
⇐ "SetCategory" (SETCAT) 3.0.57 on page 187

Exports:

any?	coerce	collect
collectUnder	collectUpper	construct
convert	copy	count
empty	empty?	eq?
eval	every?	find
hash	headRemainder	latex
less?	mainVariables	mainVariable?
map	map!	member?
members	more?	mvar
parts	reduce	remainder
remove	removeDuplicates	retract
retractIfCan	rewriteIdealWithHeadRemainder	rewriteIdealWithRemainder
roughBase?	roughEqualIdeals?	roughSubIdeal?
roughUnitIdeal?	sample	select
size?	sort	triangular?
trivialIdeal?	variables	#?
?~=?	?=?	

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These are directly exported but not implemented:

```
mvar : % -> VarSet
retract : List P -> %
retractIfCan : List P -> Union(%,"failed")
```

These are implemented by this category:

```
collect : (%,VarSet) -> %
collectUnder : (%,VarSet) -> %
collectUpper : (%,VarSet) -> %
headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
mainVariables : % -> List VarSet
mainVariable? : (VarSet,%) -> Boolean
remainder : (P,%) -> Record(rnum: R,polnum: P,den: R) if R has INTDOM
rewriteIdealWithHeadRemainder : (List P,%) -> List P if R has INTDOM
rewriteIdealWithRemainder : (List P,%) -> List P if R has INTDOM
roughBase? : % -> Boolean if R has INTDOM
roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
roughUnitIdeal? : % -> Boolean if R has INTDOM
sort : (%,VarSet) -> Record(under: %,floor: %,upper: %)
triangular? : % -> Boolean if R has INTDOM
trivialIdeal? : % -> Boolean
variables : % -> List VarSet
?=? : (%,%) -> Boolean
```

These exports come from (p187) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?~=? : (%,%) -> Boolean
```

These exports come from (p403) Collection(P)
where P:RecursivePolynomialCategory(R,E,V)
where R:Ring, E:OrderedAbelianMonoidSup, V:OrderedSet

```
any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
construct : List P -> %
copy : % -> %
count : (P,%) -> NonNegativeInteger if P has SETCAT and $ has finiteAggregate
count : ((P -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
find : ((P -> Boolean),%) -> Union(P,"failed")
less? : (%,NonNegativeInteger) -> Boolean
map : ((P -> P),%) -> %
map! : ((P -> P),%) -> % if $ has shallowlyMutable
member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
members : % -> List P if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List P if $ has finiteAggregate
reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P,P) -> P if P has SETCAT and $ has finiteAggregate
reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
sample : () -> %
select : ((P -> Boolean),%) -> % if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
```

These exports come from (p39) CoercibleTo(List(P))
where P:RecursivePolynomialCategory(R,E,V)
where R:Ring, E:OrderedAbelianMonoidSup, V:OrderedSet

```
coerce : % -> List P
convert : % -> InputForm if P has KONVERT INFORM
```

These exports come from (p1225) IntegralDomain():

— PolynomialSetCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#PSETCAT">
PolynomialSetCategory (PSETCAT)</a></h2>
</body>
```

———-

— category PSETCAT PolynomialSetCategory —

```
)abbrev category PSETCAT PolynomialSetCategory
++ Author: Marc Moreno Maza
++ Date Created: 04/26/1994
++ Date Last Updated: 12/15/1998
++ Description:
++ A category for finite subsets of a polynomial ring.
++ Such a set is only regarded as a set of polynomials and not
++ identified to the ideal it generates. So two distinct sets may
++ generate the same the ideal. Furthermore, for \spad{R} being an
++ integral domain, a set of polynomials may be viewed as a representation
++ of the ideal it generates in the polynomial ring \spad{(R)^(-1) P},
++ or the set of its zeros (described for instance by the radical of the
++ previous ideal, or a split of the associated affine variety) and so on.
++ So this category provides operations about those different notions.

PolynomialSetCategory(R,E,VarSet,P) : Category == SIG where
  R : Ring
  E : OrderedAbelianMonoidSup
  VarSet : OrderedSet
  P : RecursivePolynomialCategory(R,E,VarSet)

  SC ==> SetCategory
  CO ==> Collection(P)
  CT ==> CoercibleTo(List(P))

  SIG ==> Join(SC,CO,CT) with

    finiteAggregate

    retractIfCan : List(P) -> Union($,"failed")
      ++ \axiom{retractIfCan(lp)} returns an element of the domain
      ++ whose elements are the members of \axiom{lp} if such an element
      ++ exists, otherwise \axiom{"failed"} is returned.

    retract : List(P) -> $
      ++ \axiom{retract(lp)} returns an element of the domain whose elements
      ++ are the members of \axiom{lp} if such an element exists, otherwise
      ++ an error is produced.

    mvar : $ -> VarSet
      ++  \axiom{mvar(ps)} returns the main variable of the non constant
      ++ polynomial with the greatest main variable, if any, else an
      ++ error is returned.

    variables : $ -> List VarSet
      ++  \axiom{variables(ps)} returns the decreasingly sorted list of the
      ++  variables which are variables of some polynomial in \axiom{ps}.

    mainVariables : $  -> List VarSet
      ++ \axiom{mainVariables(ps)} returns the decreasingly sorted list
      ++ of the variables which are main variables of some polynomial
```

```
  ++ in \axiom{ps}.

mainVariable? : (VarSet,$) -> Boolean
  ++ \axiom{mainVariable?(v,ps)} returns true iff \axiom{v} is the
  ++ main variable of some polynomial in \axiom{ps}.

collectUnder : ($,VarSet) -> $
  ++ \axiom{collectUnder(ps,v)} returns the set consisting of the
  ++ polynomials of \axiom{ps} with main variable less than \axiom{v}.

collect : ($,VarSet) -> $
  ++ \axiom{collect(ps,v)}  returns the set consisting of the
  ++ polynomials of \axiom{ps} with \axiom{v} as main variable.

collectUpper : ($,VarSet) -> $
  ++ \axiom{collectUpper(ps,v)} returns the set consisting of the
  ++ polynomials of \axiom{ps} with main variable greater
  ++ than \axiom{v}.

sort : ($,VarSet) -> Record(under:$,floor:$,upper:$)
  ++ \axiom{sort(v,ps)} returns \axiom{us,vs,ws} such that \axiom{us}
  ++ is \axiom{collectUnder(ps,v)}, \axiom{vs} is \axiom{collect(ps,v)}
  ++ and \axiom{ws} is \axiom{collectUpper(ps,v)}.

trivialIdeal? : $ -> Boolean
  ++ \axiom{trivialIdeal?(ps)} returns true iff \axiom{ps} does
  ++ not contain non-zero elements.

if R has IntegralDomain then

  roughBase? : $ -> Boolean
    ++ \axiom{roughBase?(ps)} returns true iff for every pair
    ++ \axiom{{p,q}} of polynomials in \axiom{ps} their leading
    ++ monomials are relatively prime.

  roughSubIdeal?  : ($,$) -> Boolean
    ++  \axiom{roughSubIdeal?(ps1,ps2)} returns true iff it can proved
    ++ that all polynomials in  \axiom{ps1} lie in the ideal generated
    ++ by \axiom{ps2} in \axiom{\axiom{(R)^(-1) P}} without computing
    ++ Groebner bases.

  roughEqualIdeals? : ($,$) -> Boolean
    ++ \axiom{roughEqualIdeals?(ps1,ps2)} returns true iff it can
    ++ proved that \axiom{ps1} and \axiom{ps2} generate the same ideal
    ++ in \axiom{(R)^(-1) P} without computing Groebner bases.

  roughUnitIdeal? : $ -> Boolean
    ++ \axiom{roughUnitIdeal?(ps)} returns true iff \axiom{ps} contains
    ++  some non null element lying in the base ring \axiom{R}.

  headRemainder : (P,$) -> Record(num:P,den:R)
    ++ \axiom{headRemainder(a,ps)} returns \axiom{[b,r]} such that the
    ++ leading monomial of \axiom{b} is reduced in the sense of
    ++ Groebner bases w.r.t. \axiom{ps} and \axiom{r*a - b} lies in
```

```
      ++ the ideal generated by \axiom{ps}.

    remainder : (P,$) -> Record(rnum:R,polnum:P,den:R)
      ++ \axiom{remainder(a,ps)} returns \axiom{[c,b,r]} such that
      ++ \axiom{b} is fully reduced in the sense of Groebner bases
      ++ w.r.t. \axiom{ps}, \axiom{r*a - c*b} lies in the ideal
      ++ generated by \axiom{ps}. Furthermore, if \axiom{R} is a
      ++ gcd-domain, \axiom{b} is primitive.

    rewriteIdealWithHeadRemainder : (List(P),$) -> List(P)
      ++ \axiom{rewriteIdealWithHeadRemainder(lp,cs)} returns \axiom{lr}
      ++ such that the leading monomial of every polynomial in \axiom{lr}
      ++ is reduced in the sense of Groebner bases w.r.t. \axiom{cs}
      ++ and \axiom{(lp,cs)} and \axiom{(lr,cs)} generate the same
      ++ ideal in \axiom{(R)^(-1) P}.

    rewriteIdealWithRemainder : (List(P),$) -> List(P)
      ++ \axiom{rewriteIdealWithRemainder(lp,cs)} returns \axiom{lr}
      ++ such that every polynomial in \axiom{lr} is fully reduced in
      ++ the sense of Groebner bases w.r.t. \axiom{cs} and
      ++ \axiom{(lp,cs)} and \axiom{(lr,cs)} generate the same ideal
      ++ in \axiom{(R)^(-1) P}.

    triangular? : $ -> Boolean
      ++ \axiom{triangular?(ps)} returns true iff \axiom{ps} is a
      ++ triangular set, that is, two distinct polynomials have distinct
      ++ main variables and no constant lies in \axiom{ps}.

 add

   NNI ==> NonNegativeInteger
   B ==> Boolean

   elements: $ -> List(P)

   elements(ps:$):List(P) ==
     lp : List(P) := members(ps)$$

   variables1(lp:List(P)):(List VarSet) ==
     lvars : List(List(VarSet)) := [variables(p)$P for p in lp]
     sort((z1:VarSet,z2:VarSet):Boolean +-> z1 > z2,
           removeDuplicates(concat(lvars)$List(VarSet)))

   variables2(lp:List(P)):(List VarSet) ==
     lvars : List(VarSet) := [mvar(p)$P for p in lp]
     sort((z1:VarSet,z2:VarSet):Boolean +-> z1 > z2,
           removeDuplicates(lvars)$List(VarSet))

   variables (ps:$) ==
     variables1(elements(ps))

   mainVariables (ps:$) ==
     variables2(remove(ground?,elements(ps)))
```

```
mainVariable? (v,ps) ==
  lp : List(P) := remove(ground?,elements(ps))
  while (not empty? lp) and (not (mvar(first(lp)) = v)) repeat
    lp := rest lp
  (not empty? lp)

collectUnder (ps,v) ==
  lp : List P := elements(ps)
  lq : List P := []
  while (not empty? lp) repeat
    p := first lp
    lp := rest lp
    if (ground?(p)) or (mvar(p) < v)
      then
        lq := cons(p,lq)
  construct(lq)$$

collectUpper (ps,v) ==
  lp : List P := elements(ps)
  lq : List P := []
  while (not empty? lp) repeat
    p := first lp
    lp := rest lp
    if (not ground?(p)) and (mvar(p) > v)
      then
        lq := cons(p,lq)
  construct(lq)$$

collect (ps,v) ==
  lp : List P := elements(ps)
  lq : List P := []
  while (not empty? lp) repeat
    p := first lp
    lp := rest lp
    if (not ground?(p)) and (mvar(p) = v)
      then
        lq := cons(p,lq)
  construct(lq)$$

sort (ps,v) ==
  lp : List P := elements(ps)
  us : List P := []
  vs : List P := []
  ws : List P := []
  while (not empty? lp) repeat
    p := first lp
    lp := rest lp
    if (ground?(p)) or (mvar(p) < v)
      then
        us := cons(p,us)
      else
        if (mvar(p) = v)
          then
            vs := cons(p,vs)
```

```
          else
            ws := cons(p,ws)
  [construct(us)$$,_
   construct(vs)$$,_
   construct(ws)$$]$Record(under:$,floor:$,upper:$)

ps1 = ps2 ==
  {p for p in elements(ps1)} =$(Set P) {p for p in elements(ps2)}

exactQuo : (R,R) -> R

localInf? (p:P,q:P):B ==
  degree(p) <$E degree(q)

localTriangular? (lp:List(P)):B ==
  lp := remove(zero?, lp)
  empty? lp => true
  any? (ground?, lp) => false
  lp := sort((z1:P,z2:P):Boolean +-> mvar(z1)$P > mvar(z2)$P, lp)
  p,q : P
  p := first lp
  lp := rest lp
  while (not empty? lp) and (mvar(p) > mvar((q := first(lp)))) repeat
    p := q
    lp := rest lp
  empty? lp

triangular? ps ==
  localTriangular? elements ps

trivialIdeal? ps ==
  empty?(remove(zero?,elements(ps))$(List(P)))$(List(P))

if R has IntegralDomain
then

  roughUnitIdeal? ps ==
    any?(ground?,remove(zero?,elements(ps))$(List(P)))$(List P)

  relativelyPrimeLeadingMonomials? (p:P,q:P):B ==
    dp : E := degree(p)
    dq : E := degree(q)
    (sup(dp,dq)$E =$E dp +$E dq)@B

  roughBase? ps ==
    lp := remove(zero?,elements(ps))$(List(P))
    empty? lp => true
    rB? : B := true
    while (not empty? lp) and rB? repeat
      p := first lp
      lp := rest lp
      copylp := lp
      while (not empty? copylp) and rB? repeat
        rB? := relativelyPrimeLeadingMonomials?(p,first(copylp))
```

```
          copylp := rest copylp
      rB?

    roughSubIdeal?(ps1,ps2) ==
      lp: List(P) := rewriteIdealWithRemainder(elements(ps1),ps2)
      empty? (remove(zero?,lp))

    roughEqualIdeals? (ps1,ps2) ==
      ps1 =$$ ps2 => true
      roughSubIdeal?(ps1,ps2) and roughSubIdeal?(ps2,ps1)

  if (R has GcdDomain) and (VarSet has ConvertibleTo (Symbol))
  then

    LPR ==> List Polynomial R
    LS ==> List Symbol

    if R has EuclideanDomain
      then
        exactQuo(r:R,s:R):R ==
          r quo$R s
      else
        exactQuo(r:R,s:R):R ==
          (r exquo$R s)::R

    headRemainder (a,ps) ==
      lp1 : List(P) := remove(zero?, elements(ps))$(List(P))
      empty? lp1 => [a,1$R]
      any?(ground?,lp1) => [reductum(a),1$R]
      r : R := 1$R
      lp1 := sort(localInf?, reverse elements(ps))
      lp2 := lp1
      e : Union(E, "failed")
      while (not zero? a) and (not empty? lp2) repeat
        p := first lp2
        if ((e:= subtractIfCan(degree(a),degree(p))) case E)
          then
            g := gcd((lca := leadingCoefficient(a)),_
                     (lcp := leadingCoefficient(p)))$R
            (lca,lcp) := (exactQuo(lca,g),exactQuo(lcp,g))
            a := lcp * reductum(a) - monomial(lca, e::E)$P * reductum(p)
            r := r * lcp
            lp2 := lp1
          else
            lp2 := rest lp2
      [a,r]

    makeIrreducible! (frac:Record(num:P,den:R)):Record(num:P,den:R) ==
      g := gcd(frac.den,frac.num)$P
      (g = 1) => frac
      frac.num := exactQuotient!(frac.num,g)
      frac.den := exactQuo(frac.den,g)
      frac
```

```
remainder (a,ps) ==
  hRa := makeIrreducible! headRemainder (a,ps)
  a := hRa.num
  r : R := hRa.den
  zero? a => [1$R,a,r]
  b : P := monomial(1$R,degree(a))$P
  c : R := leadingCoefficient(a)
  while not zero?(a := reductum a) repeat
    hRa := makeIrreducible!  headRemainder (a,ps)
    a := hRa.num
    r := r * hRa.den
    g := gcd(c,(lca := leadingCoefficient(a)))$R
    b := ((hRa.den) * exactQuo(c,g)) * b + _
          monomial(exactQuo(lca,g),degree(a))$P
    c := g
  [c,b,r]

rewriteIdealWithHeadRemainder(ps,cs) ==
  trivialIdeal? cs => ps
  roughUnitIdeal? cs => [0$P]
  ps := remove(zero?,ps)
  empty? ps => ps
  any?(ground?,ps) => [1$P]
  rs : List P := []
  while not empty? ps repeat
    p := first ps
    ps := rest ps
    p := (headRemainder(p,cs)).num
    if not zero? p
      then
        if ground? p
          then
            ps := []
            rs := [1$P]
          else
            primitivePart! p
            rs := cons(p,rs)
  removeDuplicates rs

rewriteIdealWithRemainder(ps,cs) ==
  trivialIdeal? cs => ps
  roughUnitIdeal? cs => [0$P]
  ps := remove(zero?,ps)
  empty? ps => ps
  any?(ground?,ps) => [1$P]
  rs : List P := []
  while not empty? ps repeat
    p := first ps
    ps := rest ps
    p := (remainder(p,cs)).polnum
    if not zero? p
      then
        if ground? p
          then
```

```
                ps := []
                rs := [1$P]
              else
                rs := cons(unitCanonical(p),rs)
         removeDuplicates rs
```

———–

— COQ PSETCAT —

```
(* category PSETCAT *)
(*

     NNI ==> NonNegativeInteger
     B ==> Boolean

     elements: $ -> List(P)
     elements(ps:$):List(P) ==
       lp : List(P) := members(ps)$$

     variables1: (List(P)) -> (List VarSet)
     variables1(lp:List(P)):(List VarSet) ==
       lvars : List(List(VarSet)) := [variables(p)$P for p in lp]
       sort((z1:VarSet,z2:VarSet):Boolean +-> z1 > z2,
             removeDuplicates(concat(lvars)$List(VarSet)))

     variables2: (List(P)) -> (List VarSet)
     variables2(lp:List(P)):(List VarSet) ==
       lvars : List(VarSet) := [mvar(p)$P for p in lp]
       sort((z1:VarSet,z2:VarSet):Boolean +-> z1 > z2,
             removeDuplicates(lvars)$List(VarSet))

     variables : $ -> List VarSet
     variables (ps:$) ==
       variables1(elements(ps))

     mainVariables : $  -> List VarSet
     mainVariables (ps:$) ==
       variables2(remove(ground?,elements(ps)))

     mainVariable? : (VarSet,$) -> Boolean
     mainVariable? (v,ps) ==
       lp : List(P) := remove(ground?,elements(ps))
       while (not empty? lp) and (not (mvar(first(lp)) = v)) repeat
         lp := rest lp
       (not empty? lp)

     collectUnder : ($,VarSet) -> $
     collectUnder (ps,v) ==
       lp : List P := elements(ps)
       lq : List P := []
       while (not empty? lp) repeat
         p := first lp
```

```
    lp := rest lp
    if (ground?(p)) or (mvar(p) < v)
      then
        lq := cons(p,lq)
  construct(lq)$$

collectUpper : ($,VarSet) -> $
collectUpper (ps,v) ==
  lp : List P := elements(ps)
  lq : List P := []
  while (not empty? lp) repeat
    p := first lp
    lp := rest lp
    if (not ground?(p)) and (mvar(p) > v)
      then
        lq := cons(p,lq)
  construct(lq)$$

collect : ($,VarSet) -> $
collect (ps,v) ==
  lp : List P := elements(ps)
  lq : List P := []
  while (not empty? lp) repeat
    p := first lp
    lp := rest lp
    if (not ground?(p)) and (mvar(p) = v)
      then
        lq := cons(p,lq)
  construct(lq)$$

sort : ($,VarSet) -> Record(under:$,floor:$,upper:$)
sort (ps,v) ==
  lp : List P := elements(ps)
  us : List P := []
  vs : List P := []
  ws : List P := []
  while (not empty? lp) repeat
    p := first lp
    lp := rest lp
    if (ground?(p)) or (mvar(p) < v)
      then
        us := cons(p,us)
      else
        if (mvar(p) = v)
          then
            vs := cons(p,vs)
          else
            ws := cons(p,ws)
  [construct(us)$$,_
   construct(vs)$$,_
   construct(ws)$$]$Record(under:$,floor:$,upper:$)

?=? : (%,%) -> Boolean
ps1 = ps2 ==
```

```
      {p for p in elements(ps1)} =$(Set P) {p for p in elements(ps2)}

    localInf? (p:P,q:P):B ==
      degree(p) <$E degree(q)

    localTriangular? (lp:List(P)):B ==
      lp := remove(zero?, lp)
      empty? lp => true
      any? (ground?, lp) => false
      lp := sort((z1:P,z2:P):Boolean +-> mvar(z1)$P > mvar(z2)$P, lp)
      p,q : P
      p := first lp
      lp := rest lp
      while (not empty? lp) and (mvar(p) > mvar((q := first(lp)))) repeat
        p := q
        lp := rest lp
      empty? lp

    triangular? : $ -> Boolean
    triangular? ps ==
      localTriangular? elements ps

    trivialIdeal?: $ -> Boolean
    trivialIdeal? ps ==
      empty?(remove(zero?,elements(ps))$(List(P)))$(List(P))

    if R has IntegralDomain
    then

      roughUnitIdeal? : $ -> Boolean
      roughUnitIdeal? ps ==
        any?(ground?,remove(zero?,elements(ps))$(List(P)))$(List P)

      relativelyPrimeLeadingMonomials? (p:P,q:P):B ==
        dp : E := degree(p)
        dq : E := degree(q)
        (sup(dp,dq)$E =$E dp +$E dq)@B

      roughBase? : $ -> Boolean
      roughBase? ps ==
        lp := remove(zero?,elements(ps))$(List(P))
        empty? lp => true
        rB? : B := true
        while (not empty? lp) and rB? repeat
          p := first lp
          lp := rest lp
          copylp := lp
          while (not empty? copylp) and rB? repeat
            rB? := relativelyPrimeLeadingMonomials?(p,first(copylp))
            copylp := rest copylp
        rB?

      roughSubIdeal?  : ($,$) -> Boolean
      roughSubIdeal?(ps1,ps2) ==
```

```
    lp: List(P) := rewriteIdealWithRemainder(elements(ps1),ps2)
    empty? (remove(zero?,lp))

  roughEqualIdeals? : ($,$) -> Boolean
  roughEqualIdeals? (ps1,ps2) ==
    ps1 =$$ ps2 => true
    roughSubIdeal?(ps1,ps2) and roughSubIdeal?(ps2,ps1)

if (R has GcdDomain) and (VarSet has ConvertibleTo (Symbol))
then

  LPR ==> List Polynomial R
  LS ==> List Symbol

  if R has EuclideanDomain
    then

      exactQuo : (R,R) -> R
      exactQuo(r:R,s:R):R ==
        r quo$R s

    else

      exactQuo : (R,R) -> R
      exactQuo(r:R,s:R):R ==
        (r exquo$R s)::R

  headRemainder : (P,$) -> Record(num:P,den:R)
  headRemainder (a,ps) ==
    lp1 : List(P) := remove(zero?, elements(ps))$(List(P))
    empty? lp1 => [a,1$R]
    any?(ground?,lp1) => [reductum(a),1$R]
    r : R := 1$R
    lp1 := sort(localInf?, reverse elements(ps))
    lp2 := lp1
    e : Union(E, "failed")
    while (not zero? a) and (not empty? lp2) repeat
      p := first lp2
      if ((e:= subtractIfCan(degree(a),degree(p))) case E)
        then
          g := gcd((lca := leadingCoefficient(a)),_
                   (lcp := leadingCoefficient(p)))$R
          (lca,lcp) := (exactQuo(lca,g),exactQuo(lcp,g))
          a := lcp * reductum(a) - monomial(lca, e::E)$P * reductum(p)
          r := r * lcp
          lp2 := lp1
        else
          lp2 := rest lp2
    [a,r]

  makeIrreducible! (frac:Record(num:P,den:R)):Record(num:P,den:R) ==
    g := gcd(frac.den,frac.num)$P
    (g = 1) => frac
    frac.num := exactQuotient!(frac.num,g)
```

```
    frac.den := exactQuo(frac.den,g)
    frac

  remainder : (P,$) -> Record(rnum:R,polnum:P,den:R)
  remainder (a,ps) ==
    hRa := makeIrreducible! headRemainder (a,ps)
    a := hRa.num
    r : R := hRa.den
    zero? a => [1$R,a,r]
    b : P := monomial(1$R,degree(a))$P
    c : R := leadingCoefficient(a)
    while not zero?(a := reductum a) repeat
      hRa := makeIrreducible!  headRemainder (a,ps)
      a := hRa.num
      r := r * hRa.den
      g := gcd(c,(lca := leadingCoefficient(a)))$R
      b := ((hRa.den) * exactQuo(c,g)) * b + _
            monomial(exactQuo(lca,g),degree(a))$P
      c := g
    [c,b,r]

  rewriteIdealWithHeadRemainder : (List(P),%) -> List(P) if R has INTDOM
  rewriteIdealWithHeadRemainder(ps,cs) ==
    trivialIdeal? cs => ps
    roughUnitIdeal? cs => [0$P]
    ps := remove(zero?,ps)
    empty? ps => ps
    any?(ground?,ps) => [1$P]
    rs : List P := []
    while not empty? ps repeat
      p := first ps
      ps := rest ps
      p := (headRemainder(p,cs)).num
      if not zero? p
        then
          if ground? p
            then
              ps := []
              rs := [1$P]
            else
              primitivePart! p
              rs := cons(p,rs)
    removeDuplicates rs

  rewriteIdealWithRemainder : (List(P),%) -> List(P) if R has INTDOM
  rewriteIdealWithRemainder(ps,cs) ==
    trivialIdeal? cs => ps
    roughUnitIdeal? cs => [0$P]
    ps := remove(zero?,ps)
    empty? ps => ps
    any?(ground?,ps) => [1$P]
    rs : List P := []
    while not empty? ps repeat
      p := first ps
```

```
          ps := rest ps
          p := (remainder(p,cs)).polnum
          if not zero? p
            then
              if ground? p
                then
                  ps := []
                  rs := [1$P]
                else
                  rs := cons(unitCanonical(p),rs)
        removeDuplicates rs

*)
```

———-

— **PSETCAT.dotabb** —

```
"PSETCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PSETCAT"];
"PSETCAT" -> "KOERCE"
"PSETCAT" -> "CLAGG"
"PSETCAT" -> "SETCAT"
```

———-

— **PSETCAT.dotfull** —

```
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PSETCAT"];
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 -> "SetCategory()"
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 -> "Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet())"
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 -> "CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet()))"

"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PSETCAT"];
"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b
  -> "PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
```

———-

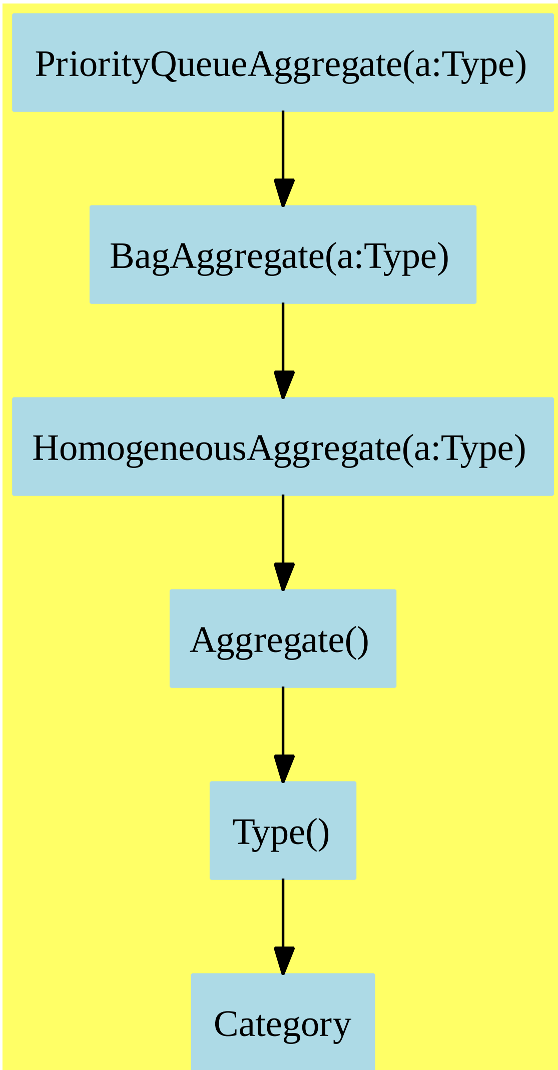
— **PSETCAT.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];
```

```
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 [color=lightblue];
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 -> "SetCategory()"
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 -> "Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet())"
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 -> "CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet()))"

"CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet()))"
 [color=seagreen];
"CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet()))"
  -> "CoercibleTo(a:Type)"

"Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet())" [color=seagreen];
"Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet())"
  -> "Collection(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"
"Collection(a:Type)" -> "ConvertibleTo(InputForm)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"ConvertibleTo(InputForm)" [color="#00EE00"];
"ConvertibleTo(InputForm)" -> "ConvertibleTo(a:Type)"

"ConvertibleTo(a:Type)" [color="#00EE00"];
"ConvertibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

———-

6.0.116 PriorityQueueAggregate (PRQAGG)



— **PriorityQueueAggregate.input** —

```
)set break resume
)sys rm -f PriorityQueueAggregate.output
)spool PriorityQueueAggregate.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show PriorityQueueAggregate
--R
--R PriorityQueueAggregate(S: OrderedSet) is a category constructor
--R Abbreviation for PriorityQueueAggregate is PRQAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PRQAGG
--R
--R------------------------------- Operations --------------------------------
--R bag : List(S) -> %                    copy : % -> %
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                extract! : % -> S
--R insert! : (S,%) -> %                  inspect : % -> S
--R latex : % -> String if S has SETCAT   map : ((S -> S),%) -> %
--R max : % -> S                          merge : (%,%) -> %
--R merge! : (%,%) -> %                   sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(S) if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

— **PriorityQueueAggregate.help** —

```
====================================================================
PriorityQueueAggregate examples
====================================================================

A priority queue is a bag of items from an ordered set where the item
extracted is always the maximum element.

See Also:
```

```
o )show PriorityQueueAggregate
```

———

See:

⇒ "OrderedMultisetAggregate" (OMSAGG) 9.0.152 on page 939
⇐ "BagAggregate" (BGAGG) 5.0.92 on page 393

Exports:

any?	bag	copy	coerce	count
empty	empty?	eq?	eval	every?
extract!	hash	insert!	inspect	latex
less?	map	map!	max	member?
members	merge	merge!	more?	parts
sample	size?	#?	?=?	?~=?

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
max : % -> S
merge : (%,%) -> %
merge! : (%,%) -> %
```

These exports come from (p393) BagAggregate(S:OrderedSet):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
bag : List S -> %
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
extract! : % -> S
hash : % -> SingleInteger if S has SETCAT
```

```
 insert! : (S,%) -> %
 inspect : % -> S
 latex : % -> String if S has SETCAT
 less? : (%,NonNegativeInteger) -> Boolean
 map : ((S -> S),%) -> %
 map! : ((S -> S),%) -> % if $ has shallowlyMutable
 member? : (S,%) -> Boolean
          if S has SETCAT and $ has finiteAggregate
 members : % -> List S if $ has finiteAggregate
 more? : (%,NonNegativeInteger) -> Boolean
 parts : % -> List S if $ has finiteAggregate
 sample : () -> %
 size? : (%,NonNegativeInteger) -> Boolean
 #? : % -> NonNegativeInteger if $ has finiteAggregate
 ?=? : (%,%) -> Boolean if S has SETCAT
 ?~=? : (%,%) -> Boolean if S has SETCAT
```

— PriorityQueueAggregate.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#PRQAGG">
PriorityQueueAggregate (PRQAGG)</a></h2>
</body>
```

— category PRQAGG PriorityQueueAggregate —

```
)abbrev category PRQAGG PriorityQueueAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ A priority queue is a bag of items from an ordered set where the item
++ extracted is always the maximum element.

PriorityQueueAggregate(S) : Category == SIG where
  S : OrdereSet

  SIG ==> BagAggregate S with

    finiteAggregate

    max : % -> S
      ++ max(q) returns the maximum element of priority queue q.

    merge : (%,%) -> %
      ++ merge(q1,q2) returns combines priority queues q1 and q2 to return
      ++ a single priority queue q.

    merge_! : (%,%) -> %
      ++ merge!(q,q1) destructively changes priority queue q to include the
      ++ values from priority queue q1.
```

— **PRQAGG.dotabb** —

```
"PRQAGG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PRQAGG"];
"PRQAGG" -> "BGAGG"
```

— **PRQAGG.dotfull** —

```
"PriorityQueueAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PRQAGG"];
"PriorityQueueAggregate(a:Type)" -> "BagAggregate(a:Type)"

"PriorityQueueAggregate(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PRQAGG"];
"PriorityQueueAggregate(a:SetCategory)" -> "PriorityQueueAggregate(a:Type)"

"PriorityQueueAggregate(a:OrderedSet)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PRQAGG"];
"PriorityQueueAggregate(a:OrderedSet)" ->
   "PriorityQueueAggregate(a:SetCategory)"
```

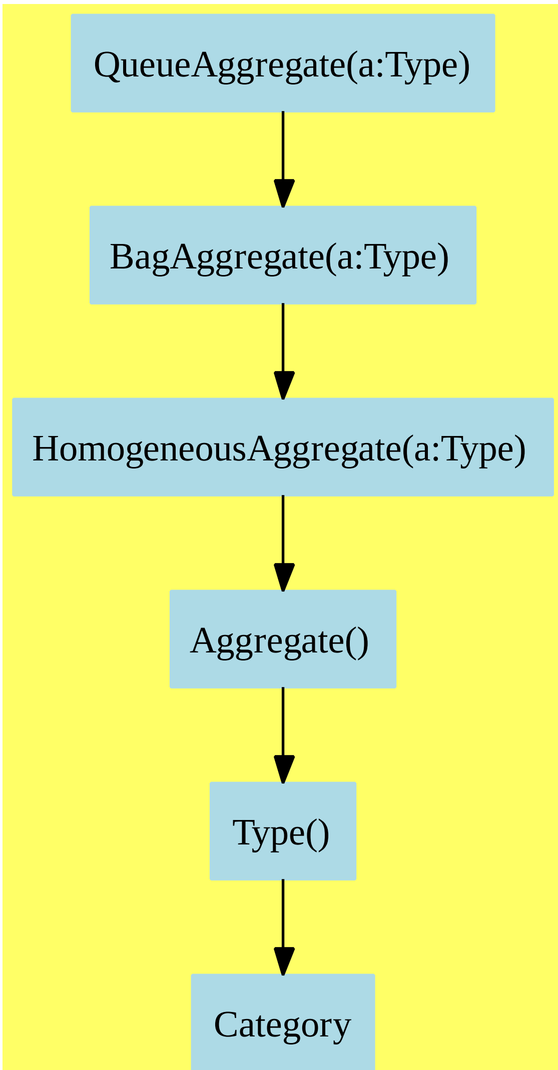
— **PRQAGG.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"PriorityQueueAggregate(a:Type)" [color=lightblue];
"PriorityQueueAggregate(a:Type)" -> "BagAggregate(a:Type)"

"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
```

```
}
```

———-

6.0.117 QueueAggregate (QUAGG)



— **QueueAggregate.input** —

```
)set break resume
)sys rm -f QueueAggregate.output
)spool QueueAggregate.output
)set message test on
)set message auto off
```

```
)clear all

--S 1 of 1
)show QueueAggregate
--R
--R QueueAggregate(S: Type) is a category constructor
--R Abbreviation for QueueAggregate is QUAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for QUAGG
--R
--R------------------------------- Operations --------------------------------
--R back : % -> S                        bag : List(S) -> %
--R copy : % -> %                        dequeue! : % -> S
--R empty : () -> %                      empty? : % -> Boolean
--R enqueue! : (S,%) -> S                eq? : (%,%) -> Boolean
--R extract! : % -> S                    front : % -> S
--R insert! : (S,%) -> %                 inspect : % -> S
--R latex : % -> String if S has SETCAT   length : % -> NonNegativeInteger
--R map : ((S -> S),%) -> %              rotate! : % -> %
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(S) if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

— **QueueAggregate.help** —

```
====================================================================
QueueAggregate examples
====================================================================
```

```
A queue is a bag where the first item inserted is the first item extracted.

See Also:
o )show QueueAggregate
```

See:

⇒ "DequeueAggregate" (DQAGG) 7.0.124 on page 694
⇐ "BagAggregate" (BGAGG) 5.0.92 on page 393

Exports:

any?	bag	back	coerce	copy
count	dequeue!	empty	empty?	enqueue!
eq?	eval	every?	extract!	front
hash	insert!	inspect	latex	length
less?	map	map!	member?	members
more?	parts	rotate!	sample	size?
#?	?=?	?~=?		

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are implemented by this category:

```
back : % -> S
dequeue! : % -> S
enqueue! : (S,%) -> S
front : % -> S
length : % -> NonNegativeInteger
rotate! : % -> %
```

These exports come from (p393) BagAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
bag : List S -> %
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
```

```
 eval : (%,Equation S) -> %
          if S has EVALAB S and S has SETCAT
 eval : (%,List Equation S) -> %
          if S has EVALAB S and S has SETCAT
 every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
 extract! : % -> S
 hash : % -> SingleInteger if S has SETCAT
 insert! : (S,%) -> %
 inspect : % -> S
 latex : % -> String if S has SETCAT
 less? : (%,NonNegativeInteger) -> Boolean
 map : ((S -> S),%) -> %
 map! : ((S -> S),%) -> % if $ has shallowlyMutable
 member? : (S,%) -> Boolean
          if S has SETCAT and $ has finiteAggregate
 members : % -> List S if $ has finiteAggregate
 more? : (%,NonNegativeInteger) -> Boolean
 parts : % -> List S if $ has finiteAggregate
 sample : () -> %
 size? : (%,NonNegativeInteger) -> Boolean
 #? : % -> NonNegativeInteger if $ has finiteAggregate
 ?=? : (%,%) -> Boolean if S has SETCAT
 ?~=? : (%,%) -> Boolean if S has SETCAT
```

— QueueAggregate.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#QUAGG">
QueueAggregate (QUAGG)</a></h2>
</body>
```

— category QUAGG QueueAggregate —

```
)abbrev category QUAGG QueueAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ A queue is a bag where the first item inserted is the first
++ item extracted.

QueueAggregate(S) : Category == SIG where
  S : Type

  SIG ==> BagAggregate S with

    finiteAggregate

    enqueue_! : (S, %) -> S
      ++ enqueue!(x,q) inserts x into the queue q at the back end.
```

```
    dequeue_! : % -> S
      ++ dequeue! s destructively extracts the first (top) element
      ++ from queue q. The element previously second in the queue becomes
      ++ the first element. Error: if q is empty.

    rotate_! : % -> %
      ++ rotate! q rotates queue q so that the element at the front of
      ++ the queue goes to the back of the queue.
      ++ Note that rotate! q is equivalent to enqueue!(dequeue!(q)).

    length : % -> NonNegativeInteger
      ++ length(q) returns the number of elements in the queue.
      ++ Note that \axiom{length(q) = #q}.

    front : % -> S
      ++ front(q) returns the element at the front of the queue.
      ++ The queue q is unchanged by this operation.
      ++ Error: if q is empty.

    back : % -> S
      ++ back(q) returns the element at the back of the queue.
      ++ The queue q is unchanged by this operation.
      ++ Error: if q is empty.
```

— **QUAGG.dotabb** —

```
"QUAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=QUAGG"];
"QUAGG" -> "BGAGG"
```

— **QUAGG.dotfull** —

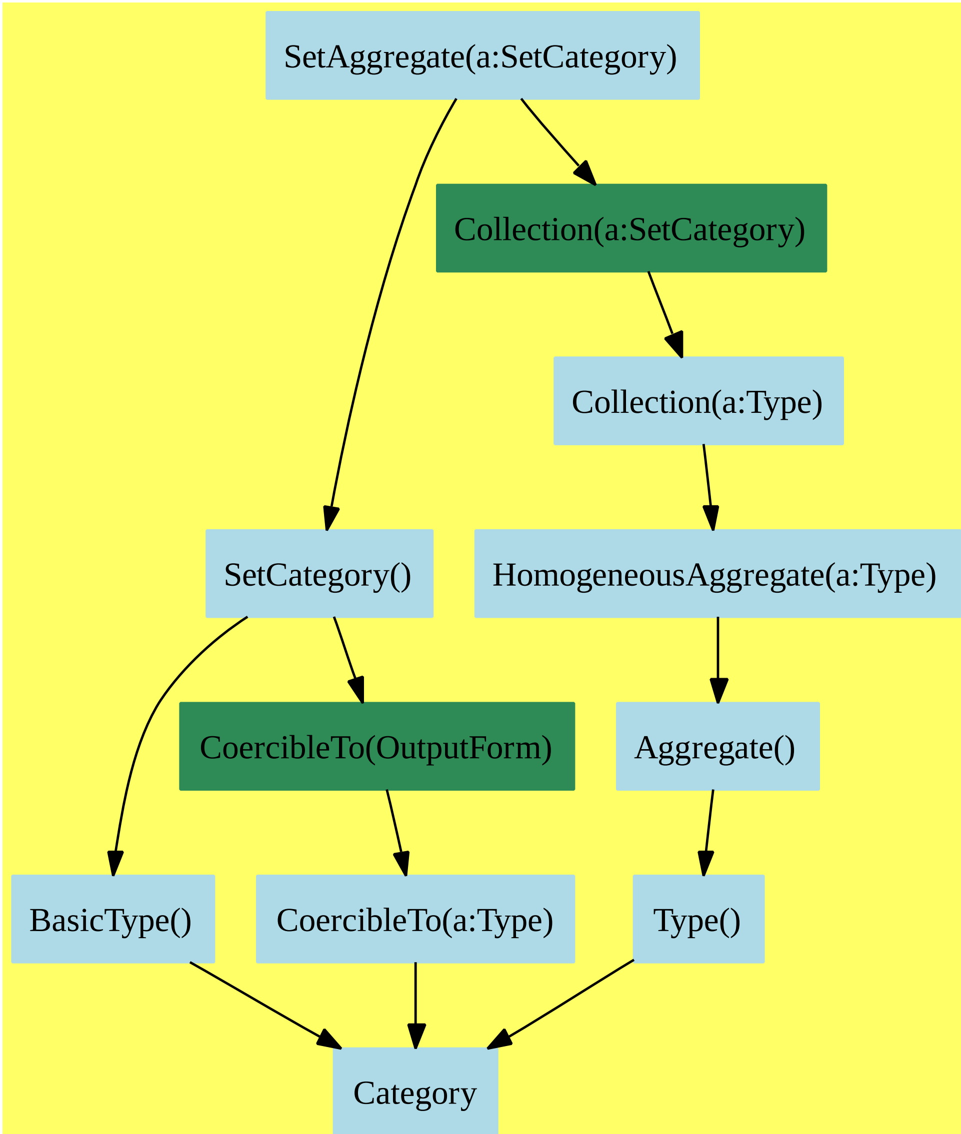
```
"QueueAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=QUAGG"];
"QueueAggregate(a:Type)" -> "BagAggregate(a:Type)"

"QueueAggregate(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=QUAGG"];
"QueueAggregate(a:SetCategory)" -> "QueueAggregate(a:Type)"
```

— **QUAGG.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];
```

```
"QueueAggregate(a:Type)" [color=lightblue];
"QueueAggregate(a:Type)" -> "BagAggregate(a:Type)"

"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

6.0.118 SetAggregate (SETAGG)



— **SetAggregate.input** —

```
)set break resume
)sys rm -f SetAggregate.output
)spool SetAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
```

```
)show SetAggregate
--R
--R SetAggregate(S: SetCategory) is a category constructor
--R Abbreviation for SetAggregate is SETAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SETAGG
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean                ?=? : (%,%) -> Boolean
--R brace : List(S) -> %                  brace : () -> %
--R coerce : % -> OutputForm              construct : List(S) -> %
--R copy : % -> %                         difference : (%,S) -> %
--R difference : (%,%) -> %               empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R hash : % -> SingleInteger             intersect : (%,%) -> %
--R latex : % -> String                   map : ((S -> S),%) -> %
--R sample : () -> %                      set : List(S) -> %
--R set : () -> %                         subset? : (%,%) -> Boolean
--R symmetricDifference : (%,%) -> %      union : (S,%) -> %
--R union : (%,S) -> %                    union : (%,%) -> %
--R ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R convert : % -> InputForm if S has KONVERT(INFORM)
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(S) if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

— **SetAggregate.help** —

```
====================================================================
SetAggregate examples
====================================================================

A set category lists a collection of set-theoretic operations useful
for both finite sets and multisets. Note however that finite sets are
distinct from multisets. Although the operations defined for set
categories are common to both, the relationship between the two cannot
be described by inclusion or inheritance.

See Also:
o )show SetAggregate
```

See:

⇒ "FiniteSetAggregate" (FSAGG) 8.0.134 on page 781
⇒ "MultisetAggregate" (MSETAGG) 8.0.139 on page 843
⇐ "Collection" (CLAGG) 5.0.94 on page 403
⇐ "SetCategory" (SETCAT) 3.0.57 on page 187

Exports:

any?	brace	coerce	construct	convert
copy	count	difference	empty	empty?
eq?	eval	every?	find	hash
intersect	latex	less?	map	map!
member?	members	more?	parts	reduce
remove	removeDuplicates	sample	select	set
size?	subset?	symmetricDifference	union	#?
?<?	?=?	?~=?		

Attributes Exported:

- **partiallyOrderedSet** is true if a set with $<$ which is transitive, but not($a < b$ or $a = b$) does not necessarily imply $b < a$.
- **nil**

These are directly exported but not implemented:

```
brace : List S -> %
brace : () -> %
difference : (%,%) -> %
intersect : (%,%) -> %
set : List S -> %
set : () -> %
subset? : (%,%) -> Boolean
union : (%,%) -> %
?<? : (%,%) -> Boolean
```

These are implemented by this category:

```
difference : (%,S) -> %
```

```
symmetricDifference : (%,%) -> %
union : (S,%) -> %
union : (%,S) -> %
```

These exports come from (p187) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

These exports come from (p403) Collection(S:SetCategory):

```
any? : ((S -> Boolean),%) -> Boolean
    if $ has finiteAggregate
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
copy : % -> %
count : ((S -> Boolean),%) -> NonNegativeInteger
    if $ has finiteAggregate
count : (S,%) -> NonNegativeInteger
    if S has SETCAT
    and $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List Equation S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,Equation S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,S,S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,List S,List S) -> %
    if S has EVALAB S
    and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
    if $ has finiteAggregate
find : ((S -> Boolean),%) -> Union(S,"failed")
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
members : % -> List S if $ has finiteAggregate
member? : (S,%) -> Boolean
    if S has SETCAT and $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
    if S has SETCAT
    and $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S
```

```
     if $ has finiteAggregate
 remove : ((S -> Boolean),%) -> %
     if $ has finiteAggregate
 remove : (S,%) -> %
     if S has SETCAT
     and $ has finiteAggregate
 removeDuplicates : % -> %
     if S has SETCAT
     and $ has finiteAggregate
 sample : () -> %
 select : ((S -> Boolean),%) -> %
     if $ has finiteAggregate
 size? : (%,NonNegativeInteger) -> Boolean
 #? : % -> NonNegativeInteger
     if $ has finiteAggregate
```

— SetAggregate.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#SETAGG">
SetAggregate (SETAGG)</a></h2>
</body>
```

— category SETAGG SetAggregate —

```
)abbrev category SETAGG SetAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: 14 Oct, 1993 by RSS
++ Description:
++ A set category lists a collection of set-theoretic operations
++ useful for both finite sets and multisets.
++ Note however that finite sets are distinct from multisets.
++ Although the operations defined for set categories are
++ common to both, the relationship between the two cannot
++ be described by inclusion or inheritance.

SetAggregate(S) : Category == SIG where
  S : SetCategory

  SIG ==> Join(SetCategory, Collection(S)) with

    partiallyOrderedSet

    "<" : (%, %) -> Boolean
      ++ s < t returns true if all elements of set aggregate s are also
      ++ elements of set aggregate t.

    brace : () -> %
      ++ brace()$D (otherwise written {}$D)
      ++ creates an empty set aggregate of type D.
```

```
  ++ This form is considered obsolete. Use \axiomFun{set} instead.

brace : List S -> %
  ++ brace([x,y,...,z])
  ++ creates a set aggregate containing items x,y,...,z.
  ++ This form is considered obsolete. Use \axiomFun{set} instead.

set : () -> %
  ++ set()$D creates an empty set aggregate of type D.

set : List S -> %
  ++ set([x,y,...,z]) creates a set aggregate containing items x,y,...,z.

intersect : (%, %) -> %
  ++ intersect(u,v) returns the set aggregate w consisting of
  ++ elements common to both set aggregates u and v.
  ++ Note that equivalent to the notation (not currently supported)
  ++ {x for x in u | member?(x,v)}.

difference  : (%, %) -> %
  ++ difference(u,v) returns the set aggregate w consisting of
  ++ elements in set aggregate u but not in set aggregate v.
  ++ If u and v have no elements in common, \axiom{difference(u,v)}
  ++ returns a copy of u.
  ++ Note that equivalent to the notation (not currently supported)
  ++ \axiom{{x for x in u | not member?(x,v)}}.

difference  : (%, S) -> %
  ++ difference(u,x) returns the set aggregate u with element x removed.
  ++ If u does not contain x, a copy of u is returned.
  ++ Note that \axiom{difference(s, x) = difference(s, {x})}.

symmetricDifference : (%, %) -> %
  ++ symmetricDifference(u,v) returns the set aggregate of elements x
  ++ which are members of set aggregate u or set aggregate v but
  ++ not both. If u and v have no elements in common,
  ++ \axiom{symmetricDifference(u,v)} returns a copy of u.
  ++ Note that \axiom{symmetricDifference(u,v) =
  ++  union(difference(u,v),difference(v,u))}

subset? : (%, %) -> Boolean
  ++ subset?(u,v) tests if u is a subset of v.
  ++ Note that equivalent to
  ++ \axiom{reduce(and,{member?(x,v) for x in u},true,false)}.

union : (%, %) -> %
  ++ union(u,v) returns the set aggregate of elements which are members
  ++ of either set aggregate u or v.

union : (%, S) -> %
  ++ union(u,x) returns the set aggregate u with the element x added.
  ++ If u already contains x, \axiom{union(u,x)} returns a copy of u.

union : (S, %) -> %
```

```
      ++ union(x,u) returns the set aggregate u with the element x added.
      ++ If u already contains x, \axiom{union(x,u)} returns a copy of u.

   add

     symmetricDifference(x, y) == union(difference(x, y), difference(y, x))

     union(s:%, x:S) == union(s, {x})

     union(x:S, s:%) == union(s, {x})

     difference(s:%, x:S) == difference(s, {x})
```

— **COQ SETAGG** —

```
(* category SETAGG *)
(*

  symmetricDifference : (%, %) -> %
  symmetricDifference(x, y)    == union(difference(x, y), difference(y, x))

  union : (%, S) -> %
  union(s:%, x:S) == union(s, {x})

  union : (S, %) -> %
  union(x:S, s:%) == union(s, {x})

  difference : (%, S) -> %
  difference(s:%, x:S) == difference(s, {x})

*)
```

— **SETAGG.dotabb** —

```
"SETAGG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SETAGG"];
"SETAGG" -> "SETCAT"
"SETAGG" -> "CLAGG"
```

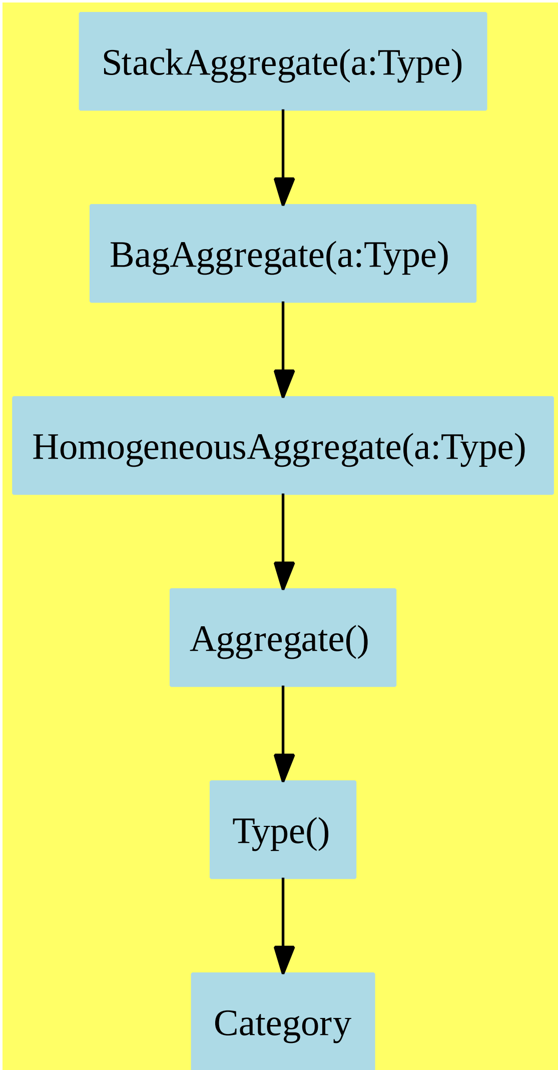
— **SETAGG.dotfull** —

```
"SetAggregate(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SETAGG"];
"SetAggregate(a:SetCategory)" -> "SetCategory()"
"SetAggregate(a:SetCategory)" -> "Collection(a:SetCategory)"
```

— **SETAGG.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"SetAggregate(a:SetCategory)" [color=lightblue];
"SetAggregate(a:SetCategory)" -> "SetCategory()"
"SetAggregate(a:SetCategory)" -> "Collection(a:SetCategory)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Collection(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=CLAGG"];
"Collection(a:SetCategory)" -> "Collection(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];

}
```

6.0.119 StackAggregate (SKAGG)



— **StackAggregate.input** —

```
)set break resume
)sys rm -f StackAggregate.output
)spool StackAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show StackAggregate
--R
--R StackAggregate(S: Type) is a category constructor
```

```
--R Abbreviation for StackAggregate is SKAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SKAGG
--R
--R------------------------------- Operations --------------------------------
--R bag : List(S) -> %                   copy : % -> %
--R depth : % -> NonNegativeInteger      empty : () -> %
--R empty? : % -> Boolean                eq? : (%,%) -> Boolean
--R extract! : % -> S                    insert! : (S,%) -> %
--R inspect : % -> S                     latex : % -> String if S has SETCAT
--R map : ((S -> S),%) -> %              pop! : % -> S
--R push! : (S,%) -> S                   sample : () -> %
--R top : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(S) if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

— **StackAggregate.help** —

```
====================================================================
StackAggregate examples
====================================================================

A stack is a bag where the last item inserted is the first item extracted.

See Also:
o )show StackAggregate
```

See:

⇒ "DequeueAggregate" (DQAGG) 7.0.124 on page 694
⇐ "BagAggregate" (BGAGG) 5.0.92 on page 393

Exports:

any?	bag	coerce	copy	count
depth	empty	empty?	eq?	eval
every?	extract!	hash	insert!	inspect
latex	less?	map	map!	member?
members	more?	parts	pop!	push!
sample	size?	top	#?	?=?
?~=?				

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
depth : % -> NonNegativeInteger
pop! : % -> S
push! : (S,%) -> S
top : % -> S
```

These exports come from (p393) BagAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
bag : List S -> %
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
extract! : % -> S
hash : % -> SingleInteger if S has SETCAT
insert! : (S,%) -> %
inspect : % -> S
```

```
 latex : % -> String if S has SETCAT
 less? : (%,NonNegativeInteger) -> Boolean
 map : ((S -> S),%) -> %
 map! : ((S -> S),%) -> % if $ has shallowlyMutable
 member? : (S,%) -> Boolean
          if S has SETCAT and $ has finiteAggregate
 members : % -> List S if $ has finiteAggregate
 more? : (%,NonNegativeInteger) -> Boolean
 parts : % -> List S if $ has finiteAggregate
 sample : () -> %
 size? : (%,NonNegativeInteger) -> Boolean
 #? : % -> NonNegativeInteger if $ has finiteAggregate
 ?=? : (%,%) -> Boolean if S has SETCAT
 ?~=? : (%,%) -> Boolean if S has SETCAT
```

— StackAggregate.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#SKAGG">
StackAggregate (SKAGG)</a></h2>
</body>
```

— category SKAGG StackAggregate —

```
)abbrev category SKAGG StackAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ A stack is a bag where the last item inserted is the first item extracted.

StackAggregate(S) : Category == SIG where
  S : Type

  SIG ==> BagAggregate S with

    finiteAggregate

    push_! : (S,%) -> S
      ++push!(x,s) pushes x onto stack s, that is, destructively changing s
      ++ so as to have a new first (top) element x.
      ++ Afterwards, pop!(s) produces x and pop!(s) produces the original s.
      ++
      ++X a:Stack INT:= stack [1,2,3,4,5]
      ++X push! a
      ++X a

    pop_! : % -> S
      ++pop!(s) returns the top element x, destructively removing x from s.
      ++ Note that Use \axiom{top(s)} to obtain x without removing it from s.
      ++ Error: if s is empty.
```

```
      ++
      ++X a:Stack INT:= stack [1,2,3,4,5]
      ++X pop! a
      ++X a

    top : % -> S
      ++top(s) returns the top element x from s; s remains unchanged.
      ++ Note that Use \axiom{pop!(s)} to obtain x and remove it from s.
      ++
      ++X a:Stack INT:= stack [1,2,3,4,5]
      ++X top a

    depth : % -> NonNegativeInteger
      ++depth(s) returns the number of elements of stack s.
      ++ Note that \axiom{depth(s) = #s}.
      ++
      ++X a:Stack INT:= stack [1,2,3,4,5]
      ++X depth a
```

———-

— SKAGG.dotabb —

```
"SKAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=SKAGG"];
"SKAGG" -> "BGAGG"
```

———-

— SKAGG.dotfull —

```
"StackAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SKAGG"];
"StackAggregate(a:Type)" -> "BagAggregate(a:Type)"

"StackAggregate(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=SKAGG"];
"StackAggregate(a:SetCategory)" -> "StackAggregate(a:Type)"
```

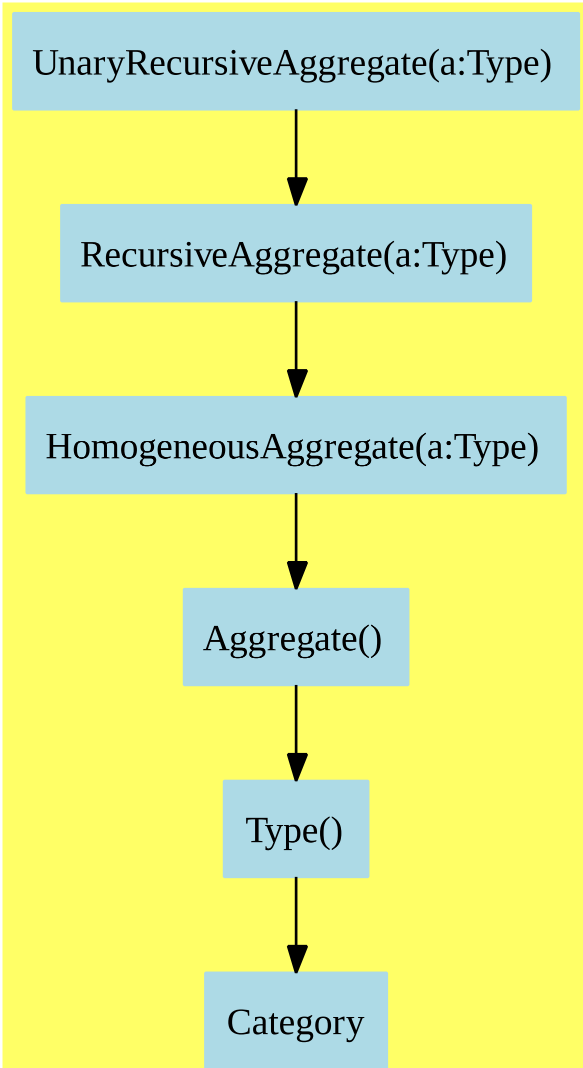
———-

— SKAGG.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"StackAggregate(a:Type)" [color=lightblue];
"StackAggregate(a:Type)" -> "BagAggregate(a:Type)"

"BagAggregate(a:Type)" [color=lightblue];
```

```
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

6.0.120 UnaryRecursiveAggregate (URAGG)



— **UnaryRecursiveAggregate.input** —

```
)set break resume
)sys rm -f UnaryRecursiveAggregate.output
)spool UnaryRecursiveAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 62
)show UnaryRecursiveAggregate
--R
--R UnaryRecursiveAggregate(S: Type) is a category constructor
```

```
--R Abbreviation for UnaryRecursiveAggregate is URAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for URAGG
--R
--R------------------------------- Operations --------------------------------
--R children : % -> List(%)              concat : (S,%) -> %
--R concat : (%,%) -> %                  copy : % -> %
--R cycleEntry : % -> %                  cycleTail : % -> %
--R cyclic? : % -> Boolean               distance : (%,%) -> Integer
--R ?.last : (%,last) -> S               ?.rest : (%,rest) -> %
--R ?.first : (%,first) -> S             ?.value : (%,value) -> S
--R empty : () -> %                      empty? : % -> Boolean
--R eq? : (%,%) -> Boolean               first : % -> S
--R last : (%,NonNegativeInteger) -> %   last : % -> S
--R latex : % -> String if S has SETCAT  leaf? : % -> Boolean
--R leaves : % -> List(S)                map : ((S -> S),%) -> %
--R nodes : % -> List(%)                 rest : (%,NonNegativeInteger) -> %
--R rest : % -> %                        sample : () -> %
--R second : % -> S                      tail : % -> %
--R third : % -> S                       value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R concat! : (%,S) -> % if $ has shallowlyMutable
--R concat! : (%,%) -> % if $ has shallowlyMutable
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R cycleLength : % -> NonNegativeInteger
--R cycleSplit! : % -> % if $ has shallowlyMutable
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R first : (%,NonNegativeInteger) -> %
--R hash : % -> SingleInteger if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List(S) if $ has finiteAggregate
--R setchildren! : (%,List(%)) -> % if $ has shallowlyMutable
--R setelt : (%,last,S) -> S if $ has shallowlyMutable
--R setelt : (%,rest,%) -> % if $ has shallowlyMutable
--R setelt : (%,first,S) -> S if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setfirst! : (%,S) -> S if $ has shallowlyMutable
--R setlast! : (%,S) -> S if $ has shallowlyMutable
--R setrest! : (%,%) -> % if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
```

```
--R size? : (%,NonNegativeInteger) -> Boolean
--R split! : (%,Integer) -> % if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

--S 2 of 62
first [1,4,2,-6,0,3,5,4,2,3]
--R
--R
--R   (1)  1
--R                                                       Type: PositiveInteger
--E 2

--S 3 of 62
t1:=[1,2,3]
--R
--R
--R   (2)  [1,2,3]
--R                                                 Type: List(PositiveInteger)
--E 3

--S 4 of 62
t1.first
--R
--R
--R   (3)  1
--R                                                       Type: PositiveInteger
--E 4

--S 5 of 62
first([1,4,2,-6,0,3,5,4,2,3],3)
--R
--R
--R   (4)  [1,4,2]
--R                                                         Type: List(Integer)
--E 5

--S 6 of 62
rest [1,4,2,-6,0,3,5,4,2,3]
--R
--R
--R   (5)  [4,2,- 6,0,3,5,4,2,3]
--R                                                         Type: List(Integer)
--E 6

--S 7 of 62
t1:=[1,2,3]
--R
--R
--R   (6)  [1,2,3]
--R                                                 Type: List(PositiveInteger)
--E 7
```

```
--S 8 of 62
t1.rest
--R
--R
--R   (7)  [2,3]
--R                                              Type: List(PositiveInteger)
--E 8

--S 9 of 62
rest([1,4,2,-6,0,3,5,4,2,3],3)
--R
--R
--R   (8)  [- 6,0,3,5,4,2,3]
--R                                                      Type: List(Integer)
--E 9

--S 10 of 62
last [1,4,2,-6,0,3,5,4,2,3]
--R
--R
--R   (9)  3
--R                                                    Type: PositiveInteger
--E 10

--S 11 of 62
t1:=[1,2,3]
--R
--R
--R   (10)  [1,2,3]
--R                                              Type: List(PositiveInteger)
--E 11

--S 12 of 62
t1.last
--R
--R
--R   (11)  3
--R                                                    Type: PositiveInteger
--E 12

--S 13 of 62
last([1,4,2,-6,0,3,5,4,2,3],3)
--R
--R
--R   (12)  [4,2,3]
--R                                                      Type: List(Integer)
--E 13

--S 14 of 62
tail [1,4,2,-6,0,3,5,4,2,3]
--R
--R
--R   (13)  [3]
--R                                                      Type: List(Integer)
```

```
--E 14

--S 15 of 62
second [1,4,2,-6,0,3,5,4,2,3]
--R
--R
--R   (14)  4
--R                                                        Type: PositiveInteger
--E 15

--S 16 of 62
third  [1,4,2,-6,0,3,5,4,2,3]
--R
--R
--R   (15)  2
--R                                                        Type: PositiveInteger
--E 16

--S 17 of 62
t1:=[1,2,3]
--R
--R
--R   (16)  [1,2,3]
--R                                                  Type: List(PositiveInteger)
--E 17

--S 18 of 62
setfirst!(t1,7)
--R
--R
--R   (17)  7
--R                                                        Type: PositiveInteger
--E 18

--S 19 of 62
t1
--R
--R
--R   (18)  [7,2,3]
--R                                                  Type: List(PositiveInteger)
--E 19

--S 20 of 62
t1:=[1,2,3]
--R
--R
--R   (19)  [1,2,3]
--R                                                  Type: List(PositiveInteger)
--E 20

--S 21 of 62
t1.first:=7
--R
--R
```

```
--R   (20)  7
--R                                                        Type: PositiveInteger
--E 21

--S 22 of 62
t1
--R
--R
--R   (21)  [7,2,3]
--R                                                  Type: List(PositiveInteger)
--E 22

--S 23 of 62
t1:=[1,2,3]
--R
--R
--R   (22)  [1,2,3]
--R                                                  Type: List(PositiveInteger)
--E 23

--S 24 of 62
setrest!(t1,[4,5,6])
--R
--R
--R   (23)  [4,5,6]
--R                                                  Type: List(PositiveInteger)
--E 24

--S 25 of 62
t1
--R
--R
--R   (24)  [1,4,5,6]
--R                                                  Type: List(PositiveInteger)
--E 25

--S 26 of 62
t1:=[1,2,3]
--R
--R
--R   (25)  [1,2,3]
--R                                                  Type: List(PositiveInteger)
--E 26

--S 27 of 62
t1.rest:=[4,5,6]
--R
--R
--R   (26)  [4,5,6]
--R                                                  Type: List(PositiveInteger)
--E 27

--S 28 of 62
t1
```

```
--R
--R
--R   (27)  [1,4,5,6]
--R                                                 Type: List(PositiveInteger)
--E 28

--S 29 of 62
t1:=[1,4,2,-6,0,3,5,4,2,3]
--R
--R
--R   (28)  [1,4,2,- 6,0,3,5,4,2,3]
--R                                                         Type: List(Integer)
--E 29

--S 30 of 62
setlast!(t1,7)
--R
--R
--R   (29)  7
--R                                                       Type: PositiveInteger
--E 30

--S 31 of 62
t1
--R
--R
--R   (30)  [1,4,2,- 6,0,3,5,4,2,7]
--R                                                         Type: List(Integer)
--E 31

--S 32 of 62
t1:=[1,2,3]
--R
--R
--R   (31)  [1,2,3]
--R                                                 Type: List(PositiveInteger)
--E 32

--S 33 of 62
t2:=concat(t1,t1)
--R
--R
--R   (32)  [1,2,3,1,2,3]
--R                                                 Type: List(PositiveInteger)
--E 33

--S 34 of 62
t1
--R
--R
--R   (33)  [1,2,3]
--R                                                 Type: List(PositiveInteger)
--E 34
```

```
--S 35 of 62
t2
--R
--R
--R   (34)  [1,2,3,1,2,3]
--R                                                 Type: List(PositiveInteger)
--E 35

--S 36 of 62
t2:=concat(4,t1)
--R
--R
--R   (35)  [4,1,2,3]
--R                                                 Type: List(PositiveInteger)
--E 36

--S 37 of 62
t1
--R
--R
--R   (36)  [1,2,3]
--R                                                 Type: List(PositiveInteger)
--E 37

--S 38 of 62
t2
--R
--R
--R   (37)  [4,1,2,3]
--R                                                 Type: List(PositiveInteger)
--E 38

--S 39 of 62
t1:=[1,2,3]
--R
--R
--R   (38)  [1,2,3]
--R                                                 Type: List(PositiveInteger)
--E 39

--S 40 of 62
t2:=[4,5,6]
--R
--R
--R   (39)  [4,5,6]
--R                                                 Type: List(PositiveInteger)
--E 40

--S 41 of 62
concat!(t1,t2)
--R
--R
--R   (40)  [1,2,3,4,5,6]
--R                                                 Type: List(PositiveInteger)
```

```
--E 41

--S 42 of 62
t1
--R
--R
--R   (41)  [1,2,3,4,5,6]
--R                                                  Type: List(PositiveInteger)
--E 42

--S 43 of 62
t2
--R
--R
--R   (42)  [4,5,6]
--R                                                  Type: List(PositiveInteger)
--E 43

--S 44 of 62
t1:=[1,2,3]
--R
--R
--R   (43)  [1,2,3]
--R                                                  Type: List(PositiveInteger)
--E 44

--S 45 of 62
concat!(t1,7)
--R
--R
--R   (44)  [1,2,3,7]
--R                                                  Type: List(PositiveInteger)
--E 45

--S 46 of 62
t1
--R
--R
--R   (45)  [1,2,3,7]
--R                                                  Type: List(PositiveInteger)
--E 46

--S 47 of 62
t1:=[1,4,2,-6,0,3,5,4,2,3]
--R
--R
--R   (46)  [1,4,2,- 6,0,3,5,4,2,3]
--R                                                          Type: List(Integer)
--E 47

--S 48 of 62
t2:=split!(t1,4)
--R
--R
```

```
--R   (47)  [0,3,5,4,2,3]
--R                                                         Type: List(Integer)
--E 48

--S 49 of 62
t1
--R
--R
--R   (48)  [1,4,2,- 6]
--R                                                         Type: List(Integer)
--E 49

--S 50 of 62
t2
--R
--R
--R   (49)  [0,3,5,4,2,3]
--R                                                         Type: List(Integer)
--E 50

--S 51 of 62
t1:=[1,2,3]
--R
--R
--R   (50)  [1,2,3]
--R                                                 Type: List(PositiveInteger)
--E 51

--S 52 of 62
t2:=concat!(t1,t1)
--R
--R
--R           _____
--R   (51)  [1,2,3]
--R                                                 Type: List(PositiveInteger)
--E 52

--S 53 of 62
cycleEntry t2
--R
--R
--R           _____
--R   (52)  [1,2,3]
--R                                                 Type: List(PositiveInteger)
--E 53

--S 54 of 62
cycleLength t2
--R
--R
--R   (53)  3
--R                                                       Type: PositiveInteger
--E 54
```

```
--S 55 of 62
cycleTail t2
--R
--R
--R          -----
--R   (54)  [3,1,2]
--R                                                  Type: List(PositiveInteger)
--E 55

--S 56 of 62
t1:=[1,2,3]
--R
--R
--R   (55)  [1,2,3]
--R                                                  Type: List(PositiveInteger)
--E 56

--S 57 of 62
t2:=concat!(t1,t1)
--R
--R
--R          -----
--R   (56)  [1,2,3]
--R                                                  Type: List(PositiveInteger)
--E 57

--S 58 of 62
t3:=[1,2,3]
--R
--R
--R   (57)  [1,2,3]
--R                                                  Type: List(PositiveInteger)
--E 58

--S 59 of 62
t4:=concat!(t3,t2)
--R
--R
--R                -----
--R   (58)  [1,2,3,1,2,3]
--R                                                  Type: List(PositiveInteger)
--E 59

--S 60 of 62
t5:=cycleSplit!(t4)
--R
--R
--R          -----
--R   (59)  [1,2,3]
--R                                                  Type: List(PositiveInteger)
--E 60

--S 61 of 62
t4
```

```
--R
--R
--R   (60)  [1,2,3]
--R                                                 Type: List(PositiveInteger)
--E 61

--S 62 of 62
t5
--R
--R
--R          -----
--R   (61)  [1,2,3]
--R                                                 Type: List(PositiveInteger)
--E 62

)spool



)lisp (bye)
```

— **UnaryRecursiveAggregate.help** —

```
====================================================================
UnaryRecursiveAggregate examples
====================================================================

A unary-recursive aggregate is a one where nodes may have either
0 or 1 children. This aggregate models, though not precisely, a linked
list possibly with a single cycle.

A node with one children models a non-empty list, with the value of the
list designating the head, or first, of the list, and the child
designating the tail, or rest, of the list. A node with no child then
designates the empty list. Since these aggregates are recursive aggregates,
they may be cyclic.

ELEMENT ACCESS:

You can get the first element by either

   first [1,4,2,-6,0,3,5,4,2,3] ==> 1

or using the subscript form

   t1:=[1,2,3]    ==> [1,2,3]
   t1.first       ==> 1

You can get multiple elements with

   first([1,4,2,-6,0,3,5,4,2,3],3)  ==> [1,4,2]
```

```
Similary, you can get the all-but-first elements with

   rest [1,4,2,-6,0,3,5,4,2,3]  ==> [4,2,- 6,0,3,5,4,2,3]

or in subscript notation

   t1:=[1,2,3]   ==> [1,2,3]
   t1.rest       ==> [2,3]

and all-but-n elements with

   rest([1,4,2,-6,0,3,5,4,2,3],3) ==> [- 6,0,3,5,4,2,3]

The last element is available in function form as

   last [1,4,2,-6,0,3,5,4,2,3] ==> 3

or subscript form

   t1:=[1,2,3]   ==>  [1,2,3]
   t1.last       ==> 3

and the last-but-n elements with

   last([1,4,2,-6,0,3,5,4,2,3],3) ==> [4,2,3]

You can get the last element as an aggregate with

   tail [1,4,2,-6,0,3,5,4,2,3]   ==> [3]

Specific elements are named with

   second [1,4,2,-6,0,3,5,4,2,3]  ==> 4

   third  [1,4,2,-6,0,3,5,4,2,3]  ==> 2

AGGREGATION:

We can destructively set positions in the aggregate in function form

   t1:=[1,2,3]       ==> [1,2,3]
   setfirst!(t1,7)   ==> 7
   t1                ==> [7,2,3]

or in subscript form

   t1:=[1,2,3]       ==> [1,2,3]
   t1.first:=7       ==> 7
   t1                ==> [7,2,3]

We can destructively set the all-but-first to a new aggregate
in function form

   t1:=[1,2,3]               ==> [1,2,3]
```

```
   setrest!(t1,[4,5,6])      ==> [4,5,6]
   t1                        ==> [1,4,5,6]
```

or subscript form

```
   t1:=[1,2,3]               ==> [1,2,3]
   t1.rest:=[4,5,6]          ==> [4,5,6]
   t1                        ==> [1,4,5,6]
```

We can destructively modify the last of the aggregate in function form

```
   t1:=[1,4,2,-6,0,3,5,4,2,3]  ==> [1,4,2,-6,0,3,5,4,2,3]
   setlast!(t1,7)              ==> 7
   t1                          ==> [1,4,2,- 6,0,3,5,4,2,7]
```

CONCATENATION:

The concat function has two forms. It accepts two aggregates

```
   t1:=[1,2,3]        ==> [1,2,3]
   t2:=concat(t1,t1)  ==> [1,2,3,1,2,3]
   t1                 ==> [1,2,3]
   t2                 ==> [1,2,3,1,2,3]
```

or it accepts an aggregate element and an aggregate

```
   t2:=concat(4,t1)   ==> [4,1,2,3]
   t1                 ==> [1,2,3]
   t2                 ==> [4,1,2,3]
```

In both cases the operation is non-destructive to t1

There is a destructive form of concatenation of aggregates

```
   t1:=[1,2,3]      ==> [1,2,3]
   t2:=[4,5,6]      ==> [4,5,6]
   concat!(t1,t2)   ==> [1,2,3,4,5,6]
   t1               ==> [1,2,3,4,5,6]
   t2               ==> [4,5,6]
```

and a destructive form for elements

```
   t1:=[1,2,3]      ==> [1,2,3]
   concat!(t1,7)    ==> [1,2,3,7]
   t1               ==> [1,2,3,7]
```

SPLITTING:

We can destructively split an aggregate into two aggregates with

```
   t1:=[1,4,2,-6,0,3,5,4,2,3]  ==> [1,4,2,- 6,0,3,5,4,2,3]
   t2:=split!(t1,4)            ==> [0,3,5,4,2,3]
   t1                          ==> [1,4,2,- 6]
   t2                          ==> [0,3,5,4,2,3]
```

```
CYCLES:

Destructive operations can create cycles in lists.
Here t1 contains t1 and we can get the start of the cycle:

   t1:=[1,2,3]         ==> [1,2,3]
                            -----
   t2:=concat!(t1,t1)  ==> [1,2,3]
                            -----
   cycleEntry t2       ==> [1,2,3]

and its length

   cycleLength t2      ==> 3

and its tail
                            -----
   cycleTail t2        ==> [3,1,2]

We can also destructively break apart at the cycle start with

   t1:=[1,2,3]         ==> [1,2,3]
                            -----
   t2:=concat!(t1,t1)  ==> [1,2,3]

   t3:=[1,2,3]         ==> [1,2,3]
                                  -----
   t4:=concat!(t3,t2)  ==> [1,2,3,1,2,3]
                            -----
   t5:=cycleSplit!(t4) ==> [1,2,3]

   t4                  ==> [1,2,3]
                            -----
   t5                  ==> [1,2,3]

See Also:
o )show UnaryRecursiveAggregate
```

See:

⇒ "StreamAggregate" (STAGG) 7.0.131 on page 739
⇐ "RecursiveAggregate" (RCAGG) 5.0.104 on page 475

Exports:

any?	children	child?	coerce	concat
concat!	copy	count	cycleEntry	cycleLength
cycleSplit!	cycleTail	cyclic?	distance	empty
empty?	eq?	eval	every?	first
hash	last	latex	leaf?	leaves
less?	map	map!	member?	members
more?	nodes	node?	parts	rest
sample	second	setchildren!	setelt	setfirst!
setlast!	setrest!	setvalue!	size?	split!
tail	third	value	#?	?.last
?.rest	?.first	?.value	?=?	?~=?

Attributes exported:

- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
concat : (S,%) -> %
concat! : (%,S) -> % if $ has shallowlyMutable
concat! : (%,%) -> % if $ has shallowlyMutable
first : % -> S
first : (%,NonNegativeInteger) -> %
rest : % -> %
setfirst! : (%,S) -> S if $ has shallowlyMutable
```

These are implemented by this category:

```
children : % -> List %
concat : (%,%) -> %
cycleEntry : % -> %
cycleLength : % -> NonNegativeInteger
cycleSplit! : % -> % if $ has shallowlyMutable
cycleTail : % -> %
cyclic? : % -> Boolean
last : % -> S
last : (%,NonNegativeInteger) -> %
leaf? : % -> Boolean
less? : (%,NonNegativeInteger) -> Boolean
more? : (%,NonNegativeInteger) -> Boolean
nodes : % -> List %
node? : (%,%) -> Boolean if S has SETCAT
rest : (%,NonNegativeInteger) -> %
second : % -> S
setchildren! : (%,List %) -> % if $ has shallowlyMutable
setelt : (%,first,S) -> S if $ has shallowlyMutable
setelt : (%,last,S) -> S if $ has shallowlyMutable
setelt : (%,rest,%) -> % if $ has shallowlyMutable
setlast! : (%,S) -> S if $ has shallowlyMutable
setvalue! : (%,S) -> S if $ has shallowlyMutable
```

```
size? : (%,NonNegativeInteger) -> Boolean
split! : (%,Integer) -> % if $ has shallowlyMutable
tail : % -> %
third : % -> S
value : % -> S
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?.first : (%,first) -> S
?.last : (%,last) -> S
?.rest : (%,rest) -> %
```

These exports come from (p475) RecursiveAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
child? : (%,%) -> Boolean if S has SETCAT
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
distance : (%,%) -> Integer
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
leaves : % -> List S
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
parts : % -> List S if $ has finiteAggregate
sample : () -> %
setelt : (%,value,S) -> S if $ has shallowlyMutable
setrest! : (%,%) -> % if $ has shallowlyMutable
?.value : (%,value) -> S
?~=? : (%,%) -> Boolean if S has SETCAT
```

— UnaryRecursiveAggregate.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#URAGG">
```

UnaryRecursiveAggregate (URAGG)</h2>
</body>

———-

— category URAGG UnaryRecursiveAggregate —

```
)abbrev category URAGG UnaryRecursiveAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ A unary-recursive aggregate is a one where nodes may have either
++ 0 or 1 children.
++ This aggregate models, though not precisely, a linked
++ list possibly with a single cycle.
++ A node with one children models a non-empty list, with the
++ \spadfun{value} of the list designating the head, or \spadfun{first},
++ of the list, and the child designating the tail, or \spadfun{rest},
++ of the list. A node with no child then designates the empty list.
++ Since these aggregates are recursive aggregates, they may be cyclic.

UnaryRecursiveAggregate(S) : Category == SIG where
  S : Type

  SIG ==> RecursiveAggregate S with

    concat : (%,%) -> %
      ++ concat(u,v) returns an aggregate w consisting of the elements of u
      ++ followed by the elements of v.
      ++ Note that \axiom{v = rest(w,#a)}.
      ++
      ++X t1:=[1,2,3]
      ++X t2:=concat(t1,t1)
      ++X t1
      ++X t2

    concat : (S,%) -> %
      ++ concat(x,u) returns aggregate consisting of x followed by
      ++ the elements of u.
      ++ Note that if \axiom{v = concat(x,u)} then \axiom{x = first v}
      ++ and \axiom{u = rest v}.
      ++
      ++X t1:=[1,2,3]
      ++X t2:=concat(4,t1)
      ++X t1
      ++X t2

    first : % -> S
      ++ first(u) returns the first element of u
      ++ (equivalently, the value at the current node).
      ++
      ++X first [1,4,2,-6,0,3,5,4,2,3]
```

```
elt : (%,"first") -> S
  ++ elt(u,"first") (also written: \axiom{u . first})
  ++ is equivalent to first u.
  ++
  ++X t1:=[1,2,3]
  ++X t1.first

first : (%,NonNegativeInteger) -> %
  ++ first(u,n) returns a copy of the first n (\axiom{n >= 0})
  ++ elements of u.
  ++
  ++ first([1,4,2,-6,0,3,5,4,2,3],3)

rest : % -> %
  ++ rest(u) returns an aggregate consisting of all but the first
  ++ element of u
  ++ (equivalently, the next node of u).
  ++
  ++X rest [1,4,2,-6,0,3,5,4,2,3]

elt : (%,"rest") -> %
  ++ elt(%,"rest") (also written: \axiom{u.rest}) is
  ++ equivalent to \axiom{rest u}.
  ++
  ++X t1:=[1,2,3]
  ++X t1.rest

rest : (%,NonNegativeInteger) -> %
  ++ rest(u,n) returns the \axiom{n}th (n >= 0) node of u.
  ++ Note that \axiom{rest(u,0) = u}.
  ++
  ++X rest([1,4,2,-6,0,3,5,4,2,3],3)

last : % -> S
  ++ last(u) resturn the last element of u.
  ++ Note that for lists, \axiom{last(u)=u . (maxIndex u)=u . (# u - 1)}.
  ++
  ++X last [1,4,2,-6,0,3,5,4,2,3]

elt : (%,"last") -> S
  ++ elt(u,"last") (also written: \axiom{u . last}) is equivalent
  ++ to last u.
  ++
  ++X t1:=[1,2,3]
  ++X t1.last

last : (%,NonNegativeInteger) -> %
  ++ last(u,n) returns a copy of the last n (\axiom{n >= 0}) nodes of u.
  ++ Note that \axiom{last(u,n)} is a list of n elements.
  ++
  ++X last([1,4,2,-6,0,3,5,4,2,3],3)

tail : % -> %
  ++ tail(u) returns the last node of u.
```

```
  ++ Note that if u is \axiom{shallowlyMutable},
  ++ \axiom{setrest(tail(u),v) = concat(u,v)}.
  ++
  ++X tail [1,4,2,-6,0,3,5,4,2,3]

second : % -> S
  ++ second(u) returns the second element of u.
  ++ Note that \axiom{second(u) = first(rest(u))}.
  ++
  ++X second [1,4,2,-6,0,3,5,4,2,3]

third : % -> S
  ++ third(u) returns the third element of u.
  ++ Note that \axiom{third(u) = first(rest(rest(u)))}.
  ++
  ++X third [1,4,2,-6,0,3,5,4,2,3]

cycleEntry : % -> %
  ++ cycleEntry(u) returns the head of a top-level cycle contained in
  ++ aggregate u, or \axiom{empty()} if none exists.
  ++
  ++X t1:=[1,2,3]
  ++X t2:=concat!(t1,t1)
  ++X cycleEntry t2

cycleLength : % -> NonNegativeInteger
  ++ cycleLength(u) returns the length of a top-level cycle
  ++ contained  in aggregate u, or 0 is u has no such cycle.
  ++
  ++X t1:=[1,2,3]
  ++X t2:=concat!(t1,t1)
  ++X cycleLength t2

cycleTail : % -> %
  ++ cycleTail(u) returns the last node in the cycle, or
  ++ empty if none exists.
  ++
  ++X t1:=[1,2,3]
  ++X t2:=concat!(t1,t1)
  ++X cycleTail t2

if % has shallowlyMutable then

  concat_! : (%,%) -> %
    ++ concat!(u,v) destructively concatenates v to the end of u.
    ++ Note that \axiom{concat!(u,v) = setlast_!(u,v)}.
    ++
    ++X t1:=[1,2,3]
    ++X t2:=[4,5,6]
    ++X concat!(t1,t2)
    ++X t1
    ++X t2

  concat_! : (%,S) -> %
```

```
    ++ concat!(u,x) destructively adds element x to the end of u.
    ++ Note that \axiom{concat!(a,x) = setlast!(a,[x])}.
    ++
    ++X t1:=[1,2,3]
    ++X concat!(t1,7)
    ++X t1

  cycleSplit_! : % -> %
    ++ cycleSplit!(u) splits the aggregate by dropping off the cycle.
    ++ The value returned is the cycle entry, or nil if none exists.
    ++ For example, if \axiom{w = concat(u,v)} is the cyclic list where
    ++ v is the head of the cycle, \axiom{cycleSplit!(w)} will drop v
    ++ off w thus destructively changing w to u, and returning v.
    ++
    ++X t1:=[1,2,3]
    ++X t2:=concat!(t1,t1)
    ++X t3:=[1,2,3]
    ++X t4:=concat!(t3,t2)
    ++X t5:=cycleSplit!(t4)
    ++X t4
    ++X t5

  setfirst_! : (%,S) -> S
    ++ setfirst!(u,x) destructively changes the first element of a to x.
    ++
    ++X t1:=[1,2,3]
    ++X setfirst!(t1,7)
    ++X t1

  setelt : (%,"first",S) -> S
    ++ setelt(u,"first",x) (also written: \axiom{u.first := x}) is
    ++ equivalent to \axiom{setfirst!(u,x)}.
    ++
    ++X t1:=[1,2,3]
    ++X t1.first:=7
    ++X t1

  setrest_! : (%,%) -> %
    ++ setrest!(u,v) destructively changes the rest of u to v.
    ++
    ++X t1:=[1,2,3]
    ++X setrest!(t1,[4,5,6])
    ++X t1

  setelt : (%,"rest",%) -> %
    ++ setelt(u,"rest",v) (also written: \axiom{u.rest := v}) is
    ++ equivalent to \axiom{setrest!(u,v)}.
    ++
    ++X t1:=[1,2,3]
    ++X t1.rest:=[4,5,6]
    ++X t1

  setlast_! : (%,S) -> S
    ++ setlast!(u,x) destructively changes the last element of u to x.
```

```
     ++
     ++X t1:=[1,4,2,-6,0,3,5,4,2,3]
     ++X setlast!(t1,7)
     ++X t1

   setelt : (%,"last",S) -> S
    ++ setelt(u,"last",x) (also written: \axiom{u.last := b})
    ++ is equivalent to \axiom{setlast!(u,v)}.
    ++
    ++X t1:=[1,4,2,-6,0,3,5,4,2,3]
    ++X t1.last := 7
    ++X t1

   split_! : (%,Integer) -> %
     ++ split!(u,n) splits u into two aggregates: \axiom{v = rest(u,n)}
     ++ and \axiom{w = first(u,n)}, returning \axiom{v}.
     ++ Note that afterwards \axiom{rest(u,n)} returns \axiom{empty()}.
     ++
     ++X t1:=[1,4,2,-6,0,3,5,4,2,3]
     ++X t2:=split!(t1,4)
     ++X t1
     ++X t2

add

  cycleMax ==> 1000

  findCycle: % -> %

  elt(x, "first") == first x

  elt(x,  "last") == last x

  elt(x,  "rest") == rest x

  second x == first rest x

  third x == first rest rest x

  cyclic? x == not empty? x and not empty? findCycle x

  last x == first tail x

  nodes x ==
    l := empty()$List(%)
    while not empty? x repeat
      l := concat(x, l)
      x := rest x
    reverse_! l

  children x ==
    l := empty()$List(%)
    empty? x => l
    concat(rest x,l)
```

```
leaf? x == empty? x

value x ==
  empty? x => error "value of empty object"
  first x

less?(l, n) ==
  i := n::Integer
  while i > 0 and not empty? l repeat (l := rest l; i := i - 1)
  i > 0

more?(l, n) ==
  i := n::Integer
  while i > 0 and not empty? l repeat (l := rest l; i := i - 1)
  zero?(i) and not empty? l

size?(l, n) ==
  i := n::Integer
  while not empty? l and i > 0 repeat (l := rest l; i := i - 1)
  empty? l and zero? i

#x ==
  for k in 0.. while not empty? x repeat
    k = cycleMax and cyclic? x => error "cyclic list"
    x := rest x
  k

tail x ==
  empty? x => error "empty list"
  y := rest x
  for k in 0.. while not empty? y repeat
    k = cycleMax and cyclic? x => error "cyclic list"
    y := rest(x := y)
  x

findCycle x ==
  y := rest x
  while not empty? y repeat
    if eq?(x, y) then return x
    x := rest x
    y := rest y
    if empty? y then return y
    if eq?(x, y) then return y
    y := rest y
  y

cycleTail x ==
  empty?(y := x := cycleEntry x) => x
  z := rest x
  while not eq?(x,z) repeat (y := z; z := rest z)
  y

cycleEntry x ==
```

```
    empty? x => x
    empty?(y := findCycle x) => y
    z := rest y
    for l in 1.. while not eq?(y,z) repeat z := rest z
    y := x
    for k in 1..l repeat y := rest y
    while not eq?(x,y) repeat (x := rest x; y := rest y)
    x

  cycleLength x ==
    empty? x => 0
    empty?(x := findCycle x) => 0
    y := rest x
    for k in 1.. while not eq?(x,y) repeat y := rest y
    k

  rest(x, n) ==
    for i in 1..n repeat
      empty? x => error "Index out of range"
      x := rest x
    x

  if % has finiteAggregate then

    last(x, n) ==
      n > (m := #x) => error "index out of range"
      copy rest(x, (m - n)::NonNegativeInteger)

  if S has SetCategory then

    x = y ==
      eq?(x, y) => true
      for k in 0.. while not empty? x and not empty? y repeat
        k = cycleMax and cyclic? x => error "cyclic list"
        first x ^= first y => return false
        x := rest x
        y := rest y
      empty? x and empty? y

    node?(u, v) ==
      for k in 0.. while not empty? v repeat
        u = v => return true
        k = cycleMax and cyclic? v => error "cyclic list"
        v := rest v
      u=v

  if % has shallowlyMutable then

    setelt(x, "first", a) == setfirst_!(x, a)

    setelt(x,  "last", a) == setlast_!(x, a)

    setelt(x,  "rest", a) == setrest_!(x, a)
```

```
    concat(x:%, y:%) == concat_!(copy x, y)

    setlast_!(x, s) ==
      empty? x => error "setlast: empty list"
      setfirst_!(tail x, s)
      s

    setchildren_!(u,lv) ==
      #lv=1 => setrest_!(u, first lv)
      error "wrong number of children specified"

    setvalue_!(u,s) == setfirst_!(u,s)

    split_!(p, n) ==
      n < 1 => error "index out of range"
      p := rest(p, (n - 1)::NonNegativeInteger)
      q := rest p
      setrest_!(p, empty())
      q

    cycleSplit_! x ==
      empty?(y := cycleEntry x) or eq?(x, y) => y
      z := rest x
      while not eq?(z, y) repeat (x := z; z := rest z)
      setrest_!(x, empty())
      y
```

— **COQ URAGG** —

```
(* category URAGG *)
(*
  cycleMax ==> 1000

  elt: (%,"first") -> S
  elt(x, "first") == first x

  elt: (%,"last") -> S
  elt(x,  "last") == last x

  elt: (%,"rest") -> %
  elt(x,  "rest") == rest x

  second: % -> S
  second x == first rest x

  third: % -> S
  third x == first rest rest x

  cyclic? : % -> Boolean
  cyclic? x == not empty? x and not empty? findCycle x
```

```
last: % -> S
last x == first tail x

nodes : % -> List(%)
nodes x ==
  l := empty()$List(%)
  while not empty? x repeat
    l := concat(x, l)
    x := rest x
  reverse_! l

children : % -> List(%)
children x ==
  l := empty()$List(%)
  empty? x => l
  concat(rest x,l)

leaf? : % -> Boolean
leaf? x == empty? x

value : % -> S
value x ==
  empty? x => error "value of empty object"
  first x

less? : (%,NonNegativeInteger) -> Boolean
less?(l, n) ==
  i := n::Integer
  while i > 0 and not empty? l repeat (l := rest l; i := i - 1)
  i > 0

more? : (%,NonNegativeInteger) -> Boolean
more?(l, n) ==
  i := n::Integer
  while i > 0 and not empty? l repeat (l := rest l; i := i - 1)
  zero?(i) and not empty? l

size? : (%,NonNegativeInteger) -> Boolean
size?(l, n) ==
  i := n::Integer
  while not empty? l and i > 0 repeat (l := rest l; i := i - 1)
  empty? l and zero? i

#? : % -> NonNegativeInteger
#x ==
  for k in 0.. while not empty? x repeat
    k = cycleMax and cyclic? x => error "cyclic list"
    x := rest x
  k

tail: % -> %
tail x ==
  empty? x => error "empty list"
  y := rest x
```

```
    for k in 0.. while not empty? y repeat
      k = cycleMax and cyclic? x => error "cyclic list"
      y := rest(x := y)
    x

  findCycle: % -> %
  findCycle x ==
    y := rest x
    while not empty? y repeat
      if eq?(x, y) then return x
      x := rest x
      y := rest y
      if empty? y then return y
      if eq?(x, y) then return y
      y := rest y
    y

  cycleTail: % -> %
  cycleTail x ==
    empty?(y := x := cycleEntry x) => x
    z := rest x
    while not eq?(x,z) repeat (y := z; z := rest z)
    y

  cycleEntry: % -> %
  cycleEntry x ==
    empty? x => x
    empty?(y := findCycle x) => y
    z := rest y
    for l in 1.. while not eq?(y,z) repeat z := rest z
    y := x
    for k in 1..l repeat y := rest y
    while not eq?(x,y) repeat (x := rest x; y := rest y)
    x

  cycleLength: % -> NonNegativeInteger
  cycleLength x ==
    empty? x => 0
    empty?(x := findCycle x) => 0
    y := rest x
    for k in 1.. while not eq?(x,y) repeat y := rest y
    k

  rest: (%,NonNegativeInteger) -> %
  rest(x, n) ==
    for i in 1..n repeat
      empty? x => error "Index out of range"
      x := rest x
    x

  if % has finiteAggregate then

    last: (%,NonNegativeInteger) -> %
    last(x, n) ==
```

```
    n > (m := #x) => error "index out of range"
    copy rest(x, (m - n)::NonNegativeInteger)

if S has SetCategory then

  ?=? : (%,%) -> Boolean
  x = y ==
    eq?(x, y) => true
    for k in 0.. while not empty? x and not empty? y repeat
      k = cycleMax and cyclic? x => error "cyclic list"
      first x ^= first y => return false
      x := rest x
      y := rest y
    empty? x and empty? y

  node? : (%,%) -> Boolean
  node?(u, v) ==
    for k in 0.. while not empty? v repeat
      u = v => return true
      k = cycleMax and cyclic? v => error "cyclic list"
      v := rest v
    u=v

if % has shallowlyMutable then

  setelt: (%,"first",S) -> S
  setelt(x, "first", a) == setfirst_!(x, a)

  setelt: (%,"last",S) -> S
  setelt(x,  "last", a) == setlast_!(x, a)

  setelt: (%,"rest",%) -> %
  setelt(x,  "rest", a) == setrest_!(x, a)

  concat : (%,%) -> %
  concat(x:%, y:%) == concat_!(copy x, y)

  setlast_!: (%,S) -> S
  setlast_!(x, s) ==
    empty? x => error "setlast: empty list"
    setfirst_!(tail x, s)
    s

  setchildren! : (%,List(%)) -> %
  setchildren_!(u,lv) ==
    #lv=1 => setrest_!(u, first lv)
    error "wrong number of children specified"

  setvalue! : (%,S) -> S
  setvalue_!(u,s) == setfirst_!(u,s)

  split_!: (%,Integer) -> %
  split_!(p, n) ==
    n < 1 => error "index out of range"
```

```
      p := rest(p, (n - 1)::NonNegativeInteger)
      q := rest p
      setrest_!(p, empty())
      q

    cycleSplit_!: % -> %
    cycleSplit_! x ==
      empty?(y := cycleEntry x) or eq?(x, y) => y
      z := rest x
      while not eq?(z, y) repeat (x := z; z := rest z)
      setrest_!(x, empty())
      y

*)
```

———-

— **URAGG.dotabb** —

```
"URAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=URAGG"];
"URAGG" -> "RCAGG"
```

———-

— **URAGG.dotfull** —

```
"UnaryRecursiveAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=URAGG"];
"UnaryRecursiveAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"
```

———-

— **URAGG.dotpic** —

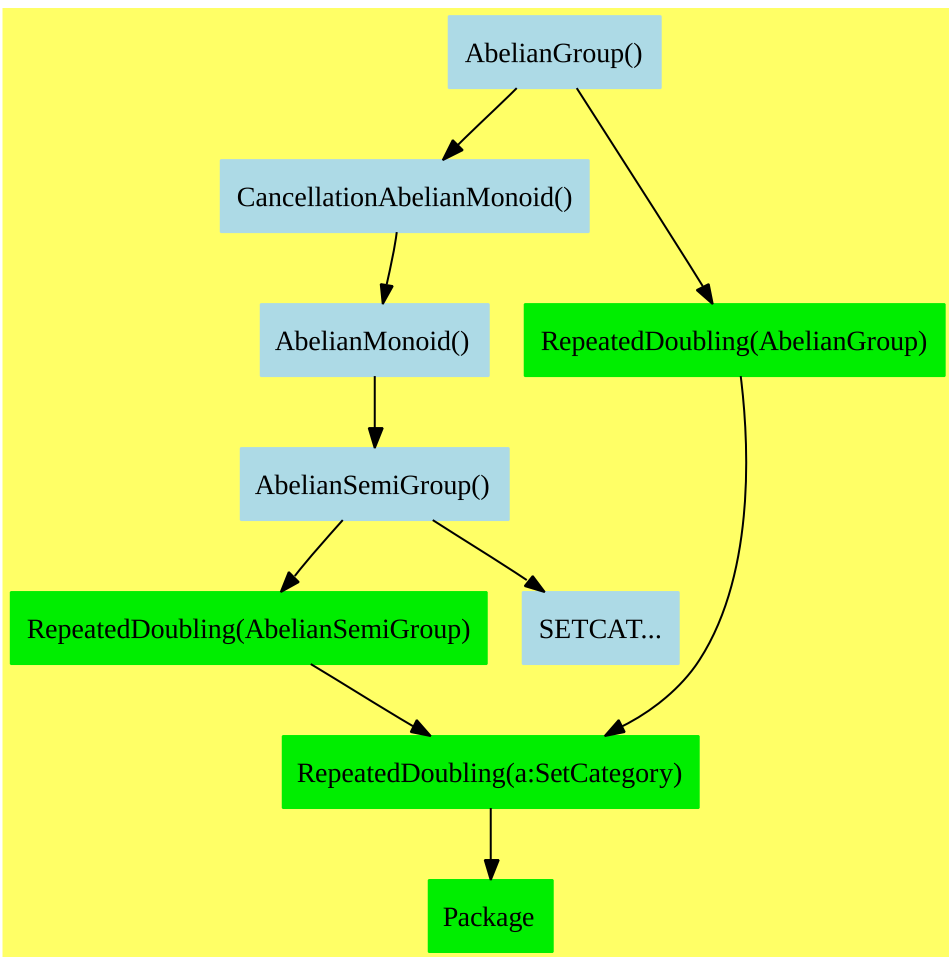
```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"UnaryRecursiveAggregate(a:Type)" [color=lightblue];
"UnaryRecursiveAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"

"RecursiveAggregate(a:Type)" [color=lightblue];
"RecursiveAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"
```

```
"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

Chapter 7

Category Layer 6

7.0.121 AbelianGroup (ABELGRP)



— **AbelianGroup.input** —

```
)set break resume
)sys rm -f AbelianGroup.output
)spool AbelianGroup.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AbelianGroup
--R
--R AbelianGroup is a category constructor
--R Abbreviation for AbelianGroup is ABELGRP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ABELGRP
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Integer,%) -> %                ?*? : (NonNegativeInteger,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?=? : (%,%) -> Boolean                0 : () -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— **AbelianGroup.help** —

```
====================================================================
AbelianGroup examples
====================================================================

The class of abelian groups, additive monoids where each element
has an additive inverse.

Axioms:
         -(-x) = x
         x+(-x) = 0

See Also:
o )show AbelianGroup
```

— **AbelianGroup.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ABELGRP">
AbelianGroup<a/></h2>

The class of abelian groups, i.e. additive monoids where each element has
an additive inverse
<br/><br/><br/>

0 : %<br/>
  0 is the additive identity element<br/><br/>

* : (NonNegativeInteger,%) -&gt; %<br/>
  n * x is left-multiplication by a non-negative integer<br/><br/>

* : (PositiveInteger,%) -&gt; %<br/>
  from AbelianSemiGroup<br/><br/>

+ : (%,%) -&gt; %<br/>
  from AbelianSemiGroup<br/><br/>

= : (%,%) -&gt; Boolean<br/>
  from BasicType<br/><br/>

~= : (%,%) -&gt; Boolean<br/>
  from BasicType<br/><br/>

coerce : % -&gt; OutputForm<br/>
  from CoercibleTo OutputForm<br/><br/>

hash : % -&gt; SingleInteger<br/>
  from SetCategory<br/><br/>

latex : % -> String<br/>
   from SetCategory<br/><br/>

sample: %<br/>
  sample yields a value of type %<br/><br/>

zero? : % -> Boolean<br/>
  zero?)x) tests if x is equal to 0<br/><br/>

See<br/><br/>
&bull; AbelianSemiGroup<br/><br/>
&bull; BasicType<br/><br/>
&bull; CoercibleTo OutputForm<br/><br/>
&bull; SetCategory
</body>
```

See:

⇒ "FiniteDivisorCategory" (FDIVCAT) 8.0.133 on page 776

⇒ "FunctionSpace" (FS) 17.0.219 on page 1544
⇒ "LeftModule" (LMODULE) 8.0.137 on page 823
⇒ "NonAssociativeRng" (NARNG) 8.0.140 on page 849
⇒ "OrderedAbelianGroup" (OAGROUP) 9.0.150 on page 932
⇒ "RightModule" (RMODULE) 8.0.144 on page 892
⇒ "Rng" (RNG) 8.0.145 on page 896
⇐ "CancellationAbelianMonoid" (CABMON) 6.0.107 on page 508

Exports:

0	coerce	hash	latex	sample
subtractIfCan	zero?	?~=?	?*?	?+?
?-?	-?	?=?		

These are directly exported but not implemented:

```
-? : % -> %
```

These are implemented by this category:

```
subtractIfCan : (%,%) -> Union(%,"failed")
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
```

These exports come from (p508) CancellationAbelianMonoid():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
```

— AbelianGroup.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ABELGRP">
AbelianGroup (ABELGRP)</a></h2>
</body>
```

— category ABELGRP AbelianGroup —

```
)abbrev category ABELGRP AbelianGroup
++ Description:
++ The class of abelian groups, additive monoids where
++ each element has an additive inverse.
++
++ Axioms\br
++ \tab{5}\spad{-(-x) = x}\br
```

```
++ \tab{5}\spad{x+(-x) = 0}
-- following domain must be compiled with subsumption disabled

AbelianGroup() : Category == SIG where

  SIG ==> CancellationAbelianMonoid with

    "-" : % -> %
      ++ -x is the additive inverse of x.

    "-" : (%,%) -> %
      ++ x-y is the difference of x and y \spad{x + (-y)}.
      -- subsumes the partial subtraction from previous

    "*" : (Integer,%) -> %
      ++ n*x is the product of x by the integer n.

   add

      (x:% - y:%):% == x+(-y)

      subtractIfCan(x:%, y:%):Union(%, "failed") == (x-y)::Union(%,"failed")

      n:NonNegativeInteger * x:% == (n::Integer) * x

      import RepeatedDoubling(%)

      if not (% has Ring) then

        n:Integer * x:% ==
          zero? n => 0
          n>0 => double(n pretend PositiveInteger,x)
          double((-n) pretend PositiveInteger,-x)
```

— COQ ABELGRP —

```
(* category ABELGRP *)
(*
Axioms
 -(-x) = x
 x+(-x) = 0

      "-": (%,%) -> %
      (x:% - y:%):% == x+(-y)

      subtractIfCan : (%,%) -> Union(%,"failed")
      subtractIfCan(x:%, y:%):Union(%, "failed") == (x-y)::Union(%,"failed")

      ?*? : (NonNegativeInteger,%) -> %
      n:NonNegativeInteger * x:% == (n::Integer) * x
```

```
      import RepeatedDoubling(%)

      if not (% has Ring) then

        "*": (Integer,%) -> %
        n:Integer * x:% ==
          zero? n => 0
          n>0 => double(n pretend PositiveInteger,x)
          double((-n) pretend PositiveInteger,-x)
*)
```

———-

— ABELGRP.dotabb —

```
"ABELGRP"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ABELGRP"];
"ABELGRP" -> "CABMON"
```

———-

— ABELGRP.dotfull —

```
"AbelianGroup()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ABELGRP"];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "RepeatedDoubling(AbelianGroup)"
```

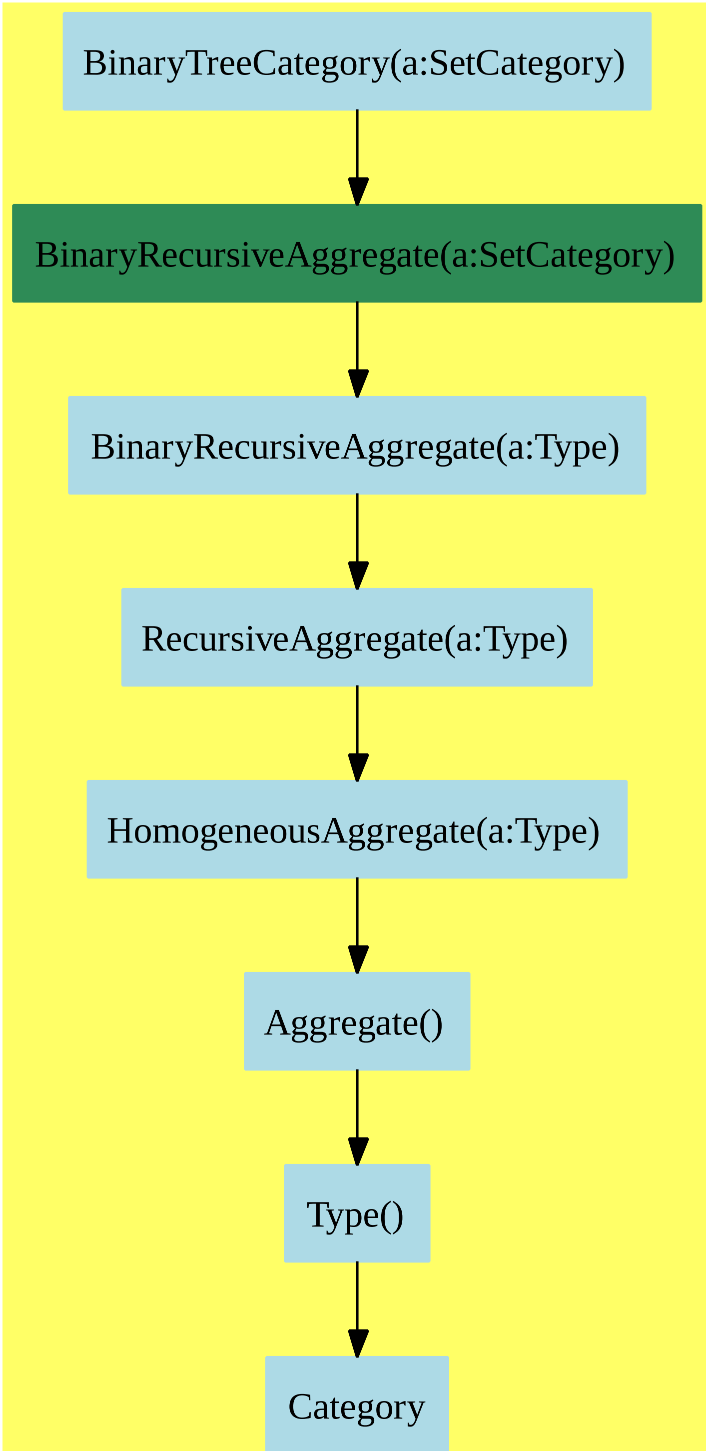
———-

— ABELGRP.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "RepeatedDoubling(AbelianGroup)"

"RepeatedDoubling(AbelianGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianGroup)" -> "RepeatedDoubling(a:SetCategory)"

"RepeatedDoubling(AbelianSemiGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianSemiGroup)" -> "RepeatedDoubling(a:SetCategory)"

"RepeatedDoubling(a:SetCategory)" [color="#00EE00"];
"RepeatedDoubling(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];
```

```
"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SETCAT..."
"AbelianSemiGroup()" -> "RepeatedDoubling(AbelianSemiGroup)"

"SETCAT..." [color=lightblue];
}
```

7.0.122 BinaryTreeCategory (BTCAT)



— **BinaryTreeCategory.input** —

```
)set break resume
)sys rm -f BinaryTreeCategory.output
)spool BinaryTreeCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BinaryTreeCategory
--R
--R BinaryTreeCategory(S: SetCategory) is a category constructor
--R Abbreviation for BinaryTreeCategory is BTCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for BTCAT
--R
--R------------------------------- Operations --------------------------------
--R children : % -> List(%)              copy : % -> %
--R cyclic? : % -> Boolean               distance : (%,%) -> Integer
--R ?.right : (%,right) -> %             ?.left : (%,left) -> %
--R ?.value : (%,value) -> S             empty : () -> %
--R empty? : % -> Boolean                eq? : (%,%) -> Boolean
--R latex : % -> String if S has SETCAT   leaf? : % -> Boolean
--R leaves : % -> List(S)                left : % -> %
--R map : ((S -> S),%) -> %              node : (%,S,%) -> %
--R nodes : % -> List(%)                 right : % -> %
--R sample : () -> %                     value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List(S) if $ has finiteAggregate
--R setchildren! : (%,List(%)) -> % if $ has shallowlyMutable
--R setelt : (%,right,%) -> % if $ has shallowlyMutable
--R setelt : (%,left,%) -> % if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setleft! : (%,%) -> % if $ has shallowlyMutable
```

```
--R setright! : (%,%) -> % if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

— BinaryTreeCategory.help —

```
====================================================================
BinaryTreeCategory examples
====================================================================

BinaryTreeCategory(S) is the category of binary trees: a tree which
is either empty or else is a node consisting of a value and a left and
right, both binary trees.

See Also:
o )show BinaryTreeCategory
```

See:

⇐ "BinaryRecursiveAggregate" (BRAGG) 6.0.106 on page 498

Exports:

any?	child?	children	coerce	copy
count	cyclic?	distance	empty	empty?
eq?	eval	every?	hash	latex
leaf?	leaves	less?	left	map
map!	member?	members	more?	node
node?	nodes	parts	right	sample
setchildren!	setelt	setleft!	setright!	setvalue!
size?	value	#?	?=?	?~=?
?.right	?.left	?.value		

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are directly exported but not implemented:

```
node : (%,S,%) -> %
```

These are implemented by this category:

```
copy : % -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
#? : % -> NonNegativeInteger if $ has finiteAggregate
```

These exports come from (p498) BinaryRecursiveAggregate(S:SetCategory):

```
any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
child? : (%,%) -> Boolean if S has SETCAT
children : % -> List %
coerce : % -> OutputForm if S has SETCAT
count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
cyclic? : % -> Boolean
distance : (%,%) -> Integer
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
leaf? : % -> Boolean
leaves : % -> List S
left : % -> %
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
right : % -> %
sample : () -> %
setchildren! : (%,List %) -> % if $ has shallowlyMutable
setelt : (%,value,S) -> S if $ has shallowlyMutable
setelt : (%,right,%) -> % if $ has shallowlyMutable
setelt : (%,left,%) -> % if $ has shallowlyMutable
setleft! : (%,%) -> % if $ has shallowlyMutable
setright! : (%,%) -> % if $ has shallowlyMutable
setvalue! : (%,S) -> S if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
value : % -> S
?~=? : (%,%) -> Boolean if S has SETCAT
?.value : (%,value) -> S
?=? : (%,%) -> Boolean if S has SETCAT
?.right : (%,right) -> %
?.left : (%,left) -> %
```

— BinaryTreeCategory.html —

```
<body>
<h2>
```

```
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#BTCAT">
BinaryTreeCategory (BTCAT)</a></h2>
</body>
```

— category BTCAT BinaryTreeCategory —

```
)abbrev category BTCAT BinaryTreeCategory
++ Author:W. H. Burge
++ Date Created:17 Feb 1992
++ Description:
++ \spadtype{BinaryTreeCategory(S)} is the category of
++ binary trees: a tree which is either empty or else is a
++ \spadfun{node} consisting of a value and a \spadfun{left} and
++ \spadfun{right}, both binary trees.

BinaryTreeCategory(S) : Category == SIG where
  S : SetCategory

  SIG ==> BinaryRecursiveAggregate(S) with

    shallowlyMutable
      ++ Binary trees have updateable components

    finiteAggregate
      ++ Binary trees have a finite number of components

    node : (%,S,%) -> %
      ++ node(left,v,right) creates a binary tree with value \spad{v}, a
      ++ binary tree \spad{left}, and a binary tree \spad{right}.

   add

     cycleTreeMax ==> 5

     copy t ==
       empty? t => empty()
       node(copy left t, value t, copy right t)

     if % has shallowlyMutable then
       map_!(f,t) ==
         empty? t => t
         t.value := f(t.value)
         map_!(f,left t)
         map_!(f,right t)
         t

     treeCount : (%, NonNegativeInteger) -> NonNegativeInteger

     #t == treeCount(t,0)

     treeCount(t,k) ==
         empty? t => k
```

```
        k := k + 1
        k = cycleTreeMax and cyclic? t => error "cyclic binary tree"
        k := treeCount(left t,k)
        treeCount(right t,k)
```

———-

— COQ BTCAT —

```
(* category BTCAT *)
(*
     cycleTreeMax ==> 5

     copy : % -> %
     copy t ==
       empty? t => empty()
       node(copy left t, value t, copy right t)

     if % has shallowlyMutable then

       map! : ((S -> S),%) -> %
       map_!(f,t) ==
         empty? t => t
         t.value := f(t.value)
         map_!(f,left t)
         map_!(f,right t)
         t

     #? : % -> NonNegativeInteger
     #t == treeCount(t,0)

     treeCount : (%, NonNegativeInteger) -> NonNegativeInteger
     treeCount(t,k) ==
         empty? t => k
         k := k + 1
         k = cycleTreeMax and cyclic? t => error "cyclic binary tree"
         k := treeCount(left t,k)
         treeCount(right t,k)

*)
```

———-

— BTCAT.dotabb —

```
"BTCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BTCAT"];
"BTCAT" -> "BRAGG"
```

———-

— **BTCAT.dotfull** —

```
"BinaryTreeCategory(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BTCAT"];
"BinaryTreeCategory(a:SetCategory)" ->
   "BinaryRecursiveAggregate(a:SetCategory)"
```

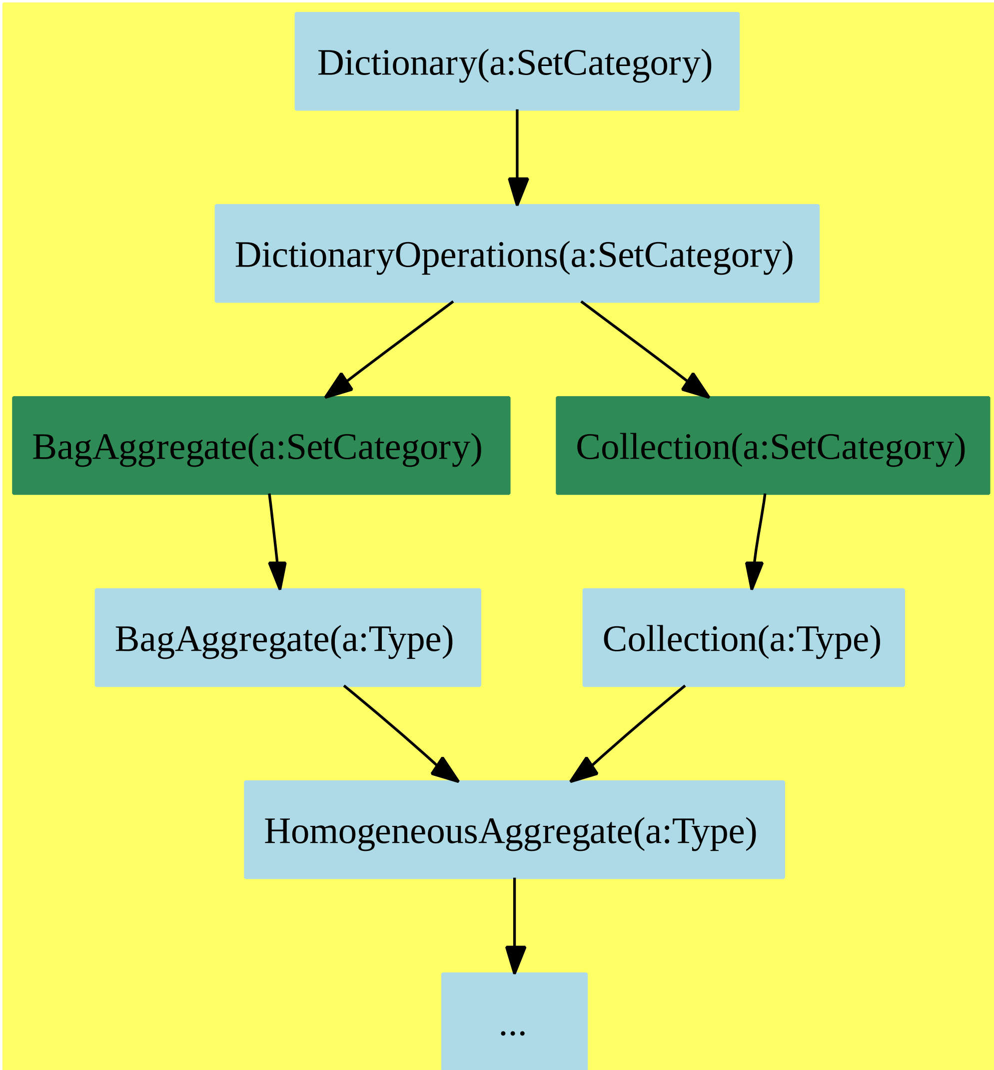
———-

— **BTCAT.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"BinaryTreeCategory(a:SetCategory)" [color=lightblue];
"BinaryTreeCategory(a:SetCategory)" ->
   "BinaryRecursiveAggregate(a:SetCategory)"

"BinaryRecursiveAggregate(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=BRAGG"];
"BinaryRecursiveAggregate(a:SetCategory)" ->
   "BinaryRecursiveAggregate(a:Type)"

"BinaryRecursiveAggregate(a:Type)" [color=lightblue];
"BinaryRecursiveAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"

"RecursiveAggregate(a:Type)" [color=lightblue];
"RecursiveAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

———-

7.0.123 Dictionary (DIAGG)



— **Dictionary.input** —

```
)set break resume
)sys rm -f Dictionary.output
)spool Dictionary.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Dictionary
--R
--R Dictionary(S: SetCategory) is a category constructor
```

```
--R Abbreviation for Dictionary is DIAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for DIAGG
--R
--R------------------------------- Operations --------------------------------
--R bag : List(S) -> %                    construct : List(S) -> %
--R copy : % -> %                         dictionary : List(S) -> %
--R dictionary : () -> %                  empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R extract! : % -> S                     insert! : (S,%) -> %
--R inspect : % -> S                      latex : % -> String if S has SETCAT
--R map : ((S -> S),%) -> %               sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT(INFORM)
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R hash : % -> SingleInteger if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(S) if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R remove! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (S,%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

— **Dictionary.help** —

```
====================================================================
Dictionary examples
====================================================================

A dictionary is an aggregate in which entries can be inserted,
searched for and removed. Duplicates are thrown away on insertion.
This category models the usual notion of dictionary which involves
large amounts of data where copying is impractical.

Principal operations are thus destructive (non-copying) ones.

See Also:
o )show Dictionary
```

See:

⇒ “FiniteSetAggregate” (FSAGG) 8.0.134 on page 781
⇒ “KeyedDictionary” (KDAGG) 8.0.135 on page 792
⇐ “DictionaryOperations” (DIOPS) 6.0.108 on page 512

Exports:

any?	bag	coerce	construct	convert
copy	count	dictionary	empty	empty?
eq?	eval	every?	extract!	find
hash	insert!	inspect	latex	less?
map	map!	member?	members	more?
parts	reduce	remove	remove!	removeDuplicates
sample	select	select!	size?	#?
?~=?	?=?			

Attributes exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.

These are implemented by this category:

```
dictionary : List S -> %
select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
remove! : ((S -> Boolean),%) -> % if $ has finiteAggregate
```

These exports come from (p512) DictionaryOperations(S:SetCategory):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
bag : List S -> %
```

```
 coerce : % -> OutputForm if S has SETCAT
 construct : List S -> %
 convert : % -> InputForm if S has KONVERT INFORM
 copy : % -> %
 count : (S,%) -> NonNegativeInteger
          if S has SETCAT and $ has finiteAggregate
 count : ((S -> Boolean),%) -> NonNegativeInteger
          if $ has finiteAggregate
 dictionary : () -> %
 empty : () -> %
 empty? : % -> Boolean
 eq? : (%,%) -> Boolean
 eval : (%,List S,List S) -> %
          if S has EVALAB S and S has SETCAT
 eval : (%,S,S) -> %
          if S has EVALAB S and S has SETCAT
 eval : (%,Equation S) -> %
          if S has EVALAB S and S has SETCAT
 eval : (%,List Equation S) -> %
          if S has EVALAB S and S has SETCAT
 every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
 extract! : % -> S
 find : ((S -> Boolean),%) -> Union(S,"failed")
 hash : % -> SingleInteger if S has SETCAT
 insert! : (S,%) -> %
 inspect : % -> S
 latex : % -> String if S has SETCAT
 less? : (%,NonNegativeInteger) -> Boolean
 map : ((S -> S),%) -> %
 map! : ((S -> S),%) -> % if $ has shallowlyMutable
 member? : (S,%) -> Boolean
          if S has SETCAT and $ has finiteAggregate
 members : % -> List S if $ has finiteAggregate
 more? : (%,NonNegativeInteger) -> Boolean
 parts : % -> List S if $ has finiteAggregate
 reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
 reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
 reduce : (((S,S) -> S),%,S,S) -> S
       if S has SETCAT and $ has finiteAggregate
 remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
 remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
 remove! : (S,%) -> % if $ has finiteAggregate
 removeDuplicates : % -> %
          if S has SETCAT and $ has finiteAggregate
 sample : () -> %
 select : ((S -> Boolean),%) -> % if $ has finiteAggregate
 size? : (%,NonNegativeInteger) -> Boolean
 #? : % -> NonNegativeInteger if $ has finiteAggregate
 ?~=? : (%,%) -> Boolean if S has SETCAT
```

— Dictionary.html —

```
<body>
<h2>
```

```
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#DIAGG">
Dictionary (DIAGG)</a></h2>
</body>
```

— category DIAGG Dictionary —

```
)abbrev category DIAGG Dictionary
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ A dictionary is an aggregate in which entries can be inserted,
++ searched for and removed. Duplicates are thrown away on insertion.
++ This category models the usual notion of dictionary which involves
++ large amounts of data where copying is impractical.
++ Principal operations are thus destructive (non-copying) ones.

Dictionary(S) : Category == SIG where
  S : SetCategory

  SIG ==> DictionaryOperations S

   add

     dictionary l ==
       d := dictionary()
       for x in l repeat insert_!(x, d)
       d

     if % has finiteAggregate then

       select_!(f, t) == remove_!((x:S):Boolean +-> not f(x), t)

       s = t ==
         eq?(s,t) => true
         #s ^= #t => false
         _and/[member?(x, t) for x in parts s]

       remove_!(f:S->Boolean, t:%) ==
         for m in parts t repeat if f m then remove_!(m, t)
         t
```

— COQ DIAGG —

```
(* category DIAGG *)
(*

   dictionary : List(S) -> %
   dictionary l ==
```

```
     d := dictionary()
     for x in l repeat insert_!(x, d)
     d

   if % has finiteAggregate then

    -- remove(f:S->Boolean,t:%)  == remove_!(f, copy t)

    -- select(f, t)           == select_!(f, copy t)

     select! : ((S -> Boolean),%) -> %
     select_!(f, t)         == remove_!((x:S):Boolean +-> not f(x), t)

     --extract_! d ==
     --         empty? d => error "empty dictionary"
     --         remove_!(x := first parts d, d, 1)
     --         x

     ?=? : (%,%) -> Boolean
     s = t ==
       eq?(s,t) => true
       #s ^= #t => false
       _and/[member?(x, t) for x in parts s]

     remove! : ((S -> Boolean),%) -> %
     remove_!(f:S->Boolean, t:%) ==
       for m in parts t repeat if f m then remove_!(m, t)
       t

*)
```

— **DIAGG.dotabb** —

```
"DIAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=DIAGG"];
"DIAGG" -> "DIOPS"
```

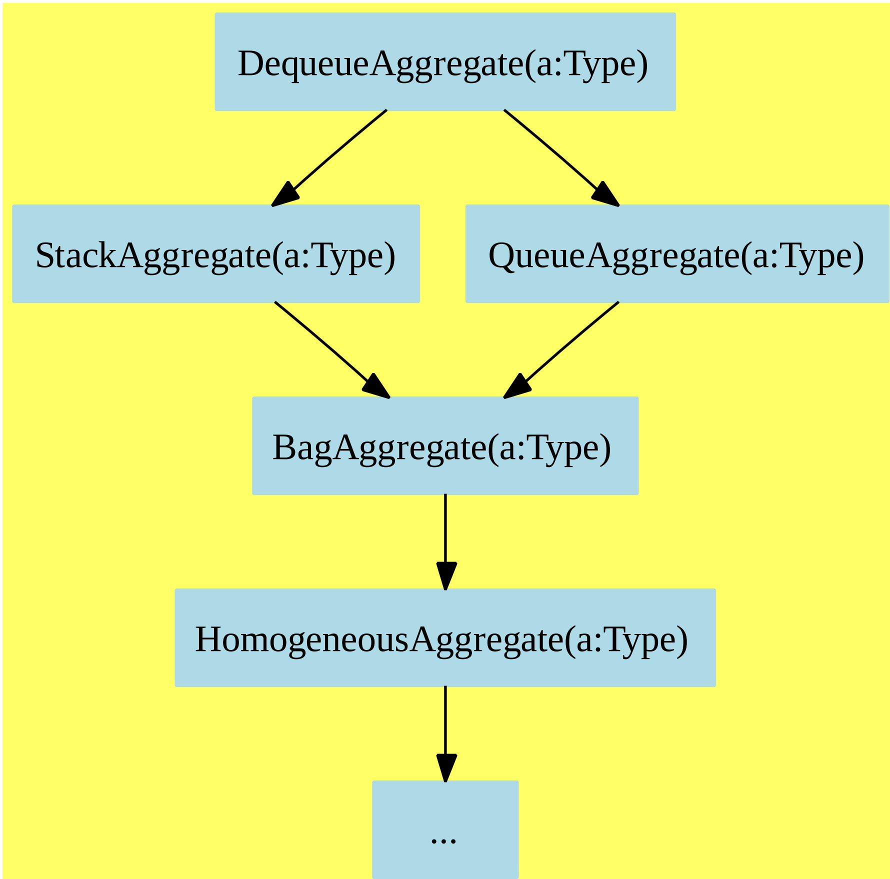
— **DIAGG.dotfull** —

```
"Dictionary(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIAGG"];
"Dictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"

"Dictionary(Record(a:SetCategory,b:SetCategory))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=DIAGG"];
"Dictionary(Record(a:SetCategory,b:SetCategory))" ->
    "Dictionary(a:SetCategory)"
```

— DIAGG.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"Dictionary(a:SetCategory)" [color=lightblue];
"Dictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"

"DictionaryOperations(a:SetCategory)" [color=lightblue];
"DictionaryOperations(a:SetCategory)" -> "BagAggregate(a:SetCategory)"
"DictionaryOperations(a:SetCategory)" -> "Collection(a:SetCategory)"

"BagAggregate(a:SetCategory)" [color=seagreen];
"BagAggregate(a:SetCategory)" -> "BagAggregate(a:Type)"

"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

"Collection(a:SetCategory)" [color=seagreen];
"Collection(a:SetCategory)" -> "Collection(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

7.0.124 DequeueAggregate (DQAGG)



— **DequeueAggregate.input** —

```
)set break resume
)sys rm -f DequeueAggregate.output
)spool DequeueAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DequeueAggregate
--R
--R DequeueAggregate(S: Type) is a category constructor
--R Abbreviation for DequeueAggregate is DQAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for DQAGG
--R
--R------------------------------- Operations --------------------------------
--R back : % -> S                         bag : List(S) -> %
```

```
--R bottom! : % -> S                       copy : % -> %
--R depth : % -> NonNegativeInteger        dequeue : List(S) -> %
--R dequeue : () -> %                      dequeue! : % -> S
--R empty : () -> %                        empty? : % -> Boolean
--R enqueue! : (S,%) -> S                  eq? : (%,%) -> Boolean
--R extract! : % -> S                      extractBottom! : % -> S
--R extractTop! : % -> S                   front : % -> S
--R height : % -> NonNegativeInteger       insert! : (S,%) -> %
--R insertBottom! : (S,%) -> S             insertTop! : (S,%) -> S
--R inspect : % -> S                       latex : % -> String if S has SETCAT
--R length : % -> NonNegativeInteger       map : ((S -> S),%) -> %
--R pop! : % -> S                          push! : (S,%) -> S
--R reverse! : % -> %                      rotate! : % -> %
--R sample : () -> %                       top : % -> S
--R top! : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R hash : % -> SingleInteger if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(S) if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

— **DequeueAggregate.help** —

```
====================================================================
DequeueAggregate examples
====================================================================

A dequeue is a doubly ended stack, that is, a bag where first items
inserted are the first items extracted, at either the front or
the back end of the data structure.

See Also:
```

```
o )show DequeueAggregate
```

———–

See:

⇐ "QueueAggregate" (QUAGG) 6.0.117 on page 622
⇐ "StackAggregate" (SKAGG) 6.0.119 on page 636

Exports:

any?	back	bag	bottom!	coerce
copy	count	depth	dequeue	dequeue!
empty	empty?	enqueue!	eq?	eval
every?	extract!	extractBottom!	extractTop!	front
hash	height	insert!	insertBottom!	insertTop!
inspect	latex	length	less?	map
map!	members	member?	more?	parts
pop!	push!	reverse!	rotate!	sample
size?	top	top!	#?	?=?
?~=?				

Attributes exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are directly exported but not implemented:

```
bottom! : % -> S
dequeue : () -> %
dequeue : List S -> %
extractBottom! : % -> S
extractTop! : % -> S
height : % -> NonNegativeInteger
insertBottom! : (S,%) -> S
insertTop! : (S,%) -> S
reverse! : % -> %
top! : % -> S
```

These exports come from (p636) StackAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
bag : List S -> %
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
depth : % -> NonNegativeInteger
```

```
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
extract! : % -> S
hash : % -> SingleInteger if S has SETCAT
insert! : (S,%) -> %
inspect : % -> S
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
members : % -> List S if $ has finiteAggregate
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
pop! : % -> S
push! : (S,%) -> S
sample : () -> %
size? : (%,NonNegativeInteger) -> Boolean
top : % -> S
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
```

These exports come from (p622) QueueAggregate(S:Type):

```
back : % -> S
dequeue! : % -> S
enqueue! : (S,%) -> S
front : % -> S
length : % -> NonNegativeInteger
rotate! : % -> %
```

— DequeueAggregate.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#DQAGG">
DequeueAggregate (DQAGG)</a></h2>
</body>
```

———-

— category DQAGG DequeueAggregate —

```
)abbrev category DQAGG DequeueAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ A dequeue is a doubly ended stack, that is, a bag where first items
++ inserted are the first items extracted, at either the front or
++ the back end of the data structure.

DequeueAggregate(S) : Category == SIG where
  S : Type

  SIG ==> Join(StackAggregate S,QueueAggregate S) with

    dequeue : () -> %
      ++ dequeue()$D creates an empty dequeue of type D.

    dequeue : List S -> %
      ++ dequeue([x,y,...,z]) creates a dequeue with first (top or front)
      ++ element x, second element y,...,and last (bottom or back) element z.

    height : % -> NonNegativeInteger
      ++ height(d) returns the number of elements in dequeue d.
      ++ Note that \axiom{height(d) = # d}.

    top_! : % -> S
      ++ top!(d) returns the element at the top (front) of the dequeue.

    bottom_! : % -> S
      ++ bottom!(d) returns the element at the bottom (back) of the dequeue.

    insertTop_! : (S,%) -> S
      ++ insertTop!(x,d) destructively inserts x into the dequeue d, that is,
      ++ at the top (front) of the dequeue.
      ++ The element previously at the top of the dequeue becomes the
      ++ second in the dequeue, and so on.

    insertBottom_! : (S,%) -> S
      ++ insertBottom!(x,d) destructively inserts x into the dequeue d
      ++ at the bottom (back) of the dequeue.

    extractTop_! : % -> S
      ++ extractTop!(d) destructively extracts the top (front) element
      ++ from the dequeue d.
      ++ Error: if d is empty.

    extractBottom_! : % -> S
      ++ extractBottom!(d) destructively extracts the bottom (back) element
      ++ from the dequeue d.
      ++ Error: if d is empty.

    reverse_! : % -> %
      ++ reverse!(d) destructively replaces d by its reverse dequeue, that is,
      ++ the top (front) element is now the bottom (back) element, and so on.
```

— **DQAGG.dotabb** —

```
"DQAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=DQAGG"];
"DQAGG" -> "SKAGG"
"DQAGG" -> "QUAGG"
```

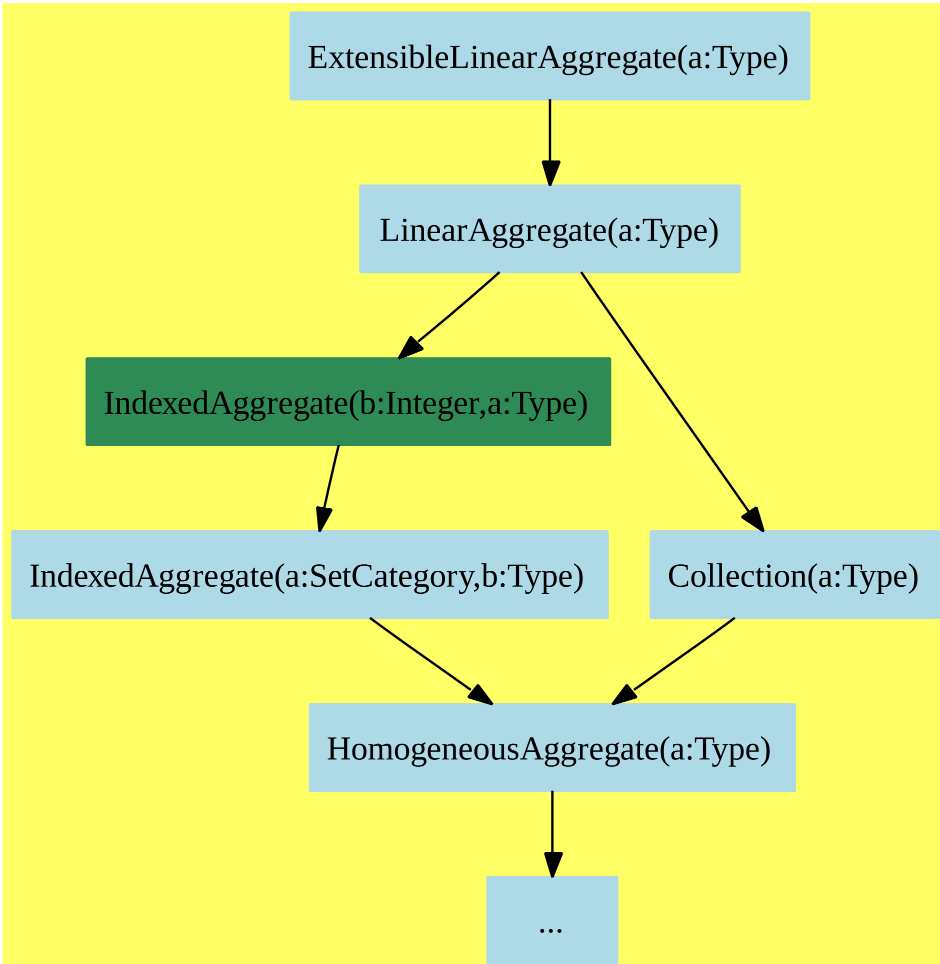
— **DQAGG.dotfull** —

```
"DequeueAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DQAGG"];
"DequeueAggregate(a:Type)" -> "StackAggregate(a:Type)"
"DequeueAggregate(a:Type)" -> "QueueAggregate(a:Type)"

"DequeueAggregate(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=DQAGG"];
"DequeueAggregate(a:SetCategory)" -> "DequeueAggregate(a:Type)"
```

— **DQAGG.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"DequeueAggregate(a:Type)" [color=lightblue];
"DequeueAggregate(a:Type)" -> "StackAggregate(a:Type)"
"DequeueAggregate(a:Type)" -> "QueueAggregate(a:Type)"

"StackAggregate(a:Type)" [color=lightblue];
"StackAggregate(a:Type)" -> "BagAggregate(a:Type)"

"QueueAggregate(a:Type)" [color=lightblue];
"QueueAggregate(a:Type)" -> "BagAggregate(a:Type)"

"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "..."

"..." [color=lightblue];
}
```

7.0.125 ExtensibleLinearAggregate (ELAGG)



— **ExtensibleLinearAggregate.input** —

```
)set break resume
)sys rm -f ExtensibleLinearAggregate.output
)spool ExtensibleLinearAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ExtensibleLinearAggregate
--R
--R ExtensibleLinearAggregate(S: Type) is a category constructor
--R Abbreviation for ExtensibleLinearAggregate is ELAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ELAGG
```

```
--R
--R------------------------------- Operations --------------------------------
--R concat : List(%) -> %                concat : (%,%) -> %
--R concat : (S,%) -> %                  concat : (%,S) -> %
--R concat! : (%,%) -> %                 concat! : (%,S) -> %
--R construct : List(S) -> %             copy : % -> %
--R delete : (%,Integer) -> %            delete! : (%,Integer) -> %
--R ?.? : (%,Integer) -> S               elt : (%,Integer,S) -> S
--R empty : () -> %                      empty? : % -> Boolean
--R entries : % -> List(S)               eq? : (%,%) -> Boolean
--R index? : (Integer,%) -> Boolean      indices : % -> List(Integer)
--R insert : (%,%,Integer) -> %          insert : (S,%,Integer) -> %
--R insert! : (%,%,Integer) -> %         insert! : (S,%,Integer) -> %
--R latex : % -> String if S has SETCAT  map : (((S,S) -> S),%,%) -> %
--R map : ((S -> S),%) -> %              new : (NonNegativeInteger,S) -> %
--R qelt : (%,Integer) -> S              remove! : ((S -> Boolean),%) -> %
--R sample : () -> %                     select! : ((S -> Boolean),%) -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT(INFORM)
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (%,UniversalSegment(Integer)) -> %
--R delete! : (%,UniversalSegment(Integer)) -> %
--R ?.? : (%,UniversalSegment(Integer)) -> %
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R first : % -> S if Integer has ORDSET
--R hash : % -> SingleInteger if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R merge! : (%,%) -> % if S has ORDSET
--R merge! : (((S,S) -> Boolean),%,%) -> %
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(S) if $ has finiteAggregate
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R remove! : (S,%) -> % if S has SETCAT
```

```
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R removeDuplicates! : % -> % if S has SETCAT
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment(Integer),S) -> S if $ has shallowlyMutable
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

— **ExtensibleLinearAggregate.help** —

```
====================================================================
ExtensibleLinearAggregate examples
====================================================================

An extensible aggregate is one which allows insertion and deletion of
entries. These aggregates are models of lists and streams which are
represented by linked structures so as to make insertion, deletion, and
concatenation efficient. However, access to elements of these
extensible aggregates is generally slow since access is made from the end.
See FlexibleArray for an exception.

See Also:
o )show ExtensibleLinearAggregate
o )show FlexibleArray
```

See:

⇒ "ListAggregate" (LSAGG) 8.0.138 on page 827
⇐ "LinearAggregate" (LNAGG) 6.0.111 on page 531

Exports:

any?	coerce	concat	concat!	construct
copy	convert	count	delete	delete!
elt	empty	empty?	entries	entry?
eval	every?	eq?	fill!	find
first	hash	index?	indices	insert
insert!	latex	less?	map	map!
maxIndex	member?	members	merge!	minIndex
more?	new	parts	qelt	qsetelt!
reduce	remove	remove!	removeDuplicates	removeDuplicates!
sample	select	select!	setelt	size?
swap!	#?	?=?	?.?	?~=?

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are directly exported but not implemented:

```
concat! : (%,S) -> %
delete! : (%,Integer) -> %
delete! : (%,UniversalSegment Integer) -> %
insert! : (%,%,Integer) -> %
insert! : (S,%,Integer) -> %
merge! : (((S,S) -> Boolean),%,%) -> %
remove! : ((S -> Boolean),%) -> %
removeDuplicates! : % -> % if S has SETCAT
select! : ((S -> Boolean),%) -> %
```

These are implemented by this category:

```
concat : (%,%) -> %
concat : (%,S) -> %
concat! : (%,%) -> %
delete : (%,Integer) -> %
delete : (%,UniversalSegment Integer) -> %
insert : (%,%,Integer) -> %
insert : (S,%,Integer) -> %
merge! : (%,%) -> % if S has ORDSET
remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
remove! : (S,%) -> % if S has SETCAT
removeDuplicates : % -> %
         if S has SETCAT and $ has finiteAggregate
select : ((S -> Boolean),%) -> % if $ has finiteAggregate
```

These exports come from (p531) LinearAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if S has SETCAT
concat : List % -> %
concat : (S,%) -> %
construct : List S -> %
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
convert : % -> InputForm if S has KONVERT INFORM
elt : (%,Integer,S) -> S
empty : () -> %
empty? : % -> Boolean
entries : % -> List S
entry? : (S,%) -> Boolean
         if $ has finiteAggregate and S has SETCAT
eq? : (%,%) -> Boolean
```

```
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
index? : (Integer,%) -> Boolean
indices : % -> List Integer
fill! : (%,S) -> % if $ has shallowlyMutable
find : ((S -> Boolean),%) -> Union(S,"failed")
first : % -> S if Integer has ORDSET
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : (((S,S) -> S),%,%) -> %
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
maxIndex : % -> Integer if Integer has ORDSET
new : (NonNegativeInteger,S) -> %
members : % -> List S if $ has finiteAggregate
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
qelt : (%,Integer) -> S
qsetelt! : (%,Integer,S) -> S
         if $ has shallowlyMutable
reduce : (((S,S) -> S),%) -> S
         if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S
         if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
         if S has SETCAT and $ has finiteAggregate
sample : () -> %
setelt : (%,Integer,S) -> S if $ has shallowlyMutable
setelt : (%,UniversalSegment Integer,S) -> S
         if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
swap! : (%,Integer,Integer) -> Void
         if $ has shallowlyMutable
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
?.? : (%,Integer) -> S
?.? : (%,UniversalSegment Integer) -> %
```

— ExtensibleLinearAggregate.html —

```
<body>
```

```
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ELAGG">
ExtensibleLinearAggregate (ELAGG)</a></h2>
</body>
```

———-

— category ELAGG ExtensibleLinearAggregate —

```
)abbrev category ELAGG ExtensibleLinearAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ An extensible aggregate is one which allows insertion and deletion of
++ entries. These aggregates are models of lists and streams which are
++ represented by linked structures so as to make insertion, deletion, and
++ concatenation efficient. However, access to elements of these
++ extensible aggregates is generally slow since access is made from the end.
++ See \spadtype{FlexibleArray} for an exception.

ExtensibleLinearAggregate(S) : Category == SIG where
  S : Type

  SIG ==> LinearAggregate S with

    shallowlyMutable

    concat_! : (%,S) -> %
      ++ concat!(u,x) destructively adds element x to the end of u.

    concat_! : (%,%) -> %
      ++ concat!(u,v) destructively appends v to the end of u.
      ++ v is unchanged

    delete_! : (%,Integer) -> %
      ++ delete!(u,i) destructively deletes the \axiom{i}th element of u.
      ++
      ++X Data:=Record(age:Integer,gender:String)
      ++X a1:AssociationList(String,Data):=table()
      ++X a1."tim":=[55,"male"]$Data
      ++X delete!(a1,1)

    delete_! : (%,UniversalSegment(Integer)) -> %
      ++ delete!(u,i..j) destructively deletes elements u.i through u.j.

    remove_! : (S->Boolean,%) -> %
      ++ remove!(p,u) destructively removes all elements x of
      ++ u such that \axiom{p(x)} is true.

    insert_! : (S,%,Integer) -> %
      ++ insert!(x,u,i) destructively inserts x into u at position i.

    insert_! : (%,%,Integer) -> %
```

```
      ++ insert!(v,u,i) destructively inserts aggregate v into u
      ++ at position i.

    merge_! : ((S,S)->Boolean,%,%) -> %
      ++ merge!(p,u,v) destructively merges u and v using predicate p.

    select_! : (S->Boolean,%) -> %
      ++ select!(p,u) destructively changes u by keeping only values
      ++ x such that \axiom{p(x)}.

    if S has SetCategory then

      remove_! : (S,%) -> %
        ++ remove!(x,u) destructively removes all values x from u.

      removeDuplicates_! : % -> %
        ++ removeDuplicates!(u) destructively removes duplicates from u.

    if S has OrderedSet then

      merge_! : (%,%) -> %
        ++ merge!(u,v) destructively merges u and v in ascending order.

   add

     delete(x:%, i:Integer) == delete_!(copy x, i)

     delete(x:%, i:UniversalSegment(Integer)) == delete_!(copy x, i)

     remove(f:S -> Boolean, x:%) == remove_!(f, copy x)

     insert(s:S, x:%, i:Integer) == insert_!(s, copy x, i)

     insert(w:%, x:%, i:Integer) == insert_!(copy w, copy x, i)

     select(f, x) == select_!(f, copy x)

     concat(x:%, y:%) == concat_!(copy x, y)

     concat(x:%, y:S) == concat_!(copy x, new(1, y))

     concat_!(x:%, y:S) == concat_!(x, new(1, y))

     if S has SetCategory then

       remove(s:S, x:%) == remove_!(s, copy x)

       remove_!(s:S, x:%) == remove_!(y +-> y = s, x)

       removeDuplicates(x:%) == removeDuplicates_!(copy x)

     if S has OrderedSet then

       merge_!(x, y) == merge_!(_<$S, x, y)
```

———-

— COQ ELAGG —

```
(* category ELAGG *)
(*

   delete : (%,Integer) -> %
   delete(x:%, i:Integer) == delete_!(copy x, i)

   delete : (%,UniversalSegment(Integer)) -> %
   delete(x:%, i:UniversalSegment(Integer)) == delete_!(copy x, i)

   remove : ((S -> Boolean),%) -> %
   remove(f:S -> Boolean, x:%) == remove_!(f, copy x)

   insert : (S,%,Integer) -> %
   insert(s:S, x:%, i:Integer) == insert_!(s, copy x, i)

   insert : (%,%,Integer) -> %
   insert(w:%, x:%, i:Integer) == insert_!(copy w, copy x, i)

   select : ((S -> Boolean),%) -> %
   select(f, x) == select_!(f, copy x)

   concat : (%,%) -> %
   concat(x:%, y:%) == concat_!(copy x, y)

   concat : (%,S) -> %
   concat(x:%, y:S) == concat_!(copy x, new(1, y))

   concat_!: (%,S) -> %
   concat_!(x:%, y:S) == concat_!(x, new(1, y))

   if S has SetCategory then

     remove : (S,%) -> %
     remove(s:S, x:%) == remove_!(s, copy x)

     remove_!: (S->Boolean,%) -> %
     remove_!(s:S, x:%) == remove_!(y +-> y = s, x)

     removeDuplicates : % -> %
     removeDuplicates(x:%) == removeDuplicates_!(copy x)

   if S has OrderedSet then

     merge! : (%,%) -> %
     merge_!(x, y) == merge_!(_<$S, x, y)

*)
```

———-

— **ELAGG.dotabb** —

```
"ELAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=ELAGG"];
"ELAGG" -> "LNAGG"
```

———-

— **ELAGG.dotfull** —

```
"ExtensibleLinearAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ELAGG"];
"ExtensibleLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"
```

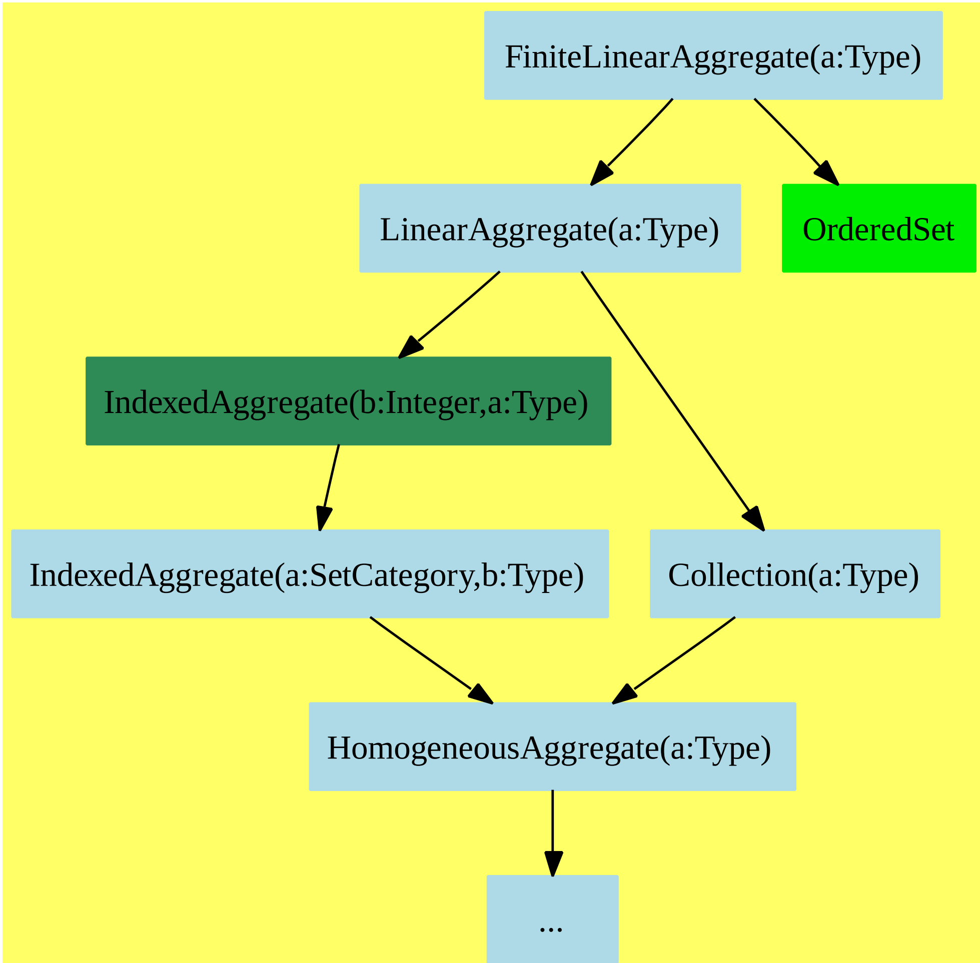
———-

— **ELAGG.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"ExtensibleLinearAggregate(a:Type)" [color=lightblue];
"ExtensibleLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "Collection(a:Type)"

"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" ->
    "IndexedAggregate(a:SetCategory,b:Type)"

"IndexedAggregate(a:SetCategory,b:Type)" [color=lightblue];
"IndexedAggregate(a:SetCategory,b:Type)" ->
    "HomogeneousAggregate(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "..."

"..." [color=lightblue];
}
```

———-

7.0.126 FiniteLinearAggregate (FLAGG)



— **FiniteLinearAggregate.input** —

```
)set break resume
)sys rm -f FiniteLinearAggregate.output
)spool FiniteLinearAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteLinearAggregate
--R
--R FiniteLinearAggregate(S: Type) is a category constructor
--R Abbreviation for FiniteLinearAggregate is FLAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FLAGG
```

```
--R
--R------------------------------- Operations --------------------------------
--R concat : List(%) -> %                concat : (%,%) -> %
--R concat : (S,%) -> %                  concat : (%,S) -> %
--R construct : List(S) -> %             copy : % -> %
--R delete : (%,Integer) -> %            ?.? : (%,Integer) -> S
--R elt : (%,Integer,S) -> S             empty : () -> %
--R empty? : % -> Boolean                entries : % -> List(S)
--R eq? : (%,%) -> Boolean               index? : (Integer,%) -> Boolean
--R indices : % -> List(Integer)         insert : (%,%,Integer) -> %
--R insert : (S,%,Integer) -> %          latex : % -> String if S has SETCAT
--R map : (((S,S) -> S),%,%) -> %        map : ((S -> S),%) -> %
--R max : (%,%) -> % if S has ORDSET     min : (%,%) -> % if S has ORDSET
--R new : (NonNegativeInteger,S) -> %    qelt : (%,Integer) -> S
--R reverse : % -> %                     sample : () -> %
--R sort : % -> % if S has ORDSET        sort : (((S,S) -> Boolean),%) -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?<? : (%,%) -> Boolean if S has ORDSET
--R ?<=? : (%,%) -> Boolean if S has ORDSET
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R ?>? : (%,%) -> Boolean if S has ORDSET
--R ?>=? : (%,%) -> Boolean if S has ORDSET
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT(INFORM)
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (%,UniversalSegment(Integer)) -> %
--R ?.? : (%,UniversalSegment(Integer)) -> %
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R first : % -> S if Integer has ORDSET
--R hash : % -> SingleInteger if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R merge : (%,%) -> % if S has ORDSET
--R merge : (((S,S) -> Boolean),%,%) -> %
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(S) if $ has finiteAggregate
--R position : (S,%,Integer) -> Integer if S has SETCAT
--R position : (S,%) -> Integer if S has SETCAT
--R position : ((S -> Boolean),%) -> Integer
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
```

```
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment(Integer),S) -> S if $ has shallowlyMutable
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort! : % -> % if S has ORDSET and $ has shallowlyMutable
--R sort! : (((S,S) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if S has ORDSET
--R sorted? : (((S,S) -> Boolean),%) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

— **FiniteLinearAggregate.help** —

```
====================================================================
FiniteLinearAggregate examples
====================================================================

A finite linear aggregate is a linear aggregate of finite length.
The finite property of the aggregate adds several exports to the
list of exports from LinearAggregate such as reverse, sort, and so on.

See Also:
o )show FiniteLinearAggregate
```

See:

⇒ "OneDimensionalArrayAggregate" (A1AGG) 8.0.141 on page 854
⇒ "ListAggregate" (LSAGG) 8.0.138 on page 827
⇐ "LinearAggregate" (LNAGG) 6.0.111 on page 531
⇐ "OrderedSet" (ORDSET) 4.0.80 on page 326

Exports:

any?	coerce	concat	construct	convert
copy	copyInto!	count	delete	elt
empty	empty?	entries	entry?	eq?
eval	every?	fill!	find	first
hash	index?	indices	insert	latex
less?	map	map!	max	maxIndex
member?	members	merge	min	minIndex
more?	new	parts	position	qelt
qsetelt!	reduce	remove	removeDuplicates	reverse
reverse!	sample	select	setelt	size?
sort	sort!	sorted?	swap!	#?
?.?	?<?	?<=?	?=?	?>?
?>=?	?~=?			

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

Attributes Used:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
copyInto! : (%,%,Integer) -> %
         if $ has shallowlyMutable
merge : (((S,S) -> Boolean),%,%) -> %
sorted? : (((S,S) -> Boolean),%) -> Boolean
position : (S,%,Integer) -> Integer if S has SETCAT
position : ((S -> Boolean),%) -> Integer
reverse! : % -> % if $ has shallowlyMutable
sort! : (((S,S) -> Boolean),%) -> %
         if $ has shallowlyMutable
```

These are implemented by this category:

```
merge : (%,%) -> % if S has ORDSET
position : (S,%) -> Integer if S has SETCAT
reverse : % -> %
sort : % -> % if S has ORDSET
sort : (((S,S) -> Boolean),%) -> %
sorted? : % -> Boolean if S has ORDSET
sort! : % -> %
         if S has ORDSET and $ has shallowlyMutable
```

These exports come from (p531) LinearAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if S has SETCAT
concat : List % -> %
concat : (%,%) -> %
concat : (S,%) -> %
concat : (%,S) -> %
construct : List S -> %
```

```
convert : % -> InputForm if S has KONVERT INFORM
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
delete : (%,UniversalSegment Integer) -> %
delete : (%,Integer) -> %
elt : (%,Integer,S) -> S
empty : () -> %
empty? : % -> Boolean
entries : % -> List S
entry? : (S,%) -> Boolean
         if $ has finiteAggregate and S has SETCAT
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
fill! : (%,S) -> % if $ has shallowlyMutable
find : ((S -> Boolean),%) -> Union(S,"failed")
first : % -> S if Integer has ORDSET
hash : % -> SingleInteger if S has SETCAT
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (S,%,Integer) -> %
insert : (%,%,Integer) -> %
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : (((S,S) -> S),%,%) -> %
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
maxIndex : % -> Integer if Integer has ORDSET
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
new : (NonNegativeInteger,S) -> %
parts : % -> List S if $ has finiteAggregate
qelt : (%,Integer) -> S
qsetelt! : (%,Integer,S) -> S
         if $ has shallowlyMutable
reduce : (((S,S) -> S),%) -> S
         if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S
         if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
```

```
          if S has SETCAT and $ has finiteAggregate
 remove : ((S -> Boolean),%) -> %
          if $ has finiteAggregate
 remove : (S,%) -> %
          if S has SETCAT and $ has finiteAggregate
 removeDuplicates : % -> %
          if S has SETCAT and $ has finiteAggregate
 sample : () -> %
 select : ((S -> Boolean),%) -> %
          if $ has finiteAggregate
 setelt : (%,Integer,S) -> S
          if $ has shallowlyMutable
 setelt : (%,UniversalSegment Integer,S) -> S
          if $ has shallowlyMutable
 size? : (%,NonNegativeInteger) -> Boolean
 swap! : (%,Integer,Integer) -> Void
          if $ has shallowlyMutable
 #? : % -> NonNegativeInteger
          if $ has finiteAggregate
 ?.? : (%,Integer) -> S
 ?.? : (%,UniversalSegment Integer) -> %
 ?~=? : (%,%) -> Boolean if S has SETCAT
 ?=? : (%,%) -> Boolean if S has SETCAT
```

These exports come from (p326) OrderedSet:

```
 max : (%,%) -> % if S has ORDSET
 min : (%,%) -> % if S has ORDSET
 ?<? : (%,%) -> Boolean if S has ORDSET
 ?<=? : (%,%) -> Boolean if S has ORDSET
 ?>? : (%,%) -> Boolean if S has ORDSET
 ?>=? : (%,%) -> Boolean if S has ORDSET
```

— FiniteLinearAggregate.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FLAGG">
FiniteLinearAggregate (FLAGG)</a></h2>
</body>
```

— category FLAGG FiniteLinearAggregate —

```
)abbrev category FLAGG FiniteLinearAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ A finite linear aggregate is a linear aggregate of finite length.
++ The finite property of the aggregate adds several exports to the
++ list of exports from \spadtype{LinearAggregate} such as
++ \spadfun{reverse}, \spadfun{sort}, and so on.
```

```
FiniteLinearAggregate(S) : Category == SIG where
  S : Type

  SIG ==> LinearAggregate S with

    finiteAggregate

    merge : ((S,S)->Boolean,%,%) -> %
      ++ merge(p,a,b) returns an aggregate c which merges \axiom{a} and b.
      ++ The result is produced by examining each element x of \axiom{a}
      ++ and y of b successively. If \axiom{p(x,y)} is true, then x is
      ++ inserted into the result; otherwise y is inserted. If x is
      ++ chosen, the next element of \axiom{a} is examined, and so on.
      ++ When all the elements of one aggregate are examined, the
      ++ remaining elements of the other are appended.
      ++ For example, \axiom{merge(<,[1,3],[2,7,5])} returns
      ++ \axiom{[1,2,3,7,5]}.

    reverse : % -> %
      ++ reverse(a) returns a copy of \axiom{a} with elements
      ++ in reverse order.

    sort : ((S,S)->Boolean,%) -> %
      ++ sort(p,a) returns a copy of \axiom{a} sorted using total ordering
      ++ predicate p.

    sorted? : ((S,S)->Boolean,%) -> Boolean
      ++ sorted?(p,a) tests if \axiom{a} is sorted according to predicate p.

    position : (S->Boolean, %) -> Integer
      ++ position(p,a) returns the index i of the first x in \axiom{a}
      ++ such that \axiom{p(x)} is true, and \axiom{minIndex(a) - 1}
      ++ if there is no such x.

    if S has SetCategory then

      position : (S, %) -> Integer
        ++ position(x,a) returns the index i of the first occurrence of
        ++ x in a, and \axiom{minIndex(a) - 1} if there is no such x.

      position : (S,%,Integer) -> Integer
        ++ position(x,a,n) returns the index i of the first occurrence of
        ++ x in \axiom{a} where \axiom{i >= n}, and \axiom{minIndex(a) - 1}
        ++ if no such x is found.

    if S has OrderedSet then

      OrderedSet

      merge : (%,%) -> %
        ++ merge(u,v) merges u and v in ascending order.
        ++ Note that \axiom{merge(u,v) = merge(<=,u,v)}.

      sort : % -> %
```

```
        ++ sort(u) returns an u with elements in ascending order.
        ++ Note that \axiom{sort(u) = sort(<=,u)}.

      sorted? : % -> Boolean
        ++ sorted?(u) tests if the elements of u are in ascending order.

    if % has shallowlyMutable then

      copyInto_! : (%,%,Integer) -> %
        ++ copyInto!(u,v,i) returns aggregate u containing a copy of
        ++ v inserted at element i.

      reverse_! : % -> %
        ++ reverse!(u) returns u with its elements in reverse order.

      sort_! : ((S,S)->Boolean,%) -> %
        ++ sort!(p,u) returns u with its elements ordered by p.

      if S has OrderedSet then

        sort_! : % -> %
          ++ sort!(u) returns u with its elements in ascending order.

   add

     if S has SetCategory then

       position(x:S, t:%) == position(x, t, minIndex t)

     if S has OrderedSet then

       sorted? l == sorted?((x,y) +-> x < y or x = y, l)

       merge(x, y) == merge(_<$S, x, y)

       sort l == sort(_<$S, l)

     if % has shallowlyMutable then

       reverse x == reverse_! copy x

       sort(f, l) == sort_!(f, copy l)

       if S has OrderedSet then

         sort_! l == sort_!(_<$S, l)
```

— COQ FLAGG —

```
(* category FLAGG *)
(*
```

```
    if S has SetCategory then

      position: (S, %) -> Integer
      position(x:S, t:%) == position(x, t, minIndex t)

    if S has OrderedSet then

      sorted?: % -> Boolean
      sorted? l          == sorted?((x,y) +-> x < y or x = y, l)

      merge: (%,%) -> %
      merge(x, y) == merge(_<$S, x, y)

      sort: % -> %
      sort l          == sort(_<$S, l)

    if % has shallowlyMutable then

      reverse: % -> %
      reverse x         == reverse_! copy x

      sort: ((S,S)->Boolean,%) -> %
      sort(f, l) == sort_!(f, copy l)

      if S has OrderedSet then

        sort_!: % -> %
        sort_! l == sort_!(_<$S, l)

*)
```

————

— FLAGG.dotabb —

```
"FLAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=FLAGG"];
"FLAGG" -> "LNAGG"
```

————

— FLAGG.dotfull —

```
"FiniteLinearAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FLAGG"];
"FiniteLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"
```

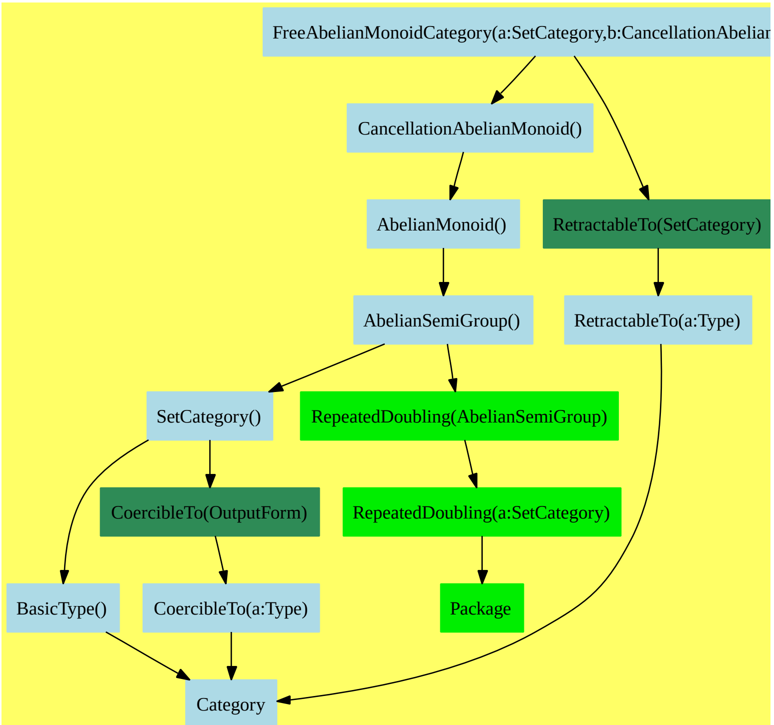
————

— FLAGG.dotpic —

```
digraph pic {
```

```
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FiniteLinearAggregate(a:Type)" [color=lightblue];
"FiniteLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"
"FiniteLinearAggregate(a:Type)" -> "OrderedSet"

"OrderedSet" [color="#00EE00"];

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "Collection(a:Type)"

"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" ->
    "IndexedAggregate(a:SetCategory,b:Type)"

"IndexedAggregate(a:SetCategory,b:Type)" [color=lightblue];
"IndexedAggregate(a:SetCategory,b:Type)" ->
    "HomogeneousAggregate(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "..."

"..." [color=lightblue];
}
```

7.0.127 FreeAbelianMonoidCategory (FAMONC)



— **FreeAbelianMonoidCategory.input** —

```
)set break resume
)sys rm -f FreeAbelianMonoidCategory.output
)spool FreeAbelianMonoidCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FreeAbelianMonoidCategory
--R
--R FreeAbelianMonoidCategory(S: SetCategory,E: CancellationAbelianMonoid) is a category constructor
--R Abbreviation for FreeAbelianMonoidCategory is FAMONC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FAMONC
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (E,S) -> %                      ?*? : (NonNegativeInteger,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?+? : (S,%) -> %
--R ?+? : (%,%) -> %                      ?=? : (%,%) -> Boolean
--R 0 : () -> %                           coefficient : (S,%) -> E
--R coerce : S -> %                       coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R mapCoef : ((E -> E),%) -> %           mapGen : ((S -> S),%) -> %
```

```
--R nthCoef : (%,Integer) -> E            nthFactor : (%,Integer) -> S
--R retract : % -> S                      sample : () -> %
--R size : % -> NonNegativeInteger        zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R highCommonTerms : (%,%) -> % if E has OAMON
--R retractIfCan : % -> Union(S,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R terms : % -> List(Record(gen: S,exp: E))
--R
--E 1

)spool
)lisp (bye)
```

— FreeAbelianMonoidCategory.help —

```
====================================================================
FreeAbelianMonoidCategory examples
====================================================================

A free abelian monoid on a set S is the monoid of finite sums of
the form reduce(+,[ni * si]) where the si's are in S, and the ni's
are in a given abelian monoid. The operation is commutative.

See Also:
o )show FreeAbelianMonoidCategory
```

See:

⇐ "CancellationAbelianMonoid" (CABMON) 6.0.107 on page 508
⇐ "RetractableTo" (RETRACT) 2.0.37 on page 110

Exports:

0	coefficient	coerce	hash	highCommonTerms
latex	mapCoef	mapGen	nthCoef	nthFactor
retract	retractIfCan	sample	size	subtractIfCan
terms	zero?	?*?	?+?	?=?
?~=?				

These are directly exported but not implemented:

```
coefficient : (S,%) -> E
highCommonTerms : (%,%) -> % if E has OAMON
mapCoef : ((E -> E),%) -> %
mapGen : ((S -> S),%) -> %
nthCoef : (%,Integer) -> E
nthFactor : (%,Integer) -> S
size : % -> NonNegativeInteger
terms : % -> List Record(gen: S,exp: E)
```

```
?+? : (S,%) -> %
?*? : (E,S) -> %
```

These exports come from (p508) CancellationAbelianMonoid():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
```

These exports come from (p110) RetractableTo(SetCategory):

```
coerce : S -> %
retract : % -> S
retractIfCan : % -> Union(S,"failed")
```

— FreeAbelianMonoidCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FAMONC">
FreeAbelianMonoidCategory (FAMONC)</a></h2>
</body>
```

———-

— category FAMONC FreeAbelianMonoidCategory —

```
)abbrev category FAMONC FreeAbelianMonoidCategory
++ Category for free abelian monoid on any set of generators
++ Author: Manuel Bronstein
++ Date Created: November 1989
++ Date Last Updated: 6 June 1991
++ Description:
++ A free abelian monoid on a set S is the monoid of finite sums of
++ the form \spad{reduce(+,[ni * si])} where the si's are in S, and the ni's
++ are in a given abelian monoid. The operation is commutative.

FreeAbelianMonoidCategory(S,E) : Category == SIG where
  S : SetCategory
  E : CancellationAbelianMonoid

  SIG ==> Join(CancellationAbelianMonoid, RetractableTo S) with

    "+" : (S, $) -> $
      ++ s + x returns the sum of s and x.

    "*" : (E, S) -> $
```

```
      ++ e * s returns e times s.

    size : $ -> NonNegativeInteger
      ++ size(x) returns the number of terms in x.
      ++ mapGen(f, a1\^e1 ... an\^en) returns
      ++\spad{f(a1)\^e1 ... f(an)\^en}.

    terms : $ -> List Record(gen: S, exp: E)
      ++ terms(e1 a1 + ... + en an) returns \spad{[[a1, e1],...,[an, en]]}.

    nthCoef : ($, Integer) -> E
      ++ nthCoef(x, n) returns the coefficient of the n^th term of x.

    nthFactor : ($, Integer) -> S
      ++ nthFactor(x, n) returns the factor of the n^th term of x.

    coefficient : (S, $) -> E
      ++ coefficient(s, e1 a1 + ... + en an) returns ei such that
      ++ ai = s, or 0 if s is not one of the ai's.

    mapCoef : (E -> E, $) -> $
      ++ mapCoef(f, e1 a1 +...+ en an) returns
      ++ \spad{f(e1) a1 +...+ f(en) an}.

    mapGen : (S -> S, $) -> $
      ++ mapGen(f, e1 a1 +...+ en an) returns
      ++ \spad{e1 f(a1) +...+ en f(an)}.

    if E has OrderedAbelianMonoid then

      highCommonTerms : ($, $) -> $
        ++ highCommonTerms(e1 a1 + ... + en an, f1 b1 + ... + fm bm)
        ++ returns \spad{reduce(+,[max(ei, fi) ci])}
        ++ where ci ranges in the intersection
        ++ of \spad{{a1,...,an}} and \spad{{b1,...,bm}}.
```

————

— FAMONC.dotabb —

```
"FAMONC"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=FAMONC"];
"FAMONC" -> "CABMON"
"FAMONC" -> "RETRACT"
```

————

— FAMONC.dotfull —

```
"FreeAbelianMonoidCategory(a:SetCategory,b:CancellationAbelianMonoid)"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=FAMONC"];
"FreeAbelianMonoidCategory(a:SetCategory,b:CancellationAbelianMonoid)" ->
```

```
   "CancellationAbelianMonoid()"
"FreeAbelianMonoidCategory(a:SetCategory,b:CancellationAbelianMonoid)" ->
   "RetractableTo(SetCategory)"
```

———-

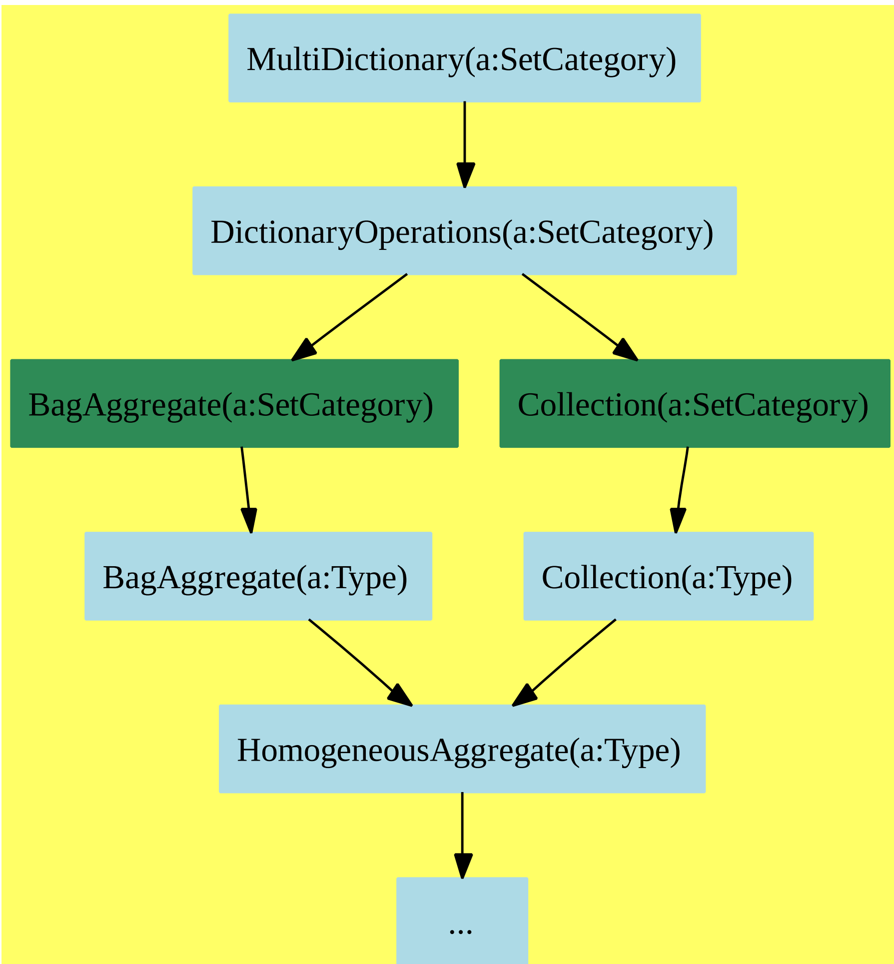
— FAMONC.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FreeAbelianMonoidCategory(a:SetCategory,b:CancellationAbelianMonoid)"
  [color=lightblue];
"FreeAbelianMonoidCategory(a:SetCategory,b:CancellationAbelianMonoid)" ->
   "CancellationAbelianMonoid()"
"FreeAbelianMonoidCategory(a:SetCategory,b:CancellationAbelianMonoid)" ->
   "RetractableTo(SetCategory)"

"RetractableTo(SetCategory)" [color=seagreen];
"RetractableTo(SetCategory)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SetCategory()"
"AbelianSemiGroup()" -> "RepeatedDoubling(AbelianSemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" ->
    "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"RepeatedDoubling(AbelianSemiGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianSemiGroup)" -> "RepeatedDoubling(a:SetCategory)"
```

```
"RepeatedDoubling(a:SetCategory)" [color="#00EE00"];
"RepeatedDoubling(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];
}
```

———-

7.0.128 MultiDictionary (MDAGG)



— **MultiDictionary.input** —

```
)set break resume
)sys rm -f MultiDictionary.output
)spool MultiDictionary.output
```

```
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MultiDictionary
--R
--R MultiDictionary(S: SetCategory) is a category constructor
--R Abbreviation for MultiDictionary is MDAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for MDAGG
--R
--R------------------------------ Operations -------------------------------
--R bag : List(S) -> %                    construct : List(S) -> %
--R copy : % -> %                         dictionary : List(S) -> %
--R dictionary : () -> %                  empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R extract! : % -> S                     insert! : (S,%) -> %
--R inspect : % -> S                      latex : % -> String if S has SETCAT
--R map : ((S -> S),%) -> %               removeDuplicates! : % -> %
--R sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT(INFORM)
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R duplicates : % -> List(Record(entry: S,count: NonNegativeInteger))
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R hash : % -> SingleInteger if S has SETCAT
--R insert! : (S,%,NonNegativeInteger) -> %
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(S) if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R remove! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (S,%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
```

```
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———-

— MultiDictionary.help —

```
====================================================================
MultiDictionary examples
====================================================================

A multi-dictionary is a dictionary which may contain duplicates.
As for any dictionary, its size is assumed large so that
copying (non-destructive) operations are generally to be avoided.

See Also:
o )show MultiDictionary
```

———-

See:

⇒ “MultisetAggregate” (MSETAGG) 8.0.139 on page 843
⇐ “DictionaryOperations” (DIOPS) 6.0.108 on page 512

Exports:

any?	bag	coerce	construct	convert
copy	count	dictionary	duplicates	empty
empty?	eq?	eval	every?	extract!
find	hash	insert!	inspect	latex
less?	map	map!	member?	members
more?	parts	reduce	remove	remove!
removeDuplicates	removeDuplicates!	sample	select	select!
size?	#?	?=?	?~=?	

Attributes exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are directly exported but not implemented:

```
insert! : (S,%,NonNegativeInteger) -> %
removeDuplicates! : % -> %
duplicates : % -> List Record(entry:S,count:NonNegativeInteger)
```

These exports come from (p512) DictionaryOperations(S:SetCategory):

```
any? : ((S -> Boolean),%) -> Boolean
```

```
        if $ has finiteAggregate
bag : List S -> %
coerce : % -> OutputForm if S has SETCAT
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
copy : % -> %
count : (S,%) -> NonNegativeInteger
        if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
        if $ has finiteAggregate
dictionary : () -> %
dictionary : List S -> %
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
        if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
        if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
        if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
        if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
        if $ has finiteAggregate
extract! : % -> S
find : ((S -> Boolean),%) -> Union(S,"failed")
hash : % -> SingleInteger if S has SETCAT
insert! : (S,%) -> %
inspect : % -> S
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
        if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
        if S has SETCAT and $ has finiteAggregate
remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
remove : (S,%) -> %
        if S has SETCAT and $ has finiteAggregate
removeDuplicates : % -> %
        if S has SETCAT and $ has finiteAggregate
remove! : ((S -> Boolean),%) -> % if $ has finiteAggregate
remove! : (S,%) -> % if $ has finiteAggregate
select : ((S -> Boolean),%) -> % if $ has finiteAggregate
sample : () -> %
select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
```

```
 #? : % -> NonNegativeInteger
          if $ has finiteAggregate
 ?=? : (%,%) -> Boolean if S has SETCAT
 ?~=? : (%,%) -> Boolean if S has SETCAT
```

— MultiDictionary.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#MDAGG">
MultiDictionary (MDAGG)</a></h2>
</body>
```

————

— category MDAGG MultiDictionary —

```
)abbrev category MDAGG MultiDictionary
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ A multi-dictionary is a dictionary which may contain duplicates.
++ As for any dictionary, its size is assumed large so that
++ copying (non-destructive) operations are generally to be avoided.

MultiDictionary(S) : Category == SIG where
  S : SetCategory

  SIG ==> DictionaryOperations S with

    insert_! : (S,%,NonNegativeInteger) -> %
      ++ insert!(x,d,n) destructively inserts n copies of x into dictionary d.

    removeDuplicates_! : % -> %
      ++ removeDuplicates!(d) destructively removes any duplicate values
      ++ in dictionary d.

    duplicates : % -> List Record(entry:S,count:NonNegativeInteger)
      ++ duplicates(d) returns a list of values which have duplicates in d
```

————

— MDAGG.dotabb —

```
"MDAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=MDAGG"];
"MDAGG" -> "DIOPS"
```

————

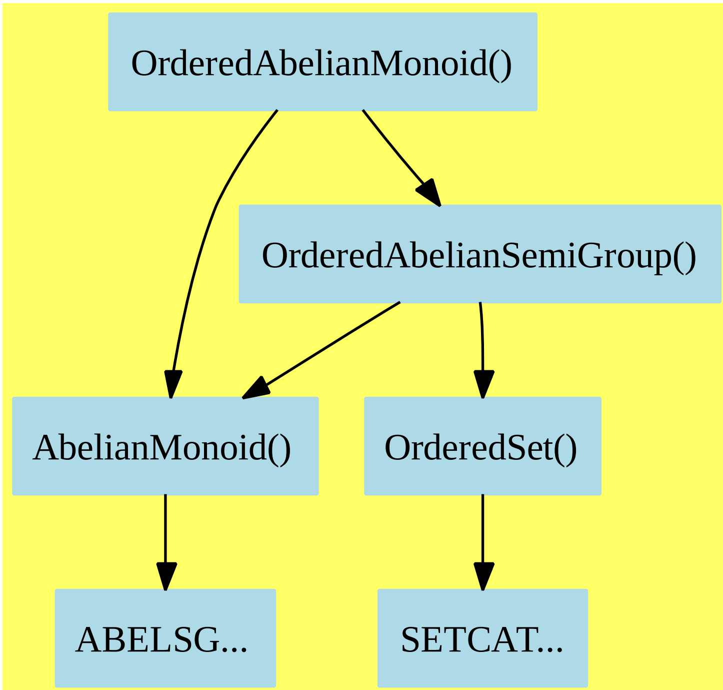
— MDAGG.dotfull —

```
"MultiDictionary(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MDAGG"];
"MultiDictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"
```

— MDAGG.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"MultiDictionary(a:SetCategory)" [color=lightblue];
"MultiDictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"

"DictionaryOperations(a:SetCategory)" [color=lightblue];
"DictionaryOperations(a:SetCategory)" -> "BagAggregate(a:SetCategory)"
"DictionaryOperations(a:SetCategory)" -> "Collection(a:SetCategory)"

"BagAggregate(a:SetCategory)" [color=seagreen];
"BagAggregate(a:SetCategory)" -> "BagAggregate(a:Type)"

"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

"Collection(a:SetCategory)" [color=seagreen];
"Collection(a:SetCategory)" -> "Collection(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "..."

"..." [color=lightblue];
}
```

7.0.129 OrderedAbelianMonoid (OAMON)



— **OrderedAbelianMonoid.input** —

```
)set break resume
)sys rm -f OrderedAbelianMonoid.output
)spool OrderedAbelianMonoid.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderedAbelianMonoid
--R
--R OrderedAbelianMonoid is a category constructor
--R Abbreviation for OrderedAbelianMonoid is OAMON
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for OAMON
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                      ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean               ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean                ?>=? : (%,%) -> Boolean
--R 0 : () -> %                           coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R max : (%,%) -> %                      min : (%,%) -> %
--R sample : () -> %                      zero? : % -> Boolean
```

```
--R ?~=? : (%,%) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

———-

— **OrderedAbelianMonoid.help** —

```
====================================================================
OrderedAbelianMonoid examples
====================================================================

Ordered sets which are also abelian monoids, such that the addition
preserves the ordering.

See Also:
o )show OrderedAbelianMonoid
```

———-

See:

⇒ "OrderedCancellationAbelianMonoid" (OCAMON) 8.0.142 on page 870
⇐ "AbelianMonoid" (ABELMON) 5.0.90 on page 383
⇐ "OrderedAbelianSemiGroup" (OASGP) 6.0.113 on page 590

Exports:

0	coerce	hash	latex	max
min	sample	zero?	?*?	?+?
?<?	?<=?	?=?	?>?	?>=?
?~=?				

These exports come from (p590) OrderedAbelianSemiGroup():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

These exports come from (p383) AbelianMonoid():

```
0 : () -> %
sample : () -> %
zero? : % -> Boolean
?*? : (PositiveInteger,%) -> %
```

```
 ?+? : (%,%) -> %
 ?*? : (NonNegativeInteger,%) -> %
```

— **OrderedAbelianMonoid.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#OAMON">
OrderedAbelianMonoid (OAMON)</a></h2>
</body>
```

————

— **category OAMON OrderedAbelianMonoid** —

```
)abbrev category OAMON OrderedAbelianMonoid
++ Description:
++ Ordered sets which are also abelian monoids, such that the addition
++ preserves the ordering.

OrderedAbelianMonoid() : Category == SIG where

  SIG ==> Join(OrderedAbelianSemiGroup, AbelianMonoid)
```

————

— **OAMON.dotabb** —

```
"OAMON" [color=lightblue,href="bookvol10.2.pdf#nameddest=OAMON"];
"OAMON" -> "OASGP"
"OAMON" -> "ABELMON"
```

————

— **OAMON.dotfull** —

```
"OrderedAbelianMonoid()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OAMON"];
"OrderedAbelianMonoid()" -> "OrderedAbelianSemiGroup()"
"OrderedAbelianMonoid()" -> "AbelianMonoid()"
```

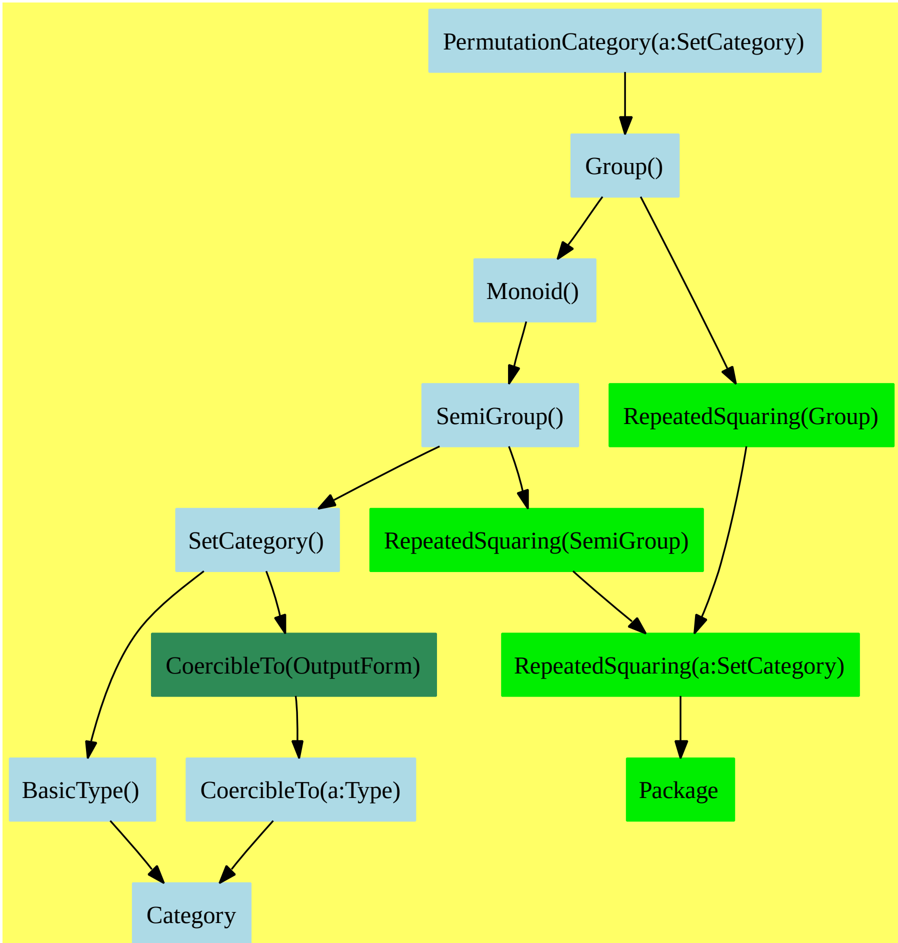
————

— **OAMON.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"OrderedAbelianMonoid()" [color=lightblue];
"OrderedAbelianMonoid()" -> "OrderedAbelianSemiGroup()"
```

```
"OrderedAbelianMonoid()" -> "AbelianMonoid()"

"OrderedAbelianSemiGroup()" [color=lightblue];
"OrderedAbelianSemiGroup()" -> "OrderedSet()"
"OrderedAbelianSemiGroup()" -> "AbelianMonoid()"

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SETCAT..."

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "ABELSG..."

"SETCAT..." [color=lightblue];
"ABELSG..." [color=lightblue];
}
```

7.0.130 PermutationCategory (PERMCAT)



— **PermutationCategory.input** —

```
)set break resume
)sys rm -f PermutationCategory.output
)spool PermutationCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PermutationCategory
--R
--R PermutationCategory(S: SetCategory) is a category constructor
--R Abbreviation for PermutationCategory is PERMCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PERMCAT
--R
```

```
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                       ?**? : (%,Integer) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?/? : (%,%) -> %                       ?<? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                 1 : () -> %
--R ?^? : (%,Integer) -> %                 ?^? : (%,NonNegativeInteger) -> %
--R ?^? : (%,PositiveInteger) -> %         coerce : % -> OutputForm
--R commutator : (%,%) -> %                conjugate : (%,%) -> %
--R cycle : List(S) -> %                   cycles : List(List(S)) -> %
--R ?.? : (%,S) -> S                       eval : (%,S) -> S
--R hash : % -> SingleInteger              inv : % -> %
--R latex : % -> String                    one? : % -> Boolean
--R orbit : (%,S) -> Set(S)                recip : % -> Union(%,"failed")
--R sample : () -> %                       ?~=? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean if S has ORDSET or S has FINITE
--R ?>? : (%,%) -> Boolean if S has ORDSET or S has FINITE
--R ?>=? : (%,%) -> Boolean if S has ORDSET or S has FINITE
--R max : (%,%) -> % if S has ORDSET or S has FINITE
--R min : (%,%) -> % if S has ORDSET or S has FINITE
--R
--E 1

)spool
)lisp (bye)
```

———-

— **PermutationCategory.help** —

```
====================================================================
PermutationCategory examples
====================================================================

PermutationCategory provides a categorial environment for subgroups
of bijections of a set (permutations)

See Also:
o )show PermutationCategory
```

———-

See:

⇐ "Group" (GROUP) 6.0.110 on page 525

Exports:

1	coerce	commutator	conjugate	cycle
cycles	eval	hash	inv	latex
max	min	one?	orbit	recip
sample	?^?	?.?	?~=?	?**?
?<?	?<=?	?>?	?>=?	?*?
?/?	?=?			

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

These are directly exported but not implemented:

```
cycle : List S -> %
cycles : List List S -> %
eval : (%,S) -> S
orbit : (%,S) -> Set S
?<? : (%,%) -> Boolean
?.? : (%,S) -> S
```

These exports come from (p525) Group():

```
1 : () -> %
coerce : % -> OutputForm
commutator : (%,%) -> %
conjugate : (%,%) -> %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
?/? : (%,%) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,Integer) -> %
?*? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,Integer) -> %
```

These exports come from (p326) OrderedSet():

```
max : (%,%) -> % if S has ORDSET or S has FINITE
min : (%,%) -> % if S has ORDSET or S has FINITE
?>? : (%,%) -> Boolean if S has ORDSET or S has FINITE
?<=? : (%,%) -> Boolean if S has ORDSET or S has FINITE
?>=? : (%,%) -> Boolean if S has ORDSET or S has FINITE
```

— **PermutationCategory.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#PERMCAT">
PermutationCategory (PERMCAT)</a></h2>
</body>
```

———-

— **category PERMCAT PermutationCategory** —

```
)abbrev category PERMCAT PermutationCategory
```

```
++ Authors:  Holger Gollan, Johannes Grabmeier, Gerhard Schneider
++ Date Created: 27 July 1989
++ Date Last Updated: 29 March 1990
++ Description:
++ PermutationCategory provides a categorial environment
++ for subgroups of bijections of a set (that is, permutations)

PermutationCategory(S) : Category == SIG where
  S : SetCategory

  SIG ==> Group with

    cycle : List S -> %
      ++ cycle(ls) coerces a cycle ls, that is, a list with not
      ++ repetitions to a permutation, which maps ls.i to
      ++ ls.i+1, indices modulo the length of the list.
      ++ Error: if repetitions occur.

    cycles : List List S -> %
      ++ cycles(lls) coerces a list list of cycles lls
      ++ to a permutation, each cycle being a list with not
      ++ repetitions, is coerced to the permutation, which maps
      ++ ls.i to ls.i+1, indices modulo the length of the list,
      ++ then these permutations are mutiplied.
      ++ Error: if repetitions occur in one cycle.

    eval : (%,S) -> S
      ++ eval(p, el) returns the image of el under the
      ++ permutation p.

    elt : (%,S) -> S
      ++ elt(p, el) returns the image of el under the
      ++ permutation p.

    orbit : (%,S) -> Set S
      ++ orbit(p, el) returns the orbit of el under the
      ++ permutation p, that is, the set which is given by applications of
      ++ the powers of p to el.

    "<" : (%,%) -> Boolean
      ++ p < q is an order relation on permutations.
      ++ Note that this order is only total if and only if S is totally ordered
      ++ or S is finite.

    if S has OrderedSet then OrderedSet

    if S has Finite then OrderedSet
```

— **PERMCAT.dotabb** —

```
"PERMCAT"
```

```
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PERMCAT"];
"PERMCAT" -> "GROUP"
```

———-

— PERMCAT.dotfull —

```
"PermutationCategory(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PERMCAT"];
"PermutationCategory(a:SetCategory)" -> "Group()"
```

———-

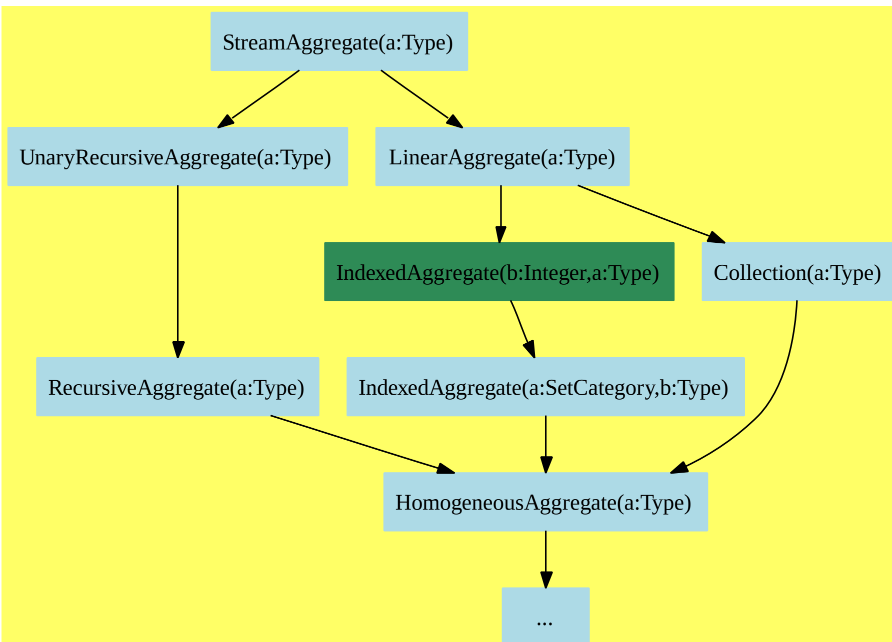
— PERMCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"PermutationCategory(a:SetCategory)" [color=lightblue];
"PermutationCategory(a:SetCategory)" -> "Group()"

"Group()" [color=lightblue];
"Group()" -> "Monoid()"
"Group()" -> "RepeatedSquaring(Group)"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SetCategory()"
"SemiGroup()" -> "RepeatedSquaring(SemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"RepeatedSquaring(Group)" [color="#00EE00"];
"RepeatedSquaring(Group)" -> "RepeatedSquaring(a:SetCategory)"

"RepeatedSquaring(SemiGroup)" [color="#00EE00"];
"RepeatedSquaring(SemiGroup)" -> "RepeatedSquaring(a:SetCategory)"
```

```
"RepeatedSquaring(a:SetCategory)" [color="#00EE00"];
"RepeatedSquaring(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];

}
```

———-

7.0.131 StreamAggregate (STAGG)



— **StreamAggregate.input** —

```
)set break resume
)sys rm -f StreamAggregate.output
)spool StreamAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show StreamAggregate
--R
--R StreamAggregate(S: Type) is a category constructor
--R Abbreviation for StreamAggregate is STAGG
```

```
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for STAGG
--R
--R------------------------------- Operations --------------------------------
--R children : % -> List(%)              concat : (%,S) -> %
--R concat : List(%) -> %                concat : (S,%) -> %
--R concat : (%,%) -> %                  construct : List(S) -> %
--R copy : % -> %                        cycleEntry : % -> %
--R cycleTail : % -> %                   cyclic? : % -> Boolean
--R delete : (%,Integer) -> %            distance : (%,%) -> Integer
--R elt : (%,Integer,S) -> S             ?.? : (%,Integer) -> S
--R ?.last : (%,last) -> S               ?.rest : (%,rest) -> %
--R ?.first : (%,first) -> S             ?.value : (%,value) -> S
--R empty : () -> %                      empty? : % -> Boolean
--R entries : % -> List(S)               eq? : (%,%) -> Boolean
--R explicitlyFinite? : % -> Boolean     first : % -> S
--R index? : (Integer,%) -> Boolean      indices : % -> List(Integer)
--R insert : (S,%,Integer) -> %          insert : (%,%,Integer) -> %
--R last : (%,NonNegativeInteger) -> %   last : % -> S
--R latex : % -> String if S has SETCAT  leaf? : % -> Boolean
--R leaves : % -> List(S)                map : (((S,S) -> S),%,%) -> %
--R map : ((S -> S),%) -> %              new : (NonNegativeInteger,S) -> %
--R nodes : % -> List(%)                 possiblyInfinite? : % -> Boolean
--R qelt : (%,Integer) -> S              rest : (%,NonNegativeInteger) -> %
--R rest : % -> %                        sample : () -> %
--R second : % -> S                      tail : % -> %
--R third : % -> S                       value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R concat! : (%,S) -> % if $ has shallowlyMutable
--R concat! : (%,%) -> % if $ has shallowlyMutable
--R convert : % -> InputForm if S has KONVERT(INFORM)
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R cycleLength : % -> NonNegativeInteger
--R cycleSplit! : % -> % if $ has shallowlyMutable
--R delete : (%,UniversalSegment(Integer)) -> %
--R ?.? : (%,UniversalSegment(Integer)) -> %
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R first : (%,NonNegativeInteger) -> %
--R hash : % -> SingleInteger if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R maxIndex : % -> Integer if Integer has ORDSET
```

```
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List(S) if $ has finiteAggregate
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R setchildren! : (%,List(%)) -> % if $ has shallowlyMutable
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R setelt : (%,UniversalSegment(Integer),S) -> S if $ has shallowlyMutable
--R setelt : (%,last,S) -> S if $ has shallowlyMutable
--R setelt : (%,rest,%) -> % if $ has shallowlyMutable
--R setelt : (%,first,S) -> S if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setfirst! : (%,S) -> S if $ has shallowlyMutable
--R setlast! : (%,S) -> S if $ has shallowlyMutable
--R setrest! : (%,%) -> % if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R split! : (%,Integer) -> % if $ has shallowlyMutable
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———

— StreamAggregate.help —

```
====================================================================
StreamAggregate examples
====================================================================

A stream aggregate is a linear aggregate which possibly has an infinite
number of elements. A basic domain constructor which builds stream
aggregates is Stream. From streams, a number of infinite structures
such power series can be built. A stream aggregate may also be infinite
since it may be cyclic. For example, see DecimalExpansion.

See Also:
o )show StreamAggregate
o )show DecimalExpansion
```

———

See:

⇒ "LazyStreamAggregate" (LZSTAGG) 8.0.136 on page 799
⇐ "LinearAggregate" (LNAGG) 6.0.111 on page 531
⇐ "UnaryRecursiveAggregate" (URAGG) 6.0.120 on page 642

Exports:

any?	children	child?	coerce	concat
concat!	construct	convert	copy	count
cycleEntry	cycleLength	cycleSplit!	cycleTail	cyclic?
delete	distance	elt	empty	empty?
entries	entry?	eq?	eval	every?
explicitlyFinite?	fill!	find	first	hash
index?	indices	insert	last	latex
leaf?	leaves	less?	map	map!
maxIndex	member?	members	minIndex	more?
new	nodes	node?	parts	possiblyInfinite?
qelt	qsetelt!	reduce	remove	removeDuplicates
rest	sample	second	select	setchildren!
setelt	setfirst!	setlast!	setrest!	setvalue!
size?	split!	swap!	tail	third
value	#?	?=?	?.?	?.first
?.last	?.rest	?.value	?~=?	

Attributes exported:

- **nil**

Attributes Used:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are implemented by this category:

```
concat : (%,%) -> %
concat : List % -> %
concat! : (%,%) -> % if $ has shallowlyMutable
fill! : (%,S) -> % if $ has shallowlyMutable
first : (%,NonNegativeInteger) -> %
explicitlyFinite? : % -> Boolean
map! : ((S -> S),%) -> % if $ has shallowlyMutable
possiblyInfinite? : % -> Boolean
setelt : (%,Integer,S) -> S if $ has shallowlyMutable
setelt : (%,UniversalSegment Integer,S) -> S
         if $ has shallowlyMutable
?.? : (%,Integer) -> S
?.? : (%,UniversalSegment Integer) -> %
```

These exports come from (p642) UnaryRecursiveAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
children : % -> List %
```

```
child? : (%,%) -> Boolean if S has SETCAT
coerce : % -> OutputForm if S has SETCAT
concat : (S,%) -> %
concat : (%,S) -> %
concat! : (%,S) -> % if $ has shallowlyMutable
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT
         and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
cycleEntry : % -> %
cycleLength : % -> NonNegativeInteger
cycleSplit! : % -> % if $ has shallowlyMutable
cycleTail : % -> %
cyclic? : % -> Boolean
distance : (%,%) -> Integer
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S
         and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S
         and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S
         and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S
         and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
first : % -> S
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
last : % -> S
last : (%,NonNegativeInteger) -> %
leaf? : % -> Boolean
leaves : % -> List S
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
member? : (S,%) -> Boolean
         if S has SETCAT
         and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
nodes : % -> List %
node? : (%,%) -> Boolean if S has SETCAT
parts : % -> List S if $ has finiteAggregate
rest : % -> %
rest : (%,NonNegativeInteger) -> %
sample : () -> %
```

```
second : % -> S
setchildren! : (%,List %) -> % if $ has shallowlyMutable
setelt : (%,first,S) -> S if $ has shallowlyMutable
setelt : (%,last,S) -> S if $ has shallowlyMutable
setelt : (%,rest,%) -> % if $ has shallowlyMutable
setelt : (%,value,S) -> S if $ has shallowlyMutable
setfirst! : (%,S) -> S if $ has shallowlyMutable
setlast! : (%,S) -> S if $ has shallowlyMutable
setrest! : (%,%) -> % if $ has shallowlyMutable
setvalue! : (%,S) -> S if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
split! : (%,Integer) -> % if $ has shallowlyMutable
tail : % -> %
third : % -> S
value : % -> S
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?.last : (%,last) -> S
?.rest : (%,rest) -> %
?.first : (%,first) -> S
?~=? : (%,%) -> Boolean if S has SETCAT
?.value : (%,value) -> S
```

These exports come from (p531) LinearAggregate(S:Type):

```
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
delete : (%,Integer) -> %
delete : (%,UniversalSegment Integer) -> %
elt : (%,Integer,S) -> S
entry? : (S,%) -> Boolean
         if $ has finiteAggregate
         and S has SETCAT
entries : % -> List S
find : ((S -> Boolean),%) -> Union(S,"failed")
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (S,%,Integer) -> %
insert : (%,%,Integer) -> %
maxIndex : % -> Integer if Integer has ORDSET
map : (((S,S) -> S),%,%) -> %
minIndex : % -> Integer if Integer has ORDSET
new : (NonNegativeInteger,S) -> %
qelt : (%,Integer) -> S
qsetelt! : (%,Integer,S) -> S
         if $ has shallowlyMutable
reduce : (((S,S) -> S),%,S,S) -> S
         if S has SETCAT
         and $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S
         if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S
         if $ has finiteAggregate
remove : (S,%) -> %
         if S has SETCAT
```

```
          and $ has finiteAggregate
 remove : ((S -> Boolean),%) -> %
          if $ has finiteAggregate
 removeDuplicates : % -> %
          if S has SETCAT
          and $ has finiteAggregate
 select : ((S -> Boolean),%) -> %
          if $ has finiteAggregate
 swap! : (%,Integer,Integer) -> Void
          if $ has shallowlyMutable
```

— **StreamAggregate.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#STAGG">
StreamAggregate (STAGG)</a></h2>
</body>
```

———-

— **category STAGG StreamAggregate** —

```
)abbrev category STAGG StreamAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ A stream aggregate is a linear aggregate which possibly has an infinite
++ number of elements. A basic domain constructor which builds stream
++ aggregates is \spadtype{Stream}. From streams, a number of infinite
++ structures such power series can be built. A stream aggregate may
++ also be infinite since it may be cyclic.
++ For example, see \spadtype{DecimalExpansion}.

StreamAggregate(S) : Category == SIG where
  S : Type

  SIG ==> Join(UnaryRecursiveAggregate S, LinearAggregate S) with

    explicitlyFinite? : % -> Boolean
      ++ explicitlyFinite?(s) tests if the stream has a finite
      ++ number of elements, and false otherwise.
      ++ Note that for many datatypes,
      ++ \axiom{explicitlyFinite?(s) = not possiblyInfinite?(s)}.

    possiblyInfinite? : % -> Boolean
      ++ possiblyInfinite?(s) tests if the stream s could possibly
      ++ have an infinite number of elements.
      ++ Note that for many datatypes,
      ++ \axiom{possiblyInfinite?(s) = not explictlyFinite?(s)}.

   add
```

```
c2: (%, %) -> S

explicitlyFinite? x == not cyclic? x

possiblyInfinite? x == cyclic? x

first(x, n) == construct [c2(x, x := rest x) for i in 1..n]

c2(x, r) ==
  empty? x => error "Index out of range"
  first x

elt(x:%, i:Integer) ==
  i := i - minIndex x
  (i < 0) or empty?(x := rest(x, i::NonNegativeInteger)) => _
      error "index out of range"
  first x

elt(x:%, i:UniversalSegment(Integer)) ==
  l := lo(i) - minIndex x
  l < 0 => error "index out of range"
  not hasHi i => copy(rest(x, l::NonNegativeInteger))
  (h := hi(i) - minIndex x) < l => empty()
  first(rest(x, l::NonNegativeInteger), (h - l + 1)::NonNegativeInteger)

if % has shallowlyMutable then

  concat(x:%, y:%) == concat_!(copy x, y)

  concat l ==
    empty? l => empty()
    concat_!(copy first l, concat rest l)

  map_!(f, l) ==
    y := l
    while not empty? l repeat
      setfirst_!(l, f first l)
      l := rest l
    y

  fill_!(x, s) ==
    y := x
    while not empty? y repeat (setfirst_!(y, s); y := rest y)
    x

  setelt(x:%, i:Integer, s:S) ==
    i := i - minIndex x
    (i < 0) or empty?(x := rest(x,i::NonNegativeInteger)) => _
        error "index out of range"
    setfirst_!(x, s)

  setelt(x:%, i:UniversalSegment(Integer), s:S) ==
    (l := lo(i) - minIndex x) < 0 => error "index out of range"
    h := if hasHi i then hi(i) - minIndex x else maxIndex x
```

```
      h < l => s
      y := rest(x, l::NonNegativeInteger)
      z := rest(y, (h - l + 1)::NonNegativeInteger)
      while not eq?(y, z) repeat (setfirst_!(y, s); y := rest y)
      s

    concat_!(x:%, y:%) ==
      empty? x => y
      setrest_!(tail x, y)
      x
```

———-

— COQ STAGG —

```
(* category STAGG *)
(*

   explicitlyFinite?: % -> Boolean
   explicitlyFinite? x == not cyclic? x

   possiblyInfinite?: % -> Boolean
   possiblyInfinite? x == cyclic? x

   first : (%,NonNegativeInteger) -> %
   first(x, n) == construct [c2(x, x := rest x) for i in 1..n]

   c2: (%, %) -> S
   c2(x, r) ==
     empty? x => error "Index out of range"
     first x

   elt : (%,Integer,S) -> S
   elt(x:%, i:Integer) ==
     i := i - minIndex x
     (i < 0) or empty?(x := rest(x, i::NonNegativeInteger)) => _
         error "index out of range"
     first x

   elt(x:%, i:UniversalSegment(Integer)) ==
     l := lo(i) - minIndex x
     l < 0 => error "index out of range"
     not hasHi i => copy(rest(x, l::NonNegativeInteger))
     (h := hi(i) - minIndex x) < l => empty()
     first(rest(x, l::NonNegativeInteger), (h - l + 1)::NonNegativeInteger)

   if % has shallowlyMutable then

     concat : (%,%) -> %
     concat(x:%, y:%) == concat_!(copy x, y)

     concat : List % -> %
     concat l ==
```

```
      empty? l => empty()
      concat_!(copy first l, concat rest l)

    map! : ((S -> S),%) -> %
    map_!(f, l) ==
      y := l
      while not empty? l repeat
        setfirst_!(l, f first l)
        l := rest l
      y

    fill! : (%,S) -> %
    fill_!(x, s) ==
      y := x
      while not empty? y repeat (setfirst_!(y, s); y := rest y)
      x

    setelt : (%,Integer,S) -> S
    setelt(x:%, i:Integer, s:S) ==
      i := i - minIndex x
      (i < 0) or empty?(x := rest(x,i::NonNegativeInteger)) => _
          error "index out of range"
      setfirst_!(x, s)

    setelt : (%,UniversalSegment Integer,S) -> S
    setelt(x:%, i:UniversalSegment(Integer), s:S) ==
      (l := lo(i) - minIndex x) < 0 => error "index out of range"
      h := if hasHi i then hi(i) - minIndex x else maxIndex x
      h < l => s
      y := rest(x, l::NonNegativeInteger)
      z := rest(y, (h - l + 1)::NonNegativeInteger)
      while not eq?(y, z) repeat (setfirst_!(y, s); y := rest y)
      s

    concat! : (%,%) -> %
    concat_!(x:%, y:%) ==
      empty? x => y
      setrest_!(tail x, y)
      x
*)
```

————

— STAGG.dotabb —

```
"STAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=STAGG"];
"STAGG" -> "RCAGG"
"STAGG" -> "LNAGG"
```

————

— STAGG.dotfull —

```
"StreamAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=STAGG"];
"StreamAggregate(a:Type)" -> "UnaryRecursiveAggregate(a:Type)"
"StreamAggregate(a:Type)" -> "LinearAggregate(a:Type)"
```

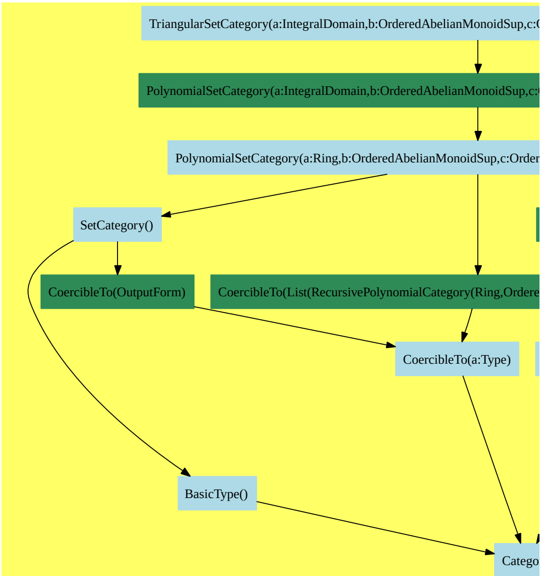
———-

— **STAGG.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"StreamAggregate(a:Type)" [color=lightblue];
"StreamAggregate(a:Type)" -> "UnaryRecursiveAggregate(a:Type)"
"StreamAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"UnaryRecursiveAggregate(a:Type)" [color=lightblue];
"UnaryRecursiveAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"

"RecursiveAggregate(a:Type)" [color=lightblue];
"RecursiveAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "Collection(a:Type)"

"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" ->
    "IndexedAggregate(a:SetCategory,b:Type)"

"IndexedAggregate(a:SetCategory,b:Type)" [color=lightblue];
"IndexedAggregate(a:SetCategory,b:Type)" ->
    "HomogeneousAggregate(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "..."

"..." [color=lightblue];

}
```

———-

7.0.132 TriangularSetCategory (TSETCAT)



— TriangularSetCategory.input —

```
)set break resume
)sys rm -f TriangularSetCategory.output
)spool TriangularSetCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show TriangularSetCategory
--R
--R TriangularSetCategory(R: IntegralDomain,E: OrderedAbelianMonoidSup,V: OrderedSet,P: RecursivePolynomialCateg
--R Abbreviation for TriangularSetCategory is TSETCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for TSETCAT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                algebraic? : (V,%) -> Boolean
--R algebraicVariables : % -> List(V)     coerce : % -> List(P)
--R coerce : % -> OutputForm              collect : (%,V) -> %
--R collectQuasiMonic : % -> %            collectUnder : (%,V) -> %
--R collectUpper : (%,V) -> %             construct : List(P) -> %
--R copy : % -> %                         degree : % -> NonNegativeInteger
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                extend : (%,P) -> %
--R first : % -> Union(P,"failed")        hash : % -> SingleInteger
--R headReduce : (P,%) -> P               headReduced? : % -> Boolean
--R headReduced? : (P,%) -> Boolean       infRittWu? : (%,%) -> Boolean
--R initiallyReduce : (P,%) -> P          initiallyReduced? : % -> Boolean
```

```
--R initials : % -> List(P)              last : % -> Union(P,"failed")
--R latex : % -> String                  mainVariable? : (V,%) -> Boolean
--R mainVariables : % -> List(V)         map : ((P -> P),%) -> %
--R mvar : % -> V                        normalized? : % -> Boolean
--R normalized? : (P,%) -> Boolean       reduceByQuasiMonic : (P,%) -> P
--R removeZero : (P,%) -> P              rest : % -> Union(%,"failed")
--R retract : List(P) -> %               sample : () -> %
--R select : (%,V) -> Union(P,"failed")  stronglyReduce : (P,%) -> P
--R stronglyReduced? : % -> Boolean      stronglyReduced? : (P,%) -> Boolean
--R trivialIdeal? : % -> Boolean         variables : % -> List(V)
--R zeroSetSplit : List(P) -> List(%)    ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R autoReduced? : (%,((P,List(P)) -> Boolean)) -> Boolean
--R basicSet : (List(P),(P -> Boolean),((P,P) -> Boolean)) -> Union(Record(bas: %,top: List(P)),"failed")
--R basicSet : (List(P),((P,P) -> Boolean)) -> Union(Record(bas: %,top: List(P)),"failed")
--R coHeight : % -> NonNegativeInteger if V has FINITE
--R convert : % -> InputForm if P has KONVERT(INFORM)
--R count : ((P -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (P,%) -> NonNegativeInteger if P has SETCAT and $ has finiteAggregate
--R eval : (%,List(Equation(P))) -> % if P has EVALAB(P) and P has SETCAT
--R eval : (%,Equation(P)) -> % if P has EVALAB(P) and P has SETCAT
--R eval : (%,P,P) -> % if P has EVALAB(P) and P has SETCAT
--R eval : (%,List(P),List(P)) -> % if P has EVALAB(P) and P has SETCAT
--R every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R extendIfCan : (%,P) -> Union(%,"failed")
--R find : ((P -> Boolean),%) -> Union(P,"failed")
--R headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
--R initiallyReduced? : (P,%) -> Boolean
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((P -> P),%) -> % if $ has shallowlyMutable
--R member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
--R members : % -> List(P) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(P) if $ has finiteAggregate
--R quasiComponent : % -> Record(close: List(P),open: List(P))
--R reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
--R reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P,P) -> P if P has SETCAT and $ has finiteAggregate
--R reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
--R remainder : (P,%) -> Record(rnum: R,polnum: P,den: R) if R has INTDOM
--R remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
--R retractIfCan : List(P) -> Union(%,"failed")
--R rewriteIdealWithHeadRemainder : (List(P),%) -> List(P) if R has INTDOM
--R rewriteIdealWithRemainder : (List(P),%) -> List(P) if R has INTDOM
--R rewriteSetWithReduction : (List(P),%,((P,P) -> P),((P,P) -> Boolean)) -> List(P)
--R roughBase? : % -> Boolean if R has INTDOM
--R roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
--R roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
--R roughUnitIdeal? : % -> Boolean if R has INTDOM
--R select : ((P -> Boolean),%) -> % if $ has finiteAggregate
```

```
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : (%,V) -> Record(under: %,floor: %,upper: %)
--R triangular? : % -> Boolean if R has INTDOM
--R zeroSetSplitIntoTriangularSystems : List(P) -> List(Record(close: %,open: List(P)))
--R
--E 1

)spool
)lisp (bye)
```

— **TriangularSetCategory.help** —

```
====================================================================
TriangularSetCategory examples
====================================================================

The category of triangular sets of multivariate polynomials with
coefficients in an integral domain.

Let R be an integral domain and V a finite ordered set of variables,
    X1 < X2 < ... < Xn

A set S of polynomials in R[X1,X2,...,Xn] is triangular if no elements
of S lies in R, and if two distinct elements of S have distinct main
variables.

Note that the empty set is a triangular set. A triangular set is not
necessarily a (lexicographical) Groebner basis and the notion of
reduction related to triangular sets is based on the recursive view
of polynomials. We recall this notion here. For details see
   P. AUBRY, D. LAZARD and M. MORENO MAZA "On the Theories
   of Triangular Sets" Journal of Symbol. Comp.

A polynomial P is reduced with respect to a non-constant polynomial Q
if the degree of P in the main variable of Q is less than the main
degree of Q. A polynomial P is reduced with respect to a triangular
set T if it is reduced with respect to every polynomial of T.

See Also:
o )show TriangularSetCategory
```

See:

⇒ "RegularTriangularSetCategory" (RSETCAT) 8.0.143 on page 873
⇐ "PolynomialSetCategory" (PSETCAT) 6.0.115 on page 598

Exports:

algebraic?	algebraicVariables
any?	autoReduced?
basicSet	coerce
coHeight	collect
collectQuasiMonic	collectUnder
collectUpper	construct
convert	copy
count	degree
empty	empty?
eq?	eval
every?	extend
extendIfCan	find
first	hash
headReduce	headReduced?
headRemainder	infRittWu?
initiallyReduce	initiallyReduced?
initials	last
latex	less?
mainVariable?	mainVariables
map	map!
member?	members
more?	mvar
normalized?	parts
quasiComponent	reduce
reduced?	reduceByQuasiMonic
remainder	remove
removeDuplicates	removeZero
rest	retract
retractIfCan	rewriteIdealWithHeadRemainder
rewriteIdealWithRemainder	rewriteSetWithReduction
roughBase?	roughEqualIdeals?
roughSubIdeal?	roughUnitIdeal?
sample	select
size?	sort
stronglyReduce	stronglyReduced?
triangular?	trivialIdeal?
variables	zeroSetSplit
zeroSetSplitIntoTriangularSystems	#?
?=?	?~=?

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These are directly exported but not implemented:

```
extendIfCan : (%,P) -> Union(%,"failed")
zeroSetSplit : List P -> List %
```

```
zeroSetSplitIntoTriangularSystems :
  List P -> List Record(close: %,open: List P)
```

These are implemented by this category:

```
algebraic? : (V,%) -> Boolean
algebraicVariables : % -> List V
autoReduced? : (%,((P,List P) -> Boolean)) -> Boolean
basicSet :
  (List P,(P -> Boolean),((P,P) -> Boolean)) ->
    Union(Record(bas: %,top: List P),"failed")
basicSet : (List P,((P,P) -> Boolean)) ->
  Union(Record(bas: %,top: List P),"failed")
coerce : % -> List P
coHeight : % -> NonNegativeInteger if V has FINITE
collectQuasiMonic : % -> %
collectUnder : (%,V) -> %x
collectUpper : (%,V) -> %
construct : List P -> %
convert : % -> InputForm if P has KONVERT INFORM
degree : % -> NonNegativeInteger
extend : (%,P) -> %
first : % -> Union(P,"failed")
headReduce : (P,%) -> P
headReduced? : % -> Boolean
headReduced? : (P,%) -> Boolean
infRittWu? : (%,%) -> Boolean
initiallyReduce : (P,%) -> P
initiallyReduced? : % -> Boolean
initiallyReduced? : (P,%) -> Boolean
initials : % -> List P
last : % -> Union(P,"failed")
mvar : % -> V
normalized? : % -> Boolean
normalized? : (P,%) -> Boolean
quasiComponent : % -> Record(close: List P,open: List P)
reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
reduceByQuasiMonic : (P,%) -> P
reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
removeZero : (P,%) -> P
rest : % -> Union(%,"failed")
retractIfCan : List P -> Union(%,"failed")
rewriteSetWithReduction :
   (List P,%,((P,P) -> P),((P,P) -> Boolean)) -> List P
select : (%,V) -> Union(P,"failed")
stronglyReduce : (P,%) -> P
stronglyReduced? : % -> Boolean
stronglyReduced? : (P,%) -> Boolean
?=? : (%,%) -> Boolean
```

These exports come from (p598) PolynomialSetCategory(R,E,V,P) where R:IntegralDomain, E:OrderedAbelianMonoidSup, V:OrderedSet, P:RecursivePolynomialCategory(R,E,V)):

```
any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
coerce : % -> OutputForm
```

```
collect : (%,V) -> %
copy : % -> %
count : ((P -> Boolean),%) -> NonNegativeInteger
  if $ has finiteAggregate
count : (P,%) -> NonNegativeInteger
  if P has SETCAT and $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
find : ((P -> Boolean),%) -> Union(P,"failed")
hash : % -> SingleInteger
headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
latex : % -> String
less? : (%,NonNegativeInteger) -> Boolean
mainVariable? : (V,%) -> Boolean
mainVariables : % -> List V
map : ((P -> P),%) -> %
map! : ((P -> P),%) -> % if $ has shallowlyMutable
member? : (P,%) -> Boolean
  if P has SETCAT and $ has finiteAggregate
members : % -> List P if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List P if $ has finiteAggregate
reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P,P) -> P
  if P has SETCAT
  and $ has finiteAggregate
remainder : (P,%) -> Record(rnum: R,polnum: P,den: R)
  if R has INTDOM
remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
removeDuplicates : % -> %
  if P has SETCAT
  and $ has finiteAggregate
retract : List P -> %
rewriteIdealWithHeadRemainder : (List P,%) -> List P
  if R has INTDOM
rewriteIdealWithRemainder : (List P,%) -> List P
  if R has INTDOM
roughBase? : % -> Boolean if R has INTDOM
roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
roughUnitIdeal? : % -> Boolean if R has INTDOM
sample : () -> %
select : ((P -> Boolean),%) -> % if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
sort : (%,V) -> Record(under: %,floor: %,upper: %)
triangular? : % -> Boolean if R has INTDOM
```

```
 trivialIdeal? : % -> Boolean
 variables : % -> List V
 #? : % -> NonNegativeInteger if $ has finiteAggregate
 ?~=? : (%,%) -> Boolean
```

See: SALSA[SALSA], Kalkbrener[Kalk91, Kalk98], Aubry[Aubr96, Aubr99, Aubr99a], Lazard[Laza91], Moreno Maza[Maza95, Maza97, Maza98, Maza00]

— **TriangularSetCategory.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#TSETCAT">
TriangularSetCategory (TSETCAT)</a></h2>
</body>
```

———-

— **category TSETCAT TriangularSetCategory** —

```
)abbrev category TSETCAT TriangularSetCategory
++ Author: Marc Moreno Maza (marc@nag.co.uk)
++ Date Created: 04/26/1994
++ Date Last Updated: 12/15/1998
++ References :
++ SALSA Solvers for Algebraic Systems and Applications
++ Kalk91 Three contributions to elimination theory
++ Kalk98 Algorithmic properties of polynomial rings
++ Aubr96 Triangular Sets for Solving Polynomial Systems:
++ Aubr99 On the Theories of Triangular Sets
++ Aubr99a Triangular Sets for Solving Polynomial Systems:
++ Laza91 A new method for solving algebraic systems of positive dimension
++ Maza95 Polynomial Gcd Computations over Towers of Algebraic Extensions
++ Maza97 Calculs de pgcd au-dessus des tours d'extensions simples et
++        resolution des systemes d'equations algebriques
++ Maza98 A new algorithm for computing triangular decomposition of
++        algebraic varieties
++ Maza00 On Triangular Decompositions of Algebraic Varieties
++ Description:
++ The category of triangular sets of multivariate polynomials
++ with coefficients in an integral domain.
++ Let \axiom{R} be an integral domain and \axiom{V} a finite ordered set of
++ variables, say \axiom{X1 < X2 < ... < Xn}.
++ A set \axiom{S} of polynomials in \axiom{R[X1,X2,...,Xn]} is triangular
++ if no elements of \axiom{S} lies in \axiom{R}, and if two distinct
++ elements of \axiom{S} have distinct main variables.
++ Note that the empty set is a triangular set. A triangular set is not
++ necessarily a (lexicographical) Groebner basis and the notion of
++ reduction related to triangular sets is based on the recursive view
++ of polynomials. We recall this notion here and refer to [1] for more
++ details.
++ A polynomial \axiom{P} is reduced w.r.t a non-constant polynomial
++ \axiom{Q} if the degree of \axiom{P} in the main variable of \axiom{Q}
++ is less than the main degree of \axiom{Q}.
++ A polynomial \axiom{P} is reduced w.r.t a triangular set \axiom{T}
++ if it is reduced w.r.t. every polynomial of \axiom{T}.
```

```
TriangularSetCategory(R,E,V,P) : Category == SIG where
  R : IntegralDomain
  E : OrderedAbelianMonoidSup
  V : OrderedSet
  P : RecursivePolynomialCategory(R,E,V)

  SIG ==> PolynomialSetCategory(R,E,V,P) with

    finiteAggregate

    shallowlyMutable

    infRittWu? : ($,$) -> Boolean
      ++ \axiom{infRittWu?(ts1,ts2)} returns true iff \axiom{ts2} has
      ++ higher rank than \axiom{ts1} in Wu Wen Tsun sense.

    basicSet : (List P,((P,P)->Boolean)) -> _
     Union(Record(bas:$,top:List P),"failed")
      ++ \axiom{basicSet(ps,redOp?)} returns \axiom{[bs,ts]} where
      ++ \axiom{concat(bs,ts)} is \axiom{ps} and \axiom{bs}
      ++ is a basic set in Wu Wen Tsun sense of \axiom{ps} w.r.t
      ++ the reduction-test \axiom{redOp?}, if no non-zero constant
      ++ polynomial lie in \axiom{ps}, otherwise \axiom{"failed"} is returned.

    basicSet : (List P,(P->Boolean),((P,P)->Boolean)) -> _
     Union(Record(bas:$,top:List P),"failed")
      ++ \axiom{basicSet(ps,pred?,redOp?)} returns the same as
      ++ \axiom{basicSet(qs,redOp?)}
      ++ where \axiom{qs} consists of the polynomials of \axiom{ps}
      ++ satisfying property \axiom{pred?}.

    initials : $ -> List P
      ++ \axiom{initials(ts)} returns the list of the non-constant initials
      ++ of the members of \axiom{ts}.

    degree : $ -> NonNegativeInteger
      ++ \axiom{degree(ts)} returns the product of main degrees of the
      ++ members of \axiom{ts}.

    quasiComponent : $ -> Record(close:List P,open:List P)
      ++ \axiom{quasiComponent(ts)} returns \axiom{[lp,lq]} where \axiom{lp}
      ++ is the list of the members of \axiom{ts} and \axiom{lq}is
      ++ \axiom{initials(ts)}.

    normalized? : (P,$) -> Boolean
      ++ \axiom{normalized?(p,ts)} returns true iff \axiom{p} and all
      ++ its iterated initials have degree zero w.r.t. the main variables
      ++ of the polynomials of \axiom{ts}

    normalized? : $  -> Boolean
      ++ \axiom{normalized?(ts)} returns true iff for every axiom{p} in
      ++ \axiom{ts} we have \axiom{normalized?(p,us)} where \axiom{us}
      ++ is \axiom{collectUnder(ts,mvar(p))}.
```

```
reduced? : (P,$,((P,P) -> Boolean)) -> Boolean
  ++ \axiom{reduced?(p,ts,redOp?)} returns true iff \axiom{p} is reduced
  ++ w.r.t.in the sense of the operation \axiom{redOp?}, that is if for
  ++ every \axiom{t} in \axiom{ts} \axiom{redOp?(p,t)} holds.

stronglyReduced? : (P,$) -> Boolean
  ++ \axiom{stronglyReduced?(p,ts)} returns true iff \axiom{p}
  ++ is reduced w.r.t. \axiom{ts}.

headReduced? : (P,$) -> Boolean
  ++ \axiom{headReduced?(p,ts)} returns true iff the head of \axiom{p} is
  ++ reduced w.r.t. \axiom{ts}.

initiallyReduced? : (P,$) -> Boolean
  ++ \axiom{initiallyReduced?(p,ts)} returns true iff \axiom{p} and all
  ++ its iterated initials are reduced w.r.t. to the elements of
  ++ \axiom{ts} with the same main variable.

autoReduced? : ($,((P,List(P)) -> Boolean)) -> Boolean
  ++ \axiom{autoReduced?(ts,redOp?)} returns true iff every element of
  ++ \axiom{ts} is reduced w.r.t to every other in the sense of
  ++ \axiom{redOp?}

stronglyReduced? : $ -> Boolean
  ++ \axiom{stronglyReduced?(ts)} returns true iff every element of
  ++ \axiom{ts} is reduced w.r.t to any other element of \axiom{ts}.

headReduced? : $ -> Boolean
  ++ headReduced?(ts) returns true iff the head of every element of
  ++ \axiom{ts} is reduced w.r.t to any other element of \axiom{ts}.

initiallyReduced? : $ -> Boolean
  ++ initiallyReduced?(ts) returns true iff for every element \axiom{p}
  ++ of \axiom{ts}. \axiom{p} and all its iterated initials are reduced
  ++ w.r.t. to the other elements of \axiom{ts} with the same main
  ++ variable.

reduce : (P,$,((P,P) -> P),((P,P) -> Boolean) ) -> P
  ++ \axiom{reduce(p,ts,redOp,redOp?)} returns a polynomial \axiom{r}
  ++ such that \axiom{redOp?(r,p)} holds for every \axiom{p} of
  ++ \axiom{ts} and there exists some product \axiom{h} of the initials
  ++ of the members of \axiom{ts} such that \axiom{h*p - r} lies in the
  ++ ideal generated by \axiom{ts}. The operation \axiom{redOp} must
  ++ satisfy the following conditions. For every \axiom{p} and \axiom{q}
  ++ we have  \axiom{redOp?(redOp(p,q),q)} and there exists an integer
  ++ \axiom{e} and a polynomial \axiom{f} such that
  ++ \axiom{init(q)^e*p = f*q + redOp(p,q)}.

rewriteSetWithReduction : (List P,$,((P,P) -> P),((P,P) -> Boolean) ) ->
 List P
  ++ \axiom{rewriteSetWithReduction(lp,ts,redOp,redOp?)} returns a list
  ++ \axiom{lq} of polynomials such that
  ++ \axiom{[reduce(p,ts,redOp,redOp?) for p in lp]} and \axiom{lp}
```

```
  ++ have the same zeros inside the regular zero set of \axiom{ts}.
  ++ Moreover, for every polynomial \axiom{q} in \axiom{lq} and every
  ++ polynomial \axiom{t} in \axiom{ts}
  ++ \axiom{redOp?(q,t)} holds and there exists a polynomial \axiom{p}
  ++ in the ideal generated by \axiom{lp} and a product \axiom{h} of
  ++ \axiom{initials(ts)} such that \axiom{h*p - r} lies in the ideal
  ++ generated by \axiom{ts}.
  ++ The operation \axiom{redOp} must satisfy the following conditions.
  ++ For every \axiom{p} and \axiom{q} we have
  ++ \axiom{redOp?(redOp(p,q),q)}
  ++ and there exists an integer \axiom{e} and a polynomial \axiom{f}
  ++ such that \axiom{init(q)^e*p = f*q + redOp(p,q)}.

stronglyReduce : (P,$) -> P
  ++ \axiom{stronglyReduce(p,ts)} returns a polynomial \axiom{r} such that
  ++ \axiom{stronglyReduced?(r,ts)} holds and there exists some product
  ++ \axiom{h} of \axiom{initials(ts)}
  ++ such that \axiom{h*p - r} lies in the ideal generated by \axiom{ts}.

headReduce : (P,$) -> P
  ++ \axiom{headReduce(p,ts)} returns a polynomial \axiom{r}  such that
  ++ \axiom{headReduce?(r,ts)} holds and there exists some product
  ++ \axiom{h} of \axiom{initials(ts)} such that \axiom{h*p - r} lies
  ++ in the ideal generated by \axiom{ts}.

initiallyReduce : (P,$) -> P
  ++ \axiom{initiallyReduce(p,ts)} returns a polynomial \axiom{r}
  ++ such that  \axiom{initiallyReduced?(r,ts)}
  ++ holds and there exists some product \axiom{h} of \axiom{initials(ts)}
  ++ such that \axiom{h*p - r} lies in the ideal generated by \axiom{ts}.

removeZero : (P, $) -> P
  ++ \axiom{removeZero(p,ts)} returns \axiom{0} if \axiom{p} reduces
  ++ to \axiom{0} by pseudo-division w.r.t \axiom{ts} otherwise
  ++ returns a polynomial \axiom{q} computed from \axiom{p}
  ++ by removing any coefficient in \axiom{p} reducing to \axiom{0}.

collectQuasiMonic : $ -> $
  ++ \axiom{collectQuasiMonic(ts)} returns the subset of \axiom{ts}
  ++ consisting of the polynomials with initial in \axiom{R}.

reduceByQuasiMonic : (P, $) -> P
  ++ \axiom{reduceByQuasiMonic(p,ts)} returns the same as
  ++ \axiom{remainder(p,collectQuasiMonic(ts)).polnum}.

zeroSetSplit : List P -> List $
  ++ \axiom{zeroSetSplit(lp)} returns a list \axiom{lts} of triangular
  ++ sets such that the zero set of \axiom{lp} is the union of the
  ++ closures of the regular zero sets  of the members of \axiom{lts}.

zeroSetSplitIntoTriangularSystems : List P -> _
 List Record(close:$,open:List P)
  ++ \axiom{zeroSetSplitIntoTriangularSystems(lp)} returns a list of
  ++ triangular systems \axiom{[[ts1,qs1],...,[tsn,qsn]]} such that the
```

```
  ++ zero set of \axiom{lp} is the union of the closures of the
  ++ \axiom{W_i} where \axiom{W_i} consists of the zeros of \axiom{ts}
  ++ which do not cancel any polynomial in \axiom{qsi}.

first : $ -> Union(P,"failed")
  ++ \axiom{first(ts)} returns the polynomial of \axiom{ts} with
  ++ greatest main variable if \axiom{ts} is not empty, otherwise
  ++ returns \axiom{"failed"}.

last : $ -> Union(P,"failed")
  ++ \axiom{last(ts)} returns the polynomial of \axiom{ts} with
  ++ smallest main variable if \axiom{ts} is not empty, otherwise
  ++ returns \axiom{"failed"}.

rest : $ -> Union($,"failed")
  ++ \axiom{rest(ts)} returns the polynomials of \axiom{ts} with smaller
  ++ main variable than \axiom{mvar(ts)} if \axiom{ts} is not empty,
  ++ otherwise returns "failed"

algebraicVariables : $ -> List(V)
  ++ \axiom{algebraicVariables(ts)} returns the decreasingly sorted
  ++ list of the main variables of the polynomials of \axiom{ts}.

algebraic? : (V,$) -> Boolean
  ++ \axiom{algebraic?(v,ts)} returns true iff \axiom{v} is the
  ++ main variable of some polynomial in \axiom{ts}.

select : ($,V) -> Union(P,"failed")
  ++ \axiom{select(ts,v)} returns the polynomial of \axiom{ts} with
  ++ \axiom{v} as main variable, if any.

extendIfCan : ($,P) -> Union($,"failed")
  ++ \axiom{extendIfCan(ts,p)} returns a triangular set which encodes
  ++ the simple extension by \axiom{p} of the extension of the base
  ++ field defined by \axiom{ts}, according
  ++ to the properties of triangular sets of the current domain.
  ++ If the required properties do not hold then "failed" is returned.
  ++ This operation encodes in some sense the properties of the
  ++ triangular sets of the current category. Is is used to implement
  ++ the \axiom{construct} operation to guarantee that every triangular
  ++ set build from a list of polynomials has the required properties.

extend : ($,P) -> $
  ++ \axiom{extend(ts,p)} returns a triangular set which encodes the
  ++ simple extension by \axiom{p} of the extension of the base field
  ++ defined by \axiom{ts}, according to the properties of triangular
  ++ sets of the current category. If the required properties do not
  ++ hold an error is returned.

if V has Finite then

  coHeight : $ -> NonNegativeInteger
    ++ \axiom{coHeight(ts)} returns \axiom{size()\$V} minus \axiom{\#ts}.
```

```
add

  GPS ==> GeneralPolynomialSet(R,E,V,P)
  B ==> Boolean
  RBT ==> Record(bas:$,top:List P)

  ts:$ = us:$ ==
    empty?(ts)$$ => empty?(us)$$
    empty?(us)$$ => false
    first(ts)::P =$P first(us)::P => rest(ts)::$ =$$ rest(us)::$
    false

  infRittWu?(ts,us) ==
    empty?(us)$$ => not empty?(ts)$$
    empty?(ts)$$ => false
    p : P := (last(ts))::P
    q : P := (last(us))::P
    infRittWu?(p,q)$P => true
    supRittWu?(p,q)$P => false
    v : V := mvar(p)
    infRittWu?(collectUpper(ts,v),collectUpper(us,v))$$

  reduced?(p,ts,redOp?) ==
    lp : List P := members(ts)
    while (not empty? lp) and (redOp?(p,first(lp))) repeat
      lp := rest lp
    empty? lp

  basicSet(ps,redOp?) ==
    ps := remove(zero?,ps)
    any?(ground?,ps) => "failed"::Union(RBT,"failed")
    ps := sort(infRittWu?,ps)
    p,b : P
    bs := empty()$$
    ts : List P := []
    while not empty? ps repeat
      b := first(ps)
      bs := extend(bs,b)$$
      ps := rest ps
      while (not empty? ps) and _
            (not reduced?((p := first(ps)),bs,redOp?)) repeat
        ts := cons(p,ts)
        ps := rest ps
    ([bs,ts]$RBT)::Union(RBT,"failed")

  basicSet(ps,pred?,redOp?) ==
    ps := remove(zero?,ps)
    any?(ground?,ps) => "failed"::Union(RBT,"failed")
    gps : List P := []
    bps : List P := []
    while not empty? ps repeat
      p := first ps
      ps := rest ps
      if pred?(p)
```

```
      then
        gps := cons(p,gps)
      else
        bps := cons(p,bps)
  gps := sort(infRittWu?,gps)
  p,b : P
  bs := empty()$$
  ts : List P := []
  while not empty? gps repeat
    b := first(gps)
    bs := extend(bs,b)$$
    gps := rest gps
    while (not empty? gps) and _
          (not reduced?((p := first(gps)),bs,redOp?)) repeat
      ts := cons(p,ts)
      gps := rest gps
  ts := sort(infRittWu?,concat(ts,bps))
  ([bs,ts]$RBT)::Union(RBT,"failed")

initials ts ==
  lip : List P := []
  empty? ts => lip
  lp := members(ts)
  while not empty? lp repeat
     p := first(lp)
     if not ground?((ip := init(p)))
       then
         lip := cons(primPartElseUnitCanonical(ip),lip)
     lp := rest lp
  removeDuplicates lip

degree ts ==
  empty? ts => 0$NonNegativeInteger
  lp := members ts
  d : NonNegativeInteger := mdeg(first lp)
  while not empty? (lp := rest lp) repeat
    d := d * mdeg(first lp)
  d

quasiComponent ts ==
  [members(ts),initials(ts)]

normalized?(p,ts) ==
  normalized?(p,members(ts))$P

stronglyReduced? (p,ts) ==
  reduced?(p,members(ts))$P

headReduced? (p,ts) ==
  stronglyReduced?(head(p),ts)

initiallyReduced? (p,ts) ==
  lp : List (P) := members(ts)
  red : Boolean := true
```

```
  while (not empty? lp) and (not ground?(p)$P) and red repeat
    while (not empty? lp) and (mvar(first(lp)) > mvar(p)) repeat
      lp := rest lp
    if (not empty? lp)
      then
        if  (mvar(first(lp)) = mvar(p))
          then
            if reduced?(p,first(lp))
              then
                lp := rest lp
                p := init(p)
              else
                red := false
          else
            p := init(p)
  red

reduce(p,ts,redOp,redOp?) ==
  (empty? ts) or (ground? p) => p
  ts0 := ts
  while (not empty? ts) and (not ground? p) repeat
     reductor := (first ts)::P
     ts := (rest ts)::$
     if not redOp?(p,reductor)
       then
         p := redOp(p,reductor)
         ts := ts0
  p

rewriteSetWithReduction(lp,ts,redOp,redOp?) ==
  trivialIdeal? ts => lp
  lp := remove(zero?,lp)
  empty? lp => lp
  any?(ground?,lp) => [1$P]
  rs : List P := []
  while not empty? lp repeat
    p := first lp
    lp := rest lp
    p := primPartElseUnitCanonical reduce(p,ts,redOp,redOp?)
    if not zero? p
      then
        if ground? p
          then
            lp := []
            rs := [1$P]
          else
            rs := cons(p,rs)
  removeDuplicates rs

stronglyReduce(p,ts) ==
  reduce (p,ts,lazyPrem,reduced?)

headReduce(p,ts) ==
  reduce (p,ts,headReduce,headReduced?)
```

```
initiallyReduce(p,ts) ==
  reduce (p,ts,initiallyReduce,initiallyReduced?)

removeZero(p,ts) ==
  (ground? p) or (empty? ts) => p
  v := mvar(p)
  ts_v_- := collectUnder(ts,v)
  if algebraic?(v,ts)
    then
      q := lazyPrem(p,select(ts,v)::P)
      zero? q => return q
      zero? removeZero(q,ts_v_-) => return 0
  empty? ts_v_- => p
  q: P := 0
  while positive? degree(p,v) repeat
     q := removeZero(init(p),ts_v_-) * mainMonomial(p) + q
     p := tail(p)
  q + removeZero(p,ts_v_-)

reduceByQuasiMonic(p, ts) ==
  (ground? p) or (empty? ts) => p
  remainder(p,collectQuasiMonic(ts)).polnum

autoReduced?(ts : $,redOp? : ((P,List(P)) -> Boolean)) ==
  empty? ts => true
  lp : List (P) := members(ts)
  p : P := first(lp)
  lp := rest lp
  while (not empty? lp) and redOp?(p,lp) repeat
     p := first lp
     lp := rest lp
  empty? lp

stronglyReduced? ts ==
  autoReduced? (ts, reduced?)

normalized? ts ==
  autoReduced? (ts,normalized?)

headReduced? ts ==
  autoReduced? (ts,headReduced?)

initiallyReduced?  ts ==
  autoReduced? (ts,initiallyReduced?)

mvar ts ==
  empty? ts => error"Error from TSETCAT in mvar : #1 is empty"
  mvar((first(ts))::P)$P

first ts ==
  empty? ts => "failed"::Union(P,"failed")
  lp : List(P) := sort(supRittWu?,members(ts))$(List P)
  first(lp)::Union(P,"failed")
```

```
last ts ==
  empty? ts => "failed"::Union(P,"failed")
  lp : List(P) := sort(infRittWu?,members(ts))$(List P)
  first(lp)::Union(P,"failed")

rest ts ==
  empty? ts => "failed"::Union($,"failed")
  lp : List(P) := sort(supRittWu?,members(ts))$(List P)
  construct(rest(lp))::Union($,"failed")

coerce (ts:$) : List(P) ==
  sort(supRittWu?,members(ts))$(List P)

algebraicVariables ts ==
  [mvar(p) for p in members(ts)]

algebraic? (v,ts) ==
  member?(v,algebraicVariables(ts))

select  (ts,v) ==
  lp : List (P) := sort(supRittWu?,members(ts))$(List P)
  while (not empty? lp) and (not (v = mvar(first lp))) repeat
    lp := rest lp
  empty? lp => "failed"::Union(P,"failed")
  (first lp)::Union(P,"failed")

collectQuasiMonic ts ==
  lp: List(P) := members(ts)
  newlp: List(P) := []
  while (not empty? lp) repeat
    if ground? init(first(lp)) then newlp := cons(first(lp),newlp)
    lp := rest lp
  construct(newlp)

collectUnder (ts,v) ==
  lp : List (P) := sort(supRittWu?,members(ts))$(List P)
  while (not empty? lp) and (not (v > mvar(first lp))) repeat
    lp := rest lp
  construct(lp)

collectUpper  (ts,v) ==
  lp1 : List(P) := sort(supRittWu?,members(ts))$(List P)
  lp2 : List(P) := []
  while (not empty? lp1) and  (mvar(first lp1) > v) repeat
    lp2 := cons(first(lp1),lp2)
    lp1 := rest lp1
  construct(reverse lp2)

construct(lp:List(P)) ==
  rif := retractIfCan(lp)@Union($,"failed")
  not (rif case $) => error"in construct : LP -> $ from TSETCAT : bad arg"
  rif::$
```

```
     retractIfCan(lp:List(P)) ==
       empty? lp => (empty()$$)::Union($,"failed")
       lp := sort(supRittWu?,lp)
       rif := retractIfCan(rest(lp))@Union($,"failed")
       not (rif case $) => _
        error "in retractIfCan : LP -> ... from TSETCAT : bad arg"
       extendIfCan(rif::$,first(lp))@Union($,"failed")

     extend(ts:$,p:P):$ ==
       eif := extendIfCan(ts,p)@Union($,"failed")
       not (eif case $) => error"in extend : ($,P) -> $ from TSETCAT : bad ars"
       eif::$

     if V has Finite then

       coHeight ts ==
         n := size()$V
         m := #(members ts)
         subtractIfCan(n,m)$NonNegativeInteger::NonNegativeInteger
```

— **COQ TSETCAT** —

```
(* category TSETCAT *)
(*

     GPS ==> GeneralPolynomialSet(R,E,V,P)
     B ==> Boolean
     RBT ==> Record(bas:$,top:List P)

     ?=? : (%,%) -> Boolean
     ts:$ = us:$ ==
       empty?(ts)$$ => empty?(us)$$
       empty?(us)$$ => false
       first(ts)::P =$P first(us)::P => rest(ts)::$ =$$ rest(us)::$
       false

     infRittWu? : ($,$) -> Boolean
     infRittWu?(ts,us) ==
       empty?(us)$$ => not empty?(ts)$$
       empty?(ts)$$ => false
       p : P := (last(ts))::P
       q : P := (last(us))::P
       infRittWu?(p,q)$P => true
       supRittWu?(p,q)$P => false
       v : V := mvar(p)
       infRittWu?(collectUpper(ts,v),collectUpper(us,v))$$

     reduced? : (P,$,((P,P) -> Boolean)) -> Boolean
     reduced?(p,ts,redOp?) ==
       lp : List P := members(ts)
       while (not empty? lp) and (redOp?(p,first(lp))) repeat
```

```
    lp := rest lp
  empty? lp

basicSet : (List P,((P,P)->Boolean)) -> _
basicSet(ps,redOp?) ==
  ps := remove(zero?,ps)
  any?(ground?,ps) => "failed"::Union(RBT,"failed")
  ps := sort(infRittWu?,ps)
  p,b : P
  bs := empty()$$
  ts : List P := []
  while not empty? ps repeat
    b := first(ps)
    bs := extend(bs,b)$$
    ps := rest ps
    while (not empty? ps) and _
          (not reduced?((p := first(ps)),bs,redOp?)) repeat
      ts := cons(p,ts)
      ps := rest ps
  ([bs,ts]$RBT)::Union(RBT,"failed")

basicSet : (List P,(P->Boolean),((P,P)->Boolean)) -> _
 Union(Record(bas:$,top:List P),"failed")
basicSet(ps,pred?,redOp?) ==
  ps := remove(zero?,ps)
  any?(ground?,ps) => "failed"::Union(RBT,"failed")
  gps : List P := []
  bps : List P := []
  while not empty? ps repeat
    p := first ps
    ps := rest ps
    if pred?(p)
      then
        gps := cons(p,gps)
      else
        bps := cons(p,bps)
  gps := sort(infRittWu?,gps)
  p,b : P
  bs := empty()$$
  ts : List P := []
  while not empty? gps repeat
    b := first(gps)
    bs := extend(bs,b)$$
    gps := rest gps
    while (not empty? gps) and _
          (not reduced?((p := first(gps)),bs,redOp?)) repeat
      ts := cons(p,ts)
      gps := rest gps
  ts := sort(infRittWu?,concat(ts,bps))
  ([bs,ts]$RBT)::Union(RBT,"failed")

initials : $ -> List P
initials ts ==
  lip : List P := []
```

```
    empty? ts => lip
    lp := members(ts)
    while not empty? lp repeat
       p := first(lp)
       if not ground?((ip := init(p)))
         then
           lip := cons(primPartElseUnitCanonical(ip),lip)
       lp := rest lp
    removeDuplicates lip

  degree : $ -> NonNegativeInteger
  degree ts ==
    empty? ts => 0$NonNegativeInteger
    lp := members ts
    d : NonNegativeInteger := mdeg(first lp)
    while not empty? (lp := rest lp) repeat
      d := d * mdeg(first lp)
    d

  quasiComponent : $ -> Record(close:List P,open:List P)
  quasiComponent ts ==
    [members(ts),initials(ts)]

  normalized? : (P,$) -> Boolean
  normalized?(p,ts) ==
    normalized?(p,members(ts))$P

  stronglyReduced? : (P,$) -> Boolean
  stronglyReduced? (p,ts) ==
    reduced?(p,members(ts))$P

  headReduced? : (P,$) -> Boolean
  headReduced? (p,ts) ==
    stronglyReduced?(head(p),ts)

  initiallyReduced? : (P,$) -> Boolean
  initiallyReduced? (p,ts) ==
    lp : List (P) := members(ts)
    red : Boolean := true
    while (not empty? lp) and (not ground?(p)$P) and red repeat
      while (not empty? lp) and (mvar(first(lp)) > mvar(p)) repeat
        lp := rest lp
      if (not empty? lp)
        then
          if  (mvar(first(lp)) = mvar(p))
            then
              if reduced?(p,first(lp))
                then
                  lp := rest lp
                  p := init(p)
                else
                  red := false
            else
              p := init(p)
```

```
  red

reduce : (P,$,((P,P) -> P),((P,P) -> Boolean) ) -> P
reduce(p,ts,redOp,redOp?) ==
  (empty? ts) or (ground? p) => p
  ts0 := ts
  while (not empty? ts) and (not ground? p) repeat
     reductor := (first ts)::P
     ts := (rest ts)::$
     if not redOp?(p,reductor)
       then
         p := redOp(p,reductor)
         ts := ts0
  p

 rewriteSetWithReduction : (List P,$,((P,P) -> P),((P,P) -> Boolean) ) ->_
   List P
 rewriteSetWithReduction(lp,ts,redOp,redOp?) ==
  trivialIdeal? ts => lp
  lp := remove(zero?,lp)
  empty? lp => lp
  any?(ground?,lp) => [1$P]
  rs : List P := []
  while not empty? lp repeat
    p := first lp
    lp := rest lp
    p := primPartElseUnitCanonical reduce(p,ts,redOp,redOp?)
    if not zero? p
      then
        if ground? p
          then
            lp := []
            rs := [1$P]
          else
            rs := cons(p,rs)
  removeDuplicates rs

stronglyReduce : (P,$) -> P
stronglyReduce(p,ts) ==
  reduce (p,ts,lazyPrem,reduced?)

headReduce : (P,$) -> P
headReduce(p,ts) ==
  reduce (p,ts,headReduce,headReduced?)

initiallyReduce : (P,$) -> P
initiallyReduce(p,ts) ==
  reduce (p,ts,initiallyReduce,initiallyReduced?)

removeZero: (P, $) -> P
removeZero(p,ts) ==
  (ground? p) or (empty? ts) => p
  v := mvar(p)
  ts_v_- := collectUnder(ts,v)
```

```
    if algebraic?(v,ts)
      then
        q := lazyPrem(p,select(ts,v)::P)
        zero? q => return q
        zero? removeZero(q,ts_v_-) => return 0
    empty? ts_v_- => p
    q: P := 0
    while positive? degree(p,v) repeat
       q := removeZero(init(p),ts_v_-) * mainMonomial(p) + q
       p := tail(p)
    q + removeZero(p,ts_v_-)

  reduceByQuasiMonic: (P, $) -> P
  reduceByQuasiMonic(p, ts) ==
    (ground? p) or (empty? ts) => p
    remainder(p,collectQuasiMonic(ts)).polnum

  autoReduced? : ($,((P,List(P)) -> Boolean)) -> Boolean
  autoReduced?(ts : $,redOp? : ((P,List(P)) -> Boolean)) ==
    empty? ts => true
    lp : List (P) := members(ts)
    p : P := first(lp)
    lp := rest lp
    while (not empty? lp) and redOp?(p,lp) repeat
       p := first lp
       lp := rest lp
    empty? lp

  stronglyReduced? : $ -> Boolean
  stronglyReduced? ts ==
    autoReduced? (ts, reduced?)

  normalized? : $  -> Boolean
  normalized? ts ==
    autoReduced? (ts,normalized?)

  headReduced? : $ -> Boolean
  headReduced? ts ==
    autoReduced? (ts,headReduced?)

  initiallyReduced? : $ -> Boolean
  initiallyReduced?  ts ==
    autoReduced? (ts,initiallyReduced?)

  mvar : % -> V
  mvar ts ==
    empty? ts => error"Error from TSETCAT in mvar : #1 is empty"
    mvar((first(ts))::P)$P

  first : $ -> Union(P,"failed")
  first ts ==
    empty? ts => "failed"::Union(P,"failed")
    lp : List(P) := sort(supRittWu?,members(ts))$(List P)
    first(lp)::Union(P,"failed")
```

```
last : $ -> Union(P,"failed")
last ts ==
  empty? ts => "failed"::Union(P,"failed")
  lp : List(P) := sort(infRittWu?,members(ts))$(List P)
  first(lp)::Union(P,"failed")

rest : $ -> Union($,"failed")
rest ts ==
  empty? ts => "failed"::Union($,"failed")
  lp : List(P) := sort(supRittWu?,members(ts))$(List P)
  construct(rest(lp))::Union($,"failed")

coerce : % -> List(P)
coerce (ts:$) : List(P) ==
  sort(supRittWu?,members(ts))$(List P)

algebraicVariables : $ -> List(V)
algebraicVariables ts ==
  [mvar(p) for p in members(ts)]

algebraic? : (V,$) -> Boolean
algebraic? (v,ts) ==
  member?(v,algebraicVariables(ts))

select : (%,V) -> Union(P,"failed")
select (ts,v) ==
  lp : List (P) := sort(supRittWu?,members(ts))$(List P)
  while (not empty? lp) and (not (v = mvar(first lp))) repeat
    lp := rest lp
  empty? lp => "failed"::Union(P,"failed")
  (first lp)::Union(P,"failed")

collectQuasiMonic : % -> %
collectQuasiMonic ts ==
  lp: List(P) := members(ts)
  newlp: List(P) := []
  while (not empty? lp) repeat
    if ground? init(first(lp)) then newlp := cons(first(lp),newlp)
    lp := rest lp
  construct(newlp)

collectUnder : (%,V) -> %
collectUnder (ts,v) ==
  lp : List (P) := sort(supRittWu?,members(ts))$(List P)
  while (not empty? lp) and (not (v > mvar(first lp))) repeat
    lp := rest lp
  construct(lp)

collectUpper : (%,V) -> %
collectUpper  (ts,v) ==
  lp1 : List(P) := sort(supRittWu?,members(ts))$(List P)
  lp2 : List(P) := []
  while (not empty? lp1) and  (mvar(first lp1) > v) repeat
```

```
        lp2 := cons(first(lp1),lp2)
        lp1 := rest lp1
      construct(reverse lp2)

    construct : List P -> %
    construct(lp:List(P)) ==
      rif := retractIfCan(lp)@Union($,"failed")
      not (rif case $) => error"in construct : LP -> $ from TSETCAT : bad arg"
      rif::$

    retractIfCan : List P -> Union(%,"failed")
    retractIfCan(lp:List(P)) ==
      empty? lp => (empty()$$)::Union($,"failed")
      lp := sort(supRittWu?,lp)
      rif := retractIfCan(rest(lp))@Union($,"failed")
      not (rif case $) => _
       error "in retractIfCan : LP -> ... from TSETCAT : bad arg"
      extendIfCan(rif::$,first(lp))@Union($,"failed")

    extend : (%,P) -> %
    extend(ts:$,p:P):$ ==
      eif := extendIfCan(ts,p)@Union($,"failed")
      not (eif case $) => error"in extend : ($,P) -> $ from TSETCAT : bad ars"
      eif::$

    if V has Finite
    then

      coHeight : % -> NonNegativeInteger
      coHeight ts ==
        n := size()$V
        m := #(members ts)
        subtractIfCan(n,m)$NonNegativeInteger::NonNegativeInteger
*)
```

——–

— TSETCAT.dotabb —

```
"TSETCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=TSETCAT"];
"TSETCAT" -> "PSETCAT"
```

——–

— TSETCAT.dotfull —

```
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b
 [color=lightblue,href="bookvol10.2.pdf#nameddest=TSETCAT"];
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b
  -> "PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategor

"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
```

```
 [color=seagreen,href="bookvol10.2.pdf#nameddest=TSETCAT"];
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 ->
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b
```

———-

— TSETCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b
 [color=lightblue];
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b
  -> "PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategor

"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b
 [color=seagreen];
"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b
  -> "PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"

"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 [color=lightblue];
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 -> "SetCategory()"
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 -> "Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet())"
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 -> "CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet()))"

"CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet()))"
 [color=seagreen];
"CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet()))"
  -> "CoercibleTo(a:Type)"

"Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet())" [color=seagreen];
"Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet())"
  -> "Collection(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"
"Collection(a:Type)" -> "ConvertibleTo(InputForm)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"
```

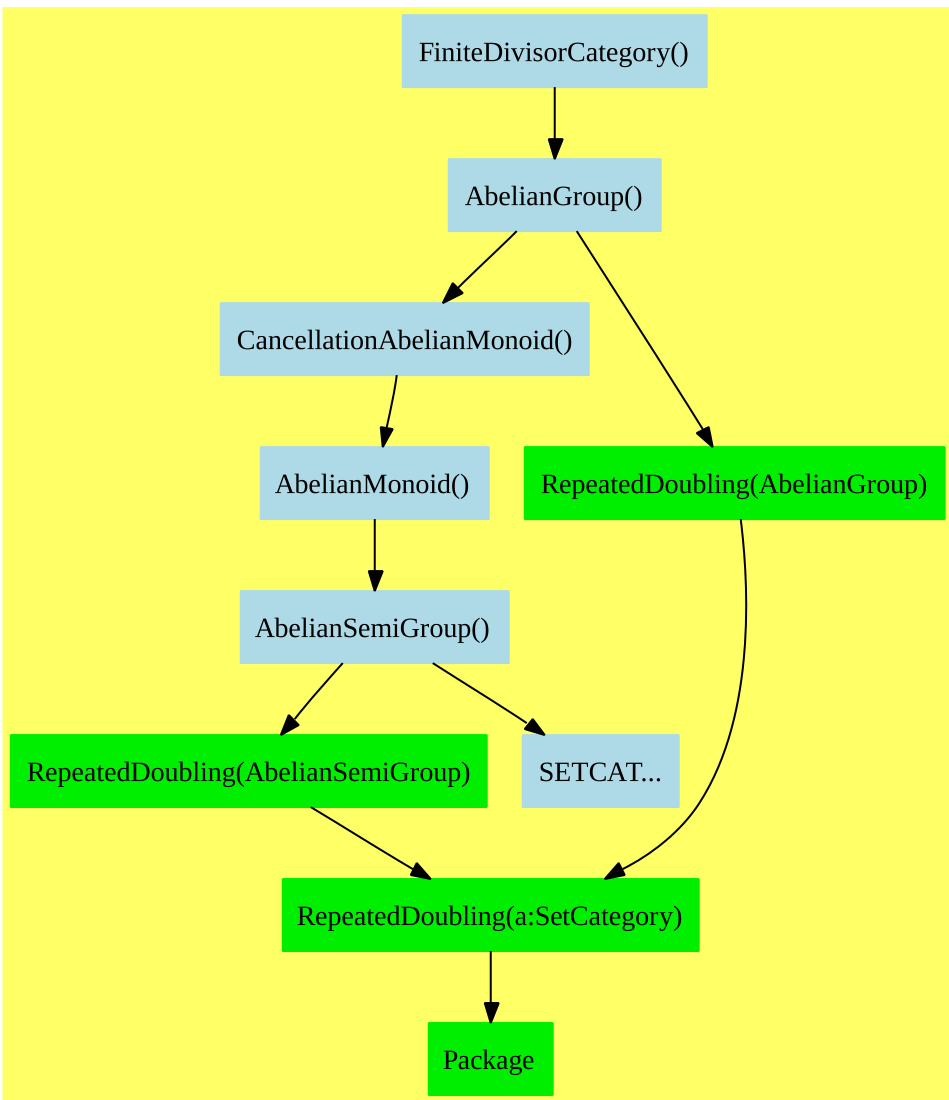
```
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"ConvertibleTo(InputForm)" [color="#00EE00"];
"ConvertibleTo(InputForm)" -> "ConvertibleTo(a:Type)"

"ConvertibleTo(a:Type)" [color="#00EE00"];
"ConvertibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

———-

Chapter 8

Category Layer 7

8.0.133 FiniteDivisorCategory (FDIVCAT)



— **FiniteDivisorCategory.input** —

```
)set break resume
)sys rm -f FiniteDivisorCategory.output
)spool FiniteDivisorCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteDivisorCategory
--R
--R FiniteDivisorCategory(F: Field,UP: UnivariatePolynomialCategory(t#1),UPUP: UnivariatePolynomialCategory(Frac
--R Abbreviation for FiniteDivisorCategory is FDIVCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FDIVCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Integer,%) -> %                ?*? : (NonNegativeInteger,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?=? : (%,%) -> Boolean                0 : () -> %
--R coerce : % -> OutputForm              divisor : (R,UP,UP,UP,F) -> %
--R divisor : (F,F,Integer) -> %          divisor : (F,F) -> %
--R divisor : R -> %                      generator : % -> Union(R,"failed")
--R hash : % -> SingleInteger             latex : % -> String
--R principal? : % -> Boolean             reduce : % -> %
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R decompose : % -> Record(id: FractionalIdeal(UP,Fraction(UP),UPUP,R),principalPart: R)
--R divisor : FractionalIdeal(UP,Fraction(UP),UPUP,R) -> %
--R ideal : % -> FractionalIdeal(UP,Fraction(UP),UPUP,R)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— **FiniteDivisorCategory.help** —

```
====================================================================
FiniteDivisorCategory examples
====================================================================

This category describes finite rational divisors on a curve, that
is finite formal sums SUM(n * P) where the n's are integers and the
P's are finite rational points on the curve.

See Also:
```

```
o )show FiniteDivisorCategory
```

———-

See:

⇐ "AbelianGroup" (ABELGRP) 7.0.121 on page 673

Exports:

0	coerce	decompose	divisor	generator
hash	ideal	latex	principal?	reduce
sample	subtractIfCan	zero?	?~=?	?*?
?+?	?-?	-?	?=?	

These are directly exported but not implemented:

```
decompose : % ->
  Record(id: FractionalIdeal(UP,Fraction UP,UPUP,R),principalPart: R)
divisor : R -> %
divisor : FractionalIdeal(UP,Fraction UP,UPUP,R) -> %
divisor : (F,F) -> %
divisor : (F,F,Integer) -> %
divisor : (R,UP,UP,UP,F) -> %
generator : % -> Union(R,"failed")
ideal : % -> FractionalIdeal(UP,Fraction UP,UPUP,R)
reduce : % -> %
```

These are implemented by this category:

```
principal? : % -> Boolean
```

These exports come from (p673) AbelianGroup():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
-? : % -> %
?-? : (%,%) -> %
?~=? : (%,%) -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
```

— **FiniteDivisorCategory.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FDIVCAT">
FiniteDivisorCategory (FDIVCAT)</a></h2>
</body>
```

— category FDIVCAT FiniteDivisorCategory —

```
)abbrev category FDIVCAT FiniteDivisorCategory
++ Category for finite rational divisors on a curve
++ Author: Manuel Bronstein
++ Date Created: 19 May 1993
++ Date Last Updated: 19 May 1993
++ Description:
++ This category describes finite rational divisors on a curve, that
++ is finite formal sums SUM(n * P) where the n's are integers and the
++ P's are finite rational points on the curve.

FiniteDivisorCategory(F, UP, UPUP, R) : Category == SIG where
  F : Field
  UP : UnivariatePolynomialCategory F
  UPUP : UnivariatePolynomialCategory Fraction UP
  R : FunctionFieldCategory(F, UP, UPUP)

  ID  ==> FractionalIdeal(UP, Fraction UP, UPUP, R)

  SIG ==> AbelianGroup with

    ideal : % -> ID
      ++ ideal(D) returns the ideal corresponding to a divisor D.

    divisor : ID -> %
      ++ divisor(I) makes a divisor D from an ideal I.

    divisor : R -> %
      ++ divisor(g) returns the divisor of the function g.

    divisor : (F, F) -> %
      ++ divisor(a, b) makes the divisor P: \spad{(x = a, y = b)}.
      ++ Error: if P is singular.

    divisor : (F, F, Integer) -> %
      ++ divisor(a, b, n) makes the divisor
      ++ \spad{nP} where P: \spad{(x = a, y = b)}.
      ++ P is allowed to be singular if n is a multiple of the rank.

    decompose : % -> Record(id:ID, principalPart: R)
      ++ decompose(d) returns \spad{[id, f]} where \spad{d = (id) + div(f)}.

    reduce : % -> %
      ++ reduce(D) converts D to some reduced form (the reduced forms can
      ++ be differents in different implementations).

    principal? : % -> Boolean
      ++ principal?(D) tests if the argument is the divisor of a function.

    generator : % -> Union(R, "failed")
      ++ generator(d) returns f if \spad{(f) = d},
```

```
      ++ "failed" if d is not principal.

    divisor : (R, UP, UP, UP, F) -> %
      ++ divisor(h, d, d', g, r) returns the sum of all the finite points
      ++ where \spad{h/d} has residue \spad{r}.
      ++ \spad{h} must be integral.
      ++ \spad{d} must be squarefree.
      ++ \spad{d'} is some derivative of \spad{d} (not necessarily dd/dx).
      ++ \spad{g = gcd(d,discriminant)} contains the ramified zeros of \spad{d}

   add

     principal? d == generator(d) case R
```

———-

— COQ FDIVCAT —

```
(* category FDIVCAT *)
(*
    principal? : % -> Boolean
    principal? d == generator(d) case R

*)
```

———-

— FDIVCAT.dotabb —

```
"FDIVCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FDIVCAT"];
"FDIVCAT" -> "ABELGRP"
```

———-

— FDIVCAT.dotfull —

```
"FiniteDivisorCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FDIVCAT"];
"FiniteDivisorCategory()" -> "AbelianGroup()"
```

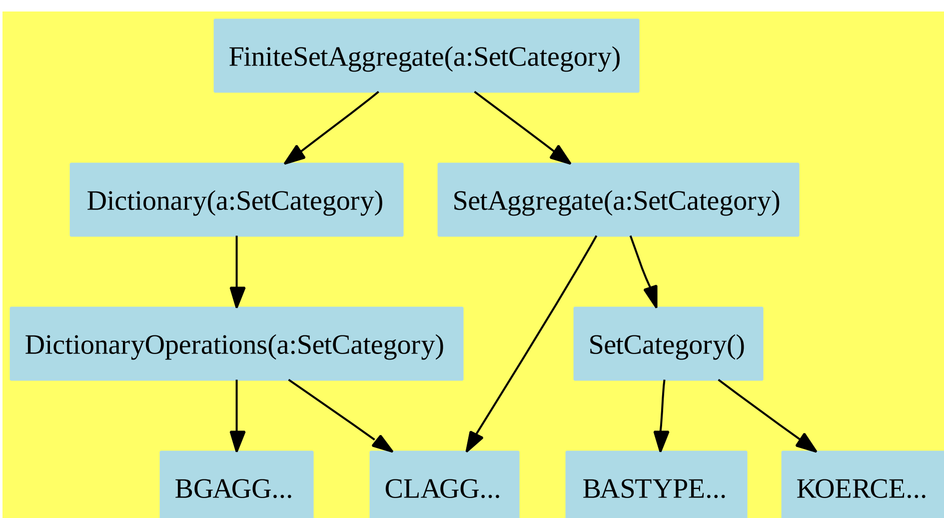
———-

— FDIVCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FiniteDivisorCategory()" [color=lightblue];
```

```
"FiniteDivisorCategory()" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "RepeatedDoubling(AbelianGroup)"

"RepeatedDoubling(AbelianGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianGroup)" -> "RepeatedDoubling(a:SetCategory)"

"RepeatedDoubling(AbelianSemiGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianSemiGroup)" -> "RepeatedDoubling(a:SetCategory)"

"RepeatedDoubling(a:SetCategory)" [color="#00EE00"];
"RepeatedDoubling(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SETCAT..."
"AbelianSemiGroup()" -> "RepeatedDoubling(AbelianSemiGroup)"

"SETCAT..." [color=lightblue];
}
```

8.0.134 FiniteSetAggregate (FSAGG)



— FiniteSetAggregate.input —

```
)set break resume
)sys rm -f FiniteSetAggregate.output
)spool FiniteSetAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteSetAggregate
--R
--R FiniteSetAggregate(S: SetCategory) is a category constructor
--R Abbreviation for FiniteSetAggregate is FSAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FSAGG
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean               ?=? : (%,%) -> Boolean
--R bag : List(S) -> %                   brace : () -> %
--R brace : List(S) -> %                 coerce : % -> OutputForm
--R construct : List(S) -> %             copy : % -> %
--R dictionary : List(S) -> %            dictionary : () -> %
--R difference : (%,%) -> %              difference : (%,S) -> %
--R empty : () -> %                      empty? : % -> Boolean
--R eq? : (%,%) -> Boolean               extract! : % -> S
--R hash : % -> SingleInteger            insert! : (S,%) -> %
--R inspect : % -> S                     intersect : (%,%) -> %
--R latex : % -> String                  map : ((S -> S),%) -> %
--R max : % -> S if S has ORDSET         min : % -> S if S has ORDSET
--R random : () -> % if S has FINITE     sample : () -> %
--R set : () -> %                        set : List(S) -> %
--R subset? : (%,%) -> Boolean           symmetricDifference : (%,%) -> %
--R union : (%,%) -> %                   union : (%,S) -> %
--R union : (S,%) -> %                   universe : () -> % if S has FINITE
--R ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R cardinality : % -> NonNegativeInteger
--R complement : % -> % if S has FINITE
--R convert : % -> InputForm if S has KONVERT(INFORM)
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R enumerate : () -> List(%) if S has FINITE
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R index : PositiveInteger -> % if S has FINITE
--R less? : (%,NonNegativeInteger) -> Boolean
```

```
--R lookup : % -> PositiveInteger if S has FINITE
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(S) if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R remove! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (S,%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R size : () -> NonNegativeInteger if S has FINITE
--R size? : (%,NonNegativeInteger) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

— **FiniteSetAggregate.help** —

```
====================================================================
FiniteSetAggregate examples
====================================================================

A finite-set aggregate models the notion of a finite set, that is,
a collection of elements characterized by membership, but not
by order or multiplicity. See Set for an example.

See Also:
o )show FiniteSetAggregate
o )show Set
```

See:

⇐ "Dictionary" (DIAGG) 7.0.123 on page 687
⇐ "SetAggregate" (SETAGG) 6.0.118 on page 628

Exports:

any?	bag	brace	cardinality	coerce
complement	construct	convert	copy	count
dictionary	difference	empty	empty?	eq?
eval	every?	extract!	find	hash
index	insert!	inspect	intersect	latex
less?	lookup	map	map!	max
member?	members	min	more?	parts
random	reduce	remove	remove!	removeDuplicates
sample	select	select!	set	size
size?	subset?	symmetricDifference	union	universe
#?	?<?	?=?	?~=?	

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **partiallyOrderedSet** is true if a set with $<$ which is transitive, but not($a < b$ or $a = b$) does not necessarily imply $b < a$.
- **nil**

These are implemented by this category:

```
brace : List S -> %
cardinality : % -> NonNegativeInteger
coerce : % -> OutputForm
complement : % -> % if S has FINITE
construct : List S -> %
count : (S,%) -> NonNegativeInteger
    if S has SETCAT and $ has finiteAggregate
difference : (%,%) -> %
index : PositiveInteger -> % if S has FINITE
intersect : (%,%) -> %
lookup : % -> PositiveInteger if S has FINITE
max : % -> S if S has ORDSET
min : % -> S if S has ORDSET
random : () -> % if S has FINITE
set : List S -> %
size : () -> NonNegativeInteger if S has FINITE
subset? : (%,%) -> Boolean
symmetricDifference : (%,%) -> %
union : (%,%) -> %
universe : () -> % if S has FINITE
?<? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
```

These exports come from (p687) Dictionary(S:SetCategory):

```
any? : ((S -> Boolean),%) -> Boolean
    if $ has finiteAggregate
bag : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
copy : % -> %
count : ((S -> Boolean),%) -> NonNegativeInteger
```

```
    if $ has finiteAggregate
dictionary : () -> %
dictionary : List S -> %
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,S,S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,Equation S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,List Equation S) -> %
    if S has EVALAB S
    and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
    if $ has finiteAggregate
extract! : % -> S
find : ((S -> Boolean),%) -> Union(S,"failed")
hash : % -> SingleInteger
insert! : (S,%) -> %
inspect : % -> S
latex : % -> String
less? : (%,NonNegativeInteger) -> Boolean
select! : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> %
    if $ has shallowlyMutable
member? : (S,%) -> Boolean
    if S has SETCAT
    and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
    if S has SETCAT
    and $ has finiteAggregate
remove : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
remove : (S,%) -> %
    if S has SETCAT
    and $ has finiteAggregate
remove! : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
remove! : (S,%) -> % if $ has finiteAggregate
removeDuplicates : % -> %
```

```
     if S has SETCAT
     and $ has finiteAggregate
 sample : () -> %
 select : ((S -> Boolean),%) -> %
     if $ has finiteAggregate
 size? : (%,NonNegativeInteger) -> Boolean
 #? : % -> NonNegativeInteger if $ has finiteAggregate
 ?~=? : (%,%) -> Boolean
```

These exports come from (p628) SetAggregate(S:SetCategory):

```
 brace : () -> %
 difference : (%,S) -> %
 set : () -> %
 union : (%,S) -> %
 union : (S,%) -> %
```

— **FiniteSetAggregate.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FSAGG">
FiniteSetAggregate (FSAGG)</a></h2>
</body>
```

— **category FSAGG FiniteSetAggregate** —

```
)abbrev category FSAGG FiniteSetAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: 14 Oct, 1993 by RSS
++ Description:
++ A finite-set aggregate models the notion of a finite set, that is,
++ a collection of elements characterized by membership, but not
++ by order or multiplicity.
++ See \spadtype{Set} for an example.

FiniteSetAggregate(S) : Category == SIG where
  S : SetCategory

  SIG ==> Join(Dictionary S, SetAggregate S) with

    finiteAggregate

    cardinality : % -> NonNegativeInteger
      ++ cardinality(u) returns the number of elements of u.
      ++ Note that \axiom{cardinality(u) = #u}.

    if S has Finite then

      Finite

      complement : % -> %
```

```
      ++ complement(u) returns the complement of the set u,
      ++ that is, the set of all values not in u.

    universe : () -> %
      ++ universe()$D returns the universal set for finite set aggregate D.

  if S has OrderedSet then

    max : % -> S
      ++ max(u) returns the largest element of aggregate u.

    min : % -> S
      ++ min(u) returns the smallest element of aggregate u.

add

  s < t == #s < #t and s = intersect(s,t)

  s = t == #s = #t and empty? difference(s,t)

  brace l == construct l

  set l == construct l

  cardinality s   == #s

  construct l == (s := set(); for x in l repeat insert_!(x,s); s)

  count(x:S, s:%) == (member?(x, s) => 1; 0)

  subset?(s, t)   == #s <= #t and _and/[member?(x, t) for x in parts s]

  coerce(s:%):OutputForm ==
    brace [x::OutputForm for x in parts s]$List(OutputForm)

  intersect(s, t) ==
    i := {}
    for x in parts s | member?(x, t) repeat insert_!(x, i)
    i

  difference(s:%, t:%) ==
    m := copy s
    for x in parts t repeat remove_!(x, m)
    m

  symmetricDifference(s, t) ==
    d := copy s
    for x in parts t repeat
      if member?(x, s) then remove_!(x, d) else insert_!(x, d)
    d

  union(s:%, t:%) ==
     u := copy s
     for x in parts t repeat insert_!(x, u)
```

```
      u

    if S has Finite then

      universe() == {index(i::PositiveInteger) for i in 1..size()$S}

      complement s == difference(universe(), s )

      size() == 2 ** size()$S

      index i ==
        {index(j::PositiveInteger)$S for j in 1..size()$S | bit?(i-1,j-1)}

      random()  ==
        index((random()$Integer rem (size()$% + 1))::PositiveInteger)

      lookup s ==
        n:PositiveInteger := 1
        for x in parts s repeat _
          n := n + 2 ** ((lookup(x) - 1)::NonNegativeInteger)
        n

    if S has OrderedSet then

      max s ==
        empty?(l := parts s) => error "Empty set"
        reduce("max", l)

      min s ==
        empty?(l := parts s) => error "Empty set"
        reduce("min", l)
```

——————

— COQ FSAGG —

```
(* category FSAGG *)
(*

   ?<? : (%,%) -> Boolean
   s < t == #s < #t and s = intersect(s,t)

   ?=? : (%,%) -> Boolean
   s = t == #s = #t and empty? difference(s,t)

   brace : List(S) -> %
   brace l == construct l

   set : List(S) -> %
   set l == construct l

   cardinality : % -> NonNegativeInteger
   cardinality s == #s
```

```
construct : List(S) -> %
construct l == (s := set(); for x in l repeat insert_!(x,s); s)

count : (S,%) -> NonNegativeInteger
count(x:S, s:%) == (member?(x, s) => 1; 0)

subset? : (%,%) -> Boolean
subset?(s, t) == #s <= #t and _and/[member?(x, t) for x in parts s]

coerce : % -> OutputForm
coerce(s:%):OutputForm ==
  brace [x::OutputForm for x in parts s]$List(OutputForm)

intersect : (%,%) -> %
intersect(s, t) ==
  i := {}
  for x in parts s | member?(x, t) repeat insert_!(x, i)
  i

difference : (%,%) -> %
difference(s:%, t:%) ==
  m := copy s
  for x in parts t repeat remove_!(x, m)
  m

symmetricDifference : (%,%) -> %
symmetricDifference(s, t) ==
  d := copy s
  for x in parts t repeat
    if member?(x, s) then remove_!(x, d) else insert_!(x, d)
  d

union : (%,%) -> %
union(s:%, t:%) ==
   u := copy s
   for x in parts t repeat insert_!(x, u)
   u

if S has Finite then

  universe : () -> %
  universe() == {index(i::PositiveInteger) for i in 1..size()$S}

  complement : % -> %
  complement s == difference(universe(), s )

  size : () -> NonNegativeInteger
  size() == 2 ** size()$S

  index : PositiveInteger -> %
  index i ==
    {index(j::PositiveInteger)$S for j in 1..size()$S | bit?(i-1,j-1)}
```

```
     random : () -> %
     random()  ==
       index((random()$Integer rem (size()$% + 1))::PositiveInteger)

     lookup : % -> PositiveInteger
     lookup s ==
       n:PositiveInteger := 1
       for x in parts s repeat _
         n := n + 2 ** ((lookup(x) - 1)::NonNegativeInteger)
       n

   if S has OrderedSet then

     max : % -> S
     max s ==
       empty?(l := parts s) => error "Empty set"
       reduce("max", l)

     min : % -> S
     min s ==
       empty?(l := parts s) => error "Empty set"
       reduce("min", l)

*)
```

— FSAGG.dotabb —

```
"FSAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=FSAGG"];
"FSAGG" -> "DIAGG"
"FSAGG" -> "SETAGG"
```

— FSAGG.dotfull —

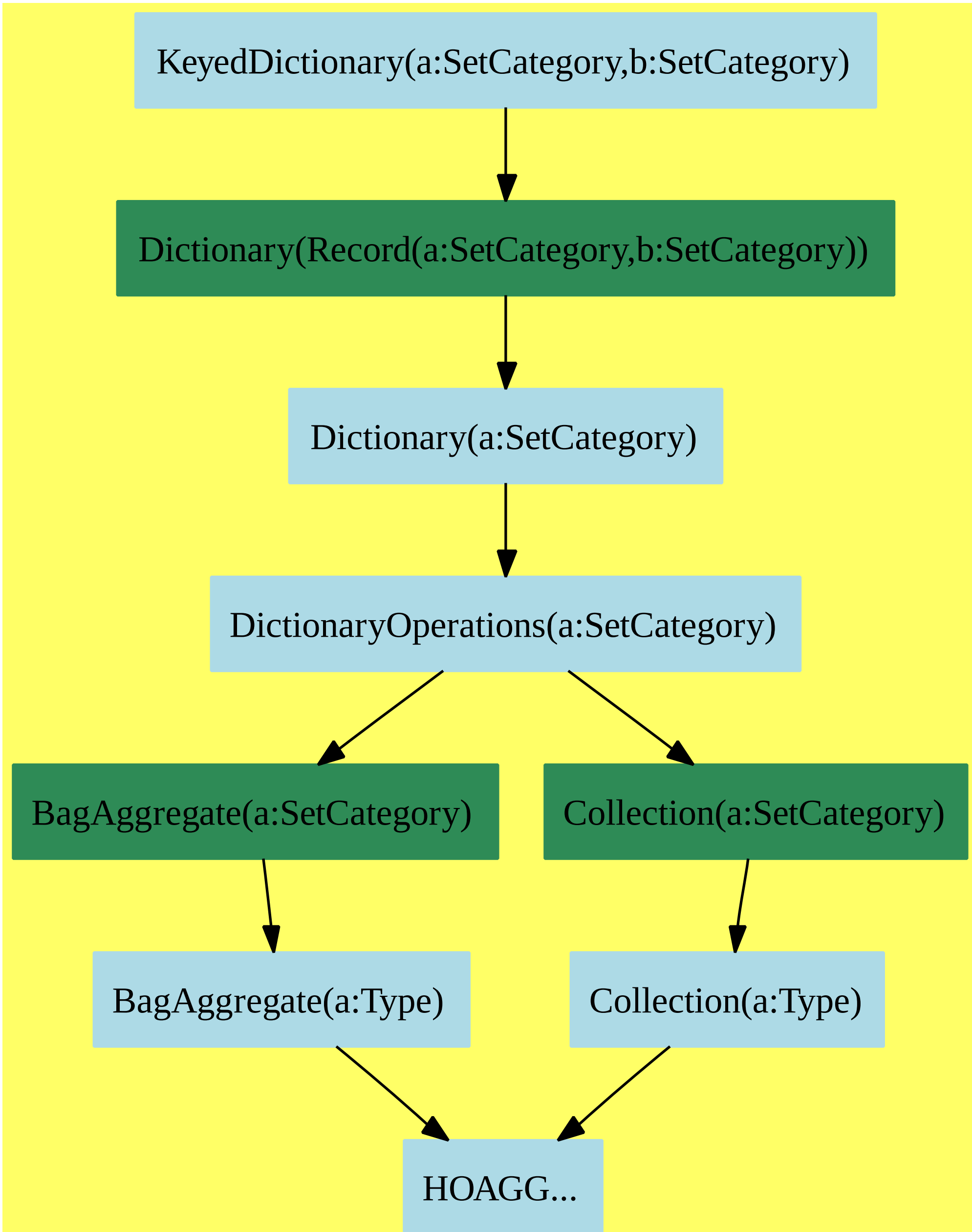
```
"FiniteSetAggregate(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FSAGG"];
"FiniteSetAggregate(a:SetCategory)" -> "Dictionary(a:SetCategory)"
"FiniteSetAggregate(a:SetCategory)" -> "SetAggregate(a:SetCategory)"
```

— FSAGG.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];
```

```
"FiniteSetAggregate(a:SetCategory)" [color=lightblue];
"FiniteSetAggregate(a:SetCategory)" -> "Dictionary(a:SetCategory)"
"FiniteSetAggregate(a:SetCategory)" -> "SetAggregate(a:SetCategory)"

"SetAggregate(a:SetCategory)" [color=lightblue];
"SetAggregate(a:SetCategory)" -> "SetCategory()"
"SetAggregate(a:SetCategory)" -> "CLAGG..."

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BASTYPE..."
"SetCategory()" -> "KOERCE..."

"Dictionary(a:SetCategory)" [color=lightblue];
"Dictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"

"DictionaryOperations(a:SetCategory)" [color=lightblue];
"DictionaryOperations(a:SetCategory)" -> "BGAGG..."
"DictionaryOperations(a:SetCategory)" -> "CLAGG..."

"BGAGG..." [color=lightblue];
"CLAGG..." [color=lightblue];
"BASTYPE..." [color=lightblue];
"KOERCE..." [color=lightblue];
}
```

8.0.135 KeyedDictionary (KDAGG)



— **KeyedDictionary.input** —

```
)set break resume
)sys rm -f KeyedDictionary.output
)spool KeyedDictionary.output
)set message test on
```

```
)set message auto off
)clear all

--S 1 of 1
)show KeyedDictionary
--R
--R KeyedDictionary(Key: SetCategory,Entry: SetCategory) is a category constructor
--R Abbreviation for KeyedDictionary is KDAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for KDAGG
--R
--R------------------------------- Operations --------------------------------
--R copy : % -> %                         dictionary : () -> %
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                key? : (Key,%) -> Boolean
--R keys : % -> List(Key)                 sample : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if Record(key: Key,entry: Entry) has SETCAT
--R any? : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R bag : List(Record(key: Key,entry: Entry)) -> %
--R coerce : % -> OutputForm if Record(key: Key,entry: Entry) has SETCAT
--R construct : List(Record(key: Key,entry: Entry)) -> %
--R convert : % -> InputForm if Record(key: Key,entry: Entry) has KONVERT(INFORM)
--R count : (Record(key: Key,entry: Entry),%) -> NonNegativeInteger if Record(key: Key,entry: Entry) has SETCAT
--R count : ((Record(key: Key,entry: Entry) -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R dictionary : List(Record(key: Key,entry: Entry)) -> %
--R eval : (%,List(Record(key: Key,entry: Entry)),List(Record(key: Key,entry: Entry))) -> % if Record(key: Key,e
--R eval : (%,Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> % if Record(key: Key,entry: Entry)
--R eval : (%,Equation(Record(key: Key,entry: Entry))) -> % if Record(key: Key,entry: Entry) has EVALAB(Record(k
--R eval : (%,List(Equation(Record(key: Key,entry: Entry)))) -> % if Record(key: Key,entry: Entry) has EVALAB(Re
--R every? : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R extract! : % -> Record(key: Key,entry: Entry)
--R find : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Union(Record(key: Key,entry: Entry),"failed")
--R hash : % -> SingleInteger if Record(key: Key,entry: Entry) has SETCAT
--R insert! : (Record(key: Key,entry: Entry),%) -> %
--R inspect : % -> Record(key: Key,entry: Entry)
--R latex : % -> String if Record(key: Key,entry: Entry) has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map : ((Record(key: Key,entry: Entry) -> Record(key: Key,entry: Entry)),%) -> %
--R map! : ((Record(key: Key,entry: Entry) -> Record(key: Key,entry: Entry)),%) -> % if $ has shallowlyMutable
--R member? : (Record(key: Key,entry: Entry),%) -> Boolean if Record(key: Key,entry: Entry) has SETCAT and $ has
--R members : % -> List(Record(key: Key,entry: Entry)) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(Record(key: Key,entry: Entry)) if $ has finiteAggregate
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry)),%
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry)),%
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry)),%
--R remove : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (Record(key: Key,entry: Entry),%) -> % if Record(key: Key,entry: Entry) has SETCAT and $ has finite
--R remove! : (Key,%) -> Union(Entry,"failed")
--R remove! : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (Record(key: Key,entry: Entry),%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if Record(key: Key,entry: Entry) has SETCAT and $ has finiteAggregate
--R search : (Key,%) -> Union(Entry,"failed")
```

```
--R select : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R ?~=? : (%,%) -> Boolean if Record(key: Key,entry: Entry) has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

— **KeyedDictionary.help** —

```
====================================================================
KeyedDictionary examples
====================================================================

A keyed dictionary is a dictionary of key-entry pairs for which there is
a unique entry for each key.

See Also:
o )show KeyedDictionary
```

See:

⇒ "TableAggregate" (TBAGG) 9.0.156 on page 972
⇐ "Dictionary" (DIAGG) 7.0.123 on page 687

Exports:

any?	bag	coerce	construct	convert
copy	count	dictionary	empty	empty?
eq?	eval	every?	extract!	find
hash	insert!	inspect	key?	keys
latex	less?	map	map!	member?
members	more?	parts	reduce	remove
remove!	removeDuplicates	sample	search	select
select!	size?	#?	?=?	?~=?

Attributes exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.

These are directly exported but not implemented:

```
remove! : (Key,%) -> Union(Entry,"failed")
```

```
search : (Key,%) -> Union(Entry,"failed")
```

These are implemented by this category:

```
key? : (Key,%) -> Boolean
member? : (Record(key: Key,entry: Entry),%) -> Boolean
    if Record(key: Key,entry: Entry) has SETCAT
    and $ has finiteAggregate
keys : % -> List Key
```

These exports come from (p687) Dictionary(R)
where R=Record(a:SetCategory,b:SetCategory))
and S=Record(key: Key,entry: Entry)

```
any? : ((S) -> Boolean),%) -> Boolean
    if $ has finiteAggregate
bag : List S -> %
coerce : % -> OutputForm if S has SETCAT
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
copy : % -> %
count : (S,%) -> NonNegativeInteger
    if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
    if $ has finiteAggregate
dictionary : () -> %
dictionary : List S -> %
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
    if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
    if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
    if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
    if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
    if $ has finiteAggregate
extract! : % -> S
find : ((S -> Boolean),%) -> Union(S,"failed")
hash : % -> SingleInteger if S has SETCAT
insert! : (S,%) -> %
inspect : % -> S
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> %
    if $ has shallowlyMutable
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S
```

```
     if $ has finiteAggregate
 reduce : (((S,S) -> S),%,S,S) -> S
     if S has SETCAT and $ has finiteAggregate
 remove : ((S -> Boolean),%) -> %
     if $ has finiteAggregate
 remove : (S,%) -> %
     if S has SETCAT and $ has finiteAggregate
 remove! : ((S -> Boolean),%) -> %
     if $ has finiteAggregate
 remove! : (S,%) -> % if $ has finiteAggregate
 removeDuplicates : % -> %
     if S has SETCAT and $ has finiteAggregate
 sample : () -> %
 select : ((S -> Boolean),%) -> %
     if $ has finiteAggregate
 select! : ((S -> Boolean),%) -> %
     if $ has finiteAggregate
 size? : (%,NonNegativeInteger) -> Boolean
 #? : % -> NonNegativeInteger
     if $ has finiteAggregate
 ?=? : (%,%) -> Boolean if S has SETCAT
 ?~=? : (%,%) -> Boolean if S has SETCAT
```

— KeyedDictionary.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#KDAGG">
KeyedDictionary (KDAGG)</a></h2>
</body>
```

— category KDAGG KeyedDictionary —

```
)abbrev category KDAGG KeyedDictionary
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ A keyed dictionary is a dictionary of key-entry pairs for which there is
++ a unique entry for each key.

KeyedDictionary(Key, Entry) : Category == SIG where
  Key : SetCategory
  Entry : SetCategory

  SIG ==> Dictionary Record(key:Key,entry:Entry) with

    key? : (Key, %) -> Boolean
      ++ key?(k,t) tests if k is a key in table t.

    keys : % -> List Key
      ++ keys(t) returns the list the keys in table t.
```

```
    remove_! : (Key, %) -> Union(Entry,"failed")
      ++ remove!(k,t) searches the table t for the key k removing
      ++ (and return) the entry if there.
      ++ If t has no such key, \axiom{remove!(k,t)} returns "failed".

    search : (Key, %) -> Union(Entry,"failed")
      ++ search(k,t) searches the table t for the key k,
      ++ returning the entry stored in t for key k.
      ++ If t has no such key, \axiom{search(k,t)} returns "failed".

   add

     key?(k, t) == search(k, t) case Entry

     member?(p, t) ==
       r := search(p.key, t)
       r case Entry and r::Entry = p.entry

     if % has finiteAggregate then

       keys t == [x.key for x in parts t]
```

— COQ KDAGG —

```
(* category KDAGG *)
(*

   key? : (Key,%) -> Boolean
   key?(k, t) == search(k, t) case Entry

   member? : (Record(key: Key,entry: Entry),%) -> Boolean
   member?(p, t) ==
     r := search(p.key, t)
     r case Entry and r::Entry = p.entry

   if % has finiteAggregate then

     keys : % -> List(Key)
     keys t == [x.key for x in parts t]

*)
```

— KDAGG.dotabb —

```
"KDAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=KDAGG"];
"KDAGG" -> "DIAGG"
```

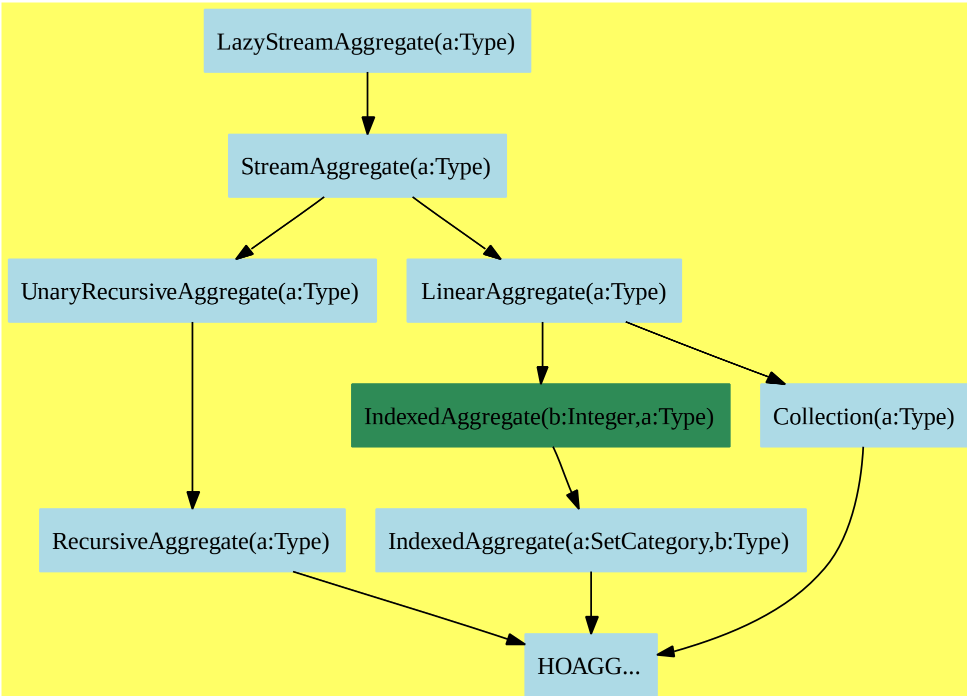
— KDAGG.dotfull —

```
"KeyedDictionary(a:SetCategory,b:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=KDAGG"];
"KeyedDictionary(a:SetCategory,b:SetCategory)" ->
    "Dictionary(Record(a:SetCategory,b:SetCategory))"
```

— KDAGG.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"KeyedDictionary(a:SetCategory,b:SetCategory)" [color=lightblue];
"KeyedDictionary(a:SetCategory,b:SetCategory)" ->
    "Dictionary(Record(a:SetCategory,b:SetCategory))"

"Dictionary(Record(a:SetCategory,b:SetCategory))" [color=seagreen];
"Dictionary(Record(a:SetCategory,b:SetCategory))" ->
    "Dictionary(a:SetCategory)"

"Dictionary(a:SetCategory)" [color=lightblue];
"Dictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"

"DictionaryOperations(a:SetCategory)" [color=lightblue];
"DictionaryOperations(a:SetCategory)" -> "BagAggregate(a:SetCategory)"
"DictionaryOperations(a:SetCategory)" -> "Collection(a:SetCategory)"

"BagAggregate(a:SetCategory)" [color=seagreen];
"BagAggregate(a:SetCategory)" -> "BagAggregate(a:Type)"

"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HOAGG..."

"Collection(a:SetCategory)" [color=seagreen];
"Collection(a:SetCategory)" -> "Collection(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HOAGG..."

"HOAGG..." [color=lightblue];
}
```

8.0.136 LazyStreamAggregate (LZSTAGG)



— **LazyStreamAggregate.input** —

```
)set break resume
)sys rm -f LazyStreamAggregate.output
)spool LazyStreamAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show LazyStreamAggregate
--R
--R LazyStreamAggregate(S: Type) is a category constructor
--R Abbreviation for LazyStreamAggregate is LZSTAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for LZSTAGG
--R
--R------------------------------- Operations --------------------------------
--R children : % -> List(%)              complete : % -> %
--R concat : (%,S) -> %                  concat : List(%) -> %
--R concat : (S,%) -> %                  concat : (%,%) -> %
--R construct : List(S) -> %             copy : % -> %
--R cycleEntry : % -> %                  cycleTail : % -> %
--R cyclic? : % -> Boolean               delete : (%,Integer) -> %
--R distance : (%,%) -> Integer          elt : (%,Integer,S) -> S
--R ?.? : (%,Integer) -> S               ?.last : (%,last) -> S
```

```
--R ?.rest : (%,rest) -> %               ?.first : (%,first) -> S
--R ?.value : (%,value) -> S             empty : () -> %
--R empty? : % -> Boolean                entries : % -> List(S)
--R eq? : (%,%) -> Boolean               explicitEntries? : % -> Boolean
--R explicitlyEmpty? : % -> Boolean      explicitlyFinite? : % -> Boolean
--R extend : (%,Integer) -> %            first : % -> S
--R frst : % -> S                        index? : (Integer,%) -> Boolean
--R indices : % -> List(Integer)         insert : (S,%,Integer) -> %
--R insert : (%,%,Integer) -> %          last : (%,NonNegativeInteger) -> %
--R last : % -> S                        latex : % -> String if S has SETCAT
--R lazy? : % -> Boolean                 lazyEvaluate : % -> %
--R leaf? : % -> Boolean                 leaves : % -> List(S)
--R map : (((S,S) -> S),%,%) -> %        map : ((S -> S),%) -> %
--R new : (NonNegativeInteger,S) -> %    nodes : % -> List(%)
--R possiblyInfinite? : % -> Boolean     qelt : (%,Integer) -> S
--R remove : ((S -> Boolean),%) -> %     rest : (%,NonNegativeInteger) -> %
--R rest : % -> %                        rst : % -> %
--R sample : () -> %                     second : % -> S
--R select : ((S -> Boolean),%) -> %     tail : % -> %
--R third : % -> S                       value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R concat! : (%,S) -> % if $ has shallowlyMutable
--R concat! : (%,%) -> % if $ has shallowlyMutable
--R convert : % -> InputForm if S has KONVERT(INFORM)
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R cycleLength : % -> NonNegativeInteger
--R cycleSplit! : % -> % if $ has shallowlyMutable
--R delete : (%,UniversalSegment(Integer)) -> %
--R ?.? : (%,UniversalSegment(Integer)) -> %
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R first : (%,NonNegativeInteger) -> %
--R hash : % -> SingleInteger if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R numberOfComputedEntries : % -> NonNegativeInteger
--R parts : % -> List(S) if $ has finiteAggregate
```

```
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R setchildren! : (%,List(%)) -> % if $ has shallowlyMutable
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R setelt : (%,UniversalSegment(Integer),S) -> S if $ has shallowlyMutable
--R setelt : (%,last,S) -> S if $ has shallowlyMutable
--R setelt : (%,rest,%) -> % if $ has shallowlyMutable
--R setelt : (%,first,S) -> S if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setfirst! : (%,S) -> S if $ has shallowlyMutable
--R setlast! : (%,S) -> S if $ has shallowlyMutable
--R setrest! : (%,%) -> % if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R split! : (%,Integer) -> % if $ has shallowlyMutable
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———-

— LazyStreamAggregate.help —

```
====================================================================
LazyStreamAggregate examples
====================================================================

LazyStreamAggregate is the category of streams with lazy
evaluation.  It is understood that the function 'empty?' will
cause lazy evaluation if necessary to determine if there are
entries.  Functions which call 'empty?', for example 'first' and 'rest',
will also cause lazy evaluation if necessary.

See Also:
o )show LazyStreamAggregate
```

———-

See:

⇐ "StreamAggregate" (STAGG) 7.0.131 on page 739

Exports:

any?	child?	children	coerce
complete	concat	concat!	construct
copy	convert	count	cycleEntry
cycleLength	cycleSplit!	cycleTail	cyclic?
delete	distance	elt	empty
empty?	entry?	entries	eq?
explicitEntries?	explicitlyEmpty?	explicitlyFinite?	extend
eval	every?	fill!	find
first	frst	hash	index?
indices	insert	last	latex
lazy?	lazyEvaluate	leaf?	leaves
less?	map	map!	maxIndex
member?	members	minIndex	more?
new	node?	nodes	numberOfComputedEntries
parts	possiblyInfinite?	qelt	qsetelt!
reduce	remove	removeDuplicates	rest
rst	sample	second	select
setchildren!	setelt	setfirst!	setlast!
setrest!	setvalue!	size?	split!
swap!	tail	third	value
#?	?=?	?.?	?.last
?.rest	?.first	?.value	?~=?

Attributes exported:

- **nil**

These are directly exported but not implemented:

```
explicitEntries? : % -> Boolean
explicitlyEmpty? : % -> Boolean
frst : % -> S
lazy? : % -> Boolean
lazyEvaluate : % -> %
numberOfComputedEntries : % -> NonNegativeInteger
remove : ((S -> Boolean),%) -> %
rst : % -> %
select : ((S -> Boolean),%) -> %
```

These are implemented by this category:

```
any? : ((S -> Boolean),%) -> Boolean
    if $ has finiteAggregate
child? : (%,%) -> Boolean if S has SETCAT
children : % -> List %
complete : % -> %
construct : List S -> %
cycleEntry : % -> %
cycleLength : % -> NonNegativeInteger
cycleTail : % -> %
cyclic? : % -> Boolean
delete : (%,Integer) -> %
delete : (%,UniversalSegment Integer) -> %
distance : (%,%) -> Integer
elt : (%,Integer,S) -> S
```

```
entries : % -> List S
every? : ((S -> Boolean),%) -> Boolean
    if $ has finiteAggregate
explicitlyFinite? : % -> Boolean
extend : (%,Integer) -> %
first : (%,NonNegativeInteger) -> %
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (S,%,Integer) -> %
insert : (%,%,Integer) -> %
last : % -> S
last : (%,NonNegativeInteger) -> %
leaf? : % -> Boolean
less? : (%,NonNegativeInteger) -> Boolean
maxIndex : % -> Integer if Integer has ORDSET
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
node? : (%,%) -> Boolean if S has SETCAT
nodes : % -> List %
possiblyInfinite? : % -> Boolean
rest : % -> %
rest : (%,NonNegativeInteger) -> %
value : % -> S
size? : (%,NonNegativeInteger) -> Boolean
tail : % -> %
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?.? : (%,UniversalSegment Integer) -> %
?.? : (%,Integer) -> S
?.first : (%,first) -> S
?.last : (%,last) -> S
?.rest : (%,rest) -> %
```

These exports come from (p739) StreamAggregate(S:Type):

```
coerce : % -> OutputForm if S has SETCAT
concat : (%,%) -> %
concat : (%,S) -> %
concat : (S,%) -> %
concat : List % -> %
concat! : (%,%) -> % if $ has shallowlyMutable
concat! : (%,S) -> % if $ has shallowlyMutable
convert : % -> InputForm if S has KONVERT INFORM
copy : % -> %
count : (S,%) -> NonNegativeInteger
    if S has SETCAT
    and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
    if $ has finiteAggregate
cycleSplit! : % -> % if $ has shallowlyMutable
empty : () -> %
empty? : % -> Boolean
entry? : (S,%) -> Boolean
    if $ has finiteAggregate
    and S has SETCAT
```

```
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,S,S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,Equation S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,List Equation S) -> %
    if S has EVALAB S
    and S has SETCAT
fill! : (%,S) -> % if $ has shallowlyMutable
find : ((S -> Boolean),%) -> Union(S,"failed")
first : % -> S
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
leaves : % -> List S
map : (((S,S) -> S),%,%) -> %
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
    if S has SETCAT
    and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
new : (NonNegativeInteger,S) -> %
parts : % -> List S if $ has finiteAggregate
qelt : (%,Integer) -> S
qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
reduce : (((S,S) -> S),%,S,S) -> S
    if S has SETCAT
    and $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
removeDuplicates : % -> %
    if S has SETCAT
    and $ has finiteAggregate
sample : () -> %
second : % -> S
setchildren! : (%,List %) -> % if $ has shallowlyMutable
setelt : (%,Integer,S) -> S if $ has shallowlyMutable
setelt : (%,UniversalSegment Integer,S) -> S
    if $ has shallowlyMutable
setelt : (%,last,S) -> S if $ has shallowlyMutable
setelt : (%,rest,%) -> % if $ has shallowlyMutable
setelt : (%,first,S) -> S if $ has shallowlyMutable
setelt : (%,value,S) -> S if $ has shallowlyMutable
setfirst! : (%,S) -> S if $ has shallowlyMutable
setlast! : (%,S) -> S if $ has shallowlyMutable
setrest! : (%,%) -> % if $ has shallowlyMutable
setvalue! : (%,S) -> S if $ has shallowlyMutable
split! : (%,Integer) -> % if $ has shallowlyMutable
```

```
 swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
 third : % -> S
 ?.value : (%,value) -> S
 ?~=? : (%,%) -> Boolean if S has SETCAT
```

— LazyStreamAggregate.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#LZSTAGG">
LazyStreamAggregate (LZSTAGG)</a></h2>
</body>
```

— category LZSTAGG LazyStreamAggregate —

```
)abbrev category LZSTAGG LazyStreamAggregate
++ Author: Clifton J. Williamson
++ Date Created: 22 November 1989
++ Date Last Updated: 20 July 1990
++ Description:
++ LazyStreamAggregate is the category of streams with lazy
++ evaluation.  It is understood that the function 'empty?' will
++ cause lazy evaluation if necessary to determine if there are
++ entries.  Functions which call 'empty?', for example 'first' and 'rest',
++ will also cause lazy evaluation if necessary.

LazyStreamAggregate(S) : Category == SIG where
  S : Type

  SIG ==> StreamAggregate(S) with

    remove : (S -> Boolean,%) -> %
      ++remove(f,st) returns a stream consisting of those elements of stream
      ++ st which do not satisfy the predicate f.
      ++ Note that \spad{remove(f,st) = [x for x in st | not f(x)]}.
      ++
      ++X m:=[i for i in 1..]
      ++X f(i:PositiveInteger):Boolean == even? i
      ++X remove(f,m)

    select : (S -> Boolean,%) -> %
      ++select(f,st) returns a stream consisting of those elements of stream
      ++ st satisfying the predicate f.
      ++ Note that \spad{select(f,st) = [x for x in st | f(x)]}.
      ++
      ++X m:=[i for i in 0..]
      ++X select(x+->prime? x,m)

    explicitEntries? : % -> Boolean
      ++explicitEntries?(s) returns true if the stream s has
      ++ explicitly computed entries, and false otherwise.
      ++
```

```
  ++X m:=[i for i in 0..]
  ++X explicitEntries? m

explicitlyEmpty? : % -> Boolean
  ++explicitlyEmpty?(s) returns true if the stream is an
  ++ (explicitly) empty stream.
  ++ Note that this is a null test which will not cause lazy evaluation.
  ++
  ++X m:=[i for i in 0..]
  ++X explicitlyEmpty? m

lazy? : % -> Boolean
  ++lazy?(s) returns true if the first node of the stream s
  ++ is a lazy evaluation mechanism which could produce an
  ++ additional entry to s.
  ++
  ++X m:=[i for i in 0..]
  ++X lazy? m

lazyEvaluate : % -> %
  ++lazyEvaluate(s) causes one lazy evaluation of stream s.
  ++ Caution: the first node must be a lazy evaluation mechanism
  ++ (satisfies \spad{lazy?(s) = true}) as there is no error check.
  ++ Note that a call to this function may
  ++ or may not produce an explicit first entry

frst : % -> S
  ++frst(s) returns the first element of stream s.
  ++ Caution: this function should only be called after a \spad{empty?}
  ++ test has been made since there no error check.
  ++
  ++X m:=[i for i in 0..]
  ++X frst m

rst : % -> %
  ++rst(s) returns a pointer to the next node of stream s.
  ++ Cautrion: this function should only be called after a \spad{empty?}
  ++ test has been made since there no error check.
  ++
  ++X m:=[i for i in 0..]
  ++X rst m

numberOfComputedEntries : % -> NonNegativeInteger
  ++numberOfComputedEntries(st) returns the number of explicitly
  ++ computed entries of stream st which exist immediately prior to the
  ++ time this function is called.
  ++
  ++X m:=[i for i in 0..]
  ++X numberOfComputedEntries m

extend : (%,Integer) -> %
  ++extend(st,n) causes entries to be computed, if necessary,
  ++ so that 'st' will have at least 'n' explicit entries or so
  ++ that all entries of 'st' will be computed if 'st' is finite
```

```
    ++ with length <= n.
    ++
    ++X m:=[i for i in 0..]
    ++X numberOfComputedEntries m
    ++X extend(m,20)
    ++X numberOfComputedEntries m

  complete : % -> %
    ++complete(st) causes all entries of 'st' to be computed.
    ++ this function should only be called on streams which are
    ++ known to be finite.
    ++
    ++X m:=[i for i in 1..]
    ++X n:=filterUntil(i+->i>100,m)
    ++X numberOfComputedEntries n
    ++X complete n
    ++X numberOfComputedEntries n

add

  MIN ==> 1  -- minimal stream index

  I   ==> Integer
  NNI ==> NonNegativeInteger
  L   ==> List
  U   ==> UniversalSegment Integer

  indexx? : (Integer,%) -> Boolean

  cycleElt : % -> Union(%,"failed")

  computeCycleLength : % -> NNI

  computeCycleEntry : (%,%) -> %

  --% SETCAT functions

  if S has SetCategory then

    x = y ==
      eq?(x,y) => true
      explicitlyFinite? x and explicitlyFinite? y =>
        entries x = entries y
      explicitEntries? x and explicitEntries? y =>
        frst x = frst y and EQ(rst x, rst y)$Lisp
      -- treat cyclic streams
      false

  --% HOAGG functions

  less?(x,n) ==
    n = 0    => false
    empty? x => true
    less?(rst x,(n-1) :: NNI)
```

```
more?(x,n) ==
  empty? x => false
  n = 0    => true
  more?(rst x,(n-1) :: NNI)

size?(x,n) ==
  empty? x => n = 0
  size?(rst x,(n-1) :: NNI)

# x ==
  -- error if stream is not finite
  y := x
  for i in 0.. repeat
    explicitlyEmpty? y  => return i
    lazy? y => error "#: infinite stream"
    y := rst y
    if odd? i then x := rst x
    eq?(x,y) => error "#: infinite stream"

--% CLAGG functions

any?(f,x) ==
  -- error message only when x is a stream with lazy
  -- evaluation and f(s) = false for all stream elements
  -- 's' which have been computed when the function is
  -- called
  y := x
  for i in 0.. repeat
    explicitlyEmpty? y  => return false
    lazy? y => error "any?: infinite stream"
    f frst y => return true
    y := rst y
    if odd? i then x := rst x
    eq?(x,y) => return false

every?(f,x) ==
  -- error message only when x is a stream with lazy
  -- evaluation and f(s) = true for all stream elements
  -- 's' which have been computed when the function is
  -- called
  y := x
  for i in 0.. repeat
    explicitlyEmpty? y => return true
    lazy? y => error "every?: infinite stream"
    not f frst y => return false
    y := rst y
    if odd? i then x := rst x
    eq?(x,y) => return true

-- following ops count and member? are only exported
-- if $ has finiteAggregate

entries x ==
```

```
  -- returns a list of elements which have been computed
  -- error if infinite
  y := x
  l : L S := empty()
  for i in 0.. repeat
    explicitlyEmpty? y  => return reverse_! l
    lazy? y => error "infinite stream"
    l := concat(frst y,l)
    y := rst y
    if odd? i then x := rst x
    eq?(x,y) => error "infinite stream"

 --% CNAGG functions

construct l ==
  empty? l => empty()
  concat(first l, construct rest l)

 --% ELTAGG functions

elt(x:%,n:I) ==
  n < MIN or empty? x => error "elt: no such element"
  n = MIN => frst x
  elt(rst x,n - 1)

elt(x:%,n:I,s:S) ==
  n < MIN or empty? x => s
  n = MIN => frst x
  elt(rst x,n - 1)

 --% IXAGG functions

indexx?(n,x) ==
  empty? x => false
  n = MIN => true
  indexx?(n-1,rst x)

index?(n,x) ==
  -- returns 'true' iff 'n' is the index of an entry which
  -- may or may not have been computed when the function is
  -- called
  -- additional entries are computed if necessary
  n < MIN => false
  indexx?(n,x)

indices x ==
  -- error if stream is not finite
  y := x
  l : L I := empty()
  for i in MIN.. repeat
    explicitlyEmpty? y  => return reverse_! l
    lazy? y => error "indices: infinite stream"
    l := concat(i,l)
    y := rst y
```

```
    if odd? i then x := rst x
    eq?(x,y) => error "indices: infinite stream"

maxIndex x ==
  -- error if stream is not finite
  empty? x =>
    error "maxIndex: no maximal index for empty stream"
  y := rst x
  for i in MIN.. repeat
    explicitlyEmpty? y  => return i
    lazy? y => error "maxIndex: infinite stream"
    y := rst y
    if odd? i then x := rst x
    eq?(x,y) => error "maxIndex: infinite stream"

minIndex x ==
  empty? x => error "minIndex: no minimal index for empty stream"
  MIN

 --% LNAGG functions

delete(x:%,n:I) ==
-- non-destructive
  not index?(n,x) => error "delete: index out of range"
  concat(first(x,(n - MIN) :: NNI), rest(x,(n - MIN + 1) :: NNI))

delete(x:%,seg:U) ==
  low := lo seg
  hasHi seg =>
    high := hi seg
    high < low => copy x
    (not index?(low,x)) or (not index?(high,x)) =>
      error "delete: index out of range"
    concat(first(x,(low - MIN) :: NNI),rest(x,(high - MIN + 1) :: NNI))
  not index?(low,x) => error "delete: index out of range"
  first(x,(low - MIN) :: NNI)

elt(x:%,seg:U) ==
  low := lo seg
  hasHi seg =>
    high := hi seg
    high < low => empty()
    (not index?(low,x)) or (not index?(high,x)) =>
      error "elt: index out of range"
    first(rest(x,(low - MIN) :: NNI),(high - low + 1) :: NNI)
  not index?(low,x) => error "elt: index out of range"
  rest(x,(low - MIN) :: NNI)

insert(s:S,x:%,n:I) ==
  not index?(n,x) => error "insert: index out of range"
  nn := (n - MIN) :: NNI
  concat([first(x,nn), concat(s, empty()), rest(x,nn)])

insert(y:%,x:%,n:I) ==
```

```
  not index?(n,x) => error "insert: index out of range"
  nn := (n - MIN) :: NNI
  concat([first(x,nn), y, rest(x,nn)])

 --% RCAGG functions

cycleElt x == cycleElt(x)$CyclicStreamTools(S,%)

cyclic? x ==
  cycleElt(x) case "failed" => false
  true

if S has SetCategory then

  child?(x,y) ==
    empty? y => error "child: no children"
    x = rst y

children x ==
  empty? x => error "children: no children"
  [rst x]

distance(x,z) ==
  y := x
  for i in 0.. repeat
    eq?(y,z) => return i
    (explicitlyEmpty? y) or (lazy? y) =>
      error "distance: 2nd arg not a descendent of the 1st"
    y := rst y
    if odd? i then x := rst x
    eq?(x,y) =>
      error "distance: 2nd arg not a descendent of the 1st"

if S has SetCategory then

  node?(z,x) ==
    -- error message only when x is a stream with lazy
    -- evaluation and 'y' is not a node of 'x'
    -- which has been computed when the function is called
    y := x
    for i in 0.. repeat
      z = y => return true
      explicitlyEmpty? y => return false
      lazy? y => error "node?: infinite stream"
      y := rst y
      if odd? i then x := rst x
      eq?(x,y) => return false

nodes x ==
  y := x
  l : L % := []
  for i in 0.. repeat
    explicitlyEmpty? y => return reverse_! l
    lazy? y => error "nodes: infinite stream"
```

```
      l := concat(y,l)
      y := rst y
      if odd? i then x := rst x
      eq?(x,y) => error "nodes: infinite stream"
    l

  leaf? x == empty? rest x

  value x == first x

   --% URAGG functions

  computeCycleLength cycElt ==
    computeCycleLength(cycElt)$CyclicStreamTools(S,%)

  computeCycleEntry(x,cycElt) ==
    computeCycleEntry(x,cycElt)$CyclicStreamTools(S,%)

  cycleEntry x ==
    cycElt := cycleElt x
    cycElt case "failed" =>
      error "cycleEntry: non-cyclic stream"
    computeCycleEntry(x,cycElt::%)

  cycleLength x ==
    cycElt := cycleElt x
    cycElt case "failed" =>
      error "cycleLength: non-cyclic stream"
    computeCycleLength(cycElt::%)

  cycleTail x ==
    cycElt := cycleElt x
    cycElt case "failed" =>
      error "cycleTail: non-cyclic stream"
    y := x := computeCycleEntry(x,cycElt::%)
    z := rst x
    repeat
      eq?(x,z) => return y
      y := z ; z := rst z

  elt(x,"first") == first x

  first(x,n) ==
  -- former name: take
    n = 0 or empty? x => empty()
    concat(frst x, first(rst x,(n-1) :: NNI))

  rest x ==
    empty? x => error "Can't take the rest of an empty stream."
    rst x

  elt(x,"rest") == rest x

  rest(x,n) ==
```

```
-- former name: drop
  n = 0 or empty? x => x
  rest(rst x,(n-1) :: NNI)

last x ==
  -- error if stream is not finite
  empty? x => error "last: empty stream"
  y1 := x
  y2 := rst x
  for i in 0.. repeat
    explicitlyEmpty? y2 => return frst y1
    lazy? y2 => error "last: infinite stream"
    y1 := y2
    y2 := rst y2
    if odd? i then x := rst x
    eq?(x,y2) => error "last: infinite stream"

if % has finiteAggregate then -- # is only defined for finiteAggregates

  last(x,n) ==
    possiblyInfinite? x => error "last: infinite stream"
    m := # x
    m < n => error "last: index out of range"
    copy rest(x,(m-n)::NNI)

elt(x,"last") == last x

tail x ==
  -- error if stream is not finite
  empty? x => error "tail: empty stream"
  y1 := x
  y2 := rst x
  for i in 0.. repeat
    explicitlyEmpty? y2 => return y1
    lazy? y2 => error "tail: infinite stream"
    y1 := y2
    y2 := rst y2
    if odd? i then x := rst x
    eq?(x,y2) => error "tail: infinite stream"

 --% STAGG functions

possiblyInfinite? x ==
  y := x
  for i in 0.. repeat
    explicitlyEmpty? y  => return false
    lazy? y => return true
    if odd? i then x := rst x
    y := rst y
    eq?(x,y) => return true

explicitlyFinite? x == not possiblyInfinite? x

 --% LZSTAGG functions
```

```
  extend(x,n) ==
    y := x
    for i in 1..n while not empty? y repeat y := rst y
    x

  complete x ==
    y := x
    while not empty? y repeat y := rst y
    x
```

———-

— COQ LZSTAGG —

```
(* category LZSTAGG *)
(*

  MIN ==> 1  -- minimal stream index

  I   ==> Integer
  NNI ==> NonNegativeInteger
  L   ==> List
  U   ==> UniversalSegment Integer

--% SETCAT functions

  if S has SetCategory then

    ?=? : (%,%) -> Boolean
    x = y ==
      eq?(x,y) => true
      explicitlyFinite? x and explicitlyFinite? y =>
        entries x = entries y
      explicitEntries? x and explicitEntries? y =>
        frst x = frst y and EQ(rst x, rst y)$Lisp
      -- treat cyclic streams
      false

--% HOAGG functions

  less? : (%,NonNegativeInteger) -> Boolean
  less?(x,n) ==
    n = 0    => false
    empty? x => true
    less?(rst x,(n-1) :: NNI)

  more? : (%,NonNegativeInteger) -> Boolean
  more?(x,n) ==
    empty? x => false
    n = 0    => true
    more?(rst x,(n-1) :: NNI)
```

```
  size? : (%,NonNegativeInteger) -> Boolean
  size?(x,n) ==
    empty? x => n = 0
    size?(rst x,(n-1) :: NNI)

  #? : % -> NonNegativeInteger
  # x ==
    -- error if stream is not finite
    y := x
    for i in 0.. repeat
      explicitlyEmpty? y  => return i
      lazy? y => error "#: infinite stream"
      y := rst y
      if odd? i then x := rst x
      eq?(x,y) => error "#: infinite stream"

--% CLAGG functions

  any? : ((S -> Boolean),%) -> Boolean
  any?(f,x) ==
    -- error message only when x is a stream with lazy
    -- evaluation and f(s) = false for all stream elements
    -- 's' which have been computed when the function is
    -- called
    y := x
    for i in 0.. repeat
      explicitlyEmpty? y  => return false
      lazy? y => error "any?: infinite stream"
      f frst y => return true
      y := rst y
      if odd? i then x := rst x
      eq?(x,y) => return false

  every? : ((S -> Boolean),%) -> Boolean
  every?(f,x) ==
    -- error message only when x is a stream with lazy
    -- evaluation and f(s) = true for all stream elements
    -- 's' which have been computed when the function is
    -- called
    y := x
    for i in 0.. repeat
      explicitlyEmpty? y => return true
      lazy? y => error "every?: infinite stream"
      not f frst y => return false
      y := rst y
      if odd? i then x := rst x
      eq?(x,y) => return true

  entries : % -> List(S)
  entries x ==
    -- returns a list of elements which have been computed
    -- error if infinite
    y := x
    l : L S := empty()
```

```
    for i in 0.. repeat
      explicitlyEmpty? y  => return reverse_! l
      lazy? y => error "infinite stream"
      l := concat(frst y,l)
      y := rst y
      if odd? i then x := rst x
      eq?(x,y) => error "infinite stream"

--% CNAGG functions

  construct : List(S) -> %
  construct l ==
    empty? l => empty()
    concat(first l, construct rest l)

--% ELTAGG functions

  elt(x:%,n:I) ==
    n < MIN or empty? x => error "elt: no such element"
    n = MIN => frst x
    elt(rst x,n - 1)

  elt : (%,Integer,S) -> S
  elt(x:%,n:I,s:S) ==
    n < MIN or empty? x => s
    n = MIN => frst x
    elt(rst x,n - 1)

--% IXAGG functions

  indexx? : (Integer,%) -> Boolean
  indexx?(n,x) ==
    empty? x => false
    n = MIN => true
    indexx?(n-1,rst x)

  index? : (Integer,%) -> Boolean
  index?(n,x) ==
    -- returns 'true' iff 'n' is the index of an entry which
    -- may or may not have been computed when the function is
    -- called
    -- additional entries are computed if necessary
    n < MIN => false
    indexx?(n,x)

  indices : % -> List(Integer)
  indices x ==
    -- error if stream is not finite
    y := x
    l : L I := empty()
    for i in MIN.. repeat
      explicitlyEmpty? y  => return reverse_! l
      lazy? y => error "indices: infinite stream"
      l := concat(i,l)
```

```
      y := rst y
      if odd? i then x := rst x
      eq?(x,y) => error "indices: infinite stream"

  maxIndex : % -> Integer
  maxIndex x ==
    -- error if stream is not finite
    empty? x =>
      error "maxIndex: no maximal index for empty stream"
    y := rst x
    for i in MIN.. repeat
      explicitlyEmpty? y  => return i
      lazy? y => error "maxIndex: infinite stream"
      y := rst y
      if odd? i then x := rst x
      eq?(x,y) => error "maxIndex: infinite stream"

  minIndex : % -> Integer
  minIndex x ==
    empty? x => error "minIndex: no minimal index for empty stream"
    MIN

--% LNAGG functions

  delete : (%,Integer) -> %
  delete(x:%,n:I) ==
  -- non-destructive
    not index?(n,x) => error "delete: index out of range"
    concat(first(x,(n - MIN) :: NNI), rest(x,(n - MIN + 1) :: NNI))

  delete : (%,UniversalSegment(Integer)) -> %
  delete(x:%,seg:U) ==
    low := lo seg
    hasHi seg =>
      high := hi seg
      high < low => copy x
      (not index?(low,x)) or (not index?(high,x)) =>
        error "delete: index out of range"
      concat(first(x,(low - MIN) :: NNI),rest(x,(high - MIN + 1) :: NNI))
    not index?(low,x) => error "delete: index out of range"
    first(x,(low - MIN) :: NNI)

  elt(x:%,seg:U) ==
    low := lo seg
    hasHi seg =>
      high := hi seg
      high < low => empty()
      (not index?(low,x)) or (not index?(high,x)) =>
        error "elt: index out of range"
      first(rest(x,(low - MIN) :: NNI),(high - low + 1) :: NNI)
    not index?(low,x) => error "elt: index out of range"
    rest(x,(low - MIN) :: NNI)

  insert : (S,%,Integer) -> %
```

```
  insert(s:S,x:%,n:I) ==
    not index?(n,x) => error "insert: index out of range"
    nn := (n - MIN) :: NNI
    concat([first(x,nn), concat(s, empty()), rest(x,nn)])

  insert : (%,%,Integer) -> %
  insert(y:%,x:%,n:I) ==
    not index?(n,x) => error "insert: index out of range"
    nn := (n - MIN) :: NNI
    concat([first(x,nn), y, rest(x,nn)])

--% RCAGG functions

  cycleElt : % -> Union(%,"failed")
  cycleElt x == cycleElt(x)$CyclicStreamTools(S,%)

  cyclic? : % -> Boolean
  cyclic? x ==
    cycleElt(x) case "failed" => false
    true

  if S has SetCategory then

    child? : (%,%) -> Boolean
    child?(x,y) ==
      empty? y => error "child: no children"
      x = rst y

  children : % -> List(%)
  children x ==
    empty? x => error "children: no children"
    [rst x]

  distance : (%,%) -> Integer
  distance(x,z) ==
    y := x
    for i in 0.. repeat
      eq?(y,z) => return i
      (explicitlyEmpty? y) or (lazy? y) =>
        error "distance: 2nd arg not a descendent of the 1st"
      y := rst y
      if odd? i then x := rst x
      eq?(x,y) =>
        error "distance: 2nd arg not a descendent of the 1st"

  if S has SetCategory then

    node? : (%,%) -> Boolean
    node?(z,x) ==
      -- error message only when x is a stream with lazy
      -- evaluation and 'y' is not a node of 'x'
      -- which has been computed when the function is called
      y := x
      for i in 0.. repeat
```

```
        z = y => return true
        explicitlyEmpty? y => return false
        lazy? y => error "node?: infinite stream"
        y := rst y
        if odd? i then x := rst x
        eq?(x,y) => return false

  nodes : % -> List(%)
  nodes x ==
    y := x
    l : L % := []
    for i in 0.. repeat
      explicitlyEmpty? y => return reverse_! l
      lazy? y => error "nodes: infinite stream"
      l := concat(y,l)
      y := rst y
      if odd? i then x := rst x
      eq?(x,y) => error "nodes: infinite stream"
    l -- @#$%^& compiler

  leaf? : % -> Boolean
  leaf? x == empty? rest x

  value : % -> S
  value x == first x

--% URAGG functions

  computeCycleLength : % -> NNI
  computeCycleLength cycElt ==
    computeCycleLength(cycElt)$CyclicStreamTools(S,%)

  computeCycleEntry : (%,%) -> %
  computeCycleEntry(x,cycElt) ==
    computeCycleEntry(x,cycElt)$CyclicStreamTools(S,%)

  cycleEntry : % -> %
  cycleEntry x ==
    cycElt := cycleElt x
    cycElt case "failed" =>
      error "cycleEntry: non-cyclic stream"
    computeCycleEntry(x,cycElt::%)

  cycleLength : % -> NonNegativeInteger
  cycleLength x ==
    cycElt := cycleElt x
    cycElt case "failed" =>
      error "cycleLength: non-cyclic stream"
    computeCycleLength(cycElt::%)

  cycleTail : % -> %
  cycleTail x ==
    cycElt := cycleElt x
    cycElt case "failed" =>
```

```
      error "cycleTail: non-cyclic stream"
    y := x := computeCycleEntry(x,cycElt::%)
    z := rst x
    repeat
      eq?(x,z) => return y
      y := z ; z := rst z

  ?.first : (%,first) -> S
  elt(x,"first") == first x

  first : (%,NonNegativeInteger) -> %
  first(x,n) ==
  -- former name: take
    n = 0 or empty? x => empty()
    concat(frst x, first(rst x,(n-1) :: NNI))

  rest : % -> %
  rest x ==
    empty? x => error "Can't take the rest of an empty stream."
    rst x

  ?.rest : (%,rest) -> %
  elt(x,"rest") == rest x

  rest : (%,NonNegativeInteger) -> %
  rest(x,n) ==
  -- former name: drop
    n = 0 or empty? x => x
    rest(rst x,(n-1) :: NNI)

  last : % -> S
  last x ==
    -- error if stream is not finite
    empty? x => error "last: empty stream"
    y1 := x
    y2 := rst x
    for i in 0.. repeat
      explicitlyEmpty? y2 => return frst y1
      lazy? y2 => error "last: infinite stream"
      y1 := y2
      y2 := rst y2
      if odd? i then x := rst x
      eq?(x,y2) => error "last: infinite stream"

  if % has finiteAggregate then -- # is only defined for finiteAggregates

    last : (%,NonNegativeInteger) -> %
    last(x,n) ==
      possiblyInfinite? x => error "last: infinite stream"
      m := # x
      m < n => error "last: index out of range"
      copy rest(x,(m-n)::NNI)

  ?.last : (%,last) -> S
```

```
  elt(x,"last") == last x

  tail : % -> %
  tail x ==
    -- error if stream is not finite
    empty? x => error "tail: empty stream"
    y1 := x
    y2 := rst x
    for i in 0.. repeat
      explicitlyEmpty? y2 => return y1
      lazy? y2 => error "tail: infinite stream"
      y1 := y2
      y2 := rst y2
      if odd? i then x := rst x
      eq?(x,y2) => error "tail: infinite stream"

--% STAGG functions

  possiblyInfinite? : % -> Boolean
  possiblyInfinite? x ==
    y := x
    for i in 0.. repeat
      explicitlyEmpty? y  => return false
      lazy? y => return true
      if odd? i then x := rst x
      y := rst y
      eq?(x,y) => return true

  explicitlyFinite? : % -> Boolean
  explicitlyFinite? x == not possiblyInfinite? x

--% LZSTAGG functions

  extend : (%,Integer) -> %
  extend(x,n) ==
    y := x
    for i in 1..n while not empty? y repeat y := rst y
    x

  complete : % -> %
  complete x ==
    y := x
    while not empty? y repeat y := rst y
    x
*)
```

— LZSTAGG.dotabb —

```
"LZSTAGG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LZSTAGG"];
"LZSTAGG" -> "STAGG"
```

———-

— **LZSTAGG.dotfull** —

```
"LazyStreamAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LZSTAGG"];
"LazyStreamAggregate(a:Type)" -> "StreamAggregate(a:Type)"
```

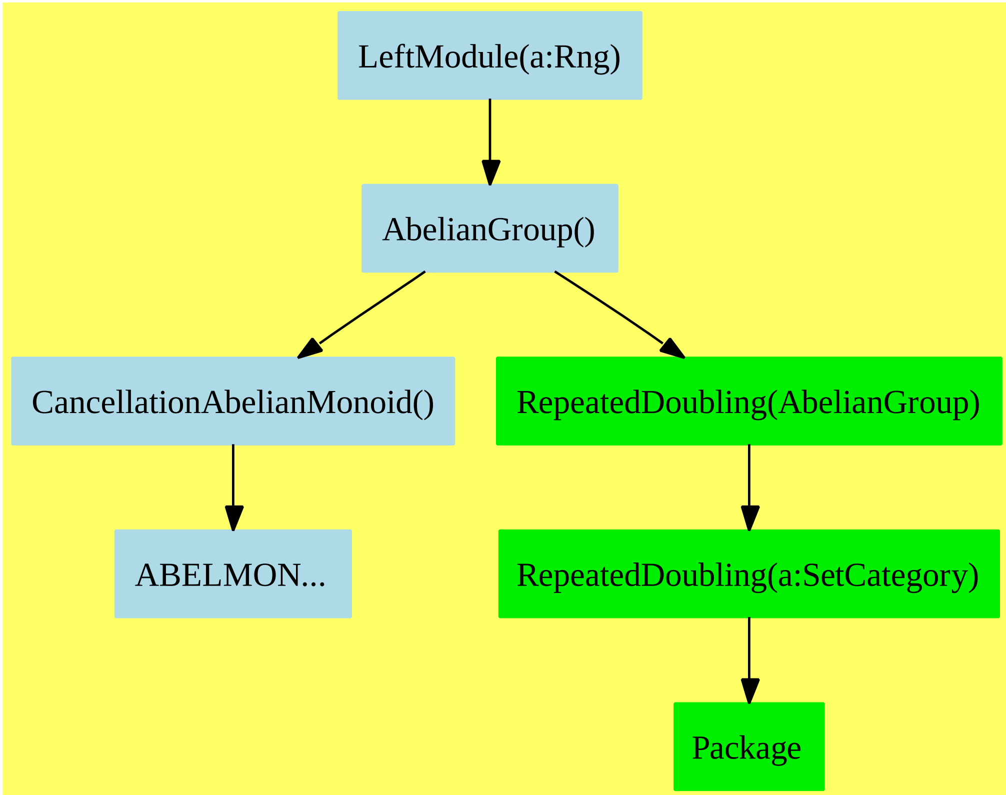
———-

— **LZSTAGG.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"LazyStreamAggregate(a:Type)" [color=lightblue];
"LazyStreamAggregate(a:Type)" -> "StreamAggregate(a:Type)"

"StreamAggregate(a:Type)" [color=lightblue];
"StreamAggregate(a:Type)" -> "UnaryRecursiveAggregate(a:Type)"
"StreamAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"UnaryRecursiveAggregate(a:Type)" [color=lightblue];
"UnaryRecursiveAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"

"RecursiveAggregate(a:Type)" [color=lightblue];
"RecursiveAggregate(a:Type)" -> "HOAGG..."

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "Collection(a:Type)"

"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" ->
    "IndexedAggregate(a:SetCategory,b:Type)"

"IndexedAggregate(a:SetCategory,b:Type)" [color=lightblue];
"IndexedAggregate(a:SetCategory,b:Type)" -> "HOAGG..."

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HOAGG..."

"HOAGG..." [color=lightblue];

}
```

———-

8.0.137 LeftModule (LMODULE)



— **LeftModule.input** —

```
)set break resume
)sys rm -f LeftModule.output
)spool LeftModule.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show LeftModule
--R
--R LeftModule(R: Rng) is a category constructor
--R Abbreviation for LeftModule is LMODULE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for LMODULE
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                          ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                          ?-? : (%,%) -> %
--R -? : % -> %                               ?=? : (%,%) -> Boolean
```

```
--R 0 : () -> %                          coerce : % -> OutputForm
--R hash : % -> SingleInteger            latex : % -> String
--R sample : () -> %                     zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— LeftModule.help —

```
====================================================================
LeftModule examples
====================================================================

The category of left modules over an rng (ring not necessarily with unit).
This is an abelian group which supports left multiplation by elements of
the rng.

See Also:
o )show LeftModule
```

See:

⇒ “BiModule” (BMODULE) 9.0.146 on page 902
⇒ “LeftAlgebra” (LALG) 10.0.165 on page 1040
⇒ “Ring” (RING) 9.0.153 on page 946
⇐ “AbelianGroup” (ABELGRP) 7.0.121 on page 673

Exports:

0	coerce	hash	latex	sample
subtractIfCan	zero?	?~=?	?*?	?+?
?-?	-?	?=?		

These are directly exported but not implemented:

```
?*? : (R,%) -> %
```

These exports come from (p673) AbelianGroup():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
```

```
?+? : (%,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Integer,%) -> %
?-? : (%,%) -> %
-? : % -> %
```

— LeftModule.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#LMODULE">
LeftModule (LMODULE)</a></h2>
</body>
```

— category LMODULE LeftModule —

```
)abbrev category LMODULE LeftModule
++ Description:
++ The category of left modules over an rng
++ \spad{(ring not necessarily with unit)}.
++ This is an abelian group which supports left multiplication by elements of
++ the rng.
++
++ Axioms\br
++ \tab{5}\spad{(a*b)*x = a*(b*x)}\br
++ \tab{5}\spad{(a+b)*x = (a*x)+(b*x)}\br
++ \tab{5}\spad{a*(x+y) = (a*x)+(a*y)}

LeftModule(R) : Category == SIG where
  R : Rng

  SIG ==> AbelianGroup with

    "*" : (R,%) -> %
      ++ r*x returns the left multiplication of the module element x
      ++ by the ring element r.
```

— LMODULE.dotabb —

```
"LMODULE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LMODULE"];
"LMODULE" -> "ABELGRP"
```

— LMODULE.dotfull —

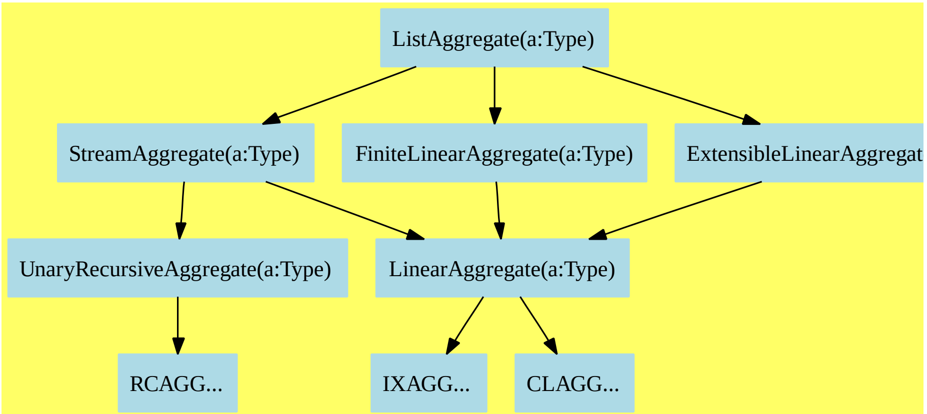
```
"LeftModule(a:Rng)"
```

```
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LMODULE"];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"LeftModule(a:Ring)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=LMODULE"];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"
```

— LMODULE.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "RepeatedDoubling(AbelianGroup)"

"RepeatedDoubling(AbelianGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianGroup)" -> "RepeatedDoubling(a:SetCategory)"

"RepeatedDoubling(a:SetCategory)" [color="#00EE00"];
"RepeatedDoubling(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "ABELMON..."

"ABELMON..." [color=lightblue];
}
```

8.0.138 ListAggregate (LSAGG)



— **ListAggregate.input** —

```
)set break resume
)sys rm -f ListAggregate.output
)spool ListAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ListAggregate
--R
--R ListAggregate(S: Type) is a category constructor
--R Abbreviation for ListAggregate is LSAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for LSAGG
--R
--R------------------------------- Operations --------------------------------
--R children : % -> List(%)              concat : (%,S) -> %
--R concat : List(%) -> %                concat : (S,%) -> %
--R concat : (%,%) -> %                  concat! : (%,S) -> %
--R concat! : (%,%) -> %                 construct : List(S) -> %
--R copy : % -> %                        cycleEntry : % -> %
--R cycleTail : % -> %                   cyclic? : % -> Boolean
--R delete : (%,Integer) -> %            delete! : (%,Integer) -> %
--R distance : (%,%) -> Integer          elt : (%,Integer,S) -> S
--R ?.? : (%,Integer) -> S               ?.last : (%,last) -> S
--R ?.rest : (%,rest) -> %               ?.first : (%,first) -> S
--R ?.value : (%,value) -> S             empty : () -> %
--R empty? : % -> Boolean                entries : % -> List(S)
--R eq? : (%,%) -> Boolean               explicitlyFinite? : % -> Boolean
--R first : % -> S                       index? : (Integer,%) -> Boolean
--R indices : % -> List(Integer)         insert : (S,%,Integer) -> %
--R insert : (%,%,Integer) -> %          insert! : (S,%,Integer) -> %
--R insert! : (%,%,Integer) -> %         last : (%,NonNegativeInteger) -> %
```

```
--R last : % -> S                       latex : % -> String if S has SETCAT
--R leaf? : % -> Boolean                leaves : % -> List(S)
--R list : S -> %                       map : (((S,S) -> S),%,%) -> %
--R map : ((S -> S),%) -> %             max : (%,%) -> % if S has ORDSET
--R min : (%,%) -> % if S has ORDSET    new : (NonNegativeInteger,S) -> %
--R nodes : % -> List(%)                possiblyInfinite? : % -> Boolean
--R qelt : (%,Integer) -> S             remove! : ((S -> Boolean),%) -> %
--R rest : (%,NonNegativeInteger) -> %  rest : % -> %
--R reverse : % -> %                    sample : () -> %
--R second : % -> S                     select! : ((S -> Boolean),%) -> %
--R sort : (((S,S) -> Boolean),%) -> %  sort : % -> % if S has ORDSET
--R tail : % -> %                       third : % -> S
--R value : % -> S
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?<? : (%,%) -> Boolean if S has ORDSET
--R ?<=? : (%,%) -> Boolean if S has ORDSET
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R ?>? : (%,%) -> Boolean if S has ORDSET
--R ?>=? : (%,%) -> Boolean if S has ORDSET
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R child? : (%,%) -> Boolean if S has SETCAT
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT(INFORM)
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R cycleLength : % -> NonNegativeInteger
--R cycleSplit! : % -> % if $ has shallowlyMutable
--R delete : (%,UniversalSegment(Integer)) -> %
--R delete! : (%,UniversalSegment(Integer)) -> %
--R ?.? : (%,UniversalSegment(Integer)) -> %
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R first : (%,NonNegativeInteger) -> %
--R hash : % -> SingleInteger if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R merge : (((S,S) -> Boolean),%,%) -> %
--R merge : (%,%) -> % if S has ORDSET
--R merge! : (((S,S) -> Boolean),%,%) -> %
--R merge! : (%,%) -> % if S has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R node? : (%,%) -> Boolean if S has SETCAT
--R parts : % -> List(S) if $ has finiteAggregate
```

```
--R position : ((S -> Boolean),%) -> Integer
--R position : (S,%) -> Integer if S has SETCAT
--R position : (S,%,Integer) -> Integer if S has SETCAT
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (S,%) -> % if S has SETCAT
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R removeDuplicates! : % -> % if S has SETCAT
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R setchildren! : (%,List(%)) -> % if $ has shallowlyMutable
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R setelt : (%,UniversalSegment(Integer),S) -> S if $ has shallowlyMutable
--R setelt : (%,last,S) -> S if $ has shallowlyMutable
--R setelt : (%,rest,%) -> % if $ has shallowlyMutable
--R setelt : (%,first,S) -> S if $ has shallowlyMutable
--R setelt : (%,value,S) -> S if $ has shallowlyMutable
--R setfirst! : (%,S) -> S if $ has shallowlyMutable
--R setlast! : (%,S) -> S if $ has shallowlyMutable
--R setrest! : (%,%) -> % if $ has shallowlyMutable
--R setvalue! : (%,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort! : (((S,S) -> Boolean),%) -> % if $ has shallowlyMutable
--R sort! : % -> % if S has ORDSET and $ has shallowlyMutable
--R sorted? : (((S,S) -> Boolean),%) -> Boolean
--R sorted? : % -> Boolean if S has ORDSET
--R split! : (%,Integer) -> % if $ has shallowlyMutable
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

— **ListAggregate.help** —

```
====================================================================
ListAggregate examples
====================================================================

A list aggregate is a model for a linked list data structure. A linked
list is a versatile data structure. Insertion and deletion are efficient
and searching is a linear operation.

See Also:
o )show ListAggregate
```

See:

⇒ "AssociationListAggregate" (ALAGG) 10.0.158 on page 997
⇐ "ExtensibleLinearAggregate" (ELAGG) 7.0.125 on page 700
⇐ "FiniteLinearAggregate" (FLAGG) 7.0.126 on page 709

Exports:

any?	children	child?	coerce
concat	concat!	construct	convert
copy	copyInto!	count	cycleEntry
cycleLength	cycleSplit!	cycleTail	cyclic?
delete	delete!	distance	elt
empty	empty?	entries	entry?
eq?	eval	every?	explicitlyFinite?
fill!	find	first	hash
index?	indices	insert	insert!
last	latex	leaf?	leaves
less?	list	map	map!
max	maxIndex	member?	members
merge	merge!	min	minIndex
more?	new	nodes	node?
parts	position	possiblyInfinite?	qelt
qsetelt!	reduce	remove	remove!
removeDuplicates	removeDuplicates!	rest	reverse
reverse!	sample	second	select
select!	setchildren!	setelt	setfirst!
setlast!	setrest!	setvalue!	size?
sort	sort!	sorted?	split!
swap!	tail	third	value
#?	?.?	?.last	?.rest
?.first	?.value	?<?	?<=?
?=?	?>?	?>=?	?~=?

Attributes exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are implemented by this category:

```
copy : % -> %
copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
delete! : (%,Integer) -> %
delete! : (%,UniversalSegment Integer) -> %
find : ((S -> Boolean),%) -> Union(S,"failed")
insert! : (S,%,Integer) -> %
insert! : (%,%,Integer) -> %
```

```
list : S -> %
map : (((S,S) -> S),%,%) -> %
merge : (((S,S) -> Boolean),%,%) -> %
merge! : (((S,S) -> Boolean),%,%) -> %
new : (NonNegativeInteger,S) -> %
position : ((S -> Boolean),%) -> Integer
position : (S,%,Integer) -> Integer if S has SETCAT
reduce : (((S,S) -> S),%,S) -> S
         if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
         if S has SETCAT
         and $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S
         if $ has finiteAggregate
remove! : ((S -> Boolean),%) -> %
removeDuplicates! : % -> % if S has SETCAT
reverse! : % -> % if $ has shallowlyMutable
select : ((S -> Boolean),%) -> %
         if $ has finiteAggregate
sort! : (((S,S) -> Boolean),%) -> %
         if $ has shallowlyMutable
sorted? : (((S,S) -> Boolean),%) -> Boolean
?<? : (%,%) -> Boolean if S has ORDSET
```

These exports come from (p739) StreamAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
children : % -> List %
child? : (%,%) -> Boolean if S has SETCAT
coerce : % -> OutputForm if S has SETCAT
concat : (%,S) -> %
concat : List % -> %
concat : (S,%) -> %
concat : (%,%) -> %
concat! : (%,S) -> %
concat! : (%,%) -> %
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
count : (S,%) -> NonNegativeInteger
         if S has SETCAT
         and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
cycleEntry : % -> %
cycleLength : % -> NonNegativeInteger
cycleSplit! : % -> % if $ has shallowlyMutable
cycleTail : % -> %
cyclic? : % -> Boolean
delete : (%,UniversalSegment Integer) -> %
delete : (%,Integer) -> %
distance : (%,%) -> Integer
elt : (%,Integer,S) -> S
empty : () -> %
empty? : % -> Boolean
```

```
entry? : (S,%) -> Boolean
         if $ has finiteAggregate
         and S has SETCAT
entries : % -> List S
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S
         and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S
         and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S
         and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S
         and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
explicitlyFinite? : % -> Boolean
fill! : (%,S) -> % if $ has shallowlyMutable
first : % -> S
first : (%,NonNegativeInteger) -> %
hash : % -> SingleInteger if S has SETCAT
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (S,%,Integer) -> %
insert : (%,%,Integer) -> %
last : % -> S
last : (%,NonNegativeInteger) -> %
latex : % -> String if S has SETCAT
leaf? : % -> Boolean
leaves : % -> List S
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
maxIndex : % -> Integer if Integer has ORDSET
member? : (S,%) -> Boolean
         if S has SETCAT
         and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
nodes : % -> List %
node? : (%,%) -> Boolean if S has SETCAT
parts : % -> List S if $ has finiteAggregate
possiblyInfinite? : % -> Boolean
qelt : (%,Integer) -> S
qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
remove : (S,%) -> %
         if S has SETCAT
         and $ has finiteAggregate
remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
removeDuplicates : % -> %
```

```
         if S has SETCAT
         and $ has finiteAggregate
rest : % -> %
rest : (%,NonNegativeInteger) -> %
sample : () -> %
second : % -> S
setchildren! : (%,List %) -> % if $ has shallowlyMutable
setelt : (%,Integer,S) -> S if $ has shallowlyMutable
setelt : (%,UniversalSegment Integer,S) -> S
         if $ has shallowlyMutable
setelt : (%,last,S) -> S if $ has shallowlyMutable
setelt : (%,rest,%) -> % if $ has shallowlyMutable
setelt : (%,first,S) -> S if $ has shallowlyMutable
setelt : (%,value,S) -> S if $ has shallowlyMutable
setfirst! : (%,S) -> S if $ has shallowlyMutable
setlast! : (%,S) -> S if $ has shallowlyMutable
setrest! : (%,%) -> % if $ has shallowlyMutable
setvalue! : (%,S) -> S if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
split! : (%,Integer) -> % if $ has shallowlyMutable
swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
tail : % -> %
third : % -> S
value : % -> S
#? : % -> NonNegativeInteger if $ has finiteAggregate
?.last : (%,last) -> S
?.rest : (%,rest) -> %
?.first : (%,first) -> S
?.value : (%,value) -> S
?.? : (%,Integer) -> S
?.? : (%,UniversalSegment Integer) -> %
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
```

These exports come from (p709) FiniteLinearAggregate(S:Type)

```
max : (%,%) -> % if S has ORDSET
merge : (%,%) -> % if S has ORDSET
min : (%,%) -> % if S has ORDSET
position : (S,%) -> Integer if S has SETCAT
reverse : % -> %
sort : (((S,S) -> Boolean),%) -> %
sort : % -> % if S has ORDSET
sort! : % -> % if S has ORDSET and $ has shallowlyMutable
sorted? : % -> Boolean if S has ORDSET
?<=? : (%,%) -> Boolean if S has ORDSET
?>? : (%,%) -> Boolean if S has ORDSET
?>=? : (%,%) -> Boolean if S has ORDSET
```

These exports come from (p700) ExtensibleLinearAggregate(S:Type):

```
merge! : (%,%) -> % if S has ORDSET
remove! : (S,%) -> % if S has SETCAT
select! : ((S -> Boolean),%) -> %
```

— ListAggregate.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#LSAGG">
ListAggregate (LSAGG)</a></h2>
</body>
```

— category LSAGG ListAggregate —

```
)abbrev category LSAGG ListAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ A list aggregate is a model for a linked list data structure.
++ A linked list is a versatile
++ data structure. Insertion and deletion are efficient and
++ searching is a linear operation.

ListAggregate(S) : Category == SIG where
  S : Type

  SA ==> StreamAggregate(S)
  FLA ==> FiniteLinearAggregate(S)
  ELA ==> ExtensibleLinearAggregate(S)

  SIG ==> Join(SA,FLA,ELA) with

    list : S -> %
      ++ list(x) returns the list of one element x.

   add

     cycleMax ==> 1000

     mergeSort: ((S, S) -> Boolean, %, Integer) -> %

     sort_!(f, l) == mergeSort(f, l, #l)

     list x == concat(x, empty())

     reduce(f, x) ==
       empty? x => _
         error "reducing over an empty list needs the 3 argument form"
       reduce(f, rest x, first x)

     merge(f, p, q) == merge_!(f, copy p, copy q)

     select_!(f, x) ==
       while not empty? x and not f first x repeat x := rest x
       empty? x => x
       y := x
       z := rest y
```

```
  while not empty? z repeat
    if f first z then (y := z; z := rest z)
                 else (z := rest z; setrest_!(y, z))
  x

merge_!(f, p, q) ==
  empty? p => q
  empty? q => p
  eq?(p, q) => error "cannot merge a list into itself"
  if f(first p, first q)
    then (r := t := p; p := rest p)
    else (r := t := q; q := rest q)
  while not empty? p and not empty? q repeat
    if f(first p, first q)
      then (setrest_!(t, p); t := p; p := rest p)
      else (setrest_!(t, q); t := q; q := rest q)
  setrest_!(t, if empty? p then q else p)
  r

insert_!(s:S, x:%, i:Integer) ==
  i < (m := minIndex x) => error "index out of range"
  i = m => concat(s, x)
  y := rest(x, (i - 1 - m)::NonNegativeInteger)
  z := rest y
  setrest_!(y, concat(s, z))
  x

insert_!(w:%, x:%, i:Integer) ==
  i < (m := minIndex x) => error "index out of range"
  i = m => concat_!(w, x)
  y := rest(x, (i - 1 - m)::NonNegativeInteger)
  z := rest y
  setrest_!(y, w)
  concat_!(y, z)
  x

remove_!(f:S -> Boolean, x:%) ==
  while not empty? x and f first x repeat x := rest x
  empty? x => x
  p := x
  q := rest x
  while not empty? q repeat
    if f first q then q := setrest_!(p, rest q)
                 else (p := q; q := rest q)
  x

delete_!(x:%, i:Integer) ==
  i < (m := minIndex x) => error "index out of range"
  i = m => rest x
  y := rest(x, (i - 1 - m)::NonNegativeInteger)
  setrest_!(y, rest(y, 2))
  x

delete_!(x:%, i:UniversalSegment(Integer)) ==
```

```
    (l := lo i) < (m := minIndex x) => error "index out of range"
    h := if hasHi i then hi i else maxIndex x
    h < l => x
    l = m => rest(x, (h + 1 - m)::NonNegativeInteger)
    t := rest(x, (l - 1 - m)::NonNegativeInteger)
    setrest_!(t, rest(t, (h - l + 2)::NonNegativeInteger))
    x

  find(f, x) ==
    while not empty? x and not f first x repeat x := rest x
    empty? x => "failed"
    first x

  position(f:S -> Boolean, x:%) ==
    for k in minIndex(x).. while not empty? x and not f first x repeat
      x := rest x
    empty? x => minIndex(x) - 1
    k

  mergeSort(f, p, n) ==
    if n = 2 and f(first rest p, first p) then p := reverse_! p
    n < 3 => p
    l := (n quo 2)::NonNegativeInteger
    q := split_!(p, l)
    p := mergeSort(f, p, l)
    q := mergeSort(f, q, n - l)
    merge_!(f, p, q)

  sorted?(f, l) ==
    empty? l => true
    p := rest l
    while not empty? p repeat
      not f(first l, first p) => return false
      p := rest(l := p)
    true

  reduce(f, x, i) ==
    r := i
    while not empty? x repeat (r := f(r, first x); x := rest x)
    r

  if S has SetCategory then

     reduce(f, x, i,a) ==
       r := i
       while not empty? x and r ^= a repeat
         r := f(r, first x)
         x := rest x
       r

  new(n, s) ==
    l := empty()
    for k in 1..n repeat l := concat(s, l)
    l
```

```
map(f, x, y) ==
  z := empty()
  while not empty? x and not empty? y repeat
    z := concat(f(first x, first y), z)
    x := rest x
    y := rest y
  reverse_! z

reverse_! x ==
  empty? x => x
  empty?(y := rest x) => x
  setrest_!(x, empty())
  while not empty? y repeat
    z := rest y
    setrest_!(y, x)
    x := y
    y := z
  x

copy x ==
  y := empty()
  for k in 0.. while not empty? x repeat
    k = cycleMax and cyclic? x => error "cyclic list"
    y := concat(first x, y)
    x := rest x
  reverse_! y

copyInto_!(y, x, s) ==
  s < (m := minIndex y) => error "index out of range"
  z := rest(y, (s - m)::NonNegativeInteger)
  while not empty? z and not empty? x repeat
    setfirst_!(z, first x)
    x := rest x
    z := rest z
  y

if S has SetCategory then

  position(w, x, s) ==
    s < (m := minIndex x) => error "index out of range"
    x := rest(x, (s - m)::NonNegativeInteger)
    for k in s.. while not empty? x and w ^= first x repeat
      x := rest x
    empty? x => minIndex x - 1
    k

  removeDuplicates_! l ==
    p := l
    while not empty? p repeat
      p := setrest_!(p, remove_!((x:S):Boolean +-> x = first p, rest p))
    l

if S has OrderedSet then
```

```
    x < y ==
       while not empty? x and not empty? y repeat
         first x ^= first y => return(first x < first y)
         x := rest x
         y := rest y
       empty? x => not empty? y
       false
```

——————

— COQ LSAGG —

```
(* category LSAGG *)
(*
   cycleMax ==> 1000

   sort! : (((S,S) -> Boolean),%) -> %
   sort_!(f, l) == mergeSort(f, l, #l)

   list : S -> %
   list x == concat(x, empty())

   reduce : (((S,S) -> S),%) -> S
   reduce(f, x) ==
     empty? x => _
       error "reducing over an empty list needs the 3 argument form"
     reduce(f, rest x, first x)

   merge : (((S,S) -> Boolean),%,%) -> %
   merge(f, p, q) == merge_!(f, copy p, copy q)

   select! : ((S -> Boolean),%) -> %
   select_!(f, x) ==
     while not empty? x and not f first x repeat x := rest x
     empty? x => x
     y := x
     z := rest y
     while not empty? z repeat
       if f first z then (y := z; z := rest z)
                    else (z := rest z; setrest_!(y, z))
     x

   merge! : (((S,S) -> Boolean),%,%) -> %
   merge_!(f, p, q) ==
     empty? p => q
     empty? q => p
     eq?(p, q) => error "cannot merge a list into itself"
     if f(first p, first q)
       then (r := t := p; p := rest p)
       else (r := t := q; q := rest q)
     while not empty? p and not empty? q repeat
       if f(first p, first q)
```

```
      then (setrest_!(t, p); t := p; p := rest p)
      else (setrest_!(t, q); t := q; q := rest q)
  setrest_!(t, if empty? p then q else p)
  r

insert! : (S,%,Integer) -> %
insert_!(s:S, x:%, i:Integer) ==
  i < (m := minIndex x) => error "index out of range"
  i = m => concat(s, x)
  y := rest(x, (i - 1 - m)::NonNegativeInteger)
  z := rest y
  setrest_!(y, concat(s, z))
  x

insert! : (%,%,Integer) -> %
insert_!(w:%, x:%, i:Integer) ==
  i < (m := minIndex x) => error "index out of range"
  i = m => concat_!(w, x)
  y := rest(x, (i - 1 - m)::NonNegativeInteger)
  z := rest y
  setrest_!(y, w)
  concat_!(y, z)
  x

remove! : ((S -> Boolean),%) -> %
remove_!(f:S -> Boolean, x:%) ==
  while not empty? x and f first x repeat x := rest x
  empty? x => x
  p := x
  q := rest x
  while not empty? q repeat
    if f first q then q := setrest_!(p, rest q)
                 else (p := q; q := rest q)
  x

delete! : (%,Integer) -> %
delete_!(x:%, i:Integer) ==
  i < (m := minIndex x) => error "index out of range"
  i = m => rest x
  y := rest(x, (i - 1 - m)::NonNegativeInteger)
  setrest_!(y, rest(y, 2))
  x

delete! : (%,UniversalSegment(Integer)) -> %
delete_!(x:%, i:UniversalSegment(Integer)) ==
  (l := lo i) < (m := minIndex x) => error "index out of range"
  h := if hasHi i then hi i else maxIndex x
  h < l => x
  l = m => rest(x, (h + 1 - m)::NonNegativeInteger)
  t := rest(x, (l - 1 - m)::NonNegativeInteger)
  setrest_!(t, rest(t, (h - l + 2)::NonNegativeInteger))
  x

find : ((S -> Boolean),%) -> Union(S,"failed")
```

```
find(f, x) ==
  while not empty? x and not f first x repeat x := rest x
  empty? x => "failed"
  first x

position : ((S -> Boolean),%) -> Integer
position(f:S -> Boolean, x:%) ==
  for k in minIndex(x).. while not empty? x and not f first x repeat
    x := rest x
  empty? x => minIndex(x) - 1
  k

mergeSort: ((S, S) -> Boolean, %, Integer) -> %
mergeSort(f, p, n) ==
  if n = 2 and f(first rest p, first p) then p := reverse_! p
  n < 3 => p
  l := (n quo 2)::NonNegativeInteger
  q := split_!(p, l)
  p := mergeSort(f, p, l)
  q := mergeSort(f, q, n - l)
  merge_!(f, p, q)

sorted? : (((S,S) -> Boolean),%) -> Boolean
sorted?(f, l) ==
  empty? l => true
  p := rest l
  while not empty? p repeat
    not f(first l, first p) => return false
    p := rest(l := p)
  true

reduce : (((S,S) -> S),%,S) -> S
reduce(f, x, i) ==
  r := i
  while not empty? x repeat (r := f(r, first x); x := rest x)
  r

if S has SetCategory then

   reduce : (((S,S) -> S),%,S,S) -> S
   reduce(f, x, i,a) ==
     r := i
     while not empty? x and r ^= a repeat
       r := f(r, first x)
       x := rest x
     r

new : (NonNegativeInteger,S) -> %
new(n, s) ==
  l := empty()
  for k in 1..n repeat l := concat(s, l)
  l

map : (((S,S) -> S),%,%) -> %
```

```
map(f, x, y) ==
  z := empty()
  while not empty? x and not empty? y repeat
    z := concat(f(first x, first y), z)
    x := rest x
    y := rest y
  reverse_! z

reverse! : % -> %
reverse_! x ==
  empty? x => x
  empty?(y := rest x) => x
  setrest_!(x, empty())
  while not empty? y repeat
    z := rest y
    setrest_!(y, x)
    x := y
    y := z
  x

copy : % -> %
copy x ==
  y := empty()
  for k in 0.. while not empty? x repeat
    k = cycleMax and cyclic? x => error "cyclic list"
    y := concat(first x, y)
    x := rest x
  reverse_! y

copyInto! : (%,%,Integer) -> %
copyInto_!(y, x, s) ==
  s < (m := minIndex y) => error "index out of range"
  z := rest(y, (s - m)::NonNegativeInteger)
  while not empty? z and not empty? x repeat
    setfirst_!(z, first x)
    x := rest x
    z := rest z
  y

if S has SetCategory then

  position : (S,%,Integer) -> Integer
  position(w, x, s) ==
    s < (m := minIndex x) => error "index out of range"
    x := rest(x, (s - m)::NonNegativeInteger)
    for k in s.. while not empty? x and w ^= first x repeat
      x := rest x
    empty? x => minIndex x - 1
    k

  removeDuplicates! : % -> %
  removeDuplicates_! l ==
    p := l
    while not empty? p repeat
```

```
        p := setrest_!(p, remove_!((x:S):Boolean +-> x = first p, rest p))
      l

   if S has OrderedSet then

     ?<? : (%,%) -> Boolean
     x < y ==
        while not empty? x and not empty? y repeat
          first x ^= first y => return(first x < first y)
          x := rest x
          y := rest y
        empty? x => not empty? y
        false

*)
```

———-

— LSAGG.dotabb —

```
"LSAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=LSAGG"];
"LSAGG" -> "FLAGG"
"LSAGG" -> "ELAGG"
```

———-

— LSAGG.dotfull —

```
"ListAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LSAGG"];
"ListAggregate(a:Type)" -> "StreamAggregate(a:Type)"
"ListAggregate(a:Type)" -> "FiniteLinearAggregate(a:Type)"
"ListAggregate(a:Type)" -> "ExtensibleLinearAggregate(a:Type)"

"ListAggregate(Record(a:SetCategory,b:SetCategory))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=LSAGG"];
"ListAggregate(Record(a:SetCategory,b:SetCategory))" ->
    "ListAggregate(a:Type)"
```

———-

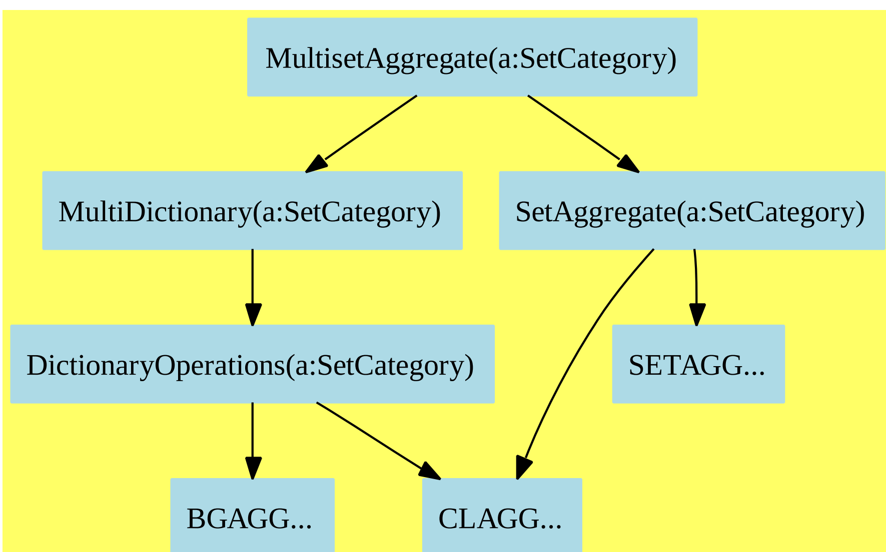
— LSAGG.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"ListAggregate(a:Type)" [color=lightblue];
"ListAggregate(a:Type)" -> "StreamAggregate(a:Type)"
"ListAggregate(a:Type)" -> "FiniteLinearAggregate(a:Type)"
```

```
"ListAggregate(a:Type)" -> "ExtensibleLinearAggregate(a:Type)"

"StreamAggregate(a:Type)" [color=lightblue];
"StreamAggregate(a:Type)" -> "UnaryRecursiveAggregate(a:Type)"
"StreamAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"FiniteLinearAggregate(a:Type)" [color=lightblue];
"FiniteLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"ExtensibleLinearAggregate(a:Type)" [color=lightblue];
"ExtensibleLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"UnaryRecursiveAggregate(a:Type)" [color=lightblue];
"UnaryRecursiveAggregate(a:Type)" -> "RCAGG..."

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IXAGG..."
"LinearAggregate(a:Type)" -> "CLAGG..."

"CLAGG..." [color=lightblue];
"IXAGG..." [color=lightblue];
"RCAGG..." [color=lightblue];
}
```

————-

8.0.139 MultisetAggregate (MSETAGG)



— **MultisetAggregate.input** —

```
)set break resume
)sys rm -f MultisetAggregate.output
)spool MultisetAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MultisetAggregate
--R
--R MultisetAggregate(S: SetCategory) is a category constructor
--R Abbreviation for MultisetAggregate is MSETAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for MSETAGG
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean                ?=? : (%,%) -> Boolean
--R bag : List(S) -> %                    brace : () -> %
--R brace : List(S) -> %                  coerce : % -> OutputForm
--R construct : List(S) -> %              copy : % -> %
--R dictionary : List(S) -> %             dictionary : () -> %
--R difference : (%,%) -> %               difference : (%,S) -> %
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                extract! : % -> S
--R hash : % -> SingleInteger             insert! : (S,%) -> %
--R inspect : % -> S                      intersect : (%,%) -> %
--R latex : % -> String                   map : ((S -> S),%) -> %
--R removeDuplicates! : % -> %            sample : () -> %
--R set : () -> %                         set : List(S) -> %
--R subset? : (%,%) -> Boolean            symmetricDifference : (%,%) -> %
--R union : (%,%) -> %                    union : (%,S) -> %
--R union : (S,%) -> %                    ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R convert : % -> InputForm if S has KONVERT(INFORM)
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R duplicates : % -> List(Record(entry: S,count: NonNegativeInteger))
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R insert! : (S,%,NonNegativeInteger) -> %
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(S) if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
```

```
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R remove! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (S,%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

— **MultisetAggregate.help** —

```
====================================================================
MultisetAggregate examples
====================================================================

A multi-set aggregate is a set which keeps track of the multiplicity
of its elements.

See Also:
o )show MultisetAggregate
```

See:

⇒ "OrderedMultisetAggregate" (OMSAGG) 9.0.152 on page 939
⇐ "MultiDictionary" (MDAGG) 7.0.128 on page 724
⇐ "SetAggregate" (SETAGG) 6.0.118 on page 628

Exports:

any?	bag	brace	coerce
construct	convert	copy	count
dictionary	difference	duplicates	empty
empty?	eq?	eval	every?
extract!	find	hash	insert!
inspect	intersect	latex	less?
map	map!	member?	members
more?	parts	reduce	remove
remove!	removeDuplicates	removeDuplicates!	sample
select	select!	set	size?
subset?	symmetricDifference	union	#?
?~=?	?<?	?=?	

Attributes exported:

- **partiallyOrderedSet** is true if a set with $<$ which is transitive, but not($a < b$ or

$a = b$) does not necessarily imply $b < a$.

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These exports come from (p724) MultiDictionary(S:SetCategory):

```
any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
bag : List S -> %
coerce : % -> OutputForm
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
copy : % -> %
count : (S,%) -> NonNegativeInteger
    if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
    if $ has finiteAggregate
dictionary : List S -> %
dictionary : () -> %
duplicates : % ->
    List Record(entry: S,count: NonNegativeInteger)
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
extract! : % -> S
find : ((S -> Boolean),%) -> Union(S,"failed")
hash : % -> SingleInteger
insert! : (S,%) -> %
insert! : (S,%,NonNegativeInteger) -> %
inspect : % -> S
latex : % -> String
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
    if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
    if S has SETCAT and $ has finiteAggregate
remove : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
remove : (S,%) -> %
```

```
    if S has SETCAT and $ has finiteAggregate
removeDuplicates : % -> %
    if S has SETCAT and $ has finiteAggregate
removeDuplicates! : % -> %
remove! : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
remove! : (S,%) -> % if $ has finiteAggregate
select : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
select! : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
?~=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
```

These exports come from (p628) SetAggregate(S:SetCategory):

```
brace : () -> %
brace : List S -> %
difference : (%,S) -> %
intersect : (%,%) -> %
sample : () -> %
set : () -> %
set : List S -> %
subset? : (%,%) -> Boolean
symmetricDifference : (%,%) -> %
union : (%,S) -> %
union : (S,%) -> %
union : (%,%) -> %
?<? : (%,%) -> Boolean
```

— MultisetAggregate.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#MSETAGG">
MultisetAggregate (MSETAGG)</a></h2>
</body>
```

— category MSETAGG MultisetAggregate —

```
)abbrev category MSETAGG MultisetAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ A multi-set aggregate is a set which keeps track of the multiplicity
++ of its elements.

MultisetAggregate(S) :  Category == SIG where
  S : SetCategory
```

```
  SIG ==> Join(MultiDictionary S, SetAggregate S)
```

———-

— **MSETAGG.dotabb** —

```
"MSETAGG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MSETAGG"];
"MSETAGG" -> "MDAGG"
"MSETAGG" -> "SETAGG"
```

———-

— **MSETAGG.dotfull** —

```
"MultisetAggregate(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MSETAGG"];
"MultisetAggregate(a:SetCategory)" -> "MultiDictionary(a:SetCategory)"
"MultisetAggregate(a:SetCategory)" -> "SetAggregate(a:SetCategory)"
```

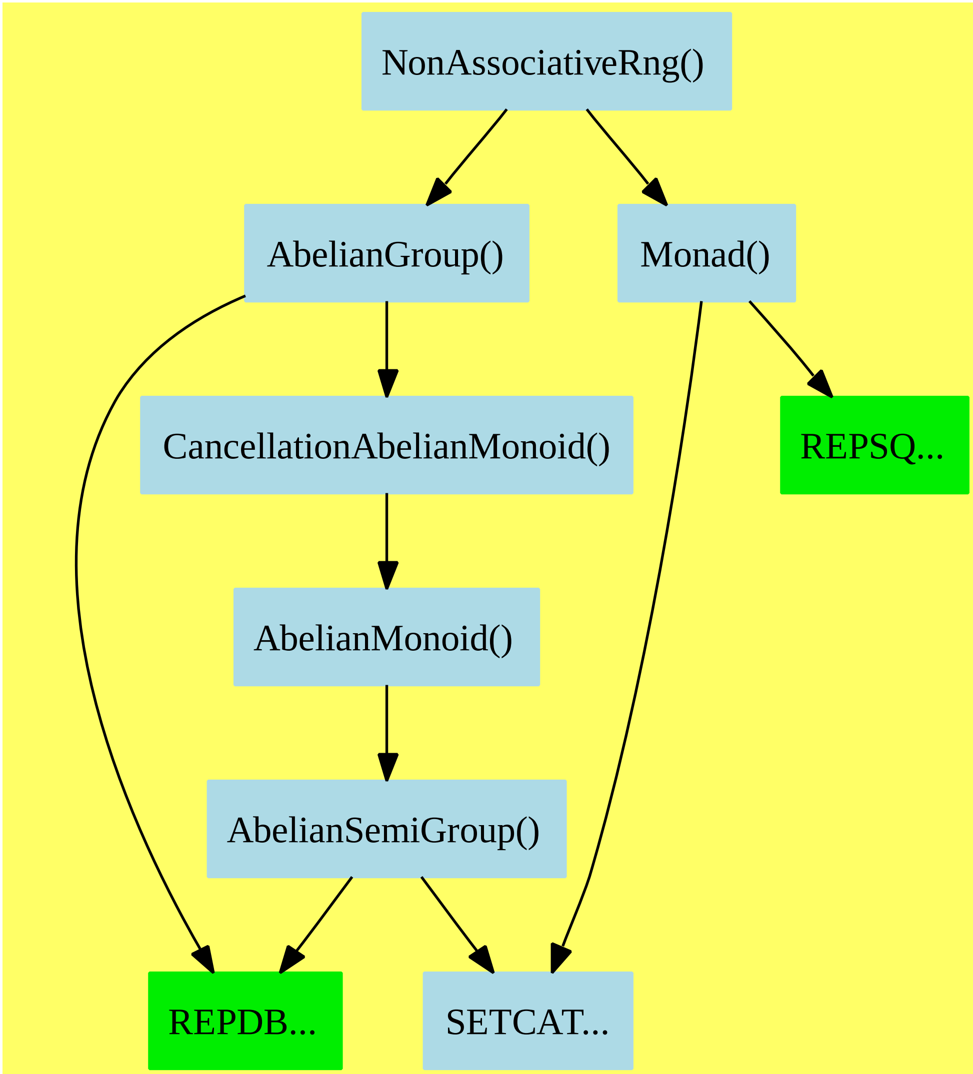
———-

— **MSETAGG.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"MultisetAggregate(a:SetCategory)" [color=lightblue];
"MultisetAggregate(a:SetCategory)" -> "MultiDictionary(a:SetCategory)"
"MultisetAggregate(a:SetCategory)" -> "SetAggregate(a:SetCategory)"

"MultiDictionary(a:SetCategory)" [color=lightblue];
"MultiDictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"

"SetAggregate(a:SetCategory)" [color=lightblue];
"SetAggregate(a:SetCategory)" -> "SETAGG..."
"SetAggregate(a:SetCategory)" -> "CLAGG..."

"DictionaryOperations(a:SetCategory)" [color=lightblue];
"DictionaryOperations(a:SetCategory)" -> "BGAGG..."
"DictionaryOperations(a:SetCategory)" -> "CLAGG..."

"BGAGG..." [color=lightblue];
"CLAGG..." [color=lightblue];
"SETAGG..." [color=lightblue];
}
```

———-

8.0.140 NonAssociativeRng (NARNG)



— **NonAssociativeRng.input** —

```
)set break resume
)sys rm -f NonAssociativeRng.output
)spool NonAssociativeRng.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NonAssociativeRng
--R
--R NonAssociativeRng is a category constructor
```

```
--R Abbreviation for NonAssociativeRng is NARNG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for NARNG
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?=? : (%,%) -> Boolean                0 : () -> %
--R antiCommutator : (%,%) -> %           associator : (%,%,%) -> %
--R coerce : % -> OutputForm              commutator : (%,%) -> %
--R hash : % -> SingleInteger             latex : % -> String
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R leftPower : (%,PositiveInteger) -> %
--R rightPower : (%,PositiveInteger) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— **NonAssociativeRng.help** —

```
====================================================================
NonAssociativeRng examples
====================================================================

NonAssociativeRng is a basic ring-type structure, not necessarily
commutative or associative, and not necessarily with unit.

Axioms:
        x*(y+z) = x*y + x*z
        (x+y)*z = x*z + y*z

Common Additional Axioms
        noZeroDivisors     ab = 0 => a=0 or b=0

See Also:
o )show NonAssociativeRng
```

See:

⇒ "NonAssociativeAlgebra" (NAALG) 11.0.181 on page 1154
⇒ "NonAssociativeRing" (NASRING) 9.0.148 on page 917
⇐ "AbelianGroup" (ABELGRP) 7.0.121 on page 673
⇐ "Monad" (MONAD) 4.0.76 on page 303

Exports:

0	antiCommutator	associator	coerce	commutator
hash	latex	leftPower	rightPower	sample
subtractIfCan	zero?	?*?	?**?	?+?
?-?	-?	?=?	?~=?	

These are implemented by this category:

```
antiCommutator : (%,%) -> %
associator : (%,%,%) -> %
commutator : (%,%) -> %
```

These exports come from (p673) AbelianGroup():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
```

These exports come from (p303) Monad():

```
leftPower : (%,PositiveInteger) -> %
rightPower : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
```

See: Schafer [Scha61][Scha66][Scha10]; Bremer [Brem08]

— NonAssociativeRng.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#NARNG">
NonAssociativeRng (NARNG)</a></h2>
</body>
```

— category NARNG NonAssociativeRng —

```
)abbrev category NARNG NonAssociativeRng
++ Author: J. Grabmeier, R. Wisbauer
++ Date Created: 01 March 1991
++ Date Last Updated: 03 July 1991
++ Reference:
++ Scha10 An Introduction to Nonassociative Algebras
++ Brem08 Nonassociative Algebras
```

```
++ Description:
++ NonAssociativeRng is a basic ring-type structure, not necessarily
++ commutative or associative, and not necessarily with unit.\br
++ Axioms\br
++ \tab{5}x*(y+z) = x*y + x*z\br
++ \tab{5}(x+y)*z = x*z + y*z\br
++
++ Common Additional Axioms\br
++ \tab{5}noZeroDivisors\tab{5} ab = 0 => a=0 or b=0

NonAssociativeRng() : Category == SIG where

  SIG ==> Join(AbelianGroup,Monad)  with

    associator : (%,%,%) -> %
      ++ associator(a,b,c) returns \spad{(a*b)*c-a*(b*c)}.

    commutator : (%,%) -> %
      ++ commutator(a,b) returns \spad{a*b-b*a}.

    antiCommutator : (%,%) -> %
      ++ antiCommutator(a,b) returns \spad{a*b+b*a}.

   add

     associator(x,y,z) == (x*y)*z - x*(y*z)

     commutator(x,y) == x*y - y*x

     antiCommutator(x,y) == x*y + y*x
```

———-

— COQ NARNG —

```
(* category NARNG *)
(*
++ \tab{5}noZeroDivisors\tab{5} ab = 0 => a=0 or b=0
    associator(x,y,z) == (x*y)*z - x*(y*z)
    commutator(x,y) == x*y - y*x
    antiCommutator(x,y) == x*y + y*x
*)
```

———-

— NARNG.dotabb —

```
"NARNG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NARNG"];
"NARNG" -> "ABELGRP"
"NARNG" -> "MONAD"
```

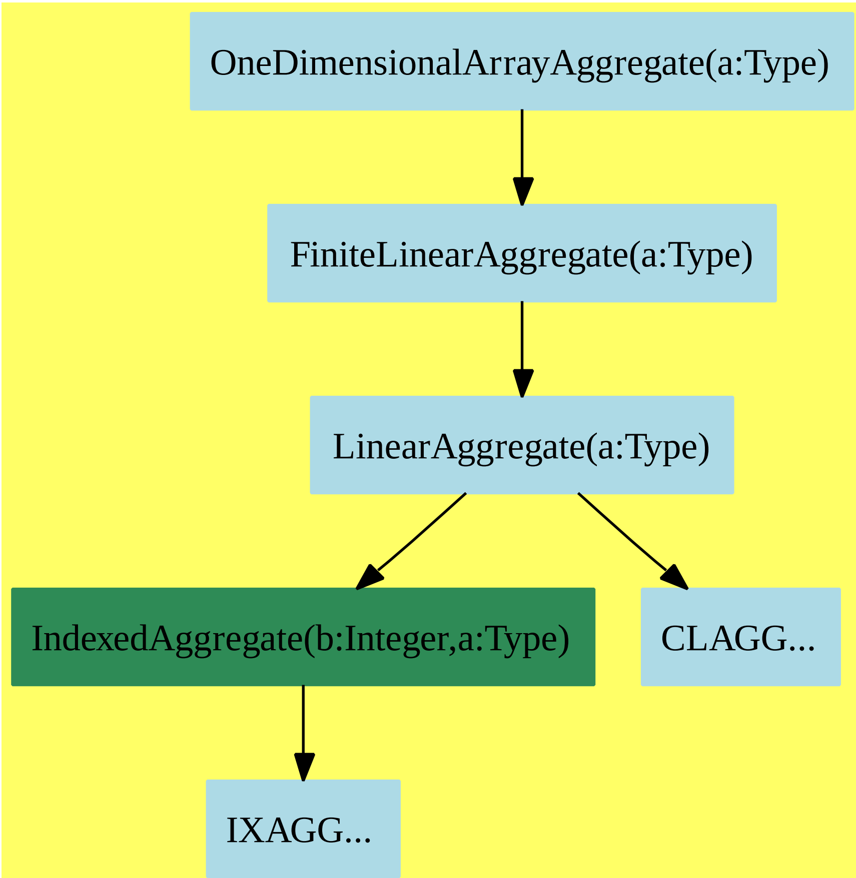
— NARNG.dotfull —

```
"NonAssociativeRng()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NARNG"];
"NonAssociativeRng()" -> "AbelianGroup()"
"NonAssociativeRng()" -> "Monad()"
```

— NARNG.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"NonAssociativeRng()" [color=lightblue];
"NonAssociativeRng()" -> "AbelianGroup()"
"NonAssociativeRng()" -> "Monad()"

"Monad()" [color=lightblue];
"Monad()" -> "SETCAT..."
"Monad()" -> "REPSQ..."

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "REPDB..."

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SETCAT..."
"AbelianSemiGroup()" -> "REPDB..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
}
```

8.0.141 OneDimensionalArrayAggregate (A1AGG)



— **OneDimensionalArrayAggregate.input** —

```
)set break resume
)sys rm -f OneDimensionalArrayAggregate.output
)spool OneDimensionalArrayAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OneDimensionalArrayAggregate
--R
--R OneDimensionalArrayAggregate(S: Type) is a category constructor
--R Abbreviation for OneDimensionalArrayAggregate is A1AGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for A1AGG
--R
--R------------------------------- Operations --------------------------------
--R concat : List(%) -> %                 concat : (%,%) -> %
```

```
--R concat : (S,%) -> %                  concat : (%,S) -> %
--R construct : List(S) -> %             copy : % -> %
--R delete : (%,Integer) -> %            ?.? : (%,Integer) -> S
--R elt : (%,Integer,S) -> S             empty : () -> %
--R empty? : % -> Boolean                entries : % -> List(S)
--R eq? : (%,%) -> Boolean               index? : (Integer,%) -> Boolean
--R indices : % -> List(Integer)         insert : (%,%,Integer) -> %
--R insert : (S,%,Integer) -> %          latex : % -> String if S has SETCAT
--R map : (((S,S) -> S),%,%) -> %        map : ((S -> S),%) -> %
--R max : (%,%) -> % if S has ORDSET     min : (%,%) -> % if S has ORDSET
--R new : (NonNegativeInteger,S) -> %    qelt : (%,Integer) -> S
--R reverse : % -> %                     sample : () -> %
--R sort : % -> % if S has ORDSET        sort : (((S,S) -> Boolean),%) -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?<? : (%,%) -> Boolean if S has ORDSET
--R ?<=? : (%,%) -> Boolean if S has ORDSET
--R ?=? : (%,%) -> Boolean if S has SETCAT
--R ?>? : (%,%) -> Boolean if S has ORDSET
--R ?>=? : (%,%) -> Boolean if S has ORDSET
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if S has SETCAT
--R convert : % -> InputForm if S has KONVERT(INFORM)
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (%,UniversalSegment(Integer)) -> %
--R ?.? : (%,UniversalSegment(Integer)) -> %
--R entry? : (S,%) -> Boolean if $ has finiteAggregate and S has SETCAT
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,S) -> % if $ has shallowlyMutable
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R first : % -> S if Integer has ORDSET
--R hash : % -> SingleInteger if S has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
--R merge : (%,%) -> % if S has ORDSET
--R merge : (((S,S) -> Boolean),%,%) -> %
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(S) if $ has finiteAggregate
--R position : (S,%,Integer) -> Integer if S has SETCAT
--R position : (S,%) -> Integer if S has SETCAT
--R position : ((S -> Boolean),%) -> Integer
--R qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
```

```
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment(Integer),S) -> S if $ has shallowlyMutable
--R setelt : (%,Integer,S) -> S if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort! : % -> % if S has ORDSET and $ has shallowlyMutable
--R sort! : (((S,S) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if S has ORDSET
--R sorted? : (((S,S) -> Boolean),%) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if S has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

— **OneDimensionalArrayAggregate.help** —

```
====================================================================
OneDimensionalArrayAggregate examples
====================================================================

One-dimensional-array aggregates serves as models for one-dimensional
arrays. Categorically, these aggregates are finite linear aggregates
with the shallowlyMutable property, that is, any component of the array
may be changed without affecting the identity of the overall array.
Array data structures are typically represented by a fixed area in
storage and cannot efficiently grow or shrink on demand as can list
structures (see however FlexibleArray for a data structure
which is a cross between a list and an array).

Iteration over, and access to, elements of arrays is extremely fast
(and often can be optimized to open-code).

Insertion and deletion however is generally slow since an entirely new
data structure must be created for the result.

See Also:
o )show OneDimensionalArrayAggregate
o )show FlexibleArray
```

See:

⇒ "BitAggregate" (BTAGG) 9.0.147 on page 907
⇒ "StringAggregate" (SRAGG) 9.0.155 on page 961

⇒ "VectorCategory" (VECTCAT) 9.0.157 on page 985
⇐ "FiniteLinearAggregate" (FLAGG) 7.0.126 on page 709

Exports:

any?	coerce	concat	construct	convert
copy	copyInto!	count	delete	elt
empty	empty?	entries	entry?	eq?
eval	every?	fill!	find	first
hash	index?	indices	insert	latex
less?	map	map!	max	maxIndex
member?	members	merge	min	minIndex
more?	new	parts	position	qelt
qsetelt!	reduce	remove	removeDuplicates	reverse
reverse!	sample	select	setelt	size?
sort	sort!	sorted?	swap!	#?
?.?	?<?	?<=?	?~=?	?=?
?>?	?>=?			

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are implemented by this category:

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if S has SETCAT
concat : (%,%) -> %
concat : List % -> %
construct : List S -> %
copy : % -> %
copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
delete : (%,UniversalSegment Integer) -> %
delete : (%,Integer) -> %
every? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
find : ((S -> Boolean),%) -> Union(S,"failed")
insert : (%,%,Integer) -> %
map : (((S,S) -> S),%,%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
merge : (((S,S) -> Boolean),%,%) -> %
parts : % -> List S if $ has finiteAggregate
position : ((S -> Boolean),%) -> Integer
position : (S,%,Integer) -> Integer
         if S has SETCAT
reduce : (((S,S) -> S),%) -> S
         if $ has finiteAggregate
```

```
reduce : (((S,S) -> S),%,S) -> S
         if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
         if S has SETCAT and $ has finiteAggregate
reverse! : % -> % if $ has shallowlyMutable
setelt : (%,UniversalSegment Integer,S) -> S
         if $ has shallowlyMutable
sort! : (((S,S) -> Boolean),%) -> %
         if $ has shallowlyMutable
sorted? : (((S,S) -> Boolean),%) -> Boolean
?.? : (%,UniversalSegment Integer) -> %
?=? : (%,%) -> Boolean if S has SETCAT
?<? : (%,%) -> Boolean if S has ORDSET
```

These exports come from (p709) FiniteLinearAggregate(S:Type):

```
concat : (S,%) -> %
concat : (%,S) -> %
convert : % -> InputForm if S has KONVERT INFORM
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
elt : (%,Integer,S) -> S
empty : () -> %
empty? : % -> Boolean
entries : % -> List S
entry? : (S,%) -> Boolean
         if $ has finiteAggregate and S has SETCAT
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
fill! : (%,S) -> % if $ has shallowlyMutable
first : % -> S if Integer has ORDSET
hash : % -> SingleInteger if S has SETCAT
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (S,%,Integer) -> %
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
max : (%,%) -> % if S has ORDSET
maxIndex : % -> Integer if Integer has ORDSET
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
merge : (%,%) -> % if S has ORDSET
min : (%,%) -> % if S has ORDSET
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
new : (NonNegativeInteger,S) -> %
```

```
 position : (S,%) -> Integer if S has SETCAT
 qelt : (%,Integer) -> S
 qsetelt! : (%,Integer,S) -> S
          if $ has shallowlyMutable
 remove : ((S -> Boolean),%) -> %
          if $ has finiteAggregate
 remove : (S,%) -> %
          if S has SETCAT and $ has finiteAggregate
 removeDuplicates : % -> %
          if S has SETCAT and $ has finiteAggregate
 reverse : % -> %
 sample : () -> %
 select : ((S -> Boolean),%) -> %
          if $ has finiteAggregate
 setelt : (%,Integer,S) -> S if $ has shallowlyMutable
 size? : (%,NonNegativeInteger) -> Boolean
 sort : % -> % if S has ORDSET
 sort : (((S,S) -> Boolean),%) -> %
 sort! : % -> %
          if S has ORDSET and $ has shallowlyMutable
 sorted? : % -> Boolean if S has ORDSET
 swap! : (%,Integer,Integer) -> Void
          if $ has shallowlyMutable
 #? : % -> NonNegativeInteger if $ has finiteAggregate
 ?.? : (%,Integer) -> S
 ?~=? : (%,%) -> Boolean if S has SETCAT
 ?>? : (%,%) -> Boolean if S has ORDSET
 ?>=? : (%,%) -> Boolean if S has ORDSET
 ?<=? : (%,%) -> Boolean if S has ORDSET
```

— OneDimensionalArrayAggregate.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#A1AGG">
OneDimensionalArrayAggregate (A1AGG)</a></h2>
</body>
```

— category A1AGG OneDimensionalArrayAggregate —

```
)abbrev category A1AGG OneDimensionalArrayAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ One-dimensional-array aggregates serves as models for one-dimensional
++ arrays. Categorically, these aggregates are finite linear aggregates
++ with the \spadatt{shallowlyMutable} property, that is, any component of
++ the array may be changed without affecting the
++ identity of the overall array.
++ Array data structures are typically represented by a fixed area in
++ storage and cannot efficiently grow or shrink on demand as can list
```

```
++ structures (see however \spadtype{FlexibleArray} for a data structure
++ which is a cross between a list and an array).
++ Iteration over, and access to, elements of arrays is extremely fast
++ (and often can be optimized to open-code).
++ Insertion and deletion however is generally slow since an entirely new
++ data structure must be created for the result.

OneDimensionalArrayAggregate(S) : Category == SIG where
  S : Type

  SIG ==> FiniteLinearAggregate S with shallowlyMutable

   add

     parts x == [qelt(x, i) for i in minIndex x .. maxIndex x]

     sort_!(f, a) == quickSort(f, a)$FiniteLinearAggregateSort(S, %)

     any?(f, a) ==
       for i in minIndex a .. maxIndex a repeat
         f qelt(a, i) => return true
       false

     every?(f, a) ==
       for i in minIndex a .. maxIndex a repeat
         not(f qelt(a, i)) => return false
       true

     position(f:S -> Boolean, a:%) ==
       for i in minIndex a .. maxIndex a repeat
         f qelt(a, i) => return i
       minIndex(a) - 1

     find(f, a) ==
       for i in minIndex a .. maxIndex a repeat
         f qelt(a, i) => return qelt(a, i)
       "failed"

     count(f:S->Boolean, a:%) ==
       n:NonNegativeInteger := 0
       for i in minIndex a .. maxIndex a repeat
         if f(qelt(a, i)) then n := n+1
       n

     map_!(f, a) ==
       for i in minIndex a .. maxIndex a repeat
         qsetelt_!(a, i, f qelt(a, i))
       a

     setelt(a:%, s:UniversalSegment(Integer), x:S) ==
       l := lo s; h := if hasHi s then hi s else maxIndex a
       l < minIndex a or h > maxIndex a => error "index out of range"
       for k in l..h repeat qsetelt_!(a, k, x)
       x
```

```
reduce(f, a) ==
  empty? a => error "cannot reduce an empty aggregate"
  r := qelt(a, m := minIndex a)
  for k in m+1 .. maxIndex a repeat r := f(r, qelt(a, k))
  r

reduce(f, a, identity) ==
  for k in minIndex a .. maxIndex a repeat
    identity := f(identity, qelt(a, k))
  identity

if S has SetCategory then

  reduce(f, a, identity,absorber) ==
    for k in minIndex a .. maxIndex a while identity ^= absorber
           repeat identity := f(identity, qelt(a, k))
    identity

-- this is necessary since new has disappeared.
stupidnew: (NonNegativeInteger, %, %) -> %

stupidget: List % -> S

-- a and b are not both empty if n > 0
stupidnew(n, a, b) ==
  zero? n => empty()
  new(n, (empty? a => qelt(b, minIndex b); qelt(a, minIndex a)))

-- at least one element of l must be non-empty
stupidget l ==
  for a in l repeat
    not empty? a => return first a
  error "Should not happen"

map(f, a, b) ==
  m := max(minIndex a, minIndex b)
  n := min(maxIndex a, maxIndex b)
  l := max(0, n - m + 1)::NonNegativeInteger
  c := stupidnew(l, a, b)
  for i in minIndex(c).. for j in m..n repeat
    qsetelt_!(c, i, f(qelt(a, j), qelt(b, j)))
  c

merge(f, a, b) ==
  r := stupidnew(#a + #b, a, b)
  i := minIndex a
  m := maxIndex a
  j := minIndex b
  n := maxIndex b
  for k in minIndex(r).. while i <= m and j <= n repeat
    if f(qelt(a, i), qelt(b, j)) then
      qsetelt_!(r, k, qelt(a, i))
      i := i+1
```

```
    else
      qsetelt_!(r, k, qelt(b, j))
      j := j+1
  for k in k.. for i in i..m repeat qsetelt_!(r, k, elt(a, i))
  for k in k.. for j in j..n repeat qsetelt_!(r, k, elt(b, j))
  r

elt(a:%, s:UniversalSegment(Integer)) ==
  l := lo s
  h := if hasHi s then hi s else maxIndex a
  l < minIndex a or h > maxIndex a => error "index out of range"
  r := stupidnew(max(0, h - l + 1)::NonNegativeInteger, a, a)
  for k in minIndex r.. for i in l..h repeat
    qsetelt_!(r, k, qelt(a, i))
  r

insert(a:%, b:%, i:Integer) ==
  m := minIndex b
  n := maxIndex b
  i < m or i > n => error "index out of range"
  y := stupidnew(#a + #b, a, b)
  for k in minIndex y.. for j in m..i-1 repeat
    qsetelt_!(y, k, qelt(b, j))
  for k in k.. for j in minIndex a .. maxIndex a repeat
    qsetelt_!(y, k, qelt(a, j))
  for k in k.. for j in i..n repeat qsetelt_!(y, k, qelt(b, j))
  y

copy x ==
  y := stupidnew(#x, x, x)
  for i in minIndex x .. maxIndex x for j in minIndex y .. repeat
    qsetelt_!(y, j, qelt(x, i))
  y

copyInto_!(y, x, s) ==
  s < minIndex y or s + #x > maxIndex y + 1 =>
                                      error "index out of range"
  for i in minIndex x .. maxIndex x for j in s.. repeat
    qsetelt_!(y, j, qelt(x, i))
  y

construct l ==
  empty? l => empty()
  a := new(#l, first l)
  for i in minIndex(a).. for x in l repeat qsetelt_!(a, i, x)
  a

delete(a:%, s:UniversalSegment(Integer)) ==
  l := lo s; h := if hasHi s then hi s else maxIndex a
  l < minIndex a or h > maxIndex a => error "index out of range"
  h < l => copy a
  r := stupidnew((#a - h + l - 1)::NonNegativeInteger, a, a)
  for k in minIndex(r).. for i in minIndex a..l-1 repeat
    qsetelt_!(r, k, qelt(a, i))
```

```
  for k in k.. for i in h+1 .. maxIndex a repeat
    qsetelt_!(r, k, qelt(a, i))
  r

delete(x:%, i:Integer) ==
  i < minIndex x or i > maxIndex x => error "index out of range"
  y := stupidnew((#x - 1)::NonNegativeInteger, x, x)
  for i in minIndex(y).. for j in minIndex x..i-1 repeat
    qsetelt_!(y, i, qelt(x, j))
  for i in i .. for j in i+1 .. maxIndex x repeat
    qsetelt_!(y, i, qelt(x, j))
  y

reverse_! x ==
  m := minIndex x
  n := maxIndex x
  for i in 0..((n-m) quo 2) repeat swap_!(x, m+i, n-i)
  x

concat l ==
  empty? l => empty()
  n := _+/[#a for a in l]
  i := minIndex(r := new(n, stupidget l))
  for a in l repeat
    copyInto_!(r, a, i)
    i := i + #a
  r

sorted?(f, a) ==
  for i in minIndex(a)..maxIndex(a)-1 repeat
    not f(qelt(a, i), qelt(a, i + 1)) => return false
  true

concat(x:%, y:%) ==
  z := stupidnew(#x + #y, x, y)
  copyInto_!(z, x, i := minIndex z)
  copyInto_!(z, y, i + #x)
  z

if S has SetCategory then

  x = y ==
    #x ^= #y => false
    for i in minIndex x .. maxIndex x repeat
      not(qelt(x, i) = qelt(y, i)) => return false
    true

  coerce(r:%):OutputForm ==
    bracket commaSeparate
          [qelt(r, k)::OutputForm for k in minIndex r .. maxIndex r]

  position(x:S, t:%, s:Integer) ==
    n := maxIndex t
    s < minIndex t or s > n => error "index out of range"
```

```
        for k in s..n repeat
          qelt(t, k) = x => return k
        minIndex(t) - 1

    if S has OrderedSet then

      a < b ==
        for i in minIndex a .. maxIndex a
          for j in minIndex b .. maxIndex b repeat
            qelt(a, i) ^= qelt(b, j) => return a.i < b.j
        #a < #b
```

— COQ A1AGG —

```
(* category A1AGG *)
(*
    parts : % -> List(S)
    parts x == [qelt(x, i) for i in minIndex x .. maxIndex x]

    sort! : (((S,S) -> Boolean),%) -> %
    sort_!(f, a) == quickSort(f, a)$FiniteLinearAggregateSort(S, %)

    any? : ((S -> Boolean),%) -> Boolean
    any?(f, a) ==
      for i in minIndex a .. maxIndex a repeat
        f qelt(a, i) => return true
      false

    every? : ((S -> Boolean),%) -> Boolean
    every?(f, a) ==
      for i in minIndex a .. maxIndex a repeat
        not(f qelt(a, i)) => return false
      true

    position : ((S -> Boolean),%) -> Integer
    position(f:S -> Boolean, a:%) ==
      for i in minIndex a .. maxIndex a repeat
        f qelt(a, i) => return i
      minIndex(a) - 1

    find : ((S -> Boolean),%) -> Union(S,"failed")
    find(f, a) ==
      for i in minIndex a .. maxIndex a repeat
        f qelt(a, i) => return qelt(a, i)
      "failed"

    count : ((S -> Boolean),%) -> NonNegativeInteger
    count(f:S->Boolean, a:%) ==
      n:NonNegativeInteger := 0
      for i in minIndex a .. maxIndex a repeat
        if f(qelt(a, i)) then n := n+1
```

```
      n

    map! : ((S -> S),%) -> %
    map_!(f, a) ==
      for i in minIndex a .. maxIndex a repeat
        qsetelt_!(a, i, f qelt(a, i))
      a

    setelt : (%,UniversalSegment(Integer),S) -> S
    setelt(a:%, s:UniversalSegment(Integer), x:S) ==
      l := lo s; h := if hasHi s then hi s else maxIndex a
      l < minIndex a or h > maxIndex a => error "index out of range"
      for k in l..h repeat qsetelt_!(a, k, x)
      x

    reduce : (((S,S) -> S),%) -> S
    reduce(f, a) ==
      empty? a => error "cannot reduce an empty aggregate"
      r := qelt(a, m := minIndex a)
      for k in m+1 .. maxIndex a repeat r := f(r, qelt(a, k))
      r

    reduce : (((S,S) -> S),%,S) -> S
    reduce(f, a, identity) ==
      for k in minIndex a .. maxIndex a repeat
        identity := f(identity, qelt(a, k))
      identity

    if S has SetCategory then

       reduce : (((S,S) -> S),%,S,S) -> S
       reduce(f, a, identity,absorber) ==
         for k in minIndex a .. maxIndex a while identity ^= absorber
                repeat identity := f(identity, qelt(a, k))
         identity

-- this is necessary since new has disappeared.
-- a and b are not both empty if n > 0

    stupidnew: (NonNegativeInteger, %, %) -> %
    stupidnew(n, a, b) ==
      zero? n => empty()
      new(n, (empty? a => qelt(b, minIndex b); qelt(a, minIndex a)))

-- at least one element of l must be non-empty

    stupidget: List % -> S
    stupidget l ==
      for a in l repeat
        not empty? a => return first a
      error "Should not happen"

    map : (((S,S) -> S),%,%) -> %
    map(f, a, b) ==
```

```
  m := max(minIndex a, minIndex b)
  n := min(maxIndex a, maxIndex b)
  l := max(0, n - m + 1)::NonNegativeInteger
  c := stupidnew(l, a, b)
  for i in minIndex(c).. for j in m..n repeat
    qsetelt_!(c, i, f(qelt(a, j), qelt(b, j)))
  c

merge : (((S,S) -> Boolean),%,%) -> %
merge(f, a, b) ==
  r := stupidnew(#a + #b, a, b)
  i := minIndex a
  m := maxIndex a
  j := minIndex b
  n := maxIndex b
  for k in minIndex(r).. while i <= m and j <= n repeat
    if f(qelt(a, i), qelt(b, j)) then
      qsetelt_!(r, k, qelt(a, i))
      i := i+1
    else
      qsetelt_!(r, k, qelt(b, j))
      j := j+1
  for k in k.. for i in i..m repeat qsetelt_!(r, k, elt(a, i))
  for k in k.. for j in j..n repeat qsetelt_!(r, k, elt(b, j))
  r

?.? : (%,UniversalSegment(Integer)) -> %
elt(a:%, s:UniversalSegment(Integer)) ==
  l := lo s
  h := if hasHi s then hi s else maxIndex a
  l < minIndex a or h > maxIndex a => error "index out of range"
  r := stupidnew(max(0, h - l + 1)::NonNegativeInteger, a, a)
  for k in minIndex r.. for i in l..h repeat
    qsetelt_!(r, k, qelt(a, i))
  r

insert : (%,%,Integer) -> %
insert(a:%, b:%, i:Integer) ==
  m := minIndex b
  n := maxIndex b
  i < m or i > n => error "index out of range"
  y := stupidnew(#a + #b, a, b)
  for k in minIndex y.. for j in m..i-1 repeat
    qsetelt_!(y, k, qelt(b, j))
  for k in k.. for j in minIndex a .. maxIndex a repeat
    qsetelt_!(y, k, qelt(a, j))
  for k in k.. for j in i..n repeat qsetelt_!(y, k, qelt(b, j))
  y

copy : % -> %
copy x ==
  y := stupidnew(#x, x, x)
  for i in minIndex x .. maxIndex x for j in minIndex y .. repeat
    qsetelt_!(y, j, qelt(x, i))
```

```
  y

copyInto! : (%,%,Integer) -> %
copyInto_!(y, x, s) ==
  s < minIndex y or s + #x > maxIndex y + 1 =>
                                          error "index out of range"
  for i in minIndex x .. maxIndex x for j in s.. repeat
    qsetelt_!(y, j, qelt(x, i))
  y

construct : List(S) -> %
construct l ==
  empty? l => empty()
  a := new(#l, first l)
  for i in minIndex(a).. for x in l repeat qsetelt_!(a, i, x)
  a

delete : (%,UniversalSegment(Integer)) -> %
delete(a:%, s:UniversalSegment(Integer)) ==
  l := lo s; h := if hasHi s then hi s else maxIndex a
  l < minIndex a or h > maxIndex a => error "index out of range"
  h < l => copy a
  r := stupidnew((#a - h + l - 1)::NonNegativeInteger, a, a)
  for k in minIndex(r).. for i in minIndex a..l-1 repeat
    qsetelt_!(r, k, qelt(a, i))
  for k in k.. for i in h+1 .. maxIndex a repeat
    qsetelt_!(r, k, qelt(a, i))
  r

delete : (%,Integer) -> %
delete(x:%, i:Integer) ==
  i < minIndex x or i > maxIndex x => error "index out of range"
  y := stupidnew((#x - 1)::NonNegativeInteger, x, x)
  for i in minIndex(y).. for j in minIndex x..i-1 repeat
    qsetelt_!(y, i, qelt(x, j))
  for i in i .. for j in i+1 .. maxIndex x repeat
    qsetelt_!(y, i, qelt(x, j))
  y

reverse! : % -> %
reverse_! x ==
  m := minIndex x
  n := maxIndex x
  for i in 0..((n-m) quo 2) repeat swap_!(x, m+i, n-i)
  x

concat : List(%) -> %
concat l ==
  empty? l => empty()
  n := _+/[#a for a in l]
  i := minIndex(r := new(n, stupidget l))
  for a in l repeat
    copyInto_!(r, a, i)
    i := i + #a
```

```
      r

    sorted? : (((S,S) -> Boolean),%) -> Boolean
    sorted?(f, a) ==
      for i in minIndex(a)..maxIndex(a)-1 repeat
        not f(qelt(a, i), qelt(a, i + 1)) => return false
      true

    concat : (%,%) -> %
    concat(x:%, y:%) ==
      z := stupidnew(#x + #y, x, y)
      copyInto_!(z, x, i := minIndex z)
      copyInto_!(z, y, i + #x)
      z

    if S has SetCategory then

      ?=? : (%,%) -> Boolean
      x = y ==
        #x ^= #y => false
        for i in minIndex x .. maxIndex x repeat
          not(qelt(x, i) = qelt(y, i)) => return false
        true

      coerce : % -> OutputForm
      coerce(r:%):OutputForm ==
        bracket commaSeparate
              [qelt(r, k)::OutputForm for k in minIndex r .. maxIndex r]

      position : (S,%,Integer) -> Integer
      position(x:S, t:%, s:Integer) ==
        n := maxIndex t
        s < minIndex t or s > n => error "index out of range"
        for k in s..n repeat
          qelt(t, k) = x => return k
        minIndex(t) - 1

    if S has OrderedSet then

      ?<? : (%,%) -> Boolean
      a < b ==
        for i in minIndex a .. maxIndex a
          for j in minIndex b .. maxIndex b repeat
            qelt(a, i) ^= qelt(b, j) => return a.i < b.j
        #a < #b
*)
```

———

— A1AGG.dotabb —

```
"A1AGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=A1AGG"];
"A1AGG" -> "FLAGG"
```

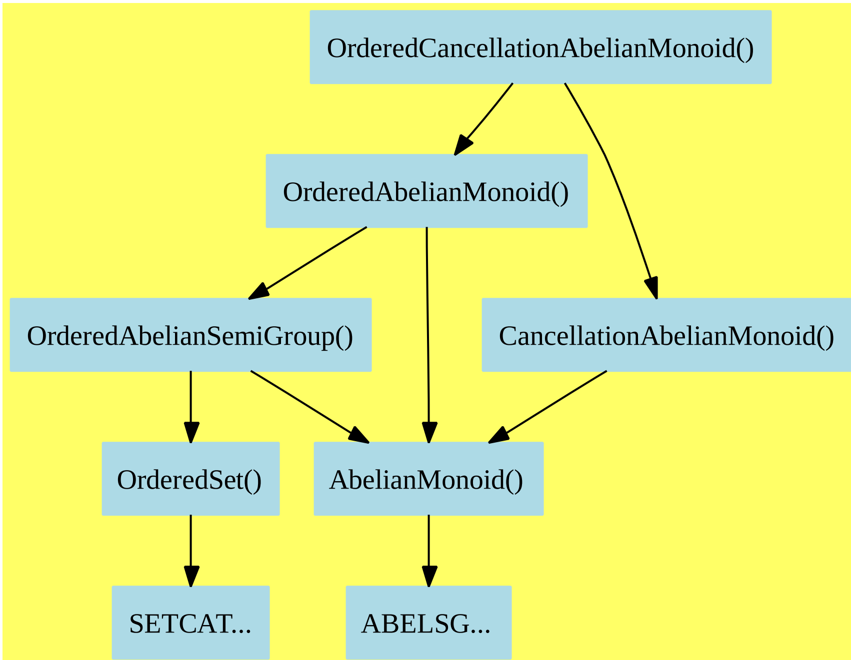
— **A1AGG.dotfull** —

```
"OneDimensionalArrayAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=A1AGG"];
"OneDimensionalArrayAggregate(a:Type)" ->
    "FiniteLinearAggregate(a:Type)"

"OneDimensionalArrayAggregate(Character)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=A1AGG"];
"OneDimensionalArrayAggregate(Character)" ->
    "OneDimensionalArrayAggregate(a:Type)"

"OneDimensionalArrayAggregate(Boolean)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=A1AGG"];
"OneDimensionalArrayAggregate(Boolean)" ->
    "OneDimensionalArrayAggregate(a:Type)"
```

— **A1AGG.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"OneDimensionalArrayAggregate(a:Type)" [color=lightblue];
"OneDimensionalArrayAggregate(a:Type)" ->
    "FiniteLinearAggregate(a:Type)"

"FiniteLinearAggregate(a:Type)" [color=lightblue];
"FiniteLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "CLAGG..."

"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" -> "IXAGG..."

"CLAGG..." [color=lightblue];
"IXAGG..." [color=lightblue];
}
```

8.0.142 OrderedCancellationAbelianMonoid (OCAMON)



— **OrderedCancellationAbelianMonoid.input** —

```
)set break resume
)sys rm -f OrderedCancellationAbelianMonoid.output
)spool OrderedCancellationAbelianMonoid.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderedCancellationAbelianMonoid
--R
--R OrderedCancellationAbelianMonoid is a category constructor
--R Abbreviation for OrderedCancellationAbelianMonoid is OCAMON
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for OCAMON
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                      ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean               ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean                ?>=? : (%,%) -> Boolean
--R 0 : () -> %                           coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R max : (%,%) -> %                      min : (%,%) -> %
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
```

```
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

— OrderedCancellationAbelianMonoid.help —

```
====================================================================
OrderedCancellationAbelianMonoid examples
====================================================================

Ordered sets which are also abelian cancellation monoids, such that
the addition preserves the ordering.

See Also:
o )show OrderedCancellationAbelianMonoid
```

———-

See:

⇒ "OrderedAbelianGroup" (OAGROUP) 9.0.150 on page 932
⇒ "OrderedAbelianMonoidSup" (OAMONS) 9.0.151 on page 935
⇐ "CancellationAbelianMonoid" (CABMON) 6.0.107 on page 508
⇐ "OrderedAbelianMonoid" (OAMON) 7.0.129 on page 730

Exports:

0	coerce	hash	latex	max
min	sample	subtractIfCan	zero?	?~=?
?*?	?+?	?<?	?<=?	?=?
?>?	?>=?			

These exports come from (p730) OrderedAbelianMonoid():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
sample : () -> %
zero? : % -> Boolean
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
```

```
?+? : (%,%) -> %
```

These exports come from (p508) CancellationAbelianMonoid():

```
subtractIfCan : (%,%) -> Union(%,"failed")
```

— OrderedCancellationAbelianMonoid.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#OCAMON">
OrderedCancellationAbelianMonoid (OCAMON)</a></h2>
</body>
```

— category OCAMON OrderedCancellationAbelianMonoid —

```
)abbrev category OCAMON OrderedCancellationAbelianMonoid
++ Description:
++ Ordered sets which are also abelian cancellation monoids,
++ such that the addition preserves the ordering.

OrderedCancellationAbelianMonoid() : Category == SIG where

  SIG ==> Join(OrderedAbelianMonoid, CancellationAbelianMonoid)
```

— OCAMON.dotabb —

```
"OCAMON"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OCAMON"];
"OCAMON" -> "OAMON"
"OCAMON" -> "CABMON"
```

— OCAMON.dotfull —

```
"OrderedCancellationAbelianMonoid()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OCAMON"];
"OrderedCancellationAbelianMonoid()" -> "OrderedAbelianMonoid()"
"OrderedCancellationAbelianMonoid()" -> "CancellationAbelianMonoid()"
```

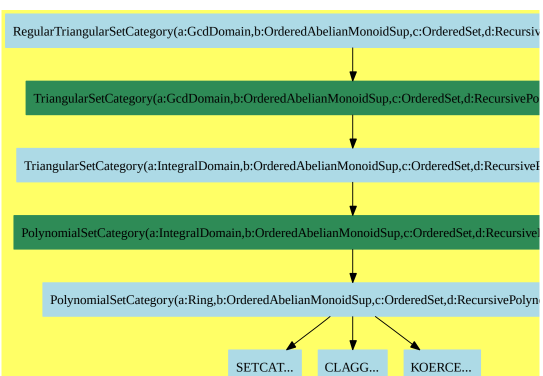
— OCAMON.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];
```

```
"OrderedCancellationAbelianMonoid()" [color=lightblue];
"OrderedCancellationAbelianMonoid()" -> "OrderedAbelianMonoid()"
"OrderedCancellationAbelianMonoid()" -> "CancellationAbelianMonoid()"

"OrderedAbelianMonoid()" [color=lightblue];
"OrderedAbelianMonoid()" -> "OrderedAbelianSemiGroup()"
"OrderedAbelianMonoid()" -> "AbelianMonoid()"

"OrderedAbelianSemiGroup()" [color=lightblue];
"OrderedAbelianSemiGroup()" -> "OrderedSet()"
"OrderedAbelianSemiGroup()" -> "AbelianMonoid()"

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SETCAT..."

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "ABELSG..."

"SETCAT..." [color=lightblue];
"ABELSG..." [color=lightblue];
}
```

———-

8.0.143 RegularTriangularSetCategory (RSETCAT)



— **RegularTriangularSetCategory.input** —

```
)set break resume
)sys rm -f RegularTriangularSetCategory.output
)spool RegularTriangularSetCategory.output
)set message test on
```

```
)set message auto off
)clear all

--S 1 of 1
)show RegularTriangularSetCategory
--R
--R RegularTriangularSetCategory(R: GcdDomain,E: OrderedAbelianMonoidSup,V: OrderedSet,P: RecursivePolynomialCat
--R Abbreviation for RegularTriangularSetCategory is RSETCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for RSETCAT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                algebraic? : (V,%) -> Boolean
--R algebraicVariables : % -> List(V)     augment : (List(P),%) -> List(%)
--R augment : (P,List(%)) -> List(%)      augment : (P,%) -> List(%)
--R coerce : % -> List(P)                 coerce : % -> OutputForm
--R collect : (%,V) -> %                  collectQuasiMonic : % -> %
--R collectUnder : (%,V) -> %             collectUpper : (%,V) -> %
--R construct : List(P) -> %              copy : % -> %
--R degree : % -> NonNegativeInteger      empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R extend : (List(P),%) -> List(%)       extend : (P,List(%)) -> List(%)
--R extend : (P,%) -> List(%)             extend : (%,P) -> %
--R first : % -> Union(P,"failed")        hash : % -> SingleInteger
--R headReduce : (P,%) -> P               headReduced? : % -> Boolean
--R headReduced? : (P,%) -> Boolean       infRittWu? : (%,%) -> Boolean
--R initiallyReduce : (P,%) -> P          initiallyReduced? : % -> Boolean
--R initials : % -> List(P)               internalAugment : (List(P),%) -> %
--R internalAugment : (P,%) -> %          intersect : (P,List(%)) -> List(%)
--R intersect : (List(P),%) -> List(%)    intersect : (P,%) -> List(%)
--R invertible? : (P,%) -> Boolean        invertibleSet : (P,%) -> List(%)
--R last : % -> Union(P,"failed")         latex : % -> String
--R mainVariable? : (V,%) -> Boolean      mainVariables : % -> List(V)
--R map : ((P -> P),%) -> %               mvar : % -> V
--R normalized? : % -> Boolean            normalized? : (P,%) -> Boolean
--R purelyAlgebraic? : % -> Boolean       purelyAlgebraic? : (P,%) -> Boolean
--R reduceByQuasiMonic : (P,%) -> P       removeZero : (P,%) -> P
--R rest : % -> Union(%,"failed")         retract : List(P) -> %
--R sample : () -> %                      select : (%,V) -> Union(P,"failed")
--R stronglyReduce : (P,%) -> P           stronglyReduced? : % -> Boolean
--R stronglyReduced? : (P,%) -> Boolean   trivialIdeal? : % -> Boolean
--R variables : % -> List(V)              zeroSetSplit : List(P) -> List(%)
--R ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R algebraicCoefficients? : (P,%) -> Boolean
--R any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R augment : (List(P),List(%)) -> List(%)
--R autoReduced? : (%,((P,List(P)) -> Boolean)) -> Boolean
--R basicSet : (List(P),(P -> Boolean),((P,P) -> Boolean)) -> Union(Record(bas: %,top: List(P)),"failed")
--R basicSet : (List(P),((P,P) -> Boolean)) -> Union(Record(bas: %,top: List(P)),"failed")
--R coHeight : % -> NonNegativeInteger if V has FINITE
--R convert : % -> InputForm if P has KONVERT(INFORM)
--R count : ((P -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (P,%) -> NonNegativeInteger if P has SETCAT and $ has finiteAggregate
```

```
--R eval : (%,List(Equation(P))) -> % if P has EVALAB(P) and P has SETCAT
--R eval : (%,Equation(P)) -> % if P has EVALAB(P) and P has SETCAT
--R eval : (%,P,P) -> % if P has EVALAB(P) and P has SETCAT
--R eval : (%,List(P),List(P)) -> % if P has EVALAB(P) and P has SETCAT
--R every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R extend : (List(P),List(%)) -> List(%)
--R extendIfCan : (%,P) -> Union(%,"failed")
--R find : ((P -> Boolean),%) -> Union(P,"failed")
--R headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
--R initiallyReduced? : (P,%) -> Boolean
--R intersect : (List(P),List(%)) -> List(%)
--R invertible? : (P,%) -> List(Record(val: Boolean,tower: %))
--R invertibleElseSplit? : (P,%) -> Union(Boolean,List(%))
--R lastSubResultant : (P,P,%) -> List(Record(val: P,tower: %))
--R lastSubResultantElseSplit : (P,P,%) -> Union(P,List(%))
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((P -> P),%) -> % if $ has shallowlyMutable
--R member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
--R members : % -> List(P) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(P) if $ has finiteAggregate
--R purelyAlgebraicLeadingMonomial? : (P,%) -> Boolean
--R purelyTranscendental? : (P,%) -> Boolean
--R quasiComponent : % -> Record(close: List(P),open: List(P))
--R reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
--R reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P,P) -> P if P has SETCAT and $ has finiteAggregate
--R reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
--R remainder : (P,%) -> Record(rnum: R,polnum: P,den: R) if R has INTDOM
--R remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
--R retractIfCan : List(P) -> Union(%,"failed")
--R rewriteIdealWithHeadRemainder : (List(P),%) -> List(P) if R has INTDOM
--R rewriteIdealWithRemainder : (List(P),%) -> List(P) if R has INTDOM
--R rewriteSetWithReduction : (List(P),%,((P,P) -> P),((P,P) -> Boolean)) -> List(P)
--R roughBase? : % -> Boolean if R has INTDOM
--R roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
--R roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
--R roughUnitIdeal? : % -> Boolean if R has INTDOM
--R select : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : (%,V) -> Record(under: %,floor: %,upper: %)
--R squareFreePart : (P,%) -> List(Record(val: P,tower: %))
--R triangular? : % -> Boolean if R has INTDOM
--R zeroSetSplit : (List(P),Boolean) -> List(%)
--R zeroSetSplitIntoTriangularSystems : List(P) -> List(Record(close: %,open: List(P)))
--R
--E 1

)spool
)lisp (bye)
```

— RegularTriangularSetCategory.help —

```
====================================================================
RegularTriangularSetCategory examples
====================================================================

The category of regular triangular sets was introduced under the name
regular chains in M. KALKBRENER "Three contributions to elimination theory".

In P. AUBRY, D. LAZARD and M. MORENO MAZA "On the Theories of Triangular Sets"
it is proved that regular triangular sets and towers of simple
extensions of a field are equivalent notions.

In the following definitions, all polynomials and ideals are taken from
the polynomial ring k[x1,...,xn] where k is the fraction field of R.

The triangular set [t1,...,tm] is regular iff for every i the initial
of ti+1 is invertible in the tower of simple extensions associated
with [t1,...,ti].

A family [T1,...,Ts] of regular triangular sets is a split of
Kalkbrener of a given ideal I iff the radical of I is equal to the
intersection of the radical ideals generated by the saturated ideals
of the [T1,...,Ti].

A family [T1,...,Ts] of regular triangular sets is a split of Kalkbrener
of a given triangular set T iff it is a split of Kalkbrener of the
saturated ideal of T. Let K be an algebraic closure of k.

Assume that V is finite with cardinality n and let A be the affine
space K^n.

For a regular triangular set T let denote by W(T) the set of regular
zeros of T. A family [T1,...,Ts] of regular triangular sets  is a split
of Lazard of a given subset S of A iff the union of the W(Ti) contains
S and is contained in the closure of S (w.r.t. Zariski topology).

A family [T1,...,Ts] of regular triangular sets is a split of Lazard
of a given triangular set T if it is a split of Lazard of W(T).
Note that if [T1,...,Ts] is a split of Lazard of T then it is also a
split of Kalkbrener of T. The converse is false.

This category provides operations related to both kinds of splits, the
former being related to ideals decomposition whereas the latter deals
with varieties decomposition. See the example illustrating the
RegularTriangularSet constructor for more explanations about
decompositions by means of regular triangular sets.

See Also:
o )show RegularTriangularSetCategory
```

See:

⇒ "NormalizedTriangularSetCategory" (NTSCAT) 9.0.149 on page 922
⇒ "SquareFreeRegularTriangularSetCategory" (SFRTCAT) 9.0.154 on page 952
⇐ "TriangularSetCategory" (TSETCAT) 7.0.132 on page 750

Exports:

algebraicCoefficients?	algebraicVariables	any?
augment	autoReduced?	basicSet
coerce	coHeight	collect
collectQuasiMonic	collectUnder	collectUpper
construct	convert	copy
count	degree	empty
empty?	eq?	eval
every?	extend	extendIfCan
find	first	hash
headReduce	headReduced?	headRemainder
infRittWu?	initiallyReduce	initiallyReduced?
initials	internalAugment	intersect
invertible?	invertibleElseSplit?	invertibleSet
last	lastSubResultant	lastSubResultantElseSplit
less?	latex	mainVariable?
mainVariables	map	map!
member?	members	more?
mvar	normalized?	parts
purelyAlgebraic?	purelyAlgebraicLeadingMonomial?	purelyTranscendental?
quasiComponent	reduce	reduceByQuasiMonic
reduced?	remainder	remove
removeDuplicates	removeZero	rest
retract	retractIfCan	rewriteIdealWithHeadRemainder
rewriteIdealWithRemainder	rewriteSetWithReduction	roughBase?
roughEqualIdeals?	roughSubIdeal?	roughUnitIdeal?
sample	select	size?
sort	squareFreePart	stronglyReduce
stronglyReduced?	triangular?	trivialIdeal?
variables	zeroSetSplit	zeroSetSplitIntoTriangularSystems
#?	?=?	?~=?

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These are directly exported but not implemented:

```
augment : (P,%) -> List %
```

```
extend : (P,%) -> List %
internalAugment : (P,%) -> %
internalAugment : (List P,%) -> %
intersect : (P,%) -> List %
invertibleElseSplit? : (P,%) -> Union(Boolean,List %)
invertible? : (P,%) -> List Record(val: Boolean,tower: %)
invertible? : (P,%) -> Boolean
invertibleSet : (P,%) -> List %
lastSubResultant : (P,P,%) -> List Record(val: P,tower: %)
lastSubResultantElseSplit : (P,P,%) -> Union(P,List %)
squareFreePart : (P,%) -> List Record(val: P,tower: %)
zeroSetSplit : (List P,Boolean) -> List %
```

These are implemented by this category:

```
algebraicCoefficients? : (P,%) -> Boolean
augment : (P,List %) -> List %
augment : (List P,%) -> List %
augment : (List P,List %) -> List %
extend : (P,List %) -> List %
extend : (List P,%) -> List %
extend : (List P,List %) -> List %
intersect : (List P,List %) -> List %
intersect : (List P,%) -> List %
intersect : (P,List %) -> List %
purelyAlgebraic? : % -> Boolean
purelyAlgebraic? : (P,%) -> Boolean
purelyAlgebraicLeadingMonomial? : (P,%) -> Boolean
purelyTranscendental? : (P,%) -> Boolean
```

These exports come from (p750) TriangularSetCategory(R,E,V,P) where R:GcdDomain, E:OrderedAbelianMonoidSup, V:OrderedSet, P:RecursivePolynomialCategory(R,E,V)):

```
algebraic? : (V,%) -> Boolean
algebraicVariables : % -> List V
any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
autoReduced? : (%,((P,List P) -> Boolean)) -> Boolean
basicSet :
  (List P,((P,P) -> Boolean)) ->
    Union(Record(bas: %,top: List P),"failed")
basicSet :
  (List P,(P -> Boolean),((P,P) -> Boolean)) ->
     Union(Record(bas: %,top: List P),"failed")
coerce : % -> List P
coerce : % -> OutputForm
coHeight : % -> NonNegativeInteger if V has FINITE
collect : (%,V) -> %
collectQuasiMonic : % -> %
collectUnder : (%,V) -> %
collectUpper : (%,V) -> %
construct : List P -> %
convert : % -> InputForm if P has KONVERT INFORM
copy : % -> %
count : ((P -> Boolean),%) -> NonNegativeInteger
  if $ has finiteAggregate
```

```
count : (P,%) -> NonNegativeInteger
  if P has SETCAT
  and $ has finiteAggregate
degree : % -> NonNegativeInteger
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
extend : (%,P) -> %
extendIfCan : (%,P) -> Union(%,"failed")
find : ((P -> Boolean),%) -> Union(P,"failed")
first : % -> Union(P,"failed")
hash : % -> SingleInteger
headReduce : (P,%) -> P
headReduced? : % -> Boolean
headReduced? : (P,%) -> Boolean
headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
infRittWu? : (%,%) -> Boolean
initiallyReduce : (P,%) -> P
initiallyReduced? : % -> Boolean
initiallyReduced? : (P,%) -> Boolean
initials : % -> List P
last : % -> Union(P,"failed")
latex : % -> String
less? : (%,NonNegativeInteger) -> Boolean
mainVariable? : (V,%) -> Boolean
mainVariables : % -> List V
map : ((P -> P),%) -> %
map! : ((P -> P),%) -> % if $ has shallowlyMutable
member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
members : % -> List P if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
mvar : % -> V
normalized? : % -> Boolean
normalized? : (P,%) -> Boolean
parts : % -> List P if $ has finiteAggregate
quasiComponent : % -> Record(close: List P,open: List P)
reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P,P) -> P
  if P has SETCAT
  and $ has finiteAggregate
reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
reduceByQuasiMonic : (P,%) -> P
reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
remainder : (P,%) -> Record(rnum: R,polnum: P,den: R) if R has INTDOM
remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
removeZero : (P,%) -> P
```

```
 rest : % -> Union(%,"failed")
 retract : List P -> %
 rewriteIdealWithHeadRemainder : (List P,%) -> List P if R has INTDOM
 rewriteIdealWithRemainder : (List P,%) -> List P if R has INTDOM
 retractIfCan : List P -> Union(%,"failed")
 rewriteSetWithReduction :
    (List P,%,((P,P) -> P),((P,P) -> Boolean)) -> List P
 roughBase? : % -> Boolean if R has INTDOM
 roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
 roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
 roughUnitIdeal? : % -> Boolean if R has INTDOM
 sample : () -> %
 select : ((P -> Boolean),%) -> % if $ has finiteAggregate
 select : (%,V) -> Union(P,"failed")
 size? : (%,NonNegativeInteger) -> Boolean
 sort : (%,V) -> Record(under: %,floor: %,upper: %)
 stronglyReduce : (P,%) -> P
 stronglyReduced? : (P,%) -> Boolean
 stronglyReduced? : % -> Boolean
 triangular? : % -> Boolean if R has INTDOM
 trivialIdeal? : % -> Boolean
 variables : % -> List V
 zeroSetSplit : List P -> List %
 zeroSetSplitIntoTriangularSystems :
    List P -> List Record(close: %,open: List P)
 #? : % -> NonNegativeInteger if $ has finiteAggregate
 ?~=? : (%,%) -> Boolean
 ?=? : (%,%) -> Boolean
```

See: SALSA[SALSA], Kalkbrener[Kalk91, Kalk98], Aubry[Aubr96, Aubr99, Aubr99a], Lazard[Laza91], Moreno Maza[Maza95, Maza97, Maza98, Maza00]

— RegularTriangularSetCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#RSETCAT">
RegularTriangularSetCategory (RSETCAT)</a></h2>
</body>
```

— category RSETCAT RegularTriangularSetCategory —

```
)abbrev category RSETCAT RegularTriangularSetCategory
++ Author: Marc Moreno Maza
++ Date Created: 09/03/1998
++ Date Last Updated: 12/15/1998
++ References :
++ SALSA Solvers for Algebraic Systems and Applications
++ Kalk91 Three contributions to elimination theory
++ Kalk98 Algorithmic properties of polynomial rings
++ Aubr96 Triangular Sets for Solving Polynomial Systems:
++ Aubr99 On the Theories of Triangular Sets
++ Aubr99a Triangular Sets for Solving Polynomial Systems:
++ Laza91 A new method for solving algebraic systems of positive dimension
```

```
++ Maza95 Polynomial Gcd Computations over Towers of Algebraic Extensions
++ Maza97 Calculs de pgcd au-dessus des tours d'extensions simples et
++        resolution des systemes d'equations algebriques
++ Maza98 A new algorithm for computing triangular decomposition of
++        algebraic varieties
++ Maza00 On Triangular Decompositions of Algebraic Varieties
++ Description:
++ The category of regular triangular sets, introduced under
++ the name regular chains in [1] (and other papers).
++ In [3] it is proved that regular triangular sets and towers of simple
++ extensions of a field are equivalent notions.
++ In the following definitions, all polynomials and ideals
++ are taken from the polynomial ring \spad{k[x1,...,xn]} where \spad{k}
++ is the fraction field of \spad{R}.
++ The triangular set \spad{[t1,...,tm]} is regular
++ iff for every \spad{i} the initial of \spad{ti+1} is invertible
++ in the tower of simple extensions associated with \spad{[t1,...,ti]}.
++ A family \spad{[T1,...,Ts]} of regular triangular sets
++ is a split of Kalkbrener of a given ideal \spad{I}
++ iff the radical of \spad{I} is equal to the intersection
++ of the radical ideals generated by the saturated ideals
++ of the \spad{[T1,...,Ti]}.
++ A family \spad{[T1,...,Ts]} of regular triangular sets
++ is a split of Kalkbrener of a given triangular set \spad{T}
++ iff it is a split of Kalkbrener of the saturated ideal of \spad{T}.
++ Let \spad{K} be an algebraic closure of \spad{k}.
++ Assume that \spad{V} is finite with cardinality
++ \spad{n} and let \spad{A} be the affine space \spad{K^n}.
++ For a regular triangular set \spad{T} let denote by \spad{W(T)} the
++ set of regular zeros of \spad{T}.
++ A family \spad{[T1,...,Ts]} of regular triangular sets
++ is a split of Lazard of a given subset \spad{S} of \spad{A}
++ iff the union of the \spad{W(Ti)} contains \spad{S} and
++ is contained in the closure of \spad{S} (w.r.t. Zariski topology).
++ A family \spad{[T1,...,Ts]} of regular triangular sets
++ is a split of Lazard of a given triangular set \spad{T}
++ if it is a split of Lazard of \spad{W(T)}.
++ Note that if \spad{[T1,...,Ts]} is a split of Lazard of
++ \spad{T} then it is also a split of Kalkbrener of \spad{T}.
++ The converse is false.
++ This category provides operations related to both kinds of
++ splits, the former being related to ideals decomposition whereas
++ the latter deals with varieties decomposition.
++ See the example illustrating the RegularTriangularSet constructor for more
++ explanations about decompositions by means of regular triangular sets.

RegularTriangularSetCategory(R,E,V,P) : Category == SIG where
  R : GcdDomain
  E : OrderedAbelianMonoidSup
  V : OrderedSet
  P : RecursivePolynomialCategory(R,E,V)

  SIG ==> TriangularSetCategory(R,E,V,P) with
```

```
purelyAlgebraic? : (P,$) -> Boolean
  ++ \spad{purelyAlgebraic?(p,ts)} returns \spad{true} iff every
  ++ variable of \spad{p} is algebraic w.r.t. \spad{ts}.

purelyTranscendental? : (P,$) -> Boolean
  ++ \spad{purelyTranscendental?(p,ts)} returns \spad{true} iff every
  ++ variable of \spad{p} is not algebraic w.r.t. \spad{ts}

algebraicCoefficients? : (P,$) -> Boolean
  ++ \spad{algebraicCoefficients?(p,ts)} returns \spad{true} iff every
  ++ variable of \spad{p} which is not the main one of \spad{p}
  ++ is algebraic w.r.t. \spad{ts}.

purelyAlgebraic? : $ -> Boolean
  ++ \spad{purelyAlgebraic?(ts)} returns true iff for every algebraic
  ++ variable \spad{v} of \spad{ts} we have
  ++ \spad{algebraicCoefficients?(t_v,ts_v_-)} where \spad{ts_v}
  ++ is select from TriangularSetCategory(ts,v) and
  ++ \spad{ts_v_-} is
  ++ collectUnder from TriangularSetCategory(ts,v).

purelyAlgebraicLeadingMonomial? : (P, $) -> Boolean
  ++ \spad{purelyAlgebraicLeadingMonomial?(p,ts)} returns true iff
  ++ the main variable of any non-constant iterarted initial
  ++ of \spad{p} is algebraic w.r.t. \spad{ts}.

invertibleElseSplit? : (P,$) -> Union(Boolean,List $)
  ++ \spad{invertibleElseSplit?(p,ts)} returns \spad{true} (resp.
  ++ \spad{false}) if \spad{p} is invertible in the tower
  ++ associated with \spad{ts} or returns a split of Kalkbrener
  ++ of \spad{ts}.

invertible? : (P,$) -> List Record(val : Boolean, tower : $)
  ++ \spad{invertible?(p,ts)} returns \spad{lbwt} where \spad{lbwt.i}
  ++ is the result of \spad{invertibleElseSplit?(p,lbwt.i.tower)} and
  ++ the list of the \spad{(lqrwt.i).tower} is a split of Kalkbrener of
  ++ \spad{ts}.

invertible? : (P,$) -> Boolean
  ++ \spad{invertible?(p,ts)} returns true iff \spad{p} is invertible
  ++ in the tower associated with \spad{ts}.

invertibleSet : (P,$) -> List $
  ++ \spad{invertibleSet(p,ts)} returns a split of Kalkbrener of the
  ++ quotient ideal of the ideal \axiom{I}  by \spad{p} where \spad{I} is
  ++ the radical of saturated of \spad{ts}.

lastSubResultantElseSplit : (P, P, $) -> Union(P,List $)
  ++ \spad{lastSubResultantElseSplit(p1,p2,ts)} returns either
  ++ \spad{g} a quasi-monic gcd of \spad{p1} and \spad{p2} w.r.t.
  ++ the \spad{ts} or a split of Kalkbrener of \spad{ts}.
  ++ This assumes that \spad{p1} and \spad{p2} have the same maim
  ++ variable and that this variable is greater that any variable
  ++ occurring in \spad{ts}.
```

```
lastSubResultant : (P, P, $) -> List Record(val : P, tower : $)
  ++ \spad{lastSubResultant(p1,p2,ts)} returns \spad{lpwt} such that
  ++ \spad{lpwt.i.val} is a quasi-monic gcd of \spad{p1} and \spad{p2}
  ++ w.r.t. \spad{lpwt.i.tower}, for every \spad{i}, and such
  ++ that the list of the \spad{lpwt.i.tower} is a split of Kalkbrener of
  ++ \spad{ts}. Moreover, if \spad{p1} and \spad{p2} do not
  ++ have a non-trivial gcd w.r.t. \spad{lpwt.i.tower} then
  ++ \spad{lpwt.i.val} is the resultant of these polynomials w.r.t.
  ++ \spad{lpwt.i.tower}. This assumes that \spad{p1} and \spad{p2} have
  ++ the same main variable and that this variable is greater that any
  ++ variable occurring in \spad{ts}.

squareFreePart : (P,$) -> List Record(val : P, tower : $)
  ++ \spad{squareFreePart(p,ts)} returns \spad{lpwt} such that
  ++ \spad{lpwt.i.val} is a square-free polynomial
  ++ w.r.t. \spad{lpwt.i.tower}, this polynomial being associated with
  ++ \spad{p} modulo \spad{lpwt.i.tower}, for every \spad{i}. Moreover,
  ++ the list of the \spad{lpwt.i.tower} is a split
  ++ of Kalkbrener of \spad{ts}.
  ++ WARNING: This assumes that \spad{p} is a non-constant polynomial
  ++ such that if \spad{p} is added to \spad{ts}, then the resulting set
  ++ is a regular triangular set.

intersect : (P,$) -> List $
  ++ \spad{intersect(p,ts)} returns the same as
  ++ \spad{intersect([p],ts)}

intersect : (List P, $) -> List $
  ++ \spad{intersect(lp,ts)} returns \spad{lts} a split of Lazard
  ++ of the intersection of the affine variety associated
  ++ with \spad{lp} and the regular zero set of \spad{ts}.

intersect : (List P, List $) -> List $
  ++ \spad{intersect(lp,lts)} returns the same as
  ++ \spad{concat([intersect(lp,ts) for ts in lts])|}

intersect : (P, List $) -> List $
  ++ \spad{intersect(p,lts)} returns the same as
  ++ \spad{intersect([p],lts)}

augment : (P,$) -> List $
  ++ \spad{augment(p,ts)} assumes that \spad{p} is a non-constant
  ++ polynomial whose main variable is greater than any variable
  ++ of \spad{ts}. This operation assumes also that if \spad{p} is
  ++ added to \spad{ts} the resulting set, say \spad{ts+p}, is a
  ++ regular triangular set. Then it returns a split of Kalkbrener
  ++ of \spad{ts+p}. This may not be \spad{ts+p} itself, if for
  ++ instance \spad{ts+p} is required to be square-free.

augment : (P,List $) -> List $
  ++ \spad{augment(p,lts)} returns the same as
  ++ \spad{concat([augment(p,ts) for ts in lts])}
```

```
augment : (List P,$) -> List $
  ++ \spad{augment(lp,ts)} returns \spad{ts} if \spad{empty? lp},
  ++ \spad{augment(p,ts)} if \spad{lp = [p]}, otherwise
  ++ \spad{augment(first lp, augment(rest lp, ts))}

augment : (List P,List $) -> List $
  ++ \spad{augment(lp,lts)} returns the same as
  ++ \spad{concat([augment(lp,ts) for ts in lts])}

internalAugment : (P, $) -> $
  ++ \spad{internalAugment(p,ts)} assumes that \spad{augment(p,ts)}
  ++ returns a singleton and returns it.

internalAugment : (List P, $) -> $
  ++ \spad{internalAugment(lp,ts)} returns \spad{ts} if \spad{lp}
  ++ is empty otherwise returns
  ++ \spad{internalAugment(rest lp, internalAugment(first lp, ts))}

extend : (P,$) -> List $
  ++ \spad{extend(p,ts)} assumes that \spad{p} is a non-constant
  ++ polynomial whose main variable is greater than any variable
  ++ of \spad{ts}. Then it returns a split of Kalkbrener
  ++ of \spad{ts+p}. This may not be \spad{ts+p} itself, if for
  ++ instance \spad{ts+p} is not a regular triangular set.

extend : (P, List $)  -> List $
  ++ \spad{extend(p,lts)} returns the same as
  ++ \spad{concat([extend(p,ts) for ts in lts])|}

extend : (List P,$) -> List $
  ++ \spad{extend(lp,ts)} returns \spad{ts} if \spad{empty? lp}
  ++ \spad{extend(p,ts)} if \spad{lp = [p]} else
  ++ \spad{extend(first lp, extend(rest lp, ts))}

extend : (List P,List $) -> List $
  ++ \spad{extend(lp,lts)} returns the same as
  ++ \spad{concat([extend(lp,ts) for ts in lts])|}

zeroSetSplit : (List P, Boolean) -> List $
  ++ \spad{zeroSetSplit(lp,clos?)} returns \spad{lts} a split of
  ++ Kalkbrener of the radical ideal associated with \spad{lp}.
  ++ If \spad{clos?} is false, it is also a decomposition of the
  ++ variety associated with \spad{lp} into the regular zero set of the
  ++ \spad{ts} in \spad{lts} (or, in other words, a split of Lazard of
  ++ this variety). See the example illustrating the
  ++ \spadtype{RegularTriangularSet} constructor for more explanations
  ++ about decompositions by means of regular triangular sets.

add

 NNI ==> NonNegativeInteger
 INT ==> Integer
 LP ==> List P
 PWT ==> Record(val : P, tower : $)
```

```
LpWT ==> Record(val : (List P), tower : $)
Split ==> List $
pack ==> PolynomialSetUtilitiesPackage(R,E,V,P)

purelyAlgebraic?(p: P, ts: $): Boolean ==
  ground? p => true
  not algebraic?(mvar(p),ts) => false
  algebraicCoefficients?(p,ts)

purelyTranscendental?(p:P,ts:$): Boolean  ==
  empty? ts => true
  lv : List V := variables(p)$P
  while (not empty? lv) and (not algebraic?(first(lv),ts)) repeat _
    lv := rest lv
  empty? lv

purelyAlgebraicLeadingMonomial?(p: P, ts: $): Boolean  ==
  ground? p => true
  algebraic?(mvar(p),ts) and purelyAlgebraicLeadingMonomial?(init(p), ts)

algebraicCoefficients?(p:P,ts:$): Boolean  ==
  ground? p => true
  (not ground? init(p)) and not (algebraic?(mvar(init(p)),ts)) => false
  algebraicCoefficients?(init(p),ts) =>
    ground? tail(p) => true
    mvar(tail(p)) = mvar(p) =>
      algebraicCoefficients?(tail(p),ts)
    algebraic?(mvar(tail(p)),ts) =>
      algebraicCoefficients?(tail(p),ts)
    false
  false

if V has Finite
then

  purelyAlgebraic?(ts: $): Boolean ==
    empty? ts => true
    size()$V = #ts => true
    lp: LP := sort(infRittWu?,members(ts))
    i: NonNegativeInteger := size()$V
    for p in lp repeat
      v: V := mvar(p)
      (i = (lookup(v)$V)::NNI) =>
        i := subtractIfCan(i,1)::NNI
      univariate?(p)$pack =>
        i := subtractIfCan(i,1)::NNI
      not algebraicCoefficients?(p,collectUnder(ts,v)) =>
        return false
      i := subtractIfCan(i,1)::NNI
    true

else

  purelyAlgebraic?(ts: $): Boolean ==
```

```
      empty? ts => true
      v: V := mvar(ts)
      p: P := select(ts,v)::P
      ts := collectUnder(ts,v)
      empty? ts => univariate?(p)$pack
      not purelyAlgebraic?(ts) => false
      algebraicCoefficients?(p,ts)

  augment(p:P,lts:List $) ==
    toSave: Split := []
    while not empty? lts repeat
      ts := first lts
      lts := rest lts
      toSave := concat(augment(p,ts),toSave)
    toSave

  augment(lp:LP,ts:$) ==
    toSave: Split := [ts]
    empty? lp => toSave
    lp := sort(infRittWu?,lp)
    while not empty? lp repeat
      p := first lp
      lp := rest lp
      toSave := augment(p,toSave)
    toSave

  augment(lp:LP,lts:List $) ==
    empty? lp => lts
    toSave: Split := []
    while not empty? lts repeat
      ts := first lts
      lts := rest lts
      toSave := concat(augment(lp,ts),toSave)
    toSave

  extend(p:P,lts:List $) ==
    toSave : Split := []
    while not empty? lts repeat
      ts := first lts
      lts := rest lts
      toSave := concat(extend(p,ts),toSave)
    toSave

  extend(lp:LP,ts:$) ==
    toSave: Split := [ts]
    empty? lp => toSave
    lp := sort(infRittWu?,lp)
    while not empty? lp repeat
      p := first lp
      lp := rest lp
      toSave := extend(p,toSave)
    toSave

  extend(lp:LP,lts:List $) ==
```

```
    empty? lp => lts
    toSave: Split := []
    while not empty? lts repeat
      ts := first lts
      lts := rest lts
      toSave := concat(extend(lp,ts),toSave)
    toSave

  intersect(lp:LP,lts:List $): List $  ==
    -- A VERY GENERAL default algorithm
    (empty? lp) or (empty? lts) => lts
    lp := [primitivePart(p) for p in lp]
    lp := removeDuplicates lp
    lp := remove(zero?,lp)
    any?(ground?,lp) => []
    toSee: List LpWT := [[lp,ts]$LpWT for ts in lts]
    toSave: List $ := []
    lp: LP
    p: P
    ts: $
    lus: List $
    while (not empty? toSee) repeat
      lpwt := first toSee; toSee := rest toSee
      lp := lpwt.val; ts := lpwt.tower
      empty? lp => toSave := cons(ts, toSave)
      p := first lp;  lp := rest lp
      lus := intersect(p,ts)
      toSee := concat([[lp,us]$LpWT for us in lus], toSee)
    toSave

  intersect(lp: LP,ts: $): List $  ==
    intersect(lp,[ts])

  intersect(p: P,lts: List $): List $  ==
    intersect([p],lts)
```

———

— **COQ RSETCAT** —

```
(* category RSETCAT *)
(*

  NNI ==> NonNegativeInteger
  INT ==> Integer
  LP ==> List P
  PWT ==> Record(val : P, tower : $)
  LpWT ==> Record(val : (List P), tower : $)
  Split ==> List $
  pack ==> PolynomialSetUtilitiesPackage(R,E,V,P)

  purelyAlgebraic? : (P,%) -> Boolean
  purelyAlgebraic?(p: P, ts: $): Boolean ==
```

```
  ground? p => true
  not algebraic?(mvar(p),ts) => false
  algebraicCoefficients?(p,ts)

purelyTranscendental? : (P,%) -> Boolean
purelyTranscendental?(p:P,ts:$): Boolean  ==
  empty? ts => true
  lv : List V := variables(p)$P
  while (not empty? lv) and (not algebraic?(first(lv),ts)) repeat _
    lv := rest lv
  empty? lv

purelyAlgebraicLeadingMonomial? : (P,%) -> Boolean
purelyAlgebraicLeadingMonomial?(p: P, ts: $): Boolean  ==
  ground? p => true
  algebraic?(mvar(p),ts) and purelyAlgebraicLeadingMonomial?(init(p), ts)

algebraicCoefficients? : (P,%) -> Boolean
algebraicCoefficients?(p:P,ts:$): Boolean  ==
  ground? p => true
  (not ground? init(p)) and not (algebraic?(mvar(init(p)),ts)) => false
  algebraicCoefficients?(init(p),ts) =>
    ground? tail(p) => true
    mvar(tail(p)) = mvar(p) =>
      algebraicCoefficients?(tail(p),ts)
    algebraic?(mvar(tail(p)),ts) =>
      algebraicCoefficients?(tail(p),ts)
    false
  false

if V has Finite
then

 purelyAlgebraic? : % -> Boolean
 purelyAlgebraic?(ts: $): Boolean ==
    empty? ts => true
    size()$V = #ts => true
    lp: LP := sort(infRittWu?,members(ts))
    i: NonNegativeInteger := size()$V
    for p in lp repeat
      v: V := mvar(p)
      (i = (lookup(v)$V)::NNI) =>
        i := subtractIfCan(i,1)::NNI
      univariate?(p)$pack =>
        i := subtractIfCan(i,1)::NNI
      not algebraicCoefficients?(p,collectUnder(ts,v)) =>
        return false
      i := subtractIfCan(i,1)::NNI
    true

else

  purelyAlgebraic? : % -> Boolean
  purelyAlgebraic?(ts: $): Boolean ==
```

```
    empty? ts => true
    v: V := mvar(ts)
    p: P := select(ts,v)::P
    ts := collectUnder(ts,v)
    empty? ts => univariate?(p)$pack
    not purelyAlgebraic?(ts) => false
    algebraicCoefficients?(p,ts)

augment : (P,List(%)) -> List(%)
augment(p:P,lts:List $) ==
  toSave: Split := []
  while not empty? lts repeat
    ts := first lts
    lts := rest lts
    toSave := concat(augment(p,ts),toSave)
  toSave

augment : (P,%) -> List(%)
augment(lp:LP,ts:$) ==
  toSave: Split := [ts]
  empty? lp => toSave
  lp := sort(infRittWu?,lp)
  while not empty? lp repeat
    p := first lp
    lp := rest lp
    toSave := augment(p,toSave)
  toSave

augment : (List(P),List(%)) -> List(%)
augment(lp:LP,lts:List $) ==
  empty? lp => lts
  toSave: Split := []
  while not empty? lts repeat
    ts := first lts
    lts := rest lts
    toSave := concat(augment(lp,ts),toSave)
  toSave

extend : (P,List(%)) -> List(%)
extend(p:P,lts:List $) ==
  toSave : Split := []
  while not empty? lts repeat
    ts := first lts
    lts := rest lts
    toSave := concat(extend(p,ts),toSave)
  toSave

extend : (List(P),List(%)) -> List(%)
extend(lp:LP,ts:$) ==
  toSave: Split := [ts]
  empty? lp => toSave
  lp := sort(infRittWu?,lp)
  while not empty? lp repeat
    p := first lp
```

```
        lp := rest lp
        toSave := extend(p,toSave)
      toSave

    extend : (List(P),%) -> List(%)
    extend(lp:LP,lts:List $) ==
      empty? lp => lts
      toSave: Split := []
      while not empty? lts repeat
        ts := first lts
        lts := rest lts
        toSave := concat(extend(lp,ts),toSave)
      toSave

    intersect : (List(P),List(%)) -> List(%)
    intersect(lp:LP,lts:List $): List $  ==
      -- A VERY GENERAL default algorithm
      (empty? lp) or (empty? lts) => lts
      lp := [primitivePart(p) for p in lp]
      lp := removeDuplicates lp
      lp := remove(zero?,lp)
      any?(ground?,lp) => []
      toSee: List LpWT := [[lp,ts]$LpWT for ts in lts]
      toSave: List $ := []
      lp: LP
      p: P
      ts: $
      lus: List $
      while (not empty? toSee) repeat
        lpwt := first toSee; toSee := rest toSee
        lp := lpwt.val; ts := lpwt.tower
        empty? lp => toSave := cons(ts, toSave)
        p := first lp;  lp := rest lp
        lus := intersect(p,ts)
        toSee := concat([[lp,us]$LpWT for us in lus], toSee)
      toSave

    intersect : (List(P),%) -> List(%)
    intersect(lp: LP,ts: $): List $  ==
      intersect(lp,[ts])

    intersect : (P,%) -> List(%)
    intersect(p: P,lts: List $): List $  ==
      intersect([p],lts)
*)
```

———-

— RSETCAT.dotabb —

```
"RSETCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RSETCAT"];
"RSETCAT" -> "TSETCAT"
```

———-

— RSETCAT.dotfull —

```
"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RSETCAT"];
"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a
  ->
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
```

———-

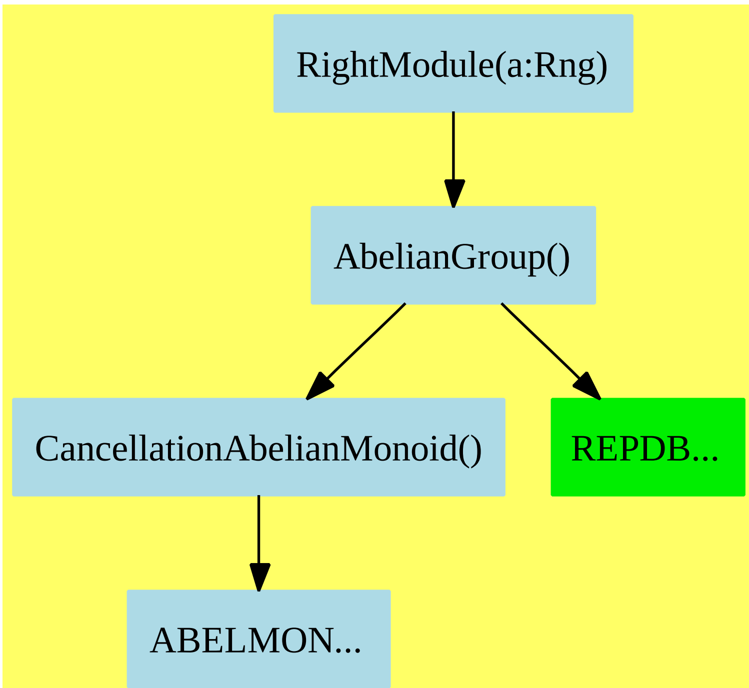
— RSETCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a
 [color=lightblue];
"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a
  ->
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"

"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 [color=seagreen];
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 ->
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b
 [color=lightblue];
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b
  -> "PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategor

"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b
 [color=seagreen];
"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b
  -> "PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"

"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 [color=lightblue];
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 -> "SETCAT..."
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 -> "CLAGG..."
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 -> "KOERCE..."

"SETCAT..." [color=lightblue];
"KOERCE..." [color=lightblue];
"CLAGG..." [color=lightblue];
```

```
}
```

———-

8.0.144 RightModule (RMODULE)



— **RightModule.input** —

```
)set break resume
)sys rm -f RightModule.output
)spool RightModule.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RightModule
--R
--R RightModule(R: Rng) is a category constructor
--R Abbreviation for RightModule is RMODULE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for RMODULE
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                      ?*? : (Integer,%) -> %
```

```
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 0 : () -> %                           coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

— RightModule.help —

```
====================================================================
RightModule examples
====================================================================

The category of right modules over an rng (ring not necessarily with unit).
This is an abelian group which supports right multiplication by elements
of the rng.

Axioms:
           x*(a*b) = (x*a)*b
           x*(a+b) = (x*a)+(x*b)
           (x+y)*x = (x*a)+(y*a)

See Also:
o )show RightModule
```

———-

See:

⇒ "BiModule" (BMODULE) 9.0.146 on page 902
⇐ "AbelianGroup" (ABELGRP) 7.0.121 on page 673

Exports:

0	coerce	hash	latex	sample
subtractIfCan	zero?	?~=?	?*?	?+?
?-?	-?	?=?		

These are directly exported but not implemented:

```
?*? : (%,R) -> %
```

These exports come from (p673) AbelianGroup():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
```

```
 latex : % -> String
 sample : () -> %
 subtractIfCan : (%,%) -> Union(%,"failed")
 zero? : % -> Boolean
 ?~=? : (%,%) -> Boolean
 ?*? : (PositiveInteger,%) -> %
 ?+? : (%,%) -> %
 ?=? : (%,%) -> Boolean
 ?*? : (NonNegativeInteger,%) -> %
 ?*? : (Integer,%) -> %
 ?-? : (%,%) -> %
 -? : % -> %
```

— RightModule.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#RMODULE">
RightModule (RMODULE)</a></h2>
</body>
```

———-

— category RMODULE RightModule —

```
)abbrev category RMODULE RightModule
++ Description:
++ The category of right modules over an rng (ring not necessarily
++ with unit). This is an abelian group which supports right
++ multiplication by elements of the rng.
++
++ Axioms\br
++ \tab{5}\spad{x*(a*b) = (x*a)*b}\br
++ \tab{5}\spad{x*(a+b) = (x*a)+(x*b)}\br
++ \tab{5}\spad{(x+y)*x = (x*a)+(y*a)}

RightModule(R) : Category == SIG where
  R : Rng

  SIG ==> AbelianGroup with

    "*" : (%,R) -> %
      ++ x*r returns the right multiplication of the module element x
      ++ by the ring element r.
```

———-

— COQ RMODULE —

```
(* category RMODULE *)
(*
Axioms
   x*(a*b) = (x*a)*b
```

```
   x*(a+b) = (x*a)+(x*b)
   (x+y)*x = (x*a)+(y*a)
*)
```

———-

— **RMODULE.dotabb** —

```
"RMODULE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RMODULE"];
"RMODULE" -> "ABELGRP"
```

———-

— **RMODULE.dotfull** —

```
"RightModule(a:Rng)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RMODULE"];
"RightModule(a:Rng)" -> "AbelianGroup()"

"RightModule(a:Ring)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RMODULE"];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"
```

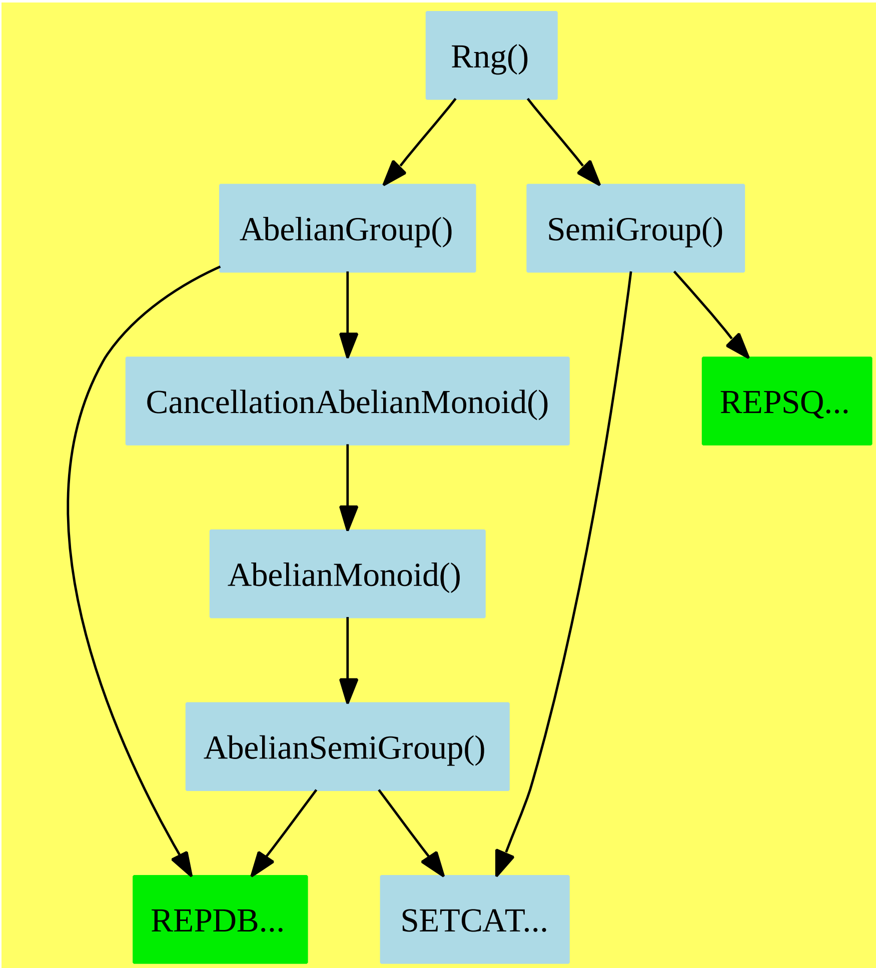
———-

— **RMODULE.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "REPDB..."

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "ABELMON..."

"ABELMON..." [color=lightblue];
"REPDB..." [color="#00EE00"];
}
```

———-

8.0.145 Rng (RNG)



Rng is a Ring that does not necessarily have a unit.

— **Rng.input** —

```
)set break resume
)sys rm -f Rng.output
)spool Rng.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Rng
--R
--R Rng is a category constructor
--R Abbreviation for Rng is RNG
--R This constructor is exposed in this frame.
```

```
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for RNG
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?=? : (%,%) -> Boolean                0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— **Rng.help** —

```
====================================================================
Rng examples
====================================================================

The category of associative rings, not necessarily commutative, and not
necessarily with a 1. This is a combination of an abelian group
and a semigroup, with multiplication distributing over addition.

Axioms:
          x*(y+z) = x*y + x*z
          (x+y)*z = x*z + y*z

Conditional attributes:
          noZeroDivisors    ab = 0 => a=0 or b=0

See Also:
o )show Rng
```

See:

⇒ “Ring” (RING) 9.0.153 on page 946
⇐ “AbelianGroup” (ABELGRP) 7.0.121 on page 673
⇐ “SemiGroup” (SGROUP) 4.0.85 on page 353

Exports:

0	coerce	hash	latex	sample
zero?	subtractIfCan	?*?	?**?	?+?
?-?	-?	?=?	?~=?	?^?

These exports come from (p673) AbelianGroup():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?-? : (%,%) -> %
-? : % -> %
```

These exports come from (p353) SemiGroup():

```
?*? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
```

— Rng.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#RNG">
Rng (RNG)</a></h2>
</body>
```

— category RNG Rng —

```
)abbrev category RNG Rng
++ Description:
++ The category of associative rings, not necessarily commutative, and not
++ necessarily with a 1. This is a combination of an abelian group
++ and a semigroup, with multiplication distributing over addition.
++
++ Axioms\br
++ \tab{5}\spad{x*(y+z) = x*y + x*z}\br
++ \tab{5}\spad{(x+y)*z = x*z + y*z}
++
++ Conditional attributes\br
++ \tab{5}noZeroDivisors\tab{5}\spad{ab = 0 => a=0 or b=0}

Rng() : Category == SIG where

  SIG ==> Join(AbelianGroup,SemiGroup)
```

———-

— COQ RNG —

```
(* category RNG *)
(*
Axioms
  x*(y+z) = x*y + x*z
  (x+y)*z = x*z + y*z

Conditional attributes
  noZeroDivisors ab = 0 => a=0 or b=0
*)
```

———-

— RNG.dotabb —

```
"RNG" [color=lightblue,href="bookvol10.2.pdf#nameddest=RNG"];
"RNG" -> "ABELGRP"
"RNG" -> "SGROUP"
```

———-

— RNG.dotfull —

```
"Rng()" [color=lightblue,href="bookvol10.2.pdf#nameddest=RNG"];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"
```

———-

— RNG.dotpic —

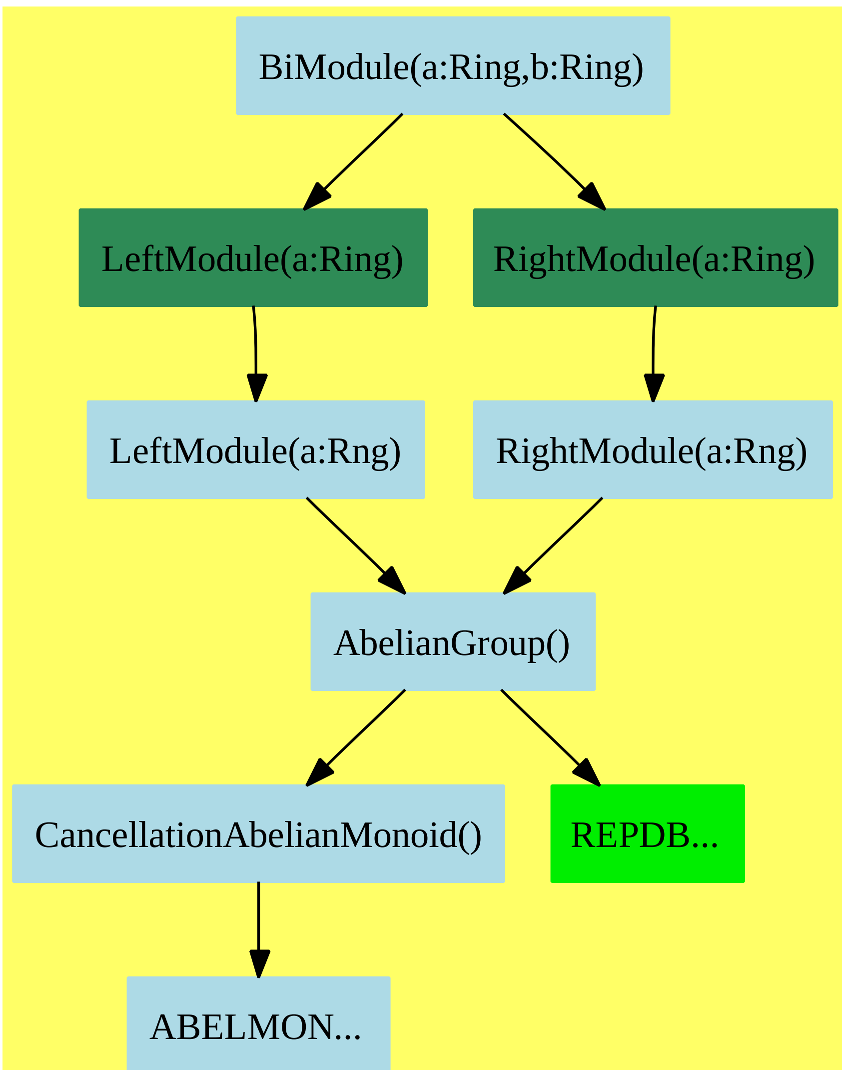
```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "REPDB..."

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"
```

```
"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SETCAT..."
"AbelianSemiGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
}
```

Chapter 9

Category Layer 8

9.0.146 BiModule (BMODULE)



— BiModule.input —

```
)set break resume
)sys rm -f BiModule.output
)spool BiModule.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BiModule
--R
--R BiModule(R: Ring,S: Ring) is a category constructor
--R Abbreviation for BiModule is BMODULE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for BMODULE
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,S) -> %                      ?*? : (R,%) -> %
--R ?*? : (Integer,%) -> %                ?*? : (NonNegativeInteger,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?=? : (%,%) -> Boolean                0 : () -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— BiModule.help —

```
====================================================================
BiModule examples
====================================================================

A BiModule is both a left and right module with respect to potentially
different rings.

Axiom:
        r*(x*s) = (r*x)*s

See Also:
o )show BiModule
```

See:

⇒ "AbelianMonoidRing" (AMR) 13.0.194 on page 1287
⇒ "CommutativeRing" (COMRING) 10.0.161 on page 1019
⇒ "DirectProductCategory" (DIRPCAT) 12.0.184 on page 1173
⇒ "EntireRing" (ENTIRER) 10.0.163 on page 1029
⇒ "FreeModuleCat" (FMCAT) 10.0.164 on page 1034
⇒ "Module" (MODULE) 10.0.167 on page 1050
⇒ "MonogenicLinearOperator" (MLO) 12.0.189 on page 1232
⇒ "RectangularMatrixCategory" (RMATCAT) 10.0.171 on page 1075
⇒ "SquareMatrixCategory" (SMATCAT) 12.0.192 on page 1266
⇒ "UnivariateSkewPolynomialCategory" (OREPCAT) 10.0.174 on page 1101
⇒ "XAlgebra" (XALG) 10.0.175 on page 1116
⇐ "LeftModule" (LMODULE) 8.0.137 on page 823
⇐ "RightModule" (RMODULE) 8.0.144 on page 892

Exports:

0	coerce	hash	latex	sample
subtractIfCan	zero?	?~=?	?*?	?+?
?-?	-?	?=?		

Attributes Exported:

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These exports come from (p823) LeftModule(R:Ring):

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (R,%) -> %
?=? : (%,%) -> Boolean
?+? : (%,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Integer,%) -> %
?-? : (%,%) -> %
-? : % -> %
```

These exports come from (p892) RightModule(S:Ring):

```
?*? : (%,S) -> %
```

— BiModule.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#BMODULE">
BiModule (BMODULE)</a></h2>
</body>
```

— category BMODULE BiModule —

```
)abbrev category BMODULE BiModule
++ Description:
++ A \spadtype{BiModule} is both a left and right module with respect
++ to potentially different rings.
++
++ Axiom\br
++ \tab{5}\spad{r*(x*s) = (r*x)*s}

BiModule(R,S) : Category == SIG where
  R : Ring
  S : Ring

  SIG ==> Join(LeftModule(R),RightModule(S)) with

    leftUnitary
      ++ \spad{1 * x = x}

    rightUnitary
      ++ \spad{x * 1 = x}
```

— COQ BMODULE —

```
(* category BMODULE *)
(*
Axiom
  r*(x*s) = (r*x)*s
leftUnitary   1 * x = x
rightUnitary  x * 1 = x
*)
```

— BMODULE.dotabb —

```
"BMODULE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BMODULE"];
"BMODULE" -> "LMODULE"
"BMODULE" -> "RMODULE"
```

— BMODULE.dotfull —

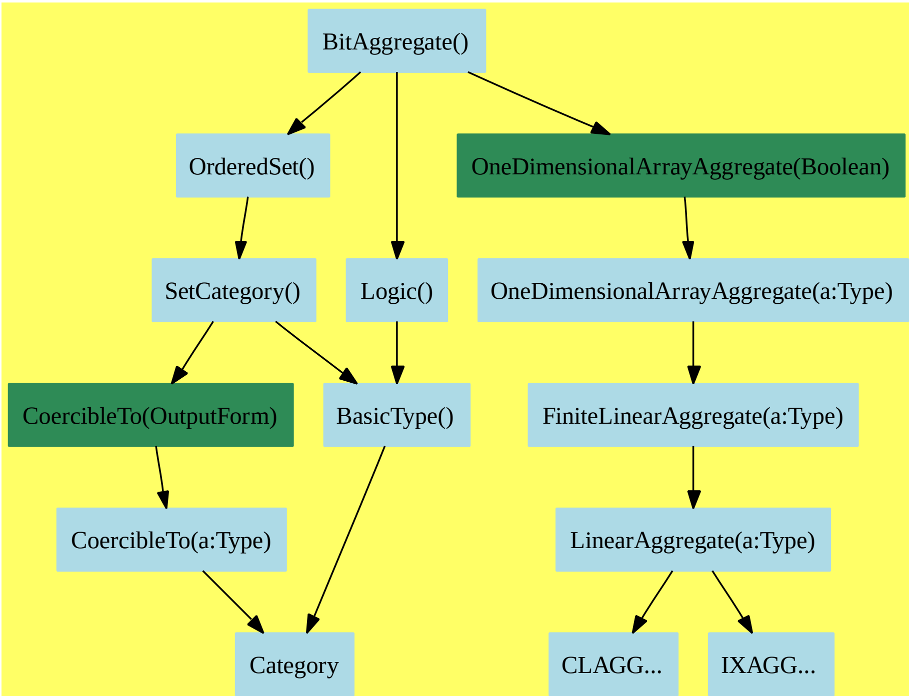
```
"BiModule(a:Ring,b:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BMODULE"];
```

```
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"BiModule(a:CommutativeRing,b:CommutativeRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=BMODULE"];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:OrderedAbelianMonoid)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=BMODULE"];
"BiModule(a:Ring,b:OrderedAbelianMonoid)" -> "BiModule(a:Ring,b:Ring)"
```

— **BMODULE.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "AbelianGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "REPDB..."

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "ABELMON..."

"ABELMON..." [color=lightblue];
"REPDB..." [color="#00EE00"];
}
```

9.0.147 BitAggregate (BTAGG)



— BitAggregate.input —

```
)set break resume
)sys rm -f BitAggregate.output
)spool BitAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show BitAggregate
--R
--R BitAggregate is a category constructor
--R Abbreviation for BitAggregate is BTAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for BTAGG
--R
--R------------------------------- Operations --------------------------------
--R ?/\? : (%,%) -> %                    ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean               ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean                ?>=? : (%,%) -> Boolean
--R ?\/? : (%,%) -> %                     ^? : % -> %
--R ?and? : (%,%) -> %                    coerce : % -> OutputForm
--R concat : (%,Boolean) -> %             concat : (Boolean,%) -> %
--R concat : (%,%) -> %                   concat : List(%) -> %
--R construct : List(Boolean) -> %        copy : % -> %
```

```
--R delete : (%,Integer) -> %             ?.? : (%,Integer) -> Boolean
--R empty : () -> %                       empty? : % -> Boolean
--R entries : % -> List(Boolean)          eq? : (%,%) -> Boolean
--R hash : % -> SingleInteger             index? : (Integer,%) -> Boolean
--R indices : % -> List(Integer)          insert : (Boolean,%,Integer) -> %
--R insert : (%,%,Integer) -> %           latex : % -> String
--R max : (%,%) -> %                      min : (%,%) -> %
--R nand : (%,%) -> %                     nor : (%,%) -> %
--R not? : % -> %                         ?or? : (%,%) -> %
--R qelt : (%,Integer) -> Boolean         reverse : % -> %
--R sample : () -> %                      xor : (%,%) -> %
--R ~? : % -> %                           ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R any? : ((Boolean -> Boolean),%) -> Boolean if $ has finiteAggregate
--R convert : % -> InputForm if Boolean has KONVERT(INFORM)
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : ((Boolean -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (Boolean,%) -> NonNegativeInteger if Boolean has SETCAT and $ has finiteAggregate
--R delete : (%,UniversalSegment(Integer)) -> %
--R elt : (%,Integer,Boolean) -> Boolean
--R ?.? : (%,UniversalSegment(Integer)) -> %
--R entry? : (Boolean,%) -> Boolean if $ has finiteAggregate and Boolean has SETCAT
--R eval : (%,List(Equation(Boolean))) -> % if Boolean has EVALAB(BOOLEAN) and Boolean has SETCAT
--R eval : (%,Equation(Boolean)) -> % if Boolean has EVALAB(BOOLEAN) and Boolean has SETCAT
--R eval : (%,Boolean,Boolean) -> % if Boolean has EVALAB(BOOLEAN) and Boolean has SETCAT
--R eval : (%,List(Boolean),List(Boolean)) -> % if Boolean has EVALAB(BOOLEAN) and Boolean has SETCAT
--R every? : ((Boolean -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,Boolean) -> % if $ has shallowlyMutable
--R find : ((Boolean -> Boolean),%) -> Union(Boolean,"failed")
--R first : % -> Boolean if Integer has ORDSET
--R less? : (%,NonNegativeInteger) -> Boolean
--R map : ((Boolean -> Boolean),%) -> %
--R map : (((Boolean,Boolean) -> Boolean),%,%) -> %
--R map! : ((Boolean -> Boolean),%) -> % if $ has shallowlyMutable
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (Boolean,%) -> Boolean if Boolean has SETCAT and $ has finiteAggregate
--R members : % -> List(Boolean) if $ has finiteAggregate
--R merge : (((Boolean,Boolean) -> Boolean),%,%) -> %
--R merge : (%,%) -> % if Boolean has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,Boolean) -> %
--R parts : % -> List(Boolean) if $ has finiteAggregate
--R position : ((Boolean -> Boolean),%) -> Integer
--R position : (Boolean,%) -> Integer if Boolean has SETCAT
--R position : (Boolean,%,Integer) -> Integer if Boolean has SETCAT
--R qsetelt! : (%,Integer,Boolean) -> Boolean if $ has shallowlyMutable
--R reduce : (((Boolean,Boolean) -> Boolean),%,Boolean,Boolean) -> Boolean if Boolean has SETCAT and $ has finit
--R reduce : (((Boolean,Boolean) -> Boolean),%,Boolean) -> Boolean if $ has finiteAggregate
--R reduce : (((Boolean,Boolean) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R remove : (Boolean,%) -> % if Boolean has SETCAT and $ has finiteAggregate
--R remove : ((Boolean -> Boolean),%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if Boolean has SETCAT and $ has finiteAggregate
--R reverse! : % -> % if $ has shallowlyMutable
```

```
--R select : ((Boolean -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,Integer,Boolean) -> Boolean if $ has shallowlyMutable
--R setelt : (%,UniversalSegment(Integer),Boolean) -> Boolean if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : (((Boolean,Boolean) -> Boolean),%) -> %
--R sort : % -> % if Boolean has ORDSET
--R sort! : (((Boolean,Boolean) -> Boolean),%) -> % if $ has shallowlyMutable
--R sort! : % -> % if Boolean has ORDSET and $ has shallowlyMutable
--R sorted? : (((Boolean,Boolean) -> Boolean),%) -> Boolean
--R sorted? : % -> Boolean if Boolean has ORDSET
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R
--E 1

)spool
)lisp (bye)
```

— **BitAggregate.help** —

```
====================================================================
BitAggregate examples
====================================================================

The bit aggregate category models aggregates representing large
quantities of Boolean data.

See Also:
o )show BitAggregate
```

See:

⇐ "Logic" (LOGIC) 3.0.52 on page 168
⇐ "OneDimensionalArrayAggregate" (A1AGG) 8.0.141 on page 854
⇐ "OrderedSet" (ORDSET) 4.0.80 on page 326

Exports:

any?	coerce	concat	construct	convert
copy	copyInto!	count	delete	elt
empty	empty?	entry?	entries	eq?
eval	every?	fill!	find	first
hash	index?	indices	insert	latex
less?	map	map!	max	maxIndex
member?	members	merge	min	minIndex
more?	nand	new	nor	not?
parts	position	qelt	qsetelt!	reduce
remove	removeDuplicates	reverse	reverse!	sample
select	setelt	size?	sort	sort!
sorted?	swap!	xor	#?	?/\?
?<?	?<=?	?=?	?>?	?>=?
?\/?	^?	?and?	?.?	?or?
~?	?~=?			

Attributes exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These are directly exported but not implemented:

```
?and? : (%,%) -> %
?or? : (%,%) -> %
xor : (%,%) -> %
```

These are implemented by this category:

```
not? : % -> %
^? : % -> %
~? : % -> %
?/\? : (%,%) -> %
?\/? : (%,%) -> %
nand : (%,%) -> %
nor : (%,%) -> %
```

These exports come from (p326) OrderedSet():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?<? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

TPDHERE: Note that none of the exports of Logic are needed. Perhaps this can be eliminated.

These exports come from (p854) OneDimensionalArrayAggregate(Boolean):

```
any? : ((Boolean -> Boolean),%) -> Boolean
    if $ has finiteAggregate
concat : (%,%) -> %
concat : List % -> %
concat : (%,Boolean) -> %
concat : (Boolean,%) -> %
construct : List Boolean -> %
convert : % -> InputForm
    if Boolean has KONVERT INFORM
copy : % -> %
copyInto! : (%,%,Integer) -> %
    if $ has shallowlyMutable
count : (Boolean,%) -> NonNegativeInteger
    if Boolean has SETCAT and $ has finiteAggregate
count : ((Boolean -> Boolean),%) -> NonNegativeInteger
    if $ has finiteAggregate
delete : (%,UniversalSegment Integer) -> %
delete : (%,Integer) -> %
elt : (%,Integer,Boolean) -> Boolean
empty : () -> %
empty? : % -> Boolean
entry? : (Boolean,%) -> Boolean
    if $ has finiteAggregate and Boolean has SETCAT
entries : % -> List Boolean
eq? : (%,%) -> Boolean
eval : (%,List Equation Boolean) -> %
    if Boolean has EVALAB BOOLEAN and Boolean has SETCAT
eval : (%,Equation Boolean) -> %
    if Boolean has EVALAB BOOLEAN and Boolean has SETCAT
eval : (%,Boolean,Boolean) -> %
    if Boolean has EVALAB BOOLEAN and Boolean has SETCAT
eval : (%,List Boolean,List Boolean) -> %
    if Boolean has EVALAB BOOLEAN and Boolean has SETCAT
every? : ((Boolean -> Boolean),%) -> Boolean
    if $ has finiteAggregate
fill! : (%,Boolean) -> %
    if $ has shallowlyMutable
find : ((Boolean -> Boolean),%) -> Union(Boolean,"failed")
first : % -> Boolean if Integer has ORDSET
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (Boolean,%,Integer) -> %
insert : (%,%,Integer) -> %
less? : (%,NonNegativeInteger) -> Boolean
map : ((Boolean -> Boolean),%) -> %
map : (((Boolean,Boolean) -> Boolean),%,%) -> %
map! : ((Boolean -> Boolean),%) -> %
    if $ has shallowlyMutable
maxIndex : % -> Integer if Integer has ORDSET
member? : (Boolean,%) -> Boolean
    if Boolean has SETCAT and $ has finiteAggregate
members : % -> List Boolean if $ has finiteAggregate
merge : (%,%) -> % if Boolean has ORDSET
```

```
 minIndex : % -> Integer if Integer has ORDSET
 more? : (%,NonNegativeInteger) -> Boolean
 new : (NonNegativeInteger,Boolean) -> %
 merge : (((Boolean,Boolean) -> Boolean),%,%) -> %
 parts : % -> List Boolean if $ has finiteAggregate
 position : ((Boolean -> Boolean),%) -> Integer
 position : (Boolean,%,Integer) -> Integer
     if Boolean has SETCAT
 position : (Boolean,%) -> Integer
     if Boolean has SETCAT
 qelt : (%,Integer) -> Boolean
 qsetelt! : (%,Integer,Boolean) -> Boolean
     if $ has shallowlyMutable
 reverse : % -> %
 reduce : (((Boolean,Boolean) -> Boolean),%) -> Boolean
     if $ has finiteAggregate
 reduce : (((Boolean,Boolean) -> Boolean),%,Boolean) -> Boolean
     if $ has finiteAggregate
 reduce :
  (((Boolean,Boolean) -> Boolean),%,Boolean,Boolean) -> Boolean
     if Boolean has SETCAT and $ has finiteAggregate
 remove : (Boolean,%) -> %
     if Boolean has SETCAT and $ has finiteAggregate
 remove : ((Boolean -> Boolean),%) -> %
     if $ has finiteAggregate
 removeDuplicates : % -> %
     if Boolean has SETCAT and $ has finiteAggregate
 reverse! : % -> % if $ has shallowlyMutable
 sample : () -> %
 setelt : (%,UniversalSegment Integer,Boolean) -> Boolean
     if $ has shallowlyMutable
 select : ((Boolean -> Boolean),%) -> %
     if $ has finiteAggregate
 setelt : (%,Integer,Boolean) -> Boolean
     if $ has shallowlyMutable
 size? : (%,NonNegativeInteger) -> Boolean
 sort : (((Boolean,Boolean) -> Boolean),%) -> %
 sort : % -> % if Boolean has ORDSET
 sort! : % -> %
     if Boolean has ORDSET and $ has shallowlyMutable
 sort! : (((Boolean,Boolean) -> Boolean),%) -> %
     if $ has shallowlyMutable
 sorted? : % -> Boolean if Boolean has ORDSET
 sorted? : (((Boolean,Boolean) -> Boolean),%) -> Boolean
 swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
 ?.? : (%,UniversalSegment Integer) -> %
 #? : % -> NonNegativeInteger if $ has finiteAggregate
 ?.? : (%,Integer) -> Boolean
```

— BitAggregate.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#BTAGG">
```

```
BitAggregate (BTAGG)</a></h2>
</body>
```

———-

— category BTAGG BitAggregate —

```
)abbrev category BTAGG BitAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ The bit aggregate category models aggregates representing large
++ quantities of Boolean data.

BitAggregate() : Category == SIG where

  SIG ==> Join(OrderedSet, Logic, OneDimensionalArrayAggregate Boolean) with

    "not" : % -> %
      ++ not(b) returns the logical not of bit aggregate
      ++ \axiom{b}.

    "^"  : % -> %
      ++ ^ b returns the logical not of bit aggregate
      ++ \axiom{b}.

    nand : (%, %) -> %
      ++ nand(a,b) returns the logical nand of bit aggregates
      ++ \axiom{a} and \axiom{b}.

    nor : (%, %) -> %
      ++ nor(a,b) returns the logical nor of bit aggregates
      ++ \axiom{a} and \axiom{b}.

    _and : (%, %) -> %
      ++ a and b returns the logical and of bit aggregates
      ++ \axiom{a} and \axiom{b}.

    _or : (%, %) -> %
      ++ a or b returns the logical or of bit aggregates
      ++ \axiom{a} and \axiom{b}.

    xor : (%, %) -> %
      ++ xor(a,b) returns the logical exclusive-or of bit aggregates
      ++ \axiom{a} and \axiom{b}.

   add

     not v == map(_not, v)

     _^ v == map(_not, v)

     _~(v) == map(_~, v)
```

```
_/_\(v, u) == map(_/_\, v, u)

_\_/(v, u) == map(_\_/, v, u)

nand(v, u) == map(nand, v, u)

nor(v, u)  == map(nor, v, u)
```

———-

— COQ BTAGG —

```
(* category BTAGG *)
(*
   ~? : % -> %
   not v == map(_not, v)

   ^? : % -> %
   _^ v == map(_not, v)

   ~? : % -> %
   _~(v) == map(_~, v)

   ?/\? : (%,%) -> %
   _/_\(v, u) == map(_/_\, v, u)

   ?\/? : (%,%) -> %
   _\_/(v, u) == map(_\_/, v, u)

   nand : (%,%) -> %
   nand(v, u) == map(nand, v, u)

   nor : (%,%) -> %
   nor(v, u)  == map(nor, v, u)

*)
```

———-

— BTAGG.dotabb —

```
"BTAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=BTAGG"];
"BTAGG" -> "ORDSET"
"BTAGG" -> "LOGIC"
"BTAGG" -> "A1AGG"
```

———-

— BTAGG.dotfull —

```
"BitAggregate()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BTAGG"];
"BitAggregate()" -> "OrderedSet()"
"BitAggregate()" -> "Logic()"
"BitAggregate()" -> "OneDimensionalArrayAggregate(Boolean)"
```

———-

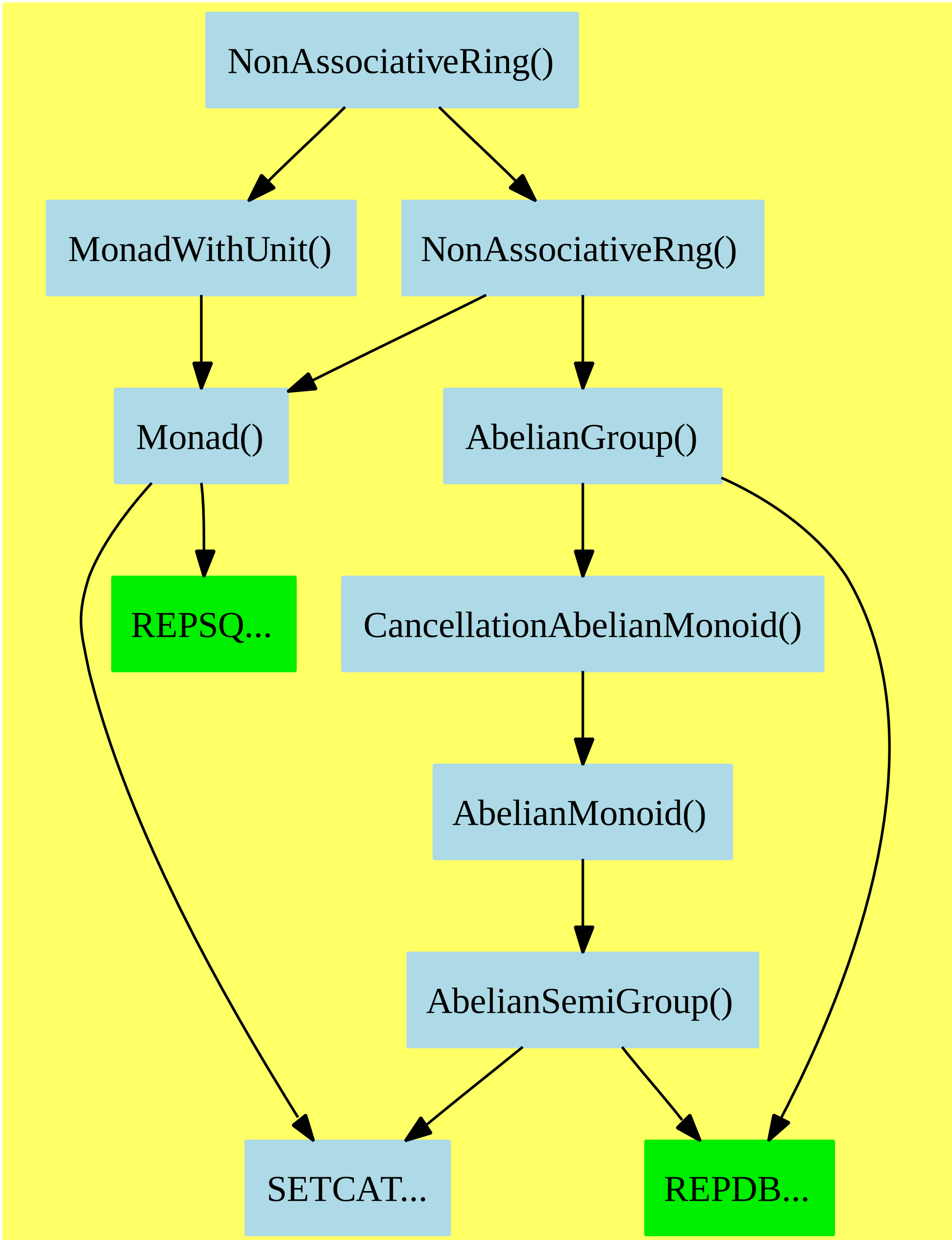
— BTAGG.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"BitAggregate()" [color=lightblue];
"BitAggregate()" -> "OrderedSet()"
"BitAggregate()" -> "Logic()"
"BitAggregate()" -> "OneDimensionalArrayAggregate(Boolean)"

"OneDimensionalArrayAggregate(Boolean)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=A1AGG"];
"OneDimensionalArrayAggregate(Boolean)" ->
    "OneDimensionalArrayAggregate(a:Type)"

"OneDimensionalArrayAggregate(a:Type)" [color=lightblue];
"OneDimensionalArrayAggregate(a:Type)" ->
    "FiniteLinearAggregate(a:Type)"

"FiniteLinearAggregate(a:Type)" [color=lightblue];
"FiniteLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IXAGG..."
"LinearAggregate(a:Type)" -> "CLAGG..."

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Logic()" [color=lightblue];
"Logic()" -> "BasicType()"

"BasicType()" [color=lightblue];
```

```
"BasicType()" -> "Category"

"Category" [color=lightblue];

"CLAGG..." [color=lightblue];
"IXAGG..." [color=lightblue];
}
```

———

9.0.148 NonAssociativeRing (NASRING)



— **NonAssociativeRing.input** —

```
)set break resume
)sys rm -f NonAssociativeRing.output
)spool NonAssociativeRing.output
)set message test on
```

```
)set message auto off
)clear all

--S 1 of 1
)show NonAssociativeRing
--R
--R NonAssociativeRing is a category constructor
--R Abbreviation for NonAssociativeRing is NASRING
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for NASRING
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R antiCommutator : (%,%) -> %           associator : (%,%,%) -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R commutator : (%,%) -> %               hash : % -> SingleInteger
--R latex : % -> String                   leftRecip : % -> Union(%,"failed")
--R one? : % -> Boolean                   recip : % -> Union(%,"failed")
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R leftPower : (%,NonNegativeInteger) -> %
--R leftPower : (%,PositiveInteger) -> %
--R rightPower : (%,NonNegativeInteger) -> %
--R rightPower : (%,PositiveInteger) -> %
--R rightRecip : % -> Union(%,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— **NonAssociativeRing.help** —

```
====================================================================
NonAssociativeRing examples
====================================================================

A NonAssociativeRing is a non associative rng which has a unit,
the multiplication is not necessarily commutative or associative.

See Also:
o )show NonAssociativeRing
```

See:

⇐ "MonadWithUnit" (MONADWU) 5.0.99 on page 449
⇐ "NonAssociativeRng" (NARNG) 8.0.140 on page 849

Exports:

0	1	antiCommutator	associator	characteristic
coerce	commutator	hash	latex	leftPower
leftRecip	one?	recip	rightPower	rightRecip
sample	subtractIfCan	zero?	?*?	?~=?
?**?	?+?	?-?	-?	?=?

These are directly exported but not implemented:

```
characteristic : () -> NonNegativeInteger
```

These are implemented by this category:

```
coerce : Integer -> %
```

These exports come from (p849) NonAssociativeRng():

```
0 : () -> %
antiCommutator : (%,%) -> %
associator : (%,%,%) -> %
coerce : % -> OutputForm
commutator : (%,%) -> %
hash : % -> SingleInteger
latex : % -> String
leftPower : (%,PositiveInteger) -> %
rightPower : (%,PositiveInteger) -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
```

These exports come from (p449) MonadWithUnit():

```
1 : () -> %
leftPower : (%,NonNegativeInteger) -> %
leftRecip : % -> Union(%,"failed")
one? : % -> Boolean
recip : % -> Union(%,"failed")
rightPower : (%,NonNegativeInteger) -> %
rightRecip : % -> Union(%,"failed")
?**? : (%,NonNegativeInteger) -> %
```

See: Schafer[Scha66]

— NonAssociativeRing.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#NASRING">
NonAssociativeRing (NASRING)</a></h2>
</body>
```

— category NASRING NonAssociativeRing —

```
)abbrev category NASRING NonAssociativeRing
++ Author: J. Grabmeier, R. Wisbauer
++ Date Created: 01 March 1991
++ Date Last Updated: 11 June 1991
++ Reference:
++ Scha66 An Introduction to Nonassociative Algebras
++ Description:
++ A NonAssociativeRing is a non associative rng which has a unit,
++ the multiplication is not necessarily commutative or associative.

NonAssociativeRing() : Category == SIG where

  SIG ==> Join(NonAssociativeRng,MonadWithUnit) with

    characteristic : -> NonNegativeInteger
      ++ characteristic() returns the characteristic of the ring.
      --we can not make this a constant, since some domains are mutable

    coerce : Integer -> %
      ++ coerce(n) coerces the integer n to an element of the ring.

   add

     n:Integer

     coerce(n) == n * 1$%
```

— COQ NASRING —

```
(* category NASRING *)
(*
      n:Integer
      coerce(n) == n * 1$%
*)
```

— NASRING.dotabb —

```
"NASRING"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NASRING"];
"NASRING" -> "MONADWU"
"NASRING" -> "NARNG"
```

——

— **NASRING.dotfull** —

```
"NonAssociativeRing()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NASRING"];
"NonAssociativeRing()" -> "NonAssociativeRng()"
"NonAssociativeRing()" -> "MonadWithUnit()"
```

——

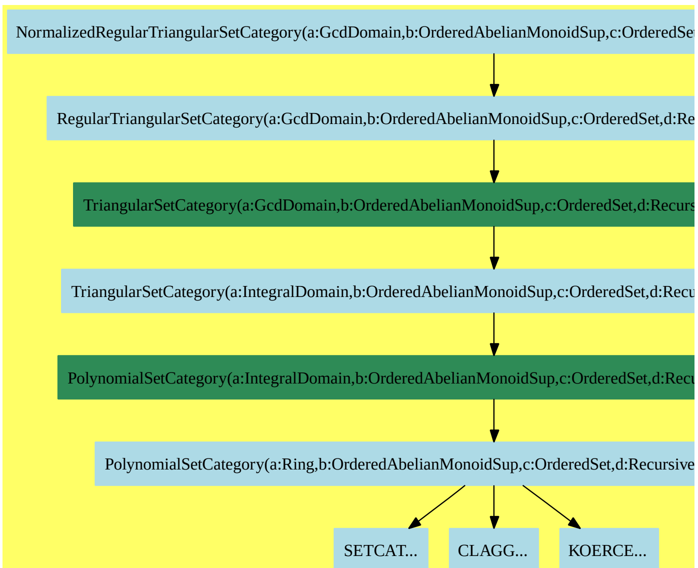
— **NASRING.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"NonAssociativeRing()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NASRING"];
"NonAssociativeRing()" -> "NonAssociativeRng()"
"NonAssociativeRing()" -> "MonadWithUnit()"

"MonadWithUnit()" [color=lightblue];
"MonadWithUnit()" -> "Monad()"

"NonAssociativeRng()" [color=lightblue];
"NonAssociativeRng()" -> "AbelianGroup()"
"NonAssociativeRng()" -> "Monad()"

"Monad()" [color=lightblue];
"Monad()" -> "SETCAT..."
"Monad()" -> "REPSQ..."

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "REPDB..."

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SETCAT..."
"AbelianSemiGroup()" -> "REPDB..."
```

```
"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
}
```

———-

9.0.149 NormalizedTriangularSetCategory (NTSCAT)



— **NormalizedTriangularSetCategory.input** —

```
)set break resume
)sys rm -f NormalizedTriangularSetCategory.output
)spool NormalizedTriangularSetCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NormalizedTriangularSetCategory
--R
--R NormalizedTriangularSetCategory(R: GcdDomain,E: OrderedAbelianMonoidSup,V: OrderedSet,P: RecursivePolynomial
--R Abbreviation for NormalizedTriangularSetCategory is NTSCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for NTSCAT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                algebraic? : (V,%) -> Boolean
--R algebraicVariables : % -> List(V)     augment : (List(P),%) -> List(%)
```

```
--R augment : (P,List(%)) -> List(%)      augment : (P,%) -> List(%)
--R coerce : % -> List(P)                 coerce : % -> OutputForm
--R collect : (%,V) -> %                  collectQuasiMonic : % -> %
--R collectUnder : (%,V) -> %             collectUpper : (%,V) -> %
--R construct : List(P) -> %              copy : % -> %
--R degree : % -> NonNegativeInteger      empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R extend : (List(P),%) -> List(%)       extend : (P,List(%)) -> List(%)
--R extend : (P,%) -> List(%)             extend : (%,P) -> %
--R first : % -> Union(P,"failed")        hash : % -> SingleInteger
--R headReduce : (P,%) -> P               headReduced? : % -> Boolean
--R headReduced? : (P,%) -> Boolean       infRittWu? : (%,%) -> Boolean
--R initiallyReduce : (P,%) -> P          initiallyReduced? : % -> Boolean
--R initials : % -> List(P)               internalAugment : (List(P),%) -> %
--R internalAugment : (P,%) -> %          intersect : (P,List(%)) -> List(%)
--R intersect : (List(P),%) -> List(%)    intersect : (P,%) -> List(%)
--R invertible? : (P,%) -> Boolean        invertibleSet : (P,%) -> List(%)
--R last : % -> Union(P,"failed")         latex : % -> String
--R mainVariable? : (V,%) -> Boolean      mainVariables : % -> List(V)
--R map : ((P -> P),%) -> %               mvar : % -> V
--R normalized? : % -> Boolean            normalized? : (P,%) -> Boolean
--R purelyAlgebraic? : % -> Boolean       purelyAlgebraic? : (P,%) -> Boolean
--R reduceByQuasiMonic : (P,%) -> P       removeZero : (P,%) -> P
--R rest : % -> Union(%,"failed")         retract : List(P) -> %
--R sample : () -> %                      select : (%,V) -> Union(P,"failed")
--R stronglyReduce : (P,%) -> P           stronglyReduced? : % -> Boolean
--R stronglyReduced? : (P,%) -> Boolean   trivialIdeal? : % -> Boolean
--R variables : % -> List(V)              zeroSetSplit : List(P) -> List(%)
--R ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R algebraicCoefficients? : (P,%) -> Boolean
--R any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R augment : (List(P),List(%)) -> List(%)
--R autoReduced? : (%,((P,List(P)) -> Boolean)) -> Boolean
--R basicSet : (List(P),(P -> Boolean),((P,P) -> Boolean)) -> Union(Record(bas: %,top: List(P)),"failed")
--R basicSet : (List(P),((P,P) -> Boolean)) -> Union(Record(bas: %,top: List(P)),"failed")
--R coHeight : % -> NonNegativeInteger if V has FINITE
--R convert : % -> InputForm if P has KONVERT(INFORM)
--R count : ((P -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (P,%) -> NonNegativeInteger if P has SETCAT and $ has finiteAggregate
--R eval : (%,List(Equation(P))) -> % if P has EVALAB(P) and P has SETCAT
--R eval : (%,Equation(P)) -> % if P has EVALAB(P) and P has SETCAT
--R eval : (%,P,P) -> % if P has EVALAB(P) and P has SETCAT
--R eval : (%,List(P),List(P)) -> % if P has EVALAB(P) and P has SETCAT
--R every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R extend : (List(P),List(%)) -> List(%)
--R extendIfCan : (%,P) -> Union(%,"failed")
--R find : ((P -> Boolean),%) -> Union(P,"failed")
--R headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
--R initiallyReduced? : (P,%) -> Boolean
--R intersect : (List(P),List(%)) -> List(%)
--R invertible? : (P,%) -> List(Record(val: Boolean,tower: %))
--R invertibleElseSplit? : (P,%) -> Union(Boolean,List(%))
--R lastSubResultant : (P,P,%) -> List(Record(val: P,tower: %))
```

```
--R lastSubResultantElseSplit : (P,P,%) -> Union(P,List(%))
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((P -> P),%) -> % if $ has shallowlyMutable
--R member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
--R members : % -> List(P) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(P) if $ has finiteAggregate
--R purelyAlgebraicLeadingMonomial? : (P,%) -> Boolean
--R purelyTranscendental? : (P,%) -> Boolean
--R quasiComponent : % -> Record(close: List(P),open: List(P))
--R reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
--R reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P,P) -> P if P has SETCAT and $ has finiteAggregate
--R reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
--R remainder : (P,%) -> Record(rnum: R,polnum: P,den: R) if R has INTDOM
--R remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
--R retractIfCan : List(P) -> Union(%,"failed")
--R rewriteIdealWithHeadRemainder : (List(P),%) -> List(P) if R has INTDOM
--R rewriteIdealWithRemainder : (List(P),%) -> List(P) if R has INTDOM
--R rewriteSetWithReduction : (List(P),%,((P,P) -> P),((P,P) -> Boolean)) -> List(P)
--R roughBase? : % -> Boolean if R has INTDOM
--R roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
--R roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
--R roughUnitIdeal? : % -> Boolean if R has INTDOM
--R select : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : (%,V) -> Record(under: %,floor: %,upper: %)
--R squareFreePart : (P,%) -> List(Record(val: P,tower: %))
--R triangular? : % -> Boolean if R has INTDOM
--R zeroSetSplit : (List(P),Boolean) -> List(%)
--R zeroSetSplitIntoTriangularSystems : List(P) -> List(Record(close: %,open: List(P)))
--R
--E 1

)spool
)lisp (bye)
```

— **NormalizedTriangularSetCategory.help** —

```
====================================================================
NormalizedTriangularSetCategory examples
====================================================================

The category of normalized triangular sets. A triangular set ts is said
normalized if for every algebraic variable v of ts the polynomial
select(ts,v) is normalized w.r.t. every polynomial in collectUnder(ts,v).

A polynomial p is said normalized w.r.t. a non-constant polynomial q
if p is constant or degree(p,mdeg(q)) = 0 and init(p) is normalized
```

```
w.r.t. q. One of the important features of normalized triangular sets
is that they are regular sets.

See Also:
o )show NormalizedTriangularSetCategory
```

———-

See:

⇒ “SquareFreeNormalizedTriangularSetCategory” (SNTSCAT) 10.0.172 on page 1085
⇐ “RegularTriangularSetCategory” (RSETCAT) 8.0.143 on page 873

Exports:

algebraic?	algebraicCoefficients?
algebraicVariables	any?
augment	autoReduced?
basicSet	coerce
coHeight	collect
collectQuasiMonic	collectUnder
collectUpper	construct
copy	convert
count	degree
empty	empty?
eq?	eval
every?	extend
extendIfCan	find
first	hash
headReduce	headReduced?
headRemainder	infRittWu?
initiallyReduce	initiallyReduced?
initials	internalAugment
intersect	invertible?
invertibleElseSplit?	invertibleSet
last	lastSubResultant
lastSubResultantElseSplit	latex
less?	mainVariable?
mainVariables	map
map!	member?
members	more?
mvar	normalized?
parts	purelyAlgebraic?
purelyAlgebraicLeadingMonomial?	purelyTranscendental?
quasiComponent	reduce
reduceByQuasiMonic	reduced?
remainder	remove
removeDuplicates	removeZero
rest	retract
retractIfCan	rewriteIdealWithHeadRemainder
rewriteIdealWithRemainder	rewriteSetWithReduction
roughBase?	roughEqualIdeals?
roughSubIdeal?	roughUnitIdeal?
sample	select
size?	sort
squareFreePart	stronglyReduce
stronglyReduced?	triangular?
trivialIdeal?	variables
zeroSetSplit	zeroSetSplitIntoTriangularSystems
#?	?=?
?~=?	

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These exports come from (p873) RegularTriangularSetCategory(R,E,V,P)
where R:GcdDomain, E:OrderedAbelianMonoidSup, V:OrderedSet,
P:RecursivePolynomialCategory(R,E,V)):

```
algebraic? : (V,%) -> Boolean
algebraicCoefficients? : (P,%) -> Boolean
algebraicVariables : % -> List V
any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
augment : (List P,List %) -> List %
augment : (List P,%) -> List %
augment : (P,List %) -> List %
augment : (P,%) -> List %
autoReduced? : (%,((P,List P) -> Boolean)) -> Boolean
basicSet :
 (List P,(P -> Boolean),((P,P) -> Boolean)) ->
   Union(Record(bas: %,top: List P),"failed")
basicSet :
 (List P,((P,P) -> Boolean)) ->
   Union(Record(bas: %,top: List P),"failed")
coerce : % -> List P
coerce : % -> OutputForm
coHeight : % -> NonNegativeInteger if V has FINITE
collect : (%,V) -> %
collectQuasiMonic : % -> %
collectUnder : (%,V) -> %
collectUpper : (%,V) -> %
construct : List P -> %
copy : % -> %
convert : % -> InputForm if P has KONVERT INFORM
count : ((P -> Boolean),%) -> NonNegativeInteger
  if $ has finiteAggregate
count : (P,%) -> NonNegativeInteger
  if P has SETCAT
  and $ has finiteAggregate
degree : % -> NonNegativeInteger
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
extend : (List P,List %) -> List %
extend : (List P,%) -> List %
extend : (P,List %) -> List %
extend : (P,%) -> List %
extend : (%,P) -> %
extendIfCan : (%,P) -> Union(%,"failed")
find : ((P -> Boolean),%) -> Union(P,"failed")
```

```
first : % -> Union(P,"failed")
hash : % -> SingleInteger
headReduce : (P,%) -> P
headReduced? : % -> Boolean
headReduced? : (P,%) -> Boolean
headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
infRittWu? : (%,%) -> Boolean
initiallyReduce : (P,%) -> P
initiallyReduced? : % -> Boolean
initiallyReduced? : (P,%) -> Boolean
initials : % -> List P
internalAugment : (P,%) -> %
internalAugment : (List P,%) -> %
intersect : (P,List %) -> List %
intersect : (List P,%) -> List %
intersect : (P,%) -> List %
intersect : (List P,List %) -> List %
invertible? : (P,%) -> Boolean
invertible? : (P,%) -> List Record(val: Boolean,tower: %)
invertibleElseSplit? : (P,%) -> Union(Boolean,List %)
invertibleSet : (P,%) -> List %
last : % -> Union(P,"failed")
lastSubResultant : (P,P,%) -> List Record(val: P,tower: %)
lastSubResultantElseSplit : (P,P,%) -> Union(P,List %)
latex : % -> String
less? : (%,NonNegativeInteger) -> Boolean
mainVariable? : (V,%) -> Boolean
mainVariables : % -> List V
map : ((P -> P),%) -> %
map! : ((P -> P),%) -> % if $ has shallowlyMutable
member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
members : % -> List P if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
mvar : % -> V
normalized? : % -> Boolean
normalized? : (P,%) -> Boolean
parts : % -> List P if $ has finiteAggregate
purelyAlgebraic? : (P,%) -> Boolean
purelyAlgebraic? : % -> Boolean
purelyAlgebraicLeadingMonomial? : (P,%) -> Boolean
purelyTranscendental? : (P,%) -> Boolean
quasiComponent : % -> Record(close: List P,open: List P)
reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P,P) -> P
  if P has SETCAT
  and $ has finiteAggregate
reduceByQuasiMonic : (P,%) -> P
reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
remainder : (P,%) -> Record(rnum: R,polnum: P,den: R)
  if R has INTDOM
remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
```

```
removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
removeZero : (P,%) -> P
rest : % -> Union(%,"failed")
retract : List P -> %
retractIfCan : List P -> Union(%,"failed")
rewriteIdealWithHeadRemainder : (List P,%) -> List P if R has INTDOM
rewriteIdealWithRemainder : (List P,%) -> List P if R has INTDOM
rewriteSetWithReduction :
   (List P,%,((P,P) -> P),((P,P) -> Boolean)) -> List P
roughBase? : % -> Boolean if R has INTDOM
roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
roughUnitIdeal? : % -> Boolean if R has INTDOM
sample : () -> %
select : (%,V) -> Union(P,"failed")
select : ((P -> Boolean),%) -> % if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
sort : (%,V) -> Record(under: %,floor: %,upper: %)
squareFreePart : (P,%) -> List Record(val: P,tower: %)
stronglyReduce : (P,%) -> P
stronglyReduced? : (P,%) -> Boolean
stronglyReduced? : % -> Boolean
triangular? : % -> Boolean if R has INTDOM
trivialIdeal? : % -> Boolean
variables : % -> List V
zeroSetSplit : List P -> List %
zeroSetSplit : (List P,Boolean) -> List %
zeroSetSplitIntoTriangularSystems :
  List P -> List Record(close: %,open: List P)
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

See: SALSA[SALSA], Kalkbrener[Kalk91, Kalk98], Aubry[Aubr96, Aubr99, Aubr99a], Lazard[Laza91], Moreno Maza[Maza95, Maza97, Maza98, Maza00]

— **NormalizedTriangularSetCategory.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#NTSCAT">
NormalizedTriangularSetCategory (NTSCAT)</a></h2>
</body>
```

————

— **category NTSCAT NormalizedTriangularSetCategory** —

```
)abbrev category NTSCAT NormalizedTriangularSetCategory
++ Author: Marc Moreno Maza
++ Date Created: 10/07/1998
++ Date Last Updated: 12/12/1998
++ References :
++ SALSA Solvers for Algebraic Systems and Applications
++ Kalk91 Three contributions to elimination theory
++ Kalk98 Algorithmic properties of polynomial rings
```

```
++ Aubr96 Triangular Sets for Solving Polynomial Systems:
++ Aubr99 On the Theories of Triangular Sets
++ Aubr99a Triangular Sets for Solving Polynomial Systems:
++ Laza91 A new method for solving algebraic systems of positive dimension
++ Maza95 Polynomial Gcd Computations over Towers of Algebraic Extensions
++ Maza97 Calculs de pgcd au-dessus des tours d'extensions simples et
++        resolution des systemes d'equations algebriques
++ Maza98 A new algorithm for computing triangular decomposition of
++        algebraic varieties
++ Maza00 On Triangular Decompositions of Algebraic Varieties
++ Description:
++ The category of normalized triangular sets. A triangular
++ set ts is said normalized if for every algebraic
++ variable v of ts the polynomial select(ts,v)
++ is normalized w.r.t. every polynomial in collectUnder(ts,v).
++ A polynomial p is said normalized w.r.t. a non-constant
++ polynomial q if p is constant or degree(p,mdeg(q)) = 0
++ and init(p) is normalized w.r.t. q. One of the important
++ features of normalized triangular sets is that they are regular sets.

NormalizedTriangularSetCategory(R,E,V,P) : Category == SIG where
  R : GcdDomain
  E : OrderedAbelianMonoidSup
  V : OrderedSet
  P : RecursivePolynomialCategory(R,E,V)

  SIG ==> RegularTriangularSetCategory(R,E,V,P)
```

— NTSCAT.dotabb —

```
"NTSCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NTSCAT"];
"NTSCAT" -> "RSETCAT"
```

— NTSCAT.dotfull —

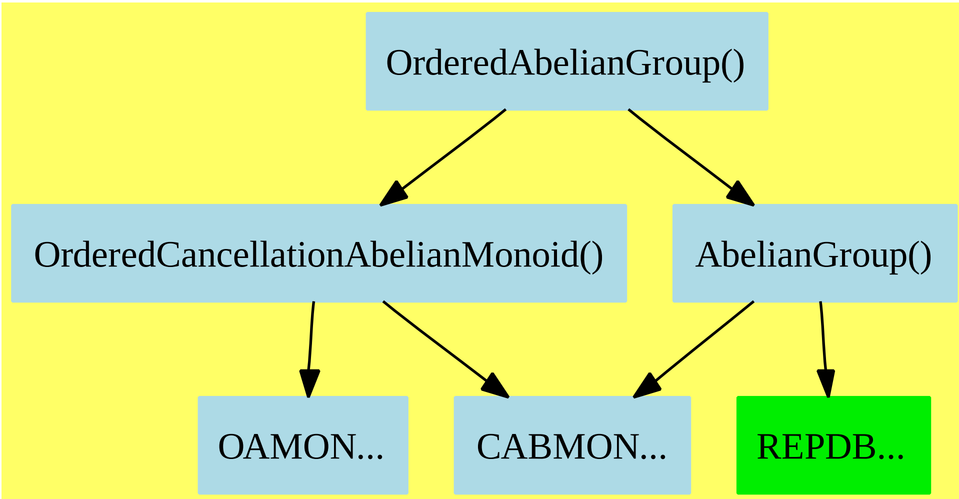
```
"NormalizedRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomial
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NTSCAT"];
"NormalizedRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomial
 ->
"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a
```

— NTSCAT.dotpic —

```
digraph pic {
 fontsize=10;
```

```
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"NormalizedRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomial
 [color=lightblue];
"NormalizedRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomial
 ->
"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a

"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a
 [color=lightblue];
"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a
  ->
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"

"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 [color=seagreen];
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 ->
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b
 [color=lightblue];
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b
  -> "PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategor

"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b
 [color=seagreen];
"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b
  -> "PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"

"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 [color=lightblue];
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 -> "SETCAT..."
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 -> "CLAGG..."
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 -> "KOERCE..."

"SETCAT..." [color=lightblue];
"KOERCE..." [color=lightblue];
"CLAGG..." [color=lightblue];
}
```

9.0.150 OrderedAbelianGroup (OAGROUP)



— **OrderedAbelianGroup.input** —

```
)set break resume
)sys rm -f OrderedAbelianGroup.output
)spool OrderedAbelianGroup.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderedAbelianGroup
--R
--R OrderedAbelianGroup is a category constructor
--R Abbreviation for OrderedAbelianGroup is OAGROUP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for OAGROUP
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Integer,%) -> %                ?*? : (NonNegativeInteger,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?+? : (%,%) -> %
--R -? : % -> %                           ?-? : (%,%) -> %
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               0 : () -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   max : (%,%) -> %
--R min : (%,%) -> %                      sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1
```

```
)spool
)lisp (bye)
```

— **OrderedAbelianGroup.help** —

```
====================================================================
OrderedAbelianGroup examples
====================================================================

Ordered sets which are also abelian groups, such that the
addition preserves the ordering.

See Also:
o )show OrderedAbelianGroup
```

See:

⇒ "OrderedRing" (ORDRING) 10.0.168 on page 1055
⇐ "AbelianGroup" (ABELGRP) 7.0.121 on page 673
⇐ "OrderedCancellationAbelianMonoid" (OCAMON) 8.0.142 on page 870

Exports:

0	coerce	hash	latex	max
min	sample	subtractIfCan	zero?	?~=?
?*?	?+?	-?	?-?	?<?
?<=?	?=?	?>?	?>=?	

These exports come from (p870) OrderedCancellationAbelianMonoid():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
```

These exports come from (p673) AbelianGroup():

```
-? : % -> %
```

```
 ?*? : (Integer,%) -> %
 ?-? : (%,%) -> %
```

— **OrderedAbelianGroup.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#OAGROUP">
OrderedAbelianGroup (OAGROUP)</a></h2>
</body>
```

———-

— **category OAGROUP OrderedAbelianGroup** —

```
)abbrev category OAGROUP OrderedAbelianGroup
++ Description:
++ Ordered sets which are also abelian groups, such that the
++ addition preserves the ordering.

OrderedAbelianGroup() : Category == SIG where

  SIG ==> Join(OrderedCancellationAbelianMonoid, AbelianGroup)
```

———-

— **OAGROUP.dotabb** —

```
"OAGROUP"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OAGROUP"];
"OAGROUP" -> "OCAMON"
"OAGROUP" -> "ABELGRP"
```

———-

— **OAGROUP.dotfull** —

```
"OrderedAbelianGroup()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OAGROUP"];
"OrderedAbelianGroup()" -> "OrderedCancellationAbelianMonoid()"
"OrderedAbelianGroup()" -> "AbelianGroup()"
```

———-

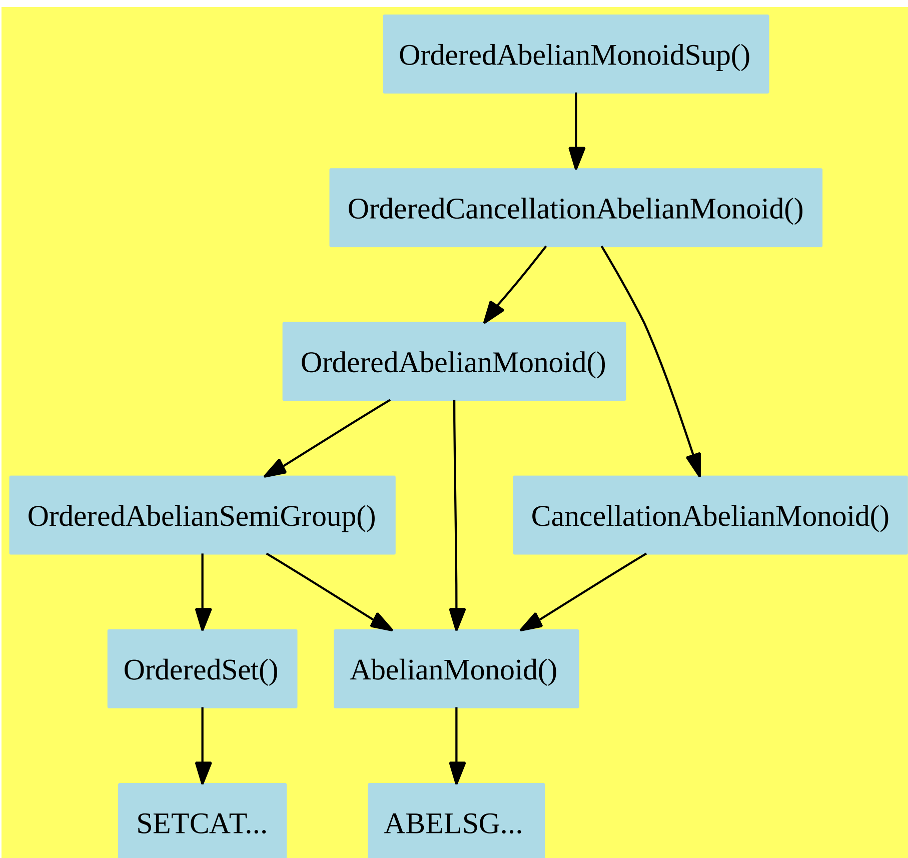
— **OAGROUP.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"OrderedAbelianGroup()" [color=lightblue];
```

```
"OrderedAbelianGroup()" -> "OrderedCancellationAbelianMonoid()"
"OrderedAbelianGroup()" -> "AbelianGroup()"

"OrderedCancellationAbelianMonoid()" [color=lightblue];
"OrderedCancellationAbelianMonoid()" -> "OAMON..."
"OrderedCancellationAbelianMonoid()" -> "CABMON..."

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"REPDB..." [color="#00EE00"];
"OAMON..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

——————

9.0.151 OrderedAbelianMonoidSup (OAMONS)



— OrderedAbelianMonoidSup.input —

```
)set break resume
)sys rm -f OrderedAbelianMonoidSup.output
)spool OrderedAbelianMonoidSup.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderedAbelianMonoidSup
--R
--R OrderedAbelianMonoidSup is a category constructor
--R Abbreviation for OrderedAbelianMonoidSup is OAMONS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for OAMONS
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                      ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean               ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean                ?>=? : (%,%) -> Boolean
--R 0 : () -> %                           coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R max : (%,%) -> %                      min : (%,%) -> %
--R sample : () -> %                      sup : (%,%) -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— OrderedAbelianMonoidSup.help —

```
====================================================================
OrderedAbelianMonoidSup examples
====================================================================

This domain is an OrderedAbelianMonoid with a sup operation added.
The purpose of the sup operator in this domain is to act as a
supremum with respect to the partial order imposed by '-', rather
than with respect to the total > order (since that is "max").

Axioms:
         sup(a,b)-a ~= "failed"
         sup(a,b)-b ~= "failed"
         x-a ~= "failed" and x-b ~= "failed" => x >= sup(a,b)

See Also:
o )show OrderedAbelianMonoidSup
```

———-

See:

⇐ "DirectProductCategory" (DIRPCAT) 12.0.184 on page 1173
⇐ "OrderedCancellationAbelianMonoid" (OCAMON) 8.0.142 on page 870

Exports:

0	coerce	hash	latex	max
min	sample	subtractIfCan	sup	zero?
?~=?	?*?	?<=?	?+?	?<?
?=?	?>?	?>=?		

These are directly exported but not implemented:

```
sup : (%,%) -> %
```

These exports come from (p870) OrderedCancellationAbelianMonoid():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

— OrderedAbelianMonoidSup.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#OAMONS">
OrderedAbelianMonoidSup (OAMONS)</a></h2>
</body>
```

———-

— category OAMONS OrderedAbelianMonoidSup —

```
)abbrev category OAMONS OrderedAbelianMonoidSup
++ Description:
++ This domain is an OrderedAbelianMonoid with a sup
++ operation added. The purpose of the sup operator
```

```
++ in this domain is to act as a supremum with respect to the
++ partial order imposed by ‘-‘, rather than with respect to
++ the total $>$ order (since that is "max").
++
++ Axioms\br
++ \tab{5}sup(a,b)-a \~~= "failed"\br
++ \tab{5}sup(a,b)-b \~~= "failed"\br
++ \tab{5}x-a \~~= "failed" and x-b \~~= "failed" => x >= sup(a,b)\br

OrderedAbelianMonoidSup() : Category == SIG where

  SIG ==> OrderedCancellationAbelianMonoid with

    sup : (%,%) -> %
      ++ sup(x,y) returns the least element from which both
      ++ x and y can be subtracted.
```

———–

— COQ OAMONS —

```
(* category OAMONS *)
(*
Axioms
  sup(a,b)-a \~~= "failed"
  sup(a,b)-b \~~= "failed"
  x-a \~~= "failed" and x-b \~~= "failed" => x >= sup(a,b)
*)
```

———–

— OAMONS.dotabb —

```
"OAMONS"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OAMONS"];
"OAMONS" -> "OCAMON"
```

———–

— OAMONS.dotfull —

```
"OrderedAbelianMonoidSup()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OAMONS"];
"OrderedAbelianMonoidSup()" -> "OrderedCancellationAbelianMonoid()"
```

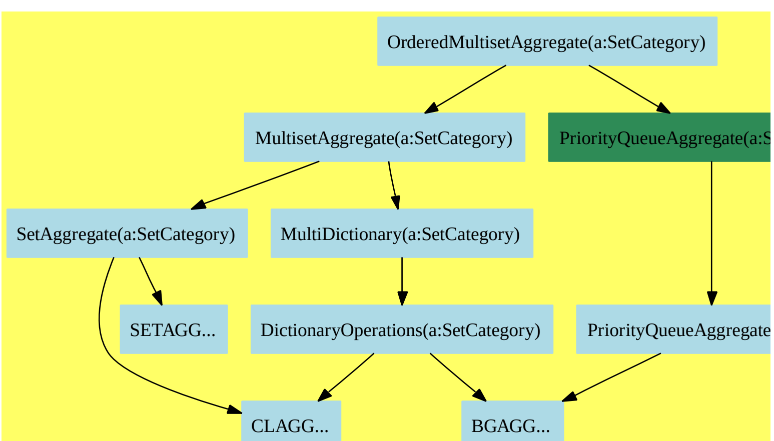
———–

— OAMONS.dotpic —

```
digraph pic {
 fontsize=10;
```

```
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"OrderedAbelianMonoidSup()" [color=lightblue];
"OrderedAbelianMonoidSup()" -> "OrderedCancellationAbelianMonoid()"

"OrderedCancellationAbelianMonoid()" [color=lightblue];
"OrderedCancellationAbelianMonoid()" -> "OrderedAbelianMonoid()"
"OrderedCancellationAbelianMonoid()" -> "CancellationAbelianMonoid()"

"OrderedAbelianMonoid()" [color=lightblue];
"OrderedAbelianMonoid()" -> "OrderedAbelianSemiGroup()"
"OrderedAbelianMonoid()" -> "AbelianMonoid()"

"OrderedAbelianSemiGroup()" [color=lightblue];
"OrderedAbelianSemiGroup()" -> "OrderedSet()"
"OrderedAbelianSemiGroup()" -> "AbelianMonoid()"

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SETCAT..."

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "ABELSG..."

"SETCAT..." [color=lightblue];
"ABELSG..." [color=lightblue];
}
```

9.0.152 OrderedMultisetAggregate (OMSAGG)



— OrderedMultisetAggregate.input —

```
)set break resume
)sys rm -f OrderedMultisetAggregate.output
)spool OrderedMultisetAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderedMultisetAggregate
--R
--R OrderedMultisetAggregate(S: OrderedSet) is a category constructor
--R Abbreviation for OrderedMultisetAggregate is OMSAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for OMSAGG
--R
--R------------------------------- Operations --------------------------------
--R ?<? : (%,%) -> Boolean                ?=? : (%,%) -> Boolean
--R bag : List(S) -> %                    brace : () -> %
--R brace : List(S) -> %                  coerce : % -> OutputForm
--R construct : List(S) -> %              copy : % -> %
--R dictionary : List(S) -> %             dictionary : () -> %
--R difference : (%,%) -> %               difference : (%,S) -> %
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                extract! : % -> S
--R hash : % -> SingleInteger             insert! : (S,%) -> %
--R inspect : % -> S                      intersect : (%,%) -> %
--R latex : % -> String                   map : ((S -> S),%) -> %
--R max : % -> S                          merge : (%,%) -> %
--R merge! : (%,%) -> %                   min : % -> S
--R removeDuplicates! : % -> %            sample : () -> %
--R set : () -> %                         set : List(S) -> %
--R subset? : (%,%) -> Boolean            symmetricDifference : (%,%) -> %
--R union : (%,%) -> %                    union : (%,S) -> %
--R union : (S,%) -> %                    ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R convert : % -> InputForm if S has KONVERT(INFORM)
--R count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
--R count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R duplicates : % -> List(Record(entry: S,count: NonNegativeInteger))
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,S,S) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,Equation(S)) -> % if S has EVALAB(S) and S has SETCAT
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S) and S has SETCAT
--R every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
--R find : ((S -> Boolean),%) -> Union(S,"failed")
--R insert! : (S,%,NonNegativeInteger) -> %
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((S -> S),%) -> % if $ has shallowlyMutable
--R member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
--R members : % -> List(S) if $ has finiteAggregate
```

```
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(S) if $ has finiteAggregate
--R reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
--R reduce : (((S,S) -> S),%,S,S) -> S if S has SETCAT and $ has finiteAggregate
--R remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
--R remove! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (S,%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if S has SETCAT and $ has finiteAggregate
--R select : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R
--E 1

)spool
)lisp (bye)
```

— **OrderedMultisetAggregate.help** —

```
====================================================================
OrderedMultisetAggregate examples
====================================================================

An ordered-multiset aggregate is a multiset built over an ordered set S
so that the relative sizes of its entries can be assessed.
These aggregates serve as models for priority queues.

See Also:
o )show OrderedMultisetAggregate
```

See:

⇐ "MultisetAggregate" (MSETAGG) 8.0.139 on page 843
⇐ "PriorityQueueAggregate" (PRQAGG) 6.0.116 on page 617

Exports:

any?	bag	brace	coerce
construct	convert	copy	count
dictionary	difference	empty	empty?
eq?	duplicates	eval	every?
extract!	find	hash	insert!
inspect	intersect	latex	less?
map	map!	max	member?
members	merge	merge!	min
more?	parts	reduce	remove
remove!	removeDuplicates	removeDuplicates!	sample
select	select!	set	size?
subset?	symmetricDifference	union	#?
?<?	?=?	?~?	

Attributes exported:

- **partiallyOrderedSet** is true if a set with $<$ which is transitive, but not$(a < b$ or $a = b)$ does not necessarily imply $b < a$.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These are directly exported but not implemented:

```
min : % -> S
```

These exports come from (p843) MultisetAggregate(S:OrderedSet):

```
any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
bag : List S -> %
brace : () -> %
brace : List S -> %
coerce : % -> OutputForm
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
copy : % -> %
count : (S,%) -> NonNegativeInteger
    if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
    if $ has finiteAggregate
dictionary : List S -> %
dictionary : () -> %
difference : (%,S) -> %
difference : (%,%) -> %
duplicates : % ->
    List Record(entry: S,count: NonNegativeInteger)
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
    if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
    if S has EVALAB S and S has SETCAT
```

```
eval : (%,Equation S) -> %
    if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
    if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
    if $ has finiteAggregate
extract! : % -> S
find : ((S -> Boolean),%) -> Union(S,"failed")
hash : % -> SingleInteger
insert! : (S,%) -> %
insert! : (S,%,NonNegativeInteger) -> %
inspect : % -> S
intersect : (%,%) -> %
latex : % -> String
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
    if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
    if S has SETCAT and $ has finiteAggregate
remove : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
remove : (S,%) -> %
    if S has SETCAT and $ has finiteAggregate
remove! : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
remove! : (S,%) -> %
    if $ has finiteAggregate
removeDuplicates : % -> %
    if S has SETCAT and $ has finiteAggregate
removeDuplicates! : % -> %
sample : () -> %
select : ((S -> Boolean),%) -> % if $ has finiteAggregate
select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
set : () -> %
set : List S -> %
size? : (%,NonNegativeInteger) -> Boolean
subset? : (%,%) -> Boolean
symmetricDifference : (%,%) -> %
union : (%,%) -> %
union : (%,S) -> %
union : (S,%) -> %
#? : % -> NonNegativeInteger if $ has finiteAggregate
?~=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?<? : (%,%) -> Boolean
```

These exports come from (p617) PriorityQueueAggregate(S:OrderedSet):

```
max : % -> S
merge : (%,%) -> %
merge! : (%,%) -> %
```

— OrderedMultisetAggregate.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#OMSAGG">
OrderedMultisetAggregate (OMSAGG)</a></h2>
</body>
```

— category OMSAGG OrderedMultisetAggregate —

```
)abbrev category OMSAGG OrderedMultisetAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ An ordered-multiset aggregate is a multiset built over an ordered set S
++ so that the relative sizes of its entries can be assessed.
++ These aggregates serve as models for priority queues.

OrderedMultisetAggregate(S) : Category == SIG where
  S : OrderedSet

  SIG ==> Join(MultisetAggregate S,PriorityQueueAggregate S) with

    min : % -> S
      ++ min(u) returns the smallest entry in the multiset aggregate u.
```

— OMSAGG.dotabb —

```
"OMSAGG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OMSAGG"];
"OMSAGG" -> "MSETAGG"
"OMSAGG" -> "PRQAGG"
```

— OMSAGG.dotfull —

```
"OrderedMultisetAggregate(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OMSAGG"];
"OrderedMultisetAggregate(a:SetCategory)" ->
    "MultisetAggregate(a:SetCategory)"
"OrderedMultisetAggregate(a:SetCategory)" ->
```

```
   "PriorityQueueAggregate(a:SetCategory)"
```

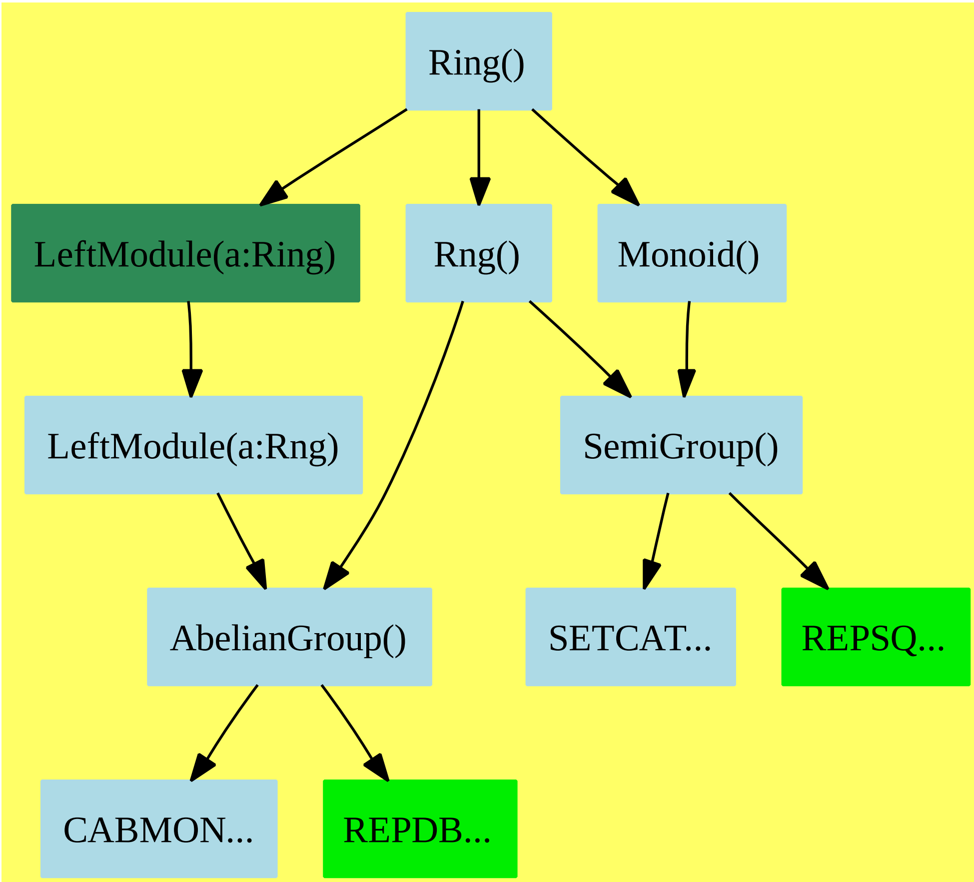
———-

— **OMSAGG.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"OrderedMultisetAggregate(a:SetCategory)" [color=lightblue];
"OrderedMultisetAggregate(a:SetCategory)" ->
    "MultisetAggregate(a:SetCategory)"
"OrderedMultisetAggregate(a:SetCategory)" ->
   "PriorityQueueAggregate(a:SetCategory)"

"MultisetAggregate(a:SetCategory)" [color=lightblue];
"MultisetAggregate(a:SetCategory)" -> "MultiDictionary(a:SetCategory)"
"MultisetAggregate(a:SetCategory)" -> "SetAggregate(a:SetCategory)"

"MultiDictionary(a:SetCategory)" [color=lightblue];
"MultiDictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"

"SetAggregate(a:SetCategory)" [color=lightblue];
"SetAggregate(a:SetCategory)" -> "SETAGG..."
"SetAggregate(a:SetCategory)" -> "CLAGG..."

"DictionaryOperations(a:SetCategory)" [color=lightblue];
"DictionaryOperations(a:SetCategory)" -> "BGAGG..."
"DictionaryOperations(a:SetCategory)" -> "CLAGG..."

"PriorityQueueAggregate(a:SetCategory)" [color=seagreen];
"PriorityQueueAggregate(a:SetCategory)" -> "PriorityQueueAggregate(a:Type)"

"PriorityQueueAggregate(a:Type)" [color=lightblue];
"PriorityQueueAggregate(a:Type)" -> "BGAGG..."

"BGAGG..." [color=lightblue];
"CLAGG..." [color=lightblue];
"SETAGG..." [color=lightblue];
}
```

———-

9.0.153 Ring (RING)



— **Ring.input** —

```
)set break resume
)sys rm -f Ring.output
)spool Ring.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Ring
--R
--R Ring is a category constructor
--R Abbreviation for Ring is RING
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for RING
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
```

```
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R one? : % -> Boolean                   recip : % -> Union(%,"failed")
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— **Ring.help** —

```
====================================================================
Ring examples
====================================================================

The category of rings with unity, always associative, but not
necessarily commutative.

Rings have two operations, '+' and '*' and satisfy 5 axioms:

1) (associative addition)        a + (b + c) = (a + b) + c
2) (commutative addition)        a + b = b + a
3) (associative multiplication)  a(bc) = (ab)c
4) (distributive mulitplication) a(b + c) = ab + ac; (b + c)a = ba + ca
5) (equation solution)           a + x = b has a solution in R

examples include

 - even integers
 - integers
 - integers module any positive integer
 - rational numbers
 - real numbers
 - complex numbers
 - real quaterions
 - a + b*sqrt(2) where a and b are rational
 - polynomials in real variables with real coefficients
 - real continuous functions in a fixed interval
 - all power series in a real variable convergent on an interval
 - all subsets of an arbitrary set
 - all square matrices with elements from a ring
```

```
Rings are not necessarily commutative. For example, square 2x2 matices:

  +-   -+ +-   -+   +-   -+
  | 0 1 | | 0 0 | = | 1 0 |
  | 0 0 | | 1 0 |   | 0 0 |
  +-   -+ +-   -+   +-   -+

  +-   -+ +-   -+   +-   -+
  | 0 0 | | 0 1 | = | 0 0 |
  | 1 0 | | 0 0 |   | 0 1 |
  +-   -+ +-   -+   +-   -+

See Also:
o )show Ring
```

See:

⇒ "Algebra" (ALGEBRA) 11.0.176 on page 1121
⇒ "AbelianMonoidRing" (AMR) 13.0.194 on page 1287
⇒ "CharacteristicNonZero" (CHARNZ) 10.0.159 on page 1010
⇒ "CharacteristicZero" (CHARZ) 10.0.160 on page 1015
⇒ "CommutativeRing" (COMRING) 10.0.161 on page 1019
⇒ "DifferentialExtension" (DIFEXT) 11.0.177 on page 1127
⇒ "DifferentialRing" (DIFRING) 10.0.162 on page 1024
⇒ "EntireRing" (ENTIRER) 10.0.163 on page 1029
⇒ "FunctionSpace" (FS) 17.0.219 on page 1544
⇒ "LeftAlgebra" (LALG) 10.0.165 on page 1040
⇒ "LinearlyExplicitRingOver" (LINEXP) 10.0.166 on page 1045
⇒ "MonogenicLinearOperator" (MLO) 12.0.189 on page 1232
⇒ "OrderedRing" (ORDRING) 10.0.168 on page 1055
⇒ "PartialDifferentialRing" (PDRING) 10.0.169 on page 1061
⇒ "UnivariateSkewPolynomialCategory" (OREPCAT) 10.0.174 on page 1101
⇒ "XAlgebra" (XALG) 10.0.175 on page 1116
⇒ "XFreeAlgebra" (XFALG) 11.0.183 on page 1164
⇐ "LeftModule" (LMODULE) 8.0.137 on page 823
⇐ "Monoid" (MONOID) 5.0.100 on page 455
⇐ "Rng" (RNG) 8.0.145 on page 896

Exports:

1	0	characteristic	coerce	hash
latex	one?	recip	sample	subtractIfCan
zero?	?~=?	?*?	?**?	?^?
?+?	?-?	-?	?=?	

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

These are directly exported but not implemented:

```
characteristic : () -> NonNegativeInteger
```

These are implemented by this category:

```
coerce : Integer -> %
```

These exports come from (p896) Rng():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p455) Monoid():

```
1 : () -> %
one? : % -> Boolean
recip : % -> Union(%,"failed")
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

— **Ring.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#RING">
Ring (RING)</a></h2>
</body>
```

———-

— **category RING Ring** —

```
)abbrev category RING Ring
++ Description:
++ The category of rings with unity, always associative, but
++ not necessarily commutative.

Ring() : Category == SIG where

  SIG ==> Join(Rng,Monoid,LeftModule(%)) with

    characteristic : () -> NonNegativeInteger
```

```
      ++ characteristic() returns the characteristic of the ring
      ++ this is the smallest positive integer n such that
      ++ \spad{n*x=0} for all x in the ring, or zero if no such n
      ++ exists.
      --We can not make this a constant, since some domains are mutable

    coerce : Integer -> %
      ++ coerce(i) converts the integer i to a member of the given domain.

    unitsKnown
      ++ recip truly yields
      ++ reciprocal or "failed" if not a unit.
      ++ Note that \spad{recip(0) = "failed"}.

   add

     n:Integer

     coerce(n) == n * 1$%
```

— **COQ RING** —

```
(* category RING *)
(*
      n:Integer
      coerce(n) == n * 1$%
*)
```

— **RING.dotabb** —

```
"RING"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RING"];
"RING" -> "RNG"
"RING" -> "MONOID"
"RING" -> "LMODULE"
```

— **RING.dotfull** —

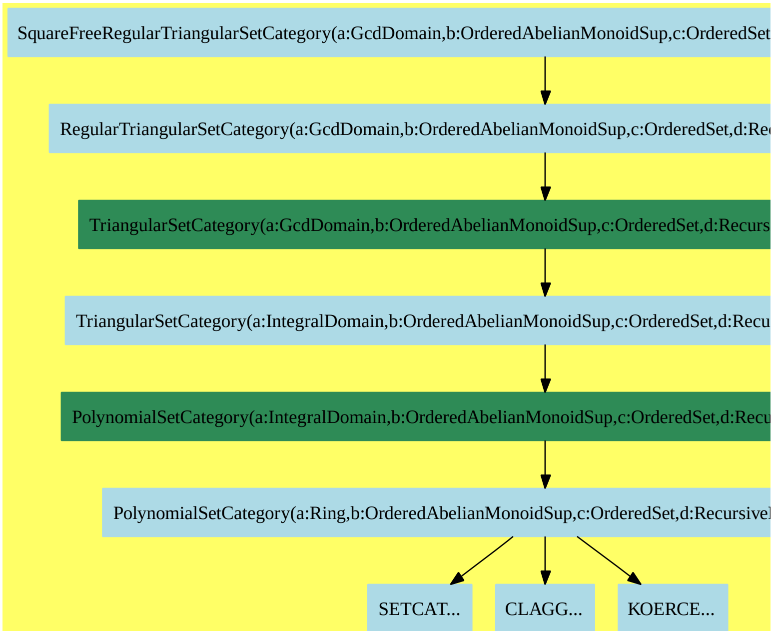
```
"Ring()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RING"];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"
```

— RING.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

———

9.0.154 SquareFreeRegularTriangularSetCategory (SFRTCAT)



— SquareFreeRegularTriangularSetCategory.input —

```
)set break resume
)sys rm -f SquareFreeRegularTriangularSetCategory.output
)spool SquareFreeRegularTriangularSetCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SquareFreeRegularTriangularSetCategory
--R
--R SquareFreeRegularTriangularSetCategory(R: GcdDomain,E: OrderedAbelianMonoidSup,V: OrderedSet,P: RecursivePol
--R Abbreviation for SquareFreeRegularTriangularSetCategory is SFRTCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SFRTCAT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                algebraic? : (V,%) -> Boolean
--R algebraicVariables : % -> List(V)     augment : (List(P),%) -> List(%)
--R augment : (P,List(%)) -> List(%)      augment : (P,%) -> List(%)
--R coerce : % -> List(P)                 coerce : % -> OutputForm
--R collect : (%,V) -> %                  collectQuasiMonic : % -> %
--R collectUnder : (%,V) -> %             collectUpper : (%,V) -> %
--R construct : List(P) -> %              copy : % -> %
--R degree : % -> NonNegativeInteger      empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R extend : (List(P),%) -> List(%)       extend : (P,List(%)) -> List(%)
```

```
--R extend : (P,%) -> List(%)            extend : (%,P) -> %
--R first : % -> Union(P,"failed")       hash : % -> SingleInteger
--R headReduce : (P,%) -> P              headReduced? : % -> Boolean
--R headReduced? : (P,%) -> Boolean      infRittWu? : (%,%) -> Boolean
--R initiallyReduce : (P,%) -> P         initiallyReduced? : % -> Boolean
--R initials : % -> List(P)              internalAugment : (List(P),%) -> %
--R internalAugment : (P,%) -> %         intersect : (P,List(%)) -> List(%)
--R intersect : (List(P),%) -> List(%)   intersect : (P,%) -> List(%)
--R invertible? : (P,%) -> Boolean       invertibleSet : (P,%) -> List(%)
--R last : % -> Union(P,"failed")        latex : % -> String
--R mainVariable? : (V,%) -> Boolean     mainVariables : % -> List(V)
--R map : ((P -> P),%) -> %              mvar : % -> V
--R normalized? : % -> Boolean           normalized? : (P,%) -> Boolean
--R purelyAlgebraic? : % -> Boolean      purelyAlgebraic? : (P,%) -> Boolean
--R reduceByQuasiMonic : (P,%) -> P      removeZero : (P,%) -> P
--R rest : % -> Union(%,"failed")        retract : List(P) -> %
--R sample : () -> %                     select : (%,V) -> Union(P,"failed")
--R stronglyReduce : (P,%) -> P          stronglyReduced? : % -> Boolean
--R stronglyReduced? : (P,%) -> Boolean  trivialIdeal? : % -> Boolean
--R variables : % -> List(V)             zeroSetSplit : List(P) -> List(%)
--R ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R algebraicCoefficients? : (P,%) -> Boolean
--R any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R augment : (List(P),List(%)) -> List(%)
--R autoReduced? : (%,((P,List(P)) -> Boolean)) -> Boolean
--R basicSet : (List(P),(P -> Boolean),((P,P) -> Boolean)) -> Union(Record(bas: %,top: List(P)),"failed")
--R basicSet : (List(P),((P,P) -> Boolean)) -> Union(Record(bas: %,top: List(P)),"failed")
--R coHeight : % -> NonNegativeInteger if V has FINITE
--R convert : % -> InputForm if P has KONVERT(INFORM)
--R count : ((P -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (P,%) -> NonNegativeInteger if P has SETCAT and $ has finiteAggregate
--R eval : (%,List(Equation(P))) -> % if P has EVALAB(P) and P has SETCAT
--R eval : (%,Equation(P)) -> % if P has EVALAB(P) and P has SETCAT
--R eval : (%,P,P) -> % if P has EVALAB(P) and P has SETCAT
--R eval : (%,List(P),List(P)) -> % if P has EVALAB(P) and P has SETCAT
--R every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R extend : (List(P),List(%)) -> List(%)
--R extendIfCan : (%,P) -> Union(%,"failed")
--R find : ((P -> Boolean),%) -> Union(P,"failed")
--R headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
--R initiallyReduced? : (P,%) -> Boolean
--R intersect : (List(P),List(%)) -> List(%)
--R invertible? : (P,%) -> List(Record(val: Boolean,tower: %))
--R invertibleElseSplit? : (P,%) -> Union(Boolean,List(%))
--R lastSubResultant : (P,P,%) -> List(Record(val: P,tower: %))
--R lastSubResultantElseSplit : (P,P,%) -> Union(P,List(%))
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((P -> P),%) -> % if $ has shallowlyMutable
--R member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
--R members : % -> List(P) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(P) if $ has finiteAggregate
--R purelyAlgebraicLeadingMonomial? : (P,%) -> Boolean
```

```
--R purelyTranscendental? : (P,%) -> Boolean
--R quasiComponent : % -> Record(close: List(P),open: List(P))
--R reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
--R reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P,P) -> P if P has SETCAT and $ has finiteAggregate
--R reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
--R remainder : (P,%) -> Record(rnum: R,polnum: P,den: R) if R has INTDOM
--R remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
--R retractIfCan : List(P) -> Union(%,"failed")
--R rewriteIdealWithHeadRemainder : (List(P),%) -> List(P) if R has INTDOM
--R rewriteIdealWithRemainder : (List(P),%) -> List(P) if R has INTDOM
--R rewriteSetWithReduction : (List(P),%,((P,P) -> P),((P,P) -> Boolean)) -> List(P)
--R roughBase? : % -> Boolean if R has INTDOM
--R roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
--R roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
--R roughUnitIdeal? : % -> Boolean if R has INTDOM
--R select : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : (%,V) -> Record(under: %,floor: %,upper: %)
--R squareFreePart : (P,%) -> List(Record(val: P,tower: %))
--R triangular? : % -> Boolean if R has INTDOM
--R zeroSetSplit : (List(P),Boolean) -> List(%)
--R zeroSetSplitIntoTriangularSystems : List(P) -> List(Record(close: %,open: List(P)))
--R
--E 1

)spool
)lisp (bye)
```

— **SquareFreeRegularTriangularSetCategory.help** —

```
====================================================================
SquareFreeRegularTriangularSetCategory examples
====================================================================

The category of square-free regular triangular sets. A regular
triangular set ts is square-free if the gcd of any polynomial p in ts
and differentiate(p,mvar(p)) w.r.t. collectUnder(ts,mvar(p))
has degree zero w.r.t. mvar(p). Thus any square-free regular
set defines a tower of square-free simple extensions.

See Also:
o )show SquareFreeRegularTriangularSetCategory
```

See:

⇒ "SquareFreeNormalizedTriangularSetCategory" (SNTSCAT) 10.0.172 on page 1085

⇐ "RegularTriangularSetCategory" (RSETCAT) 8.0.143 on page 873

Exports:

algebraic?	algebraicCoefficients?
algebraicVariables	any?
augment	autoReduced?
basicSet	coerce
coHeight	collect
collectQuasiMonic	collectUnder
collectUpper	construct
copy	convert
count	degree
empty	empty?
eq?	eval
every?	extend
extendIfCan	find
first	hash
headReduce	headReduced?
headRemainder	infRittWu?
initiallyReduce	initiallyReduced?
initials	internalAugment
intersect	invertible?
invertibleElseSplit?	invertibleSet
last	lastSubResultant
lastSubResultantElseSplit	latex
less?	mainVariable?
mainVariables	map
map!	member?
members	more?
mvar	normalized?
parts	purelyAlgebraic?
purelyAlgebraicLeadingMonomial?	purelyTranscendental?
quasiComponent	reduce
reduceByQuasiMonic	reduced?
remainder	remove
removeDuplicates	removeZero
rest	retract
retractIfCan	rewriteIdealWithHeadRemainder
rewriteIdealWithRemainder	rewriteSetWithReduction
roughBase?	roughEqualIdeals?
roughSubIdeal?	roughUnitIdeal?
sample	select
size?	sort
squareFreePart	stronglyReduce
stronglyReduced?	triangular?
trivialIdeal?	variables
zeroSetSplit	zeroSetSplitIntoTriangularSystems
#?	?=?
?~=?	

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These exports come from (p873) RegularTriangularSetCategory(R,E,V,P)
where R:GcdDomain, E:OrderedAbelianMonoidSup, V:OrderedSet,
P:RecursivePolynomialCategory(R,E,V)):

```
algebraic? : (V,%) -> Boolean
algebraicCoefficients? : (P,%) -> Boolean
algebraicVariables : % -> List V
any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
augment : (List P,List %) -> List %
augment : (List P,%) -> List %
augment : (P,List %) -> List %
augment : (P,%) -> List %
autoReduced? : (%,((P,List P) -> Boolean)) -> Boolean
basicSet :
 (List P,(P -> Boolean),((P,P) -> Boolean)) ->
   Union(Record(bas: %,top: List P),"failed")
basicSet :
 (List P,((P,P) -> Boolean)) ->
   Union(Record(bas: %,top: List P),"failed")
coerce : % -> List P
coerce : % -> OutputForm
coHeight : % -> NonNegativeInteger if V has FINITE
collect : (%,V) -> %
collectQuasiMonic : % -> %
collectUnder : (%,V) -> %
collectUpper : (%,V) -> %
construct : List P -> %
convert : % -> InputForm if P has KONVERT INFORM
copy : % -> %
count : ((P -> Boolean),%) -> NonNegativeInteger
  if $ has finiteAggregate
count : (P,%) -> NonNegativeInteger
  if P has SETCAT
  and $ has finiteAggregate
degree : % -> NonNegativeInteger
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
extend : (List P,List %) -> List %
extend : (List P,%) -> List %
extend : (P,List %) -> List %
```

```
extend : (P,%) -> List %
extend : (%,P) -> %
extendIfCan : (%,P) -> Union(%,"failed")
find : ((P -> Boolean),%) -> Union(P,"failed")
first : % -> Union(P,"failed")
hash : % -> SingleInteger
headReduce : (P,%) -> P
headReduced? : % -> Boolean
headReduced? : (P,%) -> Boolean
headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
infRittWu? : (%,%) -> Boolean
initiallyReduce : (P,%) -> P
initiallyReduced? : % -> Boolean
initiallyReduced? : (P,%) -> Boolean
initials : % -> List P
internalAugment : (P,%) -> %
internalAugment : (List P,%) -> %
intersect : (P,List %) -> List %
intersect : (List P,%) -> List %
intersect : (P,%) -> List %
intersect : (List P,List %) -> List %
invertible? : (P,%) -> Boolean
invertible? : (P,%) -> List Record(val: Boolean,tower: %)
invertibleElseSplit? : (P,%) -> Union(Boolean,List %)
invertibleSet : (P,%) -> List %
last : % -> Union(P,"failed")
lastSubResultant : (P,P,%) -> List Record(val: P,tower: %)
lastSubResultantElseSplit : (P,P,%) -> Union(P,List %)
latex : % -> String
less? : (%,NonNegativeInteger) -> Boolean
mainVariable? : (V,%) -> Boolean
mainVariables : % -> List V
map : ((P -> P),%) -> %
map! : ((P -> P),%) -> % if $ has shallowlyMutable
member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
members : % -> List P if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
mvar : % -> V
normalized? : % -> Boolean
normalized? : (P,%) -> Boolean
parts : % -> List P if $ has finiteAggregate
purelyAlgebraic? : (P,%) -> Boolean
purelyAlgebraic? : % -> Boolean
purelyAlgebraicLeadingMonomial? : (P,%) -> Boolean
purelyTranscendental? : (P,%) -> Boolean
quasiComponent : % -> Record(close: List P,open: List P)
reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P,P) -> P
  if P has SETCAT
  and $ has finiteAggregate
reduceByQuasiMonic : (P,%) -> P
reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
```

```
remainder : (P,%) -> Record(rnum: R,polnum: P,den: R)
  if R has INTDOM
remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
removeZero : (P,%) -> P
rest : % -> Union(%,"failed")
retract : List P -> %
retractIfCan : List P -> Union(%,"failed")
rewriteIdealWithHeadRemainder : (List P,%) -> List P if R has INTDOM
rewriteIdealWithRemainder : (List P,%) -> List P if R has INTDOM
rewriteSetWithReduction :
   (List P,%,((P,P) -> P),((P,P) -> Boolean)) -> List P
roughBase? : % -> Boolean if R has INTDOM
roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
roughUnitIdeal? : % -> Boolean if R has INTDOM
sample : () -> %
select : (%,V) -> Union(P,"failed")
select : ((P -> Boolean),%) -> % if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
sort : (%,V) -> Record(under: %,floor: %,upper: %)
squareFreePart : (P,%) -> List Record(val: P,tower: %)
stronglyReduce : (P,%) -> P
stronglyReduced? : (P,%) -> Boolean
stronglyReduced? : % -> Boolean
triangular? : % -> Boolean if R has INTDOM
trivialIdeal? : % -> Boolean
variables : % -> List V
zeroSetSplit : List P -> List %
zeroSetSplit : (List P,Boolean) -> List %
zeroSetSplitIntoTriangularSystems :
  List P -> List Record(close: %,open: List P)
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

See: SALSA[SALSA], Kalkbrener[Kalk91, Kalk98], Aubry[Aubr96, Aubr99, Aubr99a], Lazard[Laza91], Moreno Maza[Maza95, Maza97, Maza98, Maza00]

— **SquareFreeRegularTriangularSetCategory.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#SFRTCAT">
SquareFreeRegularTriangularSetCategory (SFRTCAT)</a></h2>
</body>
```

———-

— **category SFRTCAT SquareFreeRegularTriangularSetCategory** —

```
)abbrev category SFRTCAT SquareFreeRegularTriangularSetCategory
++ Author: Marc Moreno Maza
++ Date Created: 09/03/1996
++ Date Last Updated: 09/10/1998
```

```
++ References :
++ SALSA Solvers for Algebraic Systems and Applications
++ Kalk91 Three contributions to elimination theory
++ Kalk98 Algorithmic properties of polynomial rings
++ Aubr96 Triangular Sets for Solving Polynomial Systems:
++ Aubr99 On the Theories of Triangular Sets
++ Aubr99a Triangular Sets for Solving Polynomial Systems:
++ Laza91 A new method for solving algebraic systems of positive dimension
++ Maza95 Polynomial Gcd Computations over Towers of Algebraic Extensions
++ Maza97 Calculs de pgcd au-dessus des tours d'extensions simples et
++        resolution des systemes d'equations algebriques
++ Maza98 A new algorithm for computing triangular decomposition of
++        algebraic varieties
++ Maza00 On Triangular Decompositions of Algebraic Varieties
++ Description:
++ The category of square-free regular triangular sets.
++ A regular triangular set \spad{ts} is square-free if
++ the gcd of any polynomial \spad{p} in \spad{ts} and
++ differentiate(p,mvar(p)) w.r.t. collectUnder(ts,mvar(p))
++ has degree zero w.r.t. \spad{mvar(p)}. Thus any square-free regular
++ set defines a tower of square-free simple extensions.

SquareFreeRegularTriangularSetCategory(R,E,V,P) : Category == SIG where
  R : GcdDomain
  E : OrderedAbelianMonoidSup
  V : OrderedSet
  P : RecursivePolynomialCategory(R,E,V)

  SIG ==> RegularTriangularSetCategory(R,E,V,P)
```

———-

— **SFRTCAT.dotabb** —

```
"SFRTCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SFRTCAT"];
"SFRTCAT" -> "RSETCAT"
```

———-

— **SFRTCAT.dotfull** —

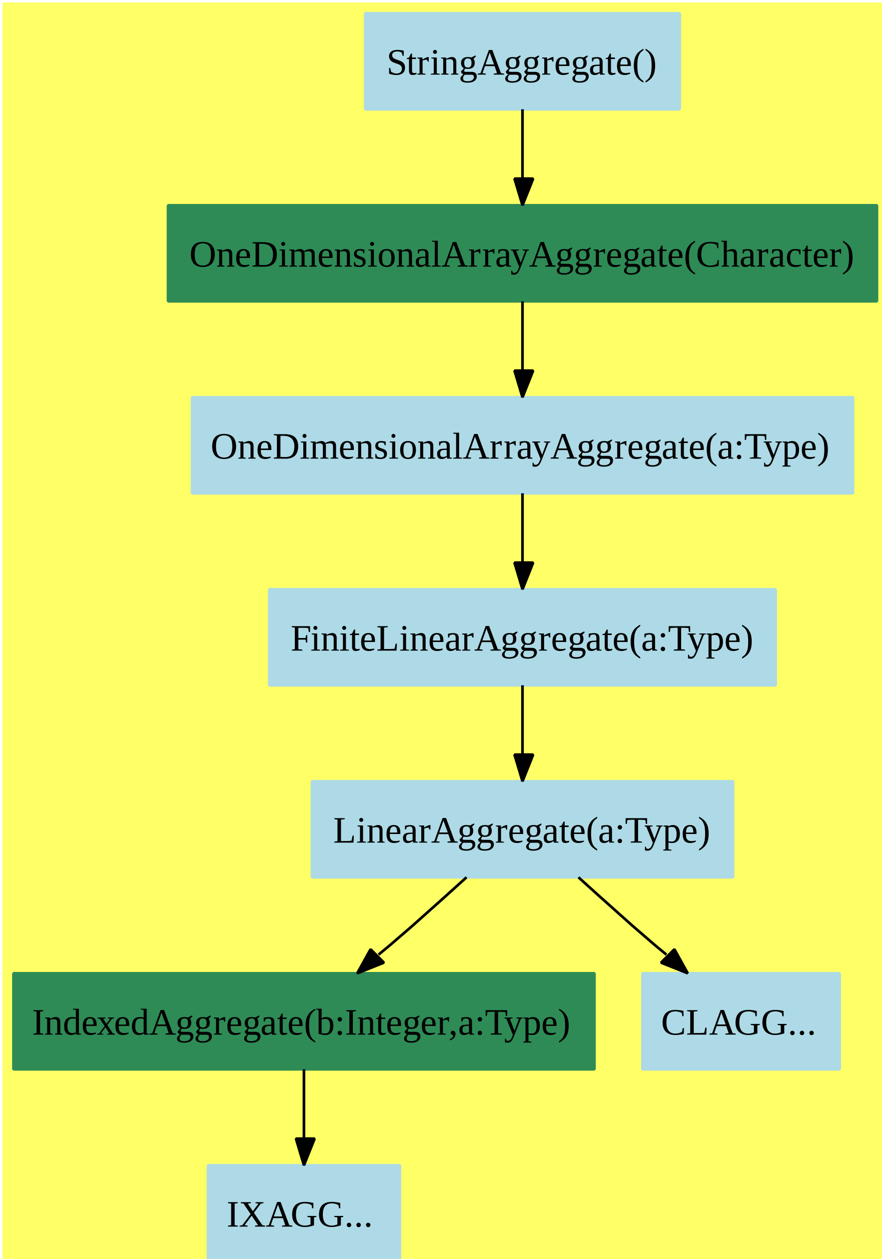
```
"SquareFreeRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomial
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SFRTCAT"];
"SquareFreeRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomial
 ->
"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a
```

———-

— **SFRTCAT.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"SquareFreeRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomial
 [color=lightblue];
"SquareFreeRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomial
 ->
"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a

"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a
 [color=lightblue];
"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a
  ->
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"

"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 [color=seagreen];
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 ->
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b
 [color=lightblue];
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b
  -> "PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategor

"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b
 [color=seagreen];
"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b
  -> "PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"

"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 [color=lightblue];
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 -> "SETCAT..."
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 -> "CLAGG..."
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCategory(a,b,c))"
 -> "KOERCE..."

"SETCAT..." [color=lightblue];
"KOERCE..." [color=lightblue];
"CLAGG..." [color=lightblue];
}
```

9.0.155 StringAggregate (SRAGG)



— **StringAggregate.input** —

```
)set break resume
)sys rm -f StringAggregate.output
)spool StringAggregate.output
)set message test on
```

```
)set message auto off
)clear all

--S 1 of 1
)show StringAggregate
--R
--R StringAggregate is a category constructor
--R Abbreviation for StringAggregate is SRAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SRAGG
--R
--R------------------------------- Operations --------------------------------
--R coerce : Character -> %              concat : List(%) -> %
--R concat : (%,%) -> %                  concat : (Character,%) -> %
--R concat : (%,Character) -> %          construct : List(Character) -> %
--R copy : % -> %                        delete : (%,Integer) -> %
--R ?.? : (%,%) -> %                     ?.? : (%,Integer) -> Character
--R empty : () -> %                      empty? : % -> Boolean
--R entries : % -> List(Character)       eq? : (%,%) -> Boolean
--R index? : (Integer,%) -> Boolean      indices : % -> List(Integer)
--R insert : (%,%,Integer) -> %          leftTrim : (%,CharacterClass) -> %
--R leftTrim : (%,Character) -> %        lowerCase : % -> %
--R lowerCase! : % -> %                  prefix? : (%,%) -> Boolean
--R qelt : (%,Integer) -> Character      reverse : % -> %
--R rightTrim : (%,Character) -> %       sample : () -> %
--R split : (%,Character) -> List(%)     suffix? : (%,%) -> Boolean
--R trim : (%,CharacterClass) -> %       trim : (%,Character) -> %
--R upperCase : % -> %                   upperCase! : % -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?<? : (%,%) -> Boolean if Character has ORDSET
--R ?<=? : (%,%) -> Boolean if Character has ORDSET
--R ?=? : (%,%) -> Boolean if Character has SETCAT
--R ?>? : (%,%) -> Boolean if Character has ORDSET
--R ?>=? : (%,%) -> Boolean if Character has ORDSET
--R any? : ((Character -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if Character has SETCAT
--R convert : % -> InputForm if Character has KONVERT(INFORM)
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (Character,%) -> NonNegativeInteger if Character has SETCAT and $ has finiteAggregate
--R count : ((Character -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (%,UniversalSegment(Integer)) -> %
--R ?.? : (%,UniversalSegment(Integer)) -> %
--R elt : (%,Integer,Character) -> Character
--R entry? : (Character,%) -> Boolean if $ has finiteAggregate and Character has SETCAT
--R eval : (%,List(Character),List(Character)) -> % if Character has EVALAB(CHAR) and Character has SETCAT
--R eval : (%,Character,Character) -> % if Character has EVALAB(CHAR) and Character has SETCAT
--R eval : (%,Equation(Character)) -> % if Character has EVALAB(CHAR) and Character has SETCAT
--R eval : (%,List(Equation(Character))) -> % if Character has EVALAB(CHAR) and Character has SETCAT
--R every? : ((Character -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,Character) -> % if $ has shallowlyMutable
--R find : ((Character -> Boolean),%) -> Union(Character,"failed")
--R first : % -> Character if Integer has ORDSET
--R hash : % -> SingleInteger if Character has SETCAT
--R insert : (Character,%,Integer) -> %
```

```
--R latex : % -> String if Character has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map : (((Character,Character) -> Character),%,%) -> %
--R map : ((Character -> Character),%) -> %
--R map! : ((Character -> Character),%) -> % if $ has shallowlyMutable
--R match : (%,%,Character) -> NonNegativeInteger
--R match? : (%,%,Character) -> Boolean
--R max : (%,%) -> % if Character has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (Character,%) -> Boolean if Character has SETCAT and $ has finiteAggregate
--R members : % -> List(Character) if $ has finiteAggregate
--R merge : (%,%) -> % if Character has ORDSET
--R merge : (((Character,Character) -> Boolean),%,%) -> %
--R min : (%,%) -> % if Character has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,Character) -> %
--R parts : % -> List(Character) if $ has finiteAggregate
--R position : (CharacterClass,%,Integer) -> Integer
--R position : (%,%,Integer) -> Integer
--R position : (Character,%,Integer) -> Integer if Character has SETCAT
--R position : (Character,%) -> Integer if Character has SETCAT
--R position : ((Character -> Boolean),%) -> Integer
--R qsetelt! : (%,Integer,Character) -> Character if $ has shallowlyMutable
--R reduce : (((Character,Character) -> Character),%) -> Character if $ has finiteAggregate
--R reduce : (((Character,Character) -> Character),%,Character) -> Character if $ has finiteAggregate
--R reduce : (((Character,Character) -> Character),%,Character,Character) -> Character if Character has SETCAT a
--R remove : ((Character -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (Character,%) -> % if Character has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if Character has SETCAT and $ has finiteAggregate
--R replace : (%,UniversalSegment(Integer),%) -> %
--R reverse! : % -> % if $ has shallowlyMutable
--R rightTrim : (%,CharacterClass) -> %
--R select : ((Character -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment(Integer),Character) -> Character if $ has shallowlyMutable
--R setelt : (%,Integer,Character) -> Character if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : % -> % if Character has ORDSET
--R sort : (((Character,Character) -> Boolean),%) -> %
--R sort! : % -> % if Character has ORDSET and $ has shallowlyMutable
--R sort! : (((Character,Character) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if Character has ORDSET
--R sorted? : (((Character,Character) -> Boolean),%) -> Boolean
--R split : (%,CharacterClass) -> List(%)
--R substring? : (%,%,Integer) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R ?~=? : (%,%) -> Boolean if Character has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

— **StringAggregate.help** —

```
====================================================================
StringAggregate examples
====================================================================

A string aggregate is a category for strings, that is, one dimensional
arrays of characters.

See Also:
o )show StringAggregate
```

———-

See:

⇒ "StringCategory" (STRICAT) 10.0.173 on page 1093
⇐ "OneDimensionalArrayAggregate" (A1AGG) 8.0.141 on page 854

Exports:

any?	coerce	concat	construct	copy
convert	copyInto!	count	delete	elt
empty	empty?	entries	entry?	eval
every?	eq?	fill!	find	first
hash	index?	indices	insert	latex
leftTrim	less?	lowerCase	lowerCase!	map
map!	match	match?	max	maxIndex
member?	members	merge	min	minIndex
more?	new	parts	position	prefix?
qelt	qsetelt!	reduce	remove	removeDuplicates
replace	reverse	reverse!	rightTrim	sample
setelt	size?	sort	sort!	sorted?
split	substring?	suffix?	swap!	trim
upperCase	upperCase!	#?	?~=?	?.?
?<?	?<=?	?=?	?>?	?>=?

Attributes exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are directly exported but not implemented:

```
leftTrim : (%,Character) -> %
leftTrim : (%,CharacterClass) -> %
lowerCase! : % -> %
match : (%,%,Character) -> NonNegativeInteger
match? : (%,%,Character) -> Boolean
position : (CharacterClass,%,Integer) -> Integer
```

```
position : (%,%,Integer) -> Integer
replace : (%,UniversalSegment Integer,%) -> %
rightTrim : (%,Character) -> %
rightTrim : (%,CharacterClass) -> %
split : (%,Character) -> List %
split : (%,CharacterClass) -> List %
substring? : (%,%,Integer) -> Boolean
suffix? : (%,%) -> Boolean
upperCase! : % -> %
```

These are implemented by this category:

```
coerce : Character -> %
lowerCase : % -> %
prefix? : (%,%) -> Boolean
trim : (%,CharacterClass) -> %
trim : (%,Character) -> %
upperCase : % -> %
?.? : (%,%) -> %
```

These exports come from (p854) OneDimensionalArrayAggregate(Character):

```
any? : ((Character -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if Character has SETCAT
concat : List % -> %
concat : (%,%) -> %
concat : (Character,%) -> %
concat : (%,Character) -> %
construct : List Character -> %
convert : % -> InputForm
         if Character has KONVERT INFORM
copy : % -> %
copyInto! : (%,%,Integer) -> %
         if $ has shallowlyMutable
count : (Character,%) -> NonNegativeInteger
         if Character has SETCAT
         and $ has finiteAggregate
count : ((Character -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
delete : (%,Integer) -> %
delete : (%,UniversalSegment Integer) -> %
elt : (%,Integer,Character) -> Character
empty : () -> %
empty? : % -> Boolean
entries : % -> List Character
entry? : (Character,%) -> Boolean
         if $ has finiteAggregate
         and Character has SETCAT
eq? : (%,%) -> Boolean
eval : (%,List Character,List Character) -> %
         if Character has EVALAB CHAR
         and Character has SETCAT
eval : (%,Character,Character) -> %
         if Character has EVALAB CHAR
         and Character has SETCAT
```

```
eval : (%,Equation Character) -> %
         if Character has EVALAB CHAR
         and Character has SETCAT
eval : (%,List Equation Character) -> %
         if Character has EVALAB CHAR
         and Character has SETCAT
every? : ((Character -> Boolean),%) -> Boolean
         if $ has finiteAggregate
fill! : (%,Character) -> %
         if $ has shallowlyMutable
find : ((Character -> Boolean),%) -> Union(Character,"failed")
first : % -> Character
         if Integer has ORDSET
hash : % -> SingleInteger
         if Character has SETCAT
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (Character,%,Integer) -> %
insert : (%,%,Integer) -> %
latex : % -> String if Character has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((Character -> Character),%) -> %
map : (((Character,Character) -> Character),%,%) -> %
map! : ((Character -> Character),%) -> %
         if $ has shallowlyMutable
max : (%,%) -> % if Character has ORDSET
maxIndex : % -> Integer if Integer has ORDSET
member? : (Character,%) -> Boolean
         if Character has SETCAT
         and $ has finiteAggregate
members : % -> List Character
         if $ has finiteAggregate
merge : (%,%) -> % if Character has ORDSET
merge : (((Character,Character) -> Boolean),%,%) -> %
min : (%,%) -> % if Character has ORDSET
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
new : (NonNegativeInteger,Character) -> %
parts : % -> List Character if $ has finiteAggregate
position : (Character,%) -> Integer
         if Character has SETCAT
position : ((Character -> Boolean),%) -> Integer
position : (Character,%,Integer) -> Integer
         if Character has SETCAT
qelt : (%,Integer) -> Character
qsetelt! : (%,Integer,Character) -> Character
         if $ has shallowlyMutable
reduce : (((Character,Character) -> Character),%) -> Character
         if $ has finiteAggregate
reduce :
  (((Character,Character) -> Character),%,Character) -> Character
         if $ has finiteAggregate
reduce :
  (((Character,Character) -> Character),%,Character,Character)
```

```
      -> Character
         if Character has SETCAT and $ has finiteAggregate
 remove : ((Character -> Boolean),%) -> %
         if $ has finiteAggregate
 remove : (Character,%) -> %
         if Character has SETCAT
         and $ has finiteAggregate
 removeDuplicates : % -> %
         if Character has SETCAT
         and $ has finiteAggregate
 reverse : % -> %
 reverse! : % -> % if $ has shallowlyMutable
 sample : () -> %
 select : ((Character -> Boolean),%) -> %
         if $ has finiteAggregate
 setelt : (%,UniversalSegment Integer,Character) -> Character
         if $ has shallowlyMutable
 sort! : (((Character,Character) -> Boolean),%) -> %
         if $ has shallowlyMutable
 sorted? : (((Character,Character) -> Boolean),%) -> Boolean
 setelt : (%,Integer,Character) -> Character
         if $ has shallowlyMutable
 size? : (%,NonNegativeInteger) -> Boolean
 sort : % -> % if Character has ORDSET
 sort : (((Character,Character) -> Boolean),%) -> %
 sort! : % -> %
         if Character has ORDSET
         and $ has shallowlyMutable
 sorted? : % -> Boolean if Character has ORDSET
 swap! : (%,Integer,Integer) -> Void
         if $ has shallowlyMutable
 #? : % -> NonNegativeInteger if $ has finiteAggregate
 ?~=? : (%,%) -> Boolean if Character has SETCAT
 ?.? : (%,UniversalSegment Integer) -> %
 ?.? : (%,Integer) -> Character
 ?<? : (%,%) -> Boolean if Character has ORDSET
 ?<=? : (%,%) -> Boolean if Character has ORDSET
 ?=? : (%,%) -> Boolean if Character has SETCAT
 ?>? : (%,%) -> Boolean if Character has ORDSET
 ?>=? : (%,%) -> Boolean if Character has ORDSET
```

— **StringAggregate.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#SRAGG">
StringAggregate (SRAGG)</a></h2>
</body>
```

———-

— **category SRAGG StringAggregate** —

```
)abbrev category SRAGG StringAggregate
```

```
++ Author: Stephen Watt and Michael Monagan.
++ revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ A string aggregate is a category for strings, that is,
++ one dimensional arrays of characters.

StringAggregate() : Category == SIG where

  SIG ==> OneDimensionalArrayAggregate Character with

    lowerCase : % -> %
      ++ lowerCase(s) returns the string with all characters in lower case.

    lowerCase_! : % -> %
      ++ lowerCase!(s) destructively replaces the alphabetic characters
      ++ in s by lower case.

    upperCase : % -> %
      ++ upperCase(s) returns the string with all characters in upper case.

    upperCase_! : % -> %
      ++ upperCase!(s) destructively replaces the alphabetic characters
      ++ in s by upper case characters.

    prefix? : (%, %) -> Boolean
      ++ prefix?(s,t) tests if the string s is the initial substring of t.
      ++ Note that \axiom{prefix?(s,t) ==
      ++   reduce(and,[s.i = t.i for i in 0..maxIndex s])}.

    suffix? : (%, %) -> Boolean
      ++ suffix?(s,t) tests if the string s is the final substring of t.
      ++ Note that \axiom{suffix?(s,t) ==
      ++  reduce(and,[s.i = t.(n - m + i) for i in 0..maxIndex s])}
      ++ where m and n denote the maxIndex of s and t respectively.

    substring? : (%, %, Integer) -> Boolean
      ++ substring?(s,t,i) tests if s is a substring of t beginning at
      ++ index i.
      ++ Note that \axiom{substring?(s,t,0) = prefix?(s,t)}.

    match : (%, %, Character) -> NonNegativeInteger
      ++ match(p,s,wc) tests if pattern \axiom{p} matches subject \axiom{s}
      ++ where \axiom{wc} is a wild card character. If no match occurs,
      ++ the index \axiom{0} is returned; otheriwse, the value returned
      ++ is the first index of the first character in the subject matching
      ++ the subject (excluding that matched by an initial wild-card).
      ++ For example, \axiom{match("*to*","yorktown","*")} returns \axiom{5}
      ++ indicating a successful match starting at index \axiom{5} of
      ++ \axiom{"yorktown"}.

    match? : (%, %, Character) -> Boolean
      ++ match?(s,t,c) tests if s matches t except perhaps for
```

```
  ++ multiple and consecutive occurrences of character c.
  ++ Typically c is the blank character.

replace : (%, UniversalSegment(Integer), %) -> %
  ++ replace(s,i..j,t) replaces the substring \axiom{s(i..j)}
  ++ of s by string t.

position : (%, %, Integer) -> Integer
  ++ position(s,t,i) returns the position j of the substring s in
  ++ string t, where \axiom{j >= i} is required.

position : (CharacterClass, %, Integer) -> Integer
  ++ position(cc,t,i) returns the position \axiom{j >= i} in t of
  ++ the first character belonging to cc.

coerce : Character -> %
  ++ coerce(c) returns c as a string s with the character c.

split : (%, Character) -> List %
  ++ split(s,c) returns a list of substrings delimited by character c.

split : (%, CharacterClass) -> List %
  ++ split(s,cc) returns a list of substrings delimited by
  ++ characters in cc.

trim : (%, Character) -> %
  ++ trim(s,c) returns s with all characters c deleted from right
  ++ and left ends.
  ++ For example, \axiom{trim(" abc ", char " ")} returns \axiom{"abc"}.

trim : (%, CharacterClass) -> %
  ++ trim(s,cc) returns s with all characters in cc deleted from right
  ++ and left ends.
  ++ For example, \axiom{trim("(abc)", charClass "()")}
  ++ returns \axiom{"abc"}.

leftTrim : (%, Character) -> %
  ++ leftTrim(s,c) returns s with all leading characters c deleted.
  ++ For example, \axiom{leftTrim("  abc  ", char " ")}
  ++ returns \axiom{"abc  "}.

leftTrim : (%, CharacterClass) -> %
  ++ leftTrim(s,cc) returns s with all leading characters in cc deleted.
  ++ For example, \axiom{leftTrim("(abc)", charClass "()")}
  ++ returns \axiom{"abc)"}.

rightTrim : (%, Character) -> %
  ++ rightTrim(s,c) returns s with all trailing occurrences of c deleted.
  ++ For example, \axiom{rightTrim("  abc  ", char " ")}
  ++ returns \axiom{"  abc"}.

rightTrim : (%, CharacterClass) -> %
  ++ rightTrim(s,cc) returns s with all trailing occurences of
  ++ characters in cc deleted.
```

```
      ++ For example, \axiom{rightTrim("(abc)", charClass "()")}
      ++ returns \axiom{"(abc"}.

    elt : (%, %) -> %
      ++ elt(s,t) returns the concatenation of s and t. It is provided to
      ++ allow juxtaposition of strings to work as concatenation.
      ++ For example, \axiom{"smoo" "shed"} returns \axiom{"smooshed"}.

   add

     trim(s: %, c:  Character) == leftTrim(rightTrim(s, c), c)

     trim(s: %, cc: CharacterClass) == leftTrim(rightTrim(s, cc), cc)

     lowerCase s == lowerCase_! copy s

     upperCase s == upperCase_! copy s

     prefix?(s, t) == substring?(s, t, minIndex t)

     coerce(c:Character):% == new(1, c)

     elt(s:%, t:%): % == concat(s,t)$%
```

— COQ SRAGG —

```
(* category SRAGG *)
(*

   trim : (%,Character) -> %
   trim(s: %, c:  Character) == leftTrim(rightTrim(s, c), c)

   trim : (%,CharacterClass) -> %
   trim(s: %, cc: CharacterClass) == leftTrim(rightTrim(s, cc), cc)

   lowerCase! : % -> %
   lowerCase s == lowerCase_! copy s

   upperCase : % -> %
   upperCase s == upperCase_! copy s

   prefix? : (%,%) -> Boolean
   prefix?(s, t) == substring?(s, t, minIndex t)

   coerce : % -> OutputForm
   coerce(c:Character):% == new(1, c)

   ?.? : (%,%) -> %
   elt(s:%, t:%): % == concat(s,t)$%

*)
```

— SRAGG.dotabb —

```
"SRAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=SRAGG"];
"SRAGG" -> "A1AGG"
```

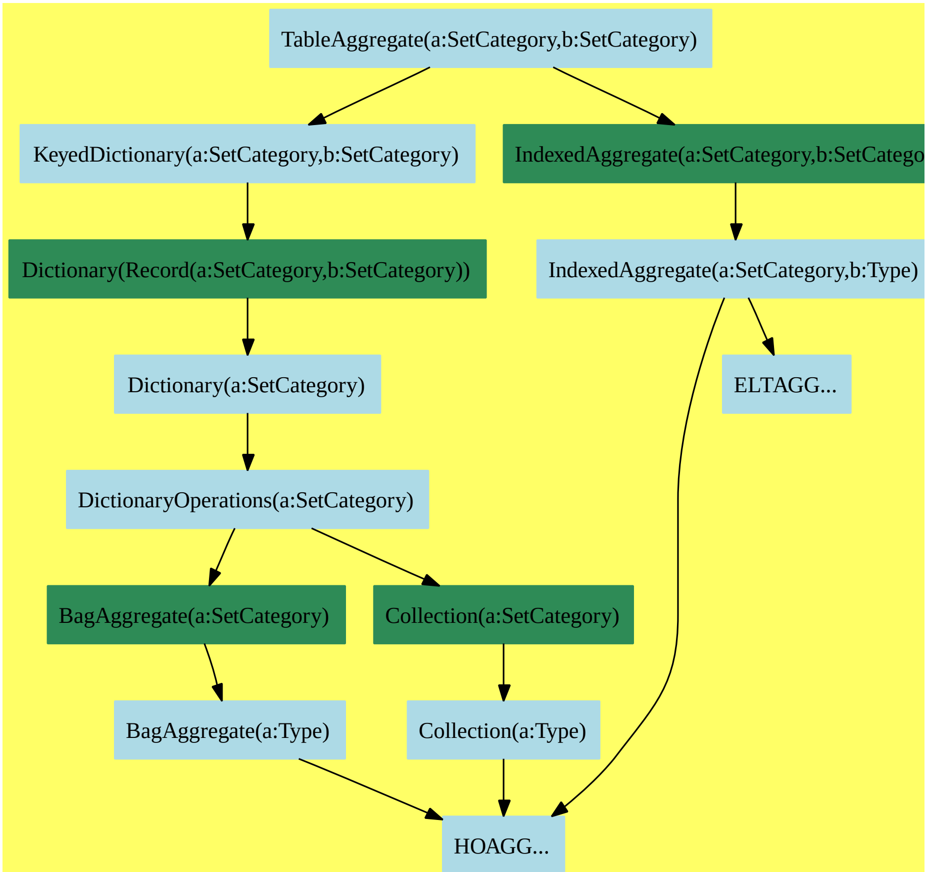
— SRAGG.dotfull —

```
"StringAggregate()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SRAGG"];
"StringAggregate()" -> "OneDimensionalArrayAggregate(Character)"
```

— SRAGG.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"StringAggregate()" [color=lightblue];
"StringAggregate()" -> "OneDimensionalArrayAggregate(Character)"

"OneDimensionalArrayAggregate(Character)" [color=seagreen];
"OneDimensionalArrayAggregate(Character)" ->
    "OneDimensionalArrayAggregate(a:Type)"

"OneDimensionalArrayAggregate(a:Type)" [color=lightblue];
"OneDimensionalArrayAggregate(a:Type)" ->
    "FiniteLinearAggregate(a:Type)"

"FiniteLinearAggregate(a:Type)" [color=lightblue];
"FiniteLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "CLAGG..."

"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" -> "IXAGG..."

"CLAGG..." [color=lightblue];
"IXAGG..." [color=lightblue];
}
```

9.0.156 TableAggregate (TBAGG)



— **TableAggregate.input** —

```
)set break resume
)sys rm -f TableAggregate.output
)spool TableAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show TableAggregate
--R
--R TableAggregate(Key: SetCategory,Entry: SetCategory) is a category constructor
--R Abbreviation for TableAggregate is TBAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for TBAGG
--R
--R------------------------------- Operations --------------------------------
--R copy : % -> %                        dictionary : () -> %
--R elt : (%,Key,Entry) -> Entry         ?.? : (%,Key) -> Entry
```

```
--R empty : () -> %                     empty? : % -> Boolean
--R entries : % -> List(Entry)          eq? : (%,%) -> Boolean
--R index? : (Key,%) -> Boolean         indices : % -> List(Key)
--R key? : (Key,%) -> Boolean           keys : % -> List(Key)
--R map : ((Entry -> Entry),%) -> %     qelt : (%,Key) -> Entry
--R sample : () -> %                    setelt : (%,Key,Entry) -> Entry
--R table : () -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?=? : (%,%) -> Boolean if Entry has SETCAT or Record(key: Key,entry: Entry) has SETCAT
--R any? : ((Entry -> Boolean),%) -> Boolean if $ has finiteAggregate
--R any? : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R bag : List(Record(key: Key,entry: Entry)) -> %
--R coerce : % -> OutputForm if Entry has SETCAT or Record(key: Key,entry: Entry) has SETCAT
--R construct : List(Record(key: Key,entry: Entry)) -> %
--R convert : % -> InputForm if Record(key: Key,entry: Entry) has KONVERT(INFORM)
--R count : ((Entry -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (Entry,%) -> NonNegativeInteger if Entry has SETCAT and $ has finiteAggregate
--R count : (Record(key: Key,entry: Entry),%) -> NonNegativeInteger if Record(key: Key,entry: Entry) has SETCAT
--R count : ((Record(key: Key,entry: Entry) -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R dictionary : List(Record(key: Key,entry: Entry)) -> %
--R entry? : (Entry,%) -> Boolean if $ has finiteAggregate and Entry has SETCAT
--R eval : (%,List(Equation(Entry))) -> % if Entry has EVALAB(Entry) and Entry has SETCAT
--R eval : (%,Equation(Entry)) -> % if Entry has EVALAB(Entry) and Entry has SETCAT
--R eval : (%,Entry,Entry) -> % if Entry has EVALAB(Entry) and Entry has SETCAT
--R eval : (%,List(Entry),List(Entry)) -> % if Entry has EVALAB(Entry) and Entry has SETCAT
--R eval : (%,List(Record(key: Key,entry: Entry)),List(Record(key: Key,entry: Entry))) -> % if Record(key: Key,e
--R eval : (%,Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> % if Record(key: Key,entry: Entry)
--R eval : (%,Equation(Record(key: Key,entry: Entry))) -> % if Record(key: Key,entry: Entry) has EVALAB(Record(k
--R eval : (%,List(Equation(Record(key: Key,entry: Entry)))) -> % if Record(key: Key,entry: Entry) has EVALAB(Re
--R every? : ((Entry -> Boolean),%) -> Boolean if $ has finiteAggregate
--R every? : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R extract! : % -> Record(key: Key,entry: Entry)
--R fill! : (%,Entry) -> % if $ has shallowlyMutable
--R find : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Union(Record(key: Key,entry: Entry),"failed")
--R first : % -> Entry if Key has ORDSET
--R hash : % -> SingleInteger if Entry has SETCAT or Record(key: Key,entry: Entry) has SETCAT
--R insert! : (Record(key: Key,entry: Entry),%) -> %
--R inspect : % -> Record(key: Key,entry: Entry)
--R latex : % -> String if Entry has SETCAT or Record(key: Key,entry: Entry) has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R map : (((Entry,Entry) -> Entry),%,%) -> %
--R map : ((Record(key: Key,entry: Entry) -> Record(key: Key,entry: Entry)),%) -> %
--R map! : ((Entry -> Entry),%) -> % if $ has shallowlyMutable
--R map! : ((Record(key: Key,entry: Entry) -> Record(key: Key,entry: Entry)),%) -> % if $ has shallowlyMutable
--R maxIndex : % -> Key if Key has ORDSET
--R member? : (Entry,%) -> Boolean if Entry has SETCAT and $ has finiteAggregate
--R member? : (Record(key: Key,entry: Entry),%) -> Boolean if Record(key: Key,entry: Entry) has SETCAT and $ has
--R members : % -> List(Entry) if $ has finiteAggregate
--R members : % -> List(Record(key: Key,entry: Entry)) if $ has finiteAggregate
--R minIndex : % -> Key if Key has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(Entry) if $ has finiteAggregate
--R parts : % -> List(Record(key: Key,entry: Entry)) if $ has finiteAggregate
--R qsetelt! : (%,Key,Entry) -> Entry if $ has shallowlyMutable
```

```
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry)),%
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry)),%
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry)),%
--R remove : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (Record(key: Key,entry: Entry),%) -> % if Record(key: Key,entry: Entry) has SETCAT and $ has finite
--R remove! : (Key,%) -> Union(Entry,"failed")
--R remove! : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (Record(key: Key,entry: Entry),%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if Record(key: Key,entry: Entry) has SETCAT and $ has finiteAggregate
--R search : (Key,%) -> Union(Entry,"failed")
--R select : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R swap! : (%,Key,Key) -> Void if $ has shallowlyMutable
--R table : List(Record(key: Key,entry: Entry)) -> %
--R ?~=? : (%,%) -> Boolean if Entry has SETCAT or Record(key: Key,entry: Entry) has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

———-

— TableAggregate.help —

```
====================================================================
TableAggregate examples
====================================================================

A table aggregate is a model of a table, a discrete many-to-one
mapping from keys to entries.

See Also:
o )show TableAggregate
```

———-

See:

⇒ "AssociationListAggregate" (ALAGG) 10.0.158 on page 997
⇐ "IndexedAggregate" (IXAGG) 5.0.98 on page 441
⇐ "KeyedDictionary" (KDAGG) 8.0.135 on page 792

Exports:

any?	bag	coerce	construct	convert
copy	count	dictionary	elt	empty
empty?	entries	entry?	eq?	eval
every?	extract!	fill!	find	first
hash	index?	indices	insert!	inspect
key?	keys	latex	less?	map
map!	maxIndex	member?	members	minIndex
more?	parts	qelt	qsetelt!	reduce
remove	remove!	removeDuplicates	sample	search
select	select!	setelt	size?	swap!
table	#?	?=?	?.?	?~=?

Attributes exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.

These are directly exported but not implemented:

```
setelt : (%,Key,Entry) -> Entry
```

These are implemented by this category:

```
any? : ((Entry -> Boolean),%) -> Boolean
    if $ has finiteAggregate
coerce : % -> OutputForm
         if Entry has SETCAT
         or Record(key: Key,entry: Entry) has SETCAT
count : ((Entry -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
elt : (%,Key,Entry) -> Entry
entries : % -> List Entry
every? : ((Entry -> Boolean),%) -> Boolean
        if $ has finiteAggregate
extract! : % -> Record(key: Key,entry: Entry)
find :
  ((Record(key: Key,entry: Entry) -> Boolean),%)
     -> Union(Record(key: Key,entry: Entry),"failed")
index? : (Key,%) -> Boolean
indices : % -> List Key
insert! : (Record(key: Key,entry: Entry),%) -> %
inspect : % -> Record(key: Key,entry: Entry)
map : (((Entry,Entry) -> Entry),%,%) -> %
map : ((Record(key: Key,entry: Entry)
      -> Record(key: Key,entry: Entry)),%) -> %
map! : ((Entry -> Entry),%) -> % if $ has shallowlyMutable
map! :
  ((Record(key: Key,entry: Entry)
      -> Record(key: Key,entry: Entry)),%)
       -> % if $ has shallowlyMutable
parts : % -> List Entry if $ has finiteAggregate
```

```
parts : % -> List Record(key: Key,entry: Entry)
         if $ has finiteAggregate
remove! : (Key,%) -> Union(Entry,"failed")
table : () -> %
table : List Record(key: Key,entry: Entry) -> %
?.? : (%,Key) -> Entry
?=? : (%,%) -> Boolean
         if Entry has SETCAT
         or Record(key: Key,entry: Entry) has SETCAT
```

These exports come from (p792) KeyedDictionary(Key,Entry)
where Key:SetCategory and Entry:SetCategory
and RecKE=Record(key: Key,entry: Entry):

```
any? : ((RecKE -> Boolean),%) -> Boolean
         if $ has finiteAggregate
bag : List RecKE -> %
construct : List RecKE -> %
convert : % -> InputForm if RecKE has KONVERT INFORM
copy : % -> %
count : (Entry,%) -> NonNegativeInteger
         if Entry has SETCAT
         and $ has finiteAggregate
count : ((RecKE -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
dictionary : () -> %
dictionary : List RecKE -> %
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List RecKE,List RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,RecKE,RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,Equation RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,List Equation RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
every? : ((RecKE -> Boolean),%) -> Boolean
         if $ has finiteAggregate
key? : (Key,%) -> Boolean
keys : % -> List Key
hash : % -> SingleInteger
         if Entry has SETCAT
         or RecKE has SETCAT
latex : % -> String
         if Entry has SETCAT
         or RecKE has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
member? : (RecKE,%) -> Boolean
    if RecKE has SETCAT
```

```
     and $ has finiteAggregate
members : % -> List RecKE if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
reduce :
 (((RecKE,RecKE) -> RecKE),%) -> RecKE
         if $ has finiteAggregate
reduce :
 (((RecKE,RecKE)->RecKE),%,RecKE) -> RecKE
         if $ has finiteAggregate
reduce :
 (((RecKE,RecKE)->RecKE),%,RecKE,RecKE)
   -> RecKE
         if RecKE has SETCAT
         and $ has finiteAggregate
remove : ((RecKE -> Boolean),%) -> %
         if $ has finiteAggregate
remove : (RecKE,%) -> %
         if RecKE has SETCAT
         and $ has finiteAggregate
remove! : ((RecKE -> Boolean),%) -> %
         if $ has finiteAggregate
remove! : (RecKE,%) -> % if $ has finiteAggregate
removeDuplicates : % -> %
         if RecKE has SETCAT
         and $ has finiteAggregate
sample : () -> %
search : (Key,%) -> Union(Entry,"failed")
select : ((RecKE -> Boolean),%) -> %
         if $ has finiteAggregate
select! : ((RecKE -> Boolean),%) -> %
         if $ has finiteAggregate
#? : % -> NonNegativeInteger if $ has finiteAggregate
?~=? : (%,%) -> Boolean
         if Entry has SETCAT
         or RecKE has SETCAT
```

These exports come from (p441) IndexedAggregate(Key,Entry))
where Key:SetCategory and Entry:SetCategory
and RecKE=Record(key: Key,entry: Entry):

```
count : (RecKE,%) -> NonNegativeInteger
         if RecKE has SETCAT
         and $ has finiteAggregate
entry? : (Entry,%) -> Boolean
         if $ has finiteAggregate
         and Entry has SETCAT
eval : (%,List Equation Entry) -> %
         if Entry has EVALAB Entry
         and Entry has SETCAT
eval : (%,Equation Entry) -> %
         if Entry has EVALAB Entry
         and Entry has SETCAT
eval : (%,Entry,Entry) -> %
         if Entry has EVALAB Entry
         and Entry has SETCAT
```

```
 eval : (%,List Entry,List Entry) -> %
          if Entry has EVALAB Entry
          and Entry has SETCAT
 fill! : (%,Entry) -> % if $ has shallowlyMutable
 first : % -> Entry if Key has ORDSET
 map : ((Entry -> Entry),%) -> %
 maxIndex : % -> Key if Key has ORDSET
 member? : (Entry,%) -> Boolean
          if Entry has SETCAT
          and $ has finiteAggregate
 members : % -> List Entry if $ has finiteAggregate
 minIndex : % -> Key if Key has ORDSET
 qelt : (%,Key) -> Entry
 qsetelt! : (%,Key,Entry) -> Entry if $ has shallowlyMutable
 size? : (%,NonNegativeInteger) -> Boolean
 swap! : (%,Key,Key) -> Void if $ has shallowlyMutable
```

— TableAggregate.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#TBAGG">
TableAggregate (TBAGG)</a></h2>
</body>
```

— category TBAGG TableAggregate —

```
)abbrev category TBAGG TableAggregate
++ Author: Michael Monagan, Stephen Watt;
++ revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ A table aggregate is a model of a table, that is, a discrete many-to-one
++ mapping from keys to entries.

TableAggregate(Key, Entry) : Category == SIG where
  Key : SetCategory
  Entry : SetCategory

  SIG ==> Join(KeyedDictionary(Key,Entry),IndexedAggregate(Key,Entry)) with

    setelt : (%,Key,Entry) -> Entry
      ++ setelt(t,k,e) (also written \axiom{t.k := e}) is equivalent
      ++ to \axiom{(insert([k,e],t); e)}.

    table : () -> %
      ++table()$T creates an empty table of type T.
      ++
      ++X Data:=Record(age:Integer,gender:String)
      ++X a1:AssociationList(String,Data):=table()
      ++X a1."tim":=[55,"male"]$Data
```

```
 table : List Record(key:Key,entry:Entry) -> %
   ++ table([x,y,...,z]) creates a table consisting of entries
   ++ \axiom{x,y,...,z}.

 map : ((Entry, Entry) -> Entry, %, %) -> %
   ++ map(fn,t1,t2) creates a new table t from given tables t1 and t2 with
   ++ elements fn(x,y) where x and y are corresponding elements from t1
   ++ and t2 respectively.

add

  table() == empty()

  table l == dictionary l

  insert_!(p, t) == (t(p.key) := p.entry; t)

  indices t == keys t

  coerce(t:%):OutputForm ==
    prefix("table"::OutputForm,
                  [k::OutputForm = (t.k)::OutputForm for k in keys t])

  elt(t, k) ==
     (r := search(k, t)) case Entry => r::Entry
     error "key not in table"

  elt(t, k, e) ==
     (r := search(k, t)) case Entry => r::Entry
     e

  map_!(f, t) ==
     for k in keys t repeat t.k := f t.k
     t

  map(f:(Entry, Entry) -> Entry, s:%, t:%) ==
     z := table()
     for k in keys s | key?(k, t) repeat z.k := f(s.k, t.k)
     z

  if % has finiteAggregate then

    parts(t:%):List Record(key:Key,entry:Entry) ==
        [[k, t.k] for k in keys t]

    parts(t:%):List Entry == [t.k for k in keys t]

    entries(t:%):List Entry == parts(t)

    s:% = t:% ==
      eq?(s,t) => true
      #s ^= #t => false
      for k in keys s repeat
```

```
    (e := search(k, t)) _
      case "failed" or (e::Entry) ^= s.k => return false
  true

map(f: Record(key:Key,entry:Entry)->Record(key:Key,entry:Entry),t:%):%==
  z := table()
  for k in keys t repeat
    ke: Record(key:Key,entry:Entry) := f [k, t.k]
    z ke.key := ke.entry
  z

map_!(f:Record(key:Key,entry:Entry)->Record(key:Key,entry:Entry),t:%):%_
 ==
  lke: List Record(key:Key,entry:Entry) := nil()
  for k in keys t repeat
    lke := cons(f [k, remove_!(k,t)::Entry], lke)
  for ke in lke repeat
    t ke.key := ke.entry
  t

inspect(t: %): Record(key:Key,entry:Entry) ==
  ks := keys t
  empty? ks => error "Cannot extract from an empty aggregate"
  [first ks, t first ks]

find(f: Record(key:Key,entry:Entry)->Boolean, t:%):_
      Union(Record(key:Key,entry:Entry), "failed") ==
  for ke in parts(t)@List(Record(key:Key,entry:Entry)) _
     repeat if f ke then return ke
  "failed"

index?(k: Key, t: %): Boolean ==
  search(k,t) case Entry

remove_!(x:Record(key:Key,entry:Entry), t:%) ==
  if member?(x, t) then remove_!(x.key, t)
  t

extract_!(t: %): Record(key:Key,entry:Entry) ==
  k: Record(key:Key,entry:Entry) := inspect t
  remove_!(k.key, t)
  k

any?(f: Entry->Boolean, t: %): Boolean ==
  for k in keys t | f t k repeat return true
  false

every?(f: Entry->Boolean, t: %): Boolean ==
  for k in keys t | not f t k repeat return false
  true

count(f: Entry->Boolean, t: %): NonNegativeInteger ==
  tally: NonNegativeInteger := 0
  for k in keys t | f t k repeat tally := tally + 1
```

```
tally
```

— COQ TBAGG —

```
(* category TBAGG *)
(*

   table : () -> %
   table() == empty()

   table : List(Record(key: Key,entry: Entry)) -> %
   table l == dictionary l

   insert! : (Record(key: Key,entry: Entry),%) -> %
   insert_!(p, t) == (t(p.key) := p.entry; t)

   indices : % -> List(Key)
   indices t               == keys t

   coerce : % -> OutputForm
   coerce(t:%):OutputForm ==
     prefix("table"::OutputForm,
                    [k::OutputForm = (t.k)::OutputForm for k in keys t])

   ?.? : (%,Key) -> Entry
   elt(t, k) ==
      (r := search(k, t)) case Entry => r::Entry
      error "key not in table"

   elt : (%,Key,Entry) -> Entry
   elt(t, k, e) ==
      (r := search(k, t)) case Entry => r::Entry
      e

   map! : ((Entry -> Entry),%) -> %
   map_!(f, t) ==
      for k in keys t repeat t.k := f t.k
      t

   map : (((Entry,Entry) -> Entry),%,%) -> %
   map(f:(Entry, Entry) -> Entry, s:%, t:%) ==
      z := table()
      for k in keys s | key?(k, t) repeat z.k := f(s.k, t.k)
      z

   if % has finiteAggregate then

     parts : % -> List(Record(key: Key,entry: Entry))
     parts(t:%):List Record(key:Key,entry:Entry) ==
         [[k, t.k] for k in keys t]
```

```
parts : % -> List(Entry)
parts(t:%):List Entry   == [t.k for k in keys t]

entries : % -> List(Entry)
entries(t:%):List Entry == parts(t)

?=? : (%,%) -> Boolean
s:% = t:% ==
  eq?(s,t) => true
  #s ^= #t => false
  for k in keys s repeat
    (e := search(k, t)) _
      case "failed" or (e::Entry) ^= s.k => return false
  true

map : ((Record(key: Key,entry: Entry) ->
         Record(key: Key,entry: Entry)),%) -> %
map(f: Record(key:Key,entry:Entry)->Record(key:Key,entry:Entry),t:%):%==
  z := table()
  for k in keys t repeat
    ke: Record(key:Key,entry:Entry) := f [k, t.k]
    z ke.key := ke.entry
  z

map! : ((Record(key: Key,entry: Entry) ->
         Record(key: Key,entry: Entry)),%) -> %
map_!(f:Record(key:Key,entry:Entry)->Record(key:Key,entry:Entry),t:%):%_
 ==
  lke: List Record(key:Key,entry:Entry) := nil()
  for k in keys t repeat
    lke := cons(f [k, remove_!(k,t)::Entry], lke)
  for ke in lke repeat
    t ke.key := ke.entry
  t

inspect : % -> Record(key: Key,entry: Entry)
inspect(t: %): Record(key:Key,entry:Entry) ==
  ks := keys t
  empty? ks => error "Cannot extract from an empty aggregate"
  [first ks, t first ks]

find : ((Record(key: Key,entry: Entry) -> Boolean),%) ->
         Union(Record(key: Key,entry: Entry),"failed")
find(f: Record(key:Key,entry:Entry)->Boolean, t:%):_
      Union(Record(key:Key,entry:Entry), "failed") ==
  for ke in parts(t)@List(Record(key:Key,entry:Entry)) _
     repeat if f ke then return ke
  "failed"

index? : (Key,%) -> Boolean
index?(k: Key, t: %): Boolean ==
  search(k,t) case Entry

remove! : (Record(key: Key,entry: Entry),%) -> %
```

```
     remove_!(x:Record(key:Key,entry:Entry), t:%) ==
       if member?(x, t) then remove_!(x.key, t)
       t

     extract! : % -> Record(key: Key,entry: Entry)
     extract_!(t: %): Record(key:Key,entry:Entry) ==
       k: Record(key:Key,entry:Entry) := inspect t
       remove_!(k.key, t)
       k

     any? : ((Entry -> Boolean),%) -> Boolean
     any?(f: Entry->Boolean, t: %): Boolean ==
       for k in keys t | f t k repeat return true
       false

     every? : ((Entry -> Boolean),%) -> Boolean
     every?(f: Entry->Boolean, t: %): Boolean ==
       for k in keys t | not f t k repeat return false
       true

     count : ((Entry -> Boolean),%) -> NonNegativeInteger
     count(f: Entry->Boolean, t: %): NonNegativeInteger ==
       tally: NonNegativeInteger := 0
       for k in keys t | f t k repeat tally := tally + 1
       tally

*)
```

———

— TBAGG.dotabb —

```
"TBAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=TBAGG"];
"TBAGG" -> "KDAGG"
"TBAGG" -> "IXAGG"
```

———

— TBAGG.dotfull —

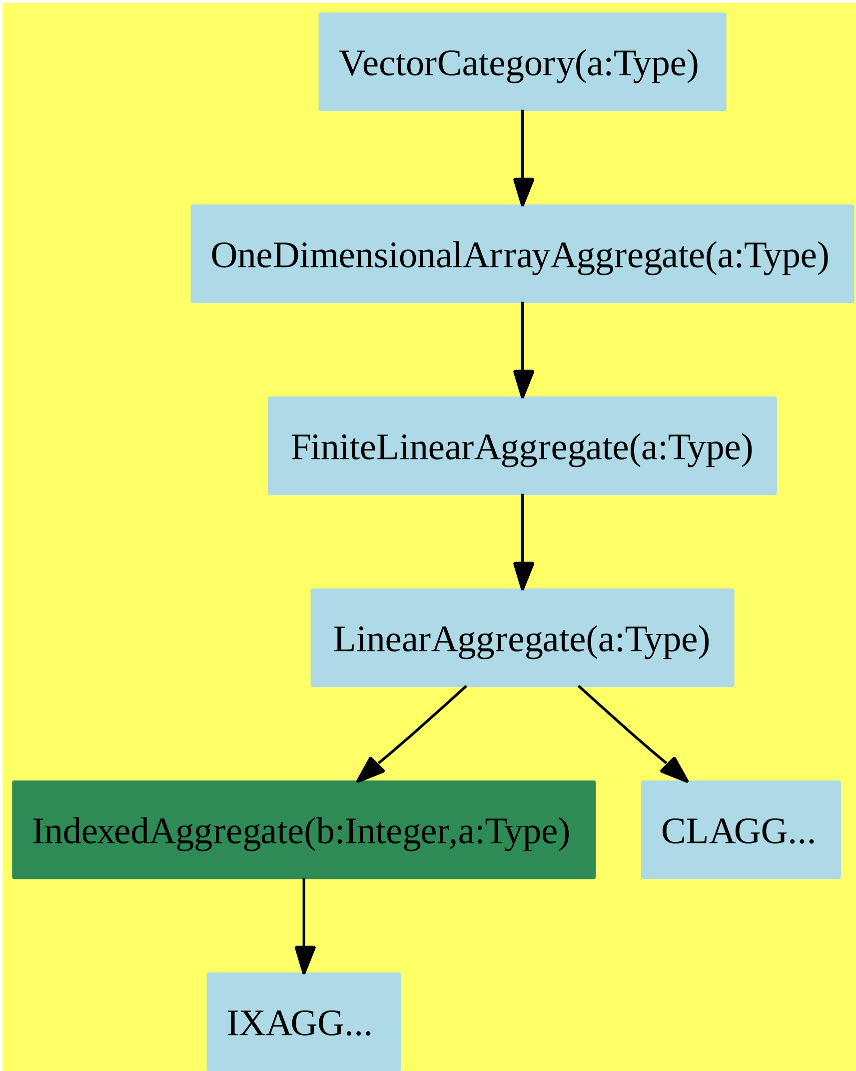
```
"TableAggregate(a:SetCategory,b:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=TBAGG"];
"TableAggregate(a:SetCategory,b:SetCategory)" ->
    "KeyedDictionary(a:SetCategory,b:SetCategory)"
"TableAggregate(a:SetCategory,b:SetCategory)" ->
    "IndexedAggregate(a:SetCategory,b:SetCategory)"
```

———

— TBAGG.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"TableAggregate(a:SetCategory,b:SetCategory)" [color=lightblue];
"TableAggregate(a:SetCategory,b:SetCategory)" ->
    "KeyedDictionary(a:SetCategory,b:SetCategory)"
"TableAggregate(a:SetCategory,b:SetCategory)" ->
    "IndexedAggregate(a:SetCategory,b:SetCategory)"

"IndexedAggregate(a:SetCategory,b:SetCategory)" [color=seagreen];
"IndexedAggregate(a:SetCategory,b:SetCategory)" ->
    "IndexedAggregate(a:SetCategory,b:Type)"

"IndexedAggregate(a:SetCategory,b:Type)" [color=lightblue];
"IndexedAggregate(a:SetCategory,b:Type)" -> "HOAGG..."
"IndexedAggregate(a:SetCategory,b:Type)" -> "ELTAGG..."

"KeyedDictionary(a:SetCategory,b:SetCategory)" [color=lightblue];
"KeyedDictionary(a:SetCategory,b:SetCategory)" ->
    "Dictionary(Record(a:SetCategory,b:SetCategory))"

"Dictionary(Record(a:SetCategory,b:SetCategory))" [color=seagreen];
"Dictionary(Record(a:SetCategory,b:SetCategory))" ->
    "Dictionary(a:SetCategory)"

"Dictionary(a:SetCategory)" [color=lightblue];
"Dictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"

"DictionaryOperations(a:SetCategory)" [color=lightblue];
"DictionaryOperations(a:SetCategory)" -> "BagAggregate(a:SetCategory)"
"DictionaryOperations(a:SetCategory)" -> "Collection(a:SetCategory)"

"BagAggregate(a:SetCategory)" [color=seagreen];
"BagAggregate(a:SetCategory)" -> "BagAggregate(a:Type)"

"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HOAGG..."

"Collection(a:SetCategory)" [color=seagreen];
"Collection(a:SetCategory)" -> "Collection(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HOAGG..."

"ELTAGG..." [color=lightblue];
"HOAGG..." [color=lightblue];
}
```

9.0.157 VectorCategory (VECTCAT)



— **VectorCategory.input** —

```
)set break resume
)sys rm -f VectorCategory.output
)spool VectorCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show VectorCategory
--R
--R VectorCategory(R: Type) is a category constructor
```

```
--R Abbreviation for VectorCategory is VECTCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for VECTCAT
--R
--R------------------------------ Operations -------------------------------
--R ?*? : (%,R) -> % if R has MONOID      ?*? : (R,%) -> % if R has MONOID
--R ?+? : (%,%) -> % if R has ABELSG      ?-? : (%,%) -> % if R has ABELGRP
--R -? : % -> % if R has ABELGRP          concat : List(%) -> %
--R concat : (%,%) -> %                   concat : (R,%) -> %
--R concat : (%,R) -> %                   construct : List(R) -> %
--R copy : % -> %                         cross : (%,%) -> % if R has RING
--R delete : (%,Integer) -> %             dot : (%,%) -> R if R has RING
--R ?.? : (%,Integer) -> R                elt : (%,Integer,R) -> R
--R empty : () -> %                       empty? : % -> Boolean
--R entries : % -> List(R)                eq? : (%,%) -> Boolean
--R index? : (Integer,%) -> Boolean       indices : % -> List(Integer)
--R insert : (%,%,Integer) -> %           insert : (R,%,Integer) -> %
--R latex : % -> String if R has SETCAT   map : (((R,R) -> R),%,%) -> %
--R map : ((R -> R),%) -> %               max : (%,%) -> % if R has ORDSET
--R min : (%,%) -> % if R has ORDSET      new : (NonNegativeInteger,R) -> %
--R qelt : (%,Integer) -> R               reverse : % -> %
--R sample : () -> %                      sort : % -> % if R has ORDSET
--R sort : (((R,R) -> Boolean),%) -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?*? : (Integer,%) -> % if R has ABELGRP
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?=? : (%,%) -> Boolean if R has SETCAT
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if R has SETCAT
--R convert : % -> InputForm if R has KONVERT(INFORM)
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (%,UniversalSegment(Integer)) -> %
--R ?.? : (%,UniversalSegment(Integer)) -> %
--R entry? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
--R eval : (%,List(R),List(R)) -> % if R has EVALAB(R) and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB(R) and R has SETCAT
--R eval : (%,Equation(R)) -> % if R has EVALAB(R) and R has SETCAT
--R eval : (%,List(Equation(R))) -> % if R has EVALAB(R) and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,R) -> % if $ has shallowlyMutable
--R find : ((R -> Boolean),%) -> Union(R,"failed")
--R first : % -> R if Integer has ORDSET
--R hash : % -> SingleInteger if R has SETCAT
--R length : % -> R if R has RING and R has RADCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R magnitude : % -> R if R has RING and R has RADCAT
--R map! : ((R -> R),%) -> % if $ has shallowlyMutable
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
```

```
--R members : % -> List(R) if $ has finiteAggregate
--R merge : (%,%) -> % if R has ORDSET
--R merge : (((R,R) -> Boolean),%,%) -> %
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R outerProduct : (%,%) -> Matrix(R) if R has RING
--R parts : % -> List(R) if $ has finiteAggregate
--R position : (R,%,Integer) -> Integer if R has SETCAT
--R position : (R,%) -> Integer if R has SETCAT
--R position : ((R -> Boolean),%) -> Integer
--R qsetelt! : (%,Integer,R) -> R if $ has shallowlyMutable
--R reduce : (((R,R) -> R),%) -> R if $ has finiteAggregate
--R reduce : (((R,R) -> R),%,R) -> R if $ has finiteAggregate
--R reduce : (((R,R) -> R),%,R,R) -> R if R has SETCAT and $ has finiteAggregate
--R remove : ((R -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (R,%) -> % if R has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if R has SETCAT and $ has finiteAggregate
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((R -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment(Integer),R) -> R if $ has shallowlyMutable
--R setelt : (%,Integer,R) -> R if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort! : % -> % if R has ORDSET and $ has shallowlyMutable
--R sort! : (((R,R) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if R has ORDSET
--R sorted? : (((R,R) -> Boolean),%) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R zero : NonNegativeInteger -> % if R has ABELMON
--R ?~=? : (%,%) -> Boolean if R has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

— **VectorCategory.help** —

```
====================================================================
VectorCategory examples
====================================================================

VectorCategory represents the type of vector like objects,
finite sequences indexed by some finite segment of the
integers. The operations available on vectors depend on the structure
of the underlying components. Many operations from the component domain
are defined for vectors componentwise. It can by assumed that extraction or
updating components can be done in constant time.

See Also:
o )show VectorCategory
```

See:

⇒ "PointCategory" (PTCAT) 10.0.170 on page 1068
⇐ "OneDimensionalArrayAggregate" (A1AGG) 8.0.141 on page 854

Exports:

any?	coerce	concat	construct	convert
copy	copyInto!	count	cross	delete
dot	entry?	elt	empty	empty?
entries	eq?	eval	every?	fill!
find	first	hash	index?	indices
insert	latex	length	less?	magnitude
map	map!	max	maxIndex	member?
members	merge	min	minIndex	more?
new	outerProduct	parts	position	qelt
qsetelt!	reduce	remove	removeDuplicates	reverse
reverse!	sample	select	setelt	size?
sort	sort!	sorted?	swap!	zero
#?	?*?	?+?	?-?	?<?
?<=?	?=?	?>?	?>=?	-?
?.?	?~=?			

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.

These are implemented by this category:

```
cross : (%,%) -> % if R has RING
dot : (%,%) -> R if R has RING
length : % -> R if R has RING and R has RADCAT
magnitude : % -> R if R has RING and R has RADCAT
outerProduct : (%,%) -> Matrix R if R has RING
zero : NonNegativeInteger -> % if R has ABELMON
?*? : (Integer,%) -> % if R has ABELGRP
?*? : (%,R) -> % if R has MONOID
?*? : (R,%) -> % if R has MONOID
?-? : (%,%) -> % if R has ABELGRP
-? : % -> % if R has ABELGRP
?+? : (%,%) -> % if R has ABELSG
```

These exports come from (p854) OneDimensionalArrayAggregate(R:Type):

```
any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
coerce : % -> OutputForm if R has SETCAT
concat : (R,%) -> %
concat : (%,R) -> %
concat : List % -> %
concat : (%,%) -> %
construct : List R -> %
convert : % -> InputForm if R has KONVERT INFORM
```

```
copy : % -> %
copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
delete : (%,Integer) -> %
delete : (%,UniversalSegment Integer) -> %
elt : (%,Integer,R) -> R
empty : () -> %
empty? : % -> Boolean
entries : % -> List R
entry? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
eq? : (%,%) -> Boolean
eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
fill! : (%,R) -> % if $ has shallowlyMutable
find : ((R -> Boolean),%) -> Union(R,"failed")
first : % -> R if Integer has ORDSET
hash : % -> SingleInteger if R has SETCAT
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (%,%,Integer) -> %
insert : (R,%,Integer) -> %
latex : % -> String if R has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((R -> R),%) -> %
map : (((R,R) -> R),%,%) -> %
map! : ((R -> R),%) -> % if $ has shallowlyMutable
max : (%,%) -> % if R has ORDSET
maxIndex : % -> Integer if Integer has ORDSET
member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
members : % -> List R if $ has finiteAggregate
merge : (%,%) -> % if R has ORDSET
merge : (((R,R) -> Boolean),%,%) -> %
min : (%,%) -> % if R has ORDSET
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
new : (NonNegativeInteger,R) -> %
parts : % -> List R if $ has finiteAggregate
position : (R,%) -> Integer if R has SETCAT
position : ((R -> Boolean),%) -> Integer
position : (R,%,Integer) -> Integer if R has SETCAT
qelt : (%,Integer) -> R
qsetelt! : (%,Integer,R) -> R if $ has shallowlyMutable
reduce : (((R,R) -> R),%) -> R if $ has finiteAggregate
reduce : (((R,R) -> R),%,R) -> R if $ has finiteAggregate
reduce : (((R,R) -> R),%,R,R) -> R if R has SETCAT and $ has finiteAggregate
remove : ((R -> Boolean),%) -> % if $ has finiteAggregate
remove : (R,%) -> % if R has SETCAT and $ has finiteAggregate
removeDuplicates : % -> % if R has SETCAT and $ has finiteAggregate
reverse : % -> %
reverse! : % -> % if $ has shallowlyMutable
```

```
 sample : () -> %
 select : ((R -> Boolean),%) -> % if $ has finiteAggregate
 setelt : (%,Integer,R) -> R if $ has shallowlyMutable
 setelt : (%,UniversalSegment Integer,R) -> R if $ has shallowlyMutable
 size? : (%,NonNegativeInteger) -> Boolean
 sort : % -> % if R has ORDSET
 sort : (((R,R) -> Boolean),%) -> %
 sort! : % -> % if R has ORDSET and $ has shallowlyMutable
 sort! : (((R,R) -> Boolean),%) -> % if $ has shallowlyMutable
 sorted? : % -> Boolean if R has ORDSET
 sorted? : (((R,R) -> Boolean),%) -> Boolean
 swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
 #? : % -> NonNegativeInteger if $ has finiteAggregate
 ?.? : (%,Integer) -> R
 ?.? : (%,UniversalSegment Integer) -> %
 ?=? : (%,%) -> Boolean if R has SETCAT
 ?<? : (%,%) -> Boolean if R has ORDSET
 ?<=? : (%,%) -> Boolean if R has ORDSET
 ?>? : (%,%) -> Boolean if R has ORDSET
 ?>=? : (%,%) -> Boolean if R has ORDSET
 ?~=? : (%,%) -> Boolean if R has SETCAT
```

— VectorCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#VECTCAT">
VectorCategory (VECTCAT)</a></h2>
</body>
```

— category VECTCAT VectorCategory —

```
)abbrev category VECTCAT VectorCategory
++ Description:
++ \spadtype{VectorCategory} represents the type of vector like objects,
++ that is, finite sequences indexed by some finite segment of the
++ integers. The operations available on vectors depend on the structure
++ of the underlying components. Many operations from the component domain
++ are defined for vectors componentwise. It can by assumed that extraction or
++ updating components can be done in constant time.

VectorCategory(R) : Category == SIG where
  R : Type

  SIG ==> OneDimensionalArrayAggregate R with

    if R has AbelianSemiGroup then

      _+ : (%, %) -> %
        ++ x + y returns the component-wise sum of the vectors x and y.
        ++ Error: if x and y are not of the same length.
```

```
 if R has AbelianMonoid then

   zero : NonNegativeInteger -> %
     ++ zero(n) creates a zero vector of length n.

 if R has AbelianGroup then

   _- : % -> %
     ++ -x negates all components of the vector x.

   _- : (%, %) -> %
     ++ x - y returns the component-wise difference of the vectors x and y.
     ++ Error: if x and y are not of the same length.

   _* : (Integer, %) -> %
     ++ n * y multiplies each component of the vector y by the integer n.

 if R has Monoid then

   _* : (R, %) -> %
     ++ r * y multiplies the element r times each component of the vector y.

   _* : (%, R) -> %
     ++ y * r multiplies each component of the vector y by the element r.

 if R has Ring then

   dot : (%, %) -> R
     ++ dot(x,y) computes the inner product of the two vectors x and y.
     ++ Error: if x and y are not of the same length.

   outerProduct : (%, %) -> Matrix R
     ++ outerProduct(u,v) constructs the matrix whose (i,j)'th element is
     ++ u(i)*v(j).

   cross : (%, %) -> %
     ++ cross(u,v) constructs the cross product of u and v.
     ++ Error: if u and v are not of length 3.

 if R has RadicalCategory and R has Ring then

   length : % -> R
     ++ length(v) computes the sqrt(dot(v,v)), that is, the magnitude

   magnitude : % -> R
     ++ magnitude(v) computes the sqrt(dot(v,v)), that is, the length

add

 if R has AbelianSemiGroup then

   u + v ==
     (n := #u) ^= #v => error "Vectors must be of the same length"
     map(_+ , u, v)
```

```
    if R has AbelianMonoid then

      zero n == new(n, 0)

    if R has AbelianGroup then

      - u == map(x +-> -x, u)

      n:Integer * u:% == map(x +-> n * x, u)

      u - v == u + (-v)

    if R has Monoid then

      u:% * r:R == map(x +-> x * r, u)

      r:R * u:% == map(x +-> r * x, u)

    if R has Ring then

      dot(u, v) ==
        #u ^= #v => error "Vectors must be of the same length"
        _+/[qelt(u, i) * qelt(v, i) for i in minIndex u .. maxIndex u]

      outerProduct(u, v) ==
        matrix [[qelt(u, i) * qelt(v,j) for i in minIndex u .. maxIndex u] _
                for j in minIndex v .. maxIndex v]

      cross(u, v) ==
        #u ^= 3 or #v ^= 3 => error "Vectors must be of length 3"
        construct [qelt(u, 2)*qelt(v, 3) - qelt(u, 3)*qelt(v, 2) , _
                   qelt(u, 3)*qelt(v, 1) - qelt(u, 1)*qelt(v, 3) , _
                   qelt(u, 1)*qelt(v, 2) - qelt(u, 2)*qelt(v, 1) ]

    if R has RadicalCategory and R has Ring then

      length p ==
         sqrt(dot(p,p))

      magnitude p ==
         sqrt(dot(p,p))
```

— COQ VECTCAT —

```
(* category VECTCAT *)
(*

    if R has AbelianSemiGroup then

      ?+? : (%,%) -> %
```

```
  u + v ==
    (n := #u) ^= #v => error "Vectors must be of the same length"
    map(_+ , u, v)

if R has AbelianMonoid then

  zero : NonNegativeInteger -> %
  zero n == new(n, 0)

if R has AbelianGroup then

  -? : % -> %
  - u == map(x +-> -x, u)

  ?*? : (Integer,%) -> %
  n:Integer * u:% == map(x +-> n * x, u)

  ?-? : (%,%) -> %
  u - v == u + (-v)

if R has Monoid then

  ?*? : (%,R) -> %
  u:% * r:R == map(x +-> x * r, u)

  ?*? : (R,%) -> %
  r:R * u:% == map(x +-> r * x, u)

if R has Ring then

  dot : (%,%) -> R
  dot(u, v) ==
    #u ^= #v => error "Vectors must be of the same length"
    _+/[qelt(u, i) * qelt(v, i) for i in minIndex u .. maxIndex u]

  outerProduct : (%,%) -> Matrix(R)
  outerProduct(u, v) ==
    matrix [[qelt(u, i) * qelt(v,j) for i in minIndex u .. maxIndex u] _
            for j in minIndex v .. maxIndex v]

  cross : (%,%) -> %
  cross(u, v) ==
    #u ^= 3 or #v ^= 3 => error "Vectors must be of length 3"
    construct [qelt(u, 2)*qelt(v, 3) - qelt(u, 3)*qelt(v, 2) , _
               qelt(u, 3)*qelt(v, 1) - qelt(u, 1)*qelt(v, 3) , _
               qelt(u, 1)*qelt(v, 2) - qelt(u, 2)*qelt(v, 1) ]

if R has RadicalCategory and R has Ring then

  length : % -> R
  length p ==
     sqrt(dot(p,p))

  magnitude : % -> R
```

```
      magnitude p ==
         sqrt(dot(p,p))

*)
```

———-

— VECTCAT.dotabb —

```
"VECTCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=VECTCAT"];
"VECTCAT" -> "A1AGG"
```

———-

— VECTCAT.dotfull —

```
"VectorCategory(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=VECTCAT"];
"VectorCategory(a:Type)" -> "OneDimensionalArrayAggregate(a:Type)"

"VectorCategory(a:Ring)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=VECTCAT"];
"VectorCategory(a:Ring)" -> "VectorCategory(a:Type)"
```

———-

— VECTCAT.dotpic —

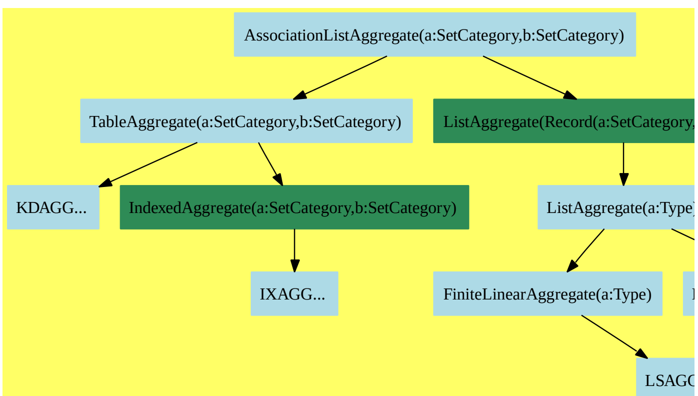
```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"VectorCategory(a:Type)" [color=lightblue];
"VectorCategory(a:Type)" -> "OneDimensionalArrayAggregate(a:Type)"

"OneDimensionalArrayAggregate(a:Type)" [color=lightblue];
"OneDimensionalArrayAggregate(a:Type)" ->
    "FiniteLinearAggregate(a:Type)"

"FiniteLinearAggregate(a:Type)" [color=lightblue];
"FiniteLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "CLAGG..."

"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" -> "IXAGG..."
```

```
"CLAGG..." [color=lightblue];
"IXAGG..." [color=lightblue];
}
```

Chapter 10

Category Layer 9

10.0.158 AssociationListAggregate (ALAGG)



— **AssociationListAggregate.input** —

```
)set break resume
)sys rm -f AssociationListAggregate.output
)spool AssociationListAggregate.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AssociationListAggregate
--R
--R AssociationListAggregate(Key: SetCategory,Entry: SetCategory) is a category constructor
--R Abbreviation for AssociationListAggregate is ALAGG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ALAGG
--R
--R------------------------------- Operations --------------------------------
```

```
--R children : % -> List(%)              concat : (%,%) -> %
--R concat : List(%) -> %                concat! : (%,%) -> %
--R copy : % -> %                        cycleEntry : % -> %
--R cycleTail : % -> %                   cyclic? : % -> Boolean
--R delete : (%,Integer) -> %            delete! : (%,Integer) -> %
--R dictionary : () -> %                 distance : (%,%) -> Integer
--R ?.rest : (%,rest) -> %               elt : (%,Key,Entry) -> Entry
--R ?.? : (%,Key) -> Entry               empty : () -> %
--R empty? : % -> Boolean                entries : % -> List(Entry)
--R eq? : (%,%) -> Boolean               explicitlyFinite? : % -> Boolean
--R index? : (Integer,%) -> Boolean      index? : (Key,%) -> Boolean
--R indices : % -> List(Integer)         indices : % -> List(Key)
--R insert : (%,%,Integer) -> %          insert! : (%,%,Integer) -> %
--R key? : (Key,%) -> Boolean            keys : % -> List(Key)
--R last : (%,NonNegativeInteger) -> %   leaf? : % -> Boolean
--R map : ((Entry -> Entry),%) -> %      nodes : % -> List(%)
--R possiblyInfinite? : % -> Boolean     qelt : (%,Key) -> Entry
--R rest : % -> %                        rest : (%,NonNegativeInteger) -> %
--R reverse : % -> %                     sample : () -> %
--R setelt : (%,Key,Entry) -> Entry      table : () -> %
--R tail : % -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?<? : (%,%) -> Boolean if Record(key: Key,entry: Entry) has ORDSET
--R ?<=? : (%,%) -> Boolean if Record(key: Key,entry: Entry) has ORDSET
--R ?=? : (%,%) -> Boolean if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCAT or Record(key: Key,en
--R ?>? : (%,%) -> Boolean if Record(key: Key,entry: Entry) has ORDSET
--R ?>=? : (%,%) -> Boolean if Record(key: Key,entry: Entry) has ORDSET
--R any? : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R any? : ((Entry -> Boolean),%) -> Boolean if $ has finiteAggregate
--R any? : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R assoc : (Key,%) -> Union(Record(key: Key,entry: Entry),"failed")
--R bag : List(Record(key: Key,entry: Entry)) -> %
--R child? : (%,%) -> Boolean if Record(key: Key,entry: Entry) has SETCAT
--R coerce : % -> OutputForm if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCAT or Record(key: Key,
--R concat : (Record(key: Key,entry: Entry),%) -> %
--R concat : (%,Record(key: Key,entry: Entry)) -> %
--R concat! : (%,Record(key: Key,entry: Entry)) -> %
--R construct : List(Record(key: Key,entry: Entry)) -> %
--R construct : List(Record(key: Key,entry: Entry)) -> %
--R convert : % -> InputForm if Record(key: Key,entry: Entry) has KONVERT(INFORM) or Record(key: Key,entry: Entr
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : ((Record(key: Key,entry: Entry) -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (Record(key: Key,entry: Entry),%) -> NonNegativeInteger if Record(key: Key,entry: Entry) has SETCAT
--R count : ((Entry -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (Entry,%) -> NonNegativeInteger if Entry has SETCAT and $ has finiteAggregate
--R count : (Record(key: Key,entry: Entry),%) -> NonNegativeInteger if Record(key: Key,entry: Entry) has SETCAT
--R count : ((Record(key: Key,entry: Entry) -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R cycleLength : % -> NonNegativeInteger
--R cycleSplit! : % -> % if $ has shallowlyMutable
--R delete : (%,UniversalSegment(Integer)) -> %
--R delete! : (%,UniversalSegment(Integer)) -> %
--R dictionary : List(Record(key: Key,entry: Entry)) -> %
--R ?.value : (%,value) -> Record(key: Key,entry: Entry)
--R ?.first : (%,first) -> Record(key: Key,entry: Entry)
```

```
--R ?.last : (%,last) -> Record(key: Key,entry: Entry)
--R ?.? : (%,UniversalSegment(Integer)) -> %
--R ?.? : (%,Integer) -> Record(key: Key,entry: Entry)
--R elt : (%,Integer,Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry)
--R entries : % -> List(Record(key: Key,entry: Entry))
--R entry? : (Record(key: Key,entry: Entry),%) -> Boolean if $ has finiteAggregate and Record(key: Key,entry: En
--R entry? : (Entry,%) -> Boolean if $ has finiteAggregate and Entry has SETCAT
--R eval : (%,List(Equation(Record(key: Key,entry: Entry)))) -> % if Record(key: Key,entry: Entry) has EVALAB(Re
--R eval : (%,Equation(Record(key: Key,entry: Entry))) -> % if Record(key: Key,entry: Entry) has EVALAB(Record(k
--R eval : (%,Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> % if Record(key: Key,entry: Entry)
--R eval : (%,List(Record(key: Key,entry: Entry)),List(Record(key: Key,entry: Entry))) -> % if Record(key: Key,e
--R eval : (%,List(Equation(Entry))) -> % if Entry has EVALAB(Entry) and Entry has SETCAT
--R eval : (%,Equation(Entry)) -> % if Entry has EVALAB(Entry) and Entry has SETCAT
--R eval : (%,Entry,Entry) -> % if Entry has EVALAB(Entry) and Entry has SETCAT
--R eval : (%,List(Entry),List(Entry)) -> % if Entry has EVALAB(Entry) and Entry has SETCAT
--R eval : (%,List(Record(key: Key,entry: Entry)),List(Record(key: Key,entry: Entry))) -> % if Record(key: Key,e
--R eval : (%,Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> % if Record(key: Key,entry: Entry)
--R eval : (%,Equation(Record(key: Key,entry: Entry))) -> % if Record(key: Key,entry: Entry) has EVALAB(Record(k
--R eval : (%,List(Equation(Record(key: Key,entry: Entry)))) -> % if Record(key: Key,entry: Entry) has EVALAB(Re
--R every? : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R every? : ((Entry -> Boolean),%) -> Boolean if $ has finiteAggregate
--R every? : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Boolean if $ has finiteAggregate
--R extract! : % -> Record(key: Key,entry: Entry)
--R fill! : (%,Record(key: Key,entry: Entry)) -> % if $ has shallowlyMutable
--R fill! : (%,Entry) -> % if $ has shallowlyMutable
--R find : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Union(Record(key: Key,entry: Entry),"failed")
--R find : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Union(Record(key: Key,entry: Entry),"failed")
--R first : % -> Record(key: Key,entry: Entry)
--R first : (%,NonNegativeInteger) -> %
--R first : % -> Entry if Key has ORDSET
--R hash : % -> SingleInteger if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCAT or Record(key: Key
--R insert : (Record(key: Key,entry: Entry),%,Integer) -> %
--R insert! : (Record(key: Key,entry: Entry),%,Integer) -> %
--R insert! : (Record(key: Key,entry: Entry),%) -> %
--R inspect : % -> Record(key: Key,entry: Entry)
--R last : % -> Record(key: Key,entry: Entry)
--R latex : % -> String if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCAT or Record(key: Key,entry
--R leaves : % -> List(Record(key: Key,entry: Entry))
--R less? : (%,NonNegativeInteger) -> Boolean
--R list : Record(key: Key,entry: Entry) -> %
--R map : ((Record(key: Key,entry: Entry) -> Record(key: Key,entry: Entry)),%) -> %
--R map : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry)),%,%)
--R map : (((Entry,Entry) -> Entry),%,%) -> %
--R map : ((Record(key: Key,entry: Entry) -> Record(key: Key,entry: Entry)),%) -> %
--R map! : ((Record(key: Key,entry: Entry) -> Record(key: Key,entry: Entry)),%) -> % if $ has shallowlyMutable
--R map! : ((Entry -> Entry),%) -> % if $ has shallowlyMutable
--R map! : ((Record(key: Key,entry: Entry) -> Record(key: Key,entry: Entry)),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if Record(key: Key,entry: Entry) has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R maxIndex : % -> Key if Key has ORDSET
--R member? : (Record(key: Key,entry: Entry),%) -> Boolean if Record(key: Key,entry: Entry) has SETCAT and $ has
--R member? : (Entry,%) -> Boolean if Entry has SETCAT and $ has finiteAggregate
--R member? : (Record(key: Key,entry: Entry),%) -> Boolean if Record(key: Key,entry: Entry) has SETCAT and $ has
--R members : % -> List(Record(key: Key,entry: Entry)) if $ has finiteAggregate
```

```
--R members : % -> List(Entry) if $ has finiteAggregate
--R members : % -> List(Record(key: Key,entry: Entry)) if $ has finiteAggregate
--R merge : (%,%) -> % if Record(key: Key,entry: Entry) has ORDSET
--R merge : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Boolean),%,%) -> %
--R merge! : (%,%) -> % if Record(key: Key,entry: Entry) has ORDSET
--R merge! : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Boolean),%,%) -> %
--R min : (%,%) -> % if Record(key: Key,entry: Entry) has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R minIndex : % -> Key if Key has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,Record(key: Key,entry: Entry)) -> %
--R node? : (%,%) -> Boolean if Record(key: Key,entry: Entry) has SETCAT
--R parts : % -> List(Record(key: Key,entry: Entry)) if $ has finiteAggregate
--R parts : % -> List(Entry) if $ has finiteAggregate
--R parts : % -> List(Record(key: Key,entry: Entry)) if $ has finiteAggregate
--R position : (Record(key: Key,entry: Entry),%,Integer) -> Integer if Record(key: Key,entry: Entry) has SETCAT
--R position : (Record(key: Key,entry: Entry),%) -> Integer if Record(key: Key,entry: Entry) has SETCAT
--R position : ((Record(key: Key,entry: Entry) -> Boolean),%) -> Integer
--R qelt : (%,Integer) -> Record(key: Key,entry: Entry)
--R qsetelt! : (%,Integer,Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry) if $ has shallowlyMuta
--R qsetelt! : (%,Key,Entry) -> Entry if $ has shallowlyMutable
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry)),%
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry)),%
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry)),%
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry)),%
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry)),%
--R reduce : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry)),%
--R remove : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (Record(key: Key,entry: Entry),%) -> % if Record(key: Key,entry: Entry) has SETCAT and $ has finite
--R remove : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (Record(key: Key,entry: Entry),%) -> % if Record(key: Key,entry: Entry) has SETCAT and $ has finite
--R remove! : (Record(key: Key,entry: Entry),%) -> % if Record(key: Key,entry: Entry) has SETCAT
--R remove! : ((Record(key: Key,entry: Entry) -> Boolean),%) -> %
--R remove! : (Key,%) -> Union(Entry,"failed")
--R remove! : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R remove! : (Record(key: Key,entry: Entry),%) -> % if $ has finiteAggregate
--R removeDuplicates : % -> % if Record(key: Key,entry: Entry) has SETCAT and $ has finiteAggregate or Record(ke
--R removeDuplicates! : % -> % if Record(key: Key,entry: Entry) has SETCAT
--R reverse! : % -> % if $ has shallowlyMutable
--R search : (Key,%) -> Union(Entry,"failed")
--R second : % -> Record(key: Key,entry: Entry)
--R select : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R select : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R select! : ((Record(key: Key,entry: Entry) -> Boolean),%) -> %
--R select! : ((Record(key: Key,entry: Entry) -> Boolean),%) -> % if $ has finiteAggregate
--R setchildren! : (%,List(%)) -> % if $ has shallowlyMutable
--R setelt : (%,value,Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry) if $ has shallowlyMutable
--R setelt : (%,first,Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry) if $ has shallowlyMutable
--R setelt : (%,rest,%) -> % if $ has shallowlyMutable
--R setelt : (%,last,Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry) if $ has shallowlyMutable
--R setelt : (%,UniversalSegment(Integer),Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry) if $ h
--R setelt : (%,Integer,Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry) if $ has shallowlyMutabl
--R setfirst! : (%,Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry) if $ has shallowlyMutable
--R setlast! : (%,Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry) if $ has shallowlyMutable
```

```
--R setrest! : (%,%) -> % if $ has shallowlyMutable
--R setvalue! : (%,Record(key: Key,entry: Entry)) -> Record(key: Key,entry: Entry) if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : % -> % if Record(key: Key,entry: Entry) has ORDSET
--R sort : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Boolean),%) -> %
--R sort! : % -> % if Record(key: Key,entry: Entry) has ORDSET and $ has shallowlyMutable
--R sort! : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Boolean),%) -> % if $ has shallow
--R sorted? : % -> Boolean if Record(key: Key,entry: Entry) has ORDSET
--R sorted? : (((Record(key: Key,entry: Entry),Record(key: Key,entry: Entry)) -> Boolean),%) -> Boolean
--R split! : (%,Integer) -> % if $ has shallowlyMutable
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R swap! : (%,Key,Key) -> Void if $ has shallowlyMutable
--R table : List(Record(key: Key,entry: Entry)) -> %
--R third : % -> Record(key: Key,entry: Entry)
--R value : % -> Record(key: Key,entry: Entry)
--R ?~=? : (%,%) -> Boolean if Record(key: Key,entry: Entry) has SETCAT or Entry has SETCAT or Record(key: Key,e
--R
--E 1

)spool
)lisp (bye)
```

— **AssociationListAggregate.help** —

```
====================================================================
AssociationListAggregate examples
====================================================================

An association list is a list of key entry pairs which may be viewed
as a table. It is a poor mans version of a table; searching for a key
is a linear operation.

See Also:
o )show AssociationListAggregate
```

See:

⇐ "ListAggregate" (LSAGG) 8.0.138 on page 827
⇐ "TableAggregate" (TBAGG) 9.0.156 on page 972

Exports:

any?	assoc	bag	children
child?	coerce	concat	concat!
construct	convert	copy	copyInto!
count	cycleEntry	cycleLength	cycleSplit!
cycleTail	cyclic?	delete	delete!
dictionary	distance	elt	empty
empty?	entries	entry?	eq?
eval	every?	explicitlyFinite?	extract!
fill!	find	first	hash
index?	indices	insert	insert!
inspect	key?	keys	last
latex	leaf?	leaves	less?
list	map	map!	max
maxIndex	member?	members	merge
merge!	min	minIndex	more?
new	nodes	node?	parts
position	possiblyInfinite?	qelt	qsetelt!
reduce	remove	remove!	removeDuplicates
removeDuplicates!	rest	reverse	reverse!
sample	search	second	select
select!	setchildren!	setelt	setfirst!
setlast!	setrest!	setvalue!	size?
sort	sort!	sorted?	split!
swap!	table	tail	third
value	#?	?<?	?<=?
?=?	?>?	?>=?	?~=?
?.rest	?.value	?.first	?.last
?.?			

Attributes exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These are directly exported but not implemented:

```
assoc : (Key,%) -> Union(Record(key: Key,entry: Entry),"failed")
```

These exports come from (p972) TableAggregate(Key, Entry)
where Key:SetCategory and Entry:SetCategory
and RecKE = Record(key: Key,entry: Entry)

```
any? : ((RecKE -> Boolean),%) -> Boolean
         if $ has finiteAggregate
any? : ((Entry -> Boolean),%) -> Boolean
         if $ has finiteAggregate
any? : ((RecKE -> Boolean),%) -> Boolean
         if $ has finiteAggregate
bag : List RecKE -> %
construct : List RecKE -> %
convert : % -> InputForm
```

```
         if RecKE has KONVERT INFORM
         or RecKE has KONVERT INFORM
copy : % -> %
count :
 ((RecKE -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
count : (RecKE,%) -> NonNegativeInteger
         if RecKE has SETCAT
         and $ has finiteAggregate
count : ((Entry -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
count : (Entry,%) -> NonNegativeInteger
         if Entry has SETCAT
         and $ has finiteAggregate
count : (RecKE,%) -> NonNegativeInteger
         if RecKE has SETCAT
         and $ has finiteAggregate
count :
 ((RecKE -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
dictionary : () -> %
dictionary : List RecKE -> %
elt : (%,Key,Entry) -> Entry
elt : (%,Integer,RecKE) -> RecKE
empty : () -> %
empty? : % -> Boolean
entries : % -> List Entry
entry? : (Entry,%) -> Boolean
         if $ has finiteAggregate
         and Entry has SETCAT
eq? : (%,%) -> Boolean
eval : (%,List Equation RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,Equation RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,RecKE,RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,List RecKE,List RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,List Equation Entry) -> %
         if Entry has EVALAB Entry
         and Entry has SETCAT
eval : (%,Equation Entry) -> %
         if Entry has EVALAB Entry
         and Entry has SETCAT
eval : (%,Entry,Entry) -> %
         if Entry has EVALAB Entry
         and Entry has SETCAT
eval : (%,List Entry,List Entry) -> %
         if Entry has EVALAB Entry
```

```
         and Entry has SETCAT
eval : (%,List RecKE,List RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,RecKE,RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,Equation RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,List Equation RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
every? : ((RecKE -> Boolean),%) -> Boolean
         if $ has finiteAggregate
every? : ((Entry -> Boolean),%) -> Boolean
         if $ has finiteAggregate
extract! : % -> RecKE
fill! : (%,Entry) -> % if $ has shallowlyMutable
find : ((RecKE -> Boolean),%) -> Union(RecKE,"failed")
first : % -> Entry if Key has ORDSET
hash : % -> SingleInteger
         if RecKE has SETCAT
         or Entry has SETCAT
         or RecKE has SETCAT
index? : (Key,%) -> Boolean
indices : % -> List Key
insert! : (RecKE,%) -> %
inspect : % -> RecKE
key? : (Key,%) -> Boolean
keys : % -> List Key
latex : % -> String
         if RecKE has SETCAT
         or Entry has SETCAT
         or RecKE has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((Entry -> Entry),%) -> %
map : ((RecKE -> RecKE),%) -> %
map : (((Entry,Entry) -> Entry),%,%) -> %
map! : ((RecKE -> RecKE),%) -> %
         if $ has shallowlyMutable
map! : ((Entry -> Entry),%) -> % if $ has shallowlyMutable
map! : ((RecKE -> RecKE),%) -> %
         if $ has shallowlyMutable
maxIndex : % -> Key if Key has ORDSET
member? : (RecKE,%) -> Boolean
         if RecKE has SETCAT
         and $ has finiteAggregate
member? : (Entry,%) -> Boolean
         if Entry has SETCAT
         and $ has finiteAggregate
members : % -> List RecKE if $ has finiteAggregate
members : % -> List Entry if $ has finiteAggregate
members : % -> List RecKE if $ has finiteAggregate
```

```
minIndex : % -> Key if Key has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List Entry if $ has finiteAggregate
parts : % -> List RecKE if $ has finiteAggregate
qelt : (%,Key) -> Entry
qsetelt! : (%,Key,Entry) -> Entry if $ has shallowlyMutable
reduce :
 (((RecKE,RecKE) -> RecKE),%)
   -> RecKE
         if $ has finiteAggregate
reduce :
 (((RecKE,RecKE) -> RecKE),%,RecKE)
   -> RecKE
         if $ has finiteAggregate
reduce :
 (((RecKE,RecKE) -> RecKE),%,RecKE,RecKE)
   -> RecKE
         if RecKE has SETCAT
         and $ has finiteAggregate
remove : ((RecKE -> Boolean),%) -> % if $ has finiteAggregate
remove : (RecKE,%) -> %
         if RecKE has SETCAT
         and $ has finiteAggregate
remove! : (Key,%) -> Union(Entry,"failed")
remove! : (RecKE,%) -> % if RecKE has SETCAT
remove! : (RecKE,%) -> % if $ has finiteAggregate
removeDuplicates : % -> %
         if RecKE has SETCAT
         and $ has finiteAggregate
         or RecKE has SETCAT
         and $ has finiteAggregate
sample : () -> %
search : (Key,%) -> Union(Entry,"failed")
select : ((RecKE -> Boolean),%) -> %
         if $ has finiteAggregate
select! : ((RecKE -> Boolean),%) -> %
         if $ has finiteAggregate
setelt : (%,Key,Entry) -> Entry
size? : (%,NonNegativeInteger) -> Boolean
swap! : (%,Key,Key) -> Void if $ has shallowlyMutable
table : () -> %
table : List RecKE -> %
?~=? : (%,%) -> Boolean
         if RecKE has SETCAT
         or Entry has SETCAT
         or RecKE has SETCAT
?.? : (%,Key) -> Entry
```

These exports come from (p827) ListAggregate(a)
where a is Record(key:Key,entry:Entry)
and RecKE=Record(key: Key,entry: Entry)

```
children : % -> List %
child? : (%,%) -> Boolean if RecKE has SETCAT
coerce : % -> OutputForm
```

```
         if RecKE has SETCAT
         or Entry has SETCAT
         or RecKE has SETCAT
concat : (%,%) -> %
concat : List % -> %
concat : (RecKE,%) -> %
concat : (%,RecKE) -> %
concat! : (%,%) -> %
concat! : (%,RecKE) -> %
copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
cycleEntry : % -> %
cycleLength : % -> NonNegativeInteger
cycleSplit! : % -> % if $ has shallowlyMutable
cycleTail : % -> %
cyclic? : % -> Boolean
delete : (%,Integer) -> %
delete! : (%,Integer) -> %
delete : (%,UniversalSegment Integer) -> %
delete! : (%,UniversalSegment Integer) -> %
distance : (%,%) -> Integer
entries : % -> List RecKE
entry? : (RecKE,%) -> Boolean
         if $ has finiteAggregate
         and RecKE has SETCAT
explicitlyFinite? : % -> Boolean
fill! : (%,RecKE) -> % if $ has shallowlyMutable
first : % -> RecKE
first : (%,NonNegativeInteger) -> %
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (%,%,Integer) -> %
insert : (RecKE,%,Integer) -> %
insert! : (%,%,Integer) -> %
insert! : (RecKE,%,Integer) -> %
last : % -> RecKE
last : (%,NonNegativeInteger) -> %
leaf? : % -> Boolean
leaves : % -> List RecKE
list : RecKE -> %
map : (((RecKE,RecKE) -> RecKE),%,%) -> %
max : (%,%) -> % if RecKE has ORDSET
maxIndex : % -> Integer if Integer has ORDSET
merge : (%,%) -> % if RecKE has ORDSET
merge : (((RecKE,RecKE) -> Boolean),%,%) -> %
merge! : (%,%) -> % if RecKE has ORDSET
merge! : (((RecKE,RecKE) -> Boolean),%,%) -> %
min : (%,%) -> % if RecKE has ORDSET
minIndex : % -> Integer if Integer has ORDSET
new : (NonNegativeInteger,RecKE) -> %
nodes : % -> List %
node? : (%,%) -> Boolean if RecKE has SETCAT
position : (RecKE,%,Integer) -> Integer
         if RecKE has SETCAT
position : (RecKE,%) -> Integer
```

```
         if RecKE has SETCAT
position : ((RecKE -> Boolean),%) -> Integer
possiblyInfinite? : % -> Boolean
qelt : (%,Integer) -> RecKE
qsetelt! : (%,Integer,RecKE) -> RecKE
         if $ has shallowlyMutable
remove! : ((RecKE -> Boolean),%) -> %
remove! : ((RecKE -> Boolean),%) -> %
         if $ has finiteAggregate
removeDuplicates! : % -> % if RecKE has SETCAT
rest : % -> %
rest : (%,NonNegativeInteger) -> %
reverse : % -> %
reverse! : % -> % if $ has shallowlyMutable
second : % -> RecKE
select! : ((RecKE -> Boolean),%) -> %
setchildren! : (%,List %) -> % if $ has shallowlyMutable
setelt : (%,value,RecKE) -> RecKE
         if $ has shallowlyMutable
setelt : (%,first,RecKE) -> RecKE
         if $ has shallowlyMutable
setelt : (%,rest,%) -> % if $ has shallowlyMutable
setelt : (%,last,RecKE) -> RecKE
         if $ has shallowlyMutable
setelt : (%,UniversalSegment Integer,RecKE) -> RecKE
         if $ has shallowlyMutable
setelt : (%,Integer,RecKE) -> RecKE
         if $ has shallowlyMutable
setfirst! : (%,RecKE) -> RecKE
         if $ has shallowlyMutable
setlast! : (%,RecKE) -> RecKE
         if $ has shallowlyMutable
setrest! : (%,%) -> % if $ has shallowlyMutable
setvalue! : (%,RecKE) -> RecKE
         if $ has shallowlyMutable
sort : % -> % if RecKE has ORDSET
sort : (((RecKE,RecKE) -> Boolean),%) -> %
sort! : % -> %
         if RecKE has ORDSET
         and $ has shallowlyMutable
sort! : (((RecKE,RecKE) -> Boolean),%) -> %
         if $ has shallowlyMutable
sorted? : % -> Boolean if RecKE has ORDSET
sorted? : (((RecKE,RecKE) -> Boolean),%) -> Boolean
split! : (%,Integer) -> % if $ has shallowlyMutable
swap! : (%,Integer,Integer) -> Void
         if $ has shallowlyMutable
tail : % -> %
third : % -> RecKE
value : % -> RecKE
#? : % -> NonNegativeInteger if $ has finiteAggregate
?<? : (%,%) -> Boolean if RecKE has ORDSET
?<=? : (%,%) -> Boolean if RecKE has ORDSET
?=? : (%,%) -> Boolean
```

```
          if RecKE has SETCAT
          or Entry has SETCAT
          or RecKE has SETCAT
 ?>? : (%,%) -> Boolean if RecKE has ORDSET
 ?>=? : (%,%) -> Boolean if RecKE has ORDSET
 ?.value : (%,value) -> RecKE
 ?.first : (%,first) -> RecKE
 ?.last : (%,last) -> RecKE
 ?.rest : (%,rest) -> %
 ?.? : (%,UniversalSegment Integer) -> %
 ?.? : (%,Integer) -> RecKE
```

— AssociationListAggregate.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ALAGG">
AssociationListAggregate (ALAGG)</a></h2>
</body>
```

— category ALAGG AssociationListAggregate —

```
)abbrev category ALAGG AssociationListAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Description:
++ An association list is a list of key entry pairs which may be viewed
++ as a table.        It is a poor mans version of a table:
++ searching for a key is a linear operation.

AssociationListAggregate(Key,Entry) : Category == SIG where
  Key : SetCategory
  Entry : SetCategory

  TA ==> TableAggregate(Key,Entry)
  LA ==> ListAggregate(Record(key:Key,entry:Entry))

  SIG ==> Join(TA,LA) with

    assoc : (Key, %) -> Union(Record(key:Key,entry:Entry), "failed")
      ++ assoc(k,u) returns the element x in association list u stored
      ++ with key k, or "failed" if u has no key k.
```

— ALAGG.dotabb —

```
"ALAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=ALAGG"];
"ALAGG" -> "TBAGG"
"ALAGG" -> "LSAGG"
```

———-

— **ALAGG.dotfull** —

```
"AssociationListAggregate(a:SetCategory,b:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ALAGG"];
"AssociationListAggregate(a:SetCategory,b:SetCategory)" ->
    "TableAggregate(a:SetCategory,b:SetCategory)"
"AssociationListAggregate(a:SetCategory,b:SetCategory)" ->
    "ListAggregate(Record(a:SetCategory,b:SetCategory))"
```

———-

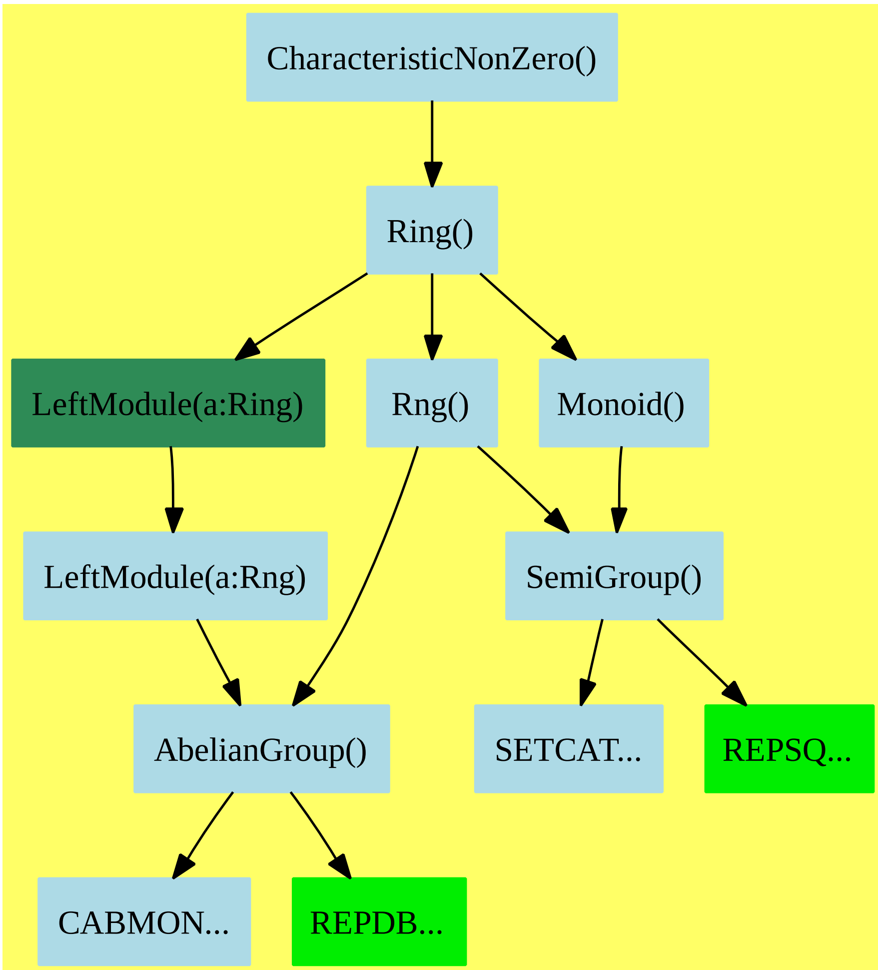
— **ALAGG.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"AssociationListAggregate(a:SetCategory,b:SetCategory)" [color=lightblue];
"AssociationListAggregate(a:SetCategory,b:SetCategory)" ->
    "TableAggregate(a:SetCategory,b:SetCategory)"
"AssociationListAggregate(a:SetCategory,b:SetCategory)" ->
    "ListAggregate(Record(a:SetCategory,b:SetCategory))"

"TableAggregate(a:SetCategory,b:SetCategory)" [color=lightblue];
"TableAggregate(a:SetCategory,b:SetCategory)" -> "KDAGG..."
"TableAggregate(a:SetCategory,b:SetCategory)" ->
    "IndexedAggregate(a:SetCategory,b:SetCategory)"

"IndexedAggregate(a:SetCategory,b:SetCategory)" [color=seagreen];
"IndexedAggregate(a:SetCategory,b:SetCategory)" -> "IXAGG..."

"ListAggregate(Record(a:SetCategory,b:SetCategory))" [color=seagreen];
"ListAggregate(Record(a:SetCategory,b:SetCategory))" ->
    "ListAggregate(a:Type)"

"ListAggregate(a:Type)" [color=lightblue];
"ListAggregate(a:Type)" -> "FiniteLinearAggregate(a:Type)"
"ListAggregate(a:Type)" -> "ExtensibleLinearAggregate(a:Type)"

"FiniteLinearAggregate(a:Type)" [color=lightblue];
"FiniteLinearAggregate(a:Type)" -> "LSAGG..."

"ExtensibleLinearAggregate(a:Type)" [color=lightblue];
"ExtensibleLinearAggregate(a:Type)" -> "LSAGG..."

"KDAGG..." [color=lightblue];
"IXAGG..." [color=lightblue];
"LSAGG..." [color=lightblue];
```

```
}
```

———-

10.0.159 CharacteristicNonZero (CHARNZ)



— **CharacteristicNonZero.input** —

```
)set break resume
)sys rm -f CharacteristicNonZero.output
)spool CharacteristicNonZero.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show CharacteristicNonZero
```

```
--R
--R CharacteristicNonZero is a category constructor
--R Abbreviation for CharacteristicNonZero is CHARNZ
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for CHARNZ
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R one? : % -> Boolean                   recip : % -> Union(%,"failed")
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

— **CharacteristicNonZero.help** —

```
====================================================================
CharacteristicNonZero examples
====================================================================

The category of Rings of Characteristic Non Zero

See Also:
o )show CharacteristicNonZero
```

———-

See:

⇒ "FieldOfPrimeCharacteristic" (FPC) 17.0.217 on page 1531
⇒ "FiniteRankAlgebra" (FINRALG) 17.0.218 on page 1537
⇒ "FunctionSpace" (FS) 17.0.219 on page 1544
⇒ "QuotientFieldCategory" (QFCAT) 17.0.222 on page 1595
⇐ "Ring" (RING) 9.0.153 on page 946

Exports:

1	0	coerce	hash	latex
one?	recip	sample	zero?	characteristic
charthRoot	subtractIfCan	?~=?	?^?	?*?
?**?	?+?	?-?	-?	?=?

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

These are directly exported but not implemented:

```
charthRoot : % -> Union(%,"failed")
```

These exports come from (p946) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
```

See: Grabmeier[Grab92]

— **CharacteristicNonZero.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#CHARNZ">
CharacteristicNonZero (CHARNZ)</a></h2>
</body>
```

——

— **category CHARNZ CharacteristicNonZero** —

```
)abbrev category CHARNZ CharacteristicNonZero
++ References:
++ Grab92 Finite Fields in Axiom
```

```
++ Description:
++ Rings of Characteristic Non Zero

CharacteristicNonZero() : Category == SIG where

  SIG ==> Ring with

    charthRoot : % -> Union(%,"failed")
      ++ charthRoot(x) returns the pth root of x
      ++ where p is the characteristic of the ring.
```

———-

— CHARNZ.dotabb —

```
"CHARNZ"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARNZ"];
"CHARNZ" -> "RING"
```

———-

— CHARNZ.dotfull —

```
"CharacteristicNonZero()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARNZ"];
"CharacteristicNonZero()" -> "Ring()"
```

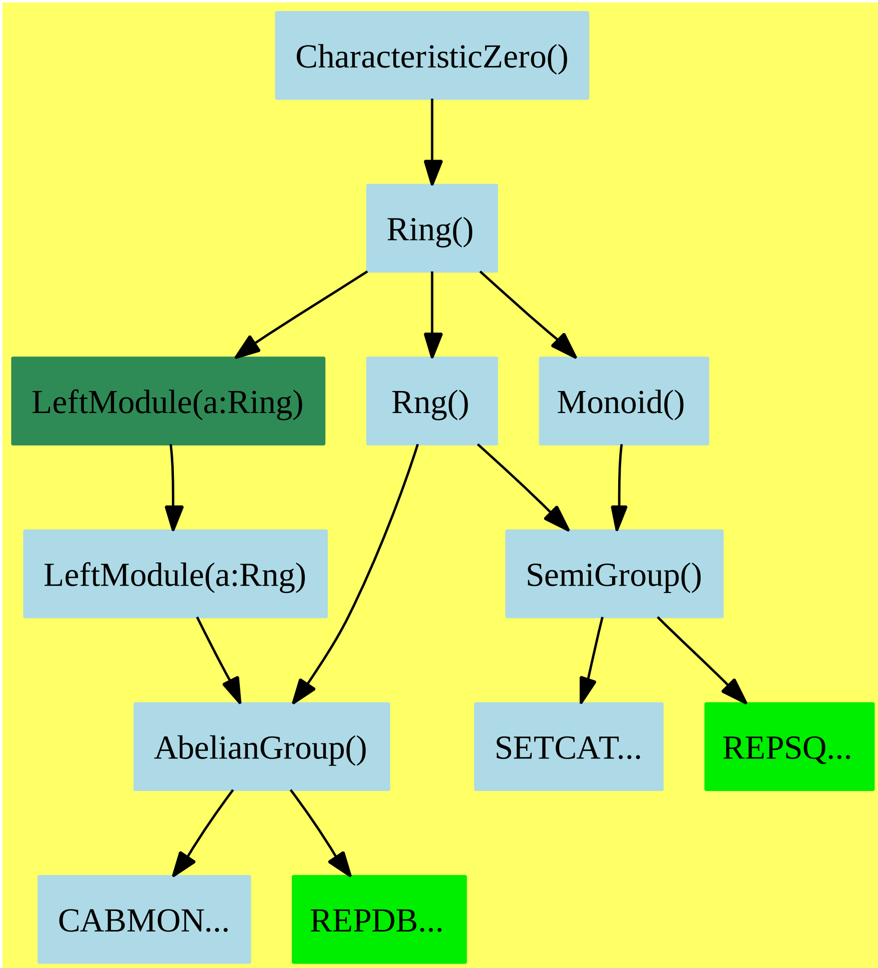
———-

— CHARNZ.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"CharacteristicNonZero()" [color=lightblue];
"CharacteristicNonZero()" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"
```

```
"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

10.0.160 CharacteristicZero (CHARZ)



— **CharacteristicZero.input** —

```
)set break resume
)sys rm -f CharacteristicZero.output
)spool CharacteristicZero.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show CharacteristicZero
--R
--R CharacteristicZero is a category constructor
--R Abbreviation for CharacteristicZero is CHARZ
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for CHARZ
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                    ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %   ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %  ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                    ?-? : (%,%) -> %
--R -? : % -> %                         ?=? : (%,%) -> Boolean
--R 1 : () -> %                         0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %   ?^? : (%,PositiveInteger) -> %
--R coerce : Integer -> %               coerce : % -> OutputForm
--R hash : % -> SingleInteger           latex : % -> String
--R one? : % -> Boolean                 recip : % -> Union(%,"failed")
--R sample : () -> %                    zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

— CharacteristicZero.help —

```
====================================================================
CharacteristicZero examples
====================================================================

The category of Rings of Characteristic Zero.

See Also:
o )show CharacteristicZero
```

———-

See:

⇒ "FiniteRankAlgebra" (FINRALG) 17.0.218 on page 1537
⇒ "FunctionSpace" (FS) 17.0.219 on page 1544
⇒ "IntegerNumberSystem" (INS) 16.0.210 on page 1420
⇒ "PAdicIntegerCategory" (PADICCT) 16.0.212 on page 1443
⇒ "QuotientFieldCategory" (QFCAT) 17.0.222 on page 1595
⇒ "RealClosedField" (RCFIELD) 17.0.223 on page 1611
⇒ "RealNumberSystem" (RNS) 17.0.224 on page 1622
⇐ "Ring" (RING) 9.0.153 on page 946

Exports:

1	0	coerce	hash	latex
one?	recip	sample	zero?	characteristic
subtractIfCan	?~=?	?^?	?*?	
?**?	?+?	?-?	-?	?=?

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

These exports come from (p946) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
```

— CharacteristicZero.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#CHARZ">
CharacteristicZero (CHARZ)</a></h2>
</body>
```

— category CHARZ CharacteristicZero —

```
)abbrev category CHARZ CharacteristicZero
++ Description:
++ Rings of Characteristic Zero.

CharacteristicZero() : Category == SIG where
```

```
  SIG ==> Ring
```

———-

— CHARZ.dotabb —

```
"CHARZ"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARZ"];
"CHARZ" -> "RING"
```

———-

— CHARZ.dotfull —

```
"CharacteristicZero()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARZ"];
"CharacteristicZero()" -> "Ring()"
```

———-

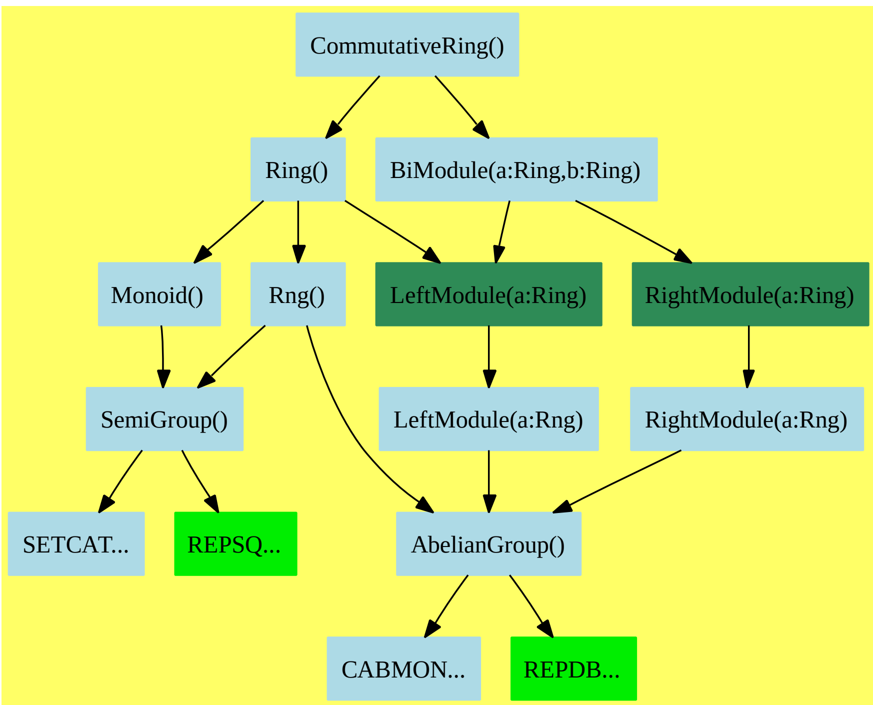
— CHARZ.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"CharacteristicZero()" [color=lightblue];
"CharacteristicZero()" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
```

```
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

———-

10.0.161 CommutativeRing (COMRING)



— **CommutativeRing.input** —

```
)set break resume
)sys rm -f CommutativeRing.output
)spool CommutativeRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show CommutativeRing
--R
--R CommutativeRing is a category constructor
```

```
--R Abbreviation for CommutativeRing is COMRING
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for COMRING
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                       ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                       ?-? : (%,%) -> %
--R -? : % -> %                            ?=? : (%,%) -> Boolean
--R 1 : () -> %                            0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R coerce : Integer -> %                  coerce : % -> OutputForm
--R hash : % -> SingleInteger              latex : % -> String
--R one? : % -> Boolean                    recip : % -> Union(%,"failed")
--R sample : () -> %                       zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

— **CommutativeRing.help** —

```
====================================================================
CommutativeRing examples
====================================================================

The category of commutative rings with unity, rings where * is
commutative, and which have a multiplicative identity element.

See Also:
o )show CommutativeRing
```

———-

Commutative Rings are a subset of IntegralDomains.
⇒ "IntegralDomain" (INTDOM) 12.0.188 on page 1225.

See:

⇒ "ComplexCategory" (COMPCAT) 20.0.240 on page 1869
⇒ "IntegralDomain" (INTDOM) 12.0.188 on page 1225
⇒ "FunctionSpace" (FS) 17.0.219 on page 1544
⇒ "MonogenicAlgebra" (MONOGEN) 19.0.238 on page 1847
⇒ "RealClosedField" (RCFIELD) 17.0.223 on page 1611
⇒ "UnivariatePolynomialCategory" (UPOLYC) 17.0.228 on page 1708
⇐ "BiModule" (BMODULE) 9.0.146 on page 902

⇐ "Ring" (RING) 9.0.153 on page 946

Exports:

1	0	coerce	hash	latex
one?	recip	sample	zero?	characteristic
subtractIfCan	?~=?	?^?	?*?	?**?
?+?	?-?	-?	?=?	

Attributes exported:

- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These exports come from (p946) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
```

— CommutativeRing.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#COMRING">
CommutativeRing (COMRING)</a></h2>
</body>
```

———-

— category COMRING CommutativeRing —

```
)abbrev category COMRING CommutativeRing
++ Description:
++ The category of commutative rings with unity, rings where
++ \spadop{*} is commutative, and which have a multiplicative identity
++ element.

CommutativeRing() : Category == SIG where

  SIG ==> Join(Ring,BiModule(%,%)) with

    commutative("*")  ++ multiplication is commutative.
```

———-

— COMRING.dotabb —

```
"COMRING"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=COMRING"];
"COMRING" -> "RING"
"COMRING" -> "BMODULE"
```

———-

— COMRING.dotfull —

```
"CommutativeRing()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=COMRING"];
"CommutativeRing()" -> "Ring()"
"CommutativeRing()" -> "BiModule(a:Ring,b:Ring)"
```

———-

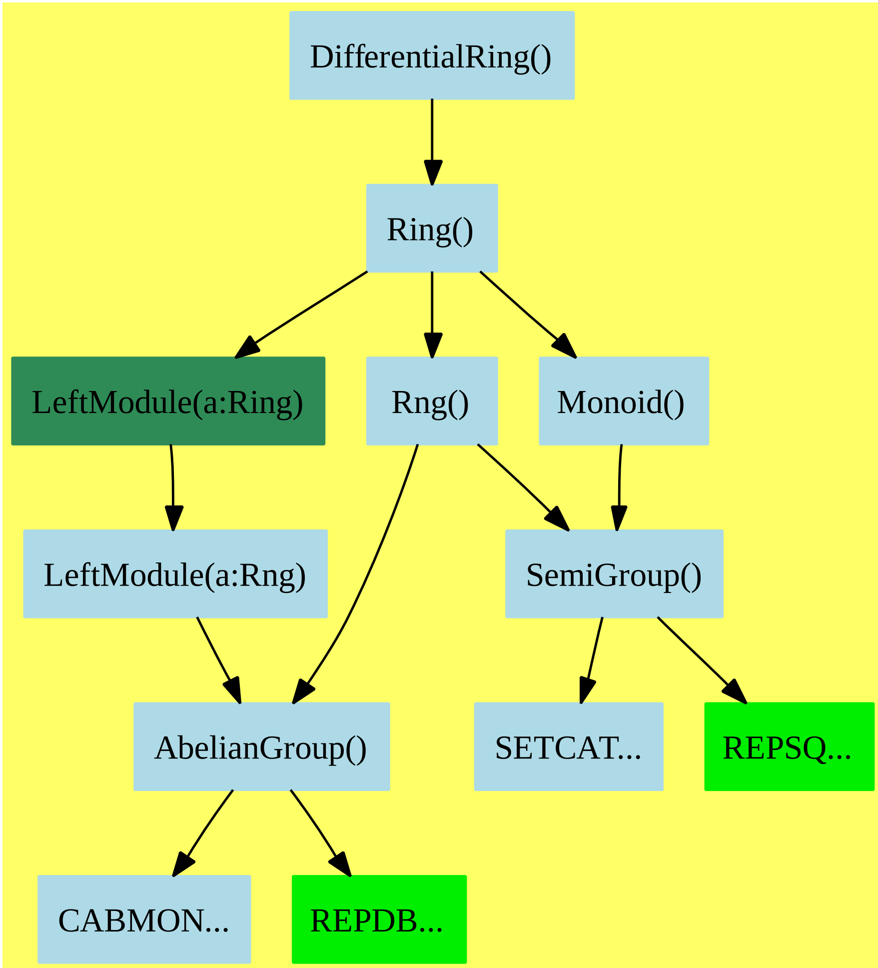
— COMRING.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "Ring()"
"CommutativeRing()" -> "BiModule(a:Ring,b:Ring)"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
```

```
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "AbelianGroup()"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

———-

10.0.162 DifferentialRing (DIFRING)



— **DifferentialRing.input** —

```
)set break resume
)sys rm -f DifferentialRing.output
)spool DifferentialRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DifferentialRing
--R
--R DifferentialRing is a category constructor
--R Abbreviation for DifferentialRing is DIFRING
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for DIFRING
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %    ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %   ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R D : (%,NonNegativeInteger) -> %      D : % -> %
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %    ?^? : (%,PositiveInteger) -> %
--R coerce : Integer -> %                coerce : % -> OutputForm
--R differentiate : % -> %               hash : % -> SingleInteger
--R latex : % -> String                  one? : % -> Boolean
--R recip : % -> Union(%,"failed")       sample : () -> %
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R differentiate : (%,NonNegativeInteger) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— DifferentialRing.help —

```
====================================================================
DifferentialRing examples
====================================================================

An ordinary differential ring, that is, a ring with an operation
differentiate.

Axioms:
        differentiate(x+y) = differentiate(x)+differentiate(y)
        differentiate(x*y) = x*differentiate(y) + differentiate(x)*y

See Also:
o )show DifferentialRing
```

See:

⇒ "IntegerNumberSystem" (INS) 16.0.210 on page 1420
⇒ "FiniteFieldCategory" (FFIELDC) 18.0.231 on page 1762
⇒ "UnivariatePolynomialCategory" (UPOLYC) 17.0.228 on page 1708
⇐ "Ring" (RING) 9.0.153 on page 946

Exports:

1	0	characteristic	coerce	D
differentiate	hash	latex	one?	recip
sample	subtractIfCan	zero?	?~=?	?**?
?^?	?*?	?+?	?-?	-?
?=?				

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

These are directly exported but not implemented:

```
differentiate : % -> %
```

These are implemented by this category:

```
D : % -> %
D : (%,NonNegativeInteger) -> %
differentiate : (%,NonNegativeInteger) -> %
```

These exports come from (p946) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
```

— **DifferentialRing.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#DIFRING">
DifferentialRing (DIFRING)</a></h2>
</body>
```

————

— category DIFRING DifferentialRing —

```
)abbrev category DIFRING DifferentialRing
++ Description:
++ An ordinary differential ring, that is, a ring with an operation
++ \spadfun{differentiate}.
++
++ Axioms\br
++ \tab{5}\spad{differentiate(x+y) = differentiate(x)+differentiate(y)}\br
++ \tab{5}\spad{differentiate(x*y) = x*differentiate(y) + differentiate(x)*y}

DifferentialRing() : Category == SIG where

  SIG ==> Ring with

    differentiate : % -> %
      ++ differentiate(x) returns the derivative of x.
      ++ This function is a simple differential operator
      ++ where no variable needs to be specified.

    D : % -> %
      ++ D(x) returns the derivative of x.
      ++ This function is a simple differential operator
      ++ where no variable needs to be specified.

    differentiate : (%, NonNegativeInteger) -> %
      ++ differentiate(x, n) returns the n-th derivative of x.

    D : (%, NonNegativeInteger) -> %
      ++ D(x, n) returns the n-th derivative of x.

   add

     D r == differentiate r

     differentiate(r, n) ==
       for i in 1..n repeat r := differentiate r
       r

     D(r,n) == differentiate(r,n)
```

———

— COQ DIFRING —

```
(* category DIFRING *)
(*

    D : % -> %
    D r == differentiate r

    differentiate : (%,NonNegativeInteger) -> %
    differentiate(r, n) ==
```

```
      for i in 1..n repeat r := differentiate r
      r

    D : (%,NonNegativeInteger) -> %
    D(r,n) == differentiate(r,n)

*)
```

———-

— DIFRING.dotabb —

```
"DIFRING"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIFRING"];
"DIFRING" -> "RING"
```

———-

— DIFRING.dotfull —

```
"DifferentialRing()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIFRING"];
"DifferentialRing()" -> "Ring()"
```

———-

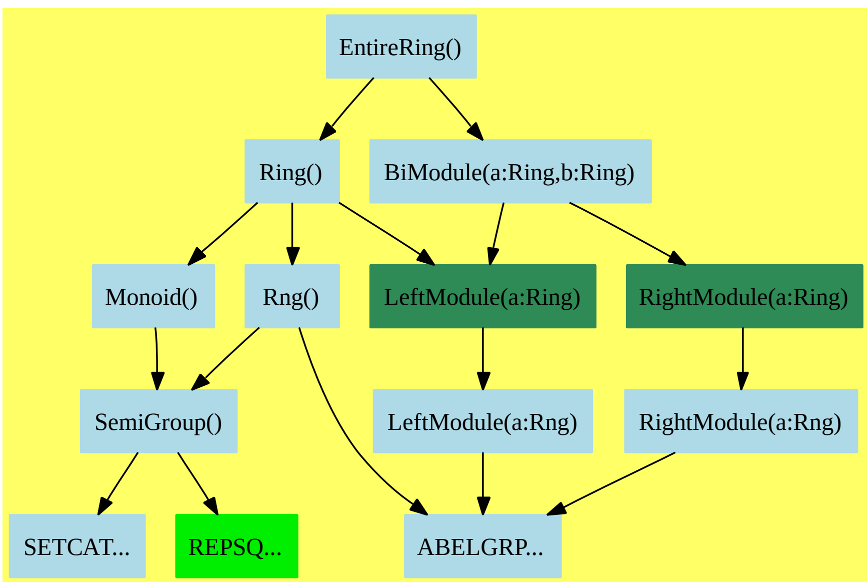
— DIFRING.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"DifferentialRing()" [color=lightblue];
"DifferentialRing()" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"
```

```
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

———-

10.0.163 EntireRing (ENTIRER)



— **EntireRing.input** —

```
)set break resume
)sys rm -f EntireRing.output
)spool EntireRing.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show EntireRing
--R
--R EntireRing is a category constructor
--R Abbreviation for EntireRing is ENTIRER
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ENTIRER
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R one? : % -> Boolean                   recip : % -> Union(%,"failed")
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

— **EntireRing.help** —

```
====================================================================
EntireRing examples
====================================================================

Entire Rings (non-commutative Integral Domains), a ring
not necessarily commutative which has no zero divisors.

Axioms:
        ab=0 => a=0 or b=0  -- known as noZeroDivisors
        not(1=0)

See Also:
o )show EntireRing
```

———-

See:

⇒ "DivisionRing" (DIVRING) 12.0.185 on page 1185
⇒ "IntegralDomain" (INTDOM) 12.0.188 on page 1225

⇐ “BiModule” (BMODULE) 9.0.146 on page 902
⇐ “Ring” (RING) 9.0.153 on page 946

Exports:

1	0	characteristic	coerce	hash
latex	one?	recip	sample	subtractIfCan
zero?	?^?	?~=?	?*?	?**?
?+?	?-?	-?	?=?	

Attributes Exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These exports come from (p946) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
```

— **EntireRing.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ENTIRER">
EntireRing (ENTIRER)</a></h2>
</body>
```

———

— category ENTIRER EntireRing —

```
)abbrev category ENTIRER EntireRing
++ Description:
++ Entire Rings (non-commutative Integral Domains), a ring
++ not necessarily commutative which has no zero divisors.
++
++ Axioms\br
++ \tab{5}\spad{ab=0 => a=0 or b=0}  -- known as noZeroDivisors\br
++ \tab{5}\spad{not(1=0)}

EntireRing() : Category == SIG where

  SIG ==> Join(Ring,BiModule(%,%)) with

    noZeroDivisors
      ++ if a product is zero then one of the factors
      ++ must be zero.
```

— COQ ENTIRER —

```
(* category ENTIRER *)
(*
Entire Rings (non-commutative Integral Domains), a ring
not necessarily commutative which has no zero divisors.

Axioms
noZeroDivisors  ab=0 => a=0 or b=0
  not(1=0)

*)
```

— ENTIRER.dotabb —

```
"ENTIRER"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ENTIRER"];
"ENTIRER" -> "RING"
"ENTIRER" -> "BMODULE"
```

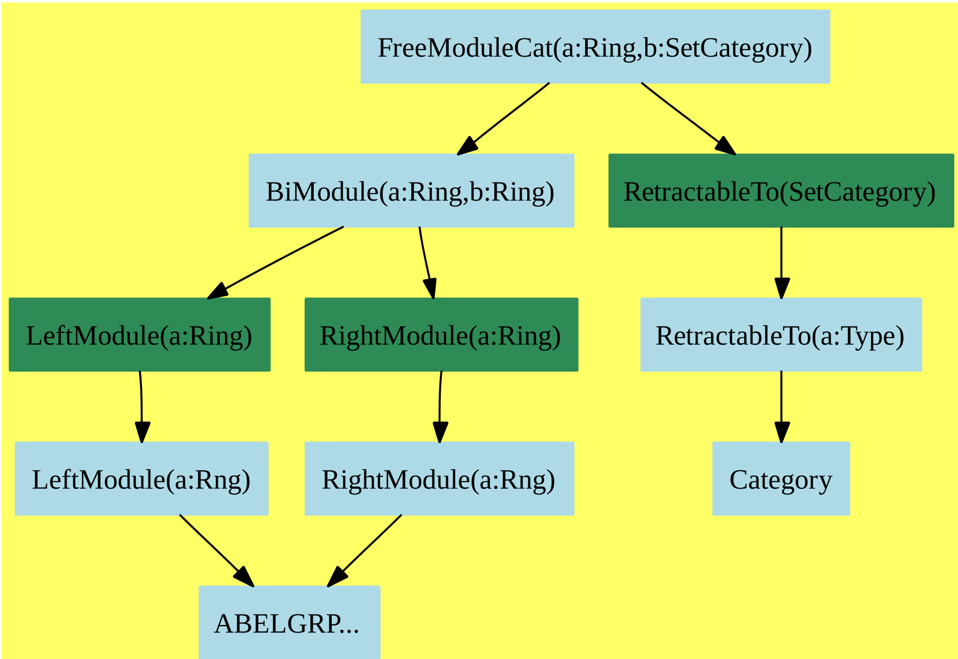
— ENTIRER.dotfull —

```
"EntireRing()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ENTIRER"];
"EntireRing()" -> "Ring()"
"EntireRing()" -> "BiModule(a:Ring,b:Ring)"
```

— **ENTIRER.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"EntireRing()" [color=lightblue];
"EntireRing()" -> "Ring()"
"EntireRing()" -> "BiModule(a:Ring,b:Ring)"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "ABELGRP..."
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"ABELGRP..." [color=lightblue];
}
```

10.0.164 FreeModuleCat (FMCAT)



— **FreeModuleCat.input** —

```
)set break resume
)sys rm -f FreeModuleCat.output
)spool FreeModuleCat.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FreeModuleCat
--R
--R FreeModuleCat(R: Ring,Basis: SetCategory) is a category constructor
--R Abbreviation for FreeModuleCat is FMCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FMCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,Basis) -> %                  ?*? : (%,R) -> %
--R ?*? : (R,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 0 : () -> %                           coefficient : (%,Basis) -> R
--R coefficients : % -> List(R)           coerce : Basis -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   leadingCoefficient : % -> R
```

```
--R leadingMonomial : % -> Basis          map : ((R -> R),%) -> %
--R monom : (Basis,R) -> %                monomial? : % -> Boolean
--R monomials : % -> List(%)              reductum : % -> %
--R retract : % -> Basis                  sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R leadingTerm : % -> Record(k: Basis,c: R)
--R listOfTerms : % -> List(Record(k: Basis,c: R))
--R numberOfMonomials : % -> NonNegativeInteger
--R retractIfCan : % -> Union(Basis,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———–

— FreeModuleCat.help —

```
====================================================================
FreeModuleCat examples
====================================================================

A domain of this category implements formal linear combinations
of elements from a domain Basis with coefficients in a domain R.
The domain Basis needs only to belong to the category SetCategory
and R to the category Ring. Thus the coefficient ring may be
non-commutative. See the XDistributedPolynomial constructor for
examples of domains built with the FreeModuleCat category constructor.

See Also:
o )show FreeModuleCat
o )show XDistributedPolynomial
```

———–

See:

⇐ “BiModule” (BMODULE) 9.0.146 on page 902
⇐ “RetractableTo” (RETRACT) 2.0.37 on page 110

Exports:

0	coefficient	coefficients	coerce
hash	latex	leadingCoefficient	leadingMonomial
leadingTerm	listOfTerms	map	monom
monomial?	monomials	numberOfMonomials	reductum
retract	retractIfCan	sample	subtractIfCan
zero?	?~=?	?*?	?+?
?-?	-?	?=?	

Attributes Exported:

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
coefficient : (%,Basis) -> R
coefficients : % -> List R
leadingCoefficient : % -> R
leadingMonomial : % -> Basis
leadingTerm : % -> Record(k: Basis,c: R)
listOfTerms : % -> List Record(k: Basis,c: R)
map : ((R -> R),%) -> %
monom : (Basis,R) -> %
monomial? : % -> Boolean
monomials : % -> List %
numberOfMonomials : % -> NonNegativeInteger
reductum : % -> %
?*? : (R,Basis) -> %
```

These exports come from (p902) BiModule(R:Ring,R:Ring):

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (R,%) -> %
?=? : (%,%) -> Boolean
?+? : (%,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Integer,%) -> %
?-? : (%,%) -> %
-? : % -> %
?*? : (%,R) -> %
```

These exports come from (p110) RetractableTo(Basis:SetCategory):

```
coerce : Basis -> %
retract : % -> Basis
retractIfCan : % -> Union(Basis,"failed")
```

See: Lambe[Lamb92]

— **FreeModuleCat.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FMCAT">
FreeModuleCat (FMCAT)</a></h2>
</body>
```

— **category FMCAT FreeModuleCat** —

```
)abbrev category FMCAT FreeModuleCat
++ Author: Michel Petitot petitot@lifl.fr
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ References:
++ Lamb92 Next Generation Computer Algebra Systems AXIOM and the
++ Scratchpad Concept: Applications to Research in Algebra
++ Description:
++ A domain of this category
++ implements formal linear combinations
++ of elements from a domain \spad{Basis} with coefficients
++ in a domain \spad{R}. The domain \spad{Basis} needs only
++ to belong to the category \spadtype{SetCategory} and \spad{R}
++ to the category \spadtype{Ring}. Thus the coefficient ring
++ may be non-commutative.
++ See the \spadtype{XDistributedPolynomial} constructor
++ for examples of domains built with the \spadtype{FreeModuleCat}
++ category constructor.

FreeModuleCat(R, Basis) : Category == SIG where
   R : Ring
   Basis : SetCategory

   TERM ==> Record(k: Basis, c: R)

   SIG ==> Join(BiModule(R,R), RetractableTo Basis) with

     "*" : (R, Basis) -> %
       ++ \spad{r*b} returns the product of \spad{r} by \spad{b}.

     coefficient : (%, Basis) -> R
       ++ \spad{coefficient(x,b)} returns the coefficient
       ++ of \spad{b} in \spad{x}.

     map : (R -> R, %) -> %
       ++ \spad{map(fn,u)} maps function \spad{fn} onto the coefficients
       ++  of the non-zero monomials of \spad{u}.

     monom : (Basis, R) -> %
       ++ \spad{monom(b,r)} returns the element with the single monomial
       ++  \spad{b} and coefficient \spad{r}.

     monomial? : % -> Boolean
       ++ \spad{monomial?(x)} returns true if \spad{x} contains a single
       ++ monomial.

     listOfTerms : % -> List TERM
       ++ \spad{listOfTerms(x)} returns a list \spad{lt} of terms with type
       ++ \spad{Record(k: Basis, c: R)} such that \spad{x} equals
       ++ \spad{reduce(+, map(x +-> monom(x.k, x.c), lt))}.

     coefficients : % -> List R
       ++ \spad{coefficients(x)} returns the list of coefficients of \spad{x}
```

```
    monomials : % -> List %
      ++ \spad{monomials(x)} returns the list of \spad{r_i*b_i}
      ++ whose sum is \spad{x}.

    numberOfMonomials : % -> NonNegativeInteger
      ++ \spad{numberOfMonomials(x)} returns the number of monomials
      ++ of \spad{x}.

    leadingMonomial : % -> Basis
      ++ \spad{leadingMonomial(x)} returns the first element from
      ++ \spad{Basis} which appears in \spad{listOfTerms(x)}.

    leadingCoefficient : % -> R
      ++ \spad{leadingCoefficient(x)} returns the first coefficient
      ++ which appears in \spad{listOfTerms(x)}.

    leadingTerm : % -> TERM
      ++ \spad{leadingTerm(x)} returns the first term which
      ++ appears in \spad{listOfTerms(x)}.

    reductum : % -> %
      ++ \spad{reductum(x)} returns \spad{x} minus its leading term.

    if R has CommutativeRing then Module(R)
```

———

— FMCAT.dotabb —

```
"FMCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FMCAT"];
"FMCAT" -> "BMODULE"
"FMCAT" -> "RETRACT"
```

———

— FMCAT.dotfull —

```
"FreeModuleCat(a:Ring,b:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FMCAT"];
"FreeModuleCat(a:Ring,b:SetCategory)" -> "BiModule(a:Ring,b:Ring)"
"FreeModuleCat(a:Ring,b:SetCategory)" -> "RetractableTo(SetCategory)"
```

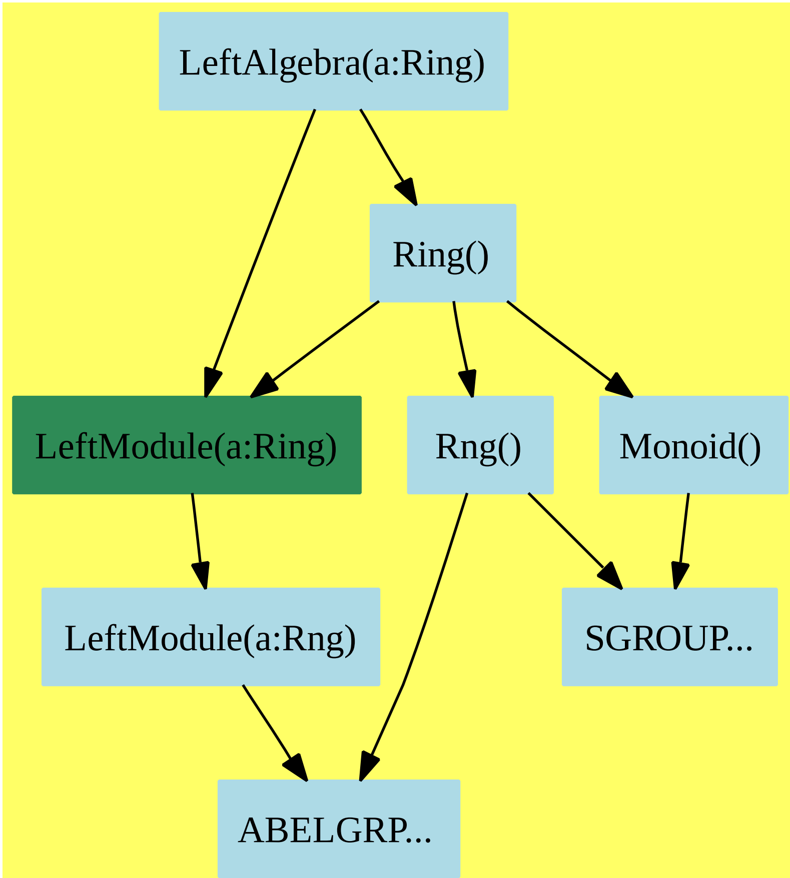
———

— FMCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];
```

```
"FreeModuleCat(a:Ring,b:SetCategory)" [color=lightblue];
"FreeModuleCat(a:Ring,b:SetCategory)" -> "BiModule(a:Ring,b:Ring)"
"FreeModuleCat(a:Ring,b:SetCategory)" -> "RetractableTo(SetCategory)"

"RetractableTo(SetCategory)" [color=seagreen];
"RetractableTo(SetCategory)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"Category" [color=lightblue];

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"ABELGRP..." [color=lightblue];
}
```

10.0.165 LeftAlgebra (LALG)



— **LeftAlgebra.input** —

```
)set break resume
)sys rm -f LeftAlgebra.output
)spool LeftAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show LeftAlgebra
--R
--R LeftAlgebra(R: Ring) is a category constructor
--R Abbreviation for LeftAlgebra is LALG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for LALG
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,%) -> %
```

```
--R ?*? : (Integer,%) -> %              ?*? : (NonNegativeInteger,%) -> %
--R ?*? : (PositiveInteger,%) -> %      ?**? : (%,NonNegativeInteger) -> %
--R ?**? : (%,PositiveInteger) -> %     ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                    -? : % -> %
--R ?=? : (%,%) -> Boolean              1 : () -> %
--R 0 : () -> %                         ?^? : (%,NonNegativeInteger) -> %
--R ?^? : (%,PositiveInteger) -> %      coerce : R -> %
--R coerce : Integer -> %               coerce : % -> OutputForm
--R hash : % -> SingleInteger           latex : % -> String
--R one? : % -> Boolean                 recip : % -> Union(%,"failed")
--R sample : () -> %                    zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— **LeftAlgebra.help** —

```
====================================================================
LeftAlgebra examples
====================================================================

The category of all left algebras over an arbitrary ring.

See Also:
o )show LeftAlgebra
```

See:

⇐ "LeftModule" (LMODULE) 8.0.137 on page 823
⇐ "Ring" (RING) 9.0.153 on page 946

Exports:

0	1	coerce	hash	latex
one?	recip	sample	zero?	characteristic
subtractIfCan	?*?	?+?	?-?	-?
?=?	?~=?	?**?	?^?	

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

These are implemented by this category:

```
coerce : R -> %
```

These exports come from (p946) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : % -> OutputForm
coerce : Integer -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p823) LeftModule(R:Type):

```
?*? : (R,%) -> %
```

— LeftAlgebra.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#LALG">
LeftAlgebra (LALG)</a></h2>
</body>
```

— category LALG LeftAlgebra —

```
)abbrev category LALG LeftAlgebra
++ Author: Larry A. Lambe
++ Date  : 03/01/89; revised 03/17/89; revised 12/02/90.
++ Description:
++ The category of all left algebras over an arbitrary ring.

LeftAlgebra(R) : Category == SIG where
  R : Ring

  SIG ==> Join(Ring, LeftModule(R)) with

    coerce: R -> %
```

```
      ++ coerce(r) returns r * 1 where 1 is the identity of the
      ++ left algebra.

   add

     coerce(x:R):% == x * 1$%
```

———-

— COQ LALG —

```
(* category LALG *)
(*
      coerce : R -> %
      coerce(x:R):% == x * 1$%

*)
```

———-

— LALG.dotabb —

```
"LALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LALG"];
"LALG" -> "LMODULE"
"LALG" -> "RING"
```

———-

— LALG.dotfull —

```
"LeftAlgebra(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LALG"];
"LeftAlgebra(a:Ring)" -> "LeftModule(a:Ring)"
"LeftAlgebra(a:Ring)" -> "Ring()"
```

———-

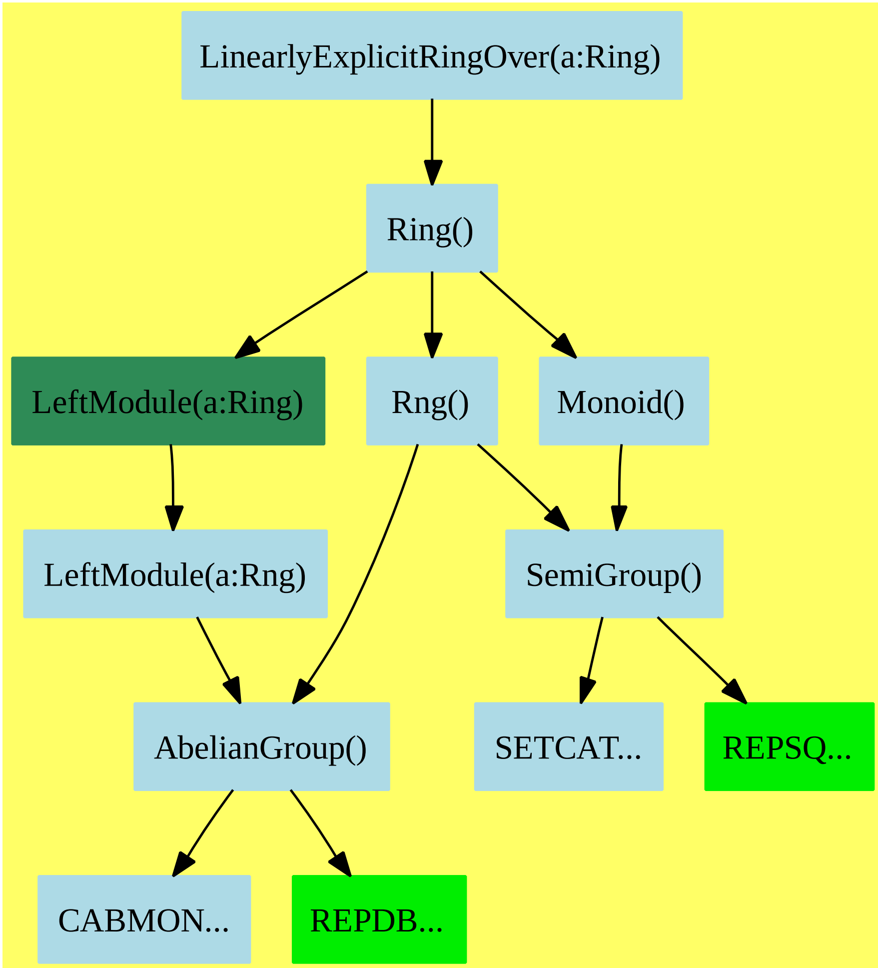
— LALG.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"LeftAlgebra(a:Ring)" [color=lightblue];
"LeftAlgebra(a:Ring)" -> "LeftModule(a:Ring)"
"LeftAlgebra(a:Ring)" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
```

```
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "ABELGRP..."
"Rng()" -> "SGROUP..."

"Monoid()" [color=lightblue];
"Monoid()" -> "SGROUP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"SGROUP..." [color=lightblue];
"ABELGRP..." [color=lightblue];
}
```

———-

10.0.166 LinearlyExplicitRingOver (LINEXP)



— **LinearlyExplicitRingOver.input** —

```
)set break resume
)sys rm -f LinearlyExplicitRingOver.output
)spool LinearlyExplicitRingOver.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show LinearlyExplicitRingOver
--R
--R LinearlyExplicitRingOver(R: Ring) is a category constructor
--R Abbreviation for LinearlyExplicitRingOver is LINEXP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for LINEXP
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %    ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %   ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %    ?^? : (%,PositiveInteger) -> %
--R coerce : Integer -> %                coerce : % -> OutputForm
--R hash : % -> SingleInteger            latex : % -> String
--R one? : % -> Boolean                  recip : % -> Union(%,"failed")
--R sample : () -> %                     zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(R),vec: Vector(R))
--R reducedSystem : Matrix(%) -> Matrix(R)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— **LinearlyExplicitRingOver.help** —

```
====================================================================
LinearlyExplicitRingOver examples
====================================================================

An extension ring with an explicit linear dependence test.

See Also:
o )show LinearlyExplicitRingOver
```

See:

⇒ "FullyLinearlyExplicitRingOver" (FLINEXP) 11.0.178 on page 1134
⇒ "IntegerNumberSystem" (INS) 16.0.210 on page 1420
⇒ "MonogenicAlgebra" (MONOGEN) 19.0.238 on page 1847
⇐ "Ring" (RING) 9.0.153 on page 946

Exports:

0	1	characteristic	coerce	hash
latex	one?	recip	reducedSystem	subtractIfCan
sample	zero?	?*?	?**?	?+?
?-?	-?	?=?	?^?	?~=?

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return

These are directly exported but not implemented:

```
reducedSystem : (Matrix %,Vector %) ->
    Record(mat: Matrix R,vec: Vector R)
reducedSystem : Matrix % -> Matrix R
```

These exports come from (p946) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

— **LinearlyExplicitRingOver.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#LINEXP">
LinearlyExplicitRingOver (LINEXP)</a></h2>
</body>
```

— **category LINEXP LinearlyExplicitRingOver** —

```
)abbrev category LINEXP LinearlyExplicitRingOver
++ Description:
++ An extension ring with an explicit linear dependence test.

LinearlyExplicitRingOver(R) : Category == SIG where
  R : Ring
```

```
  SIG ==> Ring with

    reducedSystem : Matrix % -> Matrix R
      ++ reducedSystem(A) returns a matrix B such that \spad{A x = 0}
      ++ and \spad{B x = 0} have the same solutions in R.

    reducedSystem : (Matrix %,Vector %) -> Record(mat:Matrix R,vec:Vector R)
      ++ reducedSystem(A, v) returns a matrix B and a vector w such that
      ++ \spad{A x = v} and \spad{B x = w} have the same solutions in R.
```

———-

— LINEXP.dotabb —

```
"LINEXP"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LINEXP"];
"LINEXP" -> "RING"
```

———-

— LINEXP.dotfull —

```
"LinearlyExplicitRingOver(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LINEXP"];
"LinearlyExplicitRingOver(a:Ring)" -> "Ring()"

"LinearlyExplicitRingOver(Integer)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=LINEXP"];
"LinearlyExplicitRingOver(Integer)" -> "LinearlyExplicitRingOver(a:Ring)"

"LinearlyExplicitRingOver(Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=LINEXP"];
"LinearlyExplicitRingOver(Fraction(Integer))" ->
    "LinearlyExplicitRingOver(a:Ring)"
```

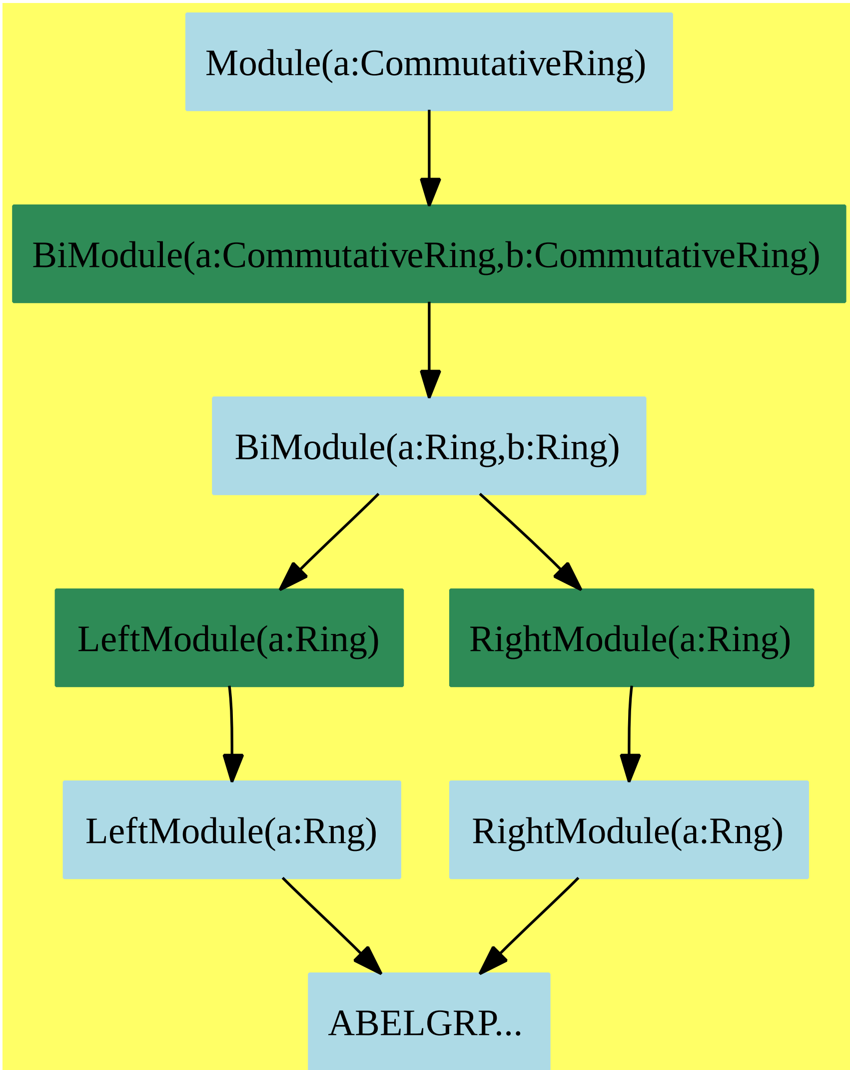
———-

— LINEXP.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"LinearlyExplicitRingOver(a:Ring)" [color=lightblue];
"LinearlyExplicitRingOver(a:Ring)" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"
```

```
"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

10.0.167 Module (MODULE)



— **Module.input** —

```
)set break resume
)sys rm -f Module.output
)spool Module.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Module
--R
--R Module(R: CommutativeRing) is a category constructor
```

```
--R Abbreviation for Module is MODULE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for MODULE
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
--R ?*? : (Integer,%) -> %                ?*? : (NonNegativeInteger,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?=? : (%,%) -> Boolean                0 : () -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— **Module.help** —

```
====================================================================
Module examples
====================================================================

The category of modules over a commutative ring.

Axioms:
         1*x = x
         (a*b)*x = a*(b*x)
         (a+b)*x = (a*x)+(b*x)
         a*(x+y) = (a*x)+(a*y)

See Also:
o )show Module
```

See:

⇒ "Algebra" (ALGEBRA) 11.0.176 on page 1121
⇒ "LieAlgebra" (LIECAT) 11.0.179 on page 1140
⇒ "NonAssociativeAlgebra" (NAALG) 11.0.181 on page 1154
⇒ "RectangularMatrixCategory" (RMATCAT) 10.0.171 on page 1075
⇒ "VectorSpace" (VSPACE) 11.0.182 on page 1160
⇐ "BiModule" (BMODULE) 9.0.146 on page 902

Exports:

0	coerce	hash	latex	sample
subtractIfCan	zero?	?~=?	?*?	?+?
?-?	-?	?=?		

Attributes exported:

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are implemented by this category:

```
?*? : (%,R) -> %
```

These exports come from (p902) BiModule(a:Ring,b:Ring):

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (R,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
```

— Module.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#MODULE">
Module (MODULE)</a></h2>
</body>
```

— category MODULE Module —

```
)abbrev category MODULE Module
++ Description:
++ The category of modules over a commutative ring.
++
++ Axioms\br
++ \tab{5}\spad{1*x = x}\br
++ \tab{5}\spad{(a*b)*x = a*(b*x)}\br
++ \tab{5}\spad{(a+b)*x = (a*x)+(b*x)}\br
++ \tab{5}\spad{a*(x+y) = (a*x)+(a*y)}

Module(R) : Category == SIG where
  R : CommutativeRing
```

```
  SIG ==> BiModule(R,R)

   add

     if not(R is %) then x:%*r:R == r*x
```

———-

— COQ MODULE —

```
(* category MODULE *)
(*
The category of modules over a commutative ring.

Axioms
  1*x = x
  (a*b)*x = a*(b*x)
  (a+b)*x = (a*x)+(b*x)
  a*(x+y) = (a*x)+(a*y)

*)
```

———-

— MODULE.dotabb —

```
"MODULE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MODULE"];
"MODULE" -> "BMODULE"
```

———-

— MODULE.dotfull —

```
"Module(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MODULE"];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"Module(Field)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=MODULE"];
"Module(Field)" -> "Module(a:CommutativeRing)"
```

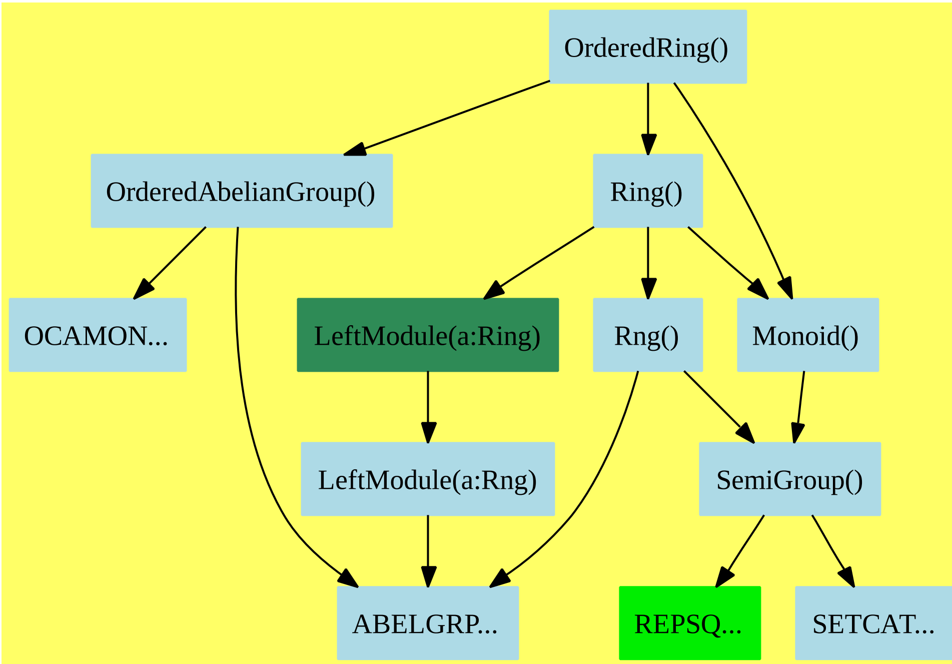
———-

— MODULE.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];
```

```
"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"ABELGRP..." [color=lightblue];
}
```

10.0.168 OrderedRing (ORDRING)



— **OrderedRing.input** —

```
)set break resume
)sys rm -f OrderedRing.output
)spool OrderedRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderedRing
--R
--R OrderedRing is a category constructor
--R Abbreviation for OrderedRing is ORDRING
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ORDRING
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                       ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,PositiveInteger) -> %       ?**? : (%,NonNegativeInteger) -> %
--R ?+? : (%,%) -> %                       -? : % -> %
--R ?-? : (%,%) -> %                       ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean                ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean                 ?>=? : (%,%) -> Boolean
--R 1 : () -> %                            0 : () -> %
--R ?^? : (%,PositiveInteger) -> %        ?^? : (%,NonNegativeInteger) -> %
```

```
--R abs : % -> %                        coerce : Integer -> %
--R coerce : % -> OutputForm            hash : % -> SingleInteger
--R latex : % -> String                 max : (%,%) -> %
--R min : (%,%) -> %                    negative? : % -> Boolean
--R one? : % -> Boolean                 positive? : % -> Boolean
--R recip : % -> Union(%,"failed")      sample : () -> %
--R sign : % -> Integer                 zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— **OrderedRing.help** —

```
====================================================================
OrderedRing examples
====================================================================

Ordered sets which are also rings, that is, domains where the ring
operations are compatible with the ordering.

Axiom:
        0<a and b<c => ab < ac

See Also:
o )show OrderedRing
```

See:

⇒ "DirectProductCategory" (DIRPCAT) 12.0.184 on page 1173
⇒ "OrderedIntegralDomain" (OINTDOM) 13.0.198 on page 1326
⇒ "RealClosedField" (RCFIELD) 17.0.223 on page 1611
⇒ "RealNumberSystem" (RNS) 17.0.224 on page 1622
⇐ "Monoid" (MONOID) 5.0.100 on page 455
⇐ "OrderedAbelianGroup" (OAGROUP) 9.0.150 on page 932
⇐ "Ring" (RING) 9.0.153 on page 946

Exports:

1	0	abs	characteristic	coerce
hash	latex	max	min	negative?
one?	positive?	recip	sample	sign
subtractIfCan	zero?	?^?	?~=?	?*?
?**?	?+?	-?	?-?	?<?
?<=?	?=?	?>?	?>=?	

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

These are implemented by this category:

```
abs : % -> %
negative? : % -> Boolean
positive? : % -> Boolean
sign : % -> Integer
```

These exports come from (p932) OrderedAbelianGroup():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?+? : (%,%) -> %
-? : % -> %
?-? : (%,%) -> %
```

These exports come from (p946) Ring():

```
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
one? : % -> Boolean
recip : % -> Union(%,"failed")
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

— OrderedRing.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ORDRING">
OrderedRing (ORDRING)</a></h2>
</body>
```

———

— category ORDRING OrderedRing —

```
)abbrev category ORDRING OrderedRing
++ Description:
++ Ordered sets which are also rings, that is, domains where the ring
++ operations are compatible with the ordering.
++
++ Axiom\br
++ \tab{5}\spad{0<a and b<c => ab< ac}

OrderedRing() : Category == SIG where

  SIG ==> Join(OrderedAbelianGroup,Ring,Monoid) with

    positive? : % -> Boolean
      ++ positive?(x) tests whether x is strictly greater than 0.

    negative? : % -> Boolean
      ++ negative?(x) tests whether x is strictly less than 0.

    sign : % -> Integer
      ++ sign(x) is 1 if x is positive, -1 if x is negative,
      ++ 0 if x equals 0.

    abs : % -> %
      ++ abs(x) returns the absolute value of x.

   add

     positive? x == x>0

     negative? x == x<0

     sign x ==
       positive? x => 1
       negative? x => -1
       zero? x => 0
       error "x satisfies neither positive?, negative? or zero?"

     abs x ==
       positive? x => x
       negative? x => -x
       zero? x => 0
       error "x satisfies neither positive?, negative? or zero?"
```

———-

— COQ ORDRING —

```
(* category ORDRING *)
(*
Axiom
  0<a and b<c => ab< ac
```

```
     positive? : % -> Boolean
     positive? x == x>0

     negative? : % -> Boolean
     negative? x == x<0

     sign : % -> Integer
     sign x ==
       positive? x => 1
       negative? x => -1
       zero? x => 0
       error "x satisfies neither positive?, negative? or zero?"

     abs : % -> %
     abs x ==
       positive? x => x
       negative? x => -x
       zero? x => 0
       error "x satisfies neither positive?, negative? or zero?"

*)
```

———-

— ORDRING.dotabb —

```
"ORDRING"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDRING"];
"ORDRING" -> "OAGROUP"
"ORDRING" -> "RING"
"ORDRING" -> "MONOID"
```

———-

— ORDRING.dotfull —

```
"OrderedRing()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDRING"];
"OrderedRing()" -> "OrderedAbelianGroup()"
"OrderedRing()" -> "Ring()"
"OrderedRing()" -> "Monoid()"
```

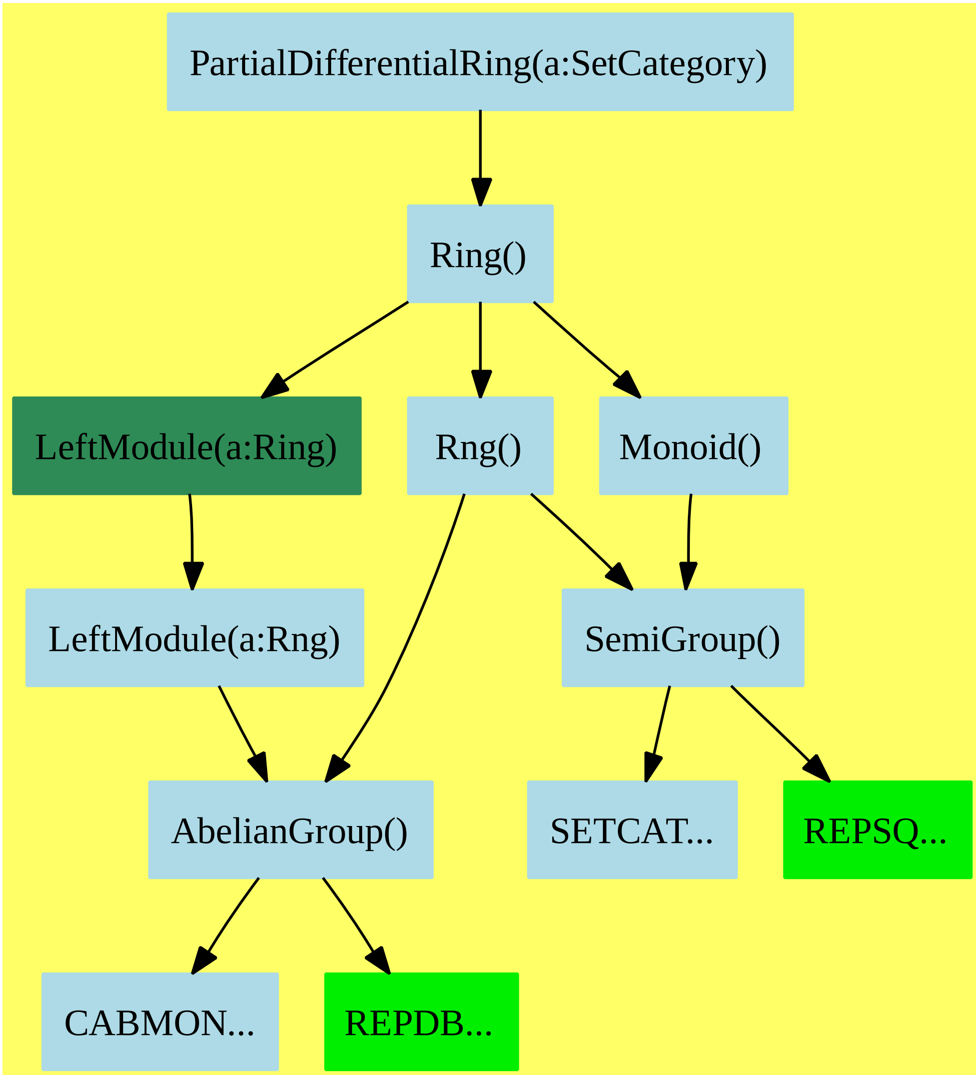
———-

— ORDRING.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];
```

```
"OrderedRing()" [color=lightblue];
"OrderedRing()" -> "OrderedAbelianGroup()"
"OrderedRing()" -> "Ring()"
"OrderedRing()" -> "Monoid()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "ABELGRP..."
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"OrderedAbelianGroup()" [color=lightblue];
"OrderedAbelianGroup()" -> "OCAMON..."
"OrderedAbelianGroup()" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPSQ..." [color="#00EE00"];
"OCAMON..." [color=lightblue];
"SETCAT..." [color=lightblue];
"ABELGRP..." [color=lightblue];
}
```

10.0.169 PartialDifferentialRing (PDRING)



— **PartialDifferentialRing.input** —

```
)set break resume
)sys rm -f PartialDifferentialRing.output
)spool PartialDifferentialRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PartialDifferentialRing
--R
--R PartialDifferentialRing(S: SetCategory) is a category constructor
```

```
--R Abbreviation for PartialDifferentialRing is PDRING
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PDRING
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                       ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %      ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %     ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                       ?-? : (%,%) -> %
--R -? : % -> %                            ?=? : (%,%) -> Boolean
--R D : (%,S,NonNegativeInteger) -> %      D : (%,List(S)) -> %
--R D : (%,S) -> %                         1 : () -> %
--R 0 : () -> %                            ?^? : (%,NonNegativeInteger) -> %
--R ?^? : (%,PositiveInteger) -> %         coerce : Integer -> %
--R coerce : % -> OutputForm               differentiate : (%,List(S)) -> %
--R differentiate : (%,S) -> %             hash : % -> SingleInteger
--R latex : % -> String                    one? : % -> Boolean
--R recip : % -> Union(%,"failed")         sample : () -> %
--R zero? : % -> Boolean                   ?~=? : (%,%) -> Boolean
--R D : (%,List(S),List(NonNegativeInteger)) -> %
--R characteristic : () -> NonNegativeInteger
--R differentiate : (%,List(S),List(NonNegativeInteger)) -> %
--R differentiate : (%,S,NonNegativeInteger) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— **PartialDifferentialRing.help** —

```
====================================================================
PartialDifferentialRing examples
====================================================================

A partial differential ring with differentiations indexed by a
parameter type S.

Axioms:
        differentiate(x+y,e)=differentiate(x,e)+differentiate(y,e)
        differentiate(x*y,e)=x*differentiate(y,e)+differentiate(x,e)*y

See Also:
o )show PartialDifferentialRing
```

See:

⇒ "DifferentialExtension" (DIFEXT) 11.0.177 on page 1127

⇒ “FunctionSpace” (FS) 17.0.219 on page 1544
⇒ “MultivariateTaylorSeriesCategory” (MTSCAT) 15.0.206 on page 1384
⇒ “PolynomialCategory” (POLYCAT) 16.0.213 on page 1449
⇐ “Ring” (RING) 9.0.153 on page 946

Exports:

1	0	characteristic	coerce	D
differentiate	hash	latex	one?	recip
sample	subtractIfCan	zero?	?^?	?*?
?~=?	?**?	?+?	?-?	-?
?=?				

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.

These are directly exported but not implemented:

```
differentiate : (%,S) -> %
```

These are implemented by this category:

```
D : (%,S) -> %
D : (%,List S) -> %
D : (%,S,NonNegativeInteger) -> %
D : (%,List S,List NonNegativeInteger) -> %
differentiate : (%,List S) -> %
differentiate : (%,S,NonNegativeInteger) -> %
differentiate : (%,List S,List NonNegativeInteger) -> %
```

These exports come from (p946) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,NonNegativeInteger) -> %
```

```
 ?**? : (%,PositiveInteger) -> %
```

— PartialDifferentialRing.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#PDRING">
PartialDifferentialRing (PDRING)</a></h2>
</body>
```

———-

— category PDRING PartialDifferentialRing —

```
)abbrev category PDRING PartialDifferentialRing
++ Description:
++ A partial differential ring with differentiations indexed by a
++ parameter type S.
++
++ Axioms\br
++ \tab{5}\spad{differentiate(x+y,e)=differentiate(x,e)+differentiate(y,e)}\br
++ \tab{5}\spad{differentiate(x*y,e)=x*differentiate(y,e)+differentiate(x,e)*y}

PartialDifferentialRing(S) : Category == SIG where
  S : SetCategory

  SIG ==> Ring with

    differentiate : (%, S) -> %
      ++ differentiate(x,v) computes the partial derivative of x
      ++ with respect to v.

    differentiate : (%, List S) -> %
      ++ differentiate(x,[s1,...sn]) computes successive partial
      ++ derivatives,
      ++ that is, \spad{differentiate(...differentiate(x, s1)..., sn)}.

    differentiate : (%, S, NonNegativeInteger) -> %
      ++ differentiate(x, s, n) computes multiple partial derivatives,
      ++ that is, n-th derivative of x with respect to s.

    differentiate : (%, List S, List NonNegativeInteger) -> %
      ++ differentiate(x, [s1,...,sn], [n1,...,nn]) computes
      ++ multiple partial derivatives, that is, \spad{D(...D(x, s1)..., sn)}.

    D : (%, S) -> %
      ++ D(x,v) computes the partial derivative of x
      ++ with respect to v.

    D : (%, List S) -> %
      ++ D(x,[s1,...sn]) computes successive partial derivatives,
      ++ that is, \spad{D(...D(x, s1)..., sn)}.

    D : (%, S, NonNegativeInteger) -> %
```

```
      ++ D(x, s, n) computes multiple partial derivatives, that is,
      ++ n-th derivative of x with respect to s.

    D : (%, List S, List NonNegativeInteger) -> %
      ++ D(x, [s1,...,sn], [n1,...,nn]) computes
      ++ multiple partial derivatives, that is,
      ++ \spad{D(...D(x, s1, n1)..., sn, nn)}.

   add

     differentiate(r:%, l:List S) ==
       for s in l repeat r := differentiate(r, s)
       r

     differentiate(r:%, s:S, n:NonNegativeInteger) ==
       for i in 1..n repeat r := differentiate(r, s)
       r

     differentiate(r:%, ls:List S, ln:List NonNegativeInteger) ==
       for s in ls for n in ln repeat r := differentiate(r, s, n)
       r

     D(r:%, v:S) == differentiate(r,v)

     D(r:%, lv:List S) == differentiate(r,lv)

     D(r:%, v:S, n:NonNegativeInteger) == differentiate(r,v,n)

     D(r:%, lv:List S, ln:List NonNegativeInteger) == differentiate(r, lv, ln)
```

— **COQ PDRING** —

```
(* category PDRING *)
(*
Axioms
  differentiate(x+y,e)=differentiate(x,e)+differentiate(y,e)
  differentiate(x*y,e)=x*differentiate(y,e)+differentiate(x,e)*y

    differentiate : (%,List(S)) -> %
    differentiate(r:%, l:List S) ==
      for s in l repeat r := differentiate(r, s)
      r

    differentiate : (%,S,NonNegativeInteger) -> %
    differentiate(r:%, s:S, n:NonNegativeInteger) ==
      for i in 1..n repeat r := differentiate(r, s)
      r

    differentiate : (%,List(S),List(NonNegativeInteger)) -> %
    differentiate(r:%, ls:List S, ln:List NonNegativeInteger) ==
      for s in ls for n in ln repeat r := differentiate(r, s, n)
```

```
      r

    D : (%,S) -> %
    D(r:%, v:S) == differentiate(r,v)

    D : (%,List(S)) -> %
    D(r:%, lv:List S) == differentiate(r,lv)

    D : (%,S,NonNegativeInteger) -> %
    D(r:%, v:S, n:NonNegativeInteger) == differentiate(r,v,n)

    D : (%,List(S),List(NonNegativeInteger)) -> %
    D(r:%, lv:List S, ln:List NonNegativeInteger) == differentiate(r, lv, ln)

*)
```

———-

— PDRING.dotabb —

```
"PDRING"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PDRING"];
"PDRING" -> "RING"
```

———-

— PDRING.dotfull —

```
"PartialDifferentialRing(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PDRING"];
"PartialDifferentialRing(a:SetCategory)" -> "Ring()"

"PartialDifferentialRing(a:OrderedSet)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PDRING"];
"PartialDifferentialRing(a:OrderedSet)" ->
    "PartialDifferentialRing(a:SetCategory)"

"PartialDifferentialRing(Symbol)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PDRING"];
"PartialDifferentialRing(Symbol)" ->
    "PartialDifferentialRing(a:SetCategory)"
```

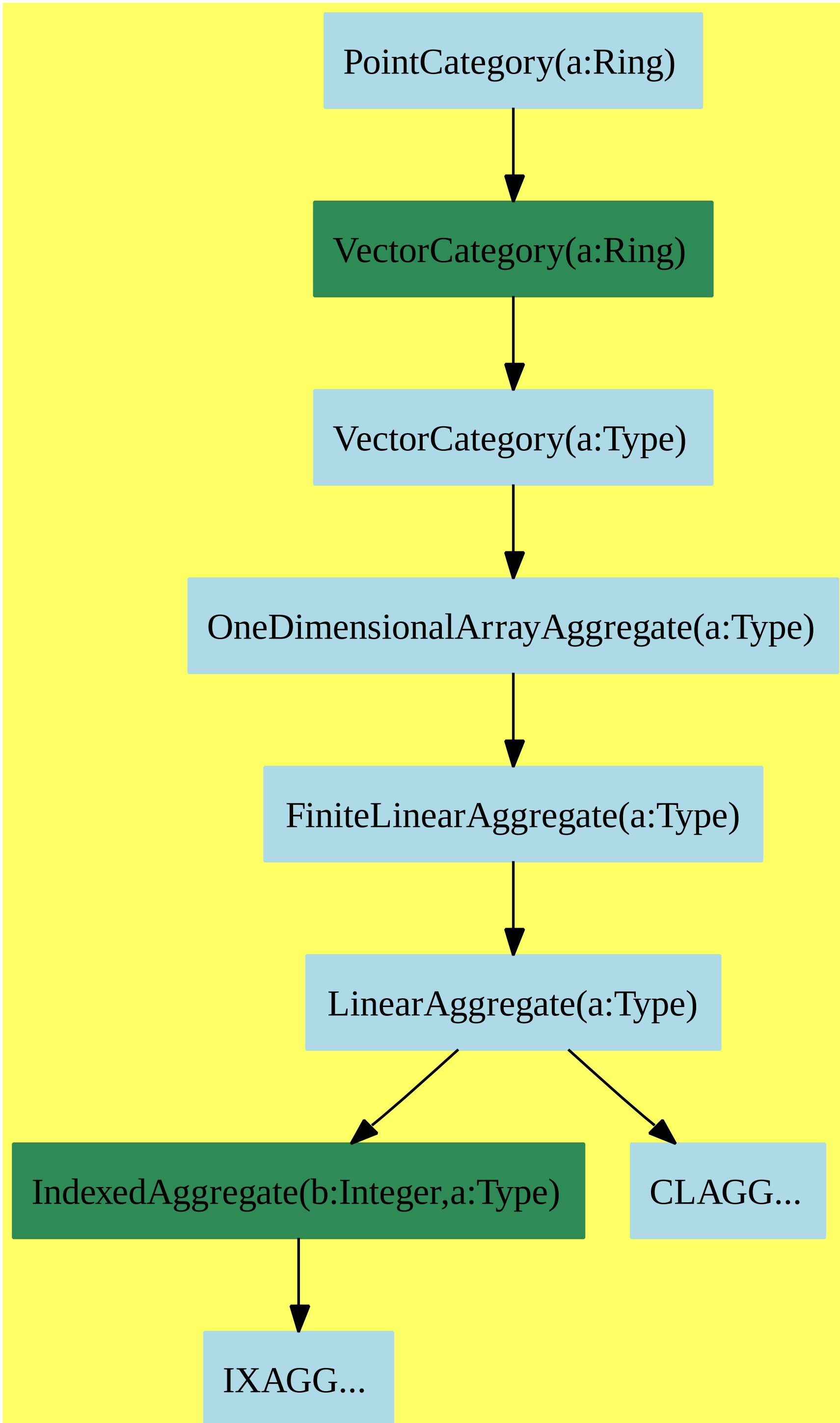
———-

— PDRING.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];
```

```
"PartialDifferentialRing(a:SetCategory)" [color=lightblue];
"PartialDifferentialRing(a:SetCategory)" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

10.0.170 PointCategory (PTCAT)



— **PointCategory.input** —

```
)set break resume
)sys rm -f PointCategory.output
)spool PointCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PointCategory
--R
--R PointCategory(R: Ring) is a category constructor
--R Abbreviation for PointCategory is PTCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PTCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> % if R has MONOID      ?*? : (R,%) -> % if R has MONOID
--R ?+? : (%,%) -> % if R has ABELSG      ?-? : (%,%) -> % if R has ABELGRP
--R -? : % -> % if R has ABELGRP          concat : List(%) -> %
--R concat : (%,%) -> %                   concat : (R,%) -> %
--R concat : (%,R) -> %                   construct : List(R) -> %
--R convert : List(R) -> %                copy : % -> %
--R cross : (%,%) -> %                    delete : (%,Integer) -> %
--R dimension : % -> PositiveInteger      dot : (%,%) -> R if R has RING
--R ?.? : (%,Integer) -> R                elt : (%,Integer,R) -> R
--R empty : () -> %                       empty? : % -> Boolean
--R entries : % -> List(R)                eq? : (%,%) -> Boolean
--R extend : (%,List(R)) -> %             index? : (Integer,%) -> Boolean
--R indices : % -> List(Integer)          insert : (%,%,Integer) -> %
--R insert : (R,%,Integer) -> %           latex : % -> String if R has SETCAT
--R map : (((R,R) -> R),%,%) -> %         map : ((R -> R),%) -> %
--R max : (%,%) -> % if R has ORDSET      min : (%,%) -> % if R has ORDSET
--R new : (NonNegativeInteger,R) -> %     point : List(R) -> %
--R qelt : (%,Integer) -> R               reverse : % -> %
--R sample : () -> %                      sort : % -> % if R has ORDSET
--R sort : (((R,R) -> Boolean),%) -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?*? : (Integer,%) -> % if R has ABELGRP
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?=? : (%,%) -> Boolean if R has SETCAT
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if R has SETCAT
--R convert : % -> InputForm if R has KONVERT(INFORM)
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (%,UniversalSegment(Integer)) -> %
--R ?.? : (%,UniversalSegment(Integer)) -> %
```

```
--R entry? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
--R eval : (%,List(R),List(R)) -> % if R has EVALAB(R) and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB(R) and R has SETCAT
--R eval : (%,Equation(R)) -> % if R has EVALAB(R) and R has SETCAT
--R eval : (%,List(Equation(R))) -> % if R has EVALAB(R) and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,R) -> % if $ has shallowlyMutable
--R find : ((R -> Boolean),%) -> Union(R,"failed")
--R first : % -> R if Integer has ORDSET
--R hash : % -> SingleInteger if R has SETCAT
--R length : % -> R if R has RING and R has RADCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R magnitude : % -> R if R has RING and R has RADCAT
--R map! : ((R -> R),%) -> % if $ has shallowlyMutable
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
--R members : % -> List(R) if $ has finiteAggregate
--R merge : (%,%) -> % if R has ORDSET
--R merge : (((R,R) -> Boolean),%,%) -> %
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R outerProduct : (%,%) -> Matrix(R) if R has RING
--R parts : % -> List(R) if $ has finiteAggregate
--R position : (R,%,Integer) -> Integer if R has SETCAT
--R position : (R,%) -> Integer if R has SETCAT
--R position : ((R -> Boolean),%) -> Integer
--R qsetelt! : (%,Integer,R) -> R if $ has shallowlyMutable
--R reduce : (((R,R) -> R),%) -> R if $ has finiteAggregate
--R reduce : (((R,R) -> R),%,R) -> R if $ has finiteAggregate
--R reduce : (((R,R) -> R),%,R,R) -> R if R has SETCAT and $ has finiteAggregate
--R remove : ((R -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (R,%) -> % if R has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if R has SETCAT and $ has finiteAggregate
--R reverse! : % -> % if $ has shallowlyMutable
--R select : ((R -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment(Integer),R) -> R if $ has shallowlyMutable
--R setelt : (%,Integer,R) -> R if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort! : % -> % if R has ORDSET and $ has shallowlyMutable
--R sort! : (((R,R) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if R has ORDSET
--R sorted? : (((R,R) -> Boolean),%) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R zero : NonNegativeInteger -> % if R has ABELMON
--R ?~=? : (%,%) -> Boolean if R has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

— **PointCategory.help** —

```
====================================================================
PointCategory examples
====================================================================

PointCategory is the category of points in space which may be plotted
via the graphics facilities.  Functions are provided for defining
points and handling elements of points.

See Also:
o )show PointCategory
```

———-

See:

⇐ "VectorCategory" (VECTCAT) 9.0.157 on page 985

Exports:

any?	coerce	concat	construct	convert
copy	copyInto!	cross	count	delete
dimension	dot	elt	empty	empty?
entry?	entries	eq?	eval	every?
extend	fill!	find	first	hash
index?	indices	insert	latex	length
less?	magnitude	map	map!	max
maxIndex	member?	members	merge	min
minIndex	more?	new	outerProduct	parts
point	position	qelt	qsetelt!	reduce
remove	removeDuplicates	reverse	reverse!	sample
select	setelt	size?	sort	sort!
sorted?	swap!	zero	#?	?.?
?~=?	-?	?*?	?+?	?-?
?<?	?<=?	?=?	?>?	?>=?

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These are directly exported but not implemented:

```
convert : List R -> %
cross : (%,%) -> %
dimension : % -> PositiveInteger
extend : (%,List R) -> %
point : List R -> %
```

These exports come from (p985) VectorCategory(R:Ring):

```
any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
coerce : % -> OutputForm if R has SETCAT
concat : List % -> %
concat : (%,%) -> %
concat : (R,%) -> %
concat : (%,R) -> %
construct : List R -> %
convert : % -> InputForm if R has KONVERT INFORM
copy : % -> %
copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
delete : (%,Integer) -> %
delete : (%,UniversalSegment Integer) -> %
dot : (%,%) -> R if R has RING
elt : (%,Integer,R) -> R
empty : () -> %
empty? : % -> Boolean
entry? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
entries : % -> List R
eq? : (%,%) -> Boolean
eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
fill! : (%,R) -> % if $ has shallowlyMutable
find : ((R -> Boolean),%) -> Union(R,"failed")
first : % -> R if Integer has ORDSET
hash : % -> SingleInteger if R has SETCAT
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (%,%,Integer) -> %
insert : (R,%,Integer) -> %
latex : % -> String if R has SETCAT
length : % -> R if R has RING and R has RADCAT
less? : (%,NonNegativeInteger) -> Boolean
magnitude : % -> R if R has RING and R has RADCAT
map : (((R,R) -> R),%,%) -> %
map : ((R -> R),%) -> %
map! : ((R -> R),%) -> % if $ has shallowlyMutable
max : (%,%) -> % if R has ORDSET
maxIndex : % -> Integer if Integer has ORDSET
member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
members : % -> List R if $ has finiteAggregate
merge : (%,%) -> % if R has ORDSET
merge : (((R,R) -> Boolean),%,%) -> %
min : (%,%) -> % if R has ORDSET
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
new : (NonNegativeInteger,R) -> %
outerProduct : (%,%) -> Matrix R if R has RING
parts : % -> List R if $ has finiteAggregate
position : (R,%,Integer) -> Integer if R has SETCAT
```

```
 position : (R,%) -> Integer if R has SETCAT
 position : ((R -> Boolean),%) -> Integer
 qelt : (%,Integer) -> R
 qsetelt! : (%,Integer,R) -> R if $ has shallowlyMutable
 reduce : (((R,R) -> R),%) -> R if $ has finiteAggregate
 reduce : (((R,R) -> R),%,R) -> R if $ has finiteAggregate
 reduce : (((R,R) -> R),%,R,R) -> R if R has SETCAT and $ has finiteAggregate
 remove : ((R -> Boolean),%) -> % if $ has finiteAggregate
 remove : (R,%) -> % if R has SETCAT and $ has finiteAggregate
 removeDuplicates : % -> % if R has SETCAT and $ has finiteAggregate
 reverse : % -> %
 reverse! : % -> % if $ has shallowlyMutable
 sample : () -> %
 select : ((R -> Boolean),%) -> % if $ has finiteAggregate
 setelt : (%,UniversalSegment Integer,R) -> R if $ has shallowlyMutable
 setelt : (%,Integer,R) -> R if $ has shallowlyMutable
 size? : (%,NonNegativeInteger) -> Boolean
 sort : % -> % if R has ORDSET
 sort : (((R,R) -> Boolean),%) -> %
 sort! : % -> % if R has ORDSET and $ has shallowlyMutable
 sort! : (((R,R) -> Boolean),%) -> % if $ has shallowlyMutable
 sorted? : % -> Boolean if R has ORDSET
 sorted? : (((R,R) -> Boolean),%) -> Boolean
 swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
 zero : NonNegativeInteger -> % if R has ABELMON
 #? : % -> NonNegativeInteger if $ has finiteAggregate
 ?.? : (%,Integer) -> R
 ?.? : (%,UniversalSegment Integer) -> %
 ?~=? : (%,%) -> Boolean if R has SETCAT
 ?<? : (%,%) -> Boolean if R has ORDSET
 ?<=? : (%,%) -> Boolean if R has ORDSET
 ?=? : (%,%) -> Boolean if R has SETCAT
 ?>? : (%,%) -> Boolean if R has ORDSET
 ?>=? : (%,%) -> Boolean if R has ORDSET
 ?*? : (Integer,%) -> % if R has ABELGRP
 ?*? : (%,R) -> % if R has MONOID
 ?*? : (R,%) -> % if R has MONOID
 ?-? : (%,%) -> % if R has ABELGRP
 -? : % -> % if R has ABELGRP
 ?+? : (%,%) -> % if R has ABELSG
```

— PointCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#PTCAT">
PointCategory (PTCAT)</a></h2>
</body>
```

— category PTCAT PointCategory —

```
)abbrev category PTCAT PointCategory
```

```
++ Description:
++ PointCategory is the category of points in space which
++ may be plotted via the graphics facilities.  Functions are provided for
++ defining points and handling elements of points.

PointCategory(R) : Category == SIG where
  R : Ring

  SIG ==> VectorCategory(R) with

    point : List R -> %
      ++ point(l) returns a point category defined by a list l of elements
      ++ from the domain R.

    dimension : % -> PositiveInteger
      ++ dimension(s) returns the dimension of the point category s.

    convert : List R -> %
      ++ convert(l) takes a list of elements, l, from the domain Ring and
      ++ returns the form of point category.

    cross : (%,%) -> %
      ++ cross(p,q) computes the cross product of the two points \spad{p}
      ++ and \spad{q}. Error if the p and q are not 3 dimensional

    extend : (%,List R) -> %
      ++ extend(x,l,r) \undocumented
```

— PTCAT.dotabb —

```
"PTCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PTCAT"];
"PTCAT" -> "VECTCAT"
```

— PTCAT.dotfull —

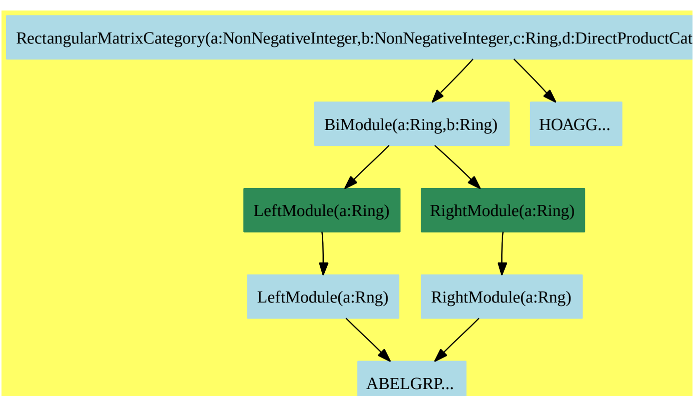
```
"PointCategory(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PTCAT"];
"PointCategory(a:Ring)" -> "VectorCategory(a:Ring)"
```

— PTCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
```

```
 node [shape=box, color=white, style=filled];

"PointCategory(a:Ring)" [color=lightblue];
"PointCategory(a:Ring)" -> "VectorCategory(a:Ring)"

"VectorCategory(a:Ring)" [color=seagreen];
"VectorCategory(a:Ring)" -> "VectorCategory(a:Type)"

"VectorCategory(a:Type)" [color=lightblue];
"VectorCategory(a:Type)" -> "OneDimensionalArrayAggregate(a:Type)"

"OneDimensionalArrayAggregate(a:Type)" [color=lightblue];
"OneDimensionalArrayAggregate(a:Type)" ->
    "FiniteLinearAggregate(a:Type)"

"FiniteLinearAggregate(a:Type)" [color=lightblue];
"FiniteLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "CLAGG..."

"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" -> "IXAGG..."

"CLAGG..." [color=lightblue];
"IXAGG..." [color=lightblue];
}
```

10.0.171 RectangularMatrixCategory (RMATCAT)



We define three categories for matrices

- MatrixCategory is the category of all matrices
- RectangularMatrixCategory is the category of all matrices of a given dimension

- SquareMatrixCategory inherits from RectangularMatrixCategory

RectangularMatrixCategory does not automatically inherit MatrixCategory. Note that domains in DirectProductCategory(n,R), which are expected as parameters of RectangularMatrixCategory do not satisfy FiniteLinearAggregate(R) as required in MatrixCategory.

The RectangularMatrix domain is matrices of fixed dimension.

— **RectangularMatrixCategory.input** —

```
)set break resume
)sys rm -f RectangularMatrixCategory.output
)spool RectangularMatrixCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RectangularMatrixCategory
--R
--R RectangularMatrixCategory(m: NonNegativeInteger,n: NonNegativeInteger,R: Ring,Row: DirectProductCategory(t#2
--R Abbreviation for RectangularMatrixCategory is RMATCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for RMATCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
--R ?*? : (Integer,%) -> %                ?*? : (NonNegativeInteger,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (%,R) -> % if R has FIELD       ?=? : (%,%) -> Boolean
--R 0 : () -> %                           antisymmetric? : % -> Boolean
--R coerce : % -> OutputForm              column : (%,Integer) -> Col
--R copy : % -> %                         diagonal? : % -> Boolean
--R elt : (%,Integer,Integer,R) -> R      elt : (%,Integer,Integer) -> R
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                hash : % -> SingleInteger
--R latex : % -> String                   listOfLists : % -> List(List(R))
--R map : (((R,R) -> R),%,%) -> %         map : ((R -> R),%) -> %
--R matrix : List(List(R)) -> %           maxColIndex : % -> Integer
--R maxRowIndex : % -> Integer            minColIndex : % -> Integer
--R minRowIndex : % -> Integer            ncols : % -> NonNegativeInteger
--R nrows : % -> NonNegativeInteger       qelt : (%,Integer,Integer) -> R
--R row : (%,Integer) -> Row              sample : () -> %
--R square? : % -> Boolean                symmetric? : % -> Boolean
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
--R eval : (%,List(Equation(R))) -> % if R has EVALAB(R) and R has SETCAT
--R eval : (%,Equation(R)) -> % if R has EVALAB(R) and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB(R) and R has SETCAT
--R eval : (%,List(R),List(R)) -> % if R has EVALAB(R) and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
```

```
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((R -> R),%) -> % if $ has shallowlyMutable
--R member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
--R members : % -> List(R) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R nullSpace : % -> List(Col) if R has INTDOM
--R nullity : % -> NonNegativeInteger if R has INTDOM
--R parts : % -> List(R) if $ has finiteAggregate
--R rank : % -> NonNegativeInteger if R has INTDOM
--R rowEchelon : % -> % if R has EUCDOM
--R size? : (%,NonNegativeInteger) -> Boolean
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— **RectangularMatrixCategory.help** —

```
====================================================================
RectangularMatrixCategory examples
====================================================================

RectangularMatrixCategory is a category of matrices of fixed dimensions.
The dimensions of the matrix will be parameters of the domain.
Domains in this category will be R-modules and will be non-mutable.

See Also:
o )show RectangularMatrixCategory
```

See:

⇒ “SquareMatrixCategory” (SMATCAT) 12.0.192 on page 1266
⇐ “BiModule” (BMODULE) 9.0.146 on page 902
⇐ “HomogeneousAggregate” (HOAGG) 4.0.73 on page 280
⇐ “MatrixCategory” (MATCAT) 6.0.112 on page 540

Exports:

0	antisymmetric?	any?	coerce	column
copy	count	diagonal?	elt	empty
empty?	eq?	eval	every?	exquo
hash	latex	less?	listOfLists	map
map!	matrix	maxColIndex	maxRowIndex	member?
members	minColIndex	minRowIndex	more?	ncols
nrows	nullSpace	nullity	parts	qelt
rank	row	rowEchelon	sample	size?
square?	subtractIfCan	symmetric?	zero?	#?
?*?	?/?	?+?	?-?	-?
?=?	?~=?			

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **nil**

These are directly exported but not implemented:

```
column : (%,Integer) -> Col
elt : (%,Integer,Integer) -> R
elt : (%,Integer,Integer,R) -> R
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
listOfLists : % -> List List R
map : (((R,R) -> R),%,%) -> %
map : ((R -> R),%) -> %
maxColIndex : % -> Integer
maxRowIndex : % -> Integer
matrix : List List R -> %
minColIndex : % -> Integer
minRowIndex : % -> Integer
nullity : % -> NonNegativeInteger if R has INTDOM
nullSpace : % -> List Col if R has INTDOM
qelt : (%,Integer,Integer) -> R
rank : % -> NonNegativeInteger if R has INTDOM
row : (%,Integer) -> Row
rowEchelon : % -> % if R has EUCDOM
?/? : (%,R) -> % if R has FIELD
```

These are implemented by this category:

```
antisymmetric? : % -> Boolean
diagonal? : % -> Boolean
ncols : % -> NonNegativeInteger
nrows : % -> NonNegativeInteger
square? : % -> Boolean
symmetric? : % -> Boolean
```

These exports come from (p902) BiModule(a:Ring,b:Ring)

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
```

```
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (R,%) -> %
?*? : (%,R) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Integer,%) -> %
?=? : (%,%) -> Boolean
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
```

These exports come from (p280) HomogeneousAggregate(Ring)"

```
any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
copy : % -> %
count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
less? : (%,NonNegativeInteger) -> Boolean
map! : ((R -> R),%) -> % if $ has shallowlyMutable
members : % -> List R if $ has finiteAggregate
member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List R if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
```

— RectangularMatrixCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#RMATCAT">
RectangularMatrixCategory (RMATCAT)</a></h2>
</body>
```

— category RMATCAT RectangularMatrixCategory —

```
)abbrev category RMATCAT RectangularMatrixCategory
++ Authors: Grabmeier, Gschnitzer, Williamson
++ Date Created: 1987
++ Date Last Updated: July 1990
++ Description:
++ \spadtype{RectangularMatrixCategory} is a category of matrices of fixed
```

```
++ dimensions. The dimensions of the matrix will be parameters of the
++ domain. Domains in this category will be R-modules and will be non-mutable.

RectangularMatrixCategory(m,n,R,Row,Col) : Category == SIG where
  m : NonNegativeInteger
  n : NonNegativeInteger
  R : Ring
  Row : DirectProductCategory(n,R)
  Col : DirectProductCategory(m,R)

  SIG ==> Join(BiModule(R,R),HomogeneousAggregate(R)) with

    finiteAggregate
      ++ matrices are finite

    if R has CommutativeRing then Module(R)

    --% Matrix creation

    matrix : List List R -> %
      ++ \spad{matrix(l)} converts the list of lists l to a matrix, where the
      ++ list of lists is viewed as a list of the rows of the matrix.

    --% Predicates

    square? : % -> Boolean
      ++ \spad{square?(m)} returns true if m is a square matrix (that is, if m
      ++ has the same number of rows as columns) and false otherwise.

    diagonal? : % -> Boolean
      ++ \spad{diagonal?(m)} returns true if the matrix m is square and
      ++ diagonal (that is, all entries of m not on the diagonal are zero) and
      ++ false otherwise.

    symmetric? : % -> Boolean
      ++ \spad{symmetric?(m)} returns true if the matrix m is square and
      ++ symmetric (that is, \spad{m[i,j] = m[j,i]} for all \spad{i} and j) and
      ++ false otherwise.

    antisymmetric? : % -> Boolean
      ++ \spad{antisymmetric?(m)} returns true if the matrix m is square and
      ++ antisymmetric. That is,
      ++ \spad{m[i,j] = -m[j,i]} for all \spad{i} and j
      ++ and false otherwise.

    --% Size inquiries

    minRowIndex : % -> Integer
      ++ \spad{minRowIndex(m)} returns the index of the 'first' row of the
      ++ matrix m.

    maxRowIndex : % -> Integer
      ++ \spad{maxRowIndex(m)} returns the index of the 'last' row of the
      ++ matrix m.
```

```
minColIndex : % -> Integer
  ++ \spad{minColIndex(m)} returns the index of the 'first' column of the
  ++ matrix m.

maxColIndex : % -> Integer
  ++ \spad{maxColIndex(m)} returns the index of the 'last' column of the
  ++ matrix m.

nrows : % -> NonNegativeInteger
  ++ \spad{nrows(m)} returns the number of rows in the matrix m.

ncols : % -> NonNegativeInteger
  ++ \spad{ncols(m)} returns the number of columns in the matrix m.

--% Part extractions

listOfLists : % -> List List R
  ++ \spad{listOfLists(m)} returns the rows of the matrix m as a list
  ++ of lists.

elt : (%,Integer,Integer) -> R
  ++ \spad{elt(m,i,j)} returns the element in the \spad{i}th row and
  ++ \spad{j}th column of the matrix m.
  ++ Error: if indices are outside the proper
  ++ ranges.

qelt : (%,Integer,Integer) -> R
  ++ \spad{qelt(m,i,j)} returns the element in the \spad{i}th row and
  ++ \spad{j}th column of
  ++ the matrix m. Note that there is NO error check to determine
  ++ if indices are in the proper ranges.

elt : (%,Integer,Integer,R) -> R
  ++ \spad{elt(m,i,j,r)} returns the element in the \spad{i}th row and
  ++ \spad{j}th column of the matrix m, if m has an \spad{i}th row and a
  ++ \spad{j}th column, and returns r otherwise.

row : (%,Integer) -> Row
  ++ \spad{row(m,i)} returns the \spad{i}th row of the matrix m.
  ++ Error: if the index is outside the proper range.

column : (%,Integer) -> Col
  ++ \spad{column(m,j)} returns the \spad{j}th column of the matrix m.
  ++ Error: if the index outside the proper range.

--% Map and Zip

map : (R -> R,%) -> %
  ++ \spad{map(f,a)} returns b, where \spad{b(i,j) = a(i,j)} for all i, j.

map : ((R,R) -> R,%,%) -> %
  ++ \spad{map(f,a,b)} returns c, where c is such that
  ++ \spad{c(i,j) = f(a(i,j),b(i,j))} for all \spad{i}, j.
```

```
    --% Arithmetic

    if R has IntegralDomain then

      "exquo" : (%,R) -> Union(%,"failed")
        ++ \spad{exquo(m,r)} computes the exact quotient of the elements
        ++ of m by r, returning \axiom{"failed"} if this is not possible.

    if R has Field then

      "/" : (%,R) -> %
        ++ \spad{m/r} divides the elements of m by r. Error: if \spad{r = 0}.

    --% Linear algebra

    if R has EuclideanDomain then

      rowEchelon : % -> %
        ++ \spad{rowEchelon(m)} returns the row echelon form of the matrix m.

    if R has IntegralDomain then

      rank : % -> NonNegativeInteger
        ++ \spad{rank(m)} returns the rank of the matrix m.

      nullity : % -> NonNegativeInteger
        ++ \spad{nullity(m)} returns the nullity of the matrix m. This is
        ++ the dimension of the null space of the matrix m.

      nullSpace : % -> List Col
        ++ \spad{nullSpace(m)}+ returns a basis for the null space of
        ++ the matrix m.
  add

   nrows x == m

   ncols x == n

   square? x == m = n

   diagonal? x ==
     not square? x => false
     for i in minRowIndex x .. maxRowIndex x repeat
       for j in minColIndex x .. maxColIndex x
         | (j - minColIndex x) ^= (i - minRowIndex x) repeat
           not zero? qelt(x, i, j) => return false
     true

   symmetric? x ==
     m ^= n => false
     mr := minRowIndex x; mc := minColIndex x
     for i in 0..(n - 1) repeat
       for j in (i + 1)..(n - 1) repeat
```

```
        qelt(x,mr + i,mc + j) ^= qelt(x,mr + j,mc + i) => return false
    true

  antisymmetric? x ==
    m ^= n => false
    mr := minRowIndex x; mc := minColIndex x
    for i in 0..(n - 1) repeat
      for j in i..(n - 1) repeat
        qelt(x,mr + i,mc + j) ^= -qelt(x,mr + j,mc + i) =>
          return false
    true
```

— **COQ RMATCAT** —

```
(* category RMATCAT *)
(*
     nrows : % -> NonNegativeInteger
     nrows x == m

     ncols : % -> NonNegativeInteger
     ncols x == n

     square? : % -> Boolean
     square? x == m = n

     diagonal? : % -> Boolean
     diagonal? x ==
       not square? x => false
       for i in minRowIndex x .. maxRowIndex x repeat
         for j in minColIndex x .. maxColIndex x
           | (j - minColIndex x) ^= (i - minRowIndex x) repeat
             not zero? qelt(x, i, j) => return false
       true

     symmetric? : % -> Boolean
     symmetric? x ==
       m ^= n => false
       mr := minRowIndex x; mc := minColIndex x
       for i in 0..(n - 1) repeat
         for j in (i + 1)..(n - 1) repeat
           qelt(x,mr + i,mc + j) ^= qelt(x,mr + j,mc + i) => return false
       true

     antisymmetric? : % -> Boolean
     antisymmetric? x ==
       m ^= n => false
       mr := minRowIndex x; mc := minColIndex x
       for i in 0..(n - 1) repeat
         for j in i..(n - 1) repeat
           qelt(x,mr + i,mc + j) ^= -qelt(x,mr + j,mc + i) =>
             return false
```

```
      true

*)
```

———-

— RMATCAT.dotabb —

```
"RMATCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RMATCAT"];
"RMATCAT" -> "BMODULE"
"RMATCAT" -> "HOAGG"
```

———-

— RMATCAT.dotfull —

```
"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProductCategory(b,c),e:Direc
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RMATCAT"];
"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProductCategory(b,c),e:Direc
  -> "BiModule(a:Ring,b:Ring)"
"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProductCategory(b,c),e:Direc
  -> "HomogeneousAggregate(Ring)"
```

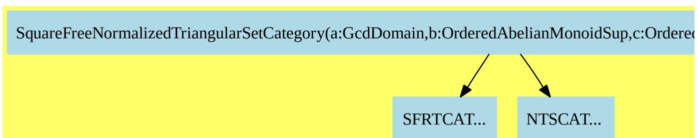
———-

— RMATCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProductCategory(b,c),e:Direc
 [color=lightblue];
"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProductCategory(b,c),e:Direc
  -> "BiModule(a:Ring,b:Ring)"
"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProductCategory(b,c),e:Direc
  -> "HOAGG..."

"HOAGG..." [color=lightblue];
"ABELGRP..." [color=lightblue];

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
```

```
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

}
```

10.0.172 SquareFreeNormalizedTriangularSetCategory (SNTSCAT)



— SquareFreeNormalizedTriangularSetCategory.input —

```
)set break resume
)sys rm -f SquareFreeNormalizedTriangularSetCategory.output
)spool SquareFreeNormalizedTriangularSetCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SquareFreeNormalizedTriangularSetCategory
--R
--R SquareFreeNormalizedTriangularSetCategory(R: GcdDomain,E: OrderedAbelianMonoidSup,V: OrderedSet,P: Recursive
--R Abbreviation for SquareFreeNormalizedTriangularSetCategory is SNTSCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SNTSCAT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                algebraic? : (V,%) -> Boolean
--R algebraicVariables : % -> List(V)     augment : (List(P),%) -> List(%)
--R augment : (P,List(%)) -> List(%)      augment : (P,%) -> List(%)
--R coerce : % -> List(P)                 coerce : % -> OutputForm
--R collect : (%,V) -> %                  collectQuasiMonic : % -> %
--R collectUnder : (%,V) -> %             collectUpper : (%,V) -> %
--R construct : List(P) -> %              copy : % -> %
--R degree : % -> NonNegativeInteger      empty : () -> %
--R empty? : % -> Boolean                 eq? : (%,%) -> Boolean
--R extend : (List(P),%) -> List(%)       extend : (P,List(%)) -> List(%)
--R extend : (P,%) -> List(%)             extend : (%,P) -> %
--R first : % -> Union(P,"failed")        hash : % -> SingleInteger
--R headReduce : (P,%) -> P               headReduced? : % -> Boolean
```

```
--R headReduced? : (P,%) -> Boolean       infRittWu? : (%,%) -> Boolean
--R initiallyReduce : (P,%) -> P          initiallyReduced? : % -> Boolean
--R initials : % -> List(P)               internalAugment : (List(P),%) -> %
--R internalAugment : (P,%) -> %          intersect : (P,List(%)) -> List(%)
--R intersect : (List(P),%) -> List(%)    intersect : (P,%) -> List(%)
--R invertible? : (P,%) -> Boolean        invertibleSet : (P,%) -> List(%)
--R last : % -> Union(P,"failed")         latex : % -> String
--R mainVariable? : (V,%) -> Boolean      mainVariables : % -> List(V)
--R map : ((P -> P),%) -> %               mvar : % -> V
--R normalized? : % -> Boolean            normalized? : (P,%) -> Boolean
--R purelyAlgebraic? : % -> Boolean       purelyAlgebraic? : (P,%) -> Boolean
--R reduceByQuasiMonic : (P,%) -> P       removeZero : (P,%) -> P
--R rest : % -> Union(%,"failed")         retract : List(P) -> %
--R sample : () -> %                      select : (%,V) -> Union(P,"failed")
--R stronglyReduce : (P,%) -> P           stronglyReduced? : % -> Boolean
--R stronglyReduced? : (P,%) -> Boolean   trivialIdeal? : % -> Boolean
--R variables : % -> List(V)              zeroSetSplit : List(P) -> List(%)
--R ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R algebraicCoefficients? : (P,%) -> Boolean
--R any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R augment : (List(P),List(%)) -> List(%)
--R autoReduced? : (%,((P,List(P)) -> Boolean)) -> Boolean
--R basicSet : (List(P),(P -> Boolean),((P,P) -> Boolean)) -> Union(Record(bas: %,top: List(P)),"failed")
--R basicSet : (List(P),((P,P) -> Boolean)) -> Union(Record(bas: %,top: List(P)),"failed")
--R coHeight : % -> NonNegativeInteger if V has FINITE
--R convert : % -> InputForm if P has KONVERT(INFORM)
--R count : ((P -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R count : (P,%) -> NonNegativeInteger if P has SETCAT and $ has finiteAggregate
--R eval : (%,List(Equation(P))) -> % if P has EVALAB(P) and P has SETCAT
--R eval : (%,Equation(P)) -> % if P has EVALAB(P) and P has SETCAT
--R eval : (%,P,P) -> % if P has EVALAB(P) and P has SETCAT
--R eval : (%,List(P),List(P)) -> % if P has EVALAB(P) and P has SETCAT
--R every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
--R extend : (List(P),List(%)) -> List(%)
--R extendIfCan : (%,P) -> Union(%,"failed")
--R find : ((P -> Boolean),%) -> Union(P,"failed")
--R headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
--R initiallyReduced? : (P,%) -> Boolean
--R intersect : (List(P),List(%)) -> List(%)
--R invertible? : (P,%) -> List(Record(val: Boolean,tower: %))
--R invertibleElseSplit? : (P,%) -> Union(Boolean,List(%))
--R lastSubResultant : (P,P,%) -> List(Record(val: P,tower: %))
--R lastSubResultantElseSplit : (P,P,%) -> Union(P,List(%))
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((P -> P),%) -> % if $ has shallowlyMutable
--R member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
--R members : % -> List(P) if $ has finiteAggregate
--R more? : (%,NonNegativeInteger) -> Boolean
--R parts : % -> List(P) if $ has finiteAggregate
--R purelyAlgebraicLeadingMonomial? : (P,%) -> Boolean
--R purelyTranscendental? : (P,%) -> Boolean
--R quasiComponent : % -> Record(close: List(P),open: List(P))
--R reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
```

```
--R reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
--R reduce : (((P,P) -> P),%,P,P) -> P if P has SETCAT and $ has finiteAggregate
--R reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
--R remainder : (P,%) -> Record(rnum: R,polnum: P,den: R) if R has INTDOM
--R remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
--R retractIfCan : List(P) -> Union(%,"failed")
--R rewriteIdealWithHeadRemainder : (List(P),%) -> List(P) if R has INTDOM
--R rewriteIdealWithRemainder : (List(P),%) -> List(P) if R has INTDOM
--R rewriteSetWithReduction : (List(P),%,((P,P) -> P),((P,P) -> Boolean)) -> List(P)
--R roughBase? : % -> Boolean if R has INTDOM
--R roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
--R roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
--R roughUnitIdeal? : % -> Boolean if R has INTDOM
--R select : ((P -> Boolean),%) -> % if $ has finiteAggregate
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : (%,V) -> Record(under: %,floor: %,upper: %)
--R squareFreePart : (P,%) -> List(Record(val: P,tower: %))
--R triangular? : % -> Boolean if R has INTDOM
--R zeroSetSplit : (List(P),Boolean) -> List(%)
--R zeroSetSplitIntoTriangularSystems : List(P) -> List(Record(close: %,open: List(P)))
--R
--E 1

)spool
)lisp (bye)
```

———

— **SquareFreeNormalizedTriangularSetCategory.help** —

```
====================================================================
SquareFreeNormalizedTriangularSetCategory examples
====================================================================

The category of square-free and normalized triangular sets.
Thus, up to the primitivity axiom of D. LAZARD
"A new method for solving algebraic systems of positive dimension",
these sets are Lazard triangular sets.

See Also:
o )show SquareFreeNormalizedTriangularSetCategory
```

———

See:

⇐ "NormalizedTriangularSetCategory" (NTSCAT) 9.0.149 on page 922
⇐ "SquareFreeRegularTriangularSetCategory" (SFRTCAT) 9.0.154 on page 952

Exports:

algebraic?	algebraicCoefficients?
algebraicVariables	any?
augment	autoReduced?
basicSet	coerce
coHeight	collect
collectQuasiMonic	collectUnder
collectUpper	construct
convert	copy
count	degree
empty	empty?
eq?	eval
every?	extend
extendIfCan	find
first	hash
headReduce	headReduced?
headRemainder	infRittWu?
initiallyReduce	initiallyReduced?
initials	internalAugment
intersect	invertible?
invertibleElseSplit?	invertibleSet
last	lastSubResultant
lastSubResultantElseSplit	latex
less?	mainVariable?
mainVariables	map
map!	member?
members	more?
mvar	normalized?
parts	purelyAlgebraic?
purelyAlgebraicLeadingMonomial?	purelyTranscendental?
quasiComponent	reduce
reduced?	reduceByQuasiMonic
remainder	remove
removeDuplicates	removeZero
rest	retract
retractIfCan	rewriteIdealWithHeadRemainder
rewriteIdealWithRemainder	rewriteSetWithReduction
roughBase?	roughEqualIdeals?
roughSubIdeal?	roughUnitIdeal?
sample	select
size?	sort
squareFreePart	stronglyReduce
stronglyReduced?	triangular?
trivialIdeal?	variables
zeroSetSplit	zeroSetSplitIntoTriangularSystems
#?	?=?
?~=?	

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These exports come from
(p952) SquareFreeRegularTriangularSetCategory(R,E,V,P)
where R:GcdDomain, E:OrderedAbelianMonoidSup, V:OrderedSet,
P:RecursivePolynomialCategory(R,E,V)):

```
algebraic? : (V,%) -> Boolean
algebraicCoefficients? : (P,%) -> Boolean
algebraicVariables : % -> List V
any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
augment : (List P,List %) -> List %
augment : (List P,%) -> List %
augment : (P,List %) -> List %
augment : (P,%) -> List %
autoReduced? : (%,((P,List P) -> Boolean)) -> Boolean
basicSet :
  (List P,(P -> Boolean),((P,P) -> Boolean)) ->
     Union(Record(bas: %,top: List P),"failed")
basicSet :
  (List P,((P,P) -> Boolean)) ->
     Union(Record(bas: %,top: List P),"failed")
coerce : % -> List P
coerce : % -> OutputForm
coHeight : % -> NonNegativeInteger if V has FINITE
collect : (%,V) -> %
collectQuasiMonic : % -> %
collectUnder : (%,V) -> %
collectUpper : (%,V) -> %
construct : List P -> %
convert : % -> InputForm if P has KONVERT INFORM
copy : % -> %
count : ((P -> Boolean),%) -> NonNegativeInteger
  if $ has finiteAggregate
count : (P,%) -> NonNegativeInteger
  if P has SETCAT
  and $ has finiteAggregate
degree : % -> NonNegativeInteger
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
extend : (List P,List %) -> List %
extend : (List P,%) -> List %
extend : (P,List %) -> List %
extend : (P,%) -> List %
extend : (%,P) -> %
extendIfCan : (%,P) -> Union(%,"failed")
```

```
find : ((P -> Boolean),%) -> Union(P,"failed")
first : % -> Union(P,"failed")
hash : % -> SingleInteger
headReduce : (P,%) -> P
headReduced? : % -> Boolean
headReduced? : (P,%) -> Boolean
headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
infRittWu? : (%,%) -> Boolean
initiallyReduce : (P,%) -> P
initiallyReduced? : % -> Boolean
initiallyReduced? : (P,%) -> Boolean
initials : % -> List P
internalAugment : (P,%) -> %
internalAugment : (List P,%) -> %
intersect : (List P,List %) -> List %
intersect : (P,List %) -> List %
intersect : (List P,%) -> List %
intersect : (P,%) -> List %
invertible? : (P,%) -> Boolean
invertible? : (P,%) -> List Record(val: Boolean,tower: %)
invertibleElseSplit? : (P,%) -> Union(Boolean,List %)
invertibleSet : (P,%) -> List %
last : % -> Union(P,"failed")
lastSubResultant : (P,P,%) -> List Record(val: P,tower: %)
lastSubResultantElseSplit : (P,P,%) -> Union(P,List %)
latex : % -> String
less? : (%,NonNegativeInteger) -> Boolean
mainVariable? : (V,%) -> Boolean
mainVariables : % -> List V
map : ((P -> P),%) -> %
map! : ((P -> P),%) -> % if $ has shallowlyMutable
member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
members : % -> List P if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
mvar : % -> V
normalized? : % -> Boolean
normalized? : (P,%) -> Boolean
parts : % -> List P if $ has finiteAggregate
purelyAlgebraic? : % -> Boolean
purelyAlgebraic? : (P,%) -> Boolean
purelyAlgebraicLeadingMonomial? : (P,%) -> Boolean
purelyTranscendental? : (P,%) -> Boolean
quasiComponent : % -> Record(close: List P,open: List P)
reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P,P) -> P
  if P has SETCAT
  and $ has finiteAggregate
reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
reduceByQuasiMonic : (P,%) -> P
remainder : (P,%) -> Record(rnum: R,polnum: P,den: R) if R has INTDOM
remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
```

```
removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
removeZero : (P,%) -> P
rest : % -> Union(%,"failed")
retract : List P -> %
retractIfCan : List P -> Union(%,"failed")
rewriteIdealWithHeadRemainder : (List P,%) -> List P if R has INTDOM
rewriteIdealWithRemainder : (List P,%) -> List P if R has INTDOM
rewriteSetWithReduction :
   (List P,%,((P,P) -> P),((P,P) -> Boolean)) -> List P
roughBase? : % -> Boolean if R has INTDOM
roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
roughUnitIdeal? : % -> Boolean if R has INTDOM
sample : () -> %
select : (%,V) -> Union(P,"failed")
select : ((P -> Boolean),%) -> % if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
sort : (%,V) -> Record(under: %,floor: %,upper: %)
squareFreePart : (P,%) -> List Record(val: P,tower: %)
stronglyReduce : (P,%) -> P
stronglyReduced? : (P,%) -> Boolean
stronglyReduced? : % -> Boolean
triangular? : % -> Boolean if R has INTDOM
trivialIdeal? : % -> Boolean
variables : % -> List V
zeroSetSplit : List P -> List %
zeroSetSplit : (List P,Boolean) -> List %
zeroSetSplitIntoTriangularSystems :
   List P -> List Record(close: %,open: List P)
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

See Lazard[Laza91]

— SquareFreeNormalizedTriangularSetCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#SNTSCAT">
SquareFreeNormalizedTriangularSetCategory (SNTSCAT)</a></h2>
</body>
```

———-

— category SNTSCAT SquareFreeNormalizedTriangularSetCategory —

```
)abbrev category SNTSCAT SquareFreeNormalizedTriangularSetCategory
++ Author: Marc Moreno Maza
++ Date Created: 10/07/1998
++ Date Last Updated: 12/16/1998
++ References :
++ Laza91 A new method for solving algebraic systems of positive dimension
++ Description:
++ The category of square-free and normalized triangular sets.
```

```
++ Thus, up to the primitivity axiom of [1], these sets are Lazard
++ triangular sets.

SquareFreeNormalizedTriangularSetCategory(R,E,V,P) : Category == SIG where
  R : GcdDomain
  E : OrderedAbelianMonoidSup
  V : OrderedSet
  P : RecursivePolynomialCategory(R,E,V)

  SFRTS ==> SquareFreeRegularTriangularSetCategory(R,E,V,P)
  NTSC ==> NormalizedTriangularSetCategory(R,E,V,P)

  SIG ==> Join(SFRTS,NTSC)
```

———-

— SNTSCAT.dotabb —

```
"SNTSCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SNTSCAT"];
"SNTSCAT" -> "NTSCAT"
"SNTSCAT" -> "SFRTCAT"
```

———-

— SNTSCAT.dotfull —

```
"SquareFreeNormalizedTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynom
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SNTSCAT"];
"SquareFreeNormalizedTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynom
 ->
"SquareFreeRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomial

"SquareFreeNormalizedTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynom
 ->
"NormalizedRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomial
```

———-

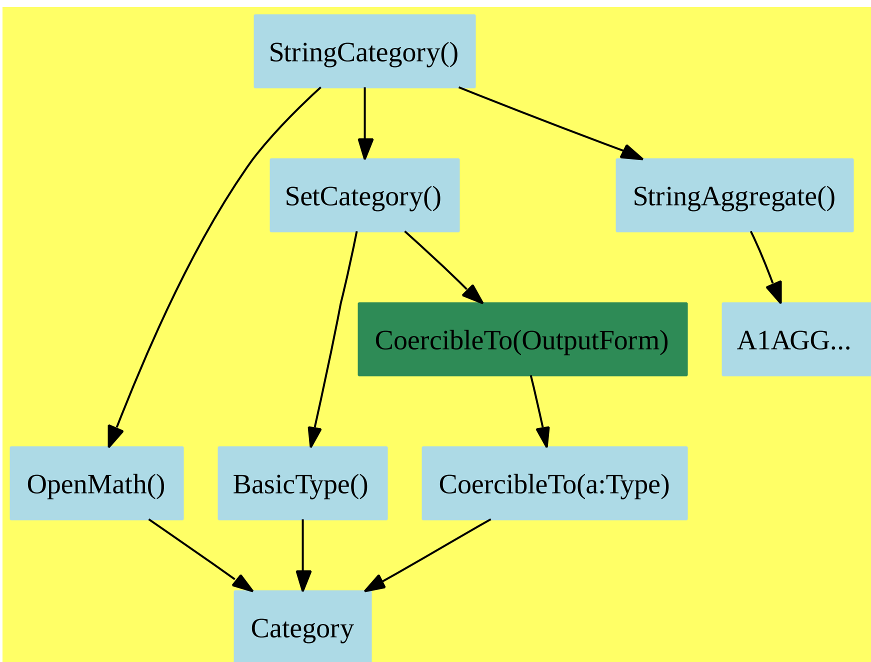
— SNTSCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"SquareFreeNormalizedTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynom
 [color=lightblue];
"SquareFreeNormalizedTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynom
 -> "SFRTCAT..."
```

```
"SquareFreeNormalizedTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynom
 -> "NTSCAT..."

"SFRTCAT..."  [color=lightblue];
"NTSCAT..."  [color=lightblue];

}
```

————-

10.0.173 StringCategory (STRICAT)



— StringCategory.input —

```
)set break resume
)sys rm -f StringCategory.output
)spool StringCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show StringCategory
--R
--R StringCategory is a category constructor
--R Abbreviation for StringCategory is STRICAT
--R This constructor is exposed in this frame.
```

```
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for STRICAT
--R
--R------------------------------- Operations --------------------------------
--R ?=? : (%,%) -> Boolean                OMwrite : (%,Boolean) -> String
--R OMwrite : % -> String                 coerce : % -> OutputForm
--R coerce : Character -> %               concat : List(%) -> %
--R concat : (%,%) -> %                   concat : (Character,%) -> %
--R concat : (%,Character) -> %           construct : List(Character) -> %
--R copy : % -> %                         delete : (%,Integer) -> %
--R ?.? : (%,%) -> %                      ?.? : (%,Integer) -> Character
--R empty : () -> %                       empty? : % -> Boolean
--R entries : % -> List(Character)        eq? : (%,%) -> Boolean
--R hash : % -> SingleInteger             index? : (Integer,%) -> Boolean
--R indices : % -> List(Integer)          insert : (%,%,Integer) -> %
--R latex : % -> String                   leftTrim : (%,CharacterClass) -> %
--R leftTrim : (%,Character) -> %         lowerCase : % -> %
--R lowerCase! : % -> %                   prefix? : (%,%) -> Boolean
--R qelt : (%,Integer) -> Character       reverse : % -> %
--R rightTrim : (%,Character) -> %        sample : () -> %
--R split : (%,Character) -> List(%)      string : Integer -> %
--R suffix? : (%,%) -> Boolean            trim : (%,CharacterClass) -> %
--R trim : (%,Character) -> %             upperCase : % -> %
--R upperCase! : % -> %                   ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?<? : (%,%) -> Boolean if Character has ORDSET
--R ?<=? : (%,%) -> Boolean if Character has ORDSET
--R ?>? : (%,%) -> Boolean if Character has ORDSET
--R ?>=? : (%,%) -> Boolean if Character has ORDSET
--R OMwrite : (OpenMathDevice,%,Boolean) -> Void
--R OMwrite : (OpenMathDevice,%) -> Void
--R any? : ((Character -> Boolean),%) -> Boolean if $ has finiteAggregate
--R convert : % -> InputForm if Character has KONVERT(INFORM)
--R copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
--R count : (Character,%) -> NonNegativeInteger if Character has SETCAT and $ has finiteAggregate
--R count : ((Character -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R delete : (%,UniversalSegment(Integer)) -> %
--R ?.? : (%,UniversalSegment(Integer)) -> %
--R elt : (%,Integer,Character) -> Character
--R entry? : (Character,%) -> Boolean if $ has finiteAggregate and Character has SETCAT
--R eval : (%,List(Character),List(Character)) -> % if Character has EVALAB(CHAR) and Character has SETCAT
--R eval : (%,Character,Character) -> % if Character has EVALAB(CHAR) and Character has SETCAT
--R eval : (%,Equation(Character)) -> % if Character has EVALAB(CHAR) and Character has SETCAT
--R eval : (%,List(Equation(Character))) -> % if Character has EVALAB(CHAR) and Character has SETCAT
--R every? : ((Character -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,Character) -> % if $ has shallowlyMutable
--R find : ((Character -> Boolean),%) -> Union(Character,"failed")
--R first : % -> Character if Integer has ORDSET
--R insert : (Character,%,Integer) -> %
--R less? : (%,NonNegativeInteger) -> Boolean
--R map : (((Character,Character) -> Character),%,%) -> %
--R map : ((Character -> Character),%) -> %
--R map! : ((Character -> Character),%) -> % if $ has shallowlyMutable
--R match : (%,%,Character) -> NonNegativeInteger
--R match? : (%,%,Character) -> Boolean
```

```
--R max : (%,%) -> % if Character has ORDSET
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (Character,%) -> Boolean if Character has SETCAT and $ has finiteAggregate
--R members : % -> List(Character) if $ has finiteAggregate
--R merge : (%,%) -> % if Character has ORDSET
--R merge : (((Character,Character) -> Boolean),%,%) -> %
--R min : (%,%) -> % if Character has ORDSET
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,Character) -> %
--R parts : % -> List(Character) if $ has finiteAggregate
--R position : (CharacterClass,%,Integer) -> Integer
--R position : (%,%,Integer) -> Integer
--R position : (Character,%,Integer) -> Integer if Character has SETCAT
--R position : (Character,%) -> Integer if Character has SETCAT
--R position : ((Character -> Boolean),%) -> Integer
--R qsetelt! : (%,Integer,Character) -> Character if $ has shallowlyMutable
--R reduce : (((Character,Character) -> Character),%) -> Character if $ has finiteAggregate
--R reduce : (((Character,Character) -> Character),%,Character) -> Character if $ has finiteAggregate
--R reduce : (((Character,Character) -> Character),%,Character,Character) -> Character if Character has SETCAT a
--R remove : ((Character -> Boolean),%) -> % if $ has finiteAggregate
--R remove : (Character,%) -> % if Character has SETCAT and $ has finiteAggregate
--R removeDuplicates : % -> % if Character has SETCAT and $ has finiteAggregate
--R replace : (%,UniversalSegment(Integer),%) -> %
--R reverse! : % -> % if $ has shallowlyMutable
--R rightTrim : (%,CharacterClass) -> %
--R select : ((Character -> Boolean),%) -> % if $ has finiteAggregate
--R setelt : (%,UniversalSegment(Integer),Character) -> Character if $ has shallowlyMutable
--R setelt : (%,Integer,Character) -> Character if $ has shallowlyMutable
--R size? : (%,NonNegativeInteger) -> Boolean
--R sort : % -> % if Character has ORDSET
--R sort : (((Character,Character) -> Boolean),%) -> %
--R sort! : % -> % if Character has ORDSET and $ has shallowlyMutable
--R sort! : (((Character,Character) -> Boolean),%) -> % if $ has shallowlyMutable
--R sorted? : % -> Boolean if Character has ORDSET
--R sorted? : (((Character,Character) -> Boolean),%) -> Boolean
--R split : (%,CharacterClass) -> List(%)
--R substring? : (%,%,Integer) -> Boolean
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R
--E 1

)spool
)lisp (bye)
```

— StringCategory.help —

```
====================================================================
StringCategory examples
====================================================================

A category for string-like objects
```

```
See Also:
o )show StringCategory
```

See:

⇐ "OpenMath" (OM) 2.0.31 on page 88
⇐ "SetCategory" (SETCAT) 3.0.57 on page 187
⇐ "StringAggregate" (SRAGG) 9.0.155 on page 961

Exports:

any?	coerce	concat	construct	convert
copy	copyInto!	count	delete	elt
empty	empty?	entry?	entries	eq?
eval	every?	fill!	find	first
hash	index?	indices	insert	latex
leftTrim	less?	lowerCase	lowerCase!	map
map!	match	match?	max	maxIndex
member?	members	merge	min	minIndex
more?	new	OMwrite	parts	position
prefix?	qelt	qsetelt!	reduce	remove
removeDuplicates	replace	reverse	reverse!	rightTrim
sample	select	setelt	size?	sort
sort!	sorted?	split	string	substring?
suffix?	swap!	trim	upperCase	upperCase!
#?	?<?	?<=?	?>?	?>=?
?=?	?.?	?~=?		

Attributes exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note that the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These are directly exported but not implemented:

```
string : Integer -> %
```

These exports come from (p961) StringAggregate():

```
any? : ((Character -> Boolean),%) -> Boolean
    if $ has finiteAggregate
coerce : % -> OutputForm
coerce : Character -> %
concat : List % -> %
concat : (%,%) -> %
concat : (Character,%) -> %
concat : (%,Character) -> %
construct : List Character -> %
```

```
convert : % -> InputForm
    if Character has KONVERT INFORM
copy : % -> %
copyInto! : (%,%,Integer) -> %
    if $ has shallowlyMutable
count : (Character,%) -> NonNegativeInteger
    if Character has SETCAT
    and $ has finiteAggregate
count : ((Character -> Boolean),%) -> NonNegativeInteger
    if $ has finiteAggregate
delete : (%,UniversalSegment Integer) -> %
delete : (%,Integer) -> %
elt : (%,Integer,Character) -> Character
empty : () -> %
empty? : % -> Boolean
entry? : (Character,%) -> Boolean
    if $ has finiteAggregate
    and Character has SETCAT
entries : % -> List Character
eq? : (%,%) -> Boolean
eval : (%,List Character,List Character) -> %
    if Character has EVALAB CHAR
    and Character has SETCAT
eval : (%,Character,Character) -> %
    if Character has EVALAB CHAR
    and Character has SETCAT
eval : (%,Equation Character) -> %
    if Character has EVALAB CHAR
    and Character has SETCAT
eval : (%,List Equation Character) -> %
    if Character has EVALAB CHAR
    and Character has SETCAT
every? : ((Character -> Boolean),%) -> Boolean
    if $ has finiteAggregate
fill! : (%,Character) -> %
    if $ has shallowlyMutable
find : ((Character -> Boolean),%) -> Union(Character,"failed")
first : % -> Character
    if Integer has ORDSET
hash : % -> SingleInteger
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (%,%,Integer) -> %
insert : (Character,%,Integer) -> %
latex : % -> String
leftTrim : (%,Character) -> %
leftTrim : (%,CharacterClass) -> %
less? : (%,NonNegativeInteger) -> Boolean
lowerCase : % -> %
lowerCase! : % -> %
map : (((Character,Character) -> Character),%,%) -> %
map : ((Character -> Character),%) -> %
map! : ((Character -> Character),%) -> %
    if $ has shallowlyMutable
```

```
match : (%,%,Character) -> NonNegativeInteger
match? : (%,%,Character) -> Boolean
max : (%,%) -> % if Character has ORDSET
maxIndex : % -> Integer if Integer has ORDSET
member? : (Character,%) -> Boolean
    if Character has SETCAT
    and $ has finiteAggregate
members : % -> List Character
    if $ has finiteAggregate
merge : (%,%) -> % if Character has ORDSET
merge : (((Character,Character) -> Boolean),%,%) -> %
min : (%,%) -> % if Character has ORDSET
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
new : (NonNegativeInteger,Character) -> %
parts : % -> List Character if $ has finiteAggregate
position : (Character,%) -> Integer
    if Character has SETCAT
position : ((Character -> Boolean),%) -> Integer
position : (Character,%,Integer) -> Integer
    if Character has SETCAT
position : (CharacterClass,%,Integer) -> Integer
position : (%,%,Integer) -> Integer
prefix? : (%,%) -> Boolean
qelt : (%,Integer) -> Character
qsetelt! : (%,Integer,Character) -> Character
    if $ has shallowlyMutable
reduce : (((Character,Character) -> Character),%)
   -> Character
    if $ has finiteAggregate
reduce : (((Character,Character) -> Character),%,Character)
   -> Character
    if $ has finiteAggregate
reduce :
 (((Character,Character) -> Character),%,Character,Character)
   -> Character
    if Character has SETCAT
    and $ has finiteAggregate
remove : ((Character -> Boolean),%) -> %
    if $ has finiteAggregate
remove : (Character,%) -> %
    if Character has SETCAT
    and $ has finiteAggregate
removeDuplicates : % -> %
    if Character has SETCAT
    and $ has finiteAggregate
replace : (%,UniversalSegment Integer,%) -> %
reverse : % -> %
reverse! : % -> % if $ has shallowlyMutable
rightTrim : (%,CharacterClass) -> %
rightTrim : (%,Character) -> %
sample : () -> %
select : ((Character -> Boolean),%) -> %
    if $ has finiteAggregate
```

```
setelt :
  (%,UniversalSegment Integer,Character) -> Character
    if $ has shallowlyMutable
setelt : (%,Integer,Character) -> Character
    if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
sort : % -> % if Character has ORDSET
sort : (((Character,Character) -> Boolean),%) -> %
sort! : % -> %
    if Character has ORDSET
    and $ has shallowlyMutable
sort! : (((Character,Character) -> Boolean),%) -> %
    if $ has shallowlyMutable
sorted? : (((Character,Character) -> Boolean),%) -> Boolean
sorted? : % -> Boolean if Character has ORDSET
split : (%,CharacterClass) -> List %
split : (%,Character) -> List %
substring? : (%,%,Integer) -> Boolean
suffix? : (%,%) -> Boolean
swap! : (%,Integer,Integer) -> Void
    if $ has shallowlyMutable
trim : (%,CharacterClass) -> %
trim : (%,Character) -> %
upperCase : % -> %
upperCase! : % -> %
#? : % -> NonNegativeInteger if $ has finiteAggregate
?<? : (%,%) -> Boolean if Character has ORDSET
?<=? : (%,%) -> Boolean if Character has ORDSET
?>? : (%,%) -> Boolean if Character has ORDSET
?>=? : (%,%) -> Boolean if Character has ORDSET
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?.? : (%,UniversalSegment Integer) -> %
?.? : (%,Integer) -> Character
?.? : (%,%) -> %
```

These exports come from (p187) SetCategory():

These exports come from (p88) OpenMath():

```
OMwrite : (%,Boolean) -> String
OMwrite : % -> String
OMwrite : (OpenMathDevice,%,Boolean) -> Void
OMwrite : (OpenMathDevice,%) -> Void
```

— **StringCategory.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#STRICAT">
StringCategory (STRICAT)</a></h2>
</body>
```

———-

— category STRICAT StringCategory —

```
)abbrev category STRICAT StringCategory
++ Description:
++ A category for string-like objects

StringCategory() : Category == SIG where

  SIG ==> Join(StringAggregate(), SetCategory, OpenMath) with

    string : Integer -> %
      ++ string(i) returns the decimal representation of i in a string
```

———-

— STRICAT.dotabb —

```
"STRICAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=STRICAT"];
"STRICAT" -> "OM"
"STRICAT" -> "SETCAT"
"STRICAT" -> "SRAGG"
```

———-

— STRICAT.dotfull —

```
"StringCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=STRICAT"];
"StringCategory()" -> "OpenMath()"
"StringCategory()" -> "SetCategory()"
"StringCategory()" -> "StringAggregate()"
```

———-

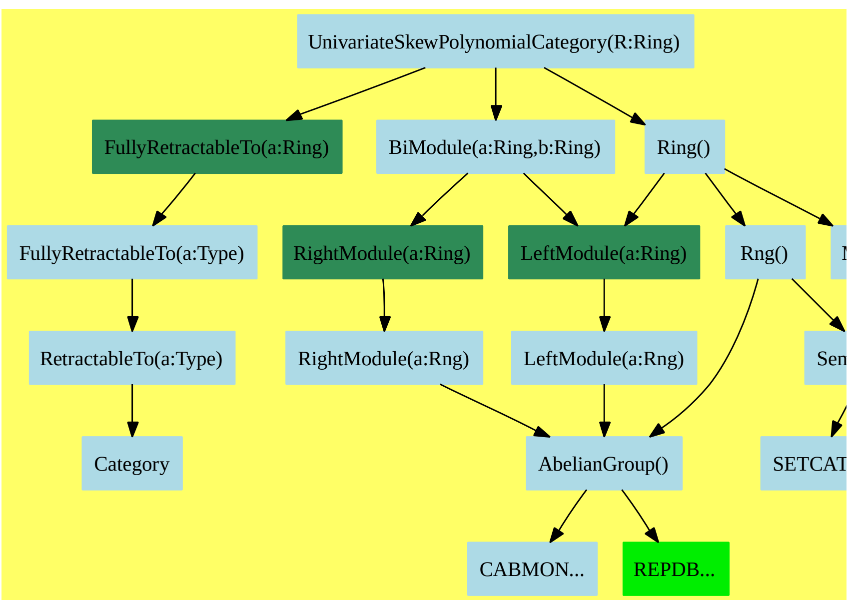
— STRICAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"StringCategory()" [color=lightblue];
"StringCategory()" -> "OpenMath()"
"StringCategory()" -> "SetCategory()"
"StringCategory()" -> "StringAggregate()"

"OpenMath()" [color=lightblue];
"OpenMath()" -> "Category"

"SetCategory()" [color=lightblue];
```

```
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"StringAggregate()" [color=lightblue];
"StringAggregate()" -> "A1AGG..."

"A1AGG..." [color=lightblue];

"Category" [color=lightblue];

}
```

————-

10.0.174 UnivariateSkewPolynomialCategory (OREPCAT)



— UnivariateSkewPolynomialCategory.input —

```
)set break resume
)sys rm -f UnivariateSkewPolynomialCategory.output
)spool UnivariateSkewPolynomialCategory.output
```

```
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariateSkewPolynomialCategory
--R
--R UnivariateSkewPolynomialCategory(R: Ring) is a category constructor
--R Abbreviation for UnivariateSkewPolynomialCategory is OREPCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for OREPCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                       ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                       ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %      ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %     ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                       ?-? : (%,%) -> %
--R -? : % -> %                            ?=? : (%,%) -> Boolean
--R 1 : () -> %                            0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %      ?^? : (%,PositiveInteger) -> %
--R apply : (%,R,R) -> R                   coefficients : % -> List(R)
--R coerce : R -> %                        coerce : Integer -> %
--R coerce : % -> OutputForm               content : % -> R if R has GCDDOM
--R degree : % -> NonNegativeInteger       hash : % -> SingleInteger
--R latex : % -> String                    leadingCoefficient : % -> R
--R one? : % -> Boolean                    recip : % -> Union(%,"failed")
--R reductum : % -> %                      retract : % -> R
--R sample : () -> %                       zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R coefficient : (%,NonNegativeInteger) -> R
--R coerce : Fraction(Integer) -> % if R has RETRACT(FRAC(INT))
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R leftDivide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R leftExactQuotient : (%,%) -> Union(%,"failed") if R has FIELD
--R leftExtendedGcd : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
--R leftGcd : (%,%) -> % if R has FIELD
--R leftLcm : (%,%) -> % if R has FIELD
--R leftQuotient : (%,%) -> % if R has FIELD
--R leftRemainder : (%,%) -> % if R has FIELD
--R minimumDegree : % -> NonNegativeInteger
--R monicLeftDivide : (%,%) -> Record(quotient: %,remainder: %) if R has INTDOM
--R monicRightDivide : (%,%) -> Record(quotient: %,remainder: %) if R has INTDOM
--R monomial : (R,NonNegativeInteger) -> %
--R primitivePart : % -> % if R has GCDDOM
--R retract : % -> Fraction(Integer) if R has RETRACT(FRAC(INT))
--R retract : % -> Integer if R has RETRACT(INT)
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction(Integer),"failed") if R has RETRACT(FRAC(INT))
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT(INT)
--R rightDivide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R rightExactQuotient : (%,%) -> Union(%,"failed") if R has FIELD
--R rightExtendedGcd : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
```

```
--R rightGcd : (%,%) -> % if R has FIELD
--R rightLcm : (%,%) -> % if R has FIELD
--R rightQuotient : (%,%) -> % if R has FIELD
--R rightRemainder : (%,%) -> % if R has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

— UnivariateSkewPolynomialCategory.help —

```
====================================================================
UnivariateSkewPolynomialCategory examples
====================================================================

This is the category of univariate skew polynomials over an Ore
coefficient ring. The multiplication is given by
     x a = \sigma(a) x + \delta a
This category is an evolution of the types MonogenicLinearOperator,
OppositeMonogenicLinearOperator, and NonCommutativeOperatorDivision

See Also:
o )show UnivariateSkewPolynomialCategory
```

———-

See:

⇒ "LinearOrdinaryDifferentialOperatorCategory" (LODOCAT) 11.0.180 on page 1145
⇐ "BiModule" (BMODULE) 9.0.146 on page 902
⇐ "FullyRetractableTo" (FRETRCT) 3.0.50 on page 158
⇐ "Ring" (RING) 9.0.153 on page 946

Exports:

0	1	apply	characteristic
coefficient	coefficients	coerce	content
degree	exquo	hash	latex
leadingCoefficient	leftDivide	leftExactQuotient	leftExtendedGcd
leftGcd	leftLcm	leftQuotient	leftRemainder
minimumDegree	monicLeftDivide	monicRightDivide	monomial
one?	primitivePart	recip	reductum
retract	retractIfCan	rightDivide	rightExactQuotient
rightExtendedGcd	rightGcd	rightLcm	rightQuotient
rightRemainder	sample	subtractIfCan	zero?
?*?	?**?	?+?	?-?
-?	?=?	?^?	?~=?

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
apply : (%,R,R) -> R
coefficient : (%,NonNegativeInteger) -> R
degree : % -> NonNegativeInteger
leadingCoefficient : % -> R
leftDivide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
minimumDegree : % -> NonNegativeInteger
monicLeftDivide : (%,%) -> Record(quotient: %,remainder: %) if R has INTDOM
monicRightDivide : (%,%) -> Record(quotient: %,remainder: %) if R has INTDOM
monomial : (R,NonNegativeInteger) -> %
reductum : % -> %
rightDivide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
```

These are implemented by this category:

```
coefficients : % -> List R
coerce : R -> %
content : % -> R if R has GCDDOM
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
leftExactQuotient : (%,%) -> Union(%,"failed") if R has FIELD
leftExtendedGcd : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
leftGcd : (%,%) -> % if R has FIELD
leftLcm : (%,%) -> % if R has FIELD
leftQuotient : (%,%) -> % if R has FIELD
leftRemainder : (%,%) -> % if R has FIELD
primitivePart : % -> % if R has GCDDOM
retractIfCan : % -> Union(R,"failed")
rightExactQuotient : (%,%) -> Union(%,"failed") if R has FIELD
rightExtendedGcd : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
rightGcd : (%,%) -> % if R has FIELD
rightLcm : (%,%) -> % if R has FIELD
rightQuotient : (%,%) -> % if R has FIELD
rightRemainder : (%,%) -> % if R has FIELD
?*? : (R,%) -> %
```

These exports come from (p946) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : % -> OutputForm
coerce : Integer -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?**? : (%,NonNegativeInteger) -> %
```

```
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p902) BiModule(R:Ring,R:Ring):

```
?*? : (%,R) -> %
```

These exports come from (p158) FullyRetractableTo(R:Ring):

```
coerce : Fraction Integer -> % if R has RETRACT FRAC INT
retract : % -> R
retract : % -> Fraction Integer if R has RETRACT FRAC INT
retract : % -> Integer if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
```

See: Bronstein[Bron95], Abramov[Abra01], Mulders[Muld95]

— UnivariateSkewPolynomialCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#OREPCAT">
UnivariateSkewPolynomialCategory (OREPCAT)</a></h2>
</body>
```

———-

— category OREPCAT UnivariateSkewPolynomialCategory —

```
)abbrev category OREPCAT UnivariateSkewPolynomialCategory
++ Author: Manuel Bronstein, Jean Della Dora, Stephen M. Watt
++ Date Created: 19 October 1993
++ Date Last Updated: 1 February 1994
++ References:
++ Bron95 On radical solutions of linear ordinary differential equations
++ Abra01 On Solutions of Linear Functional Systems
++ Muld95 Primitives: Orepoly and Lodo
++ Description:
++ This is the category of univariate skew polynomials over an Ore
++ coefficient ring.
++ The multiplication is given by \spad{x a = \sigma(a) x + \delta a}.
++ This category is an evolution of the types
++ MonogenicLinearOperator, OppositeMonogenicLinearOperator, and
++ NonCommutativeOperatorDivision

UnivariateSkewPolynomialCategory(R) : Category == SIG where
  R : Ring
```

```
SIG ==> Join(Ring, BiModule(R, R), FullyRetractableTo R) with

  degree : $ -> NonNegativeInteger
    ++ degree(l) is \spad{n} if
    ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.

  minimumDegree : $ -> NonNegativeInteger
    ++ minimumDegree(l) is the smallest \spad{k} such that
    ++ \spad{a(k) ^= 0} if
    ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.

  leadingCoefficient : $ -> R
    ++ leadingCoefficient(l) is \spad{a(n)} if
    ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.

  reductum : $ -> $
    ++ reductum(l) is \spad{l - monomial(a(n),n)} if
    ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.

  coefficient : ($, NonNegativeInteger) -> R
    ++ coefficient(l,k) is \spad{a(k)} if
    ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.

  monomial : (R, NonNegativeInteger) -> $
    ++ monomial(c,k) produces c times the k-th power of
    ++ the generating operator, \spad{monomial(1,1)}.

  coefficients : % -> List R
    ++ coefficients(l) returns the list of all the nonzero
    ++ coefficients of l.

  apply : (%, R, R) -> R
    ++ apply(p, c, m) returns \spad{p(m)} where the action is
    ++ given by \spad{x m = c sigma(m) + delta(m)}.

  if R has CommutativeRing then Algebra R

  if R has IntegralDomain then

    "exquo" : (%, R) -> Union(%, "failed")
      ++ exquo(l, a) returns the exact quotient of l by a,
      ++ returning \axiom{"failed"} if this is not possible.

    monicLeftDivide : (%, %) -> Record(quotient: %, remainder: %)
      ++ monicLeftDivide(a,b) returns the pair \spad{[q,r]} such that
      ++ \spad{a = b*q + r} and the degree of \spad{r} is
      ++ less than the degree of \spad{b}.
      ++ \spad{b} must be monic.
      ++ This process is called ``left division''.

    monicRightDivide : (%, %) -> Record(quotient: %, remainder: %)
      ++ monicRightDivide(a,b) returns the pair \spad{[q,r]} such that
      ++ \spad{a = q*b + r} and the degree of \spad{r} is
```

```
    ++ less than the degree of \spad{b}.
    ++ \spad{b} must be monic.
    ++ This process is called ``right division''.

if R has GcdDomain then

  content : % -> R
    ++ content(l) returns the gcd of all the coefficients of l.

  primitivePart : % -> %
    ++ primitivePart(l) returns l0 such that \spad{l = a * l0}
    ++ for some a in R, and \spad{content(l0) = 1}.

if R has Field then

  leftDivide : (%, %) -> Record(quotient: %, remainder: %)
    ++ leftDivide(a,b) returns the pair \spad{[q,r]} such that
    ++ \spad{a = b*q + r} and the degree of \spad{r} is
    ++ less than the degree of \spad{b}.
    ++ This process is called ``left division''.

  leftQuotient : (%, %) -> %
    ++ leftQuotient(a,b) computes the pair \spad{[q,r]} such that
    ++ \spad{a = b*q + r} and the degree of \spad{r} is
    ++ less than the degree of \spad{b}.
    ++ The value \spad{q} is returned.

  leftRemainder : (%, %) -> %
    ++ leftRemainder(a,b) computes the pair \spad{[q,r]} such that
    ++ \spad{a = b*q + r} and the degree of \spad{r} is
    ++ less than the degree of \spad{b}.
    ++ The value \spad{r} is returned.

  leftExactQuotient : (%, %) -> Union(%, "failed")
    ++ leftExactQuotient(a,b) computes the value \spad{q}, if it exists,
    ++  such that \spad{a = b*q}.

  leftGcd : (%, %) -> %
    ++ leftGcd(a,b) computes the value \spad{g} of highest degree
    ++ such that
    ++    \spad{a = g*aa}
    ++    \spad{b = g*bb}
    ++ for some values \spad{aa} and \spad{bb}.
    ++ The value \spad{g} is computed using left-division.

  leftExtendedGcd : (%, %) -> Record(coef1:%, coef2:%, generator:%)
    ++ leftExtendedGcd(a,b) returns \spad{[c,d]} such that
    ++ \spad{g = a * c + b * d = leftGcd(a, b)}.

  rightLcm : (%, %) -> %
    ++ rightLcm(a,b) computes the value \spad{m} of lowest degree
    ++ such that \spad{m = a*aa = b*bb} for some values
    ++ \spad{aa} and \spad{bb}.  The value \spad{m} is
    ++ computed using left-division.
```

```
   rightDivide : (%, %) -> Record(quotient: %, remainder: %)
     ++ rightDivide(a,b) returns the pair \spad{[q,r]} such that
     ++ \spad{a = q*b + r} and the degree of \spad{r} is
     ++ less than the degree of \spad{b}.
     ++ This process is called ``right division''.

   rightQuotient : (%, %) -> %
     ++ rightQuotient(a,b) computes the pair \spad{[q,r]} such that
     ++ \spad{a = q*b + r} and the degree of \spad{r} is
     ++ less than the degree of \spad{b}.
     ++ The value \spad{q} is returned.

   rightRemainder : (%, %) -> %
     ++ rightRemainder(a,b) computes the pair \spad{[q,r]} such that
     ++ \spad{a = q*b + r} and the degree of \spad{r} is
     ++ less than the degree of \spad{b}.
     ++ The value \spad{r} is returned.

   rightExactQuotient : (%, %) -> Union(%, "failed")
     ++ rightExactQuotient(a,b) computes the value \spad{q}, if it exists
     ++ such that \spad{a = q*b}.

   rightGcd : (%, %) -> %
     ++ rightGcd(a,b) computes the value \spad{g} of highest degree
     ++ such that
     ++    \spad{a = aa*g}
     ++    \spad{b = bb*g}
     ++ for some values \spad{aa} and \spad{bb}.
     ++ The value \spad{g} is computed using right-division.

   rightExtendedGcd : (%, %) -> Record(coef1:%, coef2:%, generator:%)
     ++ rightExtendedGcd(a,b) returns \spad{[c,d]} such that
     ++ \spad{g = c * a + d * b = rightGcd(a, b)}.

   leftLcm : (%, %) -> %
     ++ leftLcm(a,b) computes the value \spad{m} of lowest degree
     ++ such that \spad{m = aa*a = bb*b} for some values
     ++ \spad{aa} and \spad{bb}.  The value \spad{m} is
     ++ computed using right-division.

add

   coerce(x:R):% == monomial(x, 0)

   coefficients l ==
     ans:List(R) := empty()
     while l ^= 0 repeat
       ans := concat(leadingCoefficient l, ans)
       l   := reductum l
     ans

   a:R * y:% ==
     z:% := 0
```

```
  while y ^= 0 repeat
    z := z + monomial(a * leadingCoefficient y, degree y)
    y := reductum y
  z

retractIfCan(x:%):Union(R, "failed") ==
  zero? x or zero? degree x => leadingCoefficient x
  "failed"

if R has IntegralDomain then

  l exquo a ==
    ans:% := 0
    while l ^= 0 repeat
      (u := (leadingCoefficient(l) exquo a)) case "failed" =>
         return "failed"
      ans := ans + monomial(u::R, degree l)
      l   := reductum l
    ans

if R has GcdDomain then

  content l       == gcd coefficients l

  primitivePart l == (l exquo content l)::%

if R has Field then

  leftEEA:  (%, %) -> Record(gcd:%, coef1:%, coef2:%, lcm:%)

  rightEEA: (%, %) -> Record(gcd:%, coef1:%, coef2:%, lcm:%)

  ncgcd:    (%, %, (%, %) -> %) -> %

  nclcm:  (%, %, (%, %) -> Record(gcd:%, coef1:%, coef2:%, lcm:%)) -> %

  exactQuotient: Record(quotient:%, remainder:%) -> Union(%, "failed")

  extended: (%, %, (%, %) -> Record(gcd:%, coef1:%, coef2:%, lcm:%)) ->
                                Record(coef1:%, coef2:%, generator:%)

  leftQuotient(a, b) == leftDivide(a,b).quotient

  leftRemainder(a, b) == leftDivide(a,b).remainder

  leftExtendedGcd(a, b) == extended(a, b, leftEEA)

  rightLcm(a, b) == nclcm(a, b, leftEEA)

  rightQuotient(a, b) == rightDivide(a,b).quotient

  rightRemainder(a, b) == rightDivide(a,b).remainder

  rightExtendedGcd(a, b) == extended(a, b, rightEEA)
```

```
leftLcm(a, b) == nclcm(a, b, rightEEA)

leftExactQuotient(a, b) == exactQuotient leftDivide(a, b)

rightExactQuotient(a, b) == exactQuotient rightDivide(a, b)

rightGcd(a, b) == ncgcd(a, b, rightRemainder)

leftGcd(a, b) == ncgcd(a, b, leftRemainder)

exactQuotient qr == (zero?(qr.remainder) => qr.quotient; "failed")

-- returns [g = leftGcd(a, b), c, d, l = rightLcm(a, b)]
-- such that g := a c + b d
leftEEA(a, b) ==
  a0 := a
  u0:% := v:% := 1
  v0:% := u:% := 0
  while b ^= 0 repeat
    qr     := leftDivide(a, b)
    (a, b) := (b, qr.remainder)
    (u0, u):= (u, u0 - u * qr.quotient)
    (v0, v):= (v, v0 - v * qr.quotient)
  [a, u0, v0, a0 * u]

ncgcd(a, b, ncrem) ==
  zero? a => b
  zero? b => a
  degree a < degree b => ncgcd(b, a, ncrem)
  while b ^= 0 repeat (a, b) := (b, ncrem(a, b))
  a

extended(a, b, eea) ==
  zero? a => [0, 1, b]
  zero? b => [1, 0, a]
  degree a < degree b =>
    rec := eea(b, a)
    [rec.coef2, rec.coef1, rec.gcd]
  rec := eea(a, b)
  [rec.coef1, rec.coef2, rec.gcd]

nclcm(a, b, eea) ==
  zero? a or zero? b => 0
  degree a < degree b => nclcm(b, a, eea)
  rec := eea(a, b)
  rec.lcm

-- returns [g = rightGcd(a, b), c, d, l = leftLcm(a, b)]
-- such that g := a c + b d
rightEEA(a, b) ==
  a0 := a
  u0:% := v:% := 1
  v0:% := u:% := 0
```

```
while b ^= 0 repeat
  qr     := rightDivide(a, b)
  (a, b) := (b, qr.remainder)
  (u0, u):= (u, u0 - qr.quotient * u)
  (v0, v):= (v, v0 - qr.quotient * v)
[a, u0, v0, u * a0]
```

———-

— COQ OREPCAT —

```
(* category OREPCAT *)
(*
This is the category of univariate skew polynomials over an Ore
coefficient ring.

The multiplication is given by x a = \sigma(a) x + \delta a

      coerce : R -> %
      coerce(x:R):% == monomial(x, 0)

      coefficients : % -> List(R)
      coefficients l ==
        ans:List(R) := empty()
        while l ^= 0 repeat
          ans := concat(leadingCoefficient l, ans)
          l   := reductum l
        ans

      ?*? : (R,%) -> %
      a:R * y:% ==
        z:% := 0
        while y ^= 0 repeat
          z := z + monomial(a * leadingCoefficient y, degree y)
          y := reductum y
        z

      retractIfCan : % -> Union(R,"failed")
      retractIfCan(x:%):Union(R, "failed") ==
        zero? x or zero? degree x => leadingCoefficient x
        "failed"

      if R has IntegralDomain then

        exquo : (%,R) -> Union(%,"failed")
        l exquo a ==
          ans:% := 0
          while l ^= 0 repeat
            (u := (leadingCoefficient(l) exquo a)) case "failed" =>
               return "failed"
            ans := ans + monomial(u::R, degree l)
            l   := reductum l
          ans
```

```
if R has GcdDomain then

  content : % -> R
  content l == gcd coefficients l

  primitivePart : % -> %
  primitivePart l == (l exquo content l)::%

if R has Field then

  leftQuotient : (%,%) -> %
  leftQuotient(a, b) == leftDivide(a,b).quotient

  leftRemainder : (%,%) -> %
  leftRemainder(a, b) == leftDivide(a,b).remainder

  leftExtendedGcd : (%,%) -> Record(coef1: %,coef2: %,generator: %)
  leftExtendedGcd(a, b) == extended(a, b, leftEEA)

  rightLcm : (%,%) -> %
  rightLcm(a, b) == nclcm(a, b, leftEEA)

  rightQuotient : (%,%) -> %
  rightQuotient(a, b) == rightDivide(a,b).quotient

  rightRemainder : (%,%) -> %
  rightRemainder(a, b) == rightDivide(a,b).remainder

  rightExtendedGcd : (%,%) -> Record(coef1: %,coef2: %,generator: %)
  rightExtendedGcd(a, b) == extended(a, b, rightEEA)

  leftLcm : (%,%) -> %
  leftLcm(a, b) == nclcm(a, b, rightEEA)

  leftExactQuotient : (%,%) -> Union(%,"failed")
  leftExactQuotient(a, b) == exactQuotient leftDivide(a, b)

  rightExactQuotient : (%,%) -> Union(%,"failed")
  rightExactQuotient(a, b) == exactQuotient rightDivide(a, b)

  rightGcd : (%,%) -> %
  rightGcd(a, b) == ncgcd(a, b, rightRemainder)

  leftGcd : (%,%) -> %
  leftGcd(a, b) == ncgcd(a, b, leftRemainder)

  exactQuotient: Record(quotient:%, remainder:%) -> Union(%, "failed")
  exactQuotient qr == (zero?(qr.remainder) => qr.quotient; "failed")

  -- returns [g = leftGcd(a, b), c, d, l = rightLcm(a, b)]
  -- such that g := a c + b d
  leftEEA:  (%, %) -> Record(gcd:%, coef1:%, coef2:%, lcm:%)
  leftEEA(a, b) ==
```

```
    a0 := a
    u0:% := v:% := 1
    v0:% := u:% := 0
    while b ^= 0 repeat
      qr     := leftDivide(a, b)
      (a, b) := (b, qr.remainder)
      (u0, u):= (u, u0 - u * qr.quotient)
      (v0, v):= (v, v0 - v * qr.quotient)
    [a, u0, v0, a0 * u]

  ncgcd: (%, %, (%, %) -> %) -> %
  ncgcd(a, b, ncrem) ==
    zero? a => b
    zero? b => a
    degree a < degree b => ncgcd(b, a, ncrem)
    while b ^= 0 repeat (a, b) := (b, ncrem(a, b))
    a

  extended: (%, %, (%, %) -> Record(gcd:%, coef1:%, coef2:%, lcm:%)) ->
                                Record(coef1:%, coef2:%, generator:%)
  extended(a, b, eea) ==
    zero? a => [0, 1, b]
    zero? b => [1, 0, a]
    degree a < degree b =>
      rec := eea(b, a)
      [rec.coef2, rec.coef1, rec.gcd]
    rec := eea(a, b)
    [rec.coef1, rec.coef2, rec.gcd]

  nclcm: (%, %, (%, %) -> Record(gcd:%, coef1:%, coef2:%, lcm:%)) -> %
  nclcm(a, b, eea) ==
    zero? a or zero? b => 0
    degree a < degree b => nclcm(b, a, eea)
    rec := eea(a, b)
    rec.lcm

  -- returns [g = rightGcd(a, b), c, d, l = leftLcm(a, b)]
  -- such that g := a c + b d
  rightEEA: (%, %) -> Record(gcd:%, coef1:%, coef2:%, lcm:%)
  rightEEA(a, b) ==
    a0 := a
    u0:% := v:% := 1
    v0:% := u:% := 0
    while b ^= 0 repeat
      qr     := rightDivide(a, b)
      (a, b) := (b, qr.remainder)
      (u0, u):= (u, u0 - qr.quotient * u)
      (v0, v):= (v, v0 - qr.quotient * v)
    [a, u0, v0, u * a0]
*)
```

———-

— OREPCAT.dotabb —

```
"OREPCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OREPCAT"];
"OREPCAT" -> "BMODULE"
"OREPCAT" -> "FRETRCT"
"OREPCAT" -> "RING"
```

————

— OREPCAT.dotfull —

```
"UnivariateSkewPolynomialCategory(R:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OREPCAT"];
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "BiModule(a:Ring,b:Ring)"
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "FullyRetractableTo(a:Ring)"
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "Ring()"
```

————

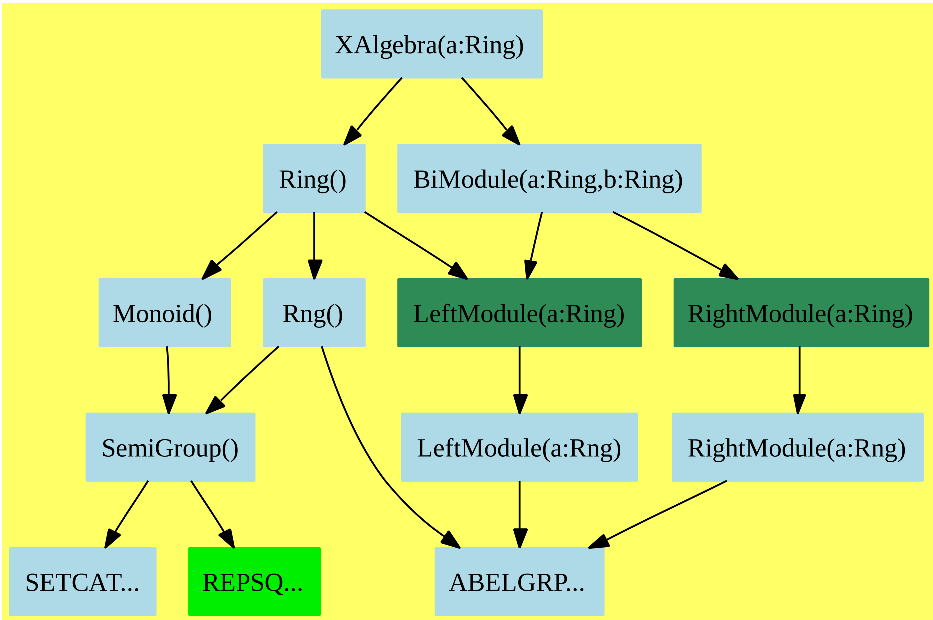
— OREPCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"UnivariateSkewPolynomialCategory(R:Ring)" [color=lightblue];
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "BiModule(a:Ring,b:Ring)"
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "FullyRetractableTo(a:Ring)"
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "Ring()"

"FullyRetractableTo(a:Ring)" [color=seagreen];
"FullyRetractableTo(a:Ring)" -> "FullyRetractableTo(a:Type)"

"FullyRetractableTo(a:Type)" [color=lightblue];
"FullyRetractableTo(a:Type)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"Category" [color=lightblue];

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"
```

```
"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "AbelianGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

———

10.0.175 XAlgebra (XALG)



— **XAlgebra.input** —

```
)set break resume
)sys rm -f XAlgebra.output
)spool XAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show XAlgebra
--R
--R XAlgebra(R: Ring) is a category constructor
--R Abbreviation for XAlgebra is XALG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for XALG
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                         ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                         ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %        ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %       ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                         ?-? : (%,%) -> %
--R -? : % -> %                              ?=? : (%,%) -> Boolean
--R 1 : () -> %                              0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %        ?^? : (%,PositiveInteger) -> %
--R coerce : R -> %                          coerce : Integer -> %
--R coerce : % -> OutputForm                 hash : % -> SingleInteger
```

```
--R latex : % -> String                    one? : % -> Boolean
--R recip : % -> Union(%,"failed")         sample : () -> %
--R zero? : % -> Boolean                   ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— **XAlgebra.help** —

```
====================================================================
XAlgebra examples
====================================================================

This is the category of algebras over non-commutative rings.
It is used by constructors of non-commutative algebras such as
XPolynomialRing and XFreeAlgebra

See Also:
o )show XAlgebra
```

See:

⇒ "XFreeAlgebra" (XFALG) 11.0.183 on page 1164
⇐ "BiModule" (BMODULE) 9.0.146 on page 902
⇐ "Ring" (RING) 9.0.153 on page 946

Exports:

0	1	characteristic	coerce	hash
latex	one?	recip	sample	subtractIfCan
zero?	?^?	?~=?	?*?	?**?
?+?	?-?	-?	?=?	

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
coerce : R -> %
```

These exports come from (p946) Ring():

```
0 : () -> %
1 : () -> %
```

```
characteristic : () -> NonNegativeInteger
coerce : % -> OutputForm
coerce : Integer -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p902) BiModule(R:Ring,R:Ring):

```
?*? : (R,%) -> %
?*? : (%,R) -> %
```

— XAlgebra.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#XALG">
XAlgebra (XALG)</a></h2>
</body>
```

— category XALG XAlgebra —

```
)abbrev category XALG XAlgebra
++ Author: Michel Petitot petitot@lifl.fr
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Description:
++ This is the category of algebras over non-commutative rings.
++ It is used by constructors of non-commutative algebras such as
++ XPolynomialRing and XFreeAlgebra

XAlgebra(R) : Category == SIG where
  R : Ring

  SIG ==> Join(Ring, BiModule(R,R)) with
```

```
    coerce : R -> %
      ++ \spad{coerce(r)} equals  \spad{r*1}.

    if R has CommutativeRing then Algebra(R)
```

————

— XALG.dotabb —

```
"XALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XALG"];
"XALG" -> "BMODULE"
"XALG" -> "RING"
```

————

— XALG.dotfull —

```
"XAlgebra(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XALG"];
"XAlgebra(a:Ring)" -> "Ring()"
"XAlgebra(a:Ring)" -> "BiModule(a:Ring,b:Ring)"
```

————

— XALG.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"XAlgebra(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XALG"];
"XAlgebra(a:Ring)" -> "Ring()"
"XAlgebra(a:Ring)" -> "BiModule(a:Ring,b:Ring)"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
```

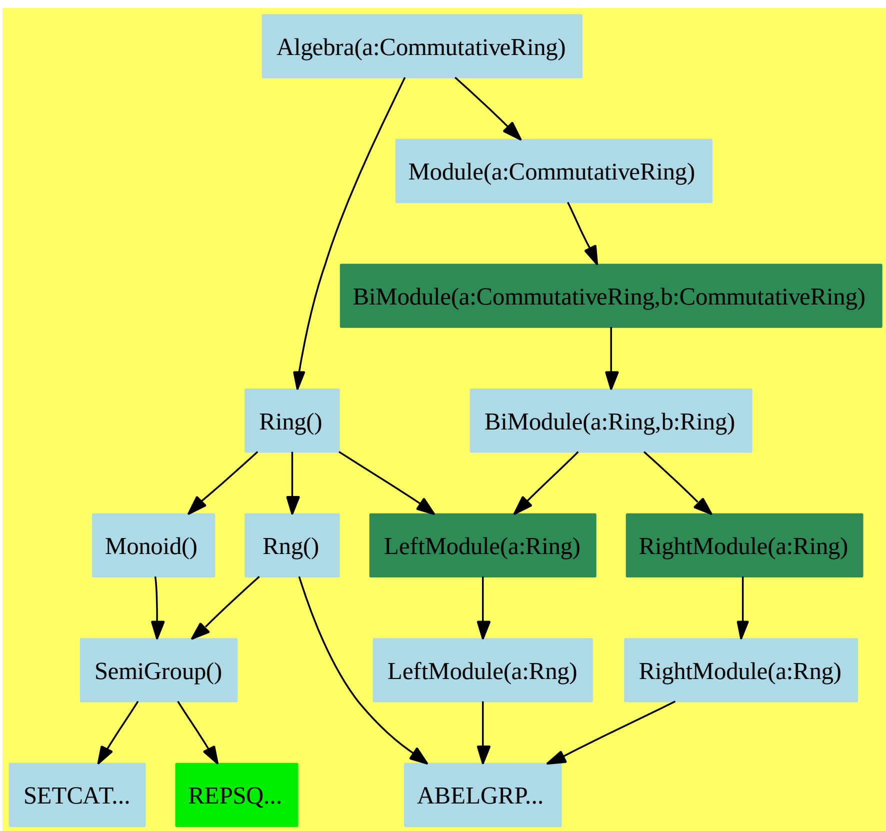
```
"RightModule(a:Rng)" -> "ABELGRP..."

"Rng()" [color=lightblue];
"Rng()" -> "ABELGRP..."
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"ABELGRP..." [color=lightblue];
}
```

———-

Chapter 11

Category Layer 10

11.0.176 Algebra (ALGEBRA)



— **Algebra.input** —

```
)set break resume
)sys rm -f Algebra.output
```

```
)spool Algebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show Algebra
--R
--R Algebra(R: CommutativeRing) is a category constructor
--R Abbreviation for Algebra is ALGEBRA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ALGEBRA
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                       ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                       ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %      ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %     ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                       ?-? : (%,%) -> %
--R -? : % -> %                            ?=? : (%,%) -> Boolean
--R 1 : () -> %                            0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %      ?^? : (%,PositiveInteger) -> %
--R coerce : R -> %                        coerce : Integer -> %
--R coerce : % -> OutputForm               hash : % -> SingleInteger
--R latex : % -> String                    one? : % -> Boolean
--R recip : % -> Union(%,"failed")         sample : () -> %
--R zero? : % -> Boolean                   ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— **Algebra.help** —

```
====================================================================
Algebra examples
====================================================================

The category of associative algebras (modules which are themselves rings).

Axioms:
         (b+c)::% = (b::%) + (c::%)
         (b*c)::% = (b::%) * (c::%)
         (1::R)::% = 1::%
         b*x = (b::%)*x
         r*(a*b) = (r*a)*b = a*(r*b)

See Also:
o )show Algebra
```

———-

See:

⇒ "DivisionRing" (DIVRING) 12.0.185 on page 1185
⇒ "FiniteRankAlgebra" (FINRALG) 17.0.218 on page 1537
⇒ "FunctionSpace" (FS) 17.0.219 on page 1544
⇒ "IntegralDomain" (INTDOM) 12.0.188 on page 1225
⇒ "MonogenicLinearOperator" (MLO) 12.0.189 on page 1232
⇒ "OctonionCategory" (OC) 12.0.190 on page 1238
⇒ "QuotientFieldCategory" (QFCAT) 17.0.222 on page 1595
⇒ "QuaternionCategory" (QUATCAT) 12.0.191 on page 1252
⇒ "RealClosedField" (RCFIELD) 17.0.223 on page 1611
⇐ "Module" (MODULE) 10.0.167 on page 1050
⇐ "Ring" (RING) 9.0.153 on page 946

Exports:

1	0	characteristic	coerce	hash
latex	one?	recip	sample	subtractIfCan
zero?	?*?	?+?	?-?	-?
?=?	?~=?	?**?	?^?	

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are implemented by this category:

```
coerce : R -> %
```

These exports come from (p946) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
```

```
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p1050) Module(R:CommutativeRing):

```
?*? : (R,%) -> %
?*? : (%,R) -> %
```

— Algebra.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ALGEBRA">
Algebra (ALGEBRA)</a></h2>
</body>
```

— category ALGEBRA Algebra —

```
)abbrev category ALGEBRA Algebra
++ Description:
++ The category of associative algebras (modules which are themselves rings).
++
++ Axioms\br
++ \tab{5}\spad{(b+c)::% = (b::%) + (c::%)}\br
++ \tab{5}\spad{(b*c)::% = (b::%) * (c::%)}\br
++ \tab{5}\spad{(1::R)::% = 1::%}\br
++ \tab{5}\spad{b*x = (b::%)*x}\br
++ \tab{5}\spad{r*(a*b) = (r*a)*b = a*(r*b)}

Algebra(R) : Category == SIG where
  R : CommutativeRing

  SIG ==> Join(Ring, Module R) with

    coerce : R -> %
      ++ coerce(r) maps the ring element r to a member of the algebra.

   add

     coerce(x:R):% == x * 1$%
```

— COQ ALGEBRA —

```
(* category ALGEBRA *)
(*
```

```
Axioms
  (b+c)::% = (b::%) + (c::%)
  (b*c)::% = (b::%) * (c::%)
  (1::R)::% = 1::%
  b*x = (b::%)*x
  r*(a*b) = (r*a)*b = a*(r*b)

  coerce : R -> %
  coerce(x:R):% == x * 1$%

*)
```

———-

— **ALGEBRA.dotabb** —

```
"ALGEBRA"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"ALGEBRA" -> "RING"
"ALGEBRA" -> "MODULE"
```

———-

— **ALGEBRA.dotfull** —

```
"Algebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(a:CommutativeRing)" -> "Ring()"
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Algebra(a:Field)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(a:Field)" -> "Algebra(a:CommutativeRing)"

"Algebra(a:CommutativeRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Algebra(Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(Fraction(Integer))" -> "Algebra(a:CommutativeRing)"

"Algebra(Integer)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(Integer)" -> "Algebra(a:CommutativeRing)"

"Algebra(IntegralDomain)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(IntegralDomain)" -> "Algebra(a:CommutativeRing)"
```

———-

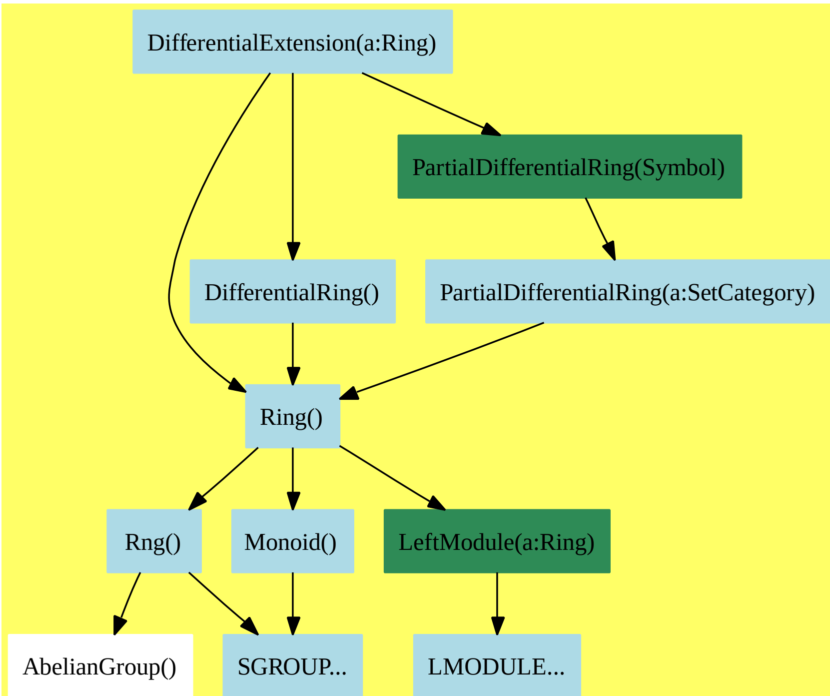
— ALGEBRA.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "Ring()"
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "ABELGRP..."
"Rng()" -> "SemiGroup()"

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"ABELGRP..." [color=lightblue];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
```

```
}
```

———-

11.0.177 DifferentialExtension (DIFEXT)



— **DifferentialExtension.input** —

```
)set break resume
)sys rm -f DifferentialExtension.output
)spool DifferentialExtension.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DifferentialExtension
--R
--R DifferentialExtension(R: Ring) is a category constructor
--R Abbreviation for DifferentialExtension is DIFEXT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for DIFEXT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
```

```
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R D : % -> % if R has DIFRING           D : (%,(R -> R)) -> %
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R differentiate : (%,(R -> R)) -> %     hash : % -> SingleInteger
--R latex : % -> String                   one? : % -> Boolean
--R recip : % -> Union(%,"failed")        sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R D : (%,NonNegativeInteger) -> % if R has DIFRING
--R D : (%,Symbol) -> % if R has PDRING(SYMBOL)
--R D : (%,List(Symbol)) -> % if R has PDRING(SYMBOL)
--R D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING(SYMBOL)
--R D : (%,List(Symbol),List(NonNegativeInteger)) -> % if R has PDRING(SYMBOL)
--R D : (%,(R -> R),NonNegativeInteger) -> %
--R characteristic : () -> NonNegativeInteger
--R differentiate : % -> % if R has DIFRING
--R differentiate : (%,NonNegativeInteger) -> % if R has DIFRING
--R differentiate : (%,Symbol) -> % if R has PDRING(SYMBOL)
--R differentiate : (%,List(Symbol)) -> % if R has PDRING(SYMBOL)
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING(SYMBOL)
--R differentiate : (%,List(Symbol),List(NonNegativeInteger)) -> % if R has PDRING(SYMBOL)
--R differentiate : (%,(R -> R),NonNegativeInteger) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— **DifferentialExtension.help** —

```
====================================================================
DifferentialExtension examples
====================================================================

Differential extensions of a ring R. Given a differentiation on R,
extend it to a differentiation on %.

See Also:
o )show DifferentialExtension
```

See:

⇒ "ComplexCategory" (COMPCAT) 20.0.240 on page 1869
⇒ "DifferentialPolynomialCategory" (DPOLCAT) 17.0.216 on page 1512
⇒ "DirectProductCategory" (DIRPCAT) 12.0.184 on page 1173

⇒ “QuaternionCategory” (QUATCAT) 12.0.191 on page 1252
⇒ “QuotientFieldCategory” (QFCAT) 17.0.222 on page 1595
⇒ “SquareMatrixCategory” (SMATCAT) 12.0.192 on page 1266
⇒ “UnivariatePolynomialCategory” (UPOLYC) 17.0.228 on page 1708
⇐ “PartialDifferentialRing” (PDRING) 10.0.169 on page 1061
⇐ “Ring” (RING) 9.0.153 on page 946

Exports:

1	0	characteristic	coerce	D
differentiate	hash	latex	one?	recip
sample	subtractIfCan	zero?	?*?	?~=?
?**?	?+?	?-?	-?	?=?
?^?				

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.

These are directly exported but not implemented:

```
differentiate : (%,(R -> R)) -> %
```

These are implemented by this category:

```
D : (%,(R -> R)) -> %
D : (%,(R -> R),NonNegativeInteger) -> %
differentiate : % -> % if R has DIFRING
differentiate : (%,(R -> R),NonNegativeInteger) -> %
differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
```

These exports come from (p946) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

```
?^? : (%,PositiveInteger) -> %
```

These exports come from (p1024) DifferentialRing():

```
D : % -> % if R has DIFRING
D : (%,NonNegativeInteger) -> % if R has DIFRING
differentiate : (%,NonNegativeInteger) -> %
    if R has DIFRING
```

These exports come from (p1061) PartialDifferentialRing(Symbol):

```
D : (%,Symbol) -> % if R has PDRING SYMBOL
D : (%,List Symbol) -> % if R has PDRING SYMBOL
D : (%,Symbol,NonNegativeInteger) -> %
    if R has PDRING SYMBOL
D : (%,List Symbol,List NonNegativeInteger) -> %
    if R has PDRING SYMBOL
differentiate : (%,List Symbol) -> %
    if R has PDRING SYMBOL
differentiate : (%,Symbol,NonNegativeInteger) -> %
    if R has PDRING SYMBOL
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
    if R has PDRING SYMBOL
```

— DifferentialExtension.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#DIFEXT">
DifferentialExtension (DIFEXT)</a></h2>
</body>
```

— category DIFEXT DifferentialExtension —

```
)abbrev category DIFEXT DifferentialExtension
++ Description:
++ Differential extensions of a ring R.
++ Given a differentiation on R, extend it to a differentiation on %.

DifferentialExtension(R) : Category == SIG where
  R : Ring

  SIG ==> Ring with

    differentiate : (%, R -> R) -> %
      ++ differentiate(x, deriv) differentiates x extending
      ++ the derivation deriv on R.

    differentiate : (%, R -> R, NonNegativeInteger) -> %
      ++ differentiate(x, deriv, n) differentiate x n times
      ++ using a derivation which extends deriv on R.

    D : (%, R -> R) -> %
      ++ D(x, deriv) differentiates x extending
```

```
      ++ the derivation deriv on R.

    D : (%, R -> R, NonNegativeInteger) -> %
      ++ D(x, deriv, n) differentiate x n times
      ++ using a derivation which extends deriv on R.

    if R has DifferentialRing then DifferentialRing

    if R has PartialDifferentialRing(Symbol) then
             PartialDifferentialRing(Symbol)

   add

     differentiate(x:%, derivation: R -> R, n:NonNegativeInteger):% ==
       for i in 1..n repeat x := differentiate(x, derivation)
       x

     D(x:%, derivation: R -> R) == differentiate(x, derivation)

     D(x:%, derivation: R -> R, n:NonNegativeInteger) ==
             differentiate(x, derivation, n)

     if R has DifferentialRing then

       differentiate x == differentiate(x, differentiate$R)

     if R has PartialDifferentialRing Symbol then

       differentiate(x:%, v:Symbol):% ==
         differentiate(x, s +-> differentiate(s, v)$R)
```

— **COQ DIFEXT** —

```
(* category DIFEXT *)
(*

    differentiate : (%,(R -> R),NonNegativeInteger) -> %
    differentiate(x:%, derivation: R -> R, n:NonNegativeInteger):% ==
      for i in 1..n repeat x := differentiate(x, derivation)
      x

    D : (%,(R -> R)) -> %
    D(x:%, derivation: R -> R) == differentiate(x, derivation)

    D : (%,(R -> R),NonNegativeInteger) -> %
    D(x:%, derivation: R -> R, n:NonNegativeInteger) ==
            differentiate(x, derivation, n)

    if R has DifferentialRing then

      differentiate : % -> %
```

```
      differentiate x == differentiate(x, differentiate$R)

    if R has PartialDifferentialRing Symbol then

      differentiate : (%,Symbol) -> %
      differentiate(x:%, v:Symbol):% ==
        differentiate(x, s +-> differentiate(s, v)$R)
*)
```

———-

— DIFEXT.dotabb —

```
"DIFEXT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIFEXT"];
"DIFEXT" -> "RING"
"DIFEXT" -> "DIFRING"
"DIFEXT" -> "PDRING"
```

———-

— DIFEXT.dotfull —

```
"DifferentialExtension(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIFEXT"];
"DifferentialExtension(a:Ring)" -> "Ring()"
"DifferentialExtension(a:Ring)" -> "DifferentialRing()"
"DifferentialExtension(a:Ring)" -> "PartialDifferentialRing(Symbol)"

"DifferentialExtension(IntegralDomain)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=DIFEXT"];
"DifferentialExtension(IntegralDomain)" ->
  "DifferentialExtension(a:Ring)"

"DifferentialExtension(CommutativeRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=DIFEXT"];
"DifferentialExtension(CommutativeRing)" ->
  "DifferentialExtension(a:Ring)"
```

———-

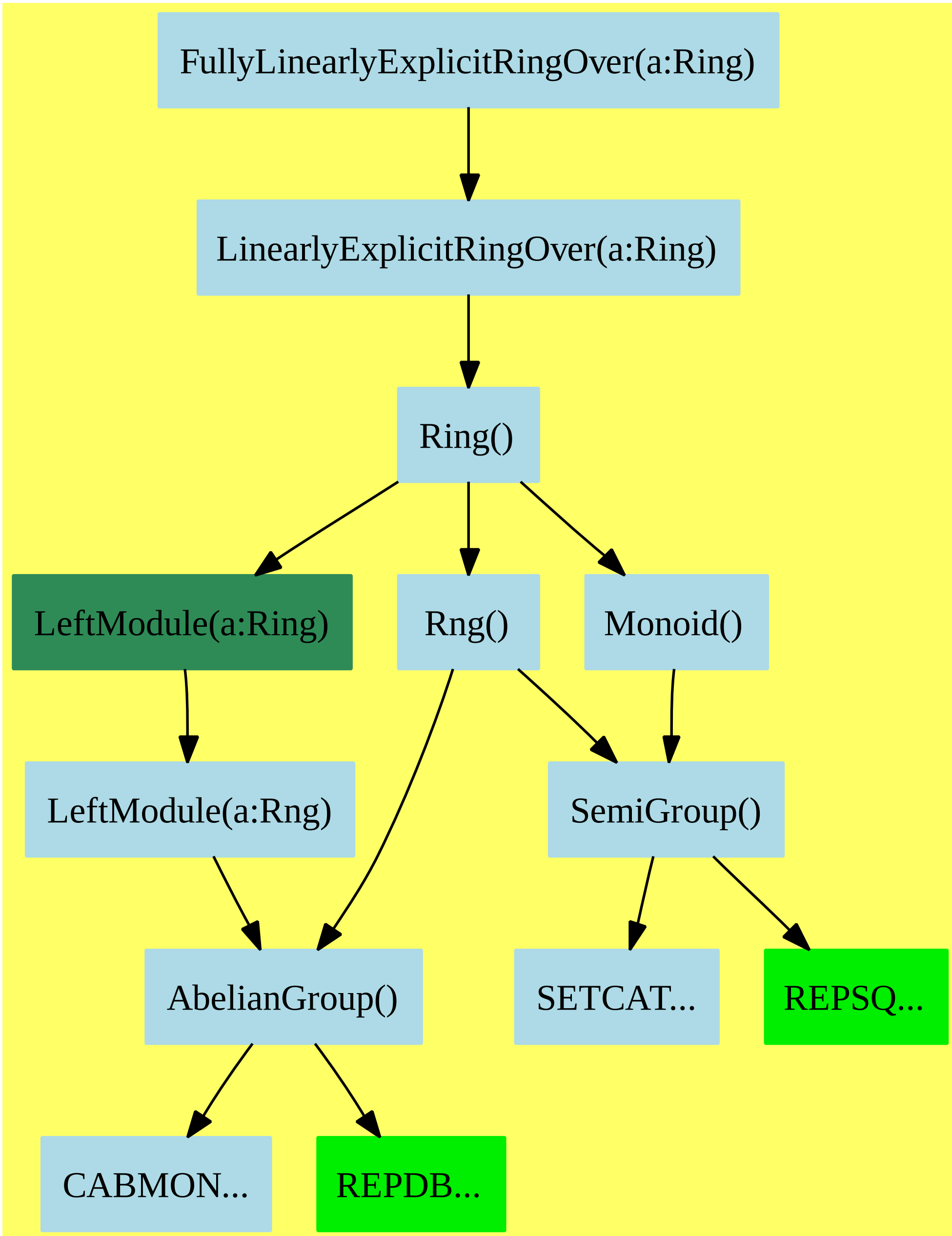
— DIFEXT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"DifferentialExtension(a:Ring)" [color=lightblue];
"DifferentialExtension(a:Ring)" -> "Ring()"
"DifferentialExtension(a:Ring)" -> "DifferentialRing()"
```

```
"DifferentialExtension(a:Ring)" -> "PartialDifferentialRing(Symbol)"

"PartialDifferentialRing(Symbol)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PDRING"];
"PartialDifferentialRing(Symbol)" ->
    "PartialDifferentialRing(a:SetCategory)"

"PartialDifferentialRing(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PDRING"];
"PartialDifferentialRing(a:SetCategory)" -> "Ring()"

"DifferentialRing()" [color=lightblue];
"DifferentialRing()" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SGROUP..."

"Monoid()" [color=lightblue];
"Monoid()" -> "SGROUP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LMODULE..."

"SGROUP..." [color=lightblue];
"LMODULE..." [color=lightblue];
}
```

———-

11.0.178 FullyLinearlyExplicitRingOver (FLINEXP)



— **FullyLinearlyExplicitRingOver.input** —

```
)set break resume
)sys rm -f FullyLinearlyExplicitRingOver.output
)spool FullyLinearlyExplicitRingOver.output
)set message test on
```

```
)set message auto off
)clear all

--S 1 of 1
)show FullyLinearlyExplicitRingOver
--R
--R FullyLinearlyExplicitRingOver(R: Ring) is a category constructor
--R Abbreviation for FullyLinearlyExplicitRingOver is FLINEXP
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FLINEXP
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R one? : % -> Boolean                   recip : % -> Union(%,"failed")
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R reducedSystem : Matrix(%) -> Matrix(Integer) if R has LINEXP(INT)
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(Integer),vec: Vector(Integer)) if R has LINEXP(I
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(R),vec: Vector(R))
--R reducedSystem : Matrix(%) -> Matrix(R)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— FullyLinearlyExplicitRingOver.help —

```
====================================================================
FullyLinearlyExplicitRingOver examples
====================================================================

S is FullyLinearlyExplicitRingOver R means that S is a
LinearlyExplicitRingOver R and, in addition, if R is a
LinearlyExplicitRingOver Integer, then so is S

See Also:
o )show FullyLinearlyExplicitRingOver
```

See:

⇒ “ComplexCategory” (COMPCAT) 20.0.240 on page 1869
⇒ “DirectProductCategory” (DIRPCAT) 12.0.184 on page 1173
⇒ “FunctionSpace” (FS) 17.0.219 on page 1544
⇒ “MonogenicAlgebra” (MONOGEN) 19.0.238 on page 1847
⇒ “PolynomialCategory” (POLYCAT) 16.0.213 on page 1449
⇒ “QuaternionCategory” (QUATCAT) 12.0.191 on page 1252
⇒ “QuotientFieldCategory” (QFCAT) 17.0.222 on page 1595
⇒ “SquareMatrixCategory” (SMATCAT) 12.0.192 on page 1266
⇐ “LinearlyExplicitRingOver” (LINEXP) 10.0.166 on page 1045

Exports:

1	0	characteristic	coerce	hash
latex	one?	recip	reducedSystem	sample
subtractIfCan	zero?	?*?	?**?	?+?
?-?	-?	?=?	?^?	?~=?

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.

These are implemented by this category:

```
reducedSystem : Matrix % ->
    Matrix Integer if R has LINEXP INT
reducedSystem : (Matrix %,Vector %) ->
    Record(mat: Matrix Integer,vec: Vector Integer)
         if R has LINEXP INT
```

These exports come from (p1045) LinearlyExplicitRingOver(a:Ring):

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
reducedSystem : (Matrix %,Vector %) ->
    Record(mat: Matrix R,vec: Vector R)
reducedSystem : Matrix % -> Matrix R
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
```

```
 -? : % -> %
 ?**? : (%,PositiveInteger) -> %
 ?**? : (%,NonNegativeInteger) -> %
 ?^? : (%,PositiveInteger) -> %
 ?^? : (%,NonNegativeInteger) -> %
```

— **FullyLinearlyExplicitRingOver.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FLINEXP">
FullyLinearlyExplicitRingOver (FLINEXP)</a></h2>
</body>
```

———

— **category FLINEXP FullyLinearlyExplicitRingOver** —

```
)abbrev category FLINEXP FullyLinearlyExplicitRingOver
++ Description:
++ S is \spadtype{FullyLinearlyExplicitRingOver R} means that S is a
++ \spadtype{LinearlyExplicitRingOver R} and, in addition, if R is a
++ \spadtype{LinearlyExplicitRingOver Integer}, then so is S

FullyLinearlyExplicitRingOver(R) : Category == SIG where
  R : Ring

  SIG ==> LinearlyExplicitRingOver R with

    if (R has LinearlyExplicitRingOver Integer) then
      LinearlyExplicitRingOver Integer

   add

     if not(R is Integer) then

       if (R has LinearlyExplicitRingOver Integer) then

         reducedSystem(m:Matrix %):Matrix(Integer) ==
           reducedSystem(reducedSystem(m)@Matrix(R))

         reducedSystem(m:Matrix %, v:Vector %):
           Record(mat:Matrix(Integer), vec:Vector(Integer)) ==
             rec := reducedSystem(m, v)@Record(mat:Matrix R, vec:Vector R)
             reducedSystem(rec.mat, rec.vec)
```

———

— **FLINEXP.dotabb** —

```
"FLINEXP"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FLINEXP"];
"FLINEXP" -> "LINEXP"
```

———-

— **FLINEXP.dotfull** —

```
"FullyLinearlyExplicitRingOver(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FLINEXP"];
"FullyLinearlyExplicitRingOver(a:Ring)" ->
     "LinearlyExplicitRingOver(a:Ring)"

"FullyLinearlyExplicitRingOver(a:CommutativeRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FLINEXP"];
"FullyLinearlyExplicitRingOver(a:CommutativeRing)" ->
    "FullyLinearlyExplicitRingOver(a:Ring)"

"FullyLinearlyExplicitRingOver(IntegralDomain)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FLINEXP"];
"FullyLinearlyExplicitRingOver(IntegralDomain)" ->
    "FullyLinearlyExplicitRingOver(a:Ring)"
```

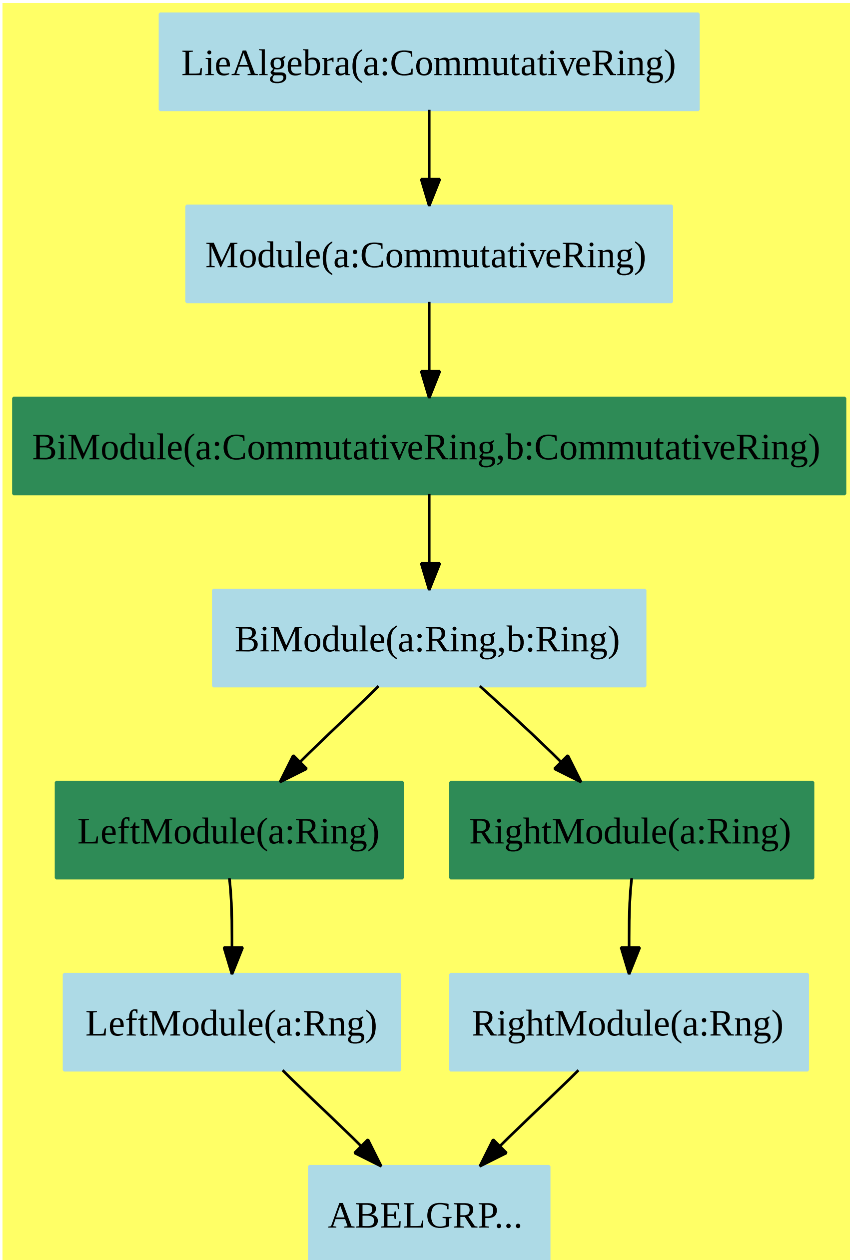
———-

— **FLINEXP.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FullyLinearlyExplicitRingOver(a:Ring)" [color=lightblue];
"FullyLinearlyExplicitRingOver(a:Ring)" ->
     "LinearlyExplicitRingOver(a:Ring)"

"LinearlyExplicitRingOver(a:Ring)" [color=lightblue];
"LinearlyExplicitRingOver(a:Ring)" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
```

```
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

11.0.179 LieAlgebra (LIECAT)



— **LieAlgebra.input** —

```
)set break resume
)sys rm -f LieAlgebra.output
)spool LieAlgebra.output
)set message test on
```

```
)set message auto off
)clear all

--S 1 of 1
)show LieAlgebra
--R
--R LieAlgebra(R: CommutativeRing) is a category constructor
--R Abbreviation for LieAlgebra is LIECAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for LIECAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                     ?*? : (R,%) -> %
--R ?*? : (Integer,%) -> %               ?*? : (NonNegativeInteger,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                     -? : % -> %
--R ?/? : (%,R) -> % if R has FIELD      ?=? : (%,%) -> Boolean
--R 0 : () -> %                          coerce : % -> OutputForm
--R construct : (%,%) -> %               hash : % -> SingleInteger
--R latex : % -> String                  sample : () -> %
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— **LieAlgebra.help** —

```
====================================================================
LieAlgebra examples
====================================================================

The category of Lie Algebras. It is used by the domains of non-commutative
algebra, LiePolynomial and XPBWPolynomial.

See Also:
o )show LieAlgebra
```

See:

⇒ "FreeLieAlgebra" (FLALG) 12.0.187 on page 1219
⇐ "Module" (MODULE) 10.0.167 on page 1050

Exports:

0	coerce	construct	hash	latex
sample	subtractIfCan	zero?	?~=?	?/?
?*?	?+?	?-?	-?	?=?

Attributes Exported:

- **NullSquare** means that $[x, x] = 0$ holds. See `LieAlgebra`.
- **JacobiIdentity** means that $[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0$ holds. See `LieAlgebra`.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
construct : (%,%) -> %
```

These are implemented by this category:

```
?/? : (%,R) -> % if R has FIELD
```

These exports come from (p1050) Module(R:Ring):

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (%,R) -> %
?*? : (R,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
```

— LieAlgebra.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#LIECAT">
LieAlgebra (LIECAT)</a></h2>
</body>
```

— category LIECAT LieAlgebra —

```
)abbrev category LIECAT LieAlgebra
++ Author: Michel Petitot (petitot@lifl.fr).
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Description:
++ The category of Lie Algebras.
++ It is used by the domains of non-commutative algebra,
++ LiePolynomial and XPBWPolynomial.

LieAlgebra(R) : Category == SIG where
```

```
  R : CommutativeRing

  SIG ==> Module(R) with

    construct :  ($,$) -> $
      ++ \axiom{construct(x,y)} returns the Lie bracket of \axiom{x}
      ++ and \axiom{y}.

    NullSquare
      ++ \axiom{NullSquare} means that \axiom{[x,x] = 0} holds.

    JacobiIdentity
      ++ \axiom{JacobiIdentity} means that
      ++ \axiom{[x,[y,z]]+[y,[z,x]]+[z,[x,y]] = 0} holds.

    if R has Field then

       "/" : ($,R) -> $
         ++ \axiom{x/r} returns the division of \axiom{x} by \axiom{r}.
   add

      if R has Field then x / r == inv(r)$R * x
```

———-

— COQ LIECAT —

```
(* category LIECAT *)
(*
    if R has Field then x / r == inv(r)$R * x
*)
```

———-

— LIECAT.dotabb —

```
"LIECAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LIECAT"];
"LIECAT" -> "MODULE"
```

———-

— LIECAT.dotfull —

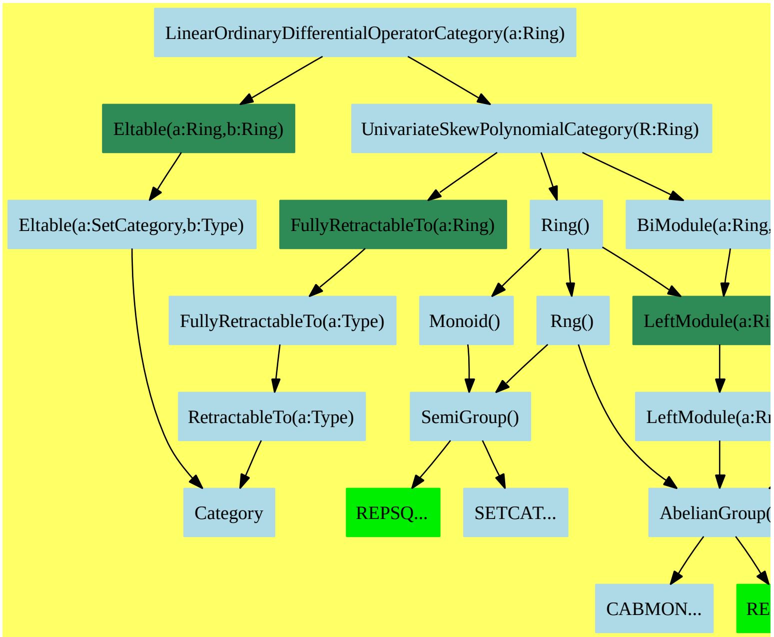
```
"LieAlgebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LIECAT"];
"LieAlgebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"
```

———-

— LIECAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"LieAlgebra(a:CommutativeRing)" [color=lightblue];
"LieAlgebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"ABELGRP..." [color=lightblue];
}
```

11.0.180 LinearOrdinaryDifferentialOperatorCategory (LODOCAT)



— **LinearOrdinaryDifferentialOperatorCategory.input** —

```
)set break resume
)sys rm -f LinearOrdinaryDifferentialOperatorCategory.output
)spool LinearOrdinaryDifferentialOperatorCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show LinearOrdinaryDifferentialOperatorCategory
--R
--R LinearOrdinaryDifferentialOperatorCategory(A: Ring) is a category constructor
--R Abbreviation for LinearOrdinaryDifferentialOperatorCategory is LODOCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for LODOCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (A,%) -> %                     ?*? : (%,A) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %    ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %   ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R D : () -> %                          1 : () -> %
--R 0 : () -> %                          ?^? : (%,NonNegativeInteger) -> %
--R ?^? : (%,PositiveInteger) -> %       adjoint : % -> %
--R apply : (%,A,A) -> A                 coefficients : % -> List(A)
```

```
--R coerce : A -> %                         coerce : Integer -> %
--R coerce : % -> OutputForm                content : % -> A if A has GCDDOM
--R degree : % -> NonNegativeInteger        ?.? : (%,A) -> A
--R hash : % -> SingleInteger               latex : % -> String
--R leadingCoefficient : % -> A             one? : % -> Boolean
--R recip : % -> Union(%,"failed")          reductum : % -> %
--R retract : % -> A                        sample : () -> %
--R zero? : % -> Boolean                    ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R coefficient : (%,NonNegativeInteger) -> A
--R coerce : Fraction(Integer) -> % if A has RETRACT(FRAC(INT))
--R directSum : (%,%) -> % if A has FIELD
--R exquo : (%,A) -> Union(%,"failed") if A has INTDOM
--R leftDivide : (%,%) -> Record(quotient: %,remainder: %) if A has FIELD
--R leftExactQuotient : (%,%) -> Union(%,"failed") if A has FIELD
--R leftExtendedGcd : (%,%) -> Record(coef1: %,coef2: %,generator: %) if A has FIELD
--R leftGcd : (%,%) -> % if A has FIELD
--R leftLcm : (%,%) -> % if A has FIELD
--R leftQuotient : (%,%) -> % if A has FIELD
--R leftRemainder : (%,%) -> % if A has FIELD
--R minimumDegree : % -> NonNegativeInteger
--R monicLeftDivide : (%,%) -> Record(quotient: %,remainder: %) if A has INTDOM
--R monicRightDivide : (%,%) -> Record(quotient: %,remainder: %) if A has INTDOM
--R monomial : (A,NonNegativeInteger) -> %
--R primitivePart : % -> % if A has GCDDOM
--R retract : % -> Fraction(Integer) if A has RETRACT(FRAC(INT))
--R retract : % -> Integer if A has RETRACT(INT)
--R retractIfCan : % -> Union(A,"failed")
--R retractIfCan : % -> Union(Fraction(Integer),"failed") if A has RETRACT(FRAC(INT))
--R retractIfCan : % -> Union(Integer,"failed") if A has RETRACT(INT)
--R rightDivide : (%,%) -> Record(quotient: %,remainder: %) if A has FIELD
--R rightExactQuotient : (%,%) -> Union(%,"failed") if A has FIELD
--R rightExtendedGcd : (%,%) -> Record(coef1: %,coef2: %,generator: %) if A has FIELD
--R rightGcd : (%,%) -> % if A has FIELD
--R rightLcm : (%,%) -> % if A has FIELD
--R rightQuotient : (%,%) -> % if A has FIELD
--R rightRemainder : (%,%) -> % if A has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R symmetricPower : (%,NonNegativeInteger) -> % if A has FIELD
--R symmetricProduct : (%,%) -> % if A has FIELD
--R symmetricSquare : % -> % if A has FIELD
--R
--E 1

)spool
)lisp (bye)
```

— **LinearOrdinaryDifferentialOperatorCategory.help** —

```
====================================================================
LinearOrdinaryDifferentialOperatorCategory examples
====================================================================
```

```
LinearOrdinaryDifferentialOperatorCategory is the category
of differential operators with coefficients in a ring A with a given
derivation.

Multiplication of operators corresponds to functional composition:
    (L1 * L2).(f) = L1 L2 f

See Also:
o )show LinearOrdinaryDifferentialOperatorCategory
```

See:

⇐ "Eltable" (ELTAB) 2.0.19 on page 54
⇐ "UnivariateSkewPolynomialCategory" (OREPCAT) 10.0.174 on page 1101

Exports:

0	1	adjoint
apply	characteristic	coefficient
coefficients	coerce	content
D	degree	directSum
exquo	hash	latex
leadingCoefficient	leftDivide	leftExactQuotient
leftExtendedGcd	leftGcd	leftLcm
leftQuotient	leftRemainder	minimumDegree
monicLeftDivide	monicRightDivide	monomial
one?	primitivePart	recip
reductum	retract	retractIfCan
rightDivide	rightExactQuotient	rightExtendedGcd
rightGcd	rightLcm	rightQuotient
rightRemainder	sample	subtractIfCan
symmetricPower	symmetricProduct	symmetricSquare
zero?	?^?	?.?
?~=?	?*?	?**?
?+?	?-?	-?
?=?		

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
directSum : (%,%) -> % if A has FIELD
symmetricPower : (%,NonNegativeInteger) -> % if A has FIELD
symmetricProduct : (%,%) -> % if A has FIELD
```

These are implemented by this category:

```
adjoint : % -> %
D : () -> %
symmetricSquare : % -> % if A has FIELD
```

These exports come from (p1101) UnivariateSkewPolynomialCategory(A:Ring):

```
0 : () -> %
1 : () -> %
apply : (%,A,A) -> A
characteristic : () -> NonNegativeInteger
coefficient : (%,NonNegativeInteger) -> A
coefficients : % -> List A
coerce : % -> OutputForm
coerce : Integer -> %
coerce : A -> %
coerce : Fraction Integer -> % if A has RETRACT FRAC INT
content : % -> A if A has GCDDOM
degree : % -> NonNegativeInteger
exquo : (%,A) -> Union(%,"failed") if A has INTDOM
hash : % -> SingleInteger
latex : % -> String
leadingCoefficient : % -> A
leftDivide : (%,%) -> Record(quotient: %,remainder: %)
   if A has FIELD
leftExactQuotient : (%,%) -> Union(%,"failed") if A has FIELD
leftExtendedGcd : (%,%) -> Record(coef1: %,coef2: %,generator: %)
   if A has FIELD
leftGcd : (%,%) -> % if A has FIELD
leftLcm : (%,%) -> % if A has FIELD
leftQuotient : (%,%) -> % if A has FIELD
leftRemainder : (%,%) -> % if A has FIELD
minimumDegree : % -> NonNegativeInteger
monicLeftDivide : (%,%) -> Record(quotient: %,remainder: %)
   if A has INTDOM
monicRightDivide : (%,%) -> Record(quotient: %,remainder: %)
   if A has INTDOM
monomial : (A,NonNegativeInteger) -> %
one? : % -> Boolean
primitivePart : % -> % if A has GCDDOM
recip : % -> Union(%,"failed")
reductum : % -> %
retract : % -> A
retract : % -> Fraction Integer if A has RETRACT FRAC INT
retract : % -> Integer if A has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
   if A has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed")
   if A has RETRACT INT
retractIfCan : % -> Union(A,"failed")
rightDivide : (%,%) -> Record(quotient: %,remainder: %)
   if A has FIELD
rightExactQuotient : (%,%) -> Union(%,"failed") if A has FIELD
rightExtendedGcd :
  (%,%) -> Record(coef1: %,coef2: %,generator: %) if A has FIELD
```

```
rightGcd : (%,%) -> % if A has FIELD
rightLcm : (%,%) -> % if A has FIELD
rightQuotient : (%,%) -> % if A has FIELD
rightRemainder : (%,%) -> % if A has FIELD
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?*? : (%,A) -> %
?*? : (A,%) -> %
?~=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p54) Eltable(A:Ring,A:Ring):

```
?.? : (%,A) -> A
```

See: Bronstein[Bron95], Abramov[Abra01], Mulders[Muld95]

— LinearOrdinaryDifferentialOperatorCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#LODOCAT">
LinearOrdinaryDifferentialOperatorCategory (LODOCAT)</a></h2>
</body>
```

— category LODOCAT LinearOrdinaryDifferentialOperatorCategory —

```
)abbrev category LODOCAT LinearOrdinaryDifferentialOperatorCategory
++ Author: Manuel Bronstein
++ Date Created: 9 December 1993
++ Date Last Updated: 15 April 1994
++ References:
++ Bron95 On radical solutions of linear ordinary differential equations
++ Abra01 On Solutions of Linear Functional Systems
++ Muld95 Primitives: Orepoly and Lodo
++ Description:
++ LinearOrdinaryDifferentialOperatorCategory is the category
++ of differential operators with coefficients in a ring A with a given
++ derivation.
++
++ Multiplication of operators corresponds to functional composition:\br
++ (L1 * L2).(f) = L1 L2 f
```

```
LinearOrdinaryDifferentialOperatorCategory(A) : Category == SIG where
  A : Ring

  SIG ==> Join(UnivariateSkewPolynomialCategory A, Eltable(A, A)) with

    D : () -> %
      ++ D() provides the operator corresponding to a derivation
      ++ in the ring \spad{A}.

    adjoint : % -> %
      ++ adjoint(a) returns the adjoint operator of a.

    if A has Field then

      symmetricProduct : (%, %) -> %
        ++ symmetricProduct(a,b) computes an operator \spad{c} of
        ++ minimal order such that the nullspace of \spad{c} is
        ++ generated by all the products of a solution of \spad{a} by
        ++ a solution of \spad{b}.

      symmetricPower : (%, NonNegativeInteger) -> %
        ++ symmetricPower(a,n) computes an operator \spad{c} of
        ++ minimal order such that the nullspace of \spad{c} is
        ++ generated by all the products of \spad{n} solutions
        ++ of \spad{a}.

      symmetricSquare : % -> %
        ++ symmetricSquare(a) computes \spad{symmetricProduct(a,a)}
        ++ using a more efficient method.

      directSum : (%, %) -> %
        ++ directSum(a,b) computes an operator \spad{c} of
        ++ minimal order such that the nullspace of \spad{c} is
        ++ generated by all the sums of a solution of \spad{a} by
        ++ a solution of \spad{b}.

   add

      m1monom: NonNegativeInteger -> %

      D() == monomial(1, 1)

      m1monom n ==
        a:A := (odd? n => -1; 1)
        monomial(a, n)

      adjoint a ==
        ans:% := 0
        while a ^= 0 repeat
          ans := ans + m1monom(degree a) * leadingCoefficient(a)::%
          a   := reductum a
        ans
```

```
    if A has Field then symmetricSquare l == symmetricPower(l, 2)
```

———-

— **COQ LODOCAT** —

```
(* category LODOCAT *)
(*
Multiplication of operators corresponds to functional composition:
(L1 * L2).(f) = L1 L2 f

        D : () -> %
        D() == monomial(1, 1)

        m1monom: NonNegativeInteger -> %
        m1monom n ==
          a:A := (odd? n => -1; 1)
          monomial(a, n)

        adjoint : % -> %
        adjoint a ==
          ans:% := 0
          while a ^= 0 repeat
            ans := ans + m1monom(degree a) * leadingCoefficient(a)::%
            a   := reductum a
          ans

        if A has Field then

          symmetricSquare : % -> %
          symmetricSquare l == symmetricPower(l, 2)
*)
```

———-

— **LODOCAT.dotabb** —

```
"LODOCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LODOCAT"];
"LODOCAT" -> "ELTAB"
"LODOCAT" -> "OREPCAT"
```

———-

— **LODOCAT.dotfull** —

```
"LinearOrdinaryDifferentialOperatorCategory(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LODOCAT"];
"LinearOrdinaryDifferentialOperatorCategory(a:Ring)"
  -> "Eltable(a:Ring,b:Ring)"
"LinearOrdinaryDifferentialOperatorCategory(a:Ring)"
```

```
  -> "UnivariateSkewPolynomialCategory(R:Ring)"
```

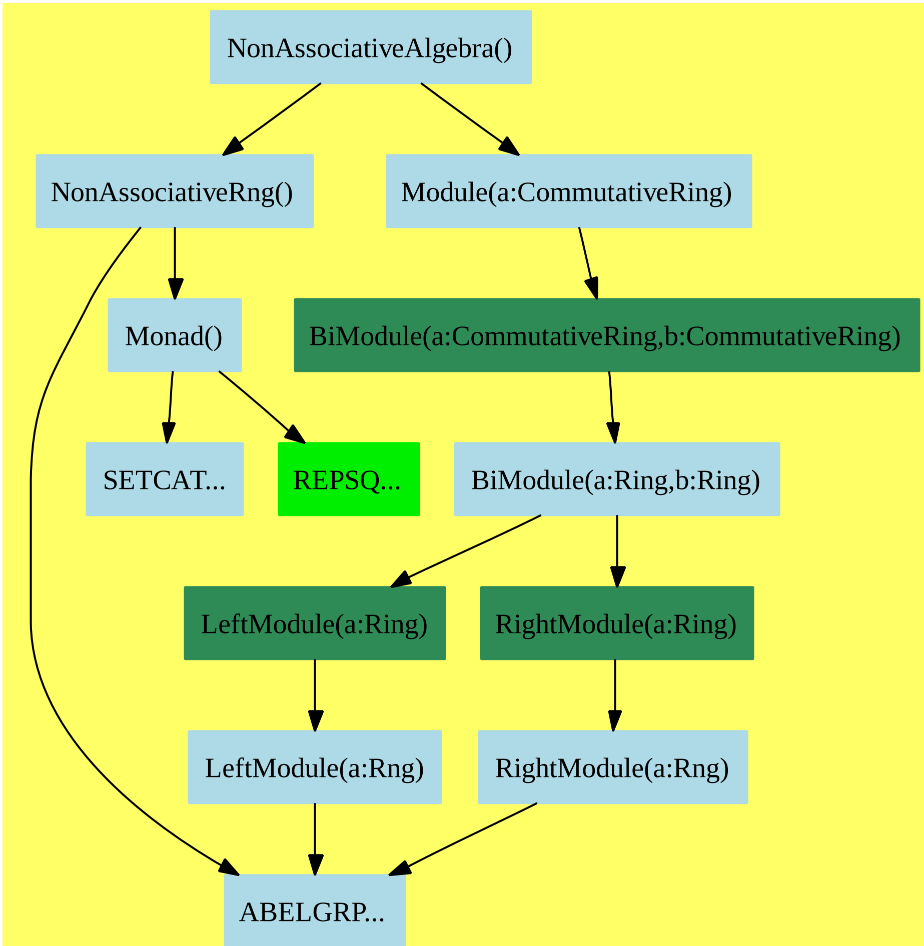
———-

— LODOCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"LinearOrdinaryDifferentialOperatorCategory(a:Ring)" [color=lightblue];
"LinearOrdinaryDifferentialOperatorCategory(a:Ring)"
  -> "Eltable(a:Ring,b:Ring)"
"LinearOrdinaryDifferentialOperatorCategory(a:Ring)"
  -> "UnivariateSkewPolynomialCategory(R:Ring)"

"Eltable(a:Ring,b:Ring)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ELTAB"];
"Eltable(a:Ring,b:Ring)" ->
   "Eltable(a:SetCategory,b:Type)"

"Eltable(a:SetCategory,b:Type)" [color=lightblue];
"Eltable(a:SetCategory,b:Type)" -> "Category"

"UnivariateSkewPolynomialCategory(R:Ring)" [color=lightblue];
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "BiModule(a:Ring,b:Ring)"
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "FullyRetractableTo(a:Ring)"
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "Ring()"

"FullyRetractableTo(a:Ring)" [color=seagreen];
"FullyRetractableTo(a:Ring)" -> "FullyRetractableTo(a:Type)"

"FullyRetractableTo(a:Type)" [color=lightblue];
"FullyRetractableTo(a:Type)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"Category" [color=lightblue];

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"
```

```
"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "AbelianGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

11.0.181 NonAssociativeAlgebra (NAALG)



— **NonAssociativeAlgebra.input** —

```
)set break resume
)sys rm -f NonAssociativeAlgebra.output
)spool NonAssociativeAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show NonAssociativeAlgebra
--R
--R NonAssociativeAlgebra(R: CommutativeRing) is a category constructor
--R Abbreviation for NonAssociativeAlgebra is NAALG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for NAALG
--R
```

```
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?=? : (%,%) -> Boolean                0 : () -> %
--R antiCommutator : (%,%) -> %           associator : (%,%,%) -> %
--R coerce : % -> OutputForm              commutator : (%,%) -> %
--R hash : % -> SingleInteger             latex : % -> String
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R leftPower : (%,PositiveInteger) -> %
--R plenaryPower : (%,PositiveInteger) -> %
--R rightPower : (%,PositiveInteger) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

— NonAssociativeAlgebra.help —

```
====================================================================
NonAssociativeAlgebra examples
====================================================================

NonAssociativeAlgebra is the category of non associative algebras
(modules which are themselves non associative rngs).\br

Axiom:
        r*(a*b) = (r*a)*b = a*(r*b)

See Also:
o )show NonAssociativeAlgebra
```

———-

See:

⇒ “FiniteRankNonAssociativeAlgebra” (FINAALG) 12.0.186 on page 1191
⇐ “Module” (MODULE) 10.0.167 on page 1050
⇐ “NonAssociativeRng” (NARNG) 8.0.140 on page 849

Exports:

0	antiCommutator	associator	coerce	commutator
hash	latex	leftPower	plenaryPower	rightPower
sample	subtractIfCan	zero?	?~=?	?*?
?**?	?+?	?-?	-?	?=?

Attributes exported:

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are implemented by this category:

```
plenaryPower : (%,PositiveInteger) -> %
```

These exports come from (p849) NonAssociativeRng():

```
0 : () -> %
antiCommutator : (%,%) -> %
associator : (%,%,%) -> %
coerce : % -> OutputForm
commutator : (%,%) -> %
hash : % -> SingleInteger
latex : % -> String
leftPower : (%,PositiveInteger) -> %
rightPower : (%,PositiveInteger) -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?*? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
```

These exports come from (p1050) Module(R:CommutativeRing):

```
?*? : (R,%) -> %
?*? : (%,R) -> %
```

— NonAssociativeAlgebra.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#NAALG">
NonAssociativeAlgebra (NAALG)</a></h2>
</body>
```

———-

— category NAALG NonAssociativeAlgebra —

```
)abbrev category NAALG NonAssociativeAlgebra
++ Author: J. Grabmeier, R. Wisbauer
++ Date Created: 01 March 1991
++ Date Last Updated: 11 June 1991
++ Reference:
++ R.D. Schafer: An Introduction to Nonassociative Algebras
++ Academic Press, New York, 1966
```

```
++ Description:
++ NonAssociativeAlgebra is the category of non associative algebras
++ (modules which are themselves non associative rngs).\br
++
++ Axioms\br
++ \tab{5}r*(a*b) = (r*a)*b = a*(r*b)

NonAssociativeAlgebra(R) : Category == SIG where
  R : CommutativeRing

  SIG ==> Join(NonAssociativeRng, Module(R)) with

    plenaryPower : (%,PositiveInteger) -> %
      ++ plenaryPower(a,n) is recursively defined to be
      ++ \spad{plenaryPower(a,n-1)*plenaryPower(a,n-1)} for \spad{n>1}
      ++ and \spad{a} for \spad{n=1}.

   add

    plenaryPower(a,n) ==
      ( n = 1 ) => a
      n1 : PositiveInteger := (n-1)::NonNegativeInteger::PositiveInteger
      plenaryPower(a,n1) * plenaryPower(a,n1)
```

———-

— COQ NAALG —

```
(* category NAALG *)
(*
Axioms
  r*(a*b) = (r*a)*b = a*(r*b)

    plenaryPower : (%,PositiveInteger) -> %
    plenaryPower(a,n) ==
      ( n = 1 ) => a
      n1 : PositiveInteger := (n-1)::NonNegativeInteger::PositiveInteger
      plenaryPower(a,n1) * plenaryPower(a,n1)
*)
```

———-

— NAALG.dotabb —

```
"NAALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NAALG"];
"NAALG" -> "NARNG"
"NAALG" -> "MODULE"
```

———-

— **NAALG.dotfull** —

```
"NonAssociativeAlgebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NAALG"];
"NonAssociativeAlgebra(a:CommutativeRing)" -> "NonAssociativeRng()"
"NonAssociativeAlgebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"
```

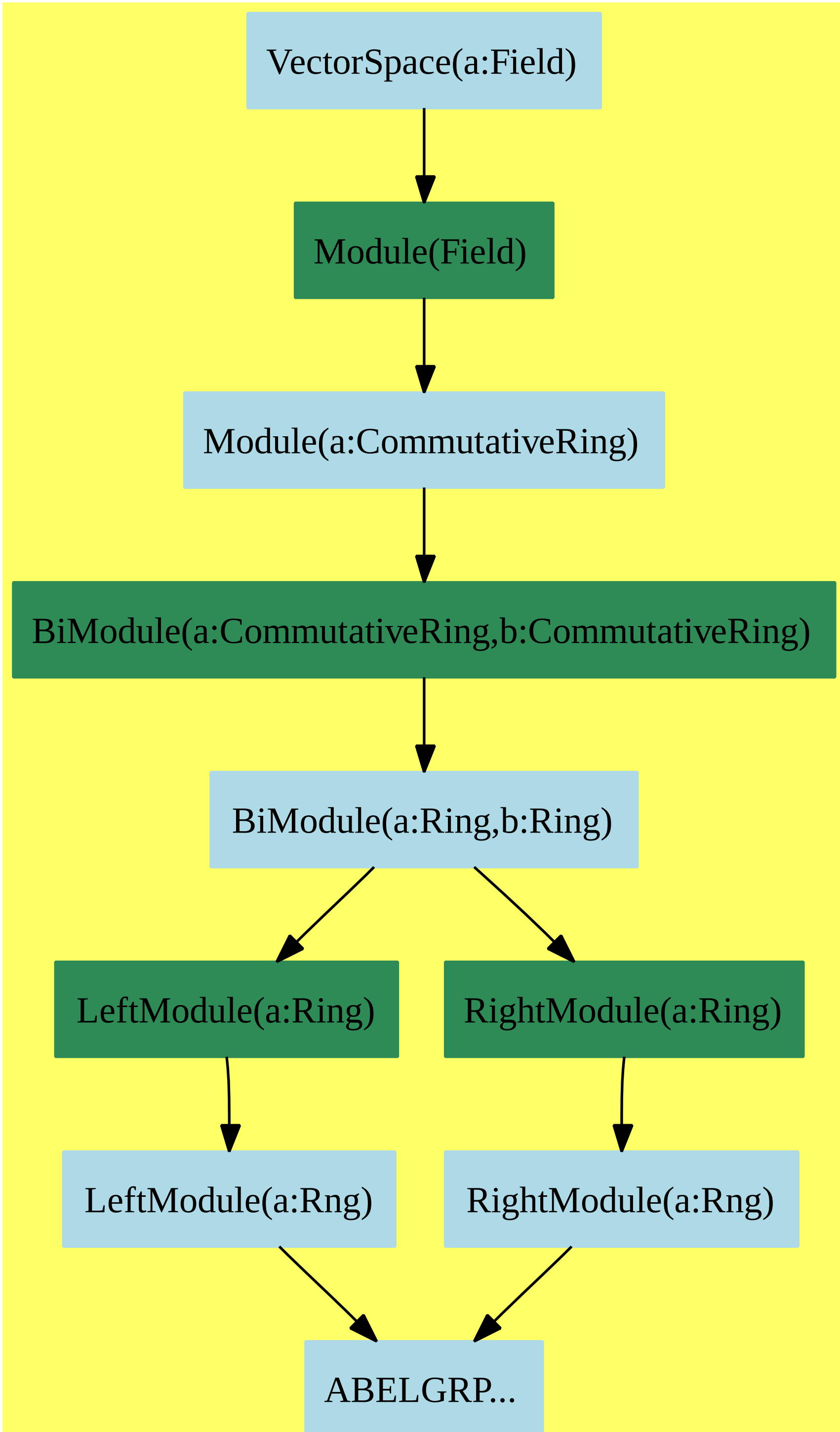
———-

— **NAALG.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"NonAssociativeAlgebra(a:CommutativeRing)" [color=lightblue];
"NonAssociativeAlgebra(a:CommutativeRing)" -> "NonAssociativeRng()"
"NonAssociativeAlgebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"NonAssociativeRng()" [color=lightblue];
"NonAssociativeRng()" -> "ABELGRP..."
"NonAssociativeRng()" -> "Monad()"

"Monad()" [color=lightblue];
"Monad()" -> "SETCAT..."
"Monad()" -> "REPSQ..."

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"ABELGRP..." [color=lightblue];
```

```
}
```

11.0.182 VectorSpace (VSPACE)



— **VectorSpace.input** —

```
)set break resume
)sys rm -f VectorSpace.output
)spool VectorSpace.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show VectorSpace
--R
--R VectorSpace(S: Field) is a category constructor
--R Abbreviation for VectorSpace is VSPACE
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for VSPACE
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,S) -> %                      ?*? : (S,%) -> %
--R ?*? : (Integer,%) -> %                ?*? : (NonNegativeInteger,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (%,S) -> %                      ?=? : (%,%) -> Boolean
--R 0 : () -> %                           coerce : % -> OutputForm
--R dimension : () -> CardinalNumber      hash : % -> SingleInteger
--R latex : % -> String                   sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— **VectorSpace.help** —

```
====================================================================
VectorSpace examples
====================================================================

Vector Spaces (not necessarily finite dimensional) over a field.

See Also:
o )show VectorSpace
```

See:

⇒ "ExtensionField" (XF) 18.0.230 on page 1754
⇐ "Module" (MODULE) 10.0.167 on page 1050

Exports:

0	coerce	dimension	hash	latex
sample	subtractIfCan	zero?	?~=?	?*?
?+?	?-?	-?	?/?	?=?

Attributes exported:

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
dimension : () -> CardinalNumber
```

These are implemented by this category:

```
?/? : (%,S) -> %
```

These exports come from (p1050) Module():

```
?*? : (%,S) -> %
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (S,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
```

— **VectorSpace.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#VSPACE">
VectorSpace (VSPACE)</a></h2>
</body>
```

————

— **category VSPACE VectorSpace** —

```
)abbrev category VSPACE VectorSpace
++ Description:
++ Vector Spaces (not necessarily finite dimensional) over a field.

VectorSpace(S) : Category == SIG where
  S : Field
```

```
  SIG ==> Module(S) with

    "/" : (%, S) -> %
      ++ x/y divides the vector x by the scalar y.

    dimension : () -> CardinalNumber
      ++ dimension() returns the dimensionality of the vector space.

   add

     (v:% / s:S):% == inv(s) * v
```

———-

— COQ VSPACE —

```
(* category VSPACE *)
(*
    ?/? : (%,S) -> %
    (v:% / s:S):% == inv(s) * v

*)
```

———-

— VSPACE.dotabb —

```
"VSPACE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=VSPACE"];
"VSPACE" -> "MODULE"
```

———-

— VSPACE.dotfull —

```
"VectorSpace(a:Field)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=VSPACE"];
"VectorSpace(a:Field)" -> "Module(Field)"
```

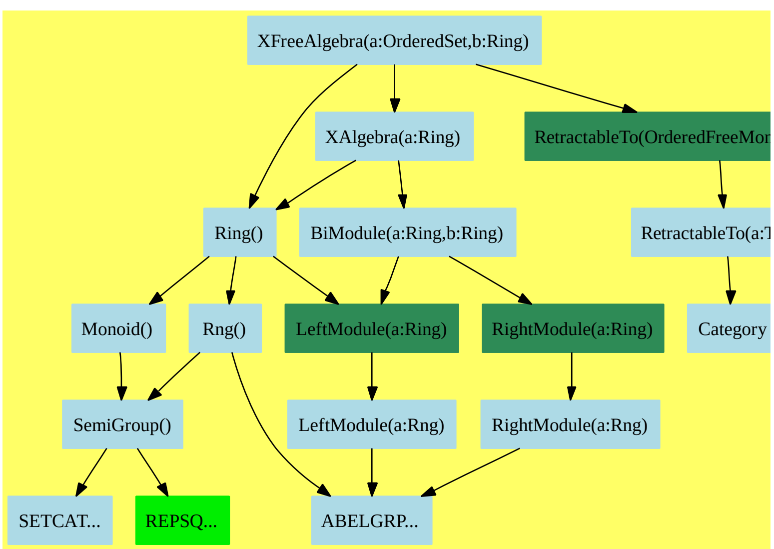
———-

— VSPACE.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"VectorSpace(a:Field)" [color=lightblue];
"VectorSpace(a:Field)" -> "Module(Field)"
```

```
"Module(Field)" [color=seagreen];
"Module(Field)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"ABELGRP..." [color=lightblue];
}
```

11.0.183 XFreeAlgebra (XFALG)



— **XFreeAlgebra.input** —

```
)set break resume
)sys rm -f XFreeAlgebra.output
)spool XFreeAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show XFreeAlgebra
--R
--R XFreeAlgebra(vl: OrderedSet,R: Ring) is a category constructor
--R Abbreviation for XFreeAlgebra is XFALG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for XFALG
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (vl,%) -> %                    ?*? : (%,R) -> %
--R ?*? : (R,%) -> %                     ?*? : (%,%) -> %
--R ?*? : (Integer,%) -> %               ?*? : (NonNegativeInteger,%) -> %
--R ?*? : (PositiveInteger,%) -> %       ?**? : (%,NonNegativeInteger) -> %
--R ?**? : (%,PositiveInteger) -> %      ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                     -? : % -> %
--R ?=? : (%,%) -> Boolean               1 : () -> %
--R 0 : () -> %                          ?^? : (%,NonNegativeInteger) -> %
--R ?^? : (%,PositiveInteger) -> %       coef : (%,%) -> R
--R coerce : vl -> %                     coerce : OrderedFreeMonoid(vl) -> %
--R coerce : R -> %                      coerce : Integer -> %
--R coerce : % -> OutputForm             constant : % -> R
--R constant? : % -> Boolean             hash : % -> SingleInteger
--R latex : % -> String                  lquo : (%,%) -> %
--R lquo : (%,vl) -> %                   map : ((R -> R),%) -> %
--R mindeg : % -> OrderedFreeMonoid(vl)  mirror : % -> %
--R monomial? : % -> Boolean             one? : % -> Boolean
--R quasiRegular : % -> %                quasiRegular? : % -> Boolean
--R recip : % -> Union(%,"failed")       rquo : (%,%) -> %
--R rquo : (%,vl) -> %                   sample : () -> %
--R sh : (%,%) -> % if R has COMRING     varList : % -> List(vl)
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R coef : (%,OrderedFreeMonoid(vl)) -> R
--R lquo : (%,OrderedFreeMonoid(vl)) -> %
--R mindegTerm : % -> Record(k: OrderedFreeMonoid(vl),c: R)
--R monom : (OrderedFreeMonoid(vl),R) -> %
--R retract : % -> OrderedFreeMonoid(vl)
--R retractIfCan : % -> Union(OrderedFreeMonoid(vl),"failed")
--R rquo : (%,OrderedFreeMonoid(vl)) -> %
--R sh : (%,NonNegativeInteger) -> % if R has COMRING
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1
```

```
)spool
)lisp (bye)
```

— XFreeAlgebra.help —

```
====================================================================
XFreeAlgebra examples
====================================================================

This category specifies opeations for  polynomials and formal series
with non-commutative variables.

See Also:
o )show XFreeAlgebra
```

See:

⇒ "XPolynomialsCat" (XPOLYC) 12.0.193 on page 1279
⇐ "RetractableTo" (RETRACT) 2.0.37 on page 110
⇐ "Ring" (RING) 9.0.153 on page 946
⇐ "XAlgebra" (XALG) 10.0.175 on page 1116

Exports:

0	1	characteristic	coef	coerce
constant	constant?	hash	latex	lquo
map	mindeg	mindegTerm	mirror	monom
monomial?	one?	quasiRegular	quasiRegular?	recip
retract	retractIfCan	rquo	sample	sh
subtractIfCan	varList	zero?	?*?	?**?
?+?	?-?	-?	?=?	?^?
?~=?				

Attributes Exported:

- if Ring has noZeroDivisors then noZeroDivisors where **noZeroDivisors** is true if $x*y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
coef : (%,OrderedFreeMonoid vl) -> R
coef : (%,%) -> R
```

```
coerce : vl -> %
constant : % -> R
constant? : % -> Boolean
lquo : (%,vl) -> %
lquo : (%,%) -> %
lquo : (%,OrderedFreeMonoid vl) -> %
map : ((R -> R),%) -> %
mindeg : % -> OrderedFreeMonoid vl
mindegTerm : % -> Record(k: OrderedFreeMonoid vl,c: R)
mirror : % -> %
monom : (OrderedFreeMonoid vl,R) -> %
monomial? : % -> Boolean
quasiRegular : % -> %
quasiRegular? : % -> Boolean
rquo : (%,OrderedFreeMonoid vl) -> %
rquo : (%,%) -> %
rquo : (%,vl) -> %
sh : (%,NonNegativeInteger) -> % if R has COMRING
sh : (%,%) -> % if R has COMRING
varList : % -> List vl
?*? : (vl,%) -> %
?*? : (%,R) -> %
```

These exports come from (p946) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : % -> OutputForm
coerce : Integer -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p1116) XAlgebra(a:Ring):

```
coerce : R -> %
?*? : (R,%) -> %
```

These exports come from (p110) RetractableTo(WORD) where WORD:OrderedFreeMonoid(OrderedSet))

```
 coerce : OrderedFreeMonoid vl -> %
 retract : % -> OrderedFreeMonoid vl
 retractIfCan : % -> Union(OrderedFreeMonoid vl,"failed")
```

— **XFreeAlgebra.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#XFALG">
XFreeAlgebra (XFALG)</a></h2>
</body>
```

— **category XFALG XFreeAlgebra** —

```
)abbrev category XFALG XFreeAlgebra
++ Author: Michel Petitot petitot@lifl.fr
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Description:
++ This category specifies opeations for  polynomials
++ and formal series with non-commutative variables.

XFreeAlgebra(vl,R) : Category == SIG where
  vl : OrderedSet
  R : Ring

  WORD ==> OrderedFreeMonoid(vl)          -- monoide libre
  NNI  ==> NonNegativeInteger
  I    ==> Integer
  TERM ==> Record(k: WORD, c: R)

  SIG ==> Join(Ring, XAlgebra(R), RetractableTo WORD) with

    "*" : (vl,%) -> %
      ++ \spad{v * x} returns the product of a variable \spad{x}
      ++ by \spad{x}.

    "*" : (%, R) -> %
      ++ \spad{x * r} returns the product of \spad{x} by \spad{r}.
      ++ Usefull if \spad{R} is a non-commutative Ring.

    mindeg : % -> WORD
      ++ \spad{mindeg(x)} returns the little word which appears
      ++ in \spad{x}. Error if \spad{x=0}.

    mindegTerm : % -> TERM
      ++ \spad{mindegTerm(x)} returns the term whose word is
      ++ \spad{mindeg(x)}.

    coef : (%,WORD) -> R
```

```
  ++ \spad{coef(x,w)} returns the coefficient of the word \spad{w}
  ++ in \spad{x}.

coef : (%,%) -> R
  ++ \spad{coef(x,y)} returns scalar product of \spad{x} by \spad{y},
  ++ the set of words being regarded as an orthogonal basis.

lquo : (%,vl) -> %
  ++ \spad{lquo(x,v)} returns the left simplification of \spad{x}
  ++ by the variable \spad{v}.

lquo : (%,WORD) -> %
  ++ \spad{lquo(x,w)} returns the left simplification of \spad{x}
  ++ by the word \spad{w}.

lquo : (%,%) -> %
  ++ \spad{lquo(x,y)} returns the left simplification of \spad{x}
  ++ by \spad{y}.

rquo : (%,vl) -> %
  ++ \spad{rquo(x,v)} returns the right simplification of \spad{x}
  ++ by the variable \spad{v}.

rquo : (%,WORD) -> %
  ++ \spad{rquo(x,w)} returns the right simplification of \spad{x}
  ++ by \spad{w}.

rquo : (%,%) -> %
  ++ \spad{rquo(x,y)} returns the right simplification of \spad{x}
  ++ by \spad{y}.

monom : (WORD , R) -> %
  ++ \spad{monom(w,r)} returns the product of the word \spad{w}
  ++ by the coefficient \spad{r}.

monomial? : % -> Boolean
  ++ \spad{monomial?(x)} returns true if \spad{x} is a monomial

mirror : % -> %
  ++ \spad{mirror(x)} returns \spad{Sum(r_i mirror(w_i))} if
  ++ \spad{x} writes \spad{Sum(r_i w_i)}.

coerce : vl -> %
  ++ \spad{coerce(v)} returns \spad{v}.

constant? : % -> Boolean
  ++ \spad{constant?(x)} returns true if \spad{x} is constant.

constant : % -> R
  ++ \spad{constant(x)} returns the constant term of \spad{x}.

quasiRegular? : % -> Boolean
  ++ \spad{quasiRegular?(x)} return true if \spad{constant(x)} is zero
```

```
    quasiRegular : % -> %
      ++ \spad{quasiRegular(x)} return \spad{x} minus its constant term.

    if R has CommutativeRing then

       sh : (%,%) -> %
         ++ \spad{sh(x,y)} returns the shuffle-product of \spad{x}
         ++ by \spad{y}.
         ++ This multiplication is associative and commutative.

       sh : (%,NNI) -> %
         ++ \spad{sh(x,n)} returns the shuffle power of \spad{x} to
         ++ the \spad{n}.

    map : (R -> R, %) -> %
      ++ \spad{map(fn,x)} returns \spad{Sum(fn(r_i) w_i)} if \spad{x}
      ++ writes \spad{Sum(r_i w_i)}.

    varList : % -> List vl
      ++ \spad{varList(x)} returns the list of variables which
      ++ appear in \spad{x}.

    if R has noZeroDivisors then noZeroDivisors
```

———-

— XFALG.dotabb —

```
"XFALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XFALG"];
"XFALG" -> "RETRACT"
"XFALG" -> "RING"
"XFALG" -> "XALG"
```

———-

— XFALG.dotfull —

```
"XFreeAlgebra(a:OrderedSet,b:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XFALG"];
"XFreeAlgebra(a:OrderedSet,b:Ring)" -> "Ring()"
"XFreeAlgebra(a:OrderedSet,b:Ring)" -> "XAlgebra(a:Ring)"
"XFreeAlgebra(a:OrderedSet,b:Ring)" ->
    "RetractableTo(OrderedFreeMonoid(OrderedSet))"
```

———-

— XFALG.dotpic —

```
digraph pic {
 fontsize=10;
```

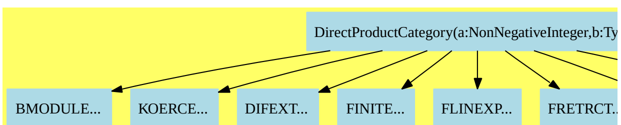
```
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"XFreeAlgebra(a:OrderedSet,b:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XFALG"];
"XFreeAlgebra(a:OrderedSet,b:Ring)" -> "Ring()"
"XFreeAlgebra(a:OrderedSet,b:Ring)" -> "XAlgebra(a:Ring)"
"XFreeAlgebra(a:OrderedSet,b:Ring)" ->
    "RetractableTo(OrderedFreeMonoid(OrderedSet))"

"RetractableTo(OrderedFreeMonoid(OrderedSet))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(OrderedFreeMonoid(OrderedSet))" -> "RetractableTo(a:Type)"

"XAlgebra(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XALG"];
"XAlgebra(a:Ring)" -> "Ring()"
"XAlgebra(a:Ring)" -> "BiModule(a:Ring,b:Ring)"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"Rng()" [color=lightblue];
"Rng()" -> "ABELGRP..."
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"
```

```
"Category" [color=lightblue];

"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"ABELGRP..." [color=lightblue];
}
```

Chapter 12

Category Layer 11

12.0.184 DirectProductCategory (DIRPCAT)



— DirectProductCategory.input —

```
)set break resume
)sys rm -f DirectProductCategory.output
)spool DirectProductCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DirectProductCategory
--R
--R DirectProductCategory(dim: NonNegativeInteger,R: Type) is a category constructor
--R Abbreviation for DirectProductCategory is DIRPCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for DIRPCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> % if R has MONOID      ?*? : (%,R) -> % if R has MONOID
--R ?*? : (%,%) -> % if R has MONOID      ?+? : (%,%) -> % if R has ABELSG
--R -? : % -> % if R has RING             ?-? : (%,%) -> % if R has RING
--R ?/? : (%,R) -> % if R has FIELD       1 : () -> % if R has MONOID
--R 0 : () -> % if R has CABMON           abs : % -> % if R has ORDRING
--R coerce : R -> % if R has SETCAT       coerce : % -> Vector(R)
--R copy : % -> %                         directProduct : Vector(R) -> %
--R dot : (%,%) -> R if R has RING        ?.? : (%,Integer) -> R
--R elt : (%,Integer,R) -> R              empty : () -> %
--R empty? : % -> Boolean                 entries : % -> List(R)
```

```
--R eq? : (%,%) -> Boolean                index? : (Integer,%) -> Boolean
--R indices : % -> List(Integer)          latex : % -> String if R has SETCAT
--R map : ((R -> R),%) -> %               one? : % -> Boolean if R has MONOID
--R qelt : (%,Integer) -> R               random : () -> % if R has FINITE
--R retract : % -> R if R has SETCAT      sample : () -> %
--R sup : (%,%) -> % if R has OAMONS
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?*? : (PositiveInteger,%) -> % if R has ABELSG
--R ?*? : (NonNegativeInteger,%) -> % if R has CABMON
--R ?*? : (Integer,%) -> % if R has RING
--R ?**? : (%,PositiveInteger) -> % if R has MONOID
--R ?**? : (%,NonNegativeInteger) -> % if R has MONOID
--R ?<? : (%,%) -> Boolean if R has ORDRING or R has OAMONS
--R ?<=? : (%,%) -> Boolean if R has ORDRING or R has OAMONS
--R ?=? : (%,%) -> Boolean if R has SETCAT
--R ?>? : (%,%) -> Boolean if R has ORDRING or R has OAMONS
--R ?>=? : (%,%) -> Boolean if R has ORDRING or R has OAMONS
--R D : (%,(R -> R)) -> % if R has RING
--R D : (%,(R -> R),NonNegativeInteger) -> % if R has RING
--R D : (%,List(Symbol),List(NonNegativeInteger)) -> % if and(has(R,PartialDifferentialRing(Symbol)),has(R,Ring)
--R D : (%,Symbol,NonNegativeInteger) -> % if and(has(R,PartialDifferentialRing(Symbol)),has(R,Ring))
--R D : (%,List(Symbol)) -> % if and(has(R,PartialDifferentialRing(Symbol)),has(R,Ring))
--R D : (%,Symbol) -> % if and(has(R,PartialDifferentialRing(Symbol)),has(R,Ring))
--R D : (%,NonNegativeInteger) -> % if and(has(R,DifferentialRing),has(R,Ring))
--R D : % -> % if and(has(R,DifferentialRing),has(R,Ring))
--R ?^? : (%,PositiveInteger) -> % if R has MONOID
--R ?^? : (%,NonNegativeInteger) -> % if R has MONOID
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R characteristic : () -> NonNegativeInteger if R has RING
--R coerce : Fraction(Integer) -> % if and(has(R,RetractableTo(Fraction(Integer))),has(R,SetCategory))
--R coerce : Integer -> % if and(has(R,RetractableTo(Integer)),has(R,SetCategory)) or R has RING
--R coerce : % -> OutputForm if R has SETCAT
--R count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R differentiate : (%,(R -> R)) -> % if R has RING
--R differentiate : (%,(R -> R),NonNegativeInteger) -> % if R has RING
--R differentiate : (%,List(Symbol),List(NonNegativeInteger)) -> % if and(has(R,PartialDifferentialRing(Symbol))
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if and(has(R,PartialDifferentialRing(Symbol)),has(R,Ring)
--R differentiate : (%,List(Symbol)) -> % if and(has(R,PartialDifferentialRing(Symbol)),has(R,Ring))
--R differentiate : (%,Symbol) -> % if and(has(R,PartialDifferentialRing(Symbol)),has(R,Ring))
--R differentiate : (%,NonNegativeInteger) -> % if and(has(R,DifferentialRing),has(R,Ring))
--R differentiate : % -> % if and(has(R,DifferentialRing),has(R,Ring))
--R dimension : () -> CardinalNumber if R has FIELD
--R entry? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
--R enumerate : () -> List(%) if R has FINITE
--R eval : (%,List(R),List(R)) -> % if R has EVALAB(R) and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB(R) and R has SETCAT
--R eval : (%,Equation(R)) -> % if R has EVALAB(R) and R has SETCAT
--R eval : (%,List(Equation(R))) -> % if R has EVALAB(R) and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R fill! : (%,R) -> % if $ has shallowlyMutable
--R first : % -> R if Integer has ORDSET
--R hash : % -> SingleInteger if R has SETCAT
--R index : PositiveInteger -> % if R has FINITE
```

```
--R less? : (%,NonNegativeInteger) -> Boolean
--R lookup : % -> PositiveInteger if R has FINITE
--R map! : ((R -> R),%) -> % if $ has shallowlyMutable
--R max : (%,%) -> % if R has ORDRING or R has OAMONS
--R maxIndex : % -> Integer if Integer has ORDSET
--R member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
--R members : % -> List(R) if $ has finiteAggregate
--R min : (%,%) -> % if R has ORDRING or R has OAMONS
--R minIndex : % -> Integer if Integer has ORDSET
--R more? : (%,NonNegativeInteger) -> Boolean
--R negative? : % -> Boolean if R has ORDRING
--R parts : % -> List(R) if $ has finiteAggregate
--R positive? : % -> Boolean if R has ORDRING
--R qsetelt! : (%,Integer,R) -> R if $ has shallowlyMutable
--R recip : % -> Union(%,"failed") if R has MONOID
--R reducedSystem : Matrix(%) -> Matrix(R) if R has RING
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(R),vec: Vector(R)) if R has RING
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(Integer),vec: Vector(Integer)) if and(has(R,Line
--R reducedSystem : Matrix(%) -> Matrix(Integer) if and(has(R,LinearlyExplicitRingOver(Integer)),has(R,Ring))
--R retract : % -> Fraction(Integer) if and(has(R,RetractableTo(Fraction(Integer))),has(R,SetCategory))
--R retract : % -> Integer if and(has(R,RetractableTo(Integer)),has(R,SetCategory))
--R retractIfCan : % -> Union(R,"failed") if R has SETCAT
--R retractIfCan : % -> Union(Fraction(Integer),"failed") if and(has(R,RetractableTo(Fraction(Integer))),has(R,S
--R retractIfCan : % -> Union(Integer,"failed") if and(has(R,RetractableTo(Integer)),has(R,SetCategory))
--R setelt : (%,Integer,R) -> R if $ has shallowlyMutable
--R sign : % -> Integer if R has ORDRING
--R size : () -> NonNegativeInteger if R has FINITE
--R size? : (%,NonNegativeInteger) -> Boolean
--R subtractIfCan : (%,%) -> Union(%,"failed") if R has CABMON
--R swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
--R unitVector : PositiveInteger -> % if R has RING
--R zero? : % -> Boolean if R has CABMON
--R ?~=? : (%,%) -> Boolean if R has SETCAT
--R
--E 1

)spool
)lisp (bye)
```

— **DirectProductCategory.help** —

```
====================================================================
DirectProductCategory examples
====================================================================

This category represents a finite cartesian product of a given type.
Many categorical properties are preserved under this construction.

See Also:
o )show DirectProductCategory
```

See:

⇐ "BiModule" (BMODULE) 9.0.146 on page 902
⇐ "CoercibleTo" (KOERCE) 2.0.14 on page 39
⇐ "DifferentialExtension" (DIFEXT) 11.0.177 on page 1127
⇐ "IndexedAggregate" (IXAGG) 5.0.98 on page 441
⇐ "Finite" (FINITE) 4.0.69 on page 258
⇐ "FullyLinearlyExplicitRingOver" (FLINEXP) 11.0.178 on page 1134
⇐ "FullyRetractableTo" (FRETRCT) 3.0.50 on page 158
⇐ "OrderedRing" (ORDRING) 10.0.168 on page 1055
⇐ "OrderedAbelianMonoidSup" (OAMONS) 9.0.151 on page 935

Exports:

0	1	abs	any?	characteristic
coerce	copy	count	D	differentiate
dimension	directProduct	dot	elt	empty
empty?	entry?	entries	eq?	eval
every?	fill!	first	hash	index
index?	indices	latex	less?	lookup
map	map!	max	maxIndex	member?
members	min	minIndex	more?	negative?
one?	parts	positive?	qelt	qsetelt!
random	recip	reducedSystem	retract	retractIfCan
sample	setelt	sign	size	size?
subtractIfCan	sup	swap!	unitVector	zero?
?~=?	-?	?.?	#?	?*?
?**?	?+?	?-?	?/? ?<?	
?<=?	?=?	?>?	?>=?	?^?

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- if #2 has commutativeRing then commutative("*") where **commutative("*")** is true if it has an operation $" * " : (D, D)- > D$ which is commutative.
- if #2 has unitsKnown then unitsKnown where **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- if #2 has Ring then rightUnitary where **leftUnitary** is true if $1 * x = x$ for all x.
- if #2 has Ring then rightUnitary where **rightUnitary** is true if $x * 1 = x$ for all x.
- **nil**

These are directly exported but not implemented:

```
directProduct : Vector R -> %
dot : (%,%) -> R if R has RING
unitVector : PositiveInteger -> % if R has RING
?*? : (R,%) -> % if R has MONOID
?*? : (%,R) -> % if R has MONOID
```

These are implemented by this category:

```
characteristic : () -> NonNegativeInteger if R has RING
coerce : Integer -> %
  if and(has(R,RetractableTo Integer),
         has(R,SetCategory))
  or R has RING
differentiate : (%,(R -> R)) -> % if R has RING
dimension : () -> CardinalNumber if R has FIELD
reducedSystem : Matrix % -> Matrix R if R has RING
reducedSystem :
   (Matrix %,Vector %) ->
      Record(mat: Matrix R,vec: Vector R)
         if R has RING
size : () -> NonNegativeInteger if R has FINITE
?/? : (%,R) -> % if R has FIELD
```

These exports come from (p441) IndexedAggregate(a:SetCategory,R:Type):

```
any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
coerce : % -> OutputForm if R has SETCAT
copy : % -> %
count : (R,%) -> NonNegativeInteger
  if R has SETCAT
  and $ has finiteAggregate
count : ((R -> Boolean),%) -> NonNegativeInteger
  if $ has finiteAggregate
elt : (%,Integer,R) -> R
empty : () -> %
empty? : % -> Boolean
entries : % -> List R
entry? : (R,%) -> Boolean
  if $ has finiteAggregate
  and R has SETCAT
eq? : (%,%) -> Boolean
eval : (%,List R,List R) -> %
  if R has EVALAB R
  and R has SETCAT
eval : (%,R,R) -> %
  if R has EVALAB R
  and R has SETCAT
eval : (%,Equation R) -> %
  if R has EVALAB R
  and R has SETCAT
eval : (%,List Equation R) -> %
  if R has EVALAB R
  and R has SETCAT
every? : ((R -> Boolean),%) -> Boolean
  if $ has finiteAggregate
fill! : (%,R) -> % if $ has shallowlyMutable
first : % -> R if Integer has ORDSET
hash : % -> SingleInteger if R has SETCAT
index? : (Integer,%) -> Boolean
indices : % -> List Integer
less? : (%,NonNegativeInteger) -> Boolean
latex : % -> String if R has SETCAT
map : ((R -> R),%) -> %
```

```
map! : ((R -> R),%) -> % if $ has shallowlyMutable
maxIndex : % -> Integer if Integer has ORDSET
member? : (R,%) -> Boolean
  if R has SETCAT
  and $ has finiteAggregate
members : % -> List R if $ has finiteAggregate
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List R if $ has finiteAggregate
qelt : (%,Integer) -> R
qsetelt! : (%,Integer,R) -> R if $ has shallowlyMutable
sample : () -> %
setelt : (%,Integer,R) -> R if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
#? : % -> NonNegativeInteger if $ has finiteAggregate
?.? : (%,Integer) -> R
?=? : (%,%) -> Boolean if R has SETCAT
?~=? : (%,%) -> Boolean if R has SETCAT
```

These exports come from (p39) CoercibleTo(Vector(R:Type)):

```
coerce : % -> Vector R
```

These exports come from (p158) FullyRetractableTo(R:SetCategory):

```
coerce : R -> % if R has SETCAT
coerce : Fraction Integer -> %
  if and(has(R,RetractableTo Fraction Integer),
         has(R,SetCategory))
retract : % -> Integer
  if and(has(R,RetractableTo Integer),
         has(R,SetCategory))
retract : % -> R if R has SETCAT
retract : % -> Fraction Integer
  if and(has(R,RetractableTo Fraction Integer),
         has(R,SetCategory))
retractIfCan : % -> Union(Integer,"failed")
  if and(has(R,RetractableTo Integer),
         has(R,SetCategory))
retractIfCan : % -> Union(R,"failed") if R has SETCAT
retractIfCan : % -> Union(Fraction Integer,"failed")
  if and(has(R,RetractableTo Fraction Integer),
         has(R,SetCategory))
```

These exports come from (p902) BiModule(R:Ring,R:Ring):

```
0 : () -> % if R has CABMON
subtractIfCan : (%,%) -> Union(%,"failed") if R has CABMON
zero? : % -> Boolean if R has CABMON
?+? : (%,%) -> % if R has ABELSG
?*? : (PositiveInteger,%) -> % if R has ABELSG
?*? : (NonNegativeInteger,%) -> % if R has CABMON
?*? : (Integer,%) -> % if R has RING
?-? : (%,%) -> % if R has RING
-? : % -> % if R has RING
```

These exports come from (p1127) DifferentialExtension(R:Ring):

```
1 : () -> % if R has RING
D : (%,(R -> R)) -> % if R has RING
D : (%,(R -> R),NonNegativeInteger) -> % if R has RING
D : % -> % if and(has(R,DifferentialRing),has(R,Ring))
D : (%,NonNegativeInteger) -> %
  if and(has(R,DifferentialRing),has(R,Ring))
differentiate : (%,NonNegativeInteger) -> %
  if and(has(R,DifferentialRing),has(R,Ring))
D : (%,List Symbol,List NonNegativeInteger) -> %
  if and(has(R,PartialDifferentialRing Symbol),has(R,Ring))
D : (%,Symbol,NonNegativeInteger) -> %
  if and(has(R,PartialDifferentialRing Symbol),has(R,Ring))
D : (%,List Symbol) -> %
  if and(has(R,PartialDifferentialRing Symbol),has(R,Ring))
D : (%,Symbol) -> %
  if and(has(R,PartialDifferentialRing Symbol),has(R,Ring))
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
  if and(has(R,PartialDifferentialRing Symbol),has(R,Ring))
differentiate : (%,Symbol,NonNegativeInteger) -> %
  if and(has(R,PartialDifferentialRing Symbol),has(R,Ring))
differentiate : (%,List Symbol) -> %
  if and(has(R,PartialDifferentialRing Symbol),has(R,Ring))
differentiate : % -> %
  if and(has(R,DifferentialRing),has(R,Ring))
differentiate : (%,(R -> R),NonNegativeInteger) -> %
  if R has RING
differentiate : (%,Symbol) -> %
  if and(has(R,PartialDifferentialRing Symbol),has(R,Ring))
one? : % -> Boolean if R has RING
recip : % -> Union(%,"failed") if R has RING
?*? : (%,%) -> % if R has RING
?**? : (%,PositiveInteger) -> % if R has RING
?**? : (%,NonNegativeInteger) -> % if R has RING
?^? : (%,PositiveInteger) -> % if R has RING
?^? : (%,NonNegativeInteger) -> % if R has RING
```

These exports come from (p1134) FullyLinearlyExplicitRingOver(R:Ring):

```
reducedSystem :
  (Matrix %,Vector %) ->
    Record(mat: Matrix Integer,vec: Vector Integer)
      if and(has(R,LinearlyExplicitRingOver Integer),has(R,Ring))
reducedSystem : Matrix % -> Matrix Integer
  if and(has(R,LinearlyExplicitRingOver Integer),has(R,Ring))
```

These exports come from (p258) Finite():

```
index : PositiveInteger -> % if R has FINITE
lookup : % -> PositiveInteger if R has FINITE
random : () -> % if R has FINITE
```

These exports come from (p1055) OrderedRing():

```
abs : % -> % if R has ORDRING
max : (%,%) -> % if R has ORDRING or R has OAMONS
min : (%,%) -> % if R has ORDRING or R has OAMONS
negative? : % -> Boolean if R has ORDRING
```

```
positive? : % -> Boolean if R has ORDRING
sign : % -> Integer if R has ORDRING
?<? : (%,%) -> Boolean if R has ORDRING or R has OAMONS
?<=? : (%,%) -> Boolean if R has ORDRING or R has OAMONS
?>? : (%,%) -> Boolean if R has ORDRING or R has OAMONS
?>=? : (%,%) -> Boolean if R has ORDRING or R has OAMONS
```

These exports come from (p935) OrderedAbelianMonoidSup():

```
sup : (%,%) -> % if R has OAMONS
```

— DirectProductCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#DIRPCAT">
DirectProductCategory (DIRPCAT)</a></h2>
</body>
```

— category DIRPCAT DirectProductCategory —

```
)abbrev category DIRPCAT DirectProductCategory
++ Description:
++ This category represents a finite cartesian product of a given type.
++ Many categorical properties are preserved under this construction.
-- all direct product category domains must be compiled
-- without subsumption, set SourceLevelSubset to EQUAL
--)bo $noSubsumption := true

DirectProductCategory(dim,R) : Category == SIG where
  dim : NonNegativeInteger
  R : Type

  SIG ==> Join(IndexedAggregate(Integer, R), CoercibleTo Vector R) with

    finiteAggregate
      ++ attribute to indicate an aggregate of finite size

    directProduct : Vector R -> %
      ++ directProduct(v) converts the vector v to become
      ++ a direct product. Error: if the length of v is
      ++ different from dim.

    if R has SetCategory then FullyRetractableTo R

    if R has Ring then

      BiModule(R, R)

      DifferentialExtension R

      FullyLinearlyExplicitRingOver R
```

```
    unitVector : PositiveInteger -> %
      ++ unitVector(n) produces a vector with 1 in position n and
      ++ zero elsewhere.

    dot : (%, %) -> R
      ++ dot(x,y) computes the inner product of the vectors x and y.

  if R has AbelianSemiGroup then AbelianSemiGroup

  if R has CancellationAbelianMonoid then CancellationAbelianMonoid

  if R has Monoid then

    Monoid

    _* : (R, %) -> %
      ++ r * y multiplies the element r times each component of the
      ++ vector y.

    _* : (%, R) -> %
      ++ y * r multiplies each component of the vector y
      ++ by the element r.

  if R has Finite then Finite

  if R has CommutativeRing then

    Algebra R

    CommutativeRing

  if R has unitsKnown then unitsKnown

  if R has OrderedRing then OrderedRing

  if R has OrderedAbelianMonoidSup then OrderedAbelianMonoidSup

  if R has Field then VectorSpace R

 add

    if R has Ring then

      equation2R: Vector % -> Matrix R

      coerce(n:Integer):%          == n::R::%

      characteristic()             == characteristic()$R

      differentiate(z:%, d:R -> R) == map(d, z)

      equation2R v ==
        ans:Matrix(R) := new(dim, #v, 0)
        for i in minRowIndex ans .. maxRowIndex ans repeat
```

```
          for j in minColIndex ans .. maxColIndex ans repeat
            qsetelt_!(ans, i, j, qelt(qelt(v, j), i))
        ans

      reducedSystem(m:Matrix %):Matrix(R) ==
        empty? m => new(0, 0, 0)
        reduce(vertConcat, [equation2R row(m, i)
               for i in minRowIndex m .. maxRowIndex m])$List(Matrix R)

      reducedSystem(m:Matrix %, v:Vector %):
        Record(mat:Matrix R, vec:Vector R) ==
          vh:Vector(R) :=
            empty? v => empty()
            rh := reducedSystem(v::Matrix %)@Matrix(R)
            column(rh, minColIndex rh)
          [reducedSystem(m)@Matrix(R), vh]

    if R has Finite then size == size$R ** dim

    if R has Field then
      x / b       == x * inv b

      dimension() == dim::CardinalNumber
```

———-

— COQ DIRPCAT —

```
(* category DIRPCAT *)
(*
      if R has Ring then

        coerce : Integer -> %
        coerce(n:Integer):% == n::R::%

        characteristic : () -> NonNegativeInteger
        characteristic() == characteristic()$R

        differentiate : (%,(R -> R)) -> %
        differentiate(z:%, d:R -> R) == map(d, z)

        equation2R: Vector % -> Matrix R
        equation2R v ==
          ans:Matrix(R) := new(dim, #v, 0)
          for i in minRowIndex ans .. maxRowIndex ans repeat
            for j in minColIndex ans .. maxColIndex ans repeat
              qsetelt_!(ans, i, j, qelt(qelt(v, j), i))
          ans

        reducedSystem : Matrix(%) -> Matrix(R)
        reducedSystem(m:Matrix %):Matrix(R) ==
          empty? m => new(0, 0, 0)
          reduce(vertConcat, [equation2R row(m, i)
```

```
                 for i in minRowIndex m .. maxRowIndex m])$List(Matrix R)

        reducedSystem : (Matrix(%),Vector(%)) ->
          Record(mat: Matrix(R),vec: Vector(R))
        reducedSystem(m:Matrix %, v:Vector %):
          Record(mat:Matrix R, vec:Vector R) ==
            vh:Vector(R) :=
              empty? v => empty()
              rh := reducedSystem(v::Matrix %)@Matrix(R)
              column(rh, minColIndex rh)
            [reducedSystem(m)@Matrix(R), vh]

      if R has Finite then

        size : () -> NonNegativeInteger
        size == size$R ** dim

      if R has Field then

        ?/? : (%,R) -> %
        x / b       == x * inv b

        dimension : () -> CardinalNumber
        dimension() == dim::CardinalNumber
*)
```

———-

— **DIRPCAT.dotabb** —

```
"DIRPCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIRPCAT"];
"DIRPCAT" -> "IXAGG"
"DIRPCAT" -> "KOERCE"
"DIRPCAT" -> "FRETRCT"
"DIRPCAT" -> "BMODULE"
"DIRPCAT" -> "DIFEXT"
"DIRPCAT" -> "FLINEXP"
"DIRPCAT" -> "FINITE"
"DIRPCAT" -> "ORDRING"
"DIRPCAT" -> "OAMONS"
```

———-

— **DIRPCAT.dotfull** —

```
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIRPCAT"];
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "IndexedAggregate(a:SetCategory,b:Type)"
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "CoercibleTo(a:Type)"
```

```
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "FullyRetractableTo(a:Type)"
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "BiModule(a:Ring,b:Ring)"
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "DifferentialExtension(a:Ring)"
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "FullyLinearlyExplicitRingOver(a:Ring)"
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "Finite()"
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "OrderedRing()"
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "OrderedAbelianMonoidSup()"
```

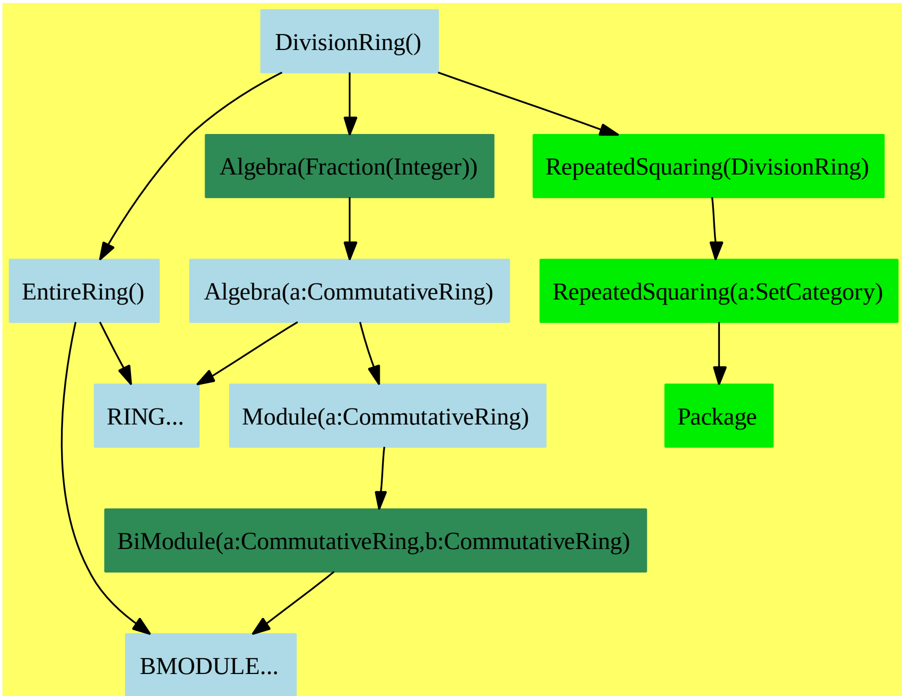
———-

— DIRPCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"DirectProductCategory(a:NonNegativeInteger,b:Type)" [color=lightblue];
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "BMODULE..."
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "KOERCE..."
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "DIFEXT..."
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "FINITE..."
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "FLINEXP..."
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "FRETRCT..."
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "IXAGG..."
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "OAMONS..."
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "ORDRING..."

"BMODULE..." [color=lightblue];
"KOERCE..." [color=lightblue];
"DIFEXT..." [color=lightblue];
"FINITE..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"FRETRCT..." [color=lightblue];
"IXAGG..." [color=lightblue];
"OAMONS..." [color=lightblue];
"ORDRING..." [color=lightblue];

}
```

———-

12.0.185 DivisionRing (DIVRING)



— **DivisionRing.input** —

```
)set break resume
)sys rm -f DivisionRing.output
)spool DivisionRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DivisionRing
--R
--R DivisionRing is a category constructor
--R Abbreviation for DivisionRing is DIVRING
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for DIVRING
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,Fraction(Integer)) -> %      ?*? : (Fraction(Integer),%) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,Integer) -> %               ?**? : (%,NonNegativeInteger) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?=? : (%,%) -> Boolean                1 : () -> %
--R 0 : () -> %                           ?^? : (%,Integer) -> %
```

```
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R coerce : Fraction(Integer) -> %       coerce : Integer -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R inv : % -> %                          latex : % -> String
--R one? : % -> Boolean                   recip : % -> Union(%,"failed")
--R sample : () -> %                      zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

— **DivisionRing.help** —

```
====================================================================
DivisionRing examples
====================================================================

A division ring (sometimes called a skew field), a not necessarily
commutative ring where all non-zero elements have multiplicative inverses.

See Also:
o )show DivisionRing
```

———-

See:

⇒ "Field" (FIELD) 16.0.209 on page 1413
⇐ "Algebra" (ALGEBRA) 11.0.176 on page 1121
⇐ "EntireRing" (ENTIRER) 10.0.163 on page 1029

Exports:

1	0	characteristic	coerce	hash
inv	latex	one?	recip	sample
subtractIfCan	zero?	?^?	?~=?	?**?
?*?	?+?	?-?	-?	?=?

Attributes exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **nil**

These are directly exported but not implemented:

```
inv : % -> %
```

These are implemented by this category:

```
?^? : (%,Integer) -> %
?**? : (%,Integer) -> %
```

These exports come from (p1029) EntireRing():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
```

These exports come from (p1121) Algebra(Fraction(Integer)):

```
coerce : Fraction Integer -> %
?*? : (%,Fraction Integer) -> %
?*? : (Fraction Integer,%) -> %
```

— DivisionRing.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#DIVRING">
DivisionRing (DIVRING)</a></h2>
</body>
```

— category DIVRING DivisionRing —

```
)abbrev category DIVRING DivisionRing
++ Description:
++ A division ring (sometimes called a skew field),
```

```
++ a not necessarily commutative ring where
++ all non-zero elements have multiplicative inverses.

DivisionRing() : Category == SIG where

  SIG ==> Join(EntireRing, Algebra Fraction Integer) with

    "**" : (%,Integer) -> %
      ++ x**n returns x raised to the integer power n.

    "^" : (%,Integer) -> %
      ++ x^n returns x raised to the integer power n.

    inv : % -> %
      ++ inv x returns the multiplicative inverse of x.
      ++ Error: if x is 0.

    -- Q-algebra is a lie, should be conditional on characteristic 0,
    -- but knownInfo cannot handle the following commented
    --    if % has CharacteristicZero then Algebra Fraction Integer

   add

      n: Integer
      x: %

      _^(x:%, n:Integer):% == x ** n

      import RepeatedSquaring(%)

      x ** n: Integer ==
         zero? n => 1
         zero? x =>
            n<0 => error "division by zero"
            x
         n<0 =>
            expt(inv x,(-n) pretend PositiveInteger)
         expt(x,n pretend PositiveInteger)

      q:Fraction(Integer) * x:% == numer(q) * inv(denom(q)::%) * x
```

———–

— COQ DIVRING —

```
(* category DIVRING *)
(*
      n: Integer
      x: %

      ?^? : (%,Integer) -> %
      _^(x:%, n:Integer):% == x ** n
```

```
      import RepeatedSquaring(%)

      ?**? : (%,Integer) -> %
      x ** n: Integer ==
         zero? n => 1
         zero? x =>
            n<0 => error "division by zero"
            x
         n<0 =>
            expt(inv x,(-n) pretend PositiveInteger)
         expt(x,n pretend PositiveInteger)

      ?*? : (Fraction(Integer),%) -> %
      q:Fraction(Integer) * x:% == numer(q) * inv(denom(q)::%) * x

*)
```

——–

— DIVRING.dotabb —

```
"DIVRING"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIVRING"];
"DIVRING" -> "ENTIRER"
"DIVRING" -> "ALGEBRA"
```

——–

— DIVRING.dotfull —

```
"DivisionRing()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIVRING"];
"DivisionRing()" -> "EntireRing()"
"DivisionRing()" -> "Algebra(Fraction(Integer))"
"DivisionRing()" -> "RepeatedSquaring(DivisionRing)"
```

——–

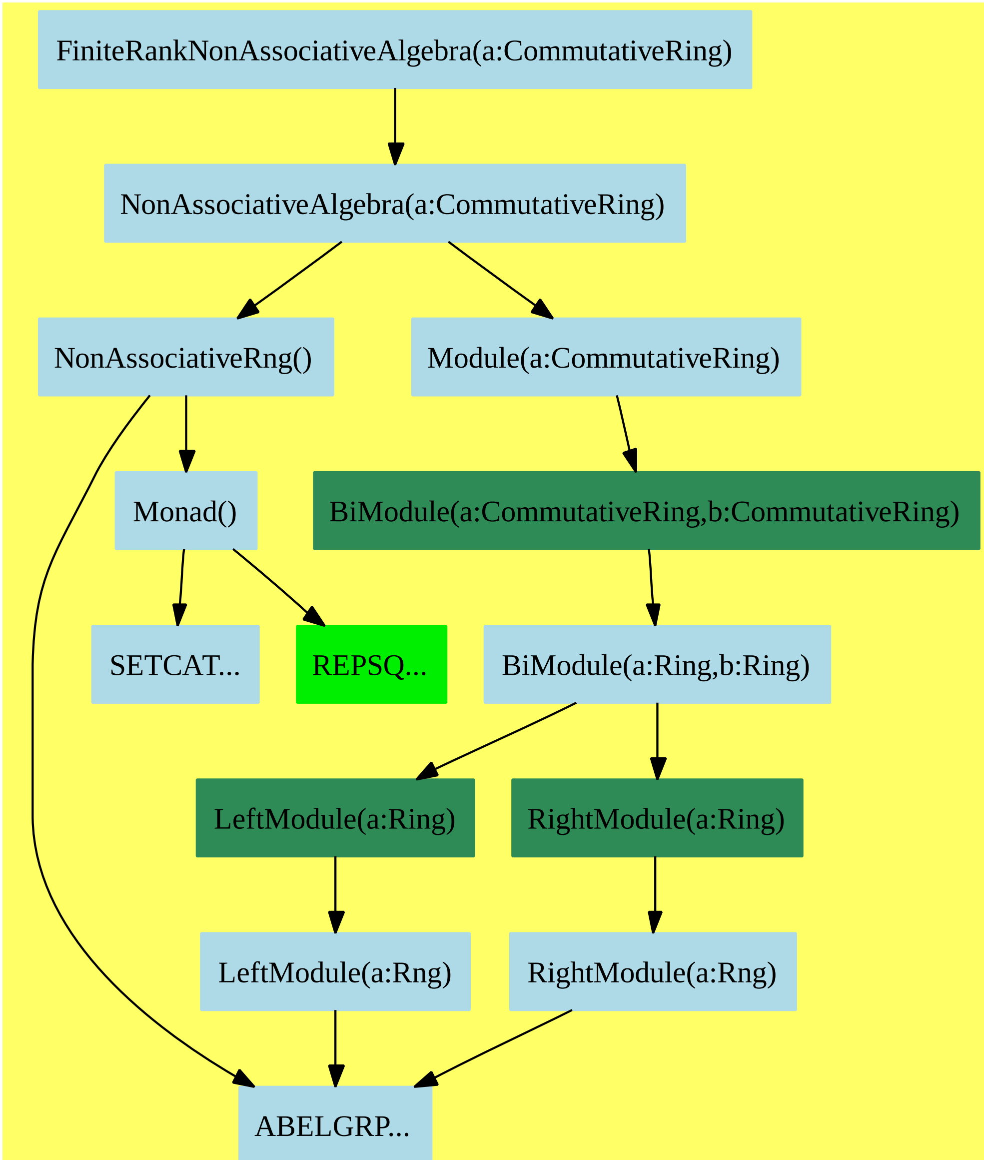
— DIVRING.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"DivisionRing()" [color=lightblue];
"DivisionRing()" -> "EntireRing()"
"DivisionRing()" -> "Algebra(Fraction(Integer))"
"DivisionRing()" -> "RepeatedSquaring(DivisionRing)"

"RepeatedSquaring(DivisionRing)" [color="#00EE00"];
```

```
"RepeatedSquaring(DivisionRing)" -> "RepeatedSquaring(a:SetCategory)"

"RepeatedSquaring(a:SetCategory)" [color="#00EE00"];
"RepeatedSquaring(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(Fraction(Integer))" [color=seagreen];
"Algebra(Fraction(Integer))" -> "Algebra(a:CommutativeRing)"

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BMODULE..."

"RING..." [color=lightblue];
"BMODULE..." [color=lightblue];
}
```

———-

12.0.186 FiniteRankNonAssociativeAlgebra (FINAALG)



— **FiniteRankNonAssociativeAlgebra.input** —

```
)set break resume
)sys rm -f FiniteRankNonAssociativeAlgebra.output
)spool FiniteRankNonAssociativeAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
```

```
)show FiniteRankNonAssociativeAlgebra
--R
--R FiniteRankNonAssociativeAlgebra(R: CommutativeRing) is a category constructor
--R Abbreviation for FiniteRankNonAssociativeAlgebra is FINAALG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FINAALG
--R
--R------------------------------ Operations -------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?=? : (%,%) -> Boolean                0 : () -> %
--R alternative? : () -> Boolean          antiAssociative? : () -> Boolean
--R antiCommutative? : () -> Boolean      antiCommutator : (%,%) -> %
--R associative? : () -> Boolean          associator : (%,%,%) -> %
--R coerce : % -> OutputForm              commutative? : () -> Boolean
--R commutator : (%,%) -> %               flexible? : () -> Boolean
--R hash : % -> SingleInteger             jacobiIdentity? : () -> Boolean
--R jordanAdmissible? : () -> Boolean     jordanAlgebra? : () -> Boolean
--R latex : % -> String                   leftAlternative? : () -> Boolean
--R leftDiscriminant : Vector(%) -> R     leftNorm : % -> R
--R leftTrace : % -> R                    lieAdmissible? : () -> Boolean
--R lieAlgebra? : () -> Boolean           powerAssociative? : () -> Boolean
--R rank : () -> PositiveInteger          rightAlternative? : () -> Boolean
--R rightDiscriminant : Vector(%) -> R    rightNorm : % -> R
--R rightTrace : % -> R                   sample : () -> %
--R someBasis : () -> Vector(%)           zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R associatorDependence : () -> List(Vector(R)) if R has INTDOM
--R conditionsForIdempotents : Vector(%) -> List(Polynomial(R))
--R coordinates : (Vector(%),Vector(%)) -> Matrix(R)
--R coordinates : (%,Vector(%)) -> Vector(R)
--R leftCharacteristicPolynomial : % -> SparseUnivariatePolynomial(R)
--R leftMinimalPolynomial : % -> SparseUnivariatePolynomial(R) if R has INTDOM
--R leftPower : (%,PositiveInteger) -> %
--R leftRecip : % -> Union(%,"failed") if R has INTDOM
--R leftRegularRepresentation : (%,Vector(%)) -> Matrix(R)
--R leftTraceMatrix : Vector(%) -> Matrix(R)
--R leftUnit : () -> Union(%,"failed") if R has INTDOM
--R leftUnits : () -> Union(Record(particular: %,basis: List(%)),"failed") if R has INTDOM
--R noncommutativeJordanAlgebra? : () -> Boolean
--R plenaryPower : (%,PositiveInteger) -> %
--R recip : % -> Union(%,"failed") if R has INTDOM
--R represents : (Vector(R),Vector(%)) -> %
--R rightCharacteristicPolynomial : % -> SparseUnivariatePolynomial(R)
--R rightMinimalPolynomial : % -> SparseUnivariatePolynomial(R) if R has INTDOM
--R rightPower : (%,PositiveInteger) -> %
--R rightRecip : % -> Union(%,"failed") if R has INTDOM
--R rightRegularRepresentation : (%,Vector(%)) -> Matrix(R)
--R rightTraceMatrix : Vector(%) -> Matrix(R)
--R rightUnit : () -> Union(%,"failed") if R has INTDOM
--R rightUnits : () -> Union(Record(particular: %,basis: List(%)),"failed") if R has INTDOM
```

```
--R structuralConstants : Vector(%) -> Vector(Matrix(R))
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unit : () -> Union(%,"failed") if R has INTDOM
--R
--E 1

)spool
)lisp (bye)
```

———-

— **FiniteRankNonAssociativeAlgebra.help** —

```
====================================================================
FiniteRankNonAssociativeAlgebra examples
====================================================================

A FiniteRankNonAssociativeAlgebra is a non associative algebra over
a commutative ring R which is a free R-module of finite rank.

See Also:
o )show FiniteRankNonAssociativeAlgebra
```

———-

See:

⇒ "FramedNonAssociativeAlgebra" (FRNAALG) 13.0.196 on page 1300
⇐ "NonAssociativeAlgebra" (NAALG) 11.0.181 on page 1154

Exports:

0	alternative?
antiAssociative?	antiCommutative?
antiCommutator	associative?
associator	associatorDependence
coerce	commutative?
commutator	conditionsForIdempotents
coordinates	flexible?
hash	jacobiIdentity?
jordanAdmissible?	jordanAlgebra?
latex	leftAlternative?
leftCharacteristicPolynomial	leftDiscriminant
leftMinimalPolynomial	leftNorm
leftPower	leftRecip
leftRegularRepresentation	leftTrace
leftTraceMatrix	leftUnit
leftUnits	lieAdmissible?
lieAlgebra?	noncommutativeJordanAlgebra?
plenaryPower	powerAssociative?
rank	recip
represents	rightAlternative?
rightCharacteristicPolynomial	rightDiscriminant
rightMinimalPolynomial	rightNorm
rightPower	rightRecip
rightRegularRepresentation	rightTrace
rightTraceMatrix	rightUnit
rightUnits	sample
someBasis	structuralConstants
subtractIfCan	unit
zero?	?*?
?**?	?+?
?-?	-?
?=?	?~=?

Attributes Exported:

- if R has IntegralDomain then unitsKnown where **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
conditionsForIdempotents : Vector % -> List Polynomial R
coordinates : (%,Vector %) -> Vector R
leftUnit : () -> Union(%,"failed") if R has INTDOM
leftUnits : () -> Union(Record(particular: %,basis: List %),"failed")
    if R has INTDOM
powerAssociative? : () -> Boolean
rank : () -> PositiveInteger
rightUnit : () -> Union(%,"failed") if R has INTDOM
rightUnits : () -> Union(Record(particular: %,basis: List %),"failed")
```

```
    if R has INTDOM
someBasis : () -> Vector %
unit : () -> Union(%,"failed") if R has INTDOM
```

These are implemented by this category:

```
alternative? : () -> Boolean
antiAssociative? : () -> Boolean
antiCommutative? : () -> Boolean
associative? : () -> Boolean
associatorDependence : () -> List Vector R if R has INTDOM
commutative? : () -> Boolean
coordinates : (Vector %,Vector %) -> Matrix R
flexible? : () -> Boolean
jacobiIdentity? : () -> Boolean
jordanAdmissible? : () -> Boolean
jordanAlgebra? : () -> Boolean
leftAlternative? : () -> Boolean
leftCharacteristicPolynomial : % -> SparseUnivariatePolynomial R
leftDiscriminant : Vector % -> R
leftMinimalPolynomial : % -> SparseUnivariatePolynomial R if R has INTDOM
leftNorm : % -> R
leftRecip : % -> Union(%,"failed") if R has INTDOM
leftRegularRepresentation : (%,Vector %) -> Matrix R
leftTrace : % -> R
leftTraceMatrix : Vector % -> Matrix R
lieAdmissible? : () -> Boolean
lieAlgebra? : () -> Boolean
noncommutativeJordanAlgebra? : () -> Boolean
recip : % -> Union(%,"failed") if R has INTDOM
represents : (Vector R,Vector %) -> %
rightAlternative? : () -> Boolean
rightCharacteristicPolynomial : % -> SparseUnivariatePolynomial R
rightDiscriminant : Vector % -> R
rightMinimalPolynomial : % -> SparseUnivariatePolynomial R if R has INTDOM
rightNorm : % -> R
rightRecip : % -> Union(%,"failed") if R has INTDOM
rightRegularRepresentation : (%,Vector %) -> Matrix R
rightTrace : % -> R
rightTraceMatrix : Vector % -> Matrix R
structuralConstants : Vector % -> Vector Matrix R
```

These exports come from (p1154) NonAssociativeAlgebra(R:CommutativeRing):

```
0 : () -> %
antiCommutator : (%,%) -> %
associator : (%,%,%) -> %
coerce : % -> OutputForm
commutator : (%,%) -> %
hash : % -> SingleInteger
latex : % -> String
leftPower : (%,PositiveInteger) -> %
plenaryPower : (%,PositiveInteger) -> %
rightPower : (%,PositiveInteger) -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
```

```
 zero? : % -> Boolean
 ?~=? : (%,%) -> Boolean
 ?*? : (PositiveInteger,%) -> %
 ?*? : (%,%) -> %
 ?*? : (Integer,%) -> %
 ?*? : (NonNegativeInteger,%) -> %
 ?*? : (R,%) -> %
 ?*? : (%,R) -> %
 ?+? : (%,%) -> %
 ?=? : (%,%) -> Boolean
 ?-? : (%,%) -> %
 -? : % -> %
 ?**? : (%,PositiveInteger) -> %
```

— **FiniteRankNonAssociativeAlgebra.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FINAALG">
FiniteRankNonAssociativeAlgebra (FINAALG)</a></h2>
</body>
```

———-

— **category FINAALG FiniteRankNonAssociativeAlgebra** —

```
)abbrev category FINAALG FiniteRankNonAssociativeAlgebra
++ Author: J. Grabmeier, R. Wisbauer
++ Date Created: 01 March 1991
++ Date Last Updated: 12 June 1991
++ References:
++   R.D. Schafer: An Introduction to Nonassociative Algebras
++   Academic Press, New York, 1966
++
++   R. Wisbauer: Bimodule Structure of Algebra
++   Lecture Notes Univ. Duesseldorf 1991
++ Description:
++ A FiniteRankNonAssociativeAlgebra is a non associative algebra over
++ a commutative ring R which is a free \spad{R}-module of finite rank.

FiniteRankNonAssociativeAlgebra(R) : Category == SIG where
  R : CommutativeRing

  SIG ==> NonAssociativeAlgebra R with

    someBasis : () -> Vector %
      ++ someBasis() returns some \spad{R}-module basis.

    rank : () -> PositiveInteger
      ++ rank() returns the rank of the algebra as \spad{R}-module.

    conditionsForIdempotents : Vector % -> List Polynomial R
      ++ conditionsForIdempotents([v1,...,vn]) determines a complete list
      ++ of polynomial equations for the coefficients of idempotents
```

```
  ++ with respect to the \spad{R}-module basis \spad{v1},...,\spad{vn}.

structuralConstants : Vector % -> Vector Matrix R
  ++ structuralConstants([v1,v2,...,vm]) calculates the structural
  ++ constants \spad{[(gammaijk) for k in 1..m]} defined by
  ++ \spad{vi * vj = gammaij1 * v1 + ... + gammaijm * vm},
  ++ where \spad{[v1,...,vm]} is an \spad{R}-module basis
  ++ of a subalgebra.

leftRegularRepresentation : (% , Vector %) -> Matrix R
  ++ leftRegularRepresentation(a,[v1,...,vn]) returns the matrix of
  ++ the linear map defined by left multiplication by \spad{a}
  ++ with respect to the \spad{R}-module basis \spad{[v1,...,vn]}.

rightRegularRepresentation : (% , Vector %) -> Matrix R
  ++ rightRegularRepresentation(a,[v1,...,vn]) returns the matrix of
  ++ the linear map defined by right multiplication by \spad{a}
  ++ with respect to the \spad{R}-module basis \spad{[v1,...,vn]}.

leftTrace : %  -> R
  ++ leftTrace(a) returns the trace of the left regular representation
  ++ of \spad{a}.

rightTrace : %  -> R
  ++ rightTrace(a) returns the trace of the right regular representation
  ++ of \spad{a}.

leftNorm : %  -> R
  ++ leftNorm(a) returns the determinant of the left regular
  ++ representation of \spad{a}.

rightNorm : %  -> R
  ++ rightNorm(a) returns the determinant of the right regular
  ++ representation of \spad{a}.

coordinates : (%, Vector %) -> Vector R
  ++ coordinates(a,[v1,...,vn]) returns the coordinates of \spad{a}
  ++ with respect to the \spad{R}-module basis \spad{v1},...,\spad{vn}.

coordinates : (Vector %, Vector %) -> Matrix R
  ++ coordinates([a1,...,am],[v1,...,vn]) returns a matrix whose
  ++ i-th row is formed by the coordinates of \spad{ai}
  ++ with respect to the \spad{R}-module basis \spad{v1},...,\spad{vn}.

represents : (Vector R, Vector %) -> %
  ++ represents([a1,...,am],[v1,...,vm]) returns the linear
  ++ combination \spad{a1*vm + ... + an*vm}.

leftDiscriminant : Vector % -> R
  ++ leftDiscriminant([v1,...,vn]) returns  the determinant of the
  ++ \spad{n}-by-\spad{n} matrix whose element at the \spad{i}-th row
  ++ and \spad{j}-th column is given by the left trace of the product
  ++ \spad{vi*vj}. Note that this is the same as
  ++ \spad{determinant(leftTraceMatrix([v1,...,vn]))}.
```

```
rightDiscriminant : Vector % -> R
  ++ rightDiscriminant([v1,...,vn]) returns  the determinant of the
  ++ \spad{n}-by-\spad{n} matrix whose element at the \spad{i}-th row
  ++ and \spad{j}-th column is given by the right trace of the product
  ++ \spad{vi*vj}. Note that this is the same as
  ++ \spad{determinant(rightTraceMatrix([v1,...,vn]))}.

leftTraceMatrix : Vector % -> Matrix R
  ++ leftTraceMatrix([v1,...,vn]) is the \spad{n}-by-\spad{n} matrix
  ++ whose element at the \spad{i}-th row and \spad{j}-th column is given
  ++ by the left trace of the product \spad{vi*vj}.

rightTraceMatrix : Vector % -> Matrix R
  ++ rightTraceMatrix([v1,...,vn]) is the \spad{n}-by-\spad{n} matrix
  ++ whose element at the \spad{i}-th row and \spad{j}-th column is given
  ++ by the right trace of the product \spad{vi*vj}.

leftCharacteristicPolynomial : % -> SparseUnivariatePolynomial R
  ++ leftCharacteristicPolynomial(a) returns the characteristic
  ++ polynomial of the left regular representation of \spad{a}
  ++ with respect to any basis.

rightCharacteristicPolynomial : % -> SparseUnivariatePolynomial R
  ++ rightCharacteristicPolynomial(a) returns the characteristic
  ++ polynomial of the right regular representation of \spad{a}
  ++ with respect to any basis.

--we not necessarily have a unit
--if R has CharacteristicZero then CharacteristicZero
--if R has CharacteristicNonZero then CharacteristicNonZero

commutative? : ()-> Boolean
  ++ commutative?() tests if multiplication in the algebra
  ++ is commutative.

antiCommutative? : ()-> Boolean
  ++ antiCommutative?() tests if \spad{a*a = 0}
  ++ for all \spad{a} in the algebra.
  ++ Note that this implies \spad{a*b + b*a = 0} for all
  ++ \spad{a} and \spad{b}.

associative? : ()-> Boolean
  ++ associative?() tests if multiplication in algebra
  ++ is associative.

antiAssociative? : ()-> Boolean
  ++ antiAssociative?() tests if multiplication in algebra
  ++ is anti-associative, that is, \spad{(a*b)*c + a*(b*c) = 0}
  ++ for all \spad{a},b,c in the algebra.

leftAlternative? : ()-> Boolean
  ++ leftAlternative?() tests if \spad{2*associator(a,a,b) = 0}
  ++ for all \spad{a}, b in the algebra.
```

```
  ++ Note that we only can test this; in general we don't know
  ++ whether \spad{2*a=0} implies \spad{a=0}.

rightAlternative? : ()-> Boolean
  ++ rightAlternative?() tests if \spad{2*associator(a,b,b) = 0}
  ++ for all \spad{a}, b in the algebra.
  ++ Note that we only can test this; in general we don't know
  ++ whether \spad{2*a=0} implies \spad{a=0}.

flexible? : ()->  Boolean
  ++ flexible?() tests if \spad{2*associator(a,b,a) = 0}
  ++ for all \spad{a}, b in the algebra.
  ++ Note that we only can test this; in general we don't know
  ++ whether \spad{2*a=0} implies \spad{a=0}.

alternative? : ()-> Boolean
  ++ alternative?() tests if
  ++ \spad{2*associator(a,a,b) = 0 = 2*associator(a,b,b)}
  ++ for all \spad{a}, b in the algebra.
  ++ Note that we only can test this; in general we don't know
  ++ whether \spad{2*a=0} implies \spad{a=0}.

powerAssociative? : ()-> Boolean
  ++ powerAssociative?() tests if all subalgebras
  ++ generated by a single element are associative.

jacobiIdentity? : () -> Boolean
  ++ jacobiIdentity?() tests if \spad{(a*b)*c + (b*c)*a + (c*a)*b = 0}
  ++ for all \spad{a},b,c in the algebra. For example, this holds
  ++ for crossed products of 3-dimensional vectors.

lieAdmissible? : () -> Boolean
  ++ lieAdmissible?() tests if the algebra defined by the commutators
  ++ is a Lie algebra, that is, satisfies the Jacobi identity.
  ++ The property of anticommutativity follows from definition.

jordanAdmissible? : () -> Boolean
  ++ jordanAdmissible?() tests if 2 is invertible in the
  ++ coefficient domain and the multiplication defined by
  ++ \spad{(1/2)(a*b+b*a)} determines a
  ++ Jordan algebra, that is, satisfies the Jordan identity.
  ++ The property of \spadatt{commutative("*")}
  ++ follows from by definition.

noncommutativeJordanAlgebra? : () -> Boolean
  ++ noncommutativeJordanAlgebra?() tests if the algebra
  ++ is flexible and Jordan admissible.

jordanAlgebra? : () -> Boolean
  ++ jordanAlgebra?() tests if the algebra is commutative,
  ++ characteristic is not 2, and \spad{(a*b)*a**2 - a*(b*a**2) = 0}
  ++ for all \spad{a},b,c in the algebra (Jordan identity).
  ++ Example:
  ++ for every associative algebra \spad{(A,+,@)} we can construct a
```

```
    ++ Jordan algebra \spad{(A,+,*)}, where \spad{a*b := (a@b+b@a)/2}.

  lieAlgebra? : () -> Boolean
    ++ lieAlgebra?() tests if the algebra is anticommutative
    ++ and \spad{(a*b)*c + (b*c)*a + (c*a)*b = 0}
    ++ for all \spad{a},b,c in the algebra (Jacobi identity).
    ++ Example:
    ++ for every associative algebra \spad{(A,+,@)} we can construct a
    ++ Lie algebra \spad{(A,+,*)}, where \spad{a*b := a@b-b@a}.

  if R has IntegralDomain then

    -- we not neccessarily have a unit, hence we don't inherit
    -- the next 3 functions and hence copy them from MonadWithUnit:

    recip : % -> Union(%,"failed")
      ++ recip(a) returns an element, which is both a left and a right
      ++ inverse of \spad{a},
      ++ or \spad{"failed"} if there is no unit element, if such an
      ++ element doesn't exist or cannot be determined (see unitsKnown).

    leftRecip : % -> Union(%,"failed")
      ++ leftRecip(a) returns an element, which is a left inverse of
      ++ \spad{a}, or \spad{"failed"} if there is no unit element, if such
      ++ an element doesn't exist or cannot be determined (see unitsKnown).

    rightRecip : % -> Union(%,"failed")
      ++ rightRecip(a) returns an element, which is a right inverse of
      ++ \spad{a},
      ++ or \spad{"failed"} if there is no unit element, if such an
      ++ element doesn't exist or cannot be determined (see unitsKnown).

    associatorDependence : () -> List Vector R
      ++ associatorDependence() looks for the associator identities, that
      ++ is, finds a basis of the solutions of the linear combinations of the
      ++ six permutations of \spad{associator(a,b,c)} which yield 0,
      ++ for all \spad{a},b,c in the algebra.
      ++ The order of the permutations is \spad{123 231 312 132 321 213}.

    leftMinimalPolynomial : % -> SparseUnivariatePolynomial R
      ++ leftMinimalPolynomial(a) returns the polynomial determined by the
      ++ smallest non-trivial linear combination of left powers of
      ++ \spad{a}. Note that the polynomial never has a constant term as in
      ++ general the algebra has no unit.

    rightMinimalPolynomial : % -> SparseUnivariatePolynomial R
      ++ rightMinimalPolynomial(a) returns the polynomial determined by the
      ++ smallest non-trivial linear
      ++ combination of right powers of \spad{a}.
      ++ Note that the polynomial never has a constant term as in general
      ++ the algebra has no unit.

    leftUnits : () -> Union(Record(particular: %, basis: List %), "failed")
      ++ leftUnits() returns the affine space of all left units of the
```

```
      ++ algebra, or \spad{"failed"} if there is none.

    rightUnits : () -> Union(Record(particular: %, basis: List %), "failed")
      ++ rightUnits() returns the affine space of all right units of the
      ++ algebra, or \spad{"failed"} if there is none.

    leftUnit : () -> Union(%, "failed")
      ++ leftUnit() returns a left unit of the algebra
      ++ (not necessarily unique), or \spad{"failed"} if there is none.

    rightUnit : () -> Union(%, "failed")
      ++ rightUnit() returns a right unit of the algebra
      ++ (not necessarily unique), or \spad{"failed"} if there is none.

    unit : () -> Union(%, "failed")
      ++ unit() returns a unit of the algebra (necessarily unique),
      ++ or \spad{"failed"} if there is none.

    -- we not necessarily have a unit, hence we can't say anything
    -- about characteristic
    -- if R has CharacteristicZero then CharacteristicZero
    -- if R has CharacteristicNonZero then CharacteristicNonZero

    unitsKnown
      ++ unitsKnown means that \spadfun{recip} truly yields reciprocal
      ++ or \spad{"failed"} if not a unit,
      ++ similarly for \spadfun{leftRecip} and
      ++ \spadfun{rightRecip}. The reason is that we use left, respectively
      ++ right, minimal polynomials to decide this question.
 add

    --n := rank()
    --b := someBasis()
    --gamma : Vector Matrix R := structuralConstants b
    -- here is a problem: there seems to be a problem having local
    -- variables in the capsule of a category, furthermore
    -- see the commented code of conditionsForIdempotents, where
    -- we call structuralConstants, which also doesn't work
    -- at runtime, is not properly inherited, hence for
    -- the moment we put the code for
    -- conditionsForIdempotents, structuralConstants, unit, leftUnit,
    -- rightUnit into the domain constructor ALGSC
    V  ==> Vector
    M  ==> Matrix
    REC  ==> Record(particular: Union(V R,"failed"),basis: List V R)
    LSMP ==> LinearSystemMatrixPackage(R,V R,V R, M R)


    SUP ==>  SparseUnivariatePolynomial
    NNI ==>  NonNegativeInteger
    -- next 2 functions: use a general characteristicPolynomial

    leftCharacteristicPolynomial a ==
       n := rank()$%
```

```
     ma : Matrix R := leftRegularRepresentation(a,someBasis()$%)
     mb : Matrix SUP R := zero(n,n)
     for i in 1..n repeat
       for j in 1..n repeat
         mb(i,j):=
           i=j => monomial(ma(i,j),0)$SUP(R) - monomial(1,1)$SUP(R)
           monomial(ma(i,j),1)$SUP(R)
     determinant mb

  rightCharacteristicPolynomial a ==
     n := rank()$%
     ma : Matrix R := rightRegularRepresentation(a,someBasis()$%)
     mb : Matrix SUP R := zero(n,n)
     for i in 1..n repeat
       for j in 1..n repeat
         mb(i,j):=
           i=j => monomial(ma(i,j),0)$SUP(R) - monomial(1,1)$SUP(R)
           monomial(ma(i,j),1)$SUP(R)
     determinant mb

  leftTrace a ==
    t : R := 0
    ma : Matrix R := leftRegularRepresentation(a,someBasis()$%)
    for i in 1..rank()$% repeat
      t := t + elt(ma,i,i)
    t

  rightTrace a ==
    t : R := 0
    ma : Matrix R := rightRegularRepresentation(a,someBasis()$%)
    for i in 1..rank()$% repeat
      t := t + elt(ma,i,i)
    t

  leftNorm a == determinant leftRegularRepresentation(a,someBasis()$%)

  rightNorm a == determinant rightRegularRepresentation(a,someBasis()$%)

  antiAssociative?() ==
    b := someBasis()
    n := rank()
    for i in 1..n repeat
      for j in 1..n repeat
        for k in 1..n repeat
          not zero? ( (b.i*b.j)*b.k + b.i*(b.j*b.k) )  =>
            messagePrint("algebra is not anti-associative")$OutputForm
            return false
    messagePrint("algebra is anti-associative")$OutputForm
    true

  jordanAdmissible?() ==
    b := someBasis()
    n := rank()
    recip(2 * 1$R) case "failed" =>
```

```
    messagePrint("this algebra is not Jordan admissible, " _
     "as 2 is not invertible in the ground ring")$OutputForm
    false
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
     for l in 1..n repeat
       not zero? ( _
         antiCommutator(antiCommutator(b.i,b.j),_
                        antiCommutator(b.l,b.k)) + _
         antiCommutator(antiCommutator(b.l,b.j),_
                        antiCommutator(b.k,b.i)) + _
         antiCommutator(antiCommutator(b.k,b.j),_
                        antiCommutator(b.i,b.l))   _
                  ) =>
          messagePrint(_
                    "this algebra is not Jordan admissible")$OutputForm
          return false
  messagePrint("this algebra is Jordan admissible")$OutputForm
  true

lieAdmissible?() ==
  n := rank()
  b := someBasis()
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
      not zero? (commutator(commutator(b.i,b.j),b.k) _
              + commutator(commutator(b.j,b.k),b.i) _
              + commutator(commutator(b.k,b.i),b.j))   =>
        messagePrint("this algebra is not Lie admissible")$OutputForm
        return false
  messagePrint("this algebra is Lie admissible")$OutputForm
  true

structuralConstants b ==
  --n := rank()
  -- be careful with the possibility that b is not a basis
  m : NonNegativeInteger := (maxIndex b) :: NonNegativeInteger
  sC : Vector Matrix R := [new(m,m,0$R) for k in 1..m]
  for i in 1..m repeat
    for j in 1..m repeat
      covec : Vector R := coordinates(b.i * b.j, b)
      for k in 1..m repeat
         setelt( sC.k, i, j, covec.k )
  sC

if R has IntegralDomain then

  leftRecip x ==
    zero? x => "failed"
    lu := leftUnit()
    lu case "failed" => "failed"
    b := someBasis()
```

```
    xx : % := (lu :: %)
    k  : PositiveInteger := 1
    cond : Matrix R := coordinates(xx,b) :: Matrix(R)
    listOfPowers : List % := [xx]
    while rank(cond) = k repeat
      k := k+1
      xx := xx*x
      listOfPowers := cons(xx,listOfPowers)
      cond := horizConcat(cond, coordinates(xx,b) :: Matrix(R) )
    vectorOfCoef : Vector R := (nullSpace(cond)$Matrix(R)).first
    invC := recip vectorOfCoef.1
    invC case "failed" => "failed"
    invCR : R :=  - (invC :: R)
    reduce(_+,[(invCR*vectorOfCoef.i)*power for i in _
     2..maxIndex vectorOfCoef for power in reverse listOfPowers])

  rightRecip x ==
    zero? x => "failed"
    ru := rightUnit()
    ru case "failed" => "failed"
    b := someBasis()
    xx : % := (ru :: %)
    k  : PositiveInteger := 1
    cond : Matrix R := coordinates(xx,b) :: Matrix(R)
    listOfPowers : List % := [xx]
    while rank(cond) = k repeat
      k := k+1
      xx := x*xx
      listOfPowers := cons(xx,listOfPowers)
      cond := horizConcat(cond, coordinates(xx,b) :: Matrix(R) )
    vectorOfCoef : Vector R := (nullSpace(cond)$Matrix(R)).first
    invC := recip vectorOfCoef.1
    invC case "failed" => "failed"
    invCR : R :=  - (invC :: R)
    reduce(_+,[(invCR*vectorOfCoef.i)*power for i in _
     2..maxIndex vectorOfCoef for power in reverse listOfPowers])

  recip x ==
    lrx := leftRecip x
    lrx case "failed" => "failed"
    rrx := rightRecip x
    rrx case "failed" => "failed"
    (lrx :: %) ^= (rrx :: %)  => "failed"
    lrx :: %

  leftMinimalPolynomial x ==
    zero? x =>  monomial(1$R,1)$(SparseUnivariatePolynomial R)
    b := someBasis()
    xx : % := x
    k  : PositiveInteger := 1
    cond : Matrix R := coordinates(xx,b) :: Matrix(R)
    while rank(cond) = k repeat
      k := k+1
      xx := x*xx
```

```
      cond := horizConcat(cond, coordinates(xx,b) :: Matrix(R) )
    vectorOfCoef : Vector R := (nullSpace(cond)$Matrix(R)).first
    res : SparseUnivariatePolynomial R := 0
    for i in 1..k repeat
      res:=res+monomial(vectorOfCoef.i,i)$(SparseUnivariatePolynomial R)
    res

  rightMinimalPolynomial x ==
    zero? x =>  monomial(1$R,1)$(SparseUnivariatePolynomial R)
    b := someBasis()
    xx : % := x
    k  : PositiveInteger := 1
    cond : Matrix R := coordinates(xx,b) :: Matrix(R)
    while rank(cond) = k repeat
      k := k+1
      xx := xx*x
      cond := horizConcat(cond, coordinates(xx,b) :: Matrix(R) )
    vectorOfCoef : Vector R := (nullSpace(cond)$Matrix(R)).first
    res : SparseUnivariatePolynomial R := 0
    for i in 1..k repeat
      res:=res+monomial(vectorOfCoef.i,i)$(SparseUnivariatePolynomial R)
    res

  associatorDependence() ==
    n := rank()
    b := someBasis()
    cond : Matrix(R) := new(n**4,6,0$R)$Matrix(R)
    z : Integer := 0
    for i in 1..n repeat
     for j in 1..n repeat
      for k in 1..n repeat
       a123 : Vector R := coordinates(associator(b.i,b.j,b.k),b)
       a231 : Vector R := coordinates(associator(b.j,b.k,b.i),b)
       a312 : Vector R := coordinates(associator(b.k,b.i,b.j),b)
       a132 : Vector R := coordinates(associator(b.i,b.k,b.j),b)
       a321 : Vector R := coordinates(associator(b.k,b.j,b.i),b)
       a213 : Vector R := coordinates(associator(b.j,b.i,b.k),b)
       for r in 1..n repeat
        z:= z+1
        setelt(cond,z,1,elt(a123,r))
        setelt(cond,z,2,elt(a231,r))
        setelt(cond,z,3,elt(a312,r))
        setelt(cond,z,4,elt(a132,r))
        setelt(cond,z,5,elt(a321,r))
        setelt(cond,z,6,elt(a213,r))
    nullSpace(cond)

jacobiIdentity?()  ==
  n := rank()
  b := someBasis()
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
      not zero? ((b.i*b.j)*b.k + (b.j*b.k)*b.i + (b.k*b.i)*b.j) =>
```

```
          messagePrint("Jacobi identity does not hold")$OutputForm
          return false
    messagePrint("Jacobi identity holds")$OutputForm
    true

  lieAlgebra?()  ==
    not antiCommutative?() =>
      messagePrint("this is not a Lie algebra")$OutputForm
      false
    not jacobiIdentity?() =>
      messagePrint("this is not a Lie algebra")$OutputForm
      false
    messagePrint("this is a Lie algebra")$OutputForm
    true

  jordanAlgebra?()  ==
    b := someBasis()
    n := rank()
    recip(2 * 1$R) case "failed" =>
      messagePrint("this is not a Jordan algebra, as 2 is not " _
       "invertible in the ground ring")$OutputForm
      false
    not commutative?() =>
      messagePrint("this is not a Jordan algebra")$OutputForm
      false
    for i in 1..n repeat
     for j in 1..n repeat
      for k in 1..n repeat
       for l in 1..n repeat
         not zero? (associator(b.i,b.j,b.l*b.k)+_
             associator(b.l,b.j,b.k*b.i)+associator(b.k,b.j,b.i*b.l)) =>
          messagePrint("not a Jordan algebra")$OutputForm
          return false
    messagePrint("this is a Jordan algebra")$OutputForm
    true

  noncommutativeJordanAlgebra?() ==
    b := someBasis()
    n := rank()
    recip(2 * 1$R) case "failed" =>
     messagePrint("this is not a noncommutative Jordan algebra,_
as 2 is not invertible in the ground ring")$OutputForm
     false
    not flexible?()$% =>
     messagePrint("this is not a noncommutative Jordan algebra,_
as it is not flexible")$OutputForm
     false
    not jordanAdmissible?()$% =>
     messagePrint("this is not a noncommutative Jordan algebra,_
as it is not Jordan admissible")$OutputForm
     false
    messagePrint("this is a noncommutative Jordan algebra")$OutputForm
    true
```

```
antiCommutative?() ==
  b := someBasis()
  n := rank()
  for i in 1..n repeat
    for j in i..n repeat
      not zero? (i=j => b.i*b.i; b.i*b.j + b.j*b.i) =>
        messagePrint("algebra is not anti-commutative")$OutputForm
        return false
  messagePrint("algebra is anti-commutative")$OutputForm
  true

commutative?() ==
  b := someBasis()
  n := rank()
  for i in 1..n repeat
   for j in i+1..n repeat
     not zero? commutator(b.i,b.j) =>
       messagePrint("algebra is not commutative")$OutputForm
       return false
  messagePrint("algebra is commutative")$OutputForm
  true

associative?() ==
  b := someBasis()
  n := rank()
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
     not zero? associator(b.i,b.j,b.k) =>
       messagePrint("algebra is not associative")$OutputForm
       return false
  messagePrint("algebra is associative")$OutputForm
  true

leftAlternative?() ==
  b := someBasis()
  n := rank()
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
     not zero? (associator(b.i,b.j,b.k) + associator(b.j,b.i,b.k)) =>
       messagePrint("algebra is not left alternative")$OutputForm
       return false
  messagePrint("algebra satisfies 2*associator(a,a,b) = 0")$OutputForm
  true

rightAlternative?() ==
  b := someBasis()
  n := rank()
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
     not zero? (associator(b.i,b.j,b.k) + associator(b.i,b.k,b.j)) =>
       messagePrint("algebra is not right alternative")$OutputForm
```

```
        return false
    messagePrint("algebra satisfies 2*associator(a,b,b) = 0")$OutputForm
    true

  flexible?() ==
    b := someBasis()
    n := rank()
    for i in 1..n repeat
     for j in 1..n repeat
      for k in 1..n repeat
       not zero? (associator(b.i,b.j,b.k) + associator(b.k,b.j,b.i)) =>
         messagePrint("algebra is not flexible")$OutputForm
         return false
    messagePrint("algebra satisfies 2*associator(a,b,a) = 0")$OutputForm
    true

  alternative?() ==
    b := someBasis()
    n := rank()
    for i in 1..n repeat
     for j in 1..n repeat
      for k in 1..n repeat
       not zero? (associator(b.i,b.j,b.k) + associator(b.j,b.i,b.k)) =>
         messagePrint("algebra is not alternative")$OutputForm
         return false
       not zero? (associator(b.i,b.j,b.k) + associator(b.i,b.k,b.j)) =>
         messagePrint("algebra is not alternative")$OutputForm
         return false
    messagePrint("algebra satisfies 2*associator(a,b,b) = 0 " _
                 "=  2*associator(a,a,b) = 0")$OutputForm
    true

  leftDiscriminant v == determinant leftTraceMatrix v
  rightDiscriminant v == determinant rightTraceMatrix v

  coordinates(v:Vector %, b:Vector %) ==
    m := new(#v, #b, 0)$Matrix(R)
    for i in minIndex v .. maxIndex v for j in minRowIndex m .. repeat
      setRow_!(m, j, coordinates(qelt(v, i), b))
    m

  represents(v, b) ==
    m := minIndex v - 1
    reduce(_+,[v(i+m) * b(i+m) for i in 1..maxIndex b])

  leftTraceMatrix v ==
    matrix [[leftTrace(v.i*v.j) for j in minIndex v..maxIndex v]$List(R)
             for i in minIndex v .. maxIndex v]$List(List R)

  rightTraceMatrix v ==
    matrix [[rightTrace(v.i*v.j) for j in minIndex v..maxIndex v]$List(R)
             for i in minIndex v .. maxIndex v]$List(List R)

  leftRegularRepresentation(x, b) ==
```

```
    m := minIndex b - 1
    matrix
     [parts coordinates(x*b(i+m),b) for i in 1..rank()]$List(List R)

  rightRegularRepresentation(x, b) ==
    m := minIndex b - 1
    matrix
     [parts coordinates(b(i+m)*x,b) for i in 1..rank()]$List(List R)
```

— COQ FINAALG —

```
(* category FINAALG *)
(*
    V  ==> Vector
    M  ==> Matrix
    REC  ==> Record(particular: Union(V R,"failed"),basis: List V R)
    LSMP ==> LinearSystemMatrixPackage(R,V R,V R, M R)
    SUP ==>  SparseUnivariatePolynomial
    NNI ==>  NonNegativeInteger

    -- next 2 functions: use a general characteristicPolynomial
    leftCharacteristicPolynomial : % -> SparseUnivariatePolynomial(R)
    leftCharacteristicPolynomial a ==
       n := rank()$%
       ma : Matrix R := leftRegularRepresentation(a,someBasis()$%)
       mb : Matrix SUP R := zero(n,n)
       for i in 1..n repeat
         for j in 1..n repeat
           mb(i,j):=
             i=j => monomial(ma(i,j),0)$SUP(R) - monomial(1,1)$SUP(R)
             monomial(ma(i,j),1)$SUP(R)
       determinant mb

    rightCharacteristicPolynomial : % -> SparseUnivariatePolynomial(R)
    rightCharacteristicPolynomial a ==
       n := rank()$%
       ma : Matrix R := rightRegularRepresentation(a,someBasis()$%)
       mb : Matrix SUP R := zero(n,n)
       for i in 1..n repeat
         for j in 1..n repeat
           mb(i,j):=
             i=j => monomial(ma(i,j),0)$SUP(R) - monomial(1,1)$SUP(R)
             monomial(ma(i,j),1)$SUP(R)
       determinant mb

    leftTrace : % -> R
    leftTrace a ==
      t : R := 0
      ma : Matrix R := leftRegularRepresentation(a,someBasis()$%)
      for i in 1..rank()$% repeat
        t := t + elt(ma,i,i)
```

```
    t

  rightTrace : % -> R
  rightTrace a ==
    t : R := 0
    ma : Matrix R := rightRegularRepresentation(a,someBasis()$%)
    for i in 1..rank()$% repeat
      t := t + elt(ma,i,i)
    t

  leftNorm : % -> R
  leftNorm a == determinant leftRegularRepresentation(a,someBasis()$%)

  rightNorm : % -> R
  rightNorm a == determinant rightRegularRepresentation(a,someBasis()$%)

  antiAssociative? : () -> Boolean
  antiAssociative?() ==
    b := someBasis()
    n := rank()
    for i in 1..n repeat
      for j in 1..n repeat
        for k in 1..n repeat
          not zero? ( (b.i*b.j)*b.k + b.i*(b.j*b.k) )  =>
            messagePrint("algebra is not anti-associative")$OutputForm
            return false
    messagePrint("algebra is anti-associative")$OutputForm
    true

  jordanAdmissible? : () -> Boolean
  jordanAdmissible?() ==
    b := someBasis()
    n := rank()
    recip(2 * 1$R) case "failed" =>
      messagePrint("this algebra is not Jordan admissible, " _
       "as 2 is not invertible in the ground ring")$OutputForm
      false
    for i in 1..n repeat
     for j in 1..n repeat
      for k in 1..n repeat
       for l in 1..n repeat
         not zero? ( _
           antiCommutator(antiCommutator(b.i,b.j),_
                          antiCommutator(b.l,b.k)) + _
           antiCommutator(antiCommutator(b.l,b.j),_
                          antiCommutator(b.k,b.i)) + _
           antiCommutator(antiCommutator(b.k,b.j),_
                          antiCommutator(b.i,b.l))   _
                    ) =>
             messagePrint(_
                       "this algebra is not Jordan admissible")$OutputForm
             return false
    messagePrint("this algebra is Jordan admissible")$OutputForm
    true
```

```
lieAdmissible? : () -> Boolean
lieAdmissible?() ==
  n := rank()
  b := someBasis()
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
      not zero? (commutator(commutator(b.i,b.j),b.k) _
              + commutator(commutator(b.j,b.k),b.i) _
              + commutator(commutator(b.k,b.i),b.j))   =>
        messagePrint("this algebra is not Lie admissible")$OutputForm
        return false
  messagePrint("this algebra is Lie admissible")$OutputForm
  true

structuralConstants : Vector(%) -> Vector(Matrix(R))
structuralConstants b ==
  --n := rank()
  -- be careful with the possibility that b is not a basis
  m : NonNegativeInteger := (maxIndex b) :: NonNegativeInteger
  sC : Vector Matrix R := [new(m,m,0$R) for k in 1..m]
  for i in 1..m repeat
    for j in 1..m repeat
      covec : Vector R := coordinates(b.i * b.j, b)
      for k in 1..m repeat
         setelt( sC.k, i, j, covec.k )
  sC

if R has IntegralDomain then

  leftRecip : % -> Union(%,"failed")
  leftRecip x ==
    zero? x => "failed"
    lu := leftUnit()
    lu case "failed" => "failed"
    b := someBasis()
    xx : % := (lu :: %)
    k  : PositiveInteger := 1
    cond : Matrix R := coordinates(xx,b) :: Matrix(R)
    listOfPowers : List % := [xx]
    while rank(cond) = k repeat
      k := k+1
      xx := xx*x
      listOfPowers := cons(xx,listOfPowers)
      cond := horizConcat(cond, coordinates(xx,b) :: Matrix(R) )
    vectorOfCoef : Vector R := (nullSpace(cond)$Matrix(R)).first
    invC := recip vectorOfCoef.1
    invC case "failed" => "failed"
    invCR : R :=  - (invC :: R)
    reduce(_+,[(invCR*vectorOfCoef.i)*power for i in _
     2..maxIndex vectorOfCoef for power in reverse listOfPowers])

  rightRecip : % -> Union(%,"failed")
```

```
rightRecip x ==
  zero? x => "failed"
  ru := rightUnit()
  ru case "failed" => "failed"
  b := someBasis()
  xx : % := (ru :: %)
  k  : PositiveInteger := 1
  cond : Matrix R := coordinates(xx,b) :: Matrix(R)
  listOfPowers : List % := [xx]
  while rank(cond) = k repeat
    k := k+1
    xx := x*xx
    listOfPowers := cons(xx,listOfPowers)
    cond := horizConcat(cond, coordinates(xx,b) :: Matrix(R) )
  vectorOfCoef : Vector R := (nullSpace(cond)$Matrix(R)).first
  invC := recip vectorOfCoef.1
  invC case "failed" => "failed"
  invCR : R :=  - (invC :: R)
  reduce(_+,[(invCR*vectorOfCoef.i)*power for i in _
   2..maxIndex vectorOfCoef for power in reverse listOfPowers])

recip : % -> Union(%,"failed")
recip x ==
  lrx := leftRecip x
  lrx case "failed" => "failed"
  rrx := rightRecip x
  rrx case "failed" => "failed"
  (lrx :: %) ^= (rrx :: %)  => "failed"
  lrx :: %

leftMinimalPolynomial : % -> SparseUnivariatePolynomial(R)
leftMinimalPolynomial x ==
  zero? x =>  monomial(1$R,1)$(SparseUnivariatePolynomial R)
  b := someBasis()
  xx : % := x
  k  : PositiveInteger := 1
  cond : Matrix R := coordinates(xx,b) :: Matrix(R)
  while rank(cond) = k repeat
    k := k+1
    xx := x*xx
    cond := horizConcat(cond, coordinates(xx,b) :: Matrix(R) )
  vectorOfCoef : Vector R := (nullSpace(cond)$Matrix(R)).first
  res : SparseUnivariatePolynomial R := 0
  for i in 1..k repeat
    res:=res+monomial(vectorOfCoef.i,i)$(SparseUnivariatePolynomial R)
  res

rightMinimalPolynomial : % -> SparseUnivariatePolynomial(R)
rightMinimalPolynomial x ==
  zero? x =>  monomial(1$R,1)$(SparseUnivariatePolynomial R)
  b := someBasis()
  xx : % := x
  k  : PositiveInteger := 1
  cond : Matrix R := coordinates(xx,b) :: Matrix(R)
```

```
    while rank(cond) = k repeat
      k := k+1
      xx := xx*x
      cond := horizConcat(cond, coordinates(xx,b) :: Matrix(R) )
    vectorOfCoef : Vector R := (nullSpace(cond)$Matrix(R)).first
    res : SparseUnivariatePolynomial R := 0
    for i in 1..k repeat
      res:=res+monomial(vectorOfCoef.i,i)$(SparseUnivariatePolynomial R)
    res

  associatorDependence : () -> List(Vector(R))
  associatorDependence() ==
    n := rank()
    b := someBasis()
    cond : Matrix(R) := new(n**4,6,0$R)$Matrix(R)
    z : Integer := 0
    for i in 1..n repeat
     for j in 1..n repeat
      for k in 1..n repeat
       a123 : Vector R := coordinates(associator(b.i,b.j,b.k),b)
       a231 : Vector R := coordinates(associator(b.j,b.k,b.i),b)
       a312 : Vector R := coordinates(associator(b.k,b.i,b.j),b)
       a132 : Vector R := coordinates(associator(b.i,b.k,b.j),b)
       a321 : Vector R := coordinates(associator(b.k,b.j,b.i),b)
       a213 : Vector R := coordinates(associator(b.j,b.i,b.k),b)
       for r in 1..n repeat
        z:= z+1
        setelt(cond,z,1,elt(a123,r))
        setelt(cond,z,2,elt(a231,r))
        setelt(cond,z,3,elt(a312,r))
        setelt(cond,z,4,elt(a132,r))
        setelt(cond,z,5,elt(a321,r))
        setelt(cond,z,6,elt(a213,r))
    nullSpace(cond)

jacobiIdentity? : () -> Boolean
jacobiIdentity?()  ==
  n := rank()
  b := someBasis()
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
      not zero? ((b.i*b.j)*b.k + (b.j*b.k)*b.i + (b.k*b.i)*b.j) =>
        messagePrint("Jacobi identity does not hold")$OutputForm
        return false
  messagePrint("Jacobi identity holds")$OutputForm
  true

lieAlgebra? : () -> Boolean
lieAlgebra?()  ==
  not antiCommutative?() =>
    messagePrint("this is not a Lie algebra")$OutputForm
    false
  not jacobiIdentity?() =>
```

```
        messagePrint("this is not a Lie algebra")$OutputForm
        false
      messagePrint("this is a Lie algebra")$OutputForm
      true

    jordanAdmissible? : () -> Boolean
    jordanAlgebra?()  ==
      b := someBasis()
      n := rank()
      recip(2 * 1$R) case "failed" =>
        messagePrint("this is not a Jordan algebra, as 2 is not " _
         "invertible in the ground ring")$OutputForm
        false
      not commutative?() =>
        messagePrint("this is not a Jordan algebra")$OutputForm
        false
      for i in 1..n repeat
       for j in 1..n repeat
        for k in 1..n repeat
         for l in 1..n repeat
           not zero? (associator(b.i,b.j,b.l*b.k)+_
               associator(b.l,b.j,b.k*b.i)+associator(b.k,b.j,b.i*b.l)) =>
             messagePrint("not a Jordan algebra")$OutputForm
             return false
      messagePrint("this is a Jordan algebra")$OutputForm
      true

    noncommutativeJordanAlgebra? : () -> Boolean
    noncommutativeJordanAlgebra?() ==
      b := someBasis()
      n := rank()
      recip(2 * 1$R) case "failed" =>
       messagePrint("this is not a noncommutative Jordan algebra,_
 as 2 is not invertible in the ground ring")$OutputForm
       false
      not flexible?()$% =>
       messagePrint("this is not a noncommutative Jordan algebra,_
 as it is not flexible")$OutputForm
       false
      not jordanAdmissible?()$% =>
       messagePrint("this is not a noncommutative Jordan algebra,_
 as it is not Jordan admissible")$OutputForm
       false
      messagePrint("this is a noncommutative Jordan algebra")$OutputForm
      true

    antiCommutative? : () -> Boolean
    antiCommutative?() ==
      b := someBasis()
      n := rank()
      for i in 1..n repeat
        for j in i..n repeat
          not zero? (i=j => b.i*b.i; b.i*b.j + b.j*b.i) =>
            messagePrint("algebra is not anti-commutative")$OutputForm
```

```
      return false
  messagePrint("algebra is anti-commutative")$OutputForm
  true

commutative? : () -> Boolean
commutative?() ==
  b := someBasis()
  n := rank()
  for i in 1..n repeat
   for j in i+1..n repeat
     not zero? commutator(b.i,b.j) =>
       messagePrint("algebra is not commutative")$OutputForm
       return false
  messagePrint("algebra is commutative")$OutputForm
  true

associative? : () -> Boolean
associative?() ==
  b := someBasis()
  n := rank()
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
     not zero? associator(b.i,b.j,b.k) =>
       messagePrint("algebra is not associative")$OutputForm
       return false
  messagePrint("algebra is associative")$OutputForm
  true

leftAlternative? : () -> Boolean
leftAlternative?() ==
  b := someBasis()
  n := rank()
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
     not zero? (associator(b.i,b.j,b.k) + associator(b.j,b.i,b.k)) =>
       messagePrint("algebra is not left alternative")$OutputForm
       return false
  messagePrint("algebra satisfies 2*associator(a,a,b) = 0")$OutputForm
  true

rightAlternative? : () -> Boolean
rightAlternative?() ==
  b := someBasis()
  n := rank()
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
     not zero? (associator(b.i,b.j,b.k) + associator(b.i,b.k,b.j)) =>
       messagePrint("algebra is not right alternative")$OutputForm
       return false
  messagePrint("algebra satisfies 2*associator(a,b,b) = 0")$OutputForm
  true
```

```
flexible? : () -> Boolean
flexible?() ==
  b := someBasis()
  n := rank()
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
     not zero? (associator(b.i,b.j,b.k) + associator(b.k,b.j,b.i)) =>
       messagePrint("algebra is not flexible")$OutputForm
       return false
  messagePrint("algebra satisfies 2*associator(a,b,a) = 0")$OutputForm
  true

alternative? : () -> Boolean
alternative?() ==
  b := someBasis()
  n := rank()
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
     not zero? (associator(b.i,b.j,b.k) + associator(b.j,b.i,b.k)) =>
       messagePrint("algebra is not alternative")$OutputForm
       return false
     not zero? (associator(b.i,b.j,b.k) + associator(b.i,b.k,b.j)) =>
       messagePrint("algebra is not alternative")$OutputForm
       return false
  messagePrint("algebra satisfies 2*associator(a,b,b) = 0 " _
               "=  2*associator(a,a,b) = 0")$OutputForm
  true

leftDiscriminant : Vector(%) -> R
leftDiscriminant v == determinant leftTraceMatrix v

rightDiscriminant : Vector(%) -> R
rightDiscriminant v == determinant rightTraceMatrix v

coordinates : (Vector(%),Vector(%)) -> Matrix(R)
coordinates(v:Vector %, b:Vector %) ==
  m := new(#v, #b, 0)$Matrix(R)
  for i in minIndex v .. maxIndex v for j in minRowIndex m .. repeat
    setRow_!(m, j, coordinates(qelt(v, i), b))
  m

represents : (Vector(R),Vector(%)) -> %
represents(v, b) ==
  m := minIndex v - 1
  reduce(_+,[v(i+m) * b(i+m) for i in 1..maxIndex b])

leftTraceMatrix : Vector(%) -> Matrix(R)
leftTraceMatrix v ==
  matrix [[leftTrace(v.i*v.j) for j in minIndex v..maxIndex v]$List(R)
           for i in minIndex v .. maxIndex v]$List(List R)
```

```
    rightTraceMatrix : Vector(%) -> Matrix(R)
    rightTraceMatrix v ==
      matrix [[rightTrace(v.i*v.j) for j in minIndex v..maxIndex v]$List(R)
               for i in minIndex v .. maxIndex v]$List(List R)

    leftRegularRepresentation : (%,Vector(%)) -> Matrix(R)
    leftRegularRepresentation(x, b) ==
      m := minIndex b - 1
      matrix
       [parts coordinates(x*b(i+m),b) for i in 1..rank()]$List(List R)

    rightRegularRepresentation : (%,Vector(%)) -> Matrix(R)
    rightRegularRepresentation(x, b) ==
      m := minIndex b - 1
      matrix
       [parts coordinates(b(i+m)*x,b) for i in 1..rank()]$List(List R)

*)
```

———-

— FINAALG.dotabb —

```
"FINAALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FINAALG"];
"FINAALG" -> "NAALG"
```

———-

— FINAALG.dotfull —

```
"FiniteRankNonAssociativeAlgebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FINAALG"];
"FiniteRankNonAssociativeAlgebra(a:CommutativeRing)" ->
    "NonAssociativeAlgebra(a:CommutativeRing)"
```

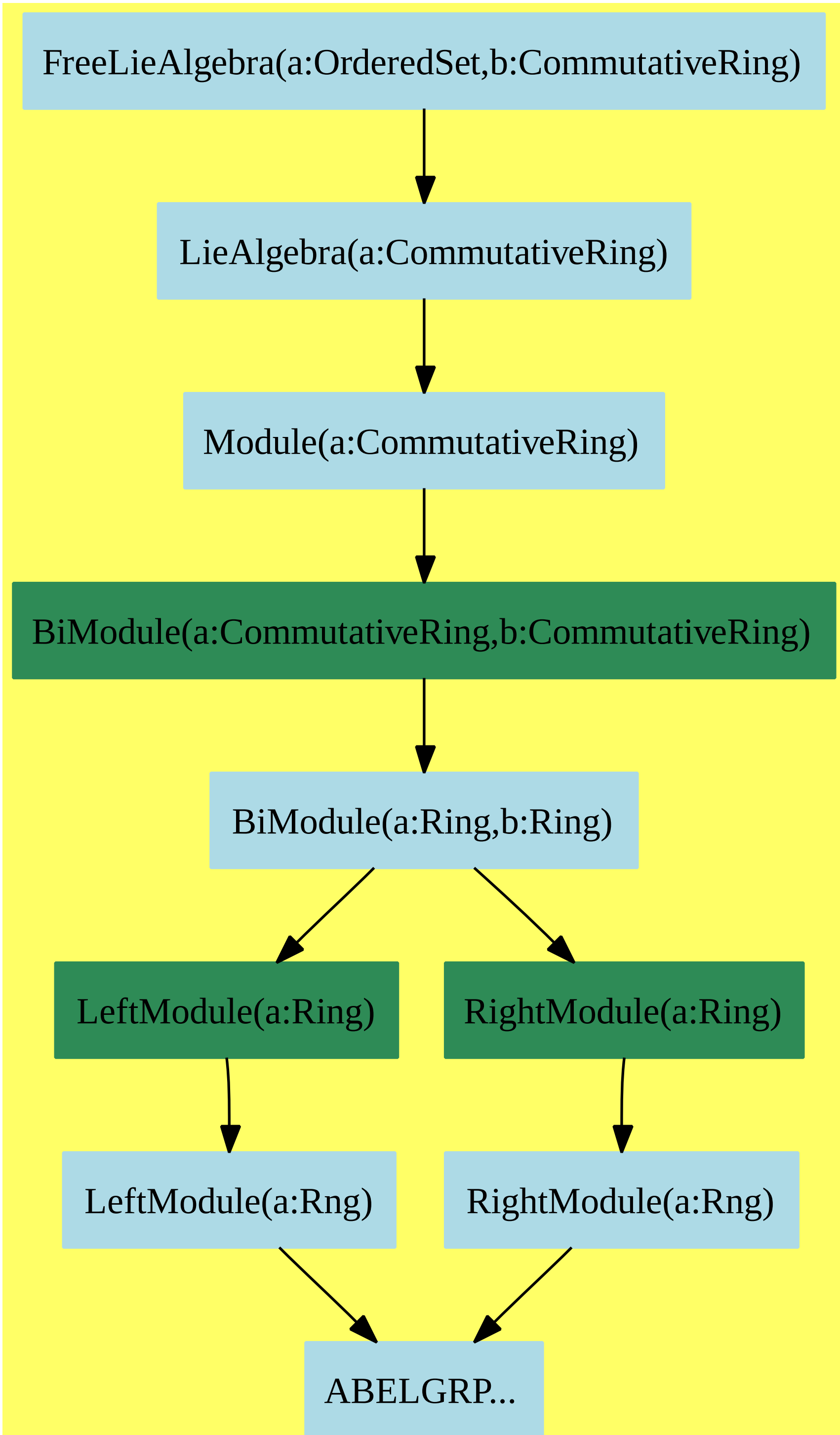
———-

— FINAALG.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FiniteRankNonAssociativeAlgebra(a:CommutativeRing)" [color=lightblue];
"FiniteRankNonAssociativeAlgebra(a:CommutativeRing)" ->
    "NonAssociativeAlgebra(a:CommutativeRing)"

"NonAssociativeAlgebra(a:CommutativeRing)" [color=lightblue];
"NonAssociativeAlgebra(a:CommutativeRing)" -> "NonAssociativeRng()"
```

```
"NonAssociativeAlgebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"NonAssociativeRng()" [color=lightblue];
"NonAssociativeRng()" -> "ABELGRP..."
"NonAssociativeRng()" -> "Monad()"

"Monad()" [color=lightblue];
"Monad()" -> "SETCAT..."
"Monad()" -> "REPSQ..."

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"ABELGRP..." [color=lightblue];
}
```

12.0.187 FreeLieAlgebra (FLALG)



— **FreeLieAlgebra.input** —

```
)set break resume
)sys rm -f FreeLieAlgebra.output
)spool FreeLieAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FreeLieAlgebra
--R
--R FreeLieAlgebra(VarSet: OrderedSet,R: CommutativeRing) is a category constructor
--R Abbreviation for FreeLieAlgebra is FLALG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FLALG
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
--R ?*? : (Integer,%) -> %                ?*? : (NonNegativeInteger,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (%,R) -> % if R has FIELD       ?=? : (%,%) -> Boolean
--R LiePoly : LyndonWord(VarSet) -> %     0 : () -> %
--R coerce : VarSet -> %                  coerce : % -> OutputForm
--R construct : (%,%) -> %                degree : % -> NonNegativeInteger
--R eval : (%,VarSet,%) -> %              hash : % -> SingleInteger
--R latex : % -> String                   mirror : % -> %
--R sample : () -> %                      varList : % -> List(VarSet)
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R coef : (XRecursivePolynomial(VarSet,R),%) -> R
--R coerce : % -> XRecursivePolynomial(VarSet,R)
--R coerce : % -> XDistributedPolynomial(VarSet,R)
--R eval : (%,List(VarSet),List(%)) -> %
--R lquo : (XRecursivePolynomial(VarSet,R),%) -> XRecursivePolynomial(VarSet,R)
--R rquo : (XRecursivePolynomial(VarSet,R),%) -> XRecursivePolynomial(VarSet,R)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R trunc : (%,NonNegativeInteger) -> %
--R
--E 1

)spool
)lisp (bye)
```

———

— **FreeLieAlgebra.help** —

```
====================================================================
FreeLieAlgebra examples
====================================================================

The category of free Lie algebras. It is used by domains of
non-commutative algebra such as LiePolynomial and XPBWPolynomial.
```

```
See Also:
o )show FreeLieAlgebra
```

———

See:

⇐ "LieAlgebra" (LIECAT) 11.0.179 on page 1140

Exports:

0	coef	coerce	construct	degree
eval	hash	latex	LiePoly	lquo
mirror	rquo	sample	subtractIfCan	trunc
varList	zero?	?~=?	?*?	?/?
?+?	?-?	-?	?=?	

Attributes Exported:

- **NullSquare** means that $[x, x] = 0$ holds. See `LieAlgebra`.
- **JacobiIdentity** means that $[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0$ holds. See `LieAlgebra`.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
coef : (XRecursivePolynomial(VarSet,R),%) -> R
coerce : VarSet -> %
coerce : % -> XRecursivePolynomial(VarSet,R)
coerce : % -> XDistributedPolynomial(VarSet,R)
degree : % -> NonNegativeInteger
eval : (%,List VarSet,List %) -> %
eval : (%,VarSet,%) -> %
LiePoly : LyndonWord VarSet -> %
lquo : (XRecursivePolynomial(VarSet,R),%) -> XRecursivePolynomial(VarSet,R)
mirror : % -> %
rquo : (XRecursivePolynomial(VarSet,R),%) -> XRecursivePolynomial(VarSet,R)
trunc : (%,NonNegativeInteger) -> %
varList : % -> List VarSet
```

These exports come from (p1140) LieAlgebra(CommutativeRing):

```
0 : () -> %
coerce : % -> OutputForm
construct : (%,%) -> %
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?/? : (%,R) -> % if R has FIELD
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (%,R) -> %
```

```
?*? : (R,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
```

— **FreeLieAlgebra.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FLALG">
FreeLieAlgebra (FLALG)</a></h2>
</body>
```

— **category FLALG FreeLieAlgebra** —

```
)abbrev category FLALG FreeLieAlgebra
++ Author: Michel Petitot (petitot@lifl.fr)
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Description:
++ The category of free Lie algebras.
++ It is used by domains of non-commutative algebra:
++ \spadtype{LiePolynomial} and
++ \spadtype{XPBWPolynomial}.

FreeLieAlgebra(VarSet,R) : Category == SIG where
  VarSet : OrderedSet
  R : CommutativeRing

  XRPOLY  ==> XRecursivePolynomial(VarSet,R)
  XDPOLY  ==> XDistributedPolynomial(VarSet,R)
  RN      ==> Fraction Integer
  LWORD   ==> LyndonWord(VarSet)

  SIG ==> Join(LieAlgebra(R)) with

    coef : (XRPOLY , $) -> R
      ++ \axiom{coef(x,y)} returns the scalar product of \axiom{x} by
      ++ \axiom{y}, the set of words being regarded as an orthogonal basis.

    coerce : VarSet -> $
      ++ \axiom{coerce(x)} returns \axiom{x} as a Lie polynomial.

    coerce : $ -> XDPOLY
      ++ \axiom{coerce(x)} returns \axiom{x} as distributed polynomial.

    coerce : $ -> XRPOLY
      ++ \axiom{coerce(x)} returns \axiom{x} as a recursive polynomial.
```

```
degree : $ -> NonNegativeInteger
  ++ \axiom{degree(x)} returns the greatest length of a word in the
  ++ support of \axiom{x}.

--if R has Module(RN) then
--  Hausdorff : ($,$,PositiveInteger) -> $

lquo : (XRPOLY , $) -> XRPOLY
  ++ \axiom{lquo(x,y)} returns the left simplification of \axiom{x}
  ++ by \axiom{y}.

rquo : (XRPOLY , $) -> XRPOLY
  ++ \axiom{rquo(x,y)} returns the right simplification of \axiom{x}
  ++ by \axiom{y}.

LiePoly : LWORD -> $
  ++ \axiom{LiePoly(l)} returns the bracketed form of \axiom{l} as
  ++ a Lie polynomial.

mirror : $ -> $
  ++ \axiom{mirror(x)} returns \axiom{Sum(r_i mirror(w_i))}
  ++ if \axiom{x} is \axiom{Sum(r_i w_i)}.

trunc : ($, NonNegativeInteger) -> $
  ++ \axiom{trunc(p,n)} returns the polynomial \axiom{p}
  ++ truncated at order \axiom{n}.

varList : $ -> List VarSet
  ++ \axiom{varList(x)} returns the list of distinct entries
  ++ of \axiom{x}.

eval : ($, VarSet, $) -> $
  ++ \axiom{eval(p, x, v)} replaces \axiom{x} by \axiom{v}
  ++ in \axiom{p}.

eval : ($, List VarSet, List $) -> $
  ++ \axiom{eval(p, [x1,...,xn], [v1,...,vn])} replaces \axiom{xi}
  ++ by \axiom{vi} in \axiom{p}.
```

— FLALG.dotabb —

```
"FLALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FLALG"];
"FLALG" -> "LIECAT"
```

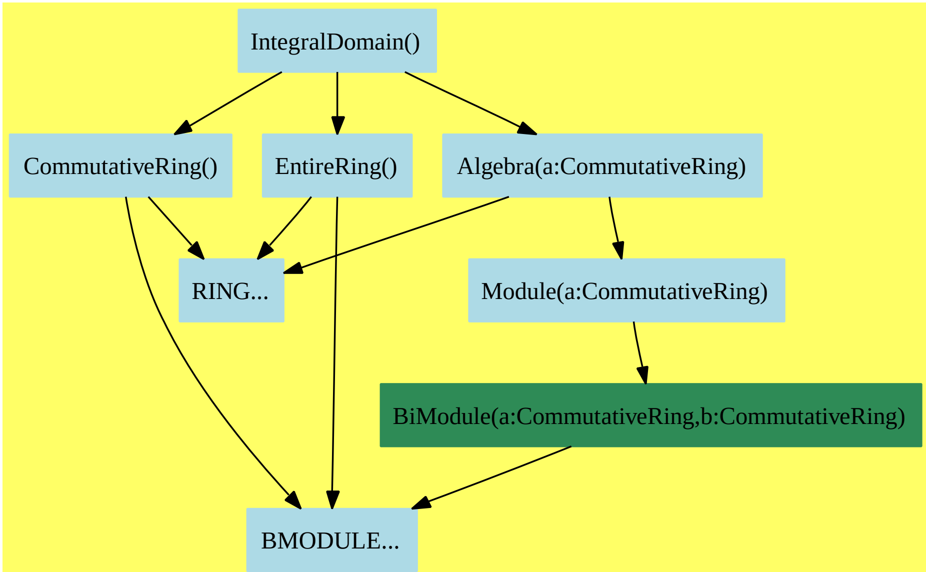
— FLALG.dotfull —

```
"FreeLieAlgebra(a:OrderedSet,b:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FLALG"];
"FreeLieAlgebra(a:OrderedSet,b:CommutativeRing)" ->
   "LieAlgebra(a:CommutativeRing)"
```

— FLALG.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FreeLieAlgebra(a:OrderedSet,b:CommutativeRing)" [color=lightblue];
"FreeLieAlgebra(a:OrderedSet,b:CommutativeRing)" ->
   "LieAlgebra(a:CommutativeRing)"

"LieAlgebra(a:CommutativeRing)" [color=lightblue];
"LieAlgebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"ABELGRP..." [color=lightblue];
}
```

12.0.188 IntegralDomain (INTDOM)



— **IntegralDomain.input** —

```
)set break resume
)sys rm -f IntegralDomain.output
)spool IntegralDomain.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IntegralDomain
--R
--R IntegralDomain is a category constructor
--R Abbreviation for IntegralDomain is INTDOM
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for INTDOM
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean        coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R hash : % -> SingleInteger             latex : % -> String
--R one? : % -> Boolean                   recip : % -> Union(%,"failed")
--R sample : () -> %                      unit? : % -> Boolean
```

```
--R unitCanonical : % -> %                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R exquo : (%,%) -> Union(%,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

— **IntegralDomain.help** —

```
====================================================================
IntegralDomain examples
====================================================================

The category of commutative integral domains, commutative
rings with no zero divisors. That means that we can show that
if ab = 0 then either a or b must be 0.

This does not have to be true. For instance, real 2x2 matrices
have zero divisors since

     A    x   B     =     0    but neither A nor B is 0:
  +-    -+ +-    -+    +-    -+
  | 0 1 |  | 0 2 |  =  | 0 0 |
  | 0 0 |  | 0 0 |     | 0 0 |
  +-    -+ +-    -+    +-    -+

Conditional attributes:
  canonicalUnitNormal - the canonical field is the same for all associates
  canonicalsClosed    - the product of two canonicals is itself canonical

 Ring -> CommutativeRing -> IntegralDomain

1) (associative addition)        a + (b + c) = (a + b) + c
2) (commutative addition)        a + b = b + a
3) (associative multiplication)  a(bc) = (ab)c
4) (distributive mulitplication) a(b + c) = ab + ac; (b + c)a = ba + ca
5) (equation solution)           a + x = b has a solution in R

See Also:
o )show IntegralDomain
```

All Commutative Rings are Integral Domains.
⇐ "CommutativeRing" (COMRING) 10.0.161 on page 1019 Integral Domains are a subset of Unique Factorization domains.
⇒ "UniqueFactorizationDomain" (UFD) 14.0.203 on page 1362.

See:

⇒ "FortranMachineTypeCategory" (FMTC) 13.0.195 on page 1295
⇒ "FunctionSpace" (FS) 17.0.219 on page 1544
⇒ "GcdDomain" (GCDDOM) 13.0.197 on page 1319
⇒ "OrderedIntegralDomain" (OINTDOM) 13.0.198 on page 1326
⇒ "UnivariatePolynomialCategory" (UPOLYC) 17.0.228 on page 1708
⇐ "Algebra" (ALGEBRA) 11.0.176 on page 1121
⇐ "CommutativeRing" (COMRING) 10.0.161 on page 1019
⇐ "EntireRing" (ENTIRER) 10.0.163 on page 1029

Exports:

0	1	associates?	characteristic	coerce
exquo	hash	latex	one?	recip
sample	subtractIfCan	unit?	unitCanonical	unitNormal
zero?	?~=?	?*?	?**?	?^?
?+?	?-?	-?	?=?	

Attributes exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

Attributes Used:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.

These are directly exported but not implemented:

```
exquo : (%,%) -> Union(%,"failed")
```

These are implemented by this category:

```
associates? : (%,%) -> Boolean
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
```

These exports come from (p1019) CommutativeRing():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
```

```
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
```

TPDHERE: Should we construct this coercion?

These exports come from (p1121) Algebra(a:IntegralDomain):

```
coerce : % -> %
```

— **IntegralDomain.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#INTDOM">
IntegralDomain (INTDOM)</a></h2>
</body>
```

———-

— **category INTDOM IntegralDomain** —

```
)abbrev category INTDOM IntegralDomain
++ Description:
++ The category of commutative integral domains, commutative
++ rings with no zero divisors.
++
++ Conditional attributes\br
++ canonicalUnitNormal\tab{5}the canonical field is the same for
++ all associates\br
++ canonicalsClosed\tab{5}the product of two canonicals is itself canonical

IntegralDomain() : Category == SIG where

  SIG ==> Join(CommutativeRing, Algebra(%), EntireRing) with

    "exquo" : (%,%) -> Union(%,"failed")
      ++ exquo(a,b) either returns an element c such that
      ++ \spad{c*b=a} or "failed" if no such element can be found.

    unitNormal : % -> Record(unit:%,canonical:%,associate:%)
      ++ unitNormal(x) tries to choose a canonical element
```

```
      ++ from the associate class of x.
      ++ The attribute canonicalUnitNormal, if asserted, means that
      ++ the "canonical" element is the same across all associates of x
      ++ if \spad{unitNormal(x) = [u,c,a]} then
      ++ \spad{u*c = x}, \spad{a*u = 1}.

    unitCanonical : % -> %
      ++ \spad{unitCanonical(x)} returns \spad{unitNormal(x).canonical}.

    associates? : (%,%) -> Boolean
      ++ associates?(x,y) tests whether x and y are associates,
      ++ differ by a unit factor.

    unit? : % -> Boolean
      ++ unit?(x) tests whether x is a unit, is invertible.

   add

      x,y: %

      UCA ==> Record(unit:%,canonical:%,associate:%)

      if not (% has Field) then
        unitNormal(x) == [1$%,x,1$%]$UCA -- the non-canonical definition

      unitCanonical(x) == unitNormal(x).canonical -- always true

      recip(x) == if zero? x then "failed" else _exquo(1$%,x)

      unit?(x) == (recip x case "failed" => false; true)

      if % has canonicalUnitNormal then

         associates?(x,y) ==
           (unitNormal x).canonical = (unitNormal y).canonical

       else

         associates?(x,y) ==
           zero? x => zero? y
           zero? y => false
           x exquo y case "failed" => false
           y exquo x case "failed" => false
           true
```

———

— COQ INTDOM —

```
(* category INTDOM *)
(*
Conditional attributes:
  canonicalUnitNormal - the canonical field is the same for all associates
```

```
  canonicalsClosed    - the product of two canonicals is itself canonical

 Ring -> CommutativeRing -> IntegralDomain

   1) (associative addition)        a + (b + c) = (a + b) + c
   2) (commutative addition)        a + b = b + a
   3) (associative multiplication)  a(bc) = (ab)c
   4) (distributive mulitplication) a(b + c) = ab + ac; (b + c)a = ba + ca
   5) (equation solution)           a + x = b has a solution in R

      x,y: %

      UCA ==> Record(unit:%,canonical:%,associate:%)

      if not (% has Field) then

        unitNormal : % -> Record(unit: %,canonical: %,associate: %)
        unitNormal(x) == [1$%,x,1$%]$UCA -- the non-canonical definition

      unitCanonical : % -> %
      unitCanonical(x) == unitNormal(x).canonical -- always true

      recip : % -> Union(%,"failed")
      recip(x) == if zero? x then "failed" else _exquo(1$%,x)

      unit? : % -> Boolean
      unit?(x) == (recip x case "failed" => false; true)

      if % has canonicalUnitNormal then

         associates? : (%,%) -> Boolean
         associates?(x,y) ==
           (unitNormal x).canonical = (unitNormal y).canonical

       else

         associates? : (%,%) -> Boolean
         associates?(x,y) ==
           zero? x => zero? y
           zero? y => false
           x exquo y case "failed" => false
           y exquo x case "failed" => false
           true

*)
```

———-

— INTDOM.dotabb —

```
"INTDOM"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=INTDOM"];
"INTDOM" -> "COMRING"
```

```
"INTDOM" -> "ALGEBRA"
"INTDOM" -> "ENTIRER"
```

——–

— **INTDOM.dotfull** —

```
"IntegralDomain()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=INTDOM"];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"
```

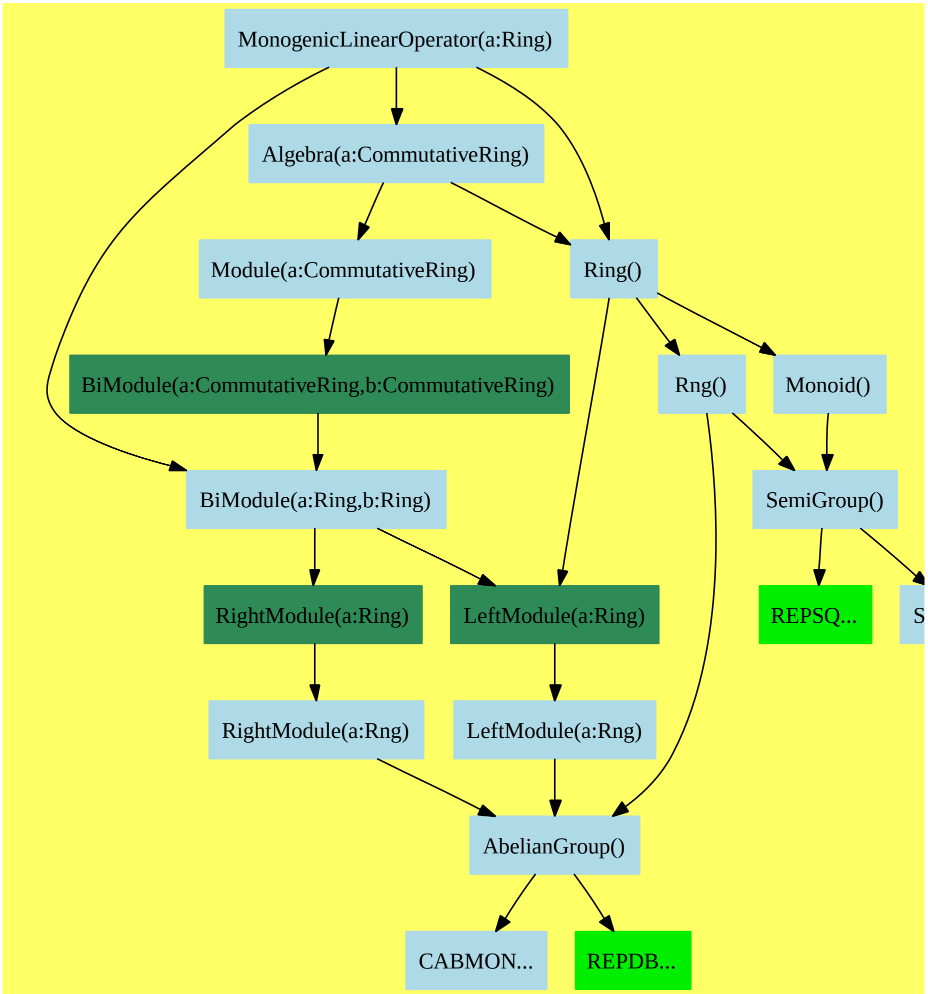
——–

— **INTDOM.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BMODULE..."

"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];
}
```

——–

12.0.189 MonogenicLinearOperator (MLO)



— **MonogenicLinearOperator.input** —

```
)set break resume
)sys rm -f MonogenicLinearOperator.output
)spool MonogenicLinearOperator.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MonogenicLinearOperator
--R
--R MonogenicLinearOperator(R: Ring) is a category constructor
--R Abbreviation for MonogenicLinearOperator is MLO
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for MLO
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                        ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                        ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %       ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %      ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                        ?-? : (%,%) -> %
--R -? : % -> %                             ?=? : (%,%) -> Boolean
--R 1 : () -> %                             0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %       ?^? : (%,PositiveInteger) -> %
--R coerce : R -> % if R has COMRING        coerce : Integer -> %
--R coerce : % -> OutputForm                degree : % -> NonNegativeInteger
--R hash : % -> SingleInteger               latex : % -> String
--R leadingCoefficient : % -> R             one? : % -> Boolean
--R recip : % -> Union(%,"failed")          reductum : % -> %
--R sample : () -> %                        zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R coefficient : (%,NonNegativeInteger) -> R
--R minimumDegree : % -> NonNegativeInteger
--R monomial : (R,NonNegativeInteger) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

— MonogenicLinearOperator.help —

```
====================================================================
MonogenicLinearOperator examples
====================================================================

This is the category of linear operator rings with one generator.
The generator is not named by the category but can always be
constructed as monomial(1,1).

For convenience, call the generator G.
Then each value is equal to
    sum(a(i)*G**i, i = 0..n)
for some unique n and a(i) in R.

Note that multiplication is not necessarily commutative.
In fact, if a is in R, it is quite normal to have a*G ^= G*a.

See Also:
o )show MonogenicLinearOperator
```

———-

See:

⇐ "Algebra" (ALGEBRA) 11.0.176 on page 1121
⇐ "BiModule" (BMODULE) 9.0.146 on page 902
⇐ "Ring" (RING) 9.0.153 on page 946

Exports:

0	1	characteristic	coefficient
coerce	degree	hash	latex
leadingCoefficient	minimumDegree	monomial	one?
recip	reductum	sample	subtractIfCan
zero?	?*?	?**?	?+?
?-?	-?	?=?	?^?
?~=?			

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
coefficient : (%,NonNegativeInteger) -> R
degree : % -> NonNegativeInteger
leadingCoefficient : % -> R
minimumDegree : % -> NonNegativeInteger
monomial : (R,NonNegativeInteger) -> %
reductum : % -> %
```

These exports come from (p946) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : % -> OutputForm
coerce : Integer -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
```

```
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

These exports come from (p902) BiModule(R:Ring,R:Ring):

```
?*? : (R,%) -> %
?*? : (%,R) -> %
```

These exports come from (p1121) Algebra(R:CommutativeRing):

```
coerce : R -> % if R has COMRING
```

— MonogenicLinearOperator.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#MLO">
MonogenicLinearOperator (MLO)</a></h2>
</body>
```

— category MLO MonogenicLinearOperator —

```
)abbrev category MLO MonogenicLinearOperator
++ Author: Stephen M. Watt
++ Date Created: 1986
++ Date Last Updated: May 30, 1991
++ Description:
++ This is the category of linear operator rings with one generator.
++ The generator is not named by the category but can always be
++ constructed as \spad{monomial(1,1)}.
++
++ For convenience, call the generator \spad{G}.
++ Then each value is equal to
++ \spad{sum(a(i)*G**i, i = 0..n)}
++ for some unique \spad{n} and \spad{a(i)} in \spad{R}.
++
++ Note that multiplication is not necessarily commutative.
++ In fact,  if \spad{a} is in \spad{R}, it is quite normal
++ to have \spad{a*G \^= G*a}.

MonogenicLinearOperator(R) : Category == SIG where
  R : Ring

  E ==> NonNegativeInteger

  SIG ==> Join(Ring, BiModule(R,R)) with

    if R has CommutativeRing then Algebra(R)

    degree : $ -> E
      ++ degree(l) is \spad{n} if
      ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.

    minimumDegree : $ -> E
```

```
      ++ minimumDegree(l) is the smallest \spad{k} such that
      ++ \spad{a(k) \^= 0} if
      ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.

    leadingCoefficient : $ -> R
      ++ leadingCoefficient(l) is \spad{a(n)} if
      ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.

    reductum : $ -> $
      ++ reductum(l) is \spad{l - monomial(a(n),n)} if
      ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.

    coefficient : ($, E) -> R
      ++ coefficient(l,k) is \spad{a(k)} if
      ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.

    monomial : (R, E) -> $
      ++ monomial(c,k) produces c times the k-th power of
      ++ the generating operator, \spad{monomial(1,1)}.
```

———-

— MLO.dotabb —

```
"MLO"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MLO"];
"MLO" -> "BMODULE"
"MLO" -> "RING"
"MLO" -> "ALGEBRA"
```

———-

— MLO.dotfull —

```
"MonogenicLinearOperator(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MLO"];
"MonogenicLinearOperator(a:Ring)" -> "Ring()"
"MonogenicLinearOperator(a:Ring)" -> "BiModule(a:Ring,b:Ring)"
"MonogenicLinearOperator(a:Ring)" -> "Algebra(a:CommutativeRing)"
```

———-

— MLO.dotpic —

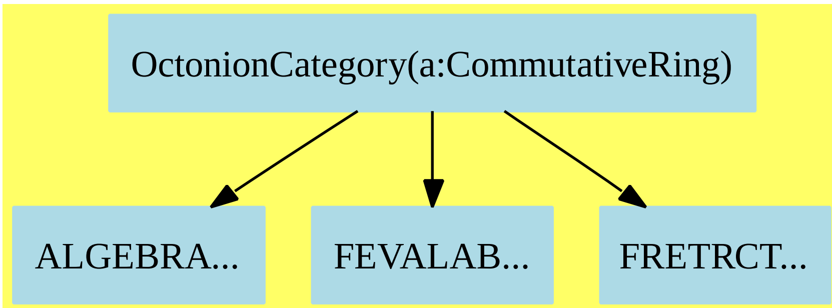
```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"MonogenicLinearOperator(a:Ring)" [color=lightblue];
"MonogenicLinearOperator(a:Ring)" -> "Ring()"
```

```
"MonogenicLinearOperator(a:Ring)" -> "BiModule(a:Ring,b:Ring)"
"MonogenicLinearOperator(a:Ring)" -> "Algebra(a:CommutativeRing)"

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "Ring()"
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "AbelianGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
```

```
}
```

———-

12.0.190 OctonionCategory (OC)



— **OctonionCategory.input** —

```
)set break resume
)sys rm -f OctonionCategory.output
)spool OctonionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OctonionCategory
--R
--R OctonionCategory(R: CommutativeRing) is a category constructor
--R Abbreviation for OctonionCategory is OC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for OC
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                     ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %    ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %   ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %    ?^? : (%,PositiveInteger) -> %
--R abs : % -> R if R has RNS            coerce : R -> %
--R coerce : Integer -> %                coerce : % -> OutputForm
--R conjugate : % -> %                   hash : % -> SingleInteger
--R imagE : % -> R                       imagI : % -> R
--R imagJ : % -> R                       imagK : % -> R
--R imagi : % -> R                       imagj : % -> R
--R imagk : % -> R                       inv : % -> % if R has FIELD
```

```
--R latex : % -> String                  map : ((R -> R),%) -> %
--R max : (%,%) -> % if R has ORDSET     min : (%,%) -> % if R has ORDSET
--R norm : % -> R                        octon : (R,R,R,R,R,R,R,R) -> %
--R one? : % -> Boolean                  random : () -> % if R has FINITE
--R real : % -> R                        recip : % -> Union(%,"failed")
--R retract : % -> R                     sample : () -> %
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
--R coerce : Fraction(Integer) -> % if R has RETRACT(FRAC(INT))
--R convert : % -> InputForm if R has KONVERT(INFORM)
--R ?.? : (%,R) -> % if R has ELTAB(R,R)
--R enumerate : () -> List(%) if R has FINITE
--R eval : (%,Symbol,R) -> % if R has IEVALAB(SYMBOL,R)
--R eval : (%,List(Symbol),List(R)) -> % if R has IEVALAB(SYMBOL,R)
--R eval : (%,List(Equation(R))) -> % if R has EVALAB(R)
--R eval : (%,Equation(R)) -> % if R has EVALAB(R)
--R eval : (%,R,R) -> % if R has EVALAB(R)
--R eval : (%,List(R),List(R)) -> % if R has EVALAB(R)
--R index : PositiveInteger -> % if R has FINITE
--R lookup : % -> PositiveInteger if R has FINITE
--R rational : % -> Fraction(Integer) if R has INS
--R rational? : % -> Boolean if R has INS
--R rationalIfCan : % -> Union(Fraction(Integer),"failed") if R has INS
--R retract : % -> Fraction(Integer) if R has RETRACT(FRAC(INT))
--R retract : % -> Integer if R has RETRACT(INT)
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction(Integer),"failed") if R has RETRACT(FRAC(INT))
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT(INT)
--R size : () -> NonNegativeInteger if R has FINITE
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— **OctonionCategory.help** —

```
====================================================================
OctonionCategory examples
====================================================================

OctonionCategory gives the categorial frame for the octonions, and
eight-dimensional non-associative algebra, doubling the the quaternions
in the same way as doubling the Complex numbers to get the quaternions.

See Also:
```

```
o )show OctonionCategory
```

———–

See:

⇐ "Algebra" (ALGEBRA) 11.0.176 on page 1121
⇐ "FullyEvalableOver" (FEVALAB) 4.0.67 on page 246
⇐ "FullyRetractableTo" (FRETRCT) 3.0.50 on page 158

Exports:

0	1	abs	characteristic	charthRoot
coerce	conjugate	convert	eval	hash
imagE	imagI	imagJ	imagK	imagi
imagj	imagk	index	inv	latex
lookup	map	max	min	norm
octon	one?	random	rational	rational?
rationalIfCan	real	recip	retract	retractIfCan
sample	size	subtractIfCan	zero?	?*?
?**?	?+?	?-?	-?	?=?
?^?	?~=?	?<?	?<=?	?>?
?>=?	?.?			

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
imagi : % -> R
imagj : % -> R
imagk : % -> R
imagE : % -> R
imagI : % -> R
imagJ : % -> R
imagK : % -> R
octon : (R,R,R,R,R,R,R,R) -> %
real : % -> R
```

These are implemented by this category:

```
abs : % -> R if R has RNS
characteristic : () -> NonNegativeInteger
coerce : R -> %
coerce : Integer -> %
coerce : % -> OutputForm
conjugate : % -> %
convert : % -> InputForm if R has KONVERT INFORM
inv : % -> % if R has FIELD
map : ((R -> R),%) -> %
norm : % -> R
```

```
rational : % -> Fraction Integer if R has INS
rational? : % -> Boolean if R has INS
rationalIfCan : % -> Union(Fraction Integer,"failed") if R has INS
retract : % -> R
retractIfCan : % -> Union(R,"failed")
zero? : % -> Boolean
?<? : (%,%) -> Boolean if R has ORDSET
?=? : (%,%) -> Boolean
?+? : (%,%) -> %
-? : % -> %
?*? : (R,%) -> %
?*? : (Integer,%) -> %
```

These exports come from (p1121) Algebra(R:CommutativeRing):

```
0 : () -> %
1 : () -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?*? : (%,R) -> %
```

These exports come from (p158) FullyRetractableTo(R:CommutativeRing):

```
coerce : Fraction Integer -> % if R has RETRACT FRAC INT
retract : % -> Fraction Integer if R has RETRACT FRAC INT
retract : % -> Integer if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
```

These exports come from (p246) FullyEvalableOver(R:CommutativeRing):

```
eval : (%,Equation R) -> % if R has EVALAB R
eval : (%,List Symbol,List R) -> % if R has IEVALAB(SYMBOL,R)
eval : (%,List Equation R) -> % if R has EVALAB R
eval : (%,R,R) -> % if R has EVALAB R
eval : (%,List R,List R) -> % if R has EVALAB R
eval : (%,Symbol,R) -> % if R has IEVALAB(SYMBOL,R)
?.? : (%,R) -> % if R has ELTAB(R,R)
```

These exports come from (p258) Finite():

```
index : PositiveInteger -> % if R has FINITE
lookup : % -> PositiveInteger if R has FINITE
random : () -> % if R has FINITE
size : () -> NonNegativeInteger if R has FINITE
```

These exports come from (p326) OrderedSet():

```
max : (%,%) -> % if R has ORDSET
min : (%,%) -> % if R has ORDSET
?<=? : (%,%) -> Boolean if R has ORDSET
?>? : (%,%) -> Boolean if R has ORDSET
?>=? : (%,%) -> Boolean if R has ORDSET
```

These exports come from (p1010) CharacteristicNonZero():

```
charthRoot : % -> Union(%,"failed") if R has CHARNZ
```

— OctonionCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#OC">
OctonionCategory (OC)</a></h2>
</body>
```

— category OC OctonionCategory —

```
)abbrev category OC OctonionCategory
++ Author: R. Wisbauer, J. Grabmeier
++ Date Created: 05 September 1990
++ Date Last Updated: 19 September 1990
++ References: I.L Kantor, A.S. Solodovnikov:
++  Hypercomplex Numbers, Springer Verlag Heidelberg, 1989,
++  ISBN 0-387-96980-2
++ Description:
++ OctonionCategory gives the categorial frame for the
++ octonions, and eight-dimensional non-associative algebra,
++ doubling the the quaternions in the same way as doubling
++ the Complex numbers to get the quaternions.

  -- we are cheating a little bit, algebras in \Language{}
  -- are mainly considered to be associative, but that's not
  -- an attribute and we can't guarantee that there is no piece
  -- of code which implicitly
  -- uses this. In a later version we shall properly combine
  -- all this code in the context of general, non-associative
  -- algebras, which are meanwhile implemented in \Language{}

OctonionCategory(R) : Category == SIG where
  R : CommutativeRing

  SIG ==> Join(Algebra R, FullyRetractableTo R, FullyEvalableOver R) with

    conjugate : % -> %
      ++ conjugate(o) negates the imaginary parts i,j,k,E,I,J,K of octonian o.

    real : % -> R
      ++ real(o) extracts real part of octonion o.
```

```
imagi : % -> R
  ++ imagi(o) extracts the i part of octonion o.

imagj : % -> R
  ++ imagj(o) extracts the j part of octonion o.

imagk : % -> R
  ++ imagk(o) extracts the k part of octonion o.

imagE : % -> R
  ++ imagE(o) extracts the imaginary E part of octonion o.

imagI : % -> R
  ++ imagI(o) extracts the imaginary I part of octonion o.

imagJ : % -> R
  ++ imagJ(o) extracts the imaginary J part of octonion o.

imagK : % -> R
  ++ imagK(o) extracts the imaginary K part of octonion o.

norm : % -> R
  ++ norm(o) returns the norm of an octonion, equal to
  ++ the sum of the squares
  ++ of its coefficients.

octon : (R,R,R,R,R,R,R,R) -> %
  ++ octon(re,ri,rj,rk,rE,rI,rJ,rK) constructs an octonion
  ++ from scalars.

if R has Finite then Finite

if R has OrderedSet then OrderedSet

if R has ConvertibleTo InputForm then ConvertibleTo InputForm

if R has CharacteristicZero then CharacteristicZero

if R has CharacteristicNonZero then CharacteristicNonZero

if R has RealNumberSystem then

  abs: % -> R
    ++ abs(o) computes the absolute value of an octonion, equal to
    ++ the square root of the \spadfunFrom{norm}{Octonion}.

if R has IntegerNumberSystem then

  rational? : % -> Boolean
    ++ rational?(o) tests if o is rational, that all seven
    ++ imaginary parts are 0.

  rational : % -> Fraction Integer
    ++ rational(o) returns the real part if all seven
```

```
      ++ imaginary parts are 0.
      ++ Error: if o is not rational.

    rationalIfCan : % -> Union(Fraction Integer, "failed")
      ++ rationalIfCan(o) returns the real part if
      ++ all seven imaginary parts are 0, and "failed" otherwise.

  if R has Field then

    inv : % -> %
      ++ inv(o) returns the inverse of o if it exists.

 add

   characteristic() ==
     characteristic()$R

   conjugate x ==
     octon(real x, - imagi x, - imagj x, - imagk x, - imagE x,_
     - imagI x, - imagJ x, - imagK x)

   map(fn, x)       ==
     octon(fn real x,fn imagi x,fn imagj x,fn imagk x, fn imagE x,_
     fn imagI x, fn imagJ x,fn imagK x)

   norm x ==
     real x * real x + imagi x * imagi x + _
     imagj x * imagj x + imagk x * imagk x + _
     imagE x * imagE x + imagI x * imagI x + _
     imagJ x * imagJ x + imagK x * imagK x

   x = y            ==
     (real x = real y) and (imagi x = imagi y) and _
     (imagj x = imagj y) and (imagk x = imagk y) and _
     (imagE x = imagE y) and (imagI x = imagI y) and _
     (imagJ x = imagJ y) and (imagK x = imagK y)

   x + y            ==
     octon(real x + real y, imagi x + imagi y,_
     imagj x + imagj y, imagk x + imagk y,_
     imagE x + imagE y, imagI x + imagI y,_
     imagJ x + imagJ y, imagK x + imagK y)

   - x              ==
     octon(- real x, - imagi x, - imagj x, - imagk x,_
     - imagE x, - imagI x, - imagJ x, - imagK x)

   r:R * x:%        ==
     octon(r * real x, r * imagi x, r * imagj x, r * imagk x,_
     r * imagE x, r * imagI x, r * imagJ x, r * imagK x)

   n:Integer * x:%  ==
     octon(n * real x, n * imagi x, n * imagj x, n * imagk x,_
     n * imagE x, n * imagI x, n * imagJ x, n * imagK x)
```

```
coerce(r:R)      ==
  octon(r,0$R,0$R,0$R,0$R,0$R,0$R,0$R)

coerce(n:Integer)      ==
  octon(n :: R,0$R,0$R,0$R,0$R,0$R,0$R,0$R)

zero? x ==
  zero? real x and zero? imagi x and _
  zero? imagj x and zero? imagk x and _
  zero? imagE x and zero? imagI x and _
  zero? imagJ x and zero? imagK x

retract(x):R ==
  not (zero? imagi x and zero? imagj x and zero? imagk x and _
  zero? imagE x and zero? imagI x and zero? imagJ x and zero? imagK x)=>
    error "Cannot retract octonion."
  real x

retractIfCan(x):Union(R,"failed") ==
  not (zero? imagi x and zero? imagj x and zero? imagk x and _
  zero? imagE x and zero? imagI x and zero? imagJ x and zero? imagK x)=>
    "failed"
  real x

coerce(x:%):OutputForm ==
    part,z : OutputForm
    y : %
    zero? x => (0$R) :: OutputForm
    not zero?(real x) =>
      y := octon(0$R,imagi(x),imagj(x),imagk(x),imagE(x),
        imagI(x),imagJ(x),imagK(x))
      zero? y => real(x) :: OutputForm
      (real(x) :: OutputForm) + (y :: OutputForm)
    -- we know that the real part is 0
    not zero?(imagi(x)) =>
      y := octon(0$R,0$R,imagj(x),imagk(x),imagE(x),
        imagI(x),imagJ(x),imagK(x))
      z :=
        part := "i"::Symbol::OutputForm
        (imagi(x) = 1) => part
        (imagi(x) :: OutputForm) * part
      zero? y => z
      z + (y :: OutputForm)
    -- we know that the real part and i part are 0
    not zero?(imagj(x)) =>
      y := octon(0$R,0$R,0$R,imagk(x),imagE(x),
        imagI(x),imagJ(x),imagK(x))
      z :=
        part := "j"::Symbol::OutputForm
        (imagj(x) = 1) => part
        (imagj(x) :: OutputForm) * part
      zero? y => z
      z + (y :: OutputForm)
```

```
    -- we know that the real part and i and j parts are 0
    not zero?(imagk(x)) =>
      y := octon(0$R,0$R,0$R,0$R,imagE(x),
        imagI(x),imagJ(x),imagK(x))
      z :=
        part := "k"::Symbol::OutputForm
        (imagk(x) = 1) => part
        (imagk(x) :: OutputForm) * part
      zero? y => z
      z + (y :: OutputForm)
    -- we know that the real part,i,j,k parts are 0
    not zero?(imagE(x)) =>
      y := octon(0$R,0$R,0$R,0$R,0$R,
        imagI(x),imagJ(x),imagK(x))
      z :=
        part := "E"::Symbol::OutputForm
        (imagE(x) = 1) => part
        (imagE(x) :: OutputForm) * part
      zero? y => z
      z + (y :: OutputForm)
    -- we know that the real part,i,j,k,E parts are 0
    not zero?(imagI(x)) =>
      y := octon(0$R,0$R,0$R,0$R,0$R,0$R,imagJ(x),imagK(x))
      z :=
        part := "I"::Symbol::OutputForm
        (imagI(x) = 1) => part
        (imagI(x) :: OutputForm) * part
      zero? y => z
      z + (y :: OutputForm)
    -- we know that the real part,i,j,k,E,I parts are 0
    not zero?(imagJ(x)) =>
      y := octon(0$R,0$R,0$R,0$R,0$R,0$R,0$R,imagK(x))
      z :=
        part := "J"::Symbol::OutputForm
        (imagJ(x) = 1) => part
        (imagJ(x) :: OutputForm) * part
      zero? y => z
      z + (y :: OutputForm)
    -- we know that the real part,i,j,k,E,I,J parts are 0
    part := "K"::Symbol::OutputForm
    (imagK(x) = 1) => part
    (imagK(x) :: OutputForm) * part

if R has Field then

  inv x ==
    (norm x) = 0 => error "This octonion is not invertible."
    (inv norm x) * conjugate x

if R has ConvertibleTo InputForm then

  convert(x:%):InputForm ==
    l : List InputForm := [convert("octon" :: Symbol),
      convert(real x)$R, convert(imagi x)$R, convert(imagj x)$R,_
```

```
            convert(imagk x)$R, convert(imagE x)$R,_
            convert(imagI x)$R, convert(imagJ x)$R,_
            convert(imagK x)$R]
        convert(l)$InputForm

    if R has OrderedSet then

      x < y ==
        real x = real y =>
         imagi x = imagi y =>
          imagj x = imagj y =>
           imagk x = imagk y =>
            imagE x = imagE y =>
             imagI x = imagI y =>
              imagJ x = imagJ y =>
               imagK x < imagK y
              imagJ x < imagJ y
             imagI x < imagI y
            imagE x < imagE y
           imagk x < imagk y
          imagj x < imagj y
         imagi x < imagi y
        real x < real y

    if R has RealNumberSystem then

      abs x == sqrt norm x

    if R has IntegerNumberSystem then

      rational? x ==
        (zero? imagi x) and (zero? imagj x) and (zero? imagk x) and _
        (zero? imagE x) and (zero? imagI x) and (zero? imagJ x) and _
        (zero? imagK x)

      rational  x ==
        rational? x => rational real x
        error "Not a rational number"

      rationalIfCan x ==
        rational? x => rational real x
        "failed"
```

——————

— **COQ OC** —

```
(* category OC *)
(*

    characteristic : () -> NonNegativeInteger
    characteristic() ==
      characteristic()$R
```

```
conjugate : % -> %
conjugate x ==
  octon(real x, - imagi x, - imagj x, - imagk x, - imagE x,_
  - imagI x, - imagJ x, - imagK x)

map : ((R -> R),%) -> %
map(fn, x)        ==
  octon(fn real x,fn imagi x,fn imagj x,fn imagk x, fn imagE x,_
  fn imagI x, fn imagJ x,fn imagK x)

norm : % -> R
norm x ==
  real x * real x + imagi x * imagi x + _
  imagj x * imagj x + imagk x * imagk x + _
  imagE x * imagE x + imagI x * imagI x + _
  imagJ x * imagJ x + imagK x * imagK x

?=? : (%,%) -> Boolean
x = y ==
  (real x = real y) and (imagi x = imagi y) and _
  (imagj x = imagj y) and (imagk x = imagk y) and _
  (imagE x = imagE y) and (imagI x = imagI y) and _
  (imagJ x = imagJ y) and (imagK x = imagK y)

?+? : (%,%) -> %
x + y ==
  octon(real x + real y, imagi x + imagi y,_
  imagj x + imagj y, imagk x + imagk y,_
  imagE x + imagE y, imagI x + imagI y,_
  imagJ x + imagJ y, imagK x + imagK y)

-? : % -> %
- x ==
  octon(- real x, - imagi x, - imagj x, - imagk x,_
  - imagE x, - imagI x, - imagJ x, - imagK x)

?*? : (R,%) -> %
r:R * x:% ==
  octon(r * real x, r * imagi x, r * imagj x, r * imagk x,_
  r * imagE x, r * imagI x, r * imagJ x, r * imagK x)

?*? : (Integer,%) -> %
n:Integer * x:%  ==
  octon(n * real x, n * imagi x, n * imagj x, n * imagk x,_
  n * imagE x, n * imagI x, n * imagJ x, n * imagK x)

coerce : R -> %
coerce(r:R) ==
  octon(r,0$R,0$R,0$R,0$R,0$R,0$R,0$R)

coerce : Integer -> %
coerce(n:Integer)      ==
  octon(n :: R,0$R,0$R,0$R,0$R,0$R,0$R,0$R)
```

```
zero? : % -> Boolean
zero? x ==
  zero? real x and zero? imagi x and _
  zero? imagj x and zero? imagk x and _
  zero? imagE x and zero? imagI x and _
  zero? imagJ x and zero? imagK x

retract : % -> R
retract(x):R ==
  not (zero? imagi x and zero? imagj x and zero? imagk x and _
  zero? imagE x and zero? imagI x and zero? imagJ x and zero? imagK x)=>
    error "Cannot retract octonion."
  real x

rationalIfCan : % -> Union(Fraction(Integer),"failed")
retractIfCan(x):Union(R,"failed") ==
  not (zero? imagi x and zero? imagj x and zero? imagk x and _
  zero? imagE x and zero? imagI x and zero? imagJ x and zero? imagK x)=>
    "failed"
  real x

coerce : % -> OutputForm
coerce(x:%):OutputForm ==
    part,z : OutputForm
    y : %
    zero? x => (0$R) :: OutputForm
    not zero?(real x) =>
      y := octon(0$R,imagi(x),imagj(x),imagk(x),imagE(x),
        imagI(x),imagJ(x),imagK(x))
      zero? y => real(x) :: OutputForm
      (real(x) :: OutputForm) + (y :: OutputForm)
    -- we know that the real part is 0
    not zero?(imagi(x)) =>
      y := octon(0$R,0$R,imagj(x),imagk(x),imagE(x),
        imagI(x),imagJ(x),imagK(x))
      z :=
        part := "i"::Symbol::OutputForm
        (imagi(x) = 1) => part
        (imagi(x) :: OutputForm) * part
      zero? y => z
      z + (y :: OutputForm)
    -- we know that the real part and i part are 0
    not zero?(imagj(x)) =>
      y := octon(0$R,0$R,0$R,imagk(x),imagE(x),
        imagI(x),imagJ(x),imagK(x))
      z :=
        part := "j"::Symbol::OutputForm
        (imagj(x) = 1) => part
        (imagj(x) :: OutputForm) * part
      zero? y => z
      z + (y :: OutputForm)
    -- we know that the real part and i and j parts are 0
    not zero?(imagk(x)) =>
```

```
      y := octon(0$R,0$R,0$R,0$R,imagE(x),
        imagI(x),imagJ(x),imagK(x))
      z :=
        part := "k"::Symbol::OutputForm
        (imagk(x) = 1) => part
        (imagk(x) :: OutputForm) * part
      zero? y => z
      z + (y :: OutputForm)
    -- we know that the real part,i,j,k parts are 0
    not zero?(imagE(x)) =>
      y := octon(0$R,0$R,0$R,0$R,0$R,
        imagI(x),imagJ(x),imagK(x))
      z :=
        part := "E"::Symbol::OutputForm
        (imagE(x) = 1) => part
        (imagE(x) :: OutputForm) * part
      zero? y => z
      z + (y :: OutputForm)
    -- we know that the real part,i,j,k,E parts are 0
    not zero?(imagI(x)) =>
      y := octon(0$R,0$R,0$R,0$R,0$R,0$R,imagJ(x),imagK(x))
      z :=
        part := "I"::Symbol::OutputForm
        (imagI(x) = 1) => part
        (imagI(x) :: OutputForm) * part
      zero? y => z
      z + (y :: OutputForm)
    -- we know that the real part,i,j,k,E,I parts are 0
    not zero?(imagJ(x)) =>
      y := octon(0$R,0$R,0$R,0$R,0$R,0$R,0$R,imagK(x))
      z :=
        part := "J"::Symbol::OutputForm
        (imagJ(x) = 1) => part
        (imagJ(x) :: OutputForm) * part
      zero? y => z
      z + (y :: OutputForm)
    -- we know that the real part,i,j,k,E,I,J parts are 0
    part := "K"::Symbol::OutputForm
    (imagK(x) = 1) => part
    (imagK(x) :: OutputForm) * part

if R has Field then

  inv : % -> %
  inv x ==
    (norm x) = 0 => error "This octonion is not invertible."
    (inv norm x) * conjugate x

if R has ConvertibleTo InputForm then

  convert : % -> InputForm
  convert(x:%):InputForm ==
    l : List InputForm := [convert("octon" :: Symbol),
      convert(real x)$R, convert(imagi x)$R, convert(imagj x)$R,_
```

```
        convert(imagk x)$R, convert(imagE x)$R,_
        convert(imagI x)$R, convert(imagJ x)$R,_
        convert(imagK x)$R]
      convert(l)$InputForm

  if R has OrderedSet then

    ?<? : (%,%) -> Boolean
    x < y ==
      real x = real y =>
       imagi x = imagi y =>
        imagj x = imagj y =>
         imagk x = imagk y =>
          imagE x = imagE y =>
           imagI x = imagI y =>
            imagJ x = imagJ y =>
             imagK x < imagK y
            imagJ x < imagJ y
           imagI x < imagI y
          imagE x < imagE y
         imagk x < imagk y
        imagj x < imagj y
       imagi x < imagi y
      real x < real y

  if R has RealNumberSystem then

    abs : % -> R
    abs x == sqrt norm x

  if R has IntegerNumberSystem then

    rational? : % -> Boolean
    rational? x ==
      (zero? imagi x) and (zero? imagj x) and (zero? imagk x) and _
      (zero? imagE x) and (zero? imagI x) and (zero? imagJ x) and _
      (zero? imagK x)

    rational : % -> Fraction(Integer)
    rational  x ==
      rational? x => rational real x
      error "Not a rational number"

    rationalIfCan : % -> Union(Fraction(Integer),"failed")
    rationalIfCan x ==
      rational? x => rational real x
      "failed"
*)
```

— OC.dotabb —

```
"OC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OC"];
"OC" -> "ALGEBRA"
"OC" -> "FEVALAB"
"OC" -> "FRETRCT"
```

————

— **OC.dotfull** —

```
"OctonionCategory(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OC"];
"OctonionCategory(a:CommutativeRing)" -> "Algebra(a:CommutativeRing)"
"OctonionCategory(a:CommutativeRing)" -> "FullyEvalableOver(CommutativeRing)"
"OctonionCategory(a:CommutativeRing)" ->
   "FullyRetractableTo(a:CommutativeRing)"
```

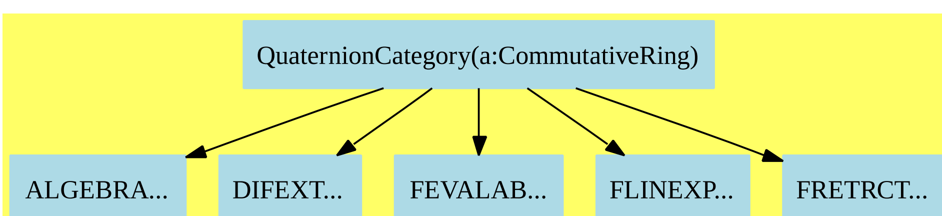
————

— **OC.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"OctonionCategory(a:CommutativeRing)" [color=lightblue];
"OctonionCategory(a:CommutativeRing)" -> "ALGEBRA..."
"OctonionCategory(a:CommutativeRing)" -> "FEVALAB..."
"OctonionCategory(a:CommutativeRing)" -> "FRETRCT..."

"ALGEBRA..." [color=lightblue];
"FEVALAB..." [color=lightblue];
"FRETRCT..." [color=lightblue];
}
```

————

12.0.191 QuaternionCategory (QUATCAT)



— **QuaternionCategory.input** —

```
)set break resume
)sys rm -f QuaternionCategory.output
)spool QuaternionCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show QuaternionCategory
--R
--R QuaternionCategory(R: CommutativeRing) is a category constructor
--R Abbreviation for QuaternionCategory is QUATCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for QUATCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R D : (%,(R -> R)) -> %                 D : % -> % if R has DIFRING
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R abs : % -> R if R has RNS             coerce : R -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R conjugate : % -> %                    differentiate : (%,(R -> R)) -> %
--R hash : % -> SingleInteger             imagI : % -> R
--R imagJ : % -> R                        imagK : % -> R
--R inv : % -> % if R has FIELD           latex : % -> String
--R map : ((R -> R),%) -> %               max : (%,%) -> % if R has ORDSET
--R min : (%,%) -> % if R has ORDSET      norm : % -> R
--R one? : % -> Boolean                   quatern : (R,R,R,R) -> %
--R real : % -> R                         recip : % -> Union(%,"failed")
--R retract : % -> R                      sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (Fraction(Integer),%) -> % if R has FIELD
--R ?*? : (%,Fraction(Integer)) -> % if R has FIELD
--R ?**? : (%,Integer) -> % if R has FIELD
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R D : (%,(R -> R),NonNegativeInteger) -> %
--R D : (%,List(Symbol),List(NonNegativeInteger)) -> % if R has PDRING(SYMBOL)
--R D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING(SYMBOL)
--R D : (%,List(Symbol)) -> % if R has PDRING(SYMBOL)
--R D : (%,Symbol) -> % if R has PDRING(SYMBOL)
--R D : (%,NonNegativeInteger) -> % if R has DIFRING
--R ?^? : (%,Integer) -> % if R has FIELD
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
```

```
--R coerce : Fraction(Integer) -> % if R has FIELD or R has RETRACT(FRAC(INT))
--R convert : % -> InputForm if R has KONVERT(INFORM)
--R differentiate : (%,(R -> R),NonNegativeInteger) -> %
--R differentiate : (%,List(Symbol),List(NonNegativeInteger)) -> % if R has PDRING(SYMBOL)
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING(SYMBOL)
--R differentiate : (%,List(Symbol)) -> % if R has PDRING(SYMBOL)
--R differentiate : (%,Symbol) -> % if R has PDRING(SYMBOL)
--R differentiate : (%,NonNegativeInteger) -> % if R has DIFRING
--R differentiate : % -> % if R has DIFRING
--R ?.? : (%,R) -> % if R has ELTAB(R,R)
--R eval : (%,Symbol,R) -> % if R has IEVALAB(SYMBOL,R)
--R eval : (%,List(Symbol),List(R)) -> % if R has IEVALAB(SYMBOL,R)
--R eval : (%,List(Equation(R))) -> % if R has EVALAB(R)
--R eval : (%,Equation(R)) -> % if R has EVALAB(R)
--R eval : (%,R,R) -> % if R has EVALAB(R)
--R eval : (%,List(R),List(R)) -> % if R has EVALAB(R)
--R rational : % -> Fraction(Integer) if R has INS
--R rational? : % -> Boolean if R has INS
--R rationalIfCan : % -> Union(Fraction(Integer),"failed") if R has INS
--R reducedSystem : Matrix(%) -> Matrix(R)
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(R),vec: Vector(R))
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(Integer),vec: Vector(Integer)) if R has LINEXP(I
--R reducedSystem : Matrix(%) -> Matrix(Integer) if R has LINEXP(INT)
--R retract : % -> Fraction(Integer) if R has RETRACT(FRAC(INT))
--R retract : % -> Integer if R has RETRACT(INT)
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction(Integer),"failed") if R has RETRACT(FRAC(INT))
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT(INT)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

— **QuaternionCategory.help** —

```
====================================================================
QuaternionCategory examples
====================================================================

QuaternionCategory describes the category of quaternions and implements
functions that are not representation specific.

See Also:
o )show QuaternionCategory
```

See:

⇐ “Algebra” (ALGEBRA) 11.0.176 on page 1121

⇐ “DifferentialExtension” (DIFEXT) 11.0.177 on page 1127
⇐ “FullyEvalableOver” (FEVALAB) 4.0.67 on page 246
⇐ “FullyLinearlyExplicitRingOver” (FLINEXP) 11.0.178 on page 1134
⇐ “FullyRetractableTo” (FRETRCT) 3.0.50 on page 158

Exports:

0	1	abs	characteristic
charthRoot	coerce	conjugate	convert
D	differentiate	eval	hash
imagI	imagJ	imagK	inv
latex	map	max	min
norm	one?	quatern	rational
rational?	rationalIfCan	real	recip
reducedSystem	retract	retractIfCan	sample
subtractIfCan	zero?	?*?	?**?
?+?	?-?	-?	?=?
?^?	?~=?	?<?	?<=?
?>?	?>=?	?.?	

Attributes Exported:

- if #1 has EntireRing then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
imagI : % -> R
imagJ : % -> R
imagK : % -> R
quatern : (R,R,R,R) -> %
real : % -> R
```

These are implemented by this category:

```
abs : % -> R if R has RNS
characteristic : () -> NonNegativeInteger
coerce : R -> %
coerce : Integer -> %
coerce : % -> OutputForm
conjugate : % -> %
convert : % -> InputForm if R has KONVERT INFORM
differentiate : (%,(R -> R)) -> %
inv : % -> % if R has FIELD
map : ((R -> R),%) -> %
norm : % -> R
one? : % -> Boolean
rational : % -> Fraction Integer if R has INS
rational? : % -> Boolean if R has INS
rationalIfCan : % -> Union(Fraction Integer,"failed") if R has INS
retract : % -> R
```

```
retractIfCan : % -> Union(R,"failed")
zero? : % -> Boolean
?=? : (%,%) -> Boolean
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?*? : (R,%) -> %
?*? : (Integer,%) -> %
?<? : (%,%) -> Boolean if R has ORDSET
```

These exports come from (p1121) Algebra(R:CommutativeRing):

```
0 : () -> %
1 : () -> %
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?*? : (%,R) -> %
```

These exports come from (p158) FullyRetractableTo(R:CommutativeRing):

```
coerce : Fraction Integer -> % if R has FIELD or R has RETRACT FRAC INT
retract : % -> Fraction Integer if R has RETRACT FRAC INT
retract : % -> Integer if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
  if R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
```

These exports come from (p1127) DifferentialExtension(R:CommutativeRing):

```
D : % -> % if R has DIFRING
D : (%,NonNegativeInteger) -> % if R has DIFRING
D : (%,(R -> R)) -> %
D : (%,(R -> R),NonNegativeInteger) -> %
D : (%,Symbol) -> % if R has PDRING SYMBOL
D : (%,List Symbol) -> % if R has PDRING SYMBOL
D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
D : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
  if R has PDRING SYMBOL
differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL
differentiate : (%,NonNegativeInteger) -> % if R has DIFRING
differentiate : % -> % if R has DIFRING
differentiate : (%,(R -> R),NonNegativeInteger) -> %
differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
?*? : (%,%) -> %
```

These exports come from (p246) FullyEvalableOver(R:CommutativeRing):

```
eval : (%,Equation R) -> % if R has EVALAB R
eval : (%,List Symbol,List R) -> % if R has IEVALAB(SYMBOL,R)
eval : (%,List Equation R) -> % if R has EVALAB R
eval : (%,R,R) -> % if R has EVALAB R
eval : (%,List R,List R) -> % if R has EVALAB R
eval : (%,Symbol,R) -> % if R has IEVALAB(SYMBOL,R)
?.? : (%,R) -> % if R has ELTAB(R,R)
```

These exports come from (p1134) FullyLinearlyExplicitRingOver(R) where R:CommutativeRing:

```
recip : % -> Union(%,"failed")
reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
reducedSystem :
  (Matrix %,Vector %) ->
     Record(mat: Matrix Integer,vec: Vector Integer)
       if R has LINEXP INT
reducedSystem : Matrix % -> Matrix R
reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
```

These exports come from (p326) OrderedSet():

```
max : (%,%) -> % if R has ORDSET
min : (%,%) -> % if R has ORDSET
?<=? : (%,%) -> Boolean if R has ORDSET
?>? : (%,%) -> Boolean if R has ORDSET
?>=? : (%,%) -> Boolean if R has ORDSET
```

These exports come from (p1185) DivisionRing():

```
?**? : (%,Integer) -> % if R has FIELD
?^? : (%,Integer) -> % if R has FIELD
?*? : (Fraction Integer,%) -> % if R has FIELD
?*? : (%,Fraction Integer) -> % if R has FIELD
```

These exports come from (p1010) CharacteristicNonZero():

```
charthRoot : % -> Union(%,"failed") if R has CHARNZ
```

— QuaternionCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#QUATCAT">
QuaternionCategory (QUATCAT)</a></h2>
</body>
```

— category QUATCAT QuaternionCategory —

```
)abbrev category QUATCAT QuaternionCategory
++ Author: Robert S. Sutor
++ Date Created: 23 May 1990
++ Change History:  10 September 1990
++ Description:
++ \spadtype{QuaternionCategory} describes the category of quaternions
++ and implements functions that are not representation specific.
```

```
QuaternionCategory(R) : Category == SIG where
  R : CommutativeRing

  AL    ==> Algebra(R)
  FRT   ==> FullyRetractableTo(R)
  DE    ==> DifferentialExtension(R)
  FEO   ==> FullyEvalableOver(R)
  FLERO ==> FullyLinearlyExplicitRingOver(R)

  SIG ==> Join(AL,FRT,DE,FEO,FLERO) with

    conjugate : $ -> $
      ++ conjugate(q) negates the imaginary parts of quaternion \spad{q}.

    imagI : $ -> R
      ++ imagI(q) extracts the imaginary i part of quaternion \spad{q}.

    imagJ : $ -> R
      ++ imagJ(q) extracts the imaginary j part of quaternion \spad{q}.

    imagK : $ -> R
      ++ imagK(q) extracts the imaginary k part of quaternion \spad{q}.

    norm : $ -> R
      ++ norm(q) computes the norm of \spad{q} (the sum of the
      ++ squares of the components).

    quatern : (R,R,R,R) -> $
      ++ quatern(r,i,j,k) constructs a quaternion from scalars.

    real : $ -> R
      ++ real(q) extracts the real part of quaternion \spad{q}.

    if R has EntireRing then EntireRing

    if R has OrderedSet then OrderedSet

    if R has Field then DivisionRing

    if R has ConvertibleTo InputForm then ConvertibleTo InputForm

    if R has CharacteristicZero then CharacteristicZero

    if R has CharacteristicNonZero then CharacteristicNonZero

    if R has RealNumberSystem then

      abs : $ -> R
        ++ abs(q) computes the absolute value of quaternion \spad{q}
        ++ (sqrt of norm).

    if R has IntegerNumberSystem then

      rational? : $ -> Boolean
```

```
    ++ rational?(q) returns {\it true} if all the imaginary
    ++ parts of \spad{q} are zero and the real part can be
    ++ converted into a rational number, and {\it false}
    ++ otherwise.

   rational : $ -> Fraction Integer
    ++ rational(q) tries to convert \spad{q} into a
    ++ rational number. Error: if this is not
    ++ possible. If \spad{rational?(q)} is true, the
    ++ conversion will be done and the rational number returned.

   rationalIfCan : $ -> Union(Fraction Integer, "failed")
    ++ rationalIfCan(q) returns \spad{q} as a rational number,
    ++ or "failed" if this is not possible.
    ++ Note that if \spad{rational?(q)} is true, the conversion
    ++ can be done and the rational number will be returned.

add

  characteristic() ==
    characteristic()$R

  conjugate x      ==
    quatern(real x, - imagI x, - imagJ x, - imagK x)

  map(fn, x)       ==
    quatern(fn real x, fn imagI x, fn imagJ x, fn imagK x)

  norm x ==
    real x * real x + imagI x * imagI x +
      imagJ x * imagJ x + imagK x * imagK x

  x = y ==
    (real x = real y) and (imagI x = imagI y) and
      (imagJ x = imagJ y) and (imagK x = imagK y)

  x + y ==
    quatern(real x + real y, imagI x + imagI y,
      imagJ x + imagJ y, imagK x + imagK y)

  x - y ==
    quatern(real x - real y, imagI x - imagI y,
      imagJ x - imagJ y, imagK x - imagK y)

  - x ==
    quatern(- real x, - imagI x, - imagJ x, - imagK x)

  r:R * x:$ ==
    quatern(r * real x, r * imagI x, r * imagJ x, r * imagK x)

  n:Integer * x:$  ==
    quatern(n * real x, n * imagI x, n * imagJ x, n * imagK x)

  differentiate(x:$, d:R -> R) ==
```

```
      quatern(d real x, d imagI x, d imagJ x, d imagK x)

    coerce(r:R) ==
      quatern(r,0$R,0$R,0$R)

    coerce(n:Integer) ==
      quatern(n :: R,0$R,0$R,0$R)

    one? x ==
      (real x) = 1 and zero? imagI x and
        zero? imagJ x and zero? imagK x

    zero? x ==
      zero? real x and zero? imagI x and
        zero? imagJ x and zero? imagK x

    retract(x):R ==
      not (zero? imagI x and zero? imagJ x and zero? imagK x) =>
        error "Cannot retract quaternion."
      real x

    retractIfCan(x):Union(R,"failed") ==
      not (zero? imagI x and zero? imagJ x and zero? imagK x) =>
        "failed"
      real x

    coerce(x:$):OutputForm ==
      part,z : OutputForm
      y : $
      zero? x => (0$R) :: OutputForm
      not zero?(real x) =>
        y := quatern(0$R,imagI(x),imagJ(x),imagK(x))
        zero? y => real(x) :: OutputForm
        (real(x) :: OutputForm) + (y :: OutputForm)
      -- we know that the real part is 0
      not zero?(imagI(x)) =>
        y := quatern(0$R,0$R,imagJ(x),imagK(x))
        z :=
          part := "i"::Symbol::OutputForm
          (imagI(x) = 1) => part
          (imagI(x) :: OutputForm) * part
        zero? y => z
        z + (y :: OutputForm)
      -- we know that the real part and i part are 0
      not zero?(imagJ(x)) =>
        y := quatern(0$R,0$R,0$R,imagK(x))
        z :=
          part := "j"::Symbol::OutputForm
          (imagJ(x) = 1) => part
          (imagJ(x) :: OutputForm) * part
        zero? y => z
        z + (y :: OutputForm)
      -- we know that the real part and i and j parts are 0
      part := "k"::Symbol::OutputForm
```

```
    (imagK(x) = 1) => part
    (imagK(x) :: OutputForm) * part

  if R has Field then

    inv x ==
      norm x = 0 => error "This quaternion is not invertible."
      (inv norm x) * conjugate x

  if R has ConvertibleTo InputForm then

    convert(x:$):InputForm ==
      l : List InputForm := [convert("quatern" :: Symbol),
        convert(real x)$R, convert(imagI x)$R, convert(imagJ x)$R,
          convert(imagK x)$R]
      convert(l)$InputForm

  if R has OrderedSet then

    x < y ==
      real x = real y =>
        imagI x = imagI y =>
          imagJ x = imagJ y =>
            imagK x < imagK y
          imagJ x < imagJ y
        imagI x < imagI y
      real x < real y

  if R has RealNumberSystem then

    abs x == sqrt norm x

  if R has IntegerNumberSystem then

    rational? x ==
      (zero? imagI x) and (zero? imagJ x) and (zero? imagK x)

    rational  x ==
      rational? x => rational real x
      error "Not a rational number"

    rationalIfCan x ==
      rational? x => rational real x
      "failed"
```

— **COQ QUATCAT** —

```
(* category QUATCAT *)
(*

       characteristic : () -> NonNegativeInteger
```

```
characteristic() ==
  characteristic()$R

conjugate : % -> %
conjugate x ==
  quatern(real x, - imagI x, - imagJ x, - imagK x)

map : ((R -> R),%) -> %
map(fn, x) ==
  quatern(fn real x, fn imagI x, fn imagJ x, fn imagK x)

norm : % -> R
norm x ==
  real x * real x + imagI x * imagI x +
    imagJ x * imagJ x + imagK x * imagK x

?=? : (%,%) -> Boolean
x = y ==
  (real x = real y) and (imagI x = imagI y) and
    (imagJ x = imagJ y) and (imagK x = imagK y)

?+? : (%,%) -> %
x + y ==
  quatern(real x + real y, imagI x + imagI y,
    imagJ x + imagJ y, imagK x + imagK y)

?-? : (%,%) -> %
x - y ==
  quatern(real x - real y, imagI x - imagI y,
    imagJ x - imagJ y, imagK x - imagK y)

-? : % -> %
- x ==
  quatern(- real x, - imagI x, - imagJ x, - imagK x)

?*? : (R,%) -> %
r:R * x:$ ==
  quatern(r * real x, r * imagI x, r * imagJ x, r * imagK x)

?*? : (Integer,%) -> %
n:Integer * x:$  ==
  quatern(n * real x, n * imagI x, n * imagJ x, n * imagK x)

differentiate : (%,(R -> R)) -> %
differentiate(x:$, d:R -> R) ==
  quatern(d real x, d imagI x, d imagJ x, d imagK x)

coerce : R -> %
coerce(r:R) ==
  quatern(r,0$R,0$R,0$R)

coerce : Integer -> %
coerce(n:Integer) ==
  quatern(n :: R,0$R,0$R,0$R)
```

```
one? : % -> Boolean
one? x ==
  (real x) = 1 and zero? imagI x and
    zero? imagJ x and zero? imagK x

zero? : % -> Boolean
zero? x ==
  zero? real x and zero? imagI x and
    zero? imagJ x and zero? imagK x

retract : % -> R
retract(x):R ==
  not (zero? imagI x and zero? imagJ x and zero? imagK x) =>
    error "Cannot retract quaternion."
  real x

rationalIfCan : % -> Union(Fraction(Integer),"failed")
retractIfCan(x):Union(R,"failed") ==
  not (zero? imagI x and zero? imagJ x and zero? imagK x) =>
    "failed"
  real x

coerce : % -> OutputForm
coerce(x:$):OutputForm ==
  part,z : OutputForm
  y : $
  zero? x => (0$R) :: OutputForm
  not zero?(real x) =>
    y := quatern(0$R,imagI(x),imagJ(x),imagK(x))
    zero? y => real(x) :: OutputForm
    (real(x) :: OutputForm) + (y :: OutputForm)
  -- we know that the real part is 0
  not zero?(imagI(x)) =>
    y := quatern(0$R,0$R,imagJ(x),imagK(x))
    z :=
      part := "i"::Symbol::OutputForm
      (imagI(x) = 1) => part
      (imagI(x) :: OutputForm) * part
    zero? y => z
    z + (y :: OutputForm)
  -- we know that the real part and i part are 0
  not zero?(imagJ(x)) =>
    y := quatern(0$R,0$R,0$R,imagK(x))
    z :=
      part := "j"::Symbol::OutputForm
      (imagJ(x) = 1) => part
      (imagJ(x) :: OutputForm) * part
    zero? y => z
    z + (y :: OutputForm)
  -- we know that the real part and i and j parts are 0
  part := "k"::Symbol::OutputForm
  (imagK(x) = 1) => part
  (imagK(x) :: OutputForm) * part
```

```
    if R has Field then

      inv : % -> %
      inv x ==
        norm x = 0 => error "This quaternion is not invertible."
        (inv norm x) * conjugate x

    if R has ConvertibleTo InputForm then

      convert : % -> InputForm
      convert(x:$):InputForm ==
        l : List InputForm := [convert("quatern" :: Symbol),
          convert(real x)$R, convert(imagI x)$R, convert(imagJ x)$R,
            convert(imagK x)$R]
        convert(l)$InputForm

    if R has OrderedSet then

      ?<? : (%,%) -> Boolean
      x < y ==
        real x = real y =>
          imagI x = imagI y =>
            imagJ x = imagJ y =>
              imagK x < imagK y
            imagJ x < imagJ y
          imagI x < imagI y
        real x < real y

    if R has RealNumberSystem then

      abs : % -> R
      abs x == sqrt norm x

    if R has IntegerNumberSystem then

      rational? : % -> Boolean
      rational? x ==
        (zero? imagI x) and (zero? imagJ x) and (zero? imagK x)

      rational : % -> Fraction(Integer)
      rational  x ==
        rational? x => rational real x
        error "Not a rational number"

      rationalIfCan : % -> Union(Fraction(Integer),"failed")
      rationalIfCan x ==
        rational? x => rational real x
        "failed"
*)
```

———-

— **QUATCAT.dotabb** —

```
"QUATCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=QUATCAT"];
"QUATCAT" -> "ALGEBRA"
"QUATCAT" -> "DIFEXT"
"QUATCAT" -> "FEVALAB"
"QUATCAT" -> "FLINEXP"
"QUATCAT" -> "FRETRCT"
```

————-

— **QUATCAT.dotfull** —

```
"QuaternionCategory(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=QUATCAT"];
"QuaternionCategory(a:CommutativeRing)" ->
   "Algebra(a:CommutativeRing)"
"QuaternionCategory(a:CommutativeRing)" ->
   "DifferentialExtension(CommutativeRing)"
"QuaternionCategory(a:CommutativeRing)" ->
   "FullyEvalableOver(CommutativeRing)"
"QuaternionCategory(a:CommutativeRing)" ->
   "FullyLinearlyExplicitRingOver(a:CommutativeRing)"
"QuaternionCategory(a:CommutativeRing)" ->
   "FullyRetractableTo(a:CommutativeRing)"
```

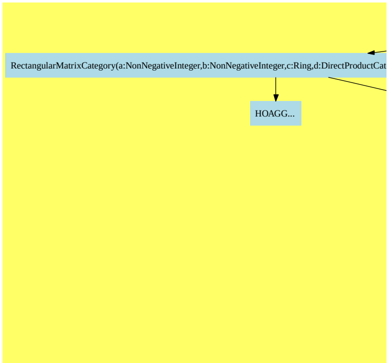
————-

— **QUATCAT.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"QuaternionCategory(a:CommutativeRing)" [color=lightblue];
"QuaternionCategory(a:CommutativeRing)" -> "ALGEBRA..."
"QuaternionCategory(a:CommutativeRing)" -> "DIFEXT..."
"QuaternionCategory(a:CommutativeRing)" -> "FEVALAB..."
"QuaternionCategory(a:CommutativeRing)" -> "FLINEXP..."
"QuaternionCategory(a:CommutativeRing)" -> "FRETRCT..."

"ALGEBRA..." [color=lightblue];
"DIFEXT..." [color=lightblue];
"FEVALAB..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"FRETRCT..." [color=lightblue];
}
```

————-

12.0.192 SquareMatrixCategory (SMATCAT)



We define three categories for matrices

- MatrixCategory is the category of all matrices
- RectangularMatrixCategory is the category of all matrices of a given dimension
- SquareMatrixCategory inherits from RectangularMatrixCategory

The SquareMatrix domain is for square matrices of fixed dimension.

— SquareMatrixCategory.input —

```
)set break resume
)sys rm -f SquareMatrixCategory.output
)spool SquareMatrixCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show SquareMatrixCategory
--R
--R SquareMatrixCategory(ndim: NonNegativeInteger,R: Ring,Row: DirectProductCategory(t#1,t#2),Col: DirectProduct
--R Abbreviation for SquareMatrixCategory is SMATCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for SMATCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Row,%) -> Row                   ?*? : (%,Col) -> Col
--R ?*? : (R,%) -> %                       ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                       ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %      ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %     ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                       ?-? : (%,%) -> %
--R -? : % -> %                            ?/? : (%,R) -> % if R has FIELD
--R ?=? : (%,%) -> Boolean                 D : % -> % if R has DIFRING
--R D : (%,(R -> R)) -> %                  1 : () -> %
--R 0 : () -> %                            ?^? : (%,NonNegativeInteger) -> %
--R ?^? : (%,PositiveInteger) -> %         antisymmetric? : % -> Boolean
--R coerce : R -> %                        coerce : Integer -> %
```

```
--R coerce : % -> OutputForm              column : (%,Integer) -> Col
--R copy : % -> %                         diagonal : % -> Row
--R diagonal? : % -> Boolean              diagonalMatrix : List(R) -> %
--R diagonalProduct : % -> R              differentiate : (%,(R -> R)) -> %
--R elt : (%,Integer,Integer) -> R        elt : (%,Integer,Integer,R) -> R
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                hash : % -> SingleInteger
--R latex : % -> String                   listOfLists : % -> List(List(R))
--R map : ((R -> R),%) -> %               map : (((R,R) -> R),%,%) -> %
--R matrix : List(List(R)) -> %           maxColIndex : % -> Integer
--R maxRowIndex : % -> Integer            minColIndex : % -> Integer
--R minRowIndex : % -> Integer            ncols : % -> NonNegativeInteger
--R nrows : % -> NonNegativeInteger       one? : % -> Boolean
--R qelt : (%,Integer,Integer) -> R       recip : % -> Union(%,"failed")
--R retract : % -> R                      row : (%,Integer) -> Row
--R sample : () -> %                      scalarMatrix : R -> %
--R square? : % -> Boolean                symmetric? : % -> Boolean
--R trace : % -> R                        zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?**? : (%,Integer) -> % if R has FIELD
--R D : (%,NonNegativeInteger) -> % if R has DIFRING
--R D : (%,Symbol) -> % if R has PDRING(SYMBOL)
--R D : (%,List(Symbol)) -> % if R has PDRING(SYMBOL)
--R D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING(SYMBOL)
--R D : (%,List(Symbol),List(NonNegativeInteger)) -> % if R has PDRING(SYMBOL)
--R D : (%,(R -> R),NonNegativeInteger) -> %
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R characteristic : () -> NonNegativeInteger
--R coerce : Fraction(Integer) -> % if R has RETRACT(FRAC(INT))
--R count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R determinant : % -> R if R has commutative(*)
--R differentiate : % -> % if R has DIFRING
--R differentiate : (%,NonNegativeInteger) -> % if R has DIFRING
--R differentiate : (%,Symbol) -> % if R has PDRING(SYMBOL)
--R differentiate : (%,List(Symbol)) -> % if R has PDRING(SYMBOL)
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING(SYMBOL)
--R differentiate : (%,List(Symbol),List(NonNegativeInteger)) -> % if R has PDRING(SYMBOL)
--R differentiate : (%,(R -> R),NonNegativeInteger) -> %
--R eval : (%,List(R),List(R)) -> % if R has EVALAB(R) and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB(R) and R has SETCAT
--R eval : (%,Equation(R)) -> % if R has EVALAB(R) and R has SETCAT
--R eval : (%,List(Equation(R))) -> % if R has EVALAB(R) and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R inverse : % -> Union(%,"failed") if R has FIELD
--R less? : (%,NonNegativeInteger) -> Boolean
--R map! : ((R -> R),%) -> % if $ has shallowlyMutable
--R member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
--R members : % -> List(R) if $ has finiteAggregate
--R minordet : % -> R if R has commutative(*)
--R more? : (%,NonNegativeInteger) -> Boolean
--R nullSpace : % -> List(Col) if R has INTDOM
```

```
--R nullity : % -> NonNegativeInteger if R has INTDOM
--R parts : % -> List(R) if $ has finiteAggregate
--R rank : % -> NonNegativeInteger if R has INTDOM
--R reducedSystem : Matrix(%) -> Matrix(R)
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(R),vec: Vector(R))
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(Integer),vec: Vector(Integer)) if R has LINEXP(I
--R reducedSystem : Matrix(%) -> Matrix(Integer) if R has LINEXP(INT)
--R retract : % -> Fraction(Integer) if R has RETRACT(FRAC(INT))
--R retract : % -> Integer if R has RETRACT(INT)
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction(Integer),"failed") if R has RETRACT(FRAC(INT))
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT(INT)
--R rowEchelon : % -> % if R has EUCDOM
--R size? : (%,NonNegativeInteger) -> Boolean
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R
--E 1

)spool
)lisp (bye)
```

———-

— SquareMatrixCategory.help —

```
====================================================================
SquareMatrixCategory examples
====================================================================

SquareMatrixCategory is a general square matrix category which allows
different representations and indexing schemes.  Rows and columns may
be extracted with rows returned as objects of type Row and colums
returned as objects of type Col.

See Also:
o )show SquareMatrixCategory
```

———-

See:

⇐ "BiModule" (BMODULE) 9.0.146 on page 902
⇐ "DifferentialExtension" (DIFEXT) 11.0.177 on page 1127
⇐ "FullyLinearlyExplicitRingOver" (FLINEXP) 11.0.178 on page 1134
⇐ "FullyRetractableTo" (FRETRCT) 3.0.50 on page 158
⇐ "MatrixCategory" (MATCAT) 6.0.112 on page 540
⇐ "RectangularMatrixCategory" (RMATCAT) 10.0.171 on page 1075

Exports:

0	1	antisymmetric?
any?	characteristic	coerce
column	copy	count
D	determinant	differentiate
diagonal	diagonal?	diagonalMatrix
diagonalProduct	elt	empty
empty?	eq?	eval
every?	exquo	hash
inverse	latex	less?
listOfLists	map	map!
matrix	maxColIndex	maxRowIndex
member?	members	minColIndex
minordet	minRowIndex	more?
ncols	nrows	nullSpace
nullity	one?	parts
qelt	rank	recip
reducedSystem	retract	retractIfCan
row	rowEchelon	sample
scalarMatrix	size?	square?
subtractIfCan	symmetric?	trace
zero?	#?	?^?
?*?	?**?	?+?
?-?	-?	?=?
?~=?	?/?	

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **nil**

TPDHERE: How did MATCAT get in the type tower?

```
determinant : % -> R if R has commutative *
inverse : % -> Union(%,"failed") if R has FIELD
```

These are directly exported but not implemented:

```
diagonalMatrix : List R -> %
minordet : % -> R if R has commutative *
scalarMatrix : R -> %
?*? : (Row,%) -> Row
?*? : (%,Col) -> Col
```

These are implemented by this category:

```
coerce : R -> %
diagonal : % -> Row
diagonalProduct : % -> R
differentiate : (%,(R -> R)) -> %
reducedSystem : Matrix % -> Matrix R
```

```
reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
retract : % -> R
retractIfCan : % -> Union(R,"failed")
trace : % -> R
?**? : (%,Integer) -> % if R has FIELD
?**? : (%,NonNegativeInteger) -> %
```

These exports come from (p1127) DifferentialExtension(R:Ring):

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
D : % -> % if R has DIFRING
D : (%,NonNegativeInteger) -> % if R has DIFRING
D : (%,(R -> R)) -> %
D : (%,(R -> R),NonNegativeInteger) -> %
D : (%,Symbol) -> % if R has PDRING SYMBOL
D : (%,List Symbol) -> % if R has PDRING SYMBOL
D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
D : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL
differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
differentiate : (%,NonNegativeInteger) -> % if R has DIFRING
differentiate : % -> % if R has DIFRING
differentiate : (%,(R -> R),NonNegativeInteger) -> %
differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p902) BiModule(R:Ring,R:Ring):

```
?*? : (R,%) -> %
?*? : (%,R) -> %
```

These exports come from
(p1075) RectangularMatrixCategory(ndim,ndim,R,Row,Col)

where ndim:NonNegativeInteger,R:Ring,Row:DirectProductCategory(ndim,R) Col:DirectProductCategory(ndim,R):

```
antisymmetric? : % -> Boolean
any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
column : (%,Integer) -> Col
copy : % -> %
count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
diagonal? : % -> Boolean
elt : (%,Integer,Integer) -> R
elt : (%,Integer,Integer,R) -> R
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
less? : (%,NonNegativeInteger) -> Boolean
listOfLists : % -> List List R
map : ((R -> R),%) -> %
map : (((R,R) -> R),%,%) -> %
map! : ((R -> R),%) -> % if $ has shallowlyMutable
matrix : List List R -> %
maxColIndex : % -> Integer
maxRowIndex : % -> Integer
minColIndex : % -> Integer
minRowIndex : % -> Integer
members : % -> List R if $ has finiteAggregate
member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
ncols : % -> NonNegativeInteger
nrows : % -> NonNegativeInteger
nullity : % -> NonNegativeInteger if R has INTDOM
nullSpace : % -> List Col if R has INTDOM
parts : % -> List R if $ has finiteAggregate
qelt : (%,Integer,Integer) -> R
rank : % -> NonNegativeInteger if R has INTDOM
row : (%,Integer) -> Row
rowEchelon : % -> % if R has EUCDOM
size? : (%,NonNegativeInteger) -> Boolean
square? : % -> Boolean
symmetric? : % -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
?/? : (%,R) -> % if R has FIELD
```

These exports come from (p158) FullyRetractableTo(R:Ring):

```
coerce : Fraction Integer -> % if R has RETRACT FRAC INT
retract : % -> Fraction Integer if R has RETRACT FRAC INT
retract : % -> Integer if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
```

These exports come from (p1134) FullyLinearlyExplicitRingOver(R:Ring):

```
reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if R has LINEXP INT
reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
```

— SquareMatrixCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#SMATCAT">
SquareMatrixCategory (SMATCAT)</a></h2>
</body>
```

———-

— category SMATCAT SquareMatrixCategory —

```
)abbrev category SMATCAT SquareMatrixCategory
++ Authors: Grabmeier, Gschnitzer, Williamson
++ Date Created: 1987
++ Date Last Updated: July 1990
++ Description:
++ \spadtype{SquareMatrixCategory} is a general square matrix category which
++ allows different representations and indexing schemes.  Rows and
++ columns may be extracted with rows returned as objects of
++ type Row and colums returned as objects of type Col.

SquareMatrixCategory(ndim,R,Row,Col) : Category == SIG where
  ndim : NonNegativeInteger
  R : Ring
  Row : DirectProductCategory(ndim,R)
  Col : DirectProductCategory(ndim,R)

  I     ==> Integer
  DE    ==> DifferentialExtension(R)
  BM    ==> BiModule(R,R)
  RMC   ==> RectangularMatrixCategory(ndim,ndim,R,Row,Col)
  FRT   ==> FullyRetractableTo(R)
  FLERO ==> FullyLinearlyExplicitRingOver(R)

  SIG ==> Join(DE,BM,RMC,FRT,FLERO) with

    if R has CommutativeRing then Module(R)

    scalarMatrix : R -> %
      ++ \spad{scalarMatrix(r)} returns an n-by-n matrix with r's on the
      ++ diagonal and zeroes elsewhere.

    diagonalMatrix : List R -> %
      ++ \spad{diagonalMatrix(l)} returns a diagonal matrix with the elements
      ++ of l on the diagonal.

    diagonal : % -> Row
      ++ \spad{diagonal(m)} returns a row consisting of the elements on the
      ++ diagonal of the matrix m.
```

```
 trace : % -> R
   ++ \spad{trace(m)} returns the trace of the matrix m. this is the sum
   ++ of the elements on the diagonal of the matrix m.

 diagonalProduct : % -> R
   ++ \spad{diagonalProduct(m)} returns the product of the elements on the
   ++ diagonal of the matrix m.

 "*" : (%,Col) -> Col
   ++ \spad{x * c} is the product of the matrix x and the column vector c.
   ++ Error: if the dimensions are incompatible.

 "*" : (Row,%) -> Row
   ++ \spad{r * x} is the product of the row vector r and the matrix x.
   ++ Error: if the dimensions are incompatible.

 --% Linear algebra

 if R has commutative("*") then

   Algebra R

   determinant : % -> R
     ++ \spad{determinant(m)} returns the determinant of the matrix m.

   minordet : % -> R
     ++ \spad{minordet(m)} computes the determinant of the matrix m
     ++ using minors.

 if R has Field then

   inverse : % -> Union(%,"failed")
     ++ \spad{inverse(m)} returns the inverse of the matrix m, if that
     ++ matrix is invertible and returns "failed" otherwise.

   "**" : (%,Integer) -> %
     ++ \spad{m**n} computes an integral power of the matrix m.
     ++ Error: if the matrix is not invertible.

add

  minr ==> minRowIndex
  maxr ==> maxRowIndex
  minc ==> minColIndex
  maxc ==> maxColIndex
  mini ==> minIndex
  maxi ==> maxIndex

  positivePower:(%,Integer) -> %
  positivePower(x,n) ==
    (n = 1) => x
    odd? n => x * positivePower(x,n - 1)
    y := positivePower(x,n quo 2)
```

```
    y * y

  x:% ** n:NonNegativeInteger ==
    zero? n => scalarMatrix 1
    positivePower(x,n)

  coerce(r:R) == scalarMatrix r

  equation2R: Vector % -> Matrix R

  differentiate(x:%,d:R -> R) == map(d,x)

  diagonal x ==
    v:Vector(R) := new(ndim,0)
    for i in minr x .. maxr x
      for j in minc x .. maxc x
        for k in minIndex v .. maxIndex v repeat
          qsetelt_!(v, k, qelt(x, i, j))
    directProduct v

  retract(x:%):R ==
    diagonal? x => retract diagonal x
    error "Not retractable"

  retractIfCan(x:%):Union(R, "failed") ==
    diagonal? x => retractIfCan diagonal x
    "failed"

  equation2R v ==
    ans:Matrix(Col) := new(ndim,#v,0)
    for i in minr ans .. maxr ans repeat
      for j in minc ans .. maxc ans repeat
        qsetelt_!(ans, i, j, column(qelt(v, j), i))
    reducedSystem ans

  reducedSystem(x:Matrix %):Matrix(R) ==
    empty? x => new(0,0,0)
    reduce(vertConcat, [equation2R row(x, i)
                         for i in minr x .. maxr x])$List(Matrix R)

  reducedSystem(m:Matrix %, v:Vector %):
   Record(mat:Matrix R, vec:Vector R) ==
    vh:Vector(R) :=
      empty? v => new(0,0)
      rh := reducedSystem(v::Matrix %)@Matrix(R)
      column(rh, minColIndex rh)
    [reducedSystem(m)@Matrix(R), vh]

  trace x ==
    tr : R := 0
    for i in minr(x)..maxr(x) for j in minc(x)..maxc(x) repeat
      tr := tr + x(i,j)
    tr
```

```
 diagonalProduct x ==
   pr : R := 1
   for i in minr(x)..maxr(x) for j in minc(x)..maxc(x) repeat
     pr := pr * x(i,j)
   pr

 if R has Field then

   x:% ** n:Integer ==
     zero? n => scalarMatrix 1
     positive? n => positivePower(x,n)
     (xInv := inverse x) case "failed" =>
       error "**: matrix must be invertible"
     positivePower(xInv :: %,-n)
```

— COQ SMATCAT —

```
(* category SMATCAT *)
(*
    minr ==> minRowIndex
    maxr ==> maxRowIndex
    minc ==> minColIndex
    maxc ==> maxColIndex
    mini ==> minIndex
    maxi ==> maxIndex

    positivePower:(%,Integer) -> %
    positivePower(x,n) ==
      (n = 1) => x
      odd? n => x * positivePower(x,n - 1)
      y := positivePower(x,n quo 2)
      y * y

    ?**? : (%,NonNegativeInteger) -> %
    x:% ** n:NonNegativeInteger ==
      zero? n => scalarMatrix 1
      positivePower(x,n)

    coerce : R -> %
    coerce(r:R) == scalarMatrix r

    differentiate : (%,(R -> R)) -> %
    differentiate(x:%,d:R -> R) == map(d,x)

    diagonal : % -> Row
    diagonal x ==
      v:Vector(R) := new(ndim,0)
      for i in minr x .. maxr x
        for j in minc x .. maxc x
          for k in minIndex v .. maxIndex v repeat
            qsetelt_!(v, k, qelt(x, i, j))
```

```
    directProduct v

  retract : % -> R
  retract(x:%):R ==
    diagonal? x => retract diagonal x
    error "Not retractable"

  retractIfCan : % -> Union(R,"failed")
  retractIfCan(x:%):Union(R, "failed") ==
    diagonal? x => retractIfCan diagonal x
    "failed"

  equation2R: Vector % -> Matrix R
  equation2R v ==
    ans:Matrix(Col) := new(ndim,#v,0)
    for i in minr ans .. maxr ans repeat
      for j in minc ans .. maxc ans repeat
        qsetelt_!(ans, i, j, column(qelt(v, j), i))
    reducedSystem ans

  reducedSystem : Matrix(%) -> Matrix(Integer)
  reducedSystem(x:Matrix %):Matrix(R) ==
    empty? x => new(0,0,0)
    reduce(vertConcat, [equation2R row(x, i)
                         for i in minr x .. maxr x])$List(Matrix R)

  reducedSystem : (Matrix(%),Vector(%)) ->
   Record(mat: Matrix(R),vec: Vector(R))
  reducedSystem(m:Matrix %, v:Vector %):
   Record(mat:Matrix R, vec:Vector R) ==
    vh:Vector(R) :=
      empty? v => new(0,0)
      rh := reducedSystem(v::Matrix %)@Matrix(R)
      column(rh, minColIndex rh)
    [reducedSystem(m)@Matrix(R), vh]

  trace : % -> R
  trace x ==
    tr : R := 0
    for i in minr(x)..maxr(x) for j in minc(x)..maxc(x) repeat
      tr := tr + x(i,j)
    tr

  diagonalProduct : % -> R
  diagonalProduct x ==
    pr : R := 1
    for i in minr(x)..maxr(x) for j in minc(x)..maxc(x) repeat
      pr := pr * x(i,j)
    pr

  if R has Field then

    ?**? : (%,Integer) -> %
    x:% ** n:Integer ==
```

```
        zero? n => scalarMatrix 1
        positive? n => positivePower(x,n)
        (xInv := inverse x) case "failed" =>
          error "**: matrix must be invertible"
        positivePower(xInv :: %,-n)
*)
```

———-

— SMATCAT.dotabb —

```
"SMATCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SMATCAT"];
"SMATCAT" -> "BMODULE"
"SMATCAT" -> "DIFEXT"
"SMATCAT" -> "FLINEXP"
"SMATCAT" -> "FRETRCT"
"SMATCAT" -> "RMATCAT"
```

———-

— SMATCAT.dotfull —

```
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:DirectProductCategory(a,b)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SMATCAT"];
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:DirectProductCategory(a,b)"
 -> "BiModule(a:Ring,b:Ring)"
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:DirectProductCategory(a,b)"
 -> "DifferentialExtension(a:Ring)"
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:DirectProductCategory(a,b)"
 -> "FullyLinearlyExplicitRingOver(a:Ring)"
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:DirectProductCategory(a,b)"
 -> "FullyRetractableTo(a:Ring)"
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:DirectProductCategory(a,b)"
 -> "RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProductCategory(b,c),e:D
```

———-

— SMATCAT.dotpic —

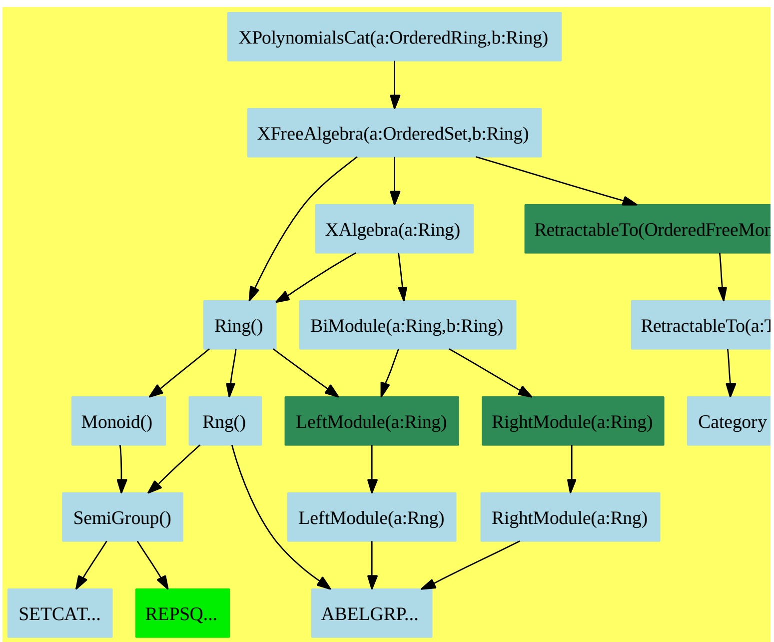
```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:DirectProductCategory(a,b)"
 [color=lightblue];
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:DirectProductCategory(a,b)"
 -> "BiModule(a:Ring,b:Ring)"
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:DirectProductCategory(a,b)"
 -> "DifferentialExtension(a:Ring)"
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:DirectProductCategory(a,b)"
```

```
 -> "FLINEXP..."
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:DirectProductCategory(a,b)"
 -> "FRETRCT..."
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:DirectProductCategory(a,b)"
 -> "RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProductCategory(b,c),e:D

"DifferentialExtension(a:Ring)" [color=lightblue];
"DifferentialExtension(a:Ring)" -> "Ring()"
"DifferentialExtension(a:Ring)" -> "DifferentialRing()"
"DifferentialExtension(a:Ring)" -> "PartialDifferentialRing(Symbol)"

"PartialDifferentialRing(Symbol)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PDRING"];
"PartialDifferentialRing(Symbol)" ->
    "PartialDifferentialRing(a:SetCategory)"

"PartialDifferentialRing(a:SetCategory)" [color=lightblue];
"PartialDifferentialRing(a:SetCategory)" -> "Ring()"

"DifferentialRing()" [color=lightblue];
"DifferentialRing()" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "ABELGRP..."
"Rng()" -> "SGROUP..."

"Monoid()" [color=lightblue];
"Monoid()" -> "SGROUP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LMODULE..."

"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProductCategory(b,c),e:Direc
 [color=lightblue];
"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProductCategory(b,c),e:Direc
  -> "BiModule(a:Ring,b:Ring)"
"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProductCategory(b,c),e:Direc
  -> "HOAGG..."

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."
```

```
"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"FRETRCT..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"SGROUP..." [color=lightblue];
"LMODULE..." [color=lightblue];
"HOAGG..." [color=lightblue];
"ABELGRP..." [color=lightblue];

}
```

———-

12.0.193 XPolynomialsCat (XPOLYC)



— **XPolynomialsCat.input** —

```
)set break resume
)sys rm -f XPolynomialsCat.output
)spool XPolynomialsCat.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show XPolynomialsCat
--R
--R XPolynomialsCat(vl: OrderedSet,R: Ring) is a category constructor
--R Abbreviation for XPolynomialsCat is XPOLYC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for XPOLYC
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (vl,%) -> %                     ?*? : (%,R) -> %
--R ?*? : (R,%) -> %                      ?*? : (%,%) -> %
--R ?*? : (Integer,%) -> %                ?*? : (NonNegativeInteger,%) -> %
--R ?*? : (PositiveInteger,%) -> %        ?**? : (%,NonNegativeInteger) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?=? : (%,%) -> Boolean                1 : () -> %
--R 0 : () -> %                           ?^? : (%,NonNegativeInteger) -> %
--R ?^? : (%,PositiveInteger) -> %        coef : (%,%) -> R
--R coerce : vl -> %                      coerce : OrderedFreeMonoid(vl) -> %
--R coerce : R -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              constant : % -> R
--R constant? : % -> Boolean              degree : % -> NonNegativeInteger
--R hash : % -> SingleInteger             latex : % -> String
--R lquo : (%,%) -> %                     lquo : (%,vl) -> %
--R map : ((R -> R),%) -> %               maxdeg : % -> OrderedFreeMonoid(vl)
--R mindeg : % -> OrderedFreeMonoid(vl)   mirror : % -> %
--R monomial? : % -> Boolean              one? : % -> Boolean
--R quasiRegular : % -> %                 quasiRegular? : % -> Boolean
--R recip : % -> Union(%,"failed")        rquo : (%,%) -> %
--R rquo : (%,vl) -> %                    sample : () -> %
--R sh : (%,%) -> % if R has COMRING      varList : % -> List(vl)
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R coef : (%,OrderedFreeMonoid(vl)) -> R
--R lquo : (%,OrderedFreeMonoid(vl)) -> %
--R mindegTerm : % -> Record(k: OrderedFreeMonoid(vl),c: R)
--R monom : (OrderedFreeMonoid(vl),R) -> %
--R retract : % -> OrderedFreeMonoid(vl)
--R retractIfCan : % -> Union(OrderedFreeMonoid(vl),"failed")
--R rquo : (%,OrderedFreeMonoid(vl)) -> %
--R sh : (%,NonNegativeInteger) -> % if R has COMRING
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R trunc : (%,NonNegativeInteger) -> %
--R
--E 1

)spool
)lisp (bye)
```

— XPolynomialsCat.help —

```
====================================================================
```

```
XPolynomialsCat examples
====================================================================

The Category of polynomial rings with non-commutative variables.
The coefficient ring may be non-commutative too.
However coefficients commute with variables.

See Also:
o )show XPolynomialsCat
```

———–

See:

⇐ "XFreeAlgebra" (XFALG) 11.0.183 on page 1164

Exports:

0	1	characteristic	coef	coerce
constant	constant?	degree	hash	latex
lquo	map	maxdeg	mindeg	mindegTerm
mirror	monom	monomial?	one?	quasiRegular
quasiRegular?	recip	retract	retractIfCan	rquo
sample	sh	subtractIfCan	trunc	varList
zero?	?*?	?**?	?+?	?-?
-?	?=?	?^?	?~=?	

Attributes Exported:

- if Ring has noZeroDivisors then noZeroDivisors where **noZeroDivisors** is true if $x*y \neq 0$ implies both x and y are non-zero.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
degree : % -> NonNegativeInteger
maxdeg : % -> OrderedFreeMonoid vl
trunc : (%,NonNegativeInteger) -> %
```

These exports come from (p133) Aggregate():

These exports come from (p1164) XFreeAlgebra(vl:OrderedSet,R:Ring):

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coef : (%,OrderedFreeMonoid vl) -> R
coef : (%,%) -> R
coerce : % -> OutputForm
coerce : R -> %
coerce : OrderedFreeMonoid vl -> %
```

```
coerce : Integer -> %
coerce : vl -> %
constant : % -> R
constant? : % -> Boolean
hash : % -> SingleInteger
latex : % -> String
lquo : (%,OrderedFreeMonoid vl) -> %
lquo : (%,%) -> %
lquo : (%,vl) -> %
map : ((R -> R),%) -> %
mindeg : % -> OrderedFreeMonoid vl
mindegTerm : % -> Record(k: OrderedFreeMonoid vl,c: R)
mirror : % -> %
monom : (OrderedFreeMonoid vl,R) -> %
monomial? : % -> Boolean
one? : % -> Boolean
quasiRegular : % -> %
quasiRegular? : % -> Boolean
recip : % -> Union(%,"failed")
retract : % -> OrderedFreeMonoid vl
retractIfCan : % -> Union(OrderedFreeMonoid vl,"failed")
rquo : (%,OrderedFreeMonoid vl) -> %
rquo : (%,%) -> %
rquo : (%,vl) -> %
sample : () -> %
sh : (%,NonNegativeInteger) -> % if R has COMRING
sh : (%,%) -> % if R has COMRING
subtractIfCan : (%,%) -> Union(%,"failed")
varList : % -> List vl
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?*? : (R,%) -> %
?*? : (%,R) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (vl,%) -> %
```

These exports come from (p187) SetCategory():

— XPolynomialsCat.html —

```
<body>
<h2>
```

```
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#XPOLYC">
XPolynomialsCat (XPOLYC)</a></h2>
</body>
```

— category XPOLYC XPolynomialsCat —

```
)abbrev category XPOLYC XPolynomialsCat
++ Author: Michel Petitot petitot@lifl.fr
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Description:
++ The Category of polynomial rings with non-commutative variables.
++ The coefficient ring may be non-commutative too.
++ However coefficients commute with variables.

XPolynomialsCat(vl,R) : Category == SIG where
  vl : OrderedSet
  R : Ring

  WORD ==> OrderedFreeMonoid(vl)

  SIG ==> XFreeAlgebra(vl,R) with

    maxdeg : % -> WORD
      ++ \spad{maxdeg(p)} returns the greatest leading word in the
      ++ support of \spad{p}.

    degree : % -> NonNegativeInteger
      ++ \spad{degree(p)} returns the degree of \spad{p}.
      ++  Note that the degree of a word is its length.

    trunc : (% , NonNegativeInteger) -> %
      ++  \spad{trunc(p,n)} returns the polynomial \spad{p} truncated
      ++ at order \spad{n}.
```

— XPOLYC.dotabb —

```
"XPOLYC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XPOLYC"];
"XPOLYC" -> "XFALG"
```

— XPOLYC.dotfull —

```
"XPolynomialsCat(a:OrderedRing,b:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XPOLYC"];
```

```
"XPolynomialsCat(a:OrderedRing,b:Ring)" ->
   "XFreeAlgebra(a:OrderedSet,b:Ring)"
```

— **XPOLYC.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"XPolynomialsCat(a:OrderedRing,b:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XPOLYC"];
"XPolynomialsCat(a:OrderedRing,b:Ring)" ->
   "XFreeAlgebra(a:OrderedSet,b:Ring)"

"XFreeAlgebra(a:OrderedSet,b:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XFALG"];
"XFreeAlgebra(a:OrderedSet,b:Ring)" -> "Ring()"
"XFreeAlgebra(a:OrderedSet,b:Ring)" -> "XAlgebra(a:Ring)"
"XFreeAlgebra(a:OrderedSet,b:Ring)" ->
    "RetractableTo(OrderedFreeMonoid(OrderedSet))"

"RetractableTo(OrderedFreeMonoid(OrderedSet))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(OrderedFreeMonoid(OrderedSet))" -> "RetractableTo(a:Type)"

"XAlgebra(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XALG"];
"XAlgebra(a:Ring)" -> "Ring()"
"XAlgebra(a:Ring)" -> "BiModule(a:Ring,b:Ring)"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"Rng()" [color=lightblue];
"Rng()" -> "ABELGRP..."
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
```

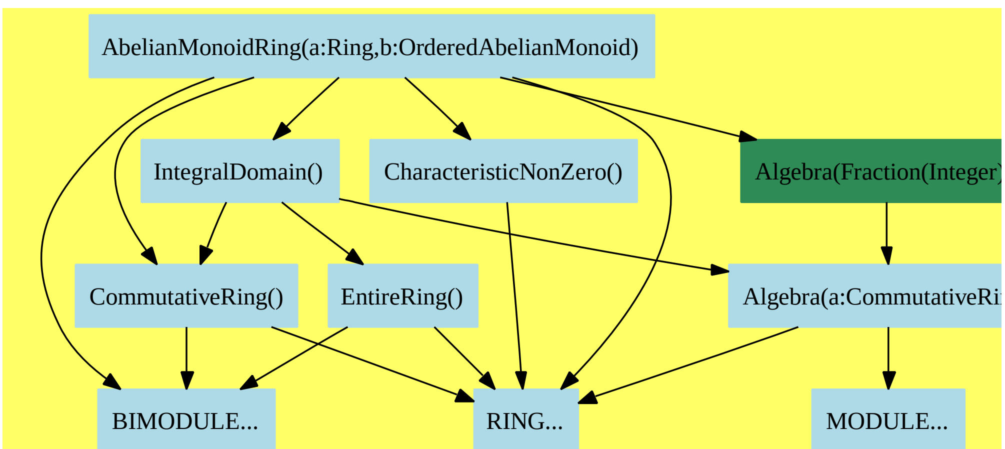
```
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"Category" [color=lightblue];

"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"ABELGRP..." [color=lightblue];
}
```

———

Chapter 13

Category Layer 12

13.0.194 AbelianMonoidRing (AMR)



— **AbelianMonoidRing.input** —

```
)set break resume
)sys rm -f AbelianMonoidRing.output
)spool AbelianMonoidRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AbelianMonoidRing
--R
--R AbelianMonoidRing(R: Ring,E: OrderedAbelianMonoid) is a category constructor
--R Abbreviation for AbelianMonoidRing is AMR
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for AMR
--R
```

```
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?/? : (%,R) -> % if R has FIELD
--R ?=? : (%,%) -> Boolean                1 : () -> %
--R 0 : () -> %                           ?^? : (%,NonNegativeInteger) -> %
--R ?^? : (%,PositiveInteger) -> %        coefficient : (%,E) -> R
--R coerce : R -> % if R has COMRING      coerce : % -> % if R has INTDOM
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R degree : % -> E                       hash : % -> SingleInteger
--R latex : % -> String                   leadingCoefficient : % -> R
--R leadingMonomial : % -> %              map : ((R -> R),%) -> %
--R monomial : (R,E) -> %                 monomial? : % -> Boolean
--R one? : % -> Boolean                   recip : % -> Union(%,"failed")
--R reductum : % -> %                     sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction(Integer)) -> % if R has ALGEBRA(FRAC(INT))
--R ?*? : (Fraction(Integer),%) -> % if R has ALGEBRA(FRAC(INT))
--R associates? : (%,%) -> Boolean if R has INTDOM
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
--R coerce : Fraction(Integer) -> % if R has ALGEBRA(FRAC(INT))
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R
--E 1

)spool
)lisp (bye)
```

— **AbelianMonoidRing.help** —

```
====================================================================
AbelianMonoidRing examples
====================================================================

Abelian monoid ring elements (not necessarily of finite support)
of this ring are of the form formal SUM (r_i * e_i)
where the r_i are coefficents and the e_i, elements of the
ordered abelian monoid, are thought of as exponents or monomials.
The monomials commute with each other, and with
the coefficients (which themselves may or may not be commutative).

See FiniteAbelianMonoidRing for the case of finite support
a useful common model for polynomials and power series.
Conceptually at least, only the non-zero terms are ever operated on.
```

```
See Also:
o )show AbelianMonoidRing
```

———-

See:

⇒ "FiniteAbelianMonoidRing" (FAMR) 14.0.199 on page 1331
⇒ "PowerSeriesCategory" (PSCAT) 14.0.201 on page 1349
⇐ "BiModule" (BMODULE) 9.0.146 on page 902
⇐ "Ring" (RING) 9.0.153 on page 946

Exports:

0	1	associates?	characteristic
charthRoot	coefficient	coerce	degree
exquo	hash	latex	leadingCoefficient
leadingMonomial	map	monomial	monomial?
one?	recip	reductum	sample
subtractIfCan	unit?	unitCanonical	unitNormal
zero?	?*?	?**?	?+?
?-?	-?	?=?	?^?
?~=?	?/?		

Attributes exported:

- if \$ has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation " $*$ " $: (D, D) -> D$ which is commutative.
- if \$ has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x*y \neq 0$ implies both x and y are non-zero.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
coefficient : (%,E) -> R
degree : % -> E
leadingCoefficient : % -> R
leadingMonomial : % -> %
monomial : (R,E) -> %
reductum : % -> %
?/? : (%,R) -> % if R has FIELD
```

These are implemented by this category:

```
map : ((R -> R),%) -> %
monomial? : % -> Boolean
?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
```

These exports come from (p946) Ring():

```
0 : () -> %
```

```
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : % -> OutputForm
coerce : Integer -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p902) BiModule(R:Ring,R:Ring):

```
?*? : (R,%) -> %
?*? : (%,R) -> %
```

These exports come from (p1225) IntegralDomain():

```
associates? : (%,%) -> Boolean if R has INTDOM
coerce : % -> % if R has INTDOM
exquo : (%,%) -> Union(%,"failed") if R has INTDOM
unit? : % -> Boolean if R has INTDOM
unitCanonical : % -> % if R has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
```

These exports come from (p1010) CharacteristicNonZero():

```
charthRoot : % -> Union(%,"failed") if R has CHARNZ
```

These exports come from (p1019) CommutativeRing():

```
coerce : R -> % if R has COMRING
```

These exports come from (p1121) Algebra(Fraction(Integer)):

```
coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT
?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
```

— AbelianMonoidRing.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#AMR">
AbelianMonoidRing (AMR)</a></h2>
</body>
```

— category AMR AbelianMonoidRing —

```
)abbrev category AMR AbelianMonoidRing
++ Description:
++ Abelian monoid ring elements (not necessarily of finite support)
++ of this ring are of the form formal SUM (r_i * e_i)
++ where the r_i are coefficents and the e_i, elements of the
++ ordered abelian monoid, are thought of as exponents or monomials.
++ The monomials commute with each other, and with
++ the coefficients (which themselves may or may not be commutative).
++ See \spadtype{FiniteAbelianMonoidRing} for the case of finite support
++ a useful common model for polynomials and power series.
++ Conceptually at least, only the non-zero terms are ever operated on.

AbelianMonoidRing(R,E) : Category == SIG where
  R : Ring
  E : OrderedAbelianMonoid

  SIG ==> Join(Ring,BiModule(R,R)) with

    leadingCoefficient : % -> R
      ++ leadingCoefficient(p) returns the coefficient highest
      ++ degree term of p.

    leadingMonomial : % -> %
      ++ leadingMonomial(p) returns the monomial of p with the highest degree.

    degree : % -> E
      ++ degree(p) returns the maximum of the exponents of the terms of p.

    map : (R -> R, %) -> %
      ++ map(fn,u) maps function fn onto the coefficients
      ++ of the non-zero monomials of u.

    monomial? : % -> Boolean
      ++ monomial?(p) tests if p is a single monomial.

    monomial : (R,E) -> %
      ++ monomial(r,e) makes a term from a coefficient r and an exponent e.

    reductum : % -> %
      ++ reductum(u) returns u minus its leading monomial
      ++ returns zero if handed the zero element.

    coefficient : (%,E) -> R
      ++ coefficient(p,e) extracts the coefficient of the monomial with
      ++ exponent e from polynomial p, or returns zero if exponent
      ++ is not present.

    if R has Field then

       "/" : (%,R) -> %
```

```
        ++ p/c divides p by the coefficient c.

    if R has CommutativeRing then
       CommutativeRing
       Algebra R

    if R has CharacteristicZero then CharacteristicZero

    if R has CharacteristicNonZero then CharacteristicNonZero

    if R has IntegralDomain then IntegralDomain

    if R has Algebra Fraction Integer then Algebra Fraction Integer

   add

     monomial? x == zero? reductum x

     map(fn:R -> R, x: %) ==
          -- this default definition assumes that reductum is cheap
        zero? x => 0
        r:=fn leadingCoefficient x
        zero? r => map(fn,reductum x)
        monomial(r, degree x) + map(fn,reductum x)

     if R has Algebra Fraction Integer then

       q:Fraction(Integer) * p:% == map(x1 +-> q * x1, p)
```

— **COQ AMR** —

```
(* category AMR *)
(*

  monomial? : % -> Boolean
  monomial? x == zero? reductum x

  map : ((R -> R),%) -> %
  map(fn:R -> R, x: %) ==
        -- this default definition assumes that reductum is cheap
     zero? x => 0
     r:=fn leadingCoefficient x
     zero? r => map(fn,reductum x)
     monomial(r, degree x) + map(fn,reductum x)

  if R has Algebra Fraction Integer then

    ?*? : (Integer,%) -> %
    q:Fraction(Integer) * p:% == map(x1 +-> q * x1, p)

*)
```

———-

— **AMR.dotabb** —

```
"AMR"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=AMR"];
"AMR" -> "RING"
"AMR" -> "BMODULE"
"AMR" -> "INTDOM"
"AMR" -> "CHARNZ"
"AMR" -> "COMRING"
"AMR" -> "ALGEBRA"
```

———-

— **AMR.dotfull** —

```
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=AMR"];
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" -> "Ring()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "BiModule(a:Ring,b:OrderedAbelianMonoid)"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "IntegralDomain()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CharacteristicNonZero()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CommutativeRing()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "Algebra(Fraction(Integer))"
```

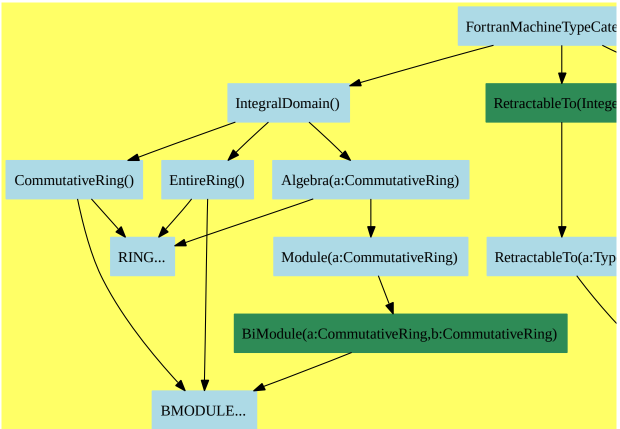
———-

— **AMR.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" [color=lightblue];
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" -> "RING..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "BIMODULE..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "IntegralDomain()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CharacteristicNonZero()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CommutativeRing()"
```

```
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "Algebra(Fraction(Integer))"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BIMODULE..."

"CharacteristicNonZero()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARNZ"];
"CharacteristicNonZero()" -> "RING..."

"Algebra(Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(Fraction(Integer))" -> "Algebra(a:CommutativeRing)"

"Algebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "MODULE..."

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BIMODULE..."

"BIMODULE..." [color=lightblue];
"RING..." [color=lightblue];
"MODULE..." [color=lightblue];
}
```

13.0.195 FortranMachineTypeCategory (FMTC)



— FortranMachineTypeCategory.input —

```
)set break resume
)sys rm -f FortranMachineTypeCategory.output
)spool FortranMachineTypeCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FortranMachineTypeCategory
--R
--R FortranMachineTypeCategory is a category constructor
--R Abbreviation for FortranMachineTypeCategory is FMTC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FMTC
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean               ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean                ?>=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean        coerce : Integer -> %
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   max : (%,%) -> %
--R min : (%,%) -> %                      one? : % -> Boolean
--R recip : % -> Union(%,"failed")        retract : % -> Integer
--R sample : () -> %                      unit? : % -> Boolean
--R unitCanonical : % -> %                zero? : % -> Boolean
```

```
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R exquo : (%,%) -> Union(%,"failed")
--R retractIfCan : % -> Union(Integer,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

— **FortranMachineTypeCategory.help** —

```
====================================================================
FortranMachineTypeCategory examples
====================================================================

A category of domains which model machine arithmetic used by machines
in the AXIOM-NAG link.

See Also:
o )show FortranMachineTypeCategory
```

See:

⇐ "IntegralDomain" (INTDOM) 12.0.188 on page 1225
⇐ "OrderedSet" (ORDSET) 4.0.80 on page 326
⇐ "RetractableTo" (RETRACT) 2.0.37 on page 110

Exports:

0	1	associates?	characteristic	coerce
exquo	hash	latex	max	min
one?	recip	retract	retractIfCan	sample
subtractIfCan	unit?	unitCanonical	unitNormal	zero?
?~=?	?^?	?*?	?**?	?+?
?-?	-?	?<?	?<=?	?=?
?>?	?>=?			

Attributes Exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.

- **rightUnitary** is true if $x * 1 = x$ for all x.

These exports come from (p1225) IntegralDomain():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
coerce : % -> %
exquo : (%,%) -> Union(%,"failed")
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
```

These exports come from (p326) OrderedSet():

```
max : (%,%) -> %
min : (%,%) -> %
?<? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

These exports come from (p110) RetractableTo(Integer):

```
coerce : Integer -> %
retract : % -> Integer
retractIfCan : % -> Union(Integer,"failed")
```

— **FortranMachineTypeCategory.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FMTC">
FortranMachineTypeCategory (FMTC)</a></h2>
```

```
</body>
```

— category FMTC FortranMachineTypeCategory —

```
)abbrev category FMTC FortranMachineTypeCategory
++ Author: Mike Dewar
++ Date Created:  December 1993
++ Description:
++ A category of domains which model machine arithmetic
++ used by machines in the AXIOM-NAG link.

FortranMachineTypeCategory() : Category == SIG where

  SIG ==> Join(IntegralDomain,OrderedSet, RetractableTo(Integer) )
```

— FMTC.dotabb —

```
"FMTC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FMTC"];
"FMTC" -> "INTDOM"
"FMTC" -> "ORDSET"
"FMTC" -> "RETRACT"
```

— FMTC.dotfull —

```
"FortranMachineTypeCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FMTC"];
"FortranMachineTypeCategory()" -> "IntegralDomain()"
"FortranMachineTypeCategory()" -> "OrderedSet()"
"FortranMachineTypeCategory()" -> "RetractableTo(Integer)"
```

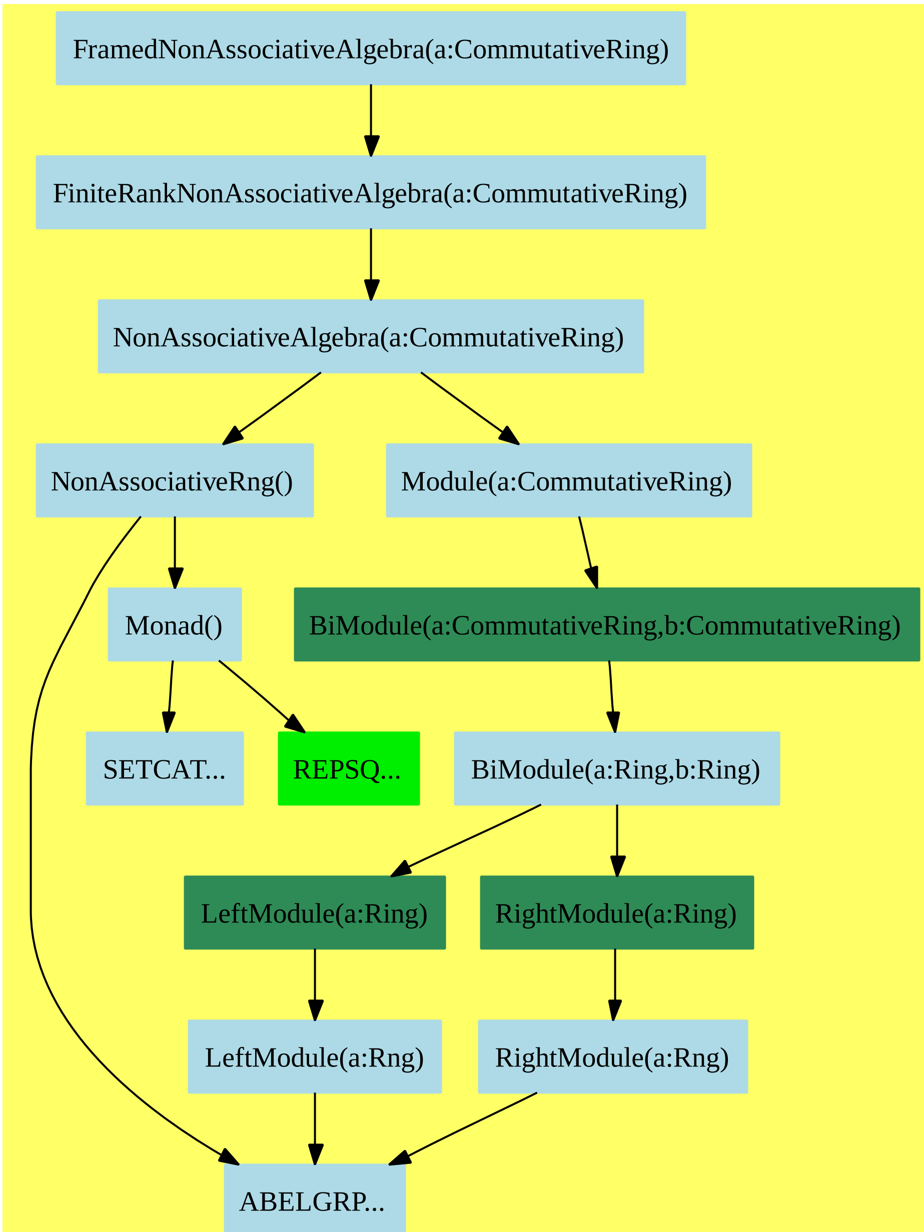
— FMTC.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FortranMachineTypeCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FMTC"];
"FortranMachineTypeCategory()" -> "IntegralDomain()"
"FortranMachineTypeCategory()" -> "OrderedSet()"
"FortranMachineTypeCategory()" -> "RetractableTo(Integer)"
```

```
"RetractableTo(Integer)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(Integer)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(a:Type)" -> "Category"

"OrderedSet()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDSET"];
"OrderedSet()" -> "SetCategory()"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BMODULE..."

"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

———-

13.0.196 FramedNonAssociativeAlgebra (FRNAALG)



— **FramedNonAssociativeAlgebra.input** —

```
)set break resume
)sys rm -f FramedNonAssociativeAlgebra.output
)spool FramedNonAssociativeAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FramedNonAssociativeAlgebra
--R
--R FramedNonAssociativeAlgebra(R: CommutativeRing) is a category constructor
--R Abbreviation for FramedNonAssociativeAlgebra is FRNAALG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FRNAALG
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?=? : (%,%) -> Boolean                0 : () -> %
--R alternative? : () -> Boolean          antiAssociative? : () -> Boolean
--R antiCommutative? : () -> Boolean      antiCommutator : (%,%) -> %
--R apply : (Matrix(R),%) -> %            associative? : () -> Boolean
--R associator : (%,%,%) -> %             basis : () -> Vector(%)
--R coerce : % -> OutputForm              commutative? : () -> Boolean
--R commutator : (%,%) -> %               convert : Vector(R) -> %
--R convert : % -> Vector(R)              coordinates : % -> Vector(R)
--R ?.? : (%,Integer) -> R                flexible? : () -> Boolean
--R hash : % -> SingleInteger             jacobiIdentity? : () -> Boolean
--R jordanAdmissible? : () -> Boolean     jordanAlgebra? : () -> Boolean
--R latex : % -> String                   leftAlternative? : () -> Boolean
--R leftDiscriminant : () -> R            leftDiscriminant : Vector(%) -> R
--R leftNorm : % -> R                     leftTrace : % -> R
--R leftTraceMatrix : () -> Matrix(R)     lieAdmissible? : () -> Boolean
--R lieAlgebra? : () -> Boolean           powerAssociative? : () -> Boolean
--R rank : () -> PositiveInteger          represents : Vector(R) -> %
--R rightAlternative? : () -> Boolean     rightDiscriminant : () -> R
--R rightDiscriminant : Vector(%) -> R    rightNorm : % -> R
--R rightTrace : % -> R                   rightTraceMatrix : () -> Matrix(R)
--R sample : () -> %                      someBasis : () -> Vector(%)
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R associatorDependence : () -> List(Vector(R)) if R has INTDOM
--R conditionsForIdempotents : () -> List(Polynomial(R))
--R conditionsForIdempotents : Vector(%) -> List(Polynomial(R))
--R coordinates : Vector(%) -> Matrix(R)
--R coordinates : (Vector(%),Vector(%)) -> Matrix(R)
--R coordinates : (%,Vector(%)) -> Vector(R)
--R leftCharacteristicPolynomial : % -> SparseUnivariatePolynomial(R)
--R leftMinimalPolynomial : % -> SparseUnivariatePolynomial(R) if R has INTDOM
--R leftPower : (%,PositiveInteger) -> %
--R leftRankPolynomial : () -> SparseUnivariatePolynomial(Polynomial(R)) if R has FIELD
--R leftRecip : % -> Union(%,"failed") if R has INTDOM
```

```
--R leftRegularRepresentation : % -> Matrix(R)
--R leftRegularRepresentation : (%,Vector(%)) -> Matrix(R)
--R leftTraceMatrix : Vector(%) -> Matrix(R)
--R leftUnit : () -> Union(%,"failed") if R has INTDOM
--R leftUnits : () -> Union(Record(particular: %,basis: List(%)),"failed") if R has INTDOM
--R noncommutativeJordanAlgebra? : () -> Boolean
--R plenaryPower : (%,PositiveInteger) -> %
--R recip : % -> Union(%,"failed") if R has INTDOM
--R represents : (Vector(R),Vector(%)) -> %
--R rightCharacteristicPolynomial : % -> SparseUnivariatePolynomial(R)
--R rightMinimalPolynomial : % -> SparseUnivariatePolynomial(R) if R has INTDOM
--R rightPower : (%,PositiveInteger) -> %
--R rightRankPolynomial : () -> SparseUnivariatePolynomial(Polynomial(R)) if R has FIELD
--R rightRecip : % -> Union(%,"failed") if R has INTDOM
--R rightRegularRepresentation : % -> Matrix(R)
--R rightRegularRepresentation : (%,Vector(%)) -> Matrix(R)
--R rightTraceMatrix : Vector(%) -> Matrix(R)
--R rightUnit : () -> Union(%,"failed") if R has INTDOM
--R rightUnits : () -> Union(Record(particular: %,basis: List(%)),"failed") if R has INTDOM
--R structuralConstants : () -> Vector(Matrix(R))
--R structuralConstants : Vector(%) -> Vector(Matrix(R))
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unit : () -> Union(%,"failed") if R has INTDOM
--R
--E 1

)spool
)lisp (bye)
```

— **FramedNonAssociativeAlgebra.help** —

```
====================================================================
FramedNonAssociativeAlgebra examples
====================================================================

FramedNonAssociativeAlgebra(R) is a FiniteRankNonAssociativeAlgebra
(a non associative algebra over R which is a free R-module of
finite rank) over a commutative ring R together with a fixed R-module basis.

See Also:
o )show FramedNonAssociativeAlgebra
```

See:

⇐ "FiniteRankNonAssociativeAlgebra" (FINAALG) 12.0.186 on page 1191

Exports:

0	alternative?
antiAssociative?	antiCommutative?
antiCommutator	apply
associative?	associator
associatorDependence	basis
coerce	commutative?
commutator	conditionsForIdempotents
convert	coordinates
flexible?	hash
jacobiIdentity?	jordanAdmissible?
jordanAlgebra?	latex
leftAlternative?	leftCharacteristicPolynomial
leftDiscriminant	leftMinimalPolynomial
leftNorm	leftPower
leftRankPolynomial	leftRecip
leftRegularRepresentation	leftTrace
leftTraceMatrix	leftUnit
leftUnits	lieAdmissible?
lieAlgebra?	noncommutativeJordanAlgebra?
plenaryPower	powerAssociative?
rank	recip
represents	rightAlternative?
rightCharacteristicPolynomial	rightDiscriminant
rightMinimalPolynomial	rightNorm
rightPower	rightRankPolynomial
rightRecip	rightRegularRepresentation
rightTrace	rightTraceMatrix
rightUnit	rightUnits
sample	someBasis
structuralConstants	subtractIfCan
unit	zero?
?*?	?**?
?+?	?-?
-?	?=?
?.?	?~=?

Attributes exported:

- if $ has IntegralDomain then unitsKnown where **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
apply : (Matrix R,%) -> %
basis : () -> Vector %
convert : Vector R -> %
?.? : (%,Integer) -> R
```

These are implemented by this category:

```
conditionsForIdempotents : () -> List Polynomial R
convert : % -> Vector R
coordinates : % -> Vector R
coordinates : Vector % -> Matrix R
leftDiscriminant : () -> R
leftRankPolynomial : () ->
  SparseUnivariatePolynomial Polynomial R
    if R has FIELD
leftRegularRepresentation : % -> Matrix R
leftTraceMatrix : () -> Matrix R
leftUnit : () -> Union(%,"failed") if R has INTDOM
leftUnits : () ->
  Union(Record(particular: %,basis: List %),"failed")
    if R has INTDOM
represents : Vector R -> %
rightDiscriminant : () -> R
rightRankPolynomial : () ->
  SparseUnivariatePolynomial Polynomial R
    if R has FIELD
rightRegularRepresentation : % -> Matrix R
rightTraceMatrix : () -> Matrix R
rightUnit : () -> Union(%,"failed") if R has INTDOM
rightUnits : () ->
  Union(Record(particular: %,basis: List %),"failed")
    if R has INTDOM
structuralConstants : () -> Vector Matrix R
unit : () -> Union(%,"failed") if R has INTDOM
```

These exports come from (p1191) FiniteRankNonAssociativeAlgebra(R) where R:CommutativeRing:

```
0 : () -> %
alternative? : () -> Boolean
antiAssociative? : () -> Boolean
antiCommutative? : () -> Boolean
antiCommutator : (%,%) -> %
associative? : () -> Boolean
associator : (%,%,%) -> %
associatorDependence : () -> List Vector R
    if R has INTDOM
coerce : % -> OutputForm
commutative? : () -> Boolean
commutator : (%,%) -> %
conditionsForIdempotents : Vector % -> List Polynomial R
coordinates : (Vector %,Vector %) -> Matrix R
coordinates : (%,Vector %) -> Vector R
flexible? : () -> Boolean
hash : % -> SingleInteger
jacobiIdentity? : () -> Boolean
jordanAdmissible? : () -> Boolean
jordanAlgebra? : () -> Boolean
latex : % -> String
leftAlternative? : () -> Boolean
leftCharacteristicPolynomial : % ->
  SparseUnivariatePolynomial R
```

```
 leftDiscriminant : Vector % -> R
 leftMinimalPolynomial : % ->
   SparseUnivariatePolynomial R
     if R has INTDOM
 leftNorm : % -> R
 leftPower : (%,PositiveInteger) -> %
 leftRecip : % -> Union(%,"failed") if R has INTDOM
 leftRegularRepresentation : (%,Vector %) -> Matrix R
 leftTrace : % -> R
 leftTraceMatrix : Vector % -> Matrix R
 lieAdmissible? : () -> Boolean
 lieAlgebra? : () -> Boolean
 noncommutativeJordanAlgebra? : () -> Boolean
 plenaryPower : (%,PositiveInteger) -> %
 powerAssociative? : () -> Boolean
 rank : () -> PositiveInteger
 recip : % -> Union(%,"failed") if R has INTDOM
 represents : (Vector R,Vector %) -> %
 rightAlternative? : () -> Boolean
 rightCharacteristicPolynomial : % ->
   SparseUnivariatePolynomial R
 rightDiscriminant : Vector % -> R
 rightMinimalPolynomial : % ->
   SparseUnivariatePolynomial R
     if R has INTDOM
 rightNorm : % -> R
 rightPower : (%,PositiveInteger) -> %
 rightRecip : % -> Union(%,"failed") if R has INTDOM
 rightRegularRepresentation : (%,Vector %) -> Matrix R
 rightTrace : % -> R
 rightTraceMatrix : Vector % -> Matrix R
 sample : () -> %
 someBasis : () -> Vector %
 structuralConstants : Vector % -> Vector Matrix R
 subtractIfCan : (%,%) -> Union(%,"failed")
 zero? : % -> Boolean
 ?~=? : (%,%) -> Boolean
 ?*? : (PositiveInteger,%) -> %
 ?*? : (%,%) -> %
 ?*? : (Integer,%) -> %
 ?*? : (NonNegativeInteger,%) -> %
 ?*? : (R,%) -> %
 ?*? : (%,R) -> %
 ?+? : (%,%) -> %
 ?=? : (%,%) -> Boolean
 ?-? : (%,%) -> %
 -? : % -> %
 ?**? : (%,PositiveInteger) -> %
```

— FramedNonAssociativeAlgebra.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FRNAALG">
```

```
FramedNonAssociativeAlgebra (FRNAALG)</a></h2>
</body>
```

— category FRNAALG FramedNonAssociativeAlgebra —

```
)abbrev category FRNAALG FramedNonAssociativeAlgebra
++ Author: J. Grabmeier, R. Wisbauer
++ Date Created: 01 March 1991
++ Date Last Updated: 11 June 1991
++ Reference:
++  R.D. Schafer: An Introduction to Nonassociative Algebras
++  Academic Press, New York, 1966
++ Description:
++ FramedNonAssociativeAlgebra(R) is a
++ \spadtype{FiniteRankNonAssociativeAlgebra} (a non associative
++ algebra over R which is a free \spad{R}-module of finite rank)
++ over a commutative ring R together with a fixed \spad{R}-module basis.

FramedNonAssociativeAlgebra(R) : Category == SIG where
  R : CommutativeRing

  SIG ==> FiniteRankNonAssociativeAlgebra(R) with

    basis : () -> Vector %
      ++ basis() returns the fixed \spad{R}-module basis.

    coordinates : % -> Vector R
      ++ coordinates(a) returns the coordinates of \spad{a}
      ++ with respect to the
      ++ fixed \spad{R}-module basis.

    coordinates : Vector % -> Matrix R
      ++ coordinates([a1,...,am]) returns a matrix whose i-th row
      ++ is formed by the coordinates of \spad{ai} with respect to the
      ++ fixed \spad{R}-module basis.

    elt : (%,Integer) -> R
      ++ elt(a,i) returns the i-th coefficient of \spad{a} with respect
      ++ to the fixed \spad{R}-module basis.

    structuralConstants : () -> Vector Matrix R
      ++ structuralConstants() calculates the structural constants
      ++ \spad{[(gammaijk) for k in 1..rank()]} defined by
      ++ \spad{vi * vj = gammaij1 * v1 + ... + gammaijn * vn},
      ++ where \spad{v1},...,\spad{vn} is the fixed \spad{R}-module basis.

    conditionsForIdempotents : () -> List Polynomial R
      ++ conditionsForIdempotents() determines a complete list
      ++ of polynomial equations for the coefficients of idempotents
      ++ with respect to the fixed \spad{R}-module basis.

    represents : Vector R -> %
```

```
  ++ represents([a1,...,an]) returns \spad{a1*v1 + ... + an*vn},
  ++ where \spad{v1}, ..., \spad{vn} are the elements of the
  ++ fixed \spad{R}-module basis.

convert : % -> Vector R
  ++ convert(a) returns the coordinates of \spad{a} with respect to the
  ++ fixed \spad{R}-module basis.

convert : Vector R -> %
  ++ convert([a1,...,an]) returns \spad{a1*v1 + ... + an*vn},
  ++ where \spad{v1}, ..., \spad{vn} are the elements of the
  ++ fixed \spad{R}-module basis.

leftDiscriminant : () -> R
  ++ leftDiscriminant() returns the
  ++ determinant of the \spad{n}-by-\spad{n}
  ++ matrix whose element at the \spad{i}-th row and \spad{j}-th column
  ++ is given by the left trace of the product \spad{vi*vj}, where
  ++ \spad{v1},...,\spad{vn} are the
  ++ elements of the fixed \spad{R}-module basis.
  ++ Note that the same as \spad{determinant(leftTraceMatrix())}.

rightDiscriminant : () -> R
  ++ rightDiscriminant() returns the determinant of the
  ++ \spad{n}-by-\spad{n} matrix whose element at the \spad{i}-th row
  ++ and \spad{j}-th column is
  ++ given by the right trace of the product \spad{vi*vj}, where
  ++ \spad{v1},...,\spad{vn} are the elements of
  ++ the fixed \spad{R}-module basis.
  ++ Note that the same as \spad{determinant(rightTraceMatrix())}.

leftTraceMatrix : () -> Matrix R
  ++ leftTraceMatrix() is the \spad{n}-by-\spad{n}
  ++ matrix whose element at the \spad{i}-th row and \spad{j}-th column
  ++ is given by left trace of the product \spad{vi*vj},
  ++ where \spad{v1},...,\spad{vn} are the
  ++ elements of the fixed \spad{R}-module basis.

rightTraceMatrix : () -> Matrix R
  ++ rightTraceMatrix() is the \spad{n}-by-\spad{n}
  ++ matrix whose element at the \spad{i}-th row and \spad{j}-th column
  ++ is given by the right trace of the product \spad{vi*vj}, where
  ++ \spad{v1},...,\spad{vn} are the elements
  ++ of the fixed \spad{R}-module basis.

leftRegularRepresentation : % -> Matrix R
  ++ leftRegularRepresentation(a) returns the matrix of the linear
  ++ map defined by left multiplication by \spad{a} with respect
  ++ to the fixed \spad{R}-module basis.

rightRegularRepresentation : % -> Matrix R
  ++ rightRegularRepresentation(a) returns the matrix of the linear
  ++ map defined by right multiplication by \spad{a} with respect
  ++ to the fixed \spad{R}-module basis.
```

```
    if R has Field then

      leftRankPolynomial : () -> SparseUnivariatePolynomial Polynomial R
        ++ leftRankPolynomial() calculates the left minimal polynomial
        ++ of the generic element in the algebra,
        ++ defined by the same structural
        ++ constants over the polynomial ring in symbolic coefficients with
        ++ respect to the fixed basis.

      rightRankPolynomial : () -> SparseUnivariatePolynomial Polynomial R
        ++ rightRankPolynomial() calculates the right minimal polynomial
        ++ of the generic element in the algebra,
        ++ defined by the same structural
        ++ constants over the polynomial ring in symbolic coefficients with
        ++ respect to the fixed basis.

    apply : (Matrix R, %) -> %
      ++ apply(m,a) defines a left operation of n by n matrices
      ++ where n is the rank of the algebra in terms of matrix-vector
      ++ multiplication, this is a substitute for a left module structure.
      ++ Error: if shape of matrix doesn't fit.

      --attributes
      --separable <=> discriminant() ^= 0

   add

    V  ==> Vector
    M  ==> Matrix
    P  ==> Polynomial
    F  ==> Fraction
    REC  ==> Record(particular: Union(V R,"failed"),basis: List V R)
    LSMP ==> LinearSystemMatrixPackage(R,V R,V R, M R)
    CVMP ==> CoerceVectorMatrixPackage(R)

    --GA ==> GenericNonAssociativeAlgebra(R,rank()$%,_
    -- [random()$Character :: String :: Symbol for i in 1..rank()$%], _
    -- structuralConstants()$%)
    --y : GA := generic()

    if R has Field then

      leftRankPolynomial() ==
        n := rank()
        b := basis()
        gamma : Vector Matrix R := structuralConstants b
        listOfNumbers : List String:= [PRINC_-TO_-STRING(q)$Lisp for q in 1..n]
        symbolsForCoef : Vector Symbol :=
          [concat("%", concat("x", i))::Symbol  for i in listOfNumbers]
        xx : M P R
        mo : P R
        x : M P R := new(1,n,0)
        for i in 1..n repeat
```

```
    mo := monomial(1, [symbolsForCoef.i], [1])$(P R)
    qsetelt_!(x,1,i,mo)
  y : M P R := copy x
  k  : PositiveInteger := 1
  cond : M P R := copy x
  -- multiplication in the generic algebra means using
  -- the structural matrices as bilinear forms.
  -- left multiplication by x, we prepare for that:
  genGamma : V M P R :=  coerceP$CVMP gamma
  x := reduce(horizConcat,[x*genGamma(i) for i in 1..#genGamma])
  while rank(cond) = k repeat
    k := k+1
    for i in 1..n repeat
      setelt(xx,[1],[i],x*transpose y)
    y := copy xx
    cond := horizConcat(cond, xx)
  vectorOfCoef : Vector P R := (nullSpace(cond)$Matrix(P R)).first
  res : SparseUnivariatePolynomial P R := 0
  for i in 1..k repeat
   res:=res+monomial(vectorOfCoef.i,i)$(SparseUnivariatePolynomial P R)
  res

rightRankPolynomial() ==
  n := rank()
  b := basis()
  gamma : Vector Matrix R := structuralConstants b
  listOfNumbers : List String :=[PRINC_-TO_-STRING(q)$Lisp for q in 1..n]
  symbolsForCoef : Vector Symbol :=
    [concat("%", concat("x", i))::Symbol  for i in listOfNumbers]
  xx : M P R
  mo : P R
  x : M P R := new(1,n,0)
  for i in 1..n repeat
    mo := monomial(1, [symbolsForCoef.i], [1])$(P R)
    qsetelt_!(x,1,i,mo)
  y : M P R := copy x
  k  : PositiveInteger := 1
  cond : M P R := copy x
  -- multiplication in the generic algebra means using
  -- the structural matrices as bilinear forms.
  -- left multiplication by x, we prepare for that:
  genGamma : V M P R :=  coerceP$CVMP gamma
  x := _
   reduce(horizConcat,[genGamma(i)*transpose x for i in 1..#genGamma])
  while rank(cond) = k repeat
    k := k+1
    for i in 1..n repeat
      setelt(xx,[1],[i],y * transpose x)
    y := copy xx
    cond := horizConcat(cond, xx)
  vectorOfCoef : Vector P R := (nullSpace(cond)$Matrix(P R)).first
  res : SparseUnivariatePolynomial P R := 0
  for i in 1..k repeat
   res := _
```

```
      res+monomial(vectorOfCoef.i,i)$(SparseUnivariatePolynomial  P R)
    res

  leftUnitsInternal : () -> REC
  leftUnitsInternal() ==
    n := rank()
    b := basis()
    gamma : Vector Matrix R := structuralConstants b
    cond : Matrix(R) := new(n**2,n,0$R)$Matrix(R)
    rhs : Vector(R) := new(n**2,0$R)$Vector(R)
    z : Integer := 0
    addOn : R := 0
    for k in 1..n repeat
     for i in 1..n repeat
       z := z+1   -- index for the rows
       addOn :=
         k=i => 1
         0
       setelt(rhs,z,addOn)$Vector(R)
       for j in 1..n repeat  -- index for the columns
         setelt(cond,z,j,elt(gamma.k,j,i))$Matrix(R)
    solve(cond,rhs)$LSMP


  leftUnit() ==
    res : REC := leftUnitsInternal()
    res.particular case "failed" =>
      messagePrint("this algebra has no left unit")$OutputForm
      "failed"
    represents (res.particular :: V R)

  leftUnits() ==
    res : REC := leftUnitsInternal()
    res.particular case "failed" =>
      messagePrint("this algebra has no left unit")$OutputForm
      "failed"
    [represents(res.particular :: V R)$%, _
      map(represents, res.basis)$ListFunctions2(Vector R, %) ]

  rightUnitsInternal : () -> REC
  rightUnitsInternal() ==
    n := rank()
    b := basis()
    gamma : Vector Matrix R := structuralConstants b
    condo : Matrix(R) := new(n**2,n,0$R)$Matrix(R)
    rhs : Vector(R) := new(n**2,0$R)$Vector(R)
    z : Integer := 0
    addOn : R := 0
    for k in 1..n repeat
     for i in 1..n repeat
       z := z+1   -- index for the rows
       addOn :=
         k=i => 1
         0
```

```
        setelt(rhs,z,addOn)$Vector(R)
        for j in 1..n repeat  -- index for the columns
          setelt(condo,z,j,elt(gamma.k,i,j))$Matrix(R)
      solve(condo,rhs)$LSMP

    rightUnit() ==
      res : REC := rightUnitsInternal()
      res.particular case "failed" =>
        messagePrint("this algebra has no right unit")$OutputForm
        "failed"
      represents (res.particular :: V R)

    rightUnits() ==
      res : REC := rightUnitsInternal()
      res.particular case "failed" =>
        messagePrint("this algebra has no right unit")$OutputForm
        "failed"
      [represents(res.particular :: V R)$%, _
        map(represents, res.basis)$ListFunctions2(Vector R, %) ]

    unit() ==
      n := rank()
      b := basis()
      gamma : Vector Matrix R := structuralConstants b
      cond : Matrix(R) := new(2*n**2,n,0$R)$Matrix(R)
      rhs : Vector(R) := new(2*n**2,0$R)$Vector(R)
      z : Integer := 0
      u : Integer := n*n
      addOn : R := 0
      for k in 1..n repeat
       for i in 1..n repeat
        z := z+1   -- index for the rows
        addOn :=
          k=i => 1
          0
        setelt(rhs,z,addOn)$Vector(R)
        setelt(rhs,u,addOn)$Vector(R)
        for j in 1..n repeat  -- index for the columns
          setelt(cond,z,j,elt(gamma.k,j,i))$Matrix(R)
          setelt(cond,u,j,elt(gamma.k,i,j))$Matrix(R)
      res : REC := solve(cond,rhs)$LSMP
      res.particular case "failed" =>
        messagePrint("this algebra has no unit")$OutputForm
        "failed"
      represents (res.particular :: V R)

  apply(m:Matrix(R),a:%) ==
    v : Vector R := coordinates(a)
    v := m *$Matrix(R) v
    convert v

  structuralConstants() == structuralConstants basis()

  conditionsForIdempotents() == conditionsForIdempotents basis()
```

```
    convert(x:%):Vector(R) == coordinates(x, basis())

    convert(v:Vector R):% == represents(v, basis())

    leftTraceMatrix() == leftTraceMatrix basis()

    rightTraceMatrix() == rightTraceMatrix basis()

    leftDiscriminant() == leftDiscriminant basis()

    rightDiscriminant() == rightDiscriminant basis()

    leftRegularRepresentation x == leftRegularRepresentation(x, basis())

    rightRegularRepresentation x == rightRegularRepresentation(x, basis())

    coordinates x == coordinates(x, basis())

    represents(v:Vector R):% == represents(v, basis())

    coordinates(v:Vector %) ==
      m := new(#v, rank(), 0)$Matrix(R)
      for i in minIndex v .. maxIndex v for j in minRowIndex m .. repeat
        setRow_!(m, j, coordinates qelt(v, i))
      m
```

— **COQ FRNAALG** —

```
(* category FRNAALG *)
(*

    V  ==> Vector
    M  ==> Matrix
    P  ==> Polynomial
    F  ==> Fraction
    REC  ==> Record(particular: Union(V R,"failed"),basis: List V R)
    LSMP ==> LinearSystemMatrixPackage(R,V R,V R, M R)
    CVMP ==> CoerceVectorMatrixPackage(R)

    --GA ==> GenericNonAssociativeAlgebra(R,rank()$%,_
    -- [random()$Character :: String :: Symbol for i in 1..rank()$%], _
    -- structuralConstants()$%)
    --y : GA := generic()
    if R has Field then

      leftRankPolynomial : () -> SparseUnivariatePolynomial(Polynomial(R))
      leftRankPolynomial() ==
        n := rank()
        b := basis()
        gamma : Vector Matrix R := structuralConstants b
```

```
  listOfNumbers : List String:= [PRINC_-TO_-STRING(q)$Lisp for q in 1..n]
  symbolsForCoef : Vector Symbol :=
    [concat("%", concat("x", i))::Symbol  for i in listOfNumbers]
  xx : M P R
  mo : P R
  x : M P R := new(1,n,0)
  for i in 1..n repeat
    mo := monomial(1, [symbolsForCoef.i], [1])$(P R)
    qsetelt_!(x,1,i,mo)
  y : M P R := copy x
  k  : PositiveInteger := 1
  cond : M P R := copy x
  -- multiplication in the generic algebra means using
  -- the structural matrices as bilinear forms.
  -- left multiplication by x, we prepare for that:
  genGamma : V M P R :=  coerceP$CVMP gamma
  x := reduce(horizConcat,[x*genGamma(i) for i in 1..#genGamma])
  while rank(cond) = k repeat
    k := k+1
    for i in 1..n repeat
      setelt(xx,[1],[i],x*transpose y)
    y := copy xx
    cond := horizConcat(cond, xx)
  vectorOfCoef : Vector P R := (nullSpace(cond)$Matrix(P R)).first
  res : SparseUnivariatePolynomial P R := 0
  for i in 1..k repeat
   res:=res+monomial(vectorOfCoef.i,i)$(SparseUnivariatePolynomial P R)
  res

rightRankPolynomial : () -> SparseUnivariatePolynomial(Polynomial(R))
rightRankPolynomial() ==
  n := rank()
  b := basis()
  gamma : Vector Matrix R := structuralConstants b
  listOfNumbers : List String :=[PRINC_-TO_-STRING(q)$Lisp for q in 1..n]
  symbolsForCoef : Vector Symbol :=
    [concat("%", concat("x", i))::Symbol  for i in listOfNumbers]
  xx : M P R
  mo : P R
  x : M P R := new(1,n,0)
  for i in 1..n repeat
    mo := monomial(1, [symbolsForCoef.i], [1])$(P R)
    qsetelt_!(x,1,i,mo)
  y : M P R := copy x
  k  : PositiveInteger := 1
  cond : M P R := copy x
  -- multiplication in the generic algebra means using
  -- the structural matrices as bilinear forms.
  -- left multiplication by x, we prepare for that:
  genGamma : V M P R :=  coerceP$CVMP gamma
  x := _
   reduce(horizConcat,[genGamma(i)*transpose x for i in 1..#genGamma])
  while rank(cond) = k repeat
    k := k+1
```

```
      for i in 1..n repeat
        setelt(xx,[1],[i],y * transpose x)
      y := copy xx
      cond := horizConcat(cond, xx)
    vectorOfCoef : Vector P R := (nullSpace(cond)$Matrix(P R)).first
    res : SparseUnivariatePolynomial P R := 0
    for i in 1..k repeat
     res := _
      res+monomial(vectorOfCoef.i,i)$(SparseUnivariatePolynomial  P R)
    res

  leftUnitsInternal : () -> REC
  leftUnitsInternal() ==
    n := rank()
    b := basis()
    gamma : Vector Matrix R := structuralConstants b
    cond : Matrix(R) := new(n**2,n,0$R)$Matrix(R)
    rhs : Vector(R) := new(n**2,0$R)$Vector(R)
    z : Integer := 0
    addOn : R := 0
    for k in 1..n repeat
     for i in 1..n repeat
       z := z+1   -- index for the rows
       addOn :=
         k=i => 1
         0
       setelt(rhs,z,addOn)$Vector(R)
       for j in 1..n repeat  -- index for the columns
         setelt(cond,z,j,elt(gamma.k,j,i))$Matrix(R)
    solve(cond,rhs)$LSMP


  leftUnit : () -> Union(%,"failed")
  leftUnit() ==
    res : REC := leftUnitsInternal()
    res.particular case "failed" =>
      messagePrint("this algebra has no left unit")$OutputForm
      "failed"
    represents (res.particular :: V R)

  leftUnits : () -> Union(Record(particular: %,basis: List(%)),"failed")
  leftUnits() ==
    res : REC := leftUnitsInternal()
    res.particular case "failed" =>
      messagePrint("this algebra has no left unit")$OutputForm
      "failed"
    [represents(res.particular :: V R)$%, _
      map(represents, res.basis)$ListFunctions2(Vector R, %) ]

  rightUnitsInternal : () -> REC
  rightUnitsInternal() ==
    n := rank()
    b := basis()
    gamma : Vector Matrix R := structuralConstants b
```

```
  condo : Matrix(R) := new(n**2,n,0$R)$Matrix(R)
  rhs : Vector(R) := new(n**2,0$R)$Vector(R)
  z : Integer := 0
  addOn : R := 0
  for k in 1..n repeat
   for i in 1..n repeat
     z := z+1   -- index for the rows
     addOn :=
       k=i => 1
       0
     setelt(rhs,z,addOn)$Vector(R)
     for j in 1..n repeat  -- index for the columns
       setelt(condo,z,j,elt(gamma.k,i,j))$Matrix(R)
  solve(condo,rhs)$LSMP

rightUnit : () -> Union(%,"failed")
rightUnit() ==
  res : REC := rightUnitsInternal()
  res.particular case "failed" =>
    messagePrint("this algebra has no right unit")$OutputForm
    "failed"
  represents (res.particular :: V R)

rightUnits : () -> Union(Record(particular: %,basis: List(%)),"failed")
rightUnits() ==
  res : REC := rightUnitsInternal()
  res.particular case "failed" =>
    messagePrint("this algebra has no right unit")$OutputForm
    "failed"
  [represents(res.particular :: V R)$%, _
    map(represents, res.basis)$ListFunctions2(Vector R, %) ]

unit : () -> Union(%,"failed")
unit() ==
  n := rank()
  b := basis()
  gamma : Vector Matrix R := structuralConstants b
  cond : Matrix(R) := new(2*n**2,n,0$R)$Matrix(R)
  rhs : Vector(R) := new(2*n**2,0$R)$Vector(R)
  z : Integer := 0
  u : Integer := n*n
  addOn : R := 0
  for k in 1..n repeat
   for i in 1..n repeat
     z := z+1   -- index for the rows
     addOn :=
       k=i => 1
       0
     setelt(rhs,z,addOn)$Vector(R)
     setelt(rhs,u,addOn)$Vector(R)
     for j in 1..n repeat  -- index for the columns
       setelt(cond,z,j,elt(gamma.k,j,i))$Matrix(R)
       setelt(cond,u,j,elt(gamma.k,i,j))$Matrix(R)
  res : REC := solve(cond,rhs)$LSMP
```

```
      res.particular case "failed" =>
        messagePrint("this algebra has no unit")$OutputForm
        "failed"
      represents (res.particular :: V R)

  apply : (Matrix(R),%) -> %
  apply(m:Matrix(R),a:%) ==
    v : Vector R := coordinates(a)
    v := m *$Matrix(R) v
    convert v

  structuralConstants : () -> Vector(Matrix(R))
  structuralConstants() == structuralConstants basis()

  conditionsForIdempotents : () -> List(Polynomial(R))
  conditionsForIdempotents() == conditionsForIdempotents basis()

  convert : % -> Vector(R)
  convert(x:%):Vector(R) == coordinates(x, basis())

  convert : Vector(R) -> %
  convert(v:Vector R):% == represents(v, basis())

  leftTraceMatrix : () -> Matrix(R)
  leftTraceMatrix() == leftTraceMatrix basis()

  rightTraceMatrix : () -> Matrix(R)
  rightTraceMatrix() == rightTraceMatrix basis()

  leftDiscriminant : () -> R
  leftDiscriminant() == leftDiscriminant basis()

  rightDiscriminant : Vector(%) -> R
  rightDiscriminant() == rightDiscriminant basis()

  leftRegularRepresentation : % -> Matrix(R)
  leftRegularRepresentation x == leftRegularRepresentation(x, basis())

  rightRegularRepresentation : % -> Matrix(R)
  rightRegularRepresentation x == rightRegularRepresentation(x, basis())

  coordinates : % -> Vector(R)
  coordinates x == coordinates(x, basis())

  represents : Vector(R) -> %
  represents(v:Vector R):%== represents(v, basis())

  coordinates : Vector(%) -> Matrix(R)
  coordinates(v:Vector %) ==
    m := new(#v, rank(), 0)$Matrix(R)
    for i in minIndex v .. maxIndex v for j in minRowIndex m .. repeat
      setRow_!(m, j, coordinates qelt(v, i))
    m
```

```
*)
```

———-

— **FRNAALG.dotabb** —

```
"FRNAALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FRNAALG"];
"FRNAALG" -> "FINAALG"
```

———-

— **FRNAALG.dotfull** —

```
"FramedNonAssociativeAlgebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FRNAALG"];
"FramedNonAssociativeAlgebra(a:CommutativeRing)" ->
    "FiniteRankNonAssociativeAlgebra(a:CommutativeRing)"
```

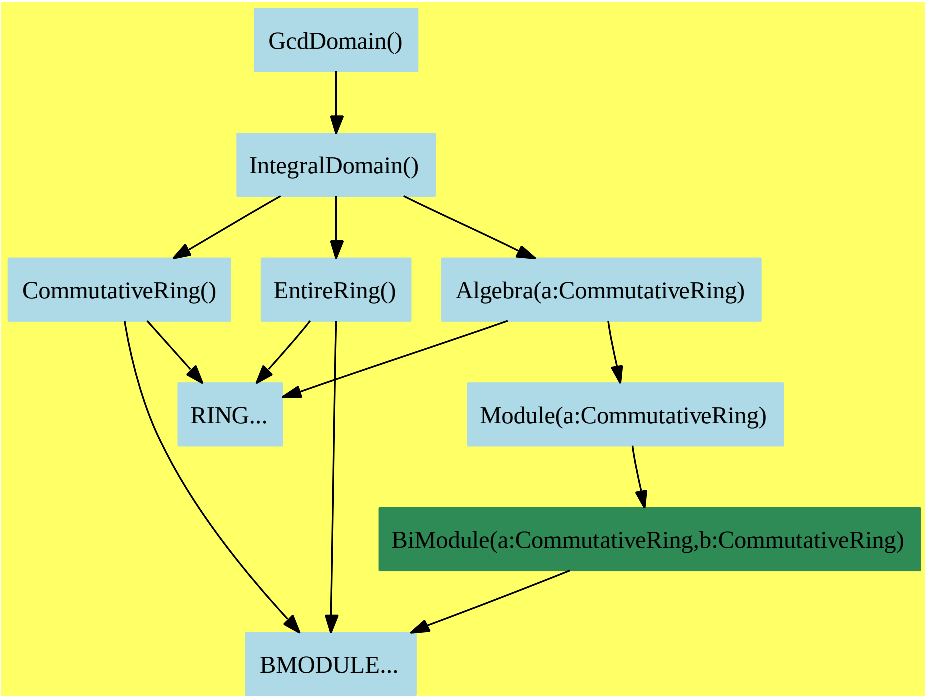
———-

— **FRNAALG.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FramedNonAssociativeAlgebra(a:CommutativeRing)" [color=lightblue];
"FramedNonAssociativeAlgebra(a:CommutativeRing)" ->
    "FiniteRankNonAssociativeAlgebra(a:CommutativeRing)"

"FiniteRankNonAssociativeAlgebra(a:CommutativeRing)" [color=lightblue];
"FiniteRankNonAssociativeAlgebra(a:CommutativeRing)" ->
    "NonAssociativeAlgebra(a:CommutativeRing)"

"NonAssociativeAlgebra(a:CommutativeRing)" [color=lightblue];
"NonAssociativeAlgebra(a:CommutativeRing)" -> "NonAssociativeRng()"
"NonAssociativeAlgebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"NonAssociativeRng()" [color=lightblue];
"NonAssociativeRng()" -> "ABELGRP..."
"NonAssociativeRng()" -> "Monad()"

"Monad()" [color=lightblue];
"Monad()" -> "SETCAT..."
"Monad()" -> "REPSQ..."

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"
```

```
"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"ABELGRP..." [color=lightblue];
}
```

13.0.197 GcdDomain (GCDDOM)



— GcdDomain.input —

```
)set break resume
)sys rm -f GcdDomain.output
)spool GcdDomain.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show GcdDomain
--R
--R GcdDomain is a category constructor
--R Abbreviation for GcdDomain is GCDDOM
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for GCDDOM
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean        coerce : % -> %
```

```
--R coerce : Integer -> %                coerce : % -> OutputForm
--R gcd : List(%) -> %                   gcd : (%,%) -> %
--R hash : % -> SingleInteger            latex : % -> String
--R lcm : List(%) -> %                   lcm : (%,%) -> %
--R one? : % -> Boolean                  recip : % -> Union(%,"failed")
--R sample : () -> %                     unit? : % -> Boolean
--R unitCanonical : % -> %               zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R exquo : (%,%) -> Union(%,"failed")
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

———-

— GcdDomain.help —

```
====================================================================
GcdDomain examples
====================================================================

This category describes domains where gcd can be computed but where
there is no guarantee of the existence of factor operation for factorisation
into irreducibles. However, if such a factor operation exist, factorization
will be unique up to order and units.

See Also:
o )show GcdDomain
```

———-

See:

⇒ "IntervalCategory" (INTCAT) 14.0.200 on page 1341
⇒ "PolynomialCategory" (POLYCAT) 16.0.213 on page 1449
⇒ "PrincipalIdealDomain" (PID) 14.0.202 on page 1357
⇒ "UniqueFactorizationDomain" (UFD) 14.0.203 on page 1362
⇒ "UnivariatePolynomialCategory" (UPOLYC) 17.0.228 on page 1708
⇐ "IntegralDomain" (INTDOM) 12.0.188 on page 1225

Exports:

0	1	associates?	characteristic	coerce
exquo	gcd	gcdPolynomial	hash	latex
lcm	one?	recip	sample	subtractIfCan
unit?	unitCanonical	unitNormal	zero?	?*?
?**?	?+?	?-?	-?	?=?
?^?	?~=?			

Attributes exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
gcd : (%,%) -> %
```

These are implemented by this category:

```
gcd : List % -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                    SparseUnivariatePolynomial %
lcm : (%,%) -> %
lcm : List % -> %
```

These exports come from (p1225) IntegralDomain():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
exquo : (%,%) -> Union(%,"failed")
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
```

```
 ?*? : (NonNegativeInteger,%) -> %
 ?-? : (%,%) -> %
 -? : % -> %
 ?**? : (%,PositiveInteger) -> %
 ?**? : (%,NonNegativeInteger) -> %
 ?^? : (%,PositiveInteger) -> %
 ?^? : (%,NonNegativeInteger) -> %
```

— GcdDomain.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#GCDDOM">
GcdDomain (GCDDOM)</a></h2>
</body>
```

— category GCDDOM GcdDomain —

```
)abbrev category GCDDOM GcdDomain
++ Description:
++ This category describes domains where
++ \spadfun{gcd} can be computed but where there is no guarantee
++ of the existence of \spadfun{factor} operation for factorisation
++ into irreducibles. However, if such a \spadfun{factor} operation exist,
++ factorization will be unique up to order and units.

GcdDomain() : Category == SIG where

  SIG ==> Join(IntegralDomain, LeftOreRing) with

    gcd : (%,%) -> %
      ++ gcd(x,y) returns the greatest common divisor of x and y.
      -- gcd(x,y) = gcd(y,x) in the presence of canonicalUnitNormal,
      -- but not necessarily elsewhere

    gcd : List(%) -> %
      ++ gcd(l) returns the common gcd of the elements in the list l.

    lcm : (%,%) -> %
      ++ lcm(x,y) returns the least common multiple of x and y.
      -- lcm(x,y) = lcm(y,x) in the presence of canonicalUnitNormal,
      -- but not necessarily elsewhere

    lcm : List(%) -> %
      ++ lcm(l) returns the least common multiple of the elements of
      ++ the list l.

    gcdPolynomial : (SparseUnivariatePolynomial %, _
                     SparseUnivariatePolynomial %) -> _
                        SparseUnivariatePolynomial %
      ++ gcdPolynomial(p,q) returns the greatest common divisor (gcd) of
      ++ univariate polynomials over the domain
```

```
add

  lcm(x: %,y: %) ==
    y = 0 => 0
    x = 0 => 0
    LCM : Union(%,"failed") := y exquo gcd(x,y)
    LCM case % =>  x * LCM
    error "bad gcd in lcm computation"

  lcm(l:List %) == reduce(lcm,l,1,0)

  gcd(l:List %) == reduce(gcd,l,0,1)

  SUP ==> SparseUnivariatePolynomial

  gcdPolynomial(p1,p2) ==
    zero? p1 => unitCanonical p2
    zero? p2 => unitCanonical p1
    c1:= content(p1); c2:= content(p2)
    p1:= (p1 exquo c1)::SUP %
    p2:= (p2 exquo c2)::SUP %
    if (e1:=minimumDegree p1) > 0 then p1:=(p1 exquo monomial(1,e1))::SUP %
    if (e2:=minimumDegree p2) > 0 then p2:=(p2 exquo monomial(1,e2))::SUP %
    e1:=min(e1,e2); c1:=gcd(c1,c2)
    p1:=
       degree p1 = 0 or degree p2 = 0 => monomial(c1,0)
       p:= subResultantGcd(p1,p2)
       degree p = 0 => monomial(c1,0)
       c2:= gcd(leadingCoefficient p1,leadingCoefficient p2)
       unitCanonical(_
         c1 * primitivePart(((c2*p) exquo leadingCoefficient p)::SUP %))
    zero? e1 => p1
    monomial(1,e1)*p1

  -- See [Delenclos 06], [Bronstein 96a]
  lcmCoef(c1, c2) ==
    g := gcd(c1, c2)
    cc1 := (c2 exquo g)::%
    cc2 := (c1 exquo g)::%
    [cc1*c1, cc1, cc2]
```

———-

— COQ GCDDOM —

```
(* category GCDDOM *)
(*

   lcm : (%,%) -> %
   lcm(x: %,y: %) ==
     y = 0 => 0
     x = 0 => 0
```

```
    LCM : Union(%,"failed") := y exquo gcd(x,y)
    LCM case % =>  x * LCM
    error "bad gcd in lcm computation"

  lcm : List(%) -> %
  lcm(l:List %) == reduce(lcm,l,1,0)

  gcd : List(%) -> %
  gcd(l:List %) == reduce(gcd,l,0,1)

  SUP ==> SparseUnivariatePolynomial

  gcdPolynomial : (SparseUnivariatePolynomial(%),
                   SparseUnivariatePolynomial(%)) ->
       SparseUnivariatePolynomial(%)
  gcdPolynomial(p1,p2) ==
    zero? p1 => unitCanonical p2
    zero? p2 => unitCanonical p1
    c1:= content(p1); c2:= content(p2)
    p1:= (p1 exquo c1)::SUP %
    p2:= (p2 exquo c2)::SUP %
    if (e1:=minimumDegree p1) > 0 then p1:=(p1 exquo monomial(1,e1))::SUP %
    if (e2:=minimumDegree p2) > 0 then p2:=(p2 exquo monomial(1,e2))::SUP %
    e1:=min(e1,e2); c1:=gcd(c1,c2)
    p1:=
       degree p1 = 0 or degree p2 = 0 => monomial(c1,0)
       p:= subResultantGcd(p1,p2)
       degree p = 0 => monomial(c1,0)
       c2:= gcd(leadingCoefficient p1,leadingCoefficient p2)
       unitCanonical(_
         c1 * primitivePart(((c2*p) exquo leadingCoefficient p)::SUP %))
    zero? e1 => p1
    monomial(1,e1)*p1

  -- See [Delenclos 06], [Bronstein 96a]
  lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
  lcmCoef(c1, c2) ==
    g := gcd(c1, c2)
    cc1 := (c2 exquo g)::%
    cc2 := (c1 exquo g)::%
    [cc1*c1, cc1, cc2]

*)
```

————

— GCDDOM.dotabb —

```
"GCDDOM"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=GCDDOM"];
"GCDDOM" -> "INTDOM"
```

————

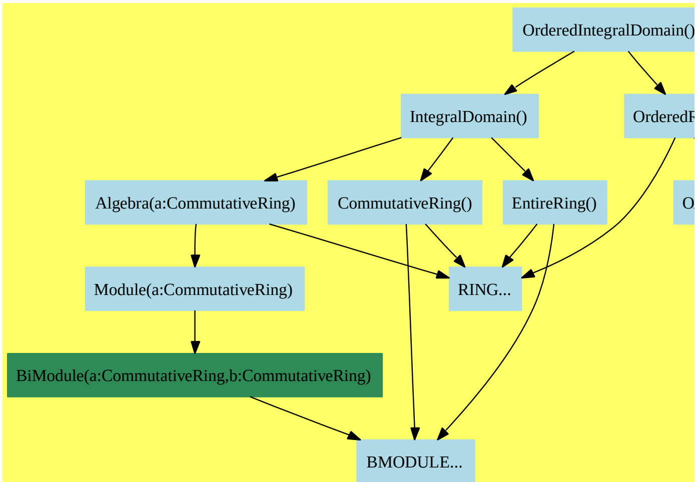
— GCDDOM.dotfull —

```
"GcdDomain()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=GCDDOM"];
"GcdDomain()" -> "IntegralDomain()"
```

— GCDDOM.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"GcdDomain()" [color=lightblue];
"GcdDomain()" -> "IntegralDomain()"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BMODULE..."

"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];
}
```

13.0.198 OrderedIntegralDomain (OINTDOM)



— OrderedIntegralDomain.input —

```
)set break resume
)sys rm -f OrderedIntegralDomain.output
)spool OrderedIntegralDomain.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show OrderedIntegralDomain
--R
--R OrderedIntegralDomain is a category constructor
--R Abbreviation for OrderedIntegralDomain is OINTDOM
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for OINTDOM
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean               ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean                ?>=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R abs : % -> %                          associates? : (%,%) -> Boolean
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              hash : % -> SingleInteger
--R latex : % -> String                   max : (%,%) -> %
--R min : (%,%) -> %                      negative? : % -> Boolean
--R one? : % -> Boolean                   positive? : % -> Boolean
```

```
--R recip : % -> Union(%,"failed")        sample : () -> %
--R sign : % -> Integer                   unit? : % -> Boolean
--R unitCanonical : % -> %                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R exquo : (%,%) -> Union(%,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

———-

— **OrderedIntegralDomain.help** —

```
====================================================================
OrderedIntegralDomain examples
====================================================================

The category of ordered commutative integral domains, where ordering
and the arithmetic operations are compatible

See Also:
o )show OrderedIntegralDomain
```

———-

See:

⇒ "IntegerNumberSystem" (INS) 16.0.210 on page 1420
⇐ "IntegralDomain" (INTDOM) 12.0.188 on page 1225
⇐ "QuotientFieldCategory" (QFCAT) 17.0.222 on page 1595
⇐ "OrderedRing" (ORDRING) 10.0.168 on page 1055

Exports:

1	0	abs	associates?	characteristic
coerce	exquo	hash	latex	max
min	negative?	one?	positive?	recip
sample	sign	subtractIfCan	unit?	unitCanonical
unitNormal	zero?	?*?	?**?	?+?
?-?	-?	?<?	?<=?	?=?
?>?	?>=?	?^?	?~=?	

Attributes exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown

means that the operation `recip` can only return "failed" if its argument is not a unit.

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These exports come from (p1225) IntegralDomain():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
exquo : (%,%) -> Union(%,"failed")
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?<? : (%,%) -> Boolean
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p1055) OrderedRing():

```
abs : % -> %
max : (%,%) -> %
min : (%,%) -> %
negative? : % -> Boolean
positive? : % -> Boolean
sign : % -> Integer
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

— **OrderedIntegralDomain.html** —

```
<body>
<h2>
```

```
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#OINTDOM">
OrderedIntegralDomain (OINTDOM)</a></h2>
</body>
```

— **category OINTDOM OrderedIntegralDomain** —

```
)abbrev category OINTDOM OrderedIntegralDomain
++ Author: JH Davenport (after L Gonzalez-Vega)
++ Date Created: 30.1.96
++ Description:
++ The category of ordered commutative integral domains, where ordering
++ and the arithmetic operations are compatible

OrderedIntegralDomain() : Category == SIG where

  SIG ==> Join(IntegralDomain, OrderedRing)
```

— **OINTDOM.dotabb** —

```
"OINTDOM"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OINTDOM"];
"OINTDOM" -> "INTDOM"
"OINTDOM" -> "ORDRING"
```

— **OINTDOM.dotfull** —

```
"OrderedIntegralDomain()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OINTDOM"];
"OrderedIntegralDomain()" -> "IntegralDomain()"
"OrderedIntegralDomain()" -> "OrderedRing()"
```

— **OINTDOM.dotpic** —

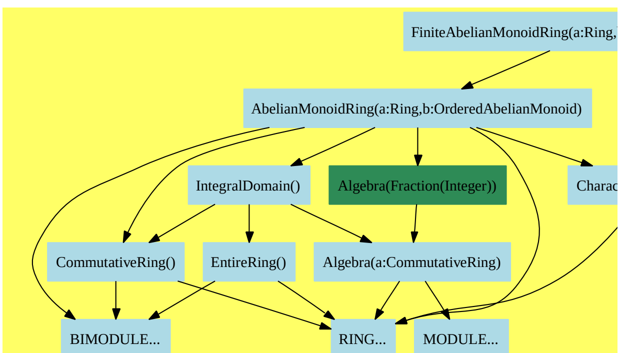
```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"OrderedIntegralDomain()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OINTDOM"];
"OrderedIntegralDomain()" -> "IntegralDomain()"
"OrderedIntegralDomain()" -> "OrderedRing()"

"IntegralDomain()" [color=lightblue];
```

```
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"OrderedRing()" [color=lightblue];
"OrderedRing()" -> "OAGROUP..."
"OrderedRing()" -> "RING..."
"OrderedRing()" -> "MONOID..."

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BMODULE..."

"OAGROUP..." [color=lightblue];
"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];
"MONOID..." [color=lightblue];
}
```

Chapter 14

Category Layer 13

14.0.199 FiniteAbelianMonoidRing (FAMR)



— **FiniteAbelianMonoidRing.input** —

```
)set break resume
)sys rm -f FiniteAbelianMonoidRing.output
)spool FiniteAbelianMonoidRing.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteAbelianMonoidRing
--R
--R FiniteAbelianMonoidRing(R: Ring,E: OrderedAbelianMonoid) is a category constructor
--R Abbreviation for FiniteAbelianMonoidRing is FAMR
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FAMR
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
```

```
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?/? : (%,R) -> % if R has FIELD
--R ?=? : (%,%) -> Boolean                1 : () -> %
--R 0 : () -> %                           ?^? : (%,NonNegativeInteger) -> %
--R ?^? : (%,PositiveInteger) -> %        coefficient : (%,E) -> R
--R coefficients : % -> List(R)           coerce : R -> %
--R coerce : % -> % if R has INTDOM       coerce : Integer -> %
--R coerce : % -> OutputForm              content : % -> R if R has GCDDOM
--R degree : % -> E                       ground : % -> R
--R ground? : % -> Boolean                hash : % -> SingleInteger
--R latex : % -> String                   leadingCoefficient : % -> R
--R leadingMonomial : % -> %              map : ((R -> R),%) -> %
--R mapExponents : ((E -> E),%) -> %      minimumDegree : % -> E
--R monomial : (R,E) -> %                 monomial? : % -> Boolean
--R one? : % -> Boolean                   pomopo! : (%,R,E,%) -> %
--R recip : % -> Union(%,"failed")        reductum : % -> %
--R retract : % -> R                      sample : () -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction(Integer)) -> % if R has ALGEBRA(FRAC(INT))
--R ?*? : (Fraction(Integer),%) -> % if R has ALGEBRA(FRAC(INT))
--R associates? : (%,%) -> Boolean if R has INTDOM
--R binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
--R coerce : Fraction(Integer) -> % if R has RETRACT(FRAC(INT)) or R has ALGEBRA(FRAC(INT))
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R numberOfMonomials : % -> NonNegativeInteger
--R primitivePart : % -> % if R has GCDDOM
--R retract : % -> Fraction(Integer) if R has RETRACT(FRAC(INT))
--R retract : % -> Integer if R has RETRACT(INT)
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction(Integer),"failed") if R has RETRACT(FRAC(INT))
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT(INT)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R
--E 1

)spool
)lisp (bye)
```

— **FiniteAbelianMonoidRing.help** —

```
====================================================================
FiniteAbelianMonoidRing examples
====================================================================
```

```
This category is similar to AbelianMonoidRing, except that the sum is
assumed to be finite. It is a useful model for polynomials, but is
somewhat more general.

See Also:
o )show FiniteAbelianMonoidRing
```

See:

⇒ "PolynomialCategory" (POLYCAT) 16.0.213 on page 1449
⇐ "AbelianMonoidRing" (AMR) 13.0.194 on page 1287
⇐ "FullyRetractableTo" (FRETRCT) 3.0.50 on page 158

Exports:

0	1	associates?	binomThmExpt
characteristic	charthRoot	coefficient	coefficients
coerce	content	degree	exquo
ground	ground?	hash	latex
leadingCoefficient	leadingMonomial	map	mapExponents
minimumDegree	monomial	monomial?	numberOfMonomials
one?	pomopo!	primitivePart	recip
reductum	retract	retractIfCan	sample
subtractIfCan	unit?	unitCanonical	unitNormal
zero?	?*?	?**?	?+?
?-?	-?	?=?	?^?
?~=?	?/?		

Attributes exported:

- if $ has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation $" * ": (D,D)->D$ which is commutative.
- if $ has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x*y \neq 0$ implies both x and y are non-zero.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
minimumDegree : % -> E
numberOfMonomials : % -> NonNegativeInteger
```

These are implemented by this category:

```
binomThmExpt : (%,%,NonNegativeInteger) -> %
    if R has COMRING
coefficients : % -> List R
content : % -> R if R has GCDDOM
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
```

```
ground : % -> R
ground? : % -> Boolean
mapExponents : ((E -> E),%) -> %
pomopo! : (%,R,E,%) -> %
primitivePart : % -> % if R has GCDDOM
?/? : (%,R) -> % if R has FIELD
```

These exports come from (p1287) AbelianMonoidRing(R,E)
where R:Ring and E:OrderedAbelianMonoid:

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean if R has INTDOM
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if R has CHARNZ
coefficient : (%,E) -> R
coerce : R -> %
coerce : Fraction Integer -> %
    if R has RETRACT FRAC INT
    or R has ALGEBRA FRAC INT
coerce : % -> % if R has INTDOM
coerce : % -> OutputForm
coerce : Integer -> %
degree : % -> E
exquo : (%,%) -> Union(%,"failed") if R has INTDOM
hash : % -> SingleInteger
latex : % -> String
leadingCoefficient : % -> R
leadingMonomial : % -> %
map : ((R -> R),%) -> %
monomial : (R,E) -> %
monomial? : % -> Boolean
one? : % -> Boolean
recip : % -> Union(%,"failed")
reductum : % -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean if R has INTDOM
unitCanonical : % -> % if R has INTDOM
unitNormal :
   % -> Record(unit: %,canonical: %,associate: %)
   if R has INTDOM
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (R,%) -> %
?*? : (%,R) -> %
?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
?**? : (%,NonNegativeInteger) -> %
```

```
?^? : (%,NonNegativeInteger) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p158) FullyRetractableTo(R:Ring):

```
retract : % -> Fraction Integer
    if R has RETRACT FRAC INT
retract : % -> Integer if R has RETRACT INT
retract : % -> R
retractIfCan : % -> Union(R,"failed")
retractIfCan : % -> Union(Integer,"failed")
    if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
    if R has RETRACT FRAC INT
```

— FiniteAbelianMonoidRing.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FAMR">
FiniteAbelianMonoidRing (FAMR)</a></h2>
</body>
```

— category FAMR FiniteAbelianMonoidRing —

```
)abbrev category FAMR FiniteAbelianMonoidRing
++ Date Last Updated: 14.08.2000 Exported pomopo! and binomThmExpt [MMM]
++ Description:
++ This category is similar to AbelianMonoidRing, except that the sum is
++ assumed to be finite. It is a useful model for polynomials,
++ but is somewhat more general.

FiniteAbelianMonoidRing(R,E) : Category == SIG where
  R : Ring
  E : OrderedAbelianMonoid

  SIG ==> Join(AbelianMonoidRing(R,E),FullyRetractableTo R) with

    ground? : % -> Boolean
      ++ ground?(p) tests if polynomial p is a member of the
      ++ coefficient ring.
      -- can't be defined earlier, since a power series
      -- might not know if there were other terms or not

    ground : % -> R
      ++ ground(p) retracts polynomial p to the coefficient ring.

    coefficients : % -> List R
      ++ coefficients(p) gives the list of non-zero coefficients
      ++ of polynomial p.
```

```
 numberOfMonomials : % -> NonNegativeInteger
   ++ numberOfMonomials(p) gives the number of non-zero monomials
   ++ in polynomial p.

 minimumDegree : % -> E
   ++ minimumDegree(p) gives the least exponent of a non-zero term
   ++ of polynomial p. Error: if applied to 0.

 mapExponents : (E -> E, %) -> %
   ++ mapExponents(fn,u) maps function fn onto the exponents
   ++ of the non-zero monomials of polynomial u.

 pomopo! : (%,R,E,%) -> %
   ++ \spad{pomopo!(p1,r,e,p2)} returns \spad{p1 + monomial(e,r) * p2}
   ++ and may use \spad{p1} as workspace. The constaant \spad{r} is
   ++ assumed to be nonzero.

 if R has CommutativeRing then

    binomThmExpt : (%,%,NonNegativeInteger) -> %
      ++ \spad{binomThmExpt(p,q,n)} returns \spad{(x+y)^n}
      ++ by means of the binomial theorem trick.

 if R has IntegralDomain then

    "exquo" : (%,R) -> Union(%,"failed")
    ++ exquo(p,r) returns the exact quotient of polynomial p by r,
    ++ or "failed" if none exists.

 if R has GcdDomain then

    content : % -> R
      ++ content(p) gives the gcd of the coefficients of polynomial p.

    primitivePart : % -> %
      ++ primitivePart(p) returns the unit normalized form of polynomial p
      ++ divided by the content of p.

add

  pomopo!(p1,r,e,p2) == p1 + r * mapExponents(x1+->x1+e,p2)

  if R has CommutativeRing then

     binomThmExpt(x,y,nn) ==
             nn = 0 => 1$%
             ans,xn,yn: %
             bincoef: Integer
             powl: List(%):= [x]
             for i in 2..nn repeat powl:=[x * powl.first, :powl]
             yn:=y; ans:=powl.first; i:=1; bincoef:=nn
             for xn in powl.rest repeat
                ans:= bincoef * xn * yn + ans
```

```
              bincoef:= (nn-i) * bincoef quo (i+1);  i:= i+1
              -- last I and BINCOEF unused
              yn:= y * yn
           ans + yn

ground? x ==
  retractIfCan(x)@Union(R,"failed") case "failed" => false
  true

ground x == retract(x)@R

mapExponents (fn:E -> E, x: %) ==
     -- this default definition assumes that reductum is cheap
   zero? x => 0
   monomial(leadingCoefficient x,fn degree x)+mapExponents(fn,reductum x)

coefficients x ==
  zero? x => empty()
  concat(leadingCoefficient x, coefficients reductum x)

if R has Field then

   x/r == map(x1+->x1/r,x)

if R has IntegralDomain then

   x exquo r ==
      -- probably not a very good definition in most special cases
      zero? x => 0
      ans:% :=0
      t:=leadingCoefficient x exquo r
      while not (t case "failed") and not zero? x repeat
        ans:=ans+monomial(t::R,degree x)
        x:=reductum x
        if not zero? x then t:=leadingCoefficient x exquo r
      t case "failed" => "failed"
      ans

if R has GcdDomain then

   content x ==       -- this assumes  reductum is cheap
      zero? x => 0
      r:=leadingCoefficient x
      x:=reductum x
      while not zero? x and not (r = 1) repeat
        r:=gcd(r,leadingCoefficient x)
        x:=reductum x
      r

   primitivePart x ==
      zero? x => x
      c := content x
      unitCanonical((x exquo c)::%)
```

———-

— **COQ FAMR** —

```
(* category FAMR *)
(*

    pomopo! : (%,R,E,%) -> %
    pomopo!(p1,r,e,p2) == p1 + r * mapExponents(x1+->x1+e,p2)

    if R has CommutativeRing then

       binomThmExpt : (%,%,NonNegativeInteger) -> %
       binomThmExpt(x,y,nn) ==
               nn = 0 => 1$%
               ans,xn,yn: %
               bincoef: Integer
               powl: List(%):= [x]
               for i in 2..nn repeat powl:=[x * powl.first, :powl]
               yn:=y; ans:=powl.first; i:=1; bincoef:=nn
               for xn in powl.rest repeat
                  ans:= bincoef * xn * yn + ans
                  bincoef:= (nn-i) * bincoef quo (i+1);  i:= i+1
                  -- last I and BINCOEF unused
                  yn:= y * yn
               ans + yn

    ground? : % -> Boolean
    ground? x ==
      retractIfCan(x)@Union(R,"failed") case "failed" => false
      true

    ground : % -> R
    ground x == retract(x)@R

    mapExponents : ((E -> E),%) -> %
    mapExponents (fn:E -> E, x: %) ==
         -- this default definition assumes that reductum is cheap
       zero? x => 0
       monomial(leadingCoefficient x,fn degree x)+mapExponents(fn,reductum x)

    coefficients : % -> List(R)
    coefficients x ==
      zero? x => empty()
      concat(leadingCoefficient x, coefficients reductum x)

    if R has Field then

       ?/? : (%,R) -> %
       x/r == map(x1+->x1/r,x)

    if R has IntegralDomain then

       exquo : (%,R) -> Union(%,"failed")
```

```
      x exquo r ==
         -- probably not a very good definition in most special cases
         zero? x => 0
         ans:% :=0
         t:=leadingCoefficient x exquo r
         while not (t case "failed") and not zero? x repeat
           ans:=ans+monomial(t::R,degree x)
           x:=reductum x
           if not zero? x then t:=leadingCoefficient x exquo r
         t case "failed" => "failed"
         ans

   if R has GcdDomain then

      content : % -> R
      content x ==       -- this assumes  reductum is cheap
         zero? x => 0
         r:=leadingCoefficient x
         x:=reductum x
         while not zero? x and not (r = 1) repeat
           r:=gcd(r,leadingCoefficient x)
           x:=reductum x
         r

      primitivePart : % -> %
      primitivePart x ==
         zero? x => x
         c := content x
         unitCanonical((x exquo c)::%)
*)
```

———-

— FAMR.dotabb —

```
"FAMR"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FAMR"];
"FAMR" -> "AMR"
"FAMR" -> "FRETRCT"
```

———-

— FAMR.dotfull —

```
"FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FAMR"];
"FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"
"FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "FullyRetractableTo(a:Ring)"

"FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoidSup)"
```

```
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FAMR"];
"FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoidSup)" ->
    "FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"
```

———-

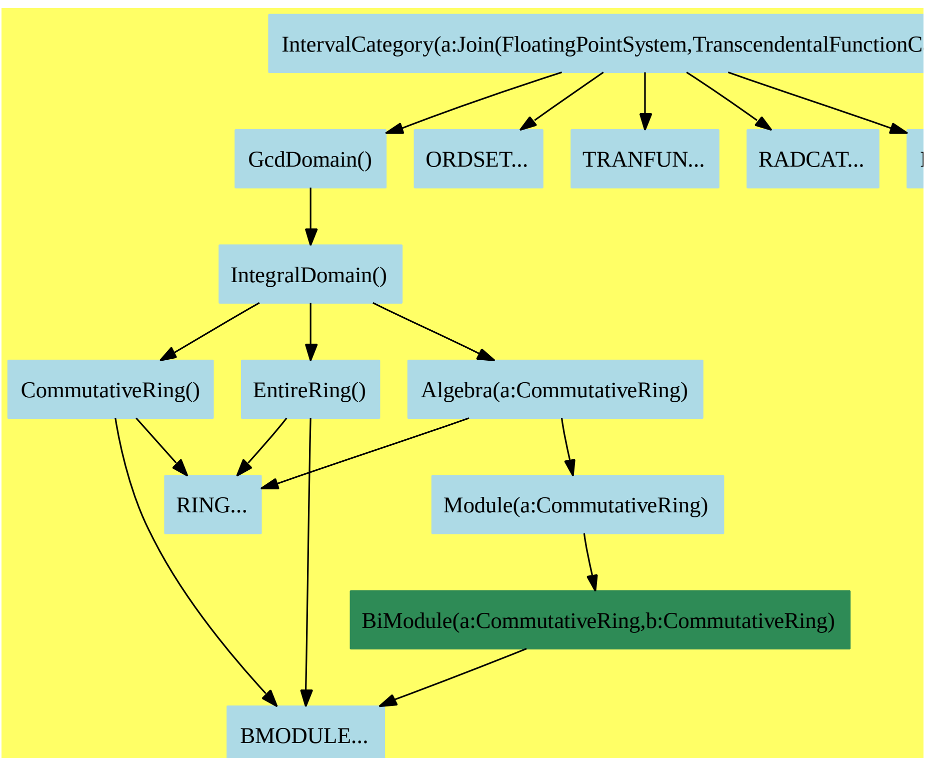
— **FAMR.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" [color=lightblue];
"FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"
"FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "FullyRetractableTo(a:Ring)"

"FullyRetractableTo(a:Ring)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FRETRCT"];
"FullyRetractableTo(a:Ring)" -> "FullyRetractableTo(a:Type)"

"FullyRetractableTo(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FRETRCT"];
"FullyRetractableTo(a:Type)" -> "RETRACT..."

"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" [color=lightblue];
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" -> "RING..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "BIMODULE..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "IntegralDomain()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CharacteristicNonZero()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CommutativeRing()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "Algebra(Fraction(Integer))"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BIMODULE..."

"CharacteristicNonZero()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARNZ"];
"CharacteristicNonZero()" -> "RING..."
```

```
"Algebra(Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(Fraction(Integer))" -> "Algebra(a:CommutativeRing)"

"Algebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "MODULE..."

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BIMODULE..."

"RETRACT..." [color=lightblue];
"BIMODULE..." [color=lightblue];
"RING..." [color=lightblue];
"MODULE..." [color=lightblue];
}
```

———–

14.0.200 IntervalCategory (INTCAT)



— **IntervalCategory.input** —

```
)set break resume
)sys rm -f IntervalCategory.output
)spool IntervalCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IntervalCategory
--R
--R IntervalCategory(R: Join(FloatingPointSystem,TranscendentalFunctionCategory)) is a category constructor
--R Abbreviation for IntervalCategory is INTCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for INTCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,Fraction(Integer)) -> %     ?**? : (%,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean               ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean                ?>=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R acos : % -> %                         acosh : % -> %
--R acot : % -> %                         acoth : % -> %
--R acsc : % -> %                         acsch : % -> %
--R asec : % -> %                         asech : % -> %
--R asin : % -> %                         asinh : % -> %
--R associates? : (%,%) -> Boolean        atan : % -> %
--R atanh : % -> %                        coerce : Integer -> %
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              contains? : (%,R) -> Boolean
--R cos : % -> %                          cosh : % -> %
--R cot : % -> %                          coth : % -> %
--R csc : % -> %                          csch : % -> %
--R exp : % -> %                          gcd : List(%) -> %
--R gcd : (%,%) -> %                      hash : % -> SingleInteger
--R inf : % -> R                          interval : Fraction(Integer) -> %
--R interval : R -> %                     interval : (R,R) -> %
--R latex : % -> String                   lcm : List(%) -> %
--R lcm : (%,%) -> %                      log : % -> %
--R max : (%,%) -> %                      min : (%,%) -> %
--R negative? : % -> Boolean              nthRoot : (%,Integer) -> %
--R one? : % -> Boolean                   pi : () -> %
--R positive? : % -> Boolean              qinterval : (R,R) -> %
--R recip : % -> Union(%,"failed")        retract : % -> Integer
--R sample : () -> %                      sec : % -> %
--R sech : % -> %                         sin : % -> %
--R sinh : % -> %                         sqrt : % -> %
--R sup : % -> R                          tan : % -> %
```

```
--R tanh : % -> %                          unit? : % -> Boolean
--R unitCanonical : % -> %                 width : % -> R
--R zero? : % -> Boolean                   ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R exquo : (%,%) -> Union(%,"failed")
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
--R retractIfCan : % -> Union(Integer,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

— **IntervalCategory.help** —

```
====================================================================
IntervalCategory examples
====================================================================

This category implements of interval arithmetic and transcendental
functions over intervals.

See Also:
o )show IntervalCategory
```

See:

⇐ “GcdDomain” (GCDDOM) 13.0.197 on page 1319
⇐ “OrderedSet” (ORDSET) 4.0.80 on page 326
⇐ “RadicalCategory” (RADCAT) 2.0.36 on page 106
⇐ “RetractableTo” (RETRACT) 2.0.37 on page 110
⇐ “TranscendentalFunctionCategory” (TRANFUN) 3.0.58 on page 192

Exports:

0	1	acos	acosh	acot
acoth	acsc	acsch	asec	asech
asin	asinh	associates?	atan	atanh
characteristic	coerce	contains?	cos	cosh
cot	coth	csc	csch	exp
exquo	gcd	gcdPolynomial	hash	inf
interval	latex	lcm	log	max
min	negative?	nthRoot	one?	pi
positive?	qinterval	recip	retract	retractIfCan
sample	sec	sech	sin	sinh
sqrt	subtractIfCan	sup	tan	tanh
unit?	unitCanonical	unitNormal	width	zero?
?*?	?**?	?+?	?-?	-?
?<?	?<=?	?=?	?>?	?>=?
?^?	?~=?			

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **approximate** means "is an approximation to the real numbers".

TPDHERE: Note that the signature `coerce : Integer -> %` shows up twice.

These are directly exported but not implemented:

```
contains? : (%,R) -> Boolean
inf : % -> R
interval : (R,R) -> %
interval : R -> %
interval : Fraction Integer -> %
negative? : % -> Boolean
positive? : % -> Boolean
qinterval : (R,R) -> %
sup : % -> R
width : % -> R
```

These exports come from (p1319) GcdDomain():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
exquo : (%,%) -> Union(%,"failed")
gcd : (%,%) -> %
gcd : List % -> %
```

```
gcdPolynomial :
 (SparseUnivariatePolynomial %,
  SparseUnivariatePolynomial %) ->
    SparseUnivariatePolynomial %
hash : % -> SingleInteger
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

These exports come from (p326) OrderedSet():

```
max : (%,%) -> %
min : (%,%) -> %
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?<? : (%,%) -> Boolean
```

These exports come from (p192) TranscendentalFunctionCategory():

```
acos : % -> %
acosh : % -> %
acot : % -> %
acoth : % -> %
acsc : % -> %
acsch : % -> %
asec : % -> %
asech : % -> %
asin : % -> %
asinh : % -> %
atan : % -> %
atanh : % -> %
cos : % -> %
cosh : % -> %
cot : % -> %
```

```
coth : % -> %
csc : % -> %
csch : % -> %
exp : % -> %
log : % -> %
pi : () -> %
sec : % -> %
sech : % -> %
sin : % -> %
sinh : % -> %
tan : % -> %
tanh : % -> %
?**? : (%,%) -> %
```

These exports come from (p106) RadicalCategory():

```
nthRoot : (%,Integer) -> %
sqrt : % -> %
?**? : (%,Fraction Integer) -> %
```

These exports come from (p110) RetractableTo(Integer):

```
coerce : Integer -> %
retract : % -> Integer
retractIfCan : % -> Union(Integer,"failed")
```

— IntervalCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#INTCAT">
IntervalCategory (INTCAT)</a></h2>
</body>
```

— category INTCAT IntervalCategory —

```
)abbrev category INTCAT IntervalCategory
++ Author: Mike Dewar
++ Date Created: November 1996
++ Description:
++ This category implements of interval arithmetic and transcendental
++ functions over intervals.

IntervalCategory(R) : Category == SIG where
  R : Join(FloatingPointSystem,TranscendentalFunctionCategory)

  GD  ==> GcdDomain
  OS  ==> OrderedSet
  TFC ==> TranscendentalFunctionCategory
  RC  ==> RadicalCategory
  RI  ==> RetractableTo(Integer)

  SIG ==> Join(GD,OS,TFC,RC,RI) with
```

```
    approximate

    interval : (R,R) -> %
      ++ interval(inf,sup) creates a new interval, either \axiom{[inf,sup]} if
      ++ \axiom{inf <= sup} or \axiom{[sup,in]} otherwise.

    qinterval : (R,R) -> %
      ++ qinterval(inf,sup) creates a new interval \axiom{[inf,sup]}, without
      ++ checking the ordering on the elements.

    interval : R -> %
      ++ interval(f) creates a new interval around f.

    interval : Fraction Integer -> %
      ++ interval(f) creates a new interval around f.

    inf : % -> R
      ++ inf(u) returns the infinum of \axiom{u}.

    sup : % -> R
      ++ sup(u) returns the supremum of \axiom{u}.

    width : % -> R
      ++ width(u) returns \axiom{sup(u) - inf(u)}.

    positive? : % -> Boolean
      ++ positive?(u) returns \axiom{true} if every element of u is positive,
      ++ \axiom{false} otherwise.

    negative? : % -> Boolean
      ++ negative?(u) returns \axiom{true} if every element of u is negative,
      ++ \axiom{false} otherwise.

    contains? : (%,R) -> Boolean
      ++ contains?(i,f) returns true if \axiom{f} is contained within the
      ++ interval \axiom{i}, false otherwise.
```

———-

— INTCAT.dotabb —

```
"INTCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=INTCAT"];
"INTCAT" -> "GCDDOM"
"INTCAT" -> "ORDSET"
"INTCAT" -> "RADCAT"
"INTCAT" -> "RETRACT"
"INTCAT" -> "TRANFUN"
```

———-

— INTCAT.dotfull —

```
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=INTCAT"];
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "GcdDomain()"
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "OrderedSet()"
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "TranscendentalFunctionCategory()"
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "RadicalCategory()"
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "RetractableTo(Integer)"
```

———-

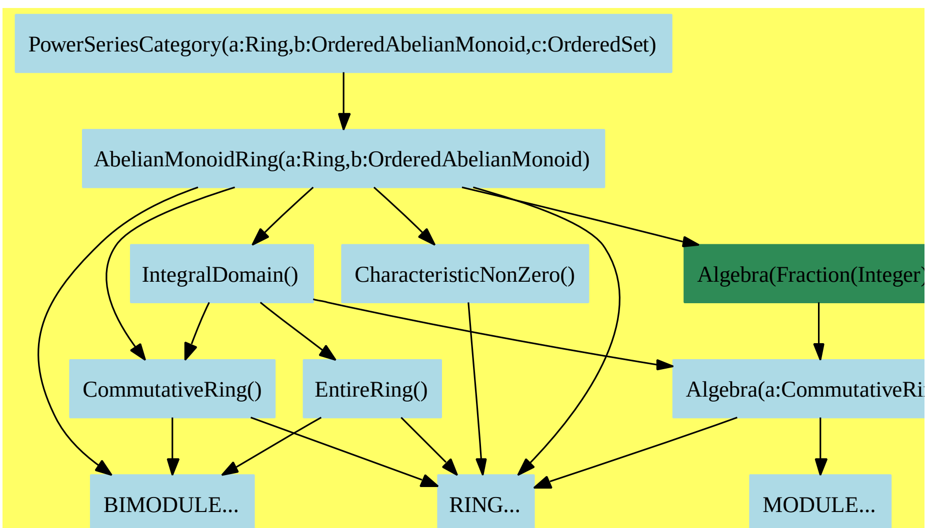
— INTCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
 [color=lightblue];
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "GcdDomain()"
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "ORDSET..."
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "TRANFUN..."
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "RADCAT..."
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "RETRACT..."

"GcdDomain()" [color=lightblue];
"GcdDomain()" -> "IntegralDomain()"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
```

```
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BMODULE..."

"RETRACT..." [color=lightblue];
"RADCAT..." [color=lightblue];
"TRANFUN..." [color=lightblue];
"ORDSET..." [color=lightblue];
"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];
}
```

———-

14.0.201 PowerSeriesCategory (PSCAT)



— **PowerSeriesCategory.input** —

```
)set break resume
)sys rm -f PowerSeriesCategory.output
)spool PowerSeriesCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PowerSeriesCategory
```

```
--R
--R PowerSeriesCategory(Coef: Ring,Expon: OrderedAbelianMonoid,Var: OrderedSet) is a category constructor
--R Abbreviation for PowerSeriesCategory is PSCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PSCAT
--R
--R------------------------------ Operations -------------------------------
--R ?*? : (Coef,%) -> %                   ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R coefficient : (%,Expon) -> Coef       coerce : % -> % if Coef has INTDOM
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R complete : % -> %                     degree : % -> Expon
--R hash : % -> SingleInteger             latex : % -> String
--R leadingCoefficient : % -> Coef        leadingMonomial : % -> %
--R map : ((Coef -> Coef),%) -> %         monomial : (%,Var,Expon) -> %
--R monomial : (Coef,Expon) -> %          monomial? : % -> Boolean
--R one? : % -> Boolean                   pole? : % -> Boolean
--R recip : % -> Union(%,"failed")        reductum : % -> %
--R sample : () -> %                      variables : % -> List(Var)
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction(Integer)) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?*? : (Fraction(Integer),%) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
--R coerce : Coef -> % if Coef has COMRING
--R coerce : Fraction(Integer) -> % if Coef has ALGEBRA(FRAC(INT))
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R monomial : (%,List(Var),List(Expon)) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R
--E 1

)spool
)lisp (bye)
```

— **PowerSeriesCategory.help** —

```
====================================================================
PowerSeriesCategory examples
====================================================================
```

```
PowerSeriesCategory is the most general power series category with
exponents in an ordered abelian monoid.

See Also:
o )show PowerSeriesCategory
```

See:

⇒ “MultivariateTaylorSeriesCategory” (MTSCAT) 15.0.206 on page 1384
⇒ “UnivariatePowerSeriesCategory” (UPSCAT) 15.0.208 on page 1401
⇐ “AbelianMonoidRing” (AMR) 13.0.194 on page 1287

Exports:

0	1	associates?	characteristic	charthRoot
coefficient	coerce	complete	degree	exquo
hash	latex	leadingCoefficient	leadingMonomial	map
monomial	monomial?	one?	pole?	recip
reductum	sample	subtractIfCan	variables	unit?
unitCanonical	unitNormal	zero?	?*?	?**?
?+?	?-?	-?	?=?	?~=?
?/?	?^?			

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- if #1 has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x*y \neq 0$ implies both x and y are non-zero.
- if #1 has CommutativeRing then commutative(“*”) where **commutative(“*”)** is true if it has an operation $" * " : (D, D)- > D$ which is commutative.

These are directly exported but not implemented:

```
complete : % -> %
degree : % -> Expon
leadingCoefficient : % -> Coef
leadingMonomial : % -> %
monomial : (%,List Var,List Expon) -> %
monomial : (%,Var,Expon) -> %
pole? : % -> Boolean
variables : % -> List Var
```

These are implemented by this category:

```
?*? : (Integer,%) -> %
?*? : (Coef,%) -> %
?*? : (%,Coef) -> %
?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
```

```
?/? : (%,Coef) -> % if Coef has FIELD
-? : % -> %
```

These exports come from (p1287) AbelianMonoidRing(Coef,Expon) where Coef:Ring and Expon:OrderedAbelianMonoid:

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean if Coef has INTDOM
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
coefficient : (%,Expon) -> Coef
coerce : Coef -> % if Coef has COMRING
coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
coerce : % -> % if Coef has INTDOM
coerce : % -> OutputForm
coerce : Integer -> %
exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
hash : % -> SingleInteger
latex : % -> String
map : ((Coef -> Coef),%) -> %
monomial : (Coef,Expon) -> %
monomial? : % -> Boolean
one? : % -> Boolean
recip : % -> Union(%,"failed")
reductum : % -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean if Coef has INTDOM
unitCanonical : % -> % if Coef has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if Coef has INTDOM
zero? : % -> Boolean
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
```

— **PowerSeriesCategory.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#PSCAT">
PowerSeriesCategory (PSCAT)</a></h2>
</body>
```

————

— category PSCAT PowerSeriesCategory —

```
)abbrev category PSCAT PowerSeriesCategory
++ Author: Clifton J. Williamson
++ Date Created: 21 December 1989
++ Date Last Updated: 25 February 1990
++ Description:
++ \spadtype{PowerSeriesCategory} is the most general power series
++ category with exponents in an ordered abelian monoid.

PowerSeriesCategory(Coef,Expon,Var) : Category == SIG where
  Coef : Ring
  Expon : OrderedAbelianMonoid
  Var : OrderedSet

  I   ==> Integer
  RN  ==> Fraction Integer

  SIG ==> AbelianMonoidRing(Coef,Expon) with

    monomial : (%,Var,Expon) -> %
      ++ \spad{monomial(a,x,n)} computes \spad{a*x**n}.

    monomial : (%,List Var,List Expon) -> %
      ++ \spad{monomial(a,[x1,..,xk],[n1,..,nk])} computes
      ++ \spad{a * x1**n1 * .. * xk**nk}.

    leadingMonomial : % -> %
      ++ leadingMonomial(f) returns the monomial of \spad{f} of lowest order.

    leadingCoefficient : % -> Coef
      ++ leadingCoefficient(f) returns the coefficient of the lowest order
      ++ term of \spad{f}

    degree : % -> Expon
      ++ degree(f) returns the exponent of the lowest order term of \spad{f}.

    variables : % -> List Var
      ++ \spad{variables(f)} returns a list of the variables occuring in the
      ++ power series f.

    pole? : % -> Boolean
      ++ \spad{pole?(f)} determines if the power series f has a pole.

    complete : % -> %
      ++ \spad{complete(f)} causes all terms of f to be computed.
      ++ Note that this results in an infinite loop
      ++ if f has infinitely many terms.

   add

     n:I * ps:% == (zero? n => 0; map((r1:Coef):Coef +-> n * r1,ps))

     r:Coef * ps:% == (zero? r => 0; map((r1:Coef):Coef +-> r * r1,ps))
```

```
 ps:% * r:Coef == (zero? r => 0; map((r1:Coef):Coef +-> r1 * r,ps))

 - ps == map((r1:Coef):Coef +-> -r1,ps)

 if Coef has Algebra Fraction Integer then

   r:RN * ps:% == (zero? r => 0; map((r1:Coef):Coef +-> r * r1,ps))

   ps:% * r:RN == (zero? r => 0; map((r1:Coef):Coef +-> r1 * r,ps))

 if Coef has Field then

   ps:% / r:Coef == map((r1:Coef):Coef +-> r1 / r,ps)
```

———-

— COQ PSCAT —

```
(* category PSCAT *)
(*

    ?*? : (Integer,%) -> %
    n:I * ps:% == (zero? n => 0; map((r1:Coef):Coef +-> n * r1,ps))

    ?*? : (Coef,%) -> %
    r:Coef * ps:% == (zero? r => 0; map((r1:Coef):Coef +-> r * r1,ps))

    ?*? : (%,Coef) -> %
    ps:% * r:Coef == (zero? r => 0; map((r1:Coef):Coef +-> r1 * r,ps))

    -? : % -> %
    - ps == map((r1:Coef):Coef +-> -r1,ps)

    if Coef has Algebra Fraction Integer then

      ?*? : (Fraction(Integer),%) -> %
      r:RN * ps:% == (zero? r => 0; map((r1:Coef):Coef +-> r * r1,ps))

      ?*? : (%,Fraction(Integer)) -> %
      ps:% * r:RN == (zero? r => 0; map((r1:Coef):Coef +-> r1 * r,ps))

    if Coef has Field then

      ?/? : (%,Coef) -> %
      ps:% / r:Coef == map((r1:Coef):Coef +-> r1 / r,ps)

*)
```

———-

— PSCAT.dotabb —

```
"PSCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PSCAT"];
"PSCAT" -> "AMR"
```

———-

— **PSCAT.dotfull** —

```
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PSCAT"];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)" ->
    "AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PSCAT"];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
  -> "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"

"PowerSeriesCategory(a:Ring,IndexedExponents(b:OrderedSet),c:OrderedSet))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PSCAT"];
"PowerSeriesCategory(a:Ring,IndexedExponents(b:OrderedSet),c:OrderedSet))"
  -> "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
```

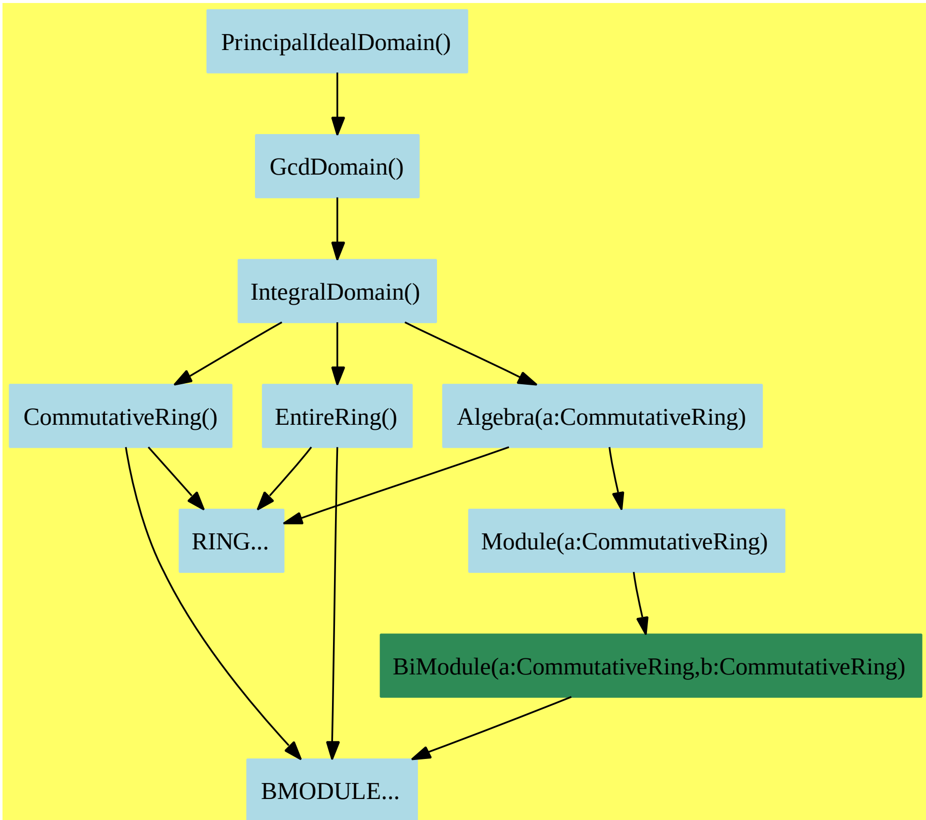
———-

— **PSCAT.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
 [color=lightblue];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)" ->
    "AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"

"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" [color=lightblue];
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" -> "RING..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "BIMODULE..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "IntegralDomain()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CharacteristicNonZero()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CommutativeRing()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "Algebra(Fraction(Integer))"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
```

```
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BIMODULE..."

"CharacteristicNonZero()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARNZ"];
"CharacteristicNonZero()" -> "RING..."

"Algebra(Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(Fraction(Integer))" -> "Algebra(a:CommutativeRing)"

"Algebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "MODULE..."

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BIMODULE..."

"BIMODULE..." [color=lightblue];
"RING..." [color=lightblue];
"MODULE..." [color=lightblue];
}
```

14.0.202 PrincipalIdealDomain (PID)



— **PrincipalIdealDomain.input** —

```
)set break resume
)sys rm -f PrincipalIdealDomain.output
)spool PrincipalIdealDomain.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PrincipalIdealDomain
--R
--R PrincipalIdealDomain is a category constructor
--R Abbreviation for PrincipalIdealDomain is PID
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PID
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
```

```
--R ?+? : (%,%) -> %                    ?-? : (%,%) -> %
--R -? : % -> %                         ?=? : (%,%) -> Boolean
--R 1 : () -> %                         0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %   ?^? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean      coerce : % -> %
--R coerce : Integer -> %               coerce : % -> OutputForm
--R gcd : List(%) -> %                  gcd : (%,%) -> %
--R hash : % -> SingleInteger           latex : % -> String
--R lcm : List(%) -> %                  lcm : (%,%) -> %
--R one? : % -> Boolean                 recip : % -> Union(%,"failed")
--R sample : () -> %                    unit? : % -> Boolean
--R unitCanonical : % -> %              zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

— **PrincipalIdealDomain.help** —

```
====================================================================
PrincipalIdealDomain examples
====================================================================

The category of constructive principal ideal domains, where a
single generator can be constructively found for any ideal given by
a finite set of generators.  Note that this constructive definition
only implies that finitely generated ideals are principal. It is not
clear what we would mean by an infinitely generated ideal.

See Also:
o )show PrincipalIdealDomain
```

See:

Unique Factorization Domains are a subset of Principal Ideal Domains.
⇐ "UniqueFactorizationDomain" (UFD) 14.0.203 on page 1362 Principal Ideal Domains are a subset of Euclidean Domains.
⇒ "EuclideanDomain" (EUCDOM) 15.0.205 on page 1374

⇐ "GcdDomain" (GCDDOM) 13.0.197 on page 1319

Exports:

0	1	associates?	characteristic	coerce
expressIdealMember	exquo	gcd	gcdPolynomial	hash
latex	lcm	one?	principalIdeal	recip
sample	subtractIfCan	unit?	unitCanonical	unitNormal
zero?	?+?	?-?	-?	?=?
?~=?	?*?	?**?	?^?	

Attributes exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
expressIdealMember : (List %,%) -> Union(List %,"failed")
principalIdeal : List % -> Record(coef: List %,generator: %)
```

These exports come from (p1319) GcdDomain():

```
associates? : (%,%) -> Boolean
exquo : (%,%) -> Union(%,"failed")
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                    SparseUnivariatePolynomial %
lcm : List % -> %
lcm : (%,%) -> %
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
```

```
 ?~=? : (%,%) -> Boolean
 ?*? : (%,%) -> %
 ?*? : (Integer,%) -> %
 ?*? : (PositiveInteger,%) -> %
 ?*? : (NonNegativeInteger,%) -> %
 ?-? : (%,%) -> %
 -? : % -> %
 ?**? : (%,PositiveInteger) -> %
 ?**? : (%,NonNegativeInteger) -> %
 ?^? : (%,PositiveInteger) -> %
 ?^? : (%,NonNegativeInteger) -> %
```

— **PrincipalIdealDomain.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#PID">
PrincipalIdealDomain (PID)</a></h2>
</body>
```

————

— **category PID PrincipalIdealDomain** —

```
)abbrev category PID PrincipalIdealDomain
++ Description:
++ The category of constructive principal ideal domains, that is,
++ where a single generator can be constructively found for
++ any ideal given by a finite set of generators.
++ Note that this constructive definition only implies that
++ finitely generated ideals are principal. It is not clear
++ what we would mean by an infinitely generated ideal.

PrincipalIdealDomain() : Category == SIG where

  SIG ==> GcdDomain with

    principalIdeal : List % -> Record(coef:List %,generator:%)
      ++ principalIdeal([f1,...,fn]) returns a record whose
      ++ generator component is a generator of the ideal
      ++ generated by \spad{[f1,...,fn]} whose coef component satisfies
      ++ \spad{generator = sum (input.i * coef.i)}

    expressIdealMember : (List %,%) -> Union(List %,"failed")
      ++ expressIdealMember([f1,...,fn],h) returns a representation
      ++ of h as a linear combination of the fi or "failed" if h
      ++ is not in the ideal generated by the fi.
```

————

— **PID.dotabb** —

```
"PID"
```

```
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PID"];
"PID" -> "GCDDOM"
```

———-

— PID.dotfull —

```
"PrincipalIdealDomain()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PID"];
"PrincipalIdealDomain()" -> "GcdDomain()"
```

———-

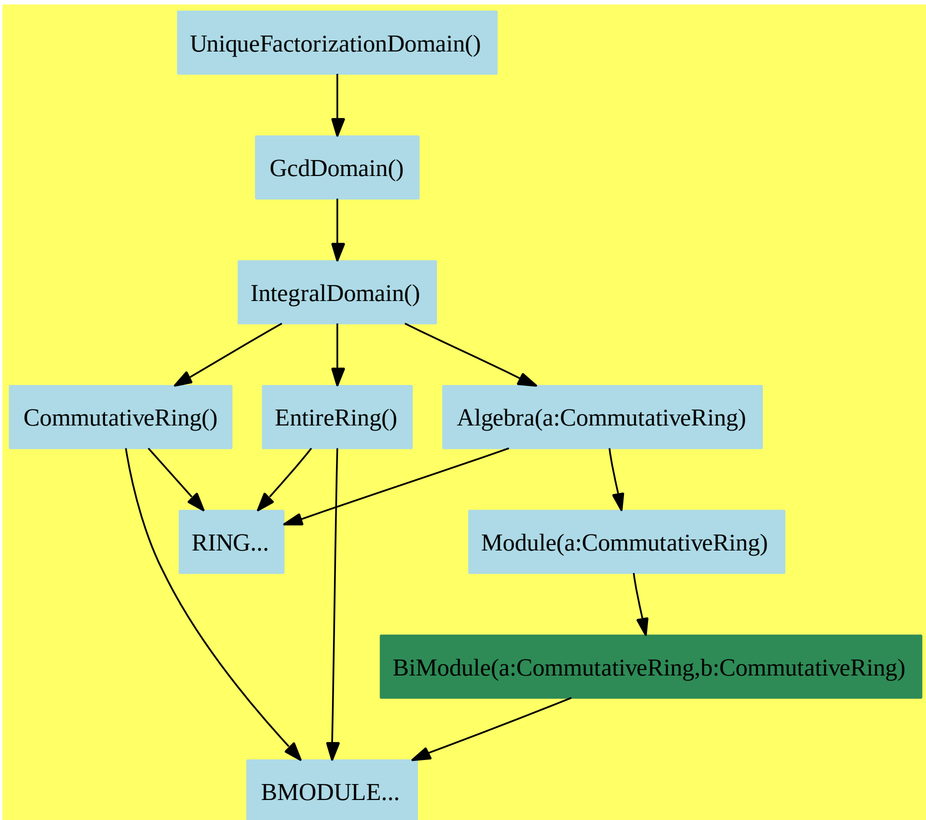
— PID.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"PrincipalIdealDomain()" [color=lightblue];
"PrincipalIdealDomain()" -> "GcdDomain()"

"GcdDomain()" [color=lightblue];
"GcdDomain()" -> "IntegralDomain()"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BMODULE..."

"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];
```

```
}
```

———-

14.0.203 UniqueFactorizationDomain (UFD)



— **UniqueFactorizationDomain.input** —

```
)set break resume
)sys rm -f UniqueFactorizationDomain.output
)spool UniqueFactorizationDomain.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UniqueFactorizationDomain
--R
--R UniqueFactorizationDomain is a category constructor
--R Abbreviation for UniqueFactorizationDomain is UFD
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for UFD
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean       coerce : % -> %
--R coerce : Integer -> %                coerce : % -> OutputForm
--R factor : % -> Factored(%)            gcd : List(%) -> %
--R gcd : (%,%) -> %                     hash : % -> SingleInteger
--R latex : % -> String                  lcm : List(%) -> %
--R lcm : (%,%) -> %                     one? : % -> Boolean
--R prime? : % -> Boolean                recip : % -> Union(%,"failed")
--R sample : () -> %                     squareFree : % -> Factored(%)
--R squareFreePart : % -> %              unit? : % -> Boolean
--R unitCanonical : % -> %               zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R exquo : (%,%) -> Union(%,"failed")
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

———-

— UniqueFactorizationDomain.help —

```
====================================================================
UniqueFactorizationDomain examples
====================================================================

A constructive unique factorization domain, where we can
constructively factor members into a product of a finite number
of irreducible elements.

See Also:
o )show UniqueFactorizationDomain
```

———-

All Integral Domains are UniqueFactorizationDomains.
⇐ "IntegralDomain" (INTDOM) 12.0.188 on page 1225. Unique Factorization Domains are a subset of Principal Ideal Domains.
⇒ "PrincipalIdealDomain" (PID) 14.0.202 on page 1357

See:

⇒ "Field" (FIELD) 16.0.209 on page 1413
⇒ "IntegerNumberSystem" (INS) 16.0.210 on page 1420
⇒ "PolynomialFactorizationExplicit" (PFECAT) 15.0.207 on page 1393
⇐ "GcdDomain" (GCDDOM) 13.0.197 on page 1319

Exports:

0	1	associates?	characteristic	coerce
exquo	factor	gcd	gcdPolynomial	hash
latex	lcm	one?	prime?	recip
sample	squareFree	squareFreePart	subtractIfCan	unit?
unitCanonical	unitNormal	zero?	?*?	?**?
?+?	?-?	-?	?=?	?~=?
?^?				

Attributes exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
factor : % -> Factored %
squareFree : % -> Factored %
```

These are implemented by this category:

```
prime? : % -> Boolean
squareFreePart : % -> %
```

These exports come from (p1319) GcdDomain():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
exquo : (%,%) -> Union(%,"failed")
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                    SparseUnivariatePolynomial %
hash : % -> SingleInteger
latex : % -> String
lcm : List % -> %
```

```
 lcm : (%,%) -> %
 one? : % -> Boolean
 recip : % -> Union(%,"failed")
 sample : () -> %
 subtractIfCan : (%,%) -> Union(%,"failed")
 unit? : % -> Boolean
 unitCanonical : % -> %
 unitNormal : % -> Record(unit: %,canonical: %,associate: %)
 zero? : % -> Boolean
 ?+? : (%,%) -> %
 ?=? : (%,%) -> Boolean
 ?~=? : (%,%) -> Boolean
 ?*? : (%,%) -> %
 ?*? : (Integer,%) -> %
 ?*? : (PositiveInteger,%) -> %
 ?*? : (NonNegativeInteger,%) -> %
 ?-? : (%,%) -> %
 -? : % -> %
 ?**? : (%,PositiveInteger) -> %
 ?**? : (%,NonNegativeInteger) -> %
 ?^? : (%,PositiveInteger) -> %
 ?^? : (%,NonNegativeInteger) -> %
```

— UniqueFactorizationDomain.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#UFD">
UniqueFactorizationDomain (UFD)</a></h2>
</body>
```

— category UFD UniqueFactorizationDomain —

```
)abbrev category UFD UniqueFactorizationDomain
++ Description:
++ A constructive unique factorization domain, where
++ we can constructively factor members into a product of
++ a finite number of irreducible elements.

UniqueFactorizationDomain() : Category == SIG where

  SIG ==> GcdDomain with

    prime? : % -> Boolean
      ++ prime?(x) tests if x can never be written as the product of two
      ++ non-units of the ring,
      ++ x is an irreducible element.

    squareFree : % -> Factored(%)
      ++ squareFree(x) returns the square-free factorization of x
      ++ such that the factors are pairwise relatively prime
      ++ and each has multiple prime factors.
```

```
    squareFreePart : % -> %
      ++ squareFreePart(x) returns a product of prime factors of
      ++ x each taken with multiplicity one.

    factor : % -> Factored(%)
      ++ factor(x) returns the factorization of x into irreducibles.

   add

     squareFreePart x ==
       unit(s := squareFree x) * _*/[f.factor for f in factors s]

     prime? x == # factorList factor x = 1
```

———-

— COQ UFD —

```
(* category UFD *)
(*

  squareFreePart : % -> %
  squareFreePart x ==
    unit(s := squareFree x) * _*/[f.factor for f in factors s]

  prime? : % -> Boolean
  prime? x == # factorList factor x = 1

*)
```

———-

— UFD.dotabb —

```
"UFD"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UFD"];
"UFD" -> "GCDDOM"
```

———-

— UFD.dotfull —

```
"UniqueFactorizationDomain()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UFD"];
"UniqueFactorizationDomain()" -> "GcdDomain()"
```

———-

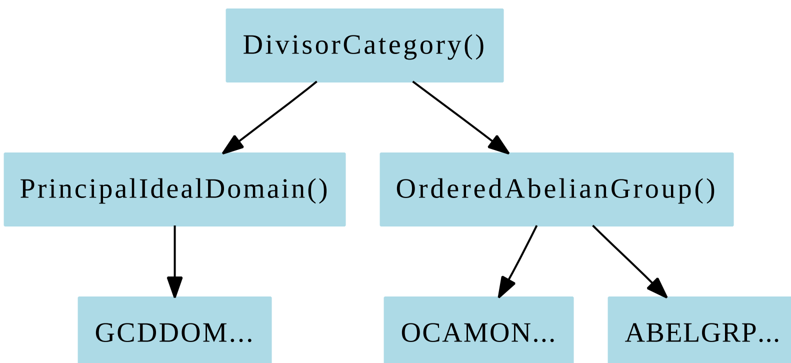
— UFD.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"UniqueFactorizationDomain()" [color=lightblue];
"UniqueFactorizationDomain()" -> "GcdDomain()"

"GcdDomain()" [color=lightblue];
"GcdDomain()" -> "IntegralDomain()"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BMODULE..."

"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];
}
```

Chapter 15

Category Layer 14

15.0.204 DivisorCategory (DIVCAT)



— **DivisorCategory.input** —

```
)set break resume
)sys rm -f DivisorCategory.output
)spool DivisorCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DivisorCategory
--R
--R DivisorCategory(S: SetCategory) is a category constructor
--R Abbreviation for DivisorCategory is DIVCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for DIVCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Integer,S) -> %                ?*? : (Integer,%) -> %
```

```
--R ?*? : (%,Integer) -> %                  ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %       ?*? : (PositiveInteger,%) -> %
--R ?+? : (S,%) -> %                        ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                        -? : % -> %
--R ?<=? : (%,%) -> Boolean                 ?=? : (%,%) -> Boolean
--R 0 : () -> %                             coefficient : (S,%) -> Integer
--R coerce : S -> %                         coerce : % -> OutputForm
--R collect : % -> %                        concat : (%,%) -> %
--R degree : % -> Integer                   divOfPole : % -> %
--R divOfZero : % -> %                      effective? : % -> Boolean
--R hash : % -> SingleInteger               incr : % -> %
--R latex : % -> String                     mapGen : ((S -> S),%) -> %
--R nthCoef : (%,Integer) -> Integer        nthFactor : (%,Integer) -> S
--R retract : % -> S                        sample : () -> %
--R size : % -> NonNegativeInteger          split : % -> List(%)
--R supp : % -> List(S)                     suppOfPole : % -> List(S)
--R suppOfZero : % -> List(S)               zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R highCommonTerms : (%,%) -> % if Integer has OAMON
--R mapCoef : ((Integer -> Integer),%) -> %
--R retractIfCan : % -> Union(S,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R terms : % -> List(Record(gen: S,exp: Integer))
--R
--E 1

)spool
)lisp (bye)
```

— **DivisorCategory.help** —

```
====================================================================
DivisorCategory examples
====================================================================

This category exports the function for domains.

See Also:
o )show DivisorCategory
```

See:

⇐ “OrderedAbelianGroup” (OAGROUP) 9.0.150 on page 932
⇐ “PrincipalIdealDomain” (PID) 14.0.202 on page 1357

Exports:

0	-?	?*?	?+?
?-?	?<=?	?=?	?~=?
coefficient	coerce	collect	concat
degree	divOfPole	divOfZero	effective?
hash	highCommonTerms	incr	latex
mapCoef	mapGen	nthCoef	nthFactor
retract	retractIfCan	sample	size
split	subtractIfCan	supp	suppOfPole
suppOfZero	terms	zero?	

These operations are exported but not implemented:

```
?<=? : (%,%) -> Boolean
collect : % -> %
concat : (%,%) -> %
degree : % -> Integer
divOfPole : % -> %
divOfZero : % -> %
effective? : % -> Boolean
incr : % -> %
split : % -> List %
supp : % -> List S
suppOfPole : % -> List S
suppOfZero : % -> List S
```

These operations come from (p673) AbelianGroup:

```
0 : () -> %
-? : % -> %
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
```

These operations come from (p1050) Module(Integer):

```
?*? : (Integer,%) -> %
?*? : (%,Integer) -> %
```

These operations come from (p719) FreeAbelianMonoidCategory(S: SetCategory,E: CancellationAbelianMonoid)

```
?*? : (E,S) -> %
?+? : (S,%) -> %
coefficient : (S,%) -> E
coerce : S -> %
highCommonTerms : (%,%) -> % if E has OAMON
mapCoef : ((E -> E),%) -> %
mapGen : ((S -> S),%) -> %
```

```
 nthCoef : (%,Integer) -> E
 nthFactor : (%,Integer) -> S
 retract : % -> S
 retractIfCan : % -> Union(S,"failed")
 size : % -> NonNegativeInteger
 terms : % -> List Record(gen: S,exp: E)
```

— DivisorCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#DIVCAT">
DivisorCategory (DIVCAT)</a></h2>
</body>
```

— category DIVCAT DivisorCategory —

```
)abbrev category DIVCAT DivisorCategory
++ Authors: Gaetan Hache
++ Date Created: may 1997
++ Date Last Updated: April 2010, by Tim Daly
++ Description: This category exports the function for domains

DivisorCategory(S) : Category == SIG where
  S : SetCategory

  INT     ==> Integer
  BOOLEAN ==> Boolean
  LIST    ==> List
  AB      ==> AbelianGroup
  MI      ==> Module(Integer)
  FAMC    ==> FreeAbelianMonoidCategory(S,Integer)

  SIG ==> Join(AB,MI,FAMC) with

    degree : % -> INT
      ++ degree(d) returns the degree of the divisor d

    split : % -> List %
      ++ split(d) splits the divisor d. For example,
      ++ \spad{split( 2 p1 + 3p2 )} returns the list \spad{[ 2 p1, 3 p2 ]}.

    "<=" : (%,%) -> BOOLEAN

    collect : % -> %
      ++ collect collects the duplicative points in the divisor.

    concat : (%,%) -> %
      ++ concat(a,b) concats the divisor a and b
      ++ without collecting the duplicative points.

    effective? : % -> BOOLEAN
```

```
      ++ effective?(d) returns true if d >= 0.

    supp: % -> LIST(S)
      ++ supp(d) returns the support of the divisor d.

    suppOfZero : % -> LIST(S)
      ++ suppOfZero(d) returns the elements of the support of d that
      ++ have a positive coefficient.

    suppOfPole : % -> LIST(S)
      ++ suppOfPole(d) returns the elements of the support of d that
      ++ have a negative coefficient.

    divOfZero : % -> %
      ++ divOfZero(d) returns the positive part of d.

    divOfPole : % -> %
      ++ divOfPole(d) returns the negative part of d.

    incr : % -> %
```

— **DIVCAT.dotabb** —

```
"DIVCAT" [color=lightblue,href="bookvol10.2.pdf#nameddest=DIVCAT"];
"OAGROUP" [color=lightblue,href="bookvol10.2.pdf#nameddest=OAGROUP"];
"PID" [color=lightblue,href="bookvol10.2.pdf#nameddest=PID"];
"DIVCAT" -> "OAGROUP"
"DIVCAT" -> "PID"
```

— **DIVCAT.dotfull** —

```
"DivisorCategory()" [color=lightblue,href="bookvol10.2.pdf#nameddest=DIVCAT"];
"DivisorCategory()" -> "PrincipalIdealDomain()"
```

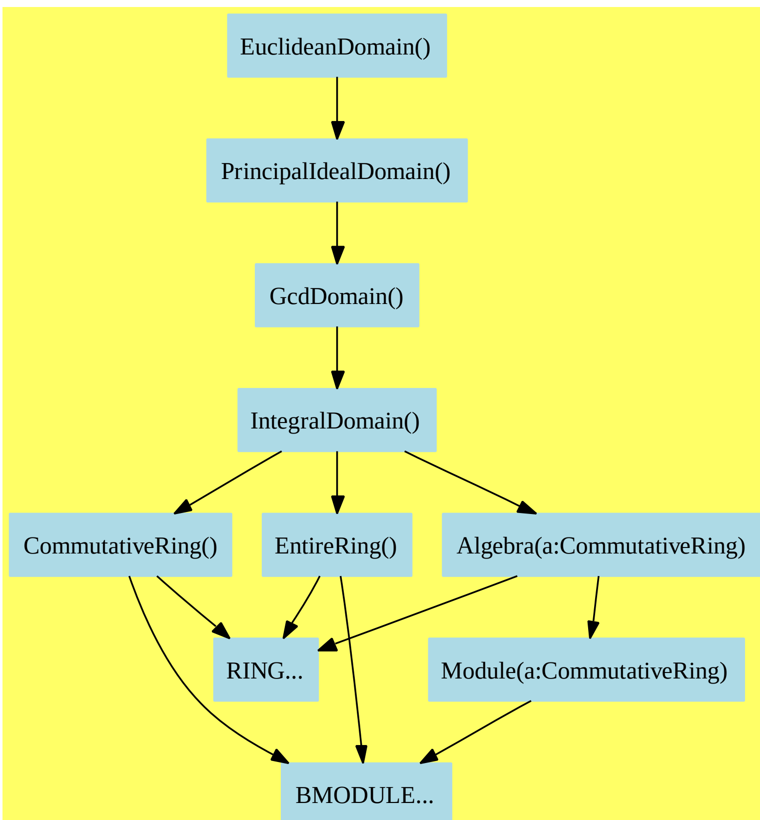
— **DIVCAT.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"DivisorCategory()" [color=lightblue];
"DivisorCategory()" -> "PrincipalIdealDomain()"
"DivisorCategory()" -> "OrderedAbelianGroup()"
```

```
"PrincipalIdealDomain()" [color=lightblue];
"PrincipalIdealDomain()" -> "GCDDOM..."

"OrderedAbelianGroup()" [color=lightblue];
"OrderedAbelianGroup()" -> "OCAMON..."
"OrderedAbelianGroup()" -> "ABELGRP..."

"GCDDOM..." [color=lightblue];
"OCAMON..." [color=lightblue];
"ABELGRP..." [color=lightblue];

}
```

————-

15.0.205 EuclideanDomain (EUCDOM)



— **EuclideanDomain.input** —

```
)set break resume
)sys rm -f EuclideanDomain.output
)spool EuclideanDomain.output
```

```
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show EuclideanDomain
--R
--R EuclideanDomain is a category constructor
--R Abbreviation for EuclideanDomain is EUCDOM
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for EUCDOM
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean        coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R gcd : List(%) -> %                    gcd : (%,%) -> %
--R hash : % -> SingleInteger             latex : % -> String
--R lcm : List(%) -> %                    lcm : (%,%) -> %
--R one? : % -> Boolean                   ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")        ?rem? : (%,%) -> %
--R sample : () -> %                      sizeLess? : (%,%) -> Boolean
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed")
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

———-

— EuclideanDomain.help —

```
====================================================================
```

```
EuclideanDomain examples
====================================================================

A constructive euclidean domain, one can divide producing
a quotient and a remainder where the remainder is either zero
or is smaller (euclideanSize) than the divisor.

Conditional attributes:
   multiplicativeValuation - Size(a*b)=Size(a)*Size(b)
   additiveValuation       - Size(a*b)=Size(a)+Size(b)

Principal Ideal Domains are a subset of Euclidean Domains.
Euclidean Domains are a subset of Fields.

See Also:
o )show EuclideanDomain
```

———-

⇐ "PrincipalIdealDomain" (PID) 14.0.202 on page 1357.
⇒ "Field" (FIELD) 16.0.209 on page 1413

See:

⇒ "Field" (FIELD) 16.0.209 on page 1413
⇒ "IntegerNumberSystem" (INS) 16.0.210 on page 1420
⇒ "PAdicIntegerCategory" (PADICCT) 16.0.212 on page 1443
⇒ "QuotientFieldCategory" (QFCAT) 17.0.222 on page 1595
⇐ "PrincipalIdealDomain" (PID) 14.0.202 on page 1357

Exports:

1	0	associates?	characteristic	coerce
divide	euclideanSize	expressIdealMember	exquo	extendedEuclidean
gcd	gcdPolynomial	hash	latex	lcm
multiEuclidean	one?	principalIdeal	recip	sample
sizeLess?	subtractIfCan	unit?	unitCanonical	unitNormal
zero?	?+?	?-?	-?	?=?
?quo?	?rem?	?~=?	?*?	?**?
?^?				

Attributes exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
```

These are implemented by this category:

```
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
gcd : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
principalIdeal : List % -> Record(coef: List %,generator: %)
sizeLess? : (%,%) -> Boolean
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p1357) PrincipalIdealDomain():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
gcd : List % -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                    SparseUnivariatePolynomial %
hash : % -> SingleInteger
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

— **EuclideanDomain.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#EUCDOM">
EuclideanDomain (EUCDOM)</a></h2>
</body>
```

———-

— **category EUCDOM EuclideanDomain** —

```
)abbrev category EUCDOM EuclideanDomain
++ Description:
++ A constructive euclidean domain, one can divide producing
++ a quotient and a remainder where the remainder is either zero
++ or is smaller (\spadfun{euclideanSize}) than the divisor.
++
++ Conditional attributes\br
++ \tab{5}multiplicativeValuation\tab{5}Size(a*b)=Size(a)*Size(b)\br
++ \tab{5}additiveValuation\tab{11}Size(a*b)=Size(a)+Size(b)

EuclideanDomain() : Category == SIG where

  SIG ==> PrincipalIdealDomain with

    sizeLess? : (%,%) -> Boolean
      ++ sizeLess?(x,y) tests whether x is strictly
      ++ smaller than y with respect to the
      ++ \spadfunFrom{euclideanSize}{EuclideanDomain}.

    euclideanSize : % -> NonNegativeInteger
      ++ euclideanSize(x) returns the euclidean size of the element x.
      ++ Error: if x is zero.

    divide : (%,%) -> Record(quotient:%,remainder:%)
      ++ divide(x,y) divides x by y producing a record containing a
      ++ \spad{quotient} and \spad{remainder},
      ++ where the remainder is smaller (see
      ++ \spadfunFrom{sizeLess?}{EuclideanDomain}) than the divisor y.

    "quo" : (%,%) -> %
      ++ x quo y is the same as \spad{divide(x,y).quotient}.
      ++ See \spadfunFrom{divide}{EuclideanDomain}.

    "rem" : (%,%) -> %
      ++ x rem y is the same as \spad{divide(x,y).remainder}.
      ++ See \spadfunFrom{divide}{EuclideanDomain}.

    extendedEuclidean : (%,%) -> Record(coef1:%,coef2:%,generator:%)
      ++ extendedEuclidean(x,y) returns a record rec where
      ++ \spad{rec.coef1*x+rec.coef2*y = rec.generator} and
      ++ rec.generator is a gcd of x and y.
      ++ The gcd is unique only
```

```
  ++ up to associates if \spadatt{canonicalUnitNormal} is not asserted.
  ++ \spadfun{principalIdeal} provides a version of this operation
  ++ which accepts an arbitrary length list of arguments.

 extendedEuclidean : (%,%,%) -> Union(Record(coef1:%,coef2:%),"failed")
  ++ extendedEuclidean(x,y,z) either returns a record rec
  ++ where \spad{rec.coef1*x+rec.coef2*y=z} or returns "failed"
  ++ if z cannot be expressed as a linear combination of x and y.

 multiEuclidean : (List %,%) -> Union(List %,"failed")
  ++ multiEuclidean([f1,...,fn],z) returns a list of coefficients
  ++ \spad{[a1, ..., an]} such that
  ++ \spad{ z / prod fi = sum aj/fj}.
  ++ If no such list of coefficients exists, "failed" is returned.

add

  x,y,z: %
  l: List %

  sizeLess?(x,y) ==
        zero? y => false
        zero? x => true
        euclideanSize(x)<euclideanSize(y)

  x quo y == divide(x,y).quotient --divide must be user-supplied

  x rem y == divide(x,y).remainder

  x exquo y ==
     zero? x => 0
     zero? y => "failed"
     qr:=divide(x,y)
     zero?(qr.remainder) => qr.quotient
     "failed"

  gcd(x,y) ==                --Euclidean Algorithm
     x:=unitCanonical x
     y:=unitCanonical y
     while not zero? y repeat
        (x,y):= (y,x rem y)
        y:=unitCanonical y   -- this doesn't affect the
                             -- correctness of Euclid's algorithm,
                             -- but
                             -- a) may improve performance
                             -- b) ensures gcd(x,y)=gcd(y,x)
                             --    if canonicalUnitNormal
     x

  IdealElt ==> Record(coef1:%,coef2:%,generator:%)

  unitNormalizeIdealElt(s:IdealElt):IdealElt ==
     (u,c,a):=unitNormal(s.generator)
     (a = 1) => s
```

```
   [a*s.coef1,a*s.coef2,c]$IdealElt

extendedEuclidean(x,y) ==         --Extended Euclidean Algorithm
   s1:=unitNormalizeIdealElt([1$%,0$%,x]$IdealElt)
   s2:=unitNormalizeIdealElt([0$%,1$%,y]$IdealElt)
   zero? y => s1
   zero? x => s2
   while not zero?(s2.generator) repeat
      qr:= divide(s1.generator, s2.generator)
      s3:=[s1.coef1 - qr.quotient * s2.coef1,
           s1.coef2 - qr.quotient * s2.coef2, qr.remainder]$IdealElt
      s1:=s2
      s2:=unitNormalizeIdealElt s3
   if not(zero?(s1.coef1)) and not sizeLess?(s1.coef1,y)
     then
        qr:= divide(s1.coef1,y)
        s1.coef1:= qr.remainder
        s1.coef2:= s1.coef2 + qr.quotient * x
        s1 := unitNormalizeIdealElt s1
   s1

TwoCoefs ==> Record(coef1:%,coef2:%)

extendedEuclidean(x,y,z) ==
   zero? z => [0,0]$TwoCoefs
   s:= extendedEuclidean(x,y)
   (w:= z exquo s.generator) case "failed" => "failed"
   zero? y => [s.coef1 * w, s.coef2 * w]$TwoCoefs
   qr:= divide((s.coef1 * w), y)
   [qr.remainder, s.coef2 * w + qr.quotient * x]$TwoCoefs

principalIdeal l ==
   l = [] => error "empty list passed to principalIdeal"
   rest l = [] =>
        uca:=unitNormal(first l)
        [[uca.unit],uca.canonical]
   rest rest l = [] =>
       u:= extendedEuclidean(first l,second l)
       [[u.coef1, u.coef2], u.generator]
   v:=principalIdeal rest l
   u:= extendedEuclidean(first l,v.generator)
   [[u.coef1,:[u.coef2*vv for vv in v.coef]],u.generator]

expressIdealMember(l,z) ==
   z = 0 => [0 for v in l]
   pid := principalIdeal l
   (q := z exquo (pid.generator)) case "failed" => "failed"
   [q*v for v in pid.coef]

multiEuclidean(l,z) ==
   n := #l
   zero? n => error "empty list passed to multiEuclidean"
   n = 1 => [z]
   l1 := copy l
```

```
l2 := split!(l1, n quo 2)
u:= extendedEuclidean(*/l1, */l2, z)
u case "failed" => "failed"
v1 := multiEuclidean(l1,u.coef2)
v1 case "failed" => "failed"
v2 := multiEuclidean(l2,u.coef1)
v2 case "failed" => "failed"
concat(v1,v2)
```

———-

— COQ EUCDOM —

```
(* category EUCDOM *)
(*
      x,y,z: %
      l: List %

      sizeLess? : (%,%) -> Boolean
      sizeLess?(x,y) ==
            zero? y => false
            zero? x => true
            euclideanSize(x)<euclideanSize(y)

      ?quo? : (%,%) -> %
      x quo y == divide(x,y).quotient --divide must be user-supplied

      ?rem? : (%,%) -> %
      x rem y == divide(x,y).remainder

      exquo : (%,%) -> Union(%,"failed")
      x exquo y ==
         zero? x => 0
         zero? y => "failed"
         qr:=divide(x,y)
         zero?(qr.remainder) => qr.quotient
         "failed"

      gcd : (%,%) -> %
      gcd(x,y) ==                  --Euclidean Algorithm
         x:=unitCanonical x
         y:=unitCanonical y
         while not zero? y repeat
            (x,y):= (y,x rem y)
            y:=unitCanonical y   -- this doesn't affect the
                                 -- correctness of Euclid's algorithm,
                                 -- but
                                 -- a) may improve performance
                                 -- b) ensures gcd(x,y)=gcd(y,x)
                                 --    if canonicalUnitNormal
         x

      IdealElt ==> Record(coef1:%,coef2:%,generator:%)
```

```
unitNormalizeIdealElt: IdealElt -> IdealElt
unitNormalizeIdealElt(s:IdealElt):IdealElt ==
   (u,c,a):=unitNormal(s.generator)
   (a = 1) => s
   [a*s.coef1,a*s.coef2,c]$IdealElt

extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
extendedEuclidean(x,y) ==         --Extended Euclidean Algorithm
   s1:=unitNormalizeIdealElt([1$%,0$%,x]$IdealElt)
   s2:=unitNormalizeIdealElt([0$%,1$%,y]$IdealElt)
   zero? y => s1
   zero? x => s2
   while not zero?(s2.generator) repeat
      qr:= divide(s1.generator, s2.generator)
      s3:=[s1.coef1 - qr.quotient * s2.coef1,
           s1.coef2 - qr.quotient * s2.coef2, qr.remainder]$IdealElt
      s1:=s2
      s2:=unitNormalizeIdealElt s3
   if not(zero?(s1.coef1)) and not sizeLess?(s1.coef1,y)
     then
        qr:= divide(s1.coef1,y)
        s1.coef1:= qr.remainder
        s1.coef2:= s1.coef2 + qr.quotient * x
        s1 := unitNormalizeIdealElt s1
   s1

TwoCoefs ==> Record(coef1:%,coef2:%)

extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean(x,y,z) ==
   zero? z => [0,0]$TwoCoefs
   s:= extendedEuclidean(x,y)
   (w:= z exquo s.generator) case "failed" => "failed"
   zero? y => [s.coef1 * w, s.coef2 * w]$TwoCoefs
   qr:= divide((s.coef1 * w), y)
   [qr.remainder, s.coef2 * w + qr.quotient * x]$TwoCoefs

principalIdeal : List(%) -> Record(coef: List(%),generator: %)
principalIdeal l ==
   l = [] => error "empty list passed to principalIdeal"
   rest l = [] =>
        uca:=unitNormal(first l)
        [[uca.unit],uca.canonical]
   rest rest l = [] =>
       u:= extendedEuclidean(first l,second l)
       [[u.coef1, u.coef2], u.generator]
   v:=principalIdeal rest l
   u:= extendedEuclidean(first l,v.generator)
   [[u.coef1,:[u.coef2*vv for vv in v.coef]],u.generator]

expressIdealMember : (List(%),%) -> Union(List(%),"failed")
expressIdealMember(l,z) ==
   z = 0 => [0 for v in l]
```

```
         pid := principalIdeal l
         (q := z exquo (pid.generator)) case "failed" => "failed"
         [q*v for v in pid.coef]

      multiEuclidean : (List(%),%) -> Union(List(%),"failed")
      multiEuclidean(l,z) ==
         n := #l
         zero? n => error "empty list passed to multiEuclidean"
         n = 1 => [z]
         l1 := copy l
         l2 := split!(l1, n quo 2)
         u:= extendedEuclidean(*/l1, */l2, z)
         u case "failed" => "failed"
         v1 := multiEuclidean(l1,u.coef2)
         v1 case "failed" => "failed"
         v2 := multiEuclidean(l2,u.coef1)
         v2 case "failed" => "failed"
         concat(v1,v2)

*)
```

———-

— EUCDOM.dotabb —

```
"EUCDOM"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=EUCDOM"];
"EUCDOM" -> "PID"
```

———-

— EUCDOM.dotfull —

```
"EuclideanDomain()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=EUCDOM"];
"EuclideanDomain()" -> "PrincipalIdealDomain()"
```

———-

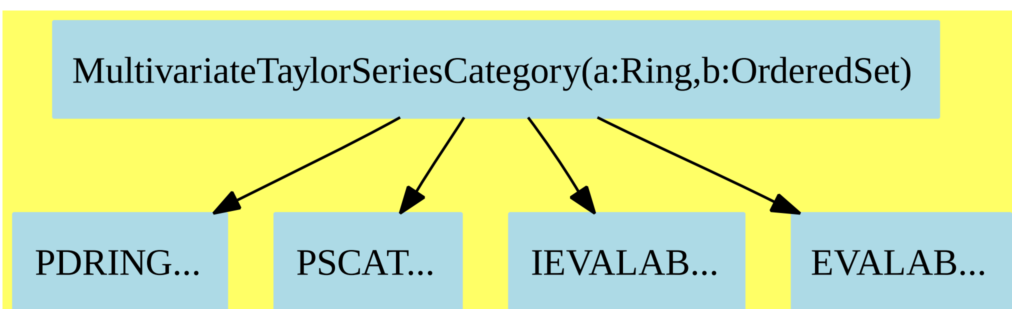
— EUCDOM.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"EuclideanDomain()" [color=lightblue];
"EuclideanDomain()" -> "PrincipalIdealDomain()"

"PrincipalIdealDomain()" [color=lightblue];
"PrincipalIdealDomain()" -> "GcdDomain()"
```

```
"GcdDomain()" [color=lightblue];
"GcdDomain()" -> "IntegralDomain()"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" -> "BMODULE..."

"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];
}
```

———-

15.0.206 MultivariateTaylorSeriesCategory (MTSCAT)



— **MultivariateTaylorSeriesCategory.input** —

```
)set break resume
)sys rm -f MultivariateTaylorSeriesCategory.output
)spool MultivariateTaylorSeriesCategory.output
)set message test on
)set message auto off
```

```
)clear all

--S 1 of 1
)show MultivariateTaylorSeriesCategory
--R
--R MultivariateTaylorSeriesCategory(Coef: Ring,Var: OrderedSet) is a category constructor
--R Abbreviation for MultivariateTaylorSeriesCategory is MTSCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for MTSCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,Coef) -> %                   ?*? : (Coef,%) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R D : (%,List(Var)) -> %                D : (%,Var) -> %
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R coerce : % -> % if Coef has INTDOM    coerce : Integer -> %
--R coerce : % -> OutputForm              complete : % -> %
--R degree : % -> IndexedExponents(Var)   differentiate : (%,List(Var)) -> %
--R differentiate : (%,Var) -> %          eval : (%,List(%),List(%)) -> %
--R eval : (%,%,%) -> %                   eval : (%,Equation(%)) -> %
--R eval : (%,List(Equation(%))) -> %     eval : (%,List(Var),List(%)) -> %
--R eval : (%,Var,%) -> %                 hash : % -> SingleInteger
--R latex : % -> String                   leadingCoefficient : % -> Coef
--R leadingMonomial : % -> %              map : ((Coef -> Coef),%) -> %
--R monomial? : % -> Boolean              one? : % -> Boolean
--R pole? : % -> Boolean                  recip : % -> Union(%,"failed")
--R reductum : % -> %                     sample : () -> %
--R variables : % -> List(Var)            zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (Fraction(Integer),%) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?*? : (%,Fraction(Integer)) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?**? : (%,Fraction(Integer)) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?**? : (%,%) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R D : (%,List(Var),List(NonNegativeInteger)) -> %
--R D : (%,Var,NonNegativeInteger) -> %
--R acos : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acosh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acot : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acoth : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acsc : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acsch : % -> % if Coef has ALGEBRA(FRAC(INT))
--R asec : % -> % if Coef has ALGEBRA(FRAC(INT))
--R asech : % -> % if Coef has ALGEBRA(FRAC(INT))
--R asin : % -> % if Coef has ALGEBRA(FRAC(INT))
--R asinh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA(FRAC(INT))
--R atanh : % -> % if Coef has ALGEBRA(FRAC(INT))
```

```
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
--R coefficient : (%,List(Var),List(NonNegativeInteger)) -> %
--R coefficient : (%,Var,NonNegativeInteger) -> %
--R coefficient : (%,IndexedExponents(Var)) -> Coef
--R coerce : Fraction(Integer) -> % if Coef has ALGEBRA(FRAC(INT))
--R coerce : Coef -> % if Coef has COMRING
--R cos : % -> % if Coef has ALGEBRA(FRAC(INT))
--R cosh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R cot : % -> % if Coef has ALGEBRA(FRAC(INT))
--R coth : % -> % if Coef has ALGEBRA(FRAC(INT))
--R csc : % -> % if Coef has ALGEBRA(FRAC(INT))
--R csch : % -> % if Coef has ALGEBRA(FRAC(INT))
--R differentiate : (%,List(Var),List(NonNegativeInteger)) -> %
--R differentiate : (%,Var,NonNegativeInteger) -> %
--R exp : % -> % if Coef has ALGEBRA(FRAC(INT))
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R extend : (%,NonNegativeInteger) -> %
--R integrate : (%,Var) -> % if Coef has ALGEBRA(FRAC(INT))
--R log : % -> % if Coef has ALGEBRA(FRAC(INT))
--R monomial : (%,List(Var),List(NonNegativeInteger)) -> %
--R monomial : (%,Var,NonNegativeInteger) -> %
--R monomial : (Coef,IndexedExponents(Var)) -> %
--R monomial : (%,Var,IndexedExponents(Var)) -> %
--R monomial : (%,List(Var),List(IndexedExponents(Var))) -> %
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA(FRAC(INT))
--R order : (%,Var,NonNegativeInteger) -> NonNegativeInteger
--R order : (%,Var) -> NonNegativeInteger
--R pi : () -> % if Coef has ALGEBRA(FRAC(INT))
--R polynomial : (%,NonNegativeInteger,NonNegativeInteger) -> Polynomial(Coef)
--R polynomial : (%,NonNegativeInteger) -> Polynomial(Coef)
--R sec : % -> % if Coef has ALGEBRA(FRAC(INT))
--R sech : % -> % if Coef has ALGEBRA(FRAC(INT))
--R sin : % -> % if Coef has ALGEBRA(FRAC(INT))
--R sinh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R sqrt : % -> % if Coef has ALGEBRA(FRAC(INT))
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA(FRAC(INT))
--R tanh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R
--E 1

)spool
)lisp (bye)
```

— **MultivariateTaylorSeriesCategory.help** —

```
====================================================================
MultivariateTaylorSeriesCategory examples
```

```
====================================================================

MultivariateTaylorSeriesCategory is the most general multivariate
Taylor series category.

See Also:
o )show MultivariateTaylorSeriesCategory
```

See:

⇐ "Evalable" (EVALAB) 3.0.48 on page 151
⇐ "InnerEvalable" (IEVALAB) 2.0.22 on page 64
⇐ "PartialDifferentialRing" (PDRING) 10.0.169 on page 1061
⇐ "PowerSeriesCategory" (PSCAT) 14.0.201 on page 1349

Exports:

0	1	acos	acosh	acot
acoth	acsc	acsch	asec	asech
asin	asinh	associates?	atan	atanh
characteristic	charthRoot	coefficient	coerce	complete
cos	cosh	cot	coth	csc
csch	D	degree	differentiate	eval
exp	exquo	extend	hash	integrate
latex	leadingCoefficient	leadingMonomial	log	map
monomial	monomial?	nthRoot	order	one?
pi	pole?	polynomial	recip	reductum
sample	sec	sech	sin	sinh
sqrt	subtractIfCan	tan	tanh	unit?
unitCanonical	unitNormal	variables	zero?	?*?
?**?	?+?	?-?	-?	?=?
?^?	?~=?	?/?		

Attributes Exported:

- if $ has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation $"*": (D,D)->D$ which is commutative.
- if $ has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x*y \neq 0$ implies both x and y are non-zero.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1*x = x$ for all x.
- **rightUnitary** is true if $x*1 = x$ for all x.

These are directly exported but not implemented:

```
coefficient : (%,Var,NonNegativeInteger) -> %
coefficient : (%,List Var,List NonNegativeInteger) -> %
extend : (%,NonNegativeInteger) -> %
integrate : (%,Var) -> % if Coef has ALGEBRA FRAC INT
```

```
monomial : (%,Var,NonNegativeInteger) -> %
monomial : (%,List Var,List NonNegativeInteger) -> %
order : (%,Var,NonNegativeInteger) -> NonNegativeInteger
order : (%,Var) -> NonNegativeInteger
polynomial : (%,NonNegativeInteger,NonNegativeInteger) -> Polynomial Coef
polynomial : (%,NonNegativeInteger) -> Polynomial Coef
```

These exports come from (p1061) PartialDifferentialRing(OrderedSet):

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
D : (%,Var) -> %
D : (%,List Var) -> %
D : (%,List Var,List NonNegativeInteger) -> %
D : (%,Var,NonNegativeInteger) -> %
differentiate : (%,Var) -> %
differentiate : (%,List Var,List NonNegativeInteger) -> %
differentiate : (%,Var,NonNegativeInteger) -> %
differentiate : (%,List Var) -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
```

These exports come from (p1349) PowerSeriesCategory(A,B,C)
where A:Ring, B:IndexedExponents(OrderedSet) and C:OrderedSet:

```
associates? : (%,%) -> Boolean if Coef has INTDOM
charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
coefficient : (%,IndexedExponents Var) -> Coef
coerce : Coef -> % if Coef has COMRING
coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
coerce : % -> % if Coef has INTDOM
complete : % -> %
degree : % -> IndexedExponents Var
exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
leadingCoefficient : % -> Coef
```

```
leadingMonomial : % -> %
map : ((Coef -> Coef),%) -> %
monomial : (Coef,IndexedExponents Var) -> %
monomial? : % -> Boolean
monomial : (%,List Var,List IndexedExponents Var) -> %
monomial : (%,Var,IndexedExponents Var) -> %
pole? : % -> Boolean
reductum : % -> %
unit? : % -> Boolean if Coef has INTDOM
unitCanonical : % -> % if Coef has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
  if Coef has INTDOM
variables : % -> List Var
?*? : (%,Coef) -> %
?*? : (Coef,%) -> %
?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?/? : (%,Coef) -> % if Coef has FIELD
```

These exports come from (p64) InnerEvalable(OrderedSet,

```
eval : (%,Var,%) -> %
eval : (%,List Var,List %) -> %
```

These exports come from (p151) Evalable(

```
eval : (%,List Equation %) -> %
eval : (%,Equation %) -> %
eval : (%,List %,List %) -> %
eval : (%,%,%) -> %
```

These exports come from (p106) RadicalCategory():

```
nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
sqrt : % -> % if Coef has ALGEBRA FRAC INT
?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
```

These exports come from (p192) TranscendentalFunctionCategory():

```
acos : % -> % if Coef has ALGEBRA FRAC INT
acosh : % -> % if Coef has ALGEBRA FRAC INT
acot : % -> % if Coef has ALGEBRA FRAC INT
acoth : % -> % if Coef has ALGEBRA FRAC INT
acsc : % -> % if Coef has ALGEBRA FRAC INT
acsch : % -> % if Coef has ALGEBRA FRAC INT
asec : % -> % if Coef has ALGEBRA FRAC INT
asech : % -> % if Coef has ALGEBRA FRAC INT
asin : % -> % if Coef has ALGEBRA FRAC INT
asinh : % -> % if Coef has ALGEBRA FRAC INT
atan : % -> % if Coef has ALGEBRA FRAC INT
atanh : % -> % if Coef has ALGEBRA FRAC INT
cos : % -> % if Coef has ALGEBRA FRAC INT
cosh : % -> % if Coef has ALGEBRA FRAC INT
cot : % -> % if Coef has ALGEBRA FRAC INT
coth : % -> % if Coef has ALGEBRA FRAC INT
csc : % -> % if Coef has ALGEBRA FRAC INT
csch : % -> % if Coef has ALGEBRA FRAC INT
exp : % -> % if Coef has ALGEBRA FRAC INT
```

```
 log : % -> % if Coef has ALGEBRA FRAC INT
 pi : () -> % if Coef has ALGEBRA FRAC INT
 sec : % -> % if Coef has ALGEBRA FRAC INT
 sech : % -> % if Coef has ALGEBRA FRAC INT
 sin : % -> % if Coef has ALGEBRA FRAC INT
 sinh : % -> % if Coef has ALGEBRA FRAC INT
 tan : % -> % if Coef has ALGEBRA FRAC INT
 tanh : % -> % if Coef has ALGEBRA FRAC INT
 ?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
```

— MultivariateTaylorSeriesCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#MTSCAT">
MultivariateTaylorSeriesCategory (MTSCAT)</a></h2>
</body>
```

— category MTSCAT MultivariateTaylorSeriesCategory —

```
)abbrev category MTSCAT MultivariateTaylorSeriesCategory
++ Author: Clifton J. Williamson
++ Date Created: 6 March 1990
++ Date Last Updated: 6 March 1990
++ Description:
++ \spadtype{MultivariateTaylorSeriesCategory} is the most general
++ multivariate Taylor series category.

MultivariateTaylorSeriesCategory(Coef,Var) : Category == SIG where
  Coef : Ring
  Var : OrderedSet

  L   ==> List
  NNI ==> NonNegativeInteger
  PDR ==> PartialDifferentialRing(Var)
  PSC ==> PowerSeriesCategory(Coef,IndexedExponents(Var),Var)
  IE  ==> InnerEvalable(Var,%)
  EV  ==> Evalable(%)

  SIG ==> Join(PDR,PSC,IE,EV) with

    coefficient : (%,Var,NNI) -> %
      ++ \spad{coefficient(f,x,n)} returns the coefficient of \spad{x^n} in f.

    coefficient : (%,L Var,L NNI) -> %
      ++ \spad{coefficient(f,[x1,x2,...,xk],[n1,n2,...,nk])} returns the
      ++ coefficient of \spad{x1^n1 * ... * xk^nk} in f.

    extend : (%,NNI) -> %
      ++ \spad{extend(f,n)} causes all terms of f of degree
      ++ \spad{<= n} to be computed.
```

```
    monomial : (%,Var,NNI) -> %
      ++ \spad{monomial(a,x,n)} returns  \spad{a*x^n}.

    monomial : (%,L Var,L NNI) -> %
      ++ \spad{monomial(a,[x1,x2,...,xk],[n1,n2,...,nk])} returns
      ++ \spad{a * x1^n1 * ... * xk^nk}.

    order : (%,Var) -> NNI
      ++ \spad{order(f,x)} returns the order of f viewed as a series in x
      ++ may result in an infinite loop if f has no non-zero terms.

    order : (%,Var,NNI) -> NNI
      ++ \spad{order(f,x,n)} returns \spad{min(n,order(f,x))}.

    polynomial : (%,NNI) -> Polynomial Coef
      ++ \spad{polynomial(f,k)} returns a polynomial consisting of the sum
      ++ of all terms of f of degree \spad{<= k}.

    polynomial : (%,NNI,NNI) -> Polynomial Coef
      ++ \spad{polynomial(f,k1,k2)} returns a polynomial consisting of the
      ++ sum of all terms of f of degree d with \spad{k1 <= d <= k2}.

    if Coef has Algebra Fraction Integer then

      integrate : (%,Var) -> %
        ++ \spad{integrate(f,x)} returns the anti-derivative of the power
        ++ series \spad{f(x)} with respect to the variable x with constant
        ++ coefficient 1.  We may integrate a series when we can divide
        ++ coefficients by integers.

      RadicalCategory
        --++ We provide rational powers when we can divide coefficients
        --++ by integers.

      TranscendentalFunctionCategory
        --++ We provide transcendental functions when we can divide
        --++ coefficients by integers.
```

———

— MTSCAT.dotabb —

```
"MTSCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MTSCAT"];
"MTSCAT" -> "PDRING"
"MTSCAT" -> "PSCAT"
"MTSCAT" -> "IEVALAB"
"MTSCAT" -> "EVALAB"
```

———

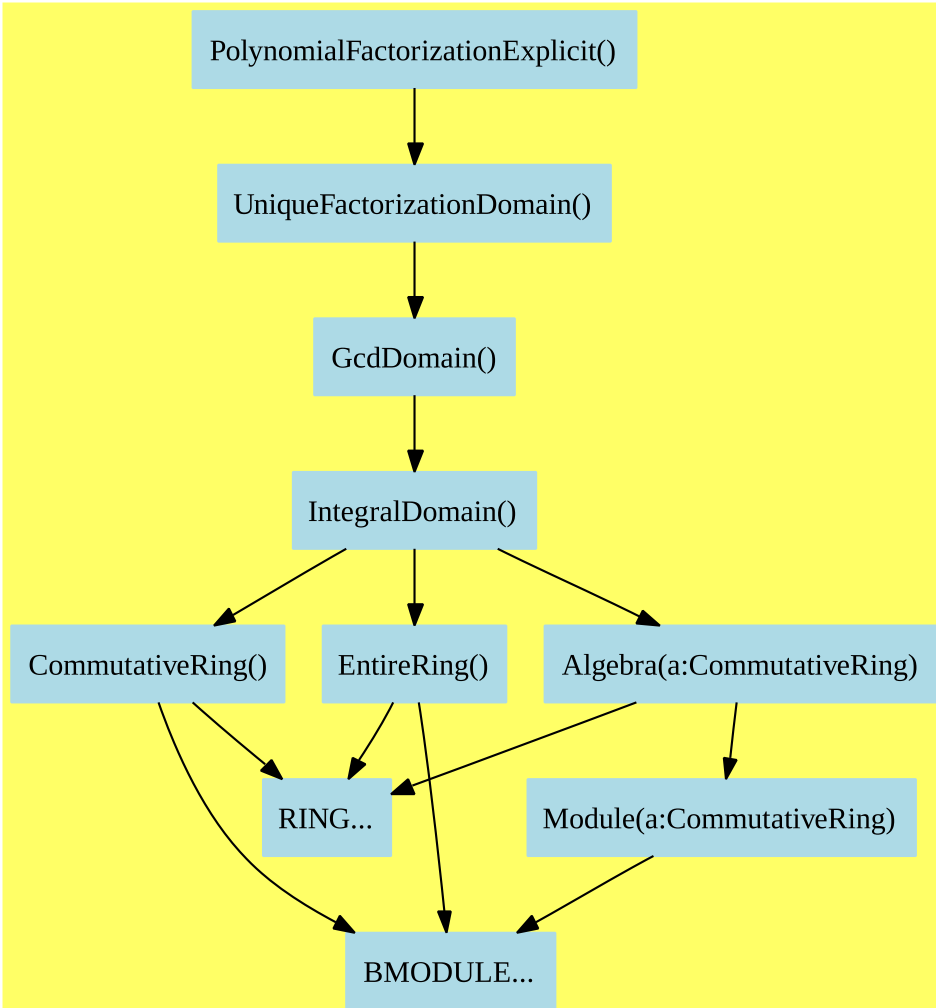
— MTSCAT.dotfull —

```
"MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MTSCAT"];
"MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" ->
    "PartialDifferentialRing(a:OrderedSet)"
"MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" ->
   "PowerSeriesCategory(a:Ring,IndexedExponents(b:OrderedSet),c:OrderedSet))"
"MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" ->
 "InnerEvalable(a:Ring,MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet))"
"MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" ->
 "Evalable(MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet))"
```

— **MTSCAT.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" [color=lightblue];
"MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" ->
    "PDRING..."
"MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" ->
   "PSCAT..."
"MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" ->
 "IEVALAB..."
"MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" ->
 "EVALAB..."

"PDRING..." [color=lightblue];
"PSCAT..." [color=lightblue];
"IEVALAB..." [color=lightblue];
"EVALAB..." [color=lightblue];

}
```

15.0.207 PolynomialFactorizationExplicit (PFECAT)



— **PolynomialFactorizationExplicit.input** —

```
)set break resume
)sys rm -f PolynomialFactorizationExplicit.output
)spool PolynomialFactorizationExplicit.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PolynomialFactorizationExplicit
--R
--R PolynomialFactorizationExplicit is a category constructor
--R Abbreviation for PolynomialFactorizationExplicit is PFECAT
--R This constructor is exposed in this frame.
```

```
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PFECAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean        coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R factor : % -> Factored(%)             gcd : List(%) -> %
--R gcd : (%,%) -> %                      hash : % -> SingleInteger
--R latex : % -> String                   lcm : List(%) -> %
--R lcm : (%,%) -> %                      one? : % -> Boolean
--R prime? : % -> Boolean                 recip : % -> Union(%,"failed")
--R sample : () -> %                      squareFree : % -> Factored(%)
--R squareFreePart : % -> %               unit? : % -> Boolean
--R unitCanonical : % -> %                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if $ has CHARNZ
--R conditionP : Matrix(%) -> Union(Vector(%),"failed") if $ has CHARNZ
--R exquo : (%,%) -> Union(%,"failed")
--R factorPolynomial : SparseUnivariatePolynomial(%) -> Factored(SparseUnivariatePolynomial(%))
--R factorSquareFreePolynomial : SparseUnivariatePolynomial(%) -> Factored(SparseUnivariatePolynomial(%))
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
--R solveLinearPolynomialEquation : (List(SparseUnivariatePolynomial(%)),SparseUnivariatePolynomial(%)) -> Union
--R squareFreePolynomial : SparseUnivariatePolynomial(%) -> Factored(SparseUnivariatePolynomial(%))
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

— **PolynomialFactorizationExplicit.help** —

```
====================================================================
PolynomialFactorizationExplicit examples
====================================================================

This is the category of domains that know "enough" about themselves
in order to factor univariate polynomials over themselves. This will
be used in future releases for supporting factorization over finitely
generated coefficient fields, it is not yet available in the current
release of Axiom.

See Also:
```

```
o )show PolynomialFactorizationExplicit
```

———-

See:

⇒ "PolynomialCategory" (POLYCAT) 16.0.213 on page 1449
⇒ "QuotientFieldCategory" (QFCAT) 17.0.222 on page 1595
⇒ "UnivariatePolynomialCategory" (UPOLYC) 17.0.228 on page 1708
⇐ "UniqueFactorizationDomain" (UFD) 14.0.203 on page 1362

Exports:

0	1	associates?
characteristic	charthRoot	coerce
conditionP	exquo	factor
factorPolynomial	factorSquareFreePolynomial	gcd
gcdPolynomial	hash	latex
lcm	one?	prime?
recip	sample	solveLinearPolynomialEquation
squareFree	squareFreePart	squareFreePolynomial
subtractIfCan	unit?	unitNormal
unitCanonical	zero?	?*?
?**?	?+?	?-?
-?	?=?	?^?
?~=?		

Attributes exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * ": (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
conditionP : Matrix % -> Union(Vector %,"failed")
    if $ has CHARNZ
factorPolynomial :
  SparseUnivariatePolynomial % ->
     Factored SparseUnivariatePolynomial %
factorSquareFreePolynomial :
  SparseUnivariatePolynomial % ->
     Factored SparseUnivariatePolynomial %
squareFreePolynomial :
  SparseUnivariatePolynomial % ->
     Factored SparseUnivariatePolynomial %
```

These are implemented by this category:

```
charthRoot : % -> Union(%,"failed") if $ has CHARNZ
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                    SparseUnivariatePolynomial %
solveLinearPolynomialEquation :
  (List SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     Union(List SparseUnivariatePolynomial %,"failed")
```

These exports come from (p1362) UniqueFactorizationDomain():

```
factor : % -> Factored %
squareFree : % -> Factored %
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
exquo : (%,%) -> Union(%,"failed")
gcd : List % -> %
gcd : (%,%) -> %
hash : % -> SingleInteger
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
one? : % -> Boolean
prime? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
unitCanonical : % -> %
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

— **PolynomialFactorizationExplicit.html** —

```
<body>
<h2>
```

```
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#PFECAT">
PolynomialFactorizationExplicit (PFECAT)</a></h2>
</body>
```

— category PFECAT PolynomialFactorizationExplicit —

```
)abbrev category PFECAT PolynomialFactorizationExplicit
++ Author: James Davenport
++ Description:
++ This is the category of domains that know "enough" about
++ themselves in order to factor univariate polynomials over themselves.
++ This will be used in future releases for supporting factorization
++ over finitely generated coefficient fields, it is not yet available
++ in the current release of axiom.

PolynomialFactorizationExplicit() : Category == SIG where

  P ==> SparseUnivariatePolynomial %

  SIG ==> UniqueFactorizationDomain with

    squareFreePolynomial : P -> Factored(P)
      ++ squareFreePolynomial(p) returns the
      ++ square-free factorization of the
      ++ univariate polynomial p.

    factorPolynomial : P -> Factored(P)
      ++ factorPolynomial(p) returns the factorization
      ++ into irreducibles of the univariate polynomial p.

    factorSquareFreePolynomial : P -> Factored(P)
      ++ factorSquareFreePolynomial(p) factors the
      ++ univariate polynomial p into irreducibles
      ++ where p is known to be square free
      ++ and primitive with respect to its main variable.

    gcdPolynomial : (P, P) -> P
      ++ gcdPolynomial(p,q) returns the gcd of the univariate
      ++ polynomials p qnd q.
      -- defaults to Euclidean, but should be implemented via
      -- modular or p-adic methods.

    solveLinearPolynomialEquation : (List P, P) -> Union(List P,"failed")
      ++ solveLinearPolynomialEquation([f1, ..., fn], g)
      ++ (where the fi are relatively prime to each other)
      ++ returns a list of ai such that
      ++ \spad{g/prod fi = sum ai/fi}
      ++ or returns "failed" if no such list of ai's exists.

    if % has CharacteristicNonZero then

      conditionP : Matrix % -> Union(Vector %,"failed")
```

```
        ++ conditionP(m) returns a vector of elements, not all zero,
        ++ whose \spad{p}-th powers (p is the characteristic of the domain)
        ++ are a solution of the homogenous linear system represented
        ++ by m, or "failed" is there is no such vector.

      charthRoot : % -> Union(%,"failed")
        ++ charthRoot(r) returns the \spad{p}-th root of r, or "failed"
        ++ if none exists in the domain.
        -- this is a special case of conditionP, but often the one we want

   add

     gcdPolynomial(f,g) ==
        zero? f => g
        zero? g => f
        cf:=content f
        if not one? cf then f:=(f exquo cf)::P
        cg:=content g
        if not one? cg then g:=(g exquo cg)::P
        ans:=subResultantGcd(f,g)$P
        gcd(cf,cg)*(ans exquo content ans)::P

     if % has CharacteristicNonZero then

       charthRoot f ==
          -- to take p'th root of f, solve the system X-fY=0,
          -- so solution is [x,y]
          -- with x^p=X and y^p=Y, then (x/y)^p = f
          zero? f => 0
          m:Matrix % := matrix [[1,-f]]
          ans:= conditionP m
          ans case "failed" => "failed"
          (ans.1) exquo (ans.2)

     if % has Field then

       solveLinearPolynomialEquation(lf,g) ==
         multiEuclidean(lf,g)$P

     else

       solveLinearPolynomialEquation(lf,g) ==
         LPE ==> LinearPolynomialEquationByFractions %
         solveLinearPolynomialEquationByFractions(lf,g)$LPE
```

— COQ PFECAT —

```
(* category PFECAT *)
(*

        gcdPolynomial :
```

```
    (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) ->
       SparseUnivariatePolynomial(%)
  gcdPolynomial(f,g) ==
     zero? f => g
     zero? g => f
     cf:=content f
     if not one? cf then f:=(f exquo cf)::P
     cg:=content g
     if not one? cg then g:=(g exquo cg)::P
     ans:=subResultantGcd(f,g)$P
     gcd(cf,cg)*(ans exquo content ans)::P

  if % has CharacteristicNonZero then

    charthRoot : % -> Union(%,"failed") if $ has CHARNZ
    charthRoot f ==
       -- to take p'th root of f, solve the system X-fY=0,
       -- so solution is [x,y]
       -- with x^p=X and y^p=Y, then (x/y)^p = f
       zero? f => 0
       m:Matrix % := matrix [[1,-f]]
       ans:= conditionP m
       ans case "failed" => "failed"
       (ans.1) exquo (ans.2)

  if % has Field then

    solveLinearPolynomialEquation :
       (List(SparseUnivariatePolynomial(%)),
        SparseUnivariatePolynomial(%)) ->
          Union(List(SparseUnivariatePolynomial(%)),"failed")
    solveLinearPolynomialEquation(lf,g) ==
      multiEuclidean(lf,g)$P

  else

    solveLinearPolynomialEquation :
       (List(SparseUnivariatePolynomial(%)),
        SparseUnivariatePolynomial(%)) ->
          Union(List(SparseUnivariatePolynomial(%)),"failed")
    solveLinearPolynomialEquation(lf,g) ==
         LPE ==> LinearPolynomialEquationByFractions %
         solveLinearPolynomialEquationByFractions(lf,g)$LPE

*)
```

——————-

— PFECAT.dotabb —

```
"PFECAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PFECAT"];
"PFECAT" -> "UFD"
```

———-

— PFECAT.dotfull —

```
"PolynomialFactorizationExplicit()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PFECAT"];
"PolynomialFactorizationExplicit()" -> "UniqueFactorizationDomain()"
```

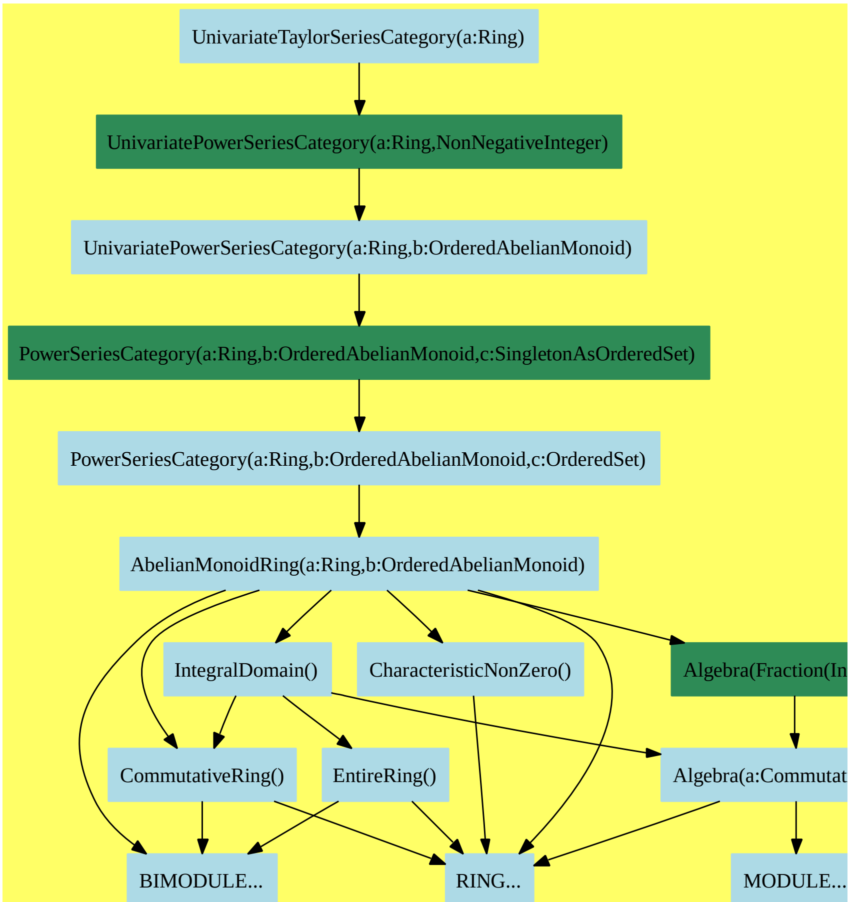
———-

— PFECAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"PolynomialFactorizationExplicit()" [color=lightblue];
"PolynomialFactorizationExplicit()" -> "UniqueFactorizationDomain()"

"UniqueFactorizationDomain()" [color=lightblue];
"UniqueFactorizationDomain()" -> "GcdDomain()"

"GcdDomain()" [color=lightblue];
"GcdDomain()" -> "IntegralDomain()"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" -> "BMODULE..."

"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];
}
```

———-

15.0.208 UnivariatePowerSeriesCategory (UPSCAT)



— **UnivariatePowerSeriesCategory.input** —

```
)set break resume
)sys rm -f UnivariatePowerSeriesCategory.output
)spool UnivariatePowerSeriesCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariatePowerSeriesCategory
--R
--R UnivariatePowerSeriesCategory(Coef: Ring,Expon: OrderedAbelianMonoid) is a category constructor
--R Abbreviation for UnivariatePowerSeriesCategory is UPSCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for UPSCAT
```

```
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Coef,%) -> %                   ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R center : % -> Coef                    coefficient : (%,Expon) -> Coef
--R coerce : % -> % if Coef has INTDOM    coerce : Integer -> %
--R coerce : % -> OutputForm              complete : % -> %
--R degree : % -> Expon                   ?.? : (%,Expon) -> Coef
--R extend : (%,Expon) -> %               hash : % -> SingleInteger
--R latex : % -> String                   leadingCoefficient : % -> Coef
--R leadingMonomial : % -> %              map : ((Coef -> Coef),%) -> %
--R monomial : (Coef,Expon) -> %          monomial? : % -> Boolean
--R one? : % -> Boolean                   order : (%,Expon) -> Expon
--R order : % -> Expon                    pole? : % -> Boolean
--R recip : % -> Union(%,"failed")        reductum : % -> %
--R sample : () -> %                      truncate : (%,Expon,Expon) -> %
--R truncate : (%,Expon) -> %             variable : % -> Symbol
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction(Integer)) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?*? : (Fraction(Integer),%) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (Expon,Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if Coef has *: (Expon,Coef) -> Coef
--R D : (%,Symbol) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Expon,Coef) -> Coef
--R D : (%,List(Symbol)) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Expon,Coef) -> Coef
--R D : (%,Symbol,NonNegativeInteger) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Expon,Coef) -> Coef
--R D : (%,List(Symbol),List(NonNegativeInteger)) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Expon,Coef) -
--R approximate : (%,Expon) -> Coef if Coef has **: (Coef,Expon) -> Coef and Coef has coerce: Symbol -> Coef
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
--R coerce : Coef -> % if Coef has COMRING
--R coerce : Fraction(Integer) -> % if Coef has ALGEBRA(FRAC(INT))
--R differentiate : % -> % if Coef has *: (Expon,Coef) -> Coef
--R differentiate : (%,NonNegativeInteger) -> % if Coef has *: (Expon,Coef) -> Coef
--R differentiate : (%,Symbol) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Expon,Coef) -> Coef
--R differentiate : (%,List(Symbol)) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Expon,Coef) -> Coef
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Expon,Coef) -
--R differentiate : (%,List(Symbol),List(NonNegativeInteger)) -> % if Coef has PDRING(SYMBOL) and Coef has *: (E
--R ?.? : (%,%) -> % if Expon has SGROUP
--R eval : (%,Coef) -> Stream(Coef) if Coef has **: (Coef,Expon) -> Coef
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R monomial : (%,List(SingletonAsOrderedSet),List(Expon)) -> %
--R monomial : (%,SingletonAsOrderedSet,Expon) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R terms : % -> Stream(Record(k: Expon,c: Coef))
--R unit? : % -> Boolean if Coef has INTDOM
```

```
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R variables : % -> List(SingletonAsOrderedSet)
--R
--E 1

)spool
)lisp (bye)
```

— UnivariatePowerSeriesCategory.help —

```
====================================================================
UnivariatePowerSeriesCategory examples
====================================================================

UnivariatePowerSeriesCategory is the most general univariate power
series category with exponents in an ordered abelian monoid. Note that
this category exports a substitution function if it is possible to
multiply exponents. Also note that this category exports a derivative
operation if it is possible to multiply coefficients by exponents.

See Also:
o )show UnivariatePowerSeriesCategory
```

See:

⇒ "LocalPowerSeriesCategory" (LOCPOWC) 16.0.211 on page 1434
⇒ "UnivariateLaurentSeriesCategory" (ULSCAT) 17.0.226 on page 1686
⇒ "UnivariatePuiseuxSeriesCategory" (UPXSCAT) 17.0.227 on page 1697
⇒ "UnivariateTaylorSeriesCategory" (UTSCAT) 16.0.214 on page 1476
⇐ "PowerSeriesCategory" (PSCAT) 14.0.201 on page 1349

Exports:

0	1	approximate	associates?	center
characteristic	charthRoot	coefficient	coerce	complete
D	degree	differentiate	eval	exquo
extend	hash	latex	leadingCoefficient	leadingMonomial
map	monomial	monomial?	multiplyExponents	one?
order	pole?	recip	reductum	sample
subtractIfCan	truncate	terms	unit?	unitCanonical
unitNormal	variable	variables	zero?	?*?
?**?	?+?	?-?	-?	?=?
?^?	?.?	?~=?	?/?	

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- if #1 has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if #1 has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation $" * " : (D, D) -> D$ which is commutative.

These are directly exported but not implemented:

```
approximate : (%,Expon) -> Coef
    if Coef has **: (Coef,Expon) -> Coef
    and Coef has coerce: Symbol -> Coef
center : % -> Coef
eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,Expon) -> Coef
extend : (%,Expon) -> %
multiplyExponents : (%,PositiveInteger) -> %
order : % -> Expon
order : (%,Expon) -> Expon
terms : % -> Stream Record(k: Expon,c: Coef)
truncate : (%,Expon) -> %
truncate : (%,Expon,Expon) -> %
variable : % -> Symbol
?.? : (%,Expon) -> Coef
```

These are implemented by this category:

```
degree : % -> Expon
leadingCoefficient : % -> Coef
leadingMonomial : % -> %
monomial : (%,SingletonAsOrderedSet,Expon) -> %
reductum : % -> %
variables : % -> List SingletonAsOrderedSet
```

These exports come from (p1349) PowerSeriesCategory(C,E,S)
where C:Ring, E:OrderedAbelianMonoid, S:SingletonAsOrderedSet:

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean if Coef has INTDOM
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
coefficient : (%,Expon) -> Coef
coerce : Coef -> % if Coef has COMRING
coerce : % -> % if Coef has INTDOM
coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
coerce : Integer -> %
coerce : % -> OutputForm
complete : % -> %
exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
hash : % -> SingleInteger
latex : % -> String
map : ((Coef -> Coef),%) -> %
monomial : (%,List Var,List Expon) -> %
monomial : (Coef,Expon) -> %
```

```
monomial? : % -> Boolean
one? : % -> Boolean
recip : % -> Union(%,"failed")
pole? : % -> Boolean
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean if Coef has INTDOM
unitCanonical : % -> % if Coef has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if Coef has INTDOM
zero? : % -> Boolean
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (Integer,%) -> %
?*? : (Coef,%) -> %
?*? : (%,Coef) -> %
?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
?/? : (%,Coef) -> % if Coef has FIELD
-? : % -> %
```

These exports come from (p54) Eltable(%,%):

```
?.? : (%,%) -> % if Expon has SGROUP
```

These exports come from (p1024) DifferentialRing():

```
D : % -> %
    if Coef has *: (Expon,Coef) -> Coef
D : (%,NonNegativeInteger) -> %
    if Coef has *: (Expon,Coef) -> Coef
differentiate : (%,NonNegativeInteger) -> %
    if Coef has *: (Expon,Coef) -> Coef
differentiate : % -> %
    if Coef has *: (Expon,Coef) -> Coef
```

These exports come from (p1061) PartialDifferentialRing(Symbol):

```
D : (%,Symbol) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (Expon,Coef) -> Coef
D : (%,List Symbol) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (Expon,Coef) -> Coef
D : (%,Symbol,NonNegativeInteger) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (Expon,Coef) -> Coef
D : (%,List Symbol,List NonNegativeInteger) -> %
```

```
     if Coef has PDRING SYMBOL
     and Coef has *: (Expon,Coef) -> Coef
 differentiate : (%,List Symbol) -> %
     if Coef has PDRING SYMBOL
     and Coef has *: (Expon,Coef) -> Coef
 differentiate : (%,Symbol,NonNegativeInteger) -> %
     if Coef has PDRING SYMBOL
     and Coef has *: (Expon,Coef) -> Coef
 differentiate : (%,List Symbol,List NonNegativeInteger) -> %
     if Coef has PDRING SYMBOL
     and Coef has *: (Expon,Coef) -> Coef
 differentiate : (%,Symbol) -> %
     if Coef has PDRING SYMBOL
     and Coef has *: (Expon,Coef) -> Coef
```

— UnivariatePowerSeriesCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#UPSCAT">
UnivariatePowerSeriesCategory (UPSCAT)</a></h2>
</body>
```

— category UPSCAT UnivariatePowerSeriesCategory —

```
)abbrev category UPSCAT UnivariatePowerSeriesCategory
++ Author: Clifton J. Williamson
++ Date Created: 21 December 1989
++ Date Last Updated: 20 September 1993
++ Description:
++ \spadtype{UnivariatePowerSeriesCategory} is the most general
++ univariate power series category with exponents in an ordered
++ abelian monoid.
++ Note that this category exports a substitution function if it is
++ possible to multiply exponents.
++ Also note that this category exports a derivative operation if it is
++ possible to multiply coefficients by exponents.

UnivariatePowerSeriesCategory(Coef,Expon) : Category == SIG where
  Coef : Ring
  Expon : OrderedAbelianMonoid

  Term ==> Record(k:Expon,c:Coef)

  SIG ==> PowerSeriesCategory(Coef,Expon,SingletonAsOrderedSet) with

    terms : % -> Stream Term
      ++ \spad{terms(f(x))} returns a stream of non-zero terms, where a
      ++ a term is an exponent-coefficient pair.  The terms in the stream
      ++ are ordered by increasing order of exponents.

    elt : (%,Expon) -> Coef
```

```
  ++ \spad{elt(f(x),r)} returns the coefficient of the term of degree r in
  ++ \spad{f(x)}.  This is the same as the function \spadfun{coefficient}.

variable : % -> Symbol
  ++ \spad{variable(f)} returns the (unique) power series variable of
  ++ the power series f.

center : % -> Coef
  ++ \spad{center(f)} returns the point about which the series f is
  ++ expanded.

multiplyExponents : (%,PositiveInteger) -> %
  ++ \spad{multiplyExponents(f,n)} multiplies all exponents of the power
  ++ series f by the positive integer n.

order : % -> Expon
  ++ \spad{order(f)} is the degree of the lowest order non-zero term in f.
  ++ This will result in an infinite loop if f has no non-zero terms.

order : (%,Expon) -> Expon
  ++ \spad{order(f,n) = min(m,n)}, where m is the degree of the
  ++ lowest order non-zero term in f.

truncate : (%,Expon) -> %
  ++ \spad{truncate(f,k)} returns a (finite) power series consisting of
  ++ the sum of all terms of f of degree \spad{<= k}.

truncate : (%,Expon,Expon) -> %
  ++ \spad{truncate(f,k1,k2)} returns a (finite) power
  ++ series consisting of
  ++ the sum of all terms of f of degree d with \spad{k1 <= d <= k2}.

if Coef has coerce: Symbol -> Coef then

  if Coef has "**":(Coef,Expon) -> Coef then

    approximate : (%,Expon) -> Coef
      ++ \spad{approximate(f)} returns a truncated power series with the
      ++ series variable viewed as an element of the coefficient domain.

extend : (%,Expon) -> %
  ++ \spad{extend(f,n)} causes all terms of f of degree <= n
  ++ to be computed.

if Expon has SemiGroup then Eltable(%,%)

if Coef has "*": (Expon,Coef) -> Coef then

  DifferentialRing
  --!! DifferentialExtension Coef

  if Coef has PartialDifferentialRing Symbol then
    PartialDifferentialRing Symbol
```

```
    if Coef has "**": (Coef,Expon) -> Coef then

      eval : (%,Coef) -> Stream Coef
        ++ \spad{eval(f,a)} evaluates a power series at a value in the
        ++ ground ring by returning a stream of partial sums.

   add

     degree f == order f

     leadingCoefficient f == coefficient(f,order f)

     leadingMonomial f ==
       ord := order f
       monomial(coefficient(f,ord),ord)

     monomial(f:%,listVar:List SingletonAsOrderedSet,listExpon:List Expon) ==
       empty? listVar or not empty? rest listVar =>
         error "monomial: variable list must have exactly one entry"
       empty? listExpon or not empty? rest listExpon =>
         error "monomial: exponent list must have exactly one entry"
       f * monomial(1,first listExpon)

     monomial(f:%,v:SingletonAsOrderedSet,n:Expon) ==
       f * monomial(1,n)

     reductum f == f - leadingMonomial f

     variables f == list create()
```

— **COQ UPSCAT** —

```
(* category UPSCAT *)
(*

    degree : % -> Expon
    degree f == order f

    leadingCoefficient : % -> Coef
    leadingCoefficient f == coefficient(f,order f)

    leadingMonomial : % -> %
    leadingMonomial f ==
      ord := order f
      monomial(coefficient(f,ord),ord)

    monomial : (%,List(SingletonAsOrderedSet),List(Expon)) -> %
    monomial(f:%,listVar:List SingletonAsOrderedSet,listExpon:List Expon) ==
      empty? listVar or not empty? rest listVar =>
        error "monomial: variable list must have exactly one entry"
      empty? listExpon or not empty? rest listExpon =>
```

```
        error "monomial: exponent list must have exactly one entry"
      f * monomial(1,first listExpon)

    monomial : (%,SingletonAsOrderedSet,Expon) -> %
    monomial(f:%,v:SingletonAsOrderedSet,n:Expon) ==
      f * monomial(1,n)

    reductum : % -> %
    reductum f == f - leadingMonomial f

    variables : % -> List(SingletonAsOrderedSet)
    variables f == list create()

*)
```

———-

— UPSCAT.dotabb —

```
"UPSCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UPSCAT"];
"UPSCAT" -> "PSCAT"
```

———-

— UPSCAT.dotfull —

```
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UPSCAT"];
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)" ->
 "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"

"UnivariatePowerSeriesCategory(a:Ring,NonNegativeInteger)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=UPSCAT"];
"UnivariatePowerSeriesCategory(a:Ring,NonNegativeInteger)" ->
    "UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"

"UnivariatePowerSeriesCategory(a:Ring,Integer)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=UPSCAT"];
"UnivariatePowerSeriesCategory(a:Ring,Integer)" ->
    "UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"

"UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=UPSCAT"];
"UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))" ->
    "UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"
```

———-

— UPSCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"
 [color=lightblue];
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)" ->
 "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
 [color=seagreen];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
  -> "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
 [color=lightblue];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)" ->
    "AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"

"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" [color=lightblue];
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" -> "RING..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "BIMODULE..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "IntegralDomain()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CharacteristicNonZero()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CommutativeRing()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "Algebra(Fraction(Integer))"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BIMODULE..."

"CharacteristicNonZero()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARNZ"];
"CharacteristicNonZero()" -> "RING..."

"Algebra(Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(Fraction(Integer))" -> "Algebra(a:CommutativeRing)"

"Algebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "MODULE..."
```

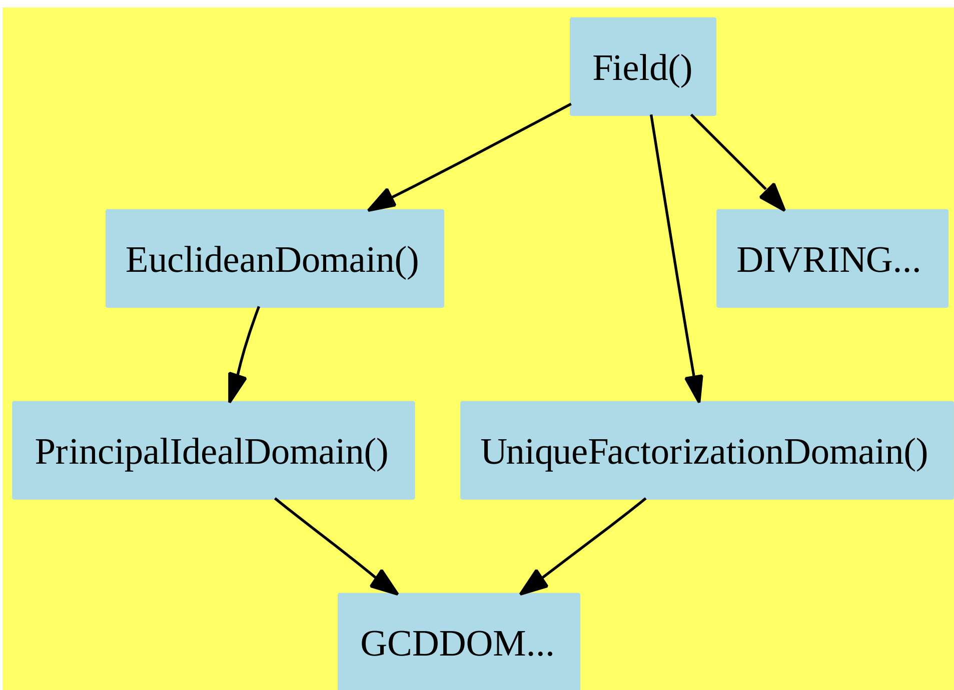
```
"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BIMODULE..."

"BIMODULE..." [color=lightblue];
"RING..." [color=lightblue];
"MODULE..." [color=lightblue];
}
```

———–

Chapter 16

Category Layer 15

16.0.209 Field (FIELD)



— **Field.input** —

```
)set break resume
)sys rm -f Field.output
)spool Field.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show Field
--R
--R Field is a category constructor
--R Abbreviation for Field is FIELD
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FIELD
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Fraction(Integer),%) -> %      ?*? : (%,Fraction(Integer)) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,Integer) -> %               ?**? : (%,NonNegativeInteger) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (%,%) -> %                      ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,Integer) -> %                ?^? : (%,NonNegativeInteger) -> %
--R ?^? : (%,PositiveInteger) -> %        associates? : (%,%) -> Boolean
--R coerce : Fraction(Integer) -> %       coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R factor : % -> Factored(%)             gcd : List(%) -> %
--R gcd : (%,%) -> %                      hash : % -> SingleInteger
--R inv : % -> %                          latex : % -> String
--R lcm : List(%) -> %                    lcm : (%,%) -> %
--R one? : % -> Boolean                   prime? : % -> Boolean
--R ?quo? : (%,%) -> %                    recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                    sample : () -> %
--R sizeLess? : (%,%) -> Boolean          squareFree : % -> Factored(%)
--R squareFreePart : % -> %               unit? : % -> Boolean
--R unitCanonical : % -> %                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed")
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

— **Field.help** —

```
====================================================================
Field examples
====================================================================

The category of commutative fields, commutative rings where all
non-zero elements have multiplicative inverses. The factor operation
while trivial is useful to have defined.

Axioms:
         a*(b/a) = b
         inv(a) = 1/a

See Also:
o )show Field
```

Euclidean Domains are a subset of Fields.
⇐ "EuclideanDomain" (EUCDOM) 15.0.205 on page 1374
⇒ "PseudoAlgebraicClosureOfPerfectFieldCategory" (PACPERC) 17.0.221 on page 1588

See:

⇒ "AlgebraicallyClosedField" (ACF) 17.0.215 on page 1497
⇒ "ExtensionField" (XF) 18.0.230 on page 1754
⇒ "FieldOfPrimeCharacteristic" (FPC) 17.0.217 on page 1531
⇒ "FiniteRankAlgebra" (FINRALG) 17.0.218 on page 1537
⇒ "FunctionSpace" (FS) 17.0.219 on page 1544
⇒ "QuotientFieldCategory" (QFCAT) 17.0.222 on page 1595
⇒ "RealClosedField" (RCFIELD) 17.0.223 on page 1611
⇒ "RealNumberSystem" (RNS) 17.0.224 on page 1622
⇒ "UnivariateLaurentSeriesCategory" (ULSCAT) 17.0.226 on page 1686
⇒ "UnivariatePolynomialCategory" (UPOLYC) 17.0.228 on page 1708
⇒ "UnivariatePuiseuxSeriesCategory" (UPXSCAT) 17.0.227 on page 1697
⇐ "DivisionRing" (DIVRING) 12.0.185 on page 1185
⇐ "EuclideanDomain" (EUCDOM) 15.0.205 on page 1374
⇐ "UniqueFactorizationDomain" (UFD) 14.0.203 on page 1362

Exports:

0	1	associates?	characteristic	coerce
divide	euclideanSize	expressIdealMember	exquo	extendedEuclidean
factor	gcd	gcdPolynomial	hash	inv
latex	lcm	multiEuclidean	one?	prime?
principalIdeal	recip	sample	sizeLess?	squareFree
squareFreePart	subtractIfCan	unit?	unitCanonical	unitNormal
zero?	?*?	?**?	?+?	?-?
-?	?/?	?=?	?^?	?quo?
?rem?	?~=?			

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are implemented by this category:

```
associates? : (%,%) -> Boolean
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
exquo : (%,%) -> Union(%,"failed")
factor : % -> Factored %
gcd : (%,%) -> %
inv : % -> %
prime? : % -> Boolean
squareFree : % -> Factored %
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
?/? : (%,%) -> %
```

These exports come from (p1374) EuclideanDomain():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
expressIdealMember : (List %,%) -> Union(List %,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
gcd : List % -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                     SparseUnivariatePolynomial %
hash : % -> SingleInteger
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
sample : () -> %
sizeLess? : (%,%) -> Boolean
```

```
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p1362) UniqueFactorizationDomain():

```
squareFreePart : % -> %
```

These exports come from (p1185) DivisionRing():

```
coerce : Fraction Integer -> %
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?**? : (%,Integer) -> %
?^? : (%,Integer) -> %
```

— Field.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FIELD">
Field (FIELD)</a></h2>
</body>
```

————

— category FIELD Field —

```
)abbrev category FIELD Field
++ Description:
++ The category of commutative fields, commutative rings
++ where all non-zero elements have multiplicative inverses.
++ The \spadfun{factor} operation while trivial is useful to have defined.
++
++ Axioms\br
++ \tab{5}\spad{a*(b/a) = b}\br
++ \tab{5}\spad{inv(a) = 1/a}

Field() : Category == SIG where

  SIG ==> Join(EuclideanDomain,UniqueFactorizationDomain,DivisionRing) with
```

```
    "/" : (%,%) -> %
      ++ x/y divides the element x by the element y.
      ++ Error: if y is 0.

    canonicalUnitNormal
      ++ either 0 or 1.

    canonicalsClosed
      ++ since \spad{0*0=0}, \spad{1*1=1}

   add

      x,y: %
      n: Integer

      UCA ==> Record(unit:%,canonical:%,associate:%)

      unitNormal(x) ==
          if zero? x then [1$%,0$%,1$%]$UCA else [x,1$%,inv(x)]$UCA

      unitCanonical(x) == if zero? x then x else 1

      associates?(x,y) == if zero? x then zero? y else not(zero? y)

      inv x ==((u:=recip x) case "failed" => error "not invertible"; u)

      x exquo y == (y=0 => "failed"; x / y)

      gcd(x,y) == 1

      euclideanSize(x) == 0

      prime? x == false

      squareFree x == x::Factored(%)

      factor x == x::Factored(%)

      x / y == (zero? y => error "catdef: division by zero"; x * inv(y))

      divide(x,y) == [x / y,0]
```

— COQ FIELD —

```
(* category FIELD *)
(*
Axioms
  a*(b/a) = b
  inv(a) = 1/a
```

```
      x,y: %
      n: Integer

      UCA ==> Record(unit:%,canonical:%,associate:%)

      unitNormal : % -> Record(unit: %,canonical: %,associate: %)
      unitNormal(x) ==
          if zero? x then [1$%,0$%,1$%]$UCA else [x,1$%,inv(x)]$UCA

      unitCanonical : % -> %
      unitCanonical(x) == if zero? x then x else 1

      associates? : (%,%) -> Boolean
      associates?(x,y) == if zero? x then zero? y else not(zero? y)

      inv : % -> %
      inv x ==((u:=recip x) case "failed" => error "not invertible"; u)

      exquo : (%,%) -> Union(%,"failed")
      x exquo y == (y=0 => "failed"; x / y)

      gcd : (%,%) -> %
      gcd(x,y) == 1

      euclideanSize : % -> NonNegativeInteger
      euclideanSize(x) == 0

      prime? : % -> Boolean
      prime? x == false

      squareFree : % -> Factored(%)
      squareFree x == x::Factored(%)

      factor : % -> Factored(%)
      factor x == x::Factored(%)

      ?/? : (%,%) -> %
      x / y == (zero? y => error "catdef: division by zero"; x * inv(y))

      divide : (%,%) -> Record(quotient: %,remainder: %)
      divide(x,y) == [x / y,0]

*)
```

— FIELD.dotabb —

```
"FIELD"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FIELD"];
"FIELD" -> "EUCDOM"
"FIELD" -> "UFD"
```

```
"FIELD" -> "DIVRING"
```

———-

— **FIELD.dotfull** —

```
"Field()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FIELD"];
"Field()" -> "EuclideanDomain()"
"Field()" -> "UniqueFactorizationDomain()"
"Field()" -> "DivisionRing()"
```

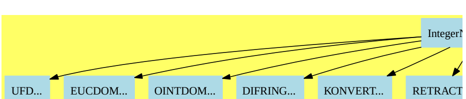
———-

— **FIELD.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"Field()" [color=lightblue];
"Field()" -> "EuclideanDomain()"
"Field()" -> "UniqueFactorizationDomain()"
"Field()" -> "DIVRING..."

"EuclideanDomain()" [color=lightblue];
"EuclideanDomain()" -> "PrincipalIdealDomain()"

"UniqueFactorizationDomain()" [color=lightblue];
"UniqueFactorizationDomain()" -> "GCDDOM..."

"PrincipalIdealDomain()" [color=lightblue];
"PrincipalIdealDomain()" -> "GCDDOM..."

"DIVRING..." [color=lightblue];
"GCDDOM..." [color=lightblue];
}
```

———-

16.0.210 IntegerNumberSystem (INS)



— **IntegerNumberSystem.input** —

```
)set break resume
)sys rm -f IntegerNumberSystem.output
)spool IntegerNumberSystem.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show IntegerNumberSystem
--R
--R IntegerNumberSystem is a category constructor
--R Abbreviation for IntegerNumberSystem is INS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for INS
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?<? : (%,%) -> Boolean
--R ?<=? : (%,%) -> Boolean               ?=? : (%,%) -> Boolean
--R ?>? : (%,%) -> Boolean                ?>=? : (%,%) -> Boolean
--R D : % -> %                            D : (%,NonNegativeInteger) -> %
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R abs : % -> %                          addmod : (%,%,%) -> %
--R associates? : (%,%) -> Boolean        base : () -> %
--R binomial : (%,%) -> %                 bit? : (%,%) -> Boolean
--R coerce : Integer -> %                 coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R convert : % -> DoubleFloat            convert : % -> Float
--R convert : % -> Pattern(Integer)       convert : % -> InputForm
--R convert : % -> Integer                copy : % -> %
--R dec : % -> %                          differentiate : % -> %
--R even? : % -> Boolean                  factor : % -> Factored(%)
--R factorial : % -> %                    gcd : List(%) -> %
--R gcd : (%,%) -> %                      hash : % -> %
--R hash : % -> SingleInteger             inc : % -> %
--R init : () -> %                        invmod : (%,%) -> %
--R latex : % -> String                   lcm : List(%) -> %
--R lcm : (%,%) -> %                      length : % -> %
--R mask : % -> %                         max : (%,%) -> %
--R min : (%,%) -> %                      mulmod : (%,%,%) -> %
--R negative? : % -> Boolean              nextItem : % -> Union(%,"failed")
--R odd? : % -> Boolean                   one? : % -> Boolean
--R permutation : (%,%) -> %              positive? : % -> Boolean
--R positiveRemainder : (%,%) -> %        powmod : (%,%,%) -> %
--R prime? : % -> Boolean                 ?quo? : (%,%) -> %
--R random : % -> %                       random : () -> %
--R rational : % -> Fraction(Integer)     rational? : % -> Boolean
--R recip : % -> Union(%,"failed")        ?rem? : (%,%) -> %
--R retract : % -> Integer                sample : () -> %
--R shift : (%,%) -> %                    sign : % -> Integer
```

```
--R sizeLess? : (%,%) -> Boolean           squareFree : % -> Factored(%)
--R squareFreePart : % -> %                submod : (%,%,%) -> %
--R symmetricRemainder : (%,%) -> %        unit? : % -> Boolean
--R unitCanonical : % -> %                 zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R differentiate : (%,NonNegativeInteger) -> %
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed")
--R patternMatch : (%,Pattern(Integer),PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%)
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %)
--R rationalIfCan : % -> Union(Fraction(Integer),"failed")
--R reducedSystem : Matrix(%) -> Matrix(Integer)
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(Integer),vec: Vector(Integer))
--R retractIfCan : % -> Union(Integer,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

— **IntegerNumberSystem.help** —

```
====================================================================
IntegerNumberSystem examples
====================================================================

An IntegerNumberSystem is a model for the integers.

See Also:
o )show IntegerNumberSystem
```

See:

⇐ “CharacteristicZero” (CHARZ) 10.0.160 on page 1015
⇐ “CombinatorialFunctionCategory” (CFCAT) 2.0.15 on page 42
⇐ “ConvertibleTo” (KONVERT) 2.0.17 on page 47
⇐ “DifferentialRing” (DIFRING) 10.0.162 on page 1024
⇐ “EuclideanDomain” (EUCDOM) 15.0.205 on page 1374
⇐ “LinearlyExplicitRingOver” (LINEXP) 10.0.166 on page 1045

⇐ "OrderedIntegralDomain" (OINTDOM) 13.0.198 on page 1326
⇐ "Patternable" (PATAB) 2.0.34 on page 100
⇐ "RealConstant" (REAL) 3.0.55 on page 180
⇐ "RetractableTo" (RETRACT) 2.0.37 on page 110
⇐ "StepThrough" (STEP) 4.0.88 on page 367
⇐ "UniqueFactorizationDomain" (UFD) 14.0.203 on page 1362

Exports:

0	1	abs	addmod
associates?	base	binomial	bit?
characteristic	coerce	convert	copy
D	dec	differentiate	divide
euclideanSize	even?	expressIdealMember	exquo
extendedEuclidean	factor	factorial	gcd
gcdPolynomial	hash	inc	init
invmod	latex	lcm	length
mask	max	min	mulmod
multiEuclidean	negative?	nextItem	odd?
one?	patternMatch	permutation	positive?
positiveRemainder	powmod	prime?	principalIdeal
random	rational	rational?	rationalIfCan
recip	reducedSystem	retract	retractIfCan
sample	shift	sign	sizeLess?
squareFree	squareFreePart	submod	subtractIfCan
symmetricRemainder	unit?	unitCanonical	unitNormal
zero?	?*?	?**?	?+?
?-?	-?	?<?	?<=?
?=?	?>?	?>=?	?^?
?~=?	?quo?	?rem?	

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **multiplicativeValuation** implies `euclideanSize(a*b)=euclideanSize(a)*euclideanSize(b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D)- > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
addmod : (%,%,%) -> %
base : () -> %
dec : % -> %
```

```
hash : % -> %
inc : % -> %
length : % -> %
mulmod : (%,%,%) -> %
odd? : % -> Boolean
positiveRemainder : (%,%) -> %
random : () -> %
random : % -> %
shift : (%,%) -> %
submod : (%,%,%) -> %
```

These are implemented by this category:

```
binomial : (%,%) -> %
bit? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
convert : % -> Float
convert : % -> DoubleFloat
convert : % -> InputForm
convert : % -> Pattern Integer
copy : % -> %
differentiate : % -> %
euclideanSize : % -> NonNegativeInteger
even? : % -> Boolean
factor : % -> Factored %
factorial : % -> %
init : () -> %
invmod : (%,%) -> %
mask : % -> %
nextItem : % -> Union(%,"failed")
patternMatch :
 (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
   PatternMatchResult(Integer,%)
permutation : (%,%) -> %
positive? : % -> Boolean
powmod : (%,%,%) -> %
prime? : % -> Boolean
rational : % -> Fraction Integer
rational? : % -> Boolean
rationalIfCan : % -> Union(Fraction Integer,"failed")
retract : % -> Integer
retractIfCan : % -> Union(Integer,"failed")
squareFree : % -> Factored %
symmetricRemainder : (%,%) -> %
```

These exports come from (p1362) UniqueFactorizationDomain():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
coerce : % -> %
coerce : Integer -> %
coerce : Integer -> %
coerce : % -> OutputForm
exquo : (%,%) -> Union(%,"failed")
gcd : List % -> %
```

```
gcd : (%,%) -> %
gcdPolynomial :
 (SparseUnivariatePolynomial %,
  SparseUnivariatePolynomial %) ->
    SparseUnivariatePolynomial %
hash : % -> SingleInteger
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

These exports come from (p1374) EuclideanDomain():

```
divide : (%,%) -> Record(quotient: %,remainder: %)
expressIdealMember : (List %,%) -> Union(List %,"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
multiEuclidean : (List %,%) -> Union(List %,"failed")
principalIdeal : List % -> Record(coef: List %,generator: %)
sizeLess? : (%,%) -> Boolean
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p1326) OrderedIntegralDomain():

```
abs : % -> %
max : (%,%) -> %
min : (%,%) -> %
negative? : % -> Boolean
sign : % -> Integer
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

These exports come from (p1024) DifferentialRing():

```
D : % -> %
D : (%,NonNegativeInteger) -> %
differentiate : (%,NonNegativeInteger) -> %
```

These exports come from (p47) ConvertibleTo(Integer):

```
convert : % -> Integer
```

These exports come from (p1045) LinearlyExplicitRingOver(Integer):

```
reducedSystem : Matrix % -> Matrix Integer
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix Integer,vec: Vector Integer)
```

— IntegerNumberSystem.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#INS">
IntegerNumberSystem (INS)</a></h2>
</body>
```

— category INS IntegerNumberSystem —

```
)abbrev category INS IntegerNumberSystem
++ Author: Stephen M. Watt
++ Date Created: January 1988
++ Description:
++ An \spad{IntegerNumberSystem} is a model for the integers.

IntegerNumberSystem() : Category == SIG where

  UFD ==> UniqueFactorizationDomain
  ED  ==> EuclideanDomain
  OID ==> OrderedIntegralDomain
  DR  ==> DifferentialRing
  CI  ==> ConvertibleTo(Integer)
  RT  ==> RetractableTo(Integer)
  LERO ==> LinearlyExplicitRingOver(Integer)
  CTIF ==> ConvertibleTo(InputForm)
  CTPI ==> ConvertibleTo(Pattern(Integer))
  PM   ==> PatternMatchable(Integer)
  CFC  ==> CombinatorialFunctionCategory
  RC   ==> RealConstant
  CZ   ==> CharacteristicZero
  ST   ==> StepThrough

  SIG ==> Join(UFD,ED,OID,DR,CI,RT,LERO,CTIF,CTPI,PM,CFC,RC,CZ,ST) with

    odd? : % -> Boolean
      ++ odd?(n) returns true if and only if n is odd.

    even? : % -> Boolean
      ++ even?(n) returns true if and only if n is even.
```

```
multiplicativeValuation
  ++ euclideanSize(a*b) returns \spad{euclideanSize(a)*euclideanSize(b)}.

base : () -> %
  ++ base() returns the base for the operations of
  ++ \spad{IntegerNumberSystem}.

length : % -> %
  ++ length(a) length of \spad{a} in digits.

shift : (%, %) -> %
  ++ shift(a,i) shift \spad{a} by i digits.

bit? : (%, %) -> Boolean
  ++ bit?(n,i) returns true if and only if i-th bit of n is a 1.

positiveRemainder : (%, %) -> %
  ++ positiveRemainder(a,b) (where \spad{b > 1}) yields r
  ++ where \spad{0 <= r < b} and \spad{r == a rem b}.

symmetricRemainder : (%, %) -> %
  ++ symmetricRemainder(a,b) (where \spad{b > 1}) yields r
  ++ where \spad{ -b/2 <= r < b/2 }.

rational? : % -> Boolean
  ++ rational?(n) tests if n is a rational number
  ++ (see \spadtype{Fraction Integer}).

rational : % -> Fraction Integer
  ++ rational(n) creates a rational number
  ++ (see \spadtype{Fraction Integer})..

rationalIfCan : % -> Union(Fraction Integer, "failed")
  ++ rationalIfCan(n) creates a rational number, or returns "failed"
  ++ if this is not possible.

random : () -> %
  ++ random() creates a random element.

random : % -> %
  ++ random(a) creates a random element from 0 to \spad{n-1}.

hash : % -> %
  ++ hash(n) returns the hash code of n.

copy : % -> %
  ++ copy(n) gives a copy of n.

inc : % -> %
  ++ inc(x) returns \spad{x + 1}.

dec : % -> %
  ++ dec(x) returns \spad{x - 1}.
```

```
 mask : % -> %
   ++ mask(n) returns \spad{2**n-1} (an n bit mask).

 addmod : (%,%,%) -> %
   ++ addmod(a,b,p), \spad{0<=a,b<p>1}, means \spad{a+b mod p}.

 submod : (%,%,%) -> %
   ++ submod(a,b,p), \spad{0<=a,b<p>1}, means \spad{a-b mod p}.

 mulmod : (%,%,%) -> %
   ++ mulmod(a,b,p), \spad{0<=a,b<p>1}, means \spad{a*b mod p}.

 powmod : (%,%,%) -> %
   ++ powmod(a,b,p), \spad{0<=a,b<p>1}, means \spad{a**b mod p}.

 invmod : (%,%) -> %
   ++ invmod(a,b), \spad{0<=a<b>1}, \spad{(a,b)=1} means \spad{1/a mod b}.

 canonicalUnitNormal
 --   commutative("*")    -- follows from the above

add

  characteristic() == 0

  differentiate x == 0

  even? x == not odd? x

  positive? x == x > 0

  copy x == x

  bit?(x, i) == odd? shift(x, -i)

  mask n == dec shift(1, n)

  rational? x == true

  euclideanSize(x) ==
       x=0 => error "euclideanSize called on zero"
       x<0 => (-convert(x)@Integer)::NonNegativeInteger
       convert(x)@Integer::NonNegativeInteger

  convert(x:%):Float == (convert(x)@Integer)::Float

  convert(x:%):DoubleFloat == (convert(x)@Integer)::DoubleFloat

  convert(x:%):InputForm == convert(convert(x)@Integer)

  retract(x:%):Integer == convert(x)@Integer

  convert(x:%):Pattern(Integer)== convert(x)@Integer ::Pattern(Integer)
```

```
factor x == factor(x)$IntegerFactorizationPackage(%)

squareFree x == squareFree(x)$IntegerFactorizationPackage(%)

prime? x == prime?(x)$IntegerPrimesPackage(%)

factorial x == factorial(x)$IntegerCombinatoricFunctions(%)

binomial(n, m) == binomial(n, m)$IntegerCombinatoricFunctions(%)

permutation(n, m) == permutation(n,m)$IntegerCombinatoricFunctions(%)

retractIfCan(x:%):Union(Integer, "failed") == convert(x)@Integer

init() == 0

-- iterates in order 0,1,-1,2,-2,3,-3,...
nextItem(n) ==
  zero? n => 1
  n>0 => -n
  1-n

patternMatch(x, p, l) ==
  patternMatch(x, p, l)$PatternMatchIntegerNumberSystem(%)

rational(x:%):Fraction(Integer) ==
  (convert(x)@Integer)::Fraction(Integer)

rationalIfCan(x:%):Union(Fraction Integer, "failed") ==
  (convert(x)@Integer)::Fraction(Integer)

symmetricRemainder(x, n) ==
   r := x rem n
   r = 0 => r
   if n < 0 then n:=-n
   r > 0 =>
      2 * r > n => r - n
      r
   2*r + n <= 0 => r + n
   r

invmod(a, b) ==
   if negative? a then a := positiveRemainder(a, b)
   c := a; c1:% := 1
   d := b; d1:% := 0
   while not zero? d repeat
      q := c quo d
      r := c-q*d
      r1 := c1-q*d1
      c := d; c1 := d1
      d := r; d1 := r1
   not (c = 1) => error "inverse does not exist"
   negative? c1 => c1 + b
```

```
        c1

     powmod(x, n, p) ==
        if negative? x then x := positiveRemainder(x, p)
        zero? x => 0
        zero? n => 1
        y:% := 1
        z := x
        repeat
           if odd? n then y := mulmod(y, z, p)
           zero?(n := shift(n, -1)) => return y
           z := mulmod(z, z, p)
```

— **COQ INS** —

```
(* category INS *)
(*

   characteristic : () -> NonNegativeInteger
   characteristic() == 0

   differentiate : % -> %
   differentiate x == 0

   even? : % -> Boolean
   even? x == not odd? x

   positive? : % -> Boolean
   positive? x == x > 0

   copy : % -> %
   copy x == x

   bit? : (%,%) -> Boolean
   bit?(x, i) == odd? shift(x, -i)

   mask : % -> %
   mask n == dec shift(1, n)

   rational? : % -> Boolean
   rational? x == true

   euclideanSize : % -> NonNegativeInteger
   euclideanSize(x) ==
        x=0 => error "euclideanSize called on zero"
        x<0 => (-convert(x)@Integer)::NonNegativeInteger
        convert(x)@Integer::NonNegativeInteger

   convert : % -> Float
   convert(x:%):Float == (convert(x)@Integer)::Float
```

```
convert : % -> DoubleFloat
convert(x:%):DoubleFloat  == (convert(x)@Integer)::DoubleFloat

convert : % -> InputForm
convert(x:%):InputForm == convert(convert(x)@Integer)

retract(x:%):Integer == convert(x)@Integer

convert : % -> Pattern(Integer)
convert(x:%):Pattern(Integer)== convert(x)@Integer ::Pattern(Integer)

factor : % -> Factored(%)
factor x == factor(x)$IntegerFactorizationPackage(%)

squareFree : % -> Factored(%)
squareFree x == squareFree(x)$IntegerFactorizationPackage(%)

prime? : % -> Boolean
prime? x == prime?(x)$IntegerPrimesPackage(%)

factorial : % -> %
factorial x == factorial(x)$IntegerCombinatoricFunctions(%)

binomial : (%,%) -> %
binomial(n, m) == binomial(n, m)$IntegerCombinatoricFunctions(%)

permutation : (%,%) -> %
permutation(n, m) == permutation(n,m)$IntegerCombinatoricFunctions(%)

rationalIfCan : % -> Union(Fraction(Integer),"failed")
retractIfCan(x:%):Union(Integer, "failed") == convert(x)@Integer

init : () -> %
init() == 0

-- iterates in order 0,1,-1,2,-2,3,-3,...
nextItem : % -> Union(%,"failed")
nextItem(n) ==
  zero? n => 1
  n>0 => -n
  1-n

patternMatch : (%,Pattern(Integer),PatternMatchResult(Integer,%)) ->
   PatternMatchResult(Integer,%)
patternMatch(x, p, l) ==
  patternMatch(x, p, l)$PatternMatchIntegerNumberSystem(%)

rational : % -> Fraction(Integer)
rational(x:%):Fraction(Integer) ==
  (convert(x)@Integer)::Fraction(Integer)

rationalIfCan : % -> Union(Fraction(Integer),"failed")
rationalIfCan(x:%):Union(Fraction Integer, "failed") ==
  (convert(x)@Integer)::Fraction(Integer)
```

```
   symmetricRemainder : (%,%) -> %
   symmetricRemainder(x, n) ==
      r := x rem n
      r = 0 => r
      if n < 0 then n:=-n
      r > 0 =>
         2 * r > n => r - n
         r
      2*r + n <= 0 => r + n
      r

   invmod : (%,%) -> %
   invmod(a, b) ==
      if negative? a then a := positiveRemainder(a, b)
      c := a; c1:% := 1
      d := b; d1:% := 0
      while not zero? d repeat
         q := c quo d
         r := c-q*d
         r1 := c1-q*d1
         c := d; c1 := d1
         d := r; d1 := r1
      not (c = 1) => error "inverse does not exist"
      negative? c1 => c1 + b
      c1

   powmod : (%,%,%) -> %
   powmod(x, n, p) ==
      if negative? x then x := positiveRemainder(x, p)
      zero? x => 0
      zero? n => 1
      y:% := 1
      z := x
      repeat
         if odd? n then y := mulmod(y, z, p)
         zero?(n := shift(n, -1)) => return y
         z := mulmod(z, z, p)
*)
```

———-

— INS.dotabb —

```
"INS"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=INS"];
"INS" -> "UFD"
"INS" -> "EUCDOM"
"INS" -> "OINTDOM"
"INS" -> "DIFRING"
"INS" -> "KONVERT"
"INS" -> "RETRACT"
"INS" -> "LINEXP"
```

```
"INS" -> "PATMAB"
"INS" -> "CFCAT"
"INS" -> "REAL"
"INS" -> "CHARZ"
"INS" -> "STEP"
```

———-

— INS.dotfull —

```
"IntegerNumberSystem()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=INS"];
"IntegerNumberSystem()" -> "UniqueFactorizationDomain()"
"IntegerNumberSystem()" -> "EuclideanDomain()"
"IntegerNumberSystem()" -> "OrderedIntegralDomain()"
"IntegerNumberSystem()" -> "DifferentialRing()"
"IntegerNumberSystem()" -> "ConvertibleTo(Integer)"
"IntegerNumberSystem()" -> "ConvertibleTo(InputForm)"
"IntegerNumberSystem()" -> "ConvertibleTo(Pattern(Integer))"
"IntegerNumberSystem()" -> "RetractableTo(Integer)"
"IntegerNumberSystem()" -> "LinearlyExplicitRingOver(Integer)"
"IntegerNumberSystem()" -> "PatternMatchable(Integer)"
"IntegerNumberSystem()" -> "CombinatorialFunctionCategory()"
"IntegerNumberSystem()" -> "RealConstant()"
"IntegerNumberSystem()" -> "CharacteristicZero()"
"IntegerNumberSystem()" -> "StepThrough()"
```

———-

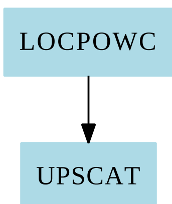
— INS.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"IntegerNumberSystem()" [color=lightblue];
"IntegerNumberSystem()" -> "UFD..."
"IntegerNumberSystem()" -> "EUCDOM..."
"IntegerNumberSystem()" -> "OINTDOM..."
"IntegerNumberSystem()" -> "DIFRING..."
"IntegerNumberSystem()" -> "KONVERT..."
"IntegerNumberSystem()" -> "RETRACT..."
"IntegerNumberSystem()" -> "LINEXP..."
"IntegerNumberSystem()" -> "PATMAB..."
"IntegerNumberSystem()" -> "CFCAT..."
"IntegerNumberSystem()" -> "REAL..."
"IntegerNumberSystem()" -> "CHARZ..."
"IntegerNumberSystem()" -> "STEP..."

"UFD..." [color=lightblue];
"EUCDOM..." [color=lightblue];
"OINTDOM..." [color=lightblue];
```

```
"DIFRING..." [color=lightblue];
"KONVERT..." [color=lightblue];
"RETRACT..." [color=lightblue];
"LINEXP..." [color=lightblue];
"PATMAB..." [color=lightblue];
"CFCAT..." [color=lightblue];
"REAL..." [color=lightblue];
"CHARZ..." [color=lightblue];
"STEP..." [color=lightblue];

}
```

———-

16.0.211 LocalPowerSeriesCategory (LOCPOWC)



— LocalPowerSeriesCategory.input —

```
)set break resume
)sys rm -f LocalPowerSeriesCategory.output
)spool LocalPowerSeriesCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show LocalPowerSeriesCategory
--R
--R LocalPowerSeriesCategory(K: Field) is a category constructor
--R Abbreviation for LocalPowerSeriesCategory is LOCPOWC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for LOCPOWC
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,K) -> %                      ?*? : (K,%) -> %
--R ?*? : (Fraction(Integer),%) -> %      ?*? : (%,Fraction(Integer)) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,Integer) -> %               ?**? : (%,NonNegativeInteger) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (%,K) -> % if K has FIELD       ?/? : (%,%) -> %
```

```
--R ?=? : (%,%) -> Boolean                1 : () -> %
--R 0 : () -> %                           ?^? : (%,Integer) -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean        center : % -> K
--R coefOfFirstNonZeroTerm : % -> K       coefficient : (%,Integer) -> K
--R coerce : K -> % if K has COMRING      coerce : Fraction(Integer) -> %
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              complete : % -> %
--R degree : % -> Integer                 delay : (() -> %) -> %
--R ?.? : (%,Integer) -> K                extend : (%,Integer) -> %
--R factor : % -> Factored(%)             filterUpTo : (%,Integer) -> %
--R findCoef : (%,Integer) -> K           gcd : List(%) -> %
--R gcd : (%,%) -> %                      hash : % -> SingleInteger
--R inv : % -> %                          latex : % -> String
--R lcm : List(%) -> %                    lcm : (%,%) -> %
--R leadingCoefficient : % -> K           leadingMonomial : % -> %
--R map : ((K -> K),%) -> %               monomial : (K,Integer) -> %
--R monomial? : % -> Boolean              one? : % -> Boolean
--R order : % -> Integer                  order : % -> Integer
--R order : (%,Integer) -> Integer        pole? : % -> Boolean
--R posExpnPart : % -> %                  prime? : % -> Boolean
--R printInfo : () -> Boolean             printInfo : Boolean -> Boolean
--R ?quo? : (%,%) -> %                    recip : % -> Union(%,"failed")
--R reductum : % -> %                     ?rem? : (%,%) -> %
--R removeFirstZeroes : % -> %            removeZeroes : % -> %
--R removeZeroes : (Integer,%) -> %       sample : () -> %
--R sbt : (%,%) -> %                      series : (Integer,K,%) -> %
--R shift : (%,Integer) -> %              sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored(%)         squareFreePart : % -> %
--R truncate : (%,Integer) -> %           unit? : % -> Boolean
--R unitCanonical : % -> %                variable : % -> Symbol
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R D : (%,List(Symbol),List(NonNegativeInteger)) -> % if K has PDRING(SYMBOL) and K has *: (Integer,K) -> K
--R D : (%,Symbol,NonNegativeInteger) -> % if K has PDRING(SYMBOL) and K has *: (Integer,K) -> K
--R D : (%,List(Symbol)) -> % if K has PDRING(SYMBOL) and K has *: (Integer,K) -> K
--R D : (%,Symbol) -> % if K has PDRING(SYMBOL) and K has *: (Integer,K) -> K
--R D : (%,NonNegativeInteger) -> % if K has *: (Integer,K) -> K
--R D : % -> % if K has *: (Integer,K) -> K
--R approximate : (%,Integer) -> K if K has **: (K,Integer) -> K and K has coerce: Symbol -> K
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if K has CHARNZ
--R coerce : % -> Stream(Record(k: Integer,c: K))
--R coerce : Stream(Record(k: Integer,c: K)) -> %
--R differentiate : (%,List(Symbol),List(NonNegativeInteger)) -> % if K has PDRING(SYMBOL) and K has *: (Integer
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if K has PDRING(SYMBOL) and K has *: (Integer,K) -> K
--R differentiate : (%,List(Symbol)) -> % if K has PDRING(SYMBOL) and K has *: (Integer,K) -> K
--R differentiate : (%,Symbol) -> % if K has PDRING(SYMBOL) and K has *: (Integer,K) -> K
--R differentiate : (%,NonNegativeInteger) -> % if K has *: (Integer,K) -> K
--R differentiate : % -> % if K has *: (Integer,K) -> K
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R ?.? : (%,%) -> % if Integer has SGROUP
--R euclideanSize : % -> NonNegativeInteger
--R eval : (%,K) -> Stream(K) if K has **: (K,Integer) -> K
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed")
```

```
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
--R monomial : (%,SingletonAsOrderedSet,Integer) -> %
--R monomial : (%,List(SingletonAsOrderedSet),List(Integer)) -> %
--R monomial2series : (List(%),List(NonNegativeInteger),Integer) -> %
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed")
--R multiplyExponents : (%,PositiveInteger) -> %
--R orderIfNegative : % -> Union(Integer,"failed")
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R terms : % -> Stream(Record(k: Integer,c: K))
--R truncate : (%,Integer,Integer) -> %
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R variables : % -> List(SingletonAsOrderedSet)
--R
--E 1

)spool
)lisp (bye)
```

— **LocalPowerSeriesCategory.help** —

```
====================================================================
LocalPowerSeriesCategory examples
====================================================================

See Also:
o )show LocalPowerSeriesCategory
```

See:

⇐ "CharacteristicZero" (CHARZ) 10.0.160 on page 1015
⇐ "EuclideanDomain" (EUCDOM) 15.0.205 on page 1374
⇐ "UnivariatePowerSeriesCategory" (UPSCAT) 15.0.208 on page 1401

Exports:

0	1	-?
?**?	?+?	?-?
?.?	?/?	?=?
?^?	?~=?	?quo?
?rem?	D	approximate
associates?	center	characteristic
charthRoot	coefOfFirstNonZeroTerm	coefficient
coerce	complete	degree
delay	differentiate	divide
euclideanSize	eval	expressIdealMember
exquo	extend	extendedEuclidean
factor	filterUpTo	findCoef
gcd	gcdPolynomial	hash
inv	latex	lcm
leadingCoefficient	leadingMonomial	map
monomial	monomial2series	monomial?
multiEuclidean	multiplyExponents	one?
order	orderIfNegative	pole?
posExpnPart	prime?	principalIdeal
printInfo	recip	reductum
removeFirstZeroes	removeZeroes	sample
sbt	series	shift
sizeLess?	squareFree	squareFreePart
subtractIfCan	terms	truncate
unit?	unitCanonical	unitNormal
variable	variables	zero?

Attributes Exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.

These are directly exported but not implemented

```
coefOfFirstNonZeroTerm : % -> K
coerce : % -> Stream Record(k: Integer,c: K)
coerce : Stream Record(k: Integer,c: K) -> %
delay : (() -> %) -> %
filterUpTo : (%,Integer) -> %
findCoef : (%,Integer) -> K
monomial2series : (List %,List NonNegativeInteger,Integer) -> %
```

```
order : % -> Integer
orderIfNegative : % -> Union(Integer,"failed")
posExpnPart : % -> %
printInfo : () -> Boolean
printInfo : Boolean -> Boolean
removeFirstZeroes : % -> %
removeZeroes : % -> %
removeZeroes : (Integer,%) -> %
sbt : (%,%) -> %
series : (Integer,K,%) -> %
shift : (%,Integer) -> %
```

These exports come from (p1401) UnivariatePowerSeriesCategory():

```
-? : % -> %
0 : () -> %
1 : () -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?*? : (%,Fraction Integer) -> %
?*? : (Fraction Integer,%) -> %
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
?.? : (%,%) -> % if Integer has SGROUP
?.? : (%,Integer) -> K
?/? : (%,%) -> %
?=? : (%,%) -> Boolean
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?~=? : (%,%) -> Boolean
D : % -> % if K has *: (Integer,K) -> K
D : (%,List Symbol) -> %
    if K has PDRING SYMBOL and K has *: (Integer,K) -> K
D : (%,List Symbol,List NonNegativeInteger) -> %
    if K has PDRING SYMBOL and K has *: (Integer,K) -> K
D : (%,NonNegativeInteger) -> % if K has *: (Integer,K) -> K
D : (%,Symbol) -> % if K has PDRING SYMBOL and K has *: (Integer,K) -> K
D : (%,Symbol,NonNegativeInteger) -> %
    if K has PDRING SYMBOL and K has *: (Integer,K) -> K
approximate : (%,Integer) -> K
    if K has **: (K,Integer) -> K and K has coerce: Symbol -> K
associates? : (%,%) -> Boolean
center : % -> K
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if K has CHARNZ
coefficient : (%,Integer) -> K
coerce : % -> %
coerce : % -> OutputForm
coerce : Fraction Integer -> %
coerce : Integer -> %
coerce : K -> % if K has COMRING
```

```
complete : % -> %
degree : % -> Integer
differentiate : % -> % if K has *: (Integer,K) -> K
differentiate : (%,List Symbol) -> %
   if K has PDRING SYMBOL and K has *: (Integer,K) -> K
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
   if K has PDRING SYMBOL and K has *: (Integer,K) -> K
differentiate : (%,NonNegativeInteger) -> %
   if K has *: (Integer,K) -> K
differentiate : (%,Symbol) -> %
   if K has PDRING SYMBOL and K has *: (Integer,K) -> K
differentiate : (%,Symbol,NonNegativeInteger) -> %
   if K has PDRING SYMBOL and K has *: (Integer,K) -> K
eval : (%,K) -> Stream K if K has **: (K,Integer) -> K
exquo : (%,%) -> Union(%,"failed")
extend : (%,Integer) -> %
hash : % -> SingleInteger
latex : % -> String
leadingCoefficient : % -> K
leadingMonomial : % -> %
map : ((K -> K),%) -> %
monomial : (%,List SingletonAsOrderedSet,List Integer) -> %
monomial : (%,SingletonAsOrderedSet,Integer) -> %
monomial : (K,Integer) -> %
monomial? : % -> Boolean
multiplyExponents : (%,PositiveInteger) -> %
one? : % -> Boolean
order : % -> Integer
order : (%,Integer) -> Integer
pole? : % -> Boolean
recip : % -> Union(%,"failed")
reductum : % -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
terms : % -> Stream Record(k: Integer,c: K)
truncate : (%,Integer) -> %
truncate : (%,Integer,Integer) -> %
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
variable : % -> Symbol
variables : % -> List SingletonAsOrderedSet
zero? : % -> Boolean
```

These exports come from (p1413) Field():

```
?**? : (%,Integer) -> %
?*? : (%,K) -> %
?*? : (K,%) -> %
?/? : (%,K) -> % if K has FIELD
?^? : (%,Integer) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
```

```
expressIdealMember : (List %,%) -> Union(List %,"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
factor : % -> Factored %
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial :
 (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) ->
   SparseUnivariatePolynomial %
inv : % -> %
lcm : (%,%) -> %
lcm : List % -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
sizeLess? : (%,%) -> Boolean
squareFree : % -> Factored %
squareFreePart : % -> %
```

— LocalPowerSeriesCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#LOCPOWC">
LocalPowerSeriesCategory (LOCPOWC)</a></h2>
</body>
```

————

— category LOCPOWC LocalPowerSeriesCategory —

```
)abbrev category LOCPOWC LocalPowerSeriesCategory
LocalPowerSeriesCategory(K) : Category == SIG where
  K : Field

  INT   ==> Integer
  TERM  ==> Record(k:INT,c:K)
  SER   ==> Stream(TERM)
  NNI   ==> NonNegativeInteger

  SIG ==> Join(Field,UnivariatePowerSeriesCategory(K,INT))  with

    order : % -> Integer
      ++ order(s) returns the order of s.

    findCoef : (%,Integer) -> K

    coerce : SER -> %

    coerce : % -> SER

    posExpnPart : % -> %
      ++ posExpnPart(s) returns the series s less the terms with
      ++ negative exponant.
```

```
    orderIfNegative : % -> Union(Integer,"failed")

    removeFirstZeroes : % -> %

    sbt : (%,%) -> %

    delay : ( () -> % ) -> %
      ++ delay delayed the computation of the next term of the series given
      ++ by the input function.

    monomial2series : (List %, List NNI, INT) -> %
      ++ monomial2series(ls,le,n) returns
      ++ t**n * reduce("*",[s ** e for s in ls for e in le])

    removeZeroes : (INT,%) -> %
      ++ removeZeroes(n,s) removes the zero terms in the first n terms of s.

    removeZeroes : % -> %
      ++ removeZeroes(s) removes the zero terms in  s.

    series : (INT,K,%) -> %
      ++ series(e,c,s) create the series c*t**e + s.

    shift : (%,INT) -> %
      ++ shift(s,n) returns t**n * s

    filterUpTo : (%,INT)  -> %
      ++ filterUpTo(s,n) returns the series consisting of the terms
      ++ of s having degree strictly less than n.

    coefOfFirstNonZeroTerm : % -> K
      ++ coefOfFirstNonZeroTerm(s) returns the first non zero coefficient
      ++ of the series.

    printInfo : Boolean -> Boolean
      ++ printInfo(b) set a flag such that when true (b <- true) prints
      ++ some information during some critical computation.

    printInfo : () -> Boolean
      ++ printInfo() returns the value of the \spad{printInfo} flag.
```

— LOCPOWC.dotabb —

```
"LOCPOWC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LOCPOWC"];
"LOCPOWC" -> "UPSCAT"
```

— **LOCPOWC.dotfull** —

```
"LocalPowerSeriesCategory(f:Field)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LOCPOWC"];
"LocalPowerSeriesCategory(f:Field)" ->
  "UnivariatePowerSeriesCategory(c:Ring,e:OrderedAbelianMonoid)"
```

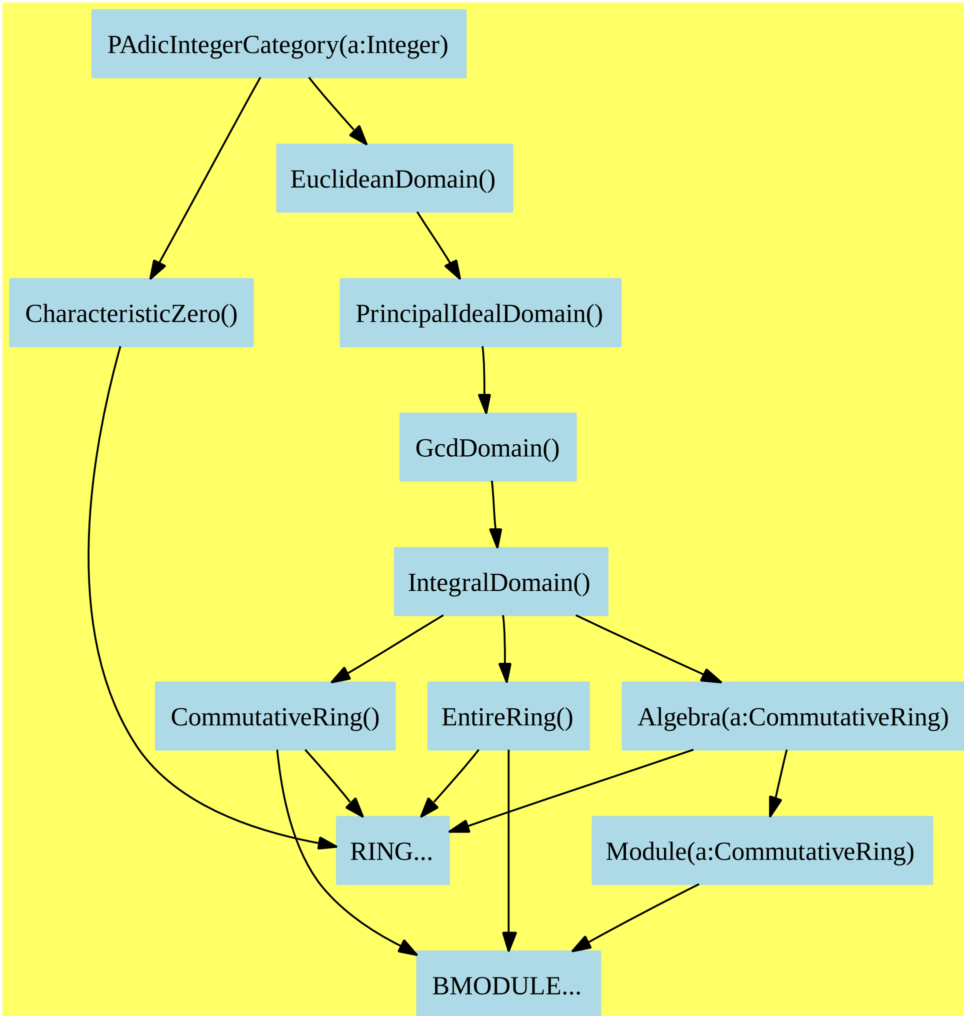
————

— **LOCPOWC.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"LocalPowerSeriesCategory(f:Field)" [color=lightblue];
"LocalPowerSeriesCategory(f:Field)" ->
  "UnivariatePowerSeriesCategory(c:Ring,e:OrderedAbelianMonoid)"

"Ring" [color=lightblue];
"OrderedAbelianMonoid"  [color=lightblue];

}
```

————

16.0.212 PAdicIntegerCategory (PADICCT)



— **PAdicIntegerCategory.input** —

```
)set break resume
)sys rm -f PAdicIntegerCategory.output
)spool PAdicIntegerCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PAdicIntegerCategory
--R
--R PAdicIntegerCategory(p: Integer) is a category constructor
--R Abbreviation for PAdicIntegerCategory is PADICCT
--R This constructor is exposed in this frame.
```

```
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PADICCT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean        coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R complete : % -> %                     digits : % -> Stream(Integer)
--R extend : (%,Integer) -> %             gcd : List(%) -> %
--R gcd : (%,%) -> %                      hash : % -> SingleInteger
--R latex : % -> String                   lcm : List(%) -> %
--R lcm : (%,%) -> %                      moduloP : % -> Integer
--R modulus : () -> Integer               one? : % -> Boolean
--R order : % -> NonNegativeInteger       ?quo? : (%,%) -> %
--R quotientByP : % -> %                  recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                    sample : () -> %
--R sizeLess? : (%,%) -> Boolean          sqrt : (%,Integer) -> %
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R approximate : (%,Integer) -> Integer
--R characteristic : () -> NonNegativeInteger
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed")
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %)
--R root : (SparseUnivariatePolynomial(Integer),Integer) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

———-

— PAdicIntegerCategory.help —

```
====================================================================
PAdicIntegerCategory examples
====================================================================

This is the category of stream-based representations of the p-adic integers.
```

```
See Also:
o )show PAdicIntegerCategory
```

See:

⇐ "CharacteristicZero" (CHARZ) 10.0.160 on page 1015
⇐ "EuclideanDomain" (EUCDOM) 15.0.205 on page 1374

Exports:

0	1	approximate	associates?
characteristic	coerce	complete	digits
divide	euclideanSize	expressIdealMember	exquo
extend	extendedEuclidean	gcd	gcdPolynomial
hash	latex	lcm	moduloP
modulus	multiEuclidean	one?	order
principalIdeal	quotientByP	recip	root
sample	sizeLess?	sqrt	subtractIfCan
unit?	unitCanonical	unitNormal	zero?
?*?	?**?	?+?	?-?
-?	?=?	?quo?	?rem?
?~=?	?^?		

Attributes Exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
approximate : (%,Integer) -> Integer
complete : % -> %
digits : % -> Stream Integer
extend : (%,Integer) -> %
moduloP : % -> Integer
modulus : () -> Integer
order : % -> NonNegativeInteger
quotientByP : % -> %
root : (SparseUnivariatePolynomial Integer,Integer) -> %
sqrt : (%,Integer) -> %
```

These exports come from (p1374) EuclideanDomain():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
```

```
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
hash : % -> SingleInteger
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
sample : () -> %
sizeLess? : (%,%) -> Boolean
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

— PAdicIntegerCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#PADICCT">
PAdicIntegerCategory (PADICCT)</a></h2>
</body>
```

— category PADICCT PAdicIntegerCategory —

```
)abbrev category PADICCT PAdicIntegerCategory
++ Author: Clifton J. Williamson
++ Date Created: 15 May 1990
++ Date Last Updated: 15 May 1990
++ Description:
++ This is the category of stream-based representations of
++ the p-adic integers.

PAdicIntegerCategory(p) : Category == SIG where
  p : Integer

  I   ==> Integer
  NNI ==> NonNegativeInteger
  ST  ==> Stream
  SUP ==> SparseUnivariatePolynomial

  SIG ==> Join(EuclideanDomain,CharacteristicZero) with

    digits : % -> ST I
      ++ \spad{digits(x)} returns a stream of p-adic digits of x.

    order : % -> NNI
      ++ \spad{order(x)} returns the exponent of the highest power of p
      ++ dividing x.

    extend : (%,I) -> %
      ++ \spad{extend(x,n)} forces the computation of digits up to order n.

    complete : % -> %
      ++ \spad{complete(x)} forces the computation of all digits.

    modulus : () -> I
      ++ \spad{modulus()} returns the value of p.

    moduloP : % -> I
      ++ \spad{modulo(x)} returns a, where \spad{x = a + b p}.

    quotientByP : % -> %
      ++ \spad{quotientByP(x)} returns b, where \spad{x = a + b p}.

    approximate : (%,I) -> I
      ++ \spad{approximate(x,n)} returns an integer y such that
      ++ \spad{y = x (mod p^n)}
      ++ when n is positive, and 0 otherwise.

    sqrt : (%,I) -> %
      ++ \spad{sqrt(b,a)} returns a square root of b.
      ++ Argument \spad{a} is a square root of b \spad{(mod p)}.

    root : (SUP I,I) -> %
```

```
++ \spad{root(f,a)} returns a root of the polynomial \spad{f}.
++ Argument \spad{a} must be a root of \spad{f} \spad{(mod p)}.
```

———-

— PADICCT.dotabb —

```
"PADICCT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PADICCT"];
"PADICCT" -> "CHARZ"
"PADICCT" -> "EUCDOM"
```

———-

— PADICCT.dotfull —

```
"PAdicIntegerCategory(a:Integer)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PADICCT"];
"PAdicIntegerCategory(a:Integer)" -> "CharacteristicZero()"
"PAdicIntegerCategory(a:Integer)" -> "EuclideanDomain()"
```

———-

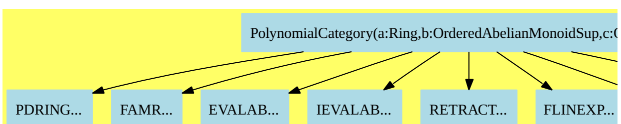
— PADICCT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"PAdicIntegerCategory(a:Integer)" [color=lightblue];
"PAdicIntegerCategory(a:Integer)" -> "CharacteristicZero()"
"PAdicIntegerCategory(a:Integer)" -> "EuclideanDomain()"

"CharacteristicZero()" [color=lightblue];
"CharacteristicZero()" -> "RING..."

"EuclideanDomain()" [color=lightblue];
"EuclideanDomain()" -> "PrincipalIdealDomain()"

"PrincipalIdealDomain()" [color=lightblue];
"PrincipalIdealDomain()" -> "GcdDomain()"

"GcdDomain()" [color=lightblue];
"GcdDomain()" -> "IntegralDomain()"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"
```

```
"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" -> "BMODULE..."

"RING..." [color=lightblue];
"BMODULE..." [color=lightblue];
}
```

———-

16.0.213 PolynomialCategory (POLYCAT)



— PolynomialCategory.input —

```
)set break resume
)sys rm -f PolynomialCategory.output
)spool PolynomialCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PolynomialCategory
--R
--R PolynomialCategory(R: Ring,E: OrderedAbelianMonoidSup,VarSet: OrderedSet) is a category constructor
--R Abbreviation for PolynomialCategory is POLYCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for POLYCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
```

```
--R -? : % -> %                         ?/? : (%,R) -> % if R has FIELD
--R ?=? : (%,%) -> Boolean              D : (%,List(VarSet)) -> %
--R D : (%,VarSet) -> %                 1 : () -> %
--R 0 : () -> %                         ?^? : (%,NonNegativeInteger) -> %
--R ?^? : (%,PositiveInteger) -> %      coefficient : (%,E) -> R
--R coefficients : % -> List(R)         coerce : % -> % if R has INTDOM
--R coerce : VarSet -> %                coerce : R -> %
--R coerce : Integer -> %               coerce : % -> OutputForm
--R content : % -> R if R has GCDDOM    degree : % -> E
--R differentiate : (%,VarSet) -> %     eval : (%,VarSet,%) -> %
--R eval : (%,VarSet,R) -> %            eval : (%,List(%),List(%)) -> %
--R eval : (%,%,%) -> %                 eval : (%,Equation(%)) -> %
--R eval : (%,List(Equation(%))) -> %   gcd : (%,%) -> % if R has GCDDOM
--R gcd : List(%) -> % if R has GCDDOM  ground : % -> R
--R ground? : % -> Boolean              hash : % -> SingleInteger
--R latex : % -> String                 lcm : (%,%) -> % if R has GCDDOM
--R lcm : List(%) -> % if R has GCDDOM  leadingCoefficient : % -> R
--R leadingMonomial : % -> %            map : ((R -> R),%) -> %
--R mapExponents : ((E -> E),%) -> %    max : (%,%) -> % if R has ORDSET
--R min : (%,%) -> % if R has ORDSET    minimumDegree : % -> E
--R monomial : (R,E) -> %               monomial? : % -> Boolean
--R monomials : % -> List(%)            one? : % -> Boolean
--R pomopo! : (%,R,E,%) -> %            primitiveMonomials : % -> List(%)
--R recip : % -> Union(%,"failed")      reductum : % -> %
--R retract : % -> VarSet               retract : % -> R
--R sample : () -> %                    variables : % -> List(VarSet)
--R zero? : % -> Boolean                ?~=? : (%,%) -> Boolean
--R ?*? : (Fraction(Integer),%) -> % if R has ALGEBRA(FRAC(INT))
--R ?*? : (%,Fraction(Integer)) -> % if R has ALGEBRA(FRAC(INT))
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R D : (%,List(VarSet),List(NonNegativeInteger)) -> %
--R D : (%,VarSet,NonNegativeInteger) -> %
--R associates? : (%,%) -> Boolean if R has INTDOM
--R binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if and(has($,CharacteristicNonZero),has(R,PolynomialFactorizationExplici
--R coefficient : (%,List(VarSet),List(NonNegativeInteger)) -> %
--R coefficient : (%,VarSet,NonNegativeInteger) -> %
--R coerce : Fraction(Integer) -> % if R has RETRACT(FRAC(INT)) or R has ALGEBRA(FRAC(INT))
--R conditionP : Matrix(%) -> Union(Vector(%),"failed") if and(has($,CharacteristicNonZero),has(R,PolynomialFact
--R content : (%,VarSet) -> % if R has GCDDOM
--R convert : % -> InputForm if VarSet has KONVERT(INFORM) and R has KONVERT(INFORM)
--R convert : % -> Pattern(Integer) if VarSet has KONVERT(PATTERN(INT)) and R has KONVERT(PATTERN(INT))
--R convert : % -> Pattern(Float) if VarSet has KONVERT(PATTERN(FLOAT)) and R has KONVERT(PATTERN(FLOAT))
--R degree : (%,List(VarSet)) -> List(NonNegativeInteger)
--R degree : (%,VarSet) -> NonNegativeInteger
--R differentiate : (%,List(VarSet),List(NonNegativeInteger)) -> %
--R differentiate : (%,VarSet,NonNegativeInteger) -> %
--R differentiate : (%,List(VarSet)) -> %
--R discriminant : (%,VarSet) -> % if R has COMRING
--R eval : (%,List(VarSet),List(%)) -> %
```

```
--R eval : (%,List(VarSet),List(R)) -> %
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R factor : % -> Factored(%) if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial(%) -> Factored(SparseUnivariatePolynomial(%)) if R has PFECAT
--R factorSquareFreePolynomial : SparseUnivariatePolynomial(%) -> Factored(SparseUnivariatePolynomial(%)) if R h
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R isExpt : % -> Union(Record(var: VarSet,exponent: NonNegativeInteger),"failed")
--R isPlus : % -> Union(List(%),"failed")
--R isTimes : % -> Union(List(%),"failed")
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %) if R has GCDDOM
--R mainVariable : % -> Union(VarSet,"failed")
--R minimumDegree : (%,List(VarSet)) -> List(NonNegativeInteger)
--R minimumDegree : (%,VarSet) -> NonNegativeInteger
--R monicDivide : (%,%,VarSet) -> Record(quotient: %,remainder: %)
--R monomial : (%,List(VarSet),List(NonNegativeInteger)) -> %
--R monomial : (%,VarSet,NonNegativeInteger) -> %
--R multivariate : (SparseUnivariatePolynomial(%),VarSet) -> %
--R multivariate : (SparseUnivariatePolynomial(R),VarSet) -> %
--R numberOfMonomials : % -> NonNegativeInteger
--R patternMatch : (%,Pattern(Integer),PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%) if VarSet
--R patternMatch : (%,Pattern(Float),PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%) if VarSet has P
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (%,VarSet) -> % if R has GCDDOM
--R primitivePart : % -> % if R has GCDDOM
--R reducedSystem : Matrix(%) -> Matrix(R)
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(R),vec: Vector(R))
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(Integer),vec: Vector(Integer)) if R has LINEXP(I
--R reducedSystem : Matrix(%) -> Matrix(Integer) if R has LINEXP(INT)
--R resultant : (%,%,VarSet) -> % if R has COMRING
--R retract : % -> Integer if R has RETRACT(INT)
--R retract : % -> Fraction(Integer) if R has RETRACT(FRAC(INT))
--R retractIfCan : % -> Union(VarSet,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT(INT)
--R retractIfCan : % -> Union(Fraction(Integer),"failed") if R has RETRACT(FRAC(INT))
--R retractIfCan : % -> Union(R,"failed")
--R solveLinearPolynomialEquation : (List(SparseUnivariatePolynomial(%)),SparseUnivariatePolynomial(%)) -> Union
--R squareFree : % -> Factored(%) if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial(%) -> Factored(SparseUnivariatePolynomial(%)) if R has PFE
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R totalDegree : (%,List(VarSet)) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial(R)
--R univariate : (%,VarSet) -> SparseUnivariatePolynomial(%)
--R
--E 1

)spool
)lisp (bye)
```

———-

— **PolynomialCategory.help** —

```
====================================================================
PolynomialCategory examples
====================================================================

The category for general multi-variate polynomials over a ring R,
in variables from VarSet, with exponents from the OrderedAbelianMonoidSup.

See Also:
o )show PolynomialCategory
```

———-

See:

⇒ "DifferentialPolynomialCategory" (DPOLCAT) 17.0.216 on page 1512
⇒ "RecursivePolynomialCategory" (RPOLCAT) 17.0.225 on page 1630
⇒ "UnivariatePolynomialCategory" (UPOLYC) 17.0.228 on page 1708

⇐ "InnerEvalable" (IEVALAB) 2.0.22 on page 64
⇐ "Evalable" (EVALAB) 3.0.48 on page 151
⇐ "FiniteAbelianMonoidRing" (FAMR) 14.0.199 on page 1331
⇐ "GcdDomain" (GCDDOM) 13.0.197 on page 1319
⇐ "FullyLinearlyExplicitRingOver" (FLINEXP) 11.0.178 on page 1134
⇐ "OrderedSet" (ORDSET) 4.0.80 on page 326
⇐ "PartialDifferentialRing" (PDRING) 10.0.169 on page 1061
⇐ "PolynomialFactorizationExplicit" (PFECAT) 15.0.207 on page 1393
⇐ "RetractableTo" (RETRACT) 2.0.37 on page 110

Exports:

0	1	associates?
binomThmExpt	characteristic	charthRoot
coefficient	coefficients	coerce
conditionP	content	convert
D	degree	differentiate
discriminant	eval	exquo
factor	factorPolynomial	factorSquareFreePolynomial
gcd	gcdPolynomial	ground
ground?	hash	isExpt
isPlus	isTimes	latex
lcm	leadingCoefficient	leadingMonomial
mainVariable	map	mapExponents
max	min	minimumDegree
monicDivide	monomial	monomial?
monomials	multivariate	numberOfMonomials
one?	patternMatch	pomopo!
prime?	primitiveMonomials	primitivePart
recip	reducedSystem	reductum
resultant	retract	retractIfCan
sample	solveLinearPolynomialEquation	squareFree
squareFreePart	squareFreePolynomial	subtractIfCan
totalDegree	unit?	unitCanonical
unitNormal	univariate	variables
zero?	?*?	?**?
?+?	?-?	-?
?=?	?^?	?~=?
?/?	?<?	?<=?
?>?	?>=?	

Attributes Exported:

-
- if R has canonicalUnitNormal then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if $ has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x*y \neq 0$ implies both x and y are non-zero.
- if $ has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation $" * " : (D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
degree : (%,VarSet) -> NonNegativeInteger
degree : (%,List VarSet) -> List NonNegativeInteger
mainVariable : % -> Union(VarSet,"failed")
```

```
minimumDegree : (%,List VarSet) -> List NonNegativeInteger
minimumDegree : (%,VarSet) -> NonNegativeInteger
monomial : (%,VarSet,NonNegativeInteger) -> %
multivariate : (SparseUnivariatePolynomial %,VarSet) -> %
multivariate : (SparseUnivariatePolynomial R,VarSet) -> %
univariate : (%,VarSet) -> SparseUnivariatePolynomial %
univariate : % -> SparseUnivariatePolynomial R
variables : % -> List VarSet
```

These are implemented by this category:

```
charthRoot : % -> Union(%,"failed")
 if
   and(has($,CharacteristicNonZero),
       has(R,PolynomialFactorizationExplicit))
   or R has CHARNZ
coefficient : (%,VarSet,NonNegativeInteger) -> %
coefficient : (%,List VarSet,List NonNegativeInteger) -> %
conditionP : Matrix % -> Union(Vector %,"failed")
 if
   and(has($,CharacteristicNonZero),
       has(R,PolynomialFactorizationExplicit))
content : (%,VarSet) -> % if R has GCDDOM
convert : % -> Pattern Integer
    if VarSet has KONVERT PATTERN INT
    and R has KONVERT PATTERN INT
convert : % -> Pattern Float
    if VarSet has KONVERT PATTERN FLOAT
    and R has KONVERT PATTERN FLOAT
convert : % -> InputForm
    if VarSet has KONVERT INFORM
    and R has KONVERT INFORM
discriminant : (%,VarSet) -> % if R has COMRING
eval : (%,List Equation %) -> %
factor : % -> Factored % if R has PFECAT
factorPolynomial :
   SparseUnivariatePolynomial % ->
      Factored SparseUnivariatePolynomial %
        if R has PFECAT
factorSquareFreePolynomial :
   SparseUnivariatePolynomial % ->
      Factored SparseUnivariatePolynomial %
        if R has PFECAT
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                    SparseUnivariatePolynomial %
    if R has GCDDOM
isExpt : % ->
  Union(Record(var: VarSet,exponent: NonNegativeInteger),"failed")
isPlus : % -> Union(List %,"failed")
isTimes : % -> Union(List %,"failed")
monicDivide : (%,%,VarSet) -> Record(quotient: %,remainder: %)
monomial : (%,List VarSet,List NonNegativeInteger) -> %
monomials : % -> List %
patternMatch :
```

```
  (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
    PatternMatchResult(Integer,%)
      if VarSet has PATMAB INT
      and R has PATMAB INT
patternMatch :
   (%,Pattern Float,PatternMatchResult(Float,%)) ->
     PatternMatchResult(Float,%)
       if VarSet has PATMAB FLOAT
       and R has PATMAB FLOAT
primitiveMonomials : % -> List %
primitivePart : % -> % if R has GCDDOM
primitivePart : (%,VarSet) -> % if R has GCDDOM
reducedSystem : Matrix % -> Matrix R
reducedSystem : (Matrix %,Vector %) ->
   Record(mat: Matrix R,vec: Vector R)
resultant : (%,%,VarSet) -> % if R has COMRING
retract : % -> VarSet
retractIfCan : % -> Union(VarSet,"failed")
solveLinearPolynomialEquation :
   (List SparseUnivariatePolynomial %,
    SparseUnivariatePolynomial %) ->
       Union(List SparseUnivariatePolynomial %,"failed")
         if R has PFECAT
squareFree : % -> Factored % if R has GCDDOM
squareFreePart : % -> % if R has GCDDOM
totalDegree : % -> NonNegativeInteger
totalDegree : (%,List VarSet) -> NonNegativeInteger
?<? : (%,%) -> Boolean if R has ORDSET
```

These exports come from (p1061) PartialDifferentialRing(VarSet) where VarSet:OrderedSet:

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
D : (%,List VarSet) -> %
D : (%,VarSet) -> %
D : (%,List VarSet,List NonNegativeInteger) -> %
D : (%,VarSet,NonNegativeInteger) -> %
differentiate : (%,VarSet) -> %
differentiate : (%,List VarSet,List NonNegativeInteger) -> %
differentiate : (%,VarSet,NonNegativeInteger) -> %
differentiate : (%,List VarSet) -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

```
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
```

These exports come from (p1331) FiniteAbelianMonoidRing(R,E)
where R:Ring and E:OrderedAbelianMonoidSup:

```
associates? : (%,%) -> Boolean if R has INTDOM
binomThmExpt : (%,%,NonNegativeInteger) -> %
    if R has COMRING
coefficient : (%,E) -> R
coefficients : % -> List R
coerce : R -> %
coerce : Fraction Integer -> %
    if R has RETRACT FRAC INT
    or R has ALGEBRA FRAC INT
coerce : % -> % if R has INTDOM
content : % -> R if R has GCDDOM
degree : % -> E
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
exquo : (%,%) -> Union(%,"failed") if R has INTDOM
ground : % -> R
ground? : % -> Boolean
leadingCoefficient : % -> R
leadingMonomial : % -> %
map : ((R -> R),%) -> %
mapExponents : ((E -> E),%) -> %
minimumDegree : % -> E
monomial : (R,E) -> %
monomial? : % -> Boolean
numberOfMonomials : % -> NonNegativeInteger
pomopo! : (%,R,E,%) -> %
reductum : % -> %
retract : % -> Integer if R has RETRACT INT
retract : % -> Fraction Integer
    if R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed")
    if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
    if R has RETRACT FRAC INT
unit? : % -> Boolean if R has INTDOM
unitCanonical : % -> % if R has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if R has INTDOM
?*? : (%,R) -> %
?*? : (R,%) -> %
?*? : (Fraction Integer,%) -> %
    if R has ALGEBRA FRAC INT
```

```
?*? : (%,Fraction Integer) -> %
    if R has ALGEBRA FRAC INT
?/? : (%,R) -> % if R has FIELD
```

These exports come from (p151) Evalable(PolynomialCategory(...)):

```
eval : (%,Equation %) -> %
eval : (%,List %,List %) -> %
eval : (%,%,%) -> %
```

These exports come from (p64) InnerEvalable(VarSet,R)
where VarSet:OrderedSet and R:Ring

```
eval : (%,VarSet,R) -> %
eval : (%,List VarSet,List R) -> %
```

These exports come from (p64) InnerEvalable(VarSet,R)
where VarSet:OrderedSet and R:PolynomialCategory(...):

```
eval : (%,VarSet,%) -> %
eval : (%,List VarSet,List %) -> %
```

These exports come from (p110) RetractableTo(VarSet)
where VarSet:OrderedSet:

```
coerce : VarSet -> %
retract : % -> R
retractIfCan : % -> Union(R,"failed")
```

These exports come from (p1134) FullyLinearlyExplicitRingOver(R)
where R:Ring:

```
reducedSystem : (Matrix %,Vector %) ->
   Record(mat: Matrix Integer,vec: Vector Integer)
      if R has LINEXP INT
reducedSystem : Matrix % -> Matrix Integer
      if R has LINEXP INT
```

These exports come from (p326) OrderedSet():

```
max : (%,%) -> % if R has ORDSET
min : (%,%) -> % if R has ORDSET
?<=? : (%,%) -> Boolean if R has ORDSET
?>? : (%,%) -> Boolean if R has ORDSET
?>=? : (%,%) -> Boolean if R has ORDSET
```

These exports come from (p1319) GcdDomain():

```
gcd : (%,%) -> % if R has GCDDOM
gcd : List % -> % if R has GCDDOM
lcm : (%,%) -> % if R has GCDDOM
lcm : List % -> % if R has GCDDOM
```

These exports come from (p1393) PolynomialFactorizationExplicit():

```
prime? : % -> Boolean if R has PFECAT
squareFreePolynomial : SparseUnivariatePolynomial % ->
   Factored SparseUnivariatePolynomial %
     if R has PFECAT
```

— **PolynomialCategory.html** —

```
<body>
```

```
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#POLYCAT">
PolynomialCategory (POLYCAT)</a></h2>
</body>
```

— category POLYCAT PolynomialCategory —

```
)abbrev category POLYCAT PolynomialCategory
++ Description:
++ The category for general multi-variate polynomials over a ring
++ R, in variables from VarSet, with exponents from the
++ \spadtype{OrderedAbelianMonoidSup}.

PolynomialCategory(R,E,VarSet) : Category == SIG where
  R : Ring
  E : OrderedAbelianMonoidSup
  VarSet : OrderedSet

  PDR   ==> PartialDifferentialRing(VarSet)
  FAMR  ==> FiniteAbelianMonoidRing(R, E)
  EV    ==> Evalable(%)
  IEVR  ==> InnerEvalable(VarSet, R)
  IEVP  ==> InnerEvalable(VarSet, %)
  RT    ==> RetractableTo(VarSet)
  FLERO ==> FullyLinearlyExplicitRingOver(R)

  SIG ==> Join(PDR,FAMR,EV,IEVR,IEVP,RT,FLERO) with

    degree : (%,VarSet) -> NonNegativeInteger
      ++ degree(p,v) gives the degree of polynomial p with respect
      ++ to the variable v.

    degree : (%,List(VarSet)) -> List(NonNegativeInteger)
      ++ degree(p,lv) gives the list of degrees of polynomial p
      ++ with respect to each of the variables in the list lv.

    coefficient : (%,VarSet,NonNegativeInteger) -> %
      ++ coefficient(p,v,n) views the polynomial p as a univariate
      ++ polynomial in v and returns the coefficient of the \spad{v**n} term.

    coefficient : (%,List VarSet,List NonNegativeInteger) -> %
      ++ coefficient(p, lv, ln) views the polynomial p as a polynomial
      ++ in the variables of lv and returns the coefficient of the term
      ++ \spad{lv**ln}, \spad{prod(lv_i ** ln_i)}.

    monomials : % -> List %
      ++ monomials(p) returns the list of non-zero monomials of
      ++ polynomial p,
      ++ \spad{monomials(sum(a_(i) X^(i))) = [a_(1) X^(1),...,a_(n) X^(n)]}.

    univariate : (%,VarSet) -> SparseUnivariatePolynomial(%)
      ++ univariate(p,v) converts the multivariate polynomial p
```

```
  ++ into a univariate polynomial in v, whose coefficients are still
  ++ multivariate polynomials (in all the other variables).

univariate : % -> SparseUnivariatePolynomial(R)
  ++ univariate(p) converts the multivariate polynomial p,
  ++ which should actually involve only one variable,
  ++ into a univariate polynomial
  ++ in that variable, whose coefficients are in the ground ring.
  ++ Error: if polynomial is genuinely multivariate

mainVariable : % -> Union(VarSet,"failed")
  ++ mainVariable(p) returns the biggest variable which actually
  ++ occurs in the polynomial p, or "failed" if no variables are
  ++ present.
  ++ fails precisely if polynomial satisfies ground?

minimumDegree : (%,VarSet) -> NonNegativeInteger
  ++ minimumDegree(p,v) gives the minimum degree of polynomial p
  ++ with respect to v, viewed a univariate polynomial in v

minimumDegree : (%,List(VarSet)) -> List(NonNegativeInteger)
  ++ minimumDegree(p, lv) gives the list of minimum degrees of the
  ++ polynomial p with respect to each of the variables in the list lv

monicDivide : (%,%,VarSet) -> Record(quotient:%,remainder:%)
  ++ monicDivide(a,b,v) divides the polynomial a by the polynomial b,
  ++ with each viewed as a univariate polynomial in v returning
  ++ both the quotient and remainder.
  ++ Error: if b is not monic with respect to v.

monomial : (%,VarSet,NonNegativeInteger) -> %
  ++ monomial(a,x,n) creates the monomial \spad{a*x**n} where \spad{a} is
  ++ a polynomial, x is a variable and n is a nonnegative integer.

monomial : (%,List VarSet,List NonNegativeInteger) -> %
  ++ monomial(a,[v1..vn],[e1..en]) returns \spad{a*prod(vi**ei)}.

multivariate : (SparseUnivariatePolynomial(R),VarSet) -> %
  ++ multivariate(sup,v) converts an anonymous univariable
  ++ polynomial sup to a polynomial in the variable v.

multivariate : (SparseUnivariatePolynomial(%),VarSet) -> %
  ++ multivariate(sup,v) converts an anonymous univariable
  ++ polynomial sup to a polynomial in the variable v.

isPlus : % -> Union(List %, "failed")
  ++ isPlus(p) returns \spad{[m1,...,mn]} if polynomial
  ++ \spad{p = m1 + ... + mn} and
  ++ \spad{n >= 2} and each mi is a nonzero monomial.

isTimes : % -> Union(List %, "failed")
  ++ isTimes(p) returns \spad{[a1,...,an]} if polynomial
  ++ \spad{p = a1 ... an} and \spad{n >= 2}, and, for each i,
  ++ ai is either a nontrivial constant in R or else of the
```

```
  ++ form \spad{x**e}, where \spad{e > 0} is an integer
  ++ and x in a member of VarSet.

isExpt : % -> Union(Record(var:VarSet, exponent:NonNegativeInteger),_
                   "failed")
  ++ isExpt(p) returns \spad{[x, n]} if polynomial p has the
  ++ form \spad{x**n} and \spad{n > 0}.

totalDegree : % -> NonNegativeInteger
  ++ totalDegree(p) returns the largest sum over all monomials
  ++ of all exponents of a monomial.

totalDegree : (%,List VarSet) -> NonNegativeInteger
  ++ totalDegree(p, lv) returns the maximum sum (over all monomials
  ++ of polynomial p) of the variables in the list lv.

variables : % -> List(VarSet)
  ++ variables(p) returns the list of those variables actually
  ++ appearing in the polynomial p.

primitiveMonomials : % -> List %
  ++ primitiveMonomials(p) gives the list of monomials of the
  ++ polynomial p with their coefficients removed. Note that
  ++ \spad{primitiveMonomials(sum(a_(i) X^(i))) = [X^(1),...,X^(n)]}.

if R has OrderedSet  then OrderedSet

-- OrderedRing view removed to allow EXPR to define abs

--if R has OrderedRing then OrderedRing

if (R has ConvertibleTo InputForm) and
   (VarSet has ConvertibleTo InputForm) then
     ConvertibleTo InputForm

if (R has ConvertibleTo Pattern Integer) and
   (VarSet has ConvertibleTo Pattern Integer) then
     ConvertibleTo Pattern Integer

if (R has ConvertibleTo Pattern Float) and
   (VarSet has ConvertibleTo Pattern Float) then
     ConvertibleTo Pattern Float

if (R has PatternMatchable Integer) and
   (VarSet has PatternMatchable Integer) then
     PatternMatchable Integer

if (R has PatternMatchable Float) and
   (VarSet has PatternMatchable Float) then
     PatternMatchable Float

if R has CommutativeRing then

  resultant : (%,%,VarSet) -> %
```

```
      ++ resultant(p,q,v) returns the resultant of the polynomials
      ++ p and q with respect to the variable v.

   discriminant : (%,VarSet) -> %
      ++ discriminant(p,v) returns the disriminant of the polynomial p
      ++ with respect to the variable v.

 if R has GcdDomain then

   GcdDomain

   content : (%,VarSet) -> %
     ++ content(p,v) is the gcd of the coefficients of the polynomial p
     ++ when p is viewed as a univariate polynomial with respect to the
     ++ variable v.
     ++ Thus, for polynomial 7*x**2*y + 14*x*y**2, the gcd of the
     ++ coefficients with respect to x is 7*y.

   primitivePart : % -> %
     ++ primitivePart(p) returns the unitCanonical associate of the
     ++ polynomial p with its content divided out.

   primitivePart : (%,VarSet) -> %
     ++ primitivePart(p,v) returns the unitCanonical associate of the
     ++ polynomial p with its content with respect to the variable v
     ++ divided out.

   squareFree : % -> Factored %
     ++ squareFree(p) returns the square free factorization of the
     ++ polynomial p.

   squareFreePart : % -> %
     ++ squareFreePart(p) returns product of all the irreducible factors
     ++ of polynomial p each taken with multiplicity one.

 -- assertions

 if R has canonicalUnitNormal then canonicalUnitNormal
          ++ we can choose a unique representative for each
          ++ associate class.
          ++ This normalization is chosen to be normalization of
          ++ leading coefficient (by default).

 if R has PolynomialFactorizationExplicit then

    PolynomialFactorizationExplicit

add

  p:%
  v:VarSet
  ln:List NonNegativeInteger
  lv:List VarSet
  n:NonNegativeInteger
```

```
pp,qq:SparseUnivariatePolynomial %

eval(p:%, l:List Equation %) ==
  empty? l => p
  for e in l repeat
    retractIfCan(lhs e)@Union(VarSet,"failed") case "failed" =>
         error "cannot find a variable to evaluate"
  lvar:=[retract(lhs e)@VarSet for e in l]
  eval(p, lvar,[rhs e for e in l]$List(%))

monomials p ==
  ml:= empty$List(%)
  while p ^= 0 repeat
    ml:=concat(leadingMonomial p, ml)
    p:= reductum p
  reverse ml

isPlus p ==
  empty? rest(l := monomials p) => "failed"
  l

isTimes p ==
  empty?(lv := variables p) or not monomial? p => "failed"
  l := [monomial(1, v, degree(p, v)) for v in lv]
  ((r := leadingCoefficient p) = 1) =>
    empty? rest lv => "failed"
    l
  concat(r::%, l)

isExpt p ==
  (u := mainVariable p) case "failed" => "failed"
  p = monomial(1, u::VarSet, d := degree(p, u::VarSet)) =>
    [u::VarSet, d]
  "failed"

coefficient(p,v,n) == coefficient(univariate(p,v),n)

coefficient(p,lv,ln) ==
   empty? lv =>
     empty? ln => p
     error "mismatched lists in coefficient"
   empty? ln  => error "mismatched lists in coefficient"
   coefficient(coefficient(univariate(p,first lv),first ln),
               rest lv,rest ln)

monomial(p,lv,ln) ==
   empty? lv =>
     empty? ln => p
     error "mismatched lists in monomial"
   empty? ln  => error "mismatched lists in monomial"
   monomial(monomial(p,first lv, first ln),rest lv, rest ln)

retract(p:%):VarSet ==
  q := mainVariable(p)::VarSet
```

```
  q::% = p => q
  error "Polynomial is not a single variable"

retractIfCan(p:%):Union(VarSet, "failed") ==
  ((q := mainVariable p) case VarSet) and (q::VarSet::% = p) => q
  "failed"

mkPrim(p:%):% == monomial(1,degree p)

primitiveMonomials p == [mkPrim q for q in monomials p]

totalDegree p ==
    ground? p => 0
    u := univariate(p, mainVariable(p)::VarSet)
    d: NonNegativeInteger := 0
    while u ^= 0 repeat
      d := max(d, degree u + totalDegree leadingCoefficient u)
      u := reductum u
    d

totalDegree(p,lv) ==
    ground? p => 0
    u := univariate(p, v:=(mainVariable(p)::VarSet))
    d: NonNegativeInteger := 0
    w: NonNegativeInteger := 0
    if member?(v, lv) then w:=1
    while u ^= 0 repeat
      d := max(d, w*(degree u) + totalDegree(leadingCoefficient u,lv))
      u := reductum u
    d

if R has CommutativeRing then

    resultant(p1,p2,mvar) ==
      resultant(univariate(p1,mvar),univariate(p2,mvar))

    discriminant(p,var) ==
      discriminant(univariate(p,var))

if R has IntegralDomain then

  allMonoms(l:List %):List(%) ==
    removeDuplicates_! concat [primitiveMonomials p for p in l]

  P2R(p:%, b:List E, n:NonNegativeInteger):Vector(R) ==
    w := new(n, 0)$Vector(R)
    for i in minIndex w .. maxIndex w for bj in b repeat
      qsetelt_!(w, i, coefficient(p, bj))
    w

  eq2R(l:List %, b:List E):Matrix(R) ==
    matrix [[coefficient(p, bj) for p in l] for bj in b]

  reducedSystem(m:Matrix %):Matrix(R) ==
```

```
    l := listOfLists m
    b := removeDuplicates_!
                      concat [allMonoms r for r in l]$List(List(%))
    d := [degree bj for bj in b]
    mm := eq2R(first l, d)
    l := rest l
    while not empty? l repeat
      mm := vertConcat(mm, eq2R(first l, d))
      l := rest l
    mm

  reducedSystem(m:Matrix %, v:Vector %):
   Record(mat:Matrix R, vec:Vector R) ==
    l := listOfLists m
    r := entries v
    b : List % := removeDuplicates_! concat(allMonoms r,
                     concat [allMonoms s for s in l]$List(List(%)))
    d := [degree bj for bj in b]
    n := #d
    mm := eq2R(first l, d)
    w := P2R(first r, d, n)
    l := rest l
    r := rest r
    while not empty? l repeat
      mm := vertConcat(mm, eq2R(first l, d))
      w := concat(w, P2R(first r, d, n))
      l := rest l
      r := rest r
    [mm, w]

 if R has PolynomialFactorizationExplicit then
  -- we might be in trouble if its actually only
  -- a univariate polynomial category - have to remember to
  -- over-ride these in UnivariatePolynomialCategory

  PFBR ==>PolynomialFactorizationByRecursion(R,E,VarSet,%)

  gcdPolynomial(pp,qq) ==
     gcdPolynomial(pp,qq)$GeneralPolynomialGcdPackage(E,VarSet,R,%)

  solveLinearPolynomialEquation(lpp,pp) ==
    solveLinearPolynomialEquationByRecursion(lpp,pp)$PFBR

  factorPolynomial(pp) ==
    factorByRecursion(pp)$PFBR

  factorSquareFreePolynomial(pp) ==
    factorSquareFreeByRecursion(pp)$PFBR

  factor p ==
    v:Union(VarSet,"failed"):=mainVariable p
    v case "failed" =>
      ansR:=factor leadingCoefficient p
      makeFR(unit(ansR)::%,
```

```
            [[w.flg,w.fctr::%,w.xpnt] for w in factorList ansR])
    up:SparseUnivariatePolynomial %:=univariate(p,v)
    ansSUP:=factorByRecursion(up)$PFBR
    makeFR(multivariate(unit(ansSUP),v),
           [[ww.flg,multivariate(ww.fctr,v),ww.xpnt]
            for ww in factorList ansSUP])

  if R has CharacteristicNonZero then

     mat: Matrix %

     conditionP mat ==
       ll:=listOfLists transpose mat  --hence each list corresponds to a
                                      --column, to one variable
       llR:List List R := [ empty() for z in first ll]
       monslist:List List % := empty()
       ch:=characteristic()$%
       for l in ll repeat
           mons:= "setUnion"/[primitiveMonomials u for u in l]
           redmons:List % :=[]
           for m in mons repeat
               vars:=variables m
               degs:=degree(m,vars)
               deg1:List NonNegativeInteger
               deg1:=[ ((nd:=d:Integer exquo ch:Integer)
                          case "failed" => return "failed" ;
                           nd::Integer::NonNegativeInteger)
                      for d in degs ]
               redmons:=[monomial(1,vars,deg1),:redmons]
               llR:=[[ground coefficient(u,vars,degs),:v]_
                       for u in l for v in llR]
           monslist:=[redmons,:monslist]
       ans:=conditionP transpose matrix llR
       ans case "failed" => "failed"
       i:NonNegativeInteger:=0
       [ +/[m*(ans.(i:=i+1))::% for m in mons ]
         for mons in monslist]

if R has CharacteristicNonZero then

     charthRootlv:(%,List VarSet,NonNegativeInteger) ->
                                          Union(%,"failed")
     charthRoot p ==
       vars:= variables p
       empty? vars =>
         ans := charthRoot ground p
         ans case "failed" => "failed"
         ans::R::%
       ch:=characteristic()$%
       charthRootlv(p,vars,ch)

     charthRootlv(p,vars,ch) ==
       empty? vars =>
         ans := charthRoot ground p
```

```
          ans case "failed" => "failed"
          ans::R::%
        v:=first vars
        vars:=rest vars
        d:=degree(p,v)
        ans:% := 0
        while (d>0) repeat
           (dd:=(d::Integer exquo ch::Integer)) case "failed" =>
                  return "failed"
           cp:=coefficient(p,v,d)
           p:=p-monomial(cp,v,d)
           ansx:=charthRootlv(cp,vars,ch)
           ansx case "failed" => return "failed"
           d:=degree(p,v)
           ans:=ans+monomial(ansx,v,dd::Integer::NonNegativeInteger)
        ansx:=charthRootlv(p,vars,ch)
        ansx case "failed" => return "failed"
        return ans+ansx

monicDivide(p1,p2,mvar) ==
  result:=monicDivide(univariate(p1,mvar),univariate(p2,mvar))
  [multivariate(result.quotient,mvar),
   multivariate(result.remainder,mvar)]

if R has GcdDomain then

  if R has EuclideanDomain and R has CharacteristicZero then

    squareFree p == squareFree(p)$MultivariateSquareFree(E,VarSet,R,%)

  else

    squareFree p == squareFree(p)$PolynomialSquareFree(VarSet,E,R,%)

  squareFreePart p ==
    unit(s := squareFree p) * */[f.factor for f in factors s]

  content(p,v) == content univariate(p,v)

  primitivePart p ==
    zero? p => p
    unitNormal((p exquo content p) ::%).canonical

  primitivePart(p,v) ==
    zero? p => p
    unitNormal((p exquo content(p,v)) ::%).canonical

if R has OrderedSet then

  p:% < q:% ==
    (dp:= degree p) < (dq := degree q) => (leadingCoefficient q) > 0
    dq < dp => (leadingCoefficient p) < 0
    leadingCoefficient(p - q) < 0
```

```
  if (R has PatternMatchable Integer) and
     (VarSet has PatternMatchable Integer) then

       patternMatch(p:%, pat:Pattern Integer,
        l:PatternMatchResult(Integer, %)) ==
          patternMatch(p, pat,
            l)$PatternMatchPolynomialCategory(Integer,E,VarSet,R,%)

  if (R has PatternMatchable Float) and
     (VarSet has PatternMatchable Float) then

       patternMatch(p:%, pat:Pattern Float,
        l:PatternMatchResult(Float, %)) ==
          patternMatch(p, pat,
            l)$PatternMatchPolynomialCategory(Float,E,VarSet,R,%)

if (R has ConvertibleTo Pattern Integer) and
   (VarSet has ConvertibleTo Pattern Integer) then

     convert(x:%):Pattern(Integer) ==
       map(convert, convert,
          x)$PolynomialCategoryLifting(E,VarSet,R,%,Pattern Integer)

if (R has ConvertibleTo Pattern Float) and
   (VarSet has ConvertibleTo Pattern Float) then

     convert(x:%):Pattern(Float) ==
       map(convert, convert,
        x)$PolynomialCategoryLifting(E, VarSet, R, %, Pattern Float)

if (R has ConvertibleTo InputForm) and
   (VarSet has ConvertibleTo InputForm) then

     convert(p:%):InputForm ==
       map(convert, convert,
                p)$PolynomialCategoryLifting(E,VarSet,R,%,InputForm)
```

— **COQ POLYCAT** —

```
(* category POLYCAT *)
(*
    p:%
    v:VarSet
    ln:List NonNegativeInteger
    lv:List VarSet
    n:NonNegativeInteger
    pp,qq:SparseUnivariatePolynomial %

    eval : (%,List(Equation(%))) -> %
    eval(p:%, l:List Equation %) ==
      empty? l => p
```

```
    for e in l repeat
      retractIfCan(lhs e)@Union(VarSet,"failed") case "failed" =>
           error "cannot find a variable to evaluate"
    lvar:=[retract(lhs e)@VarSet for e in l]
    eval(p, lvar,[rhs e for e in l]$List(%))

  monomials : % -> List(%)
  monomials p ==
    ml:= empty$List(%)
    while p ^= 0 repeat
      ml:=concat(leadingMonomial p, ml)
      p:= reductum p
    reverse ml

  isPlus : % -> Union(List(%),"failed")
  isPlus p ==
    empty? rest(l := monomials p) => "failed"
    l

  isTimes : % -> Union(List(%),"failed")
  isTimes p ==
    empty?(lv := variables p) or not monomial? p => "failed"
    l := [monomial(1, v, degree(p, v)) for v in lv]
    ((r := leadingCoefficient p) = 1) =>
      empty? rest lv => "failed"
      l
    concat(r::%, l)

  isExpt : % ->
     Union(Record(var: VarSet,exponent: NonNegativeInteger),"failed")
  isExpt p ==
    (u := mainVariable p) case "failed" => "failed"
    p = monomial(1, u::VarSet, d := degree(p, u::VarSet)) =>
      [u::VarSet, d]
    "failed"

  coefficient : (%,VarSet,NonNegativeInteger) -> %
  coefficient(p,v,n) == coefficient(univariate(p,v),n)

  coefficient : (%,List(VarSet),List(NonNegativeInteger)) -> %
  coefficient(p,lv,ln) ==
     empty? lv =>
       empty? ln => p
       error "mismatched lists in coefficient"
     empty? ln  => error "mismatched lists in coefficient"
     coefficient(coefficient(univariate(p,first lv),first ln),
                 rest lv,rest ln)

  monomial : (%,List(VarSet),List(NonNegativeInteger)) -> %
  monomial(p,lv,ln) ==
     empty? lv =>
       empty? ln => p
       error "mismatched lists in monomial"
     empty? ln  => error "mismatched lists in monomial"
```

```
   monomial(monomial(p,first lv, first ln),rest lv, rest ln)

retract : % -> VarSet
retract(p:%):VarSet ==
  q := mainVariable(p)::VarSet
  q::% = p => q
  error "Polynomial is not a single variable"

retractIfCan : % -> Union(VarSet,"failed")
retractIfCan(p:%):Union(VarSet, "failed") ==
  ((q := mainVariable p) case VarSet) and (q::VarSet::% = p) => q
  "failed"

mkPrim: % -> %
mkPrim(p:%):% == monomial(1,degree p)

primitiveMonomials : % -> List(%)
primitiveMonomials p == [mkPrim q for q in monomials p]

totalDegree : % -> NonNegativeInteger
totalDegree p ==
    ground? p => 0
    u := univariate(p, mainVariable(p)::VarSet)
    d: NonNegativeInteger := 0
    while u ^= 0 repeat
      d := max(d, degree u + totalDegree leadingCoefficient u)
      u := reductum u
    d

totalDegree : (%,List(VarSet)) -> NonNegativeInteger
totalDegree(p,lv) ==
    ground? p => 0
    u := univariate(p, v:=(mainVariable(p)::VarSet))
    d: NonNegativeInteger := 0
    w: NonNegativeInteger := 0
    if member?(v, lv) then w:=1
    while u ^= 0 repeat
      d := max(d, w*(degree u) + totalDegree(leadingCoefficient u,lv))
      u := reductum u
    d

if R has CommutativeRing then

    resultant : (%,%,VarSet) -> % if R has COMRING
    resultant(p1,p2,mvar) ==
      resultant(univariate(p1,mvar),univariate(p2,mvar))

    differentiate : (%,VarSet) -> %
    discriminant(p,var) ==
      discriminant(univariate(p,var))

if R has IntegralDomain then

  allMonoms: List(%) -> List(%)
```

```
    allMonoms(l:List %):List(%) ==
      removeDuplicates_! concat [primitiveMonomials p for p in l]

    P2R: (%, List(E), NonNegativeInteger) -> Vector(R)
    P2R(p:%, b:List E, n:NonNegativeInteger):Vector(R) ==
      w := new(n, 0)$Vector(R)
      for i in minIndex w .. maxIndex w for bj in b repeat
        qsetelt_!(w, i, coefficient(p, bj))
      w

    eq2R: (List(%), List(E)) -> Matrix(R)
    eq2R(l:List %, b:List E):Matrix(R) ==
      matrix [[coefficient(p, bj) for p in l] for bj in b]

    reducedSystem : Matrix(%) -> Matrix(R)
    reducedSystem(m:Matrix %):Matrix(R) ==
      l := listOfLists m
      b := removeDuplicates_!
                         concat [allMonoms r for r in l]$List(List(%))
      d := [degree bj for bj in b]
      mm := eq2R(first l, d)
      l := rest l
      while not empty? l repeat
        mm := vertConcat(mm, eq2R(first l, d))
        l := rest l
      mm

    reducedSystem : (Matrix(%),Vector(%)) ->
     Record(mat: Matrix(R),vec: Vector(R))
    reducedSystem(m:Matrix %, v:Vector %):
     Record(mat:Matrix R, vec:Vector R) ==
      l := listOfLists m
      r := entries v
      b : List % := removeDuplicates_! concat(allMonoms r,
                        concat [allMonoms s for s in l]$List(List(%)))
      d := [degree bj for bj in b]
      n := #d
      mm := eq2R(first l, d)
      w := P2R(first r, d, n)
      l := rest l
      r := rest r
      while not empty? l repeat
        mm := vertConcat(mm, eq2R(first l, d))
        w := concat(w, P2R(first r, d, n))
        l := rest l
        r := rest r
      [mm, w]

  if R has PolynomialFactorizationExplicit then
     -- we might be in trouble if its actually only
     -- a univariate polynomial category - have to remember to
     -- over-ride these in UnivariatePolynomialCategory

     PFBR ==>PolynomialFactorizationByRecursion(R,E,VarSet,%)
```

```
gcdPolynomial : (SparseUnivariatePolynomial(%),
                 SparseUnivariatePolynomial(%)) ->
                    SparseUnivariatePolynomial(%)
gcdPolynomial(pp,qq) ==
   gcdPolynomial(pp,qq)$GeneralPolynomialGcdPackage(E,VarSet,R,%)

solveLinearPolynomialEquation :
 (List(SparseUnivariatePolynomial(%)),SparseUnivariatePolynomial(%)) ->
   Union(List(SparseUnivariatePolynomial(%)),"failed") if R has PFECAT
solveLinearPolynomialEquation(lpp,pp) ==
  solveLinearPolynomialEquationByRecursion(lpp,pp)$PFBR

factorPolynomial : SparseUnivariatePolynomial(%) ->
  Factored(SparseUnivariatePolynomial(%))
factorPolynomial(pp) ==
  factorByRecursion(pp)$PFBR

factorSquareFreePolynomial : SparseUnivariatePolynomial(%) ->
  Factored(SparseUnivariatePolynomial(%))
factorSquareFreePolynomial(pp) ==
  factorSquareFreeByRecursion(pp)$PFBR

factor : % -> Factored(%)
factor p ==
  v:Union(VarSet,"failed"):=mainVariable p
  v case "failed" =>
    ansR:=factor leadingCoefficient p
    makeFR(unit(ansR)::%,
          [[w.flg,w.fctr::%,w.xpnt] for w in factorList ansR])
  up:SparseUnivariatePolynomial %:=univariate(p,v)
  ansSUP:=factorByRecursion(up)$PFBR
  makeFR(multivariate(unit(ansSUP),v),
         [[ww.flg,multivariate(ww.fctr,v),ww.xpnt]
          for ww in factorList ansSUP])

if R has CharacteristicNonZero then

   mat: Matrix %

   conditionP : Matrix(%) -> Union(Vector(%),"failed")
   conditionP mat ==
     ll:=listOfLists transpose mat  --hence each list corresponds to a
                                    --column, to one variable
     llR:List List R := [ empty() for z in first ll]
     monslist:List List % := empty()
     ch:=characteristic()$%
     for l in ll repeat
         mons:= "setUnion"/[primitiveMonomials u for u in l]
         redmons:List % :=[]
         for m in mons repeat
             vars:=variables m
             degs:=degree(m,vars)
             deg1:List NonNegativeInteger
```

```
                deg1:=[ ((nd:=d:Integer exquo ch:Integer)
                           case "failed" => return "failed" ;
                            nd::Integer::NonNegativeInteger)
                       for d in degs ]
                redmons:=[monomial(1,vars,deg1),:redmons]
                llR:=[[ground coefficient(u,vars,degs),:v]_
                        for u in l for v in llR]
            monslist:=[redmons,:monslist]
        ans:=conditionP transpose matrix llR
        ans case "failed" => "failed"
        i:NonNegativeInteger:=0
        [ +/[m*(ans.(i:=i+1))::% for m in mons ]
          for mons in monslist]

if R has CharacteristicNonZero then

      charthRoot : % -> Union(%,"failed")
      charthRoot p ==
        vars:= variables p
        empty? vars =>
          ans := charthRoot ground p
          ans case "failed" => "failed"
          ans::R::%
        ch:=characteristic()$%
        charthRootlv(p,vars,ch)

      charthRootlv:(%,List VarSet,NonNegativeInteger) -> Union(%,"failed")
      charthRootlv(p,vars,ch) ==
        empty? vars =>
          ans := charthRoot ground p
          ans case "failed" => "failed"
          ans::R::%
        v:=first vars
        vars:=rest vars
        d:=degree(p,v)
        ans:% := 0
        while (d>0) repeat
           (dd:=(d::Integer exquo ch::Integer)) case "failed" =>
                  return "failed"
           cp:=coefficient(p,v,d)
           p:=p-monomial(cp,v,d)
           ansx:=charthRootlv(cp,vars,ch)
           ansx case "failed" => return "failed"
           d:=degree(p,v)
           ans:=ans+monomial(ansx,v,dd::Integer::NonNegativeInteger)
        ansx:=charthRootlv(p,vars,ch)
        ansx case "failed" => return "failed"
        return ans+ansx

monicDivide : (%,%,VarSet) -> Record(quotient: %,remainder: %)
monicDivide(p1,p2,mvar) ==
   result:=monicDivide(univariate(p1,mvar),univariate(p2,mvar))
   [multivariate(result.quotient,mvar),
    multivariate(result.remainder,mvar)]
```

```
if R has GcdDomain then

  if R has EuclideanDomain and R has CharacteristicZero then

   squareFree : % -> Factored(%)
   squareFree p == squareFree(p)$MultivariateSquareFree(E,VarSet,R,%)

  else

    squareFree : % -> Factored(%)
    squareFree p == squareFree(p)$PolynomialSquareFree(VarSet,E,R,%)

  squareFreePart : % -> %
  squareFreePart p ==
    unit(s := squareFree p) * */[f.factor for f in factors s]

  content : (%,VarSet) -> %
  content(p,v) == content univariate(p,v)

  primitivePart : % -> %
  primitivePart p ==
    zero? p => p
    unitNormal((p exquo content p) ::%).canonical

  primitivePart : (%,VarSet) -> %
  primitivePart(p,v) ==
    zero? p => p
    unitNormal((p exquo content(p,v)) ::%).canonical

if R has OrderedSet then

  ?<? : (%,%) -> Boolean
  p:% < q:% ==
    (dp:= degree p) < (dq := degree q) => (leadingCoefficient q) > 0
    dq < dp => (leadingCoefficient p) < 0
    leadingCoefficient(p - q) < 0

  if (R has PatternMatchable Integer) and
     (VarSet has PatternMatchable Integer) then

       patternMatch : (%,Pattern(Integer),PatternMatchResult(Integer,%)) ->
           PatternMatchResult(Integer,%)
       patternMatch(p:%, pat:Pattern Integer,
        l:PatternMatchResult(Integer, %)) ==
          patternMatch(p, pat,
            l)$PatternMatchPolynomialCategory(Integer,E,VarSet,R,%)

  if (R has PatternMatchable Float) and
     (VarSet has PatternMatchable Float) then

       patternMatch : (%,Pattern(Float),PatternMatchResult(Float,%)) ->
          PatternMatchResult(Float,%)
       patternMatch(p:%, pat:Pattern Float,
```

```
          l:PatternMatchResult(Float, %)) ==
            patternMatch(p, pat,
              l)$PatternMatchPolynomialCategory(Float,E,VarSet,R,%)

    if (R has ConvertibleTo Pattern Integer) and
       (VarSet has ConvertibleTo Pattern Integer) then

         convert : % -> Pattern(Integer)
         convert(x:%):Pattern(Integer) ==
           map(convert, convert,
              x)$PolynomialCategoryLifting(E,VarSet,R,%,Pattern Integer)

    if (R has ConvertibleTo Pattern Float) and
       (VarSet has ConvertibleTo Pattern Float) then

         convert : % -> Pattern(Float)
         convert(x:%):Pattern(Float) ==
           map(convert, convert,
            x)$PolynomialCategoryLifting(E, VarSet, R, %, Pattern Float)

    if (R has ConvertibleTo InputForm) and
       (VarSet has ConvertibleTo InputForm) then

         convert : % -> InputForm
         convert(p:%):InputForm ==
           map(convert, convert,
                 p)$PolynomialCategoryLifting(E,VarSet,R,%,InputForm)

*)
```

———-

— **POLYCAT.dotabb** —

```
"POLYCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=POLYCAT"];
"POLYCAT" -> "PDRING"
"POLYCAT" -> "FAMR"
"POLYCAT" -> "EVALAB"
"POLYCAT" -> "IEVALAB"
"POLYCAT" -> "RETRACT"
"POLYCAT" -> "FLINEXP"
"POLYCAT" -> "ORDSET"
"POLYCAT" -> "GCDDOM"
"POLYCAT" -> "PFECAT"
```

———-

— **POLYCAT.dotfull** —

```
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=POLYCAT"];
```

```
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "PartialDifferentialRing(a:OrderedSet)"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoidSup)"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "Evalable(PolynomialCategory(...))"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "InnerEvalable(a:OrderedSet,b:Ring)"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "InnerEvalable(a:OrderedSet,b:PolynomialCategory(...))"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "RetractableTo(a:OrderedSet)"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "FullyLinearlyExplicitRingOver(a:Ring)"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "OrderedSet()"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "GcdDomain()"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "PolynomialFactorizationExplicit()"

"PolynomialCategory(a:Ring,b:NonNegativeInteger,c:SingletonAsOrderedSet)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=POLYCAT"];
"PolynomialCategory(a:Ring,b:NonNegativeInteger,c:SingletonAsOrderedSet)"
 -> "PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
```

———-

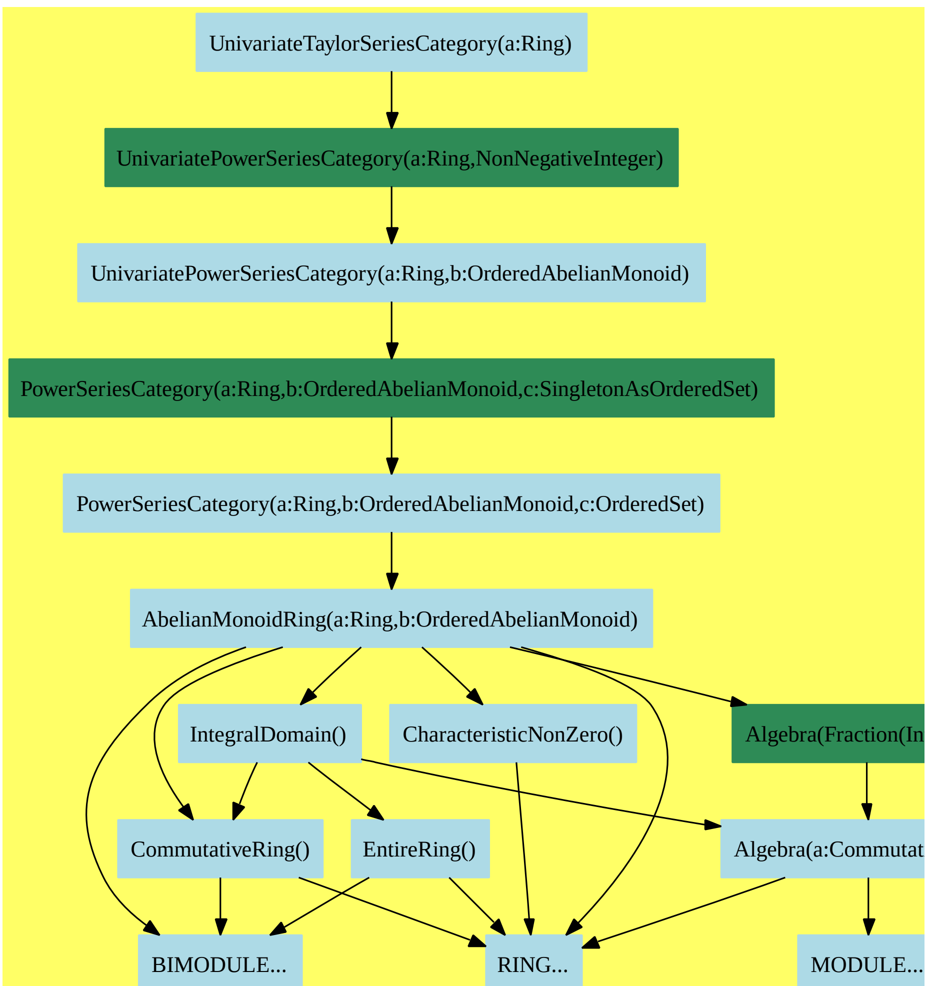
— POLYCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=POLYCAT"];
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "PDRING..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "FAMR..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "EVALAB..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "IEVALAB..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "RETRACT..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "FLINEXP..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "ORDSET..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "GCDDOM..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
```

```
  -> "PFECAT..."

"PDRING..." [color=lightblue];
"FAMR..." [color=lightblue];
"EVALAB..." [color=lightblue];
"IEVALAB..." [color=lightblue];
"RETRACT..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"ORDSET..." [color=lightblue];
"GCDDOM..." [color=lightblue];
"PFECAT..." [color=lightblue];

}
```

———–

16.0.214 UnivariateTaylorSeriesCategory (UTSCAT)



— UnivariateTaylorSeriesCategory.input —

```
)set break resume
)sys rm -f UnivariateTaylorSeriesCategory.output
)spool UnivariateTaylorSeriesCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariateTaylorSeriesCategory
--R
--R UnivariateTaylorSeriesCategory(Coef: Ring) is a category constructor
--R Abbreviation for UnivariateTaylorSeriesCategory is UTSCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for UTSCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Coef,%) -> %                  ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %    ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %   ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %    ?^? : (%,PositiveInteger) -> %
--R center : % -> Coef                   coefficients : % -> Stream(Coef)
--R coerce : % -> % if Coef has INTDOM   coerce : Integer -> %
--R coerce : % -> OutputForm             complete : % -> %
--R degree : % -> NonNegativeInteger     hash : % -> SingleInteger
--R latex : % -> String                  leadingCoefficient : % -> Coef
--R leadingMonomial : % -> %             map : ((Coef -> Coef),%) -> %
--R monomial? : % -> Boolean             one? : % -> Boolean
--R order : % -> NonNegativeInteger      pole? : % -> Boolean
--R quoByVar : % -> %                    recip : % -> Union(%,"failed")
--R reductum : % -> %                    sample : () -> %
--R series : Stream(Coef) -> %           variable : % -> Symbol
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction(Integer)) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?*? : (Fraction(Integer),%) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?**? : (%,Fraction(Integer)) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?**? : (%,%) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?**? : (%,Coef) -> % if Coef has FIELD
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R D : (%,Symbol) -> % if Coef has PDRING(SYMBOL) and Coef has *: (NonNegativeInteger,Coef) -> Coef
--R D : (%,List(Symbol)) -> % if Coef has PDRING(SYMBOL) and Coef has *: (NonNegativeInteger,Coef) -> Coef
--R D : (%,Symbol,NonNegativeInteger) -> % if Coef has PDRING(SYMBOL) and Coef has *: (NonNegativeInteger,Coef)
--R D : (%,List(Symbol),List(NonNegativeInteger)) -> % if Coef has PDRING(SYMBOL) and Coef has *: (NonNegativeIn
--R acos : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acosh : % -> % if Coef has ALGEBRA(FRAC(INT))
```

```
--R acot : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acoth : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acsc : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acsch : % -> % if Coef has ALGEBRA(FRAC(INT))
--R approximate : (%,NonNegativeInteger) -> Coef if Coef has **: (Coef,NonNegativeInteger) -> Coef and Coef has
--R asec : % -> % if Coef has ALGEBRA(FRAC(INT))
--R asech : % -> % if Coef has ALGEBRA(FRAC(INT))
--R asin : % -> % if Coef has ALGEBRA(FRAC(INT))
--R asinh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA(FRAC(INT))
--R atanh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
--R coefficient : (%,NonNegativeInteger) -> Coef
--R coerce : Coef -> % if Coef has COMRING
--R coerce : Fraction(Integer) -> % if Coef has ALGEBRA(FRAC(INT))
--R cos : % -> % if Coef has ALGEBRA(FRAC(INT))
--R cosh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R cot : % -> % if Coef has ALGEBRA(FRAC(INT))
--R coth : % -> % if Coef has ALGEBRA(FRAC(INT))
--R csc : % -> % if Coef has ALGEBRA(FRAC(INT))
--R csch : % -> % if Coef has ALGEBRA(FRAC(INT))
--R differentiate : % -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R differentiate : (%,NonNegativeInteger) -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
--R differentiate : (%,Symbol) -> % if Coef has PDRING(SYMBOL) and Coef has *: (NonNegativeInteger,Coef) -> Coef
--R differentiate : (%,List(Symbol)) -> % if Coef has PDRING(SYMBOL) and Coef has *: (NonNegativeInteger,Coef) -
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if Coef has PDRING(SYMBOL) and Coef has *: (NonNegativeIn
--R differentiate : (%,List(Symbol),List(NonNegativeInteger)) -> % if Coef has PDRING(SYMBOL) and Coef has *: (N
--R ?.? : (%,%) -> % if NonNegativeInteger has SGROUP
--R ?.? : (%,NonNegativeInteger) -> Coef
--R eval : (%,Coef) -> Stream(Coef) if Coef has **: (Coef,NonNegativeInteger) -> Coef
--R exp : % -> % if Coef has ALGEBRA(FRAC(INT))
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R extend : (%,NonNegativeInteger) -> %
--R integrate : (%,Symbol) -> % if Coef has ACFS(INT) and Coef has PRIMCAT and Coef has TRANFUN and Coef has ALG
--R integrate : % -> % if Coef has ALGEBRA(FRAC(INT))
--R log : % -> % if Coef has ALGEBRA(FRAC(INT))
--R monomial : (%,List(SingletonAsOrderedSet),List(NonNegativeInteger)) -> %
--R monomial : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R monomial : (Coef,NonNegativeInteger) -> %
--R multiplyCoefficients : ((Integer -> Coef),%) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA(FRAC(INT))
--R order : (%,NonNegativeInteger) -> NonNegativeInteger
--R pi : () -> % if Coef has ALGEBRA(FRAC(INT))
--R polynomial : (%,NonNegativeInteger,NonNegativeInteger) -> Polynomial(Coef)
--R polynomial : (%,NonNegativeInteger) -> Polynomial(Coef)
--R sec : % -> % if Coef has ALGEBRA(FRAC(INT))
--R sech : % -> % if Coef has ALGEBRA(FRAC(INT))
--R series : Stream(Record(k: NonNegativeInteger,c: Coef)) -> %
--R sin : % -> % if Coef has ALGEBRA(FRAC(INT))
--R sinh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R sqrt : % -> % if Coef has ALGEBRA(FRAC(INT))
```

```
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA(FRAC(INT))
--R tanh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R terms : % -> Stream(Record(k: NonNegativeInteger,c: Coef))
--R truncate : (%,NonNegativeInteger,NonNegativeInteger) -> %
--R truncate : (%,NonNegativeInteger) -> %
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R variables : % -> List(SingletonAsOrderedSet)
--R
--E 1

)spool
)lisp (bye)
```

— **UnivariateTaylorSeriesCategory.help** —

```
====================================================================
UnivariateTaylorSeriesCategory examples
====================================================================

UnivariateTaylorSeriesCategory is the category of Taylor series
in one variable.

See Also:
o )show UnivariateTaylorSeriesCategory
```

See:

⇐ "RadicalCategory" (RADCAT) 2.0.36 on page 106
⇐ "TranscendentalFunctionCategory" (TRANFUN) 3.0.58 on page 192
⇐ "UnivariatePowerSeriesCategory" (UPSCAT) 15.0.208 on page 1401

Exports:

0	1	acos	acosh
acot	acoth	acsc	acsch
approximate	asec	asech	asin
asinh	associates?	atan	atanh
center	characteristic	charthRoot	coefficient
coefficients	coerce	complete	cos
cosh	cot	coth	csc
csch	D	degree	differentiate
eval	exp	exquo	extend
hash	integrate	latex	leadingCoefficient
leadingMonomial	log	map	monomial
monomial?	multiplyCoefficients	multiplyExponents	nthRoot
one?	order	pi	pole?
polynomial	quoByVar	recip	reductum
sample	sec	sech	series
sin	sinh	sqrt	subtractIfCan
tan	tanh	terms	truncate
unit?	unitCanonical	unitNormal	variable
variables	zero?	?*?	?**?
?+?	?-?	-?	?=?
?^?	?~=?	?/?	?.?

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- if #1 has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x*y \neq 0$ implies both x and y are non-zero.
- if #1 has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation $" * " : (D, D) -> D$ which is commutative.

These are directly exported but not implemented:

```
coefficients : % -> Stream Coef
integrate : (%,Symbol) -> %
    if Coef has ACFS INT
    and Coef has PRIMCAT
    and Coef has TRANFUN
    and Coef has ALGEBRA FRAC INT
    or Coef has variables: Coef -> List Symbol
    and Coef has integrate: (Coef,Symbol) -> Coef
    and Coef has ALGEBRA FRAC INT
integrate : % -> % if Coef has ALGEBRA FRAC INT
multiplyCoefficients : ((Integer -> Coef),%) -> %
polynomial : (%,NonNegativeInteger,NonNegativeInteger) -> Polynomial Coef
polynomial : (%,NonNegativeInteger) -> Polynomial Coef
quoByVar : % -> %
series : Stream Coef -> %
series : Stream Record(k: NonNegativeInteger,c: Coef) -> %
```

These are implemented by this category:

```
acos : % -> % if Coef has ALGEBRA FRAC INT
acosh : % -> % if Coef has ALGEBRA FRAC INT
acot : % -> % if Coef has ALGEBRA FRAC INT
acoth : % -> % if Coef has ALGEBRA FRAC INT
acsc : % -> % if Coef has ALGEBRA FRAC INT
acsch : % -> % if Coef has ALGEBRA FRAC INT
asec : % -> % if Coef has ALGEBRA FRAC INT
asech : % -> % if Coef has ALGEBRA FRAC INT
asin : % -> % if Coef has ALGEBRA FRAC INT
asinh : % -> % if Coef has ALGEBRA FRAC INT
atan : % -> % if Coef has ALGEBRA FRAC INT
atanh : % -> % if Coef has ALGEBRA FRAC INT
coerce : % -> OutputForm
cos : % -> % if Coef has ALGEBRA FRAC INT
cosh : % -> % if Coef has ALGEBRA FRAC INT
cot : % -> % if Coef has ALGEBRA FRAC INT
coth : % -> % if Coef has ALGEBRA FRAC INT
csc : % -> % if Coef has ALGEBRA FRAC INT
csch : % -> % if Coef has ALGEBRA FRAC INT
exp : % -> % if Coef has ALGEBRA FRAC INT
log : % -> % if Coef has ALGEBRA FRAC INT
sinh : % -> % if Coef has ALGEBRA FRAC INT
sec : % -> % if Coef has ALGEBRA FRAC INT
sech : % -> % if Coef has ALGEBRA FRAC INT
sin : % -> % if Coef has ALGEBRA FRAC INT
tan : % -> % if Coef has ALGEBRA FRAC INT
tanh : % -> % if Coef has ALGEBRA FRAC INT
zero? : % -> Boolean
?**? : (%,Coef) -> % if Coef has FIELD
?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
```

These exports come from (p1401) UnivariatePowerSeriesCategory(Coef,NNI) where Coef:Ring and NNI:NonNegativeInteger:

```
0 : () -> %
1 : () -> %
approximate : (%,NonNegativeInteger) -> Coef
    if Coef has **: (Coef,NonNegativeInteger) -> Coef
    and Coef has coerce: Symbol -> Coef
associates? : (%,%) -> Boolean if Coef has INTDOM
center : % -> Coef
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
coefficient : (%,NonNegativeInteger) -> Coef
coerce : Coef -> % if Coef has COMRING
coerce : % -> % if Coef has INTDOM
coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
coerce : Integer -> %
complete : % -> %
D : % -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
D : (%,NonNegativeInteger) -> %
    if Coef has *: (NonNegativeInteger,Coef) -> Coef
D : (%,Symbol) -> %
    if Coef has PDRING SYMBOL
```

```
    and Coef has *: (NonNegativeInteger,Coef) -> Coef
D : (%,List Symbol) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (NonNegativeInteger,Coef) -> Coef
D : (%,Symbol,NonNegativeInteger) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (NonNegativeInteger,Coef) -> Coef
D : (%,List Symbol,List NonNegativeInteger) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (NonNegativeInteger,Coef) -> Coef
differentiate : (%,List Symbol) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (NonNegativeInteger,Coef) -> Coef
differentiate : (%,Symbol,NonNegativeInteger) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (NonNegativeInteger,Coef) -> Coef
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (NonNegativeInteger,Coef) -> Coef
differentiate : (%,Symbol) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (NonNegativeInteger,Coef) -> Coef
differentiate : (%,NonNegativeInteger) -> %
    if Coef has *: (NonNegativeInteger,Coef) -> Coef
differentiate : % -> %
    if Coef has *: (NonNegativeInteger,Coef) -> Coef
degree : % -> NonNegativeInteger
extend : (%,NonNegativeInteger) -> %
exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
eval : (%,Coef) -> Stream Coef
    if Coef has **: (Coef,NonNegativeInteger) -> Coef
hash : % -> SingleInteger
latex : % -> String
leadingCoefficient : % -> Coef
leadingMonomial : % -> %
map : ((Coef -> Coef),%) -> %
monomial : (Coef,NonNegativeInteger) -> %
monomial : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
monomial : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
monomial? : % -> Boolean
multiplyExponents : (%,PositiveInteger) -> %
one? : % -> Boolean
order : % -> NonNegativeInteger
order : (%,NonNegativeInteger) -> NonNegativeInteger
pole? : % -> Boolean
recip : % -> Union(%,"failed")
reductum : % -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
terms : % -> Stream Record(k: NonNegativeInteger,c: Coef)
truncate : (%,NonNegativeInteger,NonNegativeInteger) -> %
truncate : (%,NonNegativeInteger) -> %
unit? : % -> Boolean if Coef has INTDOM
unitCanonical : % -> % if Coef has INTDOM
```

```
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if Coef has INTDOM
variable : % -> Symbol
variables : % -> List SingletonAsOrderedSet
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (Integer,%) -> %
?*? : (Coef,%) -> %
?*? : (%,Coef) -> %
?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
?/? : (%,Coef) -> % if Coef has FIELD
-? : % -> %
?.? : (%,%) -> % if NonNegativeInteger has SGROUP
?.? : (%,NonNegativeInteger) -> Coef
```

These exports come from (p192) TranscendentalFunctionCategory():

```
pi : () -> % if Coef has ALGEBRA FRAC INT
```

These exports come from (p106) RadicalCategory():

```
nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
sqrt : % -> % if Coef has ALGEBRA FRAC INT
```

— UnivariateTaylorSeriesCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#UTSCAT">
UnivariateTaylorSeriesCategory (UTSCAT)</a></h2>
</body>
```

— category UTSCAT UnivariateTaylorSeriesCategory —

```
)abbrev category UTSCAT UnivariateTaylorSeriesCategory
++ Author: Clifton J. Williamson
++ Date Created: 21 December 1989
++ Date Last Updated: 26 May 1994
++ Description:
++ \spadtype{UnivariateTaylorSeriesCategory} is the category of Taylor
++ series in one variable.

UnivariateTaylorSeriesCategory(Coef) : Category == SIG where
  Coef  : Ring
```

```
I    ==> Integer
L    ==> List
NNI  ==> NonNegativeInteger
OUT  ==> OutputForm
RN   ==> Fraction Integer
STTA ==> StreamTaylorSeriesOperations Coef
STTF ==> StreamTranscendentalFunctions Coef
STNC ==> StreamTranscendentalFunctionsNonCommutative Coef
Term ==> Record(k:NNI,c:Coef)

SIG ==> UnivariatePowerSeriesCategory(Coef,NNI) with

  series : Stream Term -> %
    ++ \spad{series(st)} creates a series from a stream of non-zero terms,
    ++ where a term is an exponent-coefficient pair.  The terms in the
    ++ stream should be ordered by increasing order of exponents.

  coefficients : % -> Stream Coef
    ++ \spad{coefficients(a0 + a1 x + a2 x**2 + ...)} returns a stream
    ++ of coefficients: \spad{[a0,a1,a2,...]}. The entries of the stream
    ++ may be zero.

  series : Stream Coef -> %
    ++ \spad{series([a0,a1,a2,...])} is the Taylor series
    ++ \spad{a0 + a1 x + a2 x**2 + ...}.

  quoByVar : % -> %
    ++ \spad{quoByVar(a0 + a1 x + a2 x**2 + ...)}
    ++ returns \spad{a1 + a2 x + a3 x**2 + ...}
    ++ Thus, this function substracts the constant term and divides by
    ++ the series variable.  This function is used when Laurent series
    ++ are represented by a Taylor series and an order.

  multiplyCoefficients : (I -> Coef,%) -> %
    ++ \spad{multiplyCoefficients(f,sum(n = 0..infinity,a[n] * x**n))}
    ++ returns \spad{sum(n = 0..infinity,f(n) * a[n] * x**n)}.
    ++ This function is used when Laurent series are represented by
    ++ a Taylor series and an order.

  polynomial : (%,NNI) -> Polynomial Coef
    ++ \spad{polynomial(f,k)} returns a polynomial consisting of the sum
    ++ of all terms of f of degree \spad{<= k}.

  polynomial : (%,NNI,NNI) -> Polynomial Coef
    ++ \spad{polynomial(f,k1,k2)} returns a polynomial consisting of the
    ++ sum of all terms of f of degree d with \spad{k1 <= d <= k2}.

  if Coef has Field then

    "**" : (%,Coef) -> %
      ++ \spad{f(x) ** a} computes a power of a power series.
      ++ When the coefficient ring is a field, we may raise a series
      ++ to an exponent from the coefficient ring provided that the
```

```
      ++ constant coefficient of the series is 1.

   if Coef has Algebra Fraction Integer then

     integrate : % -> %
       ++ \spad{integrate(f(x))} returns an anti-derivative of the power
       ++ series \spad{f(x)} with constant coefficient 0.
       ++ We may integrate a series when we can divide coefficients
       ++ by integers.

     if Coef has integrate: (Coef,Symbol) -> Coef and _
        Coef has variables: Coef -> List Symbol then

       integrate : (%,Symbol) -> %
         ++ \spad{integrate(f(x),y)} returns an anti-derivative of the
         ++ power series \spad{f(x)} with respect to the variable \spad{y}.

     if Coef has TranscendentalFunctionCategory and _
        Coef has PrimitiveFunctionCategory and _
        Coef has AlgebraicallyClosedFunctionSpace Integer then

       integrate : (%,Symbol) -> %
         ++ \spad{integrate(f(x),y)} returns an anti-derivative of
         ++ the power series \spad{f(x)} with respect to the variable
         ++ \spad{y}.

     RadicalCategory
       --++ We provide rational powers when we can divide coefficients
       --++ by integers.

     TranscendentalFunctionCategory
       --++ We provide transcendental functions when we can divide
       --++ coefficients by integers.

  add

    zero? x ==
      empty? (coefs := coefficients x) => true
      (zero? frst coefs) and (empty? rst coefs) => true
      false

--% OutputForms

--  We provide defaulr output functions on UTSCAT using the functions
--  'coefficients', 'center', and 'variable'.

    factorials?: () -> Boolean
    -- check a global Lisp variable
    factorials?() == false

    termOutput: (I,Coef,OUT) -> OUT
    termOutput(k,c,vv) ==
    -- creates a term c * vv ** k
      k = 0 => c :: OUT
```

```
    mon := (k = 1 => vv; vv ** (k :: OUT))
    c = 1 => mon
    c = -1 => -mon
    (c :: OUT) * mon

  showAll?: () -> Boolean
  -- check a global Lisp variable
  showAll?() == true

  coerce(p:%):OUT ==
    empty? (uu := coefficients p) => (0$Coef) :: OUT
    var := variable p; cen := center p
    vv :=
      zero? cen => var :: OUT
      paren(var :: OUT - cen :: OUT)
    n : NNI ; count : NNI := _$streamCount$Lisp
    l : L OUT := empty()
    for n in 0..count while not empty? uu repeat
      if frst(uu) ^= 0 then
        l := concat(termOutput(n :: I,frst uu,vv),l)
      uu := rst uu
    if showAll?() then
      for n in (count + 1).. while explicitEntries? uu and _
             not eq?(uu,rst uu) repeat
        if frst(uu) ^= 0 then
          l := concat(termOutput(n :: I,frst uu,vv),l)
        uu := rst uu
    l :=
      explicitlyEmpty? uu => l
      eq?(uu,rst uu) and frst uu = 0 => l
      concat(prefix("O" :: OUT,[vv ** (n :: OUT)]),l)
    empty? l => (0$Coef) :: OUT
    reduce("+",reverse_! l)

  if Coef has Field then

    (x:%) ** (r:Coef) == series power(r,coefficients x)$STTA

  if Coef has Algebra Fraction Integer then

    if Coef has CommutativeRing then

      (x:%) ** (y:%)    == series(coefficients x **$STTF coefficients y)

      (x:%) ** (r:RN)   == series powern(r,coefficients x)$STTA

      exp x == series exp(coefficients x)$STTF

      log x == series log(coefficients x)$STTF

      sin x == series sin(coefficients x)$STTF

      cos x == series cos(coefficients x)$STTF
```

```
  tan x == series tan(coefficients x)$STTF

  cot x == series cot(coefficients x)$STTF

  sec x == series sec(coefficients x)$STTF

  csc x == series csc(coefficients x)$STTF

  asin x == series asin(coefficients x)$STTF

  acos x == series acos(coefficients x)$STTF

  atan x == series atan(coefficients x)$STTF

  acot x == series acot(coefficients x)$STTF

  asec x == series asec(coefficients x)$STTF

  acsc x == series acsc(coefficients x)$STTF

  sinh x == series sinh(coefficients x)$STTF

  cosh x == series cosh(coefficients x)$STTF

  tanh x == series tanh(coefficients x)$STTF

  coth x == series coth(coefficients x)$STTF

  sech x == series sech(coefficients x)$STTF

  csch x == series csch(coefficients x)$STTF

  asinh x == series asinh(coefficients x)$STTF

  acosh x == series acosh(coefficients x)$STTF

  atanh x == series atanh(coefficients x)$STTF

  acoth x == series acoth(coefficients x)$STTF

  asech x == series asech(coefficients x)$STTF

  acsch x == series acsch(coefficients x)$STTF

else

  (x:%) ** (y:%) == series(coefficients x **$STNC coefficients y)

  (x:%) ** (r:RN) ==
    coefs := coefficients x
    empty? coefs =>
      positive? r => 0
      zero? r => error "0**0 undefined"
      error "0 raised to a negative power"
```

```
  not (frst coefs = 1) =>
    error "**: constant coefficient should be 1"
  coefs := concat(0,rst coefs)
  onePlusX := monom(1,0)$STTA + $STTA monom(1,1)$STTA
  ratPow := powern(r,onePlusX)$STTA
  series compose(ratPow,coefs)$STTA

exp x == series exp(coefficients x)$STNC

log x == series log(coefficients x)$STNC

sin x == series sin(coefficients x)$STNC

cos x == series cos(coefficients x)$STNC

tan x == series tan(coefficients x)$STNC

cot x == series cot(coefficients x)$STNC

sec x == series sec(coefficients x)$STNC

csc x == series csc(coefficients x)$STNC

asin x == series asin(coefficients x)$STNC

acos x == series acos(coefficients x)$STNC

atan x == series atan(coefficients x)$STNC

acot x == series acot(coefficients x)$STNC

asec x == series asec(coefficients x)$STNC

acsc x == series acsc(coefficients x)$STNC

sinh x == series sinh(coefficients x)$STNC

cosh x == series cosh(coefficients x)$STNC

tanh x == series tanh(coefficients x)$STNC

coth x == series coth(coefficients x)$STNC

sech x == series sech(coefficients x)$STNC

csch x == series csch(coefficients x)$STNC

asinh x == series asinh(coefficients x)$STNC

acosh x == series acosh(coefficients x)$STNC

atanh x == series atanh(coefficients x)$STNC

acoth x == series acoth(coefficients x)$STNC
```

```
    asech x == series asech(coefficients x)$STNC

    acsch x == series acsch(coefficients x)$STNC
```

— **COQ UTSCAT** —

```
(* category UTSCAT *)
(*

    zero? : % -> Boolean
    zero? x ==
      empty? (coefs := coefficients x) => true
      (zero? frst coefs) and (empty? rst coefs) => true
      false

--% OutputForms

--  We provide defaulr output functions on UTSCAT using the functions
--  'coefficients', 'center', and 'variable'.

    -- check a global Lisp variable
    factorials?: () -> Boolean
    factorials?() == false

    termOutput: (I,Coef,OUT) -> OUT
    termOutput(k,c,vv) ==
    -- creates a term c * vv ** k
      k = 0 => c :: OUT
      mon := (k = 1 => vv; vv ** (k :: OUT))
      c = 1 => mon
      c = -1 => -mon
      (c :: OUT) * mon

    -- check a global Lisp variable
    showAll?: () -> Boolean
    showAll?() == true

    coerce : % -> OutputForm
    coerce(p:%):OUT ==
      empty? (uu := coefficients p) => (0$Coef) :: OUT
      var := variable p; cen := center p
      vv :=
        zero? cen => var :: OUT
        paren(var :: OUT - cen :: OUT)
      n : NNI ; count : NNI := _$streamCount$Lisp
      l : L OUT := empty()
      for n in 0..count while not empty? uu repeat
        if frst(uu) ^= 0 then
          l := concat(termOutput(n :: I,frst uu,vv),l)
        uu := rst uu
```

```
    if showAll?() then
      for n in (count + 1).. while explicitEntries? uu and _
             not eq?(uu,rst uu) repeat
        if frst(uu) ^= 0 then
          l := concat(termOutput(n :: I,frst uu,vv),l)
        uu := rst uu
    l :=
      explicitlyEmpty? uu => l
      eq?(uu,rst uu) and frst uu = 0 => l
      concat(prefix("O" :: OUT,[vv ** (n :: OUT)]),l)
    empty? l => (0$Coef) :: OUT
    reduce("+",reverse_! l)

  if Coef has Field then

    ?*? : (%,Coef) -> %
    (x:%) ** (r:Coef) == series power(r,coefficients x)$STTA

  if Coef has Algebra Fraction Integer then
    if Coef has CommutativeRing then

      ?**? : (%,%) -> %
      (x:%) ** (y:%)    == series(coefficients x **$STTF coefficients y)

      ?**? : (%,Fraction(Integer)) -> %
      (x:%) ** (r:RN)   == series powern(r,coefficients x)$STTA

      exp : % -> %
      exp x == series exp(coefficients x)$STTF

      log : % -> %
      log x == series log(coefficients x)$STTF

      sin : % -> %
      sin x == series sin(coefficients x)$STTF

      cos : % -> %
      cos x == series cos(coefficients x)$STTF

      tan : % -> %
      tan x == series tan(coefficients x)$STTF

      cot : % -> %
      cot x == series cot(coefficients x)$STTF

      sec : % -> %
      sec x == series sec(coefficients x)$STTF

      csc : % -> %
      csc x == series csc(coefficients x)$STTF

      asin : % -> %
      asin x == series asin(coefficients x)$STTF
```

```
  acos : % -> %
  acos x == series acos(coefficients x)$STTF

  atan : % -> %
  atan x == series atan(coefficients x)$STTF

  acot : % -> %
  acot x == series acot(coefficients x)$STTF

  asec : % -> %
  asec x == series asec(coefficients x)$STTF

  acsc : % -> %
  acsc x == series acsc(coefficients x)$STTF

  sinh : % -> %
  sinh x == series sinh(coefficients x)$STTF

  cosh : % -> %
  cosh x == series cosh(coefficients x)$STTF

  tanh : % -> %
  tanh x == series tanh(coefficients x)$STTF

  coth : % -> %
  coth x == series coth(coefficients x)$STTF

  sech : % -> %
  sech x == series sech(coefficients x)$STTF

  csch : % -> %
  csch x == series csch(coefficients x)$STTF

  asinh : % -> %
  asinh x == series asinh(coefficients x)$STTF

  acosh : % -> %
  acosh x == series acosh(coefficients x)$STTF

  atanh : % -> %
  atanh x == series atanh(coefficients x)$STTF

  acoth : % -> %
  acoth x == series acoth(coefficients x)$STTF

  asech : % -> %
  asech x == series asech(coefficients x)$STTF

  acsch : % -> %
  acsch x == series acsch(coefficients x)$STTF

else

  ?**? : (%,%) -> %
```

```
(x:%) ** (y:%) == series(coefficients x **$STNC coefficients y)

?**? : (%,Fraction(Integer)) -> %
(x:%) ** (r:RN) ==
  coefs := coefficients x
  empty? coefs =>
    positive? r => 0
    zero? r => error "0**0 undefined"
    error "0 raised to a negative power"
  not (frst coefs = 1) =>
    error "**: constant coefficient should be 1"
  coefs := concat(0,rst coefs)
  onePlusX := monom(1,0)$STTA + $STTA monom(1,1)$STTA
  ratPow := powern(r,onePlusX)$STTA
  series compose(ratPow,coefs)$STTA

exp : % -> %
exp x == series exp(coefficients x)$STNC

log : % -> %
log x == series log(coefficients x)$STNC

sin : % -> %
sin x == series sin(coefficients x)$STNC

cos : % -> %
cos x == series cos(coefficients x)$STNC

tan : % -> %
tan x == series tan(coefficients x)$STNC

cot : % -> %
cot x == series cot(coefficients x)$STNC

sec : % -> %
sec x == series sec(coefficients x)$STNC

csc : % -> %
csc x == series csc(coefficients x)$STNC

asin : % -> %
asin x == series asin(coefficients x)$STNC

acos : % -> %
acos x == series acos(coefficients x)$STNC

atan : % -> %
atan x == series atan(coefficients x)$STNC

acot : % -> %
acot x == series acot(coefficients x)$STNC

asec : % -> %
asec x == series asec(coefficients x)$STNC
```

```
        acsc : % -> %
        acsc x == series acsc(coefficients x)$STNC

        sinh : % -> %
        sinh x == series sinh(coefficients x)$STNC

        cosh : % -> %
        cosh x == series cosh(coefficients x)$STNC

        tanh : % -> %
        tanh x == series tanh(coefficients x)$STNC

        coth : % -> %
        coth x == series coth(coefficients x)$STNC

        sech : % -> %
        sech x == series sech(coefficients x)$STNC

        csch : % -> %
        csch x == series csch(coefficients x)$STNC

        asinh : % -> %
        asinh x == series asinh(coefficients x)$STNC

        acosh : % -> %
        acosh x == series acosh(coefficients x)$STNC

        atanh : % -> %
        atanh x == series atanh(coefficients x)$STNC

        acoth : % -> %
        acoth x == series acoth(coefficients x)$STNC

        asech : % -> %
        asech x == series asech(coefficients x)$STNC

        acsch : % -> %
        acsch x == series acsch(coefficients x)$STNC
*)
```

———-

— **UTSCAT.dotabb** —

```
"UTSCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UTSCAT"];
"UTSCAT" -> "UPSCAT"
```

———-

— **UTSCAT.dotfull** —

```
"UnivariateTaylorSeriesCategory(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UTSCAT"];
"UnivariateTaylorSeriesCategory(a:Ring)" ->
    "UnivariatePowerSeriesCategory(a:Ring,NonNegativeInteger)"
```

———-

— **UTSCAT.dotpic** —

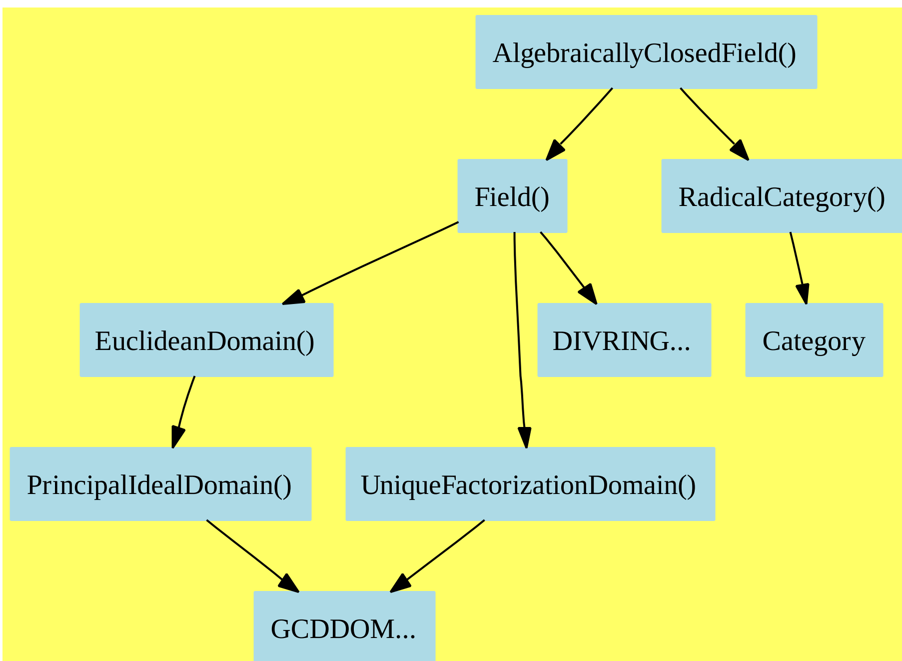
```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"UnivariateTaylorSeriesCategory(a:Ring)" [color=lightblue];
"UnivariateTaylorSeriesCategory(a:Ring)" ->
    "UnivariatePowerSeriesCategory(a:Ring,NonNegativeInteger)"

"UnivariatePowerSeriesCategory(a:Ring,NonNegativeInteger)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=UPSCAT"];
"UnivariatePowerSeriesCategory(a:Ring,NonNegativeInteger)" ->
    "UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"

"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"
 [color=lightblue];
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)" ->
 "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
 [color=seagreen];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
  -> "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
 [color=lightblue];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)" ->
    "AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"

"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" [color=lightblue];
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" -> "RING..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "BIMODULE..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "IntegralDomain()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CharacteristicNonZero()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CommutativeRing()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "Algebra(Fraction(Integer))"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
```

```
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BIMODULE..."

"CharacteristicNonZero()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARNZ"];
"CharacteristicNonZero()" -> "RING..."

"Algebra(Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(Fraction(Integer))" -> "Algebra(a:CommutativeRing)"

"Algebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "MODULE..."

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BIMODULE..."

"BIMODULE..." [color=lightblue];
"RING..." [color=lightblue];
"MODULE..." [color=lightblue];
}
```

Chapter 17

Category Layer 16

17.0.215 AlgebraicallyClosedField (ACF)



— **AlgebraicallyClosedField.input** —

```
)set break resume
)sys rm -f AlgebraicallyClosedField.output
)spool AlgebraicallyClosedField.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 15
pi:Polynomial(Integer):=-3*x^3+2*x+13
--R
--R
--R            3
--R   (1)  - 3x  + 2x + 13
--R                                                   Type: Polynomial(Integer)
--E 1

--S 2 of 15
rootOf(pi)
--R
--R
--R   (2)  x
--R                                                       Type: AlgebraicNumber
--E 2

--S 3 of 15
rootsOf(pi)
--R
--R
--R   (3)  [%x0,%x1,- %x1 - %x0]
--R                                                 Type: List(AlgebraicNumber)
--E 3

--S 4 of 15
zeroOf(pi)
--R
--R
--R   (4)  x
--R                                                       Type: AlgebraicNumber
--E 4

--S 5 of 15
zerosOf(pi)
--R
--R
--R                   +-------------+         +-------------+
--R                   |       2               |       2
--R                - \|- 27%x3  + 24  - 3%x3 \|- 27%x3  + 24  - 3%x3
--R   (5)  [%x3,-------------------------,-----------------------]
--R                            6                        6
--R                                                 Type: List(AlgebraicNumber)
--E 5

--S 6 of 15
sup:SparseUnivariatePolynomial(Integer):=-3*x^3+2*x+13
--R
--R
--R            3
--R   (6)  - 3?  + 2? + 13
--R                                  Type: SparseUnivariatePolynomial(Integer)
--E 6
```

```
--S 7 of 15
rootOf(sup)
--R
--R
--R   (7)  %B
--R                                                    Type: AlgebraicNumber
--E 7

--S 8 of 15
rootOf(sup,x)
--R
--R
--R   (8)  x
--R                                                    Type: AlgebraicNumber
--E 8

--S 9 of 15
rootsOf(sup)
--R
--R
--R   (9)  [%%C0,%%C1,- %%C1 - %%C0]
--R                                              Type: List(AlgebraicNumber)
--E 9

--S 10 of 15
rootsOf(sup,x)
--R
--R
--R   (10)  [%x6,%x7,- %x7 - %x6]
--R                                              Type: List(AlgebraicNumber)
--E 10

--S 11 of 15
zeroOf(sup)
--R
--R
--R   (11)  %D
--R                                                    Type: AlgebraicNumber
--E 11

--S 12 of 15
zeroOf(sup,x)
--R
--R
--R   (12)  x
--R                                                    Type: AlgebraicNumber
--E 12

--S 13 of 15
zerosOf(sup)
--R
--R
--R                  +---------------+          +---------------+
--R                  |        2                 |        2
```

```
--R              - \|- 27%%E0  + 24  - 3%%E0 \|- 27%%E0  + 24  - 3%%E0
--R   (13)  [%%E0,---------------------------,-------------------------]
--R                             6                          6
--R                                                  Type: List(AlgebraicNumber)
--E 13

--S 14 of 15
zerosOf(sup,x)
--R
--R
--R                 +-------------+         +-------------+
--R                 |       2               |       2
--R              - \|- 27%x9  + 24  - 3%x9 \|- 27%x9  + 24  - 3%x9
--R   (14)  [%x9,-------------------------,-----------------------]
--R                            6                         6
--R                                                  Type: List(AlgebraicNumber)
--E 14

--S 15 of 15
)show AlgebraicallyClosedField
--R
--R AlgebraicallyClosedField is a category constructor
--R Abbreviation for AlgebraicallyClosedField is ACF
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ACF
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Fraction(Integer),%) -> %      ?*? : (%,Fraction(Integer)) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,Fraction(Integer)) -> %     ?**? : (%,Integer) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                1 : () -> %
--R 0 : () -> %                           ?^? : (%,Integer) -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R associates? : (%,%) -> Boolean        coerce : Fraction(Integer) -> %
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              factor : % -> Factored(%)
--R gcd : List(%) -> %                    gcd : (%,%) -> %
--R hash : % -> SingleInteger             inv : % -> %
--R latex : % -> String                   lcm : List(%) -> %
--R lcm : (%,%) -> %                      nthRoot : (%,Integer) -> %
--R one? : % -> Boolean                   prime? : % -> Boolean
--R ?quo? : (%,%) -> %                    recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                    rootOf : Polynomial(%) -> %
--R rootsOf : Polynomial(%) -> List(%)    sample : () -> %
--R sizeLess? : (%,%) -> Boolean          sqrt : % -> %
--R squareFree : % -> Factored(%)         squareFreePart : % -> %
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R zero? : % -> Boolean                  zeroOf : Polynomial(%) -> %
--R zerosOf : Polynomial(%) -> List(%)    ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
```

```
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed")
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %)
--R rootOf : (SparseUnivariatePolynomial(%),Symbol) -> %
--R rootOf : SparseUnivariatePolynomial(%) -> %
--R rootsOf : (SparseUnivariatePolynomial(%),Symbol) -> List(%)
--R rootsOf : SparseUnivariatePolynomial(%) -> List(%)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R zeroOf : (SparseUnivariatePolynomial(%),Symbol) -> %
--R zeroOf : SparseUnivariatePolynomial(%) -> %
--R zerosOf : (SparseUnivariatePolynomial(%),Symbol) -> List(%)
--R zerosOf : SparseUnivariatePolynomial(%) -> List(%)
--R
--E 15

)spool
)lisp (bye)
```

— **AlgebraicallyClosedField.help** —

```
====================================================================
AlgebraicallyClosedField examples
====================================================================

This category is a model for algebraically closed fields.

Given the polynomial:

pi:Polynomial(Integer):=-3*x^3+2*x+13
               3
           - 3x  + 2x + 13


rootOf(pi)
           x


rootsOf(pi)
           [%x0,%x1,- %x1 - %x0]


zeroOf(pi)
           x
```

```
zerosOf(pi)
                   +-------------+         +-------------+
                   |      2                |      2
                - \|- 27%x3  + 24  - 3%x3 \|- 27%x3  + 24  - 3%x3
           [%x3,------------------------,-----------------------]
                             6                         6
```

These functions can also be applied to Sparse Univariate Polynomials:

```
sup:SparseUnivariatePolynomial(Integer):=-3*x^3+2*x+13
              3
           - 3?  + 2? + 13


rootOf(sup)
           %B


rootOf(sup,x)
           x


rootsOf(sup)
           [%%C0,%%C1,- %%C1 - %%C0]


rootsOf(sup,x)
           [%x6,%x7,- %x7 - %x6]


zeroOf(sup)
           %D


zeroOf(sup,x)
        x


zerosOf(sup)
                 +--------------+          +--------------+
                 |       2                 |       2
              - \|- 27%%E0  + 24  - 3%%E0 \|- 27%%E0  + 24  - 3%%E0
        [%%E0,--------------------------,-------------------------]
                           6                          6


zerosOf(sup,x)
                +-------------+         +-------------+
                |      2                |      2
             - \|- 27%x9  + 24  - 3%x9 \|- 27%x9  + 24  - 3%x9
        [%x9,------------------------,-----------------------]
                          6                         6
```

```
See Also:
o )show AlgebraicallyClosedField
```

———-

See:

⇒ "AlgebraicallyClosedFunctionSpace" (ACFS) 18.0.229 on page 1739
⇐ "Field" (FIELD) 16.0.209 on page 1413
⇐ "RadicalCategory" (RADCAT) 2.0.36 on page 106

Exports:

0	1	associates?	characteristic
coerce	divide	euclideanSize	expressIdealMember
exquo	extendedEuclidean	factor	gcd
gcdPolynomial	hash	inv	latex
lcm	multiEuclidean	nthRoot	one?
prime?	principalIdeal	recip	rootOf
rootsOf	sample	sizeLess?	sqrt
squareFree	squareFreePart	subtractIfCan	unit?
unitCanonical	unitNormal	zero?	zeroOf
zerosOf	?**?	?***?	?+?
?-?	-?	?/?	?=?
?quo?	?rem?	?~=?	?^?

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
rootOf : Polynomial % -> %
```

These are implemented by this category:

```
rootOf : SparseUnivariatePolynomial % -> %
rootOf : (SparseUnivariatePolynomial %,Symbol) -> %
rootsOf : Polynomial % -> List %
rootsOf : (SparseUnivariatePolynomial %,Symbol) -> List %
rootsOf : SparseUnivariatePolynomial % -> List %
zeroOf : (SparseUnivariatePolynomial %,Symbol) -> %
zeroOf : Polynomial % -> %
zeroOf : SparseUnivariatePolynomial % -> %
zerosOf : Polynomial % -> List %
zerosOf : (SparseUnivariatePolynomial %,Symbol) -> List %
zerosOf : SparseUnivariatePolynomial % -> List %
```

These exports come from (p1413) Field():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
coerce : Fraction Integer -> %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
factor : % -> Factored %
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
      SparseUnivariatePolynomial %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
sample : () -> %
sizeLess? : (%,%) -> Boolean
squareFree : % -> Factored %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?*? : (Fraction Integer,%) -> %
```

```
?*? : (%,Fraction Integer) -> %
?**? : (%,Integer) -> %
?^? : (%,Integer) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?/? : (%,%) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p106) RadicalCategory():

```
nthRoot : (%,Integer) -> %
sqrt : % -> %
?**? : (%,Fraction Integer) -> %
```

— AlgebraicallyClosedField.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ACF">
AlgebraicallyClosedField (ACF)</a></h2>
</body>
```

— category ACF AlgebraicallyClosedField —

```
)abbrev category ACF AlgebraicallyClosedField
++ Author: Manuel Bronstein
++ Date Created: 22 Mar 1988
++ Date Last Updated: 27 November 1991
++ Description:
++ Model for algebraically closed fields.

AlgebraicallyClosedField() : Category == SIG where

  SIG ==> Join(Field,RadicalCategory) with

    rootOf : Polynomial $ -> $
      ++rootOf(p) returns y such that \spad{p(y) = 0}.
      ++ Error: if p has more than one variable y.
      ++
      ++X a:Polynomial(Integer):=-3*x^3+2*x+13
      ++X rootOf(a)
```

```
rootOf : SparseUnivariatePolynomial $ -> $
  ++rootOf(p) returns y such that \spad{p(y) = 0}.
  ++
  ++X a:SparseUnivariatePolynomial(Integer):=-3*x^3+2*x+13
  ++X rootOf(a)

rootOf : (SparseUnivariatePolynomial $, Symbol) -> $
  ++rootOf(p, y) returns y such that \spad{p(y) = 0}.
  ++ The object returned displays as \spad{'y}.
  ++
  ++X a:SparseUnivariatePolynomial(Integer):=-3*x^3+2*x+13
  ++X rootOf(a,x)

rootsOf : Polynomial $ -> List $
  ++rootsOf(p) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0}.
  ++ Note that the returned symbols y1,...,yn are bound in the
  ++ interpreter to respective root values.
  ++ Error: if p has more than one variable y.
  ++
  ++X a:Polynomial(Integer):=-3*x^3+2*x+13
  ++X rootsOf(a)

rootsOf : SparseUnivariatePolynomial $ -> List $
  ++rootsOf(p) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0}.
  ++ Note that the returned symbols y1,...,yn are bound in the interpreter
  ++ to respective root values.
  ++
  ++X a:SparseUnivariatePolynomial(Integer):=-3*x^3+2*x+13
  ++X rootsOf(a)

rootsOf : (SparseUnivariatePolynomial $, Symbol) -> List $
  ++rootsOf(p, y) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0};
  ++ The returned roots display as \spad{'y1},...,\spad{'yn}.
  ++ Note that the returned symbols y1,...,yn are bound in the interpreter
  ++ to respective root values.
  ++
  ++X a:SparseUnivariatePolynomial(Integer):=-3*x^3+2*x+13
  ++X rootsOf(a,x)

zeroOf : Polynomial $ -> $
  ++zeroOf(p) returns y such that \spad{p(y) = 0}.
  ++ If possible, y is expressed in terms of radicals.
  ++ Otherwise it is an implicit algebraic quantity.
  ++ Error: if p has more than one variable y.
  ++
  ++X a:Polynomial(Integer):=-3*x^2+2*x-13
  ++X zeroOf(a)

zeroOf : SparseUnivariatePolynomial $ -> $
  ++zeroOf(p) returns y such that \spad{p(y) = 0};
  ++ if possible, y is expressed in terms of radicals.
  ++ Otherwise it is an implicit algebraic quantity.
  ++
```

```
  ++X a:SparseUnivariatePolynomial(Integer):=-3*x^3+2*x+13
  ++X zeroOf(a)

zeroOf : (SparseUnivariatePolynomial $, Symbol) -> $
  ++zeroOf(p, y) returns y such that \spad{p(y) = 0};
  ++ if possible, y is expressed in terms of radicals.
  ++ Otherwise it is an implicit algebraic quantity which
  ++ displays as \spad{'y}.
  ++
  ++X a:SparseUnivariatePolynomial(Integer):=-3*x^3+2*x+13
  ++X zeroOf(a,x)

zerosOf : Polynomial $ -> List $
  ++zerosOf(p) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0}.
  ++ The yi's are expressed in radicals if possible.
  ++ Otherwise they are implicit algebraic quantities.
  ++ The returned symbols y1,...,yn are bound in the interpreter
  ++ to respective root values.
  ++ Error: if p has more than one variable y.
  ++
  ++X a:Polynomial(Integer):=-3*x^2+2*x-13
  ++X zerosOf(a)

zerosOf : SparseUnivariatePolynomial $ -> List $
  ++zerosOf(p) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0}.
  ++ The yi's are expressed in radicals if possible, and otherwise
  ++ as implicit algebraic quantities.
  ++ The returned symbols y1,...,yn are bound in the interpreter
  ++ to respective root values.
  ++
  ++X a:SparseUnivariatePolynomial(Integer):=-3*x^3+2*x+13
  ++X zerosOf(a)

zerosOf : (SparseUnivariatePolynomial $, Symbol) -> List $
  ++zerosOf(p, y) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0}.
  ++ The yi's are expressed in radicals if possible, and otherwise
  ++ as implicit algebraic quantities
  ++ which display as \spad{'yi}.
  ++ The returned symbols y1,...,yn are bound in the interpreter
  ++ to respective root values.
  ++
  ++X a:SparseUnivariatePolynomial(Integer):=-3*x^3+2*x+13
  ++X zerosOf(a,x)

add

 SUP ==> SparseUnivariatePolynomial $

 assign  : (Symbol, $) -> $

 allroots: (SUP, Symbol, (SUP, Symbol) -> $) -> List $

 binomialRoots: (SUP, Symbol, (SUP, Symbol) -> $) -> List $
```

```
zeroOf(p:SUP) == assign(x := new(), zeroOf(p, x))

rootOf(p:SUP) == assign(x := new(), rootOf(p, x))

zerosOf(p:SUP) == zerosOf(p, new())

rootsOf(p:SUP) == rootsOf(p, new())

rootsOf(p:SUP, y:Symbol) == allroots(p, y, rootOf)

zerosOf(p:SUP, y:Symbol) == allroots(p, y, zeroOf)

assign(x, f) == (assignSymbol(x, f, $)$Lisp; f)

zeroOf(p:Polynomial $) ==
  empty?(l := variables p) => error "zeroOf: constant polynomial"
  zeroOf(univariate p, first l)

rootOf(p:Polynomial $) ==
  empty?(l := variables p) => error "rootOf: constant polynomial"
  rootOf(univariate p, first l)

zerosOf(p:Polynomial $) ==
  empty?(l := variables p) => error "zerosOf: constant polynomial"
  zerosOf(univariate p, first l)

rootsOf(p:Polynomial $) ==
  empty?(l := variables p) => error "rootsOf: constant polynomial"
  rootsOf(univariate p, first l)

zeroOf(p:SUP, y:Symbol) ==
  zero?(d := degree p) => error "zeroOf: constant polynomial"
  zero? coefficient(p, 0) => 0
  a := leadingCoefficient p
  d = 2 =>
    b := coefficient(p, 1)
    (sqrt(b**2 - 4 * a * coefficient(p, 0)) - b) / (2 * a)
  (r := retractIfCan(reductum p)@Union($,"failed")) case "failed" =>
    rootOf(p, y)
  nthRoot(- (r::$ / a), d)

binomialRoots(p, y, fn) ==
  alpha := assign(x := new(y)$Symbol, fn(p, x))
  ((n := degree p) = 1) =>  [ alpha ]
  cyclo := cyclotomic(n,monomial(1,1)$SUP)_
               $NumberTheoreticPolynomialFunctions(SUP)
  beta := assign(x := new(y)$Symbol, fn(cyclo, x))
  [alpha*beta**i for i in 0..(n-1)::NonNegativeInteger]

import PolynomialDecomposition(SUP,$)

allroots(p, y, fn) ==
  zero? p => error "allroots: polynomial must be nonzero"
  zero? coefficient(p,0) =>
```

```
     concat(0, allroots(p quo monomial(1,1), y, fn))
  zero?(p1:=reductum p) => empty()
  zero? reductum p1 => binomialRoots(p, y, fn)
  decompList := decompose(p)
  # decompList > 1 =>
      h := last decompList
      g := leftFactor(p,h) :: SUP
      groots := allroots(g, y, fn)
      "append"/[allroots(h-r::SUP, y, fn) for r in groots]
  ans := nil()$List($)
  while not ground? p repeat
    alpha := assign(x := new(y)$Symbol, fn(p, x))
    q     := monomial(1, 1)$SUP - alpha::SUP
    if not zero?(p alpha) then
      p   := p quo q
      ans := concat(alpha, ans)
    else while zero?(p alpha) repeat
      p   := (p exquo q)::SUP
      ans := concat(alpha, ans)
  reverse_! ans
```

———-

— COQ ACF —

```
(* category ACF *)
(*

    SUP ==> SparseUnivariatePolynomial $

    zeroOf : Polynomial(%) -> %
    zeroOf(p:SUP) == assign(x := new(), zeroOf(p, x))

    rootOf : Polynomial(%) -> %
    rootOf(p:SUP) == assign(x := new(), rootOf(p, x))

    zerosOf : Polynomial(%) -> List(%)
    zerosOf(p:SUP) == zerosOf(p, new())

    rootsOf : SparseUnivariatePolynomial(%) -> List(%)
    rootsOf(p:SUP) == rootsOf(p, new())

    rootsOf : (SparseUnivariatePolynomial(%),Symbol) -> List(%)
    rootsOf(p:SUP, y:Symbol) == allroots(p, y, rootOf)

    zerosOf : (SparseUnivariatePolynomial(%),Symbol) -> List(%)
    zerosOf(p:SUP, y:Symbol) == allroots(p, y, zeroOf)

    assign  : (Symbol, $) -> $
    assign(x, f) == (assignSymbol(x, f, $)$Lisp; f)

    zeroOf : Polynomial(%) -> %
    zeroOf(p:Polynomial $) ==
```

```
    empty?(l := variables p) => error "zeroOf: constant polynomial"
    zeroOf(univariate p, first l)

  rootOf : Polynomial(%) -> %
  rootOf(p:Polynomial $) ==
    empty?(l := variables p) => error "rootOf: constant polynomial"
    rootOf(univariate p, first l)

  zerosOf : Polynomial(%) -> List(%)
  zerosOf(p:Polynomial $) ==
    empty?(l := variables p) => error "zerosOf: constant polynomial"
    zerosOf(univariate p, first l)

  rootsOf : Polynomial(%) -> List(%)
  rootsOf(p:Polynomial $) ==
    empty?(l := variables p) => error "rootsOf: constant polynomial"
    rootsOf(univariate p, first l)

  zeroOf : (SparseUnivariatePolynomial(%),Symbol) -> %
  zeroOf(p:SUP, y:Symbol) ==
    zero?(d := degree p) => error "zeroOf: constant polynomial"
    zero? coefficient(p, 0) => 0
    a := leadingCoefficient p
    d = 2 =>
      b := coefficient(p, 1)
      (sqrt(b**2 - 4 * a * coefficient(p, 0)) - b) / (2 * a)
    (r := retractIfCan(reductum p)@Union($,"failed")) case "failed" =>
      rootOf(p, y)
    nthRoot(- (r::$ / a), d)

  binomialRoots: (SUP, Symbol, (SUP, Symbol) -> $) -> List $
  binomialRoots(p, y, fn) ==
    alpha := assign(x := new(y)$Symbol, fn(p, x))
    ((n := degree p) = 1) =>  [ alpha ]
    cyclo := cyclotomic(n,monomial(1,1)$SUP)_
                $NumberTheoreticPolynomialFunctions(SUP)
    beta := assign(x := new(y)$Symbol, fn(cyclo, x))
    [alpha*beta**i for i in 0..(n-1)::NonNegativeInteger]

  import PolynomialDecomposition(SUP,$)

  allroots: (SUP, Symbol, (SUP, Symbol) -> $) -> List $
  allroots(p, y, fn) ==
    zero? p => error "allroots: polynomial must be nonzero"
    zero? coefficient(p,0) =>
       concat(0, allroots(p quo monomial(1,1), y, fn))
    zero?(p1:=reductum p) => empty()
    zero? reductum p1 => binomialRoots(p, y, fn)
    decompList := decompose(p)
    # decompList > 1 =>
        h := last decompList
        g := leftFactor(p,h) :: SUP
        groots := allroots(g, y, fn)
        "append"/[allroots(h-r::SUP, y, fn) for r in groots]
```

```
      ans := nil()$List($)
      while not ground? p repeat
        alpha := assign(x := new(y)$Symbol, fn(p, x))
        q     := monomial(1, 1)$SUP - alpha::SUP
        if not zero?(p alpha) then
          p   := p quo q
          ans := concat(alpha, ans)
        else while zero?(p alpha) repeat
          p   := (p exquo q)::SUP
          ans := concat(alpha, ans)
      reverse_! ans

*)
```

— **ACF.dotabb** —

```
"ACF"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ACF"];
"ACF" -> "FIELD"
"ACF" -> "RADCAT"
```

— **ACF.dotfull** —

```
"AlgebraicallyClosedField()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ACF"];
"AlgebraicallyClosedField()" -> "Field()"
"AlgebraicallyClosedField()" -> "RadicalCategory()"
```

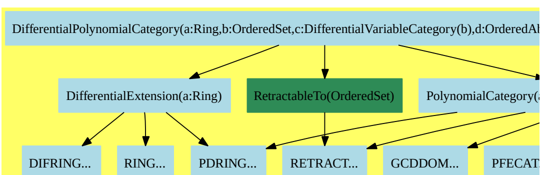
— **ACF.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"AlgebraicallyClosedField()" [color=lightblue];
"AlgebraicallyClosedField()" -> "Field()"
"AlgebraicallyClosedField()" -> "RadicalCategory()"

"Field()" [color=lightblue];
"Field()" -> "EuclideanDomain()"
"Field()" -> "UniqueFactorizationDomain()"
"Field()" -> "DIVRING..."

"EuclideanDomain()" [color=lightblue];
```

```
"EuclideanDomain()" -> "PrincipalIdealDomain()"

"UniqueFactorizationDomain()" [color=lightblue];
"UniqueFactorizationDomain()" -> "GCDDOM..."

"PrincipalIdealDomain()" [color=lightblue];
"PrincipalIdealDomain()" -> "GCDDOM..."

"DIVRING..." [color=lightblue];
"GCDDOM..." [color=lightblue];

"RadicalCategory()" [color=lightblue];
"RadicalCategory()" -> "Category"

"Category" [color=lightblue];
}
```

———-

17.0.216 DifferentialPolynomialCategory (DPOLCAT)



— DifferentialPolynomialCategory.input —

```
)set break resume
)sys rm -f DifferentialPolynomialCategory.output
)spool DifferentialPolynomialCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show DifferentialPolynomialCategory
--R
--R DifferentialPolynomialCategory(R: Ring,S: OrderedSet,V: DifferentialVariableCategory(t#2),E: OrderedAbelianM
--R Abbreviation for DifferentialPolynomialCategory is DPOLCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for DPOLCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
```

```
--R -? : % -> %                         ?/? : (%,R) -> % if R has FIELD
--R ?=? : (%,%) -> Boolean              D : (%,(R -> R)) -> %
--R D : % -> % if R has DIFRING         D : (%,V,NonNegativeInteger) -> %
--R D : (%,List(V)) -> %                D : (%,V) -> %
--R 1 : () -> %                         0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %   ?^? : (%,PositiveInteger) -> %
--R coefficient : (%,E) -> R            coefficients : % -> List(R)
--R coerce : % -> % if R has INTDOM     coerce : S -> %
--R coerce : V -> %                     coerce : R -> %
--R coerce : Integer -> %               coerce : % -> OutputForm
--R content : % -> R if R has GCDDOM    degree : % -> E
--R differentiate : (%,(R -> R)) -> %   differentiate : (%,List(V)) -> %
--R differentiate : (%,V) -> %          eval : (%,List(V),List(%)) -> %
--R eval : (%,V,%) -> %                 eval : (%,List(V),List(R)) -> %
--R eval : (%,V,R) -> %                 eval : (%,List(%),List(%)) -> %
--R eval : (%,%,%) -> %                 eval : (%,Equation(%)) -> %
--R eval : (%,List(Equation(%))) -> %   gcd : (%,%) -> % if R has GCDDOM
--R gcd : List(%) -> % if R has GCDDOM  ground : % -> R
--R ground? : % -> Boolean              hash : % -> SingleInteger
--R initial : % -> %                    isobaric? : % -> Boolean
--R latex : % -> String                 lcm : (%,%) -> % if R has GCDDOM
--R lcm : List(%) -> % if R has GCDDOM  leader : % -> V
--R leadingCoefficient : % -> R         leadingMonomial : % -> %
--R map : ((R -> R),%) -> %             mapExponents : ((E -> E),%) -> %
--R max : (%,%) -> % if R has ORDSET    min : (%,%) -> % if R has ORDSET
--R minimumDegree : % -> E              monomial : (R,E) -> %
--R monomial? : % -> Boolean            monomials : % -> List(%)
--R one? : % -> Boolean                 order : % -> NonNegativeInteger
--R order : (%,S) -> NonNegativeInteger pomopo! : (%,R,E,%) -> %
--R primitiveMonomials : % -> List(%)   recip : % -> Union(%,"failed")
--R reductum : % -> %                   retract : % -> S
--R retract : % -> V                    retract : % -> R
--R sample : () -> %                    separant : % -> %
--R variables : % -> List(V)            weight : % -> NonNegativeInteger
--R zero? : % -> Boolean                ?~=? : (%,%) -> Boolean
--R ?*? : (Fraction(Integer),%) -> % if R has ALGEBRA(FRAC(INT))
--R ?*? : (%,Fraction(Integer)) -> % if R has ALGEBRA(FRAC(INT))
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R D : (%,(R -> R),NonNegativeInteger) -> %
--R D : (%,List(Symbol),List(NonNegativeInteger)) -> % if R has PDRING(SYMBOL)
--R D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING(SYMBOL)
--R D : (%,List(Symbol)) -> % if R has PDRING(SYMBOL)
--R D : (%,Symbol) -> % if R has PDRING(SYMBOL)
--R D : (%,NonNegativeInteger) -> % if R has DIFRING
--R D : (%,List(V),List(NonNegativeInteger)) -> %
--R associates? : (%,%) -> Boolean if R has INTDOM
--R binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if and(has($,CharacteristicNonZero),has(R,PolynomialFactorizationExplici
--R coefficient : (%,List(V),List(NonNegativeInteger)) -> %
--R coefficient : (%,V,NonNegativeInteger) -> %
```

```
--R coerce : Fraction(Integer) -> % if R has RETRACT(FRAC(INT)) or R has ALGEBRA(FRAC(INT))
--R conditionP : Matrix(%) -> Union(Vector(%),"failed") if and(has($,CharacteristicNonZero),has(R,PolynomialFact
--R content : (%,V) -> % if R has GCDDOM
--R convert : % -> InputForm if V has KONVERT(INFORM) and R has KONVERT(INFORM)
--R convert : % -> Pattern(Integer) if V has KONVERT(PATTERN(INT)) and R has KONVERT(PATTERN(INT))
--R convert : % -> Pattern(Float) if V has KONVERT(PATTERN(FLOAT)) and R has KONVERT(PATTERN(FLOAT))
--R degree : (%,S) -> NonNegativeInteger
--R degree : (%,List(V)) -> List(NonNegativeInteger)
--R degree : (%,V) -> NonNegativeInteger
--R differentialVariables : % -> List(S)
--R differentiate : (%,(R -> R),NonNegativeInteger) -> %
--R differentiate : (%,List(Symbol),List(NonNegativeInteger)) -> % if R has PDRING(SYMBOL)
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING(SYMBOL)
--R differentiate : (%,List(Symbol)) -> % if R has PDRING(SYMBOL)
--R differentiate : (%,Symbol) -> % if R has PDRING(SYMBOL)
--R differentiate : (%,NonNegativeInteger) -> % if R has DIFRING
--R differentiate : % -> % if R has DIFRING
--R differentiate : (%,List(V),List(NonNegativeInteger)) -> %
--R differentiate : (%,V,NonNegativeInteger) -> %
--R discriminant : (%,V) -> % if R has COMRING
--R eval : (%,List(S),List(R)) -> % if R has DIFRING
--R eval : (%,S,R) -> % if R has DIFRING
--R eval : (%,List(S),List(%)) -> % if R has DIFRING
--R eval : (%,S,%) -> % if R has DIFRING
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R factor : % -> Factored(%) if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial(%) -> Factored(SparseUnivariatePolynomial(%)) if R has PFECAT
--R factorSquareFreePolynomial : SparseUnivariatePolynomial(%) -> Factored(SparseUnivariatePolynomial(%)) if R h
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R isExpt : % -> Union(Record(var: V,exponent: NonNegativeInteger),"failed")
--R isPlus : % -> Union(List(%),"failed")
--R isTimes : % -> Union(List(%),"failed")
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %) if R has GCDDOM
--R mainVariable : % -> Union(V,"failed")
--R makeVariable : % -> (NonNegativeInteger -> %) if R has DIFRING
--R makeVariable : S -> (NonNegativeInteger -> %)
--R minimumDegree : (%,List(V)) -> List(NonNegativeInteger)
--R minimumDegree : (%,V) -> NonNegativeInteger
--R monicDivide : (%,%,V) -> Record(quotient: %,remainder: %)
--R monomial : (%,List(V),List(NonNegativeInteger)) -> %
--R monomial : (%,V,NonNegativeInteger) -> %
--R multivariate : (SparseUnivariatePolynomial(%),V) -> %
--R multivariate : (SparseUnivariatePolynomial(R),V) -> %
--R numberOfMonomials : % -> NonNegativeInteger
--R patternMatch : (%,Pattern(Integer),PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%) if V has
--R patternMatch : (%,Pattern(Float),PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%) if V has PATMAB
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (%,V) -> % if R has GCDDOM
--R primitivePart : % -> % if R has GCDDOM
--R reducedSystem : Matrix(%) -> Matrix(R)
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(R),vec: Vector(R))
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(Integer),vec: Vector(Integer)) if R has LINEXP(I
--R reducedSystem : Matrix(%) -> Matrix(Integer) if R has LINEXP(INT)
```

```
--R resultant : (%,%,V) -> % if R has COMRING
--R retract : % -> Integer if R has RETRACT(INT)
--R retract : % -> Fraction(Integer) if R has RETRACT(FRAC(INT))
--R retractIfCan : % -> Union(S,"failed")
--R retractIfCan : % -> Union(V,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT(INT)
--R retractIfCan : % -> Union(Fraction(Integer),"failed") if R has RETRACT(FRAC(INT))
--R retractIfCan : % -> Union(R,"failed")
--R solveLinearPolynomialEquation : (List(SparseUnivariatePolynomial(%)),SparseUnivariatePolynomial(%)) -> Union
--R squareFree : % -> Factored(%) if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial(%) -> Factored(SparseUnivariatePolynomial(%)) if R has PFE
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R totalDegree : (%,List(V)) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial(R)
--R univariate : (%,V) -> SparseUnivariatePolynomial(%)
--R weight : (%,S) -> NonNegativeInteger
--R weights : (%,S) -> List(NonNegativeInteger)
--R weights : % -> List(NonNegativeInteger)
--R
--E 1

)spool
)lisp (bye)
```

— **DifferentialPolynomialCategory.help** —

```
====================================================================
DifferentialPolynomialCategory examples
====================================================================

DifferentialPolynomialCategory is a category constructor specifying
basic functions in an ordinary differential polynomial ring with a
given ordered set of differential indeterminates. In addition, it
implements defaults for the basic functions.

The functions order and weight are extended from the set of
derivatives of differential indeterminates to the set of differential
polynomials.  Other operations provided on differential polynomials are
leader, initial, separant, differentialVariables, and isobaric?.
Furthermore, if the ground ring is a differential ring, then evaluation
(substitution of differential indeterminates by elements of the ground ring
or by differential polynomials) is provided by eval.

A convenient way of referencing derivatives is provided by the functions
makeVariable.

To construct a domain using this constructor, one needs to provide a
```

```
ground ring R, an ordered set S of differential indeterminates, a ranking
V on the set of derivatives of the differential indeterminates, and a set
E of exponents in bijection with the set of differential monomials
in the given differential indeterminates.

See Also:
o )show DifferentialPolynomialCategory
```

———-

See:

⇐ "DifferentialExtension" (DIFEXT) 11.0.177 on page 1127
⇐ "PolynomialCategory" (POLYCAT) 16.0.213 on page 1449
⇐ "RetractableTo" (RETRACT) 2.0.37 on page 110

Exports:

0	1	associates?
binomThmExpt	characteristic	charthRoot
coefficient	coefficients	coerce
conditionP	content	convert
D	degree	differentialVariables
differentiate	discriminant	eval
exquo	factor	factorPolynomial
factorSquareFreePolynomial	gcd	gcdPolynomial
ground	ground?	hash
initial	isExpt	isobaric?
isPlus	isTimes	latex
lcm	leader	leadingCoefficient
leadingMonomial	makeVariable	map
mapExponents	max	min
minimumDegree	monicDivide	monomial
monomial?	monomials	multivariate
numberOfMonomials	one?	order
patternMatch	pomopo!	prime?
primitiveMonomials	primitivePart	recip
reducedSystem	reductum	resultant
retract	retractIfCan	sample
separant	solveLinearPolynomialEquation	squareFree
squareFreePart	squareFreePolynomial	subtractIfCan
totalDegree	unit?	unitCanonical
unitNormal	univariate	variables
weight	weights	zero?
?*?	?**?	?+?
?-?	-?	?=?
?^?	?~=?	?/?
?<?	?<=?	?>?
?>=?		

Attributes Exported:

-
- if R has canonicalUnitNormal then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if $ has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x*y \neq 0$ implies both x and y are non-zero.
- if $ has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation $" * ": (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are implemented by this category:

```
coerce : S -> %
degree : (%,S) -> NonNegativeInteger
differentialVariables : % -> List S
differentiate : (%,(R -> R)) -> %
eval : (%,List S,List R) -> % if R has DIFRING
eval : (%,List S,List %) -> % if R has DIFRING
eval : (%,List Equation %) -> %
initial : % -> %
isobaric? : % -> Boolean
leader : % -> V
makeVariable : S -> (NonNegativeInteger -> %)
makeVariable : % -> (NonNegativeInteger -> %) if R has DIFRING
order : % -> NonNegativeInteger
order : (%,S) -> NonNegativeInteger
retractIfCan : % -> Union(S,"failed")
separant : % -> %
weight : % -> NonNegativeInteger
weight : (%,S) -> NonNegativeInteger
weights : (%,S) -> List NonNegativeInteger
weights : % -> List NonNegativeInteger
```

These exports come from (p1449) PolynomialCategory(R,E,V)
where R:Ring, E:OrderedAbelianMonoidSup,
V:DifferentialVariableCategory(S:OrderedSet):

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean if R has INTDOM
binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed")
  if and(has($,CharacteristicNonZero),
     has(R,PolynomialFactorizationExplicit))
  or R has CHARNZ
coefficient : (%,List V,List NonNegativeInteger) -> %
```

```
coefficient : (%,V,NonNegativeInteger) -> %
coefficient : (%,E) -> R
coefficients : % -> List R
coerce : R -> %
coerce : Fraction Integer -> %
  if R has RETRACT FRAC INT
  or R has ALGEBRA FRAC INT
coerce : % -> % if R has INTDOM
coerce : Integer -> %
coerce : % -> OutputForm
coerce : V -> %
conditionP : Matrix % -> Union(Vector %,"failed")
  if and(has($,CharacteristicNonZero),
         has(R,PolynomialFactorizationExplicit))
content : % -> R if R has GCDDOM
content : (%,V) -> % if R has GCDDOM
convert : % -> Pattern Integer
  if V has KONVERT PATTERN INT
  and R has KONVERT PATTERN INT
convert : % -> Pattern Float
  if V has KONVERT PATTERN FLOAT
  and R has KONVERT PATTERN FLOAT
convert : % -> InputForm
  if V has KONVERT INFORM
  and R has KONVERT INFORM
D : (%,List V) -> %
D : (%,V) -> %
D : (%,List V,List NonNegativeInteger) -> %
D : (%,V,NonNegativeInteger) -> %
degree : % -> E
degree : (%,List V) -> List NonNegativeInteger
degree : (%,V) -> NonNegativeInteger
differentiate : (%,V) -> %
differentiate : (%,List V,List NonNegativeInteger) -> %
differentiate : (%,V,NonNegativeInteger) -> %
differentiate : (%,List V) -> %
discriminant : (%,V) -> % if R has COMRING
eval : (%,Equation %) -> %
eval : (%,List %,List %) -> %
eval : (%,%,%) -> %
eval : (%,List V,List R) -> %
eval : (%,V,R) -> %
eval : (%,List V,List %) -> %
eval : (%,V,%) -> %
exquo : (%,%) -> Union(%,"failed") if R has INTDOM
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
factor : % -> Factored % if R has PFECAT
factorPolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if R has PFECAT
factorSquareFreePolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
```

```
      if R has PFECAT
gcd : (%,%) -> % if R has GCDDOM
gcd : List % -> % if R has GCDDOM
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
       if R has GCDDOM
ground : % -> R
ground? : % -> Boolean
hash : % -> SingleInteger
isExpt : % ->
  Union(Record(var: V,exponent: NonNegativeInteger),"failed")
isPlus : % -> Union(List %,"failed")
isTimes : % -> Union(List %,"failed")
latex : % -> String
lcm : (%,%) -> % if R has GCDDOM
lcm : List % -> % if R has GCDDOM
leadingCoefficient : % -> R
leadingMonomial : % -> %
mainVariable : % -> Union(V,"failed")
map : ((R -> R),%) -> %
mapExponents : ((E -> E),%) -> %
max : (%,%) -> % if R has ORDSET
min : (%,%) -> % if R has ORDSET
minimumDegree : % -> E
minimumDegree : (%,List V) -> List NonNegativeInteger
minimumDegree : (%,V) -> NonNegativeInteger
monicDivide : (%,%,V) -> Record(quotient: %,remainder: %)
monomial : (%,V,NonNegativeInteger) -> %
monomial : (%,List V,List NonNegativeInteger) -> %
monomial : (R,E) -> %
monomial? : % -> Boolean
monomials : % -> List %
multivariate : (SparseUnivariatePolynomial %,V) -> %
multivariate : (SparseUnivariatePolynomial R,V) -> %
numberOfMonomials : % -> NonNegativeInteger
one? : % -> Boolean
patternMatch :
  (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
    PatternMatchResult(Integer,%)
      if V has PATMAB INT
      and R has PATMAB INT
patternMatch :
  (%,Pattern Float,PatternMatchResult(Float,%)) ->
    PatternMatchResult(Float,%)
      if V has PATMAB FLOAT
      and R has PATMAB FLOAT
pomopo! : (%,R,E,%) -> %
prime? : % -> Boolean if R has PFECAT
primitiveMonomials : % -> List %
primitivePart : (%,V) -> % if R has GCDDOM
primitivePart : % -> % if R has GCDDOM
recip : % -> Union(%,"failed")
```

```
reducedSystem : Matrix % -> Matrix R
reducedSystem :
  (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
reducedSystem :
  (Matrix %,Vector %) ->
    Record(mat: Matrix Integer,vec: Vector Integer)
      if R has LINEXP INT
reducedSystem : Matrix % -> Matrix Integer
    if R has LINEXP INT
reductum : % -> %
resultant : (%,%,V) -> % if R has COMRING
retract : % -> R
retract : % -> Integer if R has RETRACT INT
retract : % -> Fraction Integer if R has RETRACT FRAC INT
retract : % -> V
retractIfCan : % -> Union(R,"failed")
retractIfCan : % -> Union(Integer,"failed")
    if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
    if R has RETRACT FRAC INT
retractIfCan : % -> Union(V,"failed")
sample : () -> %
solveLinearPolynomialEquation :
  (List SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
    Union(List SparseUnivariatePolynomial %,"failed")
      if R has PFECAT
squareFree : % -> Factored % if R has GCDDOM
squareFreePart : % -> % if R has GCDDOM
squareFreePolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if R has PFECAT
subtractIfCan : (%,%) -> Union(%,"failed")
totalDegree : (%,List V) -> NonNegativeInteger
totalDegree : % -> NonNegativeInteger
unit? : % -> Boolean if R has INTDOM
unitCanonical : % -> % if R has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if R has INTDOM
univariate : % -> SparseUnivariatePolynomial R
univariate : (%,V) -> SparseUnivariatePolynomial %
variables : % -> List V
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,R) -> %
?*? : (R,%) -> %
?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
```

```
?*? : (NonNegativeInteger,%) -> %
?/? : (%,R) -> % if R has FIELD
?-? : (%,%) -> %
-? : % -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?<? : (%,%) -> Boolean if R has ORDSET
?<=? : (%,%) -> Boolean if R has ORDSET
?>? : (%,%) -> Boolean if R has ORDSET
?>=? : (%,%) -> Boolean if R has ORDSET
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
```

These exports come from (p1127) DifferentialExtension(R:Ring):

```
D : (%,(R -> R)) -> %
D : (%,(R -> R),NonNegativeInteger) -> %
D : % -> % if R has DIFRING
D : (%,NonNegativeInteger) -> % if R has DIFRING
D : (%,Symbol) -> % if R has PDRING SYMBOL
D : (%,List Symbol) -> % if R has PDRING SYMBOL
D : (%,Symbol,NonNegativeInteger) -> %
    if R has PDRING SYMBOL
D : (%,List Symbol,List NonNegativeInteger) -> %
    if R has PDRING SYMBOL
differentiate : (%,NonNegativeInteger) -> %
    if R has DIFRING
differentiate : (%,List Symbol) -> %
    if R has PDRING SYMBOL
differentiate : (%,Symbol,NonNegativeInteger) -> %
    if R has PDRING SYMBOL
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
    if R has PDRING SYMBOL
differentiate : % -> % if R has DIFRING
differentiate : (%,(R -> R),NonNegativeInteger) -> %
differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
```

These exports come from (p110) RetractableTo(S:OrderedSet):

```
retract : % -> S
```

These exports come from (p64) InnerEvalable(S,R)
where S:OrderedSet, R:Ring:

```
eval : (%,S,R) -> % if R has DIFRING
```

These exports come from (p64) InnerEvalable(S,where S:OrderedSet,

```
eval : (%,S,%) -> % if R has DIFRING
```

These exports come from (p151) Evalable(

— DifferentialPolynomialCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#DPOLCAT">
```

```
DifferentialPolynomialCategory (DPOLCAT)</a></h2>
</body>
```

—— category DPOLCAT DifferentialPolynomialCategory ——

```
)abbrev category DPOLCAT DifferentialPolynomialCategory
++ Author:  William Sit
++ Date Created: 19 July 1990
++ Date Last Updated: 13 September 1991
++ References:Kolchin, E.R. "Differential Algebra and Algebraic Groups"
++   (Academic Press, 1973).
++ Description:
++ \spadtype{DifferentialPolynomialCategory} is a category constructor
++ specifying basic functions in an ordinary differential polynomial
++ ring with a given ordered set of differential indeterminates.
++ In addition, it implements defaults for the basic functions.
++ The functions \spadfun{order} and \spadfun{weight} are extended
++ from the set of derivatives of differential indeterminates
++ to the set of differential polynomials.  Other operations
++ provided on differential polynomials are
++ \spadfun{leader}, \spadfun{initial},
++ \spadfun{separant}, \spadfun{differentialVariables}, and
++ \spadfun{isobaric?}.   Furthermore, if the ground ring is
++ a differential ring, then evaluation (substitution
++ of differential indeterminates by elements of the ground ring
++ or by differential polynomials) is
++ provided by \spadfun{eval}.
++ A convenient way of referencing derivatives is provided by
++ the functions \spadfun{makeVariable}.
++
++ To construct a domain using this constructor, one needs
++ to provide a ground ring R, an ordered set S of differential
++ indeterminates, a ranking V on the set of derivatives
++ of the differential indeterminates, and a set E of
++ exponents in bijection with the set of differential monomials
++ in the given differential indeterminates.

DifferentialPolynomialCategory(R,S,V,E) : Category == SIG where
  R : Ring
  S : OrderedSet
  V : DifferentialVariableCategory(S)
  E : OrderedAbelianMonoidSup

  PC ==> PolynomialCategory(R,E,V)
  DE ==> DifferentialExtension(R)
  RT ==> RetractableTo(S)

  SIG ==> Join(PC,DE,RT) with

    -- Examples:
    -- s:=makeVariable('s)
    -- p:= 3*(s 1)**2 + s*(s 2)**3
```

```
--  all functions below have default implementations
--  using primitives from V

makeVariable : S -> (NonNegativeInteger -> $)
   ++ makeVariable(s) views s as a differential
   ++ indeterminate,  in such a way that the n-th
   ++ derivative of s may be simply referenced as z.n
   ++ where z :=makeVariable(s).
   ++ Note that In the interpreter, z is
   ++ given as an internal map, which may be ignored.
   -- Example: makeVariable('s); %.5

differentialVariables : $ ->  List S
  ++ differentialVariables(p) returns a list of differential
  ++ indeterminates occurring in a differential polynomial p.

order : ($, S) -> NonNegativeInteger
  ++ order(p,s) returns the order of the differential
  ++ polynomial p in differential indeterminate s.

order : $   -> NonNegativeInteger
  ++ order(p) returns the order of the differential polynomial p,
  ++ which is the maximum number of differentiations of a
  ++ differential indeterminate, among all those appearing in p.

degree : ($, S) -> NonNegativeInteger
  ++ degree(p, s) returns the maximum degree of
  ++ the differential polynomial p viewed as a differential polynomial
  ++ in the differential indeterminate s alone.

weights : $ -> List NonNegativeInteger
  ++ weights(p) returns a list of weights of differential monomials
  ++ appearing in differential polynomial p.

weight : $   -> NonNegativeInteger
  ++ weight(p) returns the maximum weight of all differential monomials
  ++ appearing in the differential polynomial p.

weights : ($, S) -> List NonNegativeInteger
  ++ weights(p, s) returns a list of
  ++ weights of differential monomials
  ++ appearing in the differential polynomial p when p is viewed
  ++ as a differential polynomial in the differential indeterminate s
  ++ alone.

weight : ($, S) -> NonNegativeInteger
  ++ weight(p, s) returns the maximum weight of all differential
  ++ monomials appearing in the differential polynomial p
  ++ when p is viewed as a differential polynomial in
  ++ the differential indeterminate s alone.

isobaric? : $ -> Boolean
  ++ isobaric?(p) returns true if every differential monomial appearing
  ++ in the differential polynomial p has same weight,
```

```
      ++ and returns false otherwise.

    leader : $   -> V
      ++ leader(p) returns the derivative of the highest rank
      ++ appearing in the differential polynomial p
      ++ Note that an error occurs if p is in the ground ring.

    initial : $   -> $
      ++ initial(p) returns the
      ++ leading coefficient when the differential polynomial p
      ++ is written as a univariate polynomial in its leader.

    separant : $  -> $
      ++ separant(p) returns the
      ++ partial derivative of the differential polynomial p
      ++ with respect to its leader.

    if R has DifferentialRing then

      InnerEvalable(S, R)

      InnerEvalable(S, $)

      Evalable $

      makeVariable : $ -> (NonNegativeInteger -> $)
       ++ makeVariable(p) views p as an element of a differential
       ++ ring,  in such a way that the n-th
       ++ derivative of p may be simply referenced as z.n
       ++ where z := makeVariable(p).
       ++ Note that In the interpreter, z is
       ++ given as an internal map, which may be ignored.
       -- Example: makeVariable(p); %.5; makeVariable(%**2); %.2

  add

    p:$
    s:S

    makeVariable s == n +-> makeVariable(s,n)::$

    if R has IntegralDomain then
      differentiate(p:$, d:R -> R) ==
        ans:$ := 0
        l := variables p
        while (u:=retractIfCan(p)@Union(R, "failed")) case "failed" repeat
          t := leadingMonomial p
          lc := leadingCoefficient t
          ans := ans + d(lc)::$ * (t exquo lc)::$
              + +/[differentiate(t, v) * (differentiate v)::$ for v in l]
          p := reductum p
        ans + d(u::R)::$

    order (p:$):NonNegativeInteger ==
```

```
  ground? p => 0
  "max"/[order v for v in variables p]

order (p:$,s:S):NonNegativeInteger ==
  ground? p => 0
  empty? (vv:= [order v for v in variables p | (variable v) = s ]) =>0
  "max"/vv

degree (p, s) ==
  d:NonNegativeInteger:=0
  for lp in monomials p repeat
    lv:= [v for v in variables lp | (variable v) = s ]
    if not empty? lv then d:= max(d, +/degree(lp, lv))
  d

weights p ==
  ws:List NonNegativeInteger := nil
  empty? (mp:=monomials p) => ws
  for lp in mp repeat
    lv:= variables lp
    if not empty? lv then
      dv:= degree(lp, lv)
      w:=+/[(weight v) * d _
             for v in lv for d in dv]$(List NonNegativeInteger)
      ws:= concat(ws, w)
  ws

weight p ==
  empty? (ws:=weights p) => 0
  "max"/ws

weights (p, s) ==
  ws:List NonNegativeInteger := nil
  empty?(mp:=monomials p) => ws
  for lp in mp repeat
    lv:= [v for v in variables lp | (variable v) = s ]
    if not empty? lv then
      dv:= degree(lp, lv)
      w:=+/[(weight v) * d _
           for v in lv for d in dv]$(List NonNegativeInteger)
      ws:= concat(ws, w)
  ws

weight (p,s)  ==
  empty? (ws:=weights(p,s)) => 0
  "max"/ws

isobaric? p == (# removeDuplicates weights p) = 1

leader p ==             -- depends on the ranking
  vl:= variables p
  -- it's not enough just to look at leadingMonomial p
  -- the term-ordering need not respect the ranking
  empty? vl => error "leader is not defined "
```

```
    "max"/vl

  initial p == leadingCoefficient univariate(p,leader p)

  separant p == differentiate(p, leader p)

  coerce(s:S):$   == s::V::$

  retractIfCan(p:$):Union(S, "failed") ==
    (v := retractIfCan(p)@Union(V,"failed")) case "failed" => "failed"
    retractIfCan(v::V)

  differentialVariables p ==
    removeDuplicates [variable v for v in variables p]

  if R has DifferentialRing then

    makeVariable p == n +-> differentiate(p, n)

    eval(p:$, sl:List S, rl:List R) ==
      ordp:= order p
      vl  := concat [[makeVariable(s,j)$V for j in  0..ordp]
                              for s in sl]$List(List V)
      rrl:=nil$List(R)
      for r in rl repeat
        t:= r
        rrl:= concat(rrl,
              concat(r, [t := differentiate t for i in 1..ordp]))
      eval(p, vl, rrl)

    eval(p:$, sl:List S, rl:List $) ==
      ordp:= order p
      vl  := concat [[makeVariable(s,j)$V for j in  0..ordp]
                              for s in sl]$List(List V)
      rrl:=nil$List($)
      for r in rl repeat
        t:=r
        rrl:=concat(rrl,
             concat(r, [t:=differentiate t for i in 1..ordp]))
      eval(p, vl, rrl)

    eval(p:$, l:List Equation $) ==
      eval(p, [retract(lhs e)@S for e in l]$List(S),
            [rhs e for e in l]$List($))
```

———

— COQ DPOLCAT —

```
(* category DPOLCAT *)
(*
    p:$
    s:S
```

```
makeVariable : % -> (NonNegativeInteger -> %)
makeVariable s == n +-> makeVariable(s,n)::$

if R has IntegralDomain then

  differentiate : (%,(R -> R)) -> %
  differentiate(p:$, d:R -> R) ==
    ans:$ := 0
    l := variables p
    while (u:=retractIfCan(p)@Union(R, "failed")) case "failed" repeat
      t := leadingMonomial p
      lc := leadingCoefficient t
      ans := ans + d(lc)::$ * (t exquo lc)::$
          + +/[differentiate(t, v) * (differentiate v)::$ for v in l]
      p := reductum p
    ans + d(u::R)::$

order : % -> NonNegativeInteger
order (p:$):NonNegativeInteger ==
  ground? p => 0
  "max"/[order v for v in variables p]

order : (%,S) -> NonNegativeInteger
order (p:$,s:S):NonNegativeInteger ==
  ground? p => 0
  empty? (vv:= [order v for v in variables p | (variable v) = s ]) =>0
  "max"/vv

degree : (%,S) -> NonNegativeInteger
degree (p, s) ==
  d:NonNegativeInteger:=0
  for lp in monomials p repeat
    lv:= [v for v in variables lp | (variable v) = s ]
    if not empty? lv then d:= max(d, +/degree(lp, lv))
  d

weights : % -> List(NonNegativeInteger)
weights p ==
  ws:List NonNegativeInteger := nil
  empty? (mp:=monomials p) => ws
  for lp in mp repeat
    lv:= variables lp
    if not empty? lv then
      dv:= degree(lp, lv)
      w:=+/[(weight v) * d _
             for v in lv for d in dv]$(List NonNegativeInteger)
      ws:= concat(ws, w)
  ws

weight : % -> NonNegativeInteger
weight p ==
  empty? (ws:=weights p) => 0
  "max"/ws
```

```
weights : (%,S) -> List(NonNegativeInteger)
weights (p, s) ==
  ws:List NonNegativeInteger := nil
  empty?(mp:=monomials p) => ws
  for lp in mp repeat
    lv:= [v for v in variables lp | (variable v) = s ]
    if not empty? lv then
      dv:= degree(lp, lv)
      w:=+/[(weight v) * d _
           for v in lv for d in dv]$(List NonNegativeInteger)
      ws:= concat(ws, w)
  ws

weight : (%,S) -> NonNegativeInteger
weight (p,s)  ==
  empty? (ws:=weights(p,s)) => 0
  "max"/ws

isobaric? : % -> Boolean
isobaric? p == (# removeDuplicates weights p) = 1

leader : % -> V
leader p ==             -- depends on the ranking
  vl:= variables p
  -- it's not enough just to look at leadingMonomial p
  -- the term-ordering need not respect the ranking
  empty? vl => error "leader is not defined "
  "max"/vl

initial : % -> %
initial p == leadingCoefficient univariate(p,leader p)

separant : % -> %
separant p == differentiate(p, leader p)

coerce : S -> %
coerce(s:S):$   == s::V::$

retractIfCan : % -> Union(S,"failed")
retractIfCan(p:$):Union(S, "failed") ==
  (v := retractIfCan(p)@Union(V,"failed")) case "failed" => "failed"
  retractIfCan(v::V)

differentialVariables : % -> List(S)
differentialVariables p ==
  removeDuplicates [variable v for v in variables p]

if R has DifferentialRing then

  makeVariable : % -> (NonNegativeInteger -> %)
  makeVariable p == n +-> differentiate(p, n)

  eval : (%,List(S),List(R)) -> %
```

```
      eval(p:$, sl:List S, rl:List R) ==
        ordp:= order p
        vl  := concat [[makeVariable(s,j)$V for j in  0..ordp]
                                for s in sl]$List(List V)
        rrl:=nil$List(R)
        for r in rl repeat
          t:= r
          rrl:= concat(rrl,
                concat(r, [t := differentiate t for i in 1..ordp]))
        eval(p, vl, rrl)

      eval : (%,List(S),List(%)) -> %
      eval(p:$, sl:List S, rl:List $) ==
        ordp:= order p
        vl  := concat [[makeVariable(s,j)$V for j in  0..ordp]
                                for s in sl]$List(List V)
        rrl:=nil$List($)
        for r in rl repeat
          t:=r
          rrl:=concat(rrl,
               concat(r, [t:=differentiate t for i in 1..ordp]))
        eval(p, vl, rrl)

      eval : (%,List(Equation(%))) -> %
      eval(p:$, l:List Equation $) ==
        eval(p, [retract(lhs e)@S for e in l]$List(S),
              [rhs e for e in l]$List($))
*)
```

———-

— **DPOLCAT.dotabb** —

```
"DPOLCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DPOLCAT"];
"DPOLCAT" -> "DIFEXT"
"DPOLCAT" -> "POLYCAT"
"DPOLCAT" -> "RETRACT"
```

———-

— **DPOLCAT.dotfull** —

```
"DifferentialPolynomialCategory(a:Ring,b:OrderedSet,c:DifferentialVariableCategory(b),d:OrderedAbelianMonoidSup)
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DPOLCAT"];
"DifferentialPolynomialCategory(a:Ring,b:OrderedSet,c:DifferentialVariableCategory(b),d:OrderedAbelianMonoidSup)
 -> "DifferentialExtension(a:Ring)"
"DifferentialPolynomialCategory(a:Ring,b:OrderedSet,c:DifferentialVariableCategory(b),d:OrderedAbelianMonoidSup)
 -> "PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
"DifferentialPolynomialCategory(a:Ring,b:OrderedSet,c:DifferentialVariableCategory(b),d:OrderedAbelianMonoidSup)
 -> "RetractableTo(OrderedSet)"
```

———-

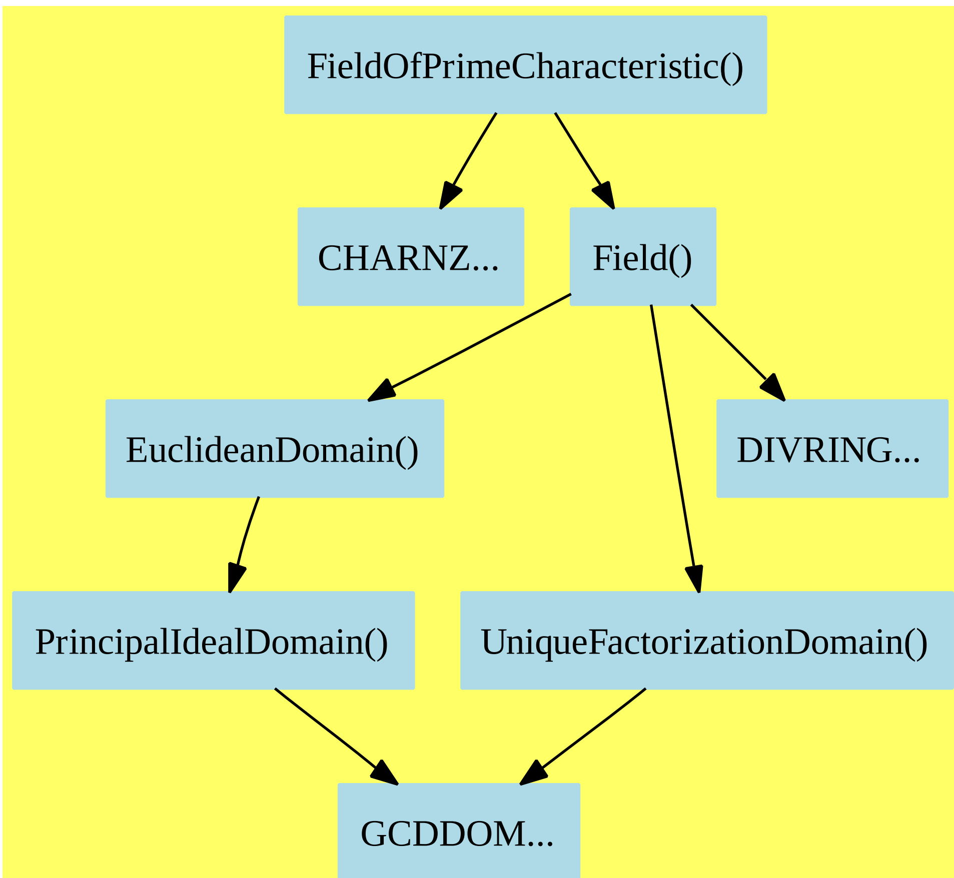
— DPOLCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"DifferentialPolynomialCategory(a:Ring,b:OrderedSet,c:DifferentialVariableCategory(b),d:OrderedAbelianMonoidSup)
 [color=lightblue];

"DifferentialPolynomialCategory(a:Ring,b:OrderedSet,c:DifferentialVariableCategory(b),d:OrderedAbelianMonoidSup)
 -> "DifferentialExtension(a:Ring)"

"DifferentialPolynomialCategory(a:Ring,b:OrderedSet,c:DifferentialVariableCategory(b),d:OrderedAbelianMonoidSup)
 -> "PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"

"DifferentialPolynomialCategory(a:Ring,b:OrderedSet,c:DifferentialVariableCategory(b),d:OrderedAbelianMonoidSup)
 -> "RetractableTo(OrderedSet)"

"DifferentialExtension(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIFEXT"];
"DifferentialExtension(a:Ring)" -> "RING..."
"DifferentialExtension(a:Ring)" -> "DIFRING..."
"DifferentialExtension(a:Ring)" -> "PDRING..."

"RetractableTo(OrderedSet)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(OrderedSet)" -> "RETRACT..."

"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=POLYCAT"];
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "PDRING..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "FAMR..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "EVALAB..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "IEVALAB..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "RETRACT..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "FLINEXP..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "ORDSET..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "GCDDOM..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "PFECAT..."

"DIFRING..." [color=lightblue];
"RING..." [color=lightblue];
```

```
"PDRING..." [color=lightblue];
"FAMR..." [color=lightblue];
"EVALAB..." [color=lightblue];
"IEVALAB..." [color=lightblue];
"RETRACT..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"ORDSET..." [color=lightblue];
"GCDDOM..." [color=lightblue];
"PFECAT..." [color=lightblue];

}
```

———-

17.0.217 FieldOfPrimeCharacteristic (FPC)



— **FieldOfPrimeCharacteristic.input** —

```
)set break resume
```

```
)sys rm -f FieldOfPrimeCharacteristic.output
)spool FieldOfPrimeCharacteristic.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FieldOfPrimeCharacteristic
--R
--R FieldOfPrimeCharacteristic is a category constructor
--R Abbreviation for FieldOfPrimeCharacteristic is FPC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FPC
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Fraction(Integer),%) -> %      ?*? : (%,Fraction(Integer)) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,Integer) -> %               ?**? : (%,NonNegativeInteger) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (%,%) -> %                      ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,Integer) -> %                ?^? : (%,NonNegativeInteger) -> %
--R ?^? : (%,PositiveInteger) -> %        associates? : (%,%) -> Boolean
--R coerce : Fraction(Integer) -> %       coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R factor : % -> Factored(%)             gcd : List(%) -> %
--R gcd : (%,%) -> %                      hash : % -> SingleInteger
--R inv : % -> %                          latex : % -> String
--R lcm : List(%) -> %                    lcm : (%,%) -> %
--R one? : % -> Boolean                   prime? : % -> Boolean
--R primeFrobenius : % -> %               ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")        ?rem? : (%,%) -> %
--R sample : () -> %                      sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored(%)         squareFreePart : % -> %
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed")
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed")
--R order : % -> OnePointCompletion(PositiveInteger)
--R primeFrobenius : (%,NonNegativeInteger) -> %
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %)
--R subtractIfCan : (%,%) -> Union(%,"failed")
```

```
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

— **FieldOfPrimeCharacteristic.help** —

```
====================================================================
FieldOfPrimeCharacteristic examples
====================================================================

FieldOfPrimeCharacteristic is the category of fields of prime
characteristic, for example, finite fields, algebraic closures of
fields of prime characteristic, transcendental extensions of
of fields of prime characteristic.

See Also:
o )show FieldOfPrimeCharacteristic
```

See:

⇒ "FiniteFieldCategory" (FFIELDC) 18.0.231 on page 1762
⇐ "CharacteristicNonZero" (CHARNZ) 10.0.159 on page 1010
⇐ "Field" (FIELD) 16.0.209 on page 1413

Exports:

0	1	associates?	characteristic
charthRoot	coerce	discreteLog	divide
euclideanSize	expressIdealMember	exquo	extendedEuclidean
factor	gcd	gcdPolynomial	hash
inv	latex	lcm	multiEuclidean
one?	order	prime?	primeFrobenius
principalIdeal	recip	sample	sizeLess?
squareFree	squareFreePart	subtractIfCan	unit?
unitCanonical	unitNormal	zero?	?*?
?**?	?+?	?-?	-?
?/?	?=?	?^?	?rem?
?quo?	?~=?		

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
order : % -> OnePointCompletion PositiveInteger
discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
```

These are implemented by this category:

```
primeFrobenius : % -> %
primeFrobenius : (%,NonNegativeInteger) -> %
```

These exports come from (p1413) Field():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
coerce : Fraction Integer -> %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
exquo : (%,%) -> Union(%,"failed")
factor : % -> Factored %
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                     SparseUnivariatePolynomial %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
sample : () -> %
sizeLess? : (%,%) -> Boolean
squareFree : % -> Factored %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
```

```
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,Integer) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,Integer) -> %
?/? : (%,%) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p1010) CharacteristicNonZero():

```
charthRoot : % -> Union(%,"failed")
```

See: Grabmeier[Grab92]

— FieldOfPrimeCharacteristic.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FPC">
FieldOfPrimeCharacteristic (FPC)</a></h2>
</body>
```

— category FPC FieldOfPrimeCharacteristic —

```
)abbrev category FPC FieldOfPrimeCharacteristic
++ Author: J. Grabmeier, A. Scheerhorn
++ Date Created: 10 March 1991
++ Date Last Updated: 31 March 1991
++ References:
++ Grab92 Finite Fields in Axiom
++ Description:
++ FieldOfPrimeCharacteristic is the category of fields of prime
++ characteristic, for example, finite fields, algebraic closures of
++ fields of prime characteristic, transcendental extensions of
++ of fields of prime characteristic.

FieldOfPrimeCharacteristic() : Category == SIG where
```

```
  SIG ==> Join(Field,CharacteristicNonZero) with

    order : $ -> OnePointCompletion PositiveInteger
      ++ order(a) computes the order of an element in the multiplicative
      ++ group of the field.
      ++ Error: if \spad{a} is 0.

    discreteLog : ($,$) -> Union(NonNegativeInteger,"failed")
      ++ discreteLog(b,a) computes s with \spad{b**s = a} if such an s exists.

    primeFrobenius : $ -> $
      ++ primeFrobenius(a) returns \spad{a**p} where p is the characteristic.

    primeFrobenius : ($,NonNegativeInteger) -> $
      ++ primeFrobenius(a,s) returns \spad{a**(p**s)} where p
      ++ is the characteristic.

   add

     primeFrobenius(a) == a ** characteristic()

     primeFrobenius(a,s) == a ** (characteristic()**s)
```

———-

— COQ FPC —

```
(* category FPC *)
(*
   primeFrobenius(a) == a ** characteristic()
   primeFrobenius(a,s) == a ** (characteristic()**s)

*)
```

———-

— FPC.dotabb —

```
"FPC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FPC"];
"FPC" -> "CHARNZ"
"FPC" -> "FIELD"
```

———-

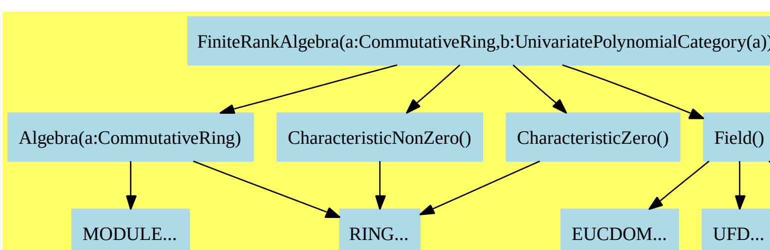
— FPC.dotfull —

```
"FieldOfPrimeCharacteristic()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FPC"];
"FieldOfPrimeCharacteristic()" -> "CharacteristicNonZero()"
```

— FPC.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FieldOfPrimeCharacteristic()" [color=lightblue];
"FieldOfPrimeCharacteristic()" -> "CHARNZ..."
"FieldOfPrimeCharacteristic()" -> "Field()"

"Field()" [color=lightblue];
"Field()" -> "EuclideanDomain()"
"Field()" -> "UniqueFactorizationDomain()"
"Field()" -> "DIVRING..."

"EuclideanDomain()" [color=lightblue];
"EuclideanDomain()" -> "PrincipalIdealDomain()"

"UniqueFactorizationDomain()" [color=lightblue];
"UniqueFactorizationDomain()" -> "GCDDOM..."

"PrincipalIdealDomain()" [color=lightblue];
"PrincipalIdealDomain()" -> "GCDDOM..."

"GCDDOM..." [color=lightblue];
"DIVRING..." [color=lightblue];
"CHARNZ..." [color=lightblue];
}
```

17.0.218 FiniteRankAlgebra (FINRALG)



— FiniteRankAlgebra.input —

```
)set break resume
)sys rm -f FiniteRankAlgebra.output
)spool FiniteRankAlgebra.output
```

```
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteRankAlgebra
--R
--R FiniteRankAlgebra(R: CommutativeRing,UP: UnivariatePolynomialCategory(t#1)) is a category constructor
--R Abbreviation for FiniteRankAlgebra is FINRALG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FINRALG
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                       ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                       ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %      ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %     ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                       ?-? : (%,%) -> %
--R -? : % -> %                            ?=? : (%,%) -> Boolean
--R 1 : () -> %                            0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %      ?^? : (%,PositiveInteger) -> %
--R characteristicPolynomial : % -> UP     coerce : R -> %
--R coerce : Integer -> %                  coerce : % -> OutputForm
--R discriminant : Vector(%) -> R          hash : % -> SingleInteger
--R latex : % -> String                    norm : % -> R
--R one? : % -> Boolean                    rank : () -> PositiveInteger
--R recip : % -> Union(%,"failed")         sample : () -> %
--R trace : % -> R                         zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
--R coordinates : (Vector(%),Vector(%)) -> Matrix(R)
--R coordinates : (%,Vector(%)) -> Vector(R)
--R minimalPolynomial : % -> UP if R has FIELD
--R regularRepresentation : (%,Vector(%)) -> Matrix(R)
--R represents : (Vector(R),Vector(%)) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R traceMatrix : Vector(%) -> Matrix(R)
--R
--E 1

)spool
)lisp (bye)
```

— **FiniteRankAlgebra.help** —

```
====================================================================
FiniteRankAlgebra examples
====================================================================

A FiniteRankAlgebra is an algebra over a commutative ring R which
is a free R-module of finite rank.
```

```
See Also:
o )show FiniteRankAlgebra
```

———–

See:

⇒ "FramedAlgebra" (FRAMALG) 18.0.233 on page 1786
⇐ "Algebra" (ALGEBRA) 11.0.176 on page 1121
⇐ "CharacteristicNonZero" (CHARNZ) 10.0.159 on page 1010
⇐ "CharacteristicZero" (CHARZ) 10.0.160 on page 1015
⇐ "Field" (FIELD) 16.0.209 on page 1413

Exports:

0	1	characteristic
characteristicPolynomial	charthRoot	coerce
coordinates	discriminant	hash
latex	minimalPolynomial	norm
one?	rank	recip
regularRepresentation	represents	sample
subtractIfCan	trace	traceMatrix
zero?	?*?	?**?
?+?	?-?	-?
?=?	?^?	?~=?

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
characteristicPolynomial : % -> UP
coordinates : (%,Vector %) -> Vector R
minimalPolynomial : % -> UP if R has FIELD
norm : % -> R
rank : () -> PositiveInteger
trace : % -> R
```

These are implemented by this category:

```
coordinates : (Vector %,Vector %) -> Matrix R
discriminant : Vector % -> R
regularRepresentation : (%,Vector %) -> Matrix R
represents : (Vector R,Vector %) -> %
traceMatrix : Vector % -> Matrix R
```

These exports come from (p1121) Algebra(R:CommutativeRing):

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
```

```
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

These exports come from (p1413) Field():

```
coerce : R -> %
?*? : (R,%) -> %
?*? : (%,R) -> %
```

These exports come from (p1010) CharacteristicNonZero():

```
charthRoot : % -> Union(%,"failed") if R has CHARNZ
```

These exports come from (p1015) CharacteristicZero():

— FiniteRankAlgebra.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FINRALG">
FiniteRankAlgebra (FINRALG)</a></h2>
</body>
```

— category FINRALG FiniteRankAlgebra —

```
)abbrev category FINRALG FiniteRankAlgebra
++ Author: Barry Trager
++ Description:
++ A FiniteRankAlgebra is an algebra over a commutative ring R which
++ is a free R-module of finite rank.

FiniteRankAlgebra(R,UP) : Category == SIG where
  R : CommutativeRing
```

```
UP : UnivariatePolynomialCategory(R)

SIG ==> Algebra R with

  rank : () -> PositiveInteger
    ++ rank() returns the rank of the algebra.

  regularRepresentation : (% , Vector %) -> Matrix R
    ++ regularRepresentation(a,basis) returns the matrix of the
    ++ linear map defined by left multiplication by \spad{a} with respect
    ++ to the basis \spad{basis}.

  trace : %  -> R
    ++ trace(a) returns the trace of the regular representation
    ++ of \spad{a} with respect to any basis.

  norm : %  -> R
    ++ norm(a) returns the determinant of the regular representation
    ++ of \spad{a} with respect to any basis.

  coordinates : (%, Vector %) -> Vector R
    ++ coordinates(a,basis) returns the coordinates of \spad{a} with
    ++ respect to the basis \spad{basis}.

  coordinates : (Vector %, Vector %) -> Matrix R
    ++ coordinates([v1,...,vm], basis) returns the coordinates of the
    ++ vi's with to the basis \spad{basis}.  The coordinates of vi are
    ++ contained in the ith row of the matrix returned by this
    ++ function.

  represents : (Vector R, Vector %) -> %
    ++ represents([a1,..,an],[v1,..,vn]) returns \spad{a1*v1+...+an*vn}.

  discriminant : Vector % -> R
    ++ discriminant([v1,..,vn]) returns
    ++ \spad{determinant(traceMatrix([v1,..,vn]))}.

  traceMatrix : Vector % -> Matrix R
    ++ traceMatrix([v1,..,vn]) is the n-by-n matrix ( Tr(vi * vj) )

  characteristicPolynomial : % -> UP
    ++ characteristicPolynomial(a) returns the characteristic
    ++ polynomial of the regular representation of \spad{a} with respect
    ++ to any basis.

  if R has Field then minimalPolynomial : % -> UP
    ++ minimalPolynomial(a) returns the minimal polynomial of \spad{a}.

  if R has CharacteristicZero then CharacteristicZero

  if R has CharacteristicNonZero then CharacteristicNonZero

 add
```

```
    discriminant v == determinant traceMatrix v

    coordinates(v:Vector %, b:Vector %) ==
      m := new(#v, #b, 0)$Matrix(R)
      for i in minIndex v .. maxIndex v for j in minRowIndex m .. repeat
        setRow_!(m, j, coordinates(qelt(v, i), b))
      m

    represents(v, b) ==
      m := minIndex v - 1
      _+/[v(i+m) * b(i+m) for i in 1..rank()]

    traceMatrix v ==
      matrix [[trace(v.i*v.j) for j in minIndex v..maxIndex v]$List(R)
               for i in minIndex v .. maxIndex v]$List(List R)

    regularRepresentation(x, b) ==
      m := minIndex b - 1
      matrix
       [parts coordinates(x*b(i+m),b) for i in 1..rank()]$List(List R)
```

———-

— COQ FINRALG —

```
(* category FINRALG *)
(*

    discriminant : Vector(%) -> R
    discriminant v == determinant traceMatrix v

    coordinates : (Vector(%),Vector(%)) -> Matrix(R)
    coordinates(v:Vector %, b:Vector %) ==
      m := new(#v, #b, 0)$Matrix(R)
      for i in minIndex v .. maxIndex v for j in minRowIndex m .. repeat
        setRow_!(m, j, coordinates(qelt(v, i), b))
      m

    represents : (Vector(R),Vector(%)) -> %
    represents(v, b) ==
      m := minIndex v - 1
      _+/[v(i+m) * b(i+m) for i in 1..rank()]

    traceMatrix : Vector(%) -> Matrix(R)
    traceMatrix v ==
      matrix [[trace(v.i*v.j) for j in minIndex v..maxIndex v]$List(R)
               for i in minIndex v .. maxIndex v]$List(List R)

    regularRepresentation : (%,Vector(%)) -> Matrix(R)
    regularRepresentation(x, b) ==
      m := minIndex b - 1
      matrix
       [parts coordinates(x*b(i+m),b) for i in 1..rank()]$List(List R)
```

```
*)
```

— **FINRALG.dotabb** —

```
"FINRALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FINRALG"];
"FINRALG" -> "ALGEBRA"
"FINRALG" -> "FIELD"
"FINRALG" -> "CHARNZ"
"FINRALG" -> "CHARZ"
```

— **FINRALG.dotfull** —

```
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FINRALG"];
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "Algebra(a:CommutativeRing)"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "Field()"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "CharacteristicNonZero()"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "CharacteristicZero()"
```

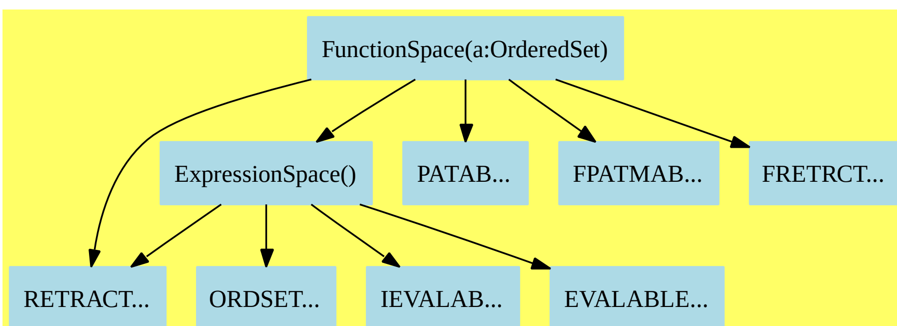
— **FINRALG.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
 [color=lightblue];
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "Algebra(a:CommutativeRing)"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "Field()"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "CharacteristicNonZero()"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "CharacteristicZero()"

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
```

```
"Algebra(a:CommutativeRing)" -> "MODULE..."

"Field()" [color=lightblue];
"Field()" -> "EUCDOM..."
"Field()" -> "UFD..."
"Field()" -> "DIVRING..."

"CharacteristicNonZero()" [color=lightblue];
"CharacteristicNonZero()" -> "RING..."

"CharacteristicZero()" [color=lightblue];
"CharacteristicZero()" -> "RING..."

"EUCDOM..." [color=lightblue];
"UFD..." [color=lightblue];
"DIVRING..." [color=lightblue];
"RING..." [color=lightblue];
"MODULE..." [color=lightblue];
}
```

————-

17.0.219 FunctionSpace (FS)



— **FunctionSpace.input** —

```
)set break resume
)sys rm -f FunctionSpace.output
)spool FunctionSpace.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FunctionSpace
--R
--R FunctionSpace(R: OrderedSet) is a category constructor
--R Abbreviation for FunctionSpace is FS
--R This constructor is exposed in this frame.
```

```
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FS
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,%) -> % if R has SGROUP      ?*? : (%,R) -> % if R has COMRING
--R ?*? : (R,%) -> % if R has COMRING     ?+? : (%,%) -> % if R has ABELSG
--R -? : % -> % if R has ABELGRP          ?-? : (%,%) -> % if R has ABELGRP
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               D : (%,Symbol) -> % if R has RING
--R 1 : () -> % if R has SGROUP           0 : () -> % if R has ABELSG
--R applyQuote : (Symbol,List(%)) -> %    applyQuote : (Symbol,%,%,%) -> %
--R applyQuote : (Symbol,%,%) -> %        applyQuote : (Symbol,%) -> %
--R belong? : BasicOperator -> Boolean    box : List(%) -> %
--R box : % -> %                          coerce : % -> % if R has INTDOM
--R coerce : R -> %                       coerce : Symbol -> %
--R coerce : Kernel(%) -> %               coerce : % -> OutputForm
--R distribute : (%,%) -> %               distribute : % -> %
--R elt : (BasicOperator,List(%)) -> %    elt : (BasicOperator,%,%,%) -> %
--R elt : (BasicOperator,%,%) -> %        elt : (BasicOperator,%) -> %
--R eval : (%,Symbol,(% -> %)) -> %       eval : (%,List(%),List(%)) -> %
--R eval : (%,%,%) -> %                   eval : (%,Equation(%)) -> %
--R eval : (%,List(Equation(%))) -> %     eval : (%,Kernel(%),%) -> %
--R freeOf? : (%,Symbol) -> Boolean       freeOf? : (%,%) -> Boolean
--R gcd : (%,%) -> % if R has INTDOM      gcd : List(%) -> % if R has INTDOM
--R ground : % -> R                       ground? : % -> Boolean
--R hash : % -> SingleInteger             height : % -> NonNegativeInteger
--R is? : (%,Symbol) -> Boolean           is? : (%,BasicOperator) -> Boolean
--R kernel : (BasicOperator,%) -> %       kernels : % -> List(Kernel(%))
--R latex : % -> String                   lcm : (%,%) -> % if R has INTDOM
--R lcm : List(%) -> % if R has INTDOM    map : ((% -> %),Kernel(%)) -> %
--R max : (%,%) -> %                      min : (%,%) -> %
--R numerator : % -> % if R has RING      one? : % -> Boolean if R has SGROUP
--R paren : List(%) -> %                  paren : % -> %
--R retract : % -> R                      retract : % -> Symbol
--R retract : % -> Kernel(%)              subst : (%,Equation(%)) -> %
--R tower : % -> List(Kernel(%))          variables : % -> List(Symbol)
--R ?~=? : (%,%) -> Boolean
--R ?*? : (PositiveInteger,%) -> % if R has ABELSG
--R ?*? : (NonNegativeInteger,%) -> % if R has ABELSG
--R ?*? : (Integer,%) -> % if R has ABELGRP
--R ?*? : (%,Fraction(Integer)) -> % if R has INTDOM
--R ?*? : (Fraction(Integer),%) -> % if R has INTDOM
--R ?**? : (%,PositiveInteger) -> % if R has SGROUP
--R ?**? : (%,NonNegativeInteger) -> % if R has SGROUP
--R ?**? : (%,Integer) -> % if R has GROUP or R has INTDOM
--R ?/? : (%,%) -> % if R has GROUP or R has INTDOM
--R ?/? : (SparseMultivariatePolynomial(R,Kernel(%)),SparseMultivariatePolynomial(R,Kernel(%))) -> % if R has IN
--R D : (%,List(Symbol)) -> % if R has RING
--R D : (%,Symbol,NonNegativeInteger) -> % if R has RING
--R D : (%,List(Symbol),List(NonNegativeInteger)) -> % if R has RING
--R ?^? : (%,PositiveInteger) -> % if R has SGROUP
--R ?^? : (%,NonNegativeInteger) -> % if R has SGROUP
--R ?^? : (%,Integer) -> % if R has GROUP or R has INTDOM
--R applyQuote : (Symbol,%,%,%,%) -> %
```

```
--R associates? : (%,%) -> Boolean if R has INTDOM
--R characteristic : () -> NonNegativeInteger if R has RING
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
--R coerce : Integer -> % if R has RING or R has RETRACT(INT)
--R coerce : Fraction(Integer) -> % if R has INTDOM or R has RETRACT(INT) and R has INTDOM or R has RETRACT(FRAC
--R coerce : Polynomial(R) -> % if R has RING
--R coerce : Fraction(Polynomial(R)) -> % if R has INTDOM
--R coerce : Fraction(Polynomial(Fraction(R))) -> % if R has INTDOM
--R coerce : Polynomial(Fraction(R)) -> % if R has INTDOM
--R coerce : Fraction(R) -> % if R has INTDOM
--R coerce : SparseMultivariatePolynomial(R,Kernel(%)) -> % if R has RING
--R commutator : (%,%) -> % if R has GROUP
--R conjugate : (%,%) -> % if R has GROUP
--R convert : % -> InputForm if R has KONVERT(INFORM)
--R convert : Factored(%) -> % if R has INTDOM
--R convert : % -> Pattern(Float) if R has KONVERT(PATTERN(FLOAT))
--R convert : % -> Pattern(Integer) if R has KONVERT(PATTERN(INT))
--R definingPolynomial : % -> % if $ has RING
--R denom : % -> SparseMultivariatePolynomial(R,Kernel(%)) if R has INTDOM
--R denominator : % -> % if R has INTDOM
--R differentiate : (%,Symbol) -> % if R has RING
--R differentiate : (%,List(Symbol)) -> % if R has RING
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has RING
--R differentiate : (%,List(Symbol),List(NonNegativeInteger)) -> % if R has RING
--R divide : (%,%) -> Record(quotient: %,remainder: %) if R has INTDOM
--R elt : (BasicOperator,%,%,%,%) -> %
--R euclideanSize : % -> NonNegativeInteger if R has INTDOM
--R eval : (%,Symbol,NonNegativeInteger,(% -> %)) -> % if R has RING
--R eval : (%,Symbol,NonNegativeInteger,(List(%) -> %)) -> % if R has RING
--R eval : (%,List(Symbol),List(NonNegativeInteger),List((List(%) -> %))) -> % if R has RING
--R eval : (%,List(Symbol),List(NonNegativeInteger),List((% -> %))) -> % if R has RING
--R eval : (%,List(BasicOperator),List(%),Symbol) -> % if R has KONVERT(INFORM)
--R eval : (%,BasicOperator,%,Symbol) -> % if R has KONVERT(INFORM)
--R eval : % -> % if R has KONVERT(INFORM)
--R eval : (%,List(Symbol)) -> % if R has KONVERT(INFORM)
--R eval : (%,Symbol) -> % if R has KONVERT(INFORM)
--R eval : (%,BasicOperator,(% -> %)) -> %
--R eval : (%,BasicOperator,(List(%) -> %)) -> %
--R eval : (%,List(BasicOperator),List((List(%) -> %))) -> %
--R eval : (%,List(BasicOperator),List((% -> %))) -> %
--R eval : (%,Symbol,(List(%) -> %)) -> %
--R eval : (%,List(Symbol),List((List(%) -> %))) -> %
--R eval : (%,List(Symbol),List((% -> %))) -> %
--R eval : (%,List(Kernel(%)),List(%)) -> %
--R even? : % -> Boolean if $ has RETRACT(INT)
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed") if R has INTDOM
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has INTDOM
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if R has INTDOM
--R factor : % -> Factored(%) if R has INTDOM
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R inv : % -> % if R has GROUP or R has INTDOM
--R isExpt : (%,Symbol) -> Union(Record(var: Kernel(%),exponent: Integer),"failed") if R has RING
--R isExpt : (%,BasicOperator) -> Union(Record(var: Kernel(%),exponent: Integer),"failed") if R has RING
```

```
--R isExpt : % -> Union(Record(var: Kernel(%),exponent: Integer),"failed") if R has SGROUP
--R isMult : % -> Union(Record(coef: Integer,var: Kernel(%)),"failed") if R has ABELSG
--R isPlus : % -> Union(List(%),"failed") if R has ABELSG
--R isPower : % -> Union(Record(val: %,exponent: Integer),"failed") if R has RING
--R isTimes : % -> Union(List(%),"failed") if R has SGROUP
--R kernel : (BasicOperator,List(%)) -> %
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %) if R has INTDOM
--R mainKernel : % -> Union(Kernel(%),"failed")
--R minPoly : Kernel(%) -> SparseUnivariatePolynomial(%) if $ has RING
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed") if R has INTDOM
--R numer : % -> SparseMultivariatePolynomial(R,Kernel(%)) if R has RING
--R odd? : % -> Boolean if $ has RETRACT(INT)
--R operator : BasicOperator -> BasicOperator
--R operators : % -> List(BasicOperator)
--R patternMatch : (%,Pattern(Float),PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%) if R has PATMAB
--R patternMatch : (%,Pattern(Integer),PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%) if R has
--R prime? : % -> Boolean if R has INTDOM
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %) if R has INTDOM
--R ?quo? : (%,%) -> % if R has INTDOM
--R recip : % -> Union(%,"failed") if R has SGROUP
--R reducedSystem : Matrix(%) -> Matrix(R) if R has RING
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(R),vec: Vector(R)) if R has RING
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(Integer),vec: Vector(Integer)) if and(has(R,Line
--R reducedSystem : Matrix(%) -> Matrix(Integer) if and(has(R,LinearlyExplicitRingOver(Integer)),has(R,Ring))
--R ?rem? : (%,%) -> % if R has INTDOM
--R retract : % -> Fraction(Integer) if R has RETRACT(INT) and R has INTDOM or R has RETRACT(FRAC(INT))
--R retract : % -> Polynomial(R) if R has RING
--R retract : % -> Fraction(Polynomial(R)) if R has INTDOM
--R retract : % -> Integer if R has RETRACT(INT)
--R retractIfCan : % -> Union(Fraction(Integer),"failed") if R has RETRACT(INT) and R has INTDOM or R has RETRAC
--R retractIfCan : % -> Union(Polynomial(R),"failed") if R has RING
--R retractIfCan : % -> Union(Fraction(Polynomial(R)),"failed") if R has INTDOM
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT(INT)
--R retractIfCan : % -> Union(Symbol,"failed")
--R retractIfCan : % -> Union(Kernel(%),"failed")
--R sample : () -> % if R has SGROUP or R has ABELSG
--R sizeLess? : (%,%) -> Boolean if R has INTDOM
--R squareFree : % -> Factored(%) if R has INTDOM
--R squareFreePart : % -> % if R has INTDOM
--R subst : (%,List(Kernel(%)),List(%)) -> %
--R subst : (%,List(Equation(%))) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed") if R has ABELGRP
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : (%,Kernel(%)) -> Fraction(SparseUnivariatePolynomial(%)) if R has INTDOM
--R zero? : % -> Boolean if R has ABELSG
--R
--E 1

)spool
)lisp (bye)
```

———-

— **FunctionSpace.help** —

```
====================================================================
FunctionSpace examples
====================================================================

This is the category for formal functions.
A space of formal functions with arguments in an arbitrary ordered set.

See Also:
o )show FunctionSpace
```

———-

See:

⇐ “AbelianGroup” (ABELGRP) 7.0.121 on page 673
⇐ “AbelianMonoid” (ABELMON) 5.0.90 on page 383
⇐ “AbelianSemiGroup” (ABELSG) 4.0.59 on page 201
⇐ “Algebra” (ALGEBRA) 11.0.176 on page 1121
⇐ “AlgebraicallyClosedFunctionSpace” (ACFS) 18.0.229 on page 1739
⇐ “CharacteristicNonZero” (CHARNZ) 10.0.159 on page 1010
⇐ “CharacteristicZero” (CHARZ) 10.0.160 on page 1015
⇐ “CommutativeRing” (COMRING) 10.0.161 on page 1019
⇐ “ConvertibleTo” (KONVERT) 2.0.17 on page 47
⇐ “ExpressionSpace” (ES) 5.0.96 on page 418
⇐ “Field” (FIELD) 16.0.209 on page 1413
⇐ “FullyLinearlyExplicitRingOver” (FLINEXP) 11.0.178 on page 1134
⇐ “FullyPatternMatchable” (FPATMAB) 3.0.51 on page 164
⇐ “FullyRetractableTo” (FRETRCT) 3.0.50 on page 158
⇐ “Group” (GROUP) 6.0.110 on page 525
⇐ “IntegralDomain” (INTDOM) 12.0.188 on page 1225
⇐ “Monoid” (MONOID) 5.0.100 on page 455
⇐ “PartialDifferentialRing” (PDRING) 10.0.169 on page 1061
⇐ “Patternable” (PATAB) 2.0.34 on page 100
⇐ “RetractableTo” (RETRACT) 2.0.37 on page 110
⇐ “Ring” (RING) 9.0.153 on page 946
⇐ “SemiGroup” (SGROUP) 4.0.85 on page 353

Exports:

0	1	applyQuote	associates?
belong?	box	characteristic	charthRoot
coerce	commutator	conjugate	convert
D	definingPolynomial	denom	denominator
differentiate	distribute	divide	elt
eval	euclideanSize	even?	expressIdealMember
exquo	extendedEuclidean	factor	freeOf?
gcd	gcdPolynomial	ground	ground?
hash	height	inv	is?
isExpt	isMult	isPlus	isPower
isTimes	kernel	kernels	latex
lcm	mainKernel	map	max
min	minPoly	multiEuclidean	numer
numerator	odd?	one?	operator
operators	paren	patternMatch	prime?
principalIdeal	recip	reducedSystem	retract
retractIfCan	sample	sizeLess?	squareFree
squareFreePart	subst	subtractIfCan	tower
unit?	unitCanonical	unitNormal	univariate
variables	zero?	-?	?<?
?<=?	?=?	?>?	?>=?
?~=?	?*?	?**?	?+?
?-?	?/?	?^?	?quo?
?rem?			

Attributes Exported:

- if $ has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x*y \neq 0$ implies both x and y are non-zero.
- if $ has IntegralDomain then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if $ has IntegralDomain then canonicalsClosed where **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- if $ has IntegralDomain then commutative("*") where **commutative("*")** is true if it has an operation $" * " : (D, D)- > D$ which is commutative.
- if $ has Ring or Group then unitsKnown where **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- if $ has CommutativeRing then leftUnitary where **leftUnitary** is true if $1 * x = x$ for all x.
- if $ has CommutativeRing then rightUnitary where **rightUnitary** is true if $x * 1 = x$ for all x.
- **nil**

These are directly exported but not implemented:

```
coerce : SparseMultivariatePolynomial(R,Kernel %) -> %
    if R has RING
```

```
coerce : Fraction R -> % if R has INTDOM
coerce : Polynomial Fraction R -> % if R has INTDOM
denom : % -> SparseMultivariatePolynomial(R,Kernel %)
    if R has INTDOM
eval : (%,List BasicOperator,List %,Symbol) -> %
    if R has KONVERT INFORM
numer : % -> SparseMultivariatePolynomial(R,Kernel %)
    if R has RING
?/? :
  (SparseMultivariatePolynomial(R,Kernel %),
   SparseMultivariatePolynomial(R,Kernel %)) -> %
    if R has INTDOM
?**? : (%,NonNegativeInteger) -> % if R has SGROUP
```

These are implemented by this category:

```
applyQuote : (Symbol,%) -> %
applyQuote : (Symbol,%,%) -> %
applyQuote : (Symbol,%,%,%) -> %
applyQuote : (Symbol,%,%,%,%) -> %
applyQuote : (Symbol,List %) -> %
belong? : BasicOperator -> Boolean
characteristic : () -> NonNegativeInteger if R has RING
coerce : Symbol -> %
coerce : Kernel % -> %
coerce : % -> OutputForm
coerce : Fraction Integer -> %
    if R has INTDOM
    or R has RETRACT INT
    and R has INTDOM
    or R has RETRACT FRAC INT
coerce : Fraction Polynomial R -> % if R has INTDOM
coerce : Fraction Polynomial Fraction R -> %
    if R has INTDOM
coerce : Polynomial R -> % if R has RING
convert : % -> Pattern Float if R has KONVERT PATTERN FLOAT
convert : % -> Pattern Integer if R has KONVERT PATTERN INT
convert : Factored % -> % if R has INTDOM
denominator : % -> % if R has INTDOM
differentiate : (%,Symbol) -> % if R has RING
elt : (BasicOperator,List %) -> %
eval : (%,List Symbol) -> % if R has KONVERT INFORM
eval : % -> % if R has KONVERT INFORM
eval : (%,Symbol) -> % if R has KONVERT INFORM
eval : (%,BasicOperator,%,Symbol) -> %
    if R has KONVERT INFORM
eval : (%,Symbol,NonNegativeInteger,(% -> %)) -> %
    if R has RING
eval : (%,Symbol,NonNegativeInteger,(List % -> %)) -> %
    if R has RING
eval : (%,List Symbol,List NonNegativeInteger,List (List % -> %)) -> %
    if R has RING
eval : (%,List Symbol,List NonNegativeInteger,List (% -> %)) -> %
    if R has RING
eval : (%,List Kernel %,List %) -> %
```

```
ground : % -> R
ground? : % -> Boolean
isExpt : (%,BasicOperator) ->
   Union(Record(var: Kernel %,exponent: Integer),"failed")
    if R has RING
isExpt : % ->
  Union(Record(var: Kernel %,exponent: Integer),"failed")
    if R has SGROUP
isExpt : (%,Symbol) ->
  Union(Record(var: Kernel %,exponent: Integer),"failed")
    if R has RING
isMult : % ->
  Union(Record(coef: Integer,var: Kernel %),"failed")
    if R has ABELSG
isPlus : % -> Union(List %,"failed") if R has ABELSG
isPower : % ->
  Union(Record(val: %,exponent: Integer),"failed")
    if R has RING
isTimes : % -> Union(List %,"failed") if R has SGROUP
kernels : % -> List Kernel %
mainKernel : % -> Union(Kernel %,"failed")
numerator : % -> % if R has RING
operator : BasicOperator -> BasicOperator
retract : % -> Fraction Polynomial R if R has INTDOM
retract : % -> Polynomial R if R has RING
retract : % -> R
retract : % -> Symbol
retractIfCan : % -> Union(R,"failed")
retractIfCan : % -> Union(Fraction Polynomial R,"failed")
    if R has INTDOM
retractIfCan : % -> Union(Polynomial R,"failed")
    if R has RING
retractIfCan : % -> Union(Symbol,"failed")
subst : (%,List Kernel %,List %) -> %
univariate : (%,Kernel %) ->
  Fraction SparseUnivariatePolynomial %
    if R has INTDOM
variables : % -> List Symbol
?*? : (%,R) -> % if R has COMRING
?*? : (R,%) -> % if R has COMRING
```

These exports come from (p418) ExpressionSpace():

```
box : List % -> %
box : % -> %
definingPolynomial : % -> % if $ has RING
distribute : (%,%) -> %
distribute : % -> %
elt : (BasicOperator,%,%,%,%) -> %
elt : (BasicOperator,%,%,%) -> %
elt : (BasicOperator,%,%) -> %
elt : (BasicOperator,%) -> %
eval : (%,List BasicOperator,List (% -> %)) -> %
eval : (%,List Equation %) -> %
eval : (%,Symbol,(% -> %)) -> %
```

```
eval : (%,Symbol,(List % -> %)) -> %
eval : (%,BasicOperator,(% -> %)) -> %
eval : (%,BasicOperator,(List % -> %)) -> %
eval : (%,List Symbol,List (% -> %)) -> %
eval : (%,List BasicOperator,List (List % -> %)) -> %
eval : (%,List Symbol,List (List % -> %)) -> %
eval : (%,List %,List %) -> %
eval : (%,%,%) -> %
eval : (%,Equation %) -> %
eval : (%,Kernel %,%) -> %
even? : % -> Boolean if $ has RETRACT INT
freeOf? : (%,Symbol) -> Boolean
freeOf? : (%,%) -> Boolean
hash : % -> SingleInteger
height : % -> NonNegativeInteger
is? : (%,BasicOperator) -> Boolean
is? : (%,Symbol) -> Boolean
kernel : (BasicOperator,%) -> %
kernel : (BasicOperator,List %) -> %
latex : % -> String
map : ((% -> %),Kernel %) -> %
max : (%,%) -> %
min : (%,%) -> %
minPoly : Kernel % -> SparseUnivariatePolynomial %
    if $ has RING
odd? : % -> Boolean if $ has RETRACT INT
operators : % -> List BasicOperator
paren : List % -> %
paren : % -> %
retract : % -> Kernel %
retractIfCan : % -> Union(Kernel %,"failed")
subst : (%,List Equation %) -> %
subst : (%,Equation %) -> %
tower : % -> List Kernel %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

These exports come from (p164) FullyPatternMatchable(OrderedSet):

```
patternMatch :
  (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
    PatternMatchResult(Integer,%)
      if R has PATMAB INT
patternMatch :
  (%,Pattern Float,PatternMatchResult(Float,%)) ->
    PatternMatchResult(Float,%)
      if R has PATMAB FLOAT
```

These exports come from (p158) FullyRetractableTo(OrderedSet):

```
coerce : Integer -> % if R has RING or R has RETRACT INT
coerce : R -> %
```

These exports come from (p47) ConvertibleTo(InputForm):

```
convert : % -> InputForm if R has KONVERT INFORM
```

These exports come from (p455) Monoid():

```
1 : () -> % if R has SGROUP
one? : % -> Boolean if R has SGROUP
recip : % -> Union(%,"failed") if R has SGROUP
sample : () -> % if R has SGROUP or R has ABELSG
?^? : (%,NonNegativeInteger) -> % if R has SGROUP
?*? : (%,%) -> % if R has SGROUP
?**? : (%,PositiveInteger) -> % if R has SGROUP
?^? : (%,PositiveInteger) -> % if R has SGROUP
```

These exports come from (p525) Group():

```
commutator : (%,%) -> % if R has GROUP
conjugate : (%,%) -> % if R has GROUP
inv : % -> % if R has GROUP or R has INTDOM
?/? : (%,%) -> % if R has GROUP or R has INTDOM
?^? : (%,Integer) -> % if R has GROUP or R has INTDOM
?**? : (%,Integer) -> % if R has GROUP or R has INTDOM
```

These exports come from (p383) AbelianMonoid():

```
0 : () -> % if R has ABELSG
zero? : % -> Boolean if R has ABELSG
?*? : (PositiveInteger,%) -> % if R has ABELSG
?*? : (NonNegativeInteger,%) -> % if R has ABELSG
?+? : (%,%) -> % if R has ABELSG
```

These exports come from (p673) AbelianGroup():

```
subtractIfCan : (%,%) -> Union(%,"failed") if R has ABELGRP
?*? : (Integer,%) -> % if R has ABELGRP
?-? : (%,%) -> % if R has ABELGRP
-? : % -> % if R has ABELGRP
```

These exports come from (p1061) PartialDifferentialRing(Symbol):

```
D : (%,Symbol) -> % if R has RING
D : (%,List Symbol) -> % if R has RING
D : (%,Symbol,NonNegativeInteger) -> % if R has RING
D : (%,List Symbol,List NonNegativeInteger) -> % if R has RING
differentiate : (%,List Symbol) -> % if R has RING
differentiate : (%,Symbol,NonNegativeInteger) -> % if R has RING
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
    if R has RING
```

These exports come from (p1134) FullyLinearlyExplicitRingOver(R) where R:OrderedSet:

```
reducedSystem : Matrix % -> Matrix R if R has RING
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix R,vec: Vector R) if R has RING
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix Integer,vec: Vector Integer)
    if and(has(R,LinearlyExplicitRingOver Integer),has(R,Ring))
reducedSystem : Matrix % -> Matrix Integer
    if and(has(R,LinearlyExplicitRingOver Integer),has(R,Ring))
```

These exports come from (p1010) CharacteristicNonZero():

```
charthRoot : % -> Union(%,"failed") if R has CHARNZ
```

These exports come from (p1225) IntegralDomain():

```
associates? : (%,%) -> Boolean if R has INTDOM
exquo : (%,%) -> Union(%,"failed") if R has INTDOM
unit? : % -> Boolean if R has INTDOM
unitCanonical : % -> % if R has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if R has INTDOM
```

These exports come from (p1413) Field():

```
coerce : % -> % if R has INTDOM
divide : (%,%) -> Record(quotient: %,remainder: %)
    if R has INTDOM
euclideanSize : % -> NonNegativeInteger if R has INTDOM
expressIdealMember : (List %,%) -> Union(List %,"failed")
    if R has INTDOM
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
    if R has INTDOM
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
    if R has INTDOM
factor : % -> Factored % if R has INTDOM
gcd : (%,%) -> % if R has INTDOM
gcd : List % -> % if R has INTDOM
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
       if R has INTDOM
lcm : (%,%) -> % if R has INTDOM
lcm : List % -> % if R has INTDOM
multiEuclidean : (List %,%) -> Union(List %,"failed")
    if R has INTDOM
prime? : % -> Boolean if R has INTDOM
principalIdeal : List % -> Record(coef: List %,generator: %)
    if R has INTDOM
sizeLess? : (%,%) -> Boolean if R has INTDOM
squareFree : % -> Factored % if R has INTDOM
squareFreePart : % -> % if R has INTDOM
?quo? : (%,%) -> % if R has INTDOM
?rem? : (%,%) -> % if R has INTDOM
```

These exports come from (p110) RetractableTo(Integer):

```
retract : % -> Integer if R has RETRACT INT
retractIfCan : % -> Union(Integer,"failed")
    if R has RETRACT INT
```

These exports come from (p110) RetractableTo(Fraction(Integer)):

```
retract : % -> Fraction Integer
    if R has RETRACT INT
    and R has INTDOM
    or R has RETRACT FRAC INT
retractIfCan : % -> Union(Fraction Integer,"failed")
```

```
     if R has RETRACT INT
     and R has INTDOM
     or R has RETRACT FRAC INT
 ?*? : (%,Fraction Integer) -> % if R has INTDOM
 ?*? : (Fraction Integer,%) -> % if R has INTDOM
```

— FunctionSpace.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FS">
FunctionSpace (FS)</a></h2>
</body>
```

———-

— category FS FunctionSpace —

```
)abbrev category FS FunctionSpace
++ Author: Manuel Bronstein
++ Date Created: 22 March 1988
++ Date Last Updated: 14 February 1994
++ Description:
++ Category for formal functions
++ A space of formal functions with arguments in an arbitrary ordered set.

FunctionSpace(R): Category == SIG where
  R : OrderedSet

  OP ==> BasicOperator
  O  ==> OutputForm
  SY ==> Symbol
  N  ==> NonNegativeInteger
  Z  ==> Integer
  K  ==> Kernel %
  Q  ==> Fraction R
  PR ==> Polynomial R
  MP ==> SparseMultivariatePolynomial(R, K)
  QF==> PolynomialCategoryQuotientFunctions(IndexedExponents K,K,R,MP,%)

  ODD  ==> "odd"
  EVEN ==> "even"

  SPECIALDIFF  ==> "%specialDiff"
  SPECIALDISP  ==> "%specialDisp"
  SPECIALEQUAL ==> "%specialEqual"
  SPECIALINPUT ==> "%specialInput"

  ES  ==> ExpressionSpace
  RT  ==> RetractableTo(SY)
  P   ==> Patternable(R)
  FPM ==> FullyPatternMatchable(R)
  FRT ==> FullyRetractableTo(R)
```

```
SIG ==> Join(ES,RT,P,FPM,FRT) with

  ground? : % -> Boolean
    ++ ground?(f) tests if f is an element of R.

  ground : % -> R
    ++ ground(f) returns f as an element of R.
    ++ An error occurs if f is not an element of R.

  variables : %  -> List SY
    ++ variables(f) returns the list of all the variables of f.

  applyQuote : (SY, %) -> %
    ++ applyQuote(foo, x) returns \spad{'foo(x)}.

  applyQuote : (SY, %, %) -> %
    ++ applyQuote(foo, x, y) returns \spad{'foo(x,y)}.

  applyQuote : (SY, %, %, %) -> %
    ++ applyQuote(foo, x, y, z) returns \spad{'foo(x,y,z)}.

  applyQuote : (SY, %, %, %, %) -> %
    ++ applyQuote(foo, x, y, z, t) returns \spad{'foo(x,y,z,t)}.

  applyQuote : (SY, List %) -> %
    ++ applyQuote(foo, [x1,...,xn]) returns \spad{'foo(x1,...,xn)}.

  if R has ConvertibleTo InputForm then

    ConvertibleTo InputForm

    eval : (%, SY) -> %
      ++ eval(f, foo) unquotes all the foo's in f.

    eval : (%, List SY) -> %
      ++ eval(f, [foo1,...,foon]) unquotes all the \spad{fooi}'s in f.

    eval : % -> %
      ++ eval(f) unquotes all the quoted operators in f.

    eval : (%, OP, %, SY) -> %
      ++ eval(x, s, f, y) replaces every \spad{s(a)} in x by \spad{f(y)}
      ++ with \spad{y} replaced by \spad{a} for any \spad{a}.

    eval : (%, List OP, List %, SY) -> %
      ++ eval(x, [s1,...,sm], [f1,...,fm], y) replaces every
      ++ \spad{si(a)} in x by \spad{fi(y)}
      ++ with \spad{y} replaced by \spad{a} for any \spad{a}.

  if R has SemiGroup then

    Monoid
    -- the following line is necessary because of a compiler bug
```

```
  "**" : (%, N) -> %
    ++ x**n returns x * x * x * ... * x (n times).

  isTimes: % -> Union(List %, "failed")
    ++ isTimes(p) returns \spad{[a1,...,an]}
    ++ if \spad{p = a1*...*an} and \spad{n > 1}.

  isExpt : % -> Union(Record(var:K,exponent:Z),"failed")
    ++ isExpt(p) returns \spad{[x, n]} if \spad{p = x**n}
    ++ and \spad{n <> 0}.

if R has Group then Group

if R has AbelianSemiGroup then

  AbelianMonoid

  isPlus: % -> Union(List %, "failed")
    ++ isPlus(p) returns \spad{[m1,...,mn]}
    ++ if \spad{p = m1 +...+ mn} and \spad{n > 1}.

  isMult: % -> Union(Record(coef:Z, var:K),"failed")
    ++ isMult(p) returns \spad{[n, x]} if \spad{p = n * x}
    ++ and \spad{n <> 0}.

if R has AbelianGroup then AbelianGroup

if R has Ring then

  Ring

  RetractableTo PR

  PartialDifferentialRing SY

  FullyLinearlyExplicitRingOver R

  coerce : MP -> %
    ++ coerce(p) returns p as an element of %.

  numer : %  -> MP
    ++ numer(f) returns the
    ++ numerator of f viewed as a polynomial in the kernels over R
    ++ if R is an integral domain. If not, then numer(f) = f viewed
    ++ as a polynomial in the kernels over R.
    -- DO NOT change this meaning of numer!  MB 1/90

  numerator : % -> %
    ++ numerator(f) returns the numerator of \spad{f} converted to %.

  isExpt:(%,OP) -> Union(Record(var:K,exponent:Z),"failed")
    ++ isExpt(p,op) returns \spad{[x, n]} if \spad{p = x**n}
    ++ and \spad{n <> 0} and \spad{x = op(a)}.
```

```
    isExpt:(%,SY) -> Union(Record(var:K,exponent:Z),"failed")
      ++ isExpt(p,f) returns \spad{[x, n]} if \spad{p = x**n}
      ++ and \spad{n <> 0} and \spad{x = f(a)}.

    isPower : % -> Union(Record(val:%,exponent:Z),"failed")
      ++ isPower(p) returns \spad{[x, n]} if \spad{p = x**n}
      ++ and \spad{n <> 0}.

    eval: (%, List SY, List N, List(% -> %)) -> %
      ++ eval(x, [s1,...,sm], [n1,...,nm], [f1,...,fm]) replaces
      ++ every \spad{si(a)**ni} in x by \spad{fi(a)} for any \spad{a}.

    eval: (%, List SY, List N, List(List % -> %)) -> %
      ++ eval(x, [s1,...,sm], [n1,...,nm], [f1,...,fm]) replaces
      ++ every \spad{si(a1,...,an)**ni} in x by \spad{fi(a1,...,an)}
      ++ for any a1,...,am.

    eval: (%, SY, N, List % -> %) -> %
      ++ eval(x, s, n, f) replaces every \spad{s(a1,...,am)**n} in x
      ++ by \spad{f(a1,...,am)} for any a1,...,am.

    eval: (%, SY, N, % -> %) -> %
      ++ eval(x, s, n, f) replaces every \spad{s(a)**n} in x
      ++ by \spad{f(a)} for any \spad{a}.

  if R has CharacteristicZero then CharacteristicZero

  if R has CharacteristicNonZero then CharacteristicNonZero

  if R has CommutativeRing then
    Algebra R

  if R has IntegralDomain then

    Field

    RetractableTo Fraction PR

    convert : Factored % -> %
      ++ convert(f1\^e1 ... fm\^em) returns \spad{(f1)\^e1 ... (fm)\^em}
      ++ as an element of %, using formal kernels
      ++ created using a \spadfunFrom{paren}{ExpressionSpace}.

    denom : %  -> MP
      ++ denom(f) returns the denominator of f viewed as a
      ++ polynomial in the kernels over R.

    denominator : % -> %
      ++ denominator(f) returns the denominator of \spad{f}
      ++ converted to %.

    "/" : (MP, MP) -> %
      ++ p1/p2 returns the quotient of p1 and p2 as an element of %.
```

```
   coerce : Q  -> %
     ++ coerce(q) returns q as an element of %.

   coerce : Polynomial Q -> %
     ++ coerce(p) returns p as an element of %.

   coerce : Fraction Polynomial Q -> %
     ++ coerce(f) returns f as an element of %.

   univariate: (%, K) -> Fraction SparseUnivariatePolynomial %
     ++ univariate(f, k) returns f viewed as a univariate fraction in k.

   if R has RetractableTo Z then RetractableTo Fraction Z

add

  import BasicOperatorFunctions1(%)

  -- these are needed in Ring only, but need to be declared here
  -- because of compiler bug: if they are declared inside the Ring
  -- case, then they are not visible inside the IntegralDomain case.
  smpIsMult : MP -> Union(Record(coef:Z, var:K),"failed")
  smpret    : MP -> Union(PR, "failed")
  smpeval   : (MP, List K, List %) -> %
  smpsubst  : (MP, List K, List %) -> %
  smpderiv  : (MP, SY) -> %
  smpunq    : (MP, List SY, Boolean) -> %
  kerderiv  : (K, SY)  -> %
  kderiv    : K -> List %
  opderiv   : (OP, N) -> List(List % -> %)
  smp2O     : MP -> O
  bestKernel: List K -> K
  worse?    : (K, K) -> Boolean
  diffArg   : (List %, OP, N) -> List %
  substArg  : (OP, List %, Z, %) -> %
  dispdiff  : List % -> Record(name:O, sub:O, arg:List O, level:N)
  ddiff     : List % -> O
  diffEval  : List % -> %
  dfeval    : (List %, K) -> %
  smprep    : (List SY, List N, List(List % -> %), MP) -> %
  diffdiff  : (List %, SY) -> %
  diffdiff0 : (List %, SY, %, K, List %) -> %
  subs      : (% -> %, K) -> %
  symsub    : (SY, Z) -> SY
  kunq      : (K, List SY, Boolean) -> %
  pushunq   : (List SY, List %) -> List %
  notfound  : (K -> %, List K, K) -> %

  equaldiff : (K,K)->Boolean

  debugA: (List % ,List %,Boolean) -> Boolean

  opdiff := operator("%diff"::SY)$CommonOperators()
```

```
opquote := operator("applyQuote"::SY)$CommonOperators

ground? x == retractIfCan(x)@Union(R,"failed") case R

ground  x == retract x

coerce(x:SY):% == kernel(x)@K :: %

retract(x:%):SY == symbolIfCan(retract(x)@K)::SY

applyQuote(s:SY, x:%) == applyQuote(s, [x])

applyQuote(s, x, y) == applyQuote(s, [x, y])

applyQuote(s, x, y, z) == applyQuote(s, [x, y, z])

applyQuote(s, x, y, z, t) == applyQuote(s, [x, y, z, t])

applyQuote(s:SY, l:List %) == opquote concat(s::%, l)

belong? op == op = opdiff or op = opquote

subs(fn, k) == kernel(operator k,[fn x for x in argument k]$List(%))

operator op ==
  is?(op, "%diff"::SY) => opdiff
  is?(op, "%quote"::SY) => opquote
  error "Unknown operator"

if R has ConvertibleTo InputForm then

  INP==>InputForm

  import MakeUnaryCompiledFunction(%, %, %)

  indiff: List % -> INP

  pint  : List INP-> INP

  differentiand: List % -> %

  differentiand l    == eval(first l, retract(second l)@K, third l)

  pint l  == convert concat(convert("D"::SY)@INP, l)

  indiff l ==
     r2:= convert([convert("::"::SY)@INP,_
                   convert(third l)@INP,_
                   convert("Symbol"::SY)@INP]@List INP)@INP
     pint [convert(differentiand l)@INP, r2]

  eval(f:%, s:SY)            == eval(f, [s])

  eval(f:%, s:OP, g:%, x:SY) == eval(f, [s], [g], x)
```

```
  eval(f:%, ls:List OP, lg:List %, x:SY) ==
    eval(f, ls, [compiledFunction(g, x) for g in lg])

  setProperty(opdiff,SPECIALINPUT,_
               indiff@(List % -> InputForm) pretend None)

variables x ==
  l := empty()$List(SY)
  for k in tower x repeat
    if ((s := symbolIfCan k) case SY) then l := concat(s::SY, l)
  reverse_! l

retractIfCan(x:%):Union(SY, "failed") ==
  (k := retractIfCan(x)@Union(K,"failed")) case "failed" => "failed"
  symbolIfCan(k::K)

if R has Ring then

  import UserDefinedPartialOrdering(SY)

  -- cannot use new()$Symbol because of possible re-instantiation
  gendiff := "%%0"::SY

  characteristic() == characteristic()$R

  coerce(k:K):% == k::MP::%

  symsub(sy, i) == concat(string sy, convert(i)@String)::SY

  numerator x == numer(x)::%

  eval(x:%, s:SY, n:N, f:% -> %) ==
    eval(x,[s],[n],[(y:List %):% +-> f(first(y))])

  eval(x:%, s:SY, n:N, f:List % -> %) == eval(x, [s], [n], [f])

  eval(x:%, l:List SY, f:List(List % -> %)) == eval(x, l, new(#l, 1), f)

  elt(op:OP, args:List %) ==
    unary? op and ((od? := has?(op, ODD)) or has?(op, EVEN)) and
      leadingCoefficient(numer first args) < 0 =>
        x := op(- first args)
        od? => -x
        x
    elt(op, args)$ExpressionSpace_&(%)

  eval(x:%, s:List SY, n:List N, l:List(% -> %)) ==
    eval(x, s, n, [y+-> f(first(y)) for f in l]$List(List % -> %))

  -- op(arg)**m ==> func(arg)**(m quo n) * op(arg)**(m rem n)
  smprep(lop, lexp, lfunc, p) ==
    (v := mainVariable p) case "failed" => p::%
    symbolIfCan(k := v::K) case SY => p::%
```

```
  g := (op := operator k)
     (arg := [eval(a,lop,lexp,lfunc) for a in argument k]$List(%))
  q := map(y+->eval(y::%, lop, lexp, lfunc),
           univariate(p, k))$SparseUnivariatePolynomialFunctions2(MP, %)
  (n := position(name op, lop)) < minIndex lop => q g
  a:%  := 0
  f    := eval((lfunc.n) arg, lop, lexp, lfunc)
  e    := lexp.n
  while q ^= 0 repeat
    m  := degree q
    qr := divide(m, e)
    t1 := f ** (qr.quotient)::N
    t2 := g ** (qr.remainder)::N
    a  := a + leadingCoefficient(q) * t1 * t2
    q  := reductum q
  a

dispdiff l ==
  s := second(l)::O
  t := third(l)::O
  a := argument(k := retract(first l)@K)
  is?(k, opdiff) =>
    rec := dispdiff a
    i   := position(s, rec.arg)
    rec.arg.i := t
    [rec.name,
       hconcat(rec.sub, hconcat(","::SY::O, (i+1-minIndex a)::O)),
                  rec.arg, (zero?(rec.level) => 0; rec.level + 1)]
  i   := position(second l, a)
  m   := [x::O for x in a]$List(O)
  m.i := t
  [name(operator k)::O, hconcat(","::SY::O, (i+1-minIndex a)::O),
                                     m, (empty? rest a => 1; 0)]

ddiff l ==
  rec := dispdiff l
  opname :=
    zero?(rec.level) => sub(rec.name, rec.sub)
    differentiate(rec.name, rec.level)
  prefix(opname, rec.arg)

substArg(op, l, i, g) ==
  z := copy l
  z.i := g
  kernel(op, z)


diffdiff(l, x) ==
  f := kernel(opdiff, l)
  diffdiff0(l, x, f, retract(f)@K, empty())

diffdiff0(l, x, expr, kd, done) ==
  op  := operator(k := retract(first l)@K)
  gg  := second l
```

```
  u   := third l
  arg := argument k
  ans:% := 0
  if (not member?(u,done)) and (ans := differentiate(u,x))^=0 then
    ans := ans * kernel(opdiff,
         [subst(expr, [kd], [kernel(opdiff, [first l, gg, gg])]),
                       gg, u])
  done := concat(gg, done)
  is?(k, opdiff) => ans + diffdiff0(arg, x, expr, k, done)
  for i in minIndex arg .. maxIndex arg for b in arg repeat
    if (not member?(b,done)) and (bp:=differentiate(b,x))^=0 then
      g   := symsub(gendiff, i)::%
      ans := ans + bp * kernel(opdiff, [subst(expr, [kd],
       [kernel(opdiff, [substArg(op, arg, i, g), gg, u])]), g, b])
  ans

dfeval(l, g) ==
  eval(differentiate(first l, symbolIfCan(g)::SY), g, third l)

diffEval l ==
  k:K
  g := retract(second l)@K
  ((u := retractIfCan(first l)@Union(K, "failed")) case "failed")
    or (u case K and symbolIfCan(k := u::K) case SY) => dfeval(l, g)
  op := operator k
  (ud := derivative op) case "failed" =>
       -- possible trouble
       -- make sure it is a dummy var
       dumm:%:=symsub(gendiff,1)::%
       ss:=subst(l.1,l.2=dumm)
       -- output(nl::OutputForm)$OutputPackage
       -- output("fixed"::OutputForm)$OutputPackage
       nl:=[ss,dumm,l.3]
       kernel(opdiff, nl)
  (n := position(second l,argument k)) < minIndex l =>
        dfeval(l,g)
  d := ud::List(List % -> %)
  eval((d.n)(argument k), g, third l)

diffArg(l, op, i) ==
  n := i - 1 + minIndex l
  z := copy l
  z.n := g := symsub(gendiff, n)::%
  [kernel(op, z), g, l.n]

opderiv(op, n) ==
  (n = 1) =>
    g := symsub(gendiff, n)::%
    [x +-> kernel(opdiff,[kernel(op, g), g, first x])]
  [y +-> kernel(opdiff, diffArg(y, op, i)) for i in 1..n]

kderiv k ==
  zero?(n := #(args := argument k)) => empty()
  op := operator k
```

```
      grad :=
        (u := derivative op) case "failed" => opderiv(op, n)
        u::List(List % -> %)
      if #grad ^= n then grad := opderiv(op, n)
      [g args for g in grad]

 -- SPECIALDIFF contains a map (List %, Symbol) -> %
 -- it is used when the usual chain rule does not apply,
 -- for instance with implicit algebraics.
    kerderiv(k, x) ==
      (v := symbolIfCan(k)) case SY =>
        v::SY = x => 1
        0
      (fn := property(operator k, SPECIALDIFF)) case None =>
         ((fn::None) pretend ((List %, SY) -> %)) (argument k, x)
      +/[g * differentiate(y,x) for g in kderiv k for y in argument k]

    smpderiv(p, x) ==
      map((s:R):R +-> retract differentiate(s::PR, x), p)::% +
       +/[differentiate(p,k)::% * kerderiv(k, x) for k in variables p]

    coerce(p:PR):% ==
      map(s +-> s::%, r +-> r::%, p)$PolynomialCategoryLifting(
                                 IndexedExponents SY, SY, R, PR, %)

    worse?(k1, k2) ==
      (u := less?(name operator k1,name operator k2)) case "failed" =>
        k1 < k2
      u::Boolean

    bestKernel l ==
      empty? rest l => first l
      a := bestKernel rest l
      worse?(first l, a) => a
      first l

    smp2O p ==
      (r:=retractIfCan(p)@Union(R,"failed")) case R =>r::R::OutputForm
      a :=
        userOrdered?() => bestKernel variables p
        mainVariable(p)::K
      outputForm(map((x:MP):% +-> x::%, univariate(p, a))_
          $SparseUnivariatePolynomialFunctions2(MP, %), a::OutputForm)

    smpsubst(p, lk, lv) ==
      map(x +-> match(lk, lv, x,
          notfound((z:K):%+->subs(s+->subst(s, lk, lv), z), lk, x))_
           $ListToMap(K,%),y+->y::%,p)_
            $PolynomialCategoryLifting(IndexedExponents K,K,R,MP,%)

    smpeval(p, lk, lv) ==
      map(x +-> match(lk, lv, x,
          notfound((z:K):%+->map(s+->eval(s,lk,lv),z),lk,x))_
           $ListToMap(K,%),y+->y::%,p)_
```

```
        $PolynomialCategoryLifting(IndexedExponents K,K,R,MP,%)

-- this is called on k when k is not a member of lk
notfound(fn, lk, k) ==
  empty? setIntersection(tower(f := k::%), lk) => f
  fn k

if R has ConvertibleTo InputForm then

  pushunq(l, arg) ==
     empty? l => [eval a for a in arg]
     [eval(a, l) for a in arg]

  kunq(k, l, givenlist?) ==
    givenlist? and empty? l => k::%
    is?(k, opquote) and
      (member?(s:=retract(first argument k)@SY, l) or empty? l) =>
        interpret(convert(concat(convert(s)@InputForm,
          [convert a for a in pushunq(l, rest argument k)
             ]@List(InputForm)))@InputForm)$InputFormFunctions1(%)
    (operator k) pushunq(l, argument k)

  smpunq(p, l, givenlist?) ==
    givenlist? and empty? l => p::%
    map(x +-> kunq(x, l, givenlist?), y+->y::%, p)_
      $PolynomialCategoryLifting(IndexedExponents K,K,R,MP,%)

smpret p ==
  "or"/[symbolIfCan(k) case "failed" for k in variables p] =>
    "failed"
  map(x+->symbolIfCan(x)::SY::PR, y+->y::PR,p)_
    $PolynomialCategoryLifting(IndexedExponents K, K, R, MP, PR)

isExpt(x:%, op:OP) ==
  (u := isExpt x) case "failed" => "failed"
  is?((u::Record(var:K, exponent:Z)).var, op) => u
  "failed"

isExpt(x:%, sy:SY) ==
  (u := isExpt x) case "failed" => "failed"
  is?((u::Record(var:K, exponent:Z)).var, sy) => u
  "failed"

if R has RetractableTo Z then

    smpIsMult p ==
      (u := mainVariable p) case K and (degree(q:=univariate(p,u::K))=1)
        and zero?(leadingCoefficient reductum q)
          and ((r:=retractIfCan(leadingCoefficient q)@Union(R,"failed"))
             case R)
              and (n := retractIfCan(r::R)@Union(Z, "failed")) case Z =>
                 [n::Z, u::K]
      "failed"
```

```
evaluate(opdiff, diffEval)

debugA(a1,a2,t) ==
   -- uncomment for debugging
   -- output(hconcat [a1::OutputForm,_
   --                 a2::OutputForm,t::OutputForm])$OutputPackage
   t

equaldiff(k1,k2) ==
  a1:=argument k1
  a2:=argument k2
  -- check the operator
  res:=operator k1 = operator k2
  not res => debugA(a1,a2,res)
  -- check the evaluation point
  res:= (a1.3 = a2.3)
  not res => debugA(a1,a2,res)
  -- check all the arguments
  res:= (a1.1 = a2.1) and (a1.2 = a2.2)
  res => debugA(a1,a2,res)
  -- check the substituted arguments
  (subst(a1.1,[retract(a1.2)@K],[a2.2]) = a2.1) => debugA(a1,a2,true)
  debugA(a1,a2,false)

setProperty(opdiff,SPECIALEQUAL,
                    equaldiff@((K,K) -> Boolean) pretend None)

setProperty(opdiff, SPECIALDIFF,
                    diffdiff@((List %, SY) -> %) pretend None)

setProperty(opdiff, SPECIALDISP,
                        ddiff@(List % -> OutputForm) pretend None)

if not(R has IntegralDomain) then

  mainKernel x == mainVariable numer x

  kernels x == variables numer x

  retract(x:%):R == retract numer x

  retract(x:%):PR == smpret(numer x)::PR

  retractIfCan(x:%):Union(R,  "failed") == retract numer x

  retractIfCan(x:%):Union(PR, "failed") == smpret numer x

  eval(x:%, lk:List K, lv:List %)  == smpeval(numer x, lk, lv)

  subst(x:%, lk:List K, lv:List %) == smpsubst(numer x, lk, lv)

  differentiate(x:%, s:SY) == smpderiv(numer x, s)

  coerce(x:%):OutputForm == smp2O numer x
```

```
    if R has ConvertibleTo InputForm then
      eval(f:%, l:List SY) == smpunq(numer f, l, true)

      eval f == smpunq(numer f, empty(), false)

    eval(x:%, s:List SY, n:List N, f:List(List % -> %)) ==
      smprep(s, n, f, numer x)

    isPlus x ==
      (u := isPlus numer x) case "failed" => "failed"
      [p::% for p in u::List(MP)]

    isTimes x ==
      (u := isTimes numer x) case "failed" => "failed"
      [p::% for p in u::List(MP)]

    isExpt x ==
      (u := isExpt numer x) case "failed" => "failed"
      r := u::Record(var:K, exponent:NonNegativeInteger)
      [r.var, r.exponent::Z]

    isPower x ==
      (u := isExpt numer x) case "failed" => "failed"
      r := u::Record(var:K, exponent:NonNegativeInteger)
      [r.var::%, r.exponent::Z]

    if R has ConvertibleTo Pattern Z then
      convert(x:%):Pattern(Z) == convert numer x

    if R has ConvertibleTo Pattern Float then
      convert(x:%):Pattern(Float) == convert numer x

    if R has RetractableTo Z then
      isMult x == smpIsMult numer x

if R has CommutativeRing then

  r:R * x:% == r::MP::% * x

if R has IntegralDomain then

  par : % -> %

  mainKernel x == mainVariable(x)$QF

  kernels x == variables(x)$QF

  univariate(x:%, k:K) == univariate(x, k)$QF

  isPlus x == isPlus(x)$QF

  isTimes x == isTimes(x)$QF
```

```
isExpt x == isExpt(x)$QF

isPower x == isPower(x)$QF

denominator x == denom(x)::%

coerce(q:Q):% == (numer q)::MP / (denom q)::MP

coerce(q:Fraction PR):% == (numer q)::% / (denom q)::%

coerce(q:Fraction Polynomial Q) == (numer q)::% / (denom q)::%

retract(x:%):PR == retract(retract(x)@Fraction(PR))

retract(x:%):Fraction(PR) == smpret(numer x)::PR / smpret(denom x)::PR

retract(x:%):R == (retract(numer x)@R exquo retract(denom x)@R)::R

coerce(x:%):OutputForm ==
  ((denom x) = 1) => smp2O numer x
  smp2O(numer x) / smp2O(denom x)

retractIfCan(x:%):Union(R, "failed") ==
  (n := retractIfCan(numer x)@Union(R, "failed")) case "failed" or
    (d := retractIfCan(denom x)@Union(R, "failed")) case "failed"
      or (r := n::R exquo d::R) case "failed" => "failed"
  r::R

eval(f:%, l:List SY) ==
  smpunq(numer f, l, true) / smpunq(denom f, l, true)

if R has ConvertibleTo InputForm then

  eval f ==
    smpunq(numer f, empty(), false) / smpunq(denom f, empty(), false)

  eval(x:%, s:List SY, n:List N, f:List(List % -> %)) ==
    smprep(s, n, f, numer x) / smprep(s, n, f, denom x)

differentiate(f:%, x:SY) ==
  (smpderiv(numer f, x) * denom(f)::% -
    numer(f)::% * smpderiv(denom f, x))
      / (denom(f)::% ** 2)

eval(x:%, lk:List K, lv:List %) ==
  smpeval(numer x, lk, lv) / smpeval(denom x, lk, lv)

subst(x:%, lk:List K, lv:List %) ==
  smpsubst(numer x, lk, lv) / smpsubst(denom x, lk, lv)

par x ==
  (r := retractIfCan(x)@Union(R, "failed")) case R => x
  paren x
```

```
convert(x:Factored %):% ==
  par(unit x) * */[par(f.factor) ** f.exponent for f in factors x]

retractIfCan(x:%):Union(PR, "failed") ==
  (u := retractIfCan(x)@Union(Fraction PR,"failed")) case "failed"
    => "failed"
  retractIfCan(u::Fraction(PR))

retractIfCan(x:%):Union(Fraction PR, "failed") ==
  (n := smpret numer x) case "failed" => "failed"
  (d := smpret denom x) case "failed" => "failed"
  n::PR / d::PR

coerce(p:Polynomial Q):% ==
  map(x+->x::%, y+->y::%,p)_
   $PolynomialCategoryLifting(IndexedExponents SY, SY,
                                             Q, Polynomial Q, %)

if R has RetractableTo Z then

  coerce(x:Fraction Z):% == numer(x)::MP / denom(x)::MP

  isMult x ==
     (u := smpIsMult numer x) case "failed"
        or (v := retractIfCan(denom x)@Union(R, "failed")) case "failed"
           or (w := retractIfCan(v::R)@Union(Z, "failed")) case "failed"
               => "failed"
     r := u::Record(coef:Z, var:K)
     (q := r.coef exquo w::Z) case "failed" => "failed"
     [q::Z, r.var]

if R has ConvertibleTo Pattern Z then

  convert(x:%):Pattern(Z) == convert(numer x) / convert(denom x)

if R has ConvertibleTo Pattern Float then

  convert(x:%):Pattern(Float) ==
    convert(numer x) / convert(denom x)
```

———-

— COQ FS —

```
(* category FS *)
(*
    import BasicOperatorFunctions1(%)

    -- these are needed in Ring only, but need to be declared here
    -- because of compiler bug: if they are declared inside the Ring
    -- case, then they are not visible inside the IntegralDomain case.

    opdiff := operator("%diff"::SY)$CommonOperators()
```

```
opquote := operator("applyQuote"::SY)$CommonOperators

ground? : % -> Boolean
ground? x == retractIfCan(x)@Union(R,"failed") case R

ground : % -> R
ground  x == retract x

coerce : Symbol -> %
coerce(x:SY):% == kernel(x)@K :: %

retract : % -> Symbol
retract(x:%):SY == symbolIfCan(retract(x)@K)::SY

applyQuote : (Symbol,%) -> %
applyQuote(s:SY, x:%) == applyQuote(s, [x])

applyQuote : (Symbol,%,%) -> %
applyQuote(s, x, y) == applyQuote(s, [x, y])

applyQuote : (Symbol,%,%,%) -> %
applyQuote(s, x, y, z) == applyQuote(s, [x, y, z])

applyQuote : (Symbol,%,%,%,%) -> %
applyQuote(s, x, y, z, t)  == applyQuote(s, [x, y, z, t])

applyQuote : (Symbol,List(%)) -> %
applyQuote(s:SY, l:List %) == opquote concat(s::%, l)

belong? : BasicOperator -> Boolean
belong? op == op = opdiff or op = opquote

subs : (% -> %, K) -> %
subs(fn, k) == kernel(operator k,[fn x for x in argument k]$List(%))

operator : BasicOperator -> BasicOperator
operator op ==
  is?(op, "%diff"::SY) => opdiff
  is?(op, "%quote"::SY) => opquote
  error "Unknown operator"

if R has ConvertibleTo InputForm then

  INP==>InputForm

  import MakeUnaryCompiledFunction(%, %, %)

  differentiand: List % -> %
  differentiand l == eval(first l, retract(second l)@K, third l)

  pint  : List INP-> INP
  pint l  == convert concat(convert("D"::SY)@INP, l)
```

```
      indiff: List % -> INP
      indiff l ==
         r2:= convert([convert("::"::SY)@INP,_
                      convert(third l)@INP,_
                      convert("Symbol"::SY)@INP]@List INP)@INP
         pint [convert(differentiand l)@INP, r2]

      eval(f:%, s:SY)            == eval(f, [s])

      eval(f:%, s:OP, g:%, x:SY) == eval(f, [s], [g], x)

      eval(f:%, ls:List OP, lg:List %, x:SY) ==
        eval(f, ls, [compiledFunction(g, x) for g in lg])

      setProperty(opdiff,SPECIALINPUT,_
                  indiff@(List % -> InputForm) pretend None)

    variables : % -> List(Symbol)
    variables x ==
      l := empty()$List(SY)
      for k in tower x repeat
        if ((s := symbolIfCan k) case SY) then l := concat(s::SY, l)
      reverse_! l

    retractIfCan : % -> Union(Symbol,"failed")
    retractIfCan(x:%):Union(SY, "failed") ==
      (k := retractIfCan(x)@Union(K,"failed")) case "failed" => "failed"
      symbolIfCan(k::K)

    if R has Ring then

      import UserDefinedPartialOrdering(SY)

-- cannot use new()$Symbol because of possible re-instantiation
      gendiff := "%%0"::SY

      characteristic : () -> NonNegativeInteger
      characteristic() == characteristic()$R

      coerce : Kernel(%) -> %
      coerce(k:K):% == k::MP::%

      symsub : (SY, Z) -> SY
      symsub(sy, i) == concat(string sy, convert(i)@String)::SY

      numerator : % -> %
      numerator x == numer(x)::%

      eval : (%,Symbol,NonNegativeInteger,(% -> %)) -> %
      eval(x:%, s:SY, n:N, f:% -> %) ==
        eval(x,[s],[n],[(y:List %):% +-> f(first(y))])

      eval : (%,Symbol,NonNegativeInteger,(List(%) -> %)) -> %
      eval(x:%, s:SY, n:N, f:List % -> %) == eval(x, [s], [n], [f])
```

```
eval : (%,List(Symbol),List((List(%) -> %))) -> %
eval(x:%, l:List SY, f:List(List % -> %)) == eval(x, l, new(#l, 1), f)

elt : (BasicOperator,List(%)) -> %
elt(op:OP, args:List %) ==
  unary? op and ((od? := has?(op, ODD)) or has?(op, EVEN)) and
    leadingCoefficient(numer first args) < 0 =>
      x := op(- first args)
      od? => -x
      x
  elt(op, args)$ExpressionSpace_&(%)

eval : (%,List(Symbol),List(NonNegativeInteger),List((% -> %))) -> %
eval(x:%, s:List SY, n:List N, l:List(% -> %)) ==
  eval(x, s, n, [y+-> f(first(y)) for f in l]$List(List % -> %))

-- op(arg)**m ==> func(arg)**(m quo n) * op(arg)**(m rem n)
smprep : (List SY, List N, List(List % -> %), MP) -> %
smprep(lop, lexp, lfunc, p) ==
  (v := mainVariable p) case "failed" => p::%
  symbolIfCan(k := v::K) case SY => p::%
  g := (op := operator k)
     (arg := [eval(a,lop,lexp,lfunc) for a in argument k]$List(%))
  q := map(y+->eval(y::%, lop, lexp, lfunc),
           univariate(p, k))$SparseUnivariatePolynomialFunctions2(MP, %)
  (n := position(name op, lop)) < minIndex lop => q g
  a:%  := 0
  f    := eval((lfunc.n) arg, lop, lexp, lfunc)
  e    := lexp.n
  while q ^= 0 repeat
    m  := degree q
    qr := divide(m, e)
    t1 := f ** (qr.quotient)::N
    t2 := g ** (qr.remainder)::N
    a  := a + leadingCoefficient(q) * t1 * t2
    q  := reductum q
  a

dispdiff : List % -> Record(name:O, sub:O, arg:List O, level:N)
dispdiff l ==
  s := second(l)::O
  t := third(l)::O
  a := argument(k := retract(first l)@K)
  is?(k, opdiff) =>
    rec := dispdiff a
    i   := position(s, rec.arg)
    rec.arg.i := t
    [rec.name,
       hconcat(rec.sub, hconcat(","::SY::O, (i+1-minIndex a)::O)),
                  rec.arg, (zero?(rec.level) => 0; rec.level + 1)]
  i   := position(second l, a)
  m   := [x::O for x in a]$List(O)
  m.i := t
```

```
  [name(operator k)::O, hconcat(","::SY::O, (i+1-minIndex a)::O),
                                    m, (empty? rest a => 1; 0)]

ddiff : List % -> O
ddiff l ==
  rec := dispdiff l
  opname :=
    zero?(rec.level) => sub(rec.name, rec.sub)
    differentiate(rec.name, rec.level)
  prefix(opname, rec.arg)

substArg : (OP, List %, Z, %) -> %
substArg(op, l, i, g) ==
  z := copy l
  z.i := g
  kernel(op, z)

diffdiff : (List %, SY) -> %
diffdiff(l, x) ==
  f := kernel(opdiff, l)
  diffdiff0(l, x, f, retract(f)@K, empty())

diffdiff0 : (List %, SY, %, K, List %) -> %
diffdiff0(l, x, expr, kd, done) ==
  op  := operator(k := retract(first l)@K)
  gg  := second l
  u   := third l
  arg := argument k
  ans:% := 0
  if (not member?(u,done)) and (ans := differentiate(u,x))^=0 then
    ans := ans * kernel(opdiff,
         [subst(expr, [kd], [kernel(opdiff, [first l, gg, gg])]),
                      gg, u])
  done := concat(gg, done)
  is?(k, opdiff) => ans + diffdiff0(arg, x, expr, k, done)
  for i in minIndex arg .. maxIndex arg for b in arg repeat
    if (not member?(b,done)) and (bp:=differentiate(b,x))^=0 then
      g   := symsub(gendiff, i)::%
      ans := ans + bp * kernel(opdiff, [subst(expr, [kd],
       [kernel(opdiff, [substArg(op, arg, i, g), gg, u])]), g, b])
  ans

dfeval : (List %, K) -> %
dfeval(l, g) ==
  eval(differentiate(first l, symbolIfCan(g)::SY), g, third l)

diffEval : List % -> %
diffEval l ==
  k:K
  g := retract(second l)@K
  ((u := retractIfCan(first l)@Union(K, "failed")) case "failed")
    or (u case K and symbolIfCan(k := u::K) case SY) => dfeval(l, g)
  op := operator k
  (ud := derivative op) case "failed" =>
```

```
          -- possible trouble
          -- make sure it is a dummy var
          dumm:%:=symsub(gendiff,1)::%
          ss:=subst(l.1,l.2=dumm)
          -- output(nl::OutputForm)$OutputPackage
          -- output("fixed"::OutputForm)$OutputPackage
          nl:=[ss,dumm,l.3]
          kernel(opdiff, nl)
     (n := position(second l,argument k)) < minIndex l =>
           dfeval(l,g)
     d := ud::List(List % -> %)
     eval((d.n)(argument k), g, third l)

   diffArg : (List %, OP, N) -> List %
   diffArg(l, op, i) ==
     n := i - 1 + minIndex l
     z := copy l
     z.n := g := symsub(gendiff, n)::%
     [kernel(op, z), g, l.n]

   opderiv : (OP, N) -> List(List % -> %)
   opderiv(op, n) ==
     (n = 1) =>
       g := symsub(gendiff, n)::%
       [x +-> kernel(opdiff,[kernel(op, g), g, first x])]
     [y +-> kernel(opdiff, diffArg(y, op, i)) for i in 1..n]

   kderiv : K -> List %
   kderiv k ==
     zero?(n := #(args := argument k)) => empty()
     op := operator k
     grad :=
       (u := derivative op) case "failed" => opderiv(op, n)
       u::List(List % -> %)
     if #grad ^= n then grad := opderiv(op, n)
     [g args for g in grad]

 -- SPECIALDIFF contains a map (List %, Symbol) -> %
 -- it is used when the usual chain rule does not apply,
 -- for instance with implicit algebraics.

   kerderiv : (K, SY)  -> %
   kerderiv(k, x) ==
     (v := symbolIfCan(k)) case SY =>
       v::SY = x => 1
       0
     (fn := property(operator k, SPECIALDIFF)) case None =>
        ((fn::None) pretend ((List %, SY) -> %)) (argument k, x)
     +/[g * differentiate(y,x) for g in kderiv k for y in argument k]

   smpderiv : (MP, SY) -> %
   smpderiv(p, x) ==
     map((s:R):R +-> retract differentiate(s::PR, x), p)::% +
      +/[differentiate(p,k)::% * kerderiv(k, x) for k in variables p]
```

```
      coerce : Polynomial(R) -> %
      coerce(p:PR):% ==
        map(s +-> s::%, r +-> r::%, p)$PolynomialCategoryLifting(
                                 IndexedExponents SY, SY, R, PR, %)

      worse? : (K, K) -> Boolean
      worse?(k1, k2) ==
        (u := less?(name operator k1,name operator k2)) case "failed" =>
          k1 < k2
        u::Boolean

      bestKernel: List K -> K
      bestKernel l ==
        empty? rest l => first l
        a := bestKernel rest l
        worse?(first l, a) => a
        first l

      smp2O : MP -> O
      smp2O p ==
        (r:=retractIfCan(p)@Union(R,"failed")) case R =>r::R::OutputForm
        a :=
          userOrdered?() => bestKernel variables p
          mainVariable(p)::K
        outputForm(map((x:MP):% +-> x::%, univariate(p, a))_
            $SparseUnivariatePolynomialFunctions2(MP, %), a::OutputForm)

      smpsubst : (MP, List K, List %) -> %
      smpsubst(p, lk, lv) ==
        map(x +-> match(lk, lv, x,
            notfound((z:K):%+->subs(s+->subst(s, lk, lv), z), lk, x))_
             $ListToMap(K,%),y+->y::%,p)_
              $PolynomialCategoryLifting(IndexedExponents K,K,R,MP,%)

      smpeval : (MP, List K, List %) -> %
      smpeval(p, lk, lv) ==
        map(x +-> match(lk, lv, x,
            notfound((z:K):%+->map(s+->eval(s,lk,lv),z),lk,x))_
             $ListToMap(K,%),y+->y::%,p)_
              $PolynomialCategoryLifting(IndexedExponents K,K,R,MP,%)

-- this is called on k when k is not a member of lk

      notfound : (K -> %, List K, K) -> %
      notfound(fn, lk, k) ==
        empty? setIntersection(tower(f := k::%), lk) => f
        fn k

      if R has ConvertibleTo InputForm then

        pushunq : (List SY, List %) -> List %
        pushunq(l, arg) ==
           empty? l => [eval a for a in arg]
```

```
      [eval(a, l) for a in arg]

    kunq : (K, List SY, Boolean) -> %
    kunq(k, l, givenlist?) ==
      givenlist? and empty? l => k::%
      is?(k, opquote) and
        (member?(s:=retract(first argument k)@SY, l) or empty? l) =>
          interpret(convert(concat(convert(s)@InputForm,
            [convert a for a in pushunq(l, rest argument k)
               ]@List(InputForm)))@InputForm)$InputFormFunctions1(%)
      (operator k) pushunq(l, argument k)

    smpunq : (MP, List SY, Boolean) -> %
    smpunq(p, l, givenlist?) ==
      givenlist? and empty? l => p::%
      map(x +-> kunq(x, l, givenlist?), y+->y::%, p)_
        $PolynomialCategoryLifting(IndexedExponents K,K,R,MP,%)

  smpret : MP -> Union(PR, "failed")
  smpret p ==
    "or"/[symbolIfCan(k) case "failed" for k in variables p] =>
      "failed"
    map(x+->symbolIfCan(x)::SY::PR, y+->y::PR,p)_
      $PolynomialCategoryLifting(IndexedExponents K, K, R, MP, PR)

  isExpt : (%,BasicOperator) ->
     Union(Record(var: Kernel(%),exponent: Integer),"failed")
  isExpt(x:%, op:OP) ==
    (u := isExpt x) case "failed" => "failed"
    is?((u::Record(var:K, exponent:Z)).var, op) => u
    "failed"

  isExpt : (%,Symbol) ->
     Union(Record(var: Kernel(%),exponent: Integer),"failed")
  isExpt(x:%, sy:SY) ==
    (u := isExpt x) case "failed" => "failed"
    is?((u::Record(var:K, exponent:Z)).var, sy) => u
    "failed"

  if R has RetractableTo Z then

      smpIsMult : MP -> Union(Record(coef:Z, var:K),"failed")
      smpIsMult p ==
        (u := mainVariable p) case K and (degree(q:=univariate(p,u::K))=1)
          and zero?(leadingCoefficient reductum q)
            and ((r:=retractIfCan(leadingCoefficient q)@Union(R,"failed"))
               case R)
                 and (n := retractIfCan(r::R)@Union(Z, "failed")) case Z =>
                   [n::Z, u::K]
        "failed"

  debugA: (List % ,List %,Boolean) -> Boolean
  debugA(a1,a2,t) ==
     -- uncomment for debugging
```

```
   -- output(hconcat [a1::OutputForm,_
   --                a2::OutputForm,t::OutputForm])$OutputPackage
   t

equaldiff : (K,K)->Boolean
equaldiff(k1,k2) ==
  a1:=argument k1
  a2:=argument k2
  -- check the operator
  res:=operator k1 = operator k2
  not res => debugA(a1,a2,res)
  -- check the evaluation point
  res:= (a1.3 = a2.3)
  not res => debugA(a1,a2,res)
  -- check all the arguments
  res:= (a1.1 = a2.1) and (a1.2 = a2.2)
  res => debugA(a1,a2,res)
  -- check the substituted arguments
  (subst(a1.1,[retract(a1.2)@K],[a2.2]) = a2.1) => debugA(a1,a2,true)
  debugA(a1,a2,false)

setProperty(opdiff,SPECIALEQUAL,
                    equaldiff@((K,K) -> Boolean) pretend None)

setProperty(opdiff, SPECIALDIFF,
                    diffdiff@((List %, SY) -> %) pretend None)

setProperty(opdiff, SPECIALDISP,
                        ddiff@(List % -> OutputForm) pretend None)

if not(R has IntegralDomain) then

  mainKernel : % -> Union(Kernel(%),"failed")
  mainKernel x == mainVariable numer x

  kernels : % -> List(Kernel(%))
  kernels x == variables numer x

  retract : % -> R
  retract(x:%):R == retract numer x

  retract : % -> Polynomial(R)
  retract(x:%):PR == smpret(numer x)::PR

  retractIfCan : % -> Union(Fraction(Integer),"failed")
  retractIfCan(x:%):Union(R, "failed") == retract numer x

  retractIfCan : % -> Union(Polynomial(R),"failed")
  retractIfCan(x:%):Union(PR, "failed") == smpret numer x

  eval : (%,List(Kernel(%)),List(%)) -> %
  eval(x:%, lk:List K, lv:List %)  == smpeval(numer x, lk, lv)

  subst : (%,List(Kernel(%)),List(%)) -> %
```

```
subst(x:%, lk:List K, lv:List %) == smpsubst(numer x, lk, lv)

differentiate : (%,Symbol) -> %
differentiate(x:%, s:SY) == smpderiv(numer x, s)

coerce : % -> OutputForm
coerce(x:%):OutputForm == smp2O numer x

if R has ConvertibleTo InputForm then

  eval(f:%, l:List SY) == smpunq(numer f, l, true)

  eval f == smpunq(numer f, empty(), false)

eval : (%,List(Symbol),List(NonNegativeInteger),List((List(%) -> %)))
    -> %
eval(x:%, s:List SY, n:List N, f:List(List % -> %)) ==
  smprep(s, n, f, numer x)

isPlus : % -> Union(List(%),"failed")
isPlus x ==
  (u := isPlus numer x) case "failed" => "failed"
  [p::% for p in u::List(MP)]

isTimes : % -> Union(List(%),"failed")
isTimes x ==
  (u := isTimes numer x) case "failed" => "failed"
  [p::% for p in u::List(MP)]

isExpt : % -> Union(Record(var: Kernel(%),exponent: Integer),"failed")
isExpt x ==
  (u := isExpt numer x) case "failed" => "failed"
  r := u::Record(var:K, exponent:NonNegativeInteger)
  [r.var, r.exponent::Z]

isPower : % -> Union(Record(val: %,exponent: Integer),"failed")
isPower x ==
  (u := isExpt numer x) case "failed" => "failed"
  r := u::Record(var:K, exponent:NonNegativeInteger)
  [r.var::%, r.exponent::Z]

if R has ConvertibleTo Pattern Z then

  convert : % -> Pattern(Integer)
  convert(x:%):Pattern(Z) == convert numer x

if R has ConvertibleTo Pattern Float then

  convert : % -> Pattern(Float)
  convert(x:%):Pattern(Float) == convert numer x

if R has RetractableTo Z then

  isMult : % -> Union(Record(coef: Integer,var: Kernel(%)),"failed")
```

```
      isMult x == smpIsMult numer x

if R has CommutativeRing then

  ?*? : (R,%) -> %
  r:R * x:% == r::MP::% * x

if R has IntegralDomain then

  mainKernel : % -> Union(Kernel(%),"failed")
  mainKernel x == mainVariable(x)$QF

  kernels : % -> List(Kernel(%))
  kernels x == variables(x)$QF

  univariate : (%,Kernel(%)) -> Fraction(SparseUnivariatePolynomial(%))
  univariate(x:%, k:K) == univariate(x, k)$QF

  isPlus : % -> Union(List(%),"failed")
  isPlus x == isPlus(x)$QF

  isTimes : % -> Union(List(%),"failed")
  isTimes x == isTimes(x)$QF

  isExpt : % -> Union(Record(var: Kernel(%),exponent: Integer),"failed")
  isExpt x == isExpt(x)$QF

  isPower : % -> Union(Record(val: %,exponent: Integer),"failed")
  isPower x == isPower(x)$QF

  denominator : % -> %
  denominator x == denom(x)::%

  coerce : Fraction(R) -> %
  coerce(q:Q):% == (numer q)::MP / (denom q)::MP

  coerce : Fraction(Polynomial(R)) -> %
  coerce(q:Fraction PR):% == (numer q)::% / (denom q)::%

  coerce : Fraction(Polynomial(Fraction(R))) -> %
  coerce(q:Fraction Polynomial Q) == (numer q)::% / (denom q)::%

  retract : % -> Polynomial(R)
  retract(x:%):PR == retract(retract(x)@Fraction(PR))

  retract : % -> Fraction(Polynomial(R))
  retract(x:%):Fraction(PR) == smpret(numer x)::PR / smpret(denom x)::PR

  retract : % -> R
  retract(x:%):R == (retract(numer x)@R exquo retract(denom x)@R)::R

  coerce : % -> OutputForm
  coerce(x:%):OutputForm ==
    ((denom x) = 1) => smp2O numer x
```

```
    smp2O(numer x) / smp2O(denom x)

  retractIfCan : % -> Union(R,"failed")
  retractIfCan(x:%):Union(R,"failed") ==
    (n := retractIfCan(numer x)@Union(R, "failed")) case "failed" or
      (d := retractIfCan(denom x)@Union(R, "failed")) case "failed"
        or (r := n::R exquo d::R) case "failed" => "failed"
    r::R

  eval : (%,Symbol) -> %
  eval(f:%, l:List SY) ==
    smpunq(numer f, l, true) / smpunq(denom f, l, true)

  if R has ConvertibleTo InputForm then

    eval : % -> %
    eval f ==
      smpunq(numer f, empty(), false) / smpunq(denom f, empty(), false)

    eval : (%,List(Symbol),List(NonNegativeInteger),List((% -> %))) -> %
    eval(x:%, s:List SY, n:List N, f:List(List % -> %)) ==
      smprep(s, n, f, numer x) / smprep(s, n, f, denom x)

  differentiate : (%,Symbol) -> %
  differentiate(f:%, x:SY) ==
    (smpderiv(numer f, x) * denom(f)::% -
      numer(f)::% * smpderiv(denom f, x))
        / (denom(f)::% ** 2)

  eval : (%,List(%),List(%)) -> %
  eval(x:%, lk:List K, lv:List %) ==
    smpeval(numer x, lk, lv) / smpeval(denom x, lk, lv)

  subst : (%,List(Kernel(%)),List(%)) -> %
  subst(x:%, lk:List K, lv:List %) ==
    smpsubst(numer x, lk, lv) / smpsubst(denom x, lk, lv)

  par : % -> %
  par x ==
    (r := retractIfCan(x)@Union(R, "failed")) case R => x
    paren x

  convert : Factored(%) -> %
  convert(x:Factored %):% ==
    par(unit x) * */[par(f.factor) ** f.exponent for f in factors x]

  retractIfCan : % -> Union(Polynomial(R),"failed")
  retractIfCan(x:%):Union(PR, "failed") ==
    (u := retractIfCan(x)@Union(Fraction PR,"failed")) case "failed"
      => "failed"
    retractIfCan(u::Fraction(PR))

  retractIfCan : % -> Union(Fraction(Polynomial(R)),"failed")
  retractIfCan(x:%):Union(Fraction PR, "failed") ==
```

```
        (n := smpret numer x) case "failed" => "failed"
        (d := smpret denom x) case "failed" => "failed"
        n::PR / d::PR

      coerce : Polynomial(Fraction(R)) ->
      coerce(p:Polynomial Q):% ==
        map(x+->x::%, y+->y::%,p)_
         $PolynomialCategoryLifting(IndexedExponents SY, SY,
                                              Q, Polynomial Q, %)

      if R has RetractableTo Z then

       coerce : Fraction(Integer) -> %
       coerce(x:Fraction Z):% == numer(x)::MP / denom(x)::MP

        isMult : % -> Union(Record(coef: Integer,var: Kernel(%)),"failed")
        isMult x ==
           (u := smpIsMult numer x) case "failed"
              or (v := retractIfCan(denom x)@Union(R, "failed")) case "failed"
                 or (w := retractIfCan(v::R)@Union(Z, "failed")) case "failed"
                     => "failed"
           r := u::Record(coef:Z, var:K)
           (q := r.coef exquo w::Z) case "failed" => "failed"
           [q::Z, r.var]

      if R has ConvertibleTo Pattern Z then

        convert : % -> Pattern(Integer)
        convert(x:%):Pattern(Z) == convert(numer x) / convert(denom x)

      if R has ConvertibleTo Pattern Float then

        convert : % -> Pattern(Float)
        convert(x:%):Pattern(Float) ==
          convert(numer x) / convert(denom x)
*)
```

———

— **FS.dotabb** —

```
"FS"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FS"];
"FS" -> "ES"
"FS" -> "FPATMAB"
"FS" -> "FRETRCT"
"FS" -> "PATAB"
"FS" -> "RETRACT"
"FS" -> "KONVERT"
"FS" -> "MONOID"
"FS" -> "GROUP"
"FS" -> "ABELMON"
"FS" -> "ABELGRP"
```

```
"FS" -> "PDRING"
"FS" -> "FLINEXP"
"FS" -> "CHARNZ"
"FS" -> "INTDOM"
"FS" -> "FIELD"
```

———-

— **FS.dotfull** —

```
"FunctionSpace(a:OrderedSet)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FS"];
"FunctionSpace(a:OrderedSet)" -> "ExpressionSpace()"
"FunctionSpace(a:OrderedSet)" -> "RetractableTo(Symbol)"
"FunctionSpace(a:OrderedSet)" -> "Patternable(OrderedSet)"
"FunctionSpace(a:OrderedSet)" -> "FullyPatternMatchable(OrderedSet)"
"FunctionSpace(a:OrderedSet)" -> "FullyRetractableTo(OrderedSet)"
"FunctionSpace(a:OrderedSet)" -> "ConvertibleTo(InputForm)"
"FunctionSpace(a:OrderedSet)" -> "Monoid()"
"FunctionSpace(a:OrderedSet)" -> "Group()"
"FunctionSpace(a:OrderedSet)" -> "AbelianMonoid()"
"FunctionSpace(a:OrderedSet)" -> "AbelianGroup()"
"FunctionSpace(a:OrderedSet)" -> "PartialDifferentialRing(Symbol)"
"FunctionSpace(a:OrderedSet)" -> "FullyLinearlyExplicitRingOver(OrderedSet)"
"FunctionSpace(a:OrderedSet)" -> "CharacteristicNonZero()"
"FunctionSpace(a:OrderedSet)" -> "IntegralDomain()"
"FunctionSpace(a:OrderedSet)" -> "Field()"
"FunctionSpace(a:OrderedSet)" -> "RetractableTo(Integer)"
"FunctionSpace(a:OrderedSet)" -> "RetractableTo(Fraction(Integer))"
```

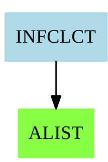
———-

— **FS.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FunctionSpace(a:OrderedSet)" [color=lightblue];
"FunctionSpace(a:OrderedSet)" -> "ES..."
"FunctionSpace(a:OrderedSet)" -> "RETRACT..."
"FunctionSpace(a:OrderedSet)" -> "PATAB..."
"FunctionSpace(a:OrderedSet)" -> "FPATMAB..."
"FunctionSpace(a:OrderedSet)" -> "FRETRCT..."
"FunctionSpace(a:OrderedSet)" -> "KONVERT..."
"FunctionSpace(a:OrderedSet)" -> "MONOID..."
"FunctionSpace(a:OrderedSet)" -> "GROUP..."
"FunctionSpace(a:OrderedSet)" -> "ABELMON..."
"FunctionSpace(a:OrderedSet)" -> "ABELGRP..."
"FunctionSpace(a:OrderedSet)" -> "PDRING..."
"FunctionSpace(a:OrderedSet)" -> "FLINEXP..."
"FunctionSpace(a:OrderedSet)" -> "CHARNZ..."
```

```
"FunctionSpace(a:OrderedSet)" -> "INTDOM..."
"FunctionSpace(a:OrderedSet)" -> "FIELD..."
"FunctionSpace(a:OrderedSet)" -> "RETRACT..."

"ES..." [color=lightblue];
"EVALABLE..." [color=lightblue];
"FRETRCT..." [color=lightblue];
"FPATMAB..." [color=lightblue];
"IEVALAB..." [color=lightblue];
"ORDSET..." [color=lightblue];
"PATAB..." [color=lightblue];
"RETRACT..." [color=lightblue];
"KONVERT..." [color=lightblue];
"MONOID..." [color=lightblue];
"GROUP..." [color=lightblue];
"ABELMON..." [color=lightblue];
"ABELGRP..." [color=lightblue];
"PDRING..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"CHARNZ..." [color=lightblue];
"INTDOM..." [color=lightblue];
"FIELD..." [color=lightblue];
"RETRACT..." [color=lightblue];
}
```

17.0.220 InfinitlyClosePointCategory (INFCLCT)



— InfinitlyClosePointCategory.input —

```
)set break resume
)sys rm -f InfinitlyClosePointCategory.output
)spool InfinitlyClosePointCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InfinitlyClosePointCategory
--R
--I InfinitlyClosePointCategory(K: Field,
--I symb: List Symbol,
--I PolyRing: PolynomialCategory(t#1,t#4,OrderedVariableList t#2),
```

```
--I E: DirectProductCategory(# t#2,NonNegativeInteger),
--I ProjPt: ProjectiveSpaceCategory t#1,
--I PCS: LocalPowerSeriesCategory t#1,
--I Plc: PlacesCategory(t#1,t#6),
--I DIVISOR: DivisorCategory t#7,
--I BLMET: BlowUpMethodCategory) is a category constructor
--I Abbreviation for InfinitlyClosePointCategory is INFCLCT
--I This constructor is exposed in this frame.
--I Issue )edit bookvol10.2.pamphlet to see algebra source code for INFCLCT
--I
--I------------------------------- Operations --------------------------------
--I ?=? : (%,%) -> Boolean                actualExtensionV : % -> K
--I chartV : % -> BLMET                   coerce : % -> OutputForm
--I create : (ProjPt,PolyRing) -> %       degree : % -> PositiveInteger
--I excpDivV : % -> DIVISOR               hash : % -> SingleInteger
--I latex : % -> String                   localParamV : % -> List PCS
--I localPointV : % -> AffinePlane K      multV : % -> NonNegativeInteger
--I pointV : % -> ProjPt                  setchart! : (%,BLMET) -> BLMET
--I setpoint! : (%,ProjPt) -> ProjPt      symbNameV : % -> Symbol
--I ?~=? : (%,%) -> Boolean
--I create : (ProjPt,
--I  DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K),
--I  AffinePlane K,
--I  NonNegativeInteger,
--I  BLMET,
--I  NonNegativeInteger,
--I  DIVISOR,
--I  K,
--I  Symbol) -> %
--I curveV : % ->
--I  DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K)
--I setcurve! :
--I  (%,DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K)) ->
--I  DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K)
--I setexcpDiv! : (%,DIVISOR) -> DIVISOR
--I setlocalParam! : (%,List PCS) -> List PCS
--I setlocalPoint! : (%,AffinePlane K) -> AffinePlane K
--I setmult! : (%,NonNegativeInteger) -> NonNegativeInteger
--I setsubmult! : (%,NonNegativeInteger) -> NonNegativeInteger
--I setsymbName! : (%,Symbol) -> Symbol
--I subMultV : % -> NonNegativeInteger
--I
--E 1

)spool
)lisp (bye)
```

— InfinitlyClosePointCategory.help —

```
====================================================================
InfinitlyClosePointCategory examples
====================================================================
```

```
This category is part of the PAFF package

See Also:
o )show InfinitlyClosePointCategory
```

———–

⇐ "SetCategoryWithDegree" (SETCATD) 4.0.86 on page 358

Exports:

?=?	?~=?	actualExtensionV	chartV
coerce	create	curveV	degree
excpDivV	hash	latex	localParamV
localPointV	multV	pointV	setchart!
setcurve!	setexcpDiv!	setlocalParam!	setlocalPoint!
setmult!	setpoint!	setsubmult!	setsymbName!
subMultV	symbNameV		

These are directly exported but not implemented:

```
actualExtensionV : % -> K
chartV : % -> BLMET
create :
 (ProjPt,DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K),
  AffinePlane K,NonNegativeInteger,BLMET,NonNegativeInteger,DIVISOR,K,Symbol)
    -> %
create : (ProjPt,PolyRing) -> %
curveV : % -> DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K)
excpDivV : % -> DIVISOR
localParamV : % -> List PCS
localPointV : % -> AffinePlane K
multV : % -> NonNegativeInteger
pointV : % -> ProjPt
setchart! : (%,BLMET) -> BLMET
setcurve! :
  (%,DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K)) ->
    DistributedMultivariatePolynomial([construct,QUOTEX,QUOTEY],K)
setexcpDiv! : (%,DIVISOR) -> DIVISOR
setlocalParam! : (%,List PCS) -> List PCS
setlocalPoint! : (%,AffinePlane K) -> AffinePlane K
setmult! : (%,NonNegativeInteger) -> NonNegativeInteger
setpoint! : (%,ProjPt) -> ProjPt
setsubmult! : (%,NonNegativeInteger) -> NonNegativeInteger
setsymbName! : (%,Symbol) -> Symbol
subMultV : % -> NonNegativeInteger
symbNameV : % -> Symbol
```

These exports come from (p358) SetCategoryWithDegree:

```
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
coerce : % -> OutputForm
degree : % -> PositiveInteger
hash : % -> SingleInteger
```

```
 latex : % -> String
```

— **InfinitlyClosePointCategory.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#INFCLCT">
InfinitlyClosePointCategory (INFCLCT)</a></h2>
</body>
```

———-

— **category INFCLCT InfinitlyClosePointCategory** —

```
)abbrev category INFCLCT InfinitlyClosePointCategory
++ Authors: Gaetan Hache
++ Date Created: may 1997
++ Date Last Updated: April 2010, by Tim Daly
++ Description:
++ This category is part of the PAFF package
InfinitlyClosePointCategory(K,symb,PolyRing,E,ProjPt,PCS,Plc,Divisor,BLMET) :
 Category == SIG where
  K : Field
  symb : List(Symbol)
  PolyRing : PolynomialCategory(K,E,OrderedVariableList(symb))
  E : DirectProductCategory(#symb,NonNegativeInteger)
  ProjPt : ProjectiveSpaceCategory(K)
  PCS : LocalPowerSeriesCategory(K)
  Plc : PlacesCategory(K,PCS)
  DIVISOR : DivisorCategory(Plc)
  BLMET : BlowUpMethodCategory

  bls ==> ['X,'Y]
  BlUpRing ==> DistributedMultivariatePolynomial(bls , K)
  AFP ==> AffinePlane(K)

  SIG ==> SetCategoryWithDegree with

    create : (ProjPt ,  BlUpRing, AFP , NonNegativeInteger,BLMET, _
              NonNegativeInteger,  DIVISOR,K,Symbol) -> %
      ++ create(p,b,a,n1,c,n2,d,k,s) an infinitly close point

    create : (ProjPt,PolyRing) -> %

    setpoint_! : (%,ProjPt) -> ProjPt

    setcurve_! : (%,BlUpRing) -> BlUpRing

    setlocalPoint_! : (%,AFP) -> AFP

    setsubmult_! : (%, NonNegativeInteger) -> NonNegativeInteger

    setmult_! : (%,NonNegativeInteger) -> NonNegativeInteger
```

```
    setchart_! : (%,BLMET) -> BLMET -- CHH

    setexcpDiv_! : (%,DIVISOR) -> DIVISOR

    setlocalParam_! : (%,List PCS) -> List(PCS)

    setsymbName_! : (%,Symbol) -> Symbol

    subMultV : % -> NonNegativeInteger

    localParamV : % -> List PCS

    symbNameV : % -> Symbol

    pointV :  % -> ProjPt
      ++ pointV(p) returns the infinitly close point.

    curveV :  % -> BlUpRing
      ++ curveV(p) returns the defining polynomial of the strict transform
      ++ on which lies the corresponding infinitly close point.

    localPointV : % -> AFP
      ++ localPointV(p) returns the coordinates of the local infinitly
      ++ close point

    multV : % -> NonNegativeInteger
      ++ multV(p) returns the multiplicity of the infinitly close point.

    chartV : % -> BLMET -- CHH
      ++ chartV(p) is the chart of the infinitly close point. The first integer
      ++ correspond to variable defining the exceptional line, the last one
      ++ the affine  neighboorhood and the second one is the
      ++ remaining integer. For example [1,2,3] means that
      ++ Z=1, X=X and Y=XY. [2,3,1] means that X=1, Y=Y and Z=YZ.

    excpDivV : % -> DIVISOR
      ++ excpDivV returns the exceptional divisor of the infinitly close point.

    actualExtensionV : % -> K
```

— INFCLCT.dotabb —

```
"INFCLCT" [color=lightblue,href="bookvol10.2.pdf#nameddest=INFCLCT"];
"ALIST" [color="#88FF44",href="bookvol10.3.pdf#nameddest=ALIST"]
"INFCLCT" -> "ALIST"
```

— INFCLCT.dotfull —

```
"InfinitlyClosePointCategory"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=INFCLCT"];
"InfinitlyClosePointCategory" -> "AssocationList(SetCategory,SetCategory)"
```

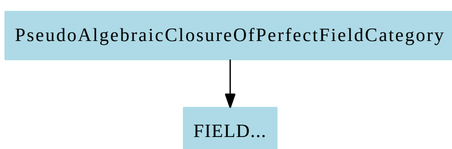
———-

— **INFCLCT.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"InfinitlyClosePointCategory" [color=lightblue];
"InfinitlyClosePointCategory" -> "AssocationList(SetCategory,SetCategory)"

"AssocationList(SetCategory,SetCategory)" -> "AssocationList()"

"AssocationList()" [color=lightblue];

}
```

———-

17.0.221 PseudoAlgebraicClosureOfPerfectFieldCategory (PACPERC)



— **PseudoAlgebraicClosureOfPerfectFieldCategory.input** —

```
)set break resume
)sys rm -f PseudoAlgebraicClosureOfPerfectFieldCategory.output
)spool PseudoAlgebraicClosureOfPerfectFieldCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PseudoAlgebraicClosureOfPerfectFieldCategory
--R
--R PseudoAlgebraicClosureOfPerfectFieldCategory is a category constructor
--R Abbreviation for PseudoAlgebraicClosureOfPerfectFieldCategory is PACPERC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PACPERC
--R
```

```
--R------------------------------- Operations --------------------------------
--R ?*? : (Fraction(Integer),%) -> %      ?*? : (%,Fraction(Integer)) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,Integer) -> %               ?**? : (%,NonNegativeInteger) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (%,%) -> %                      ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,Integer) -> %                ?^? : (%,NonNegativeInteger) -> %
--R ?^? : (%,PositiveInteger) -> %        associates? : (%,%) -> Boolean
--R coerce : Fraction(Integer) -> %       coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R conjugate : % -> %                    extDegree : % -> PositiveInteger
--R factor : % -> Factored(%)             fullOutput : % -> OutputForm
--R gcd : List(%) -> %                    gcd : (%,%) -> %
--R ground? : % -> Boolean                hash : % -> SingleInteger
--R inv : % -> %                          latex : % -> String
--R lcm : List(%) -> %                    lcm : (%,%) -> %
--R maxTower : List(%) -> %               one? : % -> Boolean
--R previousTower : % -> %                prime? : % -> Boolean
--R ?quo? : (%,%) -> %                    recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                    sample : () -> %
--R setTower! : % -> Void                 sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored(%)         squareFreePart : % -> %
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R vectorise : (%,%) -> Vector(%)        zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R definingPolynomial : % -> SparseUnivariatePolynomial(%)
--R definingPolynomial : () -> SparseUnivariatePolynomial(%)
--R distinguishedRootsOf : (SparseUnivariatePolynomial(%),%) -> List(%)
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
--R lift : (%,%) -> SparseUnivariatePolynomial(%)
--R lift : % -> SparseUnivariatePolynomial(%)
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed")
--R newElement : (SparseUnivariatePolynomial(%),Symbol) -> %
--R newElement : (SparseUnivariatePolynomial(%),%,Symbol) -> %
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %)
--R reduce : SparseUnivariatePolynomial(%) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

— PseudoAlgebraicClosureOfPerfectFieldCategory.help —

```
====================================================================
PseudoAlgebraicClosureOfPerfectFieldCategory examples
====================================================================

This category exports the function for domains which implement dynamic
extension using the simple notion of tower extensions. A tower extension
T of the ground field K is any sequence of field extensions
    (T : K_0, K_1, ..., K_i...,K_n) where K_0 = K
and for
    i =1,2,...,n, K_i is an extension of K_{i-1} of degree > 1
and defined by an irreducible polynomial p(Z) in K_{i-1}.

Two towers
    (T_1: K_01, K_11,...,K_i1,...,K_n1)
and
    (T_2: K_02, K_12,...,K_i2,...,K_n2)
are said to be related if
    T_1 <= T_2 (or T_1 >= T_2),
that is if
    K_i1 = K_i2 for i=1,2,...,n1 (or i=1,2,...,n2).

Any algebraic operations defined for several elements are only defined
if all of the concerned elements are coming from a set of related tower
extensions.

See Also:
o )show PseudoAlgebraicClosureOfPerfectFieldCategory
```

⇐ "Field" (FIELD) 16.0.209 on page 1413

Exports:

0	1	associates?	characteristic
coerce	conjugate	definingPolynomial	distinguishedRootsOf
divide	euclideanSize	expressIdealMember	exquo
extDegree	extendedEuclidean	factor	fullOutput
gcd	gcdPolynomial	ground?	hash
inv	latex	lcm	lift
maxTower	multiEuclidean	newElement	one?
previousTower	prime?	principalIdeal	?quo?
recip	reduce	?rem?	sample
setTower!	sizeLess?	squareFree	squareFreePart
subtractIfCan	unit?	unitCanonical	unitNormal
vectorise	zero?	?*?	?**?
?+?	?-?	-?	?/?
?=?	?^?	?~=?	

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
conjugate: % -> %
definingPolynomial: () -> SUP(%)
definingPolynomial: % -> SUP %
distinguishedRootsOf: (SparseUnivariatePolynomial %,%) -> List %
extDegree: % -> PI
fullOutput: % -> OutputForm
ground_? : % -> Boolean
lift: % -> SUP(%)
lift: (%,%) -> SUP(%)
maxTower: List % -> %
newElement: (SUP(%), %, Symbol) -> %
newElement: (SUP(%), Symbol) -> %
previousTower: % -> %
reduce: SUP(%) -> %
setTower_!: % -> Void
vectorise: (%,%) -> Vector(%)
```

These exports come from (p1413) Field():

```
associates? : (%,%) -> Boolean
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
exquo : (%,%) -> Union(%,"failed")
factor : % -> Factored %
gcd : (%,%) -> %
inv : % -> %
prime? : % -> Boolean
squareFree : % -> Factored %
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
?/? : (%,%) -> %
```

These exports come from (p1374) EuclideanDomain():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
```

```
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
expressIdealMember : (List %,%) -> Union(List %,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
gcd : List % -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                   SparseUnivariatePolynomial %
hash : % -> SingleInteger
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
sample : () -> %
sizeLess? : (%,%) -> Boolean
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p1362) UniqueFactorizationDomain():

```
squareFreePart : % -> %
```

These exports come from (p1185) DivisionRing():

```
coerce : Fraction Integer -> %
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?**? : (%,Integer) -> %
?^? : (%,Integer) -> %
```

— **PseudoAlgebraicClosureOfPerfectFieldCategory.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#PACPERC">
```

```
PseudoAlgebraicClosureOfPerfectFieldCategory (PACPERC)</a></h2>
</body>
```

— category PACPERC PseudoAlgebraicClosureOfPerfectFieldCategory —

```
)abbrev category PACPERC PseudoAlgebraicClosureOfPerfectFieldCategory
++ Authors: Gaetan Hache
++ Date Created: may 1997
++ Date Last Updated: April 2010, by Tim Daly
++ Description:
++ This category exports the function for domains
++ which implement dynamic extension using the simple notion of tower
++ extensions. ++ A tower extension T  of the ground
++ field K is any sequence of field extension
++ (T : K_0, K_1, ..., K_i...,K_n) where K_0 = K
++ and for i =1,2,...,n, K_i is an extension of K_{i-1} of degree > 1
++ and defined by an irreducible polynomial p(Z) in K_{i-1}.
++ Two towers (T_1: K_01, K_11,...,K_i1,...,K_n1)
++ and (T_2: K_02, K_12,...,K_i2,...,K_n2)
++ are said to be related if T_1 <= T_2 (or T_1 >= T_2),
++ that is if K_i1 = K_i2 for i=1,2,...,n1 (or i=1,2,...,n2).
++ Any algebraic operations defined for several elements
++ are only defined if all of the concerned elements are coming from
++ a set of related tower extensions.

PseudoAlgebraicClosureOfPerfectFieldCategory() : Category == SIG where

  INT      ==> Integer
  K        ==> Fraction Integer
  NNI      ==> NonNegativeInteger
  SUP      ==> SparseUnivariatePolynomial
  BOOLEAN  ==> Boolean
  PI       ==> PositiveInteger
  FFFACTSE ==> FiniteFieldFactorizationWithSizeParseBySideEffect

  SIG ==> Field with

    definingPolynomial : () -> SUP(%)

    definingPolynomial : % -> SUP %

    lift : % -> SUP(%)

    lift : (%,%) -> SUP(%)

    reduce : SUP(%) -> %

    distinguishedRootsOf : (SparseUnivariatePolynomial %,%) -> List %
      ++ distinguishedRootsOf(p,a) returns a (distinguised) root for each
      ++ irreducible factor of the polynomial p (factored over the field
      ++ defined by the element a).
```

```
    ground_? : % -> Boolean

    maxTower : List % -> %
      ++ maxTower(l) returns the tower in the list having the maximal
      ++ extension degree over the ground field. It has no meaning if the
      ++ towers not related.

    extDegree : % -> PI
      ++ extDegree(a) returns the extension degree of the extension tower
      ++ over which the element is defined.

    previousTower : % -> %
      ++ previousTower(a) returns the previous tower extension over which
      ++ the element a is defined.

    vectorise : (%,%) -> Vector(%)

    conjugate : % -> %

    newElement : (SUP(%), %, Symbol) -> %

    newElement : (SUP(%), Symbol) -> %

    setTower_! : % -> Void

    fullOutput : % -> OutputForm
```

———-

— PACPERC.dotabb —

```
"PACPERC" [color=lightblue,href="bookvol10.2.pdf#nameddest=PACPERC"];
"PACPERC" -> "FIELD"
```

———-

— PACPERC.dotfull —

```
"PseudoAlgebraicClosureOfPerfectFieldCategory"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PACPERC"];
"PseudoAlgebraicClosureOfPerfectFieldCategory" -> "Field()"
```

———-

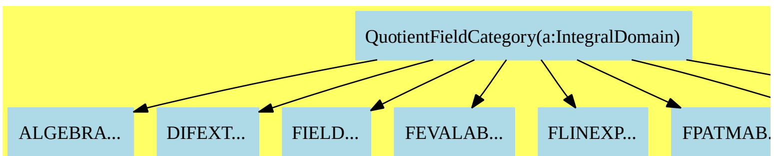
— PACPERC.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];
```

```
"PseudoAlgebraicClosureOfPerfectFieldCategory" [color=lightblue];
"PseudoAlgebraicClosureOfPerfectFieldCategory" -> "FIELD..."

"FIELD..." [color=lightblue];

}
```

———-

17.0.222 QuotientFieldCategory (QFCAT)



— QuotientFieldCategory.input —

```
)set break resume
)sys rm -f QuotientFieldCategory.output
)spool QuotientFieldCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show QuotientFieldCategory
--R
--R QuotientFieldCategory(S: IntegralDomain) is a category constructor
--R Abbreviation for QuotientFieldCategory is QFCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for QFCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,S) -> %                      ?*? : (S,%) -> %
--R ?*? : (Fraction(Integer),%) -> %      ?*? : (%,Fraction(Integer)) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,Integer) -> %               ?**? : (%,NonNegativeInteger) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (S,S) -> %                      ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                D : (%,(S -> S)) -> %
--R D : % -> % if S has DIFRING           1 : () -> %
--R 0 : () -> %                           ?^? : (%,Integer) -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R abs : % -> % if S has OINTDOM         associates? : (%,%) -> Boolean
--R ceiling : % -> S if S has INS         coerce : S -> %
```

```
--R coerce : Fraction(Integer) -> %        coerce : % -> %
--R coerce : Integer -> %                  coerce : % -> OutputForm
--R convert : % -> Float if S has REAL     denom : % -> S
--R denominator : % -> %                   differentiate : (%,(S -> S)) -> %
--R factor : % -> Factored(%)              floor : % -> S if S has INS
--R gcd : List(%) -> %                     gcd : (%,%) -> %
--R hash : % -> SingleInteger              init : () -> % if S has STEP
--R inv : % -> %                           latex : % -> String
--R lcm : List(%) -> %                     lcm : (%,%) -> %
--R map : ((S -> S),%) -> %                max : (%,%) -> % if S has ORDSET
--R min : (%,%) -> % if S has ORDSET       numer : % -> S
--R numerator : % -> %                     one? : % -> Boolean
--R prime? : % -> Boolean                  ?quo? : (%,%) -> %
--R random : () -> % if S has INS          recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                     retract : % -> S
--R sample : () -> %                       sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored(%)          squareFreePart : % -> %
--R unit? : % -> Boolean                   unitCanonical : % -> %
--R wholePart : % -> S if S has EUCDOM     zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?<? : (%,%) -> Boolean if S has ORDSET
--R ?<=? : (%,%) -> Boolean if S has ORDSET
--R ?>? : (%,%) -> Boolean if S has ORDSET
--R ?>=? : (%,%) -> Boolean if S has ORDSET
--R D : (%,(S -> S),NonNegativeInteger) -> %
--R D : (%,List(Symbol),List(NonNegativeInteger)) -> % if S has PDRING(SYMBOL)
--R D : (%,Symbol,NonNegativeInteger) -> % if S has PDRING(SYMBOL)
--R D : (%,List(Symbol)) -> % if S has PDRING(SYMBOL)
--R D : (%,Symbol) -> % if S has PDRING(SYMBOL)
--R D : (%,NonNegativeInteger) -> % if S has DIFRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if S has CHARNZ or and(has($,CharacteristicNonZero),has(S,PolynomialFact
--R coerce : Symbol -> % if S has RETRACT(SYMBOL)
--R conditionP : Matrix(%) -> Union(Vector(%),"failed") if and(has($,CharacteristicNonZero),has(S,PolynomialFact
--R convert : % -> DoubleFloat if S has REAL
--R convert : % -> InputForm if S has KONVERT(INFORM)
--R convert : % -> Pattern(Float) if S has KONVERT(PATTERN(FLOAT))
--R convert : % -> Pattern(Integer) if S has KONVERT(PATTERN(INT))
--R differentiate : (%,(S -> S),NonNegativeInteger) -> %
--R differentiate : (%,List(Symbol),List(NonNegativeInteger)) -> % if S has PDRING(SYMBOL)
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if S has PDRING(SYMBOL)
--R differentiate : (%,List(Symbol)) -> % if S has PDRING(SYMBOL)
--R differentiate : (%,Symbol) -> % if S has PDRING(SYMBOL)
--R differentiate : (%,NonNegativeInteger) -> % if S has DIFRING
--R differentiate : % -> % if S has DIFRING
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R ?.? : (%,S) -> % if S has ELTAB(S,S)
--R euclideanSize : % -> NonNegativeInteger
--R eval : (%,Symbol,S) -> % if S has IEVALAB(SYMBOL,S)
--R eval : (%,List(Symbol),List(S)) -> % if S has IEVALAB(SYMBOL,S)
--R eval : (%,List(Equation(S))) -> % if S has EVALAB(S)
--R eval : (%,Equation(S)) -> % if S has EVALAB(S)
--R eval : (%,S,S) -> % if S has EVALAB(S)
--R eval : (%,List(S),List(S)) -> % if S has EVALAB(S)
```

```
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R factorPolynomial : SparseUnivariatePolynomial(%) -> Factored(SparseUnivariatePolynomial(%)) if S has PFECAT
--R factorSquareFreePolynomial : SparseUnivariatePolynomial(%) -> Factored(SparseUnivariatePolynomial(%)) if S h
--R fractionPart : % -> % if S has EUCDOM
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed")
--R negative? : % -> Boolean if S has OINTDOM
--R nextItem : % -> Union(%,"failed") if S has STEP
--R patternMatch : (%,Pattern(Float),PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%) if S has PATMAB
--R patternMatch : (%,Pattern(Integer),PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%) if S has
--R positive? : % -> Boolean if S has OINTDOM
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %)
--R reducedSystem : Matrix(%) -> Matrix(S)
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(S),vec: Vector(S))
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(Integer),vec: Vector(Integer)) if S has LINEXP(I
--R reducedSystem : Matrix(%) -> Matrix(Integer) if S has LINEXP(INT)
--R retract : % -> Integer if S has RETRACT(INT)
--R retract : % -> Fraction(Integer) if S has RETRACT(INT)
--R retract : % -> Symbol if S has RETRACT(SYMBOL)
--R retractIfCan : % -> Union(Integer,"failed") if S has RETRACT(INT)
--R retractIfCan : % -> Union(Fraction(Integer),"failed") if S has RETRACT(INT)
--R retractIfCan : % -> Union(Symbol,"failed") if S has RETRACT(SYMBOL)
--R retractIfCan : % -> Union(S,"failed")
--R sign : % -> Integer if S has OINTDOM
--R solveLinearPolynomialEquation : (List(SparseUnivariatePolynomial(%)),SparseUnivariatePolynomial(%)) -> Union
--R squareFreePolynomial : SparseUnivariatePolynomial(%) -> Factored(SparseUnivariatePolynomial(%)) if S has PFE
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

— **QuotientFieldCategory.help** —

```
====================================================================
QuotientFieldCategory examples
====================================================================

QuotientField(S) is the category of fractions of an Integral Domain S.

See Also:
o )show QuotientFieldCategory
```

See:

⇐ “Algebra” (ALGEBRA) 11.0.176 on page 1121
⇐ “CharacteristicNonZero” (CHARNZ) 10.0.159 on page 1010
⇐ “CharacteristicZero” (CHARZ) 10.0.160 on page 1015
⇐ “ConvertibleTo” (KONVERT) 2.0.17 on page 47
⇐ “DifferentialExtension” (DIFEXT) 11.0.177 on page 1127
⇐ “EuclideanDomain” (EUCDOM) 15.0.205 on page 1374
⇐ “Field” (FIELD) 16.0.209 on page 1413
⇐ “FullyEvalableOver” (FEVALAB) 4.0.67 on page 246
⇐ “FullyLinearlyExplicitRingOver” (FLINEXP) 11.0.178 on page 1134
⇐ “FullyPatternMatchable” (FPATMAB) 3.0.51 on page 164
⇐ “OrderedIntegralDomain” (OINTDOM) 13.0.198 on page 1326
⇐ “OrderedSet” (ORDSET) 4.0.80 on page 326
⇐ “Patternable” (PATAB) 2.0.34 on page 100
⇐ “PolynomialFactorizationExplicit” (PFECAT) 15.0.207 on page 1393
⇐ “RealConstant” (REAL) 3.0.55 on page 180
⇐ “RetractableTo” (RETRACT) 2.0.37 on page 110
⇐ “StepThrough” (STEP) 4.0.88 on page 367

Exports:

0	1	abs
associates?	ceiling	characteristic
charthRoot	coerce	conditionP
convert	D	denom
denominator	differentiate	divide
euclideanSize	eval	expressIdealMember
exquo	extendedEuclidean	factor
factorPolynomial	factorSquareFreePolynomial	floor
fractionPart	gcd	gcdPolynomial
hash	init	inv
latex	lcm	map
max	min	multiEuclidean
negative?	nextItem	numer
numerator	one?	patternMatch
positive?	prime?	principalIdeal
random	recip	reducedSystem
retract	retractIfCan	sample
sign	sizeLess?	solveLinearPolynomialEquation
squareFree	squareFreePart	squareFreePolynomial
subtractIfCan	unit?	unitNormal
unitCanonical	wholePart	zero?
?.?	?*?	?**?
?+?	?-?	-?
?/?	?=?	?^?
?quo?	?rem?	?~=?
?<?	?<=?	?>?
?>=?		

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **nil**

These are directly exported but not implemented:

```
ceiling : % -> S if S has INS
denom : % -> S
floor : % -> S if S has INS
numer : % -> S
wholePart : % -> S if S has EUCDOM
?/? : (S,S) -> %
```

These are implemented by this category:

```
characteristic : () -> NonNegativeInteger
coerce : Symbol -> % if S has RETRACT SYMBOL
coerce : Fraction Integer -> %
convert : % -> InputForm if S has KONVERT INFORM
convert : % -> DoubleFloat if S has REAL
convert : % -> Float if S has REAL
convert : % -> Pattern Integer if S has KONVERT PATTERN INT
convert : % -> Pattern Float if S has KONVERT PATTERN FLOAT
denominator : % -> %
differentiate : (%,(S -> S)) -> %
fractionPart : % -> % if S has EUCDOM
init : () -> % if S has STEP
map : ((S -> S),%) -> %
nextItem : % -> Union(%,"failed") if S has STEP
numerator : % -> %
patternMatch :
  (%,Pattern Float,PatternMatchResult(Float,%)) ->
    PatternMatchResult(Float,%)
     if S has PATMAB FLOAT
patternMatch :
  (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
    PatternMatchResult(Integer,%)
      if S has PATMAB INT
random : () -> % if S has INS
reducedSystem : Matrix % -> Matrix S
reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix S,vec: Vector S)
retract : % -> Symbol if S has RETRACT SYMBOL
```

```
retract : % -> Integer if S has RETRACT INT
retractIfCan : % -> Union(Integer,"failed") if S has RETRACT INT
retractIfCan : % -> Union(Symbol,"failed") if S has RETRACT SYMBOL
?<? : (%,%) -> Boolean if S has ORDSET
```

These exports come from (p1413) Field():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
exquo : (%,%) -> Union(%,"failed")
factor : % -> Factored %
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
sample : () -> %
sizeLess? : (%,%) -> Boolean
squareFree : % -> Factored %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?**? : (%,Integer) -> %
?^? : (%,Integer) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
```

```
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?/? : (%,%) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p1121) Algebra(S:IntegralDomain):

```
coerce : S -> %
?*? : (%,S) -> %
?*? : (S,%) -> %
```

These exports come from (p110) RetractableTo(S:IntegralDomain):

```
retract : % -> S
retractIfCan : % -> Union(S,"failed")
```

These exports come from (p246) FullyEvalableOver(S:IntegralDomain):

```
?.? : (%,S) -> % if S has ELTAB(S,S)
eval : (%,Equation S) -> % if S has EVALAB S
eval : (%,List Symbol,List S) -> % if S has IEVALAB(SYMBOL,S)
eval : (%,List Equation S) -> % if S has EVALAB S
eval : (%,S,S) -> % if S has EVALAB S
eval : (%,List S,List S) -> % if S has EVALAB S
eval : (%,Symbol,S) -> % if S has IEVALAB(SYMBOL,S)
```

These exports come from (p1127) DifferentialExtension(S:IntegralDomain):

```
D : (%,(S -> S)) -> %
D : (%,(S -> S),NonNegativeInteger) -> %
D : % -> % if S has DIFRING
D : (%,NonNegativeInteger) -> % if S has DIFRING
D : (%,List Symbol,List NonNegativeInteger) -> %
    if S has PDRING SYMBOL
D : (%,Symbol,NonNegativeInteger) -> %
    if S has PDRING SYMBOL
D : (%,List Symbol) -> % if S has PDRING SYMBOL
D : (%,Symbol) -> % if S has PDRING SYMBOL
differentiate : (%,List Symbol) -> %
    if S has PDRING SYMBOL
differentiate : (%,Symbol,NonNegativeInteger) -> %
    if S has PDRING SYMBOL
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
    if S has PDRING SYMBOL
differentiate : (%,NonNegativeInteger) -> % if S has DIFRING
differentiate : % -> % if S has DIFRING
differentiate : (%,Symbol) -> % if S has PDRING SYMBOL
differentiate : (%,(S -> S),NonNegativeInteger) -> %
```

These exports come from (p1134) FullyLinearlyExplicitRingOver(S:IntegralDomain):

```
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix Integer,vec: Vector Integer)
    if S has LINEXP INT
```

```
reducedSystem : Matrix % -> Matrix Integer if S has LINEXP INT
```

These exports come from (p110) RetractableTo(Fraction(Integer)):

```
retract : % -> Fraction Integer if S has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
  if S has RETRACT INT
```

These exports come from (p326) OrderedSet():

```
max : (%,%) -> % if S has ORDSET
min : (%,%) -> % if S has ORDSET
?<=? : (%,%) -> Boolean if S has ORDSET
?>? : (%,%) -> Boolean if S has ORDSET
?>=? : (%,%) -> Boolean if S has ORDSET
```

These exports come from (p1326) OrderedIntegralDomain():

```
abs : % -> % if S has OINTDOM
negative? : % -> Boolean if S has OINTDOM
positive? : % -> Boolean if S has OINTDOM
sign : % -> Integer if S has OINTDOM
```

These exports come from (p1010) CharacteristicNonZero():

```
charthRoot : % -> Union(%,"failed")
  if S has CHARNZ
  or and(has($,CharacteristicNonZero),
         has(S,PolynomialFactorizationExplicit))
```

These exports come from (p1393) PolynomialFactorizationExplicit():

```
conditionP : Matrix % -> Union(Vector %,"failed")
  if and(has($,CharacteristicNonZero),
         has(S,PolynomialFactorizationExplicit))
factorPolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if S has PFECAT
factorSquareFreePolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if S has PFECAT
solveLinearPolynomialEquation :
  (List SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     Union(List SparseUnivariatePolynomial %,"failed")
      if S has PFECAT
squareFreePolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if S has PFECAT
```

— QuotientFieldCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#QFCAT">
QuotientFieldCategory (QFCAT)</a></h2>
</body>
```

— category QFCAT QuotientFieldCategory —

```
)abbrev category QFCAT QuotientFieldCategory
++ Date Last Updated: 5th March 1996
++ Description:
++ QuotientField(S) is the category of fractions of an Integral Domain S.

--QuotientFieldCategory(S) : Category == SIG where
--  S : IntegralDomain

--  F     ==> Field
--  A     ==> Algebra(S)
--  RT    ==> RetractableTo(S)
--  FEO   ==> FullyEvalableOver(S)
--  DE    ==> DifferentialExtension(S)
--  FLERO ==> FullyLinearlyExplicitRingOver(S)
--  P     ==> Patternable(S)
--  FPM   ==> FullyPatternMatchable(S)

--  SIG ==> Join(F,A,RT,FEO,DE,FLERO,P,FPM) with

QuotientFieldCategory(S) : Category == SIG where
  S : IntegralDomain

  SIG ==> Join(Field, Algebra S, RetractableTo S, FullyEvalableOver S,
               DifferentialExtension S, FullyLinearlyExplicitRingOver S,
                Patternable S, FullyPatternMatchable S) with

    _/ : (S, S) -> %
      ++ d1 / d2 returns the fraction d1 divided by d2.

    numer : % -> S
      ++ numer(x) returns the numerator of the fraction x.

    denom : % -> S
      ++ denom(x) returns the denominator of the fraction x.

    numerator : % -> %
      ++ numerator(x) is the numerator of the fraction x converted to %.

    denominator : % -> %
      ++ denominator(x) is the denominator of the fraction x converted to %.

    if S has StepThrough then StepThrough

    if S has RetractableTo Integer then
             RetractableTo Integer
             RetractableTo Fraction Integer

    if S has OrderedSet then OrderedSet

    if S has OrderedIntegralDomain then OrderedIntegralDomain
```

```
    if S has RealConstant then RealConstant

    if S has ConvertibleTo InputForm then ConvertibleTo InputForm

    if S has CharacteristicZero then CharacteristicZero

    if S has CharacteristicNonZero then CharacteristicNonZero

    if S has RetractableTo Symbol then RetractableTo Symbol

    if S has EuclideanDomain then

      wholePart : % -> S
        ++ wholePart(x) returns the whole part of the fraction x
        ++ the truncated quotient of the numerator by the denominator.

      fractionPart : % -> %
        ++ fractionPart(x) returns the fractional part of x.
        ++ x = wholePart(x) + fractionPart(x)

    if S has IntegerNumberSystem then

      random : () -> %
        ++ random() returns a random fraction.

      ceiling : % -> S
        ++ ceiling(x) returns the smallest integral element above x.

      floor : % -> S
        ++ floor(x) returns the largest integral element below x.

    if S has PolynomialFactorizationExplicit then
      PolynomialFactorizationExplicit

  add

    import MatrixCommonDenominator(S, %)

    numerator(x) == numer(x)::%

    denominator(x) == denom(x)::%

    if S has StepThrough then

       init() == init()$S / 1$S

       nextItem(n) ==
         m:= nextItem(numer(n))
         m case "failed" =>
           error "We seem to have a Fraction of a finite object"
         m / 1

    map(fn, x) == (fn numer x) / (fn denom x)
```

```
reducedSystem(m:Matrix %):Matrix S == clearDenominator m

characteristic() == characteristic()$S

differentiate(x:%, deriv:S -> S) ==
    n := numer x
    d := denom x
    (deriv n * d - n * deriv d) / (d**2)

if S has ConvertibleTo InputForm then

  convert(x:%):InputForm == (convert numer x) / (convert denom x)

if S has RealConstant then

  convert(x:%):Float == (convert numer x) / (convert denom x)

  convert(x:%):DoubleFloat == (convert numer x) / (convert denom x)

-- Note that being a Join(OrderedSet,IntegralDomain) is not the same
-- as being an OrderedIntegralDomain.
if S has OrderedIntegralDomain then

   if S has canonicalUnitNormal then

       x:% < y:% ==
         (numer x  * denom y) < (numer y * denom x)

     else

       x:% < y:% ==
         if denom(x) < 0 then (x,y):=(y,x)
         if denom(y) < 0 then (x,y):=(y,x)
         (numer x  * denom y) < (numer y * denom x)

else if S has OrderedSet then

   x:% < y:% ==
     (numer x  * denom y) < (numer y * denom x)

if (S has EuclideanDomain) then

  fractionPart x == x - (wholePart(x)::%)

if S has RetractableTo Symbol then

  coerce(s:Symbol):%  == s::S::%

  retract(x:%):Symbol == retract(retract(x)@S)

  retractIfCan(x:%):Union(Symbol, "failed") ==
    (r := retractIfCan(x)@Union(S,"failed")) case "failed" =>"failed"
    retractIfCan(r::S)
```

```
if (S has ConvertibleTo Pattern Integer) then

  convert(x:%):Pattern(Integer)==(convert numer x)/(convert denom x)

  if (S has PatternMatchable Integer) then

    patternMatch(x:%, p:Pattern Integer,
     l:PatternMatchResult(Integer, %)) ==
       patternMatch(x, p,
                 l)$PatternMatchQuotientFieldCategory(Integer, S, %)

if (S has ConvertibleTo Pattern Float) then

  convert(x:%):Pattern(Float) == (convert numer x)/(convert denom x)

  if (S has PatternMatchable Float) then
    patternMatch(x:%, p:Pattern Float,
     l:PatternMatchResult(Float, %)) ==
       patternMatch(x, p,
                   l)$PatternMatchQuotientFieldCategory(Float, S, %)

if S has RetractableTo Integer then

  coerce(x:Fraction Integer):% == numer(x)::% / denom(x)::%

  if not(S is Integer) then
    retract(x:%):Integer == retract(retract(x)@S)

    retractIfCan(x:%):Union(Integer, "failed") ==
      (u := retractIfCan(x)@Union(S, "failed")) case "failed" =>
        "failed"
      retractIfCan(u::S)

if S has IntegerNumberSystem then

  random():% ==
    while zero?(d:=random()$S) repeat d
    random()$S / d

reducedSystem(m:Matrix %, v:Vector %):

  Record(mat:Matrix S, vec:Vector S) ==
    n := reducedSystem(horizConcat(v::Matrix(%), m))@Matrix(S)
    [subMatrix(n, minRowIndex n, maxRowIndex n, 1 + minColIndex n,
                            maxColIndex n), column(n, minColIndex n)]
```

— COQ QFCAT —

```
(* category QFCAT *)
(*
```

```
import MatrixCommonDenominator(S, %)

numerator : % -> %
numerator(x) == numer(x)::%

denominator : % -> %
denominator(x) == denom(x) ::%

if S has StepThrough then

   init : () -> %
   init() == init()$S / 1$S

   nextItem : % -> Union(%,"failed")
   nextItem(n) ==
     m:= nextItem(numer(n))
     m case "failed" =>
       error "We seem to have a Fraction of a finite object"
     m / 1

map : ((S -> S),%) -> %
map(fn, x) == (fn numer x) / (fn denom x)

reducedSystem : Matrix(%) -> Matrix(S)
reducedSystem(m:Matrix %):Matrix S == clearDenominator m

characteristic : () -> NonNegativeInteger
characteristic() == characteristic()$S

differentiate : (%,(S -> S)) -> %
differentiate(x:%, deriv:S -> S) ==
    n := numer x
    d := denom x
    (deriv n * d - n * deriv d) / (d**2)

if S has ConvertibleTo InputForm then

  convert : % -> InputForm
  convert(x:%):InputForm == (convert numer x) / (convert denom x)

if S has RealConstant then

  convert : % -> Float
  convert(x:%):Float == (convert numer x) / (convert denom x)

  convert : % -> DoubleFloat
  convert(x:%):DoubleFloat == (convert numer x) / (convert denom x)

-- Note that being a Join(OrderedSet,IntegralDomain) is not the same
-- as being an OrderedIntegralDomain.
if S has OrderedIntegralDomain then

   if S has canonicalUnitNormal then
```

```
          ?<? : (%,%) -> Boolean
          x:% < y:% ==
            (numer x  * denom y) < (numer y * denom x)

        else

          ?<? : (%,%) -> Boolean
          x:% < y:% ==
            if denom(x) < 0 then (x,y):=(y,x)
            if denom(y) < 0 then (x,y):=(y,x)
            (numer x  * denom y) < (numer y * denom x)

    else if S has OrderedSet then

       ?<? : (%,%) -> Boolean
       x:% < y:% ==
         (numer x  * denom y) < (numer y * denom x)

    if (S has EuclideanDomain) then

      fractionPart : % -> %
      fractionPart x == x - (wholePart(x)::%)

    if S has RetractableTo Symbol then

      coerce : S -> %
      coerce(s:Symbol):%  == s::S::%

      retract : % -> S
      retract(x:%):Symbol == retract(retract(x)@S)

      retractIfCan : % -> Union(Symbol,"failed")
      retractIfCan(x:%):Union(Symbol, "failed") ==
        (r := retractIfCan(x)@Union(S,"failed")) case "failed" =>"failed"
        retractIfCan(r::S)

    if (S has ConvertibleTo Pattern Integer) then

      convert : % -> Pattern(Integer)
      convert(x:%):Pattern(Integer)==(convert numer x)/(convert denom x)

      if (S has PatternMatchable Integer) then

        patternMatch : (%,Pattern(Integer),PatternMatchResult(Integer,%)) ->
           PatternMatchResult(Integer,%)
        patternMatch(x:%, p:Pattern Integer,
         l:PatternMatchResult(Integer, %)) ==
           patternMatch(x, p,
                     l)$PatternMatchQuotientFieldCategory(Integer, S, %)

    if (S has ConvertibleTo Pattern Float) then

      convert : % -> Pattern(Float)
      convert(x:%):Pattern(Float) == (convert numer x)/(convert denom x)
```

```
      if (S has PatternMatchable Float) then

        patternMatch : (%,Pattern(Float),PatternMatchResult(Float,%)) ->
            PatternMatchResult(Float,%)
        patternMatch(x:%, p:Pattern Float,
         l:PatternMatchResult(Float, %)) ==
           patternMatch(x, p,
                       l)$PatternMatchQuotientFieldCategory(Float, S, %)

    if S has RetractableTo Integer then

      coerce : Fraction(Integer) -> %
      coerce(x:Fraction Integer):% == numer(x)::% / denom(x)::%

      if not(S is Integer) then

        retract : % -> Integer
        retract(x:%):Integer == retract(retract(x)@S)

        retractIfCan : % -> Union(Integer,"failed")
        retractIfCan(x:%):Union(Integer, "failed") ==
          (u := retractIfCan(x)@Union(S, "failed")) case "failed" =>
            "failed"
          retractIfCan(u::S)

    if S has IntegerNumberSystem then

      random : () -> %
      random():% ==
        while zero?(d:=random()$S) repeat d
        random()$S / d

    reducedSystem : (Matrix(%),Vector(%)) ->
       Record(mat: Matrix(S),vec: Vector(S))
    reducedSystem(m:Matrix %, v:Vector %):
      Record(mat:Matrix S, vec:Vector S) ==
        n := reducedSystem(horizConcat(v::Matrix(%), m))@Matrix(S)
        [subMatrix(n, minRowIndex n, maxRowIndex n, 1 + minColIndex n,
                                maxColIndex n), column(n, minColIndex n)]

*)
```

— QFCAT.dotabb —

```
"QFCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=QFCAT"];
"QFCAT" -> "ALGEBRA"
"QFCAT" -> "DIFEXT"
"QFCAT" -> "FIELD"
"QFCAT" -> "FEVALAB"
```

```
"QFCAT" -> "FLINEXP"
"QFCAT" -> "FPATMAB"
"QFCAT" -> "PATAB"
"QFCAT" -> "RETRACT"
```

———-

— **QFCAT.dotfull** —

```
"QuotientFieldCategory(a:IntegralDomain)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=QFCAT"];
"QuotientFieldCategory(a:IntegralDomain)" -> "Field()"
"QuotientFieldCategory(a:IntegralDomain)" -> "Algebra(IntegralDomain)"
"QuotientFieldCategory(a:IntegralDomain)" -> "RetractableTo(IntegralDomain)"
"QuotientFieldCategory(a:IntegralDomain)" ->
  "FullyEvalableOver(IntegralDomain)"
"QuotientFieldCategory(a:IntegralDomain)" ->
  "DifferentialExtension(IntegralDomain)"
"QuotientFieldCategory(a:IntegralDomain)" ->
  "FullyLinearlyExplicitRingOver(IntegralDomain)"
"QuotientFieldCategory(a:IntegralDomain)" ->
  "Patternable(IntegralDomain)"
"QuotientFieldCategory(a:IntegralDomain)" ->
  "FullyPatternMatchable(IntegralDomain)"
```

———-

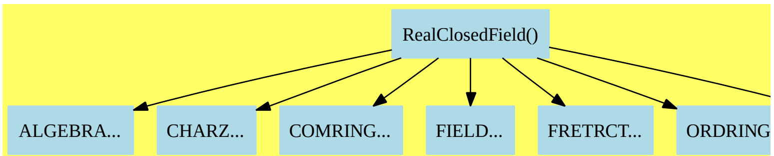
— **QFCAT.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"QuotientFieldCategory(a:IntegralDomain)" [color=lightblue];
"QuotientFieldCategory(a:IntegralDomain)" -> "ALGEBRA..."
"QuotientFieldCategory(a:IntegralDomain)" -> "DIFEXT..."
"QuotientFieldCategory(a:IntegralDomain)" -> "FIELD..."
"QuotientFieldCategory(a:IntegralDomain)" -> "FEVALAB..."
"QuotientFieldCategory(a:IntegralDomain)" -> "FLINEXP..."
"QuotientFieldCategory(a:IntegralDomain)" -> "FPATMAB..."
"QuotientFieldCategory(a:IntegralDomain)" -> "PATAB..."
"QuotientFieldCategory(a:IntegralDomain)" -> "RETRACT..."

"ALGEBRA..." [color=lightblue];
"DIFEXT..." [color=lightblue];
"FIELD..." [color=lightblue];
"FEVALAB..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"FPATMAB..." [color=lightblue];
"PATAB..." [color=lightblue];
"RETRACT..." [color=lightblue];
```

```
}
```

———-

17.0.223 RealClosedField (RCFIELD)



— RealClosedField.input —

```
)set break resume
)sys rm -f RealClosedField.output
)spool RealClosedField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RealClosedField
--R
--R RealClosedField is a category constructor
--R Abbreviation for RealClosedField is RCFIELD
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for RCFIELD
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,Fraction(Integer)) -> %      ?*? : (Fraction(Integer),%) -> %
--R ?*? : (%,Integer) -> %                ?*? : (Integer,%) -> %
--R ?*? : (%,Fraction(Integer)) -> %      ?*? : (Fraction(Integer),%) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,Fraction(Integer)) -> %     ?**? : (%,Integer) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?/? : (%,%) -> %
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               1 : () -> %
--R 0 : () -> %                           ?^? : (%,Integer) -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R abs : % -> %                          associates? : (%,%) -> Boolean
--R coerce : Fraction(Integer) -> %       coerce : Integer -> %
--R coerce : Fraction(Integer) -> %       coerce : % -> %
--R coerce : Fraction(Integer) -> %       coerce : Integer -> %
--R coerce : % -> OutputForm              factor : % -> Factored(%)
--R gcd : (%,%) -> %                      gcd : List(%) -> %
```

```
--R hash : % -> SingleInteger            inv : % -> %
--R latex : % -> String                  lcm : (%,%) -> %
--R lcm : List(%) -> %                   max : (%,%) -> %
--R min : (%,%) -> %                     negative? : % -> Boolean
--R nthRoot : (%,Integer) -> %           one? : % -> Boolean
--R positive? : % -> Boolean             prime? : % -> Boolean
--R ?quo? : (%,%) -> %                   recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                   rename : (%,OutputForm) -> %
--R rename! : (%,OutputForm) -> %        retract : % -> Fraction(Integer)
--R sample : () -> %                     sign : % -> Integer
--R sizeLess? : (%,%) -> Boolean         sqrt : Integer -> %
--R sqrt : Fraction(Integer) -> %        sqrt : (%,NonNegativeInteger) -> %
--R sqrt : % -> %                        squareFree : % -> Factored(%)
--R squareFreePart : % -> %              unit? : % -> Boolean
--R unitCanonical : % -> %               zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R allRootsOf : Polynomial(Integer) -> List(%)
--R allRootsOf : Polynomial(Fraction(Integer)) -> List(%)
--R allRootsOf : Polynomial(%) -> List(%)
--R allRootsOf : SparseUnivariatePolynomial(Integer) -> List(%)
--R allRootsOf : SparseUnivariatePolynomial(Fraction(Integer)) -> List(%)
--R allRootsOf : SparseUnivariatePolynomial(%) -> List(%)
--R approximate : (%,%) -> Fraction(Integer)
--R characteristic : () -> NonNegativeInteger
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
--R mainDefiningPolynomial : % -> Union(SparseUnivariatePolynomial(%),"failed")
--R mainForm : % -> Union(OutputForm,"failed")
--R mainValue : % -> Union(SparseUnivariatePolynomial(%),"failed")
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed")
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %)
--R retract : % -> Fraction(Integer) if Fraction(Integer) has RETRACT(FRAC(INT))
--R retract : % -> Integer if Fraction(Integer) has RETRACT(INT)
--R retractIfCan : % -> Union(Fraction(Integer),"failed")
--R retractIfCan : % -> Union(Fraction(Integer),"failed") if Fraction(Integer) has RETRACT(FRAC(INT))
--R retractIfCan : % -> Union(Integer,"failed") if Fraction(Integer) has RETRACT(INT)
--R rootOf : (SparseUnivariatePolynomial(%),PositiveInteger) -> Union(%,"failed")
--R rootOf : (SparseUnivariatePolynomial(%),PositiveInteger,OutputForm) -> Union(%,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

— **RealClosedField.help** —

```
====================================================================
RealClosedField examples
====================================================================

RealClosedField provides common access functions for all real closed fields.
It provides computations with generic real roots of polynomials.

See Also:
o )show RealClosedField
```

———-

See:

⇐ "Algebra" (ALGEBRA) 11.0.176 on page 1121
⇐ "CharacteristicZero" (CHARZ) 10.0.160 on page 1015
⇐ "CommutativeRing" (COMRING) 10.0.161 on page 1019
⇐ "Field" (FIELD) 16.0.209 on page 1413
⇐ "FullyRetractableTo" (FRETRCT) 3.0.50 on page 158
⇐ "OrderedRing" (ORDRING) 10.0.168 on page 1055
⇐ "RadicalCategory" (RADCAT) 2.0.36 on page 106

Exports:

0	1	abs	allRootsOf
approximate	associates?	characteristic	coerce
divide	euclideanSize	expressIdealMember	exquo
extendedEuclidean	factor	gcd	gcdPolynomial
hash	inv	latex	lcm
mainDefiningPolynomial	mainForm	mainValue	max
min	multiEuclidean	negative?	nthRoot
one?	positive?	prime?	principalIdeal
recip	rename	rename!	retract
retractIfCan	rootOf	sample	sign
sizeLess?	sqrt	squareFree	squareFreePart
subtractIfCan	unit?	unitCanonical	unitNormal
zero?	?*?	?**?	?+?
?-?	-?	?/?	?<?
?<=?	?=?	?>?	?>=?
?^?	?~=?	?quo?	?rem?

Attributes Exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown

means that the operation `recip` can only return "failed" if its argument is not a unit.

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.

These are directly exported but not implemented:

```
allRootsOf : SparseUnivariatePolynomial % -> List %
approximate : (%,%) -> Fraction Integer
mainDefiningPolynomial :
   % -> Union(SparseUnivariatePolynomial %,"failed")
mainForm : % -> Union(OutputForm,"failed")
mainValue : % -> Union(SparseUnivariatePolynomial %,"failed")
rename : (%,OutputForm) -> %
rename! : (%,OutputForm) -> %
```

These are implemented by this category:

```
allRootsOf : Polynomial Integer -> List %
allRootsOf : Polynomial Fraction Integer -> List %
allRootsOf : Polynomial % -> List %
allRootsOf : SparseUnivariatePolynomial Integer -> List %
allRootsOf : SparseUnivariatePolynomial Fraction Integer -> List %
characteristic : () -> NonNegativeInteger
nthRoot : (%,Integer) -> %
rootOf :
  (SparseUnivariatePolynomial %,PositiveInteger) ->
     Union(%,"failed")
rootOf :
  (SparseUnivariatePolynomial %,PositiveInteger,OutputForm) ->
     Union(%,"failed")
sqrt : (%,NonNegativeInteger) -> %
sqrt : Integer -> %
sqrt : Fraction Integer -> %
sqrt : % -> %
?**? : (%,Fraction Integer) -> %
```

These exports come from (p1015) CharacteristicZero():

```
0 : () -> %
1 : () -> %
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?*? : (NonNegativeInteger,%) -> %
```

```
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
```

These exports come from (p1055) OrderedRing():

```
abs : % -> %
coerce : Integer -> %
max : (%,%) -> %
min : (%,%) -> %
negative? : % -> Boolean
positive? : % -> Boolean
sign : % -> Integer
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?*? : (Integer,%) -> %
```

These exports come from (p1413) Field():

```
associates? : (%,%) -> Boolean
coerce : % -> %
coerce : Fraction Integer -> %
coerce : Fraction Integer -> %
coerce : Fraction Integer -> %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
factor : % -> Factored %
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
      SparseUnivariatePolynomial %
inv : % -> %
lcm : (%,%) -> %
lcm : List % -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
sizeLess? : (%,%) -> Boolean
squareFree : % -> Factored %
squareFreePart : % -> %
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
```

```
?/? : (%,%) -> %
?*? : (Fraction Integer,%) -> %
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?*? : (%,Fraction Integer) -> %
?**? : (%,Integer) -> %
?^? : (%,Integer) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p158) FullyRetractableTo(Fraction(Integer)):

```
retract : % -> Fraction Integer
retract : % -> Fraction Integer
  if Fraction Integer has RETRACT FRAC INT
retract : % -> Integer if Fraction Integer has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
retractIfCan : % -> Union(Fraction Integer,"failed")
  if Fraction Integer has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed")
  if Fraction Integer has RETRACT INT
```

These exports come from (p1121) Algebra(Integer):

```
?*? : (%,Integer) -> %
```

— RealClosedField.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#RCFIELD">
RealClosedField (RCFIELD)</a></h2>
</body>
```

— category RCFIELD RealClosedField —

```
)abbrev category RCFIELD RealClosedField
++ Author: Renaud Rioboo
++ Date Created: may 1993
++ Date Last Updated: January 2004
++ Description:
++ \axiomType{RealClosedField} provides common access
++ functions for all real closed fields.
++ provides computations with generic real roots of polynomials

RealClosedField() : Category == SIG where

  E      ==> OutputForm
  SUP    ==> SparseUnivariatePolynomial
  OFIELD ==> Join(OrderedRing,Field)
  PME    ==> SUP($)
  N      ==> NonNegativeInteger
  PI     ==> PositiveInteger
  RN     ==> Fraction(Integer)
```

```
Z      ==> Integer
POLY   ==> Polynomial
PACK   ==> SparseUnivariatePolynomialFunctions2
CZ     ==> CharacteristicZero
OR     ==> OrderedRing
CR     ==> CommutativeRing
F      ==> Field
FRT    ==> FullyRetractableTo(Fraction(Integer))
AI     ==> Algebra(Integer)
AFI    ==> Algebra(Fraction(Integer))
RC     ==> RadicalCategory

SIG ==> Join(CZ,OR,CR,F,FRT,AI,AFI,RC) with

  mainForm : $ -> Union(E,"failed")
    ++ \axiom{mainForm(x)} is the main algebraic quantity name of
    ++ \axiom{x}

  mainDefiningPolynomial : $ -> Union(PME,"failed")
    ++ \axiom{mainDefiningPolynomial(x)} is the defining
    ++ polynomial for the main algebraic quantity of \axiom{x}

  mainValue : $ -> Union(PME,"failed")
    ++ \axiom{mainValue(x)} is the expression of \axiom{x} in terms
    ++ of \axiom{SparseUnivariatePolynomial($)}

  rootOf : (PME,PI,E) -> Union($,"failed")
    ++ \axiom{rootOf(pol,n,name)} creates the nth root for the order
    ++ of \axiom{pol} and names it \axiom{name}

  rootOf : (PME,PI) -> Union($,"failed")
    ++ \axiom{rootOf(pol,n)} creates the nth root for the order
    ++ of \axiom{pol} and gives it unique name

  allRootsOf : PME -> List $
    ++ \axiom{allRootsOf(pol)} creates all the roots
    ++ of \axiom{pol} naming each uniquely

  allRootsOf : (SUP(RN)) -> List $
    ++ \axiom{allRootsOf(pol)} creates all the roots
    ++ of \axiom{pol} naming each uniquely

  allRootsOf : (SUP(Z)) -> List $
    ++ \axiom{allRootsOf(pol)} creates all the roots
    ++ of \axiom{pol} naming each uniquely

  allRootsOf : (POLY($)) -> List $
    ++ \axiom{allRootsOf(pol)} creates all the roots
    ++ of \axiom{pol} naming each uniquely

  allRootsOf : (POLY(RN)) -> List $
    ++ \axiom{allRootsOf(pol)} creates all the roots
    ++ of \axiom{pol} naming each uniquely
```

```
  allRootsOf : (POLY(Z)) ->  List $
    ++ \axiom{allRootsOf(pol)} creates all the roots
    ++ of \axiom{pol} naming each uniquely

  sqrt : ($,N) -> $
    ++ \axiom{sqrt(x,n)} is \axiom{x ** (1/n)}

  sqrt : $ -> $
    ++ \axiom{sqrt(x)} is \axiom{x ** (1/2)}

  sqrt : RN -> $
    ++ \axiom{sqrt(x)} is \axiom{x ** (1/2)}

  sqrt : Z -> $
    ++ \axiom{sqrt(x)} is \axiom{x ** (1/2)}

  rename! : ($,E) -> $
    ++ \axiom{rename!(x,name)} changes the way \axiom{x} is printed

  rename : ($,E) -> $
    ++ \axiom{rename(x,name)} gives a new number that prints as name

  approximate: ($,$) -> RN
     ++ \axiom{approximate(n,p)} gives an approximation of \axiom{n}
         ++ that has precision \axiom{p}

 add

   sqrt(a:$):$ == sqrt(a,2)

   sqrt(a:RN):$ == sqrt(a::$,2)

   sqrt(a:Z):$ == sqrt(a::$,2)

   characteristic() == 0

   rootOf(pol,n,o) ==
     r := rootOf(pol,n)
     r case "failed" => "failed"
     rename!(r,o)

   rootOf(pol,n) ==
     liste:List($):= allRootsOf(pol)
     # liste > n => "failed"
     liste.n


   sqrt(x,n) ==
     n = 0 => 1
     n = 1 => x
     zero?(x) => 0
     one?(x) => 1
     if odd?(n)
     then
```

```
        r := rootOf(monomial(1,n) - (x :: PME), 1)
      else
        r := rootOf(monomial(1,n) - (x :: PME), 2)
      r case "failed" => error "no roots"
      n = 2 => rename(r,root(x::E)$E)
      rename(r,root(x :: E, n :: E)$E)

    (x : $) ** (rn : RN) == sqrt(x**numer(rn),denom(rn)::N)

    nthRoot(x, n) ==
      zero?(n) => x
      negative?(n) => inv(sqrt(x,(-n) :: N))
      sqrt(x,n :: N)

    allRootsOf(p:SUP(RN)) == allRootsOf(map(z +-> z::$ ,p)$PACK(RN,$))

    allRootsOf(p:SUP(Z)) == allRootsOf(map(z +-> z::$ ,p)$PACK(Z,$))

    allRootsOf(p:POLY($)) == allRootsOf(univariate(p))

    allRootsOf(p:POLY(RN)) == allRootsOf(univariate(p))

    allRootsOf(p:POLY(Z)) == allRootsOf(univariate(p))
```

———-

— COQ RCFIELD —

```
(* category RCFIELD *)
(*

        sqrt : % -> %
        sqrt(a:$):$ == sqrt(a,2)

        sqrt : Fraction(Integer) -> %
        sqrt(a:RN):$ == sqrt(a::$,2)

        sqrt : Integer -> %
        sqrt(a:Z):$ == sqrt(a::$,2)

        characteristic : () -> NonNegativeInteger
        characteristic() == 0

        rootOf : (SparseUnivariatePolynomial(%),PositiveInteger,OutputForm) ->
            Union(%,"failed")
        rootOf(pol,n,o) ==
          r := rootOf(pol,n)
          r case "failed" => "failed"
          rename!(r,o)

        rootOf : (SparseUnivariatePolynomial(%),PositiveInteger) ->
            Union(%,"failed")
        rootOf(pol,n) ==
```

```
          liste:List($):= allRootsOf(pol)
          # liste > n => "failed"
          liste.n

        sqrt : Fraction(Integer) -> %
        sqrt(x,n) ==
          n = 0 => 1
          n = 1 => x
          zero?(x) => 0
          one?(x) => 1
          if odd?(n)
          then
            r := rootOf(monomial(1,n) - (x :: PME), 1)
          else
            r := rootOf(monomial(1,n) - (x :: PME), 2)
          r case "failed" => error "no roots"
          n = 2 => rename(r,root(x::E)$E)
          rename(r,root(x :: E, n :: E)$E)

        ?**? : (%,Fraction(Integer)) -> %
        (x : $) ** (rn : RN) == sqrt(x**numer(rn),denom(rn)::N)

        nthRoot : (%,Integer) -> %
        nthRoot(x, n) ==
          zero?(n) => x
          negative?(n) => inv(sqrt(x,(-n) :: N))
          sqrt(x,n :: N)

        allRootsOf : SparseUnivariatePolynomial(Fraction(Integer)) -> List(%)
        allRootsOf(p:SUP(RN)) == allRootsOf(map(z +-> z::$ ,p)$PACK(RN,$))

        allRootsOf : SparseUnivariatePolynomial(Integer) -> List(%)
        allRootsOf(p:SUP(Z)) == allRootsOf(map(z +-> z::$ ,p)$PACK(Z,$))

        allRootsOf : Polynomial(%) -> List(%)
        allRootsOf(p:POLY($)) == allRootsOf(univariate(p))

        allRootsOf : Polynomial(Fraction(Integer)) -> List(%)
        allRootsOf(p:POLY(RN)) == allRootsOf(univariate(p))

        allRootsOf : Polynomial(Integer) -> List(%)
        allRootsOf(p:POLY(Z)) == allRootsOf(univariate(p))
*)
```

———-

— RCFIELD.dotabb —

```
"RCFIELD"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RCFIELD"];
"RCFIELD" -> "ALGEBRA"
"RCFIELD" -> "CHARZ"
"RCFIELD" -> "COMRING"
```

```
"RCFIELD" -> "FIELD"
"RCFIELD" -> "FRETRCT"
"RCFIELD" -> "ORDRING"
"RCFIELD" -> "RADCAT"
```

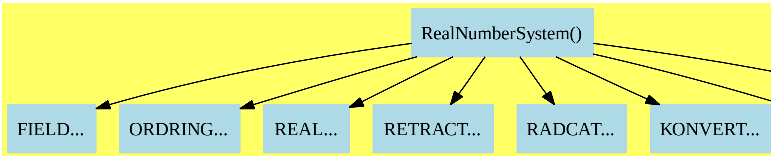
— RCFIELD.dotfull —

```
"RealClosedField()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RCFIELD"];
"RealClosedField()" -> "Algebra(Integer)"
"RealClosedField()" -> "Algebra(Fraction(Integer))"
"RealClosedField()" -> "CharacteristicZero()"
"RealClosedField()" -> "CommutativeRing()"
"RealClosedField()" -> "Field()"
"RealClosedField()" -> "FullyRetractableTo(Fraction(Integer))"
"RealClosedField()" -> "OrderedRing()"
"RealClosedField()" -> "RadicalCategory()"
```

— RCFIELD.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"RealClosedField()" [color=lightblue];
"RealClosedField()" -> "ALGEBRA..."
"RealClosedField()" -> "CHARZ..."
"RealClosedField()" -> "COMRING..."
"RealClosedField()" -> "FIELD..."
"RealClosedField()" -> "FRETRCT..."
"RealClosedField()" -> "ORDRING..."
"RealClosedField()" -> "RADCAT..."

"ALGEBRA..." [color=lightblue];
"CHARZ..." [color=lightblue];
"COMRING..." [color=lightblue];
"FIELD..." [color=lightblue];
"FRETRCT..." [color=lightblue];
"ORDRING..." [color=lightblue];
"RADCAT..." [color=lightblue];

}
```

17.0.224 RealNumberSystem (RNS)



— RealNumberSystem.input —

```
)set break resume
)sys rm -f RealNumberSystem.output
)spool RealNumberSystem.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RealNumberSystem
--R
--R RealNumberSystem is a category constructor
--R Abbreviation for RealNumberSystem is RNS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for RNS
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Fraction(Integer),%) -> %      ?*? : (%,Fraction(Integer)) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,Fraction(Integer)) -> %     ?**? : (%,Integer) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?/? : (%,%) -> %
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               1 : () -> %
--R 0 : () -> %                           ?^? : (%,Integer) -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R abs : % -> %                          associates? : (%,%) -> Boolean
--R ceiling : % -> %                      coerce : Fraction(Integer) -> %
--R coerce : Integer -> %                 coerce : Fraction(Integer) -> %
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              convert : % -> Pattern(Float)
--R convert : % -> DoubleFloat            convert : % -> Float
--R factor : % -> Factored(%)             floor : % -> %
--R fractionPart : % -> %                 gcd : List(%) -> %
--R gcd : (%,%) -> %                      hash : % -> SingleInteger
--R inv : % -> %                          latex : % -> String
--R lcm : List(%) -> %                    lcm : (%,%) -> %
--R max : (%,%) -> %                      min : (%,%) -> %
--R negative? : % -> Boolean              norm : % -> %
--R nthRoot : (%,Integer) -> %            one? : % -> Boolean
```

```
--R positive? : % -> Boolean              prime? : % -> Boolean
--R ?quo? : (%,%) -> %                    recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                    retract : % -> Fraction(Integer)
--R retract : % -> Integer                round : % -> %
--R sample : () -> %                      sign : % -> Integer
--R sizeLess? : (%,%) -> Boolean          sqrt : % -> %
--R squareFree : % -> Factored(%)         squareFreePart : % -> %
--R truncate : % -> %                     unit? : % -> Boolean
--R unitCanonical : % -> %                wholePart : % -> Integer
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed")
--R patternMatch : (%,Pattern(Float),PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%)
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %)
--R retractIfCan : % -> Union(Fraction(Integer),"failed")
--R retractIfCan : % -> Union(Integer,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

— **RealNumberSystem.help** —

```
====================================================================
RealNumberSystem examples
====================================================================

The real number system category is intended as a model for the real
numbers.  The real numbers form an ordered normed field.  Note that
we have purposely not included DifferentialRing or the elementary
functions (see TranscendentalFunctionCategory) in the definition.

See Also:
o )show RealNumberSystem
o )show TranscendentalFunctionCategory
```

See:

⇒ "FloatingPointSystem" (FPS) 18.0.232 on page 1777

⇐ "CharacteristicZero" (CHARZ) 10.0.160 on page 1015
⇐ "ConvertibleTo" (KONVERT) 2.0.17 on page 47
⇐ "Field" (FIELD) 16.0.209 on page 1413
⇐ "OrderedRing" (ORDRING) 10.0.168 on page 1055
⇐ "PatternMatchable" (PATMAB) 4.0.82 on page 337
⇐ "RadicalCategory" (RADCAT) 2.0.36 on page 106
⇐ "RealConstant" (REAL) 3.0.55 on page 180
⇐ "RetractableTo" (RETRACT) 2.0.37 on page 110

Exports:

0	1	abs	associates?
ceiling	characteristic	coerce	convert
divide	euclideanSize	expressIdealMember	exquo
extendedEuclidean	factor	floor	fractionPart
gcd	gcdPolynomial	hash	inv
latex	lcm	max	min
multiEuclidean	negative?	norm	nthRoot
one?	patternMatch	positive?	prime?
principalIdeal	recip	retract	retractIfCan
round	sample	sign	sizeLess?
sqrt	squareFree	squareFreePart	subtractIfCan
truncate	unit?	unitCanonical	unitNormal
wholePart	zero?	?*?	?**?
?+?	?-?	-?	?/?
?<?	?<=?	?=?	?>?
?>=?	?^?	?quo?	?rem?
?~=?			

These are directly exported but not implemented:

```
abs : % -> %
wholePart : % -> Integer
```

These are implemented by this category:

```
characteristic : () -> NonNegativeInteger
ceiling : % -> %
coerce : Fraction Integer -> %
convert : % -> Pattern Float
floor : % -> %
fractionPart : % -> %
norm : % -> %
patternMatch :
  (%,Pattern Float,PatternMatchResult(Float,%)) ->
    PatternMatchResult(Float,%)
round : % -> %
truncate : % -> %
```

These exports come from (p1413) Field():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
coerce : % -> %
```

```
coerce : Integer -> %
coerce : Integer -> %
coerce : Fraction Integer -> %
coerce : % -> OutputForm
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
exquo : (%,%) -> Union(%,"failed")
factor : % -> Factored %
gcd : List % -> %
gcd : (%,%) -> %
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
      SparseUnivariatePolynomial %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
sample : () -> %
sizeLess? : (%,%) -> Boolean
squareFree : % -> Factored %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?**? : (%,Fraction Integer) -> %
?^? : (%,Integer) -> %
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?/? : (%,%) -> %
```

```
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p1055) OrderedRing():

```
negative? : % -> Boolean
positive? : % -> Boolean
sign : % -> Integer
max : (%,%) -> %
min : (%,%) -> %
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

These exports come from (p180) RealConstant():

```
convert : % -> DoubleFloat
convert : % -> Float
```

These exports come from (p110) RetractableTo(Integer):

```
retract : % -> Integer
retractIfCan : % -> Union(Integer,"failed")
```

These exports come from (p110) RetractableTo(Fraction(Integer)):

```
retract : % -> Fraction Integer
retractIfCan : % -> Union(Fraction Integer,"failed")
```

These exports come from (p106) RadicalCategory():

```
nthRoot : (%,Integer) -> %
sqrt : % -> %
```

These exports come from (p47) ConvertibleTo(Pattern(Float)):

These exports come from (p337) PatternMatchable(Float):

These exports come from (p1015) CharacteristicZero():

— RealNumberSystem.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#RNS">
RealNumberSystem (RNS)</a></h2>
</body>
```

— category RNS RealNumberSystem —

```
)abbrev category RNS RealNumberSystem
++ Author: Michael Monagan and Stephen M. Watt
++ Date Created: January 1988
++ Description:
```

```
++ The real number system category is intended as a model for the real
++ numbers.  The real numbers form an ordered normed field.  Note that
++ we have purposely not included \spadtype{DifferentialRing} or
++ the elementary functions (see \spadtype{TranscendentalFunctionCategory})
++ in the definition.

RealNumberSystem() : Category == SIG where

  F   ==> Field
  OR  ==> OrderedRing
  RC  ==> RealConstant
  RI  ==> RetractableTo(Integer)
  RFI ==> RetractableTo(Fraction(Integer))
  RAC ==> RadicalCategory
  CPF ==> ConvertibleTo(Pattern(Float))
  PM  ==> PatternMatchable(Float)
  CZ  ==> CharacteristicZero

  SIG ==> Join(F,OR,RC,RI,RFI,RAC,CPF,PM,CZ) with

    norm : % -> %
      ++ norm x returns the same as absolute value.

    ceiling : % -> %
      ++ ceiling x returns the small integer \spad{>= x}.

    floor : % -> %
      ++ floor x returns the largest integer \spad{<= x}.

    wholePart : % -> Integer
      ++ wholePart x returns the integer part of x.

    fractionPart : % -> %
      ++ fractionPart x returns the fractional part of x.

    truncate : % -> %
      ++ truncate x returns the integer between x and 0 closest to x.

    round : % -> %
      ++ round x computes the integer closest to x.

    abs : % -> %
      ++ abs x returns the absolute value of x.

   add

     characteristic() == 0

     fractionPart x == x - truncate x

     truncate x == (negative? x => -floor(-x); floor x)

     round x == (negative? x => truncate(x-1/2::%); truncate(x+1/2::%))
```

```
    norm x == abs x

    coerce(x:Fraction Integer):% == numer(x)::% / denom(x)::%

    convert(x:%):Pattern(Float)  == convert(x)@Float :: Pattern(Float)

    floor x ==
       x1 := (wholePart x) :: %
       x = x1 => x
       x < 0 => (x1 - 1)
       x1

    ceiling x ==
       x1 := (wholePart x)::%
       x = x1 => x
       x >= 0 => (x1 + 1)
       x1

    patternMatch(x, p, l) ==
      generic? p => addMatch(p, x, l)
      constant? p =>
        (r := retractIfCan(p)@Union(Float, "failed")) case Float =>
          convert(x)@Float = r::Float => l
          failed()
        failed()
      failed()
```

———–

— COQ RNS —

```
(* category RNS *)
(*

   characteristic : () -> NonNegativeInteger
   characteristic() == 0

   fractionPart : % -> %
   fractionPart x == x - truncate x

   truncate : % -> %
   truncate x == (negative? x => -floor(-x); floor x)

   round : % -> %
   round x == (negative? x => truncate(x-1/2::%); truncate(x+1/2::%))

   norm : % -> %
   norm x == abs x

   coerce : Fraction(Integer) -> %
   coerce(x:Fraction Integer):% == numer(x)::% / denom(x)::%

   convert : % -> Pattern(Float)
```

```
   convert(x:%):Pattern(Float)  == convert(x)@Float :: Pattern(Float)

   floor : % -> %
   floor x ==
      x1 := (wholePart x) :: %
      x = x1 => x
      x < 0 => (x1 - 1)
      x1

   ceiling : % -> %
   ceiling x ==
      x1 := (wholePart x)::%
      x = x1 => x
      x >= 0 => (x1 + 1)
      x1

   patternMatch : (%,Pattern(Float),PatternMatchResult(Float,%)) ->
      PatternMatchResult(Float,%)
   patternMatch(x, p, l) ==
     generic? p => addMatch(p, x, l)
     constant? p =>
       (r := retractIfCan(p)@Union(Float, "failed")) case Float =>
         convert(x)@Float = r::Float => l
         failed()
       failed()
     failed()
*)
```

———-

— **RNS.dotabb** —

```
"RNS"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RNS"];
"RNS" -> "FIELD"
"RNS" -> "ORDRING"
"RNS" -> "REAL"
"RNS" -> "RETRACT"
"RNS" -> "RADCAT"
"RNS" -> "KONVERT"
"RNS" -> "PATMAB"
"RNS" -> "CHARZ"
```

———-

— **RNS.dotfull** —

```
"RealNumberSystem()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RNS"];
"RealNumberSystem()" -> "Field()"
"RealNumberSystem()" -> "OrderedRing()"
"RealNumberSystem()" -> "RealConstant()"
"RealNumberSystem()" -> "RetractableTo(Integer)"
```

```
"RealNumberSystem()" -> "RetractableTo(Fraction(Integer))"
"RealNumberSystem()" -> "RadicalCategory()"
"RealNumberSystem()" -> "ConvertibleTo(Pattern(Float))"
"RealNumberSystem()" -> "PatternMatchable(Float)"
"RealNumberSystem()" -> "CharacteristicZero()"
```

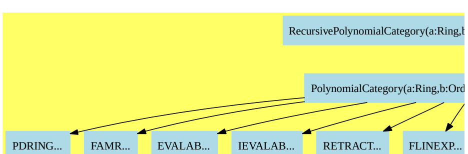
———-

— RNS.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"RealNumberSystem()" [color=lightblue];
"RealNumberSystem()" -> "FIELD..."
"RealNumberSystem()" -> "ORDRING..."
"RealNumberSystem()" -> "REAL..."
"RealNumberSystem()" -> "RETRACT..."
"RealNumberSystem()" -> "RADCAT..."
"RealNumberSystem()" -> "KONVERT..."
"RealNumberSystem()" -> "PATMAB..."
"RealNumberSystem()" -> "CHARZ..."

"FIELD..." [color=lightblue];
"ORDRING..." [color=lightblue];
"REAL..." [color=lightblue];
"RETRACT..." [color=lightblue];
"RADCAT..." [color=lightblue];
"KONVERT..." [color=lightblue];
"PATMAB..." [color=lightblue];
"CHARZ..." [color=lightblue];
}
```

———-

17.0.225 RecursivePolynomialCategory (RPOLCAT)



— RecursivePolynomialCategory.input —

```
)set break resume
)sys rm -f RecursivePolynomialCategory.output
)spool RecursivePolynomialCategory.output
```

```
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show RecursivePolynomialCategory
--R
--R RecursivePolynomialCategory(R: Ring,E: OrderedAbelianMonoidSup,V: OrderedSet) is a category constructor
--R Abbreviation for RecursivePolynomialCategory is RPOLCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for RPOLCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?/? : (%,R) -> % if R has FIELD
--R ?=? : (%,%) -> Boolean                D : (%,V,NonNegativeInteger) -> %
--R D : (%,List(V)) -> %                  D : (%,V) -> %
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R coefficient : (%,E) -> R              coefficients : % -> List(R)
--R coerce : % -> % if R has INTDOM       coerce : V -> %
--R coerce : R -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              content : % -> R if R has GCDDOM
--R deepestInitial : % -> %               deepestTail : % -> %
--R degree : % -> E                       differentiate : (%,List(V)) -> %
--R differentiate : (%,V) -> %            eval : (%,List(V),List(%)) -> %
--R eval : (%,V,%) -> %                   eval : (%,List(V),List(R)) -> %
--R eval : (%,V,R) -> %                   eval : (%,List(%),List(%)) -> %
--R eval : (%,%,%) -> %                   eval : (%,Equation(%)) -> %
--R eval : (%,List(Equation(%))) -> %     gcd : (%,%) -> % if R has GCDDOM
--R gcd : List(%) -> % if R has GCDDOM    gcd : (R,%) -> R if R has GCDDOM
--R ground : % -> R                       ground? : % -> Boolean
--R hash : % -> SingleInteger             head : % -> %
--R headReduce : (%,%) -> %               headReduced? : (%,%) -> Boolean
--R infRittWu? : (%,%) -> Boolean         init : % -> %
--R initiallyReduce : (%,%) -> %          iteratedInitials : % -> List(%)
--R latex : % -> String                   lazyPquo : (%,%,V) -> %
--R lazyPquo : (%,%) -> %                 lazyPrem : (%,%,V) -> %
--R lazyPrem : (%,%) -> %                 lcm : (%,%) -> % if R has GCDDOM
--R lcm : List(%) -> % if R has GCDDOM    leadingCoefficient : (%,V) -> %
--R leadingCoefficient : % -> R           leadingMonomial : % -> %
--R leastMonomial : % -> %                mainCoefficients : % -> List(%)
--R mainMonomial : % -> %                 mainMonomials : % -> List(%)
--R map : ((R -> R),%) -> %               mapExponents : ((E -> E),%) -> %
--R max : (%,%) -> % if R has ORDSET      mdeg : % -> NonNegativeInteger
--R min : (%,%) -> % if R has ORDSET      minimumDegree : % -> E
--R monic? : % -> Boolean                 monicModulo : (%,%) -> %
--R monomial : (R,E) -> %                 monomial? : % -> Boolean
--R monomials : % -> List(%)              mvar : % -> V
--R normalized? : (%,%) -> Boolean        one? : % -> Boolean
```

```
--R pomopo! : (%,R,E,%) -> %            pquo : (%,%,V) -> %
--R pquo : (%,%) -> %                   prem : (%,%,V) -> %
--R prem : (%,%) -> %                   primitiveMonomials : % -> List(%)
--R quasiMonic? : % -> Boolean          recip : % -> Union(%,"failed")
--R reduced? : (%,List(%)) -> Boolean   reduced? : (%,%) -> Boolean
--R reductum : (%,V) -> %               reductum : % -> %
--R retract : % -> V                    retract : % -> R
--R sample : () -> %                    supRittWu? : (%,%) -> Boolean
--R tail : % -> %                       variables : % -> List(V)
--R zero? : % -> Boolean                ?~=? : (%,%) -> Boolean
--R ?*? : (Fraction(Integer),%) -> % if R has ALGEBRA(FRAC(INT))
--R ?*? : (%,Fraction(Integer)) -> % if R has ALGEBRA(FRAC(INT))
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R D : (%,List(V),List(NonNegativeInteger)) -> %
--R LazardQuotient : (%,%,NonNegativeInteger) -> % if R has INTDOM
--R LazardQuotient2 : (%,%,%,NonNegativeInteger) -> % if R has INTDOM
--R RittWuCompare : (%,%) -> Union(Boolean,"failed")
--R associates? : (%,%) -> Boolean if R has INTDOM
--R binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if and(has($,CharacteristicNonZero),has(R,PolynomialFactorizationExplici
--R coefficient : (%,List(V),List(NonNegativeInteger)) -> %
--R coefficient : (%,V,NonNegativeInteger) -> %
--R coerce : Fraction(Integer) -> % if R has RETRACT(FRAC(INT)) or R has ALGEBRA(FRAC(INT))
--R coerce : % -> Polynomial(R) if V has KONVERT(SYMBOL)
--R conditionP : Matrix(%) -> Union(Vector(%),"failed") if and(has($,CharacteristicNonZero),has(R,PolynomialFact
--R content : (%,V) -> % if R has GCDDOM
--R convert : % -> Polynomial(R) if V has KONVERT(SYMBOL)
--R convert : % -> String if R has RETRACT(INT) and V has KONVERT(SYMBOL)
--R convert : Polynomial(R) -> % if V has KONVERT(SYMBOL)
--R convert : Polynomial(Integer) -> % if not(has(R,Algebra(Fraction(Integer)))) and R has ALGEBRA(INT) and V ha
--R convert : Polynomial(Fraction(Integer)) -> % if R has ALGEBRA(FRAC(INT)) and V has KONVERT(SYMBOL)
--R convert : % -> InputForm if V has KONVERT(INFORM) and R has KONVERT(INFORM)
--R convert : % -> Pattern(Integer) if V has KONVERT(PATTERN(INT)) and R has KONVERT(PATTERN(INT))
--R convert : % -> Pattern(Float) if V has KONVERT(PATTERN(FLOAT)) and R has KONVERT(PATTERN(FLOAT))
--R degree : (%,List(V)) -> List(NonNegativeInteger)
--R degree : (%,V) -> NonNegativeInteger
--R differentiate : (%,List(V),List(NonNegativeInteger)) -> %
--R differentiate : (%,V,NonNegativeInteger) -> %
--R discriminant : (%,V) -> % if R has COMRING
--R exactQuotient : (%,%) -> % if R has INTDOM
--R exactQuotient : (%,R) -> % if R has INTDOM
--R exactQuotient! : (%,%) -> % if R has INTDOM
--R exactQuotient! : (%,R) -> % if R has INTDOM
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R extendedSubResultantGcd : (%,%) -> Record(gcd: %,coef1: %,coef2: %) if R has INTDOM
--R factor : % -> Factored(%) if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial(%) -> Factored(SparseUnivariatePolynomial(%)) if R has PFECAT
--R factorSquareFreePolynomial : SparseUnivariatePolynomial(%) -> Factored(SparseUnivariatePolynomial(%)) if R h
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
```

```
--R halfExtendedSubResultantGcd1 : (%,%) -> Record(gcd: %,coef1: %) if R has INTDOM
--R halfExtendedSubResultantGcd2 : (%,%) -> Record(gcd: %,coef2: %) if R has INTDOM
--R headReduced? : (%,List(%)) -> Boolean
--R initiallyReduced? : (%,List(%)) -> Boolean
--R initiallyReduced? : (%,%) -> Boolean
--R isExpt : % -> Union(Record(var: V,exponent: NonNegativeInteger),"failed")
--R isPlus : % -> Union(List(%),"failed")
--R isTimes : % -> Union(List(%),"failed")
--R lastSubResultant : (%,%) -> % if R has INTDOM
--R lazyPremWithDefault : (%,%,V) -> Record(coef: %,gap: NonNegativeInteger,remainder: %)
--R lazyPremWithDefault : (%,%) -> Record(coef: %,gap: NonNegativeInteger,remainder: %)
--R lazyPseudoDivide : (%,%,V) -> Record(coef: %,gap: NonNegativeInteger,quotient: %,remainder: %)
--R lazyPseudoDivide : (%,%) -> Record(coef: %,gap: NonNegativeInteger,quotient: %,remainder: %)
--R lazyResidueClass : (%,%) -> Record(polnum: %,polden: %,power: NonNegativeInteger)
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %) if R has GCDDOM
--R mainContent : % -> % if R has GCDDOM
--R mainPrimitivePart : % -> % if R has GCDDOM
--R mainSquareFreePart : % -> % if R has GCDDOM
--R mainVariable : % -> Union(V,"failed")
--R minimumDegree : (%,List(V)) -> List(NonNegativeInteger)
--R minimumDegree : (%,V) -> NonNegativeInteger
--R monicDivide : (%,%,V) -> Record(quotient: %,remainder: %)
--R monomial : (%,List(V),List(NonNegativeInteger)) -> %
--R monomial : (%,V,NonNegativeInteger) -> %
--R multivariate : (SparseUnivariatePolynomial(%),V) -> %
--R multivariate : (SparseUnivariatePolynomial(R),V) -> %
--R nextsubResultant2 : (%,%,%,%) -> % if R has INTDOM
--R normalized? : (%,List(%)) -> Boolean
--R numberOfMonomials : % -> NonNegativeInteger
--R patternMatch : (%,Pattern(Integer),PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%) if V has
--R patternMatch : (%,Pattern(Float),PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%) if V has PATMAB
--R primPartElseUnitCanonical : % -> % if R has INTDOM
--R primPartElseUnitCanonical! : % -> % if R has INTDOM
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (%,V) -> % if R has GCDDOM
--R primitivePart : % -> % if R has GCDDOM
--R primitivePart! : % -> % if R has GCDDOM
--R pseudoDivide : (%,%) -> Record(quotient: %,remainder: %)
--R reducedSystem : Matrix(%) -> Matrix(R)
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(R),vec: Vector(R))
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(Integer),vec: Vector(Integer)) if R has LINEXP(I
--R reducedSystem : Matrix(%) -> Matrix(Integer) if R has LINEXP(INT)
--R resultant : (%,%) -> % if R has INTDOM
--R resultant : (%,%,V) -> % if R has COMRING
--R retract : Polynomial(R) -> % if not(has(R,Algebra(Fraction(Integer)))) and not(has(R,Algebra(Integer))) and
--R retract : Polynomial(Integer) -> % if not(has(R,Algebra(Fraction(Integer)))) and R has ALGEBRA(INT) and V ha
--R retract : Polynomial(Fraction(Integer)) -> % if R has ALGEBRA(FRAC(INT)) and V has KONVERT(SYMBOL)
--R retract : % -> Integer if R has RETRACT(INT)
--R retract : % -> Fraction(Integer) if R has RETRACT(FRAC(INT))
--R retractIfCan : Polynomial(R) -> Union(%,"failed") if not(has(R,Algebra(Fraction(Integer)))) and not(has(R,Al
--R retractIfCan : Polynomial(Integer) -> Union(%,"failed") if not(has(R,Algebra(Fraction(Integer)))) and R has
--R retractIfCan : Polynomial(Fraction(Integer)) -> Union(%,"failed") if R has ALGEBRA(FRAC(INT)) and V has KONV
--R retractIfCan : % -> Union(V,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT(INT)
```

```
--R retractIfCan : % -> Union(Fraction(Integer),"failed") if R has RETRACT(FRAC(INT))
--R retractIfCan : % -> Union(R,"failed")
--R solveLinearPolynomialEquation : (List(SparseUnivariatePolynomial(%)),SparseUnivariatePolynomial(%)) -> Union
--R squareFree : % -> Factored(%) if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial(%) -> Factored(SparseUnivariatePolynomial(%)) if R has PFE
--R subResultantChain : (%,%) -> List(%) if R has INTDOM
--R subResultantGcd : (%,%) -> % if R has INTDOM
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R totalDegree : (%,List(V)) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial(R)
--R univariate : (%,V) -> SparseUnivariatePolynomial(%)
--R
--E 1

)spool
)lisp (bye)
```

— **RecursivePolynomialCategory.help** —

```
====================================================================
RecursivePolynomialCategory examples
====================================================================

A category for general multi-variate polynomials with coefficients
in a ring, variables in an ordered set, and exponents from an
ordered abelian monoid, with a sup operation.

When not constant, such a polynomial is viewed as a univariate polynomial
in its main variable w. r. t. to the total ordering on the elements in
the ordered set, so that some operations usually defined for univariate
polynomials make sense here.

See Also:
o )show RecursivePolynomialCategory
```

See:

⇐ "PolynomialCategory" (POLYCAT) 16.0.213 on page 1449

Exports:

0	1	associates?
binomThmExpt	characteristic	charthRoot
coefficient	coefficients	coerce
conditionP	convert	D
deepestInitial	deepestTail	degree
differentiate	discriminant	eval
exactQuotient	exactQuotient!	exquo
extendedSubResultantGcd	factor	factorPolynomial
factorSquareFreePolynomial	gcd	gcdPolynomial
ground	ground?	halfExtendedSubResultantGcd1
halfExtendedSubResultantGcd2	hash	head
headReduce	headReduced?	infRittWu?
init	initiallyReduce	initiallyReduced?
isExpt	isPlus	isTimes
iteratedInitials	lastSubResultant	latex
LazardQuotient	LazardQuotient2	lazyPquo
lazyPrem	lazyPremWithDefault	lazyPseudoDivide
lazyResidueClass	lcm	leadingCoefficient
leadingMonomial	leastMonomial	mainCoefficients
mainContent	mainMonomial	mainPrimitivePart
mainSquareFreePart	mainVariable	map
mapExponents	max	mdeg
min	minimumDegree	monic?
monicDivide	monicModulo	monomial
monomial?	monomials	multivariate
mvar	nextsubResultant2	normalized?
numberOfMonomials	one?	patternMatch
pomopo!	pquo	prem
primPartElseUnitCanonical	primPartElseUnitCanonical!	prime?
primitiveMonomials	primitivePart	primitivePart!
pseudoDivide	quasiMonic?	recip
reduced?	reducedSystem	reductum
resultant	retract	retractIfCan
RittWuCompare	sample	solveLinearPolynomialEquation
squareFree	squareFreePart	squareFreePolynomial
subResultantChain	subResultantGcd	subtractIfCan
supRittWu?	tail	totalDegree
unit?	unitCanonical	unitNormal
univariate	variables	zero?
?*?	?**?	?+?
?-?	-?	?=?
?^?	?~=?	?/?
?<?	?<=?	?>?
?>=?		

Attributes Exported:

- if #1 has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- if #1 has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.

- if #1 has canonicalUnitNormal then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
exactQuotient! : (%,R) -> % if R has INTDOM
extendedSubResultantGcd : (%,%) -> Record(gcd: %,coef1: %,coef2: %)
  if R has INTDOM
halfExtendedSubResultantGcd1 : (%,%) -> Record(gcd: %,coef1: %)
  if R has INTDOM
halfExtendedSubResultantGcd2 : (%,%) -> Record(gcd: %,coef2: %)
  if R has INTDOM
lastSubResultant : (%,%) -> % if R has INTDOM
LazardQuotient : (%,%,NonNegativeInteger) -> %
  if R has INTDOM
LazardQuotient2 : (%,%,%,NonNegativeInteger) -> %
  if R has INTDOM
nextsubResultant2 : (%,%,%,%) -> % if R has INTDOM
resultant : (%,%) -> % if R has INTDOM
subResultantChain : (%,%) -> List % if R has INTDOM
subResultantGcd : (%,%) -> % if R has INTDOM
```

These are implemented by this category:

```
coerce : % -> OutputForm
coerce : % -> Polynomial R if V has KONVERT SYMBOL
convert : % -> String
  if R has RETRACT INT
  and V has KONVERT SYMBOL
convert : % -> Polynomial R if V has KONVERT SYMBOL
convert : Polynomial R -> % if V has KONVERT SYMBOL
convert : Polynomial Integer -> %
  if not has(R,Algebra Fraction Integer)
  and R has ALGEBRA INT
  and V has KONVERT SYMBOL
  or R has ALGEBRA FRAC INT
  and V has KONVERT SYMBOL
convert : Polynomial Fraction Integer -> %
  if R has ALGEBRA FRAC INT
  and V has KONVERT SYMBOL
deepestInitial : % -> %
deepestTail : % -> %
exactQuotient : (%,R) -> % if R has INTDOM
exactQuotient : (%,%) -> % if R has INTDOM
exactQuotient! : (%,%) -> % if R has INTDOM
gcd : (R,%) -> R if R has GCDDOM
head : % -> %
headReduce : (%,%) -> %
```

```
headReduced? : (%,List %) -> Boolean
headReduced? : (%,%) -> Boolean
infRittWu? : (%,%) -> Boolean
init : % -> %
initiallyReduce : (%,%) -> %
initiallyReduced? : (%,%) -> Boolean
initiallyReduced? : (%,List %) -> Boolean
iteratedInitials : % -> List %
lazyPremWithDefault : (%,%) ->
  Record(coef: %,gap: NonNegativeInteger,remainder: %)
lazyPremWithDefault : (%,%,V) ->
  Record(coef: %,gap: NonNegativeInteger,remainder: %)
lazyPquo : (%,%) -> %
lazyPquo : (%,%,V) -> %
lazyPrem : (%,%,V) -> %
lazyPrem : (%,%) -> %
lazyPseudoDivide : (%,%) ->
  Record(coef: %,gap: NonNegativeInteger,quotient: %,remainder: %)
lazyPseudoDivide : (%,%,V) ->
  Record(coef: %,gap: NonNegativeInteger,quotient: %,remainder: %)
lazyResidueClass : (%,%) ->
  Record(polnum: %,polden: %,power: NonNegativeInteger)
leadingCoefficient : (%,V) -> %
leastMonomial : % -> %
mainCoefficients : % -> List %
mainContent : % -> % if R has GCDDOM
mainMonomial : % -> %
mainMonomials : % -> List %
mainPrimitivePart : % -> % if R has GCDDOM
mainSquareFreePart : % -> % if R has GCDDOM
mdeg : % -> NonNegativeInteger
monic? : % -> Boolean
monicModulo : (%,%) -> %
mvar : % -> V
normalized? : (%,%) -> Boolean
normalized? : (%,List %) -> Boolean
pquo : (%,%) -> %
pquo : (%,%,V) -> %
prem : (%,%,V) -> %
prem : (%,%) -> %
primitivePart! : % -> % if R has GCDDOM
primPartElseUnitCanonical : % -> % if R has INTDOM
primPartElseUnitCanonical! : % -> % if R has INTDOM
pseudoDivide : (%,%) -> Record(quotient: %,remainder: %)
quasiMonic? : % -> Boolean
reduced? : (%,%) -> Boolean
reduced? : (%,List %) -> Boolean
reductum : (%,V) -> %
retract : Polynomial R -> %
  if not has(R,Algebra Fraction Integer)
  and not has(R,Algebra Integer)
  and V has KONVERT SYMBOL
  or not has(R,IntegerNumberSystem)
  and not has(R,Algebra Fraction Integer)
```

```
   and R has ALGEBRA INT
   and V has KONVERT SYMBOL
   or not has(R,QuotientFieldCategory Integer)
   and R has ALGEBRA FRAC INT
   and V has KONVERT SYMBOL
 retract : Polynomial Integer -> %
   if not has(R,Algebra Fraction Integer)
   and R has ALGEBRA INT
   and V has KONVERT SYMBOL
   or R has ALGEBRA FRAC INT
   and V has KONVERT SYMBOL
 retract : Polynomial Fraction Integer -> %
   if R has ALGEBRA FRAC INT and V has KONVERT SYMBOL
 retractIfCan : Polynomial R -> Union(%,"failed")
   if not has(R,Algebra Fraction Integer)
   and not has(R,Algebra Integer)
   and V has KONVERT SYMBOL
   or not has(R,IntegerNumberSystem)
   and not has(R,Algebra Fraction Integer)
   and R has ALGEBRA INT
   and V has KONVERT SYMBOL
   or not has(R,QuotientFieldCategory Integer)
   and R has ALGEBRA FRAC INT
   and V has KONVERT SYMBOL
 retractIfCan : Polynomial Fraction Integer -> Union(%,"failed")
   if R has ALGEBRA FRAC INT
   and V has KONVERT SYMBOL
 retractIfCan : Polynomial Integer -> Union(%,"failed")
   if not has(R,Algebra Fraction Integer)
   and R has ALGEBRA INT
   and V has KONVERT SYMBOL
   or R has ALGEBRA FRAC INT
   and V has KONVERT SYMBOL
 RittWuCompare : (%,%) -> Union(Boolean,"failed")
 supRittWu? : (%,%) -> Boolean
 tail : % -> %
```

These exports come from (p1449) PolynomialCategory(R,E,V)
where R:Ring, E:OrderedAbelianMonoidSup, V:OrderedSet:

```
 0 : () -> %
 1 : () -> %
 associates? : (%,%) -> Boolean
   if R has INTDOM
 binomThmExpt : (%,%,NonNegativeInteger) -> %
   if R has COMRING
 characteristic : () -> NonNegativeInteger
 charthRoot : % -> Union(%,"failed")
   if and(has($,CharacteristicNonZero),
      has(R,PolynomialFactorizationExplicit))
   or R has CHARNZ
 coefficient : (%,List V,List NonNegativeInteger) -> %
 coefficient : (%,V,NonNegativeInteger) -> %
 coefficient : (%,E) -> R
 coefficients : % -> List R
```

```
coerce : R -> %
coerce : Fraction Integer -> %
  if R has RETRACT FRAC INT or R has ALGEBRA FRAC INT
coerce : V -> %
coerce : % -> % if R has INTDOM
coerce : Integer -> %
conditionP : Matrix % -> Union(Vector %,"failed")
  if and(has($,CharacteristicNonZero),
         has(R,PolynomialFactorizationExplicit))
content : % -> R if R has GCDDOM
content : (%,V) -> % if R has GCDDOM
convert : % -> Pattern Integer
  if V has KONVERT PATTERN INT and R has KONVERT PATTERN INT
convert : % -> Pattern Float
  if V has KONVERT PATTERN FLOAT and R has KONVERT PATTERN FLOAT
convert : % -> InputForm
  if V has KONVERT INFORM and R has KONVERT INFORM
D : (%,List V) -> %
D : (%,V) -> %
D : (%,List V,List NonNegativeInteger) -> %
D : (%,V,NonNegativeInteger) -> %
degree : % -> E
degree : (%,List V) -> List NonNegativeInteger
degree : (%,V) -> NonNegativeInteger
differentiate : (%,List V,List NonNegativeInteger) -> %
differentiate : (%,V,NonNegativeInteger) -> %
differentiate : (%,List V) -> %
differentiate : (%,V) -> %
discriminant : (%,V) -> % if R has COMRING
eval : (%,List Equation %) -> %
eval : (%,Equation %) -> %
eval : (%,List %,List %) -> %
eval : (%,%,%) -> %
eval : (%,V,R) -> %
eval : (%,List V,List R) -> %
eval : (%,V,%) -> %
eval : (%,List V,List %) -> %
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
exquo : (%,%) -> Union(%,"failed") if R has INTDOM
factor : % -> Factored % if R has PFECAT
factorPolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
     if R has PFECAT
factorSquareFreePolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if R has PFECAT
gcd : (%,%) -> % if R has GCDDOM
gcd : List % -> % if R has GCDDOM
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
```

```
       if R has GCDDOM
ground : % -> R
ground? : % -> Boolean
hash : % -> SingleInteger
isExpt : % -> Union(Record(var: V,exponent: NonNegativeInteger),"failed")
isPlus : % -> Union(List %,"failed")
isTimes : % -> Union(List %,"failed")
latex : % -> String
lcm : (%,%) -> % if R has GCDDOM
lcm : List % -> % if R has GCDDOM
leadingCoefficient : % -> R
leadingMonomial : % -> %
mainVariable : % -> Union(V,"failed")
map : ((R -> R),%) -> %
mapExponents : ((E -> E),%) -> %
max : (%,%) -> % if R has ORDSET
min : (%,%) -> % if R has ORDSET
minimumDegree : % -> E
minimumDegree : (%,List V) -> List NonNegativeInteger
minimumDegree : (%,V) -> NonNegativeInteger
monicDivide : (%,%,V) -> Record(quotient: %,remainder: %)
monomial : (%,V,NonNegativeInteger) -> %
monomial : (%,List V,List NonNegativeInteger) -> %
monomial : (R,E) -> %
monomial? : % -> Boolean
monomials : % -> List %
multivariate : (SparseUnivariatePolynomial %,V) -> %
multivariate : (SparseUnivariatePolynomial R,V) -> %
numberOfMonomials : % -> NonNegativeInteger
one? : % -> Boolean
patternMatch :
  (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
    PatternMatchResult(Integer,%)
      if V has PATMAB INT and R has PATMAB INT
patternMatch :
  (%,Pattern Float,PatternMatchResult(Float,%)) ->
    PatternMatchResult(Float,%)
     if V has PATMAB FLOAT and R has PATMAB FLOAT
pomopo! : (%,R,E,%) -> %
prime? : % -> Boolean if R has PFECAT
primitiveMonomials : % -> List %
primitivePart : (%,V) -> % if R has GCDDOM
primitivePart : % -> % if R has GCDDOM
recip : % -> Union(%,"failed")
reducedSystem : Matrix % -> Matrix R
reducedSystem : (Matrix %,Vector %) ->
   Record(mat: Matrix R,vec: Vector R)
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix Integer,vec: Vector Integer)
    if R has LINEXP INT
reducedSystem : Matrix % -> Matrix Integer
  if R has LINEXP INT
reductum : % -> %
resultant : (%,%,V) -> % if R has COMRING
```

```
retract : % -> R
retract : % -> Integer if R has RETRACT INT
retract : % -> Fraction Integer
  if R has RETRACT FRAC INT
retract : % -> V
retractIfCan : % -> Union(R,"failed")
retractIfCan : % -> Union(Integer,"failed")
  if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
  if R has RETRACT FRAC INT
retractIfCan : % -> Union(V,"failed")
sample : () -> %
solveLinearPolynomialEquation :
  (List SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
    Union(List SparseUnivariatePolynomial %,"failed")
      if R has PFECAT
squareFree : % -> Factored % if R has GCDDOM
squareFreePart : % -> % if R has GCDDOM
squareFreePolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if R has PFECAT
subtractIfCan : (%,%) -> Union(%,"failed")
totalDegree : (%,List V) -> NonNegativeInteger
totalDegree : % -> NonNegativeInteger
unit? : % -> Boolean if R has INTDOM
unitCanonical : % -> % if R has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
  if R has INTDOM
univariate : % -> SparseUnivariatePolynomial R
univariate : (%,V) -> SparseUnivariatePolynomial %
variables : % -> List V
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,R) -> %
?*? : (R,%) -> %
?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?/? : (%,R) -> % if R has FIELD
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?<? : (%,%) -> Boolean if R has ORDSET
?<=? : (%,%) -> Boolean if R has ORDSET
```

```
 ?>? : (%,%) -> Boolean if R has ORDSET
 ?>=? : (%,%) -> Boolean if R has ORDSET
```

— RecursivePolynomialCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#RPOLCAT">
RecursivePolynomialCategory (RPOLCAT)</a></h2>
</body>
```

— category RPOLCAT RecursivePolynomialCategory —

```
)abbrev category RPOLCAT RecursivePolynomialCategory
++ Author: Marc Moreno Maza
++ Date Created: 04/22/1994
++ Date Last Updated: 14/12/1998
++ Description:

RecursivePolynomialCategory(R,E,V) : Category == SIG where
  R : Ring
  E : OrderedAbelianMonoidSup
  V : OrderedSet

  SIG ==> PolynomialCategory(R, E, V) with

    mvar : $ -> V
      ++ \axiom{mvar(p)} returns an error if \axiom{p} belongs to
      ++ \axiom{R}, otherwise returns its main variable w. r. t. to the
      ++ total ordering on the elements in \axiom{V}.

    mdeg : $ -> NonNegativeInteger
      ++ \axiom{mdeg(p)} returns an error if \axiom{p} is \axiom{0},
      ++ otherwise, if \axiom{p} belongs to \axiom{R} returns \axiom{0},
      ++ otherwise, returns the degree of \axiom{p} in its main variable.

    init : $ -> $
      ++ \axiom{init(p)} returns an error if \axiom{p} belongs to
      ++ \axiom{R}, otherwise returns its leading coefficient, where
      ++ \axiom{p} is viewed as a univariate polynomial in its main
      ++ variable.

    head : $ -> $
      ++ \axiom{head(p)} returns \axiom{p} if \axiom{p} belongs to
      ++ \axiom{R}, otherwise returns its leading term (monomial in the
      ++ AXIOM sense), where \axiom{p} is viewed as a univariate polynomial
      ++  in its main variable.

    tail : $ -> $
      ++ \axiom{tail(p)} returns its reductum, where \axiom{p} is viewed
      ++ as a univariate polynomial in its main variable.
```

```
deepestTail : $ -> $
  ++ \axiom{deepestTail(p)} returns \axiom{0} if \axiom{p} belongs to
  ++ \axiom{R}, otherwise returns tail(p), if \axiom{tail(p)} belongs
  ++ to  \axiom{R} or \axiom{mvar(tail(p)) < mvar(p)}, otherwise
  ++ returns \axiom{deepestTail(tail(p))}.

iteratedInitials : $ -> List $
  ++ \axiom{iteratedInitials(p)} returns \axiom{[]} if \axiom{p}
  ++ belongs to \axiom{R},
  ++ otherwise returns the list of the iterated initials of \axiom{p}.

deepestInitial : $ -> $
  ++ \axiom{deepestInitial(p)} returns an error if \axiom{p} belongs
  ++ to \axiom{R},
  ++ otherwise returns the last term of \axiom{iteratedInitials(p)}.

leadingCoefficient : ($,V) -> $
  ++ \axiom{leadingCoefficient(p,v)} returns the leading coefficient
  ++ of \axiom{p}, where \axiom{p} is viewed as A univariate
  ++ polynomial in \axiom{v}.

reductum : ($,V) -> $
  ++ \axiom{reductum(p,v)} returns the reductum of \axiom{p}, where
  ++ \axiom{p} is viewed as a univariate polynomial in \axiom{v}.

monic? : $ -> Boolean
  ++ \axiom{monic?(p)} returns false if \axiom{p} belongs to \axiom{R},
  ++ otherwise returns true iff \axiom{p} is monic as a univariate
  ++ polynomial in its main variable.

quasiMonic? : $ -> Boolean
  ++ \axiom{quasiMonic?(p)} returns false if \axiom{p} belongs to
  ++ \axiom{R}, otherwise returns true iff the initial of \axiom{p}
  ++ lies in the base ring \axiom{R}.

mainMonomial : $ -> $
  ++ \axiom{mainMonomial(p)} returns an error if \axiom{p} is
  ++ \axiom{O}, otherwise, if \axiom{p} belongs to \axiom{R} returns
  ++ \axiom{1}, otherwise, \axiom{mvar(p)} raised to the power
  ++ \axiom{mdeg(p)}.

leastMonomial : $ -> $
  ++ \axiom{leastMonomial(p)} returns an error if \axiom{p} is
  ++ \axiom{O}, otherwise, if \axiom{p} belongs to \axiom{R} returns
  ++ \axiom{1}, otherwise, the monomial of \axiom{p} with lowest
  ++ degree, where \axiom{p} is viewed as a univariate polynomial in
  ++ its main variable.

mainCoefficients : $ -> List $
  ++ \axiom{mainCoefficients(p)} returns an error if \axiom{p} is
  ++ \axiom{O}, otherwise, if \axiom{p} belongs to \axiom{R} returns
  ++ [p], otherwise returns the list of the coefficients of \axiom{p},
  ++ where \axiom{p} is viewed as a univariate polynomial in its main
  ++ variable.
```

```
mainMonomials : $ -> List $
  ++ \axiom{mainMonomials(p)} returns an error if \axiom{p} is
  ++ \axiom{O}, otherwise, if \axiom{p} belongs to \axiom{R} returns
  ++ [1], otherwise returns the list of the monomials of \axiom{p},
  ++ where \axiom{p} is viewed as a univariate polynomial in its main
  ++ variable.

RittWuCompare : ($, $) -> Union(Boolean,"failed")
  ++ \axiom{RittWuCompare(a,b)} returns \axiom{"failed"} if \axiom{a}
  ++ and \axiom{b} have same rank w.r.t.
  ++ Ritt and Wu Wen Tsun ordering using the refinement of Lazard,
  ++ otherwise returns \axiom{infRittWu?(a,b)}.

infRittWu?  : ($, $) -> Boolean
  ++ \axiom{infRittWu?(a,b)} returns true if \axiom{a} is less than
  ++ \axiom{b} w.r.t. the Ritt and Wu Wen Tsun ordering using the
  ++ refinement of Lazard.

supRittWu? : ($, $) -> Boolean
  ++ \axiom{supRittWu?(a,b)} returns true if \axiom{a} is greater
  ++ than \axiom{b} w.r.t. the Ritt and Wu Wen Tsun ordering using the
  ++ refinement of Lazard.

reduced? : ($,$) -> Boolean
  ++ \axiom{reduced?(a,b)} returns true iff
  ++ \axiom{degree(a,mvar(b)) < mdeg(b)}.

reduced? : ($,List($)) -> Boolean
  ++ \axiom{reduced?(q,lp)} returns true iff \axiom{reduced?(q,p)}
  ++ holds for every \axiom{p} in \axiom{lp}.

headReduced? : ($,$) -> Boolean
  ++ \axiom{headReduced?(a,b)} returns true iff
  ++ \axiom{degree(head(a),mvar(b)) < mdeg(b)}.

headReduced? : ($,List($)) -> Boolean
  ++ \axiom{headReduced?(q,lp)} returns true iff
  ++ \axiom{headReduced?(q,p)} holds for every \axiom{p} in \axiom{lp}.

initiallyReduced? : ($,$) -> Boolean
  ++ \axiom{initiallyReduced?(a,b)} returns false iff there exists an
  ++ iterated initial of \axiom{a} which is not reduced w.r.t \axiom{b}.

initiallyReduced? :  ($,List($)) -> Boolean
  ++ \axiom{initiallyReduced?(q,lp)} returns true iff
  ++ \axiom{initiallyReduced?(q,p)} holds for every \axiom{p} in
  ++ \axiom{lp}.

normalized? : ($,$) -> Boolean
  ++ \axiom{normalized?(a,b)} returns true iff \axiom{a} and its
  ++ iterated initials have degree zero w.r.t. the main variable of
  ++ \axiom{b}
```

```
normalized? : ($,List($)) -> Boolean
  ++ \axiom{normalized?(q,lp)} returns true iff
  ++ \axiom{normalized?(q,p)} holds
  ++ for every \axiom{p} in \axiom{lp}.

prem : ($, $) -> $
  ++ \axiom{prem(a,b)} computes the pseudo-remainder of \axiom{a} by
  ++ \axiom{b}, both viewed as univariate polynomials in the main
  ++ variable of \axiom{b}.

pquo : ($, $) -> $
  ++ \axiom{pquo(a,b)} computes the pseudo-quotient of \axiom{a} by
  ++ \axiom{b}, both viewed as univariate polynomials in the main
  ++ variable of \axiom{b}.

prem : ($, $, V) -> $
  ++ \axiom{prem(a,b,v)} computes the pseudo-remainder of \axiom{a}
  ++ by \axiom{b}, both viewed as univariate polynomials in \axiom{v}.

pquo : ($, $, V) -> $
  ++ \axiom{pquo(a,b,v)} computes the pseudo-quotient of \axiom{a} by
  ++ \axiom{b}, both viewed as univariate polynomials in \axiom{v}.

lazyPrem : ($, $) ->  $
  ++ \axiom{lazyPrem(a,b)} returns the polynomial \axiom{r} reduced
  ++ w.r.t. \axiom{b} and such that \axiom{b} divides
  ++ \axiom{init(b)^e a - r} where \axiom{e}
  ++ is the number of steps of this pseudo-division.

lazyPquo : ($, $) ->  $
  ++ \axiom{lazyPquo(a,b)} returns the polynomial \axiom{q} such that
  ++ \axiom{lazyPseudoDivide(a,b)} returns \axiom{[c,g,q,r]}.

lazyPrem : ($, $, V) -> $
  ++ \axiom{lazyPrem(a,b,v)} returns the polynomial \axiom{r}
  ++ reduced w.r.t. \axiom{b} viewed as univariate polynomials in the
  ++ variable \axiom{v} such that \axiom{b} divides
  ++ \axiom{init(b)^e a - r} where \axiom{e} is the number of steps of
  ++ this pseudo-division.

lazyPquo : ($, $, V) ->  $
  ++ \axiom{lazyPquo(a,b,v)} returns the polynomial \axiom{q} such that
  ++ \axiom{lazyPseudoDivide(a,b,v)} returns \axiom{[c,g,q,r]}.

lazyPremWithDefault : ($, $) -> _
  Record (coef : $, gap : NonNegativeInteger, remainder : $)
  ++ \axiom{lazyPremWithDefault(a,b)} returns \axiom{[c,g,r]}
  ++ such that \axiom{r = lazyPrem(a,b)} and
  ++ \axiom{(c**g)*r = prem(a,b)}.

lazyPremWithDefault : ($, $, V) -> _
  Record (coef : $, gap : NonNegativeInteger, remainder : $)
  ++ \axiom{lazyPremWithDefault(a,b,v)} returns \axiom{[c,g,r]}
  ++ such that \axiom{r = lazyPrem(a,b,v)} and
```

```
  ++ \axiom{(c**g)*r = prem(a,b,v)}.

lazyPseudoDivide : ($,$) -> _
  Record(coef:$, gap: NonNegativeInteger,quotient:$, remainder:$)
  ++ \axiom{lazyPseudoDivide(a,b)} returns \axiom{[c,g,q,r]}
  ++ such that \axiom{[c,g,r] = lazyPremWithDefault(a,b)} and
  ++ \axiom{q} is the pseudo-quotient computed in this lazy
  ++ pseudo-division.

lazyPseudoDivide : ($,$,V) -> _
  Record(coef:$, gap:NonNegativeInteger, quotient:$, remainder: $)
  ++ \axiom{lazyPseudoDivide(a,b,v)} returns \axiom{[c,g,q,r]} such
  ++ that  \axiom{r = lazyPrem(a,b,v)}, \axiom{(c**g)*r = prem(a,b,v)}
  ++ and \axiom{q} is the pseudo-quotient computed in this lazy
  ++ pseudo-division.

pseudoDivide : ($, $) -> Record (quotient : $, remainder : $)
  ++ \axiom{pseudoDivide(a,b)} computes \axiom{[pquo(a,b),prem(a,b)]},
  ++ both polynomials viewed as univariate polynomials in the main
  ++ variable of \axiom{b}, if \axiom{b} is not a constant polynomial.

monicModulo : ($, $) -> $
  ++ \axiom{monicModulo(a,b)} computes \axiom{a mod b}, if \axiom{b} is
  ++ monic as univariate polynomial in its main variable.

lazyResidueClass : ($,$) -> _
 Record(polnum:$, polden:$, power:NonNegativeInteger)
  ++ \axiom{lazyResidueClass(a,b)} returns \axiom{[p,q,n]} where
  ++ \axiom{p / q**n} represents the residue class of \axiom{a}
  ++ modulo \axiom{b} and \axiom{p} is reduced w.r.t. \axiom{b} and
  ++ \axiom{q} is \axiom{init(b)}.

headReduce: ($, $) ->  $
  ++ \axiom{headReduce(a,b)} returns a polynomial \axiom{r} such that
  ++ \axiom{headReduced?(r,b)} holds and there exists an integer
  ++ \axiom{e} such that \axiom{init(b)^e a - r} is zero modulo
  ++ \axiom{b}.

initiallyReduce: ($, $) ->  $
  ++ \axiom{initiallyReduce(a,b)} returns a polynomial \axiom{r} such
  ++ that \axiom{initiallyReduced?(r,b)} holds and there exists an
  ++ integer \axiom{e} such that \axiom{init(b)^e a - r} is zero
  ++ modulo \axiom{b}.

if (V has ConvertibleTo(Symbol)) then

  CoercibleTo(Polynomial R)

  ConvertibleTo(Polynomial R)

  if R has Algebra Fraction Integer then

      retractIfCan : Polynomial Fraction Integer -> Union($,"failed")
        ++ \axiom{retractIfCan(p)} returns \axiom{p} as an element of
```

```
      ++ the current domain if all its variables belong to \axiom{V}.

    retract : Polynomial Fraction Integer -> $
      ++ \axiom{retract(p)} returns \axiom{p} as an element of the
      ++ current domain if \axiom{retractIfCan(p)} does not return
      ++ "failed", otherwise an error is produced.

    convert : Polynomial Fraction Integer -> $
      ++ \axiom{convert(p)} returns the same as \axiom{retract(p)}.

    retractIfCan : Polynomial Integer -> Union($,"failed")
      ++ \axiom{retractIfCan(p)} returns \axiom{p} as an element of
      ++ the current domain if all its variables belong to \axiom{V}.

    retract : Polynomial Integer -> $
      ++ \axiom{retract(p)} returns \axiom{p} as an element of the
      ++ current domain if \axiom{retractIfCan(p)} does not return
      ++ "failed", otherwise an error is produced.

    convert : Polynomial Integer -> $
      ++ \axiom{convert(p)} returns the same as \axiom{retract(p)}

    if not (R has QuotientFieldCategory(Integer)) then

        retractIfCan : Polynomial R -> Union($,"failed")
          ++ \axiom{retractIfCan(p)} returns \axiom{p} as an element
          ++ of the current domain if all its variables belong to
          ++ \axiom{V}.

        retract : Polynomial R -> $
          ++ \axiom{retract(p)} returns \axiom{p} as an element of the
          ++ current domain if \axiom{retractIfCan(p)} does not
          ++ return "failed", otherwise an error is produced.

if (R has Algebra Integer) and not(R has Algebra Fraction Integer)
  then

    retractIfCan : Polynomial Integer -> Union($,"failed")
      ++ \axiom{retractIfCan(p)} returns \axiom{p} as an element of
      ++ the current domain if all its variables belong to \axiom{V}.

    retract : Polynomial Integer -> $
      ++ \axiom{retract(p)} returns \axiom{p} as an element of the
      ++ current domain if \axiom{retractIfCan(p)} does not return
      ++ "failed", otherwise an error is produced.

    convert : Polynomial Integer -> $
      ++ \axiom{convert(p)} returns the same as \axiom{retract(p)}.

    if not (R has IntegerNumberSystem) then

        retractIfCan : Polynomial R -> Union($,"failed")
          ++ \axiom{retractIfCan(p)} returns \axiom{p} as an element
          ++ of the current domain if all its variables belong to
```

```
            ++ \axiom{V}.

          retract : Polynomial R -> $
            ++ \axiom{retract(p)} returns \axiom{p} as an element of the
            ++ current domain if \axiom{retractIfCan(p)} does not
            ++ return "failed", otherwise an error is produced.

  if not(R has Algebra Integer) and not(R has Algebra Fraction Integer)
    then

      retractIfCan : Polynomial R -> Union($,"failed")
        ++ \axiom{retractIfCan(p)} returns \axiom{p} as an element of
        ++ the current domain if all its variables belong to \axiom{V}.

      retract : Polynomial R -> $
        ++ \axiom{retract(p)} returns \axiom{p} as an element of the
        ++ current domain if \axiom{retractIfCan(p)} does not return
        ++ "failed", otherwise an error is produced.

  convert : Polynomial R -> $
    ++ \axiom{convert(p)} returns \axiom{p} as an element of the current
    ++ domain if all its variables belong to \axiom{V}, otherwise an
    ++ error is produced.

  if R has RetractableTo(Integer) then

    ConvertibleTo(String)

if R has IntegralDomain then

  primPartElseUnitCanonical : $ -> $
    ++ \axiom{primPartElseUnitCanonical(p)} returns
    ++ \axiom{primitivePart(p)} if \axiom{R} is a gcd-domain,
    ++ otherwise \axiom{unitCanonical(p)}.

  primPartElseUnitCanonical! : $ -> $
    ++ \axiom{primPartElseUnitCanonical!(p)} replaces  \axiom{p}
    ++ by \axiom{primPartElseUnitCanonical(p)}.

  exactQuotient : ($,R) -> $
    ++ \axiom{exactQuotient(p,r)} computes the exact quotient of
    ++ \axiom{p} by \axiom{r}, which is assumed to be a divisor of
    ++ \axiom{p}. No error is returned if this exact quotient fails!

  exactQuotient! : ($,R) -> $
    ++ \axiom{exactQuotient!(p,r)} replaces \axiom{p} by
    ++ \axiom{exactQuotient(p,r)}.

  exactQuotient : ($,$) -> $
    ++ \axiom{exactQuotient(a,b)} computes the exact quotient of
    ++ \axiom{a} by \axiom{b}, which is assumed to be a divisor of
    ++ \axiom{a}. No error is returned if this exact quotient fails!

  exactQuotient! : ($,$) -> $
```

```
  ++ \axiom{exactQuotient!(a,b)} replaces \axiom{a} by
  ++ \axiom{exactQuotient(a,b)}

subResultantGcd : ($, $) -> $
  ++ \axiom{subResultantGcd(a,b)} computes a gcd of \axiom{a} and
  ++ \axiom{b} where \axiom{a} and \axiom{b} are assumed to have the
  ++ same main variable \axiom{v} and are viewed as univariate
  ++ polynomials in \axiom{v} with coefficients in the fraction
  ++ field of the polynomial ring generated by their other variables
  ++ over \axiom{R}.

extendedSubResultantGcd : ($, $) -> _
 Record (gcd : $, coef1 : $, coef2 : $)
  ++ \axiom{extendedSubResultantGcd(a,b)} returns \axiom{[ca,cb,r]}
  ++ such that \axiom{r} is \axiom{subResultantGcd(a,b)} and we have
  ++ \axiom{ca * a + cb * cb = r} .

halfExtendedSubResultantGcd1: ($, $) -> Record (gcd : $, coef1 : $)
  ++ \axiom{halfExtendedSubResultantGcd1(a,b)} returns \axiom{[g,ca]}
  ++ if \axiom{extendedSubResultantGcd(a,b)} returns \axiom{[g,ca,cb]}
  ++ otherwise produces an error.

halfExtendedSubResultantGcd2: ($, $) -> Record (gcd : $, coef2 : $)
  ++ \axiom{halfExtendedSubResultantGcd2(a,b)} returns \axiom{[g,cb]}
  ++ if \axiom{extendedSubResultantGcd(a,b)} returns \axiom{[g,ca,cb]}
  ++ otherwise produces an error.

resultant  : ($, $) -> $
  ++ \axiom{resultant(a,b)} computes the resultant of \axiom{a} and
  ++ \axiom{b} where \axiom{a} and \axiom{b} are assumed to have the
  ++ same main variable \axiom{v} and are viewed as univariate
  ++ polynomials in \axiom{v}.

subResultantChain : ($, $) -> List $
  ++ \axiom{subResultantChain(a,b)}, where \axiom{a} and \axiom{b}
  ++ are not contant polynomials with the same main variable, returns
  ++ the subresultant chain of \axiom{a} and \axiom{b}.

lastSubResultant: ($, $) -> $
  ++ \axiom{lastSubResultant(a,b)} returns the last non-zero
  ++ subresultant of \axiom{a} and \axiom{b} where \axiom{a} and
  ++ \axiom{b} are assumed to have the same main variable \axiom{v}
  ++ and are viewed as univariate polynomials in \axiom{v}.

LazardQuotient: ($, $, NonNegativeInteger) -> $
  ++ \axiom{LazardQuotient(a,b,n)} returns \axiom{a**n exquo b**(n-1)}
  ++ assuming that this quotient does not fail.

LazardQuotient2: ($, $, $, NonNegativeInteger) -> $
  ++ \axiom{LazardQuotient2(p,a,b,n)} returns
  ++ \axiom{(a**(n-1) * p) exquo b**(n-1)}
  ++ assuming that this quotient does not fail.

next_subResultant2: ($, $, $, $) -> $
```

```
      ++ \axiom{nextsubResultant2(p,q,z,s)} is the multivariate version
      ++ of the operation
      ++ next_sousResultant2 from PseudoRemainderSequence from
      ++ the \axiomType{PseudoRemainderSequence} constructor.

  if R has GcdDomain then

    gcd : (R,$) -> R
      ++ \axiom{gcd(r,p)} returns the gcd of \axiom{r} and the content
      ++ of \axiom{p}.

    primitivePart! : $ -> $
      ++ \axiom{primitivePart!(p)} replaces \axiom{p}  by its primitive
      ++ part.

    mainContent : $ -> $
      ++ \axiom{mainContent(p)} returns the content of \axiom{p} viewed
      ++ as a univariate polynomial in its main variable and with
      ++ coefficients in the polynomial ring generated by its other
      ++ variables over \axiom{R}.

    mainPrimitivePart : $ -> $
      ++ \axiom{mainPrimitivePart(p)} returns the primitive part of
      ++ \axiom{p} viewed as a univariate polynomial in its main
      ++ variable and with coefficients in the polynomial ring generated
      ++ by its other variables over \axiom{R}.

    mainSquareFreePart : $ -> $
      ++ \axiom{mainSquareFreePart(p)} returns the square free part of
      ++ \axiom{p} viewed as a univariate polynomial in its main
      ++ variable and with coefficients in the polynomial ring
      ++ generated by its other variables over \axiom{R}.

 add

   O ==> OutputForm
   NNI ==> NonNegativeInteger
   INT ==> Integer

   exactQuo : (R,R) -> R

   coerce(p:$):O ==
     ground? (p) => (ground(p))::O
     if (((ip := init(p))) = 1)
       then
         if zero?((tp := tail(p)))
           then
             if (((dp := mdeg(p))) = 1)
               then
                 return((mvar(p))::O)
               else
                 return(((mvar(p))::O **$O (dp::O)))
           else
             if (((dp := mdeg(p))) = 1)
```

```
            then
              return((mvar(p))::O +$O (tp::O))
            else
              return(((mvar(p))::O **$O (dp::O)) +$O (tp::O))
     else
      if zero?((tp := tail(p)))
        then
          if (((dp := mdeg(p))) = 1)
            then
              return((ip::O) *$O  (mvar(p))::O)
            else
              return((ip::O) *$O ((mvar(p))::O **$O (dp::O)))
        else
          if ((mdeg(p)) = 1)
            then
              return(((ip::O) *$O  (mvar(p))::O) +$O (tp::O))
  ((ip)::O *$O ((mvar(p))::O **$O ((mdeg(p)::O))) +$O (tail(p)::O))

mvar p ==
  ground?(p) => error"Error in mvar from RPOLCAT : #1 is constant."
  mainVariable(p)::V

mdeg p ==
  ground?(p) => 0$NNI
  degree(p,mainVariable(p)::V)

init p ==
  ground?(p) => error"Error in mvar from RPOLCAT : #1 is constant."
  v := mainVariable(p)::V
  coefficient(p,v,degree(p,v))

leadingCoefficient (p,v) ==
  zero? (d := degree(p,v)) => p
  coefficient(p,v,d)

head p ==
  ground? p => p
  v := mainVariable(p)::V
  d := degree(p,v)
  monomial(coefficient(p,v,d),v,d)

reductum(p,v) ==
  zero? (d := degree(p,v)) => 0$$
  p - monomial(coefficient(p,v,d),v,d)

tail p ==
  ground? p => 0$$
  p - head(p)

deepestTail p ==
  ground? p => 0$$
  ground? tail(p) => tail(p)
  mvar(p) > mvar(tail(p)) => tail(p)
  deepestTail(tail(p))
```

```
iteratedInitials p ==
  ground? p => []
  p := init(p)
  cons(p,iteratedInitials(p))

localDeepestInitial (p : $) : $ ==
  ground? p => p
  localDeepestInitial init p

deepestInitial p ==
  ground? p => _
    error"Error in deepestInitial from RPOLCAT : #1 is constant."
  localDeepestInitial init p

monic? p ==
  ground? p => false
  (recip(init(p))$$ case $)@Boolean

quasiMonic?  p ==
  ground? p => false
  ground?(init(p))

mainMonomial p ==
  zero? p => error"Error in mainMonomial from RPOLCAT : #1 is zero"
  ground? p => 1$$
  v := mainVariable(p)::V
  monomial(1$$,v,degree(p,v))

leastMonomial p ==
  zero? p => error"Error in leastMonomial from RPOLCAT : #1 is zero"
  ground? p => 1$$
  v := mainVariable(p)::V
  monomial(1$$,v,minimumDegree(p,v))

mainCoefficients p ==
  zero? p => error"Error in mainCoefficients from RPOLCAT : #1 is zero"
  ground? p => [p]
  v := mainVariable(p)::V
  coefficients(univariate(p,v)@SparseUnivariatePolynomial($))

mainMonomials p ==
  zero? p => error"Error in mainMonomials from RPOLCAT : #1 is zero"
  ground? p => [1$$]
  v := mainVariable(p)::V
  lm := monomials(univariate(p,v)@SparseUnivariatePolynomial($))
  [monomial(1$$,v,degree(m)) for m in lm]

RittWuCompare (a,b) ==
  (ground? b and  ground? a) => "failed"::Union(Boolean,"failed")
  ground? b => false::Union(Boolean,"failed")
  ground? a => true::Union(Boolean,"failed")
  mvar(a) < mvar(b) => true::Union(Boolean,"failed")
  mvar(a) > mvar(b) => false::Union(Boolean,"failed")
```

```
  mdeg(a) < mdeg(b) => true::Union(Boolean,"failed")
  mdeg(a) > mdeg(b) => false::Union(Boolean,"failed")
  lc := RittWuCompare(init(a),init(b))
  lc case Boolean => lc
  RittWuCompare(tail(a),tail(b))

infRittWu? (a,b) ==
  lc : Union(Boolean,"failed") := RittWuCompare(a,b)
  lc case Boolean => lc::Boolean
  false

supRittWu? (a,b) ==
  infRittWu? (b,a)

prem (a:$, b:$)  : $ ==
  cP := lazyPremWithDefault (a,b)
  ((cP.coef) ** (cP.gap)) * cP.remainder

pquo (a:$, b:$)  : $ ==
  cPS := lazyPseudoDivide (a,b)
  c := (cPS.coef) ** (cPS.gap)
  c * cPS.quotient

prem (a:$, b:$, v:V) : $ ==
  cP := lazyPremWithDefault (a,b,v)
  ((cP.coef) ** (cP.gap)) * cP.remainder

pquo (a:$, b:$, v:V)  : $ ==
  cPS := lazyPseudoDivide (a,b,v)
  c := (cPS.coef) ** (cPS.gap)
  c * cPS.quotient

lazyPrem (a:$, b:$) : $ ==
  (not ground?(b)) and (monic?(b)) => monicModulo(a,b)
  (lazyPremWithDefault (a,b)).remainder

lazyPquo (a:$, b:$) : $ ==
  (lazyPseudoDivide (a,b)).quotient

lazyPrem (a:$, b:$, v:V) : $ ==
  zero? b => _
    error"Error in lazyPrem : ($,$,V) -> $ from RPOLCAT : #2 is zero"
  ground?(b) => 0$$
  (v = mvar(b)) => lazyPrem(a,b)
  dbv : NNI := degree(b,v)
  zero? dbv => 0$$
  dav : NNI  := degree(a,v)
  zero? dav => a
  test : INT := dav::INT - dbv
  lcbv : $ := leadingCoefficient(b,v)
  while not zero?(a) and not negative?(test) repeat
    lcav := leadingCoefficient(a,v)
    term := monomial(lcav,v,test::NNI)
    a := lcbv * a - term * b
```

```
        test := degree(a,v)::INT - dbv
      a

    lazyPquo (a:$, b:$, v:V) : $ ==
      (lazyPseudoDivide (a,b,v)).quotient

    headReduce (a:$,b:$) ==
      ground? b => error _
       "Error in headReduce : ($,$) -> Boolean from TSETCAT : #2 is constant"
      ground? a => a
      mvar(a) = mvar(b) => lazyPrem(a,b)
      while not reduced?((ha := head a),b) repeat
        lrc := lazyResidueClass(ha,b)
        if zero? tail(a)
          then
            a := lrc.polnum
          else
            a := lrc.polnum +  (lrc.polden)**(lrc.power) * tail(a)
      a

    initiallyReduce(a:$,b:$) ==
      ground? b => error _
  "Error in initiallyReduce : ($,$) -> Boolean from TSETCAT : #2 is constant"
      ground? a => a
      v := mvar(b)
      mvar(a) = v => lazyPrem(a,b)
      ia := a
      ma := 1$$
      ta := 0$$
      while (not ground?(ia)) and (mvar(ia) >= mvar(b)) repeat
        if (mvar(ia) = mvar(b)) and (mdeg(ia) >= mdeg(b))
          then
            iamodb := lazyResidueClass(ia,b)
            ia := iamodb.polnum
            if not zero? ta
              then
                ta :=  (iamodb.polden)**(iamodb.power) * ta
        if zero? ia
          then
            ia := ta
            ma := 1$$
            ta := 0$$
          else
            if not ground?(ia)
              then
                ta := tail(ia) * ma + ta
                ma := mainMonomial(ia) * ma
                ia := init(ia)
      ia * ma + ta

    lazyPremWithDefault (a,b) ==
      ground?(b) => error _
        "Error in lazyPremWithDefault from RPOLCAT : #2 is constant"
      ground?(a) => [1$$,0$NNI,a]
```

```
  xa := mvar a
  xb := mvar b
  xa < xb => [1$$,0$NNI,a]
  lcb : $ := init b
  db : NNI := mdeg b
  test : INT := degree(a,xb)::INT - db
  delta : INT := max(test + 1$INT, 0$INT)
  if xa = xb
    then
      b := tail b
      while not zero?(a) and not negative?(test) repeat
        term := monomial(init(a),xb,test::NNI)
        a := lcb * tail(a) - term * b
        delta := delta - 1$INT
        test := degree(a,xb)::INT - db
    else
      while not zero?(a) and not negative?(test) repeat
        term := monomial(leadingCoefficient(a,xb),xb,test::NNI)
        a := lcb * a - term * b
        delta := delta - 1$INT
        test := degree(a,xb)::INT - db
  [lcb, (delta::NNI), a]

lazyPremWithDefault (a,b,v) ==
  zero? b =>  error _
   "Error in lazyPremWithDefault : ($,$,V) -> $ from RPOLCAT : #2 is zero"
  ground?(b) => [b,1$NNI,0$$]
  (v = mvar(b)) => lazyPremWithDefault(a,b)
  dbv : NNI := degree(b,v)
  zero? dbv => [b,1$NNI,0$$]
  dav : NNI  := degree(a,v)
  zero? dav => [1$$,0$NNI,a]
  test : INT := dav::INT - dbv
  delta : INT := max(test + 1$INT, 0$INT)
  lcbv : $ := leadingCoefficient(b,v)
  while not zero?(a) and not negative?(test) repeat
    lcav := leadingCoefficient(a,v)
    term := monomial(lcav,v,test::NNI)
    a := lcbv * a - term * b
    delta := delta - 1$INT
    test := degree(a,v)::INT - dbv
  [lcbv, (delta::NNI), a]

pseudoDivide (a,b) ==
  cPS := lazyPseudoDivide (a,b)
  c := (cPS.coef) ** (cPS.gap)
  [c * cPS.quotient, c * cPS.remainder]

lazyPseudoDivide (a,b) ==
  ground?(b) => error _
     "Error in lazyPseudoDivide from RPOLCAT : #2 is constant"
  ground?(a) => [1$$,0$NNI,0$$,a]
  xa := mvar a
  xb := mvar b
```

```
  xa < xb => [1$$,0$NNI,0$$, a]
  lcb : $ := init b
  db : NNI := mdeg b
  q := 0$$
  test : INT := degree(a,xb)::INT - db
  delta : INT := max(test + 1$INT, 0$INT)
  if xa = xb
    then
      b := tail b
      while not zero?(a) and not negative?(test) repeat
        term := monomial(init(a),xb,test::NNI)
        a := lcb * tail(a) - term * b
        q := lcb * q + term
        delta := delta - 1$INT
        test := degree(a,xb)::INT - db
    else
      while not zero?(a) and not negative?(test) repeat
        term := monomial(leadingCoefficient(a,xb),xb,test::NNI)
        a := lcb * a - term * b
        q := lcb * q + term
        delta := delta - 1$INT
        test := degree(a,xb)::INT - db
  [lcb, (delta::NNI), q, a]

lazyPseudoDivide (a,b,v) ==
  zero? b =>  error _
    "Error in lazyPseudoDivide : ($,$,V) -> $ from RPOLCAT : #2 is zero"
  ground?(b) => [b,1$NNI,a,0$$]
  (v = mvar(b)) => lazyPseudoDivide(a,b)
  dbv : NNI := degree(b,v)
  zero? dbv => [b,1$NNI,a,0$$]
  dav : NNI  := degree(a,v)
  zero? dav => [1$$,0$NNI,0$$, a]
  test : INT := dav::INT - dbv
  delta : INT := max(test + 1$INT, 0$INT)
  lcbv : $ := leadingCoefficient(b,v)
  q := 0$$
  while not zero?(a) and not negative?(test) repeat
    lcav := leadingCoefficient(a,v)
    term := monomial(lcav,v,test::NNI)
    a := lcbv * a - term * b
    q := lcbv * q + term
    delta := delta - 1$INT
    test := degree(a,v)::INT - dbv
  [lcbv, (delta::NNI), q, a]

monicModulo (a,b) ==
  ground?(b) => error"Error in monicModulo from RPOLCAT : #2 is constant"
  rec : Union($,"failed")
  rec := recip((ib := init(b)))$$
  (rec case "failed")@Boolean => error _
    "Error in monicModulo from RPOLCAT : #2 is not monic"
  ground? a => a
  ib * ((lazyPremWithDefault ((rec::$) * a,(rec::$) * b)).remainder)
```

```
  lazyResidueClass(a,b) ==
    zero? b => [a,1$$,0$NNI]
    ground? b => [0$$,1$$,0$NNI]
    ground? a => [a,1$$,0$NNI]
    xa := mvar a
    xb := mvar b
    xa < xb => [a,1$$,0$NNI]
    monic?(b) => [monicModulo(a,b),1$$,0$NNI]
    lcb : $ := init b
    db : NNI := mdeg b
    test : INT := degree(a,xb)::INT - db
    pow : NNI := 0
    if xa = xb
      then
        b := tail b
        while not zero?(a) and not negative?(test) repeat
          term := monomial(init(a),xb,test::NNI)
          a := lcb * tail(a) - term * b
          pow := pow + 1$NNI
          test := degree(a,xb)::INT - db
      else
        while not zero?(a) and not negative?(test) repeat
          term := monomial(leadingCoefficient(a,xb),xb,test::NNI)
          a := lcb * a - term * b
          pow := pow + 1$NNI
          test := degree(a,xb)::INT - db
    [a,lcb,pow]

  reduced? (a:$,b:$) : Boolean ==
    degree(a,mvar(b)) < mdeg(b)

  reduced? (p:$, lq : List($)) : Boolean ==
    ground? p => true
    while (not empty? lq) and (reduced?(p, first lq)) repeat
      lq := rest lq
    empty? lq

  headReduced? (a:$,b:$) : Boolean ==
    reduced?(head(a),b)

  headReduced? (p:$, lq : List($)) : Boolean ==
    reduced?(head(p),lq)

  initiallyReduced? (a:$,b:$) : Boolean ==
    ground? b => error _
"Error in initiallyReduced? : ($,$) -> Bool. from RPOLCAT : #2 is constant"
    ground?(a) => true
    mvar(a) < mvar(b) => true
    (mvar(a) = mvar(b)) => reduced?(a,b)
    initiallyReduced?(init(a),b)

  initiallyReduced? (p:$, lq : List($)) : Boolean ==
    ground? p => true
```

```
    while (not empty? lq) and (initiallyReduced?(p, first lq)) repeat
      lq := rest lq
    empty? lq

  normalized?(a:$,b:$) : Boolean ==
    ground? b => error _
   "Error in  normalized? : ($,$) -> Boolean from TSETCAT : #2 is constant"
    ground? a => true
    mvar(a) < mvar(b) => true
    (mvar(a) = mvar(b)) => false
    normalized?(init(a),b)

  normalized? (p:$, lq : List($)) : Boolean ==
    while (not empty? lq) and (normalized?(p, first lq)) repeat
      lq := rest lq
    empty? lq

  if R has IntegralDomain
  then

    if R has EuclideanDomain
      then
        exactQuo(r:R,s:R):R ==
          r quo$R s
      else
        exactQuo(r:R,s:R):R ==
          (r exquo$R s)::R

    exactQuotient (p:$,r:R) ==
      (p exquo$$ r)::$

    exactQuotient (a:$,b:$) ==
      ground? b => exactQuotient(a,ground(b))
      (a exquo$$ b)::$

    exactQuotient! (a:$,b:$) ==
      ground? b => exactQuotient!(a,ground(b))
      a := (a exquo$$ b)::$

    if (R has GcdDomain) and not(R has Field)
    then

      primPartElseUnitCanonical p ==
        primitivePart p

      primitivePart! p ==
        zero? p => p
        if ((cp := content(p)) = 1)
          then
            p := unitCanonical p
          else
            p := unitCanonical exactQuotient!(p,cp)
        p
```

```
    primPartElseUnitCanonical! p ==
      primitivePart! p

  else
    primPartElseUnitCanonical p ==
      unitCanonical p

    primPartElseUnitCanonical! p ==
      p := unitCanonical p


if R has GcdDomain
then

  gcd(r:R,p:$):R ==
    (r = 1) => r
    zero? p => r
    ground? p => gcd(r,ground(p))$R
    gcd(gcd(r,init(p)),tail(p))

  mainContent p ==
    zero? p => p
    "gcd"/mainCoefficients(p)

  mainPrimitivePart p ==
    zero? p => p
    (unitNormal((p exquo$$ mainContent(p))::$)).canonical

  mainSquareFreePart p ==
    ground? p => p
    v := mainVariable(p)::V
    sfp : SparseUnivariatePolynomial($)
    sfp := squareFreePart(univariate(p,v)@SparseUnivariatePolynomial($))
    multivariate(sfp,v)

if (V has ConvertibleTo(Symbol))
  then

    PR ==> Polynomial R
    PQ ==> Polynomial Fraction Integer
    PZ ==> Polynomial Integer
    IES ==> IndexedExponents(Symbol)
    Q ==> Fraction Integer
    Z ==> Integer

    convert(p:$) : PR ==
      ground? p => (ground(p)$$)::PR
      v : V := mvar(p)
      d : NNI := mdeg(p)
      convert(init(p))@PR *$PR _
                   ((convert(v)@Symbol)::PR)**d +$PR convert(tail(p))@PR

    coerce(p:$) : PR ==
      convert(p)@PR
```

```
localRetract : PR -> $
localRetractPQ : PQ -> $
localRetractPZ : PZ -> $
localRetractIfCan : PR -> Union($,"failed")
localRetractIfCanPQ : PQ -> Union($,"failed")
localRetractIfCanPZ : PZ -> Union($,"failed")

if V has Finite
  then

    sizeV : NNI := size()$V
    lv : List Symbol
    lv := _
      [convert(index(i::PositiveInteger)$V)@Symbol for i in 1..sizeV]

    localRetract(p : PR) : $ ==
      ground? p => (ground(p)$PR)::$
      mvp : Symbol := (mainVariable(p)$PR)::Symbol
      d : NNI
      imvp : PositiveInteger := _
                   (position(mvp,lv)$(List Symbol))::PositiveInteger
      vimvp : V := index(imvp)$V
      xvimvp,c : $
      newp := 0$$
      while (not zero? (d := degree(p,mvp))) repeat
        c := localRetract(coefficient(p,mvp,d)$PR)
        xvimvp := monomial(c,vimvp,d)$$
        newp := newp +$$ xvimvp
        p := p -$PR monomial(coefficient(p,mvp,d)$PR,mvp,d)$PR
      newp +$$ localRetract(p)

    if R has Algebra Fraction Integer
      then
        localRetractPQ(pq:PQ):$ ==
          ground? pq => ((ground(pq)$PQ)::R)::$
          mvp : Symbol := (mainVariable(pq)$PQ)::Symbol
          d : NNI
          imvp : PositiveInteger := _
                   (position(mvp,lv)$(List Symbol))::PositiveInteger
          vimvp : V := index(imvp)$V
          xvimvp,c : $
          newp := 0$$
          while (not zero? (d := degree(pq,mvp))) repeat
            c := localRetractPQ(coefficient(pq,mvp,d)$PQ)
            xvimvp := monomial(c,vimvp,d)$$
            newp := newp +$$ xvimvp
            pq := pq -$PQ monomial(coefficient(pq,mvp,d)$PQ,mvp,d)$PQ
          newp +$$ localRetractPQ(pq)

    if R has Algebra Integer
      then
        localRetractPZ(pz:PZ):$ ==
          ground? pz => ((ground(pz)$PZ)::R)::$
```

```
          mvp : Symbol := (mainVariable(pz)$PZ)::Symbol
          d : NNI
          imvp : PositiveInteger := _
                    (position(mvp,lv)$(List Symbol))::PositiveInteger
          vimvp : V := index(imvp)$V
          xvimvp,c : $
          newp := 0$$
          while (not zero? (d := degree(pz,mvp))) repeat
            c := localRetractPZ(coefficient(pz,mvp,d)$PZ)
            xvimvp := monomial(c,vimvp,d)$$
            newp := newp +$$ xvimvp
            pz := pz -$PZ monomial(coefficient(pz,mvp,d)$PZ,mvp,d)$PZ
          newp +$$ localRetractPZ(pz)

    retractable?(p:PR):Boolean ==
      lvp := variables(p)$PR
      while not empty? lvp and member?(first lvp,lv) repeat
        lvp := rest lvp
      empty? lvp

    retractablePQ?(p:PQ):Boolean ==
      lvp := variables(p)$PQ
      while not empty? lvp and member?(first lvp,lv) repeat
        lvp := rest lvp
      empty? lvp

    retractablePZ?(p:PZ):Boolean ==
      lvp := variables(p)$PZ
      while not empty? lvp and member?(first lvp,lv) repeat
        lvp := rest lvp
      empty? lvp

    localRetractIfCan(p : PR): Union($,"failed") ==
      not retractable?(p) => "failed"::Union($,"failed")
      localRetract(p)::Union($,"failed")

    localRetractIfCanPQ(p : PQ): Union($,"failed") ==
      not retractablePQ?(p) => "failed"::Union($,"failed")
      localRetractPQ(p)::Union($,"failed")

    localRetractIfCanPZ(p : PZ): Union($,"failed") ==
      not retractablePZ?(p) => "failed"::Union($,"failed")
      localRetractPZ(p)::Union($,"failed")

if R has Algebra Fraction Integer
  then

    mpc2Z := MPolyCatFunctions2(Symbol,IES,IES,Z,R,PZ,PR)
    mpc2Q := MPolyCatFunctions2(Symbol,IES,IES,Q,R,PQ,PR)
    ZToR (z:Z):R == coerce(z)@R
    QToR (q:Q):R == coerce(q)@R
    PZToPR (pz:PZ):PR == map(ZToR,pz)$mpc2Z
    PQToPR (pq:PQ):PR == map(QToR,pq)$mpc2Q
```

```
retract(pz:PZ) ==
  rif : Union($,"failed") := retractIfCan(pz)@Union($,"failed")
  (rif case "failed") => error _
                     "failed in retract: POLY Z -> $ from RPOLCAT"
  rif::$

convert(pz:PZ) ==
  retract(pz)@$

retract(pq:PQ) ==
  rif : Union($,"failed") := retractIfCan(pq)@Union($,"failed")
  (rif case "failed") => error _
                     "failed in retract: POLY Z -> $ from RPOLCAT"
  rif::$

convert(pq:PQ) ==
  retract(pq)@$

if not (R has QuotientFieldCategory(Integer))
  then
    -- the only operation to implement is
    -- retractIfCan : PR -> Union($,"failed")
    -- when V does not have Finite

    if V has Finite
      then
        retractIfCan(pr:PR) ==
          localRetractIfCan(pr)@Union($,"failed")

        retractIfCan(pq:PQ) ==
          localRetractIfCanPQ(pq)@Union($,"failed")
      else
        retractIfCan(pq:PQ) ==
          pr : PR := PQToPR(pq)
          retractIfCan(pr)@Union($,"failed")

    retractIfCan(pz:PZ) ==
      pr : PR := PZToPR(pz)
      retractIfCan(pr)@Union($,"failed")

    retract(pr:PR) ==
      rif : Union($,"failed") := _
                              retractIfCan(pr)@Union($,"failed")
      (rif case "failed") => error _
                   "failed in retract: POLY Z -> $ from RPOLCAT"
      rif::$

    convert(pr:PR) ==
      retract(pr)@$

  else
    -- the only operation to implement is
    -- retractIfCan : PQ -> Union($,"failed")
    -- when V does not have Finite
```

```
        mpc2ZQ := MPolyCatFunctions2(Symbol,IES,IES,Z,Q,PZ,PQ)
        mpc2RQ := MPolyCatFunctions2(Symbol,IES,IES,R,Q,PR,PQ)
        ZToQ(z:Z):Q == coerce(z)@Q
        RToQ(r:R):Q == retract(r)@Q

        PZToPQ (pz:PZ):PQ == map(ZToQ,pz)$mpc2ZQ
        PRToPQ (pr:PR):PQ == map(RToQ,pr)$mpc2RQ

        retractIfCan(pz:PZ) ==
          pq : PQ := PZToPQ(pz)
          retractIfCan(pq)@Union($,"failed")

        if V has Finite
          then
            retractIfCan(pq:PQ) ==
              localRetractIfCanPQ(pq)@Union($,"failed")

            convert(pr:PR) ==
              lrif : Union($,"failed") := _
                              localRetractIfCan(pr)@Union($,"failed")
              (lrif case "failed") => error _
                              "failed in convert: PR->$ from RPOLCAT"
              lrif::$
          else
            convert(pr:PR) ==
              pq : PQ := PRToPQ(pr)
              retract(pq)@$

if (R has Algebra Integer) and not(R has Algebra Fraction Integer)
  then

    mpc2Z := MPolyCatFunctions2(Symbol,IES,IES,Z,R,PZ,PR)
    ZToR (z:Z):R == coerce(z)@R
    PZToPR (pz:PZ):PR == map(ZToR,pz)$mpc2Z

    retract(pz:PZ) ==
      rif : Union($,"failed") := retractIfCan(pz)@Union($,"failed")
      (rif case "failed") => error _
                        "failed in retract: POLY Z -> $ from RPOLCAT"
      rif::$

    convert(pz:PZ) ==
      retract(pz)@$

    if not (R has IntegerNumberSystem)
      then
        -- the only operation to implement is
        -- retractIfCan : PR -> Union($,"failed")
        -- when V does not have Finite

        if V has Finite
          then
            retractIfCan(pr:PR) ==
              localRetractIfCan(pr)@Union($,"failed")
```

```
            retractIfCan(pz:PZ) ==
              localRetractIfCanPZ(pz)@Union($,"failed")
          else
            retractIfCan(pz:PZ) ==
              pr : PR := PZToPR(pz)
              retractIfCan(pr)@Union($,"failed")

        retract(pr:PR) ==
          rif : Union($,"failed"):=retractIfCan(pr)@Union($,"failed")
          (rif case "failed") => error _
                         "failed in retract: POLY Z -> $ from RPOLCAT"
          rif::$

        convert(pr:PR) ==
          retract(pr)@$

      else
        -- the only operation to implement is
        -- retractIfCan : PZ -> Union($,"failed")
        -- when V does not have Finite

        mpc2RZ := MPolyCatFunctions2(Symbol,IES,IES,R,Z,PR,PZ)
        RToZ(r:R):Z == retract(r)@Z
        PRToPZ (pr:PR):PZ == map(RToZ,pr)$mpc2RZ

        if V has Finite
          then
            convert(pr:PR) ==
              lrif : Union($,"failed") := _
                                localRetractIfCan(pr)@Union($,"failed")
              (lrif case "failed") => error _
                                "failed in convert: PR->$ from RPOLCAT"
              lrif::$
            retractIfCan(pz:PZ) ==
              localRetractIfCanPZ(pz)@Union($,"failed")
          else
            convert(pr:PR) ==
              pz : PZ := PRToPZ(pr)
              retract(pz)@$


if not(R has Algebra Integer) and not(R has Algebra Fraction Integer)
  then
    -- the only operation to implement is
    -- retractIfCan : PR -> Union($,"failed")

    if V has Finite
      then
        retractIfCan(pr:PR) ==
          localRetractIfCan(pr)@Union($,"failed")

    retract(pr:PR) ==
      rif : Union($,"failed") := retractIfCan(pr)@Union($,"failed")
```

```
      (rif case "failed") => error _
                      "failed in retract: POLY Z -> $ from RPOLCAT"
      rif::$

    convert(pr:PR) ==
      retract(pr)@$

if (R has RetractableTo(INT))
  then

    convert(pol:$):String ==
      ground?(pol) => convert(retract(ground(pol))@INT)@String
      ipol : $ := init(pol)
      vpol : V := mvar(pol)
      dpol : NNI := mdeg(pol)
      tpol: $  := tail(pol)
      sipol,svpol,sdpol,stpol : String
      if (ipol = 1)
        then
          sipol := empty()$String
        else
          if ((-ipol) = 1)
            then
              sipol := "-"
            else
              sipol := convert(ipol)@String
              if not monomial?(ipol)
                then
                  sipol := concat(["(",sipol,")*"])$String
                else
                  sipol := concat(sipol,"*")$String
      svpol := string(convert(vpol)@Symbol)
      if (dpol = 1)
        then
          sdpol :=  empty()$String
        else
          sdpol := _
               concat("**",convert(convert(dpol)@INT)@String )$String
      if zero? tpol
        then
          stpol :=  empty()$String
        else
          if ground?(tpol)
            then
              n := retract(ground(tpol))@INT
              if n > 0
                then
                  stpol :=  concat(" +",convert(n)@String)$String
                else
                  stpol := convert(n)@String
            else
              stpol := convert(tpol)@String
              if _
                not member?((stpol.1)::String,["+","-"])$(List String)
```

```
                    then
                      stpol :=  concat(" + ",stpol)$String
          concat([sipol,svpol,sdpol,stpol])$String
```

———-

— COQ RPOLCAT —

```
(* category RPOLCAT *)
(*
     O ==> OutputForm
     NNI ==> NonNegativeInteger
     INT ==> Integer

     coerce : % -> OutputForm
     coerce(p:$):O ==
       ground? (p) => (ground(p))::O
       if (((ip := init(p))) = 1)
         then
           if zero?((tp := tail(p)))
             then
               if (((dp := mdeg(p))) = 1)
                 then
                   return((mvar(p))::O)
                 else
                   return(((mvar(p))::O **$O (dp::O)))
             else
               if (((dp := mdeg(p))) = 1)
                 then
                   return((mvar(p))::O +$O (tp::O))
                 else
                   return(((mvar(p))::O **$O (dp::O)) +$O (tp::O))
          else
           if zero?((tp := tail(p)))
             then
               if (((dp := mdeg(p))) = 1)
                 then
                   return((ip::O) *$O  (mvar(p))::O)
                 else
                   return((ip::O) *$O ((mvar(p))::O **$O (dp::O)))
             else
               if ((mdeg(p)) = 1)
                 then
                   return(((ip::O) *$O  (mvar(p))::O) +$O (tp::O))
       ((ip)::O *$O ((mvar(p))::O **$O ((mdeg(p)::O))) +$O (tail(p)::O))

     mvar : % -> V
     mvar p ==
       ground?(p) => error"Error in mvar from RPOLCAT : #1 is constant."
       mainVariable(p)::V

     mdeg : % -> NonNegativeInteger
     mdeg p ==
```

```
  ground?(p) => 0$NNI
  degree(p,mainVariable(p)::V)

init : % -> %
init p ==
  ground?(p) => error"Error in mvar from RPOLCAT : #1 is constant."
  v := mainVariable(p)::V
  coefficient(p,v,degree(p,v))

leadingCoefficient : (%,V) -> %
leadingCoefficient (p,v) ==
  zero? (d := degree(p,v)) => p
  coefficient(p,v,d)

head : % -> %
head p ==
  ground? p => p
  v := mainVariable(p)::V
  d := degree(p,v)
  monomial(coefficient(p,v,d),v,d)

reductum : (%,V) -> %
reductum(p,v) ==
  zero? (d := degree(p,v)) => 0$$
  p - monomial(coefficient(p,v,d),v,d)

tail : % -> %
tail p ==
  ground? p => 0$$
  p - head(p)

deepestTail : % -> %
deepestTail p ==
  ground? p => 0$$
  ground? tail(p) => tail(p)
  mvar(p) > mvar(tail(p)) => tail(p)
  deepestTail(tail(p))

iteratedInitials : % -> List(%)
iteratedInitials p ==
  ground? p => []
  p := init(p)
  cons(p,iteratedInitials(p))

localDeepestInitial : $ -> $
localDeepestInitial (p : $) : $ ==
  ground? p => p
  localDeepestInitial init p

deepestInitial : % -> %
deepestInitial p ==
  ground? p => _
    error"Error in deepestInitial from RPOLCAT : #1 is constant."
  localDeepestInitial init p
```

```
monic? : % -> Boolean
monic? p ==
  ground? p => false
  (recip(init(p))$$ case $)@Boolean

quasiMonic? : % -> Boolean
quasiMonic?  p ==
  ground? p => false
  ground?(init(p))

mainMonomial : % -> %
mainMonomial p ==
  zero? p => error"Error in mainMonomial from RPOLCAT : #1 is zero"
  ground? p => 1$$
  v := mainVariable(p)::V
  monomial(1$$,v,degree(p,v))

leastMonomial : % -> %
leastMonomial p ==
  zero? p => error"Error in leastMonomial from RPOLCAT : #1 is zero"
  ground? p => 1$$
  v := mainVariable(p)::V
  monomial(1$$,v,minimumDegree(p,v))

mainCoefficients : % -> List(%)
mainCoefficients p ==
  zero? p => error"Error in mainCoefficients from RPOLCAT : #1 is zero"
  ground? p => [p]
  v := mainVariable(p)::V
  coefficients(univariate(p,v)@SparseUnivariatePolynomial($))

mainMonomials : % -> List(%)
mainMonomials p ==
  zero? p => error"Error in mainMonomials from RPOLCAT : #1 is zero"
  ground? p => [1$$]
  v := mainVariable(p)::V
  lm := monomials(univariate(p,v)@SparseUnivariatePolynomial($))
  [monomial(1$$,v,degree(m)) for m in lm]

RittWuCompare : (%,%) -> Union(Boolean,"failed")
RittWuCompare (a,b) ==
  (ground? b and  ground? a) => "failed"::Union(Boolean,"failed")
  ground? b => false::Union(Boolean,"failed")
  ground? a => true::Union(Boolean,"failed")
  mvar(a) < mvar(b) => true::Union(Boolean,"failed")
  mvar(a) > mvar(b) => false::Union(Boolean,"failed")
  mdeg(a) < mdeg(b) => true::Union(Boolean,"failed")
  mdeg(a) > mdeg(b) => false::Union(Boolean,"failed")
  lc := RittWuCompare(init(a),init(b))
  lc case Boolean => lc
  RittWuCompare(tail(a),tail(b))

infRittWu? : (%,%) -> Boolean
```

```
infRittWu? (a,b) ==
  lc : Union(Boolean,"failed") := RittWuCompare(a,b)
  lc case Boolean => lc::Boolean
  false

supRittWu? : (%,%) -> Boolean
supRittWu? (a,b) ==
  infRittWu? (b,a)

prem : (%,%) -> %
prem (a:$, b:$)  : $ ==
  cP := lazyPremWithDefault (a,b)
  ((cP.coef) ** (cP.gap)) * cP.remainder

pquo : (%,%) -> %
pquo (a:$, b:$)  : $ ==
  cPS := lazyPseudoDivide (a,b)
  c := (cPS.coef) ** (cPS.gap)
  c * cPS.quotient

prem : (%,%,V) -> %
prem (a:$, b:$, v:V) : $ ==
  cP := lazyPremWithDefault (a,b,v)
  ((cP.coef) ** (cP.gap)) * cP.remainder

pquo : (%,%,V) -> %
pquo (a:$, b:$, v:V)  : $ ==
  cPS := lazyPseudoDivide (a,b,v)
  c := (cPS.coef) ** (cPS.gap)
  c * cPS.quotient

lazyPrem : (%,%) -> %
lazyPrem (a:$, b:$) : $ ==
  (not ground?(b)) and (monic?(b)) => monicModulo(a,b)
  (lazyPremWithDefault (a,b)).remainder

lazyPquo : (%,%) -> %
lazyPquo (a:$, b:$) : $ ==
  (lazyPseudoDivide (a,b)).quotient

lazyPrem : (%,%,V) -> %
lazyPrem (a:$, b:$, v:V) : $ ==
  zero? b => _
    error"Error in lazyPrem : ($,$,V) -> $ from RPOLCAT : #2 is zero"
  ground?(b) => 0$$
  (v = mvar(b)) => lazyPrem(a,b)
  dbv : NNI := degree(b,v)
  zero? dbv => 0$$
  dav : NNI  := degree(a,v)
  zero? dav => a
  test : INT := dav::INT - dbv
  lcbv : $ := leadingCoefficient(b,v)
  while not zero?(a) and not negative?(test) repeat
    lcav := leadingCoefficient(a,v)
```

```
        term := monomial(lcav,v,test::NNI)
        a := lcbv * a - term * b
        test := degree(a,v)::INT - dbv
      a

    lazyPquo : (%,%,V) -> %
    lazyPquo (a:$, b:$, v:V) : $ ==
      (lazyPseudoDivide (a,b,v)).quotient

    headReduce : (%,%) -> %
    headReduce (a:$,b:$) ==
      ground? b => error _
       "Error in headReduce : ($,$) -> Boolean from TSETCAT : #2 is constant"
      ground? a => a
      mvar(a) = mvar(b) => lazyPrem(a,b)
      while not reduced?((ha := head a),b) repeat
        lrc := lazyResidueClass(ha,b)
        if zero? tail(a)
          then
            a := lrc.polnum
          else
            a := lrc.polnum +  (lrc.polden)**(lrc.power) * tail(a)
      a

    initiallyReduce : (%,%) -> %
    initiallyReduce(a:$,b:$) ==
      ground? b => error _
  "Error in initiallyReduce : ($,$) -> Boolean from TSETCAT : #2 is constant"
      ground? a => a
      v := mvar(b)
      mvar(a) = v => lazyPrem(a,b)
      ia := a
      ma := 1$$
      ta := 0$$
      while (not ground?(ia)) and (mvar(ia) >= mvar(b)) repeat
        if (mvar(ia) = mvar(b)) and (mdeg(ia) >= mdeg(b))
          then
            iamodb := lazyResidueClass(ia,b)
            ia := iamodb.polnum
            if not zero? ta
              then
                ta :=  (iamodb.polden)**(iamodb.power) * ta
        if zero? ia
          then
            ia := ta
            ma := 1$$
            ta := 0$$
          else
            if not ground?(ia)
              then
                ta := tail(ia) * ma + ta
                ma := mainMonomial(ia) * ma
                ia := init(ia)
      ia * ma + ta
```

```
 lazyPremWithDefault : (%,%) ->
    Record(coef: %,gap: NonNegativeInteger,remainder: %)
 lazyPremWithDefault (a,b) ==
   ground?(b) => error _
     "Error in lazyPremWithDefault from RPOLCAT : #2 is constant"
   ground?(a) => [1$$,0$NNI,a]
   xa := mvar a
   xb := mvar b
   xa < xb => [1$$,0$NNI,a]
   lcb : $ := init b
   db : NNI := mdeg b
   test : INT := degree(a,xb)::INT - db
   delta : INT := max(test + 1$INT, 0$INT)
   if xa = xb
     then
       b := tail b
       while not zero?(a) and not negative?(test) repeat
         term := monomial(init(a),xb,test::NNI)
         a := lcb * tail(a) - term * b
         delta := delta - 1$INT
         test := degree(a,xb)::INT - db
     else
       while not zero?(a) and not negative?(test) repeat
         term := monomial(leadingCoefficient(a,xb),xb,test::NNI)
         a := lcb * a - term * b
         delta := delta - 1$INT
         test := degree(a,xb)::INT - db
   [lcb, (delta::NNI), a]

lazyPremWithDefault : (%,%,V) ->
   Record(coef: %,gap: NonNegativeInteger,remainder: %)
lazyPremWithDefault (a,b,v) ==
   zero? b =>  error _
    "Error in lazyPremWithDefault : ($,$,V) -> $ from RPOLCAT : #2 is zero"
   ground?(b) => [b,1$NNI,0$$]
   (v = mvar(b)) => lazyPremWithDefault(a,b)
   dbv : NNI := degree(b,v)
   zero? dbv => [b,1$NNI,0$$]
   dav : NNI  := degree(a,v)
   zero? dav => [1$$,0$NNI,a]
   test : INT := dav::INT - dbv
   delta : INT := max(test + 1$INT, 0$INT)
   lcbv : $ := leadingCoefficient(b,v)
   while not zero?(a) and not negative?(test) repeat
     lcav := leadingCoefficient(a,v)
     term := monomial(lcav,v,test::NNI)
     a := lcbv * a - term * b
     delta := delta - 1$INT
     test := degree(a,v)::INT - dbv
   [lcbv, (delta::NNI), a]

 pseudoDivide : (%,%) -> Record(quotient: %,remainder: %)
 pseudoDivide (a,b) ==
```

```
    cPS := lazyPseudoDivide (a,b)
    c := (cPS.coef) ** (cPS.gap)
    [c * cPS.quotient, c * cPS.remainder]

  lazyPseudoDivide : (%,%) ->
     Record(coef: %,gap: NonNegativeInteger,quotient: %,remainder: %)
  lazyPseudoDivide (a,b) ==
    ground?(b) => error _
       "Error in lazyPseudoDivide from RPOLCAT : #2 is constant"
    ground?(a) => [1$$,0$NNI,0$$,a]
    xa := mvar a
    xb := mvar b
    xa < xb => [1$$,0$NNI,0$$, a]
    lcb : $ := init b
    db : NNI := mdeg b
    q := 0$$
    test : INT := degree(a,xb)::INT - db
    delta : INT := max(test + 1$INT, 0$INT)
    if xa = xb
      then
        b := tail b
        while not zero?(a) and not negative?(test) repeat
          term := monomial(init(a),xb,test::NNI)
          a := lcb * tail(a) - term * b
          q := lcb * q + term
          delta := delta - 1$INT
          test := degree(a,xb)::INT - db
      else
        while not zero?(a) and not negative?(test) repeat
          term := monomial(leadingCoefficient(a,xb),xb,test::NNI)
          a := lcb * a - term * b
          q := lcb * q + term
          delta := delta - 1$INT
          test := degree(a,xb)::INT - db
    [lcb, (delta::NNI), q, a]

  lazyPseudoDivide : (%,%,V) ->
     Record(coef: %,gap: NonNegativeInteger,quotient: %,remainder: %)
  lazyPseudoDivide (a,b,v) ==
    zero? b =>  error _
      "Error in lazyPseudoDivide : ($,$,V) -> $ from RPOLCAT : #2 is zero"
    ground?(b) => [b,1$NNI,a,0$$]
    (v = mvar(b)) => lazyPseudoDivide(a,b)
    dbv : NNI := degree(b,v)
    zero? dbv => [b,1$NNI,a,0$$]
    dav : NNI  := degree(a,v)
    zero? dav => [1$$,0$NNI,0$$, a]
    test : INT := dav::INT - dbv
    delta : INT := max(test + 1$INT, 0$INT)
    lcbv : $ := leadingCoefficient(b,v)
    q := 0$$
    while not zero?(a) and not negative?(test) repeat
      lcav := leadingCoefficient(a,v)
      term := monomial(lcav,v,test::NNI)
```

```
    a := lcbv * a - term * b
    q := lcbv * q + term
    delta := delta - 1$INT
    test := degree(a,v)::INT - dbv
  [lcbv, (delta::NNI), q, a]

monicModulo : (%,%) -> %
monicModulo (a,b) ==
  ground?(b) => error"Error in monicModulo from RPOLCAT : #2 is constant"
  rec : Union($,"failed")
  rec := recip((ib := init(b)))$$
  (rec case "failed")@Boolean => error _
    "Error in monicModulo from RPOLCAT : #2 is not monic"
  ground? a => a
  ib * ((lazyPremWithDefault ((rec::$) * a,(rec::$) * b)).remainder)

lazyResidueClass : (%,%) ->
    Record(polnum: %,polden: %,power: NonNegativeInteger)
lazyResidueClass(a,b) ==
  zero? b => [a,1$$,0$NNI]
  ground? b => [0$$,1$$,0$NNI]
  ground? a => [a,1$$,0$NNI]
  xa := mvar a
  xb := mvar b
  xa < xb => [a,1$$,0$NNI]
  monic?(b) => [monicModulo(a,b),1$$,0$NNI]
  lcb : $ := init b
  db : NNI := mdeg b
  test : INT := degree(a,xb)::INT - db
  pow : NNI := 0
  if xa = xb
    then
      b := tail b
      while not zero?(a) and not negative?(test) repeat
        term := monomial(init(a),xb,test::NNI)
        a := lcb * tail(a) - term * b
        pow := pow + 1$NNI
        test := degree(a,xb)::INT - db
    else
      while not zero?(a) and not negative?(test) repeat
        term := monomial(leadingCoefficient(a,xb),xb,test::NNI)
        a := lcb * a - term * b
        pow := pow + 1$NNI
        test := degree(a,xb)::INT - db
  [a,lcb,pow]

reduced? : (%,%) -> Boolean
reduced? (a:$,b:$) : Boolean ==
  degree(a,mvar(b)) < mdeg(b)

reduced? : (%,List(%)) -> Boolean
reduced? (p:$, lq : List($)) : Boolean ==
  ground? p => true
  while (not empty? lq) and (reduced?(p, first lq)) repeat
```

```
        lq := rest lq
      empty? lq

    headReduced? : (%,%) -> Boolean
    headReduced? (a:$,b:$) : Boolean ==
      reduced?(head(a),b)

    headReduced? : (%,List(%)) -> Boolean
    headReduced? (p:$, lq : List($)) : Boolean ==
      reduced?(head(p),lq)

    initiallyReduced? : (%,%) -> Boolean
    initiallyReduced? (a:$,b:$) : Boolean ==
      ground? b => error _
  "Error in initiallyReduced? : ($,$) -> Bool. from RPOLCAT : #2 is constant"
      ground?(a) => true
      mvar(a) < mvar(b) => true
      (mvar(a) = mvar(b)) => reduced?(a,b)
      initiallyReduced?(init(a),b)

    initiallyReduced? : (%,List(%)) -> Boolean
    initiallyReduced? (p:$, lq : List($)) : Boolean ==
      ground? p => true
      while (not empty? lq) and (initiallyReduced?(p, first lq)) repeat
        lq := rest lq
      empty? lq

    normalized? : (%,%) -> Boolean
    normalized?(a:$,b:$) : Boolean ==
      ground? b => error _
     "Error in  normalized? : ($,$) -> Boolean from TSETCAT : #2 is constant"
      ground? a => true
      mvar(a) < mvar(b) => true
      (mvar(a) = mvar(b)) => false
      normalized?(init(a),b)

    normalized? : (%,List(%)) -> Boolean
    normalized? (p:$, lq : List($)) : Boolean ==
      while (not empty? lq) and (normalized?(p, first lq)) repeat
        lq := rest lq
      empty? lq

    if R has IntegralDomain
    then

      if R has EuclideanDomain
        then

          exactQuo : (R,R) -> R
          exactQuo(r:R,s:R):R ==
            r quo$R s

        else
```

```
      exactQuo : (R,R) -> R
      exactQuo(r:R,s:R):R ==
        (r exquo$R s)::R

  exactQuotient : (%,R) -> %
  exactQuotient (p:$,r:R) ==
    (p exquo$$ r)::$

  exactQuotient : (%,%) -> %
  exactQuotient (a:$,b:$) ==
    ground? b => exactQuotient(a,ground(b))
    (a exquo$$ b)::$

  exactQuotient! : (%,%) -> %
  exactQuotient! (a:$,b:$) ==
    ground? b => exactQuotient!(a,ground(b))
    a := (a exquo$$ b)::$

  if (R has GcdDomain) and not(R has Field)
  then

    primPartElseUnitCanonical : % -> %
    primPartElseUnitCanonical p ==
      primitivePart p

    primitivePart! : % -> %
    primitivePart! p ==
      zero? p => p
      if ((cp := content(p)) = 1)
        then
          p := unitCanonical p
        else
          p := unitCanonical exactQuotient!(p,cp)
      p

    primPartElseUnitCanonical! : % -> % if R has INTDOM
    primPartElseUnitCanonical! p ==
      primitivePart! p

  else

    primPartElseUnitCanonical : % -> %
    primPartElseUnitCanonical p ==
      unitCanonical p

    primPartElseUnitCanonical! : % -> % if R has INTDOM
    primPartElseUnitCanonical! p ==
      p := unitCanonical p


if R has GcdDomain
then

  gcd : (%,%) -> %
```

```
    gcd(r:R,p:$):R ==
      (r = 1) => r
      zero? p => r
      ground? p => gcd(r,ground(p))$R
      gcd(gcd(r,init(p)),tail(p))

    mainContent : % -> %
    mainContent p ==
      zero? p => p
      "gcd"/mainCoefficients(p)

    mainPrimitivePart : % -> %
    mainPrimitivePart p ==
      zero? p => p
      (unitNormal((p exquo$$ mainContent(p))::$)).canonical

    mainSquareFreePart : % -> %
    mainSquareFreePart p ==
      ground? p => p
      v := mainVariable(p)::V
      sfp : SparseUnivariatePolynomial($)
      sfp := squareFreePart(univariate(p,v)@SparseUnivariatePolynomial($))
      multivariate(sfp,v)

  if (V has ConvertibleTo(Symbol))
    then

      PR ==> Polynomial R
      PQ ==> Polynomial Fraction Integer
      PZ ==> Polynomial Integer
      IES ==> IndexedExponents(Symbol)
      Q ==> Fraction Integer
      Z ==> Integer

      convert : % -> Polynomial(R)
      convert(p:$) : PR ==
        ground? p => (ground(p)$$)::PR
        v : V := mvar(p)
        d : NNI := mdeg(p)
        convert(init(p))@PR *$PR _
                     ((convert(v)@Symbol)::PR)**d +$PR convert(tail(p))@PR

      coerce : % -> Polynomial(R)
      coerce(p:$) : PR ==
        convert(p)@PR

      if V has Finite
        then

          sizeV : NNI := size()$V

          lv : List Symbol
          lv := _
            [convert(index(i::PositiveInteger)$V)@Symbol for i in 1..sizeV]
```

```
localRetract : PR -> $
localRetract(p : PR) : $ ==
  ground? p => (ground(p)$PR)::$
  mvp : Symbol := (mainVariable(p)$PR)::Symbol
  d : NNI
  imvp : PositiveInteger := _
              (position(mvp,lv)$(List Symbol))::PositiveInteger
  vimvp : V := index(imvp)$V
  xvimvp,c : $
  newp := 0$$
  while (not zero? (d := degree(p,mvp))) repeat
    c := localRetract(coefficient(p,mvp,d)$PR)
    xvimvp := monomial(c,vimvp,d)$$
    newp := newp +$$ xvimvp
    p := p -$PR monomial(coefficient(p,mvp,d)$PR,mvp,d)$PR
  newp +$$ localRetract(p)

if R has Algebra Fraction Integer
  then

    localRetractPQ : PQ -> $
    localRetractPQ(pq:PQ):$ ==
      ground? pq => ((ground(pq)$PQ)::R)::$
      mvp : Symbol := (mainVariable(pq)$PQ)::Symbol
      d : NNI
      imvp : PositiveInteger := _
              (position(mvp,lv)$(List Symbol))::PositiveInteger
      vimvp : V := index(imvp)$V
      xvimvp,c : $
      newp := 0$$
      while (not zero? (d := degree(pq,mvp))) repeat
        c := localRetractPQ(coefficient(pq,mvp,d)$PQ)
        xvimvp := monomial(c,vimvp,d)$$
        newp := newp +$$ xvimvp
        pq := pq -$PQ monomial(coefficient(pq,mvp,d)$PQ,mvp,d)$PQ
      newp +$$ localRetractPQ(pq)

if R has Algebra Integer
  then

    localRetractPZ : PZ -> $
    localRetractPZ(pz:PZ):$ ==
      ground? pz => ((ground(pz)$PZ)::R)::$
      mvp : Symbol := (mainVariable(pz)$PZ)::Symbol
      d : NNI
      imvp : PositiveInteger := _
              (position(mvp,lv)$(List Symbol))::PositiveInteger
      vimvp : V := index(imvp)$V
      xvimvp,c : $
      newp := 0$$
      while (not zero? (d := degree(pz,mvp))) repeat
        c := localRetractPZ(coefficient(pz,mvp,d)$PZ)
        xvimvp := monomial(c,vimvp,d)$$
```

```
          newp := newp +$$ xvimvp
          pz := pz -$PZ monomial(coefficient(pz,mvp,d)$PZ,mvp,d)$PZ
        newp +$$ localRetractPZ(pz)

  retractable? : PR -> Boolean
  retractable?(p:PR):Boolean ==
    lvp := variables(p)$PR
    while not empty? lvp and member?(first lvp,lv) repeat
      lvp := rest lvp
    empty? lvp

  retractablePQ? : PQ -> Boolean
  retractablePQ?(p:PQ):Boolean ==
    lvp := variables(p)$PQ
    while not empty? lvp and member?(first lvp,lv) repeat
      lvp := rest lvp
    empty? lvp

  retractablePZ? : PZ -> Boolean
  retractablePZ?(p:PZ):Boolean ==
    lvp := variables(p)$PZ
    while not empty? lvp and member?(first lvp,lv) repeat
      lvp := rest lvp
    empty? lvp

  localRetractIfCan : PR -> Union($,"failed")
  localRetractIfCan(p : PR): Union($,"failed") ==
    not retractable?(p) => "failed"::Union($,"failed")
    localRetract(p)::Union($,"failed")

  localRetractIfCanPQ : PQ -> Union($,"failed")
  localRetractIfCanPQ(p : PQ): Union($,"failed") ==
    not retractablePQ?(p) => "failed"::Union($,"failed")
    localRetractPQ(p)::Union($,"failed")

  localRetractIfCanPZ : PZ -> Union($,"failed")
  localRetractIfCanPZ(p : PZ): Union($,"failed") ==
    not retractablePZ?(p) => "failed"::Union($,"failed")
    localRetractPZ(p)::Union($,"failed")

if R has Algebra Fraction Integer
  then

  mpc2Z := MPolyCatFunctions2(Symbol,IES,IES,Z,R,PZ,PR)
  mpc2Q := MPolyCatFunctions2(Symbol,IES,IES,Q,R,PQ,PR)

  ZToR : Z -> R
  ZToR (z:Z):R == coerce(z)@R

  QToR : Q -> R
  QToR (q:Q):R == coerce(q)@R

  PZToPR : PZ -> PR
  PZToPR (pz:PZ):PR == map(ZToR,pz)$mpc2Z
```

```
PQToPR : PQ -> PR
PQToPR (pq:PQ):PR == map(QToR,pq)$mpc2Q

retract : Polynomial(Integer) -> %
retract(pz:PZ) ==
  rif : Union($,"failed") := retractIfCan(pz)@Union($,"failed")
  (rif case "failed") => error _
                    "failed in retract: POLY Z -> $ from RPOLCAT"
  rif::$

convert : Polynomial(Integer) -> %
convert(pz:PZ) ==
  retract(pz)@$

retract : Polynomial(Fraction(Integer)) -> %
retract(pq:PQ) ==
  rif : Union($,"failed") := retractIfCan(pq)@Union($,"failed")
  (rif case "failed") => error _
                    "failed in retract: POLY Z -> $ from RPOLCAT"
  rif::$

convert : Polynomial(Fraction(Integer)) -> %
convert(pq:PQ) ==
  retract(pq)@$

if not (R has QuotientFieldCategory(Integer))
  then
    -- the only operation to implement is
    -- retractIfCan : PR -> Union($,"failed")
    -- when V does not have Finite

    if V has Finite
      then

        retractIfCan : Polynomial(R) -> Union(%,"failed")
        retractIfCan(pr:PR) ==
          localRetractIfCan(pr)@Union($,"failed")

        retractIfCan : Polynomial(Fraction(Integer)) ->
            Union(%,"failed")
        retractIfCan(pq:PQ) ==
          localRetractIfCanPQ(pq)@Union($,"failed")
      else

        retractIfCan : Polynomial(Fraction(Integer)) ->
            Union(%,"failed")
        retractIfCan(pq:PQ) ==
          pr : PR := PQToPR(pq)
          retractIfCan(pr)@Union($,"failed")

    retractIfCan : Polynomial(Integer) -> Union(%,"failed")
    retractIfCan(pz:PZ) ==
      pr : PR := PZToPR(pz)
```

```
      retractIfCan(pr)@Union($,"failed")

    retract : Polynomial(R) -> %
    retract(pr:PR) ==
      rif : Union($,"failed") := _
                             retractIfCan(pr)@Union($,"failed")
      (rif case "failed") => error _
                  "failed in retract: POLY Z -> $ from RPOLCAT"
      rif::$

    convert : Polynomial(R) -> %
    convert(pr:PR) ==
      retract(pr)@$

  else

    -- the only operation to implement is
    -- retractIfCan : PQ -> Union($,"failed")
    -- when V does not have Finite
    mpc2ZQ := MPolyCatFunctions2(Symbol,IES,IES,Z,Q,PZ,PQ)
    mpc2RQ := MPolyCatFunctions2(Symbol,IES,IES,R,Q,PR,PQ)

    ZToQ : Z -> Q
    ZToQ(z:Z):Q == coerce(z)@Q

    RToQ : R -> Q
    RToQ(r:R):Q == retract(r)@Q

    PZToPQ : PZ -> PQ
    PZToPQ (pz:PZ):PQ == map(ZToQ,pz)$mpc2ZQ

    PRToPQ : PR -> PQ
    PRToPQ (pr:PR):PQ == map(RToQ,pr)$mpc2RQ

    retractIfCan : Polynomial(Integer) -> Union(%,"failed")
    retractIfCan(pz:PZ) ==
      pq : PQ := PZToPQ(pz)
      retractIfCan(pq)@Union($,"failed")

    if V has Finite
      then

        retractIfCan : Polynomial(Fraction(Integer)) ->
           Union(%,"failed")
        retractIfCan(pq:PQ) ==
          localRetractIfCanPQ(pq)@Union($,"failed")

        convert : Polynomial(R) -> %
        convert(pr:PR) ==
          lrif : Union($,"failed") := _
                          localRetractIfCan(pr)@Union($,"failed")
          (lrif case "failed") => error _
                          "failed in convert: PR->$ from RPOLCAT"
          lrif::$
```

```
          else

            convert : Polynomial(R) -> %
            convert(pr:PR) ==
              pq : PQ := PRToPQ(pr)
              retract(pq)@$

    if (R has Algebra Integer) and not(R has Algebra Fraction Integer)
      then

        mpc2Z := MPolyCatFunctions2(Symbol,IES,IES,Z,R,PZ,PR)

        ZToR : Z -> R
        ZToR (z:Z):R == coerce(z)@R

        PZToPR : PZ -> PR
        PZToPR (pz:PZ):PR == map(ZToR,pz)$mpc2Z

        retract : Polynomial(Integer) -> %
        retract(pz:PZ) ==
          rif : Union($,"failed") := retractIfCan(pz)@Union($,"failed")
          (rif case "failed") => error _
                           "failed in retract: POLY Z -> $ from RPOLCAT"
          rif::$

        convert : Polynomial(Integer) -> %
        convert(pz:PZ) ==
          retract(pz)@$

        if not (R has IntegerNumberSystem)
          then
            -- the only operation to implement is
            -- retractIfCan : PR -> Union($,"failed")
            -- when V does not have Finite

            if V has Finite
              then

                retractIfCan : Polynomial(R) -> Union(%,"failed")
                retractIfCan(pr:PR) ==
                  localRetractIfCan(pr)@Union($,"failed")

                retractIfCan : Polynomial(Fraction(Integer)) ->
                    Union(%,"failed")
                retractIfCan(pz:PZ) ==
                  localRetractIfCanPZ(pz)@Union($,"failed")

              else

                retractIfCan : Polynomial(Fraction(Integer)) ->
                    Union(%,"failed")
                retractIfCan(pz:PZ) ==
                  pr : PR := PZToPR(pz)
                  retractIfCan(pr)@Union($,"failed")
```

```
        retract : Polynomial(R) -> %
        retract(pr:PR) ==
          rif : Union($,"failed"):=retractIfCan(pr)@Union($,"failed")
          (rif case "failed") => error _
                         "failed in retract: POLY Z -> $ from RPOLCAT"
          rif::$

        convert : % -> Polynomial(R)
        convert(pr:PR) ==
          retract(pr)@$

      else
        -- the only operation to implement is
        -- retractIfCan : PZ -> Union($,"failed")
        -- when V does not have Finite

        mpc2RZ := MPolyCatFunctions2(Symbol,IES,IES,R,Z,PR,PZ)

        RToZ : R -> Z
        RToZ(r:R):Z == retract(r)@Z

        PRToPZ : PR -> PZ
        PRToPZ (pr:PR):PZ == map(RToZ,pr)$mpc2RZ

        if V has Finite
          then

            convert : % -> Polynomial(R)
            convert(pr:PR) ==
              lrif : Union($,"failed") := _
                               localRetractIfCan(pr)@Union($,"failed")
              (lrif case "failed") => error _
                               "failed in convert: PR->$ from RPOLCAT"
              lrif::$

            retractIfCan : Polynomial(Integer) -> Union(%,"failed")
            retractIfCan(pz:PZ) ==
              localRetractIfCanPZ(pz)@Union($,"failed")

          else

            convert : % -> Polynomial(R)
            convert(pr:PR) ==
              pz : PZ := PRToPZ(pr)
              retract(pz)@$


  if not(R has Algebra Integer) and not(R has Algebra Fraction Integer)
    then
      -- the only operation to implement is
      -- retractIfCan : PR -> Union($,"failed")

      if V has Finite
```

```
        then

          retractIfCan : Polynomial(R) -> Union(%,"failed")
          retractIfCan(pr:PR) ==
            localRetractIfCan(pr)@Union($,"failed")

      retract : Polynomial(R) -> %
      retract(pr:PR) ==
        rif : Union($,"failed") := retractIfCan(pr)@Union($,"failed")
        (rif case "failed") => error _
                        "failed in retract: POLY Z -> $ from RPOLCAT"
        rif::$

      convert : % -> Polynomial(R)
      convert(pr:PR) ==
        retract(pr)@$

  if (R has RetractableTo(INT))
    then

      convert : % -> String
      convert(pol:$):String ==
        ground?(pol) => convert(retract(ground(pol))@INT)@String
        ipol : $ := init(pol)
        vpol : V := mvar(pol)
        dpol : NNI := mdeg(pol)
        tpol: $  := tail(pol)
        sipol,svpol,sdpol,stpol : String
        if (ipol = 1)
          then
            sipol := empty()$String
          else
            if ((-ipol) = 1)
              then
                sipol := "-"
              else
                sipol := convert(ipol)@String
                if not monomial?(ipol)
                  then
                    sipol := concat(["(",sipol,")*"])$String
                  else
                    sipol := concat(sipol,"*")$String
        svpol := string(convert(vpol)@Symbol)
        if (dpol = 1)
          then
            sdpol :=  empty()$String
          else
            sdpol := _
                 concat("**",convert(convert(dpol)@INT)@String )$String
        if zero? tpol
          then
            stpol :=  empty()$String
          else
            if ground?(tpol)
```

```
                    then
                      n := retract(ground(tpol))@INT
                      if n > 0
                        then
                          stpol :=  concat(" +",convert(n)@String)$String
                        else
                          stpol := convert(n)@String
                    else
                      stpol := convert(tpol)@String
                      if _
                        not member?((stpol.1)::String,["+","-"])$(List String)
                        then
                          stpol :=  concat(" + ",stpol)$String
              concat([sipol,svpol,sdpol,stpol])$String

*)
```

———-

— RPOLCAT.dotabb —

```
"RPOLCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RPOLCAT"];
"RPOLCAT" -> "POLYCAT"
```

———-

— RPOLCAT.dotfull —

```
"RecursivePolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RPOLCAT"];
"RecursivePolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
 -> "PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
```

———-

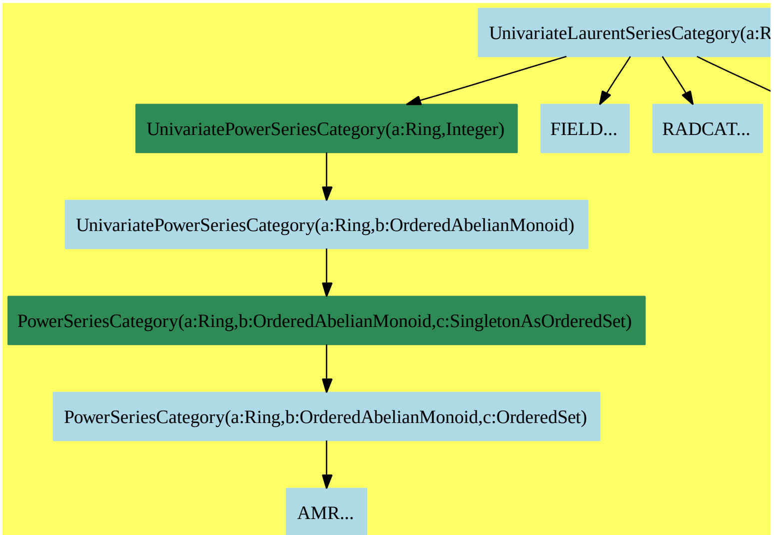
— RPOLCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"RecursivePolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
 [color=lightblue];
"RecursivePolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
 -> "PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"

"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
 [color=lightblue];
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
```

```
  -> "PDRING..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "FAMR..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "EVALAB..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "IEVALAB..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "RETRACT..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "FLINEXP..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "ORDSET..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "GCDDOM..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "PFECAT..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "KONVERT..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "PATMAB..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "COMRING..."

"PDRING..." [color=lightblue];
"FAMR..." [color=lightblue];
"EVALAB..." [color=lightblue];
"IEVALAB..." [color=lightblue];
"RETRACT..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"ORDSET..." [color=lightblue];
"GCDDOM..." [color=lightblue];
"PFECAT..." [color=lightblue];
"KONVERT..." [color=lightblue];
"PATMAB..." [color=lightblue];
"COMRING..." [color=lightblue];

}
```

———-

17.0.226 UnivariateLaurentSeriesCategory (ULSCAT)



— **UnivariateLaurentSeriesCategory.input** —

```
)set break resume
)sys rm -f UnivariateLaurentSeriesCategory.output
)spool UnivariateLaurentSeriesCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 3
)show UnivariateLaurentSeriesCategory
--R
--R UnivariateLaurentSeriesCategory(Coef: Ring) is a category constructor
--R Abbreviation for UnivariateLaurentSeriesCategory is ULSCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ULSCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Coef,%) -> %                  ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %    ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %   ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                     ?-? : (%,%) -> %
--R -? : % -> %                          ?=? : (%,%) -> Boolean
--R 1 : () -> %                          0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %    ?^? : (%,PositiveInteger) -> %
--R center : % -> Coef                   coefficient : (%,Integer) -> Coef
--R coerce : % -> % if Coef has INTDOM   coerce : Integer -> %
--R coerce : % -> OutputForm             complete : % -> %
--R degree : % -> Integer                ?.? : (%,Integer) -> Coef
--R extend : (%,Integer) -> %            hash : % -> SingleInteger
```

```
--R inv : % -> % if Coef has FIELD       latex : % -> String
--R leadingCoefficient : % -> Coef       leadingMonomial : % -> %
--R map : ((Coef -> Coef),%) -> %        monomial : (Coef,Integer) -> %
--R monomial? : % -> Boolean             one? : % -> Boolean
--R order : (%,Integer) -> Integer       order : % -> Integer
--R pole? : % -> Boolean                 recip : % -> Union(%,"failed")
--R reductum : % -> %                    sample : () -> %
--R truncate : (%,Integer) -> %          variable : % -> Symbol
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction(Integer)) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?*? : (Fraction(Integer),%) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?**? : (%,Fraction(Integer)) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?**? : (%,%) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?**? : (%,Integer) -> % if Coef has FIELD
--R ?/? : (%,%) -> % if Coef has FIELD
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (Integer,Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if Coef has *: (Integer,Coef) -> Coef
--R D : (%,Symbol) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Integer,Coef) -> Coef
--R D : (%,List(Symbol)) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Integer,Coef) -> Coef
--R D : (%,Symbol,NonNegativeInteger) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Integer,Coef) -> Coef
--R D : (%,List(Symbol),List(NonNegativeInteger)) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Integer,Coef)
--R ?^? : (%,Integer) -> % if Coef has FIELD
--R acos : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acosh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acot : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acoth : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acsc : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acsch : % -> % if Coef has ALGEBRA(FRAC(INT))
--R approximate : (%,Integer) -> Coef if Coef has **: (Coef,Integer) -> Coef and Coef has coerce: Symbol -> Coef
--R asec : % -> % if Coef has ALGEBRA(FRAC(INT))
--R asech : % -> % if Coef has ALGEBRA(FRAC(INT))
--R asin : % -> % if Coef has ALGEBRA(FRAC(INT))
--R asinh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA(FRAC(INT))
--R atanh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
--R coerce : Fraction(Integer) -> % if Coef has ALGEBRA(FRAC(INT))
--R coerce : Coef -> % if Coef has COMRING
--R cos : % -> % if Coef has ALGEBRA(FRAC(INT))
--R cosh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R cot : % -> % if Coef has ALGEBRA(FRAC(INT))
--R coth : % -> % if Coef has ALGEBRA(FRAC(INT))
--R csc : % -> % if Coef has ALGEBRA(FRAC(INT))
--R csch : % -> % if Coef has ALGEBRA(FRAC(INT))
--R differentiate : % -> % if Coef has *: (Integer,Coef) -> Coef
--R differentiate : (%,NonNegativeInteger) -> % if Coef has *: (Integer,Coef) -> Coef
--R differentiate : (%,Symbol) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Integer,Coef) -> Coef
--R differentiate : (%,List(Symbol)) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Integer,Coef) -> Coef
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Integer,Coef)
--R differentiate : (%,List(Symbol),List(NonNegativeInteger)) -> % if Coef has PDRING(SYMBOL) and Coef has *: (I
--R divide : (%,%) -> Record(quotient: %,remainder: %) if Coef has FIELD
```

```
--R ?.? : (%,%) -> % if Integer has SGROUP
--R euclideanSize : % -> NonNegativeInteger if Coef has FIELD
--R eval : (%,Coef) -> Stream(Coef) if Coef has **: (Coef,Integer) -> Coef
--R exp : % -> % if Coef has ALGEBRA(FRAC(INT))
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed") if Coef has FIELD
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if Coef has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if Coef has FIELD
--R factor : % -> Factored(%) if Coef has FIELD
--R gcd : (%,%) -> % if Coef has FIELD
--R gcd : List(%) -> % if Coef has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R integrate : (%,Symbol) -> % if Coef has ACFS(INT) and Coef has PRIMCAT and Coef has TRANFUN and Coef has ALG
--R integrate : % -> % if Coef has ALGEBRA(FRAC(INT))
--R lcm : (%,%) -> % if Coef has FIELD
--R lcm : List(%) -> % if Coef has FIELD
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %) if Coef has FIELD
--R log : % -> % if Coef has ALGEBRA(FRAC(INT))
--R monomial : (%,List(SingletonAsOrderedSet),List(Integer)) -> %
--R monomial : (%,SingletonAsOrderedSet,Integer) -> %
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed") if Coef has FIELD
--R multiplyCoefficients : ((Integer -> Coef),%) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA(FRAC(INT))
--R pi : () -> % if Coef has ALGEBRA(FRAC(INT))
--R prime? : % -> Boolean if Coef has FIELD
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %) if Coef has FIELD
--R ?quo? : (%,%) -> % if Coef has FIELD
--R rationalFunction : (%,Integer,Integer) -> Fraction(Polynomial(Coef)) if Coef has INTDOM
--R rationalFunction : (%,Integer) -> Fraction(Polynomial(Coef)) if Coef has INTDOM
--R ?rem? : (%,%) -> % if Coef has FIELD
--R sec : % -> % if Coef has ALGEBRA(FRAC(INT))
--R sech : % -> % if Coef has ALGEBRA(FRAC(INT))
--R series : Stream(Record(k: Integer,c: Coef)) -> %
--R sin : % -> % if Coef has ALGEBRA(FRAC(INT))
--R sinh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R sizeLess? : (%,%) -> Boolean if Coef has FIELD
--R sqrt : % -> % if Coef has ALGEBRA(FRAC(INT))
--R squareFree : % -> Factored(%) if Coef has FIELD
--R squareFreePart : % -> % if Coef has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA(FRAC(INT))
--R tanh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R terms : % -> Stream(Record(k: Integer,c: Coef))
--R truncate : (%,Integer,Integer) -> %
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R variables : % -> List(SingletonAsOrderedSet)
--R
--E 1

--S 2 of 3
w:SparseUnivariateLaurentSeries(Fraction(Integer),'z,0):=0
```

```
--E 2

--S 3 of 3
rationalFunction(w,0)
--E 3

)spool
)lisp (bye)
```

— UnivariateLaurentSeriesCategory.help —

```
====================================================================
UnivariateLaurentSeriesCategory examples
====================================================================

UnivariateLaurentSeriesCategory is the category of Laurent series
in one variable.

w:SparseUnivariateLaurentSeries(Fraction(Integer),'z,0):=0
rationalFunction(w,0)

See Also:
o )show UnivariateLaurentSeriesCategory
```

See:

⇒ "UnivariateLaurentSeriesConstructorCategory" (ULSCCAT) 18.0.235 on page 1801
⇐ "Field" (FIELD) 16.0.209 on page 1413
⇐ "RadicalCategory" (RADCAT) 2.0.36 on page 106
⇐ "TranscendentalFunctionCategory" (TRANFUN) 3.0.58 on page 192
⇐ "UnivariatePowerSeriesCategory" (UPSCAT) 15.0.208 on page 1401

Exports:

0	1	acos	acosh
acot	acoth	acsc	acsch
approximate	asec	asech	asin
asinh	associates?	atan	atanh
center	characteristic	charthRoot	coefficient
coerce	complete	cos	cosh
cot	coth	csc	csch
D	degree	differentiate	divide
euclideanSize	eval	exp	expressIdealMember
exquo	extend	extendedEuclidean	factor
gcd	gcdPolynomial	hash	integrate
inv	latex	lcm	leadingCoefficient
leadingMonomial	log	map	monomial
monomial?	multiEuclidean	multiplyCoefficients	multiplyExponents
nthRoot	one?	order	pi
pole?	prime?	principalIdeal	rationalFunction
recip	reductum	sample	sec
sech	series	sin	sinh
sizeLess?	sqrt	squareFree	squareFreePart
subtractIfCan	tan	tanh	terms
truncate	unit?	unitCanonical	unitNormal
variable	variables	zero?	?*?
?**?	?+?	?-? * -?	
?=?	?^?	?~=?	?/?
?.?	?quo?	?rem?	

Attributes Exported:

- if #1 has Field then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if #1 has Field then canonicalsClosed where **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- if #1 has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if #1 has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation $" * " : (D, D) -> D$ which is commutative.

These are directly exported but not implemented:

```
integrate : % -> % if Coef has ALGEBRA FRAC INT
integrate : (%,Symbol) -> %
  if Coef has ACFS INT
  and Coef has PRIMCAT
  and Coef has TRANFUN
  and Coef has ALGEBRA FRAC INT
  or Coef has variables: Coef -> List Symbol
```

```
  and Coef has integrate: (Coef,Symbol) -> Coef
  and Coef has ALGEBRA FRAC INT
multiplyCoefficients : ((Integer -> Coef),%) -> %
rationalFunction : (%,Integer) -> Fraction Polynomial Coef
  if Coef has INTDOM
rationalFunction : (%,Integer,Integer) -> Fraction Polynomial Coef
  if Coef has INTDOM
series : Stream Record(k: Integer,c: Coef) -> %
```

These exports come from (p1401) UnivariatePowerSeriesCategory(Coef,Integer) where Coef:Ring:

```
0 : () -> %
1 : () -> %
approximate : (%,Integer) -> Coef
  if Coef has **: (Coef,Integer) -> Coef
  and Coef has coerce: Symbol -> Coef
associates? : (%,%) -> Boolean if Coef has INTDOM
center : % -> Coef
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
coefficient : (%,Integer) -> Coef
coerce : % -> % if Coef has INTDOM
coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
coerce : Coef -> % if Coef has COMRING
coerce : Integer -> %
coerce : % -> OutputForm
complete : % -> %
D : % -> % if Coef has *: (Integer,Coef) -> Coef
D : (%,NonNegativeInteger) -> %
  if Coef has *: (Integer,Coef) -> Coef
D : (%,Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
D : (%,List Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
D : (%,Symbol,NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
D : (%,List Symbol,List NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
degree : % -> Integer
differentiate : (%,Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
differentiate : (%,List Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
differentiate : (%,Symbol,NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
```

```
  and Coef has *: (Integer,Coef) -> Coef
differentiate : % -> %
  if Coef has *: (Integer,Coef) -> Coef
differentiate : (%,NonNegativeInteger) -> %
  if Coef has *: (Integer,Coef) -> Coef
eval : (%,Coef) -> Stream Coef
  if Coef has **: (Coef,Integer) -> Coef
exquo : (%,%) -> Union(%,"failed")
  if Coef has INTDOM
extend : (%,Integer) -> %
hash : % -> SingleInteger
latex : % -> String
leadingCoefficient : % -> Coef
leadingMonomial : % -> %
map : ((Coef -> Coef),%) -> %
monomial : (%,SingletonAsOrderedSet,Integer) -> %
monomial : (%,List SingletonAsOrderedSet,List Integer) -> %
monomial : (Coef,Integer) -> %
monomial? : % -> Boolean
multiplyExponents : (%,PositiveInteger) -> %
one? : % -> Boolean
order : (%,Integer) -> Integer
order : % -> Integer
pole? : % -> Boolean
recip : % -> Union(%,"failed")
reductum : % -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
terms : % -> Stream Record(k: Integer,c: Coef)
truncate : (%,Integer,Integer) -> %
truncate : (%,Integer) -> %
unit? : % -> Boolean if Coef has INTDOM
unitCanonical : % -> % if Coef has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
  if Coef has INTDOM
variable : % -> Symbol
variables : % -> List SingletonAsOrderedSet
zero? : % -> Boolean
?.? : (%,Integer) -> Coef
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (Integer,%) -> %
?*? : (Coef,%) -> %
?*? : (%,Coef) -> %
?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
```

```
?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
?/? : (%,Coef) -> % if Coef has FIELD
-? : % -> %
?.? : (%,%) -> % if Integer has SGROUP
```

These exports come from (p192) TranscendentalFunctionCategory():

```
acos : % -> % if Coef has ALGEBRA FRAC INT
acosh : % -> % if Coef has ALGEBRA FRAC INT
acot : % -> % if Coef has ALGEBRA FRAC INT
acoth : % -> % if Coef has ALGEBRA FRAC INT
acsc : % -> % if Coef has ALGEBRA FRAC INT
acsch : % -> % if Coef has ALGEBRA FRAC INT
asec : % -> % if Coef has ALGEBRA FRAC INT
asech : % -> % if Coef has ALGEBRA FRAC INT
asin : % -> % if Coef has ALGEBRA FRAC INT
asinh : % -> % if Coef has ALGEBRA FRAC INT
atanh : % -> % if Coef has ALGEBRA FRAC INT
atan : % -> % if Coef has ALGEBRA FRAC INT
cos : % -> % if Coef has ALGEBRA FRAC INT
cosh : % -> % if Coef has ALGEBRA FRAC INT
cot : % -> % if Coef has ALGEBRA FRAC INT
coth : % -> % if Coef has ALGEBRA FRAC INT
csc : % -> % if Coef has ALGEBRA FRAC INT
csch : % -> % if Coef has ALGEBRA FRAC INT
exp : % -> % if Coef has ALGEBRA FRAC INT
log : % -> % if Coef has ALGEBRA FRAC INT
pi : () -> % if Coef has ALGEBRA FRAC INT
sec : % -> % if Coef has ALGEBRA FRAC INT
sech : % -> % if Coef has ALGEBRA FRAC INT
sin : % -> % if Coef has ALGEBRA FRAC INT
sinh : % -> % if Coef has ALGEBRA FRAC INT
tan : % -> % if Coef has ALGEBRA FRAC INT
tanh : % -> % if Coef has ALGEBRA FRAC INT
?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
```

These exports come from (p106) RadicalCategory():

```
nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
sqrt : % -> % if Coef has ALGEBRA FRAC INT
?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
```

These exports come from (p1413) Field():

```
divide : (%,%) -> Record(quotient: %,remainder: %)
  if Coef has FIELD
euclideanSize : % -> NonNegativeInteger
  if Coef has FIELD
expressIdealMember : (List %,%) -> Union(List %,"failed")
  if Coef has FIELD
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
  if Coef has FIELD
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
  if Coef has FIELD
factor : % -> Factored % if Coef has FIELD
gcd : (%,%) -> % if Coef has FIELD
gcd : List % -> % if Coef has FIELD
```

```
 gcdPolynomial :
   (SparseUnivariatePolynomial %,
    SparseUnivariatePolynomial %) ->
      SparseUnivariatePolynomial %
        if Coef has FIELD
 inv : % -> % if Coef has FIELD
 lcm : (%,%) -> % if Coef has FIELD
 lcm : List % -> % if Coef has FIELD
 multiEuclidean : (List %,%) -> Union(List %,"failed")
   if Coef has FIELD
 prime? : % -> Boolean if Coef has FIELD
 principalIdeal : List % -> Record(coef: List %,generator: %)
   if Coef has FIELD
 sizeLess? : (%,%) -> Boolean if Coef has FIELD
 squareFree : % -> Factored % if Coef has FIELD
 squareFreePart : % -> % if Coef has FIELD
 ?**? : (%,Integer) -> % if Coef has FIELD
 ?^? : (%,Integer) -> % if Coef has FIELD
 ?/? : (%,%) -> % if Coef has FIELD
 ?quo? : (%,%) -> % if Coef has FIELD
 ?rem? : (%,%) -> % if Coef has FIELD
```

— UnivariateLaurentSeriesCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ULSCAT">
UnivariateLaurentSeriesCategory (ULSCAT)</a></h2>
</body>
```

— category ULSCAT UnivariateLaurentSeriesCategory —

```
)abbrev category ULSCAT UnivariateLaurentSeriesCategory
++ Author: Clifton J. Williamson
++ Date Created: 21 December 1989
++ Date Last Updated: 20 September 1993
++ Description:
++ \spadtype{UnivariateLaurentSeriesCategory} is the category of
++ Laurent series in one variable.

UnivariateLaurentSeriesCategory(Coef) : Category == SIG where
  Coef : Ring

  I    ==> Integer
  NNI  ==> NonNegativeInteger
  Term ==> Record(k:I,c:Coef)

  SIG ==> UnivariatePowerSeriesCategory(Coef,Integer) with

    series : Stream Term -> %
      ++ \spad{series(st)} creates a series from a stream of non-zero terms,
      ++ where a term is an exponent-coefficient pair.  The terms in the
```

```
  ++ stream should be ordered by increasing order of exponents.

multiplyCoefficients : (I -> Coef,%) -> %
  ++ \spad{multiplyCoefficients(f,sum(n = n0..infinity,a[n] * x**n)) =
  ++ sum(n = 0..infinity,f(n) * a[n] * x**n)}.
  ++ This function is used when Puiseux series are represented by
  ++ a Laurent series and an exponent.

if Coef has IntegralDomain then

  rationalFunction : (%,I) -> Fraction Polynomial Coef
    ++ \spad{rationalFunction(f,k)} returns a rational function
    ++ consisting of the sum of all terms of f of degree <= k.
    ++
    ++X w:SparseUnivariateLaurentSeries(Fraction(Integer),'z,0):=0
    ++X rationalFunction(w,0)

  rationalFunction : (%,I,I) -> Fraction Polynomial Coef
    ++ \spad{rationalFunction(f,k1,k2)} returns a rational function
    ++ consisting of the sum of all terms of f of degree d with
    ++ \spad{k1 <= d <= k2}.

if Coef has Algebra Fraction Integer then

  integrate : % -> %
    ++ \spad{integrate(f(x))} returns an anti-derivative of the power
    ++ series \spad{f(x)} with constant coefficient 1.
    ++ We may integrate a series when we can divide coefficients
    ++ by integers.

  if Coef has integrate: (Coef,Symbol) -> Coef and _
     Coef has variables: Coef -> List Symbol then

    integrate : (%,Symbol) -> %
      ++ \spad{integrate(f(x),y)} returns an anti-derivative of the power
      ++ series \spad{f(x)} with respect to the variable \spad{y}.

  if Coef has TranscendentalFunctionCategory and _
     Coef has PrimitiveFunctionCategory and _
     Coef has AlgebraicallyClosedFunctionSpace Integer then

    integrate : (%,Symbol) -> %
      ++ \spad{integrate(f(x),y)} returns an anti-derivative of
      ++ the power series \spad{f(x)} with respect to the variable
      ++ \spad{y}.

  RadicalCategory
    --++ We provide rational powers when we can divide coefficients
    --++ by integers.

  TranscendentalFunctionCategory
    --++ We provide transcendental functions when we can divide
    --++ coefficients by integers.
```

```
    if Coef has Field then Field
        --++ Univariate Laurent series over a field form a field.
        --++ In fact, K((x)) is the quotient field of K[[x]].
```

———–

— ULSCAT.dotabb —

```
"ULSCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ULSCAT"];
"ULSCAT" -> "UPSCAT"
"ULSCAT" -> "FIELD"
"ULSCAT" -> "TRANFUN"
"ULSCAT" -> "RADCAT"
```

———–

— ULSCAT.dotfull —

```
"UnivariateLaurentSeriesCategory(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ULSCAT"];
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "Field()"
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "RadicalCategory()"
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "TranscendentalFunctionCategory()"
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "UnivariatePowerSeriesCategory(a:Ring,Integer)"
```

———–

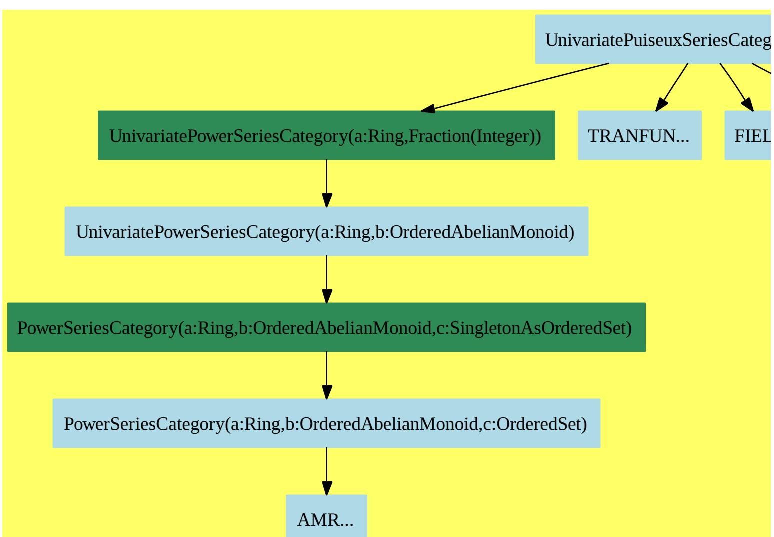
— ULSCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"UnivariateLaurentSeriesCategory(a:Ring)" [color=lightblue];
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "UnivariatePowerSeriesCategory(a:Ring,Integer)"
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "FIELD..."
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "RADCAT..."
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "TRANFUN..."

"UnivariatePowerSeriesCategory(a:Ring,Integer)" [color=seagreen];
"UnivariatePowerSeriesCategory(a:Ring,Integer)" ->
```

```
    "UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"

"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"
 [color=lightblue];
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)" ->
 "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
 [color=seagreen];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
  -> "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
 [color=lightblue];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)" ->
    "AMR..."

"AMR..." [color=lightblue];
"FIELD..." [color=lightblue];
"TRANFUN..." [color=lightblue];
"RADCAT..." [color=lightblue];
}
```

———–

17.0.227 UnivariatePuiseuxSeriesCategory (UPXSCAT)



— **UnivariatePuiseuxSeriesCategory.input** —

```
)set break resume
)sys rm -f UnivariatePuiseuxSeriesCategory.output
```

```
)spool UnivariatePuiseuxSeriesCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariatePuiseuxSeriesCategory
--R
--R UnivariatePuiseuxSeriesCategory(Coef: Ring) is a category constructor
--R Abbreviation for UnivariatePuiseuxSeriesCategory is UPXSCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for UPXSCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Coef,%) -> %                   ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R center : % -> Coef                    coerce : % -> % if Coef has INTDOM
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R complete : % -> %                     degree : % -> Fraction(Integer)
--R ?.? : (%,Fraction(Integer)) -> Coef   hash : % -> SingleInteger
--R inv : % -> % if Coef has FIELD        latex : % -> String
--R leadingCoefficient : % -> Coef        leadingMonomial : % -> %
--R map : ((Coef -> Coef),%) -> %         monomial? : % -> Boolean
--R one? : % -> Boolean                   order : % -> Fraction(Integer)
--R pole? : % -> Boolean                  recip : % -> Union(%,"failed")
--R reductum : % -> %                     sample : () -> %
--R variable : % -> Symbol                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction(Integer)) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?*? : (Fraction(Integer),%) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?**? : (%,Fraction(Integer)) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?**? : (%,%) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?**? : (%,Integer) -> % if Coef has FIELD
--R ?/? : (%,%) -> % if Coef has FIELD
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (Fraction(Integer),Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if Coef has *: (Fraction(Integer),Coef) -> Coef
--R D : (%,Symbol) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Fraction(Integer),Coef) -> Coef
--R D : (%,List(Symbol)) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Fraction(Integer),Coef) -> Coef
--R D : (%,Symbol,NonNegativeInteger) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Fraction(Integer),Coef) -
--R D : (%,List(Symbol),List(NonNegativeInteger)) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Fraction(Inte
--R ?^? : (%,Integer) -> % if Coef has FIELD
--R acos : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acosh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acot : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acoth : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acsc : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acsch : % -> % if Coef has ALGEBRA(FRAC(INT))
```

```
--R approximate : (%,Fraction(Integer)) -> Coef if Coef has **: (Coef,Fraction(Integer)) -> Coef and Coef has co
--R asec : % -> % if Coef has ALGEBRA(FRAC(INT))
--R asech : % -> % if Coef has ALGEBRA(FRAC(INT))
--R asin : % -> % if Coef has ALGEBRA(FRAC(INT))
--R asinh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA(FRAC(INT))
--R atanh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
--R coefficient : (%,Fraction(Integer)) -> Coef
--R coerce : Fraction(Integer) -> % if Coef has ALGEBRA(FRAC(INT))
--R coerce : Coef -> % if Coef has COMRING
--R cos : % -> % if Coef has ALGEBRA(FRAC(INT))
--R cosh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R cot : % -> % if Coef has ALGEBRA(FRAC(INT))
--R coth : % -> % if Coef has ALGEBRA(FRAC(INT))
--R csc : % -> % if Coef has ALGEBRA(FRAC(INT))
--R csch : % -> % if Coef has ALGEBRA(FRAC(INT))
--R differentiate : % -> % if Coef has *: (Fraction(Integer),Coef) -> Coef
--R differentiate : (%,NonNegativeInteger) -> % if Coef has *: (Fraction(Integer),Coef) -> Coef
--R differentiate : (%,Symbol) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Fraction(Integer),Coef) -> Coef
--R differentiate : (%,List(Symbol)) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Fraction(Integer),Coef) ->
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Fraction(Inte
--R differentiate : (%,List(Symbol),List(NonNegativeInteger)) -> % if Coef has PDRING(SYMBOL) and Coef has *: (F
--R divide : (%,%) -> Record(quotient: %,remainder: %) if Coef has FIELD
--R ?.? : (%,%) -> % if Fraction(Integer) has SGROUP
--R euclideanSize : % -> NonNegativeInteger if Coef has FIELD
--R eval : (%,Coef) -> Stream(Coef) if Coef has **: (Coef,Fraction(Integer)) -> Coef
--R exp : % -> % if Coef has ALGEBRA(FRAC(INT))
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed") if Coef has FIELD
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R extend : (%,Fraction(Integer)) -> %
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if Coef has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if Coef has FIELD
--R factor : % -> Factored(%) if Coef has FIELD
--R gcd : (%,%) -> % if Coef has FIELD
--R gcd : List(%) -> % if Coef has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R integrate : (%,Symbol) -> % if Coef has ACFS(INT) and Coef has PRIMCAT and Coef has TRANFUN and Coef has ALG
--R integrate : % -> % if Coef has ALGEBRA(FRAC(INT))
--R lcm : (%,%) -> % if Coef has FIELD
--R lcm : List(%) -> % if Coef has FIELD
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %) if Coef has FIELD
--R log : % -> % if Coef has ALGEBRA(FRAC(INT))
--R monomial : (%,List(SingletonAsOrderedSet),List(Fraction(Integer))) -> %
--R monomial : (%,SingletonAsOrderedSet,Fraction(Integer)) -> %
--R monomial : (Coef,Fraction(Integer)) -> %
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed") if Coef has FIELD
--R multiplyExponents : (%,Fraction(Integer)) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA(FRAC(INT))
--R order : (%,Fraction(Integer)) -> Fraction(Integer)
--R pi : () -> % if Coef has ALGEBRA(FRAC(INT))
```

```
--R prime? : % -> Boolean if Coef has FIELD
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %) if Coef has FIELD
--R ?quo? : (%,%) -> % if Coef has FIELD
--R ?rem? : (%,%) -> % if Coef has FIELD
--R sec : % -> % if Coef has ALGEBRA(FRAC(INT))
--R sech : % -> % if Coef has ALGEBRA(FRAC(INT))
--R series : (NonNegativeInteger,Stream(Record(k: Fraction(Integer),c: Coef))) -> %
--R sin : % -> % if Coef has ALGEBRA(FRAC(INT))
--R sinh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R sizeLess? : (%,%) -> Boolean if Coef has FIELD
--R sqrt : % -> % if Coef has ALGEBRA(FRAC(INT))
--R squareFree : % -> Factored(%) if Coef has FIELD
--R squareFreePart : % -> % if Coef has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA(FRAC(INT))
--R tanh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R terms : % -> Stream(Record(k: Fraction(Integer),c: Coef))
--R truncate : (%,Fraction(Integer),Fraction(Integer)) -> %
--R truncate : (%,Fraction(Integer)) -> %
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R variables : % -> List(SingletonAsOrderedSet)
--R
--E 1

)spool
)lisp (bye)
```

— UnivariatePuiseuxSeriesCategory.help —

```
====================================================================
UnivariatePuiseuxSeriesCategory examples
====================================================================

UnivariatePuiseuxSeriesCategory is the category of Puiseux series
in one variable.

See Also:
o )show UnivariatePuiseuxSeriesCategory
```

See:

⇒ "UnivariatePuiseuxSeriesConstructorCategory" (UPXSCCA) 18.0.236 on page 1817
⇐ "TranscendentalFunctionCategory" (TRANFUN) 3.0.58 on page 192
⇐ "Field" (FIELD) 16.0.209 on page 1413
⇐ "RadicalCategory" (RADCAT) 2.0.36 on page 106
⇐ "UnivariatePowerSeriesCategory" (UPSCAT) 15.0.208 on page 1401

Exports:

0	1	acos	acosh
acot	acoth	acsc	acsch
approximate	asec	asech	asin
asinh	associates?	atan	atanh
center	characteristic	charthRoot	coefficient
coerce	complete	cos	cosh
cot	coth	csc	csch
D	degree	differentiate	divide
euclideanSize	eval	exp	expressIdealMember
exquo	extend	extendedEuclidean	factor
gcd	gcdPolynomial	hash	integrate
inv	latex	lcm	leadingCoefficient
leadingMonomial	log	map	monomial
monomial?	multiEuclidean	multiplyExponents	nthRoot
one?	order	pi	pole?
prime?	principalIdeal	recip	reductum
sample	sec	sech	series
sin	sinh	sizeLess?	sqrt
squareFree	squareFreePart	subtractIfCan	tan
tanh	terms	truncate	unit?
unitCanonical	unitNormal	variable	variables
zero?	?*?	?**?	?+?
?-?	-?	?=?	?~=?
?/?	?^?	?.?	?quo?
?rem?			

Attributes Exported:

- if #1 has Field then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if #1 has Field then canonicalsClosed where **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- if #1 has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x*y \neq 0$ implies both x and y are non-zero.
- if #1 has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation $" * ": (D, D) -> D$ which is commutative.

These are directly exported but not implemented:

```
integrate : % -> % if Coef has ALGEBRA FRAC INT
integrate : (%,Symbol) -> %
  if Coef has ACFS INT
  and Coef has PRIMCAT
```

```
  and Coef has TRANFUN
  and Coef has ALGEBRA FRAC INT
  or Coef has variables: Coef -> List Symbol
  and Coef has integrate: (Coef,Symbol) -> Coef
  and Coef has ALGEBRA FRAC INT
multiplyExponents : (%,Fraction Integer) -> %
series : (NonNegativeInteger,Stream Record(k: Fraction Integer,c: Coef)) -> %
```

These exports come from (p1401) UnivariatePowerSeriesCategory(Coef,RN) where Coef:Ring and RN:Fraction(Integer):

```
0 : () -> %
1 : () -> %
approximate : (%,Fraction Integer) -> Coef
  if Coef has **: (Coef,Fraction Integer) -> Coef
  and Coef has coerce: Symbol -> Coef
associates? : (%,%) -> Boolean if Coef has INTDOM
center : % -> Coef
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
coefficient : (%,Fraction Integer) -> Coef
coerce : % -> % if Coef has INTDOM
coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
coerce : Coef -> % if Coef has COMRING
coerce : Integer -> %
coerce : % -> OutputForm
complete : % -> %
D : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,NonNegativeInteger) -> %
  if Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,List Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,Symbol,NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,List Symbol,List NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
degree : % -> Fraction Integer
differentiate : (%,Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,List Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,Symbol,NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
```

```
differentiate : % -> %
  if Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,NonNegativeInteger) -> %
  if Coef has *: (Fraction Integer,Coef) -> Coef
eval : (%,Coef) -> Stream Coef
  if Coef has **: (Coef,Fraction Integer) -> Coef
exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
extend : (%,Fraction Integer) -> %
hash : % -> SingleInteger
latex : % -> String
leadingCoefficient : % -> Coef
leadingMonomial : % -> %
map : ((Coef -> Coef),%) -> %
monomial : (%,List SingletonAsOrderedSet,List Fraction Integer) -> %
monomial : (Coef,Fraction Integer) -> %
monomial : (%,SingletonAsOrderedSet,Fraction Integer) -> %
monomial? : % -> Boolean
multiplyExponents : (%,PositiveInteger) -> %
one? : % -> Boolean
order : (%,Fraction Integer) -> Fraction Integer
order : % -> Fraction Integer
pole? : % -> Boolean
recip : % -> Union(%,"failed")
reductum : % -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
terms : % -> Stream Record(k: Fraction Integer,c: Coef)
truncate : (%,Fraction Integer,Fraction Integer) -> %
truncate : (%,Fraction Integer) -> %
unit? : % -> Boolean if Coef has INTDOM
unitCanonical : % -> % if Coef has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
  if Coef has INTDOM
variable : % -> Symbol
variables : % -> List SingletonAsOrderedSet
zero? : % -> Boolean
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (Integer,%) -> %
?*? : (Coef,%) -> %
?*? : (%,Coef) -> %
?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
?/? : (%,Coef) -> % if Coef has FIELD
-? : % -> %
```

```
?.? : (%,%) -> % if Fraction Integer has SGROUP
?.? : (%,Fraction Integer) -> Coef
```

These exports come from (p192) TranscendentalFunctionCategory():

```
acos : % -> % if Coef has ALGEBRA FRAC INT
acosh : % -> % if Coef has ALGEBRA FRAC INT
acot : % -> % if Coef has ALGEBRA FRAC INT
acoth : % -> % if Coef has ALGEBRA FRAC INT
acsc : % -> % if Coef has ALGEBRA FRAC INT
acsch : % -> % if Coef has ALGEBRA FRAC INT
asec : % -> % if Coef has ALGEBRA FRAC INT
asech : % -> % if Coef has ALGEBRA FRAC INT
asin : % -> % if Coef has ALGEBRA FRAC INT
asinh : % -> % if Coef has ALGEBRA FRAC INT
atan : % -> % if Coef has ALGEBRA FRAC INT
atanh : % -> % if Coef has ALGEBRA FRAC INT
cos : % -> % if Coef has ALGEBRA FRAC INT
cosh : % -> % if Coef has ALGEBRA FRAC INT
cot : % -> % if Coef has ALGEBRA FRAC INT
coth : % -> % if Coef has ALGEBRA FRAC INT
csc : % -> % if Coef has ALGEBRA FRAC INT
csch : % -> % if Coef has ALGEBRA FRAC INT
exp : % -> % if Coef has ALGEBRA FRAC INT
log : % -> % if Coef has ALGEBRA FRAC INT
pi : () -> % if Coef has ALGEBRA FRAC INT
sec : % -> % if Coef has ALGEBRA FRAC INT
sech : % -> % if Coef has ALGEBRA FRAC INT
sin : % -> % if Coef has ALGEBRA FRAC INT
sinh : % -> % if Coef has ALGEBRA FRAC INT
tan : % -> % if Coef has ALGEBRA FRAC INT
tanh : % -> % if Coef has ALGEBRA FRAC INT
?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
```

These exports come from (p1413) Field():

```
divide : (%,%) -> Record(quotient: %,remainder: %)
  if Coef has FIELD
euclideanSize : % -> NonNegativeInteger if Coef has FIELD
expressIdealMember : (List %,%) -> Union(List %,"failed")
  if Coef has FIELD
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
  if Coef has FIELD
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
  if Coef has FIELD
factor : % -> Factored % if Coef has FIELD
gcd : (%,%) -> % if Coef has FIELD
gcd : List % -> % if Coef has FIELD
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
       if Coef has FIELD
inv : % -> % if Coef has FIELD
lcm : (%,%) -> % if Coef has FIELD
lcm : List % -> % if Coef has FIELD
```

```
multiEuclidean : (List %,%) -> Union(List %,"failed")
  if Coef has FIELD
prime? : % -> Boolean if Coef has FIELD
principalIdeal : List % -> Record(coef: List %,generator: %)
  if Coef has FIELD
sizeLess? : (%,%) -> Boolean if Coef has FIELD
squareFree : % -> Factored % if Coef has FIELD
squareFreePart : % -> % if Coef has FIELD
?**? : (%,Integer) -> % if Coef has FIELD
?^? : (%,Integer) -> % if Coef has FIELD
?/? : (%,%) -> % if Coef has FIELD
?quo? : (%,%) -> % if Coef has FIELD
?rem? : (%,%) -> % if Coef has FIELD
```

These exports come from (p106) RadicalCategory():

```
nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
sqrt : % -> % if Coef has ALGEBRA FRAC INT
?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
```

— **UnivariatePuiseuxSeriesCategory.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#UPXSCAT">
UnivariatePuiseuxSeriesCategory (UPXSCAT)</a></h2>
</body>
```

———-

— **category UPXSCAT UnivariatePuiseuxSeriesCategory** —

```
)abbrev category UPXSCAT UnivariatePuiseuxSeriesCategory
++ Author: Clifton J. Williamson
++ Date Created: 21 December 1989
++ Date Last Updated: 20 September 1993
++ Description:
++ \spadtype{UnivariatePuiseuxSeriesCategory} is the category of Puiseux
++ series in one variable.

UnivariatePuiseuxSeriesCategory(Coef) : Category == SIG where
  Coef : Ring

  NNI  ==> NonNegativeInteger
  RN   ==> Fraction Integer
  Term ==> Record(k:RN,c:Coef)

  SIG ==> UnivariatePowerSeriesCategory(Coef,RN) with

    series : (NNI,Stream Term) -> %
      ++ \spad{series(n,st)} creates a series from a common denomiator and
      ++ a stream of non-zero terms, where a term is an exponent-coefficient
      ++ pair.  The terms in the stream should be ordered by increasing order
      ++ of exponents and \spad{n} should be a common denominator for the
      ++ exponents in the stream of terms.
```

```
    multiplyExponents : (%,Fraction Integer) -> %
      ++ \spad{multiplyExponents(f,r)} multiplies all exponents of the power
      ++ series f by the positive rational number r.

    if Coef has Algebra Fraction Integer then

      integrate : % -> %
        ++ \spad{integrate(f(x))} returns an anti-derivative of the power
        ++ series \spad{f(x)} with constant coefficient 1.
        ++ We may integrate a series when we can divide coefficients
        ++ by rational numbers.

      if Coef has integrate : (Coef,Symbol) -> Coef and _
         Coef has variables : Coef -> List Symbol then

        integrate : (%,Symbol) -> %
          ++ \spad{integrate(f(x),var)} returns an anti-derivative of the power
          ++ series \spad{f(x)} with respect to the variable \spad{var}.

      if Coef has TranscendentalFunctionCategory and _
         Coef has PrimitiveFunctionCategory and _
         Coef has AlgebraicallyClosedFunctionSpace Integer then

        integrate : (%,Symbol) -> %
          ++ \spad{integrate(f(x),y)} returns an anti-derivative of
          ++ the power series \spad{f(x)} with respect to the variable
          ++ \spad{y}.

      RadicalCategory
        --++ We provide rational powers when we can divide coefficients
        --++ by integers.

      TranscendentalFunctionCategory
        --++ We provide transcendental functions when we can divide
        --++ coefficients by integers.

    if Coef has Field then Field
        --++ Univariate Puiseux series over a field form a field.
```

———-

— UPXSCAT.dotabb —

```
"UPXSCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UPXSCAT"];
"UPXSCAT" -> "TRANFUN"
"UPXSCAT" -> "FIELD"
"UPXSCAT" -> "RADCAT"
"UPXSCAT" -> "UPSCAT"
```

———-

— UPXSCAT.dotfull —

```
"UnivariatePuiseuxSeriesCategory(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UPXSCAT"];
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))"
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "TranscendentalFunctionCategory()"
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "Field()"
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "RadicalCategory()"
```

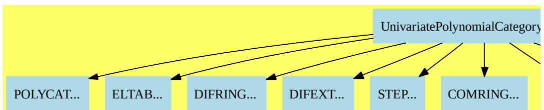
————-

— UPXSCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"UnivariatePuiseuxSeriesCategory(a:Ring)" [color=lightblue];
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))"
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "TRANFUN..."
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "FIELD..."
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "RADCAT..."

"UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))" [color=seagreen];
"UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))" ->
    "UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"

"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"
 [color=lightblue];
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)" ->
 "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
 [color=seagreen];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
  -> "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
 [color=lightblue];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)" ->
    "AMR..."

"TRANFUN..." [color=lightblue];
"FIELD..." [color=lightblue];
```

```
"RADCAT..." [color=lightblue];
"AMR..." [color=lightblue];
}
```

17.0.228 UnivariatePolynomialCategory (UPOLYC)



— UnivariatePolynomialCategory.input —

```
)set break resume
)sys rm -f UnivariatePolynomialCategory.output
)spool UnivariatePolynomialCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 4
)show UnivariatePolynomialCategory
--R
--R UnivariatePolynomialCategory(R: Ring) is a category constructor
--R Abbreviation for UnivariatePolynomialCategory is UPOLYC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for UPOLYC
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,R) -> %                      ?*? : (R,%) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?/? : (%,R) -> % if R has FIELD
--R ?=? : (%,%) -> Boolean                D : (%,(R -> R)) -> %
--R D : % -> %                            D : (%,NonNegativeInteger) -> %
--R D : (%,SingletonAsOrderedSet) -> %    1 : () -> %
--R 0 : () -> %                           ?^? : (%,NonNegativeInteger) -> %
--R ?^? : (%,PositiveInteger) -> %        coefficients : % -> List(R)
--R coerce : % -> % if R has INTDOM       coerce : SingletonAsOrderedSet -> %
--R coerce : R -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              content : % -> R if R has GCDDOM
--R degree : % -> NonNegativeInteger      differentiate : (%,(R -> R)) -> %
--R differentiate : % -> %                ?.? : (%,%) -> %
--R ?.? : (%,R) -> R                      eval : (%,List(%),List(%)) -> %
--R eval : (%,%,%) -> %                   eval : (%,Equation(%)) -> %
--R eval : (%,List(Equation(%))) -> %     gcd : (%,%) -> % if R has GCDDOM
--R gcd : List(%) -> % if R has GCDDOM    ground : % -> R
```

```
--R ground? : % -> Boolean               hash : % -> SingleInteger
--R init : () -> % if R has STEP         latex : % -> String
--R lcm : (%,%) -> % if R has GCDDOM     lcm : List(%) -> % if R has GCDDOM
--R leadingCoefficient : % -> R          leadingMonomial : % -> %
--R map : ((R -> R),%) -> %              max : (%,%) -> % if R has ORDSET
--R min : (%,%) -> % if R has ORDSET     monomial? : % -> Boolean
--R monomials : % -> List(%)             one? : % -> Boolean
--R primitiveMonomials : % -> List(%)    pseudoRemainder : (%,%) -> %
--R ?quo? : (%,%) -> % if R has FIELD    recip : % -> Union(%,"failed")
--R reductum : % -> %                    ?rem? : (%,%) -> % if R has FIELD
--R retract : % -> R                     sample : () -> %
--R unvectorise : Vector(R) -> %         zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (Fraction(Integer),%) -> % if R has ALGEBRA(FRAC(INT))
--R ?*? : (%,Fraction(Integer)) -> % if R has ALGEBRA(FRAC(INT))
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R D : (%,(R -> R),NonNegativeInteger) -> %
--R D : (%,List(Symbol),List(NonNegativeInteger)) -> % if R has PDRING(SYMBOL)
--R D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING(SYMBOL)
--R D : (%,List(Symbol)) -> % if R has PDRING(SYMBOL)
--R D : (%,Symbol) -> % if R has PDRING(SYMBOL)
--R D : (%,List(SingletonAsOrderedSet),List(NonNegativeInteger)) -> %
--R D : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R D : (%,List(SingletonAsOrderedSet)) -> %
--R associates? : (%,%) -> Boolean if R has INTDOM
--R binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if and(has($,CharacteristicNonZero),has(R,PolynomialFactorizationExplici
--R coefficient : (%,List(SingletonAsOrderedSet),List(NonNegativeInteger)) -> %
--R coefficient : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R coefficient : (%,NonNegativeInteger) -> R
--R coerce : Fraction(Integer) -> % if R has RETRACT(FRAC(INT)) or R has ALGEBRA(FRAC(INT))
--R composite : (Fraction(%),%) -> Union(Fraction(%),"failed") if R has INTDOM
--R composite : (%,%) -> Union(%,"failed") if R has INTDOM
--R conditionP : Matrix(%) -> Union(Vector(%),"failed") if and(has($,CharacteristicNonZero),has(R,PolynomialFact
--R content : (%,SingletonAsOrderedSet) -> % if R has GCDDOM
--R convert : % -> InputForm if SingletonAsOrderedSet has KONVERT(INFORM) and R has KONVERT(INFORM)
--R convert : % -> Pattern(Integer) if SingletonAsOrderedSet has KONVERT(PATTERN(INT)) and R has KONVERT(PATTERN
--R convert : % -> Pattern(Float) if SingletonAsOrderedSet has KONVERT(PATTERN(FLOAT)) and R has KONVERT(PATTERN
--R degree : (%,List(SingletonAsOrderedSet)) -> List(NonNegativeInteger)
--R degree : (%,SingletonAsOrderedSet) -> NonNegativeInteger
--R differentiate : (%,(R -> R),%) -> %
--R differentiate : (%,(R -> R),NonNegativeInteger) -> %
--R differentiate : (%,List(Symbol),List(NonNegativeInteger)) -> % if R has PDRING(SYMBOL)
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING(SYMBOL)
--R differentiate : (%,List(Symbol)) -> % if R has PDRING(SYMBOL)
--R differentiate : (%,Symbol) -> % if R has PDRING(SYMBOL)
--R differentiate : (%,NonNegativeInteger) -> %
--R differentiate : (%,List(SingletonAsOrderedSet),List(NonNegativeInteger)) -> %
--R differentiate : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R differentiate : (%,List(SingletonAsOrderedSet)) -> %
```

```
--R differentiate : (%,SingletonAsOrderedSet) -> %
--R discriminant : % -> R if R has COMRING
--R discriminant : (%,SingletonAsOrderedSet) -> % if R has COMRING
--R divide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R divideExponents : (%,NonNegativeInteger) -> Union(%,"failed")
--R ?.? : (%,Fraction(%)) -> Fraction(%) if R has INTDOM
--R elt : (Fraction(%),R) -> R if R has FIELD
--R elt : (Fraction(%),Fraction(%)) -> Fraction(%) if R has INTDOM
--R euclideanSize : % -> NonNegativeInteger if R has FIELD
--R eval : (%,List(SingletonAsOrderedSet),List(%)) -> %
--R eval : (%,SingletonAsOrderedSet,%) -> %
--R eval : (%,List(SingletonAsOrderedSet),List(R)) -> %
--R eval : (%,SingletonAsOrderedSet,R) -> %
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed") if R has FIELD
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if R has FIELD
--R factor : % -> Factored(%) if R has PFECAT
--R factorPolynomial : SparseUnivariatePolynomial(%) -> Factored(SparseUnivariatePolynomial(%)) if R has PFECAT
--R factorSquareFreePolynomial : SparseUnivariatePolynomial(%) -> Factored(SparseUnivariatePolynomial(%)) if R h
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R integrate : % -> % if R has ALGEBRA(FRAC(INT))
--R isExpt : % -> Union(Record(var: SingletonAsOrderedSet,exponent: NonNegativeInteger),"failed")
--R isPlus : % -> Union(List(%),"failed")
--R isTimes : % -> Union(List(%),"failed")
--R karatsubaDivide : (%,NonNegativeInteger) -> Record(quotient: %,remainder: %)
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %) if R has GCDDOM
--R mainVariable : % -> Union(SingletonAsOrderedSet,"failed")
--R makeSUP : % -> SparseUnivariatePolynomial(R)
--R mapExponents : ((NonNegativeInteger -> NonNegativeInteger),%) -> %
--R minimumDegree : (%,List(SingletonAsOrderedSet)) -> List(NonNegativeInteger)
--R minimumDegree : (%,SingletonAsOrderedSet) -> NonNegativeInteger
--R minimumDegree : % -> NonNegativeInteger
--R monicDivide : (%,%) -> Record(quotient: %,remainder: %)
--R monicDivide : (%,%,SingletonAsOrderedSet) -> Record(quotient: %,remainder: %)
--R monomial : (%,List(SingletonAsOrderedSet),List(NonNegativeInteger)) -> %
--R monomial : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
--R monomial : (R,NonNegativeInteger) -> %
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed") if R has FIELD
--R multiplyExponents : (%,NonNegativeInteger) -> %
--R multivariate : (SparseUnivariatePolynomial(%),SingletonAsOrderedSet) -> %
--R multivariate : (SparseUnivariatePolynomial(R),SingletonAsOrderedSet) -> %
--R nextItem : % -> Union(%,"failed") if R has STEP
--R numberOfMonomials : % -> NonNegativeInteger
--R order : (%,%) -> NonNegativeInteger if R has INTDOM
--R patternMatch : (%,Pattern(Integer),PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%) if Single
--R patternMatch : (%,Pattern(Float),PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%) if SingletonAsO
--R pomopo! : (%,R,NonNegativeInteger,%) -> %
--R prime? : % -> Boolean if R has PFECAT
--R primitivePart : (%,SingletonAsOrderedSet) -> % if R has GCDDOM
--R primitivePart : % -> % if R has GCDDOM
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %) if R has FIELD
--R pseudoDivide : (%,%) -> Record(coef: R,quotient: %,remainder: %) if R has INTDOM
```

```
--R pseudoQuotient : (%,%) -> % if R has INTDOM
--R reducedSystem : Matrix(%) -> Matrix(R)
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(R),vec: Vector(R))
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(Integer),vec: Vector(Integer)) if R has LINEXP(I
--R reducedSystem : Matrix(%) -> Matrix(Integer) if R has LINEXP(INT)
--R resultant : (%,%) -> R if R has COMRING
--R resultant : (%,%,SingletonAsOrderedSet) -> % if R has COMRING
--R retract : % -> SingletonAsOrderedSet
--R retract : % -> Integer if R has RETRACT(INT)
--R retract : % -> Fraction(Integer) if R has RETRACT(FRAC(INT))
--R retractIfCan : % -> Union(SingletonAsOrderedSet,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT(INT)
--R retractIfCan : % -> Union(Fraction(Integer),"failed") if R has RETRACT(FRAC(INT))
--R retractIfCan : % -> Union(R,"failed")
--R separate : (%,%) -> Record(primePart: %,commonPart: %) if R has GCDDOM
--R shiftLeft : (%,NonNegativeInteger) -> %
--R shiftRight : (%,NonNegativeInteger) -> %
--R sizeLess? : (%,%) -> Boolean if R has FIELD
--R solveLinearPolynomialEquation : (List(SparseUnivariatePolynomial(%)),SparseUnivariatePolynomial(%)) -> Union
--R squareFree : % -> Factored(%) if R has GCDDOM
--R squareFreePart : % -> % if R has GCDDOM
--R squareFreePolynomial : SparseUnivariatePolynomial(%) -> Factored(SparseUnivariatePolynomial(%)) if R has PFE
--R subResultantGcd : (%,%) -> % if R has INTDOM
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R totalDegree : (%,List(SingletonAsOrderedSet)) -> NonNegativeInteger
--R totalDegree : % -> NonNegativeInteger
--R unit? : % -> Boolean if R has INTDOM
--R unitCanonical : % -> % if R has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
--R univariate : % -> SparseUnivariatePolynomial(R)
--R univariate : (%,SingletonAsOrderedSet) -> SparseUnivariatePolynomial(%)
--R unmakeSUP : SparseUnivariatePolynomial(R) -> %
--R variables : % -> List(SingletonAsOrderedSet)
--R vectorise : (%,NonNegativeInteger) -> Vector(R)
--R
--E 1

--S 2 of 4
t1:UP(x,FRAC(INT)):=3*x^3+4*x^2+5*x+6
--R
--R          3     2
--R   (1)  3x  + 4x  + 5x + 6
--R                              Type: UnivariatePolynomial(x,Fraction(Integer))
--E 2

--S 3 of 4
t2:=vectorise(t1,4)
--R
--R   (2)  [6,5,4,3]
--R                                              Type: Vector(Fraction(Integer))
--E 3

--S 4 of 4
t3:UP(x,FRAC(INT)):=unvectorise(t2)
```

```
--R
--R          3     2
--R   (3)  3x  + 4x  + 5x + 6
--R                              Type: UnivariatePolynomial(x,Fraction(Integer))
--E 4

)spool
)lisp (bye)
```

———-

— **UnivariatePolynomialCategory.help** —

```
====================================================================
UnivariatePolynomialCategory examples
====================================================================

The category of univariate polynomials over a ring R. No particular
model is assumed - implementations can be either sparse or dense.

See Also:
o )show UnivariatePolynomialCategory
```

———-

See:

⇐ "Algebra" (ALGEBRA) 11.0.176 on page 1121
⇐ "CommutativeRing" (COMRING) 10.0.161 on page 1019
⇐ "DifferentialExtension" (DIFEXT) 11.0.177 on page 1127
⇐ "DifferentialRing" (DIFRING) 10.0.162 on page 1024
⇐ "Eltable" (ELTAB) 2.0.19 on page 54
⇐ "Field" (FIELD) 16.0.209 on page 1413
⇐ "GcdDomain" (GCDDOM) 13.0.197 on page 1319
⇐ "IntegralDomain" (INTDOM) 12.0.188 on page 1225
⇐ "PolynomialCategory" (POLYCAT) 16.0.213 on page 1449
⇐ "PolynomialFactorizationExplicit" (PFECAT) 15.0.207 on page 1393
⇐ "StepThrough" (STEP) 4.0.88 on page 367

Exports:

0	1	associates?
binomThmExpt	characteristic	charthRoot
coefficient	coefficients	coerce
composite	conditionP	content
convert	D	degree
differentiate	discriminant	divide
divideExponents	elt	euclideanSize
eval	expressIdealMember	exquo
extendedEuclidean	factor	factorPolynomial
factorSquareFreePolynomial	gcd	gcdPolynomial
ground	ground?	hash
init	integrate	isExpt
isPlus	isTimes	karatsubaDivide
latex	lcm	leadingCoefficient
leadingMonomial	mainVariable	makeSUP
map	mapExponents	max
min	minimumDegree	monicDivide
monomial	monomial?	monomials
multiEuclidean	multiplyExponents	multivariate
nextItem	numberOfMonomials	one?
order	patternMatch	pomopo!
prime?	primitiveMonomials	primitivePart
principalIdeal	pseudoDivide	pseudoQuotient
pseudoRemainder	recip	reducedSystem
reductum	resultant	retract
retractIfCan	sample	separate
shiftLeft	shiftRight	sizeLess?
solveLinearPolynomialEquation	squareFree	squareFreePart
squareFreePolynomial	subResultantGcd	subtractIfCan
totalDegree	unit?	unitCanonical
unitNormal	univariate	unmakeSUP
variables	vectorise	zero?
?*?	?**?	?+?
?-?	-?	?=?
?^?	?.?	?~=?
?/?	?<?	?<=?
?>?	?>=?	?quo?
?rem?		

Attributes exported:

- if $ has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation $" * " : (D,D) -> D$ which is commutative.
- if $ has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if R has Field then additiveValuation where **additiveValuation** implies `euclideanSize(a*b)=euclideanSize(a)+euclideanSize(b)`.
- if $ has canonicalUnitNormal then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) =`

unitCanonical(b).

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
discriminant : % -> R if R has COMRING
divideExponents : (%,NonNegativeInteger) -> Union(%,"failed")
monicDivide : (%,%) -> Record(quotient: %,remainder: %)
multiplyExponents : (%,NonNegativeInteger) -> %
pseudoRemainder : (%,%) -> %
resultant : (%,%) -> R if R has COMRING
subResultantGcd : (%,%) -> % if R has INTDOM
```

These are implemented by this category:

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean if R has INTDOM
binomThmExpt : (%,%,NonNegativeInteger) -> %
    if R has COMRING
characteristic : () -> NonNegativeInteger
coefficient : (%,NonNegativeInteger) -> R
coefficients : % -> List R
coerce : R -> %
coerce : Fraction Integer -> %
    if R has RETRACT FRAC INT
    or R has ALGEBRA FRAC INT
coerce : % -> % if R has INTDOM
content : % -> R if R has GCDDOM
coerce : Integer -> %
coerce : % -> OutputForm
coerce : SingletonAsOrderedSet -> %
composite : (Fraction %,%) -> Union(Fraction %,"failed")
    if R has INTDOM
composite : (%,%) -> Union(%,"failed")
    if R has INTDOM
content : (%,SingletonAsOrderedSet) -> %
    if R has GCDDOM
D : (%,List SingletonAsOrderedSet) -> %
D : (%,SingletonAsOrderedSet) -> %
D : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
D : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
degree : % -> NonNegativeInteger
degree :
  (%,List SingletonAsOrderedSet) -> List NonNegativeInteger
differentiate :
  (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
differentiate :
  (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
differentiate : (%,List SingletonAsOrderedSet) -> %
differentiate : (%,(R -> R),%) -> %
differentiate : (%,(R -> R)) -> %
```

```
differentiate : % -> %
differentiate : (%,SingletonAsOrderedSet) -> %
divide : (%,%) -> Record(quotient: %,remainder: %)
    if R has FIELD
elt : (Fraction %,Fraction %) -> Fraction %
    if R has INTDOM
elt : (Fraction %,R) -> R if R has FIELD
euclideanSize : % -> NonNegativeInteger
    if R has FIELD
eval : (%,List SingletonAsOrderedSet,List %) -> %
eval : (%,SingletonAsOrderedSet,%) -> %
eval : (%,List SingletonAsOrderedSet,List R) -> %
eval : (%,SingletonAsOrderedSet,R) -> %
eval : (%,List Equation %) -> %
eval : (%,List %,List %) -> %
eval : (%,%,%) -> %
eval : (%,Equation %) -> %
exquo : (%,R) -> Union(%,"failed")
    if R has INTDOM
exquo : (%,%) -> Union(%,"failed")
    if R has INTDOM
factor : % -> Factored % if R has PFECAT
factorPolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if R has PFECAT
factorSquareFreePolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if R has PFECAT
gcd : (%,%) -> % if R has GCDDOM
gcd : List % -> % if R has GCDDOM
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
       if R has GCDDOM
ground : % -> R
ground? : % -> Boolean
hash : % -> SingleInteger
init : () -> % if R has STEP
integrate : % -> % if R has ALGEBRA FRAC INT
karatsubaDivide :
  (%,NonNegativeInteger) -> Record(quotient: %,remainder: %)
latex : % -> String
lcm : (%,%) -> % if R has GCDDOM
lcm : List % -> % if R has GCDDOM
leadingCoefficient : % -> R
leadingMonomial : % -> %
mainVariable : % -> Union(SingletonAsOrderedSet,"failed")
makeSUP : % -> SparseUnivariatePolynomial R
map : ((R -> R),%) -> %
mapExponents :
  ((NonNegativeInteger -> NonNegativeInteger),%) -> %
```

```
max : (%,%) -> % if R has ORDSET
min : (%,%) -> % if R has ORDSET
minimumDegree :
  (%,SingletonAsOrderedSet) -> NonNegativeInteger
minimumDegree :
  (%,List SingletonAsOrderedSet) -> List NonNegativeInteger
monomial : (R,NonNegativeInteger) -> %
monomial : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
monomial? : % -> Boolean
nextItem : % -> Union(%,"failed") if R has STEP
numberOfMonomials : % -> NonNegativeInteger
one? : % -> Boolean
order : (%,%) -> NonNegativeInteger
    if R has INTDOM
pomopo! : (%,R,NonNegativeInteger,%) -> %
prime? : % -> Boolean if R has PFECAT
pseudoDivide :
  (%,%) -> Record(coef: R,quotient: %,remainder: %)
    if R has INTDOM
pseudoQuotient : (%,%) -> % if R has INTDOM
recip : % -> Union(%,"failed")
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix Integer,vec: Vector Integer)
    if R has LINEXP INT
reducedSystem : Matrix % -> Matrix Integer
    if R has LINEXP INT
reductum : % -> %
retract : % -> R
retract : % -> Integer if R has RETRACT INT
retract : % -> Fraction Integer
    if R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed")
    if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
    if R has RETRACT FRAC INT
retractIfCan : % -> Union(R,"failed")
sample : () -> %
separate : (%,%) -> Record(primePart: %,commonPart: %)
    if R has GCDDOM
shiftLeft : (%,NonNegativeInteger) -> %
shiftRight : (%,NonNegativeInteger) -> %
solveLinearPolynomialEquation :
  (List SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     Union(List SparseUnivariatePolynomial %,"failed")
       if R has PFECAT
squareFree : % -> Factored % if R has GCDDOM
squareFreePart : % -> % if R has GCDDOM
squareFreePolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
     if R has PFECAT
subtractIfCan : (%,%) -> Union(%,"failed")
totalDegree :
```

```
  (%,List SingletonAsOrderedSet) -> NonNegativeInteger
unit? : % -> Boolean if R has INTDOM
unitCanonical : % -> % if R has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if R has INTDOM
unmakeSUP : SparseUnivariatePolynomial R -> %
variables : % -> List SingletonAsOrderedSet
vectorise : (%,NonNegativeInteger) -> Vector R
zero? : % -> Boolean
?<=? : (%,%) -> Boolean if R has ORDSET
?>? : (%,%) -> Boolean if R has ORDSET
?>=? : (%,%) -> Boolean if R has ORDSET
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,R) -> %
?*? : (R,%) -> %
?*? : (Fraction Integer,%) -> %
    if R has ALGEBRA FRAC INT
?*? : (%,Fraction Integer) -> %
    if R has ALGEBRA FRAC INT
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?/? : (%,R) -> % if R has FIELD
?-? : (%,%) -> %
-? : % -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?.? : (%,Fraction %) -> Fraction %
    if R has INTDOM
```

These exports come from (p1449) PolynomialCategory(R,N,S)
where R:Ring, N:NonNegativeInteger, S:SingletonAsOrderedSet:

```
charthRoot : % -> Union(%,"failed")
  if
   and(has($,CharacteristicNonZero),
       has(R,PolynomialFactorizationExplicit))
   or R has CHARNZ
coefficient :
  (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
coefficient :
  (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
conditionP : Matrix % -> Union(Vector %,"failed")
 if and(has($,CharacteristicNonZero),
        has(R,PolynomialFactorizationExplicit))
convert : % -> Pattern Integer
    if SingletonAsOrderedSet has KONVERT PATTERN INT
    and R has KONVERT PATTERN INT
convert : % -> Pattern Float
    if SingletonAsOrderedSet has KONVERT PATTERN FLOAT
```

```
    and R has KONVERT PATTERN FLOAT
convert : % -> InputForm
    if SingletonAsOrderedSet has KONVERT INFORM
    and R has KONVERT INFORM
degree : (%,SingletonAsOrderedSet) -> NonNegativeInteger
discriminant : (%,SingletonAsOrderedSet) -> %
    if R has COMRING
isExpt : % ->
  Union(
   Record(var: SingletonAsOrderedSet,exponent: NonNegativeInteger),
  "failed")
isPlus : % -> Union(List %,"failed")
isTimes : % -> Union(List %,"failed")
minimumDegree : % -> NonNegativeInteger
monicDivide :
  (%,%,SingletonAsOrderedSet) -> Record(quotient: %,remainder: %)
monomial :
  (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
monomials : % -> List %
multivariate :
  (SparseUnivariatePolynomial %,SingletonAsOrderedSet) -> %
multivariate :
  (SparseUnivariatePolynomial R,SingletonAsOrderedSet) -> %
patternMatch :
 (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
   PatternMatchResult(Integer,%)
    if SingletonAsOrderedSet has PATMAB INT
    and R has PATMAB INT
patternMatch :
 (%,Pattern Float,PatternMatchResult(Float,%)) ->
   PatternMatchResult(Float,%)
    if SingletonAsOrderedSet has PATMAB FLOAT
    and R has PATMAB FLOAT
primitiveMonomials : % -> List %
primitivePart : (%,SingletonAsOrderedSet) -> %
    if R has GCDDOM
primitivePart : % -> % if R has GCDDOM
reducedSystem : Matrix % -> Matrix R
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix R,vec: Vector R)
resultant : (%,%,SingletonAsOrderedSet) -> %
    if R has COMRING
retract : % -> SingletonAsOrderedSet
retractIfCan : % -> Union(SingletonAsOrderedSet,"failed")
totalDegree : % -> NonNegativeInteger
univariate : % -> SparseUnivariatePolynomial R
univariate :
  (%,SingletonAsOrderedSet) -> SparseUnivariatePolynomial %
?<? : (%,%) -> Boolean if R has ORDSET
```

These exports come from (p54) Eltable(R:Ring,R:Ring):

```
?.? : (%,R) -> R
```

These exports come from (p54) Eltable(R:UPOLYC,R:UPOLYC):

```
?.? : (%,%) -> %
```

These exports come from (p1024) DifferentialRing():

```
D : % -> %
D : (%,NonNegativeInteger) -> %
differentiate : (%,NonNegativeInteger) -> %
```

These exports come from (p1127) DifferentialExtension(R:Ring):

```
D : (%,(R -> R)) -> %
D : (%,(R -> R),NonNegativeInteger) -> %
D : (%,Symbol) -> % if R has PDRING SYMBOL
D : (%,List Symbol) -> % if R has PDRING SYMBOL
D : (%,Symbol,NonNegativeInteger) -> %
    if R has PDRING SYMBOL
D : (%,List Symbol,List NonNegativeInteger) -> %
    if R has PDRING SYMBOL
differentiate : (%,(R -> R),NonNegativeInteger) -> %
differentiate : (%,Symbol) -> %
    if R has PDRING SYMBOL
differentiate : (%,List Symbol) -> %
    if R has PDRING SYMBOL
differentiate : (%,Symbol,NonNegativeInteger) -> %
    if R has PDRING SYMBOL
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
    if R has PDRING SYMBOL
```

These exports come from (p367) StepThrough()

These exports come from (p1019) CommutativeRing()

These exports come from (p1225) IntegralDomain()

These exports come from (p1319) GcdDomain()

These exports come from (p1413) Field()

```
expressIdealMember : (List %,%) -> Union(List %,"failed")
    if R has FIELD
extendedEuclidean : (%,%) ->
  Record(coef1: %,coef2: %,generator: %)
    if R has FIELD
extendedEuclidean :
  (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
    if R has FIELD
multiEuclidean : (List %,%) -> Union(List %,"failed")
    if R has FIELD
principalIdeal : List % -> Record(coef: List %,generator: %)
    if R has FIELD
sizeLess? : (%,%) -> Boolean if R has FIELD
?quo? : (%,%) -> % if R has FIELD
?rem? : (%,%) -> % if R has FIELD
```

These exports come from (p1121) Algebra(Fraction(Integer))

These exports come from (p1393) PolynomialFactorizationExplicit()

— UnivariatePolynomialCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#UPOLYC">
UnivariatePolynomialCategory (UPOLYC)</a></h2>
</body>
```

————

— category UPOLYC UnivariatePolynomialCategory —

```
)abbrev category UPOLYC UnivariatePolynomialCategory
++ Description:
++ The category of univariate polynomials over a ring R.
++ No particular model is assumed - implementations can be either
++ sparse or dense.

UnivariatePolynomialCategory(R) : Category == SIG where
  R : Ring

  RC   ==> PolynomialCategory(R, NonNegativeInteger, SingletonAsOrderedSet)
  ELT1 ==> Eltable(R,R)
  ELT2 ==> Eltable(%,%)
  DR   ==> DifferentialRing
  DE   ==> DifferentialExtension(R)

  SIG ==> Join(RC,ELT1,ELT2,DR,DE) with

    vectorise : (%,NonNegativeInteger) -> Vector R
      ++ vectorise(p, n) returns \spad{[a0,...,a(n-1)]} where
      ++ \spad{p = a0 + a1*x + ... + a(n-1)*x**(n-1)} + higher order terms.
      ++ The degree of polynomial p can be different from \spad{n-1}.
      ++
      ++X t1:UP(x,FRAC(INT)):=3*x^3+4*x^2+5*x+6
      ++X t2:=vectorise(t1,4)

    unvectorise : Vector R -> %
      ++ unvectorise(v) returns the polynomial which has for coefficients the
      ++ entries of v in the increasing order.
      ++
      ++X t1:UP(x,FRAC(INT)):=3*x^3+4*x^2+5*x+6
      ++X t2:=vectorise(t1,4)
      ++X t3:UP(x,FRAC(INT)):=unvectorise(t2)

    makeSUP : % -> SparseUnivariatePolynomial R
      ++ makeSUP(p) converts the polynomial p to be of type
      ++ SparseUnivariatePolynomial over the same coefficients.
```

```
    unmakeSUP : SparseUnivariatePolynomial R -> %
      ++ unmakeSUP(sup) converts sup of type
      ++ \spadtype{SparseUnivariatePolynomial(R)}
      ++ to be a member of the given type.
      ++ Note that converse of makeSUP.

    multiplyExponents : (%,NonNegativeInteger) -> %
      ++ multiplyExponents(p,n) returns a new polynomial resulting from
      ++ multiplying all exponents of the polynomial p by the non negative
      ++ integer n.

    divideExponents : (%,NonNegativeInteger) -> Union(%,"failed")
      ++ divideExponents(p,n) returns a new polynomial resulting from
      ++ dividing all exponents of the polynomial p by the non negative
      ++ integer n, or "failed" if some exponent is not exactly divisible
      ++ by n.

    monicDivide : (%,%) -> Record(quotient:%,remainder:%)
      ++ monicDivide(p,q) divide the polynomial p by the monic polynomial q,
      ++ returning the pair \spad{[quotient, remainder]}.
      ++ Error: if q isn't monic.

-- These three are for Karatsuba

    karatsubaDivide : (%,NonNegativeInteger) -> Record(quotient:%,remainder:%)
      ++ \spad{karatsubaDivide(p,n)} returns the same as
      ++ \spad{monicDivide(p,monomial(1,n))}

    shiftRight : (%,NonNegativeInteger) -> %
      ++ \spad{shiftRight(p,n)} returns
      ++ \spad{monicDivide(p,monomial(1,n)).quotient}

    shiftLeft : (%,NonNegativeInteger) -> %
      ++ \spad{shiftLeft(p,n)} returns \spad{p * monomial(1,n)}

    pseudoRemainder : (%,%) -> %
       ++ pseudoRemainder(p,q) = r, for polynomials p and q, returns the
       ++ remainder when
       ++ \spad{p' := p*lc(q)**(deg p - deg q + 1)}
       ++ is pseudo right-divided by q, \spad{p' = s q + r}.

    differentiate : (%, R -> R, %) -> %
       ++ differentiate(p, d, x') extends the R-derivation d to an
       ++ extension D in \spad{R[x]} where Dx is given by x', and
       ++ returns \spad{Dp}.

    if R has StepThrough then StepThrough

    if R has CommutativeRing then

      discriminant : % -> R
        ++ discriminant(p) returns the discriminant of the polynomial p.
```

```
   resultant : (%,%) -> R
     ++ resultant(p,q) returns the resultant of the polynomials p and q.

 if R has IntegralDomain then

     Eltable(Fraction %, Fraction %)

     elt : (Fraction %, Fraction %) -> Fraction %
       ++ elt(a,b) evaluates the fraction of univariate polynomials
       ++ \spad{a} with the distinguished variable replaced by b.

     order : (%, %) -> NonNegativeInteger
       ++ order(p, q) returns the largest n such that \spad{q**n}
       ++ divides polynomial p
       ++ the order of \spad{p(x)} at \spad{q(x)=0}.

     subResultantGcd : (%,%) -> %
       ++ subResultantGcd(p,q) computes the gcd of the polynomials p
       ++ and q using the SubResultant GCD algorithm.

     composite : (%, %) -> Union(%, "failed")
       ++ composite(p, q) returns h if \spad{p = h(q)}, and "failed"
       ++ no such h exists.

     composite : (Fraction %, %) -> Union(Fraction %, "failed")
       ++ composite(f, q) returns h if f = h(q), and "failed" is
       ++ no such h exists.

     pseudoQuotient : (%,%) -> %
       ++ pseudoQuotient(p,q) returns r, the quotient when
       ++ \spad{p' := p*lc(q)**(deg p - deg q + 1)}
       ++ is pseudo right-divided by q, \spad{p' = s q + r}.

     pseudoDivide : (%, %) -> Record(coef:R, quotient: %, remainder:%)
       ++ pseudoDivide(p,q) returns \spad{[c, q, r]}, when
       ++ \spad{p' := p*lc(q)**(deg p - deg q + 1) = c * p}
       ++ is pseudo right-divided by q, \spad{p' = s q + r}.

 if R has GcdDomain then

     separate : (%, %) -> Record(primePart:%, commonPart: %)
       ++ separate(p, q) returns \spad{[a, b]} such that polynomial
       ++ \spad{p = a b} and \spad{a} is relatively prime to q.

 if R has Field then

     EuclideanDomain

     additiveValuation
       ++ euclideanSize(a*b) = euclideanSize(a) + euclideanSize(b)

     elt : (Fraction %, R) -> R
       ++ elt(a,r) evaluates the fraction of univariate polynomials
       ++ \spad{a} with the distinguished variable replaced by the
```

```
      ++ constant r.

 if R has Algebra Fraction Integer then

   integrate : % -> %
     ++ integrate(p) integrates the univariate polynomial p with respect
     ++ to its distinguished variable.

add

  pp,qq: SparseUnivariatePolynomial %

  variables(p) ==
    zero? p or zero?(degree p) => []
    [create()]

  degree(p:%,v:SingletonAsOrderedSet) == degree p

  totalDegree(p:%,lv:List SingletonAsOrderedSet) ==
     empty? lv => 0
     totalDegree p

  degree(p:%,lv:List SingletonAsOrderedSet) ==
     empty? lv => []
     [degree p]

  eval(p:%,lv: List SingletonAsOrderedSet,lq: List %):% ==
    empty? lv => p
    not empty? rest lv => _
      error "can only eval a univariate polynomial once"
    eval(p,first lv,first lq)$%

  eval(p:%,v:SingletonAsOrderedSet,q:%):% == p(q)

  eval(p:%,lv: List SingletonAsOrderedSet,lr: List R):% ==
    empty? lv => p
    not empty? rest lv => _
       error "can only eval a univariate polynomial once"
    eval(p,first lv,first lr)$%

  eval(p:%,v:SingletonAsOrderedSet,r:R):% == p(r)::%

  eval(p:%,le:List Equation %):% ==
    empty? le  => p
    not empty? rest le => _
       error "can only eval a univariate polynomial once"
    mainVariable(lhs first le) case "failed" => p
    p(rhs first le)

  mainVariable(p:%) ==
    zero? degree p =>  "failed"
    create()$SingletonAsOrderedSet

  minimumDegree(p:%,v:SingletonAsOrderedSet) == minimumDegree p
```

```
minimumDegree(p:%,lv:List SingletonAsOrderedSet) ==
   empty? lv => []
   [minimumDegree p]

monomial(p:%,v:SingletonAsOrderedSet,n:NonNegativeInteger) ==
   mapExponents(x1+->x1+n,p)

coerce(v:SingletonAsOrderedSet):% == monomial(1,1)

makeSUP p ==
  zero? p => 0
  monomial(leadingCoefficient p,degree p) + makeSUP reductum p

unmakeSUP sp ==
  zero? sp => 0
  monomial(leadingCoefficient sp,degree sp) + unmakeSUP reductum sp

karatsubaDivide(p:%,n:NonNegativeInteger) == monicDivide(p,monomial(1,n))

shiftRight(p:%,n:NonNegativeInteger) ==
   monicDivide(p,monomial(1,n)).quotient

shiftLeft(p:%,n:NonNegativeInteger) == p * monomial(1,n)

if R has PolynomialFactorizationExplicit then

  PFBRU ==> PolynomialFactorizationByRecursionUnivariate(R,%)

  pp,qq:SparseUnivariatePolynomial %

  lpp:List SparseUnivariatePolynomial %

  SupR ==> SparseUnivariatePolynomial R

  sp:SupR

  solveLinearPolynomialEquation(lpp,pp) ==
    solveLinearPolynomialEquationByRecursion(lpp,pp)$PFBRU

  factorPolynomial(pp) ==
    factorByRecursion(pp)$PFBRU

  factorSquareFreePolynomial(pp) ==
    factorSquareFreeByRecursion(pp)$PFBRU

  import FactoredFunctions2(SupR,S)

  factor p ==
    zero? degree p  =>
      ansR:=factor leadingCoefficient p
      makeFR(unit(ansR)::%,
             [[w.flg,w.fctr::%,w.xpnt] for w in factorList ansR])
    map(unmakeSUP,factorPolynomial(makeSUP p)$R)
```

```
vectorise(p, n) ==
  m := minIndex(v := new(n, 0)$Vector(R))
  for i in minIndex v .. maxIndex v repeat
    qsetelt_!(v, i, coefficient(p, (i - m)::NonNegativeInteger))
  v

unvectorise(v : Vector R) : % ==
    p : % := 0
    for i in 1..#v repeat
        p := p + monomial(v(i), (i-1)::NonNegativeInteger)
    p

retract(p:%):R ==
  zero? p => 0
  zero? degree p => leadingCoefficient p
  error "Polynomial is not of degree 0"

retractIfCan(p:%):Union(R, "failed") ==
  zero? p => 0
  zero? degree p => leadingCoefficient p
  "failed"

if R has StepThrough then

  init() == init()$R::%

  nextItemInner: % -> Union(%,"failed")

  nextItemInner(n) ==
    zero? n => nextItem(0$R)::R::% -- assumed not to fail
    zero? degree n =>
      nn:=nextItem leadingCoefficient n
      nn case "failed" => "failed"
      nn::R::%
    n1:=reductum n
    n2:=nextItemInner n1 -- try stepping the reductum
    n2 case % => monomial(leadingCoefficient n,degree n) + n2
    1+degree n1 < degree n => -- there was a hole between lt n and n1
      monomial(leadingCoefficient n,degree n)+
        monomial(nextItem(init()$R)::R,1+degree n1)
    n3:=nextItem leadingCoefficient n
    n3 case "failed" => "failed"
    monomial(n3,degree n)

  nextItem(n) ==
    n1:=nextItemInner n
    n1 case "failed" => monomial(nextItem(init()$R)::R,1+degree(n))
    n1

if R has GcdDomain then

  content(p:%,v:SingletonAsOrderedSet) == content(p)::%
```

```
    primeFactor: (%, %) -> %

    primeFactor(p, q) ==
      (p1 := (p exquo gcd(p, q))::%) = p => p
      primeFactor(p1, q)

    separate(p, q) ==
      a := primeFactor(p, q)
      [a, (p exquo a)::%]

  if R has CommutativeRing then

    differentiate(x:%, deriv:R -> R, x':%) ==
      d:% := 0
      while (dg := degree x) > 0 repeat
        lc := leadingCoefficient x
        d := d + x' * monomial(dg * lc, (dg - 1)::NonNegativeInteger)
               + monomial(deriv lc, dg)
        x := reductum x
      d + deriv(leadingCoefficient x)::%

  else

    ncdiff: (NonNegativeInteger, %) -> %
    -- computes d(x**n) given dx = x', non-commutative case
    ncdiff(n, x') ==
      zero? n => 0
      zero?(n1 := (n - 1)::NonNegativeInteger) => x'
      x' * monomial(1, n1) + monomial(1, 1) * ncdiff(n1, x')

    differentiate(x:%, deriv:R -> R, x':%) ==
      d:% := 0
      while (dg := degree x) > 0 repeat
        lc := leadingCoefficient x
        d := d + monomial(deriv lc, dg) + lc * ncdiff(dg, x')
        x := reductum x
      d + deriv(leadingCoefficient x)::%

  differentiate(x:%, deriv:R -> R) == differentiate(x, deriv, 1$%)$%

  differentiate(x:%) ==
      d:% := 0
      while (dg := degree x) > 0 repeat
        d:=d+monomial(dg*leadingCoefficient x,(dg-1)::NonNegativeInteger)
        x := reductum x
      d

  differentiate(x:%,v:SingletonAsOrderedSet) == differentiate x

  if R has IntegralDomain then

    elt(g:Fraction %, f:Fraction %) == ((numer g) f) / ((denom g) f)

    pseudoQuotient(p, q) ==
```

```
    (n := degree(p)::Integer - degree q + 1) < 1 => 0
    ((leadingCoefficient(q)**(n::NonNegativeInteger) * p
      - pseudoRemainder(p, q)) exquo q)::%

  pseudoDivide(p, q) ==
    (n := degree(p)::Integer - degree q + 1) < 1 => [1, 0, p]
    prem := pseudoRemainder(p, q)
    lc   := leadingCoefficient(q)**(n::NonNegativeInteger)
    [lc,((lc*p - prem) exquo q)::%, prem]

  composite(f:Fraction %, q:%) ==
    (n := composite(numer f, q)) case "failed" => "failed"
    (d := composite(denom f, q)) case "failed" => "failed"
    n::% / d::%

  composite(p:%, q:%) ==
    ground? p => p
    cqr := pseudoDivide(p, q)
    ground?(cqr.remainder) and
      ((v := cqr.remainder exquo cqr.coef) case %) and
        ((u := composite(cqr.quotient, q)) case %) and
          ((w := (u::%) exquo cqr.coef) case %) =>
            v::% + monomial(1, 1) * w::%
    "failed"

  elt(p:%, f:Fraction %) ==
    zero? p => 0
    ans:Fraction(%) := (leadingCoefficient p)::%::Fraction(%)
    n := degree p
    while not zero?(p:=reductum p) repeat
      ans := ans * f ** (n - (n := degree p))::NonNegativeInteger +
                (leadingCoefficient p)::%::Fraction(%)
    zero? n => ans
    ans * f ** n

  order(p, q) ==
    zero? p => error "order: arguments must be nonzero"
    degree(q) < 1 => error "order: place must be non-trivial"
    ans:NonNegativeInteger := 0
    repeat
      (u  := p exquo q) case "failed" => return ans
      p   := u::%
      ans := ans + 1

if R has GcdDomain then

  squareFree(p:%) ==
    squareFree(p)$UnivariatePolynomialSquareFree(R, %)

  squareFreePart(p:%) ==
    squareFreePart(p)$UnivariatePolynomialSquareFree(R, %)

if R has PolynomialFactorizationExplicit then
```

```
      gcdPolynomial(pp,qq) ==
            zero? pp => unitCanonical qq  -- subResultantGcd can't handle 0
            zero? qq => unitCanonical pp
            unitCanonical(gcd(content (pp),content(qq))*
                   primitivePart
                      subResultantGcd(primitivePart pp,primitivePart qq))

      squareFreePolynomial pp ==
         squareFree(pp)$UnivariatePolynomialSquareFree(%,
                                    SparseUnivariatePolynomial %)

    if R has Field then

      elt(f:Fraction %, r:R) == ((numer f) r) / ((denom f) r)

      euclideanSize x ==
            zero? x =>
              error "euclideanSize called on 0 in Univariate Polynomial"
            degree x

      divide(x,y) ==
            zero? y => error "division by 0 in Univariate Polynomials"
            quot:=0
            lc := inv leadingCoefficient y
            while not zero?(x) and (degree x >= degree y) repeat
               f:=lc*leadingCoefficient x
               n:=(degree x - degree y)::NonNegativeInteger
               quot:=quot+monomial(f,n)
               x:=x-monomial(f,n)*y
            [quot,x]

    if R has Algebra Fraction Integer then

      integrate p ==
        ans:% := 0
        while p ^= 0 repeat
          l := leadingCoefficient p
          d := 1 + degree p
          ans := ans + inv(d::Fraction(Integer)) * monomial(l, d)
          p := reductum p
        ans
```

———-

— **COQ UPOLYC** —

```
(* category UPOLYC *)
(*
    pp,qq: SparseUnivariatePolynomial %

    variables : % -> List(SingletonAsOrderedSet)
    variables(p) ==
      zero? p or zero?(degree p) => []
```

```
  [create()]

degree : (%,List(SingletonAsOrderedSet)) -> List(NonNegativeInteger)
degree(p:%,v:SingletonAsOrderedSet) == degree p

totalDegree : (%,List(SingletonAsOrderedSet)) -> NonNegativeInteger
totalDegree(p:%,lv:List SingletonAsOrderedSet) ==
   empty? lv => 0
   totalDegree p

degree : (%,List(SingletonAsOrderedSet)) -> List(NonNegativeInteger)
degree(p:%,lv:List SingletonAsOrderedSet) ==
   empty? lv => []
   [degree p]

eval : (%,List(SingletonAsOrderedSet),List(%)) -> %
eval(p:%,lv: List SingletonAsOrderedSet,lq: List %):% ==
  empty? lv => p
  not empty? rest lv => _
    error "can only eval a univariate polynomial once"
  eval(p,first lv,first lq)$%

eval : (%,SingletonAsOrderedSet,%) -> %
eval(p:%,v:SingletonAsOrderedSet,q:%):% == p(q)

eval(p:%,lv: List SingletonAsOrderedSet,lr: List R):% ==
  empty? lv => p
  not empty? rest lv => _
     error "can only eval a univariate polynomial once"
  eval(p,first lv,first lr)$%

eval : (%,List(SingletonAsOrderedSet),List(R)) -> %
eval(p:%,v:SingletonAsOrderedSet,r:R):% == p(r)::%

eval : (%,List(Equation(%))) -> %
eval(p:%,le:List Equation %):% ==
  empty? le  => p
  not empty? rest le => _
     error "can only eval a univariate polynomial once"
  mainVariable(lhs first le) case "failed" => p
  p(rhs first le)

mainVariable : % -> Union(SingletonAsOrderedSet,"failed")
mainVariable(p:%) ==
  zero? degree p =>  "failed"
  create()$SingletonAsOrderedSet

minimumDegree : (%,SingletonAsOrderedSet) -> NonNegativeInteger
minimumDegree(p:%,v:SingletonAsOrderedSet) == minimumDegree p

minimumDegree : (%,List(SingletonAsOrderedSet)) -> List(NonNegativeInteger)
minimumDegree(p:%,lv:List SingletonAsOrderedSet) ==
   empty? lv => []
   [minimumDegree p]
```

```
monomial : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
monomial(p:%,v:SingletonAsOrderedSet,n:NonNegativeInteger) ==
   mapExponents(x1+->x1+n,p)

coerce : SingletonAsOrderedSet -> %
coerce(v:SingletonAsOrderedSet):% == monomial(1,1)

makeSUP : % -> SparseUnivariatePolynomial(R)
makeSUP p ==
  zero? p => 0
  monomial(leadingCoefficient p,degree p) + makeSUP reductum p

unmakeSUP : SparseUnivariatePolynomial(R) -> %
unmakeSUP sp ==
  zero? sp => 0
  monomial(leadingCoefficient sp,degree sp) + unmakeSUP reductum sp

karatsubaDivide: (%,NonNegativeInteger) -> Record(quotient: %,remainder: %)
karatsubaDivide(p:%,n:NonNegativeInteger) == monicDivide(p,monomial(1,n))

shiftRight : (%,NonNegativeInteger) -> %
shiftRight(p:%,n:NonNegativeInteger) ==
   monicDivide(p,monomial(1,n)).quotient

shiftLeft : (%,NonNegativeInteger) -> %
shiftLeft(p:%,n:NonNegativeInteger) == p * monomial(1,n)

if R has PolynomialFactorizationExplicit then

   PFBRU ==> PolynomialFactorizationByRecursionUnivariate(R,%)

   pp,qq:SparseUnivariatePolynomial %

   lpp:List SparseUnivariatePolynomial %

   SupR ==> SparseUnivariatePolynomial R

   sp:SupR

   solveLinearPolynomialEquation :
    (List(SparseUnivariatePolynomial(%)),SparseUnivariatePolynomial(%)) ->
       Union(List(SparseUnivariatePolynomial(%)),"failed") if R has PFECAT
   solveLinearPolynomialEquation(lpp,pp) ==
     solveLinearPolynomialEquationByRecursion(lpp,pp)$PFBRU

   factorPolynomial : SparseUnivariatePolynomial(%) ->
      Factored(SparseUnivariatePolynomial(%))
   factorPolynomial(pp) ==
     factorByRecursion(pp)$PFBRU

   factorSquareFreePolynomial : SparseUnivariatePolynomial(%) ->
       Factored(SparseUnivariatePolynomial(%))
   factorSquareFreePolynomial(pp) ==
```

```
      factorSquareFreeByRecursion(pp)$PFBRU

    import FactoredFunctions2(SupR,S)

    factor : % -> Factored(%)
    factor p ==
      zero? degree p  =>
        ansR:=factor leadingCoefficient p
        makeFR(unit(ansR)::%,
              [[w.flg,w.fctr::%,w.xpnt] for w in factorList ansR])
      map(unmakeSUP,factorPolynomial(makeSUP p)$R)

vectorise : (%,NonNegativeInteger) -> Vector(R)
vectorise(p, n) ==
  m := minIndex(v := new(n, 0)$Vector(R))
  for i in minIndex v .. maxIndex v repeat
    qsetelt_!(v, i, coefficient(p, (i - m)::NonNegativeInteger))
  v

unvectorise : Vector(R) -> %
unvectorise(v : Vector R) : % ==
    p : % := 0
    for i in 1..#v repeat
        p := p + monomial(v(i), (i-1)::NonNegativeInteger)
    p

retract : % -> R
retract(p:%):R ==
  zero? p => 0
  zero? degree p => leadingCoefficient p
  error "Polynomial is not of degree 0"

retractIfCan : % -> Union(R,"failed")
retractIfCan(p:%):Union(R, "failed") ==
  zero? p => 0
  zero? degree p => leadingCoefficient p
  "failed"

if R has StepThrough then

    init : () -> %
    init() == init()$R::%

    nextItemInner: % -> Union(%,"failed")
    nextItemInner(n) ==
      zero? n => nextItem(0$R)::R::% -- assumed not to fail
      zero? degree n =>
        nn:=nextItem leadingCoefficient n
        nn case "failed" => "failed"
        nn::R::%
      n1:=reductum n
      n2:=nextItemInner n1 -- try stepping the reductum
      n2 case % => monomial(leadingCoefficient n,degree n) + n2
      1+degree n1 < degree n => -- there was a hole between lt n and n1
```

```
        monomial(leadingCoefficient n,degree n)+
          monomial(nextItem(init()$R)::R,1+degree n1)
      n3:=nextItem leadingCoefficient n
      n3 case "failed" => "failed"
      monomial(n3,degree n)

    nextItem : % -> Union(%,"failed")
    nextItem(n) ==
      n1:=nextItemInner n
      n1 case "failed" => monomial(nextItem(init()$R)::R,1+degree(n))
      n1

if R has GcdDomain then

  content : (%,SingletonAsOrderedSet) -> %
  content(p:%,v:SingletonAsOrderedSet) == content(p)::%

  primeFactor: (%, %) -> %
  primeFactor(p, q) ==
    (p1 := (p exquo gcd(p, q))::%) = p => p
    primeFactor(p1, q)

  separate : (%,%) -> Record(primePart: %,commonPart: %)
  separate(p, q) ==
    a := primeFactor(p, q)
    [a, (p exquo a)::%]

if R has CommutativeRing then

  differentiate : (%,(R -> R),%) -> %
  differentiate(x:%, deriv:R -> R, x':%) ==
    d:% := 0
    while (dg := degree x) > 0 repeat
      lc := leadingCoefficient x
      d := d + x' * monomial(dg * lc, (dg - 1)::NonNegativeInteger)
             + monomial(deriv lc, dg)
      x := reductum x
    d + deriv(leadingCoefficient x)::%

else

  -- computes d(x**n) given dx = x', non-commutative case
  ncdiff: (NonNegativeInteger, %) -> %
  ncdiff(n, x') ==
    zero? n => 0
    zero?(n1 := (n - 1)::NonNegativeInteger) => x'
    x' * monomial(1, n1) + monomial(1, 1) * ncdiff(n1, x')

  differentiate : (%,(R -> R),%) -> %
  differentiate(x:%, deriv:R -> R, x':%) ==
    d:% := 0
    while (dg := degree x) > 0 repeat
      lc := leadingCoefficient x
      d := d + monomial(deriv lc, dg) + lc * ncdiff(dg, x')
```

```
      x := reductum x
    d + deriv(leadingCoefficient x)::%

differentiate : (%,(R -> R)) -> %
differentiate(x:%, deriv:R -> R) == differentiate(x, deriv, 1$%)$%

differentiate : % -> %
differentiate(x:%) ==
    d:% := 0
    while (dg := degree x) > 0 repeat
      d:=d+monomial(dg*leadingCoefficient x,(dg-1)::NonNegativeInteger)
      x := reductum x
    d

differentiate : (%,SingletonAsOrderedSet) -> %
differentiate(x:%,v:SingletonAsOrderedSet) == differentiate x

if R has IntegralDomain then

  elt : (Fraction(%),Fraction(%)) -> Fraction(%)
  elt(g:Fraction %, f:Fraction %) == ((numer g) f) / ((denom g) f)

  pseudoQuotient : (%,%) -> %
  pseudoQuotient(p, q) ==
    (n := degree(p)::Integer - degree q + 1) < 1 => 0
    ((leadingCoefficient(q)**(n::NonNegativeInteger) * p
      - pseudoRemainder(p, q)) exquo q)::%

  pseudoDivide : (%,%) -> Record(coef: R,quotient: %,remainder: %)
  pseudoDivide(p, q) ==
    (n := degree(p)::Integer - degree q + 1) < 1 => [1, 0, p]
    prem := pseudoRemainder(p, q)
    lc   := leadingCoefficient(q)**(n::NonNegativeInteger)
    [lc,((lc*p - prem) exquo q)::%, prem]

  composite : (Fraction(%),%) -> Union(Fraction(%),"failed")
  composite(f:Fraction %, q:%) ==
    (n := composite(numer f, q)) case "failed" => "failed"
    (d := composite(denom f, q)) case "failed" => "failed"
    n::% / d::%

  composite : (%,%) -> Union(%,"failed")
  composite(p:%, q:%) ==
    ground? p => p
    cqr := pseudoDivide(p, q)
    ground?(cqr.remainder) and
      ((v := cqr.remainder exquo cqr.coef) case %) and
        ((u := composite(cqr.quotient, q)) case %) and
          ((w := (u::%) exquo cqr.coef) case %) =>
            v::% + monomial(1, 1) * w::%
    "failed"

  ?.? : (%,R) -> R
  elt(p:%, f:Fraction %) ==
```

```
      zero? p => 0
      ans:Fraction(%) := (leadingCoefficient p)::%::Fraction(%)
      n := degree p
      while not zero?(p:=reductum p) repeat
        ans := ans * f ** (n - (n := degree p))::NonNegativeInteger +
                  (leadingCoefficient p)::%::Fraction(%)
      zero? n => ans
      ans * f ** n

    order : (%,%) -> NonNegativeInteger
    order(p, q) ==
      zero? p => error "order: arguments must be nonzero"
      degree(q) < 1 => error "order: place must be non-trivial"
      ans:NonNegativeInteger := 0
      repeat
        (u  := p exquo q) case "failed" => return ans
        p   := u::%
        ans := ans + 1

  if R has GcdDomain then

    squareFree : % -> Factored(%)
    squareFree(p:%) ==
      squareFree(p)$UnivariatePolynomialSquareFree(R, %)

    squareFreePart : % -> %
    squareFreePart(p:%) ==
      squareFreePart(p)$UnivariatePolynomialSquareFree(R, %)

  if R has PolynomialFactorizationExplicit then

    gcdPolynomial :
      (SparseUnivariatePolynomial(%),
       SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(%)
    gcdPolynomial(pp,qq) ==
          zero? pp => unitCanonical qq  -- subResultantGcd can't handle 0
          zero? qq => unitCanonical pp
          unitCanonical(gcd(content (pp),content(qq))*
                 primitivePart
                    subResultantGcd(primitivePart pp,primitivePart qq))

    squareFreePolynomial : SparseUnivariatePolynomial(%) ->
        Factored(SparseUnivariatePolynomial(%))
    squareFreePolynomial pp ==
       squareFree(pp)$UnivariatePolynomialSquareFree(%,
                                 SparseUnivariatePolynomial %)

  if R has Field then

    elt : (Fraction(%),R) -> R
    elt(f:Fraction %, r:R) == ((numer f) r) / ((denom f) r)

    euclideanSize : % -> NonNegativeInteger
    euclideanSize x ==
```

```
          zero? x =>
            error "euclideanSize called on 0 in Univariate Polynomial"
          degree x

    divide : (%,%) -> Record(quotient: %,remainder: %)
    divide(x,y) ==
          zero? y => error "division by 0 in Univariate Polynomials"
          quot:=0
          lc := inv leadingCoefficient y
          while not zero?(x) and (degree x >= degree y) repeat
             f:=lc*leadingCoefficient x
             n:=(degree x - degree y)::NonNegativeInteger
             quot:=quot+monomial(f,n)
             x:=x-monomial(f,n)*y
          [quot,x]

  if R has Algebra Fraction Integer then

    integrate : % -> %
    integrate p ==
      ans:% := 0
      while p ^= 0 repeat
        l := leadingCoefficient p
        d := 1 + degree p
        ans := ans + inv(d::Fraction(Integer)) * monomial(l, d)
        p := reductum p
      ans
*)
```

— UPOLYC.dotabb —

```
"UPOLYC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UPOLYC"];
"UPOLYC" -> "POLYCAT"
"UPOLYC" -> "ELTAB"
"UPOLYC" -> "DIFRING"
"UPOLYC" -> "DIFEXT"
"UPOLYC" -> "STEP"
"UPOLYC" -> "COMRING"
"UPOLYC" -> "INTDOM"
"UPOLYC" -> "GCDDOM"
"UPOLYC" -> "FIELD"
"UPOLYC" -> "ALGEBRA"
"UPOLYC" -> "PFECAT"
```

— UPOLYC.dotfull —

```
"UnivariatePolynomialCategory(a:Ring)"
```

```
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UPOLYC"];
"UnivariatePolynomialCategory(a:Ring)" ->
   "PolynomialCategory(a:Ring,b:NonNegativeInteger,c:SingletonAsOrderedSet)"
"UnivariatePolynomialCategory(a:Ring)" ->
   "Eltable(a:Ring,b:Ring)"
"UnivariatePolynomialCategory(a:Ring)" ->
   "Eltable(a:UnivariatePolynomialCategory(a:Ring),b:UnivariatePolynomialCategory(a:Ring))"
"UnivariatePolynomialCategory(a:Ring)" ->
   "DifferentialRing()"
"UnivariatePolynomialCategory(a:Ring)" ->
   "DifferentialExtension(a:Ring)"
"UnivariatePolynomialCategory(a:Ring)" ->
   "StepThrough()"
"UnivariatePolynomialCategory(a:Ring)" ->
   "CommutativeRing()"
"UnivariatePolynomialCategory(a:Ring)" ->
   "IntegralDomain()"
"UnivariatePolynomialCategory(a:Ring)" ->
   "GcdDomain()"
"UnivariatePolynomialCategory(a:Ring)" ->
   "Field()"
"UnivariatePolynomialCategory(a:Ring)" ->
   "Algebra(Fraction(Integer))"
"UnivariatePolynomialCategory(a:Ring)" ->
   "PolynomialFactorizationExplicit()"
```

———-

— UPOLYC.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"UnivariatePolynomialCategory(a:Ring)" [color=lightblue];
"UnivariatePolynomialCategory(a:Ring)" -> "POLYCAT..."
"UnivariatePolynomialCategory(a:Ring)" -> "ELTAB..."
"UnivariatePolynomialCategory(a:Ring)" -> "DIFRING..."
"UnivariatePolynomialCategory(a:Ring)" -> "DIFEXT..."
"UnivariatePolynomialCategory(a:Ring)" -> "STEP..."
"UnivariatePolynomialCategory(a:Ring)" -> "COMRING..."
"UnivariatePolynomialCategory(a:Ring)" -> "INTDOM..."
"UnivariatePolynomialCategory(a:Ring)" -> "GCDDOM..."
"UnivariatePolynomialCategory(a:Ring)" -> "FIELD..."
"UnivariatePolynomialCategory(a:Ring)" -> "ALGEBRA..."
"UnivariatePolynomialCategory(a:Ring)" -> "PFECAT..."

"POLYCAT..." [color=lightblue];
"ELTAB..." [color=lightblue];
"DIFRING..." [color=lightblue];
"DIFEXT..." [color=lightblue];
"STEP..." [color=lightblue];
```

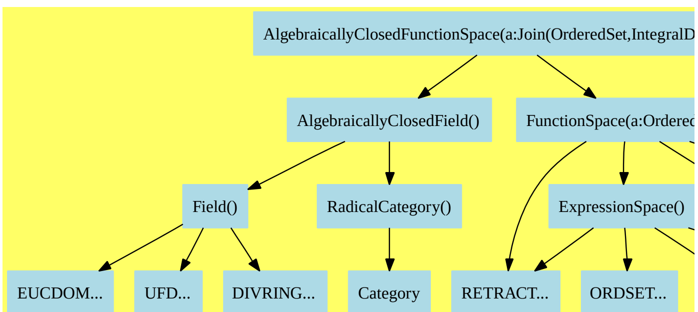
```
"COMRING..." [color=lightblue];
"INTDOM..." [color=lightblue];
"GCDDOM..." [color=lightblue];
"FIELD..." [color=lightblue];
"ALGEBRA..." [color=lightblue];
"PFECAT..." [color=lightblue];

}
```

———

Chapter 18

Category Layer 17

18.0.229 AlgebraicallyClosedFunctionSpace (ACFS)



— **AlgebraicallyClosedFunctionSpace.input** —

```
)set break resume
)sys rm -f AlgebraicallyClosedFunctionSpace.output
)spool AlgebraicallyClosedFunctionSpace.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show AlgebraicallyClosedFunctionSpace
--R
--R AlgebraicallyClosedFunctionSpace(R: Join(OrderedSet,IntegralDomain)) is a category constructor
--R Abbreviation for AlgebraicallyClosedFunctionSpace is ACFS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ACFS
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> % if R has COMRING     ?*? : (%,R) -> % if R has COMRING
--R ?*? : (Fraction(Integer),%) -> %      ?*? : (%,Fraction(Integer)) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
```

```
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,Fraction(Integer)) -> %     ?**? : (%,Integer) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?/? : (%,%) -> %
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               D : (%,Symbol) -> % if R has RING
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,Integer) -> %                ?^? : (%,NonNegativeInteger) -> %
--R ?^? : (%,PositiveInteger) -> %        applyQuote : (Symbol,%) -> %
--R applyQuote : (Symbol,%,%) -> %        applyQuote : (Symbol,%,%,%) -> %
--R applyQuote : (Symbol,List(%)) -> %    associates? : (%,%) -> Boolean
--R belong? : BasicOperator -> Boolean    box : % -> %
--R box : List(%) -> %                    coerce : Kernel(%) -> %
--R coerce : Symbol -> %                  coerce : R -> %
--R coerce : Fraction(Integer) -> %       coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R distribute : % -> %                   distribute : (%,%) -> %
--R elt : (BasicOperator,%) -> %          elt : (BasicOperator,%,%) -> %
--R elt : (BasicOperator,%,%,%) -> %      elt : (BasicOperator,List(%)) -> %
--R eval : (%,Kernel(%),%) -> %           eval : (%,List(Equation(%))) -> %
--R eval : (%,Equation(%)) -> %           eval : (%,%,%) -> %
--R eval : (%,List(%),List(%)) -> %       eval : (%,Symbol,(% -> %)) -> %
--R factor : % -> Factored(%)             freeOf? : (%,%) -> Boolean
--R freeOf? : (%,Symbol) -> Boolean       gcd : List(%) -> %
--R gcd : (%,%) -> %                      ground : % -> R
--R ground? : % -> Boolean                hash : % -> SingleInteger
--R height : % -> NonNegativeInteger      inv : % -> %
--R is? : (%,BasicOperator) -> Boolean    is? : (%,Symbol) -> Boolean
--R kernel : (BasicOperator,%) -> %       kernels : % -> List(Kernel(%))
--R latex : % -> String                   lcm : List(%) -> %
--R lcm : (%,%) -> %                      map : ((% -> %),Kernel(%)) -> %
--R max : (%,%) -> %                      min : (%,%) -> %
--R nthRoot : (%,Integer) -> %            numerator : % -> % if R has RING
--R one? : % -> Boolean                   paren : % -> %
--R paren : List(%) -> %                  prime? : % -> Boolean
--R ?quo? : (%,%) -> %                    recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                    retract : % -> Kernel(%)
--R retract : % -> Symbol                 retract : % -> R
--R rootOf : (%,Symbol) -> %              rootOf : % -> %
--R rootOf : Polynomial(%) -> %           rootsOf : (%,Symbol) -> List(%)
--R rootsOf : % -> List(%)                rootsOf : Polynomial(%) -> List(%)
--R sample : () -> %                      sizeLess? : (%,%) -> Boolean
--R sqrt : % -> %                         squareFree : % -> Factored(%)
--R squareFreePart : % -> %               subst : (%,Equation(%)) -> %
--R tower : % -> List(Kernel(%))          unit? : % -> Boolean
--R unitCanonical : % -> %                variables : % -> List(Symbol)
--R zero? : % -> Boolean                  zeroOf : (%,Symbol) -> %
--R zeroOf : % -> %                       zeroOf : Polynomial(%) -> %
--R zerosOf : (%,Symbol) -> List(%)       zerosOf : % -> List(%)
--R zerosOf : Polynomial(%) -> List(%)    ?~=? : (%,%) -> Boolean
--R ?/? : (SparseMultivariatePolynomial(R,Kernel(%)),SparseMultivariatePolynomial(R,Kernel(%))) -> % if R has IN
--R D : (%,List(Symbol),List(NonNegativeInteger)) -> % if R has RING
```

```
--R D : (%,Symbol,NonNegativeInteger) -> % if R has RING
--R D : (%,List(Symbol)) -> % if R has RING
--R applyQuote : (Symbol,%,%,%,%) -> %
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
--R coerce : SparseMultivariatePolynomial(R,Kernel(%)) -> % if R has RING
--R coerce : Fraction(R) -> % if R has INTDOM
--R coerce : Polynomial(Fraction(R)) -> % if R has INTDOM
--R coerce : Fraction(Polynomial(Fraction(R))) -> % if R has INTDOM
--R coerce : Fraction(Polynomial(R)) -> % if R has INTDOM
--R coerce : Polynomial(R) -> % if R has RING
--R commutator : (%,%) -> % if R has GROUP
--R conjugate : (%,%) -> % if R has GROUP
--R convert : % -> Pattern(Integer) if R has KONVERT(PATTERN(INT))
--R convert : % -> Pattern(Float) if R has KONVERT(PATTERN(FLOAT))
--R convert : Factored(%) -> % if R has INTDOM
--R convert : % -> InputForm if R has KONVERT(INFORM)
--R definingPolynomial : % -> % if $ has RING
--R denom : % -> SparseMultivariatePolynomial(R,Kernel(%)) if R has INTDOM
--R denominator : % -> % if R has INTDOM
--R differentiate : (%,List(Symbol),List(NonNegativeInteger)) -> % if R has RING
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has RING
--R differentiate : (%,List(Symbol)) -> % if R has RING
--R differentiate : (%,Symbol) -> % if R has RING
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R elt : (BasicOperator,%,%,%,%) -> %
--R euclideanSize : % -> NonNegativeInteger
--R eval : (%,List(Kernel(%)),List(%)) -> %
--R eval : (%,List(Symbol),List((% -> %))) -> %
--R eval : (%,List(Symbol),List((List(%) -> %))) -> %
--R eval : (%,Symbol,(List(%) -> %)) -> %
--R eval : (%,List(BasicOperator),List((% -> %))) -> %
--R eval : (%,List(BasicOperator),List((List(%) -> %))) -> %
--R eval : (%,BasicOperator,(List(%) -> %)) -> %
--R eval : (%,BasicOperator,(% -> %)) -> %
--R eval : (%,Symbol) -> % if R has KONVERT(INFORM)
--R eval : (%,List(Symbol)) -> % if R has KONVERT(INFORM)
--R eval : % -> % if R has KONVERT(INFORM)
--R eval : (%,BasicOperator,%,Symbol) -> % if R has KONVERT(INFORM)
--R eval : (%,List(BasicOperator),List(%),Symbol) -> % if R has KONVERT(INFORM)
--R eval : (%,List(Symbol),List(NonNegativeInteger),List((% -> %))) -> % if R has RING
--R eval : (%,List(Symbol),List(NonNegativeInteger),List((List(%) -> %))) -> % if R has RING
--R eval : (%,Symbol,NonNegativeInteger,(List(%) -> %)) -> % if R has RING
--R eval : (%,Symbol,NonNegativeInteger,(% -> %)) -> % if R has RING
--R even? : % -> Boolean if $ has RETRACT(INT)
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R isExpt : % -> Union(Record(var: Kernel(%),exponent: Integer),"failed") if R has SGROUP
--R isExpt : (%,BasicOperator) -> Union(Record(var: Kernel(%),exponent: Integer),"failed") if R has RING
--R isExpt : (%,Symbol) -> Union(Record(var: Kernel(%),exponent: Integer),"failed") if R has RING
--R isMult : % -> Union(Record(coef: Integer,var: Kernel(%)),"failed") if R has ABELSG
```

```
--R isPlus : % -> Union(List(%),"failed") if R has ABELSG
--R isPower : % -> Union(Record(val: %,exponent: Integer),"failed") if R has RING
--R isTimes : % -> Union(List(%),"failed") if R has SGROUP
--R kernel : (BasicOperator,List(%)) -> %
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
--R mainKernel : % -> Union(Kernel(%),"failed")
--R minPoly : Kernel(%) -> SparseUnivariatePolynomial(%) if $ has RING
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed")
--R numer : % -> SparseMultivariatePolynomial(R,Kernel(%)) if R has RING
--R odd? : % -> Boolean if $ has RETRACT(INT)
--R operator : BasicOperator -> BasicOperator
--R operators : % -> List(BasicOperator)
--R patternMatch : (%,Pattern(Integer),PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%) if R has
--R patternMatch : (%,Pattern(Float),PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%) if R has PATMAB
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %)
--R reducedSystem : Matrix(%) -> Matrix(Integer) if and(has(R,Ring),has(R,LinearlyExplicitRingOver(Integer))) or
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(Integer),vec: Vector(Integer)) if and(has(R,Ring
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(R),vec: Vector(R)) if R has RING
--R reducedSystem : Matrix(%) -> Matrix(R) if R has RING
--R retract : % -> Fraction(Integer) if R has RETRACT(INT) and R has INTDOM or R has RETRACT(FRAC(INT))
--R retract : % -> Integer if R has RETRACT(INT)
--R retract : % -> Fraction(Polynomial(R)) if R has INTDOM
--R retract : % -> Polynomial(R) if R has RING
--R retractIfCan : % -> Union(Fraction(Integer),"failed") if R has RETRACT(INT) and R has INTDOM or R has RETRAC
--R retractIfCan : % -> Union(Kernel(%),"failed")
--R retractIfCan : % -> Union(Symbol,"failed")
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT(INT)
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction(Polynomial(R)),"failed") if R has INTDOM
--R retractIfCan : % -> Union(Polynomial(R),"failed") if R has RING
--R rootOf : (SparseUnivariatePolynomial(%),Symbol) -> %
--R rootOf : SparseUnivariatePolynomial(%) -> %
--R rootsOf : (SparseUnivariatePolynomial(%),Symbol) -> List(%)
--R rootsOf : SparseUnivariatePolynomial(%) -> List(%)
--R subst : (%,List(Equation(%))) -> %
--R subst : (%,List(Kernel(%)),List(%)) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R univariate : (%,Kernel(%)) -> Fraction(SparseUnivariatePolynomial(%)) if R has INTDOM
--R zeroOf : (SparseUnivariatePolynomial(%),Symbol) -> %
--R zeroOf : SparseUnivariatePolynomial(%) -> %
--R zerosOf : (SparseUnivariatePolynomial(%),Symbol) -> List(%)
--R zerosOf : SparseUnivariatePolynomial(%) -> List(%)
--R
--E 1

)spool
)lisp (bye)
```

— **AlgebraicallyClosedFunctionSpace.help** —

```
====================================================================
```

```
AlgebraicallyClosedFunctionSpace examples
====================================================================

Model for algebraically closed function spaces.

See Also:
o )show AlgebraicallyClosedFunctionSpace
```

———-

See:

⇐ "AlgebraicallyClosedField" (ACF) 17.0.215 on page 1497
⇐ "FunctionSpace" (FS) 17.0.219 on page 1544

Exports:

0	1	applyQuote	associates?
belong?	box	characteristic	charthRoot
coerce	commutator	conjugate	convert
D	definingPolynomial	denom	denominator
differentiate	distribute	divide	elt
euclideanSize	eval	even?	expressIdealMember
exquo	extendedEuclidean	factor	freeOf?
gcd	gcdPolynomial	ground	ground?
hash	height	inv	is?
isExpt	isMult	isPlus	isPower
isTimes	kernel	kernels	latex
lcm	mainKernel	map	max
min	minPoly	multiEuclidean	nthRoot
numer	numerator	odd?	one?
operator	operators	paren	patternMatch
prime?	principalIdeal	recip	reducedSystem
retract	retractIfCan	rootOf	rootsOf
sample	sizeLess?	sqrt	squareFree
squareFreePart	subst	subtractIfCan	tower
unit?	unitCanonical	unitNormal	univariate
variables	zero?	zeroOf	zerosOf
?*?	?**?	?+?	?-?
-?	?/?	?<?	?<=?
?=?	?>?	?>=?	?^?
?~=?	?quo?	?rem?	

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **commutative("*")** is true if it has an operation $" * " : (D, D)- > D$ which is commutative.
- **nil**

These are implemented by this category:

```
rootOf : % -> %
rootOf : (%,Symbol) -> %
rootsOf : % -> List %
rootsOf : (%,Symbol) -> List %
rootsOf : (SparseUnivariatePolynomial %,Symbol) -> List %
zeroOf : % -> %
zeroOf : (%,Symbol) -> %
zeroOf : (SparseUnivariatePolynomial %,Symbol) -> %
zerosOf : % -> List %
zerosOf : (%,Symbol) -> List %
zerosOf : (SparseUnivariatePolynomial %,Symbol) -> List %
```

These exports come from (p1497) AlgebraicallyClosedField():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
coerce : Fraction Integer -> %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
factor : % -> Factored %
gcd : List % -> %
gcd : (%,%) -> %
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
nthRoot : (%,Integer) -> %
one? : % -> Boolean
```

```
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
rootOf : SparseUnivariatePolynomial % -> %
rootOf : Polynomial % -> %
rootOf : (SparseUnivariatePolynomial %,Symbol) -> %
rootsOf : SparseUnivariatePolynomial % -> List %
rootsOf : Polynomial % -> List %
sample : () -> %
sizeLess? : (%,%) -> Boolean
sqrt : % -> %
squareFree : % -> Factored %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
zeroOf : Polynomial % -> %
zeroOf : SparseUnivariatePolynomial % -> %
zerosOf : Polynomial % -> List %
zerosOf : SparseUnivariatePolynomial % -> List %
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?**? : (%,Fraction Integer) -> %
?**? : (%,Integer) -> %
?^? : (%,Integer) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?/? : (%,%) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p1544) FunctionSpace(R)
where R:Join(OrderedSet, IntegralDomain)):

```
applyQuote : (Symbol,%) -> %
applyQuote : (Symbol,%,%) -> %
applyQuote : (Symbol,%,%,%) -> %
applyQuote : (Symbol,%,%,%,%) -> %
applyQuote : (Symbol,List %) -> %
belong? : BasicOperator -> Boolean
box : % -> %
box : List % -> %
```

```
charthRoot : % -> Union(%,"failed") if R has CHARNZ
coerce : R -> %
coerce : Symbol -> %
coerce : Kernel % -> %
coerce : Fraction Polynomial R -> % if R has INTDOM
coerce : Fraction Polynomial Fraction R -> % if R has INTDOM
coerce : SparseMultivariatePolynomial(R,Kernel %) -> %
  if R has RING
coerce : Fraction R -> % if R has INTDOM
coerce : Polynomial Fraction R -> % if R has INTDOM
coerce : Polynomial R -> % if R has RING
commutator : (%,%) -> % if R has GROUP
conjugate : (%,%) -> % if R has GROUP
convert : % -> InputForm if R has KONVERT INFORM
convert : % -> Pattern Integer if R has KONVERT PATTERN INT
convert : % -> Pattern Float if R has KONVERT PATTERN FLOAT
convert : Factored % -> % if R has INTDOM
D : (%,List Symbol,List NonNegativeInteger) -> % if R has RING
D : (%,Symbol,NonNegativeInteger) -> % if R has RING
D : (%,List Symbol) -> % if R has RING
D : (%,Symbol) -> % if R has RING
definingPolynomial : % -> % if $ has RING
denom : % -> SparseMultivariatePolynomial(R,Kernel %) if R has INTDOM
denominator : % -> % if R has INTDOM
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
  if R has RING
differentiate : (%,Symbol,NonNegativeInteger) -> % if R has RING
differentiate : (%,List Symbol) -> % if R has RING
distribute : % -> %
distribute : (%,%) -> %
differentiate : (%,Symbol) -> % if R has RING
elt : (BasicOperator,%,%,%) -> %
elt : (BasicOperator,%,%,%,%) -> %
elt : (BasicOperator,%) -> %
elt : (BasicOperator,%,%) -> %
elt : (BasicOperator,List %) -> %
eval : (%,List BasicOperator,List (% -> %)) -> %
eval : (%,List Equation %) -> %
eval : (%,Symbol,(% -> %)) -> %
eval : (%,Symbol,(List % -> %)) -> %
eval : (%,BasicOperator,%,Symbol) -> % if R has KONVERT INFORM
eval : (%,BasicOperator,(List % -> %)) -> %
eval : (%,BasicOperator,(% -> %)) -> %
eval : (%,List Symbol,List (% -> %)) -> %
eval : (%,List BasicOperator,List (List % -> %)) -> %
eval : (%,List Symbol,List (List % -> %)) -> %
eval : (%,List %,List %) -> %
eval : (%,%,%) -> %
eval : (%,Equation %) -> %
eval : (%,Kernel %,%) -> %
eval : (%,List Symbol) -> % if R has KONVERT INFORM
eval : % -> % if R has KONVERT INFORM
eval : (%,Symbol) -> % if R has KONVERT INFORM
eval : (%,Symbol,NonNegativeInteger,(% -> %)) -> % if R has RING
```

```
eval : (%,Symbol,NonNegativeInteger,(List % -> %)) -> % if R has RING
eval :
  (%,List Symbol,List NonNegativeInteger,List (List % -> %)) -> %
    if R has RING
eval :
  (%,List Symbol,List NonNegativeInteger,List (% -> %)) -> %
    if R has RING
eval : (%,List Kernel %,List %) -> %
eval : (%,List BasicOperator,List %,Symbol) -> %
  if R has KONVERT INFORM
even? : % -> Boolean if $ has RETRACT INT
freeOf? : (%,%) -> Boolean
freeOf? : (%,Symbol) -> Boolean
ground : % -> R
ground? : % -> Boolean
height : % -> NonNegativeInteger
is? : (%,BasicOperator) -> Boolean
is? : (%,Symbol) -> Boolean
isExpt : % ->
  Union(Record(var: Kernel %,exponent: Integer),"failed")
    if R has SGROUP
isExpt :
  (%,BasicOperator) ->
    Union(Record(var: Kernel %,exponent: Integer),"failed")
      if R has RING
isExpt :
  (%,Symbol) ->
    Union(Record(var: Kernel %,exponent: Integer),"failed")
      if R has RING
isMult : % ->
  Union(Record(coef: Integer,var: Kernel %),"failed")
    if R has ABELSG
isPlus : % -> Union(List %,"failed") if R has ABELSG
isPower : % -> Union(Record(val: %,exponent: Integer),"failed")
    if R has RING
isTimes : % -> Union(List %,"failed") if R has SGROUP
kernel : (BasicOperator,List %) -> %
kernel : (BasicOperator,%) -> %
kernels : % -> List Kernel %
mainKernel : % -> Union(Kernel %,"failed")
map : ((% -> %),Kernel %) -> %
max : (%,%) -> %
min : (%,%) -> %
minPoly : Kernel % -> SparseUnivariatePolynomial % if $ has RING
numer : % -> SparseMultivariatePolynomial(R,Kernel %) if R has RING
numerator : % -> % if R has RING
odd? : % -> Boolean if $ has RETRACT INT
operator : BasicOperator -> BasicOperator
operators : % -> List BasicOperator
paren : % -> %
paren : List % -> %
patternMatch :
  (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
     PatternMatchResult(Integer,%)
```

```
       if R has PATMAB INT
patternMatch :
  (%,Pattern Float,PatternMatchResult(Float,%)) ->
     PatternMatchResult(Float,%)
       if R has PATMAB FLOAT
reducedSystem : Matrix % -> Matrix Integer
  if and(has(R,Ring),has(R,LinearlyExplicitRingOver Integer))
  or and(has(R,LinearlyExplicitRingOver Integer),has(R,Ring))
reducedSystem :
 (Matrix %,Vector %) ->
   Record(mat: Matrix Integer,vec: Vector Integer)
     if and(has(R,Ring),has(R,LinearlyExplicitRingOver Integer))
     or and(has(R,LinearlyExplicitRingOver Integer),has(R,Ring))
reducedSystem :
 (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
   if R has RING
reducedSystem : Matrix % -> Matrix R if R has RING
retract : % -> Kernel %
retract : % -> Fraction Polynomial R if R has INTDOM
retract : % -> Polynomial R if R has RING
retract : % -> R
retract : % -> Symbol
retract : % -> Integer if R has RETRACT INT
retract : % -> Fraction Integer
  if R has RETRACT INT
  and R has INTDOM
  or R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
  if R has RETRACT INT
  and R has INTDOM
  or R has RETRACT FRAC INT
retractIfCan : % -> Union(Kernel %,"failed")
retractIfCan : % -> Union(R,"failed")
retractIfCan : % -> Union(Fraction Polynomial R,"failed")
  if R has INTDOM
retractIfCan : % -> Union(Polynomial R,"failed") if R has RING
retractIfCan : % -> Union(Symbol,"failed")
subst : (%,List Kernel %,List %) -> %
subst : (%,List Equation %) -> %
subst : (%,Equation %) -> %
tower : % -> List Kernel %
univariate : (%,Kernel %) -> Fraction SparseUnivariatePolynomial %
  if R has INTDOM
variables : % -> List Symbol
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?*? : (R,%) -> % if R has COMRING
?*? : (%,R) -> % if R has COMRING
?/? :
 (SparseMultivariatePolynomial(R,Kernel %),
  SparseMultivariatePolynomial(R,Kernel %)) -> %
```

```
 if R has INTDOM
```

— AlgebraicallyClosedFunctionSpace.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ACFS">
AlgebraicallyClosedFunctionSpace (ACFS)</a></h2>
</body>
```

———-

— category ACFS AlgebraicallyClosedFunctionSpace —

```
)abbrev category ACFS AlgebraicallyClosedFunctionSpace
++ Author: Manuel Bronstein
++ Date Created: 31 October 1988
++ Date Last Updated: 7 October 1991
++ Description:
++ Model for algebraically closed function spaces.

AlgebraicallyClosedFunctionSpace(R): Category == SIG where
  R : Join(OrderedSet,IntegralDomain)

  SIG ==> Join(AlgebraicallyClosedField, FunctionSpace R) with

    rootOf : $ -> $
      ++ rootOf(p) returns y such that \spad{p(y) = 0}.
      ++ Error: if p has more than one variable y.

    rootsOf: $ -> List $
      ++ rootsOf(p, y) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0};
      ++ Note that the returned symbols y1,...,yn are bound in the interpreter
      ++ to respective root values.
      ++ Error: if p has more than one variable y.

    rootOf : ($, Symbol) -> $
      ++ rootOf(p,y) returns y such that \spad{p(y) = 0}.
      ++ The object returned displays as \spad{'y}.

    rootsOf: ($, Symbol) -> List $
      ++ rootsOf(p, y) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0};
      ++ The returned roots display as \spad{'y1},...,\spad{'yn}.
      ++ Note that the returned symbols y1,...,yn are bound in the interpreter
      ++ to respective root values.

    zeroOf : $ -> $
      ++ zeroOf(p) returns y such that \spad{p(y) = 0}.
      ++ The value y is expressed in terms of radicals if possible,and
      ++ otherwise as an implicit algebraic quantity.
      ++ Error: if p has more than one variable.

    zerosOf: $ -> List $
      ++ zerosOf(p) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0}.
```

```
    ++ The yi's are expressed in radicals if possible.
    ++ The returned symbols y1,...,yn are bound in the interpreter
    ++ to respective root values.
    ++ Error: if p has more than one variable.

  zeroOf : ($, Symbol) -> $
    ++ zeroOf(p, y) returns y such that \spad{p(y) = 0}.
    ++ The value y is expressed in terms of radicals if possible,and
    ++ otherwise as an implicit algebraic quantity
    ++ which displays as \spad{'y}.

  zerosOf: ($, Symbol) -> List $
    ++ zerosOf(p, y) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0}.
    ++ The yi's are expressed in radicals if possible, and otherwise
    ++ as implicit algebraic quantities
    ++ which display as \spad{'yi}.
    ++ The returned symbols y1,...,yn are bound in the interpreter
    ++ to respective root values.

 add

   rootOf(p:$) ==
     empty?(l := variables p) => error "rootOf: constant expression"
     rootOf(p, first l)

   rootsOf(p:$) ==
     empty?(l := variables p) => error "rootsOf: constant expression"
     rootsOf(p, first l)

   zeroOf(p:$) ==
     empty?(l := variables p) => error "zeroOf: constant expression"
     zeroOf(p, first l)

   zerosOf(p:$) ==
     empty?(l := variables p) => error "zerosOf: constant expression"
     zerosOf(p, first l)

   zeroOf(p:$, x:Symbol) ==
     n := numer(f := univariate(p, kernel(x)$Kernel($)))
     degree denom f > 0 => error "zeroOf: variable appears in denom"
     degree n = 0 => error "zeroOf: constant expression"
     zeroOf(n, x)

   rootOf(p:$, x:Symbol) ==
     n := numer(f := univariate(p, kernel(x)$Kernel($)))
     degree denom f > 0 => error "roofOf: variable appears in denom"
     degree n = 0 => error "rootOf: constant expression"
     rootOf(n, x)

   zerosOf(p:$, x:Symbol) ==
     n := numer(f := univariate(p, kernel(x)$Kernel($)))
     degree denom f > 0 => error "zerosOf: variable appears in denom"
     degree n = 0 => empty()
     zerosOf(n, x)
```

```
 rootsOf(p:$, x:Symbol) ==
   n := numer(f := univariate(p, kernel(x)$Kernel($)))
   degree denom f > 0 => error "roofsOf: variable appears in denom"
   degree n = 0 => empty()
   rootsOf(n, x)

 rootsOf(p:SparseUnivariatePolynomial $, y:Symbol) ==
   (r := retractIfCan(p)@Union($,"failed")) case $ => rootsOf(r::$,y)
   rootsOf(p, y)$AlgebraicallyClosedField_&($)

 zerosOf(p:SparseUnivariatePolynomial $, y:Symbol) ==
   (r := retractIfCan(p)@Union($,"failed")) case $ => zerosOf(r::$,y)
   zerosOf(p, y)$AlgebraicallyClosedField_&($)

 zeroOf(p:SparseUnivariatePolynomial $, y:Symbol) ==
   (r := retractIfCan(p)@Union($,"failed")) case $ => zeroOf(r::$, y)
   zeroOf(p, y)$AlgebraicallyClosedField_&($)
```

———-

— **COQ ACFS** —

```
(* category ACFS *)
(*

    rootOf : % -> %
    rootOf(p:$) ==
      empty?(l := variables p) => error "rootOf: constant expression"
      rootOf(p, first l)

    rootsOf : % -> List(%)
    rootsOf(p:$) ==
      empty?(l := variables p) => error "rootsOf: constant expression"
      rootsOf(p, first l)

    zerosOf : % -> List(%)
    zeroOf(p:$) ==
      empty?(l := variables p) => error "zeroOf: constant expression"
      zeroOf(p, first l)

    zerosOf : % -> List(%)
    zerosOf(p:$) ==
      empty?(l := variables p) => error "zerosOf: constant expression"
      zerosOf(p, first l)

    zeroOf : (%,Symbol) -> %
    zeroOf(p:$, x:Symbol) ==
      n := numer(f := univariate(p, kernel(x)$Kernel($)))
      degree denom f > 0 => error "zeroOf: variable appears in denom"
      degree n = 0 => error "zeroOf: constant expression"
      zeroOf(n, x)
```

```
    rootOf : (%,Symbol) -> %
    rootOf(p:$, x:Symbol) ==
      n := numer(f := univariate(p, kernel(x)$Kernel($)))
      degree denom f > 0 => error "roofOf: variable appears in denom"
      degree n = 0 => error "rootOf: constant expression"
      rootOf(n, x)

    zerosOf : (%,Symbol) -> List(%)
    zerosOf(p:$, x:Symbol) ==
      n := numer(f := univariate(p, kernel(x)$Kernel($)))
      degree denom f > 0 => error "zerosOf: variable appears in denom"
      degree n = 0 => empty()
      zerosOf(n, x)

    rootsOf : (%,Symbol) -> List(%)
    rootsOf(p:$, x:Symbol) ==
      n := numer(f := univariate(p, kernel(x)$Kernel($)))
      degree denom f > 0 => error "roofsOf: variable appears in denom"
      degree n = 0 => empty()
      rootsOf(n, x)

    rootsOf : (SparseUnivariatePolynomial(%),Symbol) -> List(%)
    rootsOf(p:SparseUnivariatePolynomial $, y:Symbol) ==
      (r := retractIfCan(p)@Union($,"failed")) case $ => rootsOf(r::$,y)
      rootsOf(p, y)$AlgebraicallyClosedField_&($)

    zerosOf : (SparseUnivariatePolynomial(%),Symbol) -> List(%)
    zerosOf(p:SparseUnivariatePolynomial $, y:Symbol) ==
      (r := retractIfCan(p)@Union($,"failed")) case $ => zerosOf(r::$,y)
      zerosOf(p, y)$AlgebraicallyClosedField_&($)

    zeroOf : (SparseUnivariatePolynomial(%),Symbol) -> %
    zeroOf(p:SparseUnivariatePolynomial $, y:Symbol) ==
      (r := retractIfCan(p)@Union($,"failed")) case $ => zeroOf(r::$, y)
      zeroOf(p, y)$AlgebraicallyClosedField_&($)

*)
```

———-

— ACFS.dotabb —

```
"ACFS"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ACFS"];
"ACFS" -> "ACF"
"ACFS" -> "FS"
```

———-

— ACFS.dotfull —

```
"AlgebraicallyClosedFunctionSpace(a:Join(OrderedSet,IntegralDomain))"
```

```
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ACFS"];
"AlgebraicallyClosedFunctionSpace(a:Join(OrderedSet,IntegralDomain))"
  -> "AlgebraicallyClosedField()"
"AlgebraicallyClosedFunctionSpace(a:Join(OrderedSet,IntegralDomain))"
  -> "FunctionSpace(a:OrderedSet)"
```

———-

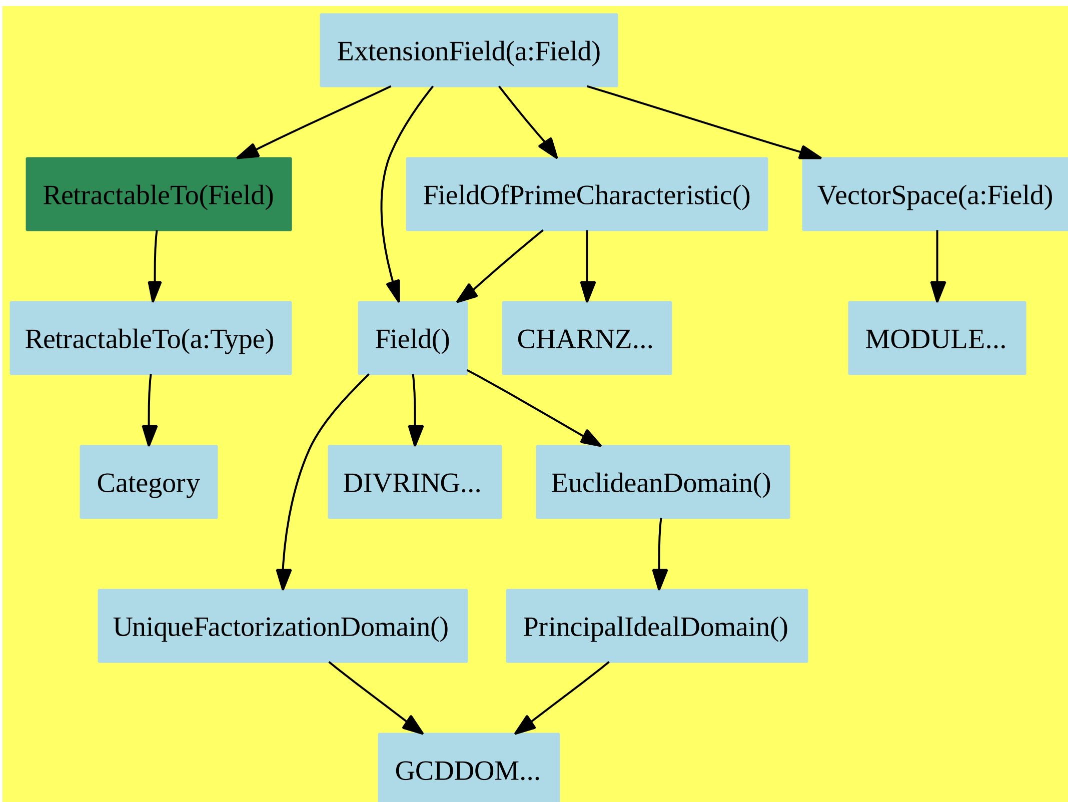
— **ACFS.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"AlgebraicallyClosedFunctionSpace(a:Join(OrderedSet,IntegralDomain))"
 [color=lightblue];
"AlgebraicallyClosedFunctionSpace(a:Join(OrderedSet,IntegralDomain))"
  -> "AlgebraicallyClosedField()"
"AlgebraicallyClosedFunctionSpace(a:Join(OrderedSet,IntegralDomain))"
  -> "FunctionSpace(a:OrderedSet)"

"AlgebraicallyClosedField()" [color=lightblue];
"AlgebraicallyClosedField()" -> "Field()"
"AlgebraicallyClosedField()" -> "RadicalCategory()"

"Field()" [color=lightblue];
"Field()" -> "EUCDOM..."
"Field()" -> "UFD..."
"Field()" -> "DIVRING..."

"RadicalCategory()" [color=lightblue];
"RadicalCategory()" -> "Category"

"Category" [color=lightblue];

"FunctionSpace(a:OrderedSet)" [color=lightblue];
"FunctionSpace(a:OrderedSet)" -> "ExpressionSpace()"
"FunctionSpace(a:OrderedSet)" -> "RETRACT..."
"FunctionSpace(a:OrderedSet)" -> "PATAB..."
"FunctionSpace(a:OrderedSet)" -> "FPATMAB..."
"FunctionSpace(a:OrderedSet)" -> "FRETRCT..."

"ExpressionSpace()" [color=lightblue];
"ExpressionSpace()" -> "ORDSET..."
"ExpressionSpace()" -> "RETRACT..."
"ExpressionSpace()" -> "IEVALAB..."
"ExpressionSpace()" -> "EVALABLE..."

"UFD..." [color=lightblue];
"EUCDOM..." [color=lightblue];
"DIVRING..." [color=lightblue];
"EVALABLE..." [color=lightblue];
```

```
"FRETRCT..." [color=lightblue];
"FPATMAB..." [color=lightblue];
"IEVALAB..." [color=lightblue];
"ORDSET..." [color=lightblue];
"PATAB..." [color=lightblue];
"RETRACT..." [color=lightblue];
}
```

———-

18.0.230 ExtensionField (XF)



— **ExtensionField.input** —

```
)set break resume
)sys rm -f ExtensionField.output
)spool ExtensionField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
```

```
)show ExtensionField
--R
--R ExtensionField(F: Field) is a category constructor
--R Abbreviation for ExtensionField is XF
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for XF
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (F,%) -> %                      ?*? : (%,F) -> %
--R ?*? : (Fraction(Integer),%) -> %      ?*? : (%,Fraction(Integer)) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,Integer) -> %               ?**? : (%,NonNegativeInteger) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (%,F) -> %                      ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                Frobenius : % -> % if F has FINITE
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,Integer) -> %                ?^? : (%,NonNegativeInteger) -> %
--R ?^? : (%,PositiveInteger) -> %        algebraic? : % -> Boolean
--R associates? : (%,%) -> Boolean        coerce : F -> %
--R coerce : Fraction(Integer) -> %       coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R dimension : () -> CardinalNumber      factor : % -> Factored(%)
--R gcd : List(%) -> %                    gcd : (%,%) -> %
--R hash : % -> SingleInteger             inGroundField? : % -> Boolean
--R inv : % -> %                          latex : % -> String
--R lcm : List(%) -> %                    lcm : (%,%) -> %
--R one? : % -> Boolean                   prime? : % -> Boolean
--R ?quo? : (%,%) -> %                    recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                    retract : % -> F
--R sample : () -> %                      sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored(%)         squareFreePart : % -> %
--R transcendent? : % -> Boolean          unit? : % -> Boolean
--R unitCanonical : % -> %                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R Frobenius : (%,NonNegativeInteger) -> % if F has FINITE
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if F has CHARNZ or F has FINITE
--R degree : % -> OnePointCompletion(PositiveInteger)
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if F has CHARNZ or F has FINITE
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extensionDegree : () -> OnePointCompletion(PositiveInteger)
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed")
--R order : % -> OnePointCompletion(PositiveInteger) if F has CHARNZ or F has FINITE
--R primeFrobenius : % -> % if F has CHARNZ or F has FINITE
--R primeFrobenius : (%,NonNegativeInteger) -> % if F has CHARNZ or F has FINITE
```

```
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %)
--R retractIfCan : % -> Union(F,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

— **ExtensionField.help** —

```
====================================================================
ExtensionField examples
====================================================================

ExtensionField F is the category of fields which extend the field F

See Also:
o )show ExtensionField
```

See:

⇒ "FiniteAlgebraicExtensionField" (FAXF) 19.0.237 on page 1829
⇐ "Field" (FIELD) 16.0.209 on page 1413
⇐ "RetractableTo" (RETRACT) 2.0.37 on page 110
⇐ "VectorSpace" (VSPACE) 11.0.182 on page 1160

Exports:

0	1	algebraic?	associates?
characteristic	charthRoot	coerce	degree
discreteLog	divide	euclideanSize	expressIdealMember
exquo	extendedEuclidean	extensionDegree	dimension
factor	Frobenius	gcd	gcdPolynomial
hash	inGroundField?	inv	latex
lcm	multiEuclidean	one?	order
prime?	primeFrbenius	principalIdeal	recip
retract	retractIfCan	sample	sizeLess?
squareFree	squareFreePart	subtractIfCan	transcendenceDegree
transcendent?	unit?	unitCanonical	unitNormal
zero?	?*?	?**?	?+?
?-?	-?	?/?	?=?
?^?	?quo?	?rem?	?~=?

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.

- **canonicalsClosed** is true if
  `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
inGroundField? : % -> Boolean
degree : % -> OnePointCompletion PositiveInteger
extensionDegree : () -> OnePointCompletion PositiveInteger
transcendenceDegree : () -> NonNegativeInteger
```

These are implemented by this category:

```
algebraic? : % -> Boolean
Frobenius : % -> % if F has FINITE
Frobenius : (%,NonNegativeInteger) -> % if F has FINITE
transcendent? : % -> Boolean
```

These exports come from (p1413) Field():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
coerce : Fraction Integer -> %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extendedEuclidean : (%,%,%) ->
  Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) ->
  Record(coef1: %,coef2: %,generator: %)
factor : % -> Factored %
gcd : List % -> %
gcd : (%,%) -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                     SparseUnivariatePolynomial %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
lcm : List % -> %
```

```
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
sample : () -> %
squareFree : % -> Factored %
squareFreePart : % -> %
sizeLess? : (%,%) -> Boolean
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?/? : (%,%) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?*? : (F,%) -> %
?*? : (%,F) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,Integer) -> %
?^? : (%,Integer) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p110) RetractableTo(F:Field):

```
coerce : F -> %
retract : % -> F
retractIfCan : % -> Union(F,"failed")
```

These exports come from (p1160) VectorSpace(F:Field):

```
dimension : () -> CardinalNumber
?/? : (%,F) -> %
```

These exports come from (p1531) FieldOfPrimeCharacteristic():

```
charthRoot : % -> Union(%,"failed")
    if F has CHARNZ or F has FINITE
discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
    if F has CHARNZ or F has FINITE
order : % -> OnePointCompletion PositiveInteger
    if F has CHARNZ or F has FINITE
```

```
 primeFrobenius : % -> %
     if F has CHARNZ or F has FINITE
 primeFrobenius : (%,NonNegativeInteger) -> %
     if F has CHARNZ or F has FINITE
```

See: Grabmeier[Grab92]

— ExtensionField.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#XF">
ExtensionField (XF)</a></h2>
</body>
```

— category XF ExtensionField —

```
)abbrev category XF ExtensionField
++ Author: J. Grabmeier, A. Scheerhorn
++ Date Created: 10 March 1991
++ Date Last Updated: 31 March 1991
++ References:
++ Grab92 Finite Fields in Axiom
++ Description:
++ ExtensionField F is the category of fields which extend the field F

ExtensionField(F) : Category  == SIG where
  F : Field

  SIG ==> Join(Field,RetractableTo F,VectorSpace F) with

    if F has CharacteristicZero then CharacteristicZero

    if F has CharacteristicNonZero then FieldOfPrimeCharacteristic

    algebraic? : $ -> Boolean
      ++ algebraic?(a) tests whether an element \spad{a} is algebraic with
      ++ respect to the ground field F.

    transcendent? : $ -> Boolean
      ++ transcendent?(a) tests whether an element \spad{a} is transcendent
      ++ with respect to the ground field F.

    inGroundField? : $ -> Boolean
      ++ inGroundField?(a) tests whether an element \spad{a}
      ++ is already in the ground field F.

    degree : $ -> OnePointCompletion PositiveInteger
      ++ degree(a) returns the degree of minimal polynomial of an element
      ++ \spad{a} if \spad{a} is algebraic
      ++ with respect to the ground field F, and \spad{infinity} otherwise.

    extensionDegree : () -> OnePointCompletion PositiveInteger
      ++ extensionDegree() returns the degree of the field extension if the
```

```
      ++ extension is algebraic, and \spad{infinity} if it is not.

    transcendenceDegree : () -> NonNegativeInteger
      ++ transcendenceDegree() returns the transcendence degree of the
      ++ field extension, 0 if the extension is algebraic.

    if F has Finite then

      FieldOfPrimeCharacteristic

      Frobenius : $ -> $
        ++ Frobenius(a) returns \spad{a ** q} where q is the \spad{size()$F}.

      Frobenius : ($,NonNegativeInteger) -> $
        ++ Frobenius(a,s) returns \spad{a**(q**s)} where q is the size()$F.
   add

     algebraic?(a) == not infinite? (degree(a)@OnePointCompletion_
       (PositiveInteger))$OnePointCompletion(PositiveInteger)

     transcendent? a == infinite?(degree(a)@OnePointCompletion _
       (PositiveInteger))$OnePointCompletion(PositiveInteger)

     if F has Finite then

       Frobenius(a) == a ** size()$F

       Frobenius(a,s) == a ** (size()$F ** s)
```

— COQ XF —

```
(* category XF *)
(*

    algebraic? : % -> Boolean
    algebraic?(a) == not infinite? (degree(a)@OnePointCompletion_
      (PositiveInteger))$OnePointCompletion(PositiveInteger)

    transcendent? : % -> Boolean
    transcendent? a == infinite?(degree(a)@OnePointCompletion _
      (PositiveInteger))$OnePointCompletion(PositiveInteger)

    if F has Finite then

      Frobenius : % -> %
      Frobenius(a) == a ** size()$F

      Frobenius : (%,NonNegativeInteger) -> %
      Frobenius(a,s) == a ** (size()$F ** s)

*)
```

———-

— **XF.dotabb** —

```
"XF"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XF"];
"XF" -> "FIELD"
"XF" -> "RETRACT"
"XF" -> "VSPACE"
"XF" -> "FPC"
```

———-

— **XF.dotfull** —

```
"ExtensionField(a:Field)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XF"];
"ExtensionField(a:Field)" -> "Field()"
"ExtensionField(a:Field)" -> "RetractableTo(Field)"
"ExtensionField(a:Field)" -> "VectorSpace(a:Field)"
"ExtensionField(a:Field)" -> "FieldOfPrimeCharacteristic()"
```

———-

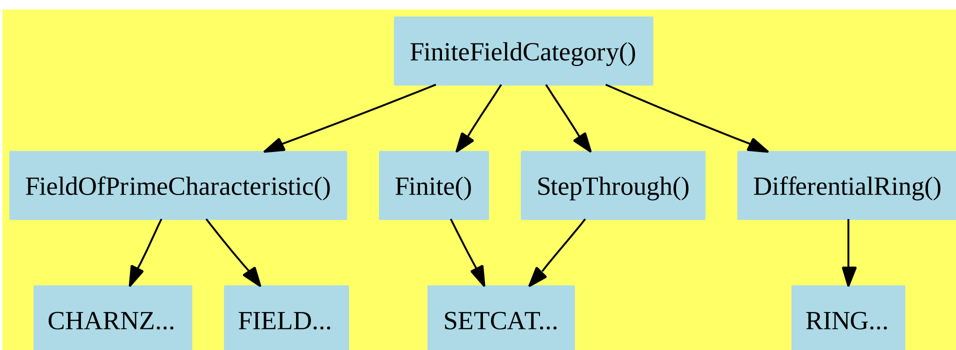
— **XF.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"ExtensionField(a:Field)" [color=lightblue];
"ExtensionField(a:Field)" -> "Field()"
"ExtensionField(a:Field)" -> "RetractableTo(Field)"
"ExtensionField(a:Field)" -> "VectorSpace(a:Field)"

"FieldOfPrimeCharacteristic()" [color=lightblue];
"FieldOfPrimeCharacteristic()" -> "CHARNZ..."
"FieldOfPrimeCharacteristic()" -> "Field()"

"Field()" [color=lightblue];
"Field()" -> "EuclideanDomain()"
"Field()" -> "UniqueFactorizationDomain()"
"Field()" -> "DIVRING..."

"EuclideanDomain()" [color=lightblue];
"EuclideanDomain()" -> "PrincipalIdealDomain()"

"UniqueFactorizationDomain()" [color=lightblue];
"UniqueFactorizationDomain()" -> "GCDDOM..."
```

```
"PrincipalIdealDomain()" [color=lightblue];
"PrincipalIdealDomain()" -> "GCDDOM..."

"RetractableTo(Field)" [color=seagreen];
"RetractableTo(Field)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"VectorSpace(a:Field)" [color=lightblue];
"VectorSpace(a:Field)" -> "MODULE..."

"MODULE..." [color=lightblue];
"DIVRING..." [color=lightblue];
"GCDDOM..." [color=lightblue];
"CHARNZ..." [color=lightblue];
"Category" [color=lightblue];
}
```

————-

18.0.231 FiniteFieldCategory (FFIELDC)



— **FiniteFieldCategory.input** —

```
)set break resume
)sys rm -f FiniteFieldCategory.output
)spool FiniteFieldCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteFieldCategory
--R
--R FiniteFieldCategory is a category constructor
--R Abbreviation for FiniteFieldCategory is FFIELDC
```

```
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FFIELDC
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Fraction(Integer),%) -> %      ?*? : (%,Fraction(Integer)) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,Integer) -> %               ?**? : (%,NonNegativeInteger) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (%,%) -> %                      ?=? : (%,%) -> Boolean
--R D : % -> %                            D : (%,NonNegativeInteger) -> %
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,Integer) -> %                ?^? : (%,NonNegativeInteger) -> %
--R ?^? : (%,PositiveInteger) -> %        associates? : (%,%) -> Boolean
--R charthRoot : % -> %                   coerce : Fraction(Integer) -> %
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              createPrimitiveElement : () -> %
--R differentiate : % -> %                enumerate : () -> List(%)
--R factor : % -> Factored(%)             gcd : List(%) -> %
--R gcd : (%,%) -> %                      hash : % -> SingleInteger
--R index : PositiveInteger -> %          init : () -> %
--R inv : % -> %                          latex : % -> String
--R lcm : List(%) -> %                    lcm : (%,%) -> %
--R lookup : % -> PositiveInteger         nextItem : % -> Union(%,"failed")
--R one? : % -> Boolean                   order : % -> PositiveInteger
--R prime? : % -> Boolean                 primeFrobenius : % -> %
--R primitive? : % -> Boolean             primitiveElement : () -> %
--R ?quo? : (%,%) -> %                    random : () -> %
--R recip : % -> Union(%,"failed")        ?rem? : (%,%) -> %
--R sample : () -> %                      size : () -> NonNegativeInteger
--R sizeLess? : (%,%) -> Boolean          squareFree : % -> Factored(%)
--R squareFreePart : % -> %               unit? : % -> Boolean
--R unitCanonical : % -> %                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed")
--R conditionP : Matrix(%) -> Union(Vector(%),"failed")
--R differentiate : (%,NonNegativeInteger) -> %
--R discreteLog : % -> NonNegativeInteger
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R factorsOfCyclicGroupSize : () -> List(Record(factor: Integer,exponent: Integer))
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed")
--R order : % -> OnePointCompletion(PositiveInteger)
--R primeFrobenius : (%,NonNegativeInteger) -> %
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %)
```

```
--R representationType : () -> Union("prime",polynomial,normal,cyclic)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger)
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

———-

— FiniteFieldCategory.help —

```
====================================================================
FiniteFieldCategory examples
====================================================================

FiniteFieldCategory is the category of finite fields

See Also:
o )show FiniteFieldCategory
```

———-

See:

⇐ "DifferentialRing" (DIFRING) 10.0.162 on page 1024
⇐ "FieldOfPrimeCharacteristic" (FPC) 17.0.217 on page 1531
⇐ "Finite" (FINITE) 4.0.69 on page 258
⇐ "StepThrough" (STEP) 4.0.88 on page 367

Exports:

0	1	associates?
characteristic	charthRoot	coerce
conditionP	createPrimitiveElement	D
differentiate	discreteLog	divide
euclideanSize	expressIdealMember	exquo
extendedEuclidean	factor	factorsOfCyclicGroupSize
gcd	gcdPolynomial	hash
index	init	inv
latex	lcm	lookup
multiEuclidean	nextItem	one?
order	prime?	primeFrobenius
primitive?	primitiveElement	principalIdeal
random	recip	representationType
sample	size	sizeLess?
squareFree	squareFreePart	subtractIfCan
tableForDiscreteLogarithm	unit?	unitCanonical
unitNormal	zero?	?*?
?**?	?+?	?-?
-?	?/?	?=?
?^?	?quo?	?rem?
?~=?		

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
factorsOfCyclicGroupSize : () ->
   List Record(factor: Integer,exponent: Integer)
primitiveElement : () -> %
representationType : () -> Union("prime",polynomial,normal,cyclic)
tableForDiscreteLogarithm : Integer ->
   Table(PositiveInteger,NonNegativeInteger)
```

These are implemented by this category:

```
charthRoot : % -> %
charthRoot : % -> Union(%,"failed")
conditionP : Matrix % -> Union(Vector %,"failed")
createPrimitiveElement : () -> %
```

```
differentiate : % -> %
discreteLog : % -> NonNegativeInteger
discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                     SparseUnivariatePolynomial %
init : () -> %
nextItem : % -> Union(%,"failed")
order : % -> OnePointCompletion PositiveInteger
order : % -> PositiveInteger
primitive? : % -> Boolean
```

These exports come from (p1531) FieldOfPrimeCharacteristic():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
coerce : Fraction Integer -> %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
exquo : (%,%) -> Union(%,"failed")
factor : % -> Factored %
gcd : List % -> %
gcd : (%,%) -> %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
prime? : % -> Boolean
primeFrobenius : % -> %
primeFrobenius : (%,NonNegativeInteger) -> %
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
sample : () -> %
sizeLess? : (%,%) -> Boolean
squareFree : % -> Factored %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

```
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,Integer) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,Integer) -> %
?/? : (%,%) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p258) Finite():

```
index : PositiveInteger -> %
lookup : % -> PositiveInteger
random : () -> %
size : () -> NonNegativeInteger
```

These exports come from (p367) StepThrough():

These exports come from (p1024) DifferentialRing():

```
D : % -> %
D : (%,NonNegativeInteger) -> %
differentiate : (%,NonNegativeInteger) -> %
```

See: Grabmeier[Grab92], Lipson[Lips81]

— **FiniteFieldCategory.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FFIELDC">
FiniteFieldCategory (FFIELDC)</a></h2>
</body>
```

———-

— **category FFIELDC FiniteFieldCategory** —

```
)abbrev category FFIELDC FiniteFieldCategory
++ Author: J. Grabmeier, A. Scheerhorn
++ Date Created: 11 March 1991
++ Date Last Updated: 31 March 1991
++ References:
++ Grab92 Finite Fields in AXIOM.
++ Lips81 Elements of Algebra and Algebraic Computing
++ Description:
++ FiniteFieldCategory is the category of finite fields
```

```
FiniteFieldCategory() : Category == SIG where

  FOPC ==> FieldOfPrimeCharacteristic
  F    ==> Finite
  ST   ==> StepThrough
  DR   ==> DifferentialRing

  SIG ==> Join(FOPC,F,ST,DR) with

    charthRoot : $ -> $
      ++ charthRoot(a) takes the characteristic'th root of a.
      ++ Note that such a root is alway defined in finite fields.

    conditionP : Matrix $ -> Union(Vector $,"failed")
      ++ conditionP(mat), given a matrix representing a homogeneous system
      ++ of equations, returns a vector whose characteristic'th powers
      ++ is a non-trivial solution, or "failed" if no such vector exists.

    -- the reason for implementing the following function is that we
    -- can implement the functions order, getGenerator and primitive? on
    -- category level without computing the, may be time intensive,
    -- factorization of size()-1 at every function call again.

    factorsOfCyclicGroupSize :_
      () -> List Record(factor:Integer,exponent:Integer)
      ++ factorsOfCyclicGroupSize() returns the factorization of size()-1

    -- the reason for implementing the function tableForDiscreteLogarithm
    -- is that we can implement the functions discreteLog and
    -- shanksDiscLogAlgorithm on category level
    -- computing the necessary exponentiation tables in the respective
    -- domains once and for all
    -- absoluteDegree : $ -> PositiveInteger
    --  ++ degree of minimal polynomial, if algebraic with respect
    --  ++ to the prime subfield

    tableForDiscreteLogarithm : Integer -> _
            Table(PositiveInteger,NonNegativeInteger)
      ++ tableForDiscreteLogarithm(a,n) returns a table of the discrete
      ++ logarithms of \spad{a**0} up to \spad{a**(n-1)} which, called with
      ++ key \spad{lookup(a**i)} returns i for i in \spad{0..n-1}.
      ++ Error: if not called for prime divisors of order of
      ++        multiplicative group.

    createPrimitiveElement : () -> $
      ++ createPrimitiveElement() computes a generator of the (cyclic)
      ++ multiplicative group of the field.
      -- RDJ: Are these next lines to be included?
      -- we run through the field and test, algorithms which construct
      -- elements of larger order were found to be too slow

    primitiveElement : () -> $
      ++ primitiveElement() returns a primitive element stored in a global
      ++ variable in the domain.
```

```
   ++ At first call, the primitive element is computed
   ++ by calling \spadfun{createPrimitiveElement}.

 primitive? : $ -> Boolean
   ++ primitive?(b) tests whether the element b is a generator of the
   ++ (cyclic) multiplicative group of the field, is a primitive
   ++ element.
   ++ Implementation Note that see ch.IX.1.3, th.2 in D. Lipson.

 discreteLog : $ -> NonNegativeInteger
   ++ discreteLog(a) computes the discrete logarithm of \spad{a}
   ++ with respect to \spad{primitiveElement()} of the field.

 order : $ -> PositiveInteger
   ++ order(b) computes the order of an element b in the multiplicative
   ++ group of the field.
   ++ Error: if b equals 0.

 representationType : () -> Union("prime","polynomial","normal","cyclic")
   ++ representationType() returns the type of the representation, one of:
   ++ \spad{prime}, \spad{polynomial}, \spad{normal}, or \spad{cyclic}.

add

  I   ==> Integer
  PI  ==> PositiveInteger
  NNI ==> NonNegativeInteger
  SUP ==> SparseUnivariatePolynomial
  DLP ==> DiscreteLogarithmPackage

  -- exported functions

  differentiate x == 0

  init() == 0

  nextItem(a) ==
    zero?(a:=index(lookup(a)+1)) => "failed"
    a

  order(e):OnePointCompletion(PositiveInteger) ==
    (order(e)@PI)::OnePointCompletion(PositiveInteger)

  conditionP(mat:Matrix $) ==
    l:=nullSpace mat
    empty? l or every?(zero?, first l) => "failed"
    map(charthRoot,first l)

  charthRoot(x:$):$ == x**(size() quo characteristic())

  charthRoot(x:%):Union($,"failed") ==
      (charthRoot(x)@$)::Union($,"failed")

  createPrimitiveElement() ==
```

```
    sm1  : PositiveInteger := (size()$$-1) pretend PositiveInteger
    start : Integer :=
      -- in the polynomial case, index from 1 to characteristic-1
      -- gives prime field elements
      representationType = "polynomial" => characteristic()::Integer
      1
    found : Boolean := false
    for i in start..  while not found repeat
      e : $ := index(i::PositiveInteger)
      found := (order(e) = sm1)
    e

  primitive? a ==
    -- add special implementation for prime field case
    zero?(a) => false
    explist := factorsOfCyclicGroupSize()
    q:=(size()-1)@Integer
    equalone : Boolean := false
    for exp in explist while not equalone repeat
      equalone := ((a**(q quo exp.factor)) = 1)
    not equalone

  order e ==
    e = 0 => error "order(0) is not defined "
    ord:Integer:= size()-1 -- order e divides ord
    a:Integer:= 0
    lof:=factorsOfCyclicGroupSize()
    for rec in lof repeat -- run through prime divisors
      a := ord quo (primeDivisor := rec.factor)
      goon := ((e**a) = 1)
      -- run through exponents of the prime divisors
      for j in 0..(rec.exponent)-2 while goon repeat
        -- as long as we get (e**ord = 1) we
        -- continue dividing by primeDivisor
        ord := a
        a := ord quo primeDivisor
        goon := ((e**a) = 1)
      if goon then ord := a
      -- as we do a top down search we have found the
      -- correct exponent of primeDivisor in order e
      -- and continue with next prime divisor
    ord pretend PositiveInteger

  discreteLog(b) ==
    zero?(b) => error "discreteLog: logarithm of zero"
    faclist:=factorsOfCyclicGroupSize()
    a:=b
    gen:=primitiveElement()
    -- in GF(2) its necessary to have discreteLog(1) = 1
    b = gen => 1
    disclog:Integer:=0
    mult:Integer:=1
    groupord := (size() - 1)@Integer
    exp:Integer:=groupord
```

```
     for f in faclist repeat
       fac:=f.factor
       for t in 0..f.exponent-1 repeat
         exp:=exp quo fac
         -- shanks discrete logarithm algorithm
         exptable:=tableForDiscreteLogarithm(fac)
         n:=#exptable
         c:=a**exp
         end:=(fac - 1) quo n
         found:=false
         disc1:Integer:=0
         for i in 0..end while not found repeat
           rho:= search(lookup(c),exptable)_
                 $Table(PositiveInteger,NNI)
           rho case NNI =>
             found := true
             disc1:=((n * i + rho)@Integer) * mult
           c:=c* gen**((groupord quo fac) * (-n))
         not found => error "discreteLog: ?? discrete logarithm"
         -- end of shanks discrete logarithm algorithm
         mult := mult * fac
         disclog:=disclog+disc1
         a:=a * (gen ** (-disc1))
     disclog pretend NonNegativeInteger

   discreteLog(logbase,b) ==
     zero?(b) =>
       messagePrint("discreteLog: logarithm of zero")$OutputForm
       "failed"
     zero?(logbase) =>
       messagePrint("discreteLog: logarithm to base zero")$OutputForm
       "failed"
     b = logbase => 1
     not zero?((groupord:=order(logbase)@PI) rem order(b)@PI) =>
        messagePrint("discreteLog: second argument not in cyclic group_
generated by first argument")$OutputForm
        "failed"
     faclist:=factors factor groupord
     a:=b
     disclog:Integer:=0
     mult:Integer:=1
     exp:Integer:= groupord
     for f in faclist repeat
       fac:=f.factor
       primroot:= logbase ** (groupord quo fac)
       for t in 0..f.exponent-1 repeat
         exp:=exp quo fac
         rhoHelp:= shanksDiscLogAlgorithm(primroot,_
               a**exp,fac pretend NonNegativeInteger)$DLP($)
         rhoHelp case "failed" => return "failed"
         rho := (rhoHelp :: NNI) * mult
         disclog := disclog + rho
         mult := mult * fac
         a:=a * (logbase ** (-rho))
```

```
      disclog pretend NonNegativeInteger

     FP ==> SparseUnivariatePolynomial($)
     FRP ==> Factored FP
     f,g:FP

 -- TPDHERE: why is this here? It isn't exported.
     squareFreePolynomial(f:FP):FRP ==
           squareFree(f)$UnivariatePolynomialSquareFree($,FP)

 -- TPDHERE: why is this here? It isn't exported.
     factorPolynomial(f:FP):FRP == factor(f)$DistinctDegreeFactorize($,FP)

 -- TPDHERE: why is this here? It isn't exported.
     factorSquareFreePolynomial(f:FP):FRP ==
         f = 0 => 0
         flist := distdfact(f,true)$DistinctDegreeFactorize($,FP)
         (flist.cont :: FP) *
             (*/[primeFactor(u.irr,u.pow) for u in flist.factors])

     gcdPolynomial(f:FP,g:FP):FP ==
          gcd(f,g)$EuclideanDomain_&(FP)
```

———-

— COQ FFIELDC —

```
(* category FFIELDC *)
(*
    I   ==> Integer
    PI  ==> PositiveInteger
    NNI ==> NonNegativeInteger
    SUP ==> SparseUnivariatePolynomial
    DLP ==> DiscreteLogarithmPackage

    -- exported functions

    differentiate : % -> %
    differentiate x == 0

    init : () -> %
    init() == 0

    nextItem : % -> Union(%,"failed")
    nextItem(a) ==
      zero?(a:=index(lookup(a)+1)) => "failed"
      a

    order : % -> OnePointCompletion(PositiveInteger)
    order(e):OnePointCompletion(PositiveInteger) ==
      (order(e)@PI)::OnePointCompletion(PositiveInteger)

    conditionP : Matrix(%) -> Union(Vector(%),"failed")
```

```
conditionP(mat:Matrix $) ==
  l:=nullSpace mat
  empty? l or every?(zero?, first l) => "failed"
  map(charthRoot,first l)

charthRoot : % -> %
charthRoot(x:$):$ == x**(size() quo characteristic())

charthRoot : % -> Union(%,"failed")
charthRoot(x:%):Union($,"failed") ==
    (charthRoot(x)@$)::Union($,"failed")

createPrimitiveElement : () -> %
createPrimitiveElement() ==
  sm1  : PositiveInteger := (size()$$-1) pretend PositiveInteger
  start : Integer :=
    -- in the polynomial case, index from 1 to characteristic-1
    -- gives prime field elements
    representationType = "polynomial" => characteristic()::Integer
    1
  found : Boolean := false
  for i in start..  while not found repeat
    e : $ := index(i::PositiveInteger)
    found := (order(e) = sm1)
  e

primitive? : % -> Boolean
primitive? a ==
  -- add special implementation for prime field case
  zero?(a) => false
  explist := factorsOfCyclicGroupSize()
  q:=(size()-1)@Integer
  equalone : Boolean := false
  for exp in explist while not equalone repeat
    equalone := ((a**(q quo exp.factor)) = 1)
  not equalone

order : % -> PositiveInteger
order e ==
  e = 0 => error "order(0) is not defined "
  ord:Integer:= size()-1 -- order e divides ord
  a:Integer:= 0
  lof:=factorsOfCyclicGroupSize()
  for rec in lof repeat -- run through prime divisors
    a := ord quo (primeDivisor := rec.factor)
    goon := ((e**a) = 1)
    -- run through exponents of the prime divisors
    for j in 0..(rec.exponent)-2 while goon repeat
      -- as long as we get (e**ord = 1) we
      -- continue dividing by primeDivisor
      ord := a
      a := ord quo primeDivisor
      goon := ((e**a) = 1)
    if goon then ord := a
```

```
        -- as we do a top down search we have found the
        -- correct exponent of primeDivisor in order e
        -- and continue with next prime divisor
      ord pretend PositiveInteger

    discreteLog : % -> NonNegativeInteger
    discreteLog(b) ==
      zero?(b) => error "discreteLog: logarithm of zero"
      faclist:=factorsOfCyclicGroupSize()
      a:=b
      gen:=primitiveElement()
      -- in GF(2) its necessary to have discreteLog(1) = 1
      b = gen => 1
      disclog:Integer:=0
      mult:Integer:=1
      groupord := (size() - 1)@Integer
      exp:Integer:=groupord
      for f in faclist repeat
        fac:=f.factor
        for t in 0..f.exponent-1 repeat
          exp:=exp quo fac
          -- shanks discrete logarithm algorithm
          exptable:=tableForDiscreteLogarithm(fac)
          n:=#exptable
          c:=a**exp
          end:=(fac - 1) quo n
          found:=false
          disc1:Integer:=0
          for i in 0..end while not found repeat
            rho:= search(lookup(c),exptable)_
                  $Table(PositiveInteger,NNI)
            rho case NNI =>
              found := true
              disc1:=((n * i + rho)@Integer) * mult
            c:=c* gen**((groupord quo fac) * (-n))
          not found => error "discreteLog: ?? discrete logarithm"
          -- end of shanks discrete logarithm algorithm
          mult := mult * fac
          disclog:=disclog+disc1
          a:=a * (gen ** (-disc1))
      disclog pretend NonNegativeInteger

    discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
    discreteLog(logbase,b) ==
      zero?(b) =>
        messagePrint("discreteLog: logarithm of zero")$OutputForm
        "failed"
      zero?(logbase) =>
        messagePrint("discreteLog: logarithm to base zero")$OutputForm
        "failed"
      b = logbase => 1
      not zero?((groupord:=order(logbase)@PI) rem order(b)@PI) =>
         messagePrint("discreteLog: second argument not in cyclic group_
generated by first argument")$OutputForm
```

```
         "failed"
      faclist:=factors factor groupord
      a:=b
      disclog:Integer:=0
      mult:Integer:=1
      exp:Integer:= groupord
      for f in faclist repeat
        fac:=f.factor
        primroot:= logbase ** (groupord quo fac)
        for t in 0..f.exponent-1 repeat
          exp:=exp quo fac
          rhoHelp:= shanksDiscLogAlgorithm(primroot,_
                a**exp,fac pretend NonNegativeInteger)$DLP($)
          rhoHelp case "failed" => return "failed"
          rho := (rhoHelp :: NNI) * mult
          disclog := disclog + rho
          mult := mult * fac
          a:=a * (logbase ** (-rho))
      disclog pretend NonNegativeInteger

    FP ==> SparseUnivariatePolynomial($)
    FRP ==> Factored FP
    f,g:FP

-- TPDHERE: why is this here? It isn't exported.
    squareFreePolynomial(f:FP):FRP ==
          squareFree(f)$UnivariatePolynomialSquareFree($,FP)

-- TPDHERE: why is this here? It isn't exported.
    factorPolynomial(f:FP):FRP == factor(f)$DistinctDegreeFactorize($,FP)

-- TPDHERE: why is this here? It isn't exported.
    factorSquareFreePolynomial(f:FP):FRP ==
        f = 0 => 0
        flist := distdfact(f,true)$DistinctDegreeFactorize($,FP)
        (flist.cont :: FP) *
            (*/[primeFactor(u.irr,u.pow) for u in flist.factors])

    gcdPolynomial : (SparseUnivariatePolynomial(%),
                     SparseUnivariatePolynomial(%)) ->
                        SparseUnivariatePolynomial(%)
    gcdPolynomial(f:FP,g:FP):FP ==
         gcd(f,g)$EuclideanDomain_&(FP)

*)
```

————

— **FFIELDC.dotabb** —

```
"FFIELDC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FFIELDC"];
"FFIELDC" -> "FPC"
```

```
"FFIELDC" -> "FINITE"
"FFIELDC" -> "STEP"
"FFIELDC" -> "DIFRING"
```

———-

— FFIELDC.dotfull —

```
"FiniteFieldCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FFIELDC"];
"FiniteFieldCategory()" -> "FieldOfPrimeCharacteristic()"
"FiniteFieldCategory()" -> "Finite()"
"FiniteFieldCategory()" -> "StepThrough()"
"FiniteFieldCategory()" -> "DifferentialRing()"
```

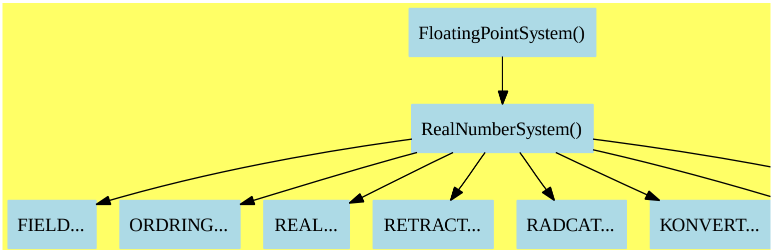
———-

— FFIELDC.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FiniteFieldCategory()" [color=lightblue];
"FiniteFieldCategory()" -> "FieldOfPrimeCharacteristic()"
"FiniteFieldCategory()" -> "Finite()"
"FiniteFieldCategory()" -> "StepThrough()"
"FiniteFieldCategory()" -> "DifferentialRing()"

"FieldOfPrimeCharacteristic()" [color=lightblue];
"FieldOfPrimeCharacteristic()" -> "CHARNZ..."
"FieldOfPrimeCharacteristic()" -> "FIELD..."

"Finite()" [color=lightblue];
"Finite()" -> "SETCAT..."

"StepThrough()" [color=lightblue];
"StepThrough()" -> "SETCAT..."

"DifferentialRing()" [color=lightblue];
"DifferentialRing()" -> "RING..."

"RING..." [color=lightblue];
"FIELD..." [color=lightblue];
"CHARNZ..." [color=lightblue];
"SETCAT..." [color=lightblue];

}
```

———-

18.0.232 FloatingPointSystem (FPS)



— FloatingPointSystem.input —

```
)set break resume
)sys rm -f FloatingPointSystem.output
)spool FloatingPointSystem.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FloatingPointSystem
--R
--R FloatingPointSystem is a category constructor
--R Abbreviation for FloatingPointSystem is FPS
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FPS
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Fraction(Integer),%) -> %      ?*? : (%,Fraction(Integer)) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,Fraction(Integer)) -> %     ?**? : (%,Integer) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?/? : (%,%) -> %
--R ?<? : (%,%) -> Boolean                ?<=? : (%,%) -> Boolean
--R ?=? : (%,%) -> Boolean                ?>? : (%,%) -> Boolean
--R ?>=? : (%,%) -> Boolean               1 : () -> %
--R 0 : () -> %                           ?^? : (%,Integer) -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R abs : % -> %                          associates? : (%,%) -> Boolean
--R base : () -> PositiveInteger          bits : () -> PositiveInteger
--R ceiling : % -> %                      coerce : Fraction(Integer) -> %
--R coerce : Integer -> %                 coerce : Fraction(Integer) -> %
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              convert : % -> Pattern(Float)
--R convert : % -> DoubleFloat            convert : % -> Float
--R digits : () -> PositiveInteger        exponent : % -> Integer
--R factor : % -> Factored(%)             float : (Integer,Integer) -> %
--R floor : % -> %                        fractionPart : % -> %
--R gcd : List(%) -> %                    gcd : (%,%) -> %
```

```
--R hash : % -> SingleInteger            inv : % -> %
--R latex : % -> String                  lcm : List(%) -> %
--R lcm : (%,%) -> %                     mantissa : % -> Integer
--R max : (%,%) -> %                     min : (%,%) -> %
--R negative? : % -> Boolean             norm : % -> %
--R nthRoot : (%,Integer) -> %           one? : % -> Boolean
--R order : % -> Integer                 positive? : % -> Boolean
--R precision : () -> PositiveInteger    prime? : % -> Boolean
--R ?quo? : (%,%) -> %                   recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                   retract : % -> Fraction(Integer)
--R retract : % -> Integer               round : % -> %
--R sample : () -> %                     sign : % -> Integer
--R sizeLess? : (%,%) -> Boolean         sqrt : % -> %
--R squareFree : % -> Factored(%)        squareFreePart : % -> %
--R truncate : % -> %                    unit? : % -> Boolean
--R unitCanonical : % -> %               wholePart : % -> Integer
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R bits : PositiveInteger -> PositiveInteger if $ has arbitraryPrecision
--R characteristic : () -> NonNegativeInteger
--R decreasePrecision : Integer -> PositiveInteger if $ has arbitraryPrecision
--R digits : PositiveInteger -> PositiveInteger if $ has arbitraryPrecision
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R float : (Integer,Integer,PositiveInteger) -> %
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R increasePrecision : Integer -> PositiveInteger if $ has arbitraryPrecision
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
--R max : () -> % if not(has($,arbitraryPrecision)) and not(has($,arbitraryExponent))
--R min : () -> % if not(has($,arbitraryPrecision)) and not(has($,arbitraryExponent))
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed")
--R patternMatch : (%,Pattern(Float),PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%)
--R precision : PositiveInteger -> PositiveInteger if $ has arbitraryPrecision
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %)
--R retractIfCan : % -> Union(Fraction(Integer),"failed")
--R retractIfCan : % -> Union(Integer,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

— **FloatingPointSystem.help** —

```
====================================================================
FloatingPointSystem examples
====================================================================
```

```
This category is intended as a model for floating point systems.
A floating point system is a model for the real numbers.  In fact,
it is an approximation in the sense that not all real numbers are
exactly representable by floating point numbers.

A floating point system is characterized by the following:

  1: base of the exponent where the actual implemenations are
     usually binary or decimal)
  2: precision of the mantissa (arbitrary or fixed)
  3: rounding error for operations
  4:  when, and what happens if exponent overflow/underflow occurs

Because a Float is an approximation to the real numbers, even though
it is defined to be a join of a Field and OrderedRing, some of
the attributes do not hold.  In particular associative("+")
does not hold.  Algorithms defined over a field need special
considerations when the field is a floating point system.

See Also:
o )show FloatingPointSystem
```

See:

⇐ "RealNumberSystem" (RNS) 17.0.224 on page 1622

Exports:

0	1	abs	associates?
base	bits	characteristic	ceiling
coerce	convert	decreasePrecision	digits
divide	euclideanSize	exponent	expressIdealMember
exquo	extendedEuclidean	factor	float
floor	fractionPart	gcd	gcdPolynomial
hash	increasePrecision	inv	latex
lcm	mantissa	max	min
multiEuclidean	negative?	norm	nthRoot
one?	order	patternMatch	positive?
precision	prime?	principalIdeal	recip
retract	retractIfCan	round	sample
sign	sizeLess?	sqrt	squareFree
squareFreePart	subtractIfCan	truncate	unit?
unitCanonical	unitNormal	wholePart	zero?
?*?	?**?	?+?	?-?
-?	?/?	?<?	?<=?
?=?	?>?	?>=?	?^?
?~=?	?quo?	?rem?	

Attributes Exported:

- **approximate** means "is an approximation to the real numbers".
- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
base : () -> PositiveInteger
bits : () -> PositiveInteger
bits : PositiveInteger -> PositiveInteger
   if $ has arbitraryPrecision
decreasePrecision : Integer -> PositiveInteger
   if $ has arbitraryPrecision
digits : PositiveInteger -> PositiveInteger
   if $ has arbitraryPrecision
exponent : % -> Integer
float : (Integer,Integer,PositiveInteger) -> %
increasePrecision : Integer -> PositiveInteger
   if $ has arbitraryPrecision
mantissa : % -> Integer
max : () -> %
   if not has($,arbitraryPrecision)
   and not has($,arbitraryExponent)
min : () -> %
   if not has($,arbitraryPrecision)
   and not has($,arbitraryExponent)
order : % -> Integer
precision : () -> PositiveInteger
precision : PositiveInteger -> PositiveInteger
   if $ has arbitraryPrecision
```

These are implemented by this category:

```
digits : () -> PositiveInteger
float : (Integer,Integer) -> %
```

These exports come from (p1622) RealNumberSystem():

```
0 : () -> %
1 : () -> %
abs : % -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
ceiling : % -> %
```

```
coerce : Fraction Integer -> %
coerce : Integer -> %
coerce : Fraction Integer -> %
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
convert : % -> Pattern Float
convert : % -> DoubleFloat
convert : % -> Float
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
factor : % -> Factored %
floor : % -> %
fractionPart : % -> %
gcd : List % -> %
gcd : (%,%) -> %
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
      SparseUnivariatePolynomial %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
max : (%,%) -> %
min : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
negative? : % -> Boolean
norm : % -> %
nthRoot : (%,Integer) -> %
one? : % -> Boolean
patternMatch :
  (%,Pattern Float,PatternMatchResult(Float,%)) ->
     PatternMatchResult(Float,%)
positive? : % -> Boolean
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
retract : % -> Fraction Integer
retract : % -> Integer
retractIfCan : % -> Union(Fraction Integer,"failed")
retractIfCan : % -> Union(Integer,"failed")
round : % -> %
sample : () -> %
sign : % -> Integer
sizeLess? : (%,%) -> Boolean
sqrt : % -> %
squareFree : % -> Factored %
squareFreePart : % -> %
```

```
 subtractIfCan : (%,%) -> Union(%,"failed")
 truncate : % -> %
 unit? : % -> Boolean
 unitCanonical : % -> %
 unitNormal : % -> Record(unit: %,canonical: %,associate: %)
 wholePart : % -> Integer
 zero? : % -> Boolean
 ?*? : (Fraction Integer,%) -> %
 ?*? : (%,Fraction Integer) -> %
 ?*? : (%,%) -> %
 ?*? : (Integer,%) -> %
 ?*? : (PositiveInteger,%) -> %
 ?**? : (%,Fraction Integer) -> %
 ?**? : (%,Integer) -> %
 ?**? : (%,PositiveInteger) -> %
 ?+? : (%,%) -> %
 ?-? : (%,%) -> %
 -? : % -> %
 ?/? : (%,%) -> %
 ?<? : (%,%) -> Boolean
 ?<=? : (%,%) -> Boolean
 ?=? : (%,%) -> Boolean
 ?>? : (%,%) -> Boolean
 ?>=? : (%,%) -> Boolean
 ?^? : (%,Integer) -> %
 ?^? : (%,PositiveInteger) -> %
 ?~=? : (%,%) -> Boolean
 ?*? : (NonNegativeInteger,%) -> %
 ?**? : (%,NonNegativeInteger) -> %
 ?^? : (%,NonNegativeInteger) -> %
 ?quo? : (%,%) -> %
 ?rem? : (%,%) -> %
```

— FloatingPointSystem.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FPS">
FloatingPointSystem (FPS)</a></h2>
</body>
```

— category FPS FloatingPointSystem —

```
)abbrev category FPS FloatingPointSystem
++ Description:
++ This category is intended as a model for floating point systems.
++ A floating point system is a model for the real numbers.  In fact,
++ it is an approximation in the sense that not all real numbers are
++ exactly representable by floating point numbers.
++ A floating point system is characterized by the following:
++
++ 1: base of the exponent where the actual implemenations are
```

```
++ usually binary or decimal)\br
++ 2: precision of the mantissa (arbitrary or fixed)\br
++ 3: rounding error for operations
--++   4:  when, and what happens if exponent overflow/underflow occurs
++
++ Because a Float is an approximation to the real numbers, even though
++ it is defined to be a join of a Field and OrderedRing, some of
++ the attributes do not hold.  In particular associative("+")
++ does not hold.  Algorithms defined over a field need special
++ considerations when the field is a floating point system.

FloatingPointSystem() : Category == SIG where

  SIG ==> RealNumberSystem() with

    approximate
       ++\spad{approximate} means "is an approximation to the real numbers".

    float : (Integer,Integer) -> %
      ++float(a,e) returns \spad{a * base() ** e}.

    float : (Integer,Integer,PositiveInteger) -> %
      ++float(a,e,b) returns \spad{a * b ** e}.

    order : % -> Integer
      ++order x is the order of magnitude of x.
      ++Note that \spad{base ** order x <= |x| < base ** (1 + order x)}.

    base : () -> PositiveInteger
      ++base() returns the base of the
      ++\spadfunFrom{exponent}{FloatingPointSystem}.

    exponent : % -> Integer
      ++ exponent(x) returns the
      ++ \spadfunFrom{exponent}{FloatingPointSystem} part of x.

    mantissa : % -> Integer
      ++ mantissa(x) returns the mantissa part of x.

    bits : () -> PositiveInteger
      ++ bits() returns ceiling's precision in bits.

    digits : () -> PositiveInteger
      ++ digits() returns ceiling's precision in decimal digits.

    precision : () -> PositiveInteger
      ++ precision() returns the precision in digits base.

    if % has arbitraryPrecision then

       bits : PositiveInteger -> PositiveInteger
         ++ bits(n) set the \spadfunFrom{precision}{FloatingPointSystem}
         ++ to n bits.
```

```
      digits : PositiveInteger -> PositiveInteger
        ++ digits(d) set the \spadfunFrom{precision}{FloatingPointSystem}
        ++ to d digits.

      precision : PositiveInteger -> PositiveInteger
        ++ precision(n) set the precision in the base to n decimal digits.

      increasePrecision : Integer -> PositiveInteger
        ++ increasePrecision(n) increases the current
        ++ \spadfunFrom{precision}{FloatingPointSystem} by n decimal digits.

      decreasePrecision : Integer -> PositiveInteger
        ++ decreasePrecision(n) decreases the current
        ++ \spadfunFrom{precision}{FloatingPointSystem} precision
        ++ by n decimal digits.

    if not (% has arbitraryExponent) then
      if not (% has arbitraryPrecision) then

         min: () -> %
          ++ min() returns the minimum floating point number.

         max: () -> %
          ++ max() returns the maximum floating point number.
   add

     float(ma, ex) == float(ma, ex, base())

     digits() == max(1,4004 * (bits()-1) quo 13301)::PositiveInteger
```

— COQ FPS —

```
(* category FPS *)
(*

   float : (Integer,Integer) -> %
   float(ma, ex) == float(ma, ex, base())

   digits : () -> PositiveInteger
   digits() == max(1,4004 * (bits()-1) quo 13301)::PositiveInteger

*)
```

— FPS.dotabb —

```
"FPS"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FPS"];
"FPS" -> "RNS"
```

———-

— **FPS.dotfull** —

```
"FloatingPointSystem()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FPS"];
"FloatingPointSystem()" -> "RealNumberSystem()"
```

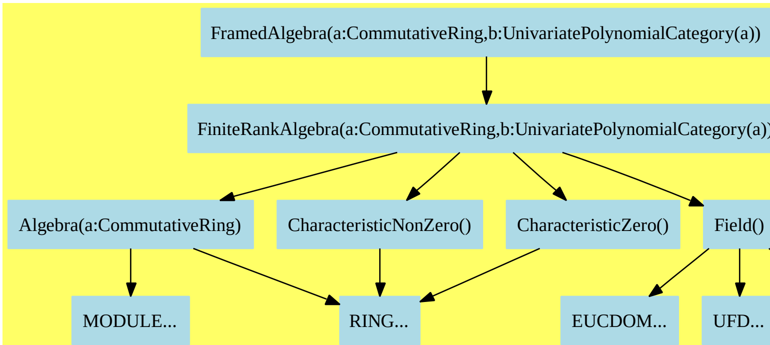
———-

— **FPS.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FloatingPointSystem()" [color=lightblue];
"FloatingPointSystem()" -> "RealNumberSystem()"

"RealNumberSystem()" [color=lightblue];
"RealNumberSystem()" -> "FIELD..."
"RealNumberSystem()" -> "ORDRING..."
"RealNumberSystem()" -> "REAL..."
"RealNumberSystem()" -> "RETRACT..."
"RealNumberSystem()" -> "RADCAT..."
"RealNumberSystem()" -> "KONVERT..."
"RealNumberSystem()" -> "PATMAB..."
"RealNumberSystem()" -> "CHARZ..."

"FIELD..." [color=lightblue];
"ORDRING..." [color=lightblue];
"REAL..." [color=lightblue];
"RETRACT..." [color=lightblue];
"RADCAT..." [color=lightblue];
"KONVERT..." [color=lightblue];
"PATMAB..." [color=lightblue];
"CHARZ..." [color=lightblue];
}
```

———-

18.0.233 FramedAlgebra (FRAMALG)



— **FramedAlgebra.input** —

```
)set break resume
)sys rm -f FramedAlgebra.output
)spool FramedAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FramedAlgebra
--R
--R FramedAlgebra(R: CommutativeRing,UP: UnivariatePolynomialCategory(t#1)) is a category constructor
--R Abbreviation for FramedAlgebra is FRAMALG
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FRAMALG
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R basis : () -> Vector(%)               characteristicPolynomial : % -> UP
--R coerce : R -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              convert : Vector(R) -> %
--R convert : % -> Vector(R)              coordinates : % -> Vector(R)
--R discriminant : () -> R                discriminant : Vector(%) -> R
--R hash : % -> SingleInteger             latex : % -> String
--R norm : % -> R                         one? : % -> Boolean
--R rank : () -> PositiveInteger          recip : % -> Union(%,"failed")
--R represents : Vector(R) -> %           sample : () -> %
--R trace : % -> R                        traceMatrix : () -> Matrix(R)
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
```

```
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
--R coordinates : Vector(%) -> Matrix(R)
--R coordinates : (Vector(%),Vector(%)) -> Matrix(R)
--R coordinates : (%,Vector(%)) -> Vector(R)
--R minimalPolynomial : % -> UP if R has FIELD
--R regularRepresentation : % -> Matrix(R)
--R regularRepresentation : (%,Vector(%)) -> Matrix(R)
--R represents : (Vector(R),Vector(%)) -> %
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R traceMatrix : Vector(%) -> Matrix(R)
--R
--E 1

)spool
)lisp (bye)
```

— **FramedAlgebra.help** —

```
====================================================================
FramedAlgebra examples
====================================================================

A FramedAlgebra is a FiniteRankAlgebra together with a fixed R-module basis.

See Also:
o )show FramedAlgebra
```

See:

⇒ "MonogenicAlgebra" (MONOGEN) 19.0.238 on page 1847
⇐ "FiniteRankAlgebra" (FINRALG) 17.0.218 on page 1537

Exports:

0	1	basis
characteristic	characteristicPolynomial	charthRoot
coerce	convert	coordinates
discriminant	hash	latex
minimalPolynomial	norm	one?
rank	recip	regularRepresentation
represents	sample	subtractIfCan
trace	traceMatrix	zero?
?*?	?**?	?+?
?-?	-?	?=?
?^?	?~=?	

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
basis : () -> Vector %
represents : Vector R -> %
```

These are implemented by this category:

```
convert : Vector R -> %
convert : % -> Vector R
coordinates : Vector % -> Matrix R
coordinates : % -> Vector R
discriminant : () -> R
regularRepresentation : % -> Matrix R
traceMatrix : () -> Matrix R
```

These exports come from (p1537) FiniteRankAlgebra(R, UP)
where R:CommutativeRing and UP:UnivariatePolynomialCategory R):

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
characteristicPolynomial : % -> UP
charthRoot : % -> Union(%,"failed") if R has CHARNZ
coerce : R -> %
coerce : Integer -> %
coerce : % -> OutputForm
coordinates : (%,Vector %) -> Vector R
coordinates : (Vector %,Vector %) -> Matrix R
discriminant : Vector % -> R
hash : % -> SingleInteger
latex : % -> String
minimalPolynomial : % -> UP if R has FIELD
norm : % -> R
one? : % -> Boolean
rank : () -> PositiveInteger
recip : % -> Union(%,"failed")
regularRepresentation : (%,Vector %) -> Matrix R
represents : (Vector R,Vector %) -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
trace : % -> R
traceMatrix : Vector % -> Matrix R
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (R,%) -> %
?*? : (%,R) -> %
?-? : (%,%) -> %
-? : % -> %
```

```
 ?**? : (%,PositiveInteger) -> %
 ?**? : (%,NonNegativeInteger) -> %
 ?^? : (%,PositiveInteger) -> %
 ?^? : (%,NonNegativeInteger) -> %
```

— FramedAlgebra.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FRAMALG">
FramedAlgebra (FRAMALG)</a></h2>
</body>
```

— category FRAMALG FramedAlgebra —

```
)abbrev category FRAMALG FramedAlgebra
++ Author: Barry Trager
++ Description:
++ A \spadtype{FramedAlgebra} is a \spadtype{FiniteRankAlgebra} together
++ with a fixed R-module basis.

FramedAlgebra(R,UP) : Category == SIG where
  R : CommutativeRing
  UP : UnivariatePolynomialCategory(R)

  SIG ==> FiniteRankAlgebra(R, UP) with

    basis : () -> Vector %
      ++ basis() returns the fixed R-module basis.

    coordinates : % -> Vector R
      ++ coordinates(a) returns the coordinates of \spad{a} with
      ++ respect to the fixed R-module basis.

    coordinates : Vector % -> Matrix R
      ++ coordinates([v1,...,vm]) returns the coordinates of the
      ++ vi's with to the fixed basis.  The coordinates of vi are
      ++ contained in the ith row of the matrix returned by this
      ++ function.

    represents : Vector R -> %
      ++ represents([a1,..,an]) returns \spad{a1*v1 + ... + an*vn}, where
      ++ v1, ..., vn are the elements of the fixed basis.

    convert : % -> Vector R
      ++ convert(a) returns the coordinates of \spad{a} with respect to the
      ++ fixed R-module basis.

    convert : Vector R -> %
      ++ convert([a1,..,an]) returns \spad{a1*v1 + ... + an*vn}, where
      ++ v1, ..., vn are the elements of the fixed basis.
```

```
  traceMatrix : () -> Matrix R
    ++ traceMatrix() is the n-by-n matrix \spad{(Tr(vi * vj))}, where
    ++ v1, ..., vn are the elements of the fixed basis.

  discriminant : () -> R
    ++ discriminant() = determinant(traceMatrix()).

  regularRepresentation : % -> Matrix R
    ++ regularRepresentation(a) returns the matrix of the linear
    ++ map defined by left multiplication by \spad{a} with respect
    ++ to the fixed basis.

  --attributes
  --separable <=> discriminant() ^= 0

 add

   convert(x:%):Vector(R) == coordinates(x)

   convert(v:Vector R):% == represents(v)

   traceMatrix() == traceMatrix basis()

   discriminant() == discriminant basis()

   regularRepresentation x == regularRepresentation(x, basis())

   coordinates x == coordinates(x, basis())

   represents x == represents(x, basis())

   coordinates(v:Vector %) ==
     m := new(#v, rank(), 0)$Matrix(R)
     for i in minIndex v .. maxIndex v for j in minRowIndex m .. repeat
       setRow_!(m, j, coordinates qelt(v, i))
     m

   regularRepresentation x ==
     m := new(n := rank(), n, 0)$Matrix(R)
     b := basis()
     for i in minIndex b .. maxIndex b for j in minRowIndex m .. repeat
       setRow_!(m, j, coordinates(x * qelt(b, i)))
     m

   characteristicPolynomial x ==
      mat00 := (regularRepresentation x)
      mat0 := map(y+->y::UP,mat00)$MatrixCategoryFunctions2(R, Vector R,
                  Vector R, Matrix R, UP, Vector UP,Vector UP, Matrix UP)
      mat1 : Matrix UP := scalarMatrix(rank(),monomial(1,1)$UP)
      determinant(mat1 - mat0)

   if R has Field then
    -- depends on the ordering of results from nullSpace, also see FFP
```

```
minimalPolynomial(x:%):UP ==
  y:%:=1
  n:=rank()
  m:Matrix R:=zero(n,n+1)
  for i in 1..n+1 repeat
    setColumn_!(m,i,coordinates(y))
    y:=y*x
  v:=first nullSpace(m)
  +/[monomial(v.(i+1),i) for i in 0..#v-1]
```

—— COQ FRAMALG ——

```
(* category FRAMALG *)
(*

   convert : % -> Vector(R)
   convert(x:%):Vector(R) == coordinates(x)

   convert : Vector(R) -> %
   convert(v:Vector R):% == represents(v)

   traceMatrix : () -> Matrix(R)
   traceMatrix() == traceMatrix basis()

   discriminant : () -> R
   discriminant() == discriminant basis()

   regularRepresentation : % -> Matrix(R)
   regularRepresentation x == regularRepresentation(x, basis())

   coordinates : % -> Vector(R)
   coordinates x == coordinates(x, basis())

   represents : Vector(R) -> %
   represents x == represents(x, basis())

   coordinates : Vector(%) -> Matrix(R)
   coordinates(v:Vector %) ==
     m := new(#v, rank(), 0)$Matrix(R)
     for i in minIndex v .. maxIndex v for j in minRowIndex m .. repeat
       setRow_!(m, j, coordinates qelt(v, i))
     m

   regularRepresentation : % -> Matrix(R)
   regularRepresentation x ==
     m := new(n := rank(), n, 0)$Matrix(R)
     b := basis()
     for i in minIndex b .. maxIndex b for j in minRowIndex m .. repeat
       setRow_!(m, j, coordinates(x * qelt(b, i)))
     m
```

```
   characteristicPolynomial : % -> UP
   characteristicPolynomial x ==
      mat00 := (regularRepresentation x)
      mat0 := map(y+->y::UP,mat00)$MatrixCategoryFunctions2(R, Vector R,
                  Vector R, Matrix R, UP, Vector UP,Vector UP, Matrix UP)
      mat1 : Matrix UP := scalarMatrix(rank(),monomial(1,1)$UP)
      determinant(mat1 - mat0)

   if R has Field then
    -- depends on the ordering of results from nullSpace, also see FFP

      minimalPolynomial : % -> UP
      minimalPolynomial(x:%):UP ==
        y:%:=1
        n:=rank()
        m:Matrix R:=zero(n,n+1)
        for i in 1..n+1 repeat
          setColumn_!(m,i,coordinates(y))
          y:=y*x
        v:=first nullSpace(m)
        +/[monomial(v.(i+1),i) for i in 0..#v-1]
*)
```

———-

— **FRAMALG.dotabb** —

```
"FRAMALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FRAMALG"];
"FRAMALG" -> "FINRALG"
```

———-

— **FRAMALG.dotfull** —

```
"FramedAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FRAMALG"];
"FramedAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
   "FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
```

———-

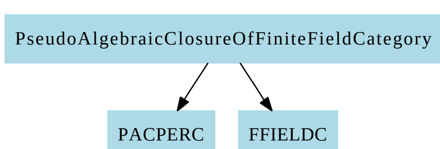
— **FRAMALG.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FramedAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
 [color=lightblue];
```

```
"FramedAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
   "FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"

"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
 [color=lightblue];
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "Algebra(a:CommutativeRing)"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "Field()"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "CharacteristicNonZero()"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "CharacteristicZero()"

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "MODULE..."

"Field()" [color=lightblue];
"Field()" -> "EUCDOM..."
"Field()" -> "UFD..."
"Field()" -> "DIVRING..."

"CharacteristicNonZero()" [color=lightblue];
"CharacteristicNonZero()" -> "RING..."

"CharacteristicZero()" [color=lightblue];
"CharacteristicZero()" -> "RING..."

"EUCDOM..." [color=lightblue];
"UFD..." [color=lightblue];
"DIVRING..." [color=lightblue];
"RING..." [color=lightblue];
"MODULE..." [color=lightblue];
}
```

———

18.0.234 PseudoAlgebraicClosureOfFiniteFieldCategory (PACFFC)



— **PseudoAlgebraicClosureOfFiniteFieldCategory.input** —

```
)set break resume
)sys rm -f PseudoAlgebraicClosureOfFiniteFieldCategory.output
```

```
)spool PseudoAlgebraicClosureOfFiniteFieldCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PseudoAlgebraicClosureOfFiniteFieldCategory
--R
--R PseudoAlgebraicClosureOfFiniteFieldCategory is a category constructor
--R Abbreviation for PseudoAlgebraicClosureOfFiniteFieldCategory is PACFFC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PACFFC
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Fraction(Integer),%) -> %      ?*? : (%,Fraction(Integer)) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,Integer) -> %               ?**? : (%,NonNegativeInteger) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (%,%) -> %                      ?=? : (%,%) -> Boolean
--R D : % -> %                            D : (%,NonNegativeInteger) -> %
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,Integer) -> %                ?^? : (%,NonNegativeInteger) -> %
--R ?^? : (%,PositiveInteger) -> %        associates? : (%,%) -> Boolean
--R charthRoot : % -> %                   coerce : Fraction(Integer) -> %
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              conjugate : % -> %
--R createPrimitiveElement : () -> %      differentiate : % -> %
--R enumerate : () -> List(%)             extDegree : % -> PositiveInteger
--R factor : % -> Factored(%)             fullOutput : % -> OutputForm
--R gcd : List(%) -> %                    gcd : (%,%) -> %
--R ground? : % -> Boolean                hash : % -> SingleInteger
--R index : PositiveInteger -> %          init : () -> %
--R inv : % -> %                          latex : % -> String
--R lcm : List(%) -> %                    lcm : (%,%) -> %
--R lookup : % -> PositiveInteger         maxTower : List(%) -> %
--R nextItem : % -> Union(%,"failed")     one? : % -> Boolean
--R order : % -> PositiveInteger          previousTower : % -> %
--R prime? : % -> Boolean                 primeFrobenius : % -> %
--R primitive? : % -> Boolean             primitiveElement : () -> %
--R ?quo? : (%,%) -> %                    random : () -> %
--R recip : % -> Union(%,"failed")        ?rem? : (%,%) -> %
--R sample : () -> %                      setTower! : % -> Void
--R size : () -> NonNegativeInteger       sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored(%)         squareFreePart : % -> %
--R unit? : % -> Boolean                  unitCanonical : % -> %
--R vectorise : (%,%) -> Vector(%)        zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed")
--R conditionP : Matrix(%) -> Union(Vector(%),"failed")
--R definingPolynomial : () -> SparseUnivariatePolynomial(%)
--R definingPolynomial : % -> SparseUnivariatePolynomial(%)
```

```
--R differentiate : (%,NonNegativeInteger) -> %
--R discreteLog : % -> NonNegativeInteger
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
--R distinguishedRootsOf : (SparseUnivariatePolynomial(%),%) -> List(%)
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R factorsOfCyclicGroupSize : () -> List(Record(factor: Integer,exponent: Integer))
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
--R lift : % -> SparseUnivariatePolynomial(%)
--R lift : (%,%) -> SparseUnivariatePolynomial(%)
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed")
--R newElement : (SparseUnivariatePolynomial(%),%,Symbol) -> %
--R newElement : (SparseUnivariatePolynomial(%),Symbol) -> %
--R order : % -> OnePointCompletion(PositiveInteger)
--R primeFrobenius : (%,NonNegativeInteger) -> %
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %)
--R reduce : SparseUnivariatePolynomial(%) -> %
--R representationType : () -> Union("prime",polynomial,normal,cyclic)
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger)
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

— **PseudoAlgebraicClosureOfFiniteFieldCategory.help** —

```
====================================================================
PseudoAlgebraicClosureOfFiniteFieldCategory examples
====================================================================

This category exports the function for the domain
PseudoAlgebraicClosureOfFiniteField which implement dynamic extension
using the simple notion of tower extensions.

A tower extension T  of the ground field K is any sequence of field extension
    (T : K_0, K_1, ..., K_i...,K_n)
where K_0 = K and for i =1,2,...,n, K_i is an extension
of K_{i-1} of degree > 1 and defined by an irreducible polynomial
p(Z) in K_{i-1}.

Two towers
   (T_1: K_01, K_11,...,K_i1,...,K_n1)
and
   (T_2: K_02, K_12,...,K_i2,...,K_n2)
```

```
are said to be related if
   T_1 <= T_2 (or T_1 >= T_2),
that is if
   K_i1 = K_i2 for i=1,2,...,n1
(or i=1,2,...,n2). Any algebraic operations defined for several elements
are only defined if all of the concerned elements are comming from
a set of related tour extensions.

See Also:
o )show PseudoAlgebraicClosureOfFiniteFieldCategory
```

———-

⇐ "FieldOfPrimeCharacteristic" (FPC) 17.0.217 on page 1531
⇐ "PseudoAlgebraicClosureOfPerfectFieldCategory" (PACPERC) 17.0.221 on page 1588

Exports:

0	1	associates?
characteristic	charthRoot	conditionP
coerce	conjugate	createPrimitiveElement
D	definingPolynomial	differentiate
discreteLog	distinguishedRootsOf	divide
euclideanSize	extendedEuclidean	expressIdealMember
exquo	extDegree	factor
factorsOfCyclicGroupSize	fullOutput	gcd
gcdPolynomial	ground?	hash
index	init	inv
latex	lcm	lift
lookup	maxTower	multiEuclidean
newElement	nextItem	one?
order	previousTower	prime?
primeFrobenius	primitive?	primitiveElement
principalIdeal	?quo?	random
recip	reduce	?rem?
representationType	sample	setTower!
size	sizeLess?	squareFree
squareFreePart	subtractIfCan	tableForDiscreteLogarithm
unit?	unitCanonical	unitNormal
vectorise	zero?	?*?
?**?	?+?	?-?
-?	?/?	?=?
?^?	?~=?	

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.

- **commutative("*")** is true if it has an operation $" * " : (D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These exports come from (p1588) PseudoAlgebraicClosureOfPerfectFieldCategory():

```
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?**? : (%,Integer) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?/? : (%,%) -> %
?=? : (%,%) -> Boolean
1 : () -> %
0 : () -> %
?^? : (%,Integer) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?~=? : (%,%) -> Boolean
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : Fraction Integer -> %
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
conjugate : % -> %
definingPolynomial : () -> SUP %
definingPolynomial : % -> SUP %
distinguishedRootsOf : (SparseUnivariatePolynomial %,%) -> List %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extDegree : % -> PI
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
factor : % -> Factored %
fullOutput : % -> OutputForm
gcd : List % -> %
gcd : (%,%) -> %
gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
ground? : % -> Boolean
hash : % -> SingleInteger
```

```
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
lift : % -> SUP %
lift : (%,%) -> SUP %
maxTower : List % -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
newElement : (SUP %,Symbol) -> %
newElement : (SUP %,%,Symbol) -> %
one? : % -> Boolean
previousTower : % -> %
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
?quo? : (%,%) -> %
recip : % -> Union(%,"failed")
reduce : SUP % -> %
?rem? : (%,%) -> %
sample : () -> %
setTower! : % -> Void
sizeLess? : (%,%) -> Boolean
squareFree : % -> Factored %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
vectorise : (%,%) -> Vector %
zero? : % -> Boolean
```

These exports come from (p1762) FiniteFieldCategory():

```
charthRoot : % -> %
charthRoot : % -> Union(%,"failed")
conditionP : Matrix % -> Union(Vector %,"failed")
createPrimitiveElement : () -> %
D : % -> %
D : (%,NonNegativeInteger) -> %
differentiate : % -> %
differentiate : (%,NonNegativeInteger) -> %
discreteLog : % -> NonNegativeInteger
discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
factorsOfCyclicGroupSize : () ->
    List Record(factor: Integer,exponent: Integer)
index : PositiveInteger -> %
init : () -> %
lookup : % -> PositiveInteger
nextItem : % -> Union(%,"failed")
order : % -> OnePointCompletion PositiveInteger
order : % -> PositiveInteger
primeFrobenius : (%,NonNegativeInteger) -> %
primeFrobenius : % -> %
primitive? : % -> Boolean
primitiveElement : () -> %
random : () -> %
```

```
 representationType : () -> Union("prime",polynomial,normal,cyclic)
 size : () -> NonNegativeInteger
 tableForDiscreteLogarithm : Integer ->
     Table(PositiveInteger,NonNegativeInteger)
```

— PseudoAlgebraicClosureOfFiniteFieldCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#PACFFC">
PseudoAlgebraicClosureOfFiniteFieldCategory (PACFFC)</a></h2>
</body>
```

——–

— category PACFFC PseudoAlgebraicClosureOfFiniteFieldCategory —

```
)abbrev category PACFFC PseudoAlgebraicClosureOfFiniteFieldCategory
-- PseudoAlgebraicClosureOfFiniteFieldCategory
++ Authors: Gaetan Hache
++ Date Created: june 1996
++ Description:
++ This category exports the function for the domain
++ PseudoAlgebraicClosureOfFiniteField which implement dynamic extension
++ using the simple notion of tower extensions.
++ A tower extension T  of the ground
++ field K is any sequence of field extension (T : K_0, K_1, ..., K_i...,K_n)
++ where K_0 = K and for i =1,2,...,n, K_i is an extension
++ of K_{i-1} of degree > 1 and defined by an irreducible polynomial
++ p(Z) in K_{i-1}.
++ Two towers (T_1: K_01, K_11,...,K_i1,...,K_n1)
++ and (T_2: K_02, K_12,...,K_i2,...,K_n2)
++ are said to be related if T_1 <= T_2 (or T_1 >= T_2),
++ that is if K_i1 = K_i2 for i=1,2,...,n1
++ (or i=1,2,...,n2). Any algebraic operations defined for several elements
++ are only defined if all of the concerned elements are comming from
++ a set of related tour extensions.
PseudoAlgebraicClosureOfFiniteFieldCategory() : Category == SIG where

  FFC ==> FiniteFieldCategory
  PAC ==> PseudoAlgebraicClosureOfPerfectFieldCategory

  SIG ==> Join(FFC,PAC)
```

——–

— PACFFC.dotabb —

```
"PACFFC" [color=lightblue,href="bookvol10.2.pdf#nameddest=PACFFC"];
"PACFFC" -> "PACPERC"
```

——–

— **PACFFC.dotfull** —

```
"PseudoAlgebraicClosureOfFiniteFieldCategory"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PACFFC"];
"PseudoAlgebraicClosureOfFiniteFieldCategory" ->
  "PseudoAlgebraicClosureOfPerfectFieldCategory()"
"PseudoAlgebraicClosureOfFiniteFieldCategory" ->
  "FiniteFieldCategory()"
```

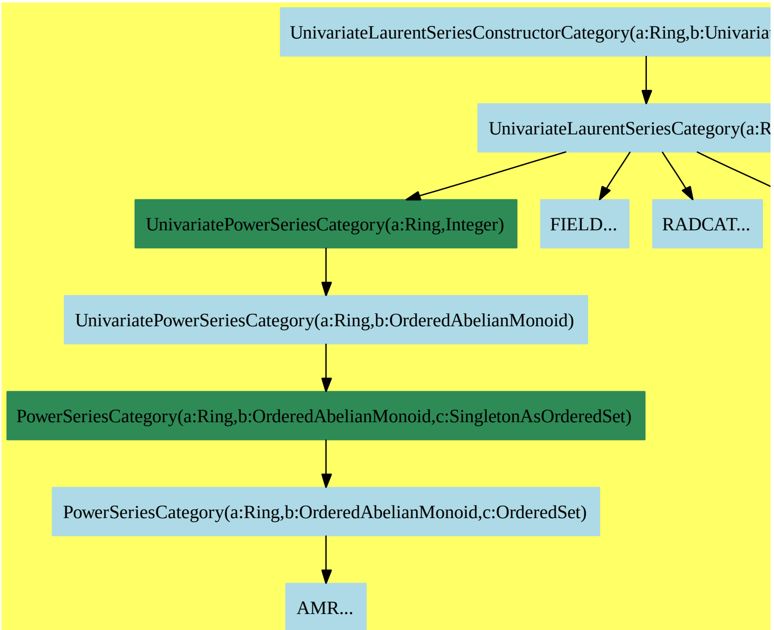
————

— **PACFFC.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"PseudoAlgebraicClosureOfFiniteFieldCategory" [color=lightblue];
"PseudoAlgebraicClosureOfFiniteFieldCategory" -> "PACPERC"
"PseudoAlgebraicClosureOfFiniteFieldCategory" -> "FFIELDC"

"PACPERC" [color=lightblue];
"FFIELDC" [color=lightblue];

}
```

————

18.0.235 UnivariateLaurentSeriesConstructorCategory (ULSCCAT)



— **UnivariateLaurentSeriesConstructorCategory.input** —

```
)set break resume
)sys rm -f UnivariateLaurentSeriesConstructorCategory.output
)spool UnivariateLaurentSeriesConstructorCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariateLaurentSeriesConstructorCategory
--R
--R UnivariateLaurentSeriesConstructorCategory(Coef: Ring,UTS: UnivariateTaylorSeriesCategory(t#1)) is a categor
--R Abbreviation for UnivariateLaurentSeriesConstructorCategory is ULSCCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for ULSCCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Coef,%) -> %                    ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                       ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %      ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %     ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                       ?-? : (%,%) -> %
--R -? : % -> %                            ?=? : (%,%) -> Boolean
--R 1 : () -> %                            0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %      ?^? : (%,PositiveInteger) -> %
--R center : % -> Coef                     coefficient : (%,Integer) -> Coef
--R coerce : % -> % if Coef has INTDOM     coerce : UTS -> %
```

```
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R complete : % -> %                     degree : % -> Integer
--R denom : % -> UTS if Coef has FIELD    ?.? : (%,Integer) -> Coef
--R extend : (%,Integer) -> %             hash : % -> SingleInteger
--R inv : % -> % if Coef has FIELD        latex : % -> String
--R laurent : (Integer,UTS) -> %          leadingCoefficient : % -> Coef
--R leadingMonomial : % -> %              map : ((Coef -> Coef),%) -> %
--R monomial : (Coef,Integer) -> %        monomial? : % -> Boolean
--R numer : % -> UTS if Coef has FIELD    one? : % -> Boolean
--R order : (%,Integer) -> Integer        order : % -> Integer
--R pole? : % -> Boolean                  recip : % -> Union(%,"failed")
--R reductum : % -> %                     removeZeroes : (Integer,%) -> %
--R removeZeroes : % -> %                 retract : % -> UTS
--R sample : () -> %                      taylor : % -> UTS
--R taylorRep : % -> UTS                  truncate : (%,Integer) -> %
--R variable : % -> Symbol                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction(Integer)) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?*? : (Fraction(Integer),%) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?*? : (UTS,%) -> % if Coef has FIELD
--R ?*? : (%,UTS) -> % if Coef has FIELD
--R ?**? : (%,Fraction(Integer)) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?**? : (%,%) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?**? : (%,Integer) -> % if Coef has FIELD
--R ?/? : (UTS,UTS) -> % if Coef has FIELD
--R ?/? : (%,%) -> % if Coef has FIELD
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R ?<? : (%,%) -> Boolean if and(has(UTS,OrderedSet),has(Coef,Field))
--R ?<=? : (%,%) -> Boolean if and(has(UTS,OrderedSet),has(Coef,Field))
--R ?>? : (%,%) -> Boolean if and(has(UTS,OrderedSet),has(Coef,Field))
--R ?>=? : (%,%) -> Boolean if and(has(UTS,OrderedSet),has(Coef,Field))
--R D : (%,Symbol) -> % if and(has(UTS,PartialDifferentialRing(Symbol)),has(Coef,Field)) or Coef has PDRING(SYMB
--R D : (%,List(Symbol)) -> % if and(has(UTS,PartialDifferentialRing(Symbol)),has(Coef,Field)) or Coef has PDRIN
--R D : (%,Symbol,NonNegativeInteger) -> % if and(has(UTS,PartialDifferentialRing(Symbol)),has(Coef,Field)) or C
--R D : (%,List(Symbol),List(NonNegativeInteger)) -> % if and(has(UTS,PartialDifferentialRing(Symbol)),has(Coef,
--R D : % -> % if and(has(UTS,DifferentialRing),has(Coef,Field)) or Coef has *: (Integer,Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if and(has(UTS,DifferentialRing),has(Coef,Field)) or Coef has *: (Integer,Co
--R D : (%,(UTS -> UTS),NonNegativeInteger) -> % if Coef has FIELD
--R D : (%,(UTS -> UTS)) -> % if Coef has FIELD
--R ?^? : (%,Integer) -> % if Coef has FIELD
--R abs : % -> % if and(has(UTS,OrderedIntegralDomain),has(Coef,Field))
--R acos : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acosh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acot : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acoth : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acsc : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acsch : % -> % if Coef has ALGEBRA(FRAC(INT))
--R approximate : (%,Integer) -> Coef if Coef has **: (Coef,Integer) -> Coef and Coef has coerce: Symbol -> Coef
--R asec : % -> % if Coef has ALGEBRA(FRAC(INT))
--R asech : % -> % if Coef has ALGEBRA(FRAC(INT))
--R asin : % -> % if Coef has ALGEBRA(FRAC(INT))
--R asinh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA(FRAC(INT))
```

```
--R atanh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R ceiling : % -> UTS if and(has(UTS,IntegerNumberSystem),has(Coef,Field))
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if and(OR(has(UTS,CharacteristicNonZero),and(has($,CharacteristicNonZero
--R coerce : Fraction(Integer) -> % if Coef has ALGEBRA(FRAC(INT))
--R coerce : Symbol -> % if and(has(UTS,RetractableTo(Symbol)),has(Coef,Field))
--R coerce : Coef -> % if Coef has COMRING
--R conditionP : Matrix(%) -> Union(Vector(%),"failed") if and(and(has($,CharacteristicNonZero),has(UTS,Polynomi
--R convert : % -> Pattern(Integer) if and(has(UTS,ConvertibleTo(Pattern(Integer))),has(Coef,Field))
--R convert : % -> Pattern(Float) if and(has(UTS,ConvertibleTo(Pattern(Float))),has(Coef,Field))
--R convert : % -> InputForm if and(has(UTS,ConvertibleTo(InputForm)),has(Coef,Field))
--R convert : % -> Float if and(has(UTS,RealConstant),has(Coef,Field))
--R convert : % -> DoubleFloat if and(has(UTS,RealConstant),has(Coef,Field))
--R cos : % -> % if Coef has ALGEBRA(FRAC(INT))
--R cosh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R cot : % -> % if Coef has ALGEBRA(FRAC(INT))
--R coth : % -> % if Coef has ALGEBRA(FRAC(INT))
--R csc : % -> % if Coef has ALGEBRA(FRAC(INT))
--R csch : % -> % if Coef has ALGEBRA(FRAC(INT))
--R denominator : % -> % if Coef has FIELD
--R differentiate : (%,Symbol) -> % if and(has(UTS,PartialDifferentialRing(Symbol)),has(Coef,Field)) or Coef has
--R differentiate : (%,List(Symbol)) -> % if and(has(UTS,PartialDifferentialRing(Symbol)),has(Coef,Field)) or Co
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if and(has(UTS,PartialDifferentialRing(Symbol)),has(Coef,
--R differentiate : (%,List(Symbol),List(NonNegativeInteger)) -> % if and(has(UTS,PartialDifferentialRing(Symbol
--R differentiate : % -> % if and(has(UTS,DifferentialRing),has(Coef,Field)) or Coef has *: (Integer,Coef) -> Co
--R differentiate : (%,NonNegativeInteger) -> % if and(has(UTS,DifferentialRing),has(Coef,Field)) or Coef has *:
--R differentiate : (%,(UTS -> UTS),NonNegativeInteger) -> % if Coef has FIELD
--R differentiate : (%,(UTS -> UTS)) -> % if Coef has FIELD
--R divide : (%,%) -> Record(quotient: %,remainder: %) if Coef has FIELD
--R ?.? : (%,UTS) -> % if and(has(UTS,Eltable(UTS,UTS)),has(Coef,Field))
--R ?.? : (%,%) -> % if Integer has SGROUP
--R euclideanSize : % -> NonNegativeInteger if Coef has FIELD
--R eval : (%,List(UTS),List(UTS)) -> % if and(has(UTS,Evalable(UTS)),has(Coef,Field))
--R eval : (%,UTS,UTS) -> % if and(has(UTS,Evalable(UTS)),has(Coef,Field))
--R eval : (%,Equation(UTS)) -> % if and(has(UTS,Evalable(UTS)),has(Coef,Field))
--R eval : (%,List(Equation(UTS))) -> % if and(has(UTS,Evalable(UTS)),has(Coef,Field))
--R eval : (%,List(Symbol),List(UTS)) -> % if and(has(UTS,InnerEvalable(Symbol,UTS)),has(Coef,Field))
--R eval : (%,Symbol,UTS) -> % if and(has(UTS,InnerEvalable(Symbol,UTS)),has(Coef,Field))
--R eval : (%,Coef) -> Stream(Coef) if Coef has **: (Coef,Integer) -> Coef
--R exp : % -> % if Coef has ALGEBRA(FRAC(INT))
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed") if Coef has FIELD
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if Coef has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if Coef has FIELD
--R factor : % -> Factored(%) if Coef has FIELD
--R factorPolynomial : SparseUnivariatePolynomial(%) -> Factored(SparseUnivariatePolynomial(%)) if and(has(UTS,P
--R factorSquareFreePolynomial : SparseUnivariatePolynomial(%) -> Factored(SparseUnivariatePolynomial(%)) if and
--R floor : % -> UTS if and(has(UTS,IntegerNumberSystem),has(Coef,Field))
--R fractionPart : % -> % if and(has(UTS,EuclideanDomain),has(Coef,Field))
--R gcd : (%,%) -> % if Coef has FIELD
--R gcd : List(%) -> % if Coef has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R init : () -> % if and(has(UTS,StepThrough),has(Coef,Field))
--R integrate : (%,Symbol) -> % if Coef has ACFS(INT) and Coef has PRIMCAT and Coef has TRANFUN and Coef has ALG
```

```
--R integrate : % -> % if Coef has ALGEBRA(FRAC(INT))
--R lcm : (%,%) -> % if Coef has FIELD
--R lcm : List(%) -> % if Coef has FIELD
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %) if Coef has FIELD
--R log : % -> % if Coef has ALGEBRA(FRAC(INT))
--R map : ((UTS -> UTS),%) -> % if Coef has FIELD
--R max : (%,%) -> % if and(has(UTS,OrderedSet),has(Coef,Field))
--R min : (%,%) -> % if and(has(UTS,OrderedSet),has(Coef,Field))
--R monomial : (%,List(SingletonAsOrderedSet),List(Integer)) -> %
--R monomial : (%,SingletonAsOrderedSet,Integer) -> %
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed") if Coef has FIELD
--R multiplyCoefficients : ((Integer -> Coef),%) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R negative? : % -> Boolean if and(has(UTS,OrderedIntegralDomain),has(Coef,Field))
--R nextItem : % -> Union(%,"failed") if and(has(UTS,StepThrough),has(Coef,Field))
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA(FRAC(INT))
--R numerator : % -> % if Coef has FIELD
--R patternMatch : (%,Pattern(Integer),PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%) if and(ha
--R patternMatch : (%,Pattern(Float),PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%) if and(has(UTS,
--R pi : () -> % if Coef has ALGEBRA(FRAC(INT))
--R positive? : % -> Boolean if and(has(UTS,OrderedIntegralDomain),has(Coef,Field))
--R prime? : % -> Boolean if Coef has FIELD
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %) if Coef has FIELD
--R ?quo? : (%,%) -> % if Coef has FIELD
--R random : () -> % if and(has(UTS,IntegerNumberSystem),has(Coef,Field))
--R rationalFunction : (%,Integer,Integer) -> Fraction(Polynomial(Coef)) if Coef has INTDOM
--R rationalFunction : (%,Integer) -> Fraction(Polynomial(Coef)) if Coef has INTDOM
--R reducedSystem : Matrix(%) -> Matrix(Integer) if and(has(UTS,LinearlyExplicitRingOver(Integer)),has(Coef,Fiel
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(Integer),vec: Vector(Integer)) if and(has(UTS,Li
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(UTS),vec: Vector(UTS)) if Coef has FIELD
--R reducedSystem : Matrix(%) -> Matrix(UTS) if Coef has FIELD
--R ?rem? : (%,%) -> % if Coef has FIELD
--R retract : % -> Symbol if and(has(UTS,RetractableTo(Symbol)),has(Coef,Field))
--R retract : % -> Fraction(Integer) if and(has(UTS,RetractableTo(Integer)),has(Coef,Field))
--R retract : % -> Integer if and(has(UTS,RetractableTo(Integer)),has(Coef,Field))
--R retractIfCan : % -> Union(Symbol,"failed") if and(has(UTS,RetractableTo(Symbol)),has(Coef,Field))
--R retractIfCan : % -> Union(Fraction(Integer),"failed") if and(has(UTS,RetractableTo(Integer)),has(Coef,Field)
--R retractIfCan : % -> Union(Integer,"failed") if and(has(UTS,RetractableTo(Integer)),has(Coef,Field))
--R retractIfCan : % -> Union(UTS,"failed")
--R sec : % -> % if Coef has ALGEBRA(FRAC(INT))
--R sech : % -> % if Coef has ALGEBRA(FRAC(INT))
--R series : Stream(Record(k: Integer,c: Coef)) -> %
--R sign : % -> Integer if and(has(UTS,OrderedIntegralDomain),has(Coef,Field))
--R sin : % -> % if Coef has ALGEBRA(FRAC(INT))
--R sinh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R sizeLess? : (%,%) -> Boolean if Coef has FIELD
--R solveLinearPolynomialEquation : (List(SparseUnivariatePolynomial(%)),SparseUnivariatePolynomial(%)) -> Union
--R sqrt : % -> % if Coef has ALGEBRA(FRAC(INT))
--R squareFree : % -> Factored(%) if Coef has FIELD
--R squareFreePart : % -> % if Coef has FIELD
--R squareFreePolynomial : SparseUnivariatePolynomial(%) -> Factored(SparseUnivariatePolynomial(%)) if and(has(U
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA(FRAC(INT))
--R tanh : % -> % if Coef has ALGEBRA(FRAC(INT))
```

```
--R taylorIfCan : % -> Union(UTS,"failed")
--R terms : % -> Stream(Record(k: Integer,c: Coef))
--R truncate : (%,Integer,Integer) -> %
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R variables : % -> List(SingletonAsOrderedSet)
--R wholePart : % -> UTS if and(has(UTS,EuclideanDomain),has(Coef,Field))
--R
--E 1

)spool
)lisp (bye)
```

— **UnivariateLaurentSeriesConstructorCategory.help** —

```
====================================================================
UnivariateLaurentSeriesConstructorCategory examples
====================================================================

This is a category of univariate Laurent series constructed from
univariate Taylor series.  A Laurent series is represented by a pair
[n,f(x)], where n is an arbitrary integer and f(x) is a Taylor series.
This pair represents the Laurent series x**n * f(x).

See Also:
o )show UnivariateLaurentSeriesConstructorCategory
```

See:

⇐ "RetractableTo" (RETRACT) 2.0.37 on page 110
⇐ "UnivariateLaurentSeriesCategory" (ULSCAT) 17.0.226 on page 1686

Exports:

0	1	abs
acos	acosh	acot
acoth	acsc	acsch
approximate	asec	asech
asin	asinh	associates?
atan	atanh	ceiling
center	characteristic	charthRoot
coefficient	coerce	complete
conditionP	convert	cos
cosh	cot	coth
csc	csch	D
degree	denom	denominator
differentiate	divide	euclideanSize
eval	exp	expressIdealMember
exquo	extend	extendedEuclidean
factor	factorPolynomial	factorSquareFreePolynomial
floor	fractionPart	gcd
gcdPolynomial	hash	init
integrate	inv	latex
laurent	lcm	leadingCoefficient
leadingMonomial	log	map
max	min	monomial
monomial?	multiEuclidean	multiplyCoefficients
multiplyExponents	negative?	nextItem
nthRoot	numer	numerator
one?	order	patternMatch
pi	pole?	positive?
prime?	principalIdeal	random
rationalFunction	recip	reducedSystem
reductum	removeZeroes	retract
retractIfCan	sample	sec
sech	series	sign
sin	sinh	sizeLess?
solveLinearPolynomialEquation	sqrt	squareFree
squareFreePart	squareFreePolynomial	subtractIfCan
tan	tanh	taylor
taylorIfCan	taylorRep	terms
truncate	unit?	unitCanonical
unitNormal	variable	variables
wholePart	zero?	?*?
?**?	?+?	?-?
-?	?=?	?^?
?~=?	?/?	?<?
?<=?	?>?	?>=?
?.?	?quo?	?rem?

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.

- **rightUnitary** is true if $x * 1 = x$ for all x.
- if #1 has Field then canonicalClosed where **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- if #1 has Field then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if #1 has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- if #1 has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if #1 has Field then nil

These are directly exported but not implemented:

```
coerce : UTS -> %
degree : % -> Integer
laurent : (Integer,UTS) -> %
removeZeroes : % -> %
removeZeroes : (Integer,%) -> %
taylor : % -> UTS
taylorIfCan : % -> Union(UTS,"failed")
taylorRep : % -> UTS
```

These are implemented by this category:

```
retract : % -> UTS
retractIfCan : % -> Union(UTS,"failed")
zero? : % -> Boolean
```

These exports come from (p1686) UnivariateLaurentSeriesCategory(Coef:Ring)

```
0 : () -> %
1 : () -> %
acos : % -> % if Coef has ALGEBRA FRAC INT
acosh : % -> % if Coef has ALGEBRA FRAC INT
acot : % -> % if Coef has ALGEBRA FRAC INT
acoth : % -> % if Coef has ALGEBRA FRAC INT
acsc : % -> % if Coef has ALGEBRA FRAC INT
acsch : % -> % if Coef has ALGEBRA FRAC INT
approximate : (%,Integer) -> Coef
  if Coef has **: (Coef,Integer) -> Coef
  and Coef has coerce: Symbol -> Coef
asec : % -> % if Coef has ALGEBRA FRAC INT
asech : % -> % if Coef has ALGEBRA FRAC INT
asin : % -> % if Coef has ALGEBRA FRAC INT
asinh : % -> % if Coef has ALGEBRA FRAC INT
associates? : (%,%) -> Boolean if Coef has INTDOM
atan : % -> % if Coef has ALGEBRA FRAC INT
atanh : % -> % if Coef has ALGEBRA FRAC INT
center : % -> Coef
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed")
  if and(OR(has(UTS,CharacteristicNonZero),
            and(has($,CharacteristicNonZero),
```

```
              has(UTS,PolynomialFactorizationExplicit))),
        has(Coef,Field))
  or Coef has CHARNZ
coefficient : (%,Integer) -> Coef
coerce : % -> % if Coef has INTDOM
coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
coerce : Coef -> % if Coef has COMRING
coerce : Integer -> %
coerce : % -> OutputForm
complete : % -> %
cos : % -> % if Coef has ALGEBRA FRAC INT
cosh : % -> % if Coef has ALGEBRA FRAC INT
cot : % -> % if Coef has ALGEBRA FRAC INT
coth : % -> % if Coef has ALGEBRA FRAC INT
csc : % -> % if Coef has ALGEBRA FRAC INT
csch : % -> % if Coef has ALGEBRA FRAC INT
D : % -> %
  if and(has(UTS,DifferentialRing),has(Coef,Field))
  or Coef has *: (Integer,Coef) -> Coef
D : (%,NonNegativeInteger) -> %
  if and(has(UTS,DifferentialRing),has(Coef,Field))
  or Coef has *: (Integer,Coef) -> Coef
D : (%,Symbol) -> %
  if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,Field))
  or Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
D : (%,List Symbol) -> %
  if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,Field))
  or Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
D : (%,Symbol,NonNegativeInteger) -> %
  if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,Field))
  or Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
D : (%,List Symbol,List NonNegativeInteger) -> %
  if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,Field))
  or Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
differentiate : (%,Symbol) -> %
  if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,Field))
  or Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
differentiate : (%,List Symbol) -> %
  if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,Field))
  or Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
differentiate : (%,Symbol,NonNegativeInteger) -> %
  if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,Field))
  or Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
  if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,Field))
  or Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
```

```
differentiate : % -> %
  if and(has(UTS,DifferentialRing),has(Coef,Field))
  or Coef has *: (Integer,Coef) -> Coef
differentiate : (%,NonNegativeInteger) -> %
  if and(has(UTS,DifferentialRing),has(Coef,Field))
  or Coef has *: (Integer,Coef) -> Coef
divide : (%,%) -> Record(quotient: %,remainder: %)
  if Coef has FIELD
euclideanSize : % -> NonNegativeInteger if Coef has FIELD
eval : (%,Coef) -> Stream Coef
  if Coef has **: (Coef,Integer) -> Coef
exp : % -> % if Coef has ALGEBRA FRAC INT
expressIdealMember : (List %,%) -> Union(List %,"failed")
  if Coef has FIELD
exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
extend : (%,Integer) -> %
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
  if Coef has FIELD
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
  if Coef has FIELD
factor : % -> Factored % if Coef has FIELD
gcd : (%,%) -> % if Coef has FIELD
gcd : List % -> % if Coef has FIELD
gcdPolynomial :
 (SparseUnivariatePolynomial %,
  SparseUnivariatePolynomial %) ->
    SparseUnivariatePolynomial %
      if Coef has FIELD
hash : % -> SingleInteger
integrate : (%,Symbol) -> %
  if Coef has ACFS INT
  and Coef has PRIMCAT
  and Coef has TRANFUN
  and Coef has ALGEBRA FRAC INT
  or Coef has variables: Coef -> List Symbol
  and Coef has integrate: (Coef,Symbol) -> Coef
  and Coef has ALGEBRA FRAC INT
integrate : % -> % if Coef has ALGEBRA FRAC INT
inv : % -> % if Coef has FIELD
latex : % -> String
lcm : (%,%) -> % if Coef has FIELD
lcm : List % -> % if Coef has FIELD
leadingCoefficient : % -> Coef
leadingMonomial : % -> %
log : % -> % if Coef has ALGEBRA FRAC INT
map : ((Coef -> Coef),%) -> %
monomial : (%,List SingletonAsOrderedSet,List Integer) -> %
monomial : (%,SingletonAsOrderedSet,Integer) -> %
monomial : (Coef,Integer) -> %
monomial? : % -> Boolean
multiEuclidean : (List %,%) -> Union(List %,"failed")
  if Coef has FIELD
multiplyCoefficients : ((Integer -> Coef),%) -> %
multiplyExponents : (%,PositiveInteger) -> %
```

```
nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
one? : % -> Boolean
order : (%,Integer) -> Integer
order : % -> Integer
pi : () -> % if Coef has ALGEBRA FRAC INT
pole? : % -> Boolean
prime? : % -> Boolean if Coef has FIELD
principalIdeal : List % -> Record(coef: List %,generator: %)
  if Coef has FIELD
rationalFunction : (%,Integer) -> Fraction Polynomial Coef
  if Coef has INTDOM
rationalFunction : (%,Integer,Integer) -> Fraction Polynomial Coef
  if Coef has INTDOM
recip : % -> Union(%,"failed")
reductum : % -> %
sample : () -> %
sec : % -> % if Coef has ALGEBRA FRAC INT
sech : % -> % if Coef has ALGEBRA FRAC INT
series : Stream Record(k: Integer,c: Coef) -> %
sin : % -> % if Coef has ALGEBRA FRAC INT
sinh : % -> % if Coef has ALGEBRA FRAC INT
sizeLess? : (%,%) -> Boolean if Coef has FIELD
squareFree : % -> Factored % if Coef has FIELD
squareFreePart : % -> % if Coef has FIELD
sqrt : % -> % if Coef has ALGEBRA FRAC INT
subtractIfCan : (%,%) -> Union(%,"failed")
tan : % -> % if Coef has ALGEBRA FRAC INT
tanh : % -> % if Coef has ALGEBRA FRAC INT
terms : % -> Stream Record(k: Integer,c: Coef)
truncate : (%,Integer,Integer) -> %
truncate : (%,Integer) -> %
unit? : % -> Boolean if Coef has INTDOM
unitCanonical : % -> % if Coef has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
  if Coef has INTDOM
variable : % -> Symbol
variables : % -> List SingletonAsOrderedSet
?.? : (%,Integer) -> Coef
?**? : (%,Integer) -> % if Coef has FIELD
?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?**? : (%,NonNegativeInteger) -> %
?^? : (%,Integer) -> % if Coef has FIELD
?^? : (%,NonNegativeInteger) -> %
?/? : (%,%) -> % if Coef has FIELD
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
```

```
?^? : (%,PositiveInteger) -> %
?*? : (Integer,%) -> %
?*? : (Coef,%) -> %
?*? : (%,Coef) -> %
?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
?/? : (%,Coef) -> % if Coef has FIELD
?.? : (%,%) -> % if Integer has SGROUP
?quo? : (%,%) -> % if Coef has FIELD
?rem? : (%,%) -> % if Coef has FIELD
```

These exports come from (p1595) QuotientFieldCategory(UTS)
where UTS:UnivariateLaurentSeriesCategory(Coef:Ring)

```
abs : % -> %
  if and(has(UTS,OrderedIntegralDomain),has(Coef,Field))
ceiling : % -> UTS
  if and(has(UTS,IntegerNumberSystem),has(Coef,Field))
conditionP : Matrix % -> Union(Vector %,"failed")
  if and(and(has($,CharacteristicNonZero),
             has(UTS,PolynomialFactorizationExplicit)),
         has(Coef,Field))
coerce : Symbol -> %
  if and(has(UTS,RetractableTo Symbol),has(Coef,Field))
convert : % -> Pattern Integer
  if and(has(UTS,ConvertibleTo Pattern Integer),has(Coef,Field))
convert : % -> Pattern Float
  if and(has(UTS,ConvertibleTo Pattern Float),has(Coef,Field)\hfill\\
convert : % -> InputForm
  if and(has(UTS,ConvertibleTo InputForm),has(Coef,Field))
convert : % -> Float
  if and(has(UTS,RealConstant),has(Coef,Field))
convert : % -> DoubleFloat
  if and(has(UTS,RealConstant),has(Coef,Field))
D : (%,(UTS -> UTS),NonNegativeInteger) -> %
  if Coef has FIELD
D : (%,(UTS -> UTS)) -> % if Coef has FIELD
denom : % -> UTS if Coef has FIELD
denominator : % -> % if Coef has FIELD
differentiate : (%,(UTS -> UTS)) -> % if Coef has FIELD
differentiate : (%,(UTS -> UTS),NonNegativeInteger) -> %
  if Coef has FIELD
eval : (%,Equation UTS) -> %
  if and(has(UTS,Evalable UTS),has(Coef,Field))
eval : (%,List Symbol,List UTS) -> %
  if and(has(UTS,InnerEvalable(Symbol,UTS)),has(Coef,Field))
eval : (%,List Equation UTS) -> %
  if and(has(UTS,Evalable UTS),has(Coef,Field))
eval : (%,UTS,UTS) -> %
  if and(has(UTS,Evalable UTS),has(Coef,Field))
eval : (%,List UTS,List UTS) -> %
  if and(has(UTS,Evalable UTS),has(Coef,Field))
eval : (%,Symbol,UTS) -> %
  if and(has(UTS,InnerEvalable(Symbol,UTS)),has(Coef,Field))
factorPolynomial :
```

```
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
     if and(has(UTS,PolynomialFactorizationExplicit),has(Coef,Field))
factorSquareFreePolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
     if and(has(UTS,PolynomialFactorizationExplicit),has(Coef,Field))
floor : % -> UTS
  if and(has(UTS,IntegerNumberSystem),has(Coef,Field))
fractionPart : % -> %
  if and(has(UTS,EuclideanDomain),has(Coef,Field))
init : () -> % if and(has(UTS,StepThrough),has(Coef,Field))
map : ((UTS -> UTS),%) -> % if Coef has FIELD
max : (%,%) -> % if and(has(UTS,OrderedSet),has(Coef,Field))
min : (%,%) -> % if and(has(UTS,OrderedSet),has(Coef,Field))
negative? : % -> Boolean
  if and(has(UTS,OrderedIntegralDomain),has(Coef,Field))
nextItem : % -> Union(%,"failed")
  if and(has(UTS,StepThrough),has(Coef,Field))
numer : % -> UTS if Coef has FIELD
numerator : % -> % if Coef has FIELD
patternMatch :
 (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
   PatternMatchResult(Integer,%)
     if and(has(UTS,PatternMatchable Integer),has(Coef,Field))
patternMatch :
 (%,Pattern Float,PatternMatchResult(Float,%)) ->
   PatternMatchResult(Float,%)
     if and(has(UTS,PatternMatchable Float),has(Coef,Field))
positive? : % -> Boolean
  if and(has(UTS,OrderedIntegralDomain),has(Coef,Field))
random : () -> %
  if and(has(UTS,IntegerNumberSystem),has(Coef,Field))
reducedSystem :
 (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer)
  if and(has(UTS,LinearlyExplicitRingOver Integer),has(Coef,Field))
reducedSystem : Matrix % -> Matrix Integer
  if and(has(UTS,LinearlyExplicitRingOver Integer),has(Coef,Field))
reducedSystem :
 (Matrix %,Vector %) -> Record(mat: Matrix UTS,vec: Vector UTS)
   if Coef has FIELD
reducedSystem : Matrix % -> Matrix UTS if Coef has FIELD
retract : % -> Symbol
  if and(has(UTS,RetractableTo Symbol),has(Coef,Field))
retract : % -> Integer
  if and(has(UTS,RetractableTo Integer),has(Coef,Field))
retract : % -> Fraction Integer
  if and(has(UTS,RetractableTo Integer),has(Coef,Field))
retractIfCan : % -> Union(Fraction Integer,"failed")
  if and(has(UTS,RetractableTo Integer),has(Coef,Field))
retractIfCan : % -> Union(Symbol,"failed")
  if and(has(UTS,RetractableTo Symbol),has(Coef,Field))
retractIfCan : % -> Union(Integer,"failed")
  if and(has(UTS,RetractableTo Integer),has(Coef,Field))
```

```
sign : % -> Integer
  if and(has(UTS,OrderedIntegralDomain),has(Coef,Field))
solveLinearPolynomialEquation :
  (List SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     Union(List SparseUnivariatePolynomial %,"failed")
      if and(has(UTS,PolynomialFactorizationExplicit),has(Coef,Field))
squareFreePolynomial :
  SparseUnivariatePolynomial % ->
     Factored SparseUnivariatePolynomial %
       if and(has(UTS,PolynomialFactorizationExplicit),has(Coef,Field))
wholePart : % -> UTS
  if and(has(UTS,EuclideanDomain),has(Coef,Field))
?*? : (UTS,%) -> % if Coef has FIELD
?*? : (%,UTS) -> % if Coef has FIELD
?<? : (%,%) -> Boolean if and(has(UTS,OrderedSet),has(Coef,Field))
?/? : (UTS,UTS) -> % if Coef has FIELD
?.? : (%,UTS) -> % if and(has(UTS,Eltable(UTS,UTS)),has(Coef,Field))
?<=? : (%,%) -> Boolean if and(has(UTS,OrderedSet),has(Coef,Field))
?>? : (%,%) -> Boolean if and(has(UTS,OrderedSet),has(Coef,Field))
?>=? : (%,%) -> Boolean if and(has(UTS,OrderedSet),has(Coef,Field))
```

— UnivariateLaurentSeriesConstructorCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#ULSCCAT">
UnivariateLaurentSeriesConstructorCategory (ULSCCAT)</a></h2>
</body>
```

— category ULSCCAT UnivariateLaurentSeriesConstructorCategory —

```
)abbrev category ULSCCAT UnivariateLaurentSeriesConstructorCategory
++ Author: Clifton J. Williamson
++ Date Created: 6 February 1990
++ Date Last Updated: 10 May 1990
++ Description:
++ This is a category of univariate Laurent series constructed from
++ univariate Taylor series.  A Laurent series is represented by a pair
++ \spad{[n,f(x)]}, where n is an arbitrary integer and \spad{f(x)}
++ is a Taylor series.  This pair represents the Laurent series
++ \spad{x**n * f(x)}.

UnivariateLaurentSeriesConstructorCategory(Coef,UTS) : Category == SIG where
  Coef: Ring
  UTS : UnivariateTaylorSeriesCategory Coef

  I ==> Integer

  SIG ==> Join(UnivariateLaurentSeriesCategory(Coef),_
                   RetractableTo UTS) with
```

```
 laurent : (I,UTS) -> %
   ++ \spad{laurent(n,f(x))} returns \spad{x**n * f(x)}.

 degree : % -> I
   ++ \spad{degree(f(x))} returns the degree of the lowest order term of
   ++ \spad{f(x)}, which may have zero as a coefficient.

 taylorRep : % -> UTS
   ++ \spad{taylorRep(f(x))} returns \spad{g(x)}, where
   ++ \spad{f = x**n * g(x)} is represented by \spad{[n,g(x)]}.

 removeZeroes : % -> %
   ++ \spad{removeZeroes(f(x))} removes leading zeroes from the
   ++ representation of the Laurent series \spad{f(x)}.
   ++ A Laurent series is represented by (1) an exponent and
   ++ (2) a Taylor series which may have leading zero coefficients.
   ++ When the Taylor series has a leading zero coefficient, the
   ++ 'leading zero' is removed from the Laurent series as follows:
   ++ the series is rewritten by increasing the exponent by 1 and
   ++ dividing the Taylor series by its variable.
   ++ Note that \spad{removeZeroes(f)} removes all leading zeroes from f

 removeZeroes : (I,%) -> %
   ++ \spad{removeZeroes(n,f(x))} removes up to n leading zeroes from
   ++ the Laurent series \spad{f(x)}.
   ++ A Laurent series is represented by (1) an exponent and
   ++ (2) a Taylor series which may have leading zero coefficients.
   ++ When the Taylor series has a leading zero coefficient, the
   ++ 'leading zero' is removed from the Laurent series as follows:
   ++ the series is rewritten by increasing the exponent by 1 and
   ++ dividing the Taylor series by its variable.

 coerce : UTS -> %
   ++ \spad{coerce(f(x))} converts the Taylor series \spad{f(x)} to a
   ++ Laurent series.

 taylor : % -> UTS
   ++ taylor(f(x)) converts the Laurent series f(x) to a Taylor series,
   ++ if possible.  Error: if this is not possible.

 taylorIfCan : % -> Union(UTS,"failed")
   ++ \spad{taylorIfCan(f(x))} converts the Laurent series \spad{f(x)}
   ++ to a Taylor series, if possible. If this is not possible,
   ++ "failed" is returned.

 if Coef has Field then QuotientFieldCategory(UTS)
   --++ the quotient field of univariate Taylor series over a field is
   --++ the field of Laurent series

add

 zero? x == zero? taylorRep x
```

```
    retract(x:%):UTS == taylor x

    retractIfCan(x:%):Union(UTS,"failed") == taylorIfCan x
```

———

— **COQ ULSCCAT** —

```
(* category ULSCCAT *)
(*

    zero? : % -> Boolean
    zero? x == zero? taylorRep x

    retract : % -> UTS
    retract(x:%):UTS == taylor x

    retractIfCan : % -> Union(Symbol,"failed")
    retractIfCan(x:%):Union(UTS,"failed") == taylorIfCan x

*)
```

———

— **ULSCCAT.dotabb** —

```
"ULSCCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ULSCCAT"];
"ULSCCAT" -> "ULSCAT"
```

———

— **ULSCCAT.dotfull** —

```
"UnivariateLaurentSeriesConstructorCategory(a:Ring,b:UnivariateTaylorSeriesCategory(Ring))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ULSCCAT"];
"UnivariateLaurentSeriesConstructorCategory(a:Ring,b:UnivariateTaylorSeriesCategory(Ring))"
  -> "UnivariateLaurentSeriesCategory(a:Ring)"
```

———

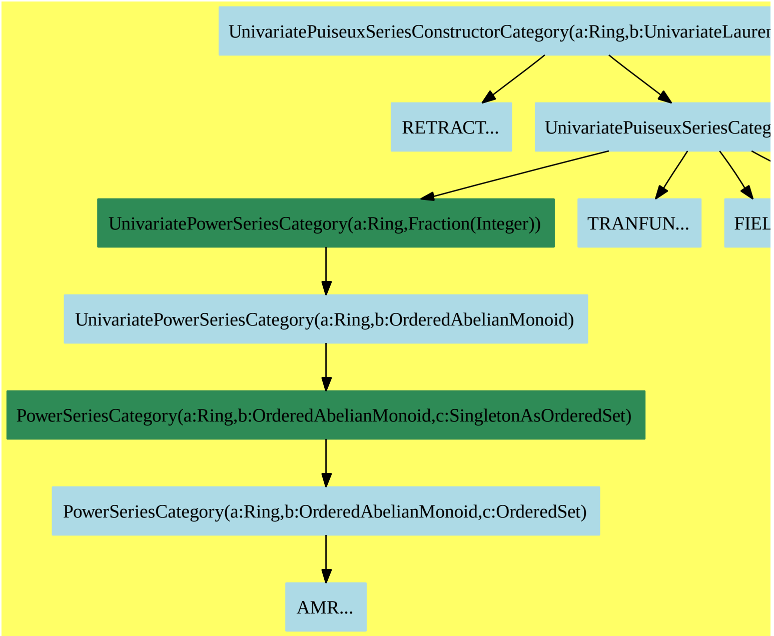
— **ULSCCAT.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"UnivariateLaurentSeriesConstructorCategory(a:Ring,b:UnivariateTaylorSeriesCategory(Ring))"
 [color=lightblue];
"UnivariateLaurentSeriesConstructorCategory(a:Ring,b:UnivariateTaylorSeriesCategory(Ring))"
```

```
  -> "UnivariateLaurentSeriesCategory(a:Ring)"

"UnivariateLaurentSeriesCategory(a:Ring)" [color=lightblue];
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "UnivariatePowerSeriesCategory(a:Ring,Integer)"
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "FIELD..."
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "RADCAT..."
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "TRANFUN..."

"UnivariatePowerSeriesCategory(a:Ring,Integer)" [color=seagreen];
"UnivariatePowerSeriesCategory(a:Ring,Integer)" ->
    "UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"

"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"
 [color=lightblue];
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)" ->
 "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
 [color=seagreen];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
  -> "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
 [color=lightblue];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)" ->
    "AMR..."

"AMR..." [color=lightblue];
"FIELD..." [color=lightblue];
"TRANFUN..." [color=lightblue];
"RADCAT..." [color=lightblue];
}
```

———-

18.0.236 UnivariatePuiseuxSeriesConstructorCategory (UPXSCCA)



— **UnivariatePuiseuxSeriesConstructorCategory.input** —

```
)set break resume
)sys rm -f UnivariatePuiseuxSeriesConstructorCategory.output
)spool UnivariatePuiseuxSeriesConstructorCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show UnivariatePuiseuxSeriesConstructorCategory
--R
--R UnivariatePuiseuxSeriesConstructorCategory(Coef: Ring,ULS: UnivariateLaurentSeriesCategory(t#1)) is a catego
--R Abbreviation for UnivariatePuiseuxSeriesConstructorCategory is UPXSCCA
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for UPXSCCA
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Coef,%) -> %                    ?*? : (%,Coef) -> %
--R ?*? : (%,%) -> %                       ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %      ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %     ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                       ?-? : (%,%) -> %
--R -? : % -> %                            ?=? : (%,%) -> Boolean
--R 1 : () -> %                            0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %      ?^? : (%,PositiveInteger) -> %
--R center : % -> Coef                     coerce : % -> % if Coef has INTDOM
--R coerce : ULS -> %                      coerce : Integer -> %
```

```
--R coerce : % -> OutputForm             complete : % -> %
--R degree : % -> Fraction(Integer)      ?.? : (%,Fraction(Integer)) -> Coef
--R hash : % -> SingleInteger            inv : % -> % if Coef has FIELD
--R latex : % -> String                  laurent : % -> ULS
--R laurentRep : % -> ULS                leadingCoefficient : % -> Coef
--R leadingMonomial : % -> %             map : ((Coef -> Coef),%) -> %
--R monomial? : % -> Boolean             one? : % -> Boolean
--R order : % -> Fraction(Integer)       pole? : % -> Boolean
--R recip : % -> Union(%,"failed")       reductum : % -> %
--R retract : % -> ULS                   sample : () -> %
--R variable : % -> Symbol               zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction(Integer)) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?*? : (Fraction(Integer),%) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?**? : (%,Fraction(Integer)) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?**? : (%,%) -> % if Coef has ALGEBRA(FRAC(INT))
--R ?**? : (%,Integer) -> % if Coef has FIELD
--R ?/? : (%,%) -> % if Coef has FIELD
--R ?/? : (%,Coef) -> % if Coef has FIELD
--R D : % -> % if Coef has *: (Fraction(Integer),Coef) -> Coef
--R D : (%,NonNegativeInteger) -> % if Coef has *: (Fraction(Integer),Coef) -> Coef
--R D : (%,Symbol) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Fraction(Integer),Coef) -> Coef
--R D : (%,List(Symbol)) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Fraction(Integer),Coef) -> Coef
--R D : (%,Symbol,NonNegativeInteger) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Fraction(Integer),Coef) -
--R D : (%,List(Symbol),List(NonNegativeInteger)) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Fraction(Inte
--R ?^? : (%,Integer) -> % if Coef has FIELD
--R acos : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acosh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acot : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acoth : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acsc : % -> % if Coef has ALGEBRA(FRAC(INT))
--R acsch : % -> % if Coef has ALGEBRA(FRAC(INT))
--R approximate : (%,Fraction(Integer)) -> Coef if Coef has **: (Coef,Fraction(Integer)) -> Coef and Coef has co
--R asec : % -> % if Coef has ALGEBRA(FRAC(INT))
--R asech : % -> % if Coef has ALGEBRA(FRAC(INT))
--R asin : % -> % if Coef has ALGEBRA(FRAC(INT))
--R asinh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R associates? : (%,%) -> Boolean if Coef has INTDOM
--R atan : % -> % if Coef has ALGEBRA(FRAC(INT))
--R atanh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
--R coefficient : (%,Fraction(Integer)) -> Coef
--R coerce : Fraction(Integer) -> % if Coef has ALGEBRA(FRAC(INT))
--R coerce : Coef -> % if Coef has COMRING
--R cos : % -> % if Coef has ALGEBRA(FRAC(INT))
--R cosh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R cot : % -> % if Coef has ALGEBRA(FRAC(INT))
--R coth : % -> % if Coef has ALGEBRA(FRAC(INT))
--R csc : % -> % if Coef has ALGEBRA(FRAC(INT))
--R csch : % -> % if Coef has ALGEBRA(FRAC(INT))
--R differentiate : % -> % if Coef has *: (Fraction(Integer),Coef) -> Coef
--R differentiate : (%,NonNegativeInteger) -> % if Coef has *: (Fraction(Integer),Coef) -> Coef
--R differentiate : (%,Symbol) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Fraction(Integer),Coef) -> Coef
```

```
--R differentiate : (%,List(Symbol)) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Fraction(Integer),Coef) ->
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if Coef has PDRING(SYMBOL) and Coef has *: (Fraction(Inte
--R differentiate : (%,List(Symbol),List(NonNegativeInteger)) -> % if Coef has PDRING(SYMBOL) and Coef has *: (F
--R divide : (%,%) -> Record(quotient: %,remainder: %) if Coef has FIELD
--R ?.? : (%,%) -> % if Fraction(Integer) has SGROUP
--R euclideanSize : % -> NonNegativeInteger if Coef has FIELD
--R eval : (%,Coef) -> Stream(Coef) if Coef has **: (Coef,Fraction(Integer)) -> Coef
--R exp : % -> % if Coef has ALGEBRA(FRAC(INT))
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed") if Coef has FIELD
--R exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
--R extend : (%,Fraction(Integer)) -> %
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if Coef has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if Coef has FIELD
--R factor : % -> Factored(%) if Coef has FIELD
--R gcd : (%,%) -> % if Coef has FIELD
--R gcd : List(%) -> % if Coef has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R integrate : (%,Symbol) -> % if Coef has ACFS(INT) and Coef has PRIMCAT and Coef has TRANFUN and Coef has ALG
--R integrate : % -> % if Coef has ALGEBRA(FRAC(INT))
--R laurentIfCan : % -> Union(ULS,"failed")
--R lcm : (%,%) -> % if Coef has FIELD
--R lcm : List(%) -> % if Coef has FIELD
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %) if Coef has FIELD
--R log : % -> % if Coef has ALGEBRA(FRAC(INT))
--R monomial : (%,List(SingletonAsOrderedSet),List(Fraction(Integer))) -> %
--R monomial : (%,SingletonAsOrderedSet,Fraction(Integer)) -> %
--R monomial : (Coef,Fraction(Integer)) -> %
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed") if Coef has FIELD
--R multiplyExponents : (%,Fraction(Integer)) -> %
--R multiplyExponents : (%,PositiveInteger) -> %
--R nthRoot : (%,Integer) -> % if Coef has ALGEBRA(FRAC(INT))
--R order : (%,Fraction(Integer)) -> Fraction(Integer)
--R pi : () -> % if Coef has ALGEBRA(FRAC(INT))
--R prime? : % -> Boolean if Coef has FIELD
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %) if Coef has FIELD
--R puiseux : (Fraction(Integer),ULS) -> %
--R ?quo? : (%,%) -> % if Coef has FIELD
--R rationalPower : % -> Fraction(Integer)
--R ?rem? : (%,%) -> % if Coef has FIELD
--R retractIfCan : % -> Union(ULS,"failed")
--R sec : % -> % if Coef has ALGEBRA(FRAC(INT))
--R sech : % -> % if Coef has ALGEBRA(FRAC(INT))
--R series : (NonNegativeInteger,Stream(Record(k: Fraction(Integer),c: Coef))) -> %
--R sin : % -> % if Coef has ALGEBRA(FRAC(INT))
--R sinh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R sizeLess? : (%,%) -> Boolean if Coef has FIELD
--R sqrt : % -> % if Coef has ALGEBRA(FRAC(INT))
--R squareFree : % -> Factored(%) if Coef has FIELD
--R squareFreePart : % -> % if Coef has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tan : % -> % if Coef has ALGEBRA(FRAC(INT))
--R tanh : % -> % if Coef has ALGEBRA(FRAC(INT))
--R terms : % -> Stream(Record(k: Fraction(Integer),c: Coef))
--R truncate : (%,Fraction(Integer),Fraction(Integer)) -> %
```

```
--R truncate : (%,Fraction(Integer)) -> %
--R unit? : % -> Boolean if Coef has INTDOM
--R unitCanonical : % -> % if Coef has INTDOM
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Coef has INTDOM
--R variables : % -> List(SingletonAsOrderedSet)
--R
--E 1

)spool
)lisp (bye)
```

— **UnivariatePuiseuxSeriesConstructorCategory.help** —

```
====================================================================
UnivariatePuiseuxSeriesConstructorCategory examples
====================================================================

This is a category of univariate Puiseux series constructed from
univariate Laurent series.  A Puiseux series is represented by a pair
[r,f(x)], where r is a positive rational number and f(x) is a Laurent
series.  This pair represents the Puiseux series f(x^r).

See Also:
o )show UnivariatePuiseuxSeriesConstructorCategory
```

See:

⇐ "RetractableTo" (RETRACT) 2.0.37 on page 110
⇐ "UnivariatePuiseuxSeriesCategory" (UPXSCAT) 17.0.227 on page 1697

Exports:

0	1	acos	acosh
acot	acoth	acsc	acsch
approximate	asec	asech	asin
asinh	associates?	atan	atanh
center	characteristic	charthRoot	coefficient
coerce	complete	cos	cosh
cot	coth	csc	csch
D	degree	differentiate	divide
euclideanSize	eval	exp	expressIdealMember
exquo	extend	extendedEuclidean	factor
gcd	gcdPolynomial	hash	integrate
inv	latex	laurent	laurentIfCan
laurentRep	lcm	leadingCoefficient	leadingMonomial
log	map	monomial	monomial?
multiEuclidean	multiplyExponents	nthRoot	one?
order	pi	pole?	prime?
principalIdeal	puiseux	rationalPower	recip
reductum	retract	retractIfCan	sample
sec	sech	series	sin
sinh	sizeLess?	sqrt	squareFree
squareFreePart	subtractIfCan	tan	tanh
terms	truncate	unit?	unitCanonical
unitNormal	variable	variables	zero?
?.?	?*?	?**?	?+?
?-?	-?	?=?	?^?
?~=?	?/?	?quo?	?rem?

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- if #1 has Field then canonicalsClosed where
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- if #1 has Field then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if #1 has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x*y \neq 0$ implies both x and y are non-zero.
- if #1 has CommutativeRing then commututive("*") where **commutative("*")** is true if it has an operation $" * " : (D, D) -> D$ which is commutative.

These are directly exported but not implemented:

```
coerce : ULS -> %
degree : % -> Fraction Integer
laurent : % -> ULS
laurentIfCan : % -> Union(ULS,"failed")
```

```
laurentRep : % -> ULS
puiseux : (Fraction Integer,ULS) -> %
rationalPower : % -> Fraction Integer
```

These are implemented by this category:

```
retract : % -> ULS
retractIfCan : % -> Union(ULS,"failed")
zero? : % -> Boolean
```

These exports come from (p1697) UnivariatePuiseuxSeriesCategory(Coef:Ring):

```
0 : () -> %
1 : () -> %
approximate : (%,Fraction Integer) -> Coef
  if Coef has **: (Coef,Fraction Integer) -> Coef
  and Coef has coerce: Symbol -> Coef
associates? : (%,%) -> Boolean if Coef has INTDOM
acos : % -> % if Coef has ALGEBRA FRAC INT
acosh : % -> % if Coef has ALGEBRA FRAC INT
acot : % -> % if Coef has ALGEBRA FRAC INT
acoth : % -> % if Coef has ALGEBRA FRAC INT
acsc : % -> % if Coef has ALGEBRA FRAC INT
acsch : % -> % if Coef has ALGEBRA FRAC INT
asec : % -> % if Coef has ALGEBRA FRAC INT
asech : % -> % if Coef has ALGEBRA FRAC INT
asin : % -> % if Coef has ALGEBRA FRAC INT
asinh : % -> % if Coef has ALGEBRA FRAC INT
atan : % -> % if Coef has ALGEBRA FRAC INT
atanh : % -> % if Coef has ALGEBRA FRAC INT
center : % -> Coef
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
coefficient : (%,Fraction Integer) -> Coef
coerce : % -> % if Coef has INTDOM
coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
coerce : Coef -> % if Coef has COMRING
coerce : Integer -> %
coerce : % -> OutputForm
complete : % -> %
cos : % -> % if Coef has ALGEBRA FRAC INT
cosh : % -> % if Coef has ALGEBRA FRAC INT
cot : % -> % if Coef has ALGEBRA FRAC INT
coth : % -> % if Coef has ALGEBRA FRAC INT
csc : % -> % if Coef has ALGEBRA FRAC INT
csch : % -> % if Coef has ALGEBRA FRAC INT
D : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,NonNegativeInteger) -> %
  if Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,List Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,Symbol,NonNegativeInteger) -> %
```

```
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,List Symbol,List NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,List Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,Symbol,NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,NonNegativeInteger) -> %
  if Coef has *: (Fraction Integer,Coef) -> Coef
divide : (%,%) -> Record(quotient: %,remainder: %) if Coef has FIELD
euclideanSize : % -> NonNegativeInteger if Coef has FIELD
eval : (%,Coef) -> Stream Coef
  if Coef has **: (Coef,Fraction Integer) -> Coef
exp : % -> % if Coef has ALGEBRA FRAC INT
expressIdealMember : (List %,%) -> Union(List %,"failed")
  if Coef has FIELD
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
  if Coef has FIELD
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
  if Coef has FIELD
exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
extend : (%,Fraction Integer) -> %
factor : % -> Factored % if Coef has FIELD
gcd : (%,%) -> % if Coef has FIELD
gcd : List % -> % if Coef has FIELD
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
    SparseUnivariatePolynomial %
      if Coef has FIELD
hash : % -> SingleInteger
integrate : (%,Symbol) -> %
  if Coef has ACFS INT
  and Coef has PRIMCAT
  and Coef has TRANFUN
  and Coef has ALGEBRA FRAC INT
  or Coef has variables: Coef -> List Symbol
  and Coef has integrate: (Coef,Symbol) -> Coef
  and Coef has ALGEBRA FRAC INT
integrate : % -> % if Coef has ALGEBRA FRAC INT
inv : % -> % if Coef has FIELD
latex : % -> String
lcm : (%,%) -> % if Coef has FIELD
```

```
lcm : List % -> % if Coef has FIELD
leadingCoefficient : % -> Coef
leadingMonomial : % -> %
log : % -> % if Coef has ALGEBRA FRAC INT
map : ((Coef -> Coef),%) -> %
monomial : (%,List SingletonAsOrderedSet,List Fraction Integer) -> %
monomial : (%,SingletonAsOrderedSet,Fraction Integer) -> %
monomial : (Coef,Fraction Integer) -> %
monomial? : % -> Boolean
multiEuclidean : (List %,%) -> Union(List %,"failed") if Coef has FIELD
multiplyExponents : (%,PositiveInteger) -> %
multiplyExponents : (%,Fraction Integer) -> %
nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
one? : % -> Boolean
order : (%,Fraction Integer) -> Fraction Integer
order : % -> Fraction Integer
pi : () -> % if Coef has ALGEBRA FRAC INT
pole? : % -> Boolean
prime? : % -> Boolean if Coef has FIELD
principalIdeal : List % -> Record(coef: List %,generator: %)
  if Coef has FIELD
recip : % -> Union(%,"failed")
reductum : % -> %
sample : () -> %
sec : % -> % if Coef has ALGEBRA FRAC INT
sech : % -> % if Coef has ALGEBRA FRAC INT
series :
  (NonNegativeInteger,Stream Record(k: Fraction Integer,c: Coef)) -> %
sin : % -> % if Coef has ALGEBRA FRAC INT
sinh : % -> % if Coef has ALGEBRA FRAC INT
sizeLess? : (%,%) -> Boolean if Coef has FIELD
sqrt : % -> % if Coef has ALGEBRA FRAC INT
squareFree : % -> Factored % if Coef has FIELD
squareFreePart : % -> % if Coef has FIELD
subtractIfCan : (%,%) -> Union(%,"failed")
tan : % -> % if Coef has ALGEBRA FRAC INT
tanh : % -> % if Coef has ALGEBRA FRAC INT
terms : % -> Stream Record(k: Fraction Integer,c: Coef)
truncate : (%,Fraction Integer,Fraction Integer) -> %
truncate : (%,Fraction Integer) -> %
unit? : % -> Boolean if Coef has INTDOM
unitCanonical : % -> % if Coef has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
  if Coef has INTDOM
variable : % -> Symbol
variables : % -> List SingletonAsOrderedSet
?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?**? : (%,Integer) -> % if Coef has FIELD
?^? : (%,Integer) -> % if Coef has FIELD
?/? : (%,%) -> % if Coef has FIELD
?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
```

```
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (Integer,%) -> %
?*? : (Coef,%) -> %
?*? : (%,Coef) -> %
?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
?/? : (%,Coef) -> % if Coef has FIELD
-? : % -> %
?.? : (%,%) -> % if Fraction Integer has SGROUP
?.? : (%,Fraction Integer) -> Coef
?quo? : (%,%) -> % if Coef has FIELD
?rem? : (%,%) -> % if Coef has FIELD
```

— UnivariatePuiseuxSeriesConstructorCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#UPXSCCA">
UnivariatePuiseuxSeriesConstructorCategory (UPXSCCA)</a></h2>
</body>
```

——

— category UPXSCCA UnivariatePuiseuxSeriesConstructorCategory —

```
)abbrev category UPXSCCA UnivariatePuiseuxSeriesConstructorCategory
++ Author: Clifton J. Williamson
++ Date Created: 6 February 1990
++ Date Last Updated: 22 March 1990
++ Description:
++ This is a category of univariate Puiseux series constructed
++ from univariate Laurent series.  A Puiseux series is represented
++ by a pair \spad{[r,f(x)]}, where r is a positive rational number and
++ \spad{f(x)} is a Laurent series.  This pair represents the Puiseux
++ series \spad{f(x^r)}.

UnivariatePuiseuxSeriesConstructorCategory(Coef,ULS) : Category == SIG where
  Coef : Ring
  ULS  : UnivariateLaurentSeriesCategory Coef

  I  ==> Integer
  RN ==> Fraction Integer

  SIG ==> Join(UnivariatePuiseuxSeriesCategory(Coef),RetractableTo ULS) with

    puiseux : (RN,ULS) -> %
```

```
      ++ \spad{puiseux(r,f(x))} returns \spad{f(x^r)}.

    rationalPower : % -> RN
      ++ \spad{rationalPower(f(x))} returns r where the Puiseux series
      ++ \spad{f(x) = g(x^r)}.

    laurentRep : % -> ULS
      ++ \spad{laurentRep(f(x))} returns \spad{g(x)} where the Puiseux series
      ++ \spad{f(x) = g(x^r)} is represented by \spad{[r,g(x)]}.

    degree : % -> RN
      ++ \spad{degree(f(x))} returns the degree of the leading term of the
      ++ Puiseux series \spad{f(x)}, which may have zero as a coefficient.

    coerce : ULS -> %
      ++ \spad{coerce(f(x))} converts the Laurent series \spad{f(x)} to a
      ++ Puiseux series.

    laurent : % -> ULS
      ++ \spad{laurent(f(x))} converts the Puiseux series \spad{f(x)} to a
      ++ Laurent series if possible. Error: if this is not possible.

    laurentIfCan : % -> Union(ULS,"failed")
      ++ \spad{laurentIfCan(f(x))} converts the Puiseux series \spad{f(x)}
      ++ to a Laurent series if possible.
      ++ If this is not possible, "failed" is returned.

   add

     zero? x == zero? laurentRep x

     retract(x:%):ULS == laurent x

     retractIfCan(x:%):Union(ULS,"failed") == laurentIfCan x
```

— COQ UPXSCCA —

```
(* category UPXSCCA *)
(*

     zero? : % -> Boolean
     zero? x == zero? laurentRep x

     retract : % -> ULS
     retract(x:%):ULS == laurent x

     retractIfCan : % -> Union(ULS,"failed")
     retractIfCan(x:%):Union(ULS,"failed") == laurentIfCan x

*)
```

— **UPXSCCA.dotabb** —

```
"UPXSCCA"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UPXSCCA"];
"UPXSCCA" -> "RETRACT"
"UPXSCCA" -> "UPXSCAT"
```

— **UPXSCCA.dotfull** —

```
"UnivariatePuiseuxSeriesConstructorCategory(a:Ring,b:UnivariateLaurentSeriesCategory(a))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UPXSCCA"];
"UnivariatePuiseuxSeriesConstructorCategory(a:Ring,b:UnivariateLaurentSeriesCategory(a))"
  -> "RetractableTo(UnivariatePuiseuxSeriesCategory(Ring))"
"UnivariatePuiseuxSeriesConstructorCategory(a:Ring,b:UnivariateLaurentSeriesCategory(a))"
  -> "UnivariatePuiseuxSeriesCategory(a:Ring)"
```

— **UPXSCCA.dotpic** —

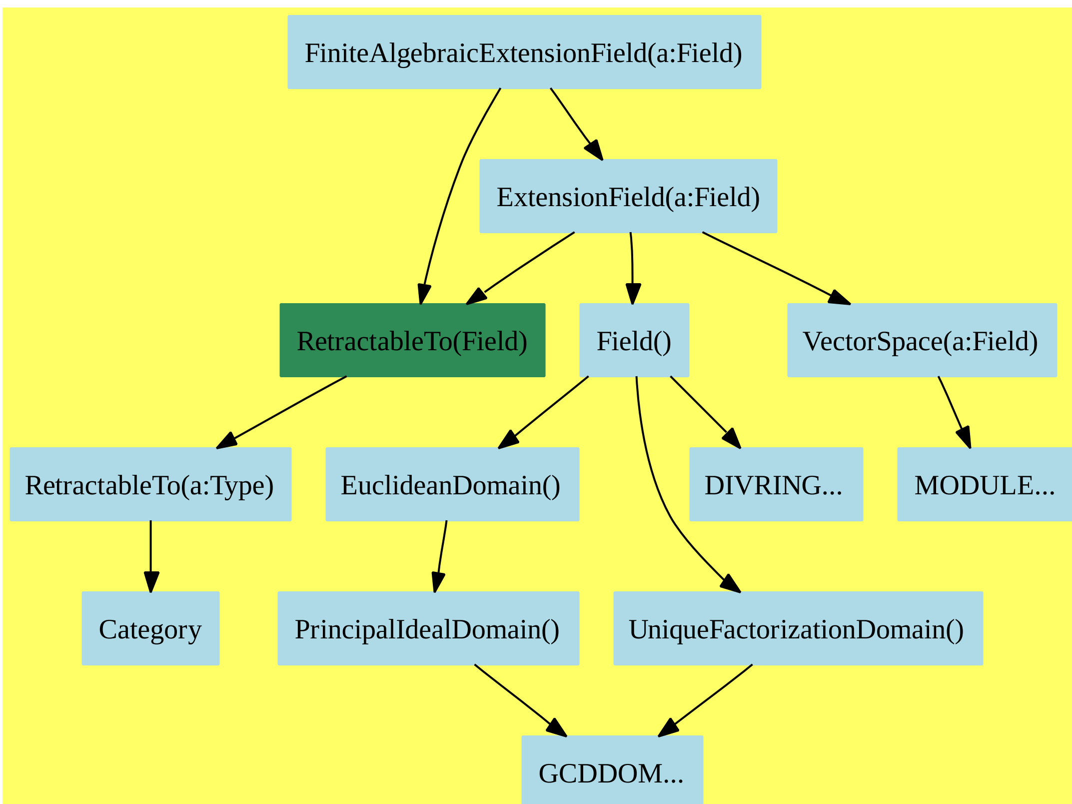
```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"UnivariatePuiseuxSeriesConstructorCategory(a:Ring,b:UnivariateLaurentSeriesCategory(a))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UPXSCCA"];
"UnivariatePuiseuxSeriesConstructorCategory(a:Ring,b:UnivariateLaurentSeriesCategory(a))"
  -> "RETRACT..."
"UnivariatePuiseuxSeriesConstructorCategory(a:Ring,b:UnivariateLaurentSeriesCategory(a))"
  -> "UnivariatePuiseuxSeriesCategory(a:Ring)"

"UnivariatePuiseuxSeriesCategory(a:Ring)" [color=lightblue];
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))"
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "TRANFUN..."
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "FIELD..."
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "RADCAT..."

"UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))" [color=seagreen];
"UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))" ->
    "UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"

"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"
 [color=lightblue];
```

```
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)" ->
 "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
 [color=seagreen];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
  -> "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
 [color=lightblue];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)" ->
    "AMR..."

"RETRACT..." [color=lightblue];
"TRANFUN..." [color=lightblue];
"FIELD..." [color=lightblue];
"RADCAT..." [color=lightblue];
"AMR..." [color=lightblue];
}
```

Chapter 19

Category Layer 18

19.0.237 FiniteAlgebraicExtensionField (FAXF)



— **FiniteAlgebraicExtensionField.input** —

```
)set break resume
)sys rm -f FiniteAlgebraicExtensionField.output
```

```
)spool FiniteAlgebraicExtensionField.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FiniteAlgebraicExtensionField
--R
--R FiniteAlgebraicExtensionField(F: Field) is a category constructor
--R Abbreviation for FiniteAlgebraicExtensionField is FAXF
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FAXF
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (F,%) -> %                      ?*? : (%,F) -> %
--R ?*? : (Fraction(Integer),%) -> %      ?*? : (%,Fraction(Integer)) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,Integer) -> %               ?**? : (%,NonNegativeInteger) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (%,F) -> %                      ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                D : % -> % if F has FINITE
--R Frobenius : % -> % if F has FINITE    1 : () -> %
--R 0 : () -> %                           ?^? : (%,Integer) -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R algebraic? : % -> Boolean             associates? : (%,%) -> Boolean
--R basis : () -> Vector(%)               coerce : F -> %
--R coerce : Fraction(Integer) -> %       coerce : % -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R coordinates : % -> Vector(F)          degree : % -> PositiveInteger
--R dimension : () -> CardinalNumber      factor : % -> Factored(%)
--R gcd : List(%) -> %                    gcd : (%,%) -> %
--R generator : () -> % if F has FINITE   hash : % -> SingleInteger
--R inGroundField? : % -> Boolean         init : () -> % if F has FINITE
--R inv : % -> %                          latex : % -> String
--R lcm : List(%) -> %                    lcm : (%,%) -> %
--R norm : % -> F                         one? : % -> Boolean
--R prime? : % -> Boolean                 ?quo? : (%,%) -> %
--R random : () -> % if F has FINITE      recip : % -> Union(%,"failed")
--R ?rem? : (%,%) -> %                    represents : Vector(F) -> %
--R retract : % -> F                      sample : () -> %
--R sizeLess? : (%,%) -> Boolean          squareFree : % -> Factored(%)
--R squareFreePart : % -> %               trace : % -> F
--R transcendent? : % -> Boolean          unit? : % -> Boolean
--R unitCanonical : % -> %                zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R D : (%,NonNegativeInteger) -> % if F has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if F has FINITE
--R basis : PositiveInteger -> Vector(%)
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if F has CHARNZ or F has FINITE
--R charthRoot : % -> % if F has FINITE
--R conditionP : Matrix(%) -> Union(Vector(%),"failed") if F has FINITE
```

```
--R coordinates : Vector(%) -> Matrix(F)
--R createNormalElement : () -> % if F has FINITE
--R createPrimitiveElement : () -> % if F has FINITE
--R definingPolynomial : () -> SparseUnivariatePolynomial(F)
--R degree : % -> OnePointCompletion(PositiveInteger)
--R differentiate : (%,NonNegativeInteger) -> % if F has FINITE
--R differentiate : % -> % if F has FINITE
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if F has CHARNZ or F has FINITE
--R discreteLog : % -> NonNegativeInteger if F has FINITE
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R enumerate : () -> List(%) if F has FINITE
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extensionDegree : () -> PositiveInteger
--R extensionDegree : () -> OnePointCompletion(PositiveInteger)
--R factorsOfCyclicGroupSize : () -> List(Record(factor: Integer,exponent: Integer)) if F has FINITE
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R index : PositiveInteger -> % if F has FINITE
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
--R linearAssociatedExp : (%,SparseUnivariatePolynomial(F)) -> % if F has FINITE
--R linearAssociatedLog : (%,%) -> Union(SparseUnivariatePolynomial(F),"failed") if F has FINITE
--R linearAssociatedLog : % -> SparseUnivariatePolynomial(F) if F has FINITE
--R linearAssociatedOrder : % -> SparseUnivariatePolynomial(F) if F has FINITE
--R lookup : % -> PositiveInteger if F has FINITE
--R minimalPolynomial : (%,PositiveInteger) -> SparseUnivariatePolynomial(%) if F has FINITE
--R minimalPolynomial : % -> SparseUnivariatePolynomial(F)
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed")
--R nextItem : % -> Union(%,"failed") if F has FINITE
--R norm : (%,PositiveInteger) -> % if F has FINITE
--R normal? : % -> Boolean if F has FINITE
--R normalElement : () -> % if F has FINITE
--R order : % -> OnePointCompletion(PositiveInteger) if F has CHARNZ or F has FINITE
--R order : % -> PositiveInteger if F has FINITE
--R primeFrobenius : % -> % if F has CHARNZ or F has FINITE
--R primeFrobenius : (%,NonNegativeInteger) -> % if F has CHARNZ or F has FINITE
--R primitive? : % -> Boolean if F has FINITE
--R primitiveElement : () -> % if F has FINITE
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %)
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if F has FINITE
--R retractIfCan : % -> Union(F,"failed")
--R size : () -> NonNegativeInteger if F has FINITE
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if F has FINITE
--R trace : (%,PositiveInteger) -> % if F has FINITE
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

— **FiniteAlgebraicExtensionField.help** —

```
====================================================================
FiniteAlgebraicExtensionField examples
====================================================================

FiniteAlgebraicExtensionField F is the category of fields
which are finite algebraic extensions of the field F.

If F is finite then any finite algebraic extension of F is finite, too.
Let K be a finite algebraic extension of the finite field F. The
exponentiation of elements of K defines a Z-module structure on the
multiplicative group of K.

The additive group of K becomes a module over the ring of polynomials
over F via the operation
    linearAssociatedExp(a:K,f:SparseUnivariatePolynomial F)
which is linear over F, for elements a from K, c,d from F and
f,g univariate polynomials over F we have linearAssociatedExp}(a,cf+dg)
equals c times linearAssociatedExp}(a,f) plus d times
linearAssociatedExp}(a,g).

Therefore linearAssociatedExp is defined completely by its action on
monomials from F[X]: linearAssociatedExp(a,monomial(1,k)\$SUP(F)) is
defined to be Frobenius(a,k) which is a**(q**k) where q=size()\$F.

The operations order and discreteLog associated with the multiplicative
exponentiation have additive analogues associated to the operation
linearAssociatedExp. These are the functions linearAssociatedOrder
and linearAssociatedLog, respectively.

See Also:
o )show FiniteAlgebraicExtensionField
```

See:

⇐ "ExtensionField" (XF) 18.0.230 on page 1754
⇐ "RetractableTo" (RETRACT) 2.0.37 on page 110

Exports:

0	1	algebraic?
associates?	basis	characteristic
charthRoot	coerce	conditionP
coordinates	createNormalElement	createPrimitiveElement
D	definingPolynomial	degree
differentiate	dimension	discreteLog
divide	euclideanSize	expressIdealMember
exquo	extendedEuclidean	extensionDegree
factor	factorsOfCyclicGroupSize	Frobenius
gcd	gcdPolynomial	generator
hash	index	inGroundField?
init	inv	latex
lcm	linearAssociatedExp	linearAssociatedLog
linearAssociatedOrder	lookup	minimalPolynomial
multiEuclidean	nextItem	norm
normal?	normalElement	one?
order	prime?	primeFrobenius
primitive?	primitiveElement	principalIdeal
random	recip	representationType
represents	retract	retractIfCan
sample	size	sizeLess?
squareFree	squareFreePart	subtractIfCan
tableForDiscreteLogarithm	trace	transcendenceDegree
transcendent?	unit?	unitCanonical
unitNormal	zero?	?*?
?**?	?+?	?-?
-?	?/?	?=?
?^?	?quo?	?rem?
?~=?		

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
basis : () -> Vector %
basis : PositiveInteger -> Vector %
coordinates : % -> Vector F
```

```
definingPolynomial : () -> SparseUnivariatePolynomial F
generator : () -> % if F has FINITE
minimalPolynomial : (%,PositiveInteger) ->
   SparseUnivariatePolynomial %
     if F has FINITE
normalElement : () -> % if F has FINITE
```

These are implemented by this category:

```
algebraic? : % -> Boolean
charthRoot : % -> Union(%,"failed")
    if F has CHARNZ or F has FINITE
coordinates : Vector % -> Matrix F
createNormalElement : () -> % if F has FINITE
degree : % -> PositiveInteger
dimension : () -> CardinalNumber
extensionDegree : () -> PositiveInteger
linearAssociatedExp : (%,SparseUnivariatePolynomial F) -> %
    if F has FINITE
linearAssociatedLog : (%,%) ->
   Union(SparseUnivariatePolynomial F,"failed")
     if F has FINITE
linearAssociatedLog : % -> SparseUnivariatePolynomial F
    if F has FINITE
linearAssociatedOrder : % -> SparseUnivariatePolynomial F
    if F has FINITE
minimalPolynomial : % -> SparseUnivariatePolynomial F
norm : % -> F
norm : (%,PositiveInteger) -> % if F has FINITE
normal? : % -> Boolean if F has FINITE
represents : Vector F -> %
size : () -> NonNegativeInteger if F has FINITE
trace : % -> F
trace : (%,PositiveInteger) -> % if F has FINITE
transcendenceDegree : () -> NonNegativeInteger
transcendent? : % -> Boolean
```

These exports come from (p1754) ExtensionField(F:Field):

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : F -> %
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
coerce : Fraction Integer -> %
discreteLog : (%,%) ->
   Union(NonNegativeInteger,"failed")
     if F has CHARNZ or F has FINITE
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extendedEuclidean : (%,%,%) ->
```

```
   Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) ->
   Record(coef1: %,coef2: %,generator: %)
factor : % -> Factored %
Frobenius : (%,NonNegativeInteger) -> % if F has FINITE
Frobenius : % -> % if F has FINITE
gcd : List % -> %
gcd : (%,%) -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                    SparseUnivariatePolynomial %
hash : % -> SingleInteger
inGroundField? : % -> Boolean
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
order : % -> OnePointCompletion PositiveInteger
     if F has CHARNZ or F has FINITE
prime? : % -> Boolean
primeFrobenius : % -> %
    if F has CHARNZ or F has FINITE
primeFrobenius : (%,NonNegativeInteger) -> %
    if F has CHARNZ or F has FINITE
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
retract : % -> F
retractIfCan : % -> Union(F,"failed")
sample : () -> %
squareFree : % -> Factored %
squareFreePart : % -> %
sizeLess? : (%,%) -> Boolean
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?/? : (%,%) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?*? : (F,%) -> %
?*? : (%,F) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,NonNegativeInteger) -> %
```

```
?**? : (%,PositiveInteger) -> %
?**? : (%,Integer) -> %
?^? : (%,Integer) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
?/? : (%,F) -> %
```

These exports come from (p110) RetractableTo(F:Field):

These exports come from (p1762) FiniteFieldCategory():

```
charthRoot : % -> % if F has FINITE
conditionP : Matrix % -> Union(Vector %,"failed")
    if F has FINITE
createPrimitiveElement : () -> % if F has FINITE
D : % -> % if F has FINITE
D : (%,NonNegativeInteger) -> % if F has FINITE
differentiate : % -> % if F has FINITE
differentiate : (%,NonNegativeInteger) -> %
    if F has FINITE
discreteLog : % -> NonNegativeInteger if F has FINITE
factorsOfCyclicGroupSize : () ->
   List Record(factor: Integer,exponent: Integer)
     if F has FINITE
index : PositiveInteger -> % if F has FINITE
init : () -> % if F has FINITE
lookup : % -> PositiveInteger if F has FINITE
nextItem : % -> Union(%,"failed") if F has FINITE
order : % -> PositiveInteger if F has FINITE
primitive? : % -> Boolean if F has FINITE
primitiveElement : () -> % if F has FINITE
random : () -> % if F has FINITE
representationType : () ->
   Union("prime",polynomial,normal,cyclic)
     if F has FINITE
tableForDiscreteLogarithm : Integer ->
   Table(PositiveInteger,NonNegativeInteger)
     if F has FINITE
```

See: Grabmeier[Grab92], Lidl[lidl83]

— **FiniteAlgebraicExtensionField.html** —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FAXF">
FiniteAlgebraicExtensionField (FAXF)</a></h2>
</body>
```

———-

— **category FAXF FiniteAlgebraicExtensionField** —

```
)abbrev category FAXF FiniteAlgebraicExtensionField
```

```
++ Author: J. Grabmeier, A. Scheerhorn
++ Date Created: 11 March 1991
++ Date Last Updated: 31 March 1991
++ References:
++ Grab92 Finite Fields in Axiom
++ Lidl83 Finite Field, Encyclopedia of Mathematics and Its Applications
++ Description:
++ FiniteAlgebraicExtensionField F is the category of fields
++ which are finite algebraic extensions of the field F.
++ If F is finite then any finite algebraic extension of F
++ is finite, too. Let K be a finite algebraic extension of the
++ finite field F. The exponentiation of elements of K
++ defines a Z-module structure on the multiplicative group of K.
++ The additive group of K becomes a module over the ring of
++ polynomials over F via the operation
++ \spadfun{linearAssociatedExp}(a:K,f:SparseUnivariatePolynomial F)
++ which is linear over F, that is, for elements a from K,
++ c,d from F and f,g univariate polynomials over F
++ we have \spadfun{linearAssociatedExp}(a,cf+dg) equals c times
++ \spadfun{linearAssociatedExp}(a,f) plus d times
++ \spadfun{linearAssociatedExp}(a,g).
++ Therefore \spadfun{linearAssociatedExp} is defined completely by
++ its action on  monomials from F[X]:
++ \spadfun{linearAssociatedExp}(a,monomial(1,k)\$SUP(F)) is defined to be
++ \spadfun{Frobenius}(a,k) which is a**(q**k) where q=size()\$F.
++ The operations order and discreteLog associated with the multiplicative
++ exponentiation have additive analogues associated to the operation
++ \spadfun{linearAssociatedExp}. These are the functions
++ \spadfun{linearAssociatedOrder} and \spadfun{linearAssociatedLog},
++ respectively.

FiniteAlgebraicExtensionField(F) : Category == SIG where
  F : Field

  SIG ==> Join(ExtensionField F, RetractableTo F) with

    -- should be unified with algebras
    -- Join(ExtensionField F, FramedAlgebra F, RetractableTo F) with

    basis : () -> Vector $
      ++ basis() returns a fixed basis of \$ as \spad{F}-vectorspace.

    basis : PositiveInteger -> Vector $
      ++ basis(n) returns a fixed basis of a subfield of \$ as
      ++ \spad{F}-vectorspace.

    coordinates : $ -> Vector F
      ++ coordinates(a) returns the coordinates of \spad{a} with respect
      ++ to the fixed \spad{F}-vectorspace basis.

    coordinates : Vector $ -> Matrix F
      ++ coordinates([v1,...,vm]) returns the coordinates of the
      ++ vi's with to the fixed basis.  The coordinates of vi are
      ++ contained in the ith row of the matrix returned by this
```

```
    ++ function.

  represents : Vector F -> $
    ++ represents([a1,..,an]) returns \spad{a1*v1 + ... + an*vn}, where
    ++ v1,...,vn are the elements of the fixed basis.

  minimalPolynomial : $ -> SparseUnivariatePolynomial F
    ++ minimalPolynomial(a) returns the minimal polynomial of an
    ++ element \spad{a} over the ground field F.

  definingPolynomial : () -> SparseUnivariatePolynomial F
    ++ definingPolynomial() returns the polynomial used to define
    ++ the field extension.

  extensionDegree : () ->  PositiveInteger
    ++ extensionDegree() returns the degree of field extension.

  degree : $ -> PositiveInteger
    ++ degree(a) returns the degree of the minimal polynomial of an
    ++ element \spad{a} over the ground field F.

  norm : $  -> F
    ++ norm(a) computes the norm of \spad{a} with respect to the
    ++ field considered as an algebra with 1 over the ground field F.

  trace : $ -> F
    ++ trace(a) computes the trace of \spad{a} with respect to
    ++ the field considered as an algebra with 1 over the ground field F.

  if F has Finite then

    FiniteFieldCategory

    minimalPolynomial : ($,PositiveInteger) -> SparseUnivariatePolynomial $
      ++ minimalPolynomial(x,n) computes the minimal polynomial of x over
      ++ the field of extension degree n over the ground field F.

    norm : ($,PositiveInteger)  -> $
      ++ norm(a,d) computes the norm of \spad{a} with respect to the field
      ++ of extension degree d over the ground field of size.
      ++ Error: if d does not divide the extension degree of \spad{a}.
      ++ Note that norm(a,d) = reduce(*,[a**(q**(d*i)) for i in 0..n/d])

    trace : ($,PositiveInteger)   -> $
      ++ trace(a,d) computes the trace of \spad{a} with respect to the
      ++ field of extension degree d over the ground field of size q.
      ++ Error: if d does not divide the extension degree of \spad{a}.
      ++ Note that
      ++ \spad{trace(a,d)=reduce(+,[a**(q**(d*i)) for i in 0..n/d])}.

    createNormalElement : () -> $
      ++ createNormalElement() computes a normal element over the ground
      ++ field F, that is,
      ++ \spad{a**(q**i), 0 <= i < extensionDegree()} is an F-basis,
```

```
    ++ where \spad{q = size()\$F}.
    ++ Reference: Such an element exists Lidl/Niederreiter: Theorem 2.35.

  normalElement : () -> $
    ++ normalElement() returns a element, normal over the ground field F,
    ++ thus \spad{a**(q**i), 0 <= i < extensionDegree()} is an F-basis,
    ++ where \spad{q = size()\$F}.
    ++ At the first call, the element is computed by
    ++ \spadfunFrom{createNormalElement}{FiniteAlgebraicExtensionField}
    ++ then cached in a global variable.
    ++ On subsequent calls, the element is retrieved by referencing the
    ++ global variable.

  normal? : $ -> Boolean
    ++ normal?(a) tests whether the element \spad{a} is normal over the
    ++ ground field F, that is,
    ++ \spad{a**(q**i), 0 <= i <= extensionDegree()-1} is an F-basis,
    ++ where \spad{q = size()\$F}.
    ++ Implementation according to Lidl/Niederreiter: Theorem 2.39.

  generator : () -> $
    ++ generator() returns a root of the defining polynomial.
    ++ This element generates the field as an algebra over the ground
    ++ field.

  linearAssociatedExp : ($,SparseUnivariatePolynomial F) -> $
    ++ linearAssociatedExp(a,f) is linear over F, that is,
    ++ for elements a from \$, c,d form F and
    ++ f,g univariate polynomials over F we have
    ++ \spadfun{linearAssociatedExp}(a,cf+dg) equals c times
    ++ \spadfun{linearAssociatedExp}(a,f) plus d times
    ++ \spadfun{linearAssociatedExp}(a,g). Therefore
    ++ \spadfun{linearAssociatedExp} is defined completely by its
    ++ action on monomials from F[X]:
    ++ \spadfun{linearAssociatedExp}(a,monomial(1,k)\$SUP(F)) is
    ++ defined to be \spadfun{Frobenius}(a,k) which is a**(q**k),
    ++ where q=size()\$F.

  linearAssociatedOrder : $ -> SparseUnivariatePolynomial F
    ++ linearAssociatedOrder(a) retruns the monic polynomial g of
    ++ least degree, such that \spadfun{linearAssociatedExp}(a,g) is 0.

  linearAssociatedLog : $ -> SparseUnivariatePolynomial F
    ++ linearAssociatedLog(a) returns a polynomial g, such that
    ++ \spadfun{linearAssociatedExp}(normalElement(),g) equals a.

  linearAssociatedLog : ($,$) -> _
    Union(SparseUnivariatePolynomial F,"failed")
    ++ linearAssociatedLog(b,a) returns a polynomial g, such
    ++ that the \spadfun{linearAssociatedExp}(b,g) equals a.
    ++ If there is no such polynomial g, then
    ++ \spadfun{linearAssociatedLog} fails.

add
```

```
I   ==> Integer
PI  ==> PositiveInteger
NNI ==> NonNegativeInteger
SUP ==> SparseUnivariatePolynomial
DLP ==> DiscreteLogarithmPackage

represents(v) ==
  a:$:=0
  b:=basis()
  for i in 1..extensionDegree()@PI repeat
    a:=a+(v.i)*(b.i)
  a

transcendenceDegree() == 0$NNI

dimension() == (#basis()) ::NonNegativeInteger::CardinalNumber

coordinates(v:Vector $) ==
  m := new(#v, extensionDegree(), 0)$Matrix(F)
  for i in minIndex v .. maxIndex v for j in minRowIndex m .. repeat
    setRow_!(m, j, coordinates qelt(v, i))
  m

algebraic? a == true

transcendent? a == false

extensionDegree() == (#basis()) :: PositiveInteger

trace a ==
  b := basis()
  abs : F := 0
  for i in 1..#b repeat
    abs := abs + coordinates(a*b.i).i
  abs

norm a ==
  b := basis()
  m := new(#b,#b, 0)$Matrix(F)
  for i in 1..#b repeat
    setRow_!(m,i, coordinates(a*b.i))
  determinant(m)

if F has Finite then
  linearAssociatedExp(x,f) ==
    erg:$:=0
    y:=x
    for i in 0..degree(f) repeat
      erg:=erg + coefficient(f,i) * y
      y:=Frobenius(y)
    erg

  linearAssociatedLog(b,x) ==
```

```
  x=0 => 0
  l:List List F:=[entries coordinates b]
  a:$:=b
  extdeg:NNI:=extensionDegree()@PI
  for i in 2..extdeg repeat
    a:=Frobenius(a)
    l:=concat(l,entries coordinates a)$(List List F)
  l:=concat(l,entries coordinates x)$(List List F)
  m1:=rowEchelon transpose matrix(l)$(Matrix F)
  v:=zero(extdeg)$(Vector F)
  rown:I:=1
  for i in 1..extdeg repeat
    if qelt(m1,rown,i) = 1$F then
      v.i:=qelt(m1,rown,extdeg+1)
      rown:=rown+1
  p:=+/[monomial(v.(i+1),i::NNI) for i in 0..(#v-1)]
  p=0 =>
   messagePrint("linearAssociatedLog: second argument not in_
                 group generated by first argument")$OutputForm
   "failed"
  p

linearAssociatedLog(x) == linearAssociatedLog(normalElement(),x) ::
                        SparseUnivariatePolynomial(F)

linearAssociatedOrder(x) ==
  x=0 => 0
  l:List List F:=[entries coordinates x]
  a:$:=x
  for i in 1..extensionDegree()@PI repeat
    a:=Frobenius(a)
    l:=concat(l,entries coordinates a)$(List List F)
  v:=first nullSpace transpose matrix(l)$(Matrix F)
  +/[monomial(v.(i+1),i::NNI) for i in 0..(#v-1)]

charthRoot(x):Union($,"failed") ==
  (charthRoot(x)@$)::Union($,"failed")

minimalPolynomial(a,n) ==
  extensionDegree()@PI rem n ^= 0 =>
    error "minimalPolynomial: 2. argument must divide extension degree"
  f:SUP $:=monomial(1,1)$(SUP $) - monomial(a,0)$(SUP $)
  u:$:=Frobenius(a,n)
  while not(u = a) repeat
    f:=f * (monomial(1,1)$(SUP $) - monomial(u,0)$(SUP $))
    u:=Frobenius(u,n)
  f

norm(e,s) ==
  qr := divide(extensionDegree(), s)
  zero?(qr.remainder) =>
    pow := (size()-1) quo (size()$F ** s - 1)
    e ** (pow::NonNegativeInteger)
  error "norm: second argument must divide degree of extension"
```

```
trace(e,s) ==
  qr:=divide(extensionDegree(),s)
  q:=size()$F
  zero?(qr.remainder) =>
    a:$:=0
    for i in 0..qr.quotient-1 repeat
      a:=a + e**(q**(s*i))
    a
  error "trace: second argument must divide degree of extension"

size() == size()$F ** extensionDegree()

createNormalElement() ==
  characteristic() = size() => 1
  res : $
  for i in 1.. repeat
    res := index(i :: PI)
    not inGroundField? res =>
      normal? res => return res
  -- theorem: there exists a normal element, this theorem is
  -- unknown to the compiler
  res

normal?(x:$) ==
  p:SUP $:=(monomial(1,extensionDegree()) - monomial(1,0))@(SUP $)
  f:SUP $:= +/[monomial(Frobenius(x,i),i)$(SUP $) _
              for i in 0..extensionDegree()-1]
  gcd(p,f) = 1 => true
  false

degree a ==
  y:$:=Frobenius a
  deg:PI:=1
  while y^=a repeat
    y := Frobenius(y)
    deg:=deg+1
  deg
```

———-

— COQ FAXF —

```
(* category FAXF *)
(*
  I   ==> Integer
  PI  ==> PositiveInteger
  NNI ==> NonNegativeInteger
  SUP ==> SparseUnivariatePolynomial
  DLP ==> DiscreteLogarithmPackage

  represents : Vector(F) -> %
  represents(v) ==
```

```
    a:$:=0
    b:=basis()
    for i in 1..extensionDegree()@PI repeat
      a:=a+(v.i)*(b.i)
    a

  transcendenceDegree : () -> NonNegativeInteger
  transcendenceDegree() == 0$NNI

  dimension : () -> CardinalNumber
  dimension() == (#basis()) ::NonNegativeInteger::CardinalNumber

  coordinates : Vector(%) -> Matrix(F)
  coordinates(v:Vector $) ==
    m := new(#v, extensionDegree(), 0)$Matrix(F)
    for i in minIndex v .. maxIndex v for j in minRowIndex m .. repeat
      setRow_!(m, j, coordinates qelt(v, i))
    m

  algebraic? : % -> Boolean
  algebraic? a == true

  transcendent? : % -> Boolean
  transcendent? a == false

  extensionDegree : () -> PositiveInteger
  extensionDegree() == (#basis()) :: PositiveInteger

  trace : % -> F
  trace a ==
    b := basis()
    abs : F := 0
    for i in 1..#b repeat
      abs := abs + coordinates(a*b.i).i
    abs

  norm : % -> F
  norm a ==
    b := basis()
    m := new(#b,#b, 0)$Matrix(F)
    for i in 1..#b repeat
      setRow_!(m,i, coordinates(a*b.i))
    determinant(m)

  if F has Finite then

    linearAssociatedExp : (%,SparseUnivariatePolynomial(F)) -> %
    linearAssociatedExp(x,f) ==
      erg:$:=0
      y:=x
      for i in 0..degree(f) repeat
        erg:=erg + coefficient(f,i) * y
        y:=Frobenius(y)
      erg
```

```
linearAssociatedLog : (%,%) ->
   Union(SparseUnivariatePolynomial(F),"failed")
linearAssociatedLog(b,x) ==
  x=0 => 0
  l:List List F:=[entries coordinates b]
  a:$:=b
  extdeg:NNI:=extensionDegree()@PI
  for i in 2..extdeg repeat
    a:=Frobenius(a)
    l:=concat(l,entries coordinates a)$(List List F)
  l:=concat(l,entries coordinates x)$(List List F)
  m1:=rowEchelon transpose matrix(l)$(Matrix F)
  v:=zero(extdeg)$(Vector F)
  rown:I:=1
  for i in 1..extdeg repeat
    if qelt(m1,rown,i) = 1$F then
      v.i:=qelt(m1,rown,extdeg+1)
      rown:=rown+1
  p:=+/[monomial(v.(i+1),i::NNI) for i in 0..(#v-1)]
  p=0 =>
   messagePrint("linearAssociatedLog: second argument not in_
                 group generated by first argument")$OutputForm
   "failed"
  p

linearAssociatedLog : % -> SparseUnivariatePolynomial(F)
linearAssociatedLog(x) == linearAssociatedLog(normalElement(),x) ::
                        SparseUnivariatePolynomial(F)

linearAssociatedOrder : % -> SparseUnivariatePolynomial(F)
linearAssociatedOrder(x) ==
  x=0 => 0
  l:List List F:=[entries coordinates x]
  a:$:=x
  for i in 1..extensionDegree()@PI repeat
    a:=Frobenius(a)
    l:=concat(l,entries coordinates a)$(List List F)
  v:=first nullSpace transpose matrix(l)$(Matrix F)
  +/[monomial(v.(i+1),i::NNI) for i in 0..(#v-1)]

charthRoot : % -> Union(%,"failed")
charthRoot(x):Union($,"failed") ==
  (charthRoot(x)@$)::Union($,"failed")
-- norm(e) == norm(e,1) pretend F
-- trace(e) == trace(e,1) pretend F

minimalPolynomial : (%,PositiveInteger) -> SparseUnivariatePolynomial(%)
minimalPolynomial(a,n) ==
  extensionDegree()@PI rem n ^= 0 =>
    error "minimalPolynomial: 2. argument must divide extension degree"
  f:SUP $:=monomial(1,1)$(SUP $) - monomial(a,0)$(SUP $)
  u:$:=Frobenius(a,n)
  while not(u = a) repeat
```

```
    f:=f * (monomial(1,1)$(SUP $) - monomial(u,0)$(SUP $))
    u:=Frobenius(u,n)
  f

norm : (%,PositiveInteger) -> %
norm(e,s) ==
  qr := divide(extensionDegree(), s)
  zero?(qr.remainder) =>
    pow := (size()-1) quo (size()$F ** s - 1)
    e ** (pow::NonNegativeInteger)
  error "norm: second argument must divide degree of extension"

trace : (%,PositiveInteger) -> %
trace(e,s) ==
  qr:=divide(extensionDegree(),s)
  q:=size()$F
  zero?(qr.remainder) =>
    a:$:=0
    for i in 0..qr.quotient-1 repeat
      a:=a + e**(q**(s*i))
    a
  error "trace: second argument must divide degree of extension"

size : () -> NonNegativeInteger
size() == size()$F ** extensionDegree()

createNormalElement : () -> %
createNormalElement() ==
  characteristic() = size() => 1
  res : $
  for i in 1.. repeat
    res := index(i :: PI)
    not inGroundField? res =>
      normal? res => return res
  -- theorem: there exists a normal element, this theorem is
  -- unknown to the compiler
  res

normal? : % -> Boolean
normal?(x:$) ==
  p:SUP $:=(monomial(1,extensionDegree()) - monomial(1,0))@(SUP $)
  f:SUP $:= +/[monomial(Frobenius(x,i),i)$(SUP $) _
             for i in 0..extensionDegree()-1]
  gcd(p,f) = 1 => true
  false

degree : % -> PositiveInteger
degree a ==
  y:$:=Frobenius a
  deg:PI:=1
  while y^=a repeat
    y := Frobenius(y)
    deg:=deg+1
  deg
```

```
*)
```

———-

— **FAXF.dotabb** —

```
"FAXF"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FAXF"];
"FAXF" -> "XF"
"FAXF" -> "RETRACT"
```

———-

— **FAXF.dotfull** —

```
"FiniteAlgebraicExtensionField(a:Field)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FAXF"];
"FiniteAlgebraicExtensionField(a:Field)" -> "ExtensionField(a:Field)"
"FiniteAlgebraicExtensionField(a:Field)" -> "RetractableTo(a:Field)"
```

———-

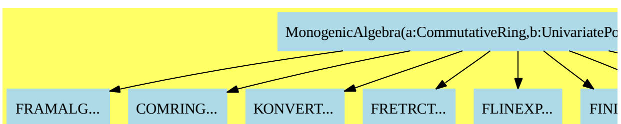
— **FAXF.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FiniteAlgebraicExtensionField(a:Field)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FAXF"];
"FiniteAlgebraicExtensionField(a:Field)" -> "ExtensionField(a:Field)"
"FiniteAlgebraicExtensionField(a:Field)" -> "RetractableTo(Field)"

"ExtensionField(a:Field)" [color=lightblue];
"ExtensionField(a:Field)" -> "Field()"
"ExtensionField(a:Field)" -> "RetractableTo(Field)"
"ExtensionField(a:Field)" -> "VectorSpace(a:Field)"

"Field()" [color=lightblue];
"Field()" -> "EuclideanDomain()"
"Field()" -> "UniqueFactorizationDomain()"
"Field()" -> "DIVRING..."

"EuclideanDomain()" [color=lightblue];
"EuclideanDomain()" -> "PrincipalIdealDomain()"

"UniqueFactorizationDomain()" [color=lightblue];
"UniqueFactorizationDomain()" -> "GCDDOM..."

"PrincipalIdealDomain()" [color=lightblue];
```

```
"PrincipalIdealDomain()" -> "GCDDOM..."

"RetractableTo(Field)" [color=seagreen];
"RetractableTo(Field)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"VectorSpace(a:Field)" [color=lightblue];
"VectorSpace(a:Field)" -> "MODULE..."

"MODULE..." [color=lightblue];
"DIVRING..." [color=lightblue];
"GCDDOM..." [color=lightblue];
"Category" [color=lightblue];
}
```

———-

19.0.238 MonogenicAlgebra (MONOGEN)



— MonogenicAlgebra.input —

```
)set break resume
)sys rm -f MonogenicAlgebra.output
)spool MonogenicAlgebra.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show MonogenicAlgebra
--R
--R MonogenicAlgebra(R: CommutativeRing,UP: UnivariatePolynomialCategory(t#1)) is a category constructor
--R Abbreviation for MonogenicAlgebra is MONOGEN
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for MONOGEN
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?/? : (%,%) -> % if R has FIELD
--R ?=? : (%,%) -> Boolean                1 : () -> %
```

```
--R 0 : () -> %                          ?^? : (%,NonNegativeInteger) -> %
--R ?^? : (%,PositiveInteger) -> %       basis : () -> Vector(%)
--R characteristicPolynomial : % -> UP   coerce : % -> % if R has FIELD
--R coerce : R -> %                      coerce : Integer -> %
--R coerce : % -> OutputForm             convert : UP -> %
--R convert : % -> UP                    convert : Vector(R) -> %
--R convert : % -> Vector(R)             coordinates : % -> Vector(R)
--R definingPolynomial : () -> UP        discriminant : () -> R
--R discriminant : Vector(%) -> R        gcd : (%,%) -> % if R has FIELD
--R gcd : List(%) -> % if R has FIELD    generator : () -> %
--R hash : % -> SingleInteger            init : () -> % if R has FFIELDC
--R inv : % -> % if R has FIELD          latex : % -> String
--R lcm : (%,%) -> % if R has FIELD      lcm : List(%) -> % if R has FIELD
--R lift : % -> UP                       norm : % -> R
--R one? : % -> Boolean                  ?quo? : (%,%) -> % if R has FIELD
--R random : () -> % if R has FINITE     rank : () -> PositiveInteger
--R recip : % -> Union(%,"failed")       reduce : UP -> %
--R ?rem? : (%,%) -> % if R has FIELD    represents : Vector(R) -> %
--R retract : % -> R                     sample : () -> %
--R trace : % -> R                       traceMatrix : () -> Matrix(R)
--R unit? : % -> Boolean if R has FIELD  zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction(Integer)) -> % if R has FIELD
--R ?*? : (Fraction(Integer),%) -> % if R has FIELD
--R ?**? : (%,Integer) -> % if R has FIELD
--R D : (%,(R -> R)) -> % if R has FIELD
--R D : (%,(R -> R),NonNegativeInteger) -> % if R has FIELD
--R D : (%,List(Symbol),List(NonNegativeInteger)) -> % if and(has(R,PartialDifferentialRing(Symbol)),has(R,Field
--R D : (%,Symbol,NonNegativeInteger) -> % if and(has(R,PartialDifferentialRing(Symbol)),has(R,Field))
--R D : (%,List(Symbol)) -> % if and(has(R,PartialDifferentialRing(Symbol)),has(R,Field))
--R D : (%,Symbol) -> % if and(has(R,PartialDifferentialRing(Symbol)),has(R,Field))
--R D : (%,NonNegativeInteger) -> % if and(has(R,DifferentialRing),has(R,Field)) or R has FFIELDC
--R D : % -> % if and(has(R,DifferentialRing),has(R,Field)) or R has FFIELDC
--R ?^? : (%,Integer) -> % if R has FIELD
--R associates? : (%,%) -> Boolean if R has FIELD
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if R has CHARNZ
--R charthRoot : % -> % if R has FFIELDC
--R coerce : Fraction(Integer) -> % if R has FIELD or R has RETRACT(FRAC(INT))
--R conditionP : Matrix(%) -> Union(Vector(%),"failed") if R has FFIELDC
--R coordinates : Vector(%) -> Matrix(R)
--R coordinates : (Vector(%),Vector(%)) -> Matrix(R)
--R coordinates : (%,Vector(%)) -> Vector(R)
--R createPrimitiveElement : () -> % if R has FFIELDC
--R derivationCoordinates : (Vector(%),(R -> R)) -> Matrix(R) if R has FIELD
--R differentiate : (%,(R -> R)) -> % if R has FIELD
--R differentiate : (%,(R -> R),NonNegativeInteger) -> % if R has FIELD
--R differentiate : (%,List(Symbol),List(NonNegativeInteger)) -> % if and(has(R,PartialDifferentialRing(Symbol))
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if and(has(R,PartialDifferentialRing(Symbol)),has(R,Field
--R differentiate : (%,List(Symbol)) -> % if and(has(R,PartialDifferentialRing(Symbol)),has(R,Field))
--R differentiate : (%,Symbol) -> % if and(has(R,PartialDifferentialRing(Symbol)),has(R,Field))
--R differentiate : (%,NonNegativeInteger) -> % if and(has(R,DifferentialRing),has(R,Field)) or R has FFIELDC
--R differentiate : % -> % if and(has(R,DifferentialRing),has(R,Field)) or R has FFIELDC
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if R has FFIELDC
```

```
--R discreteLog : % -> NonNegativeInteger if R has FFIELDC
--R divide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
--R enumerate : () -> List(%) if R has FINITE
--R euclideanSize : % -> NonNegativeInteger if R has FIELD
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed") if R has FIELD
--R exquo : (%,%) -> Union(%,"failed") if R has FIELD
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if R has FIELD
--R factor : % -> Factored(%) if R has FIELD
--R factorsOfCyclicGroupSize : () -> List(Record(factor: Integer,exponent: Integer)) if R has FFIELDC
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R index : PositiveInteger -> % if R has FINITE
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %) if R has FIELD
--R lookup : % -> PositiveInteger if R has FINITE
--R minimalPolynomial : % -> UP if R has FIELD
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed") if R has FIELD
--R nextItem : % -> Union(%,"failed") if R has FFIELDC
--R order : % -> OnePointCompletion(PositiveInteger) if R has FFIELDC
--R order : % -> PositiveInteger if R has FFIELDC
--R prime? : % -> Boolean if R has FIELD
--R primeFrobenius : % -> % if R has FFIELDC
--R primeFrobenius : (%,NonNegativeInteger) -> % if R has FFIELDC
--R primitive? : % -> Boolean if R has FFIELDC
--R primitiveElement : () -> % if R has FFIELDC
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %) if R has FIELD
--R reduce : Fraction(UP) -> Union(%,"failed") if R has FIELD
--R reducedSystem : Matrix(%) -> Matrix(R)
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(R),vec: Vector(R))
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(Integer),vec: Vector(Integer)) if R has LINEXP(I
--R reducedSystem : Matrix(%) -> Matrix(Integer) if R has LINEXP(INT)
--R regularRepresentation : % -> Matrix(R)
--R regularRepresentation : (%,Vector(%)) -> Matrix(R)
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if R has FFIELDC
--R represents : (Vector(R),Vector(%)) -> %
--R retract : % -> Fraction(Integer) if R has RETRACT(FRAC(INT))
--R retract : % -> Integer if R has RETRACT(INT)
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction(Integer),"failed") if R has RETRACT(FRAC(INT))
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT(INT)
--R size : () -> NonNegativeInteger if R has FINITE
--R sizeLess? : (%,%) -> Boolean if R has FIELD
--R squareFree : % -> Factored(%) if R has FIELD
--R squareFreePart : % -> % if R has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if R has FFIELDC
--R traceMatrix : Vector(%) -> Matrix(R)
--R unitCanonical : % -> % if R has FIELD
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has FIELD
--R
--E 1

)spool
)lisp (bye)
```

— MonogenicAlgebra.help —

```
====================================================================
MonogenicAlgebra examples
====================================================================

A MonogenicAlgebra is an algebra of finite rank which can be
generated by a single element.

See Also:
o )show MonogenicAlgebra
```

See:

⇒ “FunctionFieldCategory” (FFCAT) 20.0.241 on page 1900
⇐ “CommutativeRing” (COMRING) 10.0.161 on page 1019
⇐ “ConvertibleTo” (KONVERT) 2.0.17 on page 47
⇐ “FramedAlgebra” (FRAMALG) 18.0.233 on page 1786
⇐ “FullyRetractableTo” (FRETRCT) 3.0.50 on page 158
⇐ “LinearlyExplicitRingOver” (LINEXP) 10.0.166 on page 1045

Exports:

0	1	associates?
basis	characteristic	characteristicPolynomial
charthRoot	coerce	conditionP
convert	coordinates	createPrimitiveElement
D	definingPolynomial	derivationCoordinates
differentiate	discreteLog	discriminant
divide	euclideanSize	expressIdealMember
exquo	extendedEuclidean	factor
factorsOfCyclicGroupSize	generator	gcd
gcdPolynomial	hash	index
init	inv	latex
lcm	lift	lookup
minimalPolynomial	multiEuclidean	nextItem
norm	one?	order
prime?	primeFrobenius	primitive?
primitiveElement	principalIdeal	random
rank	recip	reduce
reducedSystem	regularRepresentation	represents
representationType	retract	retractIfCan
sample	size	sizeLess?
squareFree	squareFreePart	subtractIfCan
tableForDiscreteLogarithm	trace	traceMatrix
unit?	unitCanonical	unitNormal
zero?	?*?	?**?
?+?	?-?	-?
?=?	?^?	?~=?
?/?	?quo?	?rem?

Attributes Exported:

- if $ has Field then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if $ has Field then canonicalClosed where **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- if $ has Field then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * "\ :\ (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
definingPolynomial : () -> UP
lift : % -> UP
reduce : UP -> %
```

These are implemented by this category:

```
basis : () -> Vector %
characteristicPolynomial : % -> UP
convert : % -> UP
convert : UP -> %
derivationCoordinates : (Vector %,(R -> R)) -> Matrix R
    if R has FIELD
differentiate : (%,(R -> R)) -> % if R has FIELD
generator : () -> %
norm : % -> R
random : () -> % if R has FINITE
recip : % -> Union(%,"failed")
reduce : Fraction UP -> Union(%,"failed") if R has FIELD
retract : % -> R
retractIfCan : % -> Union(R,"failed")
size : () -> NonNegativeInteger if R has FINITE
```

These exports come from (p1786) FramedAlgebra(R,UP)
where R:CommutativeRing and UP:UnivariatePolynomialCategory(a)

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if R has CHARNZ
coerce : R -> %
coerce : Integer -> %
coerce : % -> OutputForm
convert : Vector R -> %
convert : % -> Vector R
coordinates : (%,Vector %) -> Vector R
coordinates : (Vector %,Vector %) -> Matrix R
coordinates : Vector % -> Matrix R
coordinates : % -> Vector R
discriminant : Vector % -> R
discriminant : () -> R
hash : % -> SingleInteger
latex : % -> String
minimalPolynomial : % -> UP if R has FIELD
one? : % -> Boolean
rank : () -> PositiveInteger
regularRepresentation : (%,Vector %) -> Matrix R
regularRepresentation : % -> Matrix R
represents : (Vector R,Vector %) -> %
represents : Vector R -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
trace : % -> R
traceMatrix : Vector % -> Matrix R
traceMatrix : () -> Matrix R
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
```

```
?*? : (NonNegativeInteger,%) -> %
?*? : (R,%) -> %
?*? : (%,R) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

These exports come from (p158) FullyRetractableTo(R)
where R:CommutativeRing

```
coerce : Fraction Integer -> %
    if R has FIELD or R has RETRACT FRAC INT
retract : % -> Integer if R has RETRACT INT
retract : % -> Fraction Integer
    if R has RETRACT FRAC INT
retractIfCan : % -> Union(Fraction Integer,"failed")
    if R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed")
    if R has RETRACT INT
```

These exports come from (p1134) FullyLinearlyExplicitRingOver(R)
where R:CommutativeRing

```
reducedSystem : Matrix % -> Matrix R
reducedSystem :
  (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
reducedSystem :
  (Matrix %,Vector %) ->
     Record(mat: Matrix Integer,vec: Vector Integer)
       if R has LINEXP INT
reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
```

These exports come from (p258) Finite()

```
index : PositiveInteger -> % if R has FINITE
lookup : % -> PositiveInteger if R has FINITE
```

These exports come from (p1413) Field()

```
associates? : (%,%) -> Boolean if R has FIELD
coerce : % -> % if R has FIELD
divide : (%,%) -> Record(quotient: %,remainder: %)
    if R has FIELD
euclideanSize : % -> NonNegativeInteger if R has FIELD
expressIdealMember : (List %,%) -> Union(List %,"failed")
    if R has FIELD
exquo : (%,%) -> Union(%,"failed") if R has FIELD
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
    if R has FIELD
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
    if R has FIELD
factor : % -> Factored % if R has FIELD
gcd : (%,%) -> % if R has FIELD
gcd : List % -> % if R has FIELD
gcdPolynomial :
```

```
   (SparseUnivariatePolynomial %,
    SparseUnivariatePolynomial %) ->
       SparseUnivariatePolynomial % if R has FIELD
inv : % -> % if R has FIELD
lcm : (%,%) -> % if R has FIELD
lcm : List % -> % if R has FIELD
multiEuclidean : (List %,%) -> Union(List %,"failed")
    if R has FIELD
prime? : % -> Boolean if R has FIELD
principalIdeal : List % -> Record(coef: List %,generator: %)
    if R has FIELD
sizeLess? : (%,%) -> Boolean if R has FIELD
squareFree : % -> Factored % if R has FIELD
squareFreePart : % -> % if R has FIELD
unit? : % -> Boolean if R has FIELD
unitCanonical : % -> % if R has FIELD
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if R has FIELD
?/? : (%,%) -> % if R has FIELD
?*? : (%,Fraction Integer) -> % if R has FIELD
?*? : (Fraction Integer,%) -> % if R has FIELD
?**? : (%,Integer) -> % if R has FIELD
?^? : (%,Integer) -> % if R has FIELD
?quo? : (%,%) -> % if R has FIELD
?rem? : (%,%) -> % if R has FIELD
```

These exports come from (p1127) DifferentialExtension(R)
where R:CommutativeRing

```
D : (%,(R -> R)) -> % if R has FIELD
D : (%,(R -> R),NonNegativeInteger) -> % if R has FIELD
D : % -> %
    if and(has(R,DifferentialRing),has(R,Field))
    or R has FFIELDC
D : (%,NonNegativeInteger) -> %
    if and(has(R,DifferentialRing),has(R,Field))
    or R has FFIELDC
D : (%,List Symbol,List NonNegativeInteger) -> %
    if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
D : (%,Symbol,NonNegativeInteger) -> %
    if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
D : (%,List Symbol) -> %
    if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
D : (%,Symbol) -> %
    if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
differentiate : % -> %
    if and(has(R,DifferentialRing),has(R,Field))
    or R has FFIELDC
differentiate : (%,NonNegativeInteger) -> %
    if and(has(R,DifferentialRing),has(R,Field))
    or R has FFIELDC
differentiate : (%,List Symbol) -> %
    if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
differentiate : (%,Symbol,NonNegativeInteger) -> %
    if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
```

```
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
    if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
differentiate : (%,(R -> R),NonNegativeInteger) -> %
    if R has FIELD
differentiate : (%,Symbol) -> %
    if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
```

These exports come from (p1762) FiniteFieldCategory():

```
charthRoot : % -> % if R has FFIELDC
conditionP : Matrix % -> Union(Vector %,"failed")
    if R has FFIELDC
createPrimitiveElement : () -> % if R has FFIELDC
discreteLog : % -> NonNegativeInteger if R has FFIELDC
discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
    if R has FFIELDC
factorsOfCyclicGroupSize : () ->
   List Record(factor: Integer,exponent: Integer)
     if R has FFIELDC
init : () -> % if R has FFIELDC
nextItem : % -> Union(%,"failed") if R has FFIELDC
order : % -> OnePointCompletion PositiveInteger
    if R has FFIELDC
order : % -> PositiveInteger if R has FFIELDC
primeFrobenius : % -> % if R has FFIELDC
primeFrobenius : (%,NonNegativeInteger) -> % if R has FFIELDC
primitive? : % -> Boolean if R has FFIELDC
primitiveElement : () -> % if R has FFIELDC
representationType : () ->
   Union("prime",polynomial,normal,cyclic)
     if R has FFIELDC
tableForDiscreteLogarithm :
   Integer -> Table(PositiveInteger,NonNegativeInteger)
     if R has FFIELDC
```

— MonogenicAlgebra.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#MONOGEN">
MonogenicAlgebra (MONOGEN)</a></h2>
</body>
```

———-

— category MONOGEN MonogenicAlgebra —

```
)abbrev category MONOGEN MonogenicAlgebra
++ Author: Barry Trager
++ Description:
++ A \spadtype{MonogenicAlgebra} is an algebra of finite rank which
++ can be generated by a single element.

MonogenicAlgebra(R,UP) : Category == SIG where
  R : CommutativeRing
```

```
UP : UnivariatePolynomialCategory(R)

FA ==> FramedAlgebra(R,UP)
CR ==> CommutativeRing
CT ==> ConvertibleTo(UP)
FRT ==> FullyRetractableTo(R)
FLERO ==> FullyLinearlyExplicitRingOver(R)

SIG ==> Join(FA,CR,CT,FRT,FLERO) with

    generator : () -> %
      ++ generator() returns the generator for this domain.

    definingPolynomial : () -> UP
      ++ definingPolynomial() returns the minimal polynomial which
      ++ \spad{generator()} satisfies.

    reduce : UP -> %
      ++ reduce(up) converts the univariate polynomial up to an algebra
      ++ element, reducing by the \spad{definingPolynomial()} if necessary.

    convert : UP -> %
      ++ convert(up) converts the univariate polynomial up to an algebra
      ++ element, reducing by the \spad{definingPolynomial()} if necessary.

    lift : % -> UP
      ++ lift(z) returns a minimal degree univariate polynomial up such that
      ++ \spad{z=reduce up}.

    if R has Finite then Finite

    if R has Field then

      Field

      DifferentialExtension R

      reduce : Fraction UP -> Union(%, "failed")
        ++ reduce(frac) converts the fraction frac to an algebra element.

      derivationCoordinates: (Vector %, R -> R) -> Matrix R
        ++ derivationCoordinates(b, ') returns M such that \spad{b' = M b}.

    if R has FiniteFieldCategory then FiniteFieldCategory

 add

   convert(x:%):UP == lift x

   convert(p:UP):% == reduce p

   generator() == reduce monomial(1, 1)$UP

   norm x == resultant(definingPolynomial(), lift x)
```

```
    retract(x:%):R  == retract lift x

    retractIfCan(x:%):Union(R, "failed") == retractIfCan lift x

    basis() ==
      [reduce monomial(1,i)$UP for i in 0..(rank()-1)::NonNegativeInteger]

    characteristicPolynomial(x:%):UP ==
      characteristicPolynomial(x)$CharacteristicPolynomialInMonogenicalAlgebra(R,UP,%)

    if R has Finite then
      size()   == size()$R ** rank()
      random() == represents [random()$R for i in 1..rank()]$Vector(R)

    if R has Field then
      reduce(x:Fraction UP) == reduce(numer x) exquo reduce(denom x)

      differentiate(x:%, d:R -> R) ==
        p := definingPolynomial()
        yprime := - reduce(map(d, p)) / reduce(differentiate p)
        reduce(map(d, lift x)) + yprime * reduce differentiate lift x

      derivationCoordinates(b, d) ==
        coordinates(map(x +-> differentiate(x, d), b), b)

      recip x ==
        (bc := extendedEuclidean(lift x, definingPolynomial(), 1))
                                                  case "failed" => "failed"
        reduce(bc.coef1)
```

———-

— COQ MONOGEN —

```
(* category MONOGEN *)
(*

   convert : % -> UP
   convert(x:%):UP == lift x

   convert : UP -> %
   convert(p:UP):% == reduce p

   generator : () -> %
   generator() == reduce monomial(1, 1)$UP

   norm : % -> R
   norm x == resultant(definingPolynomial(), lift x)

   retract : % -> R
   retract(x:%):R  == retract lift x
```

```
   retractIfCan : % -> Union(R,"failed")
   retractIfCan(x:%):Union(R, "failed") == retractIfCan lift x

   basis : () -> Vector(%)
   basis() ==
     [reduce monomial(1,i)$UP for i in 0..(rank()-1)::NonNegativeInteger]

   characteristicPolynomial : % -> UP
   characteristicPolynomial(x:%):UP ==
     characteristicPolynomial(x)$CharacteristicPolynomialInMonogenicalAlgebra(R,UP,%)

   if R has Finite then

     size : () -> NonNegativeInteger
     size()   == size()$R ** rank()

     random : () -> %
     random() == represents [random()$R for i in 1..rank()]$Vector(R)

   if R has Field then

     reduce : UP -> %
     reduce(x:Fraction UP) == reduce(numer x) exquo reduce(denom x)

     differentiate : (%,(R -> R)) -> %
     differentiate(x:%, d:R -> R) ==
       p := definingPolynomial()
       yprime := - reduce(map(d, p)) / reduce(differentiate p)
       reduce(map(d, lift x)) + yprime * reduce differentiate lift x

     derivationCoordinates : (Vector(%),(R -> R))
     derivationCoordinates(b, d) ==
       coordinates(map(x +-> differentiate(x, d), b), b)

     recip : % -> Union(%,"failed")
     recip x ==
       (bc := extendedEuclidean(lift x, definingPolynomial(), 1))
                                                 case "failed" => "failed"
       reduce(bc.coef1)
*)
```

— MONOGEN.dotabb —

```
"MONOGEN"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MONOGEN"];
"MONOGEN" -> "FRAMALG"
"MONOGEN" -> "COMRING"
"MONOGEN" -> "KONVERT"
"MONOGEN" -> "FRETRCT"
"MONOGEN" -> "FLINEXP"
"MONOGEN" -> "FINITE"
```

```
"MONOGEN" -> "FIELD"
"MONOGEN" -> "DIFEXT"
"MONOGEN" -> "FFIELDC"
```

———-

— **MONOGEN.dotfull** —

```
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MONOGEN"];
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FramedAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "CommutativeRing()"
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "ConvertibleTo(UnivariatePolynomialCategory(CommutativeRing))"
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FullyRetractableTo(a:CommutativeRing)"
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FullyLinearlyExplicitRingOver(a:CommutativeRing)"

"MonogenicAlgebra(a:FRAC(UPOLYC(UFD)),b:UPOLYC(FRAC(UPOLYC(UFD))))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=MONOGEN"];
"MonogenicAlgebra(a:FRAC(UPOLYC(UFD)),b:UPOLYC(FRAC(UPOLYC(UFD))))" ->
    "MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
```

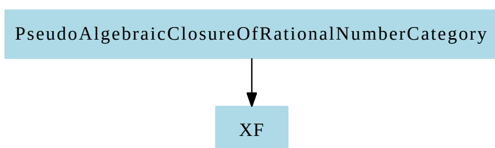
———-

— **MONOGEN.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
 [color=lightblue];
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FRAMALG..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "COMRING..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "KONVERT..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FRETRCT..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FLINEXP..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FINITE..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FIELD..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
```

```
    "DIFEXT..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FFIELDC..."

"FRAMALG..." [color=lightblue];
"COMRING..." [color=lightblue];
"KONVERT..." [color=lightblue];
"FRETRCT..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"FINITE..." [color=lightblue];
"FIELD..." [color=lightblue];
"DIFEXT..." [color=lightblue];
"FFIELDC..." [color=lightblue];
}
```

19.0.239 PseudoAlgebraicClosureOfRationalNumberCategory (PACRATC)



— **PseudoAlgebraicClosureOfRationalNumberCategory.input** —

```
)set break resume
)sys rm -f PseudoAlgebraicClosureOfRationalNumberCategory.output
)spool PseudoAlgebraicClosureOfRationalNumberCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PseudoAlgebraicClosureOfRationalNumberCategory
--R
--R PseudoAlgebraicClosureOfRationalNumberCategory is a category constructor
--R Abbreviation for PseudoAlgebraicClosureOfRationalNumberCategory is PACRATC
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PACRATC
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,Fraction(Integer)) -> %      ?*? : (Fraction(Integer),%) -> %
--R ?*? : (Fraction(Integer),%) -> %      ?*? : (%,Fraction(Integer)) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,Integer) -> %               ?**? : (%,NonNegativeInteger) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
```

```
--R ?-? : (%,%) -> %                       -? : % -> %
--R ?/? : (%,Fraction(Integer)) -> %       ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                 1 : () -> %
--R 0 : () -> %                            ?^? : (%,Integer) -> %
--R ?^? : (%,NonNegativeInteger) -> %      ?^? : (%,PositiveInteger) -> %
--R algebraic? : % -> Boolean              associates? : (%,%) -> Boolean
--R coerce : Fraction(Integer) -> %        coerce : Fraction(Integer) -> %
--R coerce : Integer -> %                  coerce : Fraction(Integer) -> %
--R coerce : % -> %                        coerce : Integer -> %
--R coerce : % -> OutputForm               conjugate : % -> %
--R dimension : () -> CardinalNumber       extDegree : % -> PositiveInteger
--R factor : % -> Factored(%)              fullOutput : % -> OutputForm
--R gcd : List(%) -> %                     gcd : (%,%) -> %
--R ground? : % -> Boolean                 hash : % -> SingleInteger
--R inGroundField? : % -> Boolean          inv : % -> %
--R latex : % -> String                    lcm : List(%) -> %
--R lcm : (%,%) -> %                       maxTower : List(%) -> %
--R one? : % -> Boolean                    previousTower : % -> %
--R prime? : % -> Boolean                  ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")         ?rem? : (%,%) -> %
--R retract : % -> Fraction(Integer)       retract : % -> Fraction(Integer)
--R retract : % -> Integer                 sample : () -> %
--R setTower! : % -> Void                  sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored(%)          squareFreePart : % -> %
--R transcendent? : % -> Boolean           unit? : % -> Boolean
--R unitCanonical : % -> %                 vectorise : (%,%) -> Vector(%)
--R zero? : % -> Boolean                   ?~=? : (%,%) -> Boolean
--R Frobenius : % -> % if Fraction(Integer) has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if Fraction(Integer) has FINITE
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if Fraction(Integer) has CHARNZ or Fraction(Integer) has FINITE
--R definingPolynomial : % -> SparseUnivariatePolynomial(%)
--R definingPolynomial : () -> SparseUnivariatePolynomial(%)
--R degree : % -> OnePointCompletion(PositiveInteger)
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if Fraction(Integer) has CHARNZ or Fraction(Intege
--R distinguishedRootsOf : (SparseUnivariatePolynomial(%),%) -> List(%)
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extensionDegree : () -> OnePointCompletion(PositiveInteger)
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
--R lift : (%,%) -> SparseUnivariatePolynomial(%)
--R lift : % -> SparseUnivariatePolynomial(%)
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed")
--R newElement : (SparseUnivariatePolynomial(%),Symbol) -> %
--R newElement : (SparseUnivariatePolynomial(%),%,Symbol) -> %
--R order : % -> OnePointCompletion(PositiveInteger) if Fraction(Integer) has CHARNZ or Fraction(Integer) has FI
--R primeFrobenius : % -> % if Fraction(Integer) has CHARNZ or Fraction(Integer) has FINITE
--R primeFrobenius : (%,NonNegativeInteger) -> % if Fraction(Integer) has CHARNZ or Fraction(Integer) has FINITE
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %)
```

```
--R reduce : SparseUnivariatePolynomial(%) -> %
--R retractIfCan : % -> Union(Fraction(Integer),"failed")
--R retractIfCan : % -> Union(Fraction(Integer),"failed")
--R retractIfCan : % -> Union(Integer,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--E 1

)spool
)lisp (bye)
```

— **PseudoAlgebraicClosureOfRationalNumberCategory.help** —

```
====================================================================
PseudoAlgebraicClosureOfRationalNumberCategory examples
====================================================================

This category exports the function for the domain
PseudoAlgebraicClosureOfRationalNumber which implement dynamic extension
using the simple notion of tower extensions. A tower extension T of the
ground field K is any sequence of field extension
    (T : K_0, K_1, ..., K_i...,K_n)
where K_0 = K and for i =1,2,...,n, K_i is an extension
of K_{i-1} of degree > 1 and defined by an irreducible polynomial
p(Z) in K_{i-1}.

Two towers
    (T_1: K_01, K_11,...,K_i1,...,K_n1)
and
    (T_2: K_02, K_12,...,K_i2,...,K_n2)
are said to be related if
    T_1 <= T_2 (or T_1 >= T_2),
that is if
    K_i1 = K_i2 for i=1,2,...,n1
(or i=1,2,...,n2). Any algebraic operations defined for several elements
are only defined if all of the concerned elements are comming from
a set of related tour extensions.

See Also:
o )show PseudoAlgebraicClosureOfRationalNumberCategory
```

See:

⇒ "PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory" (PACEXTC) 20.0.242 on page 1927
⇐ "ExtensionField" (XF) 18.0.230 on page 1754

Exports:

0	1	algebraic?
associates?	characteristic	charthRoot
coerce	conjugate	definingPolynomial
degree	dimension	discreteLog
distinguishedRootsOf	divide	euclideanSize
expressIdealMember	exquo	extDegree
extendedEuclidean	extensionDegree	factor
Frobenius	fullOutput	gcd
gcdPolynomial	ground?	hash
inGroundField?	inv	latex
lcm	lift	maxTower
multiEuclidean	newElement	one?
order	previousTower	prime?
primeFrobenius	principalIdeal	?quo?
recip	reduce	?rem?
retract	retractIfCan	sample
setTower!	sizeLess?	squareFree
squareFreePart	subtractIfCan	transcendenceDegree
transcendent?	unit?	unitCanonical
unitNormal	vectorise	zero?
?*?	?**?	?+?
?-?	-?	?/?
?=?	?~=?	?^?

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * " : (D, D)- > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These exports come from (p1588) PseudoAlgebraicClosureOfPerfectFieldCategory():

```
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?**? : (%,Integer) -> %
?**? : (%,PositiveInteger) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
```

```
-? : % -> %
?/? : (%,%) -> %
?=? : (%,%) -> Boolean
1 : () -> %
0 : () -> %
?^? : (%,Integer) -> %
?^? : (%,PositiveInteger) -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : Fraction Integer -> %
coerce : Integer -> %
coerce : % -> %
coerce : % -> OutputForm
conjugate : % -> %
definingPolynomial : % -> SUP %
definingPolynomial : () -> SUP %
distinguishedRootsOf : (SparseUnivariatePolynomial %,%) -> List %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extDegree : % -> PI
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
factor : % -> Factored %
fullOutput : % -> OutputForm
gcd : List % -> %
gcd : (%,%) -> %
gcdPolynomial :
  (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
ground? : % -> Boolean
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
lift : (%,%) -> SUP %
lift : % -> SUP %
maxTower : List % -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
newElement : (SUP %,Symbol) -> %
newElement : (SUP %,%,Symbol) -> %
one? : % -> Boolean
previousTower : % -> %
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
?quo? : (%,%) -> %
recip : % -> Union(%,"failed")
reduce : SUP % -> %
?rem? : (%,%) -> %
sample : () -> %
setTower! : % -> Void
sizeLess? : (%,%) -> Boolean
```

```
squareFree : % -> Factored %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
vectorise : (%,%) -> Vector %
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

These exports come from (p1015) CharacteristicZero():

These exports come from (p110) RetractableTo(Integer):

```
retract : % -> Integer
retractIfCan : % -> Union(Integer,"failed")
```

These exports come from (p110) RetractableTo(Fraction(Integer)):

```
retract : % -> Fraction Integer
retractIfCan : % -> Union(Fraction Integer,"failed")
```

These exports come from (p1754) ExtensionField(Fraction(Integer)):

```
?/? : (%,Fraction Integer) -> %
algebraic? : % -> Boolean
charthRoot : % -> Union(%,"failed")
  if Fraction Integer has CHARNZ or Fraction Integer has FINITE
degree : % -> OnePointCompletion PositiveInteger
dimension : () -> CardinalNumber
discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
  if Fraction Integer has CHARNZ or Fraction Integer has FINITE
extensionDegree : () -> OnePointCompletion PositiveInteger
Frobenius : % -> % if Fraction Integer has FINITE
Frobenius : (%,NonNegativeInteger) -> % if Fraction Integer has FINITE
inGroundField? : % -> Boolean
order : % -> OnePointCompletion PositiveInteger
  if Fraction Integer has CHARNZ or Fraction Integer has FINITE
primeFrobenius : % -> %
  if Fraction Integer has CHARNZ or Fraction Integer has FINITE
transcendent? : % -> Boolean
primeFrobenius : (%,NonNegativeInteger) -> %
  if Fraction Integer has CHARNZ or Fraction Integer has FINITE
retractIfCan : % -> Union(Fraction Integer,"failed")
transcendenceDegree : () -> NonNegativeInteger
```

— PseudoAlgebraicClosureOfRationalNumberCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#PACRATC">
PseudoAlgebraicClosureOfRationalNumberCategory (PACRATC)</a></h2>
</body>
```

———-

— category PACRATC PseudoAlgebraicClosureOfRationalNumberCategory —

```
)abbrev category PACRATC PseudoAlgebraicClosureOfRationalNumberCategory
++ Authors: Gaetan Hache
++ Date Created: feb 1997
++ Description:
++ This category exports the function for the domain
++ PseudoAlgebraicClosureOfRationalNumber
++ which implement dynamic extension using the simple notion of tower
++ extensions. A tower extension T  of the ground
++ field K is any sequence of field extension (T : K_0, K_1, ..., K_i...,K_n)
++ where K_0 = K and for i =1,2,...,n, K_i is an extension
++ of K_{i-1} of degree > 1 and defined by an irreducible polynomial
++ p(Z) in K_{i-1}.
++ Two towers (T_1: K_01, K_11,...,K_i1,...,K_n1)
++ and (T_2: K_02, K_12,...,K_i2,...,K_n2)
++ are said to be related if T_1 <= T_2 (or T_1 >= T_2),
++ that is if K_i1 = K_i2 for i=1,2,...,n1
++ (or i=1,2,...,n2). Any algebraic operations defined for several elements
++ are only defined if all of the concerned elements are comming from
++ a set of related tour extensions.
PseudoAlgebraicClosureOfRationalNumberCategory() : Category == SIG where

  PAC  ==> PseudoAlgebraicClosureOfPerfectFieldCategory
  CZ   ==> CharacteristicZero
  RTI  ==> RetractableTo(Integer)
  RTFI ==> RetractableTo(Fraction(Integer))
  EF   ==> ExtensionField(Fraction(Integer))

  SIG ==> Join(PAC,CZ,RTI,RTFI,EF)
```

———-

— PACRATC.dotabb —

```
"PACRATC" [color=lightblue,href="bookvol10.2.pdf#nameddest=PACRATC"];
"PACRATC" -> "XF"
```

———-

— PACRATC.dotfull —

```
"PseudoAlgebraicClosureOfRationalNumberCategory"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PACRATC"];
"PseudoAlgebraicClosureOfRationalNumberCategory" -> "ExtensionField(F:Field)"
```

———-

— PACRATC.dotpic —

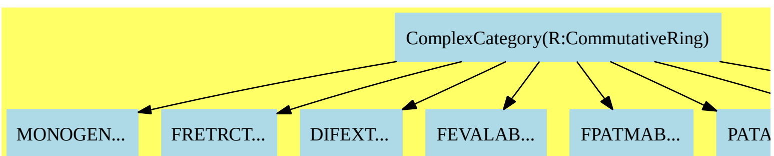
```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"PseudoAlgebraicClosureOfRationalNumberCategory" [color=lightblue];
"PseudoAlgebraicClosureOfRationalNumberCategory" -> "XF"

"XF" [color=lightblue];

}
```

———-

Chapter 20

Category Layer 19

20.0.240 ComplexCategory (COMPCAT)



— ComplexCategory.input —

```
)set break resume
)sys rm -f ComplexCategory.output
)spool ComplexCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show ComplexCategory
--R
--R ComplexCategory(R: CommutativeRing) is a category constructor
--R Abbreviation for ComplexCategory is COMPCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for COMPCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (R,%) -> %                      ?*? : (%,R) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?/? : (%,%) -> % if R has FIELD
--R ?=? : (%,%) -> Boolean                D : (%,(R -> R)) -> %
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
```

```
--R abs : % -> % if R has RNS            acos : % -> % if R has TRANFUN
--R acosh : % -> % if R has TRANFUN      acot : % -> % if R has TRANFUN
--R acoth : % -> % if R has TRANFUN      acsc : % -> % if R has TRANFUN
--R acsch : % -> % if R has TRANFUN      argument : % -> R if R has TRANFUN
--R asec : % -> % if R has TRANFUN       asech : % -> % if R has TRANFUN
--R asin : % -> % if R has TRANFUN       asinh : % -> % if R has TRANFUN
--R atan : % -> % if R has TRANFUN       atanh : % -> % if R has TRANFUN
--R basis : () -> Vector(%)              coerce : R -> %
--R coerce : Integer -> %                coerce : % -> OutputForm
--R complex : (R,R) -> %                 conjugate : % -> %
--R convert : Vector(R) -> %             convert : % -> Vector(R)
--R coordinates : % -> Vector(R)         cos : % -> % if R has TRANFUN
--R cosh : % -> % if R has TRANFUN       cot : % -> % if R has TRANFUN
--R coth : % -> % if R has TRANFUN       csc : % -> % if R has TRANFUN
--R csch : % -> % if R has TRANFUN       differentiate : (%,(R -> R)) -> %
--R discriminant : () -> R               discriminant : Vector(%) -> R
--R exp : % -> % if R has TRANFUN        generator : () -> %
--R hash : % -> SingleInteger            imag : % -> R
--R imaginary : () -> %                  init : () -> % if R has FFIELDC
--R inv : % -> % if R has FIELD          latex : % -> String
--R log : % -> % if R has TRANFUN        map : ((R -> R),%) -> %
--R max : (%,%) -> % if R has ORDSET     min : (%,%) -> % if R has ORDSET
--R norm : % -> R                        one? : % -> Boolean
--R pi : () -> % if R has TRANFUN        random : () -> % if R has FINITE
--R rank : () -> PositiveInteger         real : % -> R
--R recip : % -> Union(%,"failed")       represents : Vector(R) -> %
--R retract : % -> R                     sample : () -> %
--R sec : % -> % if R has TRANFUN        sech : % -> % if R has TRANFUN
--R sin : % -> % if R has TRANFUN        sinh : % -> % if R has TRANFUN
--R tan : % -> % if R has TRANFUN        tanh : % -> % if R has TRANFUN
--R trace : % -> R                       traceMatrix : () -> Matrix(R)
--R zero? : % -> Boolean                 ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction(Integer)) -> % if R has FIELD
--R ?*? : (Fraction(Integer),%) -> % if R has FIELD
--R ?**? : (%,Integer) -> % if R has FIELD
--R ?**? : (%,%) -> % if R has TRANFUN
--R ?**? : (%,Fraction(Integer)) -> % if R has RADCAT and R has TRANFUN
--R ?<? : (%,%) -> Boolean if R has ORDSET
--R ?<=? : (%,%) -> Boolean if R has ORDSET
--R ?>? : (%,%) -> Boolean if R has ORDSET
--R ?>=? : (%,%) -> Boolean if R has ORDSET
--R D : % -> % if and(has(R,Field),has(R,DifferentialRing)) or R has DIFRING or and(has(R,DifferentialRing),has(
--R D : (%,NonNegativeInteger) -> % if and(has(R,Field),has(R,DifferentialRing)) or R has DIFRING or and(has(R,D
--R D : (%,Symbol) -> % if R has PDRING(SYMBOL)
--R D : (%,List(Symbol)) -> % if R has PDRING(SYMBOL)
--R D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING(SYMBOL)
--R D : (%,List(Symbol),List(NonNegativeInteger)) -> % if R has PDRING(SYMBOL)
--R D : (%,(R -> R),NonNegativeInteger) -> %
--R ?^? : (%,Integer) -> % if R has FIELD
--R associates? : (%,%) -> Boolean if R has INTDOM or R has EUCDOM and R has PFECAT
--R characteristic : () -> NonNegativeInteger
--R characteristicPolynomial : % -> SparseUnivariatePolynomial(R)
--R charthRoot : % -> Union(%,"failed") if and(has($,CharacteristicNonZero),AND(has(R,EuclideanDomain),has(R,Pol
--R charthRoot : % -> % if R has FFIELDC
```

```
--R coerce : % -> % if R has INTDOM or R has EUCDOM and R has PFECAT
--R coerce : Fraction(Integer) -> % if R has FIELD or R has RETRACT(FRAC(INT))
--R conditionP : Matrix(%) -> Union(Vector(%),"failed") if and(has($,CharacteristicNonZero),AND(has(R,EuclideanD
--R convert : % -> InputForm if R has KONVERT(INFORM)
--R convert : % -> Complex(DoubleFloat) if R has REAL
--R convert : % -> Complex(Float) if R has REAL
--R convert : % -> Pattern(Float) if R has KONVERT(PATTERN(FLOAT))
--R convert : % -> Pattern(Integer) if R has KONVERT(PATTERN(INT))
--R convert : SparseUnivariatePolynomial(R) -> %
--R convert : % -> SparseUnivariatePolynomial(R)
--R coordinates : Vector(%) -> Matrix(R)
--R coordinates : (Vector(%),Vector(%)) -> Matrix(R)
--R coordinates : (%,Vector(%)) -> Vector(R)
--R createPrimitiveElement : () -> % if R has FFIELDC
--R definingPolynomial : () -> SparseUnivariatePolynomial(R)
--R derivationCoordinates : (Vector(%),(R -> R)) -> Matrix(R) if R has FIELD
--R differentiate : % -> % if and(has(R,Field),has(R,DifferentialRing)) or R has DIFRING or and(has(R,Differenti
--R differentiate : (%,NonNegativeInteger) -> % if and(has(R,Field),has(R,DifferentialRing)) or R has DIFRING or
--R differentiate : (%,Symbol) -> % if R has PDRING(SYMBOL)
--R differentiate : (%,List(Symbol)) -> % if R has PDRING(SYMBOL)
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING(SYMBOL)
--R differentiate : (%,List(Symbol),List(NonNegativeInteger)) -> % if R has PDRING(SYMBOL)
--R differentiate : (%,(R -> R),NonNegativeInteger) -> %
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if R has FFIELDC
--R discreteLog : % -> NonNegativeInteger if R has FFIELDC
--R divide : (%,%) -> Record(quotient: %,remainder: %) if R has EUCDOM
--R ?.? : (%,R) -> % if R has ELTAB(R,R)
--R enumerate : () -> List(%) if R has FINITE
--R euclideanSize : % -> NonNegativeInteger if R has EUCDOM
--R eval : (%,Symbol,R) -> % if R has IEVALAB(SYMBOL,R)
--R eval : (%,List(Symbol),List(R)) -> % if R has IEVALAB(SYMBOL,R)
--R eval : (%,List(Equation(R))) -> % if R has EVALAB(R)
--R eval : (%,Equation(R)) -> % if R has EVALAB(R)
--R eval : (%,R,R) -> % if R has EVALAB(R)
--R eval : (%,List(R),List(R)) -> % if R has EVALAB(R)
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed") if R has EUCDOM
--R exquo : (%,%) -> Union(%,"failed") if R has INTDOM or R has EUCDOM and R has PFECAT
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has EUCDOM
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if R has EUCDOM
--R factor : % -> Factored(%) if R has EUCDOM and R has PFECAT or R has FIELD
--R factorPolynomial : SparseUnivariatePolynomial(%) -> Factored(SparseUnivariatePolynomial(%)) if R has EUCDOM
--R factorSquareFreePolynomial : SparseUnivariatePolynomial(%) -> Factored(SparseUnivariatePolynomial(%)) if R h
--R factorsOfCyclicGroupSize : () -> List(Record(factor: Integer,exponent: Integer)) if R has FFIELDC
--R gcd : (%,%) -> % if R has EUCDOM or R has EUCDOM and R has PFECAT
--R gcd : List(%) -> % if R has EUCDOM or R has EUCDOM and R has PFECAT
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R index : PositiveInteger -> % if R has FINITE
--R lcm : (%,%) -> % if R has EUCDOM or R has EUCDOM and R has PFECAT
--R lcm : List(%) -> % if R has EUCDOM or R has EUCDOM and R has PFECAT
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %) if R has EUCDOM or R has EUCDOM and R has PFECAT
--R lift : % -> SparseUnivariatePolynomial(R)
--R lookup : % -> PositiveInteger if R has FINITE
--R minimalPolynomial : % -> SparseUnivariatePolynomial(R) if R has FIELD
```

```
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed") if R has EUCDOM
--R nextItem : % -> Union(%,"failed") if R has FFIELDC
--R nthRoot : (%,Integer) -> % if R has RADCAT and R has TRANFUN
--R order : % -> OnePointCompletion(PositiveInteger) if R has FFIELDC
--R order : % -> PositiveInteger if R has FFIELDC
--R patternMatch : (%,Pattern(Float),PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%) if R has PATMAB
--R patternMatch : (%,Pattern(Integer),PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%) if R has
--R polarCoordinates : % -> Record(r: R,phi: R) if R has RNS and R has TRANFUN
--R prime? : % -> Boolean if R has EUCDOM and R has PFECAT or R has FIELD
--R primeFrobenius : % -> % if R has FFIELDC
--R primeFrobenius : (%,NonNegativeInteger) -> % if R has FFIELDC
--R primitive? : % -> Boolean if R has FFIELDC
--R primitiveElement : () -> % if R has FFIELDC
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %) if R has EUCDOM
--R ?quo? : (%,%) -> % if R has EUCDOM
--R rational : % -> Fraction(Integer) if R has INS
--R rational? : % -> Boolean if R has INS
--R rationalIfCan : % -> Union(Fraction(Integer),"failed") if R has INS
--R reduce : Fraction(SparseUnivariatePolynomial(R)) -> Union(%,"failed") if R has FIELD
--R reduce : SparseUnivariatePolynomial(R) -> %
--R reducedSystem : Matrix(%) -> Matrix(R)
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(R),vec: Vector(R))
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(Integer),vec: Vector(Integer)) if R has LINEXP(I
--R reducedSystem : Matrix(%) -> Matrix(Integer) if R has LINEXP(INT)
--R regularRepresentation : % -> Matrix(R)
--R regularRepresentation : (%,Vector(%)) -> Matrix(R)
--R ?rem? : (%,%) -> % if R has EUCDOM
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if R has FFIELDC
--R represents : (Vector(R),Vector(%)) -> %
--R retract : % -> Fraction(Integer) if R has RETRACT(FRAC(INT))
--R retract : % -> Integer if R has RETRACT(INT)
--R retractIfCan : % -> Union(R,"failed")
--R retractIfCan : % -> Union(Fraction(Integer),"failed") if R has RETRACT(FRAC(INT))
--R retractIfCan : % -> Union(Integer,"failed") if R has RETRACT(INT)
--R size : () -> NonNegativeInteger if R has FINITE
--R sizeLess? : (%,%) -> Boolean if R has EUCDOM
--R solveLinearPolynomialEquation : (List(SparseUnivariatePolynomial(%)),SparseUnivariatePolynomial(%)) -> Union
--R sqrt : % -> % if R has RADCAT and R has TRANFUN
--R squareFree : % -> Factored(%) if R has EUCDOM and R has PFECAT or R has FIELD
--R squareFreePart : % -> % if R has EUCDOM and R has PFECAT or R has FIELD
--R squareFreePolynomial : SparseUnivariatePolynomial(%) -> Factored(SparseUnivariatePolynomial(%)) if R has EUC
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if R has FFIELDC
--R traceMatrix : Vector(%) -> Matrix(R)
--R unit? : % -> Boolean if R has INTDOM or R has EUCDOM and R has PFECAT
--R unitCanonical : % -> % if R has INTDOM or R has EUCDOM and R has PFECAT
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM or R has EUCDOM and R has PFECAT
--R
--E 1

)spool
)lisp (bye)
```

— ComplexCategory.help —

```
====================================================================
ComplexCategory examples
====================================================================

This category represents the extension of a ring by a square root of -1.

See Also:
o )show ComplexCategory
```

See:

⇐ "CommutativeRing" (COMRING) 10.0.161 on page 1019
⇐ "DifferentialExtension" (DIFEXT) 11.0.177 on page 1127
⇐ "FullyEvalableOver" (FEVALAB) 4.0.67 on page 246
⇐ "FullyPatternMatchable" (FPATMAB) 3.0.51 on page 164
⇐ "FullyRetractableTo" (FRETRCT) 3.0.50 on page 158
⇐ "MonogenicAlgebra" (MONOGEN) 19.0.238 on page 1847
⇐ "Patternable" (PATAB) 2.0.34 on page 100
⇐ "FullyLinearlyExplicitRingOver" (FLINEXP) 11.0.178 on page 1134

Exports:

0	1	abs
acos	acosh	acot
acoth	acsc	acsch
argument	asec	asech
asin	asinh	associates?
atan	atanh	basis
characteristic	characteristicPolynomial	charthRoot
coerce	complex	conditionP
conjugate	convert	coordinates
cos	cosh	cot
coth	createPrimitiveElement	csc
csch	D	definingPolynomial
derivationCoordinates	differentiate	discreteLog
discriminant	divide	euclideanSize
eval	exp	expressIdealMember
exquo	extendedEuclidean	factor
factorPolynomial	factorSquareFreePolynomial	factorsOfCyclicGroupSize
gcd	gcdPolynomial	generator
hash	imag	imaginary
index	init	inv
latex	lcm	lift
log	lookup	map
max	min	minimalPolynomial
multiEuclidean	nextItem	norm
nthRoot	one?	order
patternMatch	pi	polarCoordinates
prime?	primeFrobenius	primitive?
primitiveElement	principalIdeal	random
rank	rational	rational?
rationalIfCan	real	recip
reduce	reducedSystem	regularRepresentation
represents	representationType	retract
retractIfCan	sample	sec
sech	sin	sinh
size	sizeLess?	solveLinearPolynomialEquation
sqrt	squareFree	squareFreePart
squareFreePolynomial	subtractIfCan	tableForDiscreteLogarithm
tan	tanh	trace
traceMatrix	unit?	unitCanonical
unitNormal	zero?	?*?
?**?	?+?	?-?
-?	?=?	?^?
?~=?	?/?	?<?
?<=?	?>?	?>=?
?.?	?quo?	?rem?

Attributes Exported:

- if #1 has multiplicativeValuation then multiplicativeValuation where **multiplicative-Valuation** implies `euclideanSize(a*b)=euclideanSize(a)*euclideanSize(b)`.

- if #1 has additiveValuation then additiveValuation where **additiveValuation** implies `euclideanSize(a*b)=euclideanSize(a)+euclideanSize(b)`.
- if #1 has Field then canonicalsClosed where **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- if #1 has Field then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if #1 has IntegralDomain or #1 has both EuclideanDomain and PolynomialFactorizationExplicit then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation $" * "\ :\ (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **complex** means that this domain has $\sqrt{-1}$
- **nil**

These are directly exported but not implemented:

```
abs : % -> % if R has RNS
argument : % -> R if R has TRANFUN
complex : (R,R) -> %
conjugate : % -> %
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
imag : % -> R
imaginary : () -> %
norm : % -> R
polarCoordinates : % -> Record(r: R,phi: R) if R has RNS and R has TRANFUN
rational : % -> Fraction Integer if R has INS
rational? : % -> Boolean if R has INS
rationalIfCan : % -> Union(Fraction Integer,"failed") if R has INS
real : % -> R
```

These are implemented by this category:

```
acos : % -> % if R has TRANFUN
acosh : % -> % if R has TRANFUN
asin : % -> % if R has TRANFUN
asinh : % -> % if R has TRANFUN
atan : % -> % if R has TRANFUN
atanh : % -> % if R has TRANFUN
characteristic : () -> NonNegativeInteger
characteristicPolynomial : % -> SparseUnivariatePolynomial R
coerce : % -> OutputForm
convert : % -> Complex DoubleFloat if R has REAL
convert : % -> Complex Float if R has REAL
convert : % -> InputForm if R has KONVERT INFORM
convert : % -> Pattern Float if R has KONVERT PATTERN FLOAT
convert : % -> Pattern Integer if R has KONVERT PATTERN INT
```

```
coordinates : % -> Vector R
coordinates : (%,Vector %) -> Vector R
cos : % -> % if R has TRANFUN
cosh : % -> % if R has TRANFUN
definingPolynomial : () -> SparseUnivariatePolynomial R
differentiate : (%,(R -> R)) -> %
discriminant : () -> R
divide : (%,%) -> Record(quotient: %,remainder: %) if R has EUCDOM
euclideanSize : % -> NonNegativeInteger if R has EUCDOM
exp : % -> % if R has TRANFUN
exquo : (%,%) -> Union(%,"failed") if R has INTDOM or R has EUCDOM and R has PFECAT
factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if R has EUCDOM and R
factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if R has EUC
inv : % -> % if R has FIELD
lift : % -> SparseUnivariatePolynomial R
log : % -> % if R has TRANFUN
map : ((R -> R),%) -> %
minimalPolynomial : % -> SparseUnivariatePolynomial R if R has FIELD
patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(Integer,%) if R has PATM
patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%) if R has PATMAB FLO
pi : () -> % if R has TRANFUN
rank : () -> PositiveInteger
recip : % -> Union(%,"failed")
reduce : SparseUnivariatePolynomial R -> %
reducedSystem : Matrix % -> Matrix R
reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
retract : % -> R
retractIfCan : % -> Union(R,"failed")
sin : % -> % if R has TRANFUN
sinh : % -> % if R has TRANFUN
solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> Union(List
tan : % -> % if R has TRANFUN
tanh : % -> % if R has TRANFUN
trace : % -> R
unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM or R has EUCDOM and R has PFECAT
?=? : (%,%) -> Boolean
?+? : (%,%) -> %
-? : % -> %
?*? : (%,%) -> %
?*? : (R,%) -> %
?*? : (Integer,%) -> %
?<? : (%,%) -> Boolean if R has ORDSET
?**? : (%,Fraction Integer) -> % if R has RADCAT and R has TRANFUN
?rem? : (%,%) -> % if R has EUCDOM
?quo? : (%,%) -> % if R has EUCDOM
```

These exports come from (p1847) MonogenicAlgebra(R,S)
where R:CommutativeRing and S:SparseUnivariatePolynomial(R):

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean if R has INTDOM or R has EUCDOM and R has PFECAT
basis : () -> Vector %
charthRoot : % -> Union(%,"failed") if and(has($,CharacteristicNonZero),AND(has(R,EuclideanDomain),has(R,Polyno
charthRoot : % -> % if R has FFIELDC
```

```
coerce : R -> %
coerce : % -> % if R has INTDOM or R has EUCDOM and R has PFECAT
coerce : Integer -> %
coerce : Fraction Integer -> % if R has FIELD or R has RETRACT FRAC INT
conditionP : Matrix % -> Union(Vector %,"failed") if and(has($,CharacteristicNonZero),AND(has(R,EuclideanDomain
convert : SparseUnivariatePolynomial R -> %
convert : Vector R -> %
convert : % -> Vector R
convert : % -> SparseUnivariatePolynomial R
coordinates : Vector % -> Matrix R
coordinates : (Vector %,Vector %) -> Matrix R
createPrimitiveElement : () -> % if R has FFIELDC
D : (%,(R -> R)) -> %
D : (%,(R -> R),NonNegativeInteger) -> %
D : % -> % if and(has(R,Field),has(R,DifferentialRing)) or R has DIFRING or and(has(R,DifferentialRing),has(R,F
D : (%,NonNegativeInteger) -> % if and(has(R,Field),has(R,DifferentialRing)) or R has DIFRING or and(has(R,Diff
D : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
D : (%,List Symbol) -> % if R has PDRING SYMBOL
D : (%,Symbol) -> % if R has PDRING SYMBOL
derivationCoordinates : (Vector %,(R -> R)) -> Matrix R if R has FIELD
differentiate : (%,NonNegativeInteger) -> % if and(has(R,Field),has(R,DifferentialRing)) or R has DIFRING or an
differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL
differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
differentiate : (%,(R -> R),NonNegativeInteger) -> %
differentiate : % -> % if and(has(R,Field),has(R,DifferentialRing)) or R has DIFRING or and(has(R,DifferentialR
differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if R has FFIELDC
discreteLog : % -> NonNegativeInteger if R has FFIELDC
discriminant : Vector % -> R
expressIdealMember : (List %,%) -> Union(List %,"failed") if R has EUCDOM
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has EUCDOM
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if R has EUCDOM
factor : % -> Factored % if R has EUCDOM and R has PFECAT or R has FIELD
factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer) if R has FFIELDC
gcd : (%,%) -> % if R has EUCDOM or R has EUCDOM and R has PFECAT
gcd : List % -> % if R has EUCDOM or R has EUCDOM and R has PFECAT
gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial % if
generator : () -> %
hash : % -> SingleInteger
index : PositiveInteger -> % if R has FINITE
init : () -> % if R has FFIELDC
latex : % -> String
lcm : (%,%) -> % if R has EUCDOM or R has EUCDOM and R has PFECAT
lcm : List % -> % if R has EUCDOM or R has EUCDOM and R has PFECAT
lookup : % -> PositiveInteger if R has FINITE
multiEuclidean : (List %,%) -> Union(List %,"failed") if R has EUCDOM
nextItem : % -> Union(%,"failed") if R has FFIELDC
one? : % -> Boolean
order : % -> OnePointCompletion PositiveInteger if R has FFIELDC
order : % -> PositiveInteger if R has FFIELDC
prime? : % -> Boolean if R has EUCDOM and R has PFECAT or R has FIELD
primeFrobenius : % -> % if R has FFIELDC
```

```
primeFrobenius : (%,NonNegativeInteger) -> % if R has FFIELDC
primitive? : % -> Boolean if R has FFIELDC
primitiveElement : () -> % if R has FFIELDC
principalIdeal : List % -> Record(coef: List %,generator: %) if R has EUCDOM
random : () -> % if R has FINITE
reduce : Fraction SparseUnivariatePolynomial R -> Union(%,"failed") if R has FIELD
reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if R has LINEXP INT
regularRepresentation : % -> Matrix R
regularRepresentation : (%,Vector %) -> Matrix R
representationType : () -> Union("prime",polynomial,normal,cyclic) if R has FFIELDC
represents : (Vector R,Vector %) -> %
represents : Vector R -> %
retract : % -> Fraction Integer if R has RETRACT FRAC INT
retract : % -> Integer if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
sample : () -> %
size : () -> NonNegativeInteger if R has FINITE
sizeLess? : (%,%) -> Boolean if R has EUCDOM
subtractIfCan : (%,%) -> Union(%,"failed")
squareFree : % -> Factored % if R has EUCDOM and R has PFECAT or R has FIELD
squareFreePart : % -> % if R has EUCDOM and R has PFECAT or R has FIELD
tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if R has FFIELDC
traceMatrix : () -> Matrix R
traceMatrix : Vector % -> Matrix R
unit? : % -> Boolean if R has INTDOM or R has EUCDOM and R has PFECAT
unitCanonical : % -> % if R has INTDOM or R has EUCDOM and R has PFECAT
zero? : % -> Boolean
?/? : (%,%) -> % if R has FIELD
?*? : (%,Fraction Integer) -> % if R has FIELD
?*? : (Fraction Integer,%) -> % if R has FIELD
?**? : (%,Integer) -> % if R has FIELD
?^? : (%,Integer) -> % if R has FIELD
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (%,R) -> %
?-? : (%,%) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p246) FullyEvalableOver(R:CommutativeRing)

```
?.? : (%,R) -> % if R has ELTAB(R,R)
eval : (%,Equation R) -> % if R has EVALAB R
eval : (%,List Symbol,List R) -> % if R has IEVALAB(SYMBOL,R)
eval : (%,List Equation R) -> % if R has EVALAB R
eval : (%,R,R) -> % if R has EVALAB R
eval : (%,List R,List R) -> % if R has EVALAB R
eval : (%,Symbol,R) -> % if R has IEVALAB(SYMBOL,R)
```

These exports come from (p1019) CommutativeRing():

```
?~=? : (%,%) -> Boolean
```

These exports come from (p326) OrderedSet():

```
max : (%,%) -> % if R has ORDSET
min : (%,%) -> % if R has ORDSET
?<=? : (%,%) -> Boolean if R has ORDSET
?>? : (%,%) -> Boolean if R has ORDSET
?>=? : (%,%) -> Boolean if R has ORDSET
```

These exports come from (p106) RadicalCategory():

```
nthRoot : (%,Integer) -> % if R has RADCAT and R has TRANFUN
sqrt : % -> % if R has RADCAT and R has TRANFUN
```

These exports come from (p192) TranscendentalFunctionCategory():

```
acot : % -> % if R has TRANFUN
acoth : % -> % if R has TRANFUN
acsc : % -> % if R has TRANFUN
acsch : % -> % if R has TRANFUN
asec : % -> % if R has TRANFUN
asech : % -> % if R has TRANFUN
cot : % -> % if R has TRANFUN
coth : % -> % if R has TRANFUN
csc : % -> % if R has TRANFUN
csch : % -> % if R has TRANFUN
sec : % -> % if R has TRANFUN
sech : % -> % if R has TRANFUN
?**? : (%,%) -> % if R has TRANFUN
```

These exports come from (p1393) PolynomialFactorizationExplicit():

```
squareFreePolynomial : SparseUnivariatePolynomial % ->
  Factored SparseUnivariatePolynomial % if R has EUCDOM and R has PFECAT
```

— ComplexCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#COMPCAT">
ComplexCategory (COMPCAT)</a></h2>
</body>
```

— category COMPCAT ComplexCategory —

```
)abbrev category COMPCAT ComplexCategory
++ Date Last Updated: 18 March 1994
++ Description:
++ This category represents the extension of a ring by a square root of -1.

ComplexCategory(R) : Category == SIG where
  R : CommutativeRing

  MA    ==> MonogenicAlgebra(R, SparseUnivariatePolynomial(R))
  FRT   ==> FullyRetractableTo(R)
  DE    ==> DifferentialExtension(R)
  FEO   ==> FullyEvalableOver(R)
```

```
FPM   ==> FullyPatternMatchable(R)
P     ==> Patternable(R)
FLERO ==> FullyLinearlyExplicitRingOver(R)
CR    ==> CommutativeRing

SIG ==> Join(MA,FRT,DE,FEO,FPM,P,FLERO,CR) with

   complex
     ++ indicates that % has sqrt(-1)

   imaginary : () -> %
     ++ imaginary() = sqrt(-1) = %i.

   conjugate : % -> %
     ++ conjugate(x + %i y) returns x - %i y.

   complex : (R, R) -> %
     ++ complex(x,y) constructs x + %i*y.

   imag : % -> R
     ++ imag(x) returns imaginary part of x.

   real : % -> R
     ++ real(x) returns real part of x.

   norm : % -> R
     ++ norm(x) returns x * conjugate(x)

   if R has OrderedSet then OrderedSet

   if R has IntegralDomain then

     IntegralDomain

     _exquo : (%,R) -> Union(%,"failed")
       ++ exquo(x, r) returns the exact quotient of x by r, or
       ++ "failed" if r does not divide x exactly.

   if R has EuclideanDomain then EuclideanDomain

   if R has multiplicativeValuation then multiplicativeValuation

   if R has additiveValuation then additiveValuation

   if R has Field then        -- this is a lie; we must know that
     Field                    -- x**2+1 is irreducible in R

   if R has ConvertibleTo InputForm then ConvertibleTo InputForm

   if R has CharacteristicZero then CharacteristicZero

   if R has CharacteristicNonZero then CharacteristicNonZero

   if R has RealConstant then
```

```
    ConvertibleTo Complex DoubleFloat
    ConvertibleTo Complex Float

  if R has RealNumberSystem then

    abs : % -> %
      ++ abs(x) returns the absolute value of x = sqrt(norm(x)).

  if R has TranscendentalFunctionCategory then

    TranscendentalFunctionCategory

    argument : % -> R
      ++ argument(x) returns the angle made by (0,1) and (0,x).

    if R has RadicalCategory then RadicalCategory

    if R has RealNumberSystem then

      polarCoordinates : % -> Record(r:R, phi:R)
        ++ polarCoordinates(x) returns (r, phi) such that
        ++ x = r * exp(%i * phi).

  if R has IntegerNumberSystem then

    rational? : % -> Boolean
      ++ rational?(x) tests if x is a rational number.

    rational : % -> Fraction Integer
      ++ rational(x) returns x as a rational number.
      ++ Error: if x is not a rational number.

    rationalIfCan : % -> Union(Fraction Integer, "failed")
      ++ rationalIfCan(x) returns x as a rational number, or
      ++ "failed" if x is not a rational number.

  if R has PolynomialFactorizationExplicit and R has EuclideanDomain then
     PolynomialFactorizationExplicit

add

    import MatrixCategoryFunctions2(%, Vector %, Vector %, Matrix %,
                                    R, Vector R, Vector R, Matrix R)
    SUP ==> SparseUnivariatePolynomial
    characteristicPolynomial x ==
       v := monomial(1,1)$SUP(R)
       v**2 - trace(x)*v**1 + norm(x)*v**0
    if R has PolynomialFactorizationExplicit and R has EuclideanDomain then
       SupR ==> SparseUnivariatePolynomial R
       Sup ==> SparseUnivariatePolynomial %
       import FactoredFunctionUtilities Sup
       import UnivariatePolynomialCategoryFunctions2(R,SupR,%,Sup)
       import UnivariatePolynomialCategoryFunctions2(%,Sup,R,SupR)
       pp,qq:Sup
```

```
    if R has IntegerNumberSystem then
       myNextPrime: (%,NonNegativeInteger) -> %
       myNextPrime(x,n ) == -- prime is actually in R, and = 3(mod 4)
          xr:=real(x)-4::R
          while not prime? xr repeat
             xr:=xr-4::R
          complex(xr,0)
       --!TT:=InnerModularGcd(%,Sup,32719 :: %,myNextPrime)
       --!gcdPolynomial(pp,qq) == modularGcd(pp,qq)$TT
       solveLinearPolynomialEquation(lp:List Sup,p:Sup) ==
         solveLinearPolynomialEquation(lp,p)$ComplexIntegerSolveLinearPolynomialEquation(R,%)

    normPolynomial: Sup -> SupR
    normPolynomial pp ==
        map(z+->retract(z@%)::R,pp * map(conjugate,pp))

    factorPolynomial pp ==
        refine(squareFree pp,factorSquareFreePolynomial)

    factorSquareFreePolynomial pp ==
        pnorm:=normPolynomial pp
        k:R:=0
        while degree gcd(pnorm,differentiate pnorm)>0 repeat
           k:=k+1
           pnorm:=normPolynomial
                    elt(pp,monomial(1,1)-monomial(complex(0,k),0))
        fR:=factorSquareFreePolynomial pnorm
        numberOfFactors fR = 1 =>
            makeFR(1,[["irred",pp,1]])
        lF:List Record(flg:Union("nil", "sqfr", "irred", "prime"),
                       fctr:Sup, xpnt:Integer):=[]
        for u in factorList fR repeat
            p1:=map((z:R):%+->z::%,u.fctr)
            if not zero? k then
               p1:=elt(p1,monomial(1,1)+monomial(complex(0,k),0))
            p2:=gcd(p1,pp)
            lF:=cons(["irred",p2,1],lF)
            pp:=(pp exquo p2)::Sup
        makeFR(pp,lF)

 rank() == 2

 discriminant() == -4 :: R

 norm x == real(x)**2 + imag(x)**2

 trace x == 2 * real x

 imaginary() == complex(0, 1)

 conjugate x == complex(real x, - imag x)

 characteristic() == characteristic()$R
```

```
map(fn, x) == complex(fn real x, fn imag x)

x = y == real(x) = real(y) and imag(x) = imag(y)

x + y == complex(real x + real y, imag x + imag y)

- x == complex(- real x, - imag x)

r:R * x:% == complex(r * real x, r * imag x)

coordinates(x:%) == [real x, imag x]

n:Integer * x:%  == complex(n * real x, n * imag x)

differentiate(x:%, d:R -> R) == complex(d real x, d imag x)

definingPolynomial() ==
  monomial(1,2)$(SUP R) + monomial(1,0)$(SUP R)

reduce(pol:SUP R) ==
  part:= (monicDivide(pol,definingPolynomial())).remainder
  complex(coefficient(part,0),coefficient(part,1))

lift(x) == monomial(real x,0)$(SUP R)+monomial(imag x,1)$(SUP R)

minimalPolynomial x ==
  zero? imag x =>
    monomial(1, 1)$(SUP R) - monomial(real x, 0)$(SUP R)
  monomial(1, 2)$(SUP R) - monomial(trace x, 1)$(SUP R)
    + monomial(norm x, 0)$(SUP R)

coordinates(x:%, v:Vector %):Vector(R) ==
  ra := real(a := v(minIndex v))
  rb := real(b := v(maxIndex v))
  (#v ^= 2) or
    ((d := recip(ra * (ib := imag b) - (ia := imag a) * rb))
      case "failed") =>error "coordinates: vector is not a basis"
  rx := real x
  ix := imag x
  [d::R * (rx * ib - ix * rb), d::R * (ra * ix - ia * rx)]

coerce(x:%):OutputForm ==
  re := (r := real x)::OutputForm
  ie := (i := imag x)::OutputForm
  zero? i => re
  outi := "%i"::Symbol::OutputForm
  ip :=
    (i = 1) => outi
    ((-i) = 1) => -outi
    ie * outi
  zero? r => ip
  re + ip

retract(x:%):R ==
```

```
  not zero?(imag x) =>
    error "Imaginary part is nonzero. Cannot retract."
  real x

retractIfCan(x:%):Union(R, "failed") ==
  not zero?(imag x) => "failed"
  real x

x:% * y:% ==
  complex(real x * real y - imag x * imag y,
           imag x * real y + imag y * real x)

reducedSystem(m:Matrix %):Matrix R ==
  vertConcat(map(real, m), map(imag, m))

reducedSystem(m:Matrix %, v:Vector %):
 Record(mat:Matrix R, vec:Vector R) ==
  rh := reducedSystem(v::Matrix %)@Matrix(R)
  [reducedSystem(m)@Matrix(R), column(rh, minColIndex rh)]

if R has RealNumberSystem then
  abs(x:%):%        == (sqrt norm x)::%

if R has RealConstant then
  convert(x:%):Complex(DoubleFloat) ==
   complex(convert(real x)@DoubleFloat,convert(imag x)@DoubleFloat)

  convert(x:%):Complex(Float) ==
    complex(convert(real x)@Float, convert(imag x)@Float)

if R has ConvertibleTo InputForm then
  convert(x:%):InputForm ==
    convert([convert("complex"::Symbol), convert real x,
             convert imag x]$List(InputForm))@InputForm

if R has ConvertibleTo Pattern Integer then
  convert(x:%):Pattern Integer ==
     convert(x)$ComplexPattern(Integer, R, %)
if R has ConvertibleTo Pattern Float then
  convert(x:%):Pattern Float ==
     convert(x)$ComplexPattern(Float, R, %)

if R has PatternMatchable Integer then
  patternMatch(x:%, p:Pattern Integer,
   l:PatternMatchResult(Integer, %)) ==
    patternMatch(x, p, l)$ComplexPatternMatch(Integer, R, %)

if R has PatternMatchable Float then
  patternMatch(x:%, p:Pattern Float,
   l:PatternMatchResult(Float, %)) ==
    patternMatch(x, p, l)$ComplexPatternMatch(Float, R, %)


if R has OrderedSet then
```

```
      x < y ==
        real x = real y => imag x < imag y
        real x < real y

    if R has IntegerNumberSystem then
      rational? x == zero? imag x

      rational x ==
        zero? imag x => rational real x
        error "Not a rational number"

      rationalIfCan x ==
        zero? imag x => rational real x
        "failed"

    if R has Field then
      inv x ==
        zero? imag x => (inv real x)::%
        r := norm x
        complex(real(x) / r, - imag(x) / r)

    if R has IntegralDomain then
      _exquo(x:%, r:R) ==
        (r = 1) => x
        (r1 := real(x) exquo r) case "failed" => "failed"
        (r2 := imag(x) exquo r) case "failed" => "failed"
        complex(r1, r2)

      _exquo(x:%, y:%) ==
        zero? imag y => x exquo real y
        x * conjugate(y) exquo norm(y)

      recip(x:%) == 1 exquo x

      if R has OrderedRing then
        unitNormal x ==
          zero? x => [1,x,1]
          (u := recip x) case % => [x, 1, u]
          zero? real x =>
            c := unitNormal imag x
            [complex(0, c.unit), (c.associate * imag x)::%,
                                        complex(0, - c.associate)]
          c := unitNormal real x
          x := c.associate * x
          imag x < 0 =>
            x := complex(- imag x, real x)
            [- c.unit * imaginary(), x, c.associate * imaginary()]
          [c.unit ::%, x, c.associate ::%]
      else
        unitNormal x ==
          zero? x => [1,x,1]
          (u := recip x) case % => [x, 1, u]
          zero? real x =>
            c := unitNormal imag x
```

```
          [complex(0, c.unit), (c.associate * imag x)::%,
                                          complex(0, - c.associate)]
         c := unitNormal real x
         x := c.associate * x
         [c.unit ::%, x, c.associate ::%]

    if R has EuclideanDomain then
       if R has additiveValuation then
           euclideanSize x == max(euclideanSize real x,
                                  euclideanSize imag x)
       else
           euclideanSize x == euclideanSize(real(x)**2 + imag(x)**2)$R
       if R has IntegerNumberSystem then
         x rem y ==
           zero? imag y =>
             yr:=real y
             complex(symmetricRemainder(real(x), yr),
                     symmetricRemainder(imag(x), yr))
           divide(x, y).remainder
         x quo y ==
           zero? imag y =>
             yr:= real y
             xr:= real x
             xi:= imag x
             complex((xr-symmetricRemainder(xr,yr)) quo yr,
                     (xi-symmetricRemainder(xi,yr)) quo yr)
           divide(x, y).quotient

       else
         x rem y ==
           zero? imag y =>
             yr:=real y
             complex(real(x) rem yr,imag(x) rem yr)
           divide(x, y).remainder
         x quo y ==
           zero? imag y => complex(real x quo real y,imag x quo real y)
           divide(x, y).quotient

       divide(x, y) ==
         r := norm y
         y1 := conjugate y
         xx := x * y1
         x1 := real(xx) rem r
         a  := x1
         if x1^=0 and sizeLess?(r, 2 * x1) then
           a := x1 - r
           if sizeLess?(x1, a) then a := x1 + r
         x2 := imag(xx) rem r
         b  := x2
         if x2^=0 and sizeLess?(r, 2 * x2) then
           b := x2 - r
           if sizeLess?(x2, b) then b := x2 + r
         y1 := (complex(a, b) exquo y1)::%
         [((x - y1) exquo y)::%, y1]
```

```
if R has TranscendentalFunctionCategory then
  half := recip(2::R)::R

  if R has RealNumberSystem then
    atan2loc(y: R, x: R): R ==
        pi1 := pi()$R
        pi2 := pi1 * half
        x = 0 => if y >= 0 then pi2 else -pi2

        -- Atan in (-pi/2,pi/2]
        theta := atan(y * recip(x)::R)
        while theta <= -pi2 repeat theta := theta + pi1
        while theta >   pi2 repeat theta := theta - pi1

        x >= 0 => theta      -- I or IV

        if y >= 0 then
            theta + pi1      -- II
        else
            theta - pi1      -- III

    argument x == atan2loc(imag x, real x)

  else
    -- Not ordered so dictate two quadrants
    argument x ==
      zero? real x => pi()$R * half
      atan(imag(x) * recip(real x)::R)

  pi()  == pi()$R :: %

  if R is DoubleFloat then
     stoc ==> S_-TO_-C$Lisp
     ctos ==> C_-TO_-S$Lisp

     exp   x == ctos EXP(stoc x)$Lisp
     log   x == ctos LOG(stoc x)$Lisp

     sin   x == ctos SIN(stoc x)$Lisp
     cos   x == ctos COS(stoc x)$Lisp
     tan   x == ctos TAN(stoc x)$Lisp
     asin  x == ctos ASIN(stoc x)$Lisp
     acos  x == ctos ACOS(stoc x)$Lisp
     atan  x == ctos ATAN(stoc x)$Lisp

     sinh  x == ctos SINH(stoc x)$Lisp
     cosh  x == ctos COSH(stoc x)$Lisp
     tanh  x == ctos TANH(stoc x)$Lisp
     asinh x == ctos ASINH(stoc x)$Lisp
     acosh x == ctos ACOSH(stoc x)$Lisp
     atanh x == ctos ATANH(stoc x)$Lisp

  else
```

```
      atan x ==
        ix := imaginary()*x
        - imaginary() * half * (log(1 + ix) - log(1 - ix))

      log x ==
        complex(log(norm x) * half, argument x)

      exp x ==
        e := exp real x
        complex(e * cos imag x, e * sin imag x)

      cos x ==
        e := exp(imaginary() * x)
        half * (e + recip(e)::%)

      sin x ==
        e := exp(imaginary() * x)
        - imaginary() * half * (e - recip(e)::%)

    if R has RealNumberSystem then
      polarCoordinates x ==
        [sqrt norm x, (negative?(t := argument x) => t + 2 * pi(); t)]

      x:% ** q:Fraction(Integer) ==
        zero? q =>
          zero? x => error "0 ** 0 is undefined"
          1
        zero? x => 0
        rx := real x
        zero? imag x and positive? rx => (rx ** q)::%
        zero? imag x and denom q = 2 => complex(0, (-rx)**q)
        ax := sqrt(norm x) ** q
        tx := q::R * argument x
        complex(ax * cos tx, ax * sin tx)

    else if R has RadicalCategory then
      x:% ** q:Fraction(Integer) ==
        zero? q =>
          zero? x => error "0 ** 0 is undefined"
          1
        r := real x
        zero?(i := imag x) => (r ** q)::%
        t := numer(q) * recip(denom(q)::R)::R * argument x
        e:R :=
          zero? r => i ** q
          norm(x) ** (q / (2::Fraction(Integer)))
        complex(e * cos t, e * sin t)
```

— COQ COMPCAT —

```
(* category COMPCAT *)
```

```
(*
        import MatrixCategoryFunctions2(%, Vector %, Vector %, Matrix %,
                                        R, Vector R, Vector R, Matrix R)

        SUP ==> SparseUnivariatePolynomial

        characteristicPolynomial : % -> SparseUnivariatePolynomial(R)
        characteristicPolynomial x ==
           v := monomial(1,1)$SUP(R)
           v**2 - trace(x)*v**1 + norm(x)*v**0

        if R has PolynomialFactorizationExplicit and R has EuclideanDomain then

           SupR ==> SparseUnivariatePolynomial R

           Sup ==> SparseUnivariatePolynomial %

           import FactoredFunctionUtilities Sup
           import UnivariatePolynomialCategoryFunctions2(R,SupR,%,Sup)
           import UnivariatePolynomialCategoryFunctions2(%,Sup,R,SupR)

           pp,qq:Sup

           if R has IntegerNumberSystem then

              myNextPrime: (%,NonNegativeInteger) -> %
              myNextPrime(x,n ) == -- prime is actually in R, and = 3(mod 4)
                 xr:=real(x)-4::R
                 while not prime? xr repeat
                    xr:=xr-4::R
                 complex(xr,0)
              --!TT:=InnerModularGcd(%,Sup,32719 :: %,myNextPrime)
              --!gcdPolynomial(pp,qq) == modularGcd(pp,qq)$TT

              solveLinearPolynomialEquation :
                (List(SparseUnivariatePolynomial(%)),
                 SparseUnivariatePolynomial(%)) ->
                    Union(List(SparseUnivariatePolynomial(%)),"failed")
              solveLinearPolynomialEquation(lp:List Sup,p:Sup) ==
                solveLinearPolynomialEquation(lp,p)$ComplexIntegerSolveLinearPolynomialEquation(R,%)

           normPolynomial: Sup -> SupR
           normPolynomial pp ==
               map(z+->retract(z@%)::R,pp * map(conjugate,pp))

           factorPolynomial : SparseUnivariatePolynomial(%) ->
              Factored(SparseUnivariatePolynomial(%))
           factorPolynomial pp ==
               refine(squareFree pp,factorSquareFreePolynomial)

           factorSquareFreePolynomial : SparseUnivariatePolynomial(%) ->
              Factored(SparseUnivariatePolynomial(%))
           factorSquareFreePolynomial pp ==
               pnorm:=normPolynomial pp
```

```
        k:R:=0
        while degree gcd(pnorm,differentiate pnorm)>0 repeat
           k:=k+1
           pnorm:=normPolynomial
                    elt(pp,monomial(1,1)-monomial(complex(0,k),0))
        fR:=factorSquareFreePolynomial pnorm
        numberOfFactors fR = 1 =>
            makeFR(1,[["irred",pp,1]])
        lF:List Record(flg:Union("nil", "sqfr", "irred", "prime"),
                       fctr:Sup, xpnt:Integer):=[]
        for u in factorList fR repeat
            p1:=map((z:R):%+->z::%,u.fctr)
            if not zero? k then
               p1:=elt(p1,monomial(1,1)+monomial(complex(0,k),0))
            p2:=gcd(p1,pp)
            lF:=cons(["irred",p2,1],lF)
            pp:=(pp exquo p2)::Sup
        makeFR(pp,lF)

rank : () -> PositiveInteger
rank() == 2

discriminant : () -> R
discriminant() == -4 :: R

norm : % -> R
norm x == real(x)**2 + imag(x)**2

trace : % -> R
trace x == 2 * real x

imaginary : () -> %
imaginary() == complex(0, 1)

conjugate : % -> %
conjugate x == complex(real x, - imag x)

characteristic : () -> NonNegativeInteger
characteristic() == characteristic()$R

map : ((R -> R),%) -> %
map(fn, x) == complex(fn real x, fn imag x)

?=? : (%,%) -> Boolean
x = y == real(x) = real(y) and imag(x) = imag(y)

?+? : (%,%) -> %
x + y == complex(real x + real y, imag x + imag y)

-? : % -> %
- x == complex(- real x, - imag x)

?*? : (R,%) -> %
r:R * x:% == complex(r * real x, r * imag x)
```

```
coordinates : % -> Vector(R)
coordinates(x:%) == [real x, imag x]

?*? : (Integer,%) -> %
n:Integer * x:%  == complex(n * real x, n * imag x)

differentiate : (%,(R -> R)) -> %
differentiate(x:%, d:R -> R) == complex(d real x, d imag x)

definingPolynomial : () -> SparseUnivariatePolynomial(R)
definingPolynomial() ==
  monomial(1,2)$(SUP R) + monomial(1,0)$(SUP R)

reduce : SparseUnivariatePolynomial(R) -> %
reduce(pol:SUP R) ==
  part:= (monicDivide(pol,definingPolynomial())).remainder
  complex(coefficient(part,0),coefficient(part,1))

lift : % -> SparseUnivariatePolynomial(R)
lift(x) == monomial(real x,0)$(SUP R)+monomial(imag x,1)$(SUP R)

minimalPolynomial : % -> SparseUnivariatePolynomial(R)
minimalPolynomial x ==
  zero? imag x =>
    monomial(1, 1)$(SUP R) - monomial(real x, 0)$(SUP R)
  monomial(1, 2)$(SUP R) - monomial(trace x, 1)$(SUP R)
    + monomial(norm x, 0)$(SUP R)

coordinates : (%,Vector(%)) -> Vector(R)
coordinates(x:%, v:Vector %):Vector(R) ==
  ra := real(a := v(minIndex v))
  rb := real(b := v(maxIndex v))
  (#v ^= 2) or
    ((d := recip(ra * (ib := imag b) - (ia := imag a) * rb))
      case "failed") =>error "coordinates: vector is not a basis"
  rx := real x
  ix := imag x
  [d::R * (rx * ib - ix * rb), d::R * (ra * ix - ia * rx)]

coerce : % -> OutputForm
coerce(x:%):OutputForm ==
  re := (r := real x)::OutputForm
  ie := (i := imag x)::OutputForm
  zero? i => re
  outi := "%i"::Symbol::OutputForm
  ip :=
    (i = 1) => outi
    ((-i) = 1) => -outi
    ie * outi
  zero? r => ip
  re + ip

retract : % -> R
```

```
retract(x:%):R ==
  not zero?(imag x) =>
    error "Imaginary part is nonzero. Cannot retract."
  real x

retractIfCan : % -> Union(R,"failed")
retractIfCan(x:%):Union(R, "failed") ==
  not zero?(imag x) => "failed"
  real x

?*? : (%,%) -> %
x:% * y:% ==
  complex(real x * real y - imag x * imag y,
           imag x * real y + imag y * real x)

reducedSystem : Matrix(%) -> Matrix(R)
reducedSystem(m:Matrix %):Matrix R ==
  vertConcat(map(real, m), map(imag, m))

reducedSystem : (Matrix(%),Vector(%)) ->
   Record(mat: Matrix(R),vec: Vector(R))
reducedSystem(m:Matrix %, v:Vector %):
 Record(mat:Matrix R, vec:Vector R) ==
  rh := reducedSystem(v::Matrix %)@Matrix(R)
  [reducedSystem(m)@Matrix(R), column(rh, minColIndex rh)]

if R has RealNumberSystem then

  abs : % -> %
  abs(x:%):%        == (sqrt norm x)::%

if R has RealConstant then

  convert : % -> Complex(DoubleFloat)
  convert(x:%):Complex(DoubleFloat) ==
   complex(convert(real x)@DoubleFloat,convert(imag x)@DoubleFloat)

  convert : % -> Complex(Float)
  convert(x:%):Complex(Float) ==
    complex(convert(real x)@Float, convert(imag x)@Float)

if R has ConvertibleTo InputForm then

  convert : % -> InputForm
  convert(x:%):InputForm ==
    convert([convert("complex"::Symbol), convert real x,
             convert imag x]$List(InputForm))@InputForm

if R has ConvertibleTo Pattern Integer then

  convert : % -> Pattern(Integer)
  convert(x:%):Pattern Integer ==
     convert(x)$ComplexPattern(Integer, R, %)
```

```
if R has ConvertibleTo Pattern Float then

  convert : % -> Pattern(Float)
  convert(x:%):Pattern Float ==
     convert(x)$ComplexPattern(Float, R, %)

if R has PatternMatchable Integer then

  patternMatch : (%,Pattern(Integer),PatternMatchResult(Integer,%)) ->
     PatternMatchResult(Integer,%)
  patternMatch(x:%, p:Pattern Integer,
   l:PatternMatchResult(Integer, %)) ==
    patternMatch(x, p, l)$ComplexPatternMatch(Integer, R, %)

if R has PatternMatchable Float then

  patternMatch : (%,Pattern(Float),PatternMatchResult(Float,%)) ->
     PatternMatchResult(Float,%)
  patternMatch(x:%, p:Pattern Float,
   l:PatternMatchResult(Float, %)) ==
    patternMatch(x, p, l)$ComplexPatternMatch(Float, R, %)


if R has OrderedSet then

  ?<? : (%,%) -> Boolean
  x < y ==
    real x = real y => imag x < imag y
    real x < real y

if R has IntegerNumberSystem then

  rational? : % -> Boolean
  rational? x == zero? imag x

  rational : % -> Fraction(Integer)
  rational x ==
    zero? imag x => rational real x
    error "Not a rational number"

  rationalIfCan : % -> Union(Fraction(Integer),"failed")
  rationalIfCan x ==
    zero? imag x => rational real x
    "failed"

if R has Field then

  inv : % -> %
  inv x ==
    zero? imag x => (inv real x)::%
    r := norm x
    complex(real(x) / r, - imag(x) / r)

if R has IntegralDomain then
```

```
    exquo : (%,R) -> Union(%,"failed")
    _exquo(x:%, r:R) ==
      (r = 1) => x
      (r1 := real(x) exquo r) case "failed" => "failed"
      (r2 := imag(x) exquo r) case "failed" => "failed"
      complex(r1, r2)

    exquo : (%,%) -> Union(%,"failed")
    _exquo(x:%, y:%) ==
      zero? imag y => x exquo real y
      x * conjugate(y) exquo norm(y)

    recip : % -> Union(%,"failed")
    recip(x:%) == 1 exquo x

    if R has OrderedRing then

      unitNormal : % -> Record(unit: %,canonical: %,associate: %)
      unitNormal x ==
        zero? x => [1,x,1]
        (u := recip x) case % => [x, 1, u]
        zero? real x =>
          c := unitNormal imag x
          [complex(0, c.unit), (c.associate * imag x)::%,
                                 complex(0, - c.associate)]
        c := unitNormal real x
        x := c.associate * x
        imag x < 0 =>
          x := complex(- imag x, real x)
          [- c.unit * imaginary(), x, c.associate * imaginary()]
        [c.unit ::%, x, c.associate ::%]

    else

      unitNormal : % -> Record(unit: %,canonical: %,associate: %)
      unitNormal x ==
        zero? x => [1,x,1]
        (u := recip x) case % => [x, 1, u]
        zero? real x =>
          c := unitNormal imag x
          [complex(0, c.unit), (c.associate * imag x)::%,
                                 complex(0, - c.associate)]
        c := unitNormal real x
        x := c.associate * x
        [c.unit ::%, x, c.associate ::%]

  if R has EuclideanDomain then

     if R has additiveValuation then

         euclideanSize : % -> NonNegativeInteger
         euclideanSize x == max(euclideanSize real x,
                               euclideanSize imag x)
```

```
else

    euclideanSize : % -> NonNegativeInteger
    euclideanSize x == euclideanSize(real(x)**2 + imag(x)**2)$R

if R has IntegerNumberSystem then

  ?rem? : (%,%) -> % if R has EUCDOM
  x rem y ==
    zero? imag y =>
      yr:=real y
      complex(symmetricRemainder(real(x), yr),
              symmetricRemainder(imag(x), yr))
    divide(x, y).remainder

  ?quo? : (%,%) -> % if R has EUCDOM
  x quo y ==
    zero? imag y =>
      yr:= real y
      xr:= real x
      xi:= imag x
      complex((xr-symmetricRemainder(xr,yr)) quo yr,
              (xi-symmetricRemainder(xi,yr)) quo yr)
    divide(x, y).quotient

else

  ?rem? : (%,%) -> % if R has EUCDOM
  x rem y ==
    zero? imag y =>
      yr:=real y
      complex(real(x) rem yr,imag(x) rem yr)
    divide(x, y).remainder

  ?quo? : (%,%) -> % if R has EUCDOM
  x quo y ==
    zero? imag y => complex(real x quo real y,imag x quo real y)
    divide(x, y).quotient

divide : (%,%) -> Record(quotient: %,remainder: %)
divide(x, y) ==
  r := norm y
  y1 := conjugate y
  xx := x * y1
  x1 := real(xx) rem r
  a  := x1
  if x1^=0 and sizeLess?(r, 2 * x1) then
    a := x1 - r
    if sizeLess?(x1, a) then a := x1 + r
  x2 := imag(xx) rem r
  b  := x2
  if x2^=0 and sizeLess?(r, 2 * x2) then
    b := x2 - r
```

```
          if sizeLess?(x2, b) then b := x2 + r
        y1 := (complex(a, b) exquo y1)::%
        [((x - y1) exquo y)::%, y1]

  if R has TranscendentalFunctionCategory then

    half := recip(2::R)::R

    if R has RealNumberSystem then

      atan2loc : R -> R
      atan2loc(y: R, x: R): R ==
          pi1 := pi()$R
          pi2 := pi1 * half
          x = 0 => if y >= 0 then pi2 else -pi2

          -- Atan in (-pi/2,pi/2]
          theta := atan(y * recip(x)::R)
          while theta <= -pi2 repeat theta := theta + pi1
          while theta >   pi2 repeat theta := theta - pi1

          x >= 0 => theta      -- I or IV

          if y >= 0 then
              theta + pi1      -- II
          else
              theta - pi1      -- III

      argument : % -> R
      argument x == atan2loc(imag x, real x)

    else

      -- Not ordered so dictate two quadrants
      argument : % -> R
      argument x ==
        zero? real x => pi()$R * half
        atan(imag(x) * recip(real x)::R)

    pi : () -> %
    pi()  == pi()$R :: %

    if R is DoubleFloat then

       stoc ==> S_-TO_-C$Lisp
       ctos ==> C_-TO_-S$Lisp

       exp : % -> %
       exp x == ctos EXP(stoc x)$Lisp

       log : % -> %
       log x == ctos LOG(stoc x)$Lisp

       sin : % -> %
```

```
   sin x == ctos SIN(stoc x)$Lisp

   cos : % -> %
   cos x == ctos COS(stoc x)$Lisp

   tan : % -> %
   tan x == ctos TAN(stoc x)$Lisp

   asin : % -> %
   asin x == ctos ASIN(stoc x)$Lisp

   acos : % -> %
   acos x == ctos ACOS(stoc x)$Lisp

   atan : % -> %
   atan x == ctos ATAN(stoc x)$Lisp

   sinh : % -> %
   sinh x == ctos SINH(stoc x)$Lisp

   cosh : % -> %
   cosh x == ctos COSH(stoc x)$Lisp

   tanh : % -> %
   tanh x == ctos TANH(stoc x)$Lisp

   asinh : % -> %
   asinh x == ctos ASINH(stoc x)$Lisp

   acosh : % -> %
   acosh x == ctos ACOSH(stoc x)$Lisp

   atanh : % -> %
   atanh x == ctos ATANH(stoc x)$Lisp

else

  atan : % -> %
  atan x ==
    ix := imaginary()*x
    - imaginary() * half * (log(1 + ix) - log(1 - ix))

  log : % -> %
  log x ==
    complex(log(norm x) * half, argument x)

  exp : % -> %
  exp x ==
    e := exp real x
    complex(e * cos imag x, e * sin imag x)

  cos : % -> %
  cos x ==
    e := exp(imaginary() * x)
```

```
            half * (e + recip(e)::%)

          sin : % -> %
          sin x ==
            e := exp(imaginary() * x)
            - imaginary() * half * (e - recip(e)::%)

        if R has RealNumberSystem then

          polarCoordinates : % -> Record(r: R,phi: R)
          polarCoordinates x ==
            [sqrt norm x, (negative?(t := argument x) => t + 2 * pi(); t)]

          ?**? : (%,Fraction(Integer)) -> %
          x:% ** q:Fraction(Integer) ==
            zero? q =>
              zero? x => error "0 ** 0 is undefined"
              1
            zero? x => 0
            rx := real x
            zero? imag x and positive? rx => (rx ** q)::%
            zero? imag x and denom q = 2 => complex(0, (-rx)**q)
            ax := sqrt(norm x) ** q
            tx := q::R * argument x
            complex(ax * cos tx, ax * sin tx)

        else if R has RadicalCategory then

          ?**? : (%,Fraction(Integer)) -> %
          x:% ** q:Fraction(Integer) ==
            zero? q =>
              zero? x => error "0 ** 0 is undefined"
              1
            r := real x
            zero?(i := imag x) => (r ** q)::%
            t := numer(q) * recip(denom(q)::R)::R * argument x
            e:R :=
              zero? r => i ** q
              norm(x) ** (q / (2::Fraction(Integer)))
            complex(e * cos t, e * sin t)
*)
```

— COMPCAT.dotabb —

```
"COMPCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=COMPCAT"];
"COMPCAT" -> "COMRING"
"COMPCAT" -> "DIFEXT"
"COMPCAT" -> "FEVALAB"
"COMPCAT" -> "FPATMAB"
"COMPCAT" -> "FRETRCT"
```

```
"COMPCAT" -> "MONOGEN"
"COMPCAT" -> "PATAB"
"COMPCAT" -> "FLINEXP"
"COMPCAT" -> "ORDSET"
```

———-

— COMPCAT.dotfull —

```
"ComplexCategory(R:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=COMPCAT"];
"ComplexCategory(R:CommutativeRing)" ->
  "MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
"ComplexCategory(R:CommutativeRing)" ->
  "FullyRetractableTo(a:CommutativeRing)"
"ComplexCategory(R:CommutativeRing)" ->
  "DifferentialExtension(CommutativeRing)"
"ComplexCategory(R:CommutativeRing)" ->
  "FullyEvalableOver(CommutativeRing)"
"ComplexCategory(R:CommutativeRing)" ->
  "FullyPatternMatchable(CommutativeRing)"
"ComplexCategory(R:CommutativeRing)" ->
  "Patternable(CommutativeRing)"
"ComplexCategory(R:CommutativeRing)" ->
  "FullyLinearlyExplicitRingOver(a:CommutativeRing)"
"ComplexCategory(R:CommutativeRing)" ->
  "CommutativeRing()"
"ComplexCategory(R:CommutativeRing)" ->
  "OrderedSet()"
```

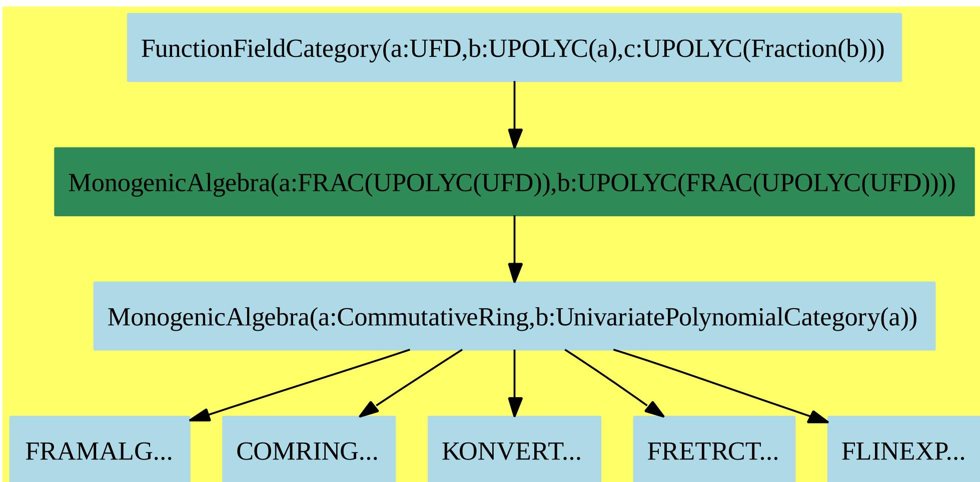
———-

— COMPCAT.dotpic —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"ComplexCategory(R:CommutativeRing)" [color=lightblue];
"ComplexCategory(R:CommutativeRing)" -> "MONOGEN..."
"ComplexCategory(R:CommutativeRing)" -> "FRETRCT..."
"ComplexCategory(R:CommutativeRing)" -> "DIFEXT..."
"ComplexCategory(R:CommutativeRing)" -> "FEVALAB..."
"ComplexCategory(R:CommutativeRing)" -> "FPATMAB..."
"ComplexCategory(R:CommutativeRing)" -> "PATAB..."
"ComplexCategory(R:CommutativeRing)" -> "FLINEXP..."
"ComplexCategory(R:CommutativeRing)" -> "COMRING..."
"ComplexCategory(R:CommutativeRing)" -> "ORDSET..."

"MONOGEN..." [color=lightblue];
"FRETRCT..." [color=lightblue];
```

```
"DIFEXT..." [color=lightblue];
"FEVALAB..." [color=lightblue];
"COMRING..." [color=lightblue];
"FPATMAB..." [color=lightblue];
"PATAB..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"ORDSET..." [color=lightblue];
}
```

20.0.241 FunctionFieldCategory (FFCAT)



— **FunctionFieldCategory.input** —

```
)set break resume
)sys rm -f FunctionFieldCategory.output
)spool FunctionFieldCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show FunctionFieldCategory
--R
--R FunctionFieldCategory(F: UniqueFactorizationDomain,UP: UnivariatePolynomialCategory(t#1),UPUP: UnivariatePol
--R Abbreviation for FunctionFieldCategory is FFCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for FFCAT
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (Fraction(UP),%) -> %          ?*? : (%,Fraction(UP)) -> %
--R ?*? : (%,%) -> %                     ?*? : (Integer,%) -> %
```

```
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,NonNegativeInteger) -> %    ?**? : (%,PositiveInteger) -> %
--R ?+? : (%,%) -> %                      ?-? : (%,%) -> %
--R -? : % -> %                           ?=? : (%,%) -> Boolean
--R 1 : () -> %                           0 : () -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R basis : () -> Vector(%)               branchPoint? : UP -> Boolean
--R branchPoint? : F -> Boolean           coerce : Fraction(UP) -> %
--R coerce : Integer -> %                 coerce : % -> OutputForm
--R convert : UPUP -> %                   convert : % -> UPUP
--R convert : Vector(Fraction(UP)) -> %   convert : % -> Vector(Fraction(UP))
--R definingPolynomial : () -> UPUP       discriminant : () -> Fraction(UP)
--R elliptic : () -> Union(UP,"failed")   elt : (%,F,F) -> F
--R generator : () -> %                   genus : () -> NonNegativeInteger
--R hash : % -> SingleInteger             integral? : (%,UP) -> Boolean
--R integral? : (%,F) -> Boolean          integral? : % -> Boolean
--R integralAtInfinity? : % -> Boolean    integralBasis : () -> Vector(%)
--R latex : % -> String                   lift : % -> UPUP
--R norm : % -> Fraction(UP)              one? : % -> Boolean
--R primitivePart : % -> %                ramified? : UP -> Boolean
--R ramified? : F -> Boolean              ramifiedAtInfinity? : () -> Boolean
--R rank : () -> PositiveInteger          rationalPoint? : (F,F) -> Boolean
--R recip : % -> Union(%,"failed")        reduce : UPUP -> %
--R represents : (Vector(UP),UP) -> %     retract : % -> Fraction(UP)
--R sample : () -> %                      singular? : UP -> Boolean
--R singular? : F -> Boolean              singularAtInfinity? : () -> Boolean
--R trace : % -> Fraction(UP)             zero? : % -> Boolean
--R ?~=? : (%,%) -> Boolean
--R ?*? : (%,Fraction(Integer)) -> % if Fraction(UP) has FIELD
--R ?*? : (Fraction(Integer),%) -> % if Fraction(UP) has FIELD
--R ?**? : (%,Integer) -> % if Fraction(UP) has FIELD
--R ?/? : (%,%) -> % if Fraction(UP) has FIELD
--R D : % -> % if and(has(Fraction(UP),Field),has(Fraction(UP),DifferentialRing)) or and(has(Fraction(UP),Differ
--R D : (%,NonNegativeInteger) -> % if and(has(Fraction(UP),Field),has(Fraction(UP),DifferentialRing)) or and(ha
--R D : (%,Symbol) -> % if and(has(Fraction(UP),Field),has(Fraction(UP),PartialDifferentialRing(Symbol))) or and
--R D : (%,List(Symbol)) -> % if and(has(Fraction(UP),Field),has(Fraction(UP),PartialDifferentialRing(Symbol)))
--R D : (%,Symbol,NonNegativeInteger) -> % if and(has(Fraction(UP),Field),has(Fraction(UP),PartialDifferentialRi
--R D : (%,List(Symbol),List(NonNegativeInteger)) -> % if and(has(Fraction(UP),Field),has(Fraction(UP),PartialDi
--R D : (%,(Fraction(UP) -> Fraction(UP))) -> % if Fraction(UP) has FIELD
--R D : (%,(Fraction(UP) -> Fraction(UP)),NonNegativeInteger) -> % if Fraction(UP) has FIELD
--R ?^? : (%,Integer) -> % if Fraction(UP) has FIELD
--R absolutelyIrreducible? : () -> Boolean
--R algSplitSimple : (%,(UP -> UP)) -> Record(num: %,den: UP,derivden: UP,gd: UP)
--R associates? : (%,%) -> Boolean if Fraction(UP) has FIELD
--R branchPointAtInfinity? : () -> Boolean
--R characteristic : () -> NonNegativeInteger
--R characteristicPolynomial : % -> UPUP
--R charthRoot : % -> Union(%,"failed") if Fraction(UP) has CHARNZ
--R charthRoot : % -> % if Fraction(UP) has FFIELDC
--R coerce : % -> % if Fraction(UP) has FIELD
--R coerce : Fraction(Integer) -> % if Fraction(UP) has FIELD or Fraction(UP) has RETRACT(FRAC(INT))
--R complementaryBasis : Vector(%) -> Vector(%)
--R conditionP : Matrix(%) -> Union(Vector(%),"failed") if Fraction(UP) has FFIELDC
--R coordinates : Vector(%) -> Matrix(Fraction(UP))
```

```
--R coordinates : % -> Vector(Fraction(UP))
--R coordinates : (Vector(%),Vector(%)) -> Matrix(Fraction(UP))
--R coordinates : (%,Vector(%)) -> Vector(Fraction(UP))
--R createPrimitiveElement : () -> % if Fraction(UP) has FFIELDC
--R derivationCoordinates : (Vector(%),(Fraction(UP) -> Fraction(UP))) -> Matrix(Fraction(UP)) if Fraction(UP) h
--R differentiate : % -> % if and(has(Fraction(UP),Field),has(Fraction(UP),DifferentialRing)) or and(has(Fractio
--R differentiate : (%,NonNegativeInteger) -> % if and(has(Fraction(UP),Field),has(Fraction(UP),DifferentialRing
--R differentiate : (%,Symbol) -> % if and(has(Fraction(UP),Field),has(Fraction(UP),PartialDifferentialRing(Symb
--R differentiate : (%,List(Symbol)) -> % if and(has(Fraction(UP),Field),has(Fraction(UP),PartialDifferentialRin
--R differentiate : (%,Symbol,NonNegativeInteger) -> % if and(has(Fraction(UP),Field),has(Fraction(UP),PartialDi
--R differentiate : (%,List(Symbol),List(NonNegativeInteger)) -> % if and(has(Fraction(UP),Field),has(Fraction(U
--R differentiate : (%,(UP -> UP)) -> %
--R differentiate : (%,(Fraction(UP) -> Fraction(UP))) -> % if Fraction(UP) has FIELD
--R differentiate : (%,(Fraction(UP) -> Fraction(UP)),NonNegativeInteger) -> % if Fraction(UP) has FIELD
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if Fraction(UP) has FFIELDC
--R discreteLog : % -> NonNegativeInteger if Fraction(UP) has FFIELDC
--R discriminant : Vector(%) -> Fraction(UP)
--R divide : (%,%) -> Record(quotient: %,remainder: %) if Fraction(UP) has FIELD
--R enumerate : () -> List(%) if Fraction(UP) has FINITE
--R euclideanSize : % -> NonNegativeInteger if Fraction(UP) has FIELD
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed") if Fraction(UP) has FIELD
--R exquo : (%,%) -> Union(%,"failed") if Fraction(UP) has FIELD
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if Fraction(UP) has FIELD
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if Fraction(UP) has FIELD
--R factor : % -> Factored(%) if Fraction(UP) has FIELD
--R factorsOfCyclicGroupSize : () -> List(Record(factor: Integer,exponent: Integer)) if Fraction(UP) has FFIELDC
--R gcd : (%,%) -> % if Fraction(UP) has FIELD
--R gcd : List(%) -> % if Fraction(UP) has FIELD
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R hyperelliptic : () -> Union(UP,"failed")
--R index : PositiveInteger -> % if Fraction(UP) has FINITE
--R init : () -> % if Fraction(UP) has FFIELDC
--R integralBasisAtInfinity : () -> Vector(%)
--R integralCoordinates : % -> Record(num: Vector(UP),den: UP)
--R integralDerivationMatrix : (UP -> UP) -> Record(num: Matrix(UP),den: UP)
--R integralMatrix : () -> Matrix(Fraction(UP))
--R integralMatrixAtInfinity : () -> Matrix(Fraction(UP))
--R integralRepresents : (Vector(UP),UP) -> %
--R inv : % -> % if Fraction(UP) has FIELD
--R inverseIntegralMatrix : () -> Matrix(Fraction(UP))
--R inverseIntegralMatrixAtInfinity : () -> Matrix(Fraction(UP))
--R lcm : (%,%) -> % if Fraction(UP) has FIELD
--R lcm : List(%) -> % if Fraction(UP) has FIELD
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %) if Fraction(UP) has FIELD
--R lookup : % -> PositiveInteger if Fraction(UP) has FINITE
--R minimalPolynomial : % -> UPUP if Fraction(UP) has FIELD
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed") if Fraction(UP) has FIELD
--R nextItem : % -> Union(%,"failed") if Fraction(UP) has FFIELDC
--R nonSingularModel : Symbol -> List(Polynomial(F)) if F has FIELD
--R normalizeAtInfinity : Vector(%) -> Vector(%)
--R numberOfComponents : () -> NonNegativeInteger
--R order : % -> OnePointCompletion(PositiveInteger) if Fraction(UP) has FFIELDC
--R order : % -> PositiveInteger if Fraction(UP) has FFIELDC
--R prime? : % -> Boolean if Fraction(UP) has FIELD
```

```
--R primeFrobenius : % -> % if Fraction(UP) has FFIELDC
--R primeFrobenius : (%,NonNegativeInteger) -> % if Fraction(UP) has FFIELDC
--R primitive? : % -> Boolean if Fraction(UP) has FFIELDC
--R primitiveElement : () -> % if Fraction(UP) has FFIELDC
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %) if Fraction(UP) has FIELD
--R ?quo? : (%,%) -> % if Fraction(UP) has FIELD
--R random : () -> % if Fraction(UP) has FINITE
--R rationalPoints : () -> List(List(F)) if F has FINITE
--R reduce : Fraction(UPUP) -> Union(%,"failed") if Fraction(UP) has FIELD
--R reduceBasisAtInfinity : Vector(%) -> Vector(%)
--R reducedSystem : Matrix(%) -> Matrix(Fraction(UP))
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(Fraction(UP)),vec: Vector(Fraction(UP)))
--R reducedSystem : (Matrix(%),Vector(%)) -> Record(mat: Matrix(Integer),vec: Vector(Integer)) if Fraction(UP) h
--R reducedSystem : Matrix(%) -> Matrix(Integer) if Fraction(UP) has LINEXP(INT)
--R regularRepresentation : % -> Matrix(Fraction(UP))
--R regularRepresentation : (%,Vector(%)) -> Matrix(Fraction(UP))
--R ?rem? : (%,%) -> % if Fraction(UP) has FIELD
--R representationType : () -> Union("prime",polynomial,normal,cyclic) if Fraction(UP) has FFIELDC
--R represents : Vector(Fraction(UP)) -> %
--R represents : (Vector(Fraction(UP)),Vector(%)) -> %
--R retract : % -> Fraction(Integer) if Fraction(UP) has RETRACT(FRAC(INT))
--R retract : % -> Integer if Fraction(UP) has RETRACT(INT)
--R retractIfCan : % -> Union(Fraction(UP),"failed")
--R retractIfCan : % -> Union(Fraction(Integer),"failed") if Fraction(UP) has RETRACT(FRAC(INT))
--R retractIfCan : % -> Union(Integer,"failed") if Fraction(UP) has RETRACT(INT)
--R size : () -> NonNegativeInteger if Fraction(UP) has FINITE
--R sizeLess? : (%,%) -> Boolean if Fraction(UP) has FIELD
--R squareFree : % -> Factored(%) if Fraction(UP) has FIELD
--R squareFreePart : % -> % if Fraction(UP) has FIELD
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if Fraction(UP) has FFIELDC
--R traceMatrix : () -> Matrix(Fraction(UP))
--R traceMatrix : Vector(%) -> Matrix(Fraction(UP))
--R unit? : % -> Boolean if Fraction(UP) has FIELD
--R unitCanonical : % -> % if Fraction(UP) has FIELD
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %) if Fraction(UP) has FIELD
--R yCoordinates : % -> Record(num: Vector(UP),den: UP)
--R
--E 1

)spool
)lisp (bye)
```

— **FunctionFieldCategory.help** —

```
====================================================================
FunctionFieldCategory examples
====================================================================

This category is a model for the function field of a plane algebraic curve.

See Also:
```

```
o )show FunctionFieldCategory
```

———–

See:

⇐ "MonogenicAlgebra" (MONOGEN) 19.0.238 on page 1847

Exports:

0	1
absolutelyIrreducible?	algSplitSimple
associates?	basis
branchPoint?	branchPointAtInfinity?
characteristic	characteristicPolynomial
charthRoot	coerce
complementaryBasis	conditionP
convert	coordinates
createPrimitiveElement	D
definingPolynomial	derivationCoordinates
differentiate	discreteLog
discriminant	divide
elliptic	elt
euclideanSize	expressIdealMember
exquo	extendedEuclidean
factor	factorsOfCyclicGroupSize
gcd	gcdPolynomial
generator	genus
hash	hyperelliptic
index	init
integral?	integralAtInfinity?
integralBasis	integralBasisAtInfinity
integralCoordinates	integralDerivationMatrix
integralMatrix	integralMatrixAtInfinity
integralRepresents	inv
inverseIntegralMatrix	inverseIntegralMatrixAtInfinity
latex	lcm
lift	lookup
minimalPolynomial	multiEuclidean
nextItem	nonSingularModel
norm	normalizeAtInfinity
numberOfComponents	one?
order	prime?
primeFrobenius	primitive?
primitiveElement	primitivePart
principalIdeal	ramified?
ramifiedAtInfinity?	rank
random	rationalPoints
rationalPoint?	recip
reduce	reduceBasisAtInfinity
reducedSystem	regularRepresentation
representationType	represents
retract	retractIfCan
sample	singular?
singularAtInfinity?	size
sizeLess?	squareFree
squareFreePart	subtractIfCan
tableForDiscreteLogarithm	trace
traceMatrix	unit?
unitCanonical	unitNormal
yCoordinates	zero?
?*?	?**?
?+?	?-?
-?	?=?
?^?	?~=?
?/?	?quo?
?rem?	

Attributes Exported:

- if $ has Fraction(UPOLYC(UFD)) and Field then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if $ has Fraction(UPOLYC(UFD)) and Field then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if $ has Fraction(UPOLYC(UFD)) and Field then canonicalsClosed where **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **commutative("*")** is true if it has an operation $" * " : (D, D)- > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
branchPointAtInfinity? : () -> Boolean
branchPoint? : UP -> Boolean
branchPoint? : F -> Boolean
integralBasis : () -> Vector %
integralBasisAtInfinity : () -> Vector %
ramifiedAtInfinity? : () -> Boolean
ramified? : UP -> Boolean
ramified? : F -> Boolean
singularAtInfinity? : () -> Boolean
singular? : F -> Boolean
singular? : UP -> Boolean
```

These are implemented by this category:

```
absolutelyIrreducible? : () -> Boolean
algSplitSimple : (%,(UP -> UP)) ->
     Record(num: %,den: UP,derivden: UP,gd: UP)
complementaryBasis : Vector % -> Vector %
differentiate : (%,(UP -> UP)) -> %
elliptic : () -> Union(UP,"failed")
elt : (%,F,F) -> F
genus : () -> NonNegativeInteger
hyperelliptic : () -> Union(UP,"failed")
integral? : % -> Boolean
integral? : (%,F) -> Boolean
integral? : (%,UP) -> Boolean
integralAtInfinity? : % -> Boolean
normalizeAtInfinity : Vector % -> Vector %
numberOfComponents : () -> NonNegativeInteger
primitivePart : % -> %
rationalPoint? : (F,F) -> Boolean
rationalPoints : () -> List List F if F has FINITE
reduceBasisAtInfinity : Vector % -> Vector %
```

```
represents : (Vector UP,UP) -> %
yCoordinates : % -> Record(num: Vector UP,den: UP)
```

These exports come from (p1847) MonogenicAlgebra(RF, UPUP)
where RF:Fraction UP, UP:UnivariatePolynomialCategory F
F:UniqueFactorizationDomain, and
UPUP:UnivariatePolynomialCategory Fraction UP

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
    if Fraction UP has FIELD
basis : () -> Vector %
characteristic : () -> NonNegativeInteger
characteristicPolynomial : % -> UPUP
charthRoot : % -> Union(%,"failed")
    if Fraction UP has CHARNZ
charthRoot : % -> %
    if Fraction UP has FFIELDC
coerce : % -> %
    if Fraction UP has FIELD
coerce : Fraction Integer -> %
    if Fraction UP has FIELD
    or Fraction UP has RETRACT FRAC INT
coerce : Fraction UP -> %
coerce : Integer -> %
coerce : % -> OutputForm
conditionP : Matrix % -> Union(Vector %,"failed")
    if Fraction UP has FFIELDC
convert : UPUP -> %
convert : % -> UPUP
convert : Vector Fraction UP -> %
convert : % -> Vector Fraction UP
coordinates : Vector % -> Matrix Fraction UP
coordinates : % -> Vector Fraction UP
coordinates : (Vector %,Vector %) -> Matrix Fraction UP
coordinates : (%,Vector %) -> Vector Fraction UP
createPrimitiveElement : () -> %
    if Fraction UP has FFIELDC
D : % -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,DifferentialRing))
     or
       and(
         has(Fraction UP,DifferentialRing),
         has(Fraction UP,Field))
     or Fraction UP has FFIELDC
D : (%,NonNegativeInteger) -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,DifferentialRing))
     or
```

```
       and(
         has(Fraction UP,DifferentialRing),
         has(Fraction UP,Field))
     or Fraction UP has FFIELDC
D : (%,Symbol) -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,PartialDifferentialRing Symbol))
     or
       and(
         has(Fraction UP,PartialDifferentialRing Symbol),
         has(Fraction UP,Field))
D : (%,List Symbol) -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,PartialDifferentialRing Symbol))
     or
       and(
         has(Fraction UP,PartialDifferentialRing Symbol),
         has(Fraction UP,Field))
D : (%,Symbol,NonNegativeInteger) -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,PartialDifferentialRing Symbol))
     or
       and(
         has(Fraction UP,PartialDifferentialRing Symbol),
         has(Fraction UP,Field))
D : (%,List Symbol,List NonNegativeInteger) -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,PartialDifferentialRing Symbol))
     or
       and(
         has(Fraction UP,PartialDifferentialRing Symbol),
         has(Fraction UP,Field))
D : (%,(Fraction UP -> Fraction UP)) -> %
    if Fraction UP has FIELD
D : (%,(Fraction UP -> Fraction UP),NonNegativeInteger) -> %
    if Fraction UP has FIELD
definingPolynomial : () -> UPUP
derivationCoordinates : (Vector %,(Fraction UP -> Fraction UP))
  -> Matrix Fraction UP
    if Fraction UP has FIELD
differentiate : % -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,DifferentialRing))
     or
```

```
       and(
         has(Fraction UP,DifferentialRing),
         has(Fraction UP,Field))
     or Fraction UP has FFIELDC
differentiate : (%,NonNegativeInteger) -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,DifferentialRing))
     or
       and(
         has(Fraction UP,DifferentialRing),
         has(Fraction UP,Field))
     or Fraction UP has FFIELDC
differentiate : (%,Symbol) -> %
    if
      and(
        has(Fraction UP,Field),
        has(Fraction UP,PartialDifferentialRing Symbol))
    or
     and(
       has(Fraction UP,PartialDifferentialRing Symbol),
       has(Fraction UP,Field))
differentiate : (%,List Symbol) -> %
    if
      and(
        has(Fraction UP,Field),
        has(Fraction UP,PartialDifferentialRing Symbol))
     or
      and(
        has(Fraction UP,PartialDifferentialRing Symbol),
        has(Fraction UP,Field))
differentiate : (%,Symbol,NonNegativeInteger) -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,PartialDifferentialRing Symbol))
     or
       and(
         has(Fraction UP,PartialDifferentialRing Symbol),
         has(Fraction UP,Field))
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,PartialDifferentialRing Symbol))
      or
       and(
         has(Fraction UP,PartialDifferentialRing Symbol),
         has(Fraction UP,Field))
differentiate : (%,(Fraction UP -> Fraction UP)) -> %
    if Fraction UP has FIELD
differentiate :
  (%,(Fraction UP -> Fraction UP),NonNegativeInteger) -> %
```

```
    if Fraction UP has FIELD
discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
    if Fraction UP has FFIELDC
discreteLog : % -> NonNegativeInteger
    if Fraction UP has FFIELDC
discriminant : Vector % -> Fraction UP
discriminant : () -> Fraction UP
divide : (%,%) -> Record(quotient: %,remainder: %)
    if Fraction UP has FIELD
euclideanSize : % -> NonNegativeInteger
    if Fraction UP has FIELD
expressIdealMember : (List %,%) -> Union(List %,"failed")
    if Fraction UP has FIELD
exquo : (%,%) -> Union(%,"failed")
    if Fraction UP has FIELD
extendedEuclidean : (%,%) ->
   Record(coef1: %,coef2: %,generator: %)
    if Fraction UP has FIELD
extendedEuclidean : (%,%,%) ->
   Union(Record(coef1: %,coef2: %),"failed")
    if Fraction UP has FIELD
factor : % -> Factored %
    if Fraction UP has FIELD
factorsOfCyclicGroupSize : () ->
   List Record(factor: Integer,exponent: Integer)
    if Fraction UP has FFIELDC
gcd : (%,%) -> %
    if Fraction UP has FIELD
gcd : List % -> %
    if Fraction UP has FIELD
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                    SparseUnivariatePolynomial %
    if Fraction UP has FIELD
generator : () -> %
hash : % -> SingleInteger
index : PositiveInteger -> %
    if Fraction UP has FINITE
init : () -> %
    if Fraction UP has FFIELDC
integralCoordinates : % ->
   Record(num: Vector UP,den: UP)
integralDerivationMatrix : (UP -> UP) ->
   Record(num: Matrix UP,den: UP)
integralMatrix : () -> Matrix Fraction UP
integralMatrixAtInfinity : () -> Matrix Fraction UP
integralRepresents : (Vector UP,UP) -> %
inv : % -> %
    if Fraction UP has FIELD
inverseIntegralMatrix : () -> Matrix Fraction UP
inverseIntegralMatrixAtInfinity : () ->
   Matrix Fraction UP
latex : % -> String
lcm : (%,%) -> %
```

```
    if Fraction UP has FIELD
lcm : List % -> %
    if Fraction UP has FIELD
lift : % -> UPUP
lookup : % -> PositiveInteger
    if Fraction UP has FINITE
minimalPolynomial : % -> UPUP
    if Fraction UP has FIELD
multiEuclidean : (List %,%) -> Union(List %,"failed")
    if Fraction UP has FIELD
nextItem : % -> Union(%,"failed")
    if Fraction UP has FFIELDC
nonSingularModel : Symbol -> List Polynomial F
    if F has FIELD
norm : % -> Fraction UP
one? : % -> Boolean
order : % -> OnePointCompletion PositiveInteger
    if Fraction UP has FFIELDC
order : % -> PositiveInteger
    if Fraction UP has FFIELDC
prime? : % -> Boolean
    if Fraction UP has FIELD
primeFrobenius : % -> %
    if Fraction UP has FFIELDC
primeFrobenius : (%,NonNegativeInteger) -> %
    if Fraction UP has FFIELDC
primitive? : % -> Boolean
    if Fraction UP has FFIELDC
primitiveElement : () -> %
    if Fraction UP has FFIELDC
principalIdeal : List % ->
  Record(coef: List %,generator: %)
    if Fraction UP has FIELD
rank : () -> PositiveInteger
random : () -> %
    if Fraction UP has FINITE
recip : % -> Union(%,"failed")
reduce : UPUP -> %
reduce : Fraction UPUP -> Union(%,"failed")
    if Fraction UP has FIELD
reducedSystem : Matrix % -> Matrix Fraction UP
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix Fraction UP,vec: Vector Fraction UP)
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix Integer,vec: Vector Integer)
    if Fraction UP has LINEXP INT
reducedSystem : Matrix % -> Matrix Integer
    if Fraction UP has LINEXP INT
regularRepresentation : % -> Matrix Fraction UP
regularRepresentation : (%,Vector %) -> Matrix Fraction UP
representationType : () ->
  Union("prime",polynomial,normal,cyclic)
    if Fraction UP has FFIELDC
represents : Vector Fraction UP -> %
```

```
represents : (Vector Fraction UP,Vector %) -> %
retract : % -> Fraction Integer
    if Fraction UP has RETRACT FRAC INT
retract : % -> Integer
    if Fraction UP has RETRACT INT
retract : % -> Fraction UP
retractIfCan : % -> Union(Fraction UP,"failed")
retractIfCan : % -> Union(Fraction Integer,"failed")
    if Fraction UP has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed")
    if Fraction UP has RETRACT INT
sample : () -> %
size : () -> NonNegativeInteger
    if Fraction UP has FINITE
sizeLess? : (%,%) -> Boolean
    if Fraction UP has FIELD
squareFree : % -> Factored %
    if Fraction UP has FIELD
squareFreePart : % -> %
    if Fraction UP has FIELD
subtractIfCan : (%,%) -> Union(%,"failed")
tableForDiscreteLogarithm : Integer ->
  Table(PositiveInteger,NonNegativeInteger)
    if Fraction UP has FFIELDC
trace : % -> Fraction UP
traceMatrix : () -> Matrix Fraction UP
traceMatrix : Vector % -> Matrix Fraction UP
unit? : % -> Boolean
    if Fraction UP has FIELD
unitCanonical : % -> %
    if Fraction UP has FIELD
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if Fraction UP has FIELD
zero? : % -> Boolean
?*? : (Fraction UP,%) -> %
?*? : (%,Fraction UP) -> %
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?**? : (%,PositiveInteger) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
?^? : (%,PositiveInteger) -> %
?~=? : (%,%) -> Boolean
?*? : (%,Fraction Integer) -> % if Fraction UP has FIELD
?*? : (Fraction Integer,%) -> % if Fraction UP has FIELD
?*? : (NonNegativeInteger,%) -> %
?**? : (%,Integer) -> % if Fraction UP has FIELD
?**? : (%,NonNegativeInteger) -> %
?/? : (%,%) -> % if Fraction UP has FIELD
?^? : (%,Integer) -> % if Fraction UP has FIELD
?^? : (%,NonNegativeInteger) -> %
```

```
 ?quo? : (%,%) -> % if Fraction UP has FIELD
 ?rem? : (%,%) -> % if Fraction UP has FIELD
```

— FunctionFieldCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#FFCAT">
FunctionFieldCategory (FFCAT)</a></h2>
</body>
```

—————

— category FFCAT FunctionFieldCategory —

```
)abbrev category FFCAT FunctionFieldCategory
++ Author: Manuel Bronstein
++ Date Created: 1987
++ Date Last Updated: 19 Mai 1993
++ Description:
++ Function field of a curve
++ This category is a model for the function field of a
++ plane algebraic curve.

FunctionFieldCategory(F, UP, UPUP) : Category == SIG where
  F   : UniqueFactorizationDomain
  UP  : UnivariatePolynomialCategory F
  UPUP: UnivariatePolynomialCategory Fraction UP

  Z   ==> Integer
  Q   ==> Fraction F
  P   ==> Polynomial F
  RF  ==> Fraction UP
  QF  ==> Fraction UPUP
  SY  ==> Symbol
  REC ==> Record(num:$, den:UP, derivden:UP, gd:UP)

  SIG ==> MonogenicAlgebra(RF, UPUP) with

    numberOfComponents : () -> NonNegativeInteger
      ++numberOfComponents() returns the number of absolutely irreducible
      ++ components.
      ++
      ++X P0 := UnivariatePolynomial(x, Integer)
      ++X P1 := UnivariatePolynomial(y, Fraction P0)
      ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
      ++X numberOfComponents()$R

    genus : () -> NonNegativeInteger
      ++genus() returns the genus of one absolutely irreducible component
      ++
      ++X P0 := UnivariatePolynomial(x, Integer)
      ++X P1 := UnivariatePolynomial(y, Fraction P0)
      ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
```

```
    ++X genus()$R

  absolutelyIrreducible? : () -> Boolean
    ++absolutelyIrreducible?() tests if the curve absolutely irreducible?
    ++
    ++X P0 := UnivariatePolynomial(x, Integer)
    ++X P1 := UnivariatePolynomial(y, Fraction P0)
    ++X R2 := RadicalFunctionField(INT, P0, P1, 2 * x**2, 4)
    ++X absolutelyIrreducible?()$R2

  rationalPoint? : (F, F) -> Boolean
    ++rationalPoint?(a, b) tests if \spad{(x=a,y=b)} is on the curve.
    ++
    ++X P0 := UnivariatePolynomial(x, Integer)
    ++X P1 := UnivariatePolynomial(y, Fraction P0)
    ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
    ++X rationalPoint?(0,0)$R
    ++X R2 := RadicalFunctionField(INT, P0, P1, 2 * x**2, 4)
    ++X rationalPoint?(0,0)$R2

  branchPointAtInfinity? : () -> Boolean
    ++branchPointAtInfinity?() tests if there is a branch point
    ++ at infinity.
    ++
    ++X P0 := UnivariatePolynomial(x, Integer)
    ++X P1 := UnivariatePolynomial(y, Fraction P0)
    ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
    ++X branchPointAtInfinity?()$R
    ++X R2 := RadicalFunctionField(INT, P0, P1, 2 * x**2, 4)
    ++X branchPointAtInfinity?()$R

  branchPoint? : F -> Boolean
    ++branchPoint?(a) tests whether \spad{x = a} is a branch point.

  branchPoint? : UP -> Boolean
    ++branchPoint?(p) tests whether \spad{p(x) = 0} is a branch point.

  singularAtInfinity? : () -> Boolean
    ++singularAtInfinity?() tests if there is a singularity at infinity.

  singular? : F -> Boolean
    ++singular?(a) tests whether \spad{x = a} is singular.

  singular? : UP -> Boolean
    ++singular?(p) tests whether \spad{p(x) = 0} is singular.

  ramifiedAtInfinity? : () -> Boolean
    ++ramifiedAtInfinity?() tests if infinity is ramified.

  ramified? : F -> Boolean
    ++ramified?(a) tests whether \spad{x = a} is ramified.

  ramified? : UP -> Boolean
    ++ramified?(p) tests whether \spad{p(x) = 0} is ramified.
```

```
integralBasis : () -> Vector $
  ++integralBasis() returns the integral basis for the curve.
  ++
  ++X P0 := UnivariatePolynomial(x, Integer)
  ++X P1 := UnivariatePolynomial(y, Fraction P0)
  ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
  ++X integralBasis()$R

integralBasisAtInfinity: () -> Vector $
  ++integralBasisAtInfinity() returns the local integral basis
  ++ at infinity
  ++
  ++X P0 := UnivariatePolynomial(x, Integer)
  ++X P1 := UnivariatePolynomial(y, Fraction P0)
  ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
  ++X integralBasisAtInfinity()$R

integralAtInfinity? : $  -> Boolean
  ++integralAtInfinity?() tests if f is locally integral at infinity.

integral? : $  -> Boolean
  ++integral?() tests if f is integral over \spad{k[x]}.

complementaryBasis : Vector $ -> Vector $
  ++complementaryBasis(b1,...,bn) returns the complementary basis
  ++ \spad{(b1',...,bn')} of \spad{(b1,...,bn)}.

normalizeAtInfinity : Vector $ -> Vector $
  ++normalizeAtInfinity(v) makes v normal at infinity.

reduceBasisAtInfinity : Vector $ -> Vector $
  ++reduceBasisAtInfinity(b1,...,bn) returns \spad{(x**i * bj)}
  ++ for all i,j such that \spad{x**i*bj} is locally integral
  ++ at infinity.

integralMatrix : () -> Matrix RF
  ++integralMatrix() returns M such that
  ++ \spad{(w1,...,wn) = M (1, y, ..., y**(n-1))},
  ++ where \spad{(w1,...,wn)} is the integral basis of
  ++ \spadfunFrom{integralBasis}{FunctionFieldCategory}.
  ++
  ++X P0 := UnivariatePolynomial(x, Integer)
  ++X P1 := UnivariatePolynomial(y, Fraction P0)
  ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
  ++X integralMatrix()$R

inverseIntegralMatrix : () -> Matrix RF
  ++inverseIntegralMatrix() returns M such that
  ++ \spad{M (w1,...,wn) = (1, y, ..., y**(n-1))}
  ++ where \spad{(w1,...,wn)} is the integral basis of
  ++ \spadfunFrom{integralBasis}{FunctionFieldCategory}.
  ++
  ++X P0 := UnivariatePolynomial(x, Integer)
```

```
    ++X P1 := UnivariatePolynomial(y, Fraction P0)
    ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
    ++X inverseIntegralMatrix()$R

  integralMatrixAtInfinity : () -> Matrix RF
    ++integralMatrixAtInfinity() returns M such that
    ++ \spad{(v1,...,vn) = M (1, y, ..., y**(n-1))}
    ++ where \spad{(v1,...,vn)} is the local integral basis at infinity
    ++ returned by \spad{infIntBasis()}.
    ++
    ++X P0 := UnivariatePolynomial(x, Integer)
    ++X P1 := UnivariatePolynomial(y, Fraction P0)
    ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
    ++X integralMatrixAtInfinity()$R

  inverseIntegralMatrixAtInfinity: () -> Matrix RF
    ++inverseIntegralMatrixAtInfinity() returns M such
    ++ that \spad{M (v1,...,vn) = (1, y, ..., y**(n-1))}
    ++ where \spad{(v1,...,vn)} is the local integral basis at infinity
    ++ returned by \spad{infIntBasis()}.
    ++
    ++X P0 := UnivariatePolynomial(x, Integer)
    ++X P1 := UnivariatePolynomial(y, Fraction P0)
    ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
    ++X inverseIntegralMatrixAtInfinity()$R

  yCoordinates : $ -> Record(num:Vector(UP), den:UP)
    ++yCoordinates(f) returns \spad{[[A1,...,An], D]} such that
    ++ \spad{f = (A1 + A2 y +...+ An y**(n-1)) / D}.

  represents : (Vector UP, UP) -> $
    ++represents([A0,...,A(n-1)],D) returns
    ++ \spad{(A0 + A1 y +...+ A(n-1)*y**(n-1))/D}.

  integralCoordinates : $ -> Record(num:Vector(UP), den:UP)
    ++integralCoordinates(f) returns \spad{[[A1,...,An], D]} such that
    ++ \spad{f = (A1 w1 +...+ An wn) / D}  where \spad{(w1,...,wn)} is the
    ++ integral basis returned by \spad{integralBasis()}.

  integralRepresents : (Vector UP, UP) -> $
    ++integralRepresents([A1,...,An], D) returns
    ++ \spad{(A1 w1+...+An wn)/D}
    ++ where \spad{(w1,...,wn)} is the integral
    ++ basis of \spad{integralBasis()}.

  integralDerivationMatrix : (UP -> UP) -> Record(num:Matrix(UP),den:UP)
    ++integralDerivationMatrix(d) extends the derivation d from UP to $
    ++ and returns (M, Q) such that the i^th row of M divided by Q form
    ++ the coordinates of \spad{d(wi)} with respect to \spad{(w1,...,wn)}
    ++ where \spad{(w1,...,wn)} is the integral basis returned
    ++ by integralBasis().

  integral? : ($,  F) -> Boolean
    ++integral?(f, a) tests whether f is locally integral at \spad{x = a}.
```

```
 integral? : ($, UP) -> Boolean
   ++integral?(f, p) tests whether f is locally integral at
   ++ \spad{p(x) = 0}

 differentiate : ($, UP -> UP) -> $
   ++differentiate(x, d) extends the derivation d from UP to $ and
   ++ applies it to x.

 represents : (Vector UP, UP) -> $
   ++represents([A0,...,A(n-1)],D) returns
   ++ \spad{(A0 + A1 y +...+ A(n-1)*y**(n-1))/D}.

 primitivePart : $ -> $
   ++primitivePart(f) removes the content of the denominator and
   ++ the common content of the numerator of f.

 elt : ($, F, F) -> F
   ++elt(f,a,b) or f(a, b) returns the value of f
   ++ at the point \spad{(x = a, y = b)}
   ++ if it is not singular.

 elliptic : () -> Union(UP, "failed")
   ++elliptic() returns \spad{p(x)} if the curve is the elliptic
   ++ defined by \spad{y**2 = p(x)}, "failed" otherwise.

 hyperelliptic : () -> Union(UP, "failed")
   ++hyperelliptic() returns \spad{p(x)} if the curve is the
   ++ hyperelliptic
   ++ defined by \spad{y**2 = p(x)}, "failed" otherwise.

 algSplitSimple : ($, UP -> UP) -> REC
   ++algSplitSimple(f, D) returns \spad{[h,d,d',g]} such that
   ++ \spad{f=h/d},
   ++ \spad{h} is integral at all the normal places w.r.t. \spad{D},
   ++ \spad{d' = Dd}, \spad{g = gcd(d, discriminant())} and \spad{D}
   ++ is the derivation to use. \spad{f} must have at most simple finite
   ++ poles.

 if F has Field then

   nonSingularModel : SY -> List Polynomial F
     ++nonSingularModel(u) returns the equations in u1,...,un of
     ++ an affine non-singular model for the curve.

 if F has Finite then

   rationalPoints: () -> List List F
     ++rationalPoints() returns the list of all the affine
     ++rational points.
add

 import InnerCommonDenominator(UP, RF, Vector UP, Vector RF)
 import UnivariatePolynomialCommonDenominator(UP, RF, UPUP)
```

```
repOrder: (Matrix RF, Z) -> Z
Q2RF    : Q  -> RF
infOrder: RF -> Z
infValue: RF -> Fraction F
intvalue: (Vector UP, F, F) -> F
rfmonom : Z  -> RF
kmin    : (Matrix RF,Vector Q) -> Union(Record(pos:Z,km:Z),"failed")

Q2RF q                 == numer(q)::UP / denom(q)::UP
infOrder f             == (degree denom f)::Z - (degree numer f)::Z
integral? f            == ground?(integralCoordinates(f).den)
integral?(f:$, a:F)    == (integralCoordinates(f).den)(a) ^= 0
absolutelyIrreducible? == numberOfComponents() = 1
yCoordinates f         == splitDenominator coordinates f

hyperelliptic() ==
  degree(f := definingPolynomial()) ^= 2 => "failed"
  (u:=retractIfCan(reductum f)@Union(RF,"failed"))
    case "failed" => "failed"
  (v:=retractIfCan(-(u::RF) / leadingCoefficient f)@Union(UP, "failed"))
    case "failed" => "failed"
  odd? degree(p := v::UP) => p
  "failed"

algSplitSimple(f, derivation) ==
  cd := splitDenominator lift f
  dd := (cd.den exquo (g := gcd(cd.den, derivation(cd.den))))::UP
  [reduce(inv(g::RF) * cd.num), dd, derivation dd,
                               gcd(dd, retract(discriminant())@UP)]

elliptic() ==
  (u := hyperelliptic()) case "failed" => "failed"
  degree(p := u::UP) = 3 => p
  "failed"

rationalPoint?(x, y)   ==
  zero?((definingPolynomial() (y::UP::RF)) (x::UP::RF))

if F has Field then
  import PolyGroebner(F)
  import MatrixCommonDenominator(UP, RF)

  UP2P  : (UP,   P)    -> P
  UPUP2P: (UPUP, P, P) -> P

  UP2P(p, x) ==
    (map((s:F):P +-> s::P, p)_
      $UnivariatePolynomialCategoryFunctions2(F, UP,
                                P, SparseUnivariatePolynomial P)) x

  UPUP2P(p, x, y) ==
    (map((s:RF):P +-> UP2P(retract(s)@UP, x),p)_
      $UnivariatePolynomialCategoryFunctions2(RF, UPUP,
```

```
                                P, SparseUnivariatePolynomial P)) y

      nonSingularModel u ==
        d    := commonDenominator(coordinates(w := integralBasis()))::RF
        vars := [concat(string u, string i)::SY for i in 1..(n := #w)]
        x    := "%%dummy1"::SY
        y    := "%%dummy2"::SY
        select_!(s+->zero?(degree(s, x)) and zero?(degree(s, y)),
                 lexGroebner([v::P - UPUP2P(lift(d * w.i), x::P, y::P)
                    for v in vars for i in 1..n], concat([x, y], vars)))

    if F has Finite then
      ispoint: (UPUP, F, F) -> List F

-- must use the 'elt function explicitely or the compiler takes 45 mins
-- on that function    MB 5/90
-- still takes ages : I split the expression up. JHD 6/Aug/90
      ispoint(p, x, y) ==
        jhd:RF:=p(y::UP::RF)
        zero?(jhd (x::UP::RF)) => [x, y]
        empty()

      rationalPoints() ==
        p := definingPolynomial()
        concat [[pt for y in 1..size()$F | not empty?(pt :=
          ispoint(p, index(x::PositiveInteger)$F,
                     index(y::PositiveInteger)$F))]$List(List F)
                                for x in 1..size()$F]$List(List(List F))

    intvalue(v, x, y) ==
      singular? x => error "Point is singular"
      mini := minIndex(w := integralBasis())
      rec := yCoordinates(+/[qelt(v, i)::RF * qelt(w, i)
                           for i in mini .. maxIndex w])
      n   := +/[(qelt(rec.num, i) x) *
                (y ** ((i - mini)::NonNegativeInteger))
                           for i in mini .. maxIndex w]
      zero?(d := (rec.den) x) =>
        zero? n => error "0/0 -- cannot compute value yet"
        error "Shouldn't happen"
      (n exquo d)::F

    elt(f, x, y) ==
      rec := integralCoordinates f
      n   := intvalue(rec.num, x, y)
      zero?(d := (rec.den) x) =>
        zero? n => error "0/0 -- cannot compute value yet"
        error "Function has a pole at the given point"
      (n exquo d)::F

    primitivePart f ==
      cd := yCoordinates f
      d  := gcd([content qelt(cd.num, i)
                 for i in minIndex(cd.num) .. maxIndex(cd.num)]$List(F))
```

```
              * primitivePart(cd.den)
    represents [qelt(cd.num, i) / d
             for i in minIndex(cd.num) .. maxIndex(cd.num)]$Vector(RF)

  reduceBasisAtInfinity b ==
    x := monomial(1, 1)$UP ::RF
    concat([[f for j in 0.. while
              integralAtInfinity?(f := x**j * qelt(b, i))]$Vector($)
                    for i in minIndex b .. maxIndex b]$List(Vector $))

  complementaryBasis b ==
    m := inverse(traceMatrix b)::Matrix(RF)
    [represents row(m, i) for i in minRowIndex m .. maxRowIndex m]

  integralAtInfinity? f ==
    not any?(s +-> infOrder(s) < 0,
       coordinates(f) * inverseIntegralMatrixAtInfinity())$Vector(RF)

  numberOfComponents() ==
    count(integralAtInfinity?, integralBasis())$Vector($)

  represents(v:Vector UP, d:UP) ==
    represents
      [qelt(v, i) / d for i in minIndex v .. maxIndex v]$Vector(RF)

  genus() ==
    ds := discriminant()
    d  := degree(retract(ds)@UP) + infOrder(ds * determinant(
           integralMatrixAtInfinity() * inverseIntegralMatrix()) ** 2)
    dd := (((d exquo 2)::Z - rank()) exquo numberOfComponents())::Z
    (dd + 1)::NonNegativeInteger

  repOrder(m, i) ==
    nostart:Boolean := true
    ans:Z := 0
    r := row(m, i)
    for j in minIndex r .. maxIndex r | qelt(r, j) ^= 0 repeat
      ans :=
        nostart => (nostart := false; infOrder qelt(r, j))
        min(ans, infOrder qelt(r,j))
    nostart => error "Null row"
    ans

  infValue f ==
    zero? f => 0
    (n := infOrder f) > 0 => 0
    zero? n =>
      (leadingCoefficient numer f) / (leadingCoefficient denom f)
    error "f not locally integral at infinity"

  rfmonom n ==
    n < 0 => inv(monomial(1, (-n)::NonNegativeInteger)$UP :: RF)
    monomial(1, n::NonNegativeInteger)$UP :: RF
```

```
    kmin(m, v) ==
      nostart:Boolean := true
      k:Z := 0
      ii  := minRowIndex m - (i0  := minIndex v)
      for i in minIndex v .. maxIndex v | qelt(v, i) ^= 0 repeat
        nk := repOrder(m, i + ii)
        if nostart then (nostart := false; k := nk; i0 := i)
        else
          if nk < k then (k := nk; i0 := i)
      nostart => "failed"
      [i0, k]

    normalizeAtInfinity w ==
      ans   := copy w
      infm  := inverseIntegralMatrixAtInfinity()
      mhat  := zero(rank(), rank())$Matrix(RF)
      ii    := minIndex w - minRowIndex mhat
      repeat
        m := coordinates(ans) * infm
        r := [rfmonom repOrder(m, i)
                     for i in minRowIndex m .. maxRowIndex m]$Vector(RF)
        for i in minRowIndex m .. maxRowIndex m repeat
          for j in minColIndex m .. maxColIndex m repeat
            qsetelt_!(mhat, i, j, qelt(r, i + ii) * qelt(m, i, j))
        sol := first nullSpace transpose map(infValue,
                mhat)$MatrixCategoryFunctions2(RF, Vector RF, Vector RF,
                             Matrix RF, Q, Vector Q, Vector Q, Matrix Q)
        (pr := kmin(m, sol)) case "failed" => return ans
        qsetelt_!(ans, pr.pos,
         +/[Q2RF(qelt(sol, i)) * rfmonom(repOrder(m, i - ii) - pr.km)
                 * qelt(ans, i) for i in minIndex sol .. maxIndex sol])

    integral?(f:$, p:UP) ==
      (r:=retractIfCan(p)@Union(F,"failed")) case F => integral?(f,r::F)
      (integralCoordinates(f).den exquo p) case "failed"

    differentiate(f:$, d:UP -> UP) ==
      differentiate(f, x +-> differentiate(x, d)$RF)
```

——————

— COQ FFCAT —

```
(* category FFCAT *)
(*
    import InnerCommonDenominator(UP, RF, Vector UP, Vector RF)
    import UnivariatePolynomialCommonDenominator(UP, RF, UPUP)

    Q2RF : Q  -> RF
    Q2RF q == numer(q)::UP / denom(q)::UP

    infOrder: RF -> Z
    infOrder f == (degree denom f)::Z - (degree numer f)::Z
```

```
integral? : % -> Boolean
integral? f == ground?(integralCoordinates(f).den)

integral? : (%,F) -> Boolean
integral?(f:$, a:F) == (integralCoordinates(f).den)(a) ^= 0

absolutelyIrreducible? : () -> Boolean
absolutelyIrreducible? == numberOfComponents() = 1

yCoordinates : % -> Record(num: Vector(UP),den: UP)
yCoordinates f == splitDenominator coordinates f

hyperelliptic : () -> Union(UP,"failed")
hyperelliptic() ==
  degree(f := definingPolynomial()) ^= 2 => "failed"
  (u:=retractIfCan(reductum f)@Union(RF,"failed"))
    case "failed" => "failed"
  (v:=retractIfCan(-(u::RF) / leadingCoefficient f)@Union(UP, "failed"))
    case "failed" => "failed"
  odd? degree(p := v::UP) => p
  "failed"

algSplitSimple : (%,(UP -> UP)) ->
   Record(num: %,den: UP,derivden: UP,gd: UP)
algSplitSimple(f, derivation) ==
  cd := splitDenominator lift f
  dd := (cd.den exquo (g := gcd(cd.den, derivation(cd.den))))::UP
  [reduce(inv(g::RF) * cd.num), dd, derivation dd,
                              gcd(dd, retract(discriminant())@UP)]

elliptic : () -> Union(UP,"failed")
elliptic() ==
  (u := hyperelliptic()) case "failed" => "failed"
  degree(p := u::UP) = 3 => p
  "failed"

rationalPoint? : (F,F) -> Boolean
rationalPoint?(x, y)   ==
  zero?((definingPolynomial() (y::UP::RF)) (x::UP::RF))

if F has Field then
  import PolyGroebner(F)
  import MatrixCommonDenominator(UP, RF)

  UP2P : (UP, P) -> P
  UP2P(p, x) ==
    (map((s:F):P +-> s::P, p)_
      $UnivariatePolynomialCategoryFunctions2(F, UP,
                               P, SparseUnivariatePolynomial P)) x

  UPUP2P: (UPUP, P, P) -> P
  UPUP2P(p, x, y) ==
    (map((s:RF):P +-> UP2P(retract(s)@UP, x),p)_
```

```
      $UnivariatePolynomialCategoryFunctions2(RF, UPUP,
                                 P, SparseUnivariatePolynomial P)) y

  nonSingularModel : Symbol -> List(Polynomial(F))
  nonSingularModel u ==
    d    := commonDenominator(coordinates(w := integralBasis()))::RF
    vars := [concat(string u, string i)::SY for i in 1..(n := #w)]
    x    := "%%dummy1"::SY
    y    := "%%dummy2"::SY
    select_!(s+->zero?(degree(s, x)) and zero?(degree(s, y)),
             lexGroebner([v::P - UPUP2P(lift(d * w.i), x::P, y::P)
                for v in vars for i in 1..n], concat([x, y], vars)))

if F has Finite then

  ispoint: (UPUP, F, F) -> List F
  ispoint(p, x, y) ==
    jhd:RF:=p(y::UP::RF)
    zero?(jhd (x::UP::RF)) => [x, y]
    empty()

  rationalPoints : () -> List(List(F))
  rationalPoints() ==
    p := definingPolynomial()
    concat [[pt for y in 1..size()$F | not empty?(pt :=
      ispoint(p, index(x::PositiveInteger)$F,
                 index(y::PositiveInteger)$F))]$List(List F)
                           for x in 1..size()$F]$List(List(List F))

intvalue: (Vector UP, F, F) -> F
intvalue(v, x, y) ==
  singular? x => error "Point is singular"
  mini := minIndex(w := integralBasis())
  rec := yCoordinates(+/[qelt(v, i)::RF * qelt(w, i)
                       for i in mini .. maxIndex w])
  n   := +/[(qelt(rec.num, i) x) *
            (y ** ((i - mini)::NonNegativeInteger))
                       for i in mini .. maxIndex w]
  zero?(d := (rec.den) x) =>
    zero? n => error "0/0 -- cannot compute value yet"
    error "Shouldn't happen"
  (n exquo d)::F

elt : (%,F,F) -> F
elt(f, x, y) ==
  rec := integralCoordinates f
  n   := intvalue(rec.num, x, y)
  zero?(d := (rec.den) x) =>
    zero? n => error "0/0 -- cannot compute value yet"
    error "Function has a pole at the given point"
  (n exquo d)::F

primitivePart : % -> %
primitivePart f ==
```

```
    cd := yCoordinates f
    d  := gcd([content qelt(cd.num, i)
              for i in minIndex(cd.num) .. maxIndex(cd.num)]$List(F))
                * primitivePart(cd.den)
    represents [qelt(cd.num, i) / d
            for i in minIndex(cd.num) .. maxIndex(cd.num)]$Vector(RF)

  reduceBasisAtInfinity : Vector(%) -> Vector(%)
  reduceBasisAtInfinity b ==
    x := monomial(1, 1)$UP ::RF
    concat([[f for j in 0.. while
             integralAtInfinity?(f := x**j * qelt(b, i))]$Vector($)
                   for i in minIndex b .. maxIndex b]$List(Vector $))

  complementaryBasis : Vector(%) -> Vector(%)
  complementaryBasis b ==
    m := inverse(traceMatrix b)::Matrix(RF)
    [represents row(m, i) for i in minRowIndex m .. maxRowIndex m]

  integralAtInfinity? : % -> Boolean
  integralAtInfinity? f ==
    not any?(s +-> infOrder(s) < 0,
       coordinates(f) * inverseIntegralMatrixAtInfinity())$Vector(RF)

  numberOfComponents : () -> NonNegativeInteger
  numberOfComponents() ==
    count(integralAtInfinity?, integralBasis())$Vector($)

  represents : (Vector(UP),UP) -> %
  represents(v:Vector UP, d:UP) ==
    represents
      [qelt(v, i) / d for i in minIndex v .. maxIndex v]$Vector(RF)

  genus : () -> NonNegativeInteger
  genus() ==
    ds := discriminant()
    d  := degree(retract(ds)@UP) + infOrder(ds * determinant(
           integralMatrixAtInfinity() * inverseIntegralMatrix()) ** 2)
    dd := (((d exquo 2)::Z - rank()) exquo numberOfComponents())::Z
    (dd + 1)::NonNegativeInteger

  repOrder: (Matrix RF, Z) -> Z
  repOrder(m, i) ==
    nostart:Boolean := true
    ans:Z := 0
    r := row(m, i)
    for j in minIndex r .. maxIndex r | qelt(r, j) ^= 0 repeat
      ans :=
        nostart => (nostart := false; infOrder qelt(r, j))
        min(ans, infOrder qelt(r,j))
    nostart => error "Null row"
    ans

  infValue: RF -> Fraction F
```

```
infValue f ==
  zero? f => 0
  (n := infOrder f) > 0 => 0
  zero? n =>
    (leadingCoefficient numer f) / (leadingCoefficient denom f)
  error "f not locally integral at infinity"

rfmonom : Z  -> RF
rfmonom n ==
  n < 0 => inv(monomial(1, (-n)::NonNegativeInteger)$UP :: RF)
  monomial(1, n::NonNegativeInteger)$UP :: RF

kmin : (Matrix RF,Vector Q) -> Union(Record(pos:Z,km:Z),"failed")
kmin(m, v) ==
  nostart:Boolean := true
  k:Z := 0
  ii  := minRowIndex m - (i0  := minIndex v)
  for i in minIndex v .. maxIndex v | qelt(v, i) ^= 0 repeat
    nk := repOrder(m, i + ii)
    if nostart then (nostart := false; k := nk; i0 := i)
    else
      if nk < k then (k := nk; i0 := i)
  nostart => "failed"
  [i0, k]

normalizeAtInfinity : Vector(%) -> Vector(%)
normalizeAtInfinity w ==
  ans   := copy w
  infm  := inverseIntegralMatrixAtInfinity()
  mhat  := zero(rank(), rank())$Matrix(RF)
  ii    := minIndex w - minRowIndex mhat
  repeat
    m := coordinates(ans) * infm
    r := [rfmonom repOrder(m, i)
                 for i in minRowIndex m .. maxRowIndex m]$Vector(RF)
    for i in minRowIndex m .. maxRowIndex m repeat
      for j in minColIndex m .. maxColIndex m repeat
        qsetelt_!(mhat, i, j, qelt(r, i + ii) * qelt(m, i, j))
    sol := first nullSpace transpose map(infValue,
            mhat)$MatrixCategoryFunctions2(RF, Vector RF, Vector RF,
                        Matrix RF, Q, Vector Q, Vector Q, Matrix Q)
    (pr := kmin(m, sol)) case "failed" => return ans
    qsetelt_!(ans, pr.pos,
     +/[Q2RF(qelt(sol, i)) * rfmonom(repOrder(m, i - ii) - pr.km)
              * qelt(ans, i) for i in minIndex sol .. maxIndex sol])

integral? : (%,UP) -> Boolean
integral?(f:$, p:UP) ==
  (r:=retractIfCan(p)@Union(F,"failed")) case F => integral?(f,r::F)
  (integralCoordinates(f).den exquo p) case "failed"

differentiate : (%,(UP -> UP)) -> %
differentiate(f:$, d:UP -> UP) ==
  differentiate(f, x +-> differentiate(x, d)$RF)
```

```
*)
```

———-

— **FFCAT.dotabb** —

```
"FFCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FFCAT"];
"FFCAT" -> "MONOGEN"
```

———-

— **FFCAT.dotfull** —

```
"FunctionFieldCategory(a:UFD,b:UPOLYC(a),c:UPOLYC(Fraction(b)))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FFCAT"];
"FunctionFieldCategory(a:UFD,b:UPOLYC(a),c:UPOLYC(Fraction(b)))"
   -> "MonogenicAlgebra(a:FRAC(UPOLYC(UFD)),b:UPOLYC(FRAC(UPOLYC(UFD))))"
```

———-

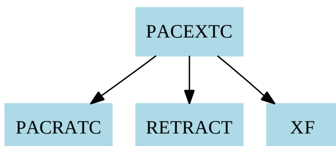
— **FFCAT.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"FunctionFieldCategory(a:UFD,b:UPOLYC(a),c:UPOLYC(Fraction(b)))"
 [color=lightblue];
"FunctionFieldCategory(a:UFD,b:UPOLYC(a),c:UPOLYC(Fraction(b)))"
   -> "MonogenicAlgebra(a:FRAC(UPOLYC(UFD)),b:UPOLYC(FRAC(UPOLYC(UFD))))"

"MonogenicAlgebra(a:FRAC(UPOLYC(UFD)),b:UPOLYC(FRAC(UPOLYC(UFD))))"
 [color=seagreen];
"MonogenicAlgebra(a:FRAC(UPOLYC(UFD)),b:UPOLYC(FRAC(UPOLYC(UFD))))" ->
    "MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"

"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
 [color=lightblue];
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FRAMALG..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "COMRING..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "KONVERT..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FRETRCT..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FLINEXP..."
```

```
"FRAMALG..." [color=lightblue];
"COMRING..." [color=lightblue];
"KONVERT..." [color=lightblue];
"FRETRCT..." [color=lightblue];
"FLINEXP..." [color=lightblue];
}
```

———-

20.0.242 PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory (PACEXTC)



— PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory.input —

```
)set break resume
)sys rm -f PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory.output
)spool PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory
--R
--R PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory is a category constructor
--R Abbreviation for PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory is PACEXTC
--R This constructor is not exposed in this frame.
--R Issue )edit bookvol10.2.pamphlet to see algebra source code for PACEXTC
--R
--R------------------------------- Operations --------------------------------
--R ?*? : (%,Fraction(Integer)) -> %      ?*? : (Fraction(Integer),%) -> %
--R ?*? : (Fraction(Integer),%) -> %      ?*? : (%,Fraction(Integer)) -> %
--R ?*? : (%,%) -> %                      ?*? : (Integer,%) -> %
--R ?*? : (NonNegativeInteger,%) -> %     ?*? : (PositiveInteger,%) -> %
--R ?**? : (%,Integer) -> %               ?**? : (%,NonNegativeInteger) -> %
--R ?**? : (%,PositiveInteger) -> %       ?+? : (%,%) -> %
--R ?-? : (%,%) -> %                      -? : % -> %
--R ?/? : (%,Fraction(Integer)) -> %      ?/? : (%,%) -> %
--R ?=? : (%,%) -> Boolean                1 : () -> %
--R 0 : () -> %                           ?^? : (%,Integer) -> %
--R ?^? : (%,NonNegativeInteger) -> %     ?^? : (%,PositiveInteger) -> %
--R algebraic? : % -> Boolean             associates? : (%,%) -> Boolean
```

```
--R coerce : Fraction(Integer) -> %       coerce : Fraction(Integer) -> %
--R coerce : Integer -> %                 coerce : Fraction(Integer) -> %
--R coerce : % -> %                       coerce : Integer -> %
--R coerce : % -> OutputForm              conjugate : % -> %
--R dimension : () -> CardinalNumber      extDegree : % -> PositiveInteger
--R factor : % -> Factored(%)             fullOutput : % -> OutputForm
--R gcd : List(%) -> %                    gcd : (%,%) -> %
--R ground? : % -> Boolean                hash : % -> SingleInteger
--R inGroundField? : % -> Boolean         inv : % -> %
--R latex : % -> String                   lcm : List(%) -> %
--R lcm : (%,%) -> %                      maxTower : List(%) -> %
--R one? : % -> Boolean                   previousTower : % -> %
--R prime? : % -> Boolean                 ?quo? : (%,%) -> %
--R recip : % -> Union(%,"failed")        ?rem? : (%,%) -> %
--R retract : % -> Fraction(Integer)      retract : % -> Fraction(Integer)
--R retract : % -> Integer                sample : () -> %
--R setTower! : % -> Void                 sizeLess? : (%,%) -> Boolean
--R squareFree : % -> Factored(%)         squareFreePart : % -> %
--R transcendent? : % -> Boolean          unit? : % -> Boolean
--R unitCanonical : % -> %                vectorise : (%,%) -> Vector(%)
--R zero? : % -> Boolean                  ?~=? : (%,%) -> Boolean
--R ?*? : (%,PseudoAlgebraicClosureOfRationalNumber) -> %
--R ?*? : (PseudoAlgebraicClosureOfRationalNumber,%) -> %
--R ?/? : (%,PseudoAlgebraicClosureOfRationalNumber) -> %
--R Frobenius : % -> % if PseudoAlgebraicClosureOfRationalNumber has FINITE or Fraction(Integer) has FINITE
--R Frobenius : (%,NonNegativeInteger) -> % if PseudoAlgebraicClosureOfRationalNumber has FINITE or Fraction(Int
--R characteristic : () -> NonNegativeInteger
--R charthRoot : % -> Union(%,"failed") if PseudoAlgebraicClosureOfRationalNumber has CHARNZ or PseudoAlgebraicC
--R coerce : PseudoAlgebraicClosureOfRationalNumber -> %
--R coerce : PseudoAlgebraicClosureOfRationalNumber -> %
--R definingPolynomial : % -> SparseUnivariatePolynomial(%)
--R definingPolynomial : () -> SparseUnivariatePolynomial(%)
--R degree : % -> OnePointCompletion(PositiveInteger)
--R discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if PseudoAlgebraicClosureOfRationalNumber has CHAR
--R distinguishedRootsOf : (SparseUnivariatePolynomial(%),%) -> List(%)
--R divide : (%,%) -> Record(quotient: %,remainder: %)
--R euclideanSize : % -> NonNegativeInteger
--R expressIdealMember : (List(%),%) -> Union(List(%),"failed")
--R exquo : (%,%) -> Union(%,"failed")
--R extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
--R extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
--R extensionDegree : () -> OnePointCompletion(PositiveInteger)
--R gcdPolynomial : (SparseUnivariatePolynomial(%),SparseUnivariatePolynomial(%)) -> SparseUnivariatePolynomial(
--R lcmCoef : (%,%) -> Record(llcmres: %,coeff1: %,coeff2: %)
--R lift : (%,%) -> SparseUnivariatePolynomial(%)
--R lift : % -> SparseUnivariatePolynomial(%)
--R multiEuclidean : (List(%),%) -> Union(List(%),"failed")
--R newElement : (SparseUnivariatePolynomial(%),Symbol) -> %
--R newElement : (SparseUnivariatePolynomial(%),%,Symbol) -> %
--R order : % -> OnePointCompletion(PositiveInteger) if PseudoAlgebraicClosureOfRationalNumber has CHARNZ or Pse
--R primeFrobenius : % -> % if PseudoAlgebraicClosureOfRationalNumber has CHARNZ or PseudoAlgebraicClosureOfRati
--R primeFrobenius : (%,NonNegativeInteger) -> % if PseudoAlgebraicClosureOfRationalNumber has CHARNZ or PseudoA
--R principalIdeal : List(%) -> Record(coef: List(%),generator: %)
--R reduce : SparseUnivariatePolynomial(%) -> %
```

```
--R retract : % -> PseudoAlgebraicClosureOfRationalNumber
--R retract : % -> PseudoAlgebraicClosureOfRationalNumber
--R retractIfCan : % -> Union(PseudoAlgebraicClosureOfRationalNumber,"failed")
--R retractIfCan : % -> Union(PseudoAlgebraicClosureOfRationalNumber,"failed")
--R retractIfCan : % -> Union(Fraction(Integer),"failed")
--R retractIfCan : % -> Union(Fraction(Integer),"failed")
--R retractIfCan : % -> Union(Integer,"failed")
--R subtractIfCan : (%,%) -> Union(%,"failed")
--R transcendenceDegree : () -> NonNegativeInteger
--R unitNormal : % -> Record(unit: %,canonical: %,associate: %)
--R
--R
--E 1

)spool
)lisp (bye)
```

— PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory.help —

```
====================================================================
PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory examples
====================================================================

This category exports the function for the domain
PseudoAlgebraicClosureOfAlgExtOfRationalNumber which implement dynamic
extension using the simple notion of tower extensions. A tower extension
T of the ground field K is any sequence of field extension
   (T : K_0, K_1, ..., K_i...,K_n)
where K_0 = K and for i =1,2,...,n,
      K_i is an extension of K_{i-1} of degree > 1
and defined by an  irreducible polynomial p(Z) in K_{i-1}.

Two towers
    (T_1: K_01, K_11,...,K_i1,...,K_n1)
and
    (T_2: K_02, K_12,...,K_i2,...,K_n2)
are said to be related if
    T_1 <= T_2 (or T_1 >= T_2),
that is if
    K_i1 = K_i2 for i=1,2,...,n1 (or i=1,2,...,n2).
Any algebraic operations defined for several elements
are only defined if all of the concerned elements are comming from
a set of related tour extensions.

See Also:
o )show PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory
```

See:

⇐ "PseudoAlgebraicClosureOfRationalNumberCategory" (PACRATC) 19.0.239 on page 1860

Exports:

0	1	-?	?**?
?*?	?+?	?-?	?/?
?=?	?^?	?~=?	?quo?
?rem?	algebraic?	associates?	characteristic
charthRoot	coerce	conjugate	definingPolynomial
degree	dimension	discreteLog	distinguishedRootsOf
divide	euclideanSize	expressIdealMember	exquo
extDegree	extendedEuclidean	extensionDegree	factor
Frobenius	fullOutput	gcd	gcdPolynomial
ground?	hash	inGroundField?	inv
latex	lcm	lift	maxTower
multiEuclidean	newElement	one?	order
previousTower	prime?	primeFrobenius	principalIdeal
recip	reduce	retract	retractIfCan
sample	setTower!	sizeLess?	squareFree
squareFreePart	subtractIfCan	transcendenceDegree	transcendent?
unit?	unitCanonical	unitNormal	vectorise
zero?			

Attributes Exported:

- **commutative("*")** is true if it has an operation $" * " : (D, D) - > D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.

These exports come from (p1860) PseudoAlgebraicClosureOfRationalNumberCategory

```
-? : % -> %
0 : () -> %
1 : () -> %
?**? : (%,Integer) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?*? : (%,Fraction Integer) -> %
?*? : (%,Fraction Integer) -> %
?*? : (Fraction Integer,%) -> %
?*? : (Fraction Integer,%) -> %
```

```
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
?/? : (%,%) -> %
?/? : (%,Fraction Integer) -> %
?=? : (%,%) -> Boolean
?^? : (%,Integer) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
?~=? : (%,%) -> Boolean
Frobenius : % -> % if Fraction Integer has FINITE
Frobenius : (%,NonNegativeInteger) -> % if Fraction Integer has FINITE
algebraic? : % -> Boolean
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed")
   if Fraction Integer has CHARNZ or Fraction Integer has FINITE
coerce : % -> %
coerce : % -> OutputForm
coerce : Fraction Integer -> %
coerce : Fraction Integer -> %
coerce : Fraction Integer -> %
coerce : Integer -> %
coerce : Integer -> %
conjugate : % -> %
definingPolynomial : % -> SparseUnivariatePolynomial %
definingPolynomial : () -> SparseUnivariatePolynomial %
degree : % -> OnePointCompletion PositiveInteger
dimension : () -> CardinalNumber
discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
   if Fraction Integer has CHARNZ or Fraction Integer has FINITE
distinguishedRootsOf : (SparseUnivariatePolynomial %,%) -> List %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extDegree : % -> PositiveInteger
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extensionDegree : () -> OnePointCompletion PositiveInteger
factor : % -> Factored %
fullOutput : % -> OutputForm
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
             SparseUnivariatePolynomial %) -> SparseUnivariatePolynomial %
ground? : % -> Boolean
hash : % -> SingleInteger
inGroundField? : % -> Boolean
inv : % -> %
```

```
 latex : % -> String
 lcm : (%,%) -> %
 lcm : List % -> %
 lift : % -> SparseUnivariatePolynomial %
 lift : (%,%) -> SparseUnivariatePolynomial %
 maxTower : List % -> %
 multiEuclidean : (List %,%) -> Union(List %,"failed")
 newElement : (SparseUnivariatePolynomial %,%,Symbol) -> %
 newElement : (SparseUnivariatePolynomial %,Symbol) -> %
 one? : % -> Boolean
 order : % -> OnePointCompletion PositiveInteger
    if Fraction Integer has CHARNZ or Fraction Integer has FINITE
 previousTower : % -> %
 prime? : % -> Boolean
 primeFrobenius : % -> %
    if Fraction Integer has CHARNZ or Fraction Integer has FINITE
 primeFrobenius : (%,NonNegativeInteger) -> %
    if Fraction Integer has CHARNZ or Fraction Integer has FINITE
 principalIdeal : List % -> Record(coef: List %,generator: %)
 recip : % -> Union(%,"failed")
 reduce : SparseUnivariatePolynomial % -> %
 retract : % -> Fraction Integer
 retract : % -> Fraction Integer
 retract : % -> Integer
 retractIfCan : % -> Union(Fraction Integer,"failed")
 retractIfCan : % -> Union(Fraction Integer,"failed")
 retractIfCan : % -> Union(Integer,"failed")
 sample : () -> %
 setTower! : % -> Void
 sizeLess? : (%,%) -> Boolean
 squareFree : % -> Factored %
 squareFreePart : % -> %
 subtractIfCan : (%,%) -> Union(%,"failed")
 transcendenceDegree : () -> NonNegativeInteger
 transcendent? : % -> Boolean
 unit? : % -> Boolean
 unitCanonical : % -> %
 unitNormal : % -> Record(unit: %,canonical: %,associate: %)
 vectorise : (%,%) -> Vector %
 zero? : % -> Boolean

These exports come from \refto{RetractableTo}
\begin{verbatim}
 coerce : PseudoAlgebraicClosureOfRationalNumber -> %
 coerce : PseudoAlgebraicClosureOfRationalNumber -> %
 retract : % -> PseudoAlgebraicClosureOfRationalNumber
 retract : % -> PseudoAlgebraicClosureOfRationalNumber
 retractIfCan : % -> Union(PseudoAlgebraicClosureOfRationalNumber,"failed")
 retractIfCan : % -> Union(PseudoAlgebraicClosureOfRationalNumber,"failed")
```

These exports come from (p1754) ExtensionField

```
 ?*? : (%,PseudoAlgebraicClosureOfRationalNumber) -> %
 ?*? : (PseudoAlgebraicClosureOfRationalNumber,%) -> %
 ?/? : (%,PseudoAlgebraicClosureOfRationalNumber) -> %
```

```
 charthRoot : % -> Union(%,"failed") if PseudoAlgebraicClosureOfRationalNumber has CHARNZ or PseudoAlgebraicClos
 discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if PseudoAlgebraicClosureOfRationalNumber has CHARNZ
 Frobenius : % -> % if PseudoAlgebraicClosureOfRationalNumber has FINITE or Fraction Integer has FINITE
 Frobenius : (%,NonNegativeInteger) -> % if PseudoAlgebraicClosureOfRationalNumber has FINITE or Fraction Intege
 order : % -> OnePointCompletion PositiveInteger if PseudoAlgebraicClosureOfRationalNumber has CHARNZ or PseudoA
 primeFrobenius : % -> % if PseudoAlgebraicClosureOfRationalNumber has CHARNZ or PseudoAlgebraicClosureOfRationa
 primeFrobenius : (%,NonNegativeInteger) -> % if PseudoAlgebraicClosureOfRationalNumber has CHARNZ or PseudoAlge
```

— PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory.html —

```
<body>
<h2>
<a href="http://axiom-developer.org/axiom-website/bookvol10.2.pdf#PACEXTC">
PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory (PACEXTC)</a></h2>
</body>
```

————

— category PACEXTC PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory —

```
)abbrev category PACEXTC PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory
++ Authors: Gaetan Hache
++ Date Created: jan 1998
++ Date Last Updated: May 2010 by Tim Daly
++ Description:
++ This category exports the function for the domain
++ PseudoAlgebraicClosureOfAlgExtOfRationalNumber which implement dynamic
++ extension using the simple notion of tower extensions. A tower extension
++ T of the ground field K is any sequence of field extension
++ (T : K0, K1, ..., Ki...,Kn) where K0 = K and for i =1,2,...,n,
++ Ki is an extension of K{i-1} of degree > 1 and defined by an
++ irreducible polynomial p(Z) in K{i-1}.
++ Two towers (T1: K01, K11,...,Ki1,...,Kn1) and
++ (T2: K02, K12,...,Ki2,...,Kn2)
++ are said to be related if T1 <= T2 (or T1 >= T2),
++ that is if Ki1 = Ki2 for i=1,2,...,n1 (or i=1,2,...,n2).
++ Any algebraic operations defined for several elements
++ are only defined if all of the concerned elements are comming from
++ a set of related tour extensions.

PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory() :
 Category == SIG where

  Q   ==> PseudoAlgebraicClosureOfRationalNumber
  PAC ==> PseudoAlgebraicClosureOfRationalNumberCategory
  RT  ==> RetractableTo(Q)
  EF  ==> ExtensionField(Q)

  SIG ==> Join(PAC,RT,EF)
```

————

— **PACEXTC.dotabb** —

```
"PACEXTC" [color=lightblue,href="bookvol10.2.pdf#nameddest=PACEXTC"];
"PACEXTC" -> "PACRATC"
"PACEXTC" -> "RETRACT"
"PACEXTC" -> "XF"
```

———-

— **PACEXTC.dotfull** —

```
"PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PACEXTC"];
   -> "PseudoAlgebraicClosureOfRationalNumberCategory"
"PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory"
   -> "RetractableTo(PseudoAlgebraicClosureOfRationalNumber)"
"PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory"
   -> "ExtensionField(PseudoAlgebraicClosureOfRationalNumber)"

"PseudoAlgebraicClosureOfRationalNumberCategory"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PACRATC"];

"RetractableTo(PseudoAlgebraicClosureOfRationalNumber)"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=RETRACT"];
 ->  "RetractableTo(a:Type)"

"RetractableTo(a:Type)"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=RETRACT"];

"ExtensionField(PseudoAlgebraicClosureOfRationalNumber)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XF"];
"ExtensionField(PseudoAlgebraicClosureOfRationalNumber)" ->
 ->  "ExtensionField(a:Field)"

"ExtensionField(a:Field)"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=XF"];
```

———-

— **PACEXTC.dotpic** —

```
digraph pic {
 fontsize=10;
 bgcolor="#ECEA81";
 node [shape=box, color=white, style=filled];

"PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PACEXTC"];
   -> "PseudoAlgebraicClosureOfRationalNumberCategory"
"PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory"
   -> "RetractableTo(PseudoAlgebraicClosureOfRationalNumber)"
"PseudoAlgebraicClosureOfAlgExtOfRationalNumberCategory"
```

```
   -> "ExtensionField(PseudoAlgebraicClosureOfRationalNumber)"

"RetractableTo(PseudoAlgebraicClosureOfRationalNumber)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(PseudoAlgebraicClosureOfRationalNumber)"
  -> "RETRACT..."

"ExtensionField(PseudoAlgebraicClosureOfRationalNumber)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XF"];
"ExtensionField(PseudoAlgebraicClosureOfRationalNumber)" ->
 ->  "XF..."

"XF..." [color=lightblue];
"RETRACT..." [color=lightblue];

}
```

Chapter 21

The bootstrap code

21.1 ABELGRP.lsp BOOTSTRAP

ABELGRP depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ABELGRP** category which we can write into the **MID** directory. We compile the lisp code and copy the **ABELGRP.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— ABELGRP.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |AbelianGroup;AL| (QUOTE NIL))

(DEFUN |AbelianGroup| NIL
 (declare (special |AbelianGroup;AL|))
   (COND
     (|AbelianGroup;AL|)
     (T (SETQ |AbelianGroup;AL| (|AbelianGroup;|)))))

(DEFUN |AbelianGroup;| NIL
  (PROG (#1=#:G82662)
   (RETURN
     (PROG1
       (LETT #1#
         (|Join|
           (|CancellationAbelianMonoid|)
           (|mkCategory|
             (QUOTE |domain|)
             (QUOTE (
               ((|-| (|$| |$|)) T)
               ((|-| (|$| |$| |$|)) T)
               ((|*| (|$| (|Integer|) |$|)) T)))
             NIL
             (QUOTE ((|Integer|)))
```

```
              NIL))
           |AbelianGroup|)
         (SETELT #1# 0 (QUOTE (|AbelianGroup|)))))))

(setf (get (quote |AbelianGroup|) (quote niladic)) t)
```

————-

21.2 ABELGRP-.lsp BOOTSTRAP

ABELGRP- depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ABELGRP-** category which we can write into the **MID** directory. We compile the lisp code and copy the **ABELGRP-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— ABELGRP-.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(DEFUN |ABELGRP-;-;3S;1| (|x| |y| |$|)
  (SPADCALL |x| (SPADCALL |y| (QREFELT |$| 7)) (QREFELT |$| 8)))

(DEFUN |ABELGRP-;subtractIfCan;2SU;2| (|x| |y| |$|)
  (CONS 0 (SPADCALL |x| |y| (QREFELT |$| 10))))

(DEFUN |ABELGRP-;*;Nni2S;3| (|n| |x| |$|)
  (SPADCALL |n| |x| (QREFELT |$| 14)))

(DEFUN |ABELGRP-;*;I2S;4| (|n| |x| |$|)
  (COND
    ((ZEROP |n|) (|spadConstant| |$| 17))
    ((|<| 0 |n|) (SPADCALL |n| |x| (QREFELT |$| 20)))
    ((QUOTE T)
      (SPADCALL (|-| |n|) (SPADCALL |x| (QREFELT |$| 7)) (QREFELT |$| 20)))))

(DEFUN |AbelianGroup&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|AbelianGroup&|))
        (LETT |dv$| (LIST (QUOTE |AbelianGroup&|) |DV$1|) . #1#)
        (LETT |$| (make-array 22) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        (COND
          ((|HasCategory| |#1| (QUOTE (|Ring|))))
          ((QUOTE T)
            (QSETREFV |$| 21
```

```
            (CONS (|dispatchFunction| |ABELGRP-;*;I2S;4|) |$|))))
      |$|))))

(setf (get
  (QUOTE |AbelianGroup&|)
  (QUOTE |infovec|))
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (0 . |-|)
        (5 . |+|)
        |ABELGRP-;-;3S;1|
        (11 . |-|)
        (|Union| |$| (QUOTE "failed"))
        |ABELGRP-;subtractIfCan;2SU;2|
        (|Integer|)
        (17 . |*|)
        (|NonNegativeInteger|)
        |ABELGRP-;*;Nni2S;3|
        (23 . |Zero|)
        (|PositiveInteger|)
        (|RepeatedDoubling| 6)
        (27 . |double|)
        (33 . |*|)))
    (QUOTE #(|subtractIfCan| 39 |-| 45 |*| 51))
    (QUOTE NIL)
    (CONS
      (|makeByteWordVec2| 1 (QUOTE NIL))
      (CONS
        (QUOTE #())
        (CONS
          (QUOTE #())
          (|makeByteWordVec2| 21
            (QUOTE (1 6 0 0 7 2 6 0 0 0 8 2 6 0 0 0 10 2 6 0 13 0 14 0 6 0 17
                    2 19 6 18 6 20 2 0 0 13 0 21 2 0 11 0 0 12 2 0 0 0 0 9 2
                    0 0 13 0 21 2 0 0 15 0 16))))))
    (QUOTE |lookupComplete|)))
```

———

21.3 ABELMON.lsp BOOTSTRAP

ABELMON which needs **ABELSG** which needs **SETCAT** which needs **SINT** which needs **UFD** which needs **GCDDOM** which needs **COMRING** which needs **RING** which needs **RNG** which needs **ABELGRP** which needs **CABMON** which needs **ABELMON**. We break this chain with **ABELMON.lsp** which we cache here. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ABELMON** category which we can write into the **MID** directory. We compile the lisp code and copy the **ABELMON.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— ABELMON.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |AbelianMonoid;AL| (QUOTE NIL))

(DEFUN |AbelianMonoid| NIL
 (declare (special |AbelianMonoid;AL|))
   (COND
     (|AbelianMonoid;AL|)
     (T (SETQ |AbelianMonoid;AL| (|AbelianMonoid;|)))))

(DEFUN |AbelianMonoid;| NIL
  (PROG (#1=#:G82595)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|AbelianSemiGroup|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|Zero| (|$|) |constant|) T)
                ((|sample| (|$|) |constant|) T)
                ((|zero?| ((|Boolean|) |$|)) T)
                ((|*| (|$| (|NonNegativeInteger|) |$|)) T)))
              NIL
              (QUOTE ((|NonNegativeInteger|) (|Boolean|)))
              NIL))
            |AbelianMonoid|)
        (SETELT #1# 0 (QUOTE (|AbelianMonoid|)))))))

(setf (get (QUOTE |AbelianMonoid|) (QUOTE NILADIC)) T)
```

———-

21.4 ABELMON-.lsp BOOTSTRAP

ABELMON- which needs **ABELSG** which needs **SETCAT** which needs **SINT** which needs **UFD** which needs **GCDDOM** which needs **COMRING** which needs **RING** which needs **RNG** which needs **ABELGRP** which needs **CABMON** which needs **ABELMON-**. We break this chain with **ABELMON-.lsp** which we cache here. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ABELMON-** category which we can write into the **MID** directory. We compile the lisp code and copy the **ABELMON-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— ABELMON-.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(DEFUN |ABELMON-;zero?;SB;1| (|x| |$|)
  (SPADCALL |x| (|spadConstant| |$| 7) (QREFELT |$| 9)))

(DEFUN |ABELMON-;*;Pi2S;2| (|n| |x| |$|)
  (SPADCALL |n| |x| (QREFELT |$| 12)))

(DEFUN |ABELMON-;sample;S;3| (|$|)
  (|spadConstant| |$| 7))

(DEFUN |ABELMON-;*;Nni2S;4| (|n| |x| |$|)
  (COND
    ((ZEROP |n|) (|spadConstant| |$| 7))
    ((QUOTE T) (SPADCALL |n| |x| (QREFELT |$| 17)))))

(DEFUN |AbelianMonoid&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|AbelianMonoid&|))
        (LETT |dv$| (LIST (QUOTE |AbelianMonoid&|) |DV$1|) . #1#)
        (LETT |$| (make-array 19) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        (COND
          ((|HasCategory| |#1| (QUOTE (|Ring|))))
          ((QUOTE T)
            (QSETREFV |$| 18
              (CONS (|dispatchFunction| |ABELMON-;*;Nni2S;4|) |$|)))) |$|))))

(setf (get
  (QUOTE |AbelianMonoid&|)
  (QUOTE |infovec|))
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (0 . |Zero|)
        (|Boolean|)
        (4 . |=|)
        |ABELMON-;zero?;SB;1|
        (|NonNegativeInteger|)
        (10 . |*|)
        (|PositiveInteger|)
        |ABELMON-;*;Pi2S;2|
        |ABELMON-;sample;S;3|
        (|RepeatedDoubling| 6)
        (16 . |double|)
        (22 . |*|)))
    (QUOTE #(|zero?| 28 |sample| 33 |*| 37))
    (QUOTE NIL)
```

```
  (CONS
    (|makeByteWordVec2| 1 (QUOTE NIL))
    (CONS
      (QUOTE #())
      (CONS
        (QUOTE #())
        (|makeByteWordVec2| 18
          (QUOTE (0 6 0 7 2 6 8 0 0 9 2 6 0 11 0 12 2 16 6 13 6 17 2 0 0 11
                  0 18 1 0 8 0 10 0 0 0 15 2 0 0 11 0 18 2 0 0 13 0 14))))))
 (QUOTE |lookupComplete|)))
```

———-

21.5 ABELSG.lsp BOOTSTRAP

ABELSG needs **SETCAT** which needs **SINT** which needs **UFD** which needs **GCDDOM** which needs **COMRING** which needs **RING** which needs **RNG** which needs **ABELGRP** which needs **CABMON** which needs **ABELMON** which needs **ABELSG**. We break this chain with **ABELSG.lsp** which we cache here. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ABELSG** category which we can write into the **MID** directory. We compile the lisp code and copy the **ABELSG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— ABELSG.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |AbelianSemiGroup;AL| (QUOTE NIL))

(DEFUN |AbelianSemiGroup| NIL
  (declare (special |AbelianSemiGroup;AL|))
    (COND
      (|AbelianSemiGroup;AL|)
      (T (SETQ |AbelianSemiGroup;AL| (|AbelianSemiGroup;|)))))

(DEFUN |AbelianSemiGroup;| NIL
  (PROG (#1=#:G82566)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|SetCategory|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|+| (|$| |$| |$|)) T)
                ((|*| (|$| (|PositiveInteger|) |$|)) T)))
              NIL
              (QUOTE ((|PositiveInteger|)))
              NIL))
```

```
      |AbelianSemiGroup|)
    (SETELT #1# 0 (QUOTE (|AbelianSemiGroup|)))))))

(setf (get (QUOTE |AbelianSemiGroup|) (QUOTE NILADIC)) T)
```

———-

21.6 ABELSG-.lsp BOOTSTRAP

ABELSG- needs **SETCAT** which needs **SINT** which needs **UFD** which needs **GCDDOM** which needs **COMRING** which needs **RING** which needs **RNG** which needs **ABELGRP** which needs **CABMON** which needs **ABELMON** which needs **ABELSG-**. We break this chain with **ABELSG-.lsp** which we cache here. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ABELSG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **ABELSG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— ABELSG-.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(DEFUN |ABELSG-;*;Pi2S;1| (|n| |x| |$|) (SPADCALL |n| |x| (QREFELT |$| 9)))

(DEFUN |AbelianSemiGroup&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|AbelianSemiGroup&|))
        (LETT |dv$| (LIST (QUOTE |AbelianSemiGroup&|) |DV$1|) . #1#)
        (LETT |$| (make-array 11) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        (COND
          ((|HasCategory| |#1| (QUOTE (|Ring|))))
          ((QUOTE T)
            (QSETREFV |$| 10
              (CONS (|dispatchFunction| |ABELSG-;*;Pi2S;1|) |$|))))
        |$|))))

(setf (get
  (QUOTE |AbelianSemiGroup&|)
  (QUOTE |infovec|))
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (|PositiveInteger|)
        (|RepeatedDoubling| 6)
        (0 . |double|)
```

```
        (6 . |*|)))
    (QUOTE #(|*| 12))
    (QUOTE NIL)
    (CONS
      (|makeByteWordVec2| 1 (QUOTE NIL))
      (CONS
        (QUOTE #())
        (CONS
          (QUOTE #())
          (|makeByteWordVec2| 10
            (QUOTE (2 8 6 7 6 9 2 0 0 7 0 10 2 0 0 7 0 10))))))
    (QUOTE |lookupComplete|)))
```

21.7 ALAGG.lsp BOOTSTRAP

ALAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ALAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **ALAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— ALAGG.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |AssociationListAggregate;CAT| (QUOTE NIL))

(SETQ |AssociationListAggregate;AL| (QUOTE NIL))

(DEFUN |AssociationListAggregate|
 (|&REST| #1=#:G88404 |&AUX| #2=#:G88402)
  (declare (special |AssociationListAggregate;AL|))
  (DSETQ #2# #1#)
  (LET (#3=#:G88403)
   (COND
    ((SETQ #3# (|assoc| (|devaluateList| #2#) |AssociationListAggregate;AL|))
      (CDR #3#))
    (T
      (SETQ |AssociationListAggregate;AL|
       (|cons5|
        (CONS
         (|devaluateList| #2#)
         (SETQ #3# (APPLY (FUNCTION |AssociationListAggregate;|) #2#)))
        |AssociationListAggregate;AL|)) #3#))))

(DEFUN |AssociationListAggregate;| (|t#1| |t#2|)
  (declare (special |AssociationListAggregate;CAT|))
 (PROG (#1=#:G88401)
  (RETURN
   (PROG1
```

```
    (LETT #1#
     (|sublisV|
      (PAIR
       (QUOTE (|t#1| |t#2|)) (LIST (|devaluate| |t#1|) (|devaluate| |t#2|)))
      (|sublisV|
       (PAIR
        (QUOTE (#2=#:G88400))
        (LIST (QUOTE (|Record| (|:| |key| |t#1|) (|:| |entry| |t#2|)))))
       (COND
        (|AssociationListAggregate;CAT|)
        ((QUOTE T)
         (LETT |AssociationListAggregate;CAT|
          (|Join|
           (|TableAggregate| (QUOTE |t#1|) (QUOTE |t#2|))
           (|ListAggregate| (QUOTE #2#))
           (|mkCategory|
            (QUOTE |domain|)
            (QUOTE
             (((|assoc|
                ((|Union|
                  (|Record| (|:| |key| |t#1|) (|:| |entry| |t#2|)) "failed")
                 |t#1| |$|))
                T)))
            NIL (QUOTE NIL) NIL))
          . #3=(|AssociationListAggregate|))))))
      . #3#)
    (SETELT #1# 0
     (LIST
      (QUOTE |AssociationListAggregate|)
      (|devaluate| |t#1|)
      (|devaluate| |t#2|)))))))
```

———-

21.8 CABMON.lsp BOOTSTRAP

CABMON which needs **ABELMON** which needs **ABELSG** which needs **SETCAT** which needs **SINT** which needs **UFD** which needs **GCDDOM** which needs **COMRING** which needs **RING** which needs **RNG** which needs **ABELGRP** which needs **CABMON**. We break this chain with **CABMON.lsp** which we cache here. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **CABMON** category which we can write into the **MID** directory. We compile the lisp code and copy the **CABMON.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— CABMON.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |CancellationAbelianMonoid;AL| (QUOTE NIL))

(DEFUN |CancellationAbelianMonoid| NIL
```

```
  (declare (special |CancellationAbelianMonoid;AL|))
    (COND
      (|CancellationAbelianMonoid;AL|)
      (T
        (SETQ
          |CancellationAbelianMonoid;AL|
          (|CancellationAbelianMonoid;|)))))

(DEFUN |CancellationAbelianMonoid;| NIL
  (PROG (#1=#:G82644)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|AbelianMonoid|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE
               (((|subtractIfCan| ((|Union| |$| "failed") |$| |$|)) T)))
              NIL
             (QUOTE NIL)
             NIL))
           |CancellationAbelianMonoid|)
        (SETELT #1# 0 (QUOTE (|CancellationAbelianMonoid|)))))))

(setf (get (QUOTE |CancellationAbelianMonoid|) (QUOTE NILADIC)) T)
```

———-

21.9 CLAGG.lsp BOOTSTRAP

CLAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **CLAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **CLAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— CLAGG.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |Collection;CAT| (QUOTE NIL))

(SETQ |Collection;AL| (QUOTE NIL))

(DEFUN |Collection| (#1=#:G82618)
 (declare (special |Collection;AL|))
 (LET (#2=#:G82619)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |Collection;AL|)) (CDR #2#))
   (T
```

```
    (SETQ |Collection;AL|
     (|cons5|
      (CONS
       (|devaluate| #1#)
       (SETQ #2# (|Collection;| #1#)))
      |Collection;AL|))
     #2#))))

(DEFUN |Collection;| (|t#1|)
 (declare (special |Collection;CAT|))
 (PROG (#1=#:G82617)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (COND
       (|Collection;CAT|)
       ((QUOTE T)
        (LETT |Collection;CAT|
         (|Join|
          (|HomogeneousAggregate| (QUOTE |t#1|))
          (|mkCategory|
           (QUOTE |domain|)
           (QUOTE (
            ((|construct| (|$| (|List| |t#1|))) T)
            ((|find| ((|Union| |t#1| "failed")
                      (|Mapping| (|Boolean|) |t#1|) |$|))
              T)
            ((|reduce| (|t#1| (|Mapping| |t#1| |t#1| |t#1|) |$|))
             (|has| |$| (ATTRIBUTE |finiteAggregate|)))
            ((|reduce| (|t#1| (|Mapping| |t#1| |t#1| |t#1|) |$| |t#1|))
             (|has| |$| (ATTRIBUTE |finiteAggregate|)))
            ((|remove| (|$| (|Mapping| (|Boolean|) |t#1|) |$|))
             (|has| |$| (ATTRIBUTE |finiteAggregate|)))
            ((|select| (|$| (|Mapping| (|Boolean|) |t#1|) |$|))
             (|has| |$| (ATTRIBUTE |finiteAggregate|)))
            ((|reduce| (|t#1| (|Mapping| |t#1| |t#1| |t#1|) |$| |t#1| |t#1|))
             (AND
              (|has| |t#1| (|SetCategory|))
              (|has| |$| (ATTRIBUTE |finiteAggregate|))))
            ((|remove| (|$| |t#1| |$|))
             (AND
              (|has| |t#1| (|SetCategory|))
              (|has| |$| (ATTRIBUTE |finiteAggregate|))))
            ((|removeDuplicates| (|$| |$|))
             (AND
              (|has| |t#1| (|SetCategory|))
              (|has| |$| (ATTRIBUTE |finiteAggregate|))))))
           (QUOTE (((|ConvertibleTo| (|InputForm|))
                    (|has| |t#1| (|ConvertibleTo| (|InputForm|))))))
           (QUOTE ((|List| |t#1|))) NIL))
         . #2=(|Collection|)))))
      . #2#)
```

```
(SETELT #1# 0 (LIST (QUOTE |Collection|) (|devaluate| |t#1|)))))))
```

———-

21.10 CLAGG-.lsp BOOTSTRAP

CLAGG- depends on **CLAGG**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **CLAGG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **CLAGG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— CLAGG-.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(DEFUN |CLAGG-;#;ANni;1| (|c| |$|) (LENGTH (SPADCALL |c| (QREFELT |$| 9))))

(DEFUN |CLAGG-;count;MANni;2| (|f| |c| |$|)
 (PROG (|x| #1=#:G82637 #2=#:G82634 #3=#:G82632 #4=#:G82633)
  (RETURN
   (SEQ
    (PROGN
     (LETT #4# NIL |CLAGG-;count;MANni;2|)
     (SEQ
      (LETT |x| NIL |CLAGG-;count;MANni;2|)
      (LETT #1# (SPADCALL |c| (QREFELT |$| 9)) |CLAGG-;count;MANni;2|)
      G190
      (COND
       ((OR (ATOM #1#) (PROGN (LETT |x| (CAR #1#) |CLAGG-;count;MANni;2|) NIL))
        (GO G191)))
      (SEQ
       (EXIT
        (COND
         ((SPADCALL |x| |f|)
          (PROGN
           (LETT #2# 1 |CLAGG-;count;MANni;2|)
           (COND
            (#4# (LETT #3# (|+| #3# #2#) |CLAGG-;count;MANni;2|))
            ((QUOTE T)
             (PROGN
              (LETT #3# #2# |CLAGG-;count;MANni;2|)
              (LETT #4# (QUOTE T) |CLAGG-;count;MANni;2|)))))))))
      (LETT #1# (CDR #1#) |CLAGG-;count;MANni;2|)
      (GO G190)
      G191
      (EXIT NIL))
     (COND (#4# #3#) ((QUOTE T) 0)))))))

(DEFUN |CLAGG-;any?;MAB;3| (|f| |c| |$|)
 (PROG (|x| #1=#:G82642 #2=#:G82640 #3=#:G82638 #4=#:G82639)
  (RETURN
```

```
   (SEQ
    (PROGN
     (LETT #4# NIL |CLAGG-;any?;MAB;3|)
     (SEQ
      (LETT |x| NIL |CLAGG-;any?;MAB;3|)
      (LETT #1# (SPADCALL |c| (QREFELT |$| 9)) |CLAGG-;any?;MAB;3|)
      G190
      (COND
       ((OR (ATOM #1#) (PROGN (LETT |x| (CAR #1#) |CLAGG-;any?;MAB;3|) NIL))
        (GO G191)))
      (SEQ
       (EXIT
        (PROGN
         (LETT #2# (SPADCALL |x| |f|) |CLAGG-;any?;MAB;3|)
         (COND
          (#4#
           (LETT #3#
            (COND (#3# (QUOTE T)) ((QUOTE T) #2#))
            |CLAGG-;any?;MAB;3|))
          ((QUOTE T)
           (PROGN
            (LETT #3# #2# |CLAGG-;any?;MAB;3|)
            (LETT #4# (QUOTE T) |CLAGG-;any?;MAB;3|)))))))
      (LETT #1# (CDR #1#) |CLAGG-;any?;MAB;3|)
      (GO G190)
      G191
      (EXIT NIL))
     (COND (#4# #3#) ((QUOTE T) (QUOTE NIL))))))))

(DEFUN |CLAGG-;every?;MAB;4| (|f| |c| |$|)
 (PROG (|x| #1=#:G82647 #2=#:G82645 #3=#:G82643 #4=#:G82644)
  (RETURN
   (SEQ
    (PROGN
     (LETT #4# NIL |CLAGG-;every?;MAB;4|)
     (SEQ
      (LETT |x| NIL |CLAGG-;every?;MAB;4|)
      (LETT #1# (SPADCALL |c| (QREFELT |$| 9)) |CLAGG-;every?;MAB;4|)
      G190
      (COND
       ((OR (ATOM #1#) (PROGN (LETT |x| (CAR #1#) |CLAGG-;every?;MAB;4|) NIL))
        (GO G191)))
      (SEQ
       (EXIT
        (PROGN
         (LETT #2# (SPADCALL |x| |f|) |CLAGG-;every?;MAB;4|)
         (COND
          (#4#
           (LETT #3#
            (COND (#3# #2#) ((QUOTE T) (QUOTE NIL)))
            |CLAGG-;every?;MAB;4|))
          ((QUOTE T)
           (PROGN
            (LETT #3# #2# |CLAGG-;every?;MAB;4|)
```

```
           (LETT #4# (QUOTE T) |CLAGG-;every?;MAB;4|)))))))
      (LETT #1# (CDR #1#) |CLAGG-;every?;MAB;4|)
      (GO G190)
      G191
      (EXIT NIL))
     (COND (#4# #3#) ((QUOTE T) (QUOTE T))))))))

(DEFUN |CLAGG-;find;MAU;5| (|f| |c| |$|)
 (SPADCALL |f| (SPADCALL |c| (QREFELT |$| 9)) (QREFELT |$| 18)))

(DEFUN |CLAGG-;reduce;MAS;6| (|f| |x| |$|)
 (SPADCALL |f| (SPADCALL |x| (QREFELT |$| 9)) (QREFELT |$| 21)))

(DEFUN |CLAGG-;reduce;MA2S;7| (|f| |x| |s| |$|)
 (SPADCALL |f| (SPADCALL |x| (QREFELT |$| 9)) |s| (QREFELT |$| 23)))

(DEFUN |CLAGG-;remove;M2A;8| (|f| |x| |$|)
 (SPADCALL
  (SPADCALL |f| (SPADCALL |x| (QREFELT |$| 9)) (QREFELT |$| 25))
  (QREFELT |$| 26)))

(DEFUN |CLAGG-;select;M2A;9| (|f| |x| |$|)
 (SPADCALL
  (SPADCALL |f| (SPADCALL |x| (QREFELT |$| 9)) (QREFELT |$| 28))
  (QREFELT |$| 26)))

(DEFUN |CLAGG-;remove;S2A;10| (|s| |x| |$|)
 (SPADCALL
  (CONS (FUNCTION |CLAGG-;remove;S2A;10!0|) (VECTOR |$| |s|))
  |x|
  (QREFELT |$| 31)))

(DEFUN |CLAGG-;remove;S2A;10!0| (|#1| |$$|)
 (SPADCALL |#1| (QREFELT |$$| 1) (QREFELT (QREFELT |$$| 0) 30)))

(DEFUN |CLAGG-;reduce;MA3S;11| (|f| |x| |s1| |s2| |$|)
 (SPADCALL |f| (SPADCALL |x| (QREFELT |$| 9)) |s1| |s2| (QREFELT |$| 33)))

(DEFUN |CLAGG-;removeDuplicates;2A;12| (|x| |$|)
 (SPADCALL
  (SPADCALL (SPADCALL |x| (QREFELT |$| 9)) (QREFELT |$| 35))
  (QREFELT |$| 26)))

(DEFUN |Collection&| (|#1| |#2|)
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|Collection&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |Collection&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (make-array 37) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3
     (LETT |pv$|
```

```
      (|buildPredVector| 0 0
       (LIST
        (|HasCategory| |#2| (QUOTE (|ConvertibleTo| (|InputForm|))))
        (|HasCategory| |#2| (QUOTE (|SetCategory|)))
        (|HasAttribute| |#1| (QUOTE |finiteAggregate|))))
      . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
    (COND
     ((|testBitVector| |pv$| 3)
      (PROGN
       (QSETREFV |$| 11 (CONS (|dispatchFunction| |CLAGG-;#;ANni;1|) |$|))
       (QSETREFV |$| 13 (CONS (|dispatchFunction| |CLAGG-;count;MANni;2|) |$|))
       (QSETREFV |$| 15 (CONS (|dispatchFunction| |CLAGG-;any?;MAB;3|) |$|))
       (QSETREFV |$| 16 (CONS (|dispatchFunction| |CLAGG-;every?;MAB;4|) |$|))
       (QSETREFV |$| 19 (CONS (|dispatchFunction| |CLAGG-;find;MAU;5|) |$|))
       (QSETREFV |$| 22 (CONS (|dispatchFunction| |CLAGG-;reduce;MAS;6|) |$|))
       (QSETREFV |$| 24 (CONS (|dispatchFunction| |CLAGG-;reduce;MA2S;7|) |$|))
       (QSETREFV |$| 27 (CONS (|dispatchFunction| |CLAGG-;remove;M2A;8|) |$|))
       (QSETREFV |$| 29 (CONS (|dispatchFunction| |CLAGG-;select;M2A;9|) |$|))
       (COND
        ((|testBitVector| |pv$| 2)
         (PROGN
          (QSETREFV |$| 32
           (CONS (|dispatchFunction| |CLAGG-;remove;S2A;10|) |$|))
          (QSETREFV |$| 34
           (CONS (|dispatchFunction| |CLAGG-;reduce;MA3S;11|) |$|))
          (QSETREFV |$| 36
           (CONS (|dispatchFunction| |CLAGG-;removeDuplicates;2A;12|)
                 |$|))))))))
    |$|))))

(setf (get
 (QUOTE |Collection&|)
 (QUOTE |infovec|))
 (LIST (QUOTE
  #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|) (|List| 7)
   (0 . |parts|) (|NonNegativeInteger|) (5 . |#|) (|Mapping| 14 7)
   (10 . |count|) (|Boolean|) (16 . |any?|) (22 . |every?|)
   (|Union| 7 (QUOTE "failed")) (28 . |find|) (34 . |find|)
   (|Mapping| 7 7 7) (40 . |reduce|) (46 . |reduce|) (52 . |reduce|)
   (59 . |reduce|) (66 . |remove|) (72 . |construct|) (77 . |remove|)
   (83 . |select|) (89 . |select|) (95 . |=|) (101 . |remove|)
   (107 . |remove|) (113 . |reduce|) (121 . |reduce|)
   (129 . |removeDuplicates|) (134 . |removeDuplicates|)))
   (QUOTE #(|select| 139 |removeDuplicates| 145 |remove| 150 |reduce|
   162 |find| 183 |every?| 189 |count| 195 |any?| 201 |#| 207))
   (QUOTE NIL)
   (CONS
    (|makeByteWordVec2| 1 (QUOTE NIL))
    (CONS
     (QUOTE #())
     (CONS
```

```
      (QUOTE #())
      (|makeByteWordVec2| 36
       (QUOTE (1 6 8 0 9 1 0 10 0 11 2 0 10 12 0 13 2 0 14 12 0 15 2 0 14
        12 0 16 2 8 17 12 0 18 2 0 17 12 0 19 2 8 7 20 0 21 2 0 7 20 0 22
        3 8 7 20 0 7 23 3 0 7 20 0 7 24 2 8 0 12 0 25 1 6 0 8 26 2 0 0 12
        0 27 2 8 0 12 0 28 2 0 0 12 0 29 2 7 14 0 0 30 2 6 0 12 0 31 2 0 0
        7 0 32 4 8 7 20 0 7 7 33 4 0 7 20 0 7 7 34 1 8 0 0 35 1 0 0 0 36 2
        0 0 12 0 29 1 0 0 0 36 2 0 0 7 0 32 2 0 0 12 0 27 4 0 7 20 0 7 7 34
        3 0 7 20 0 7 24 2 0 7 20 0 22 2 0 17 12 0 19 2 0 14 12 0 16 2 0 10
        12 0 13 2 0 14 12 0 15 1 0 10 0 11))))))
    (QUOTE |lookupComplete|)))
```

21.11 COMRING.lsp BOOTSTRAP

COMRING depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **COMRING** category which we can write into the **MID** directory. We compile the lisp code and copy the **COMRING.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— COMRING.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |CommutativeRing;AL| (QUOTE NIL))

(DEFUN |CommutativeRing| NIL
  (declare (special |CommutativeRing;AL|))
    (COND
      (|CommutativeRing;AL|)
      (T (SETQ |CommutativeRing;AL| (|CommutativeRing;|)))))

(DEFUN |CommutativeRing;| NIL
  (PROG (#1=#:G82890)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|Ring|)
            (|BiModule| (QUOTE |$|) (QUOTE |$|))
            (|mkCategory|
              (QUOTE |package|)
              NIL
              (QUOTE (((|commutative| "*") T)))
              (QUOTE NIL)
              NIL))
           |CommutativeRing|)
        (SETELT #1# 0 (QUOTE (|CommutativeRing|)))))))

(setf (get (QUOTE |CommutativeRing|) (QUOTE NILADIC)) T)
```

——–

21.12 DIFRING.lsp BOOTSTRAP

DIFRING needs **INT** which needs **DIFRING**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **DIFRING** category which we can write into the **MID** directory. We compile the lisp code and copy the **DIFRING.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— DIFRING.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |DifferentialRing;AL| (QUOTE NIL))

(DEFUN |DifferentialRing| NIL
  (declare (special |DifferentialRing;AL|))
    (COND
      (|DifferentialRing;AL|)
      (T (SETQ |DifferentialRing;AL| (|DifferentialRing;|)))))

(DEFUN |DifferentialRing;| NIL
  (PROG (#1=#:G84563)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|Ring|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE
                (((|differentiate| (|$| |$|)) T)
                 ((D (|$| |$|)) T)
                 ((|differentiate| (|$| |$| (|NonNegativeInteger|))) T)
                 ((D (|$| |$| (|NonNegativeInteger|))) T)))
              NIL
              (QUOTE ((|NonNegativeInteger|)))
              NIL))
          |DifferentialRing|)
        (SETELT #1# 0 (QUOTE (|DifferentialRing|)))))))

(setf (get (QUOTE |DifferentialRing|) (QUOTE NILADIC)) T)
```

——–

21.13 DIFRING-.lsp BOOTSTRAP

DIFRING- needs **DIFRING**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **DIFRING-** category which we can write into the **MID** directory. We compile the lisp code and copy the **DIFRING-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— DIFRING-.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(DEFUN |DIFRING-;D;2S;1| (|r| |$|)
  (SPADCALL |r| (QREFELT |$| 7)))

(DEFUN |DIFRING-;differentiate;SNniS;2| (|r| |n| |$|)
  (PROG (|i|)
    (RETURN
      (SEQ
        (SEQ
          (LETT |i| 1 |DIFRING-;differentiate;SNniS;2|)
          G190
          (COND ((QSGREATERP |i| |n|) (GO G191)))
          (SEQ
            (EXIT
              (LETT |r|
                (SPADCALL |r| (QREFELT |$| 7))
                |DIFRING-;differentiate;SNniS;2|)))
          (LETT |i| (QSADD1 |i|) |DIFRING-;differentiate;SNniS;2|)
          (GO G190)
          G191
          (EXIT NIL))
        (EXIT |r|)))))

(DEFUN |DIFRING-;D;SNniS;3| (|r| |n| |$|)
  (SPADCALL |r| |n| (QREFELT |$| 11)))

(DEFUN |DifferentialRing&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|DifferentialRing&|))
        (LETT |dv$| (LIST (QUOTE |DifferentialRing&|) |DV$1|) . #1#)
        (LETT |$| (make-array 13) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        |$|))))

(setf (get
  (QUOTE |DifferentialRing&|)
  (QUOTE |infovec|))
```

```
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (0 . |differentiate|)
        |DIFRING-;D;2S;1|
        (|NonNegativeInteger|)
        |DIFRING-;differentiate;SNniS;2|
        (5 . |differentiate|)
        |DIFRING-;D;SNniS;3|))
    (QUOTE #(|differentiate| 11 D 17))
    (QUOTE NIL)
    (CONS
      (|makeByteWordVec2| 1 (QUOTE NIL))
      (CONS
        (QUOTE #())
        (CONS
          (QUOTE #())
          (|makeByteWordVec2| 12
            (QUOTE
             (1 6 0 0 7 2 6 0 0 9 11 2 0 0 0 9 10 2 0 0 0 9 12 1 0 0 0 8))))))
    (QUOTE |lookupComplete|)))
```

————-

21.14 DIVRING.lsp BOOTSTRAP

DIVRING depends on **QFCAT** which eventually depends on **DIVRING**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **DIVRING** category which we can write into the **MID** directory. We compile the lisp code and copy the **DIVRING.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— DIVRING.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |DivisionRing;AL| (QUOTE NIL))

(DEFUN |DivisionRing| NIL
  (declare (special |DivisionRing;AL|))
    (COND
      (|DivisionRing;AL|)
      (T (SETQ |DivisionRing;AL| (|DivisionRing;|)))))

(DEFUN |DivisionRing;| NIL
  (PROG (#1=#:G84033)
    (RETURN
      (PROG1
        (LETT #1#
```

```
          (|sublisV|
            (PAIR
              (QUOTE (#2=#:G84032))
              (LIST (QUOTE (|Fraction| (|Integer|)))))
            (|Join|
              (|EntireRing|)
              (|Algebra| (QUOTE #2#))
              (|mkCategory|
                (QUOTE |domain|)
                (QUOTE (
                  ((|**| (|$| |$| (|Integer|))) T)
                  ((|^| (|$| |$| (|Integer|))) T)
                  ((|inv| (|$| |$|)) T)))
                NIL
                (QUOTE ((|Integer|)))
                NIL)))
          |DivisionRing|)
        (SETELT #1# 0 (QUOTE (|DivisionRing|)))))))

(setf (get (QUOTE |DivisionRing|) (QUOTE NILADIC)) T)
```

———-

21.15 DIVRING-.lsp BOOTSTRAP

DIVRING- depends on **DIVRING**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **DIVRING-** category which we can write into the **MID** directory. We compile the lisp code and copy the **DIVRING-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— DIVRING-.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(DEFUN |DIVRING-;^;SIS;1| (|x| |n| |$|)
  (SPADCALL |x| |n| (QREFELT |$| 8)))

(DEFUN |DIVRING-;**;SIS;2| (|x| |n| |$|)
  (COND
    ((ZEROP |n|) (|spadConstant| |$| 10))
    ((SPADCALL |x| (QREFELT |$| 12))
      (COND
        ((|<| |n| 0) (|error| "division by zero"))
        ((QUOTE T) |x|)))
    ((|<| |n| 0)
      (SPADCALL (SPADCALL |x| (QREFELT |$| 14)) (|-| |n|) (QREFELT |$| 17)))
    ((QUOTE T) (SPADCALL |x| |n| (QREFELT |$| 17)))))

(DEFUN |DIVRING-;*;F2S;3| (|q| |x| |$|)
  (SPADCALL
```

```
    (SPADCALL
      (SPADCALL |q| (QREFELT |$| 20))
      (SPADCALL
        (SPADCALL (SPADCALL |q| (QREFELT |$| 21)) (QREFELT |$| 22))
        (QREFELT |$| 14))
      (QREFELT |$| 23))
    |x|
    (QREFELT |$| 24)))

(DEFUN |DivisionRing&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|DivisionRing&|))
        (LETT |dv$| (LIST (QUOTE |DivisionRing&|) |DV$1|) . #1#)
        (LETT |$| (make-array 27) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        |$|))))

(setf (get
  (QUOTE |DivisionRing&|)
  (QUOTE |infovec|))
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (|Integer|)
        (0 . |**|)
        |DIVRING-;^;SIS;1|
        (6 . |One|)
        (|Boolean|)
        (10 . |zero?|)
        (15 . |Zero|)
        (19 . |inv|)
        (|PositiveInteger|)
        (|RepeatedSquaring| 6)
        (24 . |expt|)
        |DIVRING-;**;SIS;2|
        (|Fraction| 7)
        (30 . |numer|)
        (35 . |denom|)
        (40 . |coerce|)
        (45 . |*|)
        (51 . |*|)
        |DIVRING-;*;F2S;3|
        (|NonNegativeInteger|)))
    (QUOTE #(|^| 57 |**| 63 |*| 69))
    (QUOTE NIL)
    (CONS
      (|makeByteWordVec2| 1 (QUOTE NIL))
      (CONS
```

```
        (QUOTE #())
        (CONS
          (QUOTE #())
            (|makeByteWordVec2| 25
              (QUOTE
                (2 6 0 0 7 8 0 6 0 10 1 6 11 0 12 0 6 0 13 1 6 0 0 14 2 16 6
                 6 15 17 1 19 7 0 20 1 19 7 0 21 1 6 0 7 22 2 6 0 7 0 23 2 6
                 0 0 0 24 2 0 0 0 7 9 2 0 0 0 7 18 2 0 0 19 0 25))))))
    (QUOTE |lookupComplete|)))
```

———-

21.16 ES.lsp BOOTSTRAP

ES depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ES** category which we can write into the **MID** directory. We compile the lisp code and copy the **ES.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— ES.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |ExpressionSpace;AL| (QUOTE NIL))

(DEFUN |ExpressionSpace| NIL
 (declare (special |ExpressionSpace;AL|))
  (COND
   (|ExpressionSpace;AL|)
   (T (SETQ |ExpressionSpace;AL| (|ExpressionSpace;|)))))

(DEFUN |ExpressionSpace;| NIL
 (PROG (#1=#:G82342)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR
       (QUOTE (#2=#:G82340 #3=#:G82341))
       (LIST (QUOTE (|Kernel| |$|)) (QUOTE (|Kernel| |$|))))
      (|Join|
       (|OrderedSet|)
       (|RetractableTo| (QUOTE #2#))
       (|InnerEvalable| (QUOTE #3#) (QUOTE |$|))
       (|Evalable| (QUOTE |$|))
       (|mkCategory|
        (QUOTE |domain|)
        (QUOTE (
         ((|elt| (|$| (|BasicOperator|) |$|)) T)
         ((|elt| (|$| (|BasicOperator|) |$| |$|)) T)
```

```
  ((|elt| (|$| (|BasicOperator|) |$| |$| |$|)) T)
  ((|elt| (|$| (|BasicOperator|) |$| |$| |$| |$|)) T)
  ((|elt| (|$| (|BasicOperator|) (|List| |$|))) T)
  ((|subst| (|$| |$| (|Equation| |$|))) T)
  ((|subst| (|$| |$| (|List| (|Equation| |$|)))) T)
  ((|subst| (|$| |$| (|List| (|Kernel| |$|)) (|List| |$|))) T)
  ((|box| (|$| |$|)) T)
  ((|box| (|$| (|List| |$|))) T)
  ((|paren| (|$| |$|)) T)
  ((|paren| (|$| (|List| |$|))) T)
  ((|distribute| (|$| |$|)) T)
  ((|distribute| (|$| |$| |$|)) T)
  ((|height| ((|NonNegativeInteger|) |$|)) T)
  ((|mainKernel| ((|Union| (|Kernel| |$|) "failed") |$|)) T)
  ((|kernels| ((|List| (|Kernel| |$|)) |$|)) T)
  ((|tower| ((|List| (|Kernel| |$|)) |$|)) T)
  ((|operators| ((|List| (|BasicOperator|)) |$|)) T)
  ((|operator| ((|BasicOperator|) (|BasicOperator|))) T)
  ((|belong?| ((|Boolean|) (|BasicOperator|))) T)
  ((|is?| ((|Boolean|) |$| (|BasicOperator|))) T)
  ((|is?| ((|Boolean|) |$| (|Symbol|))) T)
  ((|kernel| (|$| (|BasicOperator|) |$|)) T)
  ((|kernel| (|$| (|BasicOperator|) (|List| |$|))) T)
  ((|map| (|$| (|Mapping| |$| |$|) (|Kernel| |$|))) T)
  ((|freeOf?| ((|Boolean|) |$| |$|)) T)
  ((|freeOf?| ((|Boolean|) |$| (|Symbol|))) T)
  ((|eval| (|$| |$| (|List| (|Symbol|)) (|List| (|Mapping| |$| |$|))))
    T)
  ((|eval|
   (|$| |$| (|List| (|Symbol|)) (|List| (|Mapping| |$| (|List| |$|)))))
   T)
  ((|eval| (|$| |$| (|Symbol|) (|Mapping| |$| (|List| |$|)))) T)
  ((|eval| (|$| |$| (|Symbol|) (|Mapping| |$| |$|))) T)
  ((|eval|
   (|$| |$| (|List| (|BasicOperator|)) (|List| (|Mapping| |$| |$|))))
   T)
  ((|eval|
   (|$| |$| (|List| (|BasicOperator|))
     (|List| (|Mapping| |$| (|List| |$|)))))
   T)
  ((|eval| (|$| |$| (|BasicOperator|) (|Mapping| |$| (|List| |$|)))) T)
  ((|eval| (|$| |$| (|BasicOperator|) (|Mapping| |$| |$|))) T)
  ((|minPoly|
   ((|SparseUnivariatePolynomial| |$|) (|Kernel| |$|)))
   (|has| |$| (|Ring|)))
  ((|definingPolynomial| (|$| |$|)) (|has| |$| (|Ring|)))
  ((|even?|
   ((|Boolean|) |$|)) (|has| |$| (|RetractableTo| (|Integer|))))
  ((|odd?|
   ((|Boolean|) |$|)) (|has| |$| (|RetractableTo| (|Integer|))))))
 NIL
 (QUOTE (
  (|Boolean|)
  (|SparseUnivariatePolynomial| |$|)
```

```
         (|Kernel| |$|)
         (|BasicOperator|)
         (|List| (|BasicOperator|))
         (|List| (|Mapping| |$| (|List| |$|)))
         (|List| (|Mapping| |$| |$|))
         (|Symbol|)
         (|List| (|Symbol|))
         (|List| |$|)
         (|List| (|Kernel| |$|))
         (|NonNegativeInteger|)
         (|List| (|Equation| |$|))
         (|Equation| |$|)))
        NIL)))
     |ExpressionSpace|)
    (SETELT #1# 0 (QUOTE (|ExpressionSpace|)))))))

(setf (get (QUOTE |ExpressionSpace|) (QUOTE NILADIC)) T)
```

————-

21.17 ES-.lsp BOOTSTRAP

ES- depends on **ES**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ES-** category which we can write into the **MID** directory. We compile the lisp code and copy the **ES-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— ES-.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(DEFUN |ES-;box;2S;1| (|x| |$|)
 (SPADCALL (LIST |x|) (QREFELT |$| 16)))

(DEFUN |ES-;paren;2S;2| (|x| |$|)
 (SPADCALL (LIST |x|) (QREFELT |$| 18)))

(DEFUN |ES-;belong?;BoB;3| (|op| |$|)
 (COND
  ((SPADCALL |op| (QREFELT |$| 13) (QREFELT |$| 21)) (QUOTE T))
  ((QUOTE T) (SPADCALL |op| (QREFELT |$| 14) (QREFELT |$| 21)))))

(DEFUN |ES-;listk| (|f| |$|)
 (SPADCALL (|ES-;allKernels| |f| |$|) (QREFELT |$| 25)))

(DEFUN |ES-;tower;SL;5| (|f| |$|)
 (SPADCALL (|ES-;listk| |f| |$|) (QREFELT |$| 26)))

(DEFUN |ES-;allk| (|l| |$|)
 (PROG (#1=#:G82361 |f| #2=#:G82362)
  (RETURN
```

```
   (SEQ
    (SPADCALL
     (ELT |$| 30)
     (PROGN
      (LETT #1# NIL |ES-;allk|)
      (SEQ
       (LETT |f| NIL |ES-;allk|)
       (LETT #2# |l| |ES-;allk|)
       G190
       (COND
        ((OR (ATOM #2#)
             (PROGN (LETT |f| (CAR #2#) |ES-;allk|) NIL))
         (GO G191)))
       (SEQ (EXIT (LETT #1# (CONS (|ES-;allKernels| |f| |$|) #1#) |ES-;allk|)))
       (LETT #2# (CDR #2#) |ES-;allk|) (GO G190) G191 (EXIT (NREVERSE0 #1#))))
     (SPADCALL NIL (QREFELT |$| 29))
     (QREFELT |$| 33))))))

(DEFUN |ES-;operators;SL;7| (|f| |$|)
 (PROG (#1=#:G82365 |k| #2=#:G82366)
  (RETURN
   (SEQ
    (PROGN
     (LETT #1# NIL |ES-;operators;SL;7|)
     (SEQ
      (LETT |k| NIL |ES-;operators;SL;7|)
      (LETT #2# (|ES-;listk| |f| |$|) |ES-;operators;SL;7|)
      G190
      (COND
       ((OR (ATOM #2#) (PROGN (LETT |k| (CAR #2#) |ES-;operators;SL;7|) NIL))
         (GO G191)))
      (SEQ
       (EXIT
        (LETT #1#
         (CONS (SPADCALL |k| (QREFELT |$| 35)) #1#) |ES-;operators;SL;7|)))
      (LETT #2# (CDR #2#) |ES-;operators;SL;7|)
      (GO G190)
      G191
      (EXIT (NREVERSE0 #1#))))))))

(DEFUN |ES-;height;SNni;8| (|f| |$|)
 (PROG (#1=#:G82371 |k| #2=#:G82372)
  (RETURN
   (SEQ
    (SPADCALL
     (ELT |$| 41)
     (PROGN
      (LETT #1# NIL |ES-;height;SNni;8|)
      (SEQ
       (LETT |k| NIL |ES-;height;SNni;8|)
       (LETT #2# (SPADCALL |f| (QREFELT |$| 38)) |ES-;height;SNni;8|)
       G190
       (COND
        ((OR (ATOM #2#) (PROGN (LETT |k| (CAR #2#) |ES-;height;SNni;8|) NIL))
```

```
          (GO G191)))
       (SEQ
        (EXIT
         (LETT #1#
          (CONS (SPADCALL |k| (QREFELT |$| 40)) #1#) |ES-;height;SNni;8|)))
       (LETT #2# (CDR #2#) |ES-;height;SNni;8|)
       (GO G190)
       G191
       (EXIT (NREVERSEO #1#))))
     0
    (QREFELT |$| 44))))))

(DEFUN |ES-;freeOf?;SSB;9| (|x| |s| |$|)
 (PROG (#1=#:G82377 |k| #2=#:G82378)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |s|
      (PROGN
       (LETT #1# NIL |ES-;freeOf?;SSB;9|)
       (SEQ
        (LETT |k| NIL |ES-;freeOf?;SSB;9|)
        (LETT #2# (|ES-;listk| |x| |$|) |ES-;freeOf?;SSB;9|)
        G190
        (COND
         ((OR (ATOM #2#) (PROGN (LETT |k| (CAR #2#) |ES-;freeOf?;SSB;9|) NIL))
           (GO G191)))
        (SEQ
         (EXIT
          (LETT #1#
           (CONS (SPADCALL |k| (QREFELT |$| 46)) #1#) |ES-;freeOf?;SSB;9|)))
        (LETT #2# (CDR #2#) |ES-;freeOf?;SSB;9|)
        (GO G190)
        G191
        (EXIT (NREVERSEO #1#))))
       (QREFELT |$| 48))
      (QUOTE NIL))
     ((QUOTE T) (QUOTE T)))))))

(DEFUN |ES-;distribute;2S;10| (|x| |$|)
 (PROG (#1=#:G82381 |k| #2=#:G82382)
  (RETURN
   (SEQ
    (|ES-;unwrap|
     (PROGN (LETT #1# NIL |ES-;distribute;2S;10|)
      (SEQ
       (LETT |k| NIL |ES-;distribute;2S;10|)
       (LETT #2# (|ES-;listk| |x| |$|) |ES-;distribute;2S;10|)
       G190
       (COND
        ((OR
          (ATOM #2#)
          (PROGN (LETT |k| (CAR #2#) |ES-;distribute;2S;10|) NIL))
           (GO G191)))
```

```
       (SEQ
        (EXIT
         (COND
          ((SPADCALL |k| (QREFELT |$| 13) (QREFELT |$| 50))
            (LETT #1# (CONS |k| #1#) |ES-;distribute;2S;10|)))))
       (LETT #2# (CDR #2#) |ES-;distribute;2S;10|)
       (GO G190)
       G191
       (EXIT (NREVERSE0 #1#))))
     |x|
     |$|)))))

(DEFUN |ES-;box;LS;11| (|l| |$|)
 (SPADCALL (QREFELT |$| 14) |l| (QREFELT |$| 52)))

(DEFUN |ES-;paren;LS;12| (|l| |$|)
 (SPADCALL (QREFELT |$| 13) |l| (QREFELT |$| 52)))

(DEFUN |ES-;freeOf?;2SB;13| (|x| |k| |$|)
 (COND
  ((SPADCALL
    (SPADCALL |k| (QREFELT |$| 56))
    (|ES-;listk| |x| |$|)
    (QREFELT |$| 57))
     (QUOTE NIL))
  ((QUOTE T) (QUOTE T))))

(DEFUN |ES-;kernel;Bo2S;14| (|op| |arg| |$|)
 (SPADCALL |op| (LIST |arg|) (QREFELT |$| 59)))

(DEFUN |ES-;elt;Bo2S;15| (|op| |x| |$|)
 (SPADCALL |op| (LIST |x|) (QREFELT |$| 52)))

(DEFUN |ES-;elt;Bo3S;16| (|op| |x| |y| |$|)
 (SPADCALL |op| (LIST |x| |y|) (QREFELT |$| 52)))

(DEFUN |ES-;elt;Bo4S;17| (|op| |x| |y| |z| |$|)
 (SPADCALL |op| (LIST |x| |y| |z|) (QREFELT |$| 52)))

(DEFUN |ES-;elt;Bo5S;18| (|op| |x| |y| |z| |t| |$|)
 (SPADCALL |op| (LIST |x| |y| |z| |t|) (QREFELT |$| 52)))

(DEFUN |ES-;eval;SSMS;19| (|x| |s| |f| |$|)
 (SPADCALL |x| (LIST |s|) (LIST |f|) (QREFELT |$| 67)))

(DEFUN |ES-;eval;SBoMS;20| (|x| |s| |f| |$|)
 (SPADCALL
  |x|
  (LIST (SPADCALL |s| (QREFELT |$| 69)))
  (LIST |f|)
  (QREFELT |$| 67)))

(DEFUN |ES-;eval;SSMS;21| (|x| |s| |f| |$|)
 (SPADCALL
```

```
  |x|
  (LIST |s|)
  (LIST (CONS (FUNCTION |ES-;eval;SSMS;21!0|) (VECTOR |f| |$|)))
  (QREFELT |$| 67)))

(DEFUN |ES-;eval;SSMS;21!0| (|#1| |$$|)
 (SPADCALL (SPADCALL |#1| (QREFELT (QREFELT |$$| 1) 72)) (QREFELT |$$| 0)))

(DEFUN |ES-;eval;SBoMS;22| (|x| |s| |f| |$|)
 (SPADCALL
  |x|
  (LIST |s|)
  (LIST (CONS (FUNCTION |ES-;eval;SBoMS;22!0|) (VECTOR |f| |$|)))
  (QREFELT |$| 75)))

(DEFUN |ES-;eval;SBoMS;22!0| (|#1| |$$|)
 (SPADCALL (SPADCALL |#1| (QREFELT (QREFELT |$$| 1) 72)) (QREFELT |$$| 0)))

(DEFUN |ES-;subst;SES;23| (|x| |e| |$|)
 (SPADCALL |x| (LIST |e|) (QREFELT |$| 78)))

(DEFUN |ES-;eval;SLLS;24| (|x| |ls| |lf| |$|)
 (PROG (#1=#:G82403 |f| #2=#:G82404)
  (RETURN
   (SEQ
    (SPADCALL
     |x|
     |ls|
     (PROGN
      (LETT #1# NIL |ES-;eval;SLLS;24|)
      (SEQ
       (LETT |f| NIL |ES-;eval;SLLS;24|)
       (LETT #2# |lf| |ES-;eval;SLLS;24|)
       G190
       (COND
        ((OR (ATOM #2#) (PROGN (LETT |f| (CAR #2#) |ES-;eval;SLLS;24|) NIL))
          (GO G191)))
       (SEQ
        (EXIT
         (LETT #1#
          (CONS (CONS (FUNCTION |ES-;eval;SLLS;24!0|) (VECTOR |f| |$|)) #1#)
          |ES-;eval;SLLS;24|)))
       (LETT #2# (CDR #2#) |ES-;eval;SLLS;24|)
       (GO G190)
       G191
       (EXIT (NREVERSE0 #1#))))
     (QREFELT |$| 75))))))

(DEFUN |ES-;eval;SLLS;24!0| (|#1| |$$|)
 (SPADCALL (SPADCALL |#1| (QREFELT (QREFELT |$$| 1) 72)) (QREFELT |$$| 0)))

(DEFUN |ES-;eval;SLLS;25| (|x| |ls| |lf| |$|)
 (PROG (#1=#:G82407 |f| #2=#:G82408)
  (RETURN
```

```
   (SEQ
    (SPADCALL
     |x|
     |ls|
     (PROGN
      (LETT #1# NIL |ES-;eval;SLLS;25|)
      (SEQ
       (LETT |f| NIL |ES-;eval;SLLS;25|)
       (LETT #2# |lf| |ES-;eval;SLLS;25|)
       G190
       (COND
        ((OR (ATOM #2#) (PROGN (LETT |f| (CAR #2#) |ES-;eval;SLLS;25|) NIL))
          (GO G191)))
       (SEQ
        (EXIT
         (LETT #1#
          (CONS (CONS (FUNCTION |ES-;eval;SLLS;25!0|) (VECTOR |f| |$|)) #1#)
          |ES-;eval;SLLS;25|)))
       (LETT #2# (CDR #2#) |ES-;eval;SLLS;25|)
       (GO G190)
       G191
       (EXIT (NREVERSE0 #1#))))
     (QREFELT |$| 67))))))

(DEFUN |ES-;eval;SLLS;25!0| (|#1| |$$|)
 (SPADCALL (SPADCALL |#1| (QREFELT (QREFELT |$$| 1) 72)) (QREFELT |$$| 0)))

(DEFUN |ES-;eval;SLLS;26| (|x| |ls| |lf| |$|)
 (PROG (#1=#:G82412 |s| #2=#:G82413)
  (RETURN
   (SEQ
    (SPADCALL
     |x|
     (PROGN
      (LETT #1# NIL |ES-;eval;SLLS;26|)
      (SEQ
       (LETT |s| NIL |ES-;eval;SLLS;26|)
       (LETT #2# |ls| |ES-;eval;SLLS;26|)
       G190
       (COND
        ((OR (ATOM #2#) (PROGN (LETT |s| (CAR #2#) |ES-;eval;SLLS;26|) NIL))
          (GO G191)))
       (SEQ
        (EXIT
         (LETT #1#
          (CONS (SPADCALL |s| (QREFELT |$| 69)) #1#) |ES-;eval;SLLS;26|)))
       (LETT #2# (CDR #2#) |ES-;eval;SLLS;26|)
       (GO G190)
       G191
       (EXIT (NREVERSE0 #1#))))
     |lf|
     (QREFELT |$| 67))))))

(DEFUN |ES-;map;MKS;27| (|fn| |k| |$|)
```

```
 (PROG (#1=#:G82428 |x| #2=#:G82429 |l|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL
       (LETT |l|
        (PROGN
         (LETT #1# NIL |ES-;map;MKS;27|)
         (SEQ
          (LETT |x| NIL |ES-;map;MKS;27|)
          (LETT #2# (SPADCALL |k| (QREFELT |$| 85)) |ES-;map;MKS;27|)
          G190
          (COND
           ((OR (ATOM #2#) (PROGN (LETT |x| (CAR #2#) |ES-;map;MKS;27|) NIL))
             (GO G191)))
          (SEQ
           (EXIT
            (LETT #1# (CONS (SPADCALL |x| |fn|) #1#) |ES-;map;MKS;27|)))
          (LETT #2# (CDR #2#) |ES-;map;MKS;27|)
          (GO G190)
          G191
          (EXIT (NREVERSE0 #1#))))
        |ES-;map;MKS;27|)
        (SPADCALL |k| (QREFELT |$| 85)) (QREFELT |$| 86))
       (SPADCALL |k| (QREFELT |$| 87)))
     ((QUOTE T)
       (SPADCALL (SPADCALL |k| (QREFELT |$| 35)) |l| (QREFELT |$| 52))))))))

(DEFUN |ES-;operator;2Bo;28| (|op| |$|)
 (COND
  ((SPADCALL |op| (SPADCALL "%paren" (QREFELT |$| 9)) (QREFELT |$| 89))
    (QREFELT |$| 13))
  ((SPADCALL |op| (SPADCALL "%box" (QREFELT |$| 9)) (QREFELT |$| 89))
    (QREFELT |$| 14))
  ((QUOTE T) (|error| "Unknown operator"))))

(DEFUN |ES-;mainKernel;SU;29| (|x| |$|)
 (PROG (|l| |kk| #1=#:G82445 |n| |k|)
  (RETURN
   (SEQ
    (COND
     ((NULL (LETT |l| (SPADCALL |x| (QREFELT |$| 38)) |ES-;mainKernel;SU;29|))
       (CONS 1 "failed"))
     ((QUOTE T)
      (SEQ
       (LETT |n|
        (SPADCALL
         (LETT |k| (|SPADfirst| |l|) |ES-;mainKernel;SU;29|) (QREFELT |$| 40))
        |ES-;mainKernel;SU;29|)
       (SEQ
        (LETT |kk| NIL |ES-;mainKernel;SU;29|)
        (LETT #1# (CDR |l|) |ES-;mainKernel;SU;29|)
        G190
        (COND
```

```
         ((OR
           (ATOM #1#)
           (PROGN (LETT |kk| (CAR #1#) |ES-;mainKernel;SU;29|) NIL))
            (GO G191)))
        (SEQ
         (EXIT
          (COND
           ((|<| |n| (SPADCALL |kk| (QREFELT |$| 40)))
            (SEQ
             (LETT |n| (SPADCALL |kk| (QREFELT |$| 40)) |ES-;mainKernel;SU;29|)
             (EXIT (LETT |k| |kk| |ES-;mainKernel;SU;29|)))))))
        (LETT #1# (CDR #1#) |ES-;mainKernel;SU;29|) (GO G190) G191 (EXIT NIL))
       (EXIT (CONS 0 |k|)))))))))

(DEFUN |ES-;allKernels| (|f| |$|)
 (PROG (|l| |k| #1=#:G82458 |u| |s0| |n| |arg| |t| |s|)
  (RETURN
   (SEQ
    (LETT |s|
     (SPADCALL
      (LETT |l|
       (SPADCALL |f| (QREFELT |$| 38))
       |ES-;allKernels|)
      (QREFELT |$| 29))
     |ES-;allKernels|)
    (SEQ
     (LETT |k| NIL |ES-;allKernels|)
     (LETT #1# |l| |ES-;allKernels|)
     G190
     (COND
      ((OR (ATOM #1#) (PROGN (LETT |k| (CAR #1#) |ES-;allKernels|) NIL))
       (GO G191)))
     (SEQ
      (LETT |t|
       (SEQ
        (LETT |u|
         (SPADCALL
          (SPADCALL |k| (QREFELT |$| 35))
           "%dummyVar"
           (QREFELT |$| 94))
          |ES-;allKernels|)
        (EXIT
         (COND
          ((QEQCAR |u| 0)
           (SEQ
            (LETT |arg| (SPADCALL |k| (QREFELT |$| 85)) |ES-;allKernels|)
            (LETT |s0|
             (SPADCALL
              (SPADCALL
               (SPADCALL |arg| (QREFELT |$| 95))
               (QREFELT |$| 56))
              (|ES-;allKernels| (|SPADfirst| |arg|) |$|)
              (QREFELT |$| 96))
             |ES-;allKernels|)
```

```
            (LETT |arg| (CDR (CDR |arg|)) |ES-;allKernels|)
            (LETT |n| (QCDR |u|) |ES-;allKernels|)
            (COND ((|<| 1 |n|) (LETT |arg| (CDR |arg|) |ES-;allKernels|)))
            (EXIT (SPADCALL |s0| (|ES-;allk| |arg| |$|) (QREFELT |$| 30)))))
          ((QUOTE T) (|ES-;allk| (SPADCALL |k| (QREFELT |$| 85)) |$|)))))
       |ES-;allKernels|)
      (EXIT
       (LETT |s| (SPADCALL |s| |t| (QREFELT |$| 30)) |ES-;allKernels|)))
     (LETT #1# (CDR #1#) |ES-;allKernels|)
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |s|)))))

(DEFUN |ES-;kernel;BoLS;31| (|op| |args| |$|)
 (COND
  ((NULL (SPADCALL |op| (QREFELT |$| 97))) (|error| "Unknown operator"))
  ((QUOTE T) (|ES-;okkernel| |op| |args| |$|))))

(DEFUN |ES-;okkernel| (|op| |l| |$|)
 (PROG (#1=#:G82465 |f| #2=#:G82466)
  (RETURN
   (SEQ
    (SPADCALL
     (SPADCALL |op| |l|
      (|+| 1
       (SPADCALL
        (ELT |$| 41)
        (PROGN
         (LETT #1# NIL |ES-;okkernel|)
         (SEQ
          (LETT |f| NIL |ES-;okkernel|)
          (LETT #2# |l| |ES-;okkernel|)
          G190
          (COND
           ((OR (ATOM #2#) (PROGN (LETT |f| (CAR #2#) |ES-;okkernel|) NIL))
             (GO G191)))
          (SEQ
           (EXIT
            (LETT #1#
             (CONS (SPADCALL |f| (QREFELT |$| 99)) #1#) |ES-;okkernel|)))
          (LETT #2# (CDR #2#) |ES-;okkernel|)
          (GO G190)
          G191
          (EXIT (NREVERSE0 #1#))))
        0
        (QREFELT |$| 44)))
      (QREFELT |$| 100))
     (QREFELT |$| 87))))))

(DEFUN |ES-;elt;BoLS;33| (|op| |args| |$|)
 (PROG (|u| #1=#:G82482 |v|)
  (RETURN
   (SEQ
```

```
    (EXIT
     (COND
      ((NULL (SPADCALL |op| (QREFELT |$| 97))) (|error| "Unknown operator"))
      ((QUOTE T)
       (SEQ
        (SEQ
         (LETT |u| (SPADCALL |op| (QREFELT |$| 102)) |ES-;elt;BoLS;33|)
         (EXIT
          (COND
           ((QEQCAR |u| 0)
            (COND
             ((NULL (EQL (LENGTH |args|) (QCDR |u|)))
              (PROGN
               (LETT #1#
                (|error| "Wrong number of arguments")
                |ES-;elt;BoLS;33|)
                (GO #1#)))))))
        (LETT |v| (SPADCALL |op| |args| (QREFELT |$| 105)) |ES-;elt;BoLS;33|)
        (EXIT
         (COND
          ((QEQCAR |v| 0) (QCDR |v|))
          ((QUOTE T) (|ES-;okkernel| |op| |args| |$|))))))))
    #1#
    (EXIT #1#)))))

(DEFUN |ES-;retract;SK;34| (|f| |$|)
 (PROG (|k|)
  (RETURN
   (SEQ
    (LETT |k| (SPADCALL |f| (QREFELT |$| 107)) |ES-;retract;SK;34|)
    (EXIT
     (COND
      ((OR
        (QEQCAR |k| 1)
        (NULL
         (SPADCALL
          (SPADCALL (QCDR |k|) (QREFELT |$| 87))
          |f|
          (QREFELT |$| 108))))
        (|error| "not a kernel"))
      ((QUOTE T) (QCDR |k|))))))))

(DEFUN |ES-;retractIfCan;SU;35| (|f| |$|)
 (PROG (|k|)
  (RETURN
   (SEQ
    (LETT |k| (SPADCALL |f| (QREFELT |$| 107)) |ES-;retractIfCan;SU;35|)
    (EXIT
     (COND
      ((OR
        (QEQCAR |k| 1)
        (NULL
         (SPADCALL
          (SPADCALL (QCDR |k|) (QREFELT |$| 87))
```

```
          |f|
          (QREFELT |$| 108))))
        (CONS 1 "failed"))
      ((QUOTE T) |k|)))))))

(DEFUN |ES-;is?;SSB;36| (|f| |s| |$|)
 (PROG (|k|)
  (RETURN
   (SEQ
    (LETT |k| (SPADCALL |f| (QREFELT |$| 111)) |ES-;is?;SSB;36|)
    (EXIT
     (COND
      ((QEQCAR |k| 1) (QUOTE NIL))
      ((QUOTE T) (SPADCALL (QCDR |k|) |s| (QREFELT |$| 112)))))))))

(DEFUN |ES-;is?;SBoB;37| (|f| |op| |$|)
 (PROG (|k|)
  (RETURN
   (SEQ
    (LETT |k| (SPADCALL |f| (QREFELT |$| 111)) |ES-;is?;SBoB;37|)
    (EXIT
     (COND
      ((QEQCAR |k| 1) (QUOTE NIL))
      ((QUOTE T) (SPADCALL (QCDR |k|) |op| (QREFELT |$| 50)))))))))

(DEFUN |ES-;unwrap| (|l| |x| |$|)
 (PROG (|k| #1=#:G82507)
  (RETURN
   (SEQ
    (SEQ
     (LETT |k| NIL |ES-;unwrap|)
     (LETT #1# (NREVERSE |l|) |ES-;unwrap|)
     G190
     (COND
      ((OR (ATOM #1#) (PROGN (LETT |k| (CAR #1#) |ES-;unwrap|) NIL))
        (GO G191)))
     (SEQ
      (EXIT
       (LETT |x|
        (SPADCALL |x| |k|
         (|SPADfirst| (SPADCALL |k| (QREFELT |$| 85)))
         (QREFELT |$| 115))
        |ES-;unwrap|)))
     (LETT #1# (CDR #1#) |ES-;unwrap|)
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |x|)))))

(DEFUN |ES-;distribute;3S;39| (|x| |y| |$|)
 (PROG (|ky| #1=#:G82512 |k| #2=#:G82513)
  (RETURN
   (SEQ
    (LETT |ky| (SPADCALL |y| (QREFELT |$| 56)) |ES-;distribute;3S;39|)
```

```
    (EXIT
     (|ES-;unwrap|
      (PROGN
       (LETT #1# NIL |ES-;distribute;3S;39|)
       (SEQ
        (LETT |k| NIL |ES-;distribute;3S;39|)
        (LETT #2# (|ES-;listk| |x| |$|) |ES-;distribute;3S;39|)
        G190
        (COND
         ((OR
           (ATOM #2#)
           (PROGN (LETT |k| (CAR #2#) |ES-;distribute;3S;39|) NIL))
            (GO G191)))
        (SEQ
         (EXIT
          (COND
           ((COND
             ((SPADCALL |k|
               (SPADCALL "%paren" (QREFELT |$| 9))
                (QREFELT |$| 112))
               (SPADCALL |ky|
                (|ES-;listk| (SPADCALL |k| (QREFELT |$| 87)) |$|)
                (QREFELT |$| 57)))
             ((QUOTE T) (QUOTE NIL)))
              (LETT #1# (CONS |k| #1#) |ES-;distribute;3S;39|)))))
        (LETT #2# (CDR #2#) |ES-;distribute;3S;39|)
        (GO G190)
        G191
        (EXIT (NREVERSE0 #1#))))
      |x|
      |$|))))))

(DEFUN |ES-;eval;SLS;40| (|f| |leq| |$|)
 (PROG (|rec|)
  (RETURN
   (SEQ
    (LETT |rec| (|ES-;mkKerLists| |leq| |$|) |ES-;eval;SLS;40|)
    (EXIT (SPADCALL |f| (QCAR |rec|) (QCDR |rec|) (QREFELT |$| 117)))))))

(DEFUN |ES-;subst;SLS;41| (|f| |leq| |$|)
 (PROG (|rec|)
  (RETURN
   (SEQ
    (LETT |rec| (|ES-;mkKerLists| |leq| |$|) |ES-;subst;SLS;41|)
    (EXIT (SPADCALL |f| (QCAR |rec|) (QCDR |rec|) (QREFELT |$| 119)))))))

(DEFUN |ES-;mkKerLists| (|leq| |$|)
 (PROG (|eq| #1=#:G82530 |k| |lk| |lv|)
  (RETURN
   (SEQ
    (LETT |lk| NIL |ES-;mkKerLists|)
    (LETT |lv| NIL |ES-;mkKerLists|)
    (SEQ
     (LETT |eq| NIL |ES-;mkKerLists|)
```

```
     (LETT #1# |leq| |ES-;mkKerLists|)
     G190
     (COND
      ((OR (ATOM #1#) (PROGN (LETT |eq| (CAR #1#) |ES-;mkKerLists|) NIL))
        (GO G191)))
     (SEQ
      (LETT |k|
       (SPADCALL (SPADCALL |eq| (QREFELT |$| 122)) (QREFELT |$| 111))
       |ES-;mkKerLists|)
      (EXIT
       (COND
        ((QEQCAR |k| 1) (|error| "left hand side must be a single kernel"))
        ((NULL (SPADCALL (QCDR |k|) |lk| (QREFELT |$| 57)))
         (SEQ
          (LETT |lk| (CONS (QCDR |k|) |lk|) |ES-;mkKerLists|)
          (EXIT
           (LETT |lv|
            (CONS (SPADCALL |eq| (QREFELT |$| 123)) |lv|)
            |ES-;mkKerLists|)))))))
     (LETT #1# (CDR #1#) |ES-;mkKerLists|)
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT (CONS |lk| |lv|))))))

(DEFUN |ES-;even?;SB;43| (|x| |$|)
 (|ES-;intpred?| |x| (ELT |$| 125) |$|))

(DEFUN |ES-;odd?;SB;44| (|x| |$|)
 (|ES-;intpred?| |x| (ELT |$| 127) |$|))

(DEFUN |ES-;intpred?| (|x| |pred?| |$|)
 (PROG (|u|)
  (RETURN
   (SEQ
    (LETT |u| (SPADCALL |x| (QREFELT |$| 130)) |ES-;intpred?|)
    (EXIT
     (COND
      ((QEQCAR |u| 0) (SPADCALL (QCDR |u|) |pred?|))
      ((QUOTE T) (QUOTE NIL))))))))

(DEFUN |ExpressionSpace&| (|#1|)
 (PROG (|DV$1| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|ExpressionSpace&|))
    (LETT |dv$| (LIST (QUOTE |ExpressionSpace&|) |DV$1|) . #1#)
    (LETT |$| (make-array 131) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3
     (LETT |pv$|
      (|buildPredVector| 0 0
       (LIST
        (|HasCategory| |#1| (QUOTE (|RetractableTo| (|Integer|))))
```

```
        (|HasCategory| |#1| (QUOTE (|Ring|))))) . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 13
     (SPADCALL (SPADCALL "%paren" (QREFELT |$| 9)) (QREFELT |$| 12)))
    (QSETREFV |$| 14
     (SPADCALL (SPADCALL "%box" (QREFELT |$| 9)) (QREFELT |$| 12)))
    (COND
     ((|testBitVector| |pv$| 1)
      (PROGN
       (QSETREFV |$| 126 (CONS (|dispatchFunction| |ES-;even?;SB;43|) |$|))
       (QSETREFV |$| 128 (CONS (|dispatchFunction| |ES-;odd?;SB;44|) |$|)))))
    |$|))))

(setf (get
 (QUOTE |ExpressionSpace&|)
 (QUOTE |infovec|))
 (LIST
  (QUOTE #(
   NIL NIL NIL NIL NIL NIL (|local| |#1|) (|String|) (|Symbol|)
   (0 . |coerce|) (|BasicOperator|) (|CommonOperators|) (5 . |operator|)
   (QUOTE |oppren|) (QUOTE |opbox|) (|List| |$|) (10 . |box|)
   |ES-;box;2S;1| (15 . |paren|) |ES-;paren;2S;2| (|Boolean|)
   (20 . |=|) |ES-;belong?;BoB;3| (|List| 34) (|Set| 34)
   (26 . |parts|) (31 . |sort!|) (|List| 55) |ES-;tower;SL;5|
   (36 . |brace|) (41 . |union|) (|Mapping| 24 24 24) (|List| 24)
   (47 . |reduce|) (|Kernel| 6) (54 . |operator|) (|List| 10)
   |ES-;operators;SL;7| (59 . |kernels|) (|NonNegativeInteger|)
   (64 . |height|) (69 . |max|) (|Mapping| 39 39 39) (|List| 39)
   (75 . |reduce|) |ES-;height;SNni;8| (82 . |name|) (|List| 8)
   (87 . |member?|) |ES-;freeOf?;SSB;9| (93 . |is?|)
   |ES-;distribute;2S;10| (99 . |elt|) |ES-;box;LS;11|
   |ES-;paren;LS;12| (|Kernel| |$|) (105 . |retract|)
   (110 . |member?|) |ES-;freeOf?;2SB;13| (116 . |kernel|)
   |ES-;kernel;Bo2S;14| |ES-;elt;Bo2S;15| |ES-;elt;Bo3S;16|
   |ES-;elt;Bo4S;17| |ES-;elt;Bo5S;18| (|Mapping| |$| 15)
   (|List| 65) (122 . |eval|) |ES-;eval;SSMS;19| (129 . |name|)
   |ES-;eval;SBoMS;20| (|List| 6) (134 . |first|)
   (|Mapping| |$| |$|) |ES-;eval;SSMS;21| (139 . |eval|)
   |ES-;eval;SBoMS;22| (|List| 79) (146 . |subst|) (|Equation| |$|)
   |ES-;subst;SES;23| (|List| 73) |ES-;eval;SLLS;24|
   |ES-;eval;SLLS;25| |ES-;eval;SLLS;26| (152 . |argument|)
   (157 . |=|) (163 . |coerce|) |ES-;map;MKS;27| (168 . |is?|)
   |ES-;operator;2Bo;28| (|Union| 55 (QUOTE "failed"))
   |ES-;mainKernel;SU;29| (|Union| (|None|) (QUOTE "failed"))
   (174 . |property|) (180 . |second|) (185 . |remove!|)
   (191 . |belong?|) |ES-;kernel;BoLS;31| (196 . |height|)
   (201 . |kernel|) (|Union| 39 (QUOTE "failed")) (208 . |arity|)
   (|Union| 6 (QUOTE "failed")) (|BasicOperatorFunctions1| 6)
   (213 . |evaluate|) |ES-;elt;BoLS;33| (219 . |mainKernel|)
   (224 . |=|) |ES-;retract;SK;34| |ES-;retractIfCan;SU;35|
   (230 . |retractIfCan|) (235 . |is?|) |ES-;is?;SSB;36|
   |ES-;is?;SBoB;37| (241 . |eval|) |ES-;distribute;3S;39|
   (248 . |eval|) |ES-;eval;SLS;40| (255 . |subst|)
```

```
 |ES-;subst;SLS;41| (|Equation| 6) (262 . |lhs|) (267 . |rhs|)
 (|Integer|) (272 . |even?|) (277 . |even?|) (282 . |odd?|)
 (287 . |odd?|) (|Union| 124 (QUOTE "failed")) (292 . |retractIfCan|)))
(QUOTE #(
 |tower| 297 |subst| 302 |retractIfCan| 314 |retract| 319 |paren| 324
 |operators| 334 |operator| 339 |odd?| 344 |map| 349 |mainKernel| 355
 |kernel| 360 |is?| 372 |height| 384 |freeOf?| 389 |even?| 401 |eval| 406
 |elt| 461 |distribute| 497 |box| 508 |belong?| 518))
(QUOTE NIL)
(CONS
 (|makeByteWordVec2| 1 (QUOTE NIL))
 (CONS
  (QUOTE #())
  (CONS
   (QUOTE #())
   (|makeByteWordVec2| 130 (QUOTE
    (1 8 0 7 9 1 11 10 8 12 1 6 0 15 16 1 6 0 15 18 2 10 20 0 0 21 1 24
     23 0 25 1 23 0 0 26 1 24 0 23 29 2 24 0 0 0 30 3 32 24 31 0 24 33
     1 34 10 0 35 1 6 27 0 38 1 34 39 0 40 2 39 0 0 0 41 3 43 39 42 0
     39 44 1 34 8 0 46 2 47 20 8 0 48 2 34 20 0 10 50 2 6 0 10 15 52 1
     6 55 0 56 2 23 20 34 0 57 2 6 0 10 15 59 3 6 0 0 47 66 67 1 10 8
     0 69 1 71 6 0 72 3 6 0 0 36 66 75 2 6 0 0 77 78 1 34 71 0 85 2 71
     20 0 0 86 1 6 0 55 87 2 10 20 0 8 89 2 10 93 0 7 94 1 71 6 0 95
     2 24 0 34 0 96 1 6 20 10 97 1 6 39 0 99 3 34 0 10 71 39 100 1 10
     101 0 102 2 104 103 10 71 105 1 6 91 0 107 2 6 20 0 0 108 1 6 91
     0 111 2 34 20 0 8 112 3 6 0 0 55 0 115 3 6 0 0 27 15 117 3 6 0 0
     27 15 119 1 121 6 0 122 1 121 6 0 123 1 124 20 0 125 1 0 20 0 126
     1 124 20 0 127 1 0 20 0 128 1 6 129 0 130 1 0 27 0 28 2 0 0 0 77
     120 2 0 0 0 79 80 1 0 91 0 110 1 0 55 0 109 1 0 0 0 19 1 0 0 15
     54 1 0 36 0 37 1 0 10 10 90 1 0 20 0 128 2 0 0 73 55 88 1 0 91 0
     92 2 0 0 10 15 98 2 0 0 10 0 60 2 0 20 0 8 113 2 0 20 0 10 114 1
     0 39 0 45 2 0 20 0 8 49 2 0 20 0 0 58 1 0 20 0 126 3 0 0 0 10 73
     76 3 0 0 0 36 66 84 3 0 0 0 10 65 70 3 0 0 0 36 81 82 3 0 0 0 8
     65 68 3 0 0 0 8 73 74 3 0 0 0 47 81 83 2 0 0 0 77 118 2 0 0 10
     15 106 5 0 0 10 0 0 0 0 64 3 0 0 10 0 0 62 4 0 0 10 0 0 0 63 2
     0 0 10 0 61 2 0 0 0 0 116 1 0 0 0 51 1 0 0 15 53 1 0 0 0 17 1 0
     20 10 22))))))
(QUOTE |lookupComplete|)))
```

21.18 EUCDOM.lsp BOOTSTRAP

EUCDOM depends on **INT** which depends on **EUCDOM**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **EUCDOM** category which we can write into the **MID** directory. We compile the lisp code and copy the **EUCDOM.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

21.18.1 The Lisp Implementation

EUCDOM;VersionCheck

This implements the bootstrap code for **EuclideanDomain**. The call to **VERSIONCHECK** is a legacy check to ensure that we did not load algebra code from a previous system version (which would not run due to major surgical changes in the system) without recompiling.

— **EUCDOM;VersionCheck** —

```
(|/VERSIONCHECK| 2)
```

————

The Domain Cache Variable

We create a variable which is formed by concatenating the string "**;AL**" to the domain name forming, in this case, "**EuclideanDomain;AL**". The variable has the initial value at load time of a list of one element, **NIL**. This list is a data structure that will be modified to hold an executable function. This function is created the first time the domain is used which it replaces the **NIL**.

— **EuclideanDomain;AL** —

```
(SETQ |EuclideanDomain;AL| (QUOTE NIL))
```

————

The Domain Function

When you call a domain the code is pretty simple at the top level. This code will check to see if this domain has ever been used. It does this by checking the value of the cached domain variable (which is the domain name **EuclideanDomain** concatenated with the string "**;AL**" to form the cache variable name which is **EuclideanDomain;AL**).

If this value is NIL we have never executed this function before. If it is not NIL we have executed this function before and we need only return the cached function which was stored in the cache variable.

If this is the first time this function is called, the cache variable is NIL and we execute the other branch of the conditional. This calls a function which

1. creates a procedure
2. returns the procedure as a value.

This procedure replaces the cached variable **EuclideanDomain;AL** value so it will be non-NIL the second time this domain is used. Thus the work of building the domain only happens once.

If this function has never been called before we call the

— **EuclideanDomain** —

```
(DEFUN |EuclideanDomain| NIL
  (declare (special |EuclideanDomain;AL|))
    (COND
      (|EuclideanDomain;AL|)
```

```
    (T (SETQ |EuclideanDomain;AL| (|EuclideanDomain;|)))))
```

———–

The First Call Domain Function

— EuclideanDomain; —

```
(DEFUN |EuclideanDomain;| NIL
  (PROG (#1=#:G83583)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|PrincipalIdealDomain|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|sizeLess?| ((|Boolean|) |$| |$|)) T)
                ((|euclideanSize| ((|NonNegativeInteger|) |$|)) T)
                ((|divide|
                  ((|Record|
                    (|:| |quotient| |$|)
                    (|:| |remainder| |$|))
                  |$| |$|)) T)
                ((|quo| (|$| |$| |$|)) T)
                ((|rem| (|$| |$| |$|)) T)
                ((|extendedEuclidean|
                  ((|Record|
                    (|:| |coef1| |$|)
                    (|:| |coef2| |$|)
                    (|:| |generator| |$|))
                  |$| |$|)) T)
                ((|extendedEuclidean|
                  ((|Union|
                      (|Record| (|:| |coef1| |$|) (|:| |coef2| |$|))
                      "failed")
                    |$| |$| |$|)) T)
                ((|multiEuclidean|
                  ((|Union|
                      (|List| |$|)
                      "failed")
                   (|List| |$|) |$|)) T)))
              NIL
              (QUOTE ((|List| |$|) (|NonNegativeInteger|) (|Boolean|)))
              NIL))
            |EuclideanDomain|)
        (SETELT #1# 0 (QUOTE (|EuclideanDomain|)))))))
```

———–

EUCDOM;NILADIC

— **EUCDOM;NILADIC** —

```
(setf (get (QUOTE |EuclideanDomain|) (QUOTE NILADIC)) T)
```

———-

— **EUCDOM.lsp BOOTSTRAP** —

```
\getchunk{EUCDOM;VersionCheck}
\getchunk{EuclideanDomain;AL}
\getchunk{EuclideanDomain}
\getchunk{EuclideanDomain;}
\getchunk{EUCDOM;NILADIC}
```

———-

21.19 EUCDOM-.lsp BOOTSTRAP

EUCDOM- depends on **EUCDOM**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **EUCDOM-** category which we can write into the **MID** directory. We compile the lisp code and copy the **EUCDOM-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

21.19.1 The Lisp Implementation

EUCDOM-;VersionCheck

This implements the bootstrap code for **EuclideanDomain**. The call to **VERSIONCHECK** is a legacy check to ensure that we did not load algebra code from a previous system version (which would not run due to major surgical changes in the system) without recompiling.

— **EUCDOM-;VersionCheck** —

```
(|/VERSIONCHECK| 2)
```

———-

EUCDOM-;sizeLess?;2SB;1

— **EUCDOM-;sizeLess?;2SB;1** —

```
(DEFUN |EUCDOM-;sizeLess?;2SB;1| (|x| |y| $)
 (COND
  ((SPADCALL |y| (QREFELT $ 8)) (QUOTE NIL))
  ((SPADCALL |x| (QREFELT $ 8)) (QUOTE T))
  ((QUOTE T)
```

```
(< (SPADCALL |x| (QREFELT $ 10)) (SPADCALL |y| (QREFELT $ 10))))))
```

———-

EUCDOM-;quo;3S;2

— EUCDOM-;quo;3S;2 —

```
(DEFUN |EUCDOM-;quo;3S;2| (|x| |y| $)
 (QCAR (SPADCALL |x| |y| (QREFELT $ 13))))
```

———-

EUCDOM-;rem;3S;3

— EUCDOM-;rem;3S;3 —

```
(DEFUN |EUCDOM-;rem;3S;3| (|x| |y| $)
 (QCDR (SPADCALL |x| |y| (QREFELT $ 13))))
```

———-

EUCDOM-;exquo;2SU;4

— EUCDOM-;exquo;2SU;4 —

```
(DEFUN |EUCDOM-;exquo;2SU;4| (|x| |y| $)
 (PROG (|qr|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |x| (QREFELT $ 8)) (CONS 0 (|spadConstant| $ 16)))
     ((SPADCALL |y| (QREFELT $ 8)) (CONS 1 "failed"))
     ((QUOTE T)
      (SEQ
       (LETT |qr|
        (SPADCALL |x| |y| (QREFELT $ 13))
        |EUCDOM-;exquo;2SU;4|)
       (EXIT
        (COND
         ((SPADCALL (QCDR |qr|) (QREFELT $ 8)) (CONS 0 (QCAR |qr|)))
         ((QUOTE T) (CONS 1 "failed"))))))))))
```

———-

EUCDOM-;gcd;3S;5

— EUCDOM-;gcd;3S;5 —

```
(DEFUN |EUCDOM-;gcd;3S;5| (|x| |y| $)
 (PROG (|#G13| |#G14|)
  (RETURN
   (SEQ
    (LETT |x| (SPADCALL |x| (QREFELT $ 19)) |EUCDOM-;gcd;3S;5|)
    (LETT |y| (SPADCALL |y| (QREFELT $ 19)) |EUCDOM-;gcd;3S;5|)
    (SEQ G190
     (COND
      ((NULL
        (COND
         ((SPADCALL |y| (QREFELT $ 8)) (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
        (GO G191)))
     (SEQ
      (PROGN
       (LETT |#G13| |y| |EUCDOM-;gcd;3S;5|)
       (LETT |#G14| (SPADCALL |x| |y| (QREFELT $ 20)) |EUCDOM-;gcd;3S;5|)
       (LETT |x| |#G13| |EUCDOM-;gcd;3S;5|)
       (LETT |y| |#G14| |EUCDOM-;gcd;3S;5|))
      (EXIT
       (LETT |y| (SPADCALL |y| (QREFELT $ 19)) |EUCDOM-;gcd;3S;5|)))
     NIL
     (GO G190)
    G191
   (EXIT NIL))
  (EXIT |x|)))))
```

———-

EUCDOM-;unitNormalizeIdealElt

— EUCDOM-;unitNormalizeIdealElt —

```
(DEFUN |EUCDOM-;unitNormalizeIdealElt| (|s| $)
 (PROG (|#G16| |u| |c| |a|)
  (RETURN
   (SEQ
    (PROGN
     (LETT |#G16|
       (SPADCALL (QVELT |s| 2) (QREFELT $ 23))
       |EUCDOM-;unitNormalizeIdealElt|)
     (LETT |u| (QVELT |#G16| 0) |EUCDOM-;unitNormalizeIdealElt|)
     (LETT |c| (QVELT |#G16| 1) |EUCDOM-;unitNormalizeIdealElt|)
     (LETT |a| (QVELT |#G16| 2) |EUCDOM-;unitNormalizeIdealElt|)
     |#G16|)
    (EXIT
     (COND
      ((SPADCALL |a| (|spadConstant| $ 24) (QREFELT $ 25)) |s|)
```

```
      ((QUOTE T)
       (VECTOR
        (SPADCALL |a| (QVELT |s| 0) (QREFELT $ 26))
        (SPADCALL |a| (QVELT |s| 1) (QREFELT $ 26))
        |c|))))))))
```

——————-

EUCDOM-;extendedEuclidean;2SR;7

— EUCDOM-;extendedEuclidean;2SR;7 —

```
(DEFUN |EUCDOM-;extendedEuclidean;2SR;7| (|x| |y| $)
 (PROG (|s3| |s2| |qr| |s1|)
  (RETURN
   (SEQ
    (LETT |s1|
     (|EUCDOM-;unitNormalizeIdealElt|
      (VECTOR (|spadConstant| $ 24) (|spadConstant| $ 16) |x|)
       $)
     |EUCDOM-;extendedEuclidean;2SR;7|)
    (LETT |s2|
     (|EUCDOM-;unitNormalizeIdealElt|
      (VECTOR (|spadConstant| $ 16) (|spadConstant| $ 24) |y|)
      $)
      |EUCDOM-;extendedEuclidean;2SR;7|)
    (EXIT
     (COND
      ((SPADCALL |y| (QREFELT $ 8)) |s1|)
      ((SPADCALL |x| (QREFELT $ 8)) |s2|)
      ((QUOTE T)
       (SEQ
        (SEQ
         G190
         (COND
          ((NULL
            (COND
             ((SPADCALL (QVELT |s2| 2) (QREFELT $ 8)) (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
           (GO G191)))
         (SEQ
          (LETT |qr|
           (SPADCALL (QVELT |s1| 2) (QVELT |s2| 2) (QREFELT $ 13))
           |EUCDOM-;extendedEuclidean;2SR;7|)
          (LETT |s3|
           (VECTOR
            (SPADCALL (QVELT |s1| 0)
             (SPADCALL (QCAR |qr|) (QVELT |s2| 0) (QREFELT $ 26))
             (QREFELT $ 27))
            (SPADCALL (QVELT |s1| 1)
             (SPADCALL (QCAR |qr|) (QVELT |s2| 1) (QREFELT $ 26))
             (QREFELT $ 27))
```

```
          (QCDR |qr|))
         |EUCDOM-;extendedEuclidean;2SR;7|)
        (LETT |s1| |s2| |EUCDOM-;extendedEuclidean;2SR;7|)
        (EXIT
         (LETT |s2|
          (|EUCDOM-;unitNormalizeIdealElt| |s3| $)
          |EUCDOM-;extendedEuclidean;2SR;7|)))
       NIL
       (GO G190)
       G191
       (EXIT NIL))
      (COND
       ((NULL (SPADCALL (QVELT |s1| 0) (QREFELT $ 8)))
         (COND
          ((NULL (SPADCALL (QVELT |s1| 0) |y| (QREFELT $ 28)))
           (SEQ
            (LETT |qr|
             (SPADCALL (QVELT |s1| 0) |y| (QREFELT $ 13))
             |EUCDOM-;extendedEuclidean;2SR;7|)
            (QSETVELT |s1| 0 (QCDR |qr|))
            (QSETVELT |s1| 1
             (SPADCALL (QVELT |s1| 1)
              (SPADCALL (QCAR |qr|) |x| (QREFELT $ 26)) (QREFELT $ 29)))
            (EXIT
             (LETT |s1|
              (|EUCDOM-;unitNormalizeIdealElt| |s1| $)
              |EUCDOM-;extendedEuclidean;2SR;7|)))))))
      (EXIT |s1|)))))))))
```

———-

EUCDOM-;extendedEuclidean;3SU;8

— **EUCDOM-;extendedEuclidean;3SU;8** —

```
(DEFUN |EUCDOM-;extendedEuclidean;3SU;8| (|x| |y| |z| $)
 (PROG (|s| |w| |qr|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |z| (QREFELT $ 8))
       (CONS 0 (CONS (|spadConstant| $ 16) (|spadConstant| $ 16))))
     ((QUOTE T)
       (SEQ
        (LETT |s|
         (SPADCALL |x| |y| (QREFELT $ 32))
         |EUCDOM-;extendedEuclidean;3SU;8|)
        (LETT |w|
         (SPADCALL |z| (QVELT |s| 2) (QREFELT $ 33))
         |EUCDOM-;extendedEuclidean;3SU;8|)
        (EXIT
         (COND
```

```
          ((QEQCAR |w| 1) (CONS 1 "failed"))
          ((SPADCALL |y| (QREFELT $ 8))
            (CONS 0
             (CONS (SPADCALL (QVELT |s| 0) (QCDR |w|) (QREFELT $ 26))
              (SPADCALL (QVELT |s| 1) (QCDR |w|) (QREFELT $ 26)))))
          ((QUOTE T)
           (SEQ
            (LETT |qr|
             (SPADCALL
              (SPADCALL (QVELT |s| 0) (QCDR |w|) (QREFELT $ 26))
              |y|
              (QREFELT $ 13))
             |EUCDOM-;extendedEuclidean;3SU;8|)
            (EXIT
             (CONS 0
              (CONS (QCDR |qr|)
               (SPADCALL
                (SPADCALL (QVELT |s| 1) (QCDR |w|) (QREFELT $ 26))
                (SPADCALL (QCAR |qr|) |x| (QREFELT $ 26))
                (QREFELT $ 29)))))))))))))))))
```

———-

EUCDOM-;principalIdeal;LR;9

— EUCDOM-;principalIdeal;LR;9 —

```
(DEFUN |EUCDOM-;principalIdeal;LR;9| (|l| $)
 (PROG (|uca| |v| |u| #0=#:G1497 |vv| #1=#:G1498)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |l| NIL (QREFELT $ 38))
       (|error| "empty list passed to principalIdeal"))
     ((SPADCALL (CDR |l|) NIL (QREFELT $ 38))
       (SEQ
        (LETT |uca|
         (SPADCALL (|SPADfirst| |l|) (QREFELT $ 23))
         |EUCDOM-;principalIdeal;LR;9|)
        (EXIT (CONS (LIST (QVELT |uca| 0)) (QVELT |uca| 1)))))
     ((SPADCALL (CDR (CDR |l|)) NIL (QREFELT $ 38))
       (SEQ
        (LETT |u|
         (SPADCALL (|SPADfirst| |l|)
          (SPADCALL |l| (QREFELT $ 39)) (QREFELT $ 32))
         |EUCDOM-;principalIdeal;LR;9|)
        (EXIT (CONS (LIST (QVELT |u| 0) (QVELT |u| 1)) (QVELT |u| 2)))))
     ((QUOTE T)
       (SEQ
        (LETT |v|
         (SPADCALL (CDR |l|) (QREFELT $ 42))
         |EUCDOM-;principalIdeal;LR;9|)
```

```
        (LETT |u|
         (SPADCALL (|SPADfirst| |l|) (QCDR |v|) (QREFELT $ 32))
         |EUCDOM-;principalIdeal;LR;9|)
        (EXIT
         (CONS
          (CONS (QVELT |u| 0)
           (PROGN
            (LETT #0# NIL |EUCDOM-;principalIdeal;LR;9|)
            (SEQ
             (LETT |vv| NIL |EUCDOM-;principalIdeal;LR;9|)
             (LETT #1# (QCAR |v|) |EUCDOM-;principalIdeal;LR;9|)
             G190
             (COND
              ((OR (ATOM #1#)
                   (PROGN
                    (LETT |vv| (CAR #1#) |EUCDOM-;principalIdeal;LR;9|) NIL))
               (GO G191)))
             (SEQ
              (EXIT
               (LETT #0#
                (CONS (SPADCALL (QVELT |u| 1) |vv| (QREFELT $ 26))
                   #0#)
                |EUCDOM-;principalIdeal;LR;9|)))
             (LETT #1# (CDR #1#)
              |EUCDOM-;principalIdeal;LR;9|)
             (GO G190)
             G191
             (EXIT (NREVERSE0 #0#)))))
          (QVELT |u| 2)))))))))
```

———-

EUCDOM-;expressIdealMember;LSU;10

— **EUCDOM-;expressIdealMember;LSU;10** —

```
(DEFUN |EUCDOM-;expressIdealMember;LSU;10| (|l| |z| $)
 (PROG (#0=#:G1513 #1=#:G1514 |pid| |q| #2=#:G1515 |v| #3=#:G1516)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |z| (|spadConstant| $ 16) (QREFELT $ 25))
      (CONS 0
       (PROGN
        (LETT #0# NIL |EUCDOM-;expressIdealMember;LSU;10|)
        (SEQ
         (LETT |v| NIL |EUCDOM-;expressIdealMember;LSU;10|)
         (LETT #1# |l| |EUCDOM-;expressIdealMember;LSU;10|)
         G190
         (COND
          ((OR (ATOM #1#)
               (PROGN
```

```
              (LETT |v| (CAR #1#) |EUCDOM-;expressIdealMember;LSU;10|) NIL))
         (GO G191)))
       (SEQ
        (EXIT
         (LETT #0#
          (CONS (|spadConstant| $ 16) #0#)
          |EUCDOM-;expressIdealMember;LSU;10|)))
       (LETT #1# (CDR #1#) |EUCDOM-;expressIdealMember;LSU;10|)
       (GO G190)
       G191
      (EXIT (NREVERSE0 #0#))))))
  ((QUOTE T)
   (SEQ
    (LETT |pid|
     (SPADCALL |l| (QREFELT $ 42))
     |EUCDOM-;expressIdealMember;LSU;10|)
    (LETT |q|
     (SPADCALL |z| (QCDR |pid|) (QREFELT $ 33))
     |EUCDOM-;expressIdealMember;LSU;10|)
    (EXIT
     (COND
      ((QEQCAR |q| 1) (CONS 1 "failed"))
      ((QUOTE T)
       (CONS 0
        (PROGN
         (LETT #2# NIL |EUCDOM-;expressIdealMember;LSU;10|)
         (SEQ
          (LETT |v| NIL |EUCDOM-;expressIdealMember;LSU;10|)
          (LETT #3# (QCAR |pid|) |EUCDOM-;expressIdealMember;LSU;10|)
          G190
          (COND
           ((OR (ATOM #3#)
                (PROGN
                 (LETT |v| (CAR #3#) |EUCDOM-;expressIdealMember;LSU;10|)
                 NIL))
             (GO G191)))
          (SEQ
           (EXIT
           (LETT #2#
            (CONS (SPADCALL (QCDR |q|) |v| (QREFELT $ 26))
               #2#)
            |EUCDOM-;expressIdealMember;LSU;10|)))
          (LETT #3# (CDR #3#) |EUCDOM-;expressIdealMember;LSU;10|)
          (GO G190)
          G191
          (EXIT (NREVERSE0 #2#))))))))))))))
```

———-

EUCDOM-;multiEuclidean;LSU;11

— EUCDOM-;multiEuclidean;LSU;11 —

```
(DEFUN |EUCDOM-;multiEuclidean;LSU;11| (|l| |z| $)
 (PROG (|n| |l1| |l2| #0=#:G1405 #1=#:G1535 #2=#:G1522 #3=#:G1520
        #4=#:G1521 #5=#:G1406 #6=#:G1536 #7=#:G1525 #8=#:G1523 #9=#:G1524
        |u| |v1| |v2|)
  (RETURN
   (SEQ
    (LETT |n| (LENGTH |l|) |EUCDOM-;multiEuclidean;LSU;11|)
    (EXIT
     (COND
      ((ZEROP |n|) (|error| "empty list passed to multiEuclidean"))
      ((EQL |n| 1) (CONS 0 (LIST |z|)))
      ((QUOTE T)
       (SEQ
        (LETT |l1|
         (SPADCALL |l| (QREFELT $ 46)) |EUCDOM-;multiEuclidean;LSU;11|)
        (LETT |l2|
         (SPADCALL |l1| (QUOTIENT2 |n| 2) (QREFELT $ 48))
         |EUCDOM-;multiEuclidean;LSU;11|)
        (LETT |u|
         (SPADCALL
          (PROGN
           (LETT #4# NIL |EUCDOM-;multiEuclidean;LSU;11|)
           (SEQ
            (LETT #0# NIL |EUCDOM-;multiEuclidean;LSU;11|)
            (LETT #1# |l1| |EUCDOM-;multiEuclidean;LSU;11|)
            G190
            (COND
             ((OR (ATOM #1#)
                  (PROGN
                   (LETT #0# (CAR #1#) |EUCDOM-;multiEuclidean;LSU;11|)
                   NIL))
              (GO G191)))
            (SEQ
             (EXIT
              (PROGN
               (LETT #2# #0# |EUCDOM-;multiEuclidean;LSU;11|)
               (COND
                (#4#
                 (LETT #3#
                  (SPADCALL #3# #2# (QREFELT $ 26))
                  |EUCDOM-;multiEuclidean;LSU;11|))
                ((QUOTE T)
                 (PROGN
                  (LETT #3# #2# |EUCDOM-;multiEuclidean;LSU;11|)
                  (LETT #4# (QUOTE T) |EUCDOM-;multiEuclidean;LSU;11|)))))))
            (LETT #1# (CDR #1#) |EUCDOM-;multiEuclidean;LSU;11|)
            (GO G190)
            G191
            (EXIT NIL))
           (COND (#4# #3#) ((QUOTE T) (|spadConstant| $ 24))))
          (PROGN
           (LETT #9# NIL |EUCDOM-;multiEuclidean;LSU;11|)
           (SEQ
            (LETT #5# NIL |EUCDOM-;multiEuclidean;LSU;11|)
```

```
          (LETT #6# |l2| |EUCDOM-;multiEuclidean;LSU;11|)
          G190
          (COND
           ((OR (ATOM #6#)
                (PROGN
                 (LETT #5# (CAR #6#) |EUCDOM-;multiEuclidean;LSU;11|)
                 NIL))
            (GO G191)))
          (SEQ
           (EXIT
            (PROGN
             (LETT #7# #5# |EUCDOM-;multiEuclidean;LSU;11|)
             (COND
              (#9#
               (LETT #8#
                (SPADCALL #8# #7# (QREFELT $ 26))
                |EUCDOM-;multiEuclidean;LSU;11|))
              ((QUOTE T)
               (PROGN
                (LETT #8# #7# |EUCDOM-;multiEuclidean;LSU;11|)
                (LETT #9# (QUOTE T) |EUCDOM-;multiEuclidean;LSU;11|)))))))
          (LETT #6# (CDR #6#) |EUCDOM-;multiEuclidean;LSU;11|)
          (GO G190)
          G191
          (EXIT NIL))
         (COND (#9# #8#) ((QUOTE T) (|spadConstant| $ 24))))
        |z| (QREFELT $ 49))
       |EUCDOM-;multiEuclidean;LSU;11|)
      (EXIT
       (COND
        ((QEQCAR |u| 1) (CONS 1 "failed"))
        ((QUOTE T)
         (SEQ
          (LETT |v1|
           (SPADCALL |l1| (QCDR (QCDR |u|)) (QREFELT $ 50))
           |EUCDOM-;multiEuclidean;LSU;11|)
          (EXIT
           (COND
            ((QEQCAR |v1| 1) (CONS 1 "failed"))
            ((QUOTE T)
             (SEQ
              (LETT |v2|
               (SPADCALL |l2| (QCAR (QCDR |u|)) (QREFELT $ 50))
               |EUCDOM-;multiEuclidean;LSU;11|)
              (EXIT
               (COND
                ((QEQCAR |v2| 1) (CONS 1 "failed"))
                ((QUOTE T)
                 (CONS 0
                  (SPADCALL
                   (QCDR |v1|)
                   (QCDR |v2|)
                   (QREFELT $ 51))))))))))))))))))))))
```

———-

EuclideanDomain&

— EuclideanDomainAmp —

```
(DEFUN |EuclideanDomain&| (|#1|)
 (PROG (DV$1 |dv$| $ |pv$|)
  (RETURN
   (PROGN
    (LETT DV$1 (|devaluate| |#1|) . #0=(|EuclideanDomain&|))
    (LETT |dv$| (LIST (QUOTE |EuclideanDomain&|) DV$1) . #0#)
    (LETT $ (make-array 53) . #0#)
    (QSETREFV $ 0 |dv$|)
    (QSETREFV $ 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #0#))
    (|stuffDomainSlots| $)
    (QSETREFV $ 6 |#1|)
    $))))
```

———-

EUCDOM-;INFOVEC

— EUCDOM-;INFOVEC —

```
(setf (get
 (QUOTE |EuclideanDomain&|)
 (QUOTE |infovec|))
 (LIST
  (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|Boolean|) (0 . |zero?|)
  (|NonNegativeInteger|) (5 . |euclideanSize|) |EUCDOM-;sizeLess?;2SB;1|
  (|Record| (|:| |quotient| $) (|:| |remainder| $)) (10 . |divide|)
  |EUCDOM-;quo;3S;2| |EUCDOM-;rem;3S;3| (16 . |Zero|)
  (|Union| $ (QUOTE "failed")) |EUCDOM-;exquo;2SU;4| (20 . |unitCanonical|)
  (25 . |rem|) |EUCDOM-;gcd;3S;5|
  (|Record| (|:| |unit| $) (|:| |canonical| $) (|:| |associate| $))
  (31 . |unitNormal|) (36 . |One|) (40 . =) (46 . *) (52 . -)
  (58 . |sizeLess?|) (64 . +)
  (|Record| (|:| |coef1| $) (|:| |coef2| $) (|:| |generator| $))
  |EUCDOM-;extendedEuclidean;2SR;7|
  (70 . |extendedEuclidean|) (76 . |exquo|)
  (|Record| (|:| |coef1| $) (|:| |coef2| $))
  (|Union| 34 (QUOTE "failed")) |EUCDOM-;extendedEuclidean;3SU;8|
  (|List| 6) (82 . =) (88 . |second|)
  (|Record| (|:| |coef| 41) (|:| |generator| $))
  (|List| $) (93 . |principalIdeal|) |EUCDOM-;principalIdeal;LR;9|
  (|Union| 41 (QUOTE "failed")) |EUCDOM-;expressIdealMember;LSU;10|
  (98 . |copy|) (|Integer|) (103 . |split!|) (109 . |extendedEuclidean|)
  (116 . |multiEuclidean|) (122 . |concat|) |EUCDOM-;multiEuclidean;LSU;11|))
  (QUOTE
   #(|sizeLess?| 128 |rem| 134 |quo| 140 |principalIdeal| 146
```

```
     |multiEuclidean| 151 |gcd| 157 |extendedEuclidean| 163
     |exquo| 176 |expressIdealMember| 182))
  (QUOTE NIL)
  (CONS (|makeByteWordVec2| 1 (QUOTE NIL))
   (CONS (QUOTE #())
    (CONS (QUOTE #())
     (|makeByteWordVec2| 52 (QUOTE (1 6 7 0 8 1 6 9 0 10 2 6 12 0 0 13 0
       6 0 16 1 6 0 0 19 2 6 0 0 0 20 1 6 22 0 23 0 6 0 24 2 6 7 0 0 25 2 6 0
       0 0 26 2 6 0 0 0 27 2 6 7 0 0 28 2 6 0 0 0 29 2 6 30 0 0 32 2 6 17 0 0
       33 2 37 7 0 0 38 1 37 6 0 39 1 6 40 41 42 1 37 0 0 46 2 37 0 0 47 48 3
       6 35 0 0 0 49 2 6 44 41 0 50 2 37 0 0 0 51 2 0 7 0 0 11 2 0 0 0 0 15 2
       0 0 0 0 14 1 0 40 41 43 2 0 44 41 0 52 2 0 0 0 0 21 3 0 35 0 0 0 36 2 0
       30 0 0 31 2 0 17 0 0 18 2 0 44 41 0 45))))))
  (QUOTE |lookupComplete|)))
```

— EUCDOM-.lsp BOOTSTRAP —

```
\getchunk{EUCDOM-;VersionCheck}
\getchunk{EUCDOM-;sizeLess?;2SB;1}
\getchunk{EUCDOM-;quo;3S;2}
\getchunk{EUCDOM-;rem;3S;3}
\getchunk{EUCDOM-;exquo;2SU;4}
\getchunk{EUCDOM-;gcd;3S;5}
\getchunk{EUCDOM-;unitNormalizeIdealElt}
\getchunk{EUCDOM-;extendedEuclidean;2SR;7}
\getchunk{EUCDOM-;extendedEuclidean;3SU;8}
\getchunk{EUCDOM-;principalIdeal;LR;9}
\getchunk{EUCDOM-;expressIdealMember;LSU;10}
\getchunk{EUCDOM-;multiEuclidean;LSU;11}
\getchunk{EuclideanDomainAmp}
\getchunk{EUCDOM-;INFOVEC}
```

21.20 ENTIRER.lsp BOOTSTRAP

ENTIRER depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ENTIRER** category which we can write into the **MID** directory. We compile the lisp code and copy the **ENTIRER.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— ENTIRER.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |EntireRing;AL| (QUOTE NIL))

(DEFUN |EntireRing| NIL
  (declare (special |EntireRing;AL|))
```

```
    (COND
      ((|EntireRing;AL|)
      (T (SETQ |EntireRing;AL| (|EntireRing;|)))))

(DEFUN |EntireRing;| NIL
  (PROG (#1=#:G82839)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|Ring|)
            (|BiModule| (QUOTE |$|) (QUOTE |$|))
            (|mkCategory|
              (QUOTE |package|)
              NIL
              (QUOTE ((|noZeroDivisors| T)))
              (QUOTE NIL)
              NIL))
          |EntireRing|)
        (SETELT #1# 0 (QUOTE (|EntireRing|)))))))

(setf (get (QUOTE |EntireRing|) (QUOTE NILADIC)) T)
```

21.21 FFIELDC.lsp BOOTSTRAP

FFIELDC depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **FFIELDC** category which we can write into the **MID** directory. We compile the lisp code and copy the **FFIELDC.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— FFIELDC.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |FiniteFieldCategory;AL| (QUOTE NIL))

(DEFUN |FiniteFieldCategory| NIL
 (declare (special |FiniteFieldCategory;AL|))
  (COND
   ((|FiniteFieldCategory;AL|)
   (T (SETQ |FiniteFieldCategory;AL| (|FiniteFieldCategory;|)))))

(DEFUN |FiniteFieldCategory;| NIL
 (PROG (#1=#:G83127)
  (RETURN
   (PROG1
    (LETT #1#
     (|Join|
```

```
      (|FieldOfPrimeCharacteristic|)
      (|Finite|)
      (|StepThrough|)
      (|DifferentialRing|)
      (|mkCategory|
       (QUOTE |domain|)
       (QUOTE (
        ((|charthRoot| ((|$| |$|)) T)
        ((|conditionP| ((|Union| (|Vector| |$|) "failed") (|Matrix| |$|))) T)
        ((|factorsOfCyclicGroupSize|
         ((|List| (|Record|
                   (|:| |factor| (|Integer|))
                   (|:| |exponent| (|Integer|))))))
          T)
        ((|tableForDiscreteLogarithm|
          ((|Table| (|PositiveInteger|) (|NonNegativeInteger|))
           (|Integer|))) T)
        ((|createPrimitiveElement| (|$|)) T)
        ((|primitiveElement| (|$|)) T)
        ((|primitive?| ((|Boolean|) |$|)) T)
        ((|discreteLog| ((|NonNegativeInteger|) |$|)) T)
        ((|order| ((|PositiveInteger|) |$|)) T)
        ((|representationType|
          ((|Union| "prime" "polynomial" "normal" "cyclic"))) T)))
       NIL
       (QUOTE (
        (|PositiveInteger|)
        (|NonNegativeInteger|)
        (|Boolean|)
        (|Table| (|PositiveInteger|) (|NonNegativeInteger|))
        (|Integer|)
        (|List|
         (|Record| (|:| |factor| (|Integer|)) (|:| |exponent| (|Integer|))))
        (|Matrix| |$|)))
       NIL))
     |FiniteFieldCategory|)
    (SETELT #1# 0 (QUOTE (|FiniteFieldCategory|)))))))

(setf (get (QUOTE |FiniteFieldCategory|) (QUOTE NILADIC)) T)
```

21.22 FFIELDC-.lsp BOOTSTRAP

FFIELDC- depends on **FFIELDC**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **FFIELDC-** category which we can write into the **MID** directory. We compile the lisp code and copy the **FFIELDC-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— **FFIELDC-.lsp BOOTSTRAP** —

```
(|/VERSIONCHECK| 2)

(DEFUN |FFIELDC-;differentiate;2S;1| (|x| |$|)
 (declare (ignore |x|))
 (|spadConstant| |$| 7))

(DEFUN |FFIELDC-;init;S;2| (|$|) (|spadConstant| |$| 7))

(DEFUN |FFIELDC-;nextItem;SU;3| (|a| |$|)
 (COND
  ((SPADCALL
    (LETT |a|
     (SPADCALL (|+| (SPADCALL |a| (QREFELT |$| 11)) 1) (QREFELT |$| 12))
     |FFIELDC-;nextItem;SU;3|)
    (QREFELT |$| 14))
     (CONS 1 "failed"))
  ((QUOTE T) (CONS 0 |a|))))

(DEFUN |FFIELDC-;order;SOpc;4| (|e| |$|)
 (SPADCALL (SPADCALL |e| (QREFELT |$| 17)) (QREFELT |$| 20)))

(DEFUN |FFIELDC-;conditionP;MU;5| (|mat| |$|)
 (PROG (|l|)
  (RETURN
   (SEQ
    (LETT |l| (SPADCALL |mat| (QREFELT |$| 24)) |FFIELDC-;conditionP;MU;5|)
    (COND
     ((OR
       (NULL |l|)
       (SPADCALL (ELT |$| 14) (|SPADfirst| |l|) (QREFELT |$| 27)))
         (EXIT (CONS 1 "failed"))))
    (EXIT
     (CONS 0
      (SPADCALL (ELT |$| 28) (|SPADfirst| |l|) (QREFELT |$| 30))))))))

(DEFUN |FFIELDC-;charthRoot;2S;6| (|x| |$|)
 (SPADCALL |x|
  (QUOTIENT2 (SPADCALL (QREFELT |$| 35)) (SPADCALL (QREFELT |$| 36)))
  (QREFELT |$| 37)))

(DEFUN |FFIELDC-;charthRoot;SU;7| (|x| |$|)
 (CONS 0 (SPADCALL |x| (QREFELT |$| 28))))

(DEFUN |FFIELDC-;createPrimitiveElement;S;8| (|$|)
 (PROG (|sm1| |start| |i| #1=#:G83175 |e| |found|)
  (RETURN
   (SEQ
    (LETT |sm1|
     (|-| (SPADCALL (QREFELT |$| 35)) 1)
     |FFIELDC-;createPrimitiveElement;S;8|)
    (LETT |start|
     (COND
      ((SPADCALL
```

```
        (SPADCALL (QREFELT |$| 42))
        (CONS 1 "polynomial")
        (QREFELT |$| 43))
       (SPADCALL (QREFELT |$| 36)))
      ((QUOTE T) 1))
     |FFIELDC-;createPrimitiveElement;S;8|)
    (LETT |found| (QUOTE NIL) |FFIELDC-;createPrimitiveElement;S;8|)
    (SEQ
     (LETT |i| |start| |FFIELDC-;createPrimitiveElement;S;8|)
     G190
     (COND
      ((NULL (COND (|found| (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
        (GO G191)))
     (SEQ
      (LETT |e|
       (SPADCALL
        (PROG1
         (LETT #1# |i| |FFIELDC-;createPrimitiveElement;S;8|)
         (|check-subtype| (|>| #1# 0) (QUOTE (|PositiveInteger|)) #1#))
        (QREFELT |$| 12))
       |FFIELDC-;createPrimitiveElement;S;8|)
      (EXIT
       (LETT |found|
        (EQL (SPADCALL |e| (QREFELT |$| 17)) |sm1|)
        |FFIELDC-;createPrimitiveElement;S;8|)))
     (LETT |i| (|+| |i| 1) |FFIELDC-;createPrimitiveElement;S;8|)
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |e|)))))

(DEFUN |FFIELDC-;primitive?;SB;9| (|a| |$|)
 (PROG (|explist| |q| |exp| #1=#:G83187 |equalone|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |a| (QREFELT |$| 14)) (QUOTE NIL))
     ((QUOTE T)
      (SEQ
       (LETT |explist|
        (SPADCALL (QREFELT |$| 47)) |FFIELDC-;primitive?;SB;9|)
       (LETT |q|
        (|-| (SPADCALL (QREFELT |$| 35)) 1) |FFIELDC-;primitive?;SB;9|)
       (LETT |equalone| (QUOTE NIL) |FFIELDC-;primitive?;SB;9|)
       (SEQ
        (LETT |exp| NIL |FFIELDC-;primitive?;SB;9|)
        (LETT #1# |explist| |FFIELDC-;primitive?;SB;9|)
        G190
        (COND
         ((OR
           (ATOM #1#)
           (PROGN (LETT |exp| (CAR #1#) |FFIELDC-;primitive?;SB;9|) NIL)
           (NULL (COND (|equalone| (QUOTE NIL)) ((QUOTE T) (QUOTE T)))))
          (GO G191)))
```

```
        (SEQ
         (EXIT
          (LETT |equalone|
           (SPADCALL
            (SPADCALL |a| (QUOTIENT2 |q| (QCAR |exp|)) (QREFELT |$| 48))
            (QREFELT |$| 49))
           |FFIELDC-;primitive?;SB;9|)))
        (LETT #1# (CDR #1#) |FFIELDC-;primitive?;SB;9|)
        (GO G190)
        G191
        (EXIT NIL))
       (EXIT (COND (|equalone| (QUOTE NIL)) ((QUOTE T) (QUOTE T))))))))))

(DEFUN |FFIELDC-;order;SPi;10| (|e| |$|)
 (PROG (|lof| |rec| #1=#:G83195 |primeDivisor|
        |j| #2=#:G83196 |a| |goon| |ord|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |e| (|spadConstant| |$| 7) (QREFELT |$| 51))
       (|error| "order(0) is not defined "))
     ((QUOTE T)
      (SEQ
       (LETT |ord|
        (|-| (SPADCALL (QREFELT |$| 35)) 1) |FFIELDC-;order;SPi;10|)
       (LETT |a| 0 |FFIELDC-;order;SPi;10|)
       (LETT |lof| (SPADCALL (QREFELT |$| 47)) |FFIELDC-;order;SPi;10|)
       (SEQ
        (LETT |rec| NIL |FFIELDC-;order;SPi;10|)
        (LETT #1# |lof| |FFIELDC-;order;SPi;10|)
        G190
        (COND
         ((OR
           (ATOM #1#)
           (PROGN (LETT |rec| (CAR #1#) |FFIELDC-;order;SPi;10|) NIL))
          (GO G191)))
        (SEQ
         (LETT |a|
          (QUOTIENT2 |ord|
           (LETT |primeDivisor| (QCAR |rec|) |FFIELDC-;order;SPi;10|))
          |FFIELDC-;order;SPi;10|)
         (LETT |goon|
          (SPADCALL (SPADCALL |e| |a| (QREFELT |$| 48)) (QREFELT |$| 49))
          |FFIELDC-;order;SPi;10|)
         (SEQ
          (LETT |j| 0 |FFIELDC-;order;SPi;10|)
          (LETT #2# (|-| (QCDR |rec|) 2) |FFIELDC-;order;SPi;10|)
          G190
          (COND ((OR (QSGREATERP |j| #2#) (NULL |goon|)) (GO G191)))
          (SEQ
           (LETT |ord| |a| |FFIELDC-;order;SPi;10|)
           (LETT |a|
            (QUOTIENT2 |ord| |primeDivisor|)
            |FFIELDC-;order;SPi;10|)
```

```
          (EXIT
           (LETT |goon|
            (SPADCALL (SPADCALL |e| |a| (QREFELT |$| 48)) (QREFELT |$| 49))
            |FFIELDC-;order;SPi;10|)))
         (LETT |j| (QSADD1 |j|) |FFIELDC-;order;SPi;10|)
         (GO G190)
         G191
         (EXIT NIL))
        (EXIT (COND (|goon| (LETT |ord| |a| |FFIELDC-;order;SPi;10|)))))
       (LETT #1# (CDR #1#) |FFIELDC-;order;SPi;10|)
       (GO G190)
       G191
       (EXIT NIL))
      (EXIT |ord|)))))))

(DEFUN |FFIELDC-;discreteLog;SNni;11| (|b| |$|)
 (PROG (|faclist| |gen| |groupord| |f| #1=#:G83216 |fac| |t| #2=#:G83217
        |exp| |exptable| |n| |end| |i| |rho| |found| |disc1| |c| |mult|
        |disclog| |a|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |b| (QREFELT |$| 14))
       (|error| "discreteLog: logarithm of zero"))
     ((QUOTE T)
      (SEQ
       (LETT |faclist|
        (SPADCALL (QREFELT |$| 47))
        |FFIELDC-;discreteLog;SNni;11|)
       (LETT |a| |b| |FFIELDC-;discreteLog;SNni;11|)
       (LETT |gen|
        (SPADCALL (QREFELT |$| 53))
        |FFIELDC-;discreteLog;SNni;11|)
       (EXIT
        (COND
         ((SPADCALL |b| |gen| (QREFELT |$| 51)) 1)
         ((QUOTE T)
          (SEQ
           (LETT |disclog| 0 |FFIELDC-;discreteLog;SNni;11|)
           (LETT |mult| 1 |FFIELDC-;discreteLog;SNni;11|)
           (LETT |groupord|
            (|-| (SPADCALL (QREFELT |$| 35)) 1)
            |FFIELDC-;discreteLog;SNni;11|)
           (LETT |exp| |groupord| |FFIELDC-;discreteLog;SNni;11|)
           (SEQ
            (LETT |f| NIL |FFIELDC-;discreteLog;SNni;11|)
            (LETT #1# |faclist| |FFIELDC-;discreteLog;SNni;11|)
            G190
            (COND
             ((OR
               (ATOM #1#)
               (PROGN
                (LETT |f| (CAR #1#) |FFIELDC-;discreteLog;SNni;11|)
                NIL))
```

```
 (GO G191)))
(SEQ
 (LETT |fac| (QCAR |f|) |FFIELDC-;discreteLog;SNni;11|)
 (EXIT
  (SEQ
   (LETT |t| 0 |FFIELDC-;discreteLog;SNni;11|)
   (LETT #2# (|-| (QCDR |f|) 1) |FFIELDC-;discreteLog;SNni;11|)
   G190
   (COND ((QSGREATERP |t| #2#) (GO G191)))
   (SEQ
    (LETT |exp|
     (QUOTIENT2 |exp| |fac|)
     |FFIELDC-;discreteLog;SNni;11|)
    (LETT |exptable|
     (SPADCALL |fac| (QREFELT |$| 55))
     |FFIELDC-;discreteLog;SNni;11|)
    (LETT |n|
     (SPADCALL |exptable| (QREFELT |$| 56))
     |FFIELDC-;discreteLog;SNni;11|)
    (LETT |c|
     (SPADCALL |a| |exp| (QREFELT |$| 48))
     |FFIELDC-;discreteLog;SNni;11|)
    (LETT |end|
     (QUOTIENT2 (|-| |fac| 1) |n|)
     |FFIELDC-;discreteLog;SNni;11|)
    (LETT |found| (QUOTE NIL) |FFIELDC-;discreteLog;SNni;11|)
    (LETT |disc1| 0 |FFIELDC-;discreteLog;SNni;11|)
    (SEQ
     (LETT |i| 0 |FFIELDC-;discreteLog;SNni;11|)
     G190
     (COND
      ((OR
        (QSGREATERP |i| |end|)
        (NULL
         (COND (|found| (QUOTE NIL)) ((QUOTE T) (QUOTE T)))))
       (GO G191)))
     (SEQ
      (LETT |rho|
       (SPADCALL
        (SPADCALL |c| (QREFELT |$| 11))
        |exptable|
        (QREFELT |$| 58))
       |FFIELDC-;discreteLog;SNni;11|)
      (EXIT
       (COND
        ((QEQCAR |rho| 0)
         (SEQ
          (LETT |found| (QUOTE T) |FFIELDC-;discreteLog;SNni;11|)
          (EXIT
           (LETT |disc1|
            (|*| (|+| (|*| |n| |i|) (QCDR |rho|)) |mult|)
            |FFIELDC-;discreteLog;SNni;11|))))
        ((QUOTE T)
         (LETT |c|
```

```
                    (SPADCALL |c|
                     (SPADCALL |gen|
                      (|*| (QUOTIENT2 |groupord| |fac|) (|-| |n|))
                      (QREFELT |$| 48))
                     (QREFELT |$| 59))
                    |FFIELDC-;discreteLog;SNni;11|)))))
               (LETT |i| (QSADD1 |i|) |FFIELDC-;discreteLog;SNni;11|)
               (GO G190)
               G191
               (EXIT NIL))
              (EXIT
               (COND
                (|found|
                 (SEQ
                  (LETT |mult|
                   (|*| |mult| |fac|)
                   |FFIELDC-;discreteLog;SNni;11|)
                  (LETT |disclog|
                   (|+| |disclog| |disc1|)
                   |FFIELDC-;discreteLog;SNni;11|)
                  (EXIT
                   (LETT |a|
                    (SPADCALL |a|
                     (SPADCALL |gen| (|-| |disc1|) (QREFELT |$| 48))
                     (QREFELT |$| 59))
                    |FFIELDC-;discreteLog;SNni;11|))))
                ((QUOTE T)
                  (|error| "discreteLog: ?? discrete logarithm")))))
             (LETT |t|
              (QSADD1 |t|)
              |FFIELDC-;discreteLog;SNni;11|)
             (GO G190)
             G191
             (EXIT NIL))))
          (LETT #1#
           (CDR #1#)
           |FFIELDC-;discreteLog;SNni;11|)
          (GO G190)
          G191
          (EXIT NIL))
         (EXIT |disclog|))))))))))))

(DEFUN |FFIELDC-;discreteLog;2SU;12| (|logbase| |b| |$|)
 (PROG (|groupord| |faclist| |f| #1=#:G83235 |fac| |primroot|
       |t| #2=#:G83236 |exp| |rhoHelp| #3=#:G83234 |rho| |disclog|
       |mult| |a|)
  (RETURN
   (SEQ
    (EXIT
     (COND
      ((SPADCALL |b| (QREFELT |$| 14))
       (SEQ
        (SPADCALL "discreteLog: logarithm of zero" (QREFELT |$| 64))
        (EXIT (CONS 1 "failed"))))
```

```
      ((SPADCALL |logbase| (QREFELT |$| 14))
       (SEQ
        (SPADCALL "discreteLog: logarithm to base zero" (QREFELT |$| 64))
        (EXIT (CONS 1 "failed"))))
      ((SPADCALL |b| |logbase| (QREFELT |$| 51)) (CONS 0 1))
      ((QUOTE T)
       (COND
        ((NULL
          (ZEROP
           (REMAINDER2
            (LETT |groupord|
             (SPADCALL |logbase| (QREFELT |$| 17))
             |FFIELDC-;discreteLog;2SU;12|)
            (SPADCALL |b| (QREFELT |$| 17)))))
         (SEQ
          (SPADCALL
"discreteLog: second argument not in cyclic group generated by first argument"
           (QREFELT |$| 64))
          (EXIT (CONS 1 "failed"))))
        ((QUOTE T)
         (SEQ
          (LETT |faclist|
           (SPADCALL (SPADCALL |groupord| (QREFELT |$| 66)) (QREFELT |$| 68))
           |FFIELDC-;discreteLog;2SU;12|)
          (LETT |a| |b| |FFIELDC-;discreteLog;2SU;12|)
          (LETT |disclog| 0 |FFIELDC-;discreteLog;2SU;12|)
          (LETT |mult| 1 |FFIELDC-;discreteLog;2SU;12|)
          (LETT |exp| |groupord| |FFIELDC-;discreteLog;2SU;12|)
          (SEQ
           (LETT |f| NIL |FFIELDC-;discreteLog;2SU;12|)
           (LETT #1# |faclist| |FFIELDC-;discreteLog;2SU;12|)
           G190
           (COND
            ((OR
              (ATOM #1#)
              (PROGN (LETT |f| (CAR #1#) |FFIELDC-;discreteLog;2SU;12|) NIL))
             (GO G191)))
           (SEQ
            (LETT |fac| (QCAR |f|) |FFIELDC-;discreteLog;2SU;12|)
            (LETT |primroot|
             (SPADCALL |logbase|
              (QUOTIENT2 |groupord| |fac|)
              (QREFELT |$| 48))
             |FFIELDC-;discreteLog;2SU;12|)
            (EXIT
             (SEQ
              (LETT |t| 0 |FFIELDC-;discreteLog;2SU;12|)
              (LETT #2# (|-| (QCDR |f|) 1) |FFIELDC-;discreteLog;2SU;12|)
              G190
              (COND ((QSGREATERP |t| #2#) (GO G191)))
              (SEQ
               (LETT |exp|
                (QUOTIENT2 |exp| |fac|)
                |FFIELDC-;discreteLog;2SU;12|)
```

```
               (LETT |rhoHelp|
                (SPADCALL |primroot|
                 (SPADCALL |a| |exp| (QREFELT |$| 48))
                 |fac|
                 (QREFELT |$| 70))
                 |FFIELDC-;discreteLog;2SU;12|)
               (EXIT
                (COND
                 ((QEQCAR |rhoHelp| 1)
                  (PROGN
                   (LETT #3# (CONS 1 "failed") |FFIELDC-;discreteLog;2SU;12|)
                   (GO #3#)))
                 ((QUOTE T)
                  (SEQ
                   (LETT |rho|
                    (|*| (QCDR |rhoHelp|) |mult|)
                    |FFIELDC-;discreteLog;2SU;12|)
                   (LETT |disclog|
                    (|+| |disclog| |rho|)
                    |FFIELDC-;discreteLog;2SU;12|)
                   (LETT |mult|
                    (|*| |mult| |fac|)
                    |FFIELDC-;discreteLog;2SU;12|)
                   (EXIT
                    (LETT |a|
                     (SPADCALL |a|
                      (SPADCALL |logbase| (|-| |rho|) (QREFELT |$| 48))
                      (QREFELT |$| 59))
                     |FFIELDC-;discreteLog;2SU;12|)))))))
              (LETT |t| (QSADD1 |t|) |FFIELDC-;discreteLog;2SU;12|)
              (GO G190)
              G191
              (EXIT NIL))))
           (LETT #1# (CDR #1#) |FFIELDC-;discreteLog;2SU;12|)
           (GO G190)
           G191
           (EXIT NIL))
          (EXIT (CONS 0 |disclog|))))))))
    #3#
    (EXIT #3#)))))

(DEFUN |FFIELDC-;squareFreePolynomial| (|f| |$|)
 (SPADCALL |f| (QREFELT |$| 75)))

(DEFUN |FFIELDC-;factorPolynomial| (|f| |$|)
 (SPADCALL |f| (QREFELT |$| 77)))

(DEFUN |FFIELDC-;factorSquareFreePolynomial| (|f| |$|)
 (PROG (|flist| |u| #1=#:G83248 #2=#:G83245 #3=#:G83243 #4=#:G83244)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |f| (|spadConstant| |$| 78) (QREFELT |$| 79))
       (|spadConstant| |$| 80))
```

```
     ((QUOTE T)
      (SEQ
       (LETT |flist|
        (SPADCALL |f| (QUOTE T) (QREFELT |$| 83))
        |FFIELDC-;factorSquareFreePolynomial|)
       (EXIT
        (SPADCALL
         (SPADCALL (QCAR |flist|) (QREFELT |$| 84))
         (PROGN
          (LETT #4# NIL |FFIELDC-;factorSquareFreePolynomial|)
          (SEQ
           (LETT |u| NIL |FFIELDC-;factorSquareFreePolynomial|)
           (LETT #1# (QCDR |flist|) |FFIELDC-;factorSquareFreePolynomial|)
           G190
           (COND
            ((OR
              (ATOM #1#)
              (PROGN
               (LETT |u| (CAR #1#) |FFIELDC-;factorSquareFreePolynomial|)
               NIL))
             (GO G191)))
           (SEQ
            (EXIT
             (PROGN
              (LETT #2#
               (SPADCALL (QCAR |u|) (QCDR |u|) (QREFELT |$| 85))
               |FFIELDC-;factorSquareFreePolynomial|)
              (COND
               (#4#
                (LETT #3#
                 (SPADCALL #3# #2# (QREFELT |$| 86))
                 |FFIELDC-;factorSquareFreePolynomial|))
               ((QUOTE T)
                (PROGN
                 (LETT #3# #2# |FFIELDC-;factorSquareFreePolynomial|)
                 (LETT #4#
                  (QUOTE T)
                  |FFIELDC-;factorSquareFreePolynomial|)))))))
           (LETT #1# (CDR #1#) |FFIELDC-;factorSquareFreePolynomial|)
           (GO G190)
           G191
          (EXIT NIL))
          (COND (#4# #3#) ((QUOTE T) (|spadConstant| |$| 87))))
         (QREFELT |$| 88)))))))))

(DEFUN |FFIELDC-;gcdPolynomial;3Sup;16| (|f| |g| |$|)
 (SPADCALL |f| |g| (QREFELT |$| 90)))

(DEFUN |FiniteFieldCategory&| (|#1|)
 (PROG (|DV$1| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|FiniteFieldCategory&|))
    (LETT |dv$| (LIST (QUOTE |FiniteFieldCategory&|) |DV$1|) . #1#)
```

```
    (LETT |$| (make-array 93) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|) |$|))))

(setf (get
 (QUOTE |FiniteFieldCategory&|)
 (QUOTE |infovec|))
 (LIST
  (QUOTE
  #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (0 . |Zero|)
    |FFIELDC-;differentiate;2S;1| |FFIELDC-;init;S;2| (|PositiveInteger|)
    (4 . |lookup|) (9 . |index|) (|Boolean|) (14 . |zero?|)
    (|Union| |$| (QUOTE "failed")) |FFIELDC-;nextItem;SU;3| (19 . |order|)
    (|Integer|) (|OnePointCompletion| 10) (24 . |coerce|)
    |FFIELDC-;order;SOpc;4| (|List| 26) (|Matrix| 6) (29 . |nullSpace|)
    (|Mapping| 13 6) (|Vector| 6) (34 . |every?|) (40 . |charthRoot|)
    (|Mapping| 6 6) (45 . |map|) (|Union| (|Vector| |$|) (QUOTE "failed"))
    (|Matrix| |$|) |FFIELDC-;conditionP;MU;5| (|NonNegativeInteger|)
    (51 . |size|) (55 . |characteristic|) (59 . |**|)
    |FFIELDC-;charthRoot;2S;6| |FFIELDC-;charthRoot;SU;7| (65 . |One|)
    (|Union| (QUOTE "prime") (QUOTE "polynomial") (QUOTE "normal")
    (QUOTE "cyclic")) (69 . |representationType|) (73 . |=|)
    |FFIELDC-;createPrimitiveElement;S;8| (|Record| (|:| |factor| 18)
    (|:| |exponent| 18)) (|List| 45) (79 . |factorsOfCyclicGroupSize|)
    (83 . |**|) (89 . |one?|) |FFIELDC-;primitive?;SB;9| (94 . |=|)
    |FFIELDC-;order;SPi;10| (100 . |primitiveElement|) (|Table| 10 34)
    (104 . |tableForDiscreteLogarithm|) (109 . |#|)
    (|Union| 34 (QUOTE "failed")) (114 . |search|) (120 . |*|)
    |FFIELDC-;discreteLog;SNni;11| (|Void|) (|String|) (|OutputForm|)
    (126 . |messagePrint|) (|Factored| |$|) (131 . |factor|)
    (|Factored| 18) (136 . |factors|) (|DiscreteLogarithmPackage| 6)
    (141 . |shanksDiscLogAlgorithm|) |FFIELDC-;discreteLog;2SU;12|
    (|Factored| 73) (|SparseUnivariatePolynomial| 6)
    (|UnivariatePolynomialSquareFree| 6 73) (148 . |squareFree|)
    (|DistinctDegreeFactorize| 6 73) (153 . |factor|) (158 . |Zero|)
    (162 . |=|) (168 . |Zero|) (|Record| (|:| |irr| 73) (|:| |pow| 18))
    (|Record| (|:| |cont| 6) (|:| |factors| (|List| 81)))
    (172 . |distdfact|) (178 . |coerce|) (183 . |primeFactor|)
    (189 . |*|) (195 . |One|) (199 . |*|) (|EuclideanDomain&| 73)
    (205 . |gcd|) (|SparseUnivariatePolynomial| |$|)
    |FFIELDC-;gcdPolynomial;3Sup;16|))
   (QUOTE
    #(|primitive?| 211 |order| 216 |nextItem| 226 |init| 231
      |gcdPolynomial| 235 |discreteLog| 241 |differentiate| 252
      |createPrimitiveElement| 257 |conditionP| 261 |charthRoot| 266))
  (QUOTE NIL)
  (CONS
   (|makeByteWordVec2| 1 (QUOTE NIL))
   (CONS
    (QUOTE #())
    (CONS
     (QUOTE #())
```

```
     (|makeByteWordVec2| 92
      (QUOTE
       (0 6 0 7 1 6 10 0 11 1 6 0 10 12 1 6 13 0 14 1 6 10 0 17 1 19
        0 18 20 1 23 22 0 24 2 26 13 25 0 27 1 6 0 0 28 2 26 0 29 0 30
        0 6 34 35 0 6 34 36 2 6 0 0 34 37 0 6 0 40 0 6 41 42 2 41 13
        0 0 43 0 6 46 47 2 6 0 0 18 48 1 6 13 0 49 2 6 13 0 0 51 0 6 0
        53 1 6 54 18 55 1 54 34 0 56 2 54 57 10 0 58 2 6 0 0 0 59 1
        63 61 62 64 1 18 65 0 66 1 67 46 0 68 3 69 57 6 6 34 70 1 74
        72 73 75 1 76 72 73 77 0 73 0 78 2 73 13 0 0 79 0 72 0 80 2
        76 82 73 13 83 1 73 0 6 84 2 72 0 73 18 85 2 72 0 0 0 86 0 72
        0 87 2 72 0 73 0 88 2 89 0 0 0 90 1 0 13 0 50 1 0 10 0 52 1 0
        19 0 21 1 0 15 0 16 0 0 0 9 2 0 91 91 91 92 1 0 34 0 60 2 0 57
        0 0 71 1 0 0 0 8 0 0 0 44 1 0 31 32 33 1 0 0 0 38 1 0 15 0 39))))))
  (QUOTE |lookupComplete|)))
```

21.23 FPS.lsp BOOTSTRAP

FPS depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **FPS** category which we can write into the **MID** directory. We compile the lisp code and copy the **FPS.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— FPS.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |FloatingPointSystem;AL| (QUOTE NIL))

(DEFUN |FloatingPointSystem| NIL
  (declare (special |FloatingPointSystem;AL|))
    (COND
      (|FloatingPointSystem;AL|)
      (T (SETQ |FloatingPointSystem;AL| (|FloatingPointSystem;|)))))

(DEFUN |FloatingPointSystem;| NIL
  (PROG (#1=#:G105643)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|RealNumberSystem|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|float| (|$| (|Integer|) (|Integer|))) T)
                ((|float| (|$| (|Integer|) (|Integer|) (|PositiveInteger|))) T)
                ((|order| ((|Integer|) |$|)) T)
                ((|base| ((|PositiveInteger|))) T)
                ((|exponent| ((|Integer|) |$|)) T)
```

```
((|mantissa| ((|Integer|) |$|)) T)
((|bits| ((|PositiveInteger|))) T)
((|digits| ((|PositiveInteger|))) T)
((|precision| ((|PositiveInteger|))) T)
((|bits| ((|PositiveInteger|) (|PositiveInteger|)))
  (|has| |$| (ATTRIBUTE |arbitraryPrecision|)))
((|digits| ((|PositiveInteger|) (|PositiveInteger|)))
  (|has| |$| (ATTRIBUTE |arbitraryPrecision|)))
((|precision| ((|PositiveInteger|) (|PositiveInteger|)))
  (|has| |$| (ATTRIBUTE |arbitraryPrecision|)))
((|increasePrecision| ((|PositiveInteger|) (|Integer|)))
  (|has| |$| (ATTRIBUTE |arbitraryPrecision|)))
((|decreasePrecision| ((|PositiveInteger|) (|Integer|)))
  (|has| |$| (ATTRIBUTE |arbitraryPrecision|)))
((|min| (|$|))
  (AND
    (|not| (|has| |$| (ATTRIBUTE |arbitraryPrecision|)))
    (|not| (|has| |$| (ATTRIBUTE |arbitraryExponent|)))))
((|max| (|$|))
  (AND
    (|not| (|has| |$| (ATTRIBUTE |arbitraryPrecision|)))
    (|not| (|has| |$| (ATTRIBUTE |arbitraryExponent|)))))))
(QUOTE ((|approximate| T)))
(QUOTE ((|PositiveInteger|) (|Integer|)))
NIL))
|FloatingPointSystem|)
(SETELT #1# 0 (QUOTE (|FloatingPointSystem|)))))))

(setf (get (QUOTE |FloatingPointSystem|) (QUOTE NILADIC)) T)
```

21.24 FPS-.lsp BOOTSTRAP

FPS- depends **FPS**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **FPS-** category which we can write into the **MID** directory. We compile the lisp code and copy the **FPS-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— FPS-.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(DEFUN |FPS-;float;2IS;1| (|ma| |ex| |$|)
  (SPADCALL |ma| |ex| (SPADCALL (QREFELT |$| 8)) (QREFELT |$| 10)))

(DEFUN |FPS-;digits;Pi;2| (|$|)
  (PROG (#1=#:G105654)
    (RETURN
      (PROG1
```

```
        (LETT #1#
          (MAX 1
            (QUOTIENT2
              (SPADCALL 4004
                (|-| (SPADCALL (QREFELT |$| 13)) 1)
                (QREFELT |$| 14))
              13301))
          |FPS-;digits;Pi;2|)
        (|check-subtype| (|>| #1# 0) (QUOTE (|PositiveInteger|)) #1#)))))

(DEFUN |FloatingPointSystem&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|FloatingPointSystem&|))
        (LETT |dv$| (LIST (QUOTE |FloatingPointSystem&|) |DV$1|) . #1#)
        (LETT |$| (make-array 17) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3
          (LETT |pv$|
            (|buildPredVector| 0 0
              (LIST
                (|HasAttribute| |#1| (QUOTE |arbitraryExponent|))
                (|HasAttribute| |#1| (QUOTE |arbitraryPrecision|)))) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        |$|))))

(setf (get
  (QUOTE |FloatingPointSystem&|)
  (QUOTE |infovec|))
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (|PositiveInteger|)
        (0 . |base|)
        (|Integer|)
        (4 . |float|)
        |FPS-;float;2IS;1|
        (11 . |One|)
        (15 . |bits|)
        (19 . |*|)
        (25 . |max|)
        |FPS-;digits;Pi;2|))
    (QUOTE #(|float| 29 |digits| 35))
    (QUOTE NIL)
    (CONS
      (|makeByteWordVec2| 1 (QUOTE NIL))
      (CONS
        (QUOTE #())
        (CONS
          (QUOTE #())
          (|makeByteWordVec2| 16
```

```
        (QUOTE
          (0 6 7 8 3 6 0 9 9 7 10 0 6 0 12 0 6 7 13 2 9 0 7 0 14 0 6 0 15
           2 0 0 9 9 11 0 0 7 16))))))
     (QUOTE |lookupComplete|)))
```

———-

21.25 GCDDOM.lsp BOOTSTRAP

GCDDOM needs **COMRING** which needs **RING** which needs **RNG** which needs **ABELGRP** which needs **CABMON** which needs **ABELMON** which needs **ABELSG** which needs **SETCAT** which needs **SINT** which needs **UFD** which needs **GCDDOM**. We break this chain with **GCDDOM.lsp** which we cache here. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **GCDDOM** category which we can write into the **MID** directory. We compile the lisp code and copy the **GCDDOM.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— GCDDOM.lsp BOOTSTRAP —

```
(/VERSIONCHECK 2)

(SETQ |GcdDomain;AL| (QUOTE NIL))

(DEFUN |GcdDomain| NIL
 (declare (special |GcdDomain;AL|))
  (COND
   (|GcdDomain;AL|)
   (T (SETQ |GcdDomain;AL| (|GcdDomain;|)))))

(DEFUN |GcdDomain;| NIL
 (PROG (#0=#:G22289)
  (RETURN
   (PROG1
    (LETT #0#
     (|Join|
      (|IntegralDomain|)
      (|LeftOreRing|)
      (|mkCategory|
       (QUOTE |domain|)
       (QUOTE (
        ((|gcd| ($ $ $)) T)
        ((|gcd| ($ (|List| $))) T)
        ((|lcm| ($ $ $)) T)
        ((|lcm| ($ (|List| $))) T)
        ((|gcdPolynomial|
         ((|SparseUnivariatePolynomial| $)
          (|SparseUnivariatePolynomial| $)
          (|SparseUnivariatePolynomial| $))) T)))
       NIL
       (QUOTE ((|SparseUnivariatePolynomial| $) (|List| $)))
```

```
      NIL))
     |GcdDomain|)
    (SETELT #0# 0 (QUOTE (|GcdDomain|))))))

(SETF (GET (QUOTE |GcdDomain|) (QUOTE NILADIC)) T)
```

———-

21.26 GCDDOM-.lsp BOOTSTRAP

GCDDOM- depends on **GCDDOM**. We break this chain with **GCDDOM-.lsp** which we cache here. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **GCDDOM-** category which we can write into the **MID** directory. We compile the lisp code and copy the **GCDDOM-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— GCDDOM-.lsp BOOTSTRAP —

```
(/VERSIONCHECK 2)

(DEFUN |GCDDOM-;lcm;3S;1| (|x| |y| $)
 (PROG (LCM)
  (RETURN
   (SEQ
    (COND
     ((OR (SPADCALL |y| (|spadConstant| $ 7) (QREFELT $ 9))
          (SPADCALL |x| (|spadConstant| $ 7) (QREFELT $ 9)))
      (|spadConstant| $ 7))
     ((QUOTE T)
      (SEQ
       (LETT LCM
        (SPADCALL |y| (SPADCALL |x| |y| (QREFELT $ 10)) (QREFELT $ 12))
        |GCDDOM-;lcm;3S;1|)
       (EXIT
        (COND
         ((QEQCAR LCM 0) (SPADCALL |x| (QCDR LCM) (QREFELT $ 13)))
         ((QUOTE T) (|error| "bad gcd in lcm computation")))))))))))

(DEFUN |GCDDOM-;lcm;LS;2| (|l| $)
 (SPADCALL (ELT $ 15) |l| (|spadConstant| $ 16) (|spadConstant| $ 7)
  (QREFELT $ 19)))

(DEFUN |GCDDOM-;gcd;LS;3| (|l| $)
 (SPADCALL (ELT $ 10) |l| (|spadConstant| $ 7) (|spadConstant| $ 16)
  (QREFELT $ 19)))

(DEFUN |GCDDOM-;gcdPolynomial;3Sup;4| (|p1| |p2| $)
 (PROG (|e2| |e1| |c1| |p| |c2| #0=#:G22304)
  (RETURN
   (SEQ
```

```
(COND
 ((SPADCALL |p1| (QREFELT $ 24)) (SPADCALL |p2| (QREFELT $ 25)))
 ((SPADCALL |p2| (QREFELT $ 24)) (SPADCALL |p1| (QREFELT $ 25)))
 ((QUOTE T)
  (SEQ
   (LETT |c1|
    (SPADCALL |p1| (QREFELT $ 26)) |GCDDOM-;gcdPolynomial;3Sup;4|)
   (LETT |c2|
    (SPADCALL |p2| (QREFELT $ 26)) |GCDDOM-;gcdPolynomial;3Sup;4|)
   (LETT |p1|
    (PROG2
     (LETT #0# (SPADCALL |p1| |c1| (QREFELT $ 27)) |GCDDOM-;gcdPolynomial;3Sup;4|)
     (QCDR #0#)
     (|check-union| (QEQCAR #0# 0)
      (|SparseUnivariatePolynomial| (QREFELT $ 6)) #0#))
    |GCDDOM-;gcdPolynomial;3Sup;4|)
   (LETT |p2|
    (PROG2
     (LETT #0#
      (SPADCALL |p2| |c2| (QREFELT $ 27))
      |GCDDOM-;gcdPolynomial;3Sup;4|)
     (QCDR #0#)
     (|check-union| (QEQCAR #0# 0)
       (|SparseUnivariatePolynomial| (QREFELT $ 6)) #0#))
    |GCDDOM-;gcdPolynomial;3Sup;4|)
   (SEQ
    (LETT |e1|
     (SPADCALL |p1| (QREFELT $ 29))
     |GCDDOM-;gcdPolynomial;3Sup;4|)
    (EXIT
     (COND
      ((< 0 |e1|)
       (LETT |p1|
        (PROG2
         (LETT #0#
          (SPADCALL |p1|
           (SPADCALL (|spadConstant| $ 16) |e1| (QREFELT $ 32))
           (QREFELT $ 33))
          |GCDDOM-;gcdPolynomial;3Sup;4|)
         (QCDR #0#)
         (|check-union| (QEQCAR #0# 0)
          (|SparseUnivariatePolynomial| (QREFELT $ 6)) #0#))
        |GCDDOM-;gcdPolynomial;3Sup;4|)))))
   (SEQ
    (LETT |e2|
     (SPADCALL |p2| (QREFELT $ 29))
     |GCDDOM-;gcdPolynomial;3Sup;4|)
    (EXIT
     (COND
      ((< 0 |e2|)
       (LETT |p2|
        (PROG2
         (LETT #0#
          (SPADCALL |p2|
```

```
        (SPADCALL (|spadConstant| $ 16) |e2| (QREFELT $ 32))
        (QREFELT $ 33))
       |GCDDOM-;gcdPolynomial;3Sup;4|)
      (QCDR #0#)
      (|check-union| (QEQCAR #0# 0)
       (|SparseUnivariatePolynomial| (QREFELT $ 6)) #0#))
     |GCDDOM-;gcdPolynomial;3Sup;4|)))))
(LETT |e1| (MIN |e1| |e2|) |GCDDOM-;gcdPolynomial;3Sup;4|)
(LETT |c1|
 (SPADCALL |c1| |c2| (QREFELT $ 10))
 |GCDDOM-;gcdPolynomial;3Sup;4|)
(LETT |p1|
 (COND
  ((OR (EQL (SPADCALL |p1| (QREFELT $ 34)) 0)
       (EQL (SPADCALL |p2| (QREFELT $ 34)) 0))
   (SPADCALL |c1| 0 (QREFELT $ 32)))
  ((QUOTE T)
   (SEQ
    (LETT |p|
     (SPADCALL |p1| |p2| (QREFELT $ 35))
     |GCDDOM-;gcdPolynomial;3Sup;4|)
   (EXIT
    (COND
     ((EQL (SPADCALL |p| (QREFELT $ 34)) 0)
       (SPADCALL |c1| 0 (QREFELT $ 32)))
     ((QUOTE T)
      (SEQ
       (LETT |c2|
        (SPADCALL
         (SPADCALL |p1| (QREFELT $ 36))
         (SPADCALL |p2| (QREFELT $ 36))
         (QREFELT $ 10))
        |GCDDOM-;gcdPolynomial;3Sup;4|)
       (EXIT
        (SPADCALL
         (SPADCALL |c1|
          (SPADCALL
           (PROG2
            (LETT #0#
             (SPADCALL
              (SPADCALL |c2| |p| (QREFELT $ 37))
              (SPADCALL |p| (QREFELT $ 36))
              (QREFELT $ 27))
             |GCDDOM-;gcdPolynomial;3Sup;4|)
            (QCDR #0#)
            (|check-union| (QEQCAR #0# 0)
              (|SparseUnivariatePolynomial| (QREFELT $ 6)) #0#))
           (QREFELT $ 38))
          (QREFELT $ 37))
         (QREFELT $ 25))))))))))
 |GCDDOM-;gcdPolynomial;3Sup;4|)
(EXIT
 (COND
  ((ZEROP |e1|) |p1|)
```

```
         ((QUOTE T)
          (SPADCALL
           (SPADCALL (|spadConstant| $ 16) |e1| (QREFELT $ 32))
           |p1| (QREFELT $ 39))))))))))))

(DEFUN |GCDDOM-;lcmCoef;2SR;5| (|c1| |c2| $)
 (PROG (|g| |cc1| #0=#:G22318 |cc2|)
  (RETURN
   (SEQ
    (LETT |g| (SPADCALL |c1| |c2| (QREFELT $ 10)) |GCDDOM-;lcmCoef;2SR;5|)
    (LETT |cc1|
     (PROG2
      (LETT #0# (SPADCALL |c2| |g| (QREFELT $ 12)) |GCDDOM-;lcmCoef;2SR;5|)
      (QCDR #0#)
      (|check-union| (QEQCAR #0# 0) (QREFELT $ 6) #0#))
     |GCDDOM-;lcmCoef;2SR;5|)
    (LETT |cc2|
     (PROG2
      (LETT #0#
       (SPADCALL |c1| |g| (QREFELT $ 12))
       |GCDDOM-;lcmCoef;2SR;5|)
      (QCDR #0#)
      (|check-union| (QEQCAR #0# 0) (QREFELT $ 6) #0#))
     |GCDDOM-;lcmCoef;2SR;5|)
    (EXIT (VECTOR (SPADCALL |cc1| |c1| (QREFELT $ 13)) |cc1| |cc2|))))))

(DEFUN |GcdDomain&| (|#1|)
 (PROG (DV$1 |dv$| $ |pv$|)
  (RETURN
   (PROGN
    (LETT DV$1 (|devaluate| |#1|) . #0=(|GcdDomain&|))
    (LETT |dv$| (LIST (QUOTE |GcdDomain&|) DV$1) . #0#)
    (LETT $ (MAKE-ARRAY 44) . #0#)
    (QSETREFV $ 0 |dv$|)
    (QSETREFV $ 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #0#))
    (|stuffDomainSlots| $)
    (QSETREFV $ 6 |#1|)
    $))))

(SETF (GET (QUOTE |GcdDomain&|) (QUOTE |infovec|))
 (LIST
  (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (0 . |Zero|) (|Boolean|)
    (4 . =) (10 . |gcd|) (|Union| $ (QUOTE "failed")) (16 . |exquo|)
    (22 . *) |GCDDOM-;lcm;3S;1| (28 . |lcm|) (34 . |One|) (|Mapping| 6 6 6)
    (|List| 6) (38 . |reduce|) (|List| $) |GCDDOM-;lcm;LS;2|
    |GCDDOM-;gcd;LS;3| (|SparseUnivariatePolynomial| 6) (46 . |zero?|)
    (51 . |unitCanonical|) (56 . |content|) (61 . |exquo|)
    (|NonNegativeInteger|) (67 . |minimumDegree|) (72 . |Zero|) (76 . |One|)
    (80 . |monomial|) (86 . |exquo|) (92 . |degree|) (97 . |subResultantGcd|)
    (103 . |leadingCoefficient|) (108 . *) (114 . |primitivePart|) (119 . *)
    (|SparseUnivariatePolynomial| $) |GCDDOM-;gcdPolynomial;3Sup;4|
    (|Record| (|:| |llcmres| $) (|:| |coeff1| $) (|:| |coeff2| $))
    |GCDDOM-;lcmCoef;2SR;5|))
  (QUOTE #(|lcmCoef| 125 |lcm| 131 |gcdPolynomial| 142 |gcd| 148))
```

```
  (QUOTE NIL)
  (CONS
   (|makeByteWordVec2| 1 (QUOTE NIL))
   (CONS
    (QUOTE #())
    (CONS
     (QUOTE #())
     (|makeByteWordVec2| 43 (QUOTE (0 6 0 7 2 6 8 0 0 9 2 6 0 0 0 10 2 6
       11 0 0 12 2 6 0 0 0 13 2 6 0 0 0 15 0 6 0 16 4 18 6 17 0 6 6 19 1
       23 8 0 24 1 23 0 0 25 1 23 6 0 26 2 23 11 0 6 27 1 23 28 0 29 0
       23 0 30 0 23 0 31 2 23 0 6 28 32 2 23 11 0 0 33 1 23 28 0 34 2
       23 0 0 0 35 1 23 6 0 36 2 23 0 6 0 37 1 23 0 0 38 2 23 0 0 0 39
       2 0 42 0 0 43 2 0 0 0 0 14 1 0 0 20 21 2 0 40 40 40 41 1 0 0 20 22))))))
  (QUOTE |lookupComplete|)))
```

21.27 HOAGG.lsp BOOTSTRAP

HOAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **HOAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **HOAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— HOAGG.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |HomogeneousAggregate;CAT| (QUOTE NIL))

(SETQ |HomogeneousAggregate;AL| (QUOTE NIL))

(DEFUN |HomogeneousAggregate| (#1=#:G82375)
  (declare (special |HomogeneousAggregate;AL|))
  (LET (#2=#:G82376)
    (COND
      ((SETQ #2# (|assoc| (|devaluate| #1#) |HomogeneousAggregate;AL|))
        (CDR #2#))
      (T
        (SETQ |HomogeneousAggregate;AL|
          (|cons5|
            (CONS (|devaluate| #1#) (SETQ #2# (|HomogeneousAggregate;| #1#)))
            |HomogeneousAggregate;AL|))
        #2#))))

(DEFUN |HomogeneousAggregate;| (|t#1|)
  (declare (special |HomogeneousAggregate;CAT|))
  (PROG (#1=#:G82374)
    (RETURN
      (PROG1
```

```
(LETT #1#
  (|sublisV|
    (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
    (COND
      (|HomogeneousAggregate;CAT|)
      ((QUOTE T)
        (LETT |HomogeneousAggregate;CAT|
          (|Join|
            (|Aggregate|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|map| (|$| (|Mapping| |t#1| |t#1|) |$|)) T)
                ((|map!| (|$| (|Mapping| |t#1| |t#1|) |$|))
                  (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
                ((|any?|
                   ((|Boolean|) (|Mapping| (|Boolean|) |t#1|) |$|))
                  (|has| |$| (ATTRIBUTE |finiteAggregate|)))
                ((|every?|
                   ((|Boolean|) (|Mapping| (|Boolean|) |t#1|) |$|))
                  (|has| |$| (ATTRIBUTE |finiteAggregate|)))
                ((|count|
                   ((|NonNegativeInteger|)
                    (|Mapping| (|Boolean|) |t#1|) |$|))
                  (|has| |$| (ATTRIBUTE |finiteAggregate|)))
                ((|parts| ((|List| |t#1|) |$|))
                  (|has| |$| (ATTRIBUTE |finiteAggregate|)))
                ((|members| ((|List| |t#1|) |$|))
                  (|has| |$| (ATTRIBUTE |finiteAggregate|)))
                ((|count| ((|NonNegativeInteger|) |t#1| |$|))
                  (AND
                    (|has| |t#1| (|SetCategory|))
                    (|has| |$| (ATTRIBUTE |finiteAggregate|))))
                ((|member?| ((|Boolean|) |t#1| |$|))
                  (AND
                    (|has| |t#1| (|SetCategory|))
                    (|has| |$| (ATTRIBUTE |finiteAggregate|))))))
             (QUOTE (
              ((|SetCategory|) (|has| |t#1| (|SetCategory|)))
              ((|Evalable| |t#1|)
                (AND
                  (|has| |t#1| (|Evalable| |t#1|))
                  (|has| |t#1| (|SetCategory|))))))
            (QUOTE (
              (|Boolean|)
              (|NonNegativeInteger|)
              (|List| |t#1|)))
            NIL))
        . #2=(|HomogeneousAggregate|))))) . #2#)
(SETELT #1# 0
  (LIST (QUOTE |HomogeneousAggregate|) (|devaluate| |t#1|))))))
```

———-

21.28 HOAGG-.lsp BOOTSTRAP

HOAGG- depends on **HOAGG**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **HOAGG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **HOAGG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— HOAGG-.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(DEFUN |HOAGG-;eval;ALA;1| (|u| |l| |$|)
 (SPADCALL
  (CONS (FUNCTION |HOAGG-;eval;ALA;1!0|) (VECTOR |$| |l|))
  |u|
  (QREFELT |$| 11)))

(DEFUN |HOAGG-;eval;ALA;1!0| (|#1| |$$|)
 (SPADCALL |#1| (QREFELT |$$| 1) (QREFELT (QREFELT |$$| 0) 9)))

(DEFUN |HOAGG-;#;ANni;2| (|c| |$|)
 (LENGTH (SPADCALL |c| (QREFELT |$| 14))))

(DEFUN |HOAGG-;any?;MAB;3| (|f| |c| |$|)
 (PROG (|x| #1=#:G82396 #2=#:G82393 #3=#:G82391 #4=#:G82392)
  (RETURN
   (SEQ
    (PROGN
     (LETT #4# NIL |HOAGG-;any?;MAB;3|)
     (SEQ
      (LETT |x| NIL |HOAGG-;any?;MAB;3|)
      (LETT #1# (SPADCALL |c| (QREFELT |$| 14)) |HOAGG-;any?;MAB;3|)
      G190
      (COND
       ((OR (ATOM #1#) (PROGN (LETT |x| (CAR #1#) |HOAGG-;any?;MAB;3|) NIL))
        (GO G191)))
      (SEQ
       (EXIT
        (PROGN
         (LETT #2# (SPADCALL |x| |f|) |HOAGG-;any?;MAB;3|)
         (COND
          (#4#
           (LETT #3#
             (COND
              (#3# (QUOTE T))
              ((QUOTE T) #2#))
             |HOAGG-;any?;MAB;3|))
          ((QUOTE T)
           (PROGN
            (LETT #3# #2# |HOAGG-;any?;MAB;3|)
            (LETT #4# (QUOTE T) |HOAGG-;any?;MAB;3|)))))))
      (LETT #1# (CDR #1#) |HOAGG-;any?;MAB;3|) (GO G190) G191 (EXIT NIL))
```

```
     (COND (#4# #3#) ((QUOTE T) (QUOTE NIL)))))))

(DEFUN |HOAGG-;every?;MAB;4| (|f| |c| |$|)
 (PROG (|x| #1=#:G82401 #2=#:G82399 #3=#:G82397 #4=#:G82398)
  (RETURN
   (SEQ
    (PROGN
     (LETT #4# NIL |HOAGG-;every?;MAB;4|)
     (SEQ
      (LETT |x| NIL |HOAGG-;every?;MAB;4|)
      (LETT #1# (SPADCALL |c| (QREFELT |$| 14)) |HOAGG-;every?;MAB;4|)
      G190
      (COND
        ((OR (ATOM #1#) (PROGN (LETT |x| (CAR #1#) |HOAGG-;every?;MAB;4|) NIL))
         (GO G191)))
      (SEQ
       (EXIT
        (PROGN
         (LETT #2# (SPADCALL |x| |f|) |HOAGG-;every?;MAB;4|)
         (COND
          (#4#
           (LETT #3#
            (COND (#3# #2#) ((QUOTE T) (QUOTE NIL)))
            |HOAGG-;every?;MAB;4|))
          ((QUOTE T)
           (PROGN
            (LETT #3# #2# |HOAGG-;every?;MAB;4|)
            (LETT #4# (QUOTE T) |HOAGG-;every?;MAB;4|)))))))
      (LETT #1# (CDR #1#) |HOAGG-;every?;MAB;4|)
      (GO G190)
      G191
      (EXIT NIL))
     (COND (#4# #3#) ((QUOTE T) (QUOTE T)))))))

(DEFUN |HOAGG-;count;MANni;5| (|f| |c| |$|)
 (PROG (|x| #1=#:G82406 #2=#:G82404 #3=#:G82402 #4=#:G82403)
  (RETURN
   (SEQ
    (PROGN
     (LETT #4# NIL |HOAGG-;count;MANni;5|)
     (SEQ
      (LETT |x| NIL |HOAGG-;count;MANni;5|)
      (LETT #1# (SPADCALL |c| (QREFELT |$| 14)) |HOAGG-;count;MANni;5|)
      G190
      (COND
       ((OR (ATOM #1#) (PROGN (LETT |x| (CAR #1#) |HOAGG-;count;MANni;5|) NIL))
        (GO G191)))
      (SEQ
       (EXIT
        (COND
         ((SPADCALL |x| |f|)
          (PROGN
           (LETT #2# 1 |HOAGG-;count;MANni;5|)
           (COND
```

```
            (#4# (LETT #3# (|+| #3# #2#) |HOAGG-;count;MANni;5|))
            ((QUOTE T)
             (PROGN
              (LETT #3# #2# |HOAGG-;count;MANni;5|)
              (LETT #4# (QUOTE T) |HOAGG-;count;MANni;5|)))))))))
      (LETT #1# (CDR #1#) |HOAGG-;count;MANni;5|)
      (GO G190)
      G191
      (EXIT NIL))
     (COND (#4# #3#) ((QUOTE T) 0)))))))

(DEFUN |HOAGG-;members;AL;6| (|x| |$|) (SPADCALL |x| (QREFELT |$| 14)))

(DEFUN |HOAGG-;count;SANni;7| (|s| |x| |$|)
 (SPADCALL
  (CONS (FUNCTION |HOAGG-;count;SANni;7!0|) (VECTOR |$| |s|))
  |x|
  (QREFELT |$| 24)))

(DEFUN |HOAGG-;count;SANni;7!0| (|#1| |$$|)
 (SPADCALL (QREFELT |$$| 1) |#1| (QREFELT (QREFELT |$$| 0) 23)))

(DEFUN |HOAGG-;member?;SAB;8| (|e| |c| |$|)
 (SPADCALL
  (CONS (FUNCTION |HOAGG-;member?;SAB;8!0|) (VECTOR |$| |e|))
  |c|
  (QREFELT |$| 26)))

(DEFUN |HOAGG-;member?;SAB;8!0| (|#1| |$$|)
 (SPADCALL (QREFELT |$$| 1) |#1| (QREFELT (QREFELT |$$| 0) 23)))

(DEFUN |HOAGG-;=;2AB;9| (|x| |y| |$|)
 (PROG (|b| #1=#:G82416 |a| #2=#:G82415 #3=#:G82412 #4=#:G82410 #5=#:G82411)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |x| (SPADCALL |y| (QREFELT |$| 28)) (QREFELT |$| 29))
      (PROGN
       (LETT #5# NIL |HOAGG-;=;2AB;9|)
       (SEQ
        (LETT |b| NIL |HOAGG-;=;2AB;9|)
        (LETT #1# (SPADCALL |y| (QREFELT |$| 14)) |HOAGG-;=;2AB;9|)
        (LETT |a| NIL |HOAGG-;=;2AB;9|)
        (LETT #2# (SPADCALL |x| (QREFELT |$| 14)) |HOAGG-;=;2AB;9|)
        G190
        (COND
         ((OR
           (ATOM #2#)
           (PROGN (LETT |a| (CAR #2#) |HOAGG-;=;2AB;9|) NIL)
           (ATOM #1#)
           (PROGN (LETT |b| (CAR #1#) |HOAGG-;=;2AB;9|) NIL))
          (GO G191)))
        (SEQ
         (EXIT
```

```
          (PROGN
           (LETT #3# (SPADCALL |a| |b| (QREFELT |$| 23)) |HOAGG-;=;2AB;9|)
           (COND
             (#5#
              (LETT #4#
               (COND (#4# #3#) ((QUOTE T) (QUOTE NIL)))
               |HOAGG-;=;2AB;9|))
             ((QUOTE T)
              (PROGN
               (LETT #4# #3# |HOAGG-;=;2AB;9|)
               (LETT #5# (QUOTE T) |HOAGG-;=;2AB;9|)))))))
        (LETT #2#
         (PROG1
          (CDR #2#)
          (LETT #1# (CDR #1#) |HOAGG-;=;2AB;9|))
         |HOAGG-;=;2AB;9|)
        (GO G190)
        G191
        (EXIT NIL))
       (COND (#5# #4#) ((QUOTE T) (QUOTE T)))))
     ((QUOTE T) (QUOTE NIL)))))))

(DEFUN |HOAGG-;coerce;AOf;10| (|x| |$|)
 (PROG (#1=#:G82420 |a| #2=#:G82421)
  (RETURN
   (SEQ
    (SPADCALL
     (SPADCALL
      (PROGN
       (LETT #1# NIL |HOAGG-;coerce;AOf;10|)
       (SEQ
        (LETT |a| NIL |HOAGG-;coerce;AOf;10|)
        (LETT #2# (SPADCALL |x| (QREFELT |$| 14)) |HOAGG-;coerce;AOf;10|)
        G190
        (COND
         ((OR
           (ATOM #2#)
           (PROGN (LETT |a| (CAR #2#) |HOAGG-;coerce;AOf;10|) NIL))
          (GO G191)))
        (SEQ
         (EXIT
          (LETT #1#
           (CONS (SPADCALL |a| (QREFELT |$| 32)) #1#)
           |HOAGG-;coerce;AOf;10|)))
        (LETT #2# (CDR #2#) |HOAGG-;coerce;AOf;10|)
        (GO G190)
        G191
        (EXIT (NREVERSE0 #1#))))
      (QREFELT |$| 34))
     (QREFELT |$| 35))))))

(DEFUN |HomogeneousAggregate&| (|#1| |#2|)
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
```

```
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|HomogeneousAggregate&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |HomogeneousAggregate&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (make-array 38) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3
     (LETT |pv$|
      (|buildPredVector| 0 0
       (LIST
        (|HasAttribute| |#1| (QUOTE |finiteAggregate|))
        (|HasAttribute| |#1| (QUOTE |shallowlyMutable|))
        (|HasCategory| |#2| (LIST (QUOTE |Evalable|) (|devaluate| |#2|)))
        (|HasCategory| |#2| (QUOTE (|SetCategory|)))))
      . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
    (COND
     ((|testBitVector| |pv$| 3)
       (QSETREFV |$| 12 (CONS (|dispatchFunction| |HOAGG-;eval;ALA;1|) |$|))))
    (COND
     ((|testBitVector| |pv$| 1)
      (PROGN
       (QSETREFV |$| 16 (CONS (|dispatchFunction| |HOAGG-;#;ANni;2|) |$|))
       (QSETREFV |$| 19 (CONS (|dispatchFunction| |HOAGG-;any?;MAB;3|) |$|))
       (QSETREFV |$| 20 (CONS (|dispatchFunction| |HOAGG-;every?;MAB;4|) |$|))
       (QSETREFV |$| 21 (CONS (|dispatchFunction| |HOAGG-;count;MANni;5|) |$|))
       (QSETREFV |$| 22 (CONS (|dispatchFunction| |HOAGG-;members;AL;6|) |$|))
       (COND
        ((|testBitVector| |pv$| 4)
         (PROGN
          (QSETREFV |$| 25
           (CONS (|dispatchFunction| |HOAGG-;count;SANni;7|) |$|))
          (QSETREFV |$| 27
           (CONS (|dispatchFunction| |HOAGG-;member?;SAB;8|) |$|))
          (QSETREFV |$| 30
           (CONS (|dispatchFunction| |HOAGG-;=;2AB;9|) |$|))
          (QSETREFV |$| 36
           (CONS (|dispatchFunction| |HOAGG-;coerce;AOf;10|) |$|))))))))
    |$|))))

(setf (get
 (QUOTE |HomogeneousAggregate&|)
 (QUOTE |infovec|))
 (LIST
  (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|) (|List| 37)
   (0 . |eval|) (|Mapping| 7 7) (6 . |map|) (12 . |eval|) (|List| 7)
   (18 . |parts|) (|NonNegativeInteger|) (23 . |#|) (|Boolean|)
   (|Mapping| 17 7) (28 . |any?|) (34 . |every?|) (40 . |count|)
   (46 . |members|) (51 . |=|) (57 . |count|) (63 . |count|) (69 . |any?|)
   (75 . |member?|) (81 . |#|) (86 . |size?|) (92 . |=|) (|OutputForm|)
   (98 . |coerce|) (|List| |$|) (103 . |commaSeparate|) (108 . |bracket|)
   (113 . |coerce|) (|Equation| 7)))
```

```
  (QUOTE #(|members| 118 |member?| 123 |every?| 129 |eval| 135 |count| 141
   |coerce| 153 |any?| 158 |=| 164 |#| 170))
  (QUOTE NIL)
  (CONS
   (|makeByteWordVec2| 1 (QUOTE NIL))
   (CONS
    (QUOTE #())
    (CONS
     (QUOTE #())
     (|makeByteWordVec2| 36
      (QUOTE (2 7 0 0 8 9 2 6 0 10 0 11 2 0 0 0 8 12 1 6 13 0 14 1 0 15 0
        16 2 0 17 18 0 19 2 0 17 18 0 20 2 0 15 18 0 21 1 0 13 0 22 2 7 17
        0 0 23 2 6 15 18 0 24 2 0 15 7 0 25 2 6 17 18 0 26 2 0 17 7 0 27 1
        6 15 0 28 2 6 17 0 15 29 2 0 17 0 0 30 1 7 31 0 32 1 31 0 33 34 1
        31 0 0 35 1 0 31 0 36 1 0 13 0 22 2 0 17 7 0 27 2 0 17 18 0 20 2 0
        0 0 8 12 2 0 15 7 0 25 2 0 15 18 0 21 1 0 31 0 36 2 0 17 18 0 19 2
        0 17 0 0 30 1 0 15 0 16))))))
  (QUOTE |lookupComplete|)))
```

21.29 INS.lsp BOOTSTRAP

INS depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **INS** category which we can write into the **MID** directory. We compile the lisp code and copy the **INS.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

— INS.lsp BOOTSTRAP —

```
(/VERSIONCHECK 2)

(SETQ |IntegerNumberSystem;AL| (QUOTE NIL))

(DEFUN |IntegerNumberSystem| NIL
  (declare (special |IntegerNumberSystem;AL|))
    (COND
      (|IntegerNumberSystem;AL|)
      (T (SETQ |IntegerNumberSystem;AL| (|IntegerNumberSystem;|)))))

(DEFUN |IntegerNumberSystem;| NIL (PROG (#0=#:G1066)
  (RETURN
    (PROG1
      (LETT #0#
        (|sublisV|
          (PAIR
            (QUOTE (#1=#:G1060 #2=#:G1061 #3=#:G1062
                    #4=#:G1063 #5=#:G1064 #6=#:G1065))
            (LIST
              (QUOTE (|Integer|))
              (QUOTE (|Integer|))
              (QUOTE (|Integer|))
              (QUOTE (|InputForm|))
              (QUOTE (|Pattern| (|Integer|)))
```

```
            (QUOTE (|Integer|))))
          (|Join|
            (|UniqueFactorizationDomain|)
            (|EuclideanDomain|)
            (|OrderedIntegralDomain|)
            (|DifferentialRing|)
            (|ConvertibleTo| (QUOTE #1#))
            (|RetractableTo| (QUOTE #2#))
            (|LinearlyExplicitRingOver| (QUOTE #3#))
            (|ConvertibleTo| (QUOTE #4#))
            (|ConvertibleTo| (QUOTE #5#))
            (|PatternMatchable| (QUOTE #6#))
            (|CombinatorialFunctionCategory|)
            (|RealConstant|)
            (|CharacteristicZero|)
            (|StepThrough|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|odd?| ((|Boolean|) $)) T)
                ((|even?| ((|Boolean|) $)) T)
                ((|base| ($)) T)
                ((|length| ($ $)) T)
                ((|shift| ($ $ $)) T)
                ((|bit?| ((|Boolean|) $ $)) T)
                ((|positiveRemainder| ($ $ $)) T)
                ((|symmetricRemainder| ($ $ $)) T)
                ((|rational?| ((|Boolean|) $)) T)
                ((|rational| ((|Fraction| (|Integer|)) $)) T)
                ((|rationalIfCan|
                  ((|Union| (|Fraction| (|Integer|)) "failed") $)) T)
                ((|random| ($)) T)
                ((|random| ($ $)) T)
                ((|hash| ($ $)) T)
                ((|copy| ($ $)) T)
                ((|inc| ($ $)) T)
                ((|dec| ($ $)) T)
                ((|mask| ($ $)) T)
                ((|addmod| ($ $ $ $)) T)
                ((|submod| ($ $ $ $)) T)
                ((|mulmod| ($ $ $ $)) T)
                ((|powmod| ($ $ $ $)) T)
                ((|invmod| ($ $ $)) T)))
              (QUOTE ((|multiplicativeValuation| T) (|canonicalUnitNormal| T)))
              (QUOTE ((|Fraction| (|Integer|)) (|Boolean|))) NIL)))
         |IntegerNumberSystem|)
       (SETELT #0# 0 (QUOTE (|IntegerNumberSystem|)))))))

(setf (get (QUOTE |IntegerNumberSystem|) (QUOTE NILADIC)) T)
```

———-

21.30 INS-.lsp BOOTSTRAP

INS- depends on **INS**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **INS-** category which we can write into the **MID** directory. We compile the lisp code and copy the **INS-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

— **INS-.lsp BOOTSTRAP** —

```
(/VERSIONCHECK 2)

(PUT
  (QUOTE |INS-;characteristic;Nni;1|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL 0)))

(DEFUN |INS-;characteristic;Nni;1| ($)
 (declare (ignore $))
 0)

(DEFUN |INS-;differentiate;2S;2| (|x| $)
 (declare (ignore |x|))
  (|spadConstant| $ 9))

(DEFUN |INS-;even?;SB;3| (|x| $)
 (COND
   ((SPADCALL |x| (QREFELT $ 12)) (QUOTE NIL))
   ((QUOTE T) (QUOTE T))))

(DEFUN |INS-;positive?;SB;4| (|x| $)
  (SPADCALL (|spadConstant| $ 9) |x| (QREFELT $ 14)))

(PUT
  (QUOTE |INS-;copy;2S;5|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|x|) |x|)))

(DEFUN |INS-;copy;2S;5| (|x| $)
 (declare (ignore $))
 |x|)

(DEFUN |INS-;bit?;2SB;6| (|x| |i| $)
  (SPADCALL
    (SPADCALL |x|
      (SPADCALL |i| (QREFELT $ 17))
      (QREFELT $ 18))
    (QREFELT $ 12)))

(DEFUN |INS-;mask;2S;7| (|n| $)
  (SPADCALL
    (SPADCALL (|spadConstant| $ 20) |n| (QREFELT $ 18))
    (QREFELT $ 21)))

(PUT
  (QUOTE |INS-;rational?;SB;8|)
```

```
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|x|) (QUOTE T))))

(DEFUN |INS-;rational?;SB;8| (|x| $)
 (declare (ignore |x|))
 (declare (ignore $))
  (QUOTE T))

(DEFUN |INS-;euclideanSize;SNni;9| (|x| $)
  (PROG (#0=#:G1078 #1=#:G1079)
    (RETURN
      (COND
        ((SPADCALL |x| (|spadConstant| $ 9) (QREFELT $ 24))
          (|error| "euclideanSize called on zero"))
        ((SPADCALL |x| (|spadConstant| $ 9) (QREFELT $ 14))
          (PROG1
            (LETT #0#
              (- (SPADCALL |x| (QREFELT $ 26)))
              |INS-;euclideanSize;SNni;9|)
            (|check-subtype|
              (>= #0# 0)
              (QUOTE (|NonNegativeInteger|))
              #0#)))
        ((QUOTE T)
          (PROG1
            (LETT #1#
              (SPADCALL |x| (QREFELT $ 26))
              |INS-;euclideanSize;SNni;9|)
            (|check-subtype|
              (>= #1# 0)
              (QUOTE (|NonNegativeInteger|))
              #1#)))))))

(DEFUN |INS-;convert;SF;10| (|x| $)
  (SPADCALL (SPADCALL |x| (QREFELT $ 26)) (QREFELT $ 29)))

(DEFUN |INS-;convert;SDf;11| (|x| $)
  (FLOAT (SPADCALL |x| (QREFELT $ 26)) MOST-POSITIVE-LONG-FLOAT))

(DEFUN |INS-;convert;SIf;12| (|x| $)
  (SPADCALL (SPADCALL |x| (QREFELT $ 26)) (QREFELT $ 34)))

(DEFUN |INS-;retract;SI;13| (|x| $)
  (SPADCALL |x| (QREFELT $ 26)))

(DEFUN |INS-;convert;SP;14| (|x| $)
  (SPADCALL (SPADCALL |x| (QREFELT $ 26)) (QREFELT $ 38)))

(DEFUN |INS-;factor;SF;15| (|x| $)
  (SPADCALL |x| (QREFELT $ 42)))

(DEFUN |INS-;squareFree;SF;16| (|x| $)
  (SPADCALL |x| (QREFELT $ 45)))
```

```
(DEFUN |INS-;prime?;SB;17| (|x| $)
  (SPADCALL |x| (QREFELT $ 48)))

(DEFUN |INS-;factorial;2S;18| (|x| $)
  (SPADCALL |x| (QREFELT $ 51)))

(DEFUN |INS-;binomial;3S;19| (|n| |m| $)
  (SPADCALL |n| |m| (QREFELT $ 53)))

(DEFUN |INS-;permutation;3S;20| (|n| |m| $)
  (SPADCALL |n| |m| (QREFELT $ 55)))

(DEFUN |INS-;retractIfCan;SU;21| (|x| $)
  (CONS 0 (SPADCALL |x| (QREFELT $ 26))))

(DEFUN |INS-;init;S;22| ($)
  (|spadConstant| $ 9))

(DEFUN |INS-;nextItem;SU;23| (|n| $)
  (COND
    ((SPADCALL |n| (QREFELT $ 60))
       (CONS 0 (|spadConstant| $ 20)))
    ((SPADCALL (|spadConstant| $ 9) |n| (QREFELT $ 14))
       (CONS 0 (SPADCALL |n| (QREFELT $ 17))))
    ((QUOTE T)
       (CONS 0 (SPADCALL (|spadConstant| $ 20) |n| (QREFELT $ 61))))))

(DEFUN |INS-;patternMatch;SP2Pmr;24| (|x| |p| |l| $)
  (SPADCALL |x| |p| |l| (QREFELT $ 66)))

(DEFUN |INS-;rational;SF;25| (|x| $)
  (SPADCALL (SPADCALL |x| (QREFELT $ 26)) (QREFELT $ 70)))

(DEFUN |INS-;rationalIfCan;SU;26| (|x| $)
  (CONS 0 (SPADCALL (SPADCALL |x| (QREFELT $ 26)) (QREFELT $ 70))))

(DEFUN |INS-;symmetricRemainder;3S;27| (|x| |n| $)
  (PROG (|r|)
    (RETURN
      (SEQ
        (LETT |r|
          (SPADCALL |x| |n| (QREFELT $ 74))
          |INS-;symmetricRemainder;3S;27|)
        (EXIT
          (COND
            ((SPADCALL |r| (|spadConstant| $ 9) (QREFELT $ 24)) |r|)
            ((QUOTE T)
              (SEQ
                (COND
                  ((SPADCALL |n| (|spadConstant| $ 9) (QREFELT $ 14))
                    (LETT |n|
                      (SPADCALL |n| (QREFELT $ 17))
                      |INS-;symmetricRemainder;3S;27|)))
                (EXIT
```

```
                  (COND
                    ((SPADCALL (|spadConstant| $ 9) |r| (QREFELT $ 14))
                      (COND
                        ((SPADCALL |n|
                            (SPADCALL 2 |r| (QREFELT $ 76))
                            (QREFELT $ 14))
                          (SPADCALL |r| |n| (QREFELT $ 61)))
                        ((QUOTE T) |r|)))
                    ((NULL
                      (SPADCALL
                         (|spadConstant| $ 9)
                         (SPADCALL
                           (SPADCALL 2 |r| (QREFELT $ 76))
                           |n|
                           (QREFELT $ 77))
                         (QREFELT $ 14)))
                       (SPADCALL |r| |n| (QREFELT $ 77)))
                    ((QUOTE T) |r|))))))))))

(DEFUN |INS-;invmod;3S;28| (|a| |b| $)
  (PROG (|q| |r| |r1| |c| |c1| |d| |d1|)
    (RETURN
      (SEQ
        (COND
          ((SPADCALL |a| (QREFELT $ 79))
            (LETT |a| (SPADCALL |a| |b| (QREFELT $ 80)) |INS-;invmod;3S;28|)))
        (LETT |c| |a| |INS-;invmod;3S;28|)
        (LETT |c1| (|spadConstant| $ 20) |INS-;invmod;3S;28|)
        (LETT |d| |b| |INS-;invmod;3S;28|)
        (LETT |d1| (|spadConstant| $ 9) |INS-;invmod;3S;28|)
        (SEQ G190
          (COND
            ((NULL
              (COND
                ((SPADCALL |d| (QREFELT $ 60)) (QUOTE NIL))
                ((QUOTE T) (QUOTE T))))
              (GO G191)))
          (SEQ
            (LETT |q| (SPADCALL |c| |d| (QREFELT $ 81)) |INS-;invmod;3S;28|)
            (LETT |r|
              (SPADCALL |c| (SPADCALL |q| |d| (QREFELT $ 82)) (QREFELT $ 61))
              |INS-;invmod;3S;28|)
            (LETT |r1|
              (SPADCALL |c1| (SPADCALL |q| |d1| (QREFELT $ 82)) (QREFELT $ 61))
              |INS-;invmod;3S;28|)
            (LETT |c| |d| |INS-;invmod;3S;28|)
            (LETT |c1| |d1| |INS-;invmod;3S;28|)
            (LETT |d| |r| |INS-;invmod;3S;28|)
            (EXIT (LETT |d1| |r1| |INS-;invmod;3S;28|)))
          NIL
          (GO G190)
          G191
          (EXIT NIL))
        (COND
```

```
          ((NULL (SPADCALL |c| (QREFELT $ 83)))
            (EXIT (|error| "inverse does not exist"))))
        (EXIT
          (COND
            ((SPADCALL |c1| (QREFELT $ 79)) (SPADCALL |c1| |b| (QREFELT $ 77)))
            ((QUOTE T) |c1|)))))))

(DEFUN |INS-;powmod;4S;29| (|x| |n| |p| $)
  (PROG (|y| #0=#:G1137 |z|)
    (RETURN
      (SEQ
        (EXIT
          (SEQ
            (COND
              ((SPADCALL |x| (QREFELT $ 79))
                (LETT |x|
                  (SPADCALL |x| |p| (QREFELT $ 80))
                  |INS-;powmod;4S;29|)))
           (EXIT
             (COND
               ((SPADCALL |x| (QREFELT $ 60)) (|spadConstant| $ 9))
               ((SPADCALL |n| (QREFELT $ 60)) (|spadConstant| $ 20))
               ((QUOTE T)
                 (SEQ
                   (LETT |y| (|spadConstant| $ 20) |INS-;powmod;4S;29|)
                   (LETT |z| |x| |INS-;powmod;4S;29|)
                   (EXIT
                     (SEQ G190
                       NIL
                       (SEQ
                         (COND
                           ((SPADCALL |n| (QREFELT $ 12))
                             (LETT |y|
                               (SPADCALL |y| |z| |p| (QREFELT $ 85))
                               |INS-;powmod;4S;29|)))
                         (EXIT
                           (COND
                             ((SPADCALL
                               (LETT |n|
                                 (SPADCALL |n|
                                   (SPADCALL
                                     (|spadConstant| $ 20)
                                     (QREFELT $ 17))
                                   (QREFELT $ 18))
                                  |INS-;powmod;4S;29|)
                               (QREFELT $ 60))
                              (PROGN
                                (LETT #0# |y| |INS-;powmod;4S;29|)
                                (GO #0#)))
                             ((QUOTE T)
                               (LETT |z|
                                 (SPADCALL |z| |z| |p| (QREFELT $ 85))
                                 |INS-;powmod;4S;29|)))))
                       NIL
```

```
                       (GO G190)
                       G191
                       (EXIT NIL))))))))
        #0#
        (EXIT #0#)))))

(DEFUN |IntegerNumberSystem&| (|#1|)
  (PROG (DV$1 |dv$| $ |pv$|)
    (RETURN
      (PROGN
        (LETT DV$1 (|devaluate| |#1|) . #0=(|IntegerNumberSystem&|))
        (LETT |dv$| (LIST (QUOTE |IntegerNumberSystem&|) DV$1) . #0#)
        (LETT $ (make-array 87) . #0#)
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #0#))
        (|stuffDomainSlots| $)
        (QSETREFV $ 6 |#1|)
        $))))

(setf (get
  (QUOTE |IntegerNumberSystem&|)
  (QUOTE |infovec|))
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (|NonNegativeInteger|)
        |INS-;characteristic;Nni;1|
        (0 . |Zero|)
        |INS-;differentiate;2S;2|
        (|Boolean|)
        (4 . |odd?|)
        |INS-;even?;SB;3|
        (9 . <)
        |INS-;positive?;SB;4|
        |INS-;copy;2S;5|
        (15 . -)
        (20 . |shift|)
        |INS-;bit?;2SB;6|
        (26 . |One|)
        (30 . |dec|)
        |INS-;mask;2S;7|
        |INS-;rational?;SB;8|
        (35 . =)
        (|Integer|)
        (41 . |convert|)
        |INS-;euclideanSize;SNni;9|
        (|Float|)
        (46 . |coerce|)
        |INS-;convert;SF;10|
        (|DoubleFloat|)
        |INS-;convert;SDf;11|
        (|InputForm|)
        (51 . |convert|)
```

```
    |INS-;convert;SIf;12|
    |INS-;retract;SI;13|
    (|Pattern| 25)
    (56 . |coerce|)
    |INS-;convert;SP;14|
    (|Factored| 6)
    (|IntegerFactorizationPackage| 6)
    (61 . |factor|)
    (|Factored| $)
    |INS-;factor;SF;15|
    (66 . |squareFree|)
    |INS-;squareFree;SF;16|
    (|IntegerPrimesPackage| 6)
    (71 . |prime?|)
    |INS-;prime?;SB;17|
    (|IntegerCombinatoricFunctions| 6)
    (76 . |factorial|)
    |INS-;factorial;2S;18|
    (81 . |binomial|)
    |INS-;binomial;3S;19|
    (87 . |permutation|)
    |INS-;permutation;3S;20|
    (|Union| 25 (QUOTE "failed"))
    |INS-;retractIfCan;SU;21|
    |INS-;init;S;22|
    (93 . |zero?|)
    (98 . -)
    (|Union| $ (QUOTE "failed"))
    |INS-;nextItem;SU;23|
    (|PatternMatchResult| 25 6)
    (|PatternMatchIntegerNumberSystem| 6)
    (104 . |patternMatch|)
    (|PatternMatchResult| 25 $)
    |INS-;patternMatch;SP2Pmr;24|
    (|Fraction| 25)
    (111 . |coerce|)
    |INS-;rational;SF;25|
    (|Union| 69 (QUOTE "failed"))
    |INS-;rationalIfCan;SU;26|
    (116 . |rem|)
    (|PositiveInteger|)
    (122 . *)
    (128 . +)
    |INS-;symmetricRemainder;3S;27|
    (134 . |negative?|)
    (139 . |positiveRemainder|)
    (145 . |quo|)
    (151 . *)
    (157 . |one?|)
    |INS-;invmod;3S;28|
    (162 . |mulmod|)
    |INS-;powmod;4S;29|))
 (QUOTE
  #(|symmetricRemainder| 169 |squareFree| 175 |retractIfCan| 180
```

```
        |retract| 185 |rationalIfCan| 190 |rational?| 195 |rational| 200
        |prime?| 205 |powmod| 210 |positive?| 217 |permutation| 222
        |patternMatch| 228 |nextItem| 235 |mask| 240 |invmod| 245 |init| 251
        |factorial| 255 |factor| 260 |even?| 265 |euclideanSize| 270
        |differentiate| 275 |copy| 280 |convert| 285 |characteristic| 305
        |bit?| 309 |binomial| 315))
    (QUOTE NIL)
    (CONS
      (|makeByteWordVec2| 1 (QUOTE NIL))
      (CONS
        (QUOTE #())
        (CONS
          (QUOTE #())
          (|makeByteWordVec2| 86
            (QUOTE
              (0 6 0 9 1 6 11 0 12 2 6 11 0 0 14 1 6 0 0 17 2 6 0 0 0 18 0 6
               0 20 1 6 0 0 21 2 6 11 0 0 24 1 6 25 0 26 1 28 0 25 29 1 33 0
               25 34 1 37 0 25 38 1 41 40 6 42 1 41 40 6 45 1 47 11 6 48 1 50
               6 6 51 2 50 6 6 6 53 2 50 6 6 6 55 1 6 11 0 60 2 6 0 0 0 61 3
               65 64 6 37 64 66 1 69 0 25 70 2 6 0 0 0 74 2 6 0 75 0 76 2 6 0
               0 0 77 1 6 11 0 79 2 6 0 0 0 80 2 6 0 0 0 81 2 6 0 0 0 82 1 6
               11 0 83 3 6 0 0 0 0 85 2 0 0 0 0 78 1 0 43 0 46 1 0 57 0 58 1
               0 25 0 36 1 0 72 0 73 1 0 11 0 23 1 0 69 0 71 1 0 11 0 49 3 0
               0 0 0 0 86 1 0 11 0 15 2 0 0 0 0 56 3 0 67 0 37 67 68 1 0 62
               0 63 1 0 0 0 22 2 0 0 0 0 84 0 0 0 59 1 0 0 0 52 1 0 43 0 44
               1 0 11 0 13 1 0 7 0 27 1 0 0 0 10 1 0 0 0 16 1 0 31 0 32 1 0
               28 0 30 1 0 37 0 39 1 0 33 0 35 0 0 7 8 2 0 11 0 0 19 2 0 0
               0 0 54))))))
  (QUOTE |lookupComplete|)))
```

21.31 INTDOM.lsp BOOTSTRAP

INTDOM depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **INTDOM** category which we can write into the **MID** directory. We compile the lisp code and copy the **INTDOM.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— INTDOM.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |IntegralDomain;AL| (QUOTE NIL))

(DEFUN |IntegralDomain| NIL
  (declare (special |IntegralDomain;AL|))
    (COND
      (|IntegralDomain;AL|)
      (T (SETQ |IntegralDomain;AL| (|IntegralDomain;|)))))
```

```
(DEFUN |IntegralDomain;| NIL
  (PROG (#1=#:G83058)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|CommutativeRing|)
            (|Algebra| (QUOTE |$|))
            (|EntireRing|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|exquo| ((|Union| |$| "failed") |$| |$|)) T)
                ((|unitNormal|
                  ((|Record|
                     (|:| |unit| |$|)
                     (|:| |canonical| |$|)
                     (|:| |associate| |$|)) |$|)) T)
                ((|unitCanonical| (|$| |$|)) T)
                ((|associates?| ((|Boolean|) |$| |$|)) T)
                ((|unit?| ((|Boolean|) |$|)) T)))
               NIL
               (QUOTE ((|Boolean|)))
               NIL))
           |IntegralDomain|)
        (SETELT #1# 0 (QUOTE (|IntegralDomain|)))))))

(setf (get (QUOTE |IntegralDomain|) (QUOTE NILADIC)) T)
```

21.32 INTDOM-.lsp BOOTSTRAP

INTDOM- depends on **INTDOM**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **INTDOM-** category which we can write into the **MID** directory. We compile the lisp code and copy the **INTDOM-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— INTDOM-.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(DEFUN |INTDOM-;unitNormal;SR;1| (|x| |$|)
  (VECTOR (|spadConstant| |$| 7) |x| (|spadConstant| |$| 7)))

(DEFUN |INTDOM-;unitCanonical;2S;2| (|x| |$|)
  (QVELT (SPADCALL |x| (QREFELT |$| 10)) 1))

(DEFUN |INTDOM-;recip;SU;3| (|x| |$|)
```

```
  (COND
    ((SPADCALL |x| (QREFELT |$| 13)) (CONS 1 "failed"))
    ((QUOTE T) (SPADCALL (|spadConstant| |$| 7) |x| (QREFELT |$| 15)))))

(DEFUN |INTDOM-;unit?;SB;4| (|x| |$|)
  (COND
    ((QEQCAR (SPADCALL |x| (QREFELT |$| 17)) 1) (QUOTE NIL))
    ((QUOTE T) (QUOTE T))))

(DEFUN |INTDOM-;associates?;2SB;5| (|x| |y| |$|)
  (SPADCALL
    (QVELT (SPADCALL |x| (QREFELT |$| 10)) 1)
    (QVELT (SPADCALL |y| (QREFELT |$| 10)) 1)
    (QREFELT |$| 19)))

(DEFUN |INTDOM-;associates?;2SB;6| (|x| |y| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 13)) (SPADCALL |y| (QREFELT |$| 13)))
    ((OR
        (SPADCALL |y| (QREFELT |$| 13))
        (OR
          (QEQCAR (SPADCALL |x| |y| (QREFELT |$| 15)) 1)
          (QEQCAR (SPADCALL |y| |x| (QREFELT |$| 15)) 1)))
      (QUOTE NIL))
    ((QUOTE T) (QUOTE T))))

(DEFUN |IntegralDomain&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|IntegralDomain&|))
        (LETT |dv$| (LIST (QUOTE |IntegralDomain&|) |DV$1|) . #1#)
        (LETT |$| (make-array 21) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        (COND
          ((|HasCategory| |#1| (QUOTE (|Field|))))
          ((QUOTE T)
            (QSETREFV |$| 9
              (CONS (|dispatchFunction| |INTDOM-;unitNormal;SR;1|) |$|))))
        (COND
          ((|HasAttribute| |#1| (QUOTE |canonicalUnitNormal|))
            (QSETREFV |$| 20
              (CONS (|dispatchFunction| |INTDOM-;associates?;2SB;5|) |$|)))
          ((QUOTE T)
            (QSETREFV |$| 20
              (CONS (|dispatchFunction| |INTDOM-;associates?;2SB;6|) |$|))))
        |$|))))

(setf (get
  (QUOTE |IntegralDomain&|)
  (QUOTE |infovec|))
```

```
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (0 . |One|)
        (|Record|
          (|:| |unit| |$|)
          (|:| |canonical| |$|)
          (|:| |associate| |$|))
        (4 . |unitNormal|)
        (9 . |unitNormal|)
        |INTDOM-;unitCanonical;2S;2|
        (|Boolean|)
        (14 . |zero?|)
        (|Union| |$| (QUOTE "failed"))
        (19 . |exquo|)
        |INTDOM-;recip;SU;3|
        (25 . |recip|)
        |INTDOM-;unit?;SB;4|
        (30 . |=|)
        (36 . |associates?|)))
    (QUOTE
      #(|unitNormal| 42 |unitCanonical| 47 |unit?| 52 |recip| 57
        |associates?| 62))
    (QUOTE NIL)
    (CONS
      (|makeByteWordVec2| 1 (QUOTE NIL))
      (CONS
        (QUOTE #())
        (CONS
          (QUOTE #())
          (|makeByteWordVec2| 20
            (QUOTE
              (0 6 0 7 1 0 8 0 9 1 6 8 0 10 1 6 12 0 13 2 6 14 0 0 15 1 6 14
               0 17 2 6 12 0 0 19 2 0 12 0 0 20 1 0 8 0 9 1 0 0 0 11 1 0 12 0
               18 1 0 14 0 16 2 0 12 0 0 20))))))
   (QUOTE |lookupComplete|)))
```

———-

21.33 LNAGG.lsp BOOTSTRAP

LNAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **LNAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **LNAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— LNAGG.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)
```

```
(SETQ |LinearAggregate;CAT| (QUOTE NIL))

(SETQ |LinearAggregate;AL| (QUOTE NIL))

(DEFUN |LinearAggregate| (#1=#:G85818)
 (declare (special |LinearAggregate;AL|))
 (LET (#2=#:G85819)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |LinearAggregate;AL|)) (CDR #2#))
   (T
    (SETQ |LinearAggregate;AL|
     (|cons5|
      (CONS (|devaluate| #1#) (SETQ #2# (|LinearAggregate;| #1#)))
      |LinearAggregate;AL|))
    #2#))))

(DEFUN |LinearAggregate;| (|t#1|)
 (declare (special |LinearAggregate;CAT|))
 (PROG (#1=#:G85817)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (|sublisV|
       (PAIR (QUOTE (#2=#:G85816)) (LIST (QUOTE (|Integer|))))
       (COND
        (|LinearAggregate;CAT|)
        ((QUOTE T)
         (LETT |LinearAggregate;CAT|
          (|Join|
           (|IndexedAggregate| (QUOTE #2#) (QUOTE |t#1|))
           (|Collection| (QUOTE |t#1|))
           (|mkCategory|
            (QUOTE |domain|)
            (QUOTE (
             ((|new| (|$| (|NonNegativeInteger|) |t#1|)) T)
             ((|concat| (|$| |$| |t#1|)) T)
             ((|concat| (|$| |t#1| |$|)) T)
             ((|concat| (|$| |$| |$|)) T)
             ((|concat| (|$| (|List| |$|))) T)
             ((|map| (|$| (|Mapping| |t#1| |t#1| |t#1|) |$| |$|)) T)
             ((|elt| (|$| |$| (|UniversalSegment| (|Integer|)))) T)
             ((|delete| (|$| |$| (|Integer|))) T)
             ((|delete| (|$| |$| (|UniversalSegment| (|Integer|)))) T)
             ((|insert| (|$| |t#1| |$| (|Integer|))) T)
             ((|insert| (|$| |$| |$| (|Integer|))) T)
             ((|setelt| (|t#1| |$| (|UniversalSegment| (|Integer|)) |t#1|))
              (|has| |$| (ATTRIBUTE |shallowlyMutable|)))))
            NIL
            (QUOTE (
              (|UniversalSegment| (|Integer|))
              (|Integer|)
```

```
              (|List| |$|)
              (|NonNegativeInteger|)))
             NIL))
          . #3=(|LinearAggregate|))))))
      . #3#)
     (SETELT #1# 0 (LIST (QUOTE |LinearAggregate|) (|devaluate| |t#1|)))))))
```

21.34 LNAGG-.lsp BOOTSTRAP

LNAGG- depends on **LNAGG**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **LNAGG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **LNAGG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— LNAGG-.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(DEFUN |LNAGG-;indices;AL;1| (|a| |$|)
 (PROG (#1=#:G85833 |i| #2=#:G85834)
  (RETURN
   (SEQ
    (PROGN
     (LETT #1# NIL |LNAGG-;indices;AL;1|)
     (SEQ
      (LETT |i| (SPADCALL |a| (QREFELT |$| 9)) |LNAGG-;indices;AL;1|)
      (LETT #2# (SPADCALL |a| (QREFELT |$| 10)) |LNAGG-;indices;AL;1|)
      G190
      (COND ((|>| |i| #2#) (GO G191)))
      (SEQ (EXIT (LETT #1# (CONS |i| #1#) |LNAGG-;indices;AL;1|)))
      (LETT |i| (|+| |i| 1) |LNAGG-;indices;AL;1|)
      (GO G190)
      G191
      (EXIT (NREVERSE0 #1#))))))))

(DEFUN |LNAGG-;index?;IAB;2| (|i| |a| |$|)
 (COND
  ((OR
     (|<| |i| (SPADCALL |a| (QREFELT |$| 9)))
     (|<| (SPADCALL |a| (QREFELT |$| 10)) |i|))
    (QUOTE NIL))
  ((QUOTE T) (QUOTE T))))

(DEFUN |LNAGG-;concat;ASA;3| (|a| |x| |$|)
 (SPADCALL |a| (SPADCALL 1 |x| (QREFELT |$| 16)) (QREFELT |$| 17)))

(DEFUN |LNAGG-;concat;S2A;4| (|x| |y| |$|)
 (SPADCALL (SPADCALL 1 |x| (QREFELT |$| 16)) |y| (QREFELT |$| 17)))
```

```
(DEFUN |LNAGG-;insert;SAIA;5| (|x| |a| |i| |$|)
 (SPADCALL (SPADCALL 1 |x| (QREFELT |$| 16)) |a| |i| (QREFELT |$| 20)))

(DEFUN |LNAGG-;maxIndex;AI;6| (|l| |$|)
 (|+| (|-| (SPADCALL |l| (QREFELT |$| 22)) 1) (SPADCALL |l| (QREFELT |$| 9))))

(DEFUN |LinearAggregate&| (|#1| |#2|)
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|LinearAggregate&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |LinearAggregate&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (make-array 25) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3
     (LETT |pv$|
      (|buildPredVector| 0 0
       (LIST (|HasAttribute| |#1| (QUOTE |shallowlyMutable|))))
      . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
    (COND
     ((|HasAttribute| |#1| (QUOTE |finiteAggregate|))
      (QSETREFV |$| 23
       (CONS (|dispatchFunction| |LNAGG-;maxIndex;AI;6|) |$|))))
    |$|))))

(setf (get
 (QUOTE |LinearAggregate&|)
 (QUOTE |infovec|))
 (LIST
  (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|) (|Integer|)
    (0 . |minIndex|) (5 . |maxIndex|) (|List| 8) |LNAGG-;indices;AL;1|
    (|Boolean|) |LNAGG-;index?;IAB;2| (|NonNegativeInteger|) (10 . |new|)
    (16 . |concat|) |LNAGG-;concat;ASA;3| |LNAGG-;concat;S2A;4|
    (22 . |insert|) |LNAGG-;insert;SAIA;5| (29 . |#|) (34 . |maxIndex|)
    (|List| |$|)))
  (QUOTE #(|maxIndex| 39 |insert| 44 |indices| 51 |index?| 56 |concat| 62))
  (QUOTE NIL)
  (CONS
   (|makeByteWordVec2| 1 (QUOTE NIL))
   (CONS
    (QUOTE #())
    (CONS
     (QUOTE #())
     (|makeByteWordVec2| 23 (QUOTE (1 6 8 0 9 1 6 8 0 10 2 6 0 15 7
      16 2 6 0 0 0 17 3 6 0 0 0 8 20 1 6 15 0 22 1 0 8 0 23 1 0 8 0 23 3 0
      0 7 0 8 21 1 0 11 0 12 2 0 13 8 0 14 2 0 0 0 7 18 2 0 0 7 0 19))))))
  (QUOTE |lookupComplete|)))
```

———-

21.35 LSAGG.lsp BOOTSTRAP

LSAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **LSAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **LSAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— LSAGG.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |ListAggregate;CAT| (QUOTE NIL))

(SETQ |ListAggregate;AL| (QUOTE NIL))

(DEFUN |ListAggregate| (#1=#:G87500)
 (declare (special |ListAggregate;AL|))
 (LET (#2=#:G87501)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |ListAggregate;AL|)) (CDR #2#))
   (T
    (SETQ |ListAggregate;AL|
     (|cons5|
      (CONS (|devaluate| #1#) (SETQ #2# (|ListAggregate;| #1#)))
      |ListAggregate;AL|))
    #2#))))

(DEFUN |ListAggregate;| (|t#1|)
 (declare (special |ListAggregate;CAT|))
 (PROG (#1=#:G87499)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (COND
       (|ListAggregate;CAT|)
       ((QUOTE T)
        (LETT |ListAggregate;CAT|
         (|Join|
          (|StreamAggregate| (QUOTE |t#1|))
          (|FiniteLinearAggregate| (QUOTE |t#1|))
          (|ExtensibleLinearAggregate| (QUOTE |t#1|))
          (|mkCategory|
           (QUOTE |domain|)
           (QUOTE (((|list| (|$| |t#1|)) T)))
           NIL
           (QUOTE NIL)
            NIL))
          . #2=(|ListAggregate|)))))
     . #2#)
    (SETELT #1# 0 (LIST (QUOTE |ListAggregate|) (|devaluate| |t#1|)))))))
```

——–

21.36 LSAGG-.lsp BOOTSTRAP

LSAGG- depends on **LSAGG**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **LSAGG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **LSAGG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— LSAGG-.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(DEFUN |LSAGG-;sort!;M2A;1| (|f| |l| |$|)
 (|LSAGG-;mergeSort| |f| |l| (SPADCALL |l| (QREFELT |$| 9)) |$|))

(DEFUN |LSAGG-;list;SA;2| (|x| |$|)
 (SPADCALL |x| (SPADCALL (QREFELT |$| 12)) (QREFELT |$| 13)))

(DEFUN |LSAGG-;reduce;MAS;3| (|f| |x| |$|)
 (COND
  ((SPADCALL |x| (QREFELT |$| 16))
    (|error| "reducing over an empty list needs the 3 argument form"))
  ((QUOTE T)
    (SPADCALL |f|
     (SPADCALL |x| (QREFELT |$| 17))
     (SPADCALL |x| (QREFELT |$| 18))
     (QREFELT |$| 20)))))

(DEFUN |LSAGG-;merge;M3A;4| (|f| |p| |q| |$|)
 (SPADCALL |f|
  (SPADCALL |p| (QREFELT |$| 22))
  (SPADCALL |q| (QREFELT |$| 22))
  (QREFELT |$| 23)))

(DEFUN |LSAGG-;select!;M2A;5| (|f| |x| |$|)
 (PROG (|y| |z|)
  (RETURN
   (SEQ
    (SEQ G190
     (COND
      ((NULL
        (COND
         ((OR
           (SPADCALL |x| (QREFELT |$| 16))
           (SPADCALL (SPADCALL |x| (QREFELT |$| 18)) |f|))
          (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
```

```
      (EXIT
       (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;select!;M2A;5|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT
     (COND
      ((SPADCALL |x| (QREFELT |$| 16)) |x|)
      ((QUOTE T)
       (SEQ
        (LETT |y| |x| |LSAGG-;select!;M2A;5|)
        (LETT |z| (SPADCALL |y| (QREFELT |$| 17)) |LSAGG-;select!;M2A;5|)
        (SEQ
         G190
         (COND
          ((NULL
            (COND
             ((SPADCALL |z| (QREFELT |$| 16)) (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
           (GO G191)))
         (SEQ
          (EXIT
           (COND
            ((SPADCALL (SPADCALL |z| (QREFELT |$| 18)) |f|)
              (SEQ
               (LETT |y| |z| |LSAGG-;select!;M2A;5|)
               (EXIT
                (LETT |z|
                 (SPADCALL |z| (QREFELT |$| 17))
                 |LSAGG-;select!;M2A;5|))))
            ((QUOTE T)
             (SEQ
              (LETT |z| (SPADCALL |z| (QREFELT |$| 17)) |LSAGG-;select!;M2A;5|)
              (EXIT (SPADCALL |y| |z| (QREFELT |$| 25))))))))
         NIL
         (GO G190)
         G191
         (EXIT NIL))
        (EXIT |x|)))))))))

(DEFUN |LSAGG-;merge!;M3A;6| (|f| |p| |q| |$|)
 (PROG (|r| |t|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |p| (QREFELT |$| 16)) |q|)
     ((SPADCALL |q| (QREFELT |$| 16)) |p|)
     ((SPADCALL |p| |q| (QREFELT |$| 28))
       (|error| "cannot merge a list into itself"))
     ((QUOTE T)
      (SEQ
       (COND
        ((SPADCALL
```

```
    (SPADCALL |p| (QREFELT |$| 18))
    (SPADCALL |q| (QREFELT |$| 18))
    |f|)
   (SEQ
    (LETT |r| (LETT |t| |p| |LSAGG-;merge!;M3A;6|) |LSAGG-;merge!;M3A;6|)
    (EXIT
     (LETT |p| (SPADCALL |p| (QREFELT |$| 17)) |LSAGG-;merge!;M3A;6|))))
  ((QUOTE T)
   (SEQ
    (LETT |r| (LETT |t| |q| |LSAGG-;merge!;M3A;6|) |LSAGG-;merge!;M3A;6|)
    (EXIT
     (LETT |q| (SPADCALL |q| (QREFELT |$| 17)) |LSAGG-;merge!;M3A;6|)))))
 (SEQ
  G190
  (COND
   ((NULL
      (COND
       ((OR
         (SPADCALL |p| (QREFELT |$| 16))
         (SPADCALL |q| (QREFELT |$| 16)))
        (QUOTE NIL))
       ((QUOTE T) (QUOTE T))))
    (GO G191)))
  (SEQ
   (EXIT
    (COND
     ((SPADCALL
        (SPADCALL |p| (QREFELT |$| 18))
        (SPADCALL |q| (QREFELT |$| 18))
        |f|)
      (SEQ
       (SPADCALL |t| |p| (QREFELT |$| 25))
       (LETT |t| |p| |LSAGG-;merge!;M3A;6|)
       (EXIT
        (LETT |p|
         (SPADCALL |p| (QREFELT |$| 17))
         |LSAGG-;merge!;M3A;6|))))
     ((QUOTE T)
      (SEQ
       (SPADCALL |t| |q| (QREFELT |$| 25))
       (LETT |t| |q| |LSAGG-;merge!;M3A;6|)
       (EXIT
        (LETT |q|
         (SPADCALL |q| (QREFELT |$| 17))
         |LSAGG-;merge!;M3A;6|)))))))
  NIL
  (GO G190)
  G191
  (EXIT NIL))
 (SPADCALL |t|
  (COND
   ((SPADCALL |p| (QREFELT |$| 16)) |q|)
   ((QUOTE T) |p|))
  (QREFELT |$| 25))
```

```
      (EXIT |r|))))))))

(DEFUN |LSAGG-;insert!;SAIA;7| (|s| |x| |i| |$|)
 (PROG (|m| #1=#:G87547 |y| |z|)
  (RETURN
   (SEQ
    (LETT |m| (SPADCALL |x| (QREFELT |$| 31)) |LSAGG-;insert!;SAIA;7|)
    (EXIT
     (COND
      ((|<| |i| |m|) (|error| "index out of range"))
      ((EQL |i| |m|) (SPADCALL |s| |x| (QREFELT |$| 13)))
      ((QUOTE T)
       (SEQ
        (LETT |y|
         (SPADCALL |x|
          (PROG1
           (LETT #1# (|-| (|-| |i| 1) |m|) |LSAGG-;insert!;SAIA;7|)
           (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
          (QREFELT |$| 32))
         |LSAGG-;insert!;SAIA;7|)
        (LETT |z| (SPADCALL |y| (QREFELT |$| 17)) |LSAGG-;insert!;SAIA;7|)
        (SPADCALL |y| (SPADCALL |s| |z| (QREFELT |$| 13)) (QREFELT |$| 25))
        (EXIT |x|)))))))))

(DEFUN |LSAGG-;insert!;2AIA;8| (|w| |x| |i| |$|)
 (PROG (|m| #1=#:G87551 |y| |z|)
  (RETURN
   (SEQ
    (LETT |m| (SPADCALL |x| (QREFELT |$| 31)) |LSAGG-;insert!;2AIA;8|)
    (EXIT
     (COND
      ((|<| |i| |m|) (|error| "index out of range"))
      ((EQL |i| |m|) (SPADCALL |w| |x| (QREFELT |$| 34)))
      ((QUOTE T)
       (SEQ
        (LETT |y|
         (SPADCALL |x|
          (PROG1
           (LETT #1# (|-| (|-| |i| 1) |m|) |LSAGG-;insert!;2AIA;8|)
           (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
          (QREFELT |$| 32))
         |LSAGG-;insert!;2AIA;8|)
        (LETT |z| (SPADCALL |y| (QREFELT |$| 17)) |LSAGG-;insert!;2AIA;8|)
        (SPADCALL |y| |w| (QREFELT |$| 25))
        (SPADCALL |y| |z| (QREFELT |$| 34))
        (EXIT |x|)))))))))

(DEFUN |LSAGG-;remove!;M2A;9| (|f| |x| |$|)
 (PROG (|p| |q|)
  (RETURN
   (SEQ
    (SEQ
     G190
     (COND
```

```
      ((NULL
        (COND
         ((SPADCALL |x| (QREFELT |$| 16)) (QUOTE NIL))
         ((QUOTE T) (SPADCALL (SPADCALL |x| (QREFELT |$| 18)) |f|))))
       (GO G191)))
     (SEQ
      (EXIT
       (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;remove!;M2A;9|)))
      NIL
      (GO G190)
      G191
      (EXIT NIL))
    (EXIT
     (COND
      ((SPADCALL |x| (QREFELT |$| 16)) |x|)
      ((QUOTE T)
       (SEQ
        (LETT |p| |x| |LSAGG-;remove!;M2A;9|)
        (LETT |q| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;remove!;M2A;9|)
        (SEQ
         G190
         (COND
          ((NULL
             (COND
              ((SPADCALL |q| (QREFELT |$| 16)) (QUOTE NIL))
              ((QUOTE T) (QUOTE T))))
            (GO G191)))
         (SEQ
          (EXIT
           (COND
            ((SPADCALL (SPADCALL |q| (QREFELT |$| 18)) |f|)
              (LETT |q|
               (SPADCALL |p| (SPADCALL |q| (QREFELT |$| 17)) (QREFELT |$| 25))
               |LSAGG-;remove!;M2A;9|))
            ((QUOTE T)
              (SEQ
               (LETT |p| |q| |LSAGG-;remove!;M2A;9|)
               (EXIT
                (LETT |q|
                  (SPADCALL |q| (QREFELT |$| 17))
                  |LSAGG-;remove!;M2A;9|)))))))
         NIL
         (GO G190)
         G191
         (EXIT NIL))
        (EXIT |x|)))))))))

(DEFUN |LSAGG-;delete!;AIA;10| (|x| |i| |$|)
 (PROG (|m| #1=#:G87564 |y|)
  (RETURN
   (SEQ
    (LETT |m| (SPADCALL |x| (QREFELT |$| 31)) |LSAGG-;delete!;AIA;10|)
    (EXIT
     (COND
```

```
      ((|<| |i| |m|) (|error| "index out of range"))
      ((EQL |i| |m|) (SPADCALL |x| (QREFELT |$| 17)))
      ((QUOTE T)
       (SEQ
        (LETT |y|
         (SPADCALL |x|
          (PROG1
           (LETT #1# (|-| (|-| |i| 1) |m|) |LSAGG-;delete!;AIA;10|)
           (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
          (QREFELT |$| 32))
         |LSAGG-;delete!;AIA;10|)
        (SPADCALL |y| (SPADCALL |y| 2 (QREFELT |$| 32)) (QREFELT |$| 25))
        (EXIT |x|)))))))))

(DEFUN |LSAGG-;delete!;AUsA;11| (|x| |i| |$|)
 (PROG (|l| |m| |h| #1=#:G87569 #2=#:G87570 |t| #3=#:G87571)
  (RETURN
   (SEQ
    (LETT |l| (SPADCALL |i| (QREFELT |$| 39)) |LSAGG-;delete!;AUsA;11|)
    (LETT |m| (SPADCALL |x| (QREFELT |$| 31)) |LSAGG-;delete!;AUsA;11|)
    (EXIT
     (COND
      ((|<| |l| |m|) (|error| "index out of range"))
      ((QUOTE T)
       (SEQ
        (LETT |h|
         (COND
          ((SPADCALL |i| (QREFELT |$| 40)) (SPADCALL |i| (QREFELT |$| 41)))
          ((QUOTE T) (SPADCALL |x| (QREFELT |$| 42))))
         |LSAGG-;delete!;AUsA;11|)
        (EXIT
         (COND
          ((|<| |h| |l|) |x|)
          ((EQL |l| |m|)
           (SPADCALL |x|
            (PROG1
             (LETT #1# (|-| (|+| |h| 1) |m|) |LSAGG-;delete!;AUsA;11|)
             (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
            (QREFELT |$| 32)))
          ((QUOTE T)
           (SEQ
            (LETT |t|
             (SPADCALL |x|
              (PROG1
               (LETT #2# (|-| (|-| |l| 1) |m|) |LSAGG-;delete!;AUsA;11|)
               (|check-subtype| (|>=| #2# 0)
                 (QUOTE (|NonNegativeInteger|)) #2#))
              (QREFELT |$| 32))
             |LSAGG-;delete!;AUsA;11|)
            (SPADCALL |t|
             (SPADCALL |t|
              (PROG1
               (LETT #3# (|+| (|-| |h| |l|) 2) |LSAGG-;delete!;AUsA;11|)
               (|check-subtype| (|>=| #3# 0)
```

```
                (QUOTE (|NonNegativeInteger|)) #3#))
              (QREFELT |$| 32))
             (QREFELT |$| 25))
            (EXIT |x|)))))))))))))

(DEFUN |LSAGG-;find;MAU;12| (|f| |x| |$|)
 (SEQ
  (SEQ
   G190
   (COND
    ((NULL
     (COND
      ((OR
         (SPADCALL |x| (QREFELT |$| 16))
         (SPADCALL (SPADCALL |x| (QREFELT |$| 18)) |f|))
        (QUOTE NIL))
      ((QUOTE T) (QUOTE T))))
     (GO G191)))
   (SEQ (EXIT (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;find;MAU;12|)))
   NIL
   (GO G190)
   G191
   (EXIT NIL))
  (EXIT
   (COND
    ((SPADCALL |x| (QREFELT |$| 16)) (CONS 1 "failed"))
    ((QUOTE T) (CONS 0 (SPADCALL |x| (QREFELT |$| 18))))))))

(DEFUN |LSAGG-;position;MAI;13| (|f| |x| |$|)
 (PROG (|k|)
  (RETURN
   (SEQ
    (SEQ
     (LETT |k| (SPADCALL |x| (QREFELT |$| 31)) |LSAGG-;position;MAI;13|)
     G190
     (COND
      ((NULL
        (COND
         ((OR
           (SPADCALL |x| (QREFELT |$| 16))
           (SPADCALL (SPADCALL |x| (QREFELT |$| 18)) |f|))
          (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (EXIT
       (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;position;MAI;13|)))
     (LETT |k| (|+| |k| 1) |LSAGG-;position;MAI;13|)
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT
     (COND
      ((SPADCALL |x| (QREFELT |$| 16)) (|-| (SPADCALL |x| (QREFELT |$| 31)) 1))
```

```
      ((QUOTE T) |k|)))))))

(DEFUN |LSAGG-;mergeSort| (|f| |p| |n| |$|)
 (PROG (#1=#:G87593 |l| |q|)
  (RETURN
   (SEQ
    (COND
     ((EQL |n| 2)
      (COND
       ((SPADCALL
         (SPADCALL (SPADCALL |p| (QREFELT |$| 17)) (QREFELT |$| 18))
         (SPADCALL |p| (QREFELT |$| 18)) |f|)
        (LETT |p| (SPADCALL |p| (QREFELT |$| 47)) |LSAGG-;mergeSort|)))))
    (EXIT
     (COND
      ((|<| |n| 3) |p|)
      ((QUOTE T)
       (SEQ
        (LETT |l|
         (PROG1
          (LETT #1# (QUOTIENT2 |n| 2) |LSAGG-;mergeSort|)
          (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
         |LSAGG-;mergeSort|)
        (LETT |q| (SPADCALL |p| |l| (QREFELT |$| 48)) |LSAGG-;mergeSort|)
        (LETT |p| (|LSAGG-;mergeSort| |f| |p| |l| |$|) |LSAGG-;mergeSort|)
        (LETT |q|
          (|LSAGG-;mergeSort| |f| |q| (|-| |n| |l|) |$|)
          |LSAGG-;mergeSort|)
        (EXIT (SPADCALL |f| |p| |q| (QREFELT |$| 23))))))))))

(DEFUN |LSAGG-;sorted?;MAB;15| (|f| |l| |$|)
 (PROG (#1=#:G87603 |p|)
  (RETURN
   (SEQ
    (EXIT
     (COND
      ((SPADCALL |l| (QREFELT |$| 16)) (QUOTE T))
      ((QUOTE T)
       (SEQ
        (LETT |p| (SPADCALL |l| (QREFELT |$| 17)) |LSAGG-;sorted?;MAB;15|)
        (SEQ
         G190
         (COND
          ((NULL
            (COND
             ((SPADCALL |p| (QREFELT |$| 16)) (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
           (GO G191)))
         (SEQ
          (EXIT
           (COND
            ((NULL
              (SPADCALL
               (SPADCALL |l| (QREFELT |$| 18))
```

```
               (SPADCALL |p| (QREFELT |$| 18))
               |f|))
              (PROGN (LETT #1# (QUOTE NIL) |LSAGG-;sorted?;MAB;15|) (GO #1#)))
            ((QUOTE T)
             (LETT |p|
              (SPADCALL
               (LETT |l| |p| |LSAGG-;sorted?;MAB;15|)
               (QREFELT |$| 17))
              |LSAGG-;sorted?;MAB;15|)))))
          NIL
          (GO G190)
          G191
          (EXIT NIL))
        (EXIT (QUOTE T))))))
    #1#
    (EXIT #1#)))))

(DEFUN |LSAGG-;reduce;MA2S;16| (|f| |x| |i| |$|)
 (PROG (|r|)
  (RETURN
   (SEQ
    (LETT |r| |i| |LSAGG-;reduce;MA2S;16|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((SPADCALL |x| (QREFELT |$| 16)) (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (LETT |r|
       (SPADCALL |r| (SPADCALL |x| (QREFELT |$| 18)) |f|)
       |LSAGG-;reduce;MA2S;16|)
      (EXIT
       (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;reduce;MA2S;16|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |r|)))))

(DEFUN |LSAGG-;reduce;MA3S;17| (|f| |x| |i| |a| |$|)
 (PROG (|r|)
  (RETURN
   (SEQ
    (LETT |r| |i| |LSAGG-;reduce;MA3S;17|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((OR
           (SPADCALL |x| (QREFELT |$| 16))
```

```
            (SPADCALL |r| |a| (QREFELT |$| 51)))
           (QUOTE NIL))
         ((QUOTE T) (QUOTE T)))) (GO G191)))
     (SEQ
      (LETT |r|
       (SPADCALL |r| (SPADCALL |x| (QREFELT |$| 18)) |f|)
       |LSAGG-;reduce;MA3S;17|)
      (EXIT
       (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;reduce;MA3S;17|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |r|)))))

(DEFUN |LSAGG-;new;NniSA;18| (|n| |s| |$|)
 (PROG (|k| |l|)
  (RETURN
   (SEQ
    (LETT |l| (SPADCALL (QREFELT |$| 12)) |LSAGG-;new;NniSA;18|)
    (SEQ
     (LETT |k| 1 |LSAGG-;new;NniSA;18|)
     G190
     (COND ((QSGREATERP |k| |n|) (GO G191)))
     (SEQ
      (EXIT
       (LETT |l| (SPADCALL |s| |l| (QREFELT |$| 13)) |LSAGG-;new;NniSA;18|)))
     (LETT |k| (QSADD1 |k|) |LSAGG-;new;NniSA;18|)
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |l|)))))

(DEFUN |LSAGG-;map;M3A;19| (|f| |x| |y| |$|)
 (PROG (|z|)
  (RETURN
   (SEQ
    (LETT |z| (SPADCALL (QREFELT |$| 12)) |LSAGG-;map;M3A;19|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((OR (SPADCALL |x| (QREFELT |$| 16)) (SPADCALL |y| (QREFELT |$| 16)))
           (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (LETT |z|
       (SPADCALL
        (SPADCALL
          (SPADCALL |x| (QREFELT |$| 18))
          (SPADCALL |y| (QREFELT |$| 18))
          |f|)
```

```
        |z|
        (QREFELT |$| 13))
      |LSAGG-;map;M3A;19|)
     (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;map;M3A;19|)
     (EXIT (LETT |y| (SPADCALL |y| (QREFELT |$| 17)) |LSAGG-;map;M3A;19|)))
    NIL
    (GO G190)
    G191
    (EXIT NIL))
   (EXIT (SPADCALL |z| (QREFELT |$| 47)))))))

(DEFUN |LSAGG-;reverse!;2A;20| (|x| |$|)
 (PROG (|z| |y|)
  (RETURN
   (SEQ
    (COND
     ((OR
       (SPADCALL |x| (QREFELT |$| 16))
       (SPADCALL
        (LETT |y| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;reverse!;2A;20|)
        (QREFELT |$| 16)))
      |x|)
     ((QUOTE T)
      (SEQ
       (SPADCALL |x| (SPADCALL (QREFELT |$| 12)) (QREFELT |$| 25))
       (SEQ
        G190
        (COND
         ((NULL
           (COND
            ((SPADCALL |y| (QREFELT |$| 16)) (QUOTE NIL))
            ((QUOTE T) (QUOTE T))))
          (GO G191)))
        (SEQ
         (LETT |z| (SPADCALL |y| (QREFELT |$| 17)) |LSAGG-;reverse!;2A;20|)
         (SPADCALL |y| |x| (QREFELT |$| 25))
         (LETT |x| |y| |LSAGG-;reverse!;2A;20|)
         (EXIT (LETT |y| |z| |LSAGG-;reverse!;2A;20|)))
         NIL
         (GO G190)
         G191
         (EXIT NIL))
       (EXIT |x|))))))))

(DEFUN |LSAGG-;copy;2A;21| (|x| |$|)
 (PROG (|k| |y|)
  (RETURN
   (SEQ
    (LETT |y| (SPADCALL (QREFELT |$| 12)) |LSAGG-;copy;2A;21|)
    (SEQ
     (LETT |k| 0 |LSAGG-;copy;2A;21|)
     G190
     (COND
      ((NULL
```

```
        (COND
         ((SPADCALL |x| (QREFELT |$| 16)) (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
      (SEQ
       (COND
        ((EQL |k| 1000)
          (COND
            ((SPADCALL |x| (QREFELT |$| 56)) (EXIT (|error| "cyclic list"))))))
       (LETT |y|
        (SPADCALL (SPADCALL |x| (QREFELT |$| 18)) |y| (QREFELT |$| 13))
        |LSAGG-;copy;2A;21|)
       (EXIT (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;copy;2A;21|)))
      (LETT |k| (QSADD1 |k|) |LSAGG-;copy;2A;21|)
      (GO G190)
      G191
      (EXIT NIL))
     (EXIT (SPADCALL |y| (QREFELT |$| 47)))))))

(DEFUN |LSAGG-;copyInto!;2AIA;22| (|y| |x| |s| |$|)
 (PROG (|m| #1=#:G87636 |z|)
  (RETURN
   (SEQ
    (LETT |m| (SPADCALL |y| (QREFELT |$| 31)) |LSAGG-;copyInto!;2AIA;22|)
    (EXIT
     (COND
      ((|<| |s| |m|) (|error| "index out of range"))
      ((QUOTE T)
       (SEQ
        (LETT |z|
         (SPADCALL |y|
          (PROG1
           (LETT #1# (|-| |s| |m|) |LSAGG-;copyInto!;2AIA;22|)
           (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
          (QREFELT |$| 32))
         |LSAGG-;copyInto!;2AIA;22|)
        (SEQ
         G190
         (COND
          ((NULL
            (COND
             ((OR
               (SPADCALL |z| (QREFELT |$| 16))
               (SPADCALL |x| (QREFELT |$| 16)))
              (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
           (GO G191)))
         (SEQ
          (SPADCALL |z| (SPADCALL |x| (QREFELT |$| 18)) (QREFELT |$| 58))
          (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;copyInto!;2AIA;22|)
          (EXIT
           (LETT |z|
            (SPADCALL |z| (QREFELT |$| 17))
            |LSAGG-;copyInto!;2AIA;22|)))
```

```
        NIL
        (GO G190)
        G191
        (EXIT NIL))
       (EXIT |y|)))))))))

(DEFUN |LSAGG-;position;SA2I;23| (|w| |x| |s| |$|)
 (PROG (|m| #1=#:G87644 |k|)
  (RETURN
   (SEQ
    (LETT |m| (SPADCALL |x| (QREFELT |$| 31)) |LSAGG-;position;SA2I;23|)
    (EXIT
     (COND
      ((|<| |s| |m|) (|error| "index out of range"))
      ((QUOTE T)
       (SEQ
        (LETT |x|
         (SPADCALL |x|
          (PROG1
           (LETT #1# (|-| |s| |m|) |LSAGG-;position;SA2I;23|)
           (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
          (QREFELT |$| 32))
         |LSAGG-;position;SA2I;23|)
        (SEQ
         (LETT |k| |s| |LSAGG-;position;SA2I;23|)
         G190
         (COND
          ((NULL
            (COND
             ((OR
               (SPADCALL |x| (QREFELT |$| 16))
               (SPADCALL |w| (SPADCALL |x| (QREFELT |$| 18)) (QREFELT |$| 51)))
              (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
           (GO G191)))
         (SEQ
          (EXIT
           (LETT |x|
            (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;position;SA2I;23|)))
         (LETT |k| (|+| |k| 1) |LSAGG-;position;SA2I;23|)
         (GO G190)
         G191
         (EXIT NIL))
        (EXIT
         (COND
          ((SPADCALL |x| (QREFELT |$| 16))
            (|-| (SPADCALL |x| (QREFELT |$| 31)) 1))
          ((QUOTE T) |k|)))))))))))

(DEFUN |LSAGG-;removeDuplicates!;2A;24| (|l| |$|)
 (PROG (|p|)
  (RETURN
   (SEQ
    (LETT |p| |l| |LSAGG-;removeDuplicates!;2A;24|)
```

```
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((SPADCALL |p| (QREFELT |$| 16)) (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (EXIT
       (LETT |p|
        (SPADCALL |p|
         (SPADCALL
          (CONS
           (FUNCTION |LSAGG-;removeDuplicates!;2A;24!0|)
           (VECTOR |$| |p|))
          (SPADCALL |p| (QREFELT |$| 17))
          (QREFELT |$| 61))
         (QREFELT |$| 25))
        |LSAGG-;removeDuplicates!;2A;24|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |l|)))))

(DEFUN |LSAGG-;removeDuplicates!;2A;24!0| (|#1| |$$|)
 (PROG (|$|)
  (LETT |$| (QREFELT |$$| 0) |LSAGG-;removeDuplicates!;2A;24|)
  (RETURN
   (PROGN
    (SPADCALL |#1|
     (SPADCALL (QREFELT |$$| 1) (QREFELT |$| 18))
     (QREFELT |$| 51))))))

(DEFUN |LSAGG-;<;2AB;25| (|x| |y| |$|)
 (PROG (#1=#:G87662)
  (RETURN
   (SEQ
    (EXIT
     (SEQ
      (SEQ
       G190
       (COND
        ((NULL
          (COND
           ((OR
              (SPADCALL |x| (QREFELT |$| 16))
              (SPADCALL |y| (QREFELT |$| 16)))
             (QUOTE NIL))
           ((QUOTE T) (QUOTE T))))
         (GO G191)))
       (SEQ
        (EXIT
```

```
         (COND
          ((NULL
            (SPADCALL
             (SPADCALL |x| (QREFELT |$| 18))
             (SPADCALL |y| (QREFELT |$| 18))
             (QREFELT |$| 51)))
            (PROGN
             (LETT #1#
               (SPADCALL
                (SPADCALL |x| (QREFELT |$| 18))
                (SPADCALL |y| (QREFELT |$| 18))
               (QREFELT |$| 63))
              |LSAGG-;<;2AB;25|)
             (GO #1#)))
          ((QUOTE T)
           (SEQ
            (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;<;2AB;25|)
            (EXIT
             (LETT |y| (SPADCALL |y| (QREFELT |$| 17)) |LSAGG-;<;2AB;25|)))))))
       NIL
       (GO G190)
       G191
       (EXIT NIL))
      (EXIT
       (COND
        ((SPADCALL |x| (QREFELT |$| 16))
          (COND
           ((SPADCALL |y| (QREFELT |$| 16)) (QUOTE NIL))
           ((QUOTE T) (QUOTE T))))
        ((QUOTE T) (QUOTE NIL))))))
    #1#
    (EXIT #1#)))))

(DEFUN |ListAggregate&| (|#1| |#2|)
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|ListAggregate&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |ListAggregate&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (make-array 66) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
    (COND
     ((|HasCategory| |#2| (QUOTE (|SetCategory|)))
       (QSETREFV |$| 52
         (CONS (|dispatchFunction| |LSAGG-;reduce;MA3S;17|) |$|))))
    (COND
     ((|HasCategory| |#2| (QUOTE (|SetCategory|)))
       (PROGN
        (QSETREFV |$| 60
```

```
        (CONS (|dispatchFunction| |LSAGG-;position;SA2I;23|) |$|))
       (QSETREFV |$| 62
        (CONS (|dispatchFunction| |LSAGG-;removeDuplicates!;2A;24|) |$|)))))
   (COND
    ((|HasCategory| |#2| (QUOTE (|OrderedSet|)))
      (QSETREFV |$| 64 (CONS (|dispatchFunction| |LSAGG-;<;2AB;25|) |$|))))
   |$|))))

(setf (get
 (QUOTE |ListAggregate&|)
 (QUOTE |infovec|))
 (LIST
  (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|)
   (|NonNegativeInteger|) (0 . |#|) (|Mapping| 15 7 7) |LSAGG-;sort!;M2A;1|
   (5 . |empty|) (9 . |concat|) |LSAGG-;list;SA;2| (|Boolean|)
   (15 . |empty?|) (20 . |rest|) (25 . |first|) (|Mapping| 7 7 7)
   (30 . |reduce|) |LSAGG-;reduce;MAS;3| (37 . |copy|) (42 . |merge!|)
   |LSAGG-;merge;M3A;4| (49 . |setrest!|) (|Mapping| 15 7)
   |LSAGG-;select!;M2A;5| (55 . |eq?|) |LSAGG-;merge!;M3A;6|
   (|Integer|) (61 . |minIndex|) (66 . |rest|) |LSAGG-;insert!;SAIA;7|
   (72 . |concat!|) |LSAGG-;insert!;2AIA;8| |LSAGG-;remove!;M2A;9|
   |LSAGG-;delete!;AIA;10| (|UniversalSegment| 30) (78 . |lo|)
   (83 . |hasHi|) (88 . |hi|) (93 . |maxIndex|) |LSAGG-;delete!;AUsA;11|
   (|Union| 7 (QUOTE "failed")) |LSAGG-;find;MAU;12|
   |LSAGG-;position;MAI;13| (98 . |reverse!|) (103 . |split!|)
   |LSAGG-;sorted?;MAB;15| |LSAGG-;reduce;MA2S;16| (109 . |=|)
   (115 . |reduce|) |LSAGG-;new;NniSA;18| |LSAGG-;map;M3A;19|
   |LSAGG-;reverse!;2A;20| (123 . |cyclic?|) |LSAGG-;copy;2A;21|
   (128 . |setfirst!|) |LSAGG-;copyInto!;2AIA;22| (134 . |position|)
   (141 . |remove!|) (147 . |removeDuplicates!|) (152 . |<|) (158 . |<|)
   (|Mapping| 7 7)))
  (QUOTE #(|sorted?| 164 |sort!| 170 |select!| 176 |reverse!| 182
    |removeDuplicates!| 187 |remove!| 192 |reduce| 198 |position| 219
    |new| 232 |merge!| 238 |merge| 245 |map| 252 |list| 259 |insert!|
    264 |find| 278 |delete!| 284 |copyInto!| 296 |copy| 303 |<| 308))
   (QUOTE NIL)
   (CONS
    (|makeByteWordVec2| 1 (QUOTE NIL))
    (CONS
     (QUOTE #())
     (CONS
      (QUOTE #())
      (|makeByteWordVec2| 64 (QUOTE (1 6 8 0 9 0 6 0 12 2 6 0 7 0 13 1 6
       15 0 16 1 6 0 0 17 1 6 7 0 18 3 6 7 19 0 7 20 1 6 0 0 22 3 6 0 10
       0 0 23 2 6 0 0 0 25 2 6 15 0 0 28 1 6 30 0 31 2 6 0 0 8 32 2 6 0 0
       0 34 1 38 30 0 39 1 38 15 0 40 1 38 30 0 41 1 6 30 0 42 1 6 0 0 47
       2 6 0 0 30 48 2 7 15 0 0 51 4 0 7 19 0 7 7 52 1 6 15 0 56 2 6 7 0
       7 58 3 0 30 7 0 30 60 2 6 0 26 0 61 1 0 0 0 62 2 7 15 0 0 63 2 0 15
       0 0 64 2 0 15 10 0 49 2 0 0 10 0 11 2 0 0 26 0 27 1 0 0 0 55 1 0 0
       0 62 2 0 0 26 0 36 3 0 7 19 0 7 50 4 0 7 19 0 7 7 52 2 0 7 19 0 21
       2 0 30 26 0 46 3 0 30 7 0 30 60 2 0 0 8 7 53 3 0 0 10 0 0 29 3 0 0
       10 0 0 24 3 0 0 19 0 0 54 1 0 0 7 14 3 0 0 7 0 30 33 3 0 0 0 0 30
       35 2 0 44 26 0 45 2 0 0 0 38 43 2 0 0 0 30 37 3 0 0 0 0 30 59 1 0
       0 0 57 2 0 15 0 0 64))))))
```

```
   (QUOTE |lookupComplete|)))
```

———–

21.37 MONOID.lsp BOOTSTRAP

MONOID depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **MONOID** category which we can write into the **MID** directory. We compile the lisp code and copy the **MONOID.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— MONOID.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |Monoid;AL| (QUOTE NIL))

(DEFUN |Monoid| NIL
  (declare (special |Monoid;AL|))
    (COND
      (|Monoid;AL|)
      (T (SETQ |Monoid;AL| (|Monoid;|)))))

(DEFUN |Monoid;| NIL
  (PROG (#1=#:G82432)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|SemiGroup|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|One| (|$|) |constant|) T)
                ((|sample| (|$|) |constant|) T)
                ((|one?| ((|Boolean|) |$|)) T)
                ((|**| (|$| |$| (|NonNegativeInteger|))) T)
                ((|^| (|$| |$| (|NonNegativeInteger|))) T)
                ((|recip| ((|Union| |$| "failed") |$|)) T)))
              NIL
              (QUOTE ((|NonNegativeInteger|) (|Boolean|)))
              NIL))
            |Monoid|)
         (SETELT #1# 0 (QUOTE (|Monoid|)))))))

(setf (get (QUOTE |Monoid|) (QUOTE NILADIC)) T)
```

———–

21.38 MONOID-.lsp BOOTSTRAP

MONOID- depends on **MONOID**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **MONOID-** category which we can write into the **MID** directory. We compile the lisp code and copy the **MONOID-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— MONOID-.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(DEFUN |MONOID-;^;SNniS;1| (|x| |n| |$|)
  (SPADCALL |x| |n| (QREFELT |$| 8)))

(DEFUN |MONOID-;one?;SB;2| (|x| |$|)
  (SPADCALL |x| (|spadConstant| |$| 10) (QREFELT |$| 12)))

(DEFUN |MONOID-;sample;S;3| (|$|)
  (|spadConstant| |$| 10))

(DEFUN |MONOID-;recip;SU;4| (|x| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 15)) (CONS 0 |x|))
    ((QUOTE T) (CONS 1 "failed"))))

(DEFUN |MONOID-;**;SNniS;5| (|x| |n| |$|)
  (COND
    ((ZEROP |n|) (|spadConstant| |$| 10))
    ((QUOTE T) (SPADCALL |x| |n| (QREFELT |$| 20)))))

(DEFUN |Monoid&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|Monoid&|))
        (LETT |dv$| (LIST (QUOTE |Monoid&|) |DV$1|) . #1#)
        (LETT |$| (make-array 22) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        |$|))))

(setf (get
  (QUOTE |Monoid&|)
  (QUOTE |infovec|))
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (|NonNegativeInteger|)
        (0 . |**|)
```

```
        |MONOID-;^;SNniS;1|
        (6 . |One|)
        (|Boolean|)
        (10 . |=|)
        |MONOID-;one?;SB;2|
        |MONOID-;sample;S;3|
        (16 . |one?|)
        (|Union| |$| (QUOTE "failed"))
        |MONOID-;recip;SU;4|
        (|PositiveInteger|)
        (|RepeatedSquaring| 6)
        (21 . |expt|)
        |MONOID-;**;SNniS;5|))
    (QUOTE #(|sample| 27 |recip| 31 |one?| 36 |^| 41 |**| 47))
    (QUOTE NIL)
    (CONS
      (|makeByteWordVec2| 1 (QUOTE NIL))
      (CONS
        (QUOTE #())
        (CONS
          (QUOTE #())
          (|makeByteWordVec2| 21
            (QUOTE
              (2 6 0 0 7 8 0 6 0 10 2 6 11 0 0 12 1 6 11 0 15 2 19 6 6 18 20
              0 0 0 14 1 0 16 0 17 1 0 11 0 13 2 0 0 0 7 9 2 0 0 0 7 21))))))
  (QUOTE |lookupComplete|)))
```

21.39 MTSCAT.lsp BOOTSTRAP

MTSCAT depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **MTSCAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **MTSCAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— MTSCAT.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |MultivariateTaylorSeriesCategory;CAT| (QUOTE NIL))

(SETQ |MultivariateTaylorSeriesCategory;AL| (QUOTE NIL))

(DEFUN |MultivariateTaylorSeriesCategory|
 (|&REST| #1=#:G83334 |&AUX| #2=#:G83332)
 (declare (special |MultivariateTaylorSeriesCategory;AL|))
   (DSETQ #2# #1#)
   (LET (#3=#:G83333)
    (COND
```

```
     ((SETQ #3#
       (|assoc| (|devaluateList| #2#) |MultivariateTaylorSeriesCategory;AL|))
        (CDR #3#))
     (T
      (SETQ |MultivariateTaylorSeriesCategory;AL|
       (|cons5|
        (CONS
         (|devaluateList| #2#)
         (SETQ #3# (APPLY (FUNCTION |MultivariateTaylorSeriesCategory;|) #2#)))
        |MultivariateTaylorSeriesCategory;AL|))
       #3#))))

(DEFUN |MultivariateTaylorSeriesCategory;| (|t#1| |t#2|)
 (declare (special |MultivariateTaylorSeriesCategory;CAT|))
 (PROG (#1=#:G83331)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1| |t#2|)) (LIST (|devaluate| |t#1|) (|devaluate| |t#2|)))
      (|sublisV|
       (PAIR (QUOTE (#2=#:G83330)) (LIST (QUOTE (|IndexedExponents| |t#2|))))
       (COND
        (|MultivariateTaylorSeriesCategory;CAT|)
        ((QUOTE T)
         (LETT |MultivariateTaylorSeriesCategory;CAT|
          (|Join|
           (|PartialDifferentialRing| (QUOTE |t#2|))
           (|PowerSeriesCategory| (QUOTE |t#1|) (QUOTE #2#) (QUOTE |t#2|))
           (|InnerEvalable| (QUOTE |t#2|) (QUOTE |$|))
           (|Evalable| (QUOTE |$|))
           (|mkCategory|
            (QUOTE |domain|)
            (QUOTE (
             ((|coefficient| (|$| |$| |t#2| (|NonNegativeInteger|))) T)
             ((|coefficient|
                 (|$| |$| (|List| |t#2|) (|List| (|NonNegativeInteger|))))
                 T)
             ((|extend| (|$| |$| (|NonNegativeInteger|))) T)
             ((|monomial| (|$| |$| |t#2| (|NonNegativeInteger|))) T)
             ((|monomial|
                (|$| |$| (|List| |t#2|) (|List| (|NonNegativeInteger|))))
                 T)
             ((|order| ((|NonNegativeInteger|) |$| |t#2|)) T)
             ((|order|
                ((|NonNegativeInteger|) |$| |t#2| (|NonNegativeInteger|)))
                 T)
             ((|polynomial|
               ((|Polynomial| |t#1|) |$| (|NonNegativeInteger|)))
                T)
             ((|polynomial|
               ((|Polynomial| |t#1|)
                |$|
                (|NonNegativeInteger|)
```

```
                (|NonNegativeInteger|)))
                 T)
            ((|integrate| (|$| |$| |t#2|))
                (|has| |t#1| (|Algebra| (|Fraction| (|Integer|)))))))
           (QUOTE (
            ((|RadicalCategory|)
                (|has| |t#1| (|Algebra| (|Fraction| (|Integer|)))))
            ((|TranscendentalFunctionCategory|)
                (|has| |t#1| (|Algebra| (|Fraction| (|Integer|)))))))
           (QUOTE
            ((|Polynomial| |t#1|)
             (|NonNegativeInteger|)
             (|List| |t#2|)
             (|List| (|NonNegativeInteger|))))
             NIL))
         . #3=(|MultivariateTaylorSeriesCategory|))))))
        . #3#)
    (SETELT #1# 0
     (LIST
      (QUOTE |MultivariateTaylorSeriesCategory|)
      (|devaluate| |t#1|)
      (|devaluate| |t#2|)))))))
```

21.40 OINTDOM.lsp BOOTSTRAP

OINTDOM depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **OINTDOM** category which we can write into the **MID** directory. We compile the lisp code and copy the **OINTDOM.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— OINTDOM.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |OrderedIntegralDomain;AL| (QUOTE NIL))

(DEFUN |OrderedIntegralDomain| NIL
  (declare (special |OrderedIntegralDomain;AL|))
    (COND
      (|OrderedIntegralDomain;AL|)
      (T (SETQ |OrderedIntegralDomain;AL| (|OrderedIntegralDomain;|)))))

(DEFUN |OrderedIntegralDomain;| NIL
  (PROG (#1=#:G84529)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join| (|IntegralDomain|) (|OrderedRing|)) |OrderedIntegralDomain|)
        (SETELT #1# 0 (QUOTE (|OrderedIntegralDomain|)))))))
```

```
(setf (get (QUOTE |OrderedIntegralDomain|) (QUOTE NILADIC)) T)
```

———-

21.41 ORDRING.lsp BOOTSTRAP

ORDRING depends on **INT**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ORDRING** category which we can write into the **MID** directory. We compile the lisp code and copy the **ORDRING.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Technically I can't justify this bootstrap stanza based on the lattice since **INT** is already bootstrapped. However using **INT** naked generates a "value stack overflow" error suggesting an infinite recursive loop. This code is here to experiment with breaking that loop.

Note that this code is not included in the generated catdef.spad file.

— ORDRING.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |OrderedRing;AL| (QUOTE NIL))

(DEFUN |OrderedRing| NIL
  (declare (special |OrderedRing;AL|))
    (COND
      (|OrderedRing;AL|)
      (T (SETQ |OrderedRing;AL| (|OrderedRing;|)))))

(DEFUN |OrderedRing;| NIL
  (PROG (#1=#:G84455)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|OrderedAbelianGroup|)
            (|Ring|)
            (|Monoid|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|positive?| ((|Boolean|) |$|)) T)
                ((|negative?| ((|Boolean|) |$|)) T)
                ((|sign| ((|Integer|) |$|)) T)
                ((|abs| (|$| |$|)) T)))
              NIL
              (QUOTE ((|Integer|) (|Boolean|)))
              NIL))
           |OrderedRing|)
        (SETELT #1# 0 (QUOTE (|OrderedRing|)))))))
```

```
(setf (get (QUOTE |OrderedRing|) (QUOTE NILADIC)) T)
```

———-

21.42 ORDRING-.lsp BOOTSTRAP

ORDRING- depends on **ORDRING**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ORDRING-** category which we can write into the **MID** directory. We compile the lisp code and copy the **ORDRING-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— ORDRING-.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(DEFUN |ORDRING-;positive?;SB;1| (|x| |$|)
  (SPADCALL (|spadConstant| |$| 7) |x| (QREFELT |$| 9)))

(DEFUN |ORDRING-;negative?;SB;2| (|x| |$|)
  (SPADCALL |x| (|spadConstant| |$| 7) (QREFELT |$| 9)))

(DEFUN |ORDRING-;sign;SI;3| (|x| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 12)) 1)
    ((SPADCALL |x| (QREFELT |$| 13)) -1)
    ((SPADCALL |x| (QREFELT |$| 15)) 0)
    ((QUOTE T)
      (|error| "x satisfies neither positive?, negative? or zero?"))))

(DEFUN |ORDRING-;abs;2S;4| (|x| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 12)) |x|)
    ((SPADCALL |x| (QREFELT |$| 13)) (SPADCALL |x| (QREFELT |$| 18)))
    ((SPADCALL |x| (QREFELT |$| 15)) (|spadConstant| |$| 7))
    ((QUOTE T)
      (|error| "x satisfies neither positive?, negative? or zero?"))))

(DEFUN |OrderedRing&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|OrderedRing&|))
        (LETT |dv$| (LIST (QUOTE |OrderedRing&|) |DV$1|) . #1#)
        (LETT |$| (make-array 20) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        |$|))))
```

```
(setf (get
  (QUOTE |OrderedRing&|)
  (QUOTE |infovec|))
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (0 . |Zero|)
        (|Boolean|)
        (4 . |<|)
        |ORDRING-;positive?;SB;1|
        |ORDRING-;negative?;SB;2|
        (10 . |positive?|)
        (15 . |negative?|)
        (20 . |One|)
        (24 . |zero?|)
        (|Integer|)
        |ORDRING-;sign;SI;3|
        (29 . |-|)
        |ORDRING-;abs;2S;4|))
    (QUOTE #(|sign| 34 |positive?| 39 |negative?| 44 |abs| 49))
    (QUOTE NIL)
    (CONS
      (|makeByteWordVec2| 1 (QUOTE NIL))
      (CONS
        (QUOTE #())
        (CONS
          (QUOTE #())
          (|makeByteWordVec2| 19
            (QUOTE
              (0 6 0 7 2 6 8 0 0 9 1 6 8 0 12 1 6 8 0 13 0 6 0 14 1 6 8 0 15
               1 6 0 0 18 1 0 16 0 17 1 0 8 0 10 1 0 8 0 11 1 0 0 0 19))))))
   (QUOTE |lookupComplete|)))
```

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21.43 POLYCAT.lsp BOOTSTRAP

POLYCAT depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **POLYCAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **POLYCAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— POLYCAT.lsp BOOTSTRAP —

```
(/VERSIONCHECK 2)

(SETQ |PolynomialCategory;CAT| (QUOTE NIL))

(SETQ |PolynomialCategory;AL| (QUOTE NIL))
```

```
(DEFUN |PolynomialCategory| (&REST #0=#:G1430 &AUX #1=#:G1428)
 (declare (special |PolynomialCategory;AL|))
 (DSETQ #1# #0#)
 (LET (#2=#:G1429)
  (COND
   ((SETQ #2# (|assoc| (|devaluateList| #1#) |PolynomialCategory;AL|))
     (CDR #2#))
   (T
     (SETQ |PolynomialCategory;AL|
      (|cons5|
       (CONS (|devaluateList| #1#)
        (SETQ #2# (APPLY (FUNCTION |PolynomialCategory;|) #1#)))
       |PolynomialCategory;AL|))
     #2#))))

(DEFUN |PolynomialCategory;| (|t#1| |t#2| |t#3|)
 (declare (special |PolynomialCategory;CAT|))
 (PROG (#0=#:G1427)
  (RETURN
   (PROG1
    (LETT #0#
     (|sublisV|
      (PAIR (QUOTE (|t#1| |t#2| |t#3|))
            (LIST (|devaluate| |t#1|) (|devaluate| |t#2|) (|devaluate| |t#3|)))
      (COND
       (|PolynomialCategory;CAT|)
       ((QUOTE T)
        (LETT |PolynomialCategory;CAT|
         (|Join|
          (|PartialDifferentialRing| (QUOTE |t#3|))
          (|FiniteAbelianMonoidRing| (QUOTE |t#1|) (QUOTE |t#2|))
          (|Evalable| (QUOTE $))
          (|InnerEvalable| (QUOTE |t#3|) (QUOTE |t#1|))
          (|InnerEvalable| (QUOTE |t#3|) (QUOTE $))
          (|RetractableTo| (QUOTE |t#3|))
          (|FullyLinearlyExplicitRingOver| (QUOTE |t#1|))
          (|mkCategory| (QUOTE |domain|)
           (QUOTE
            (((|degree| ((|NonNegativeInteger|) $ |t#3|)) T)
             ((|degree| ((|List| (|NonNegativeInteger|)) $ (|List| |t#3|))) T)
             ((|coefficient| ($ $ |t#3| (|NonNegativeInteger|))) T)
             ((|coefficient| ($ $ (|List| |t#3|)
               (|List| (|NonNegativeInteger|)))) T)
             ((|monomials| ((|List| $) $)) T)
             ((|univariate| ((|SparseUnivariatePolynomial| $) $ |t#3|)) T)
             ((|univariate| ((|SparseUnivariatePolynomial| |t#1|) $)) T)
             ((|mainVariable| ((|Union| |t#3| "failed") $)) T)
             ((|minimumDegree| ((|NonNegativeInteger|) $ |t#3|)) T)
             ((|minimumDegree| ((|List| (|NonNegativeInteger|)) $
               (|List| |t#3|))) T)
             ((|monicDivide|
               ((|Record| (|:| |quotient| $) (|:| |remainder| $)) $ $ |t#3|))
               T)
             ((|monomial| ($ $ |t#3| (|NonNegativeInteger|))) T)
```

```
  ((|monomial| ($ $ (|List| |t#3|) (|List| (|NonNegativeInteger|))))
    T)
  ((|multivariate| ($ (|SparseUnivariatePolynomial| |t#1|) |t#3|))
    T)
  ((|multivariate| ($ (|SparseUnivariatePolynomial| $) |t#3|)) T)
  ((|isPlus| ((|Union| (|List| $) "failed") $)) T)
  ((|isTimes| ((|Union| (|List| $) "failed") $)) T)
  ((|isExpt|
    ((|Union|
     (|Record| (|:| |var| |t#3|)
               (|:| |exponent| (|NonNegativeInteger|)))
     "failed") $))
    T)
  ((|totalDegree| ((|NonNegativeInteger|) $)) T)
  ((|totalDegree| ((|NonNegativeInteger|) $ (|List| |t#3|))) T)
  ((|variables| ((|List| |t#3|) $)) T)
  ((|primitiveMonomials| ((|List| $) $)) T)
  ((|resultant| ($ $ $ |t#3|)) (|has| |t#1| (|CommutativeRing|)))
  ((|discriminant| ($ $ |t#3|)) (|has| |t#1| (|CommutativeRing|)))
  ((|content| ($ $ |t#3|)) (|has| |t#1| (|GcdDomain|)))
  ((|primitivePart| ($ $)) (|has| |t#1| (|GcdDomain|)))
  ((|primitivePart| ($ $ |t#3|)) (|has| |t#1| (|GcdDomain|)))
  ((|squareFree| ((|Factored| $) $)) (|has| |t#1| (|GcdDomain|)))
  ((|squareFreePart| ($ $)) (|has| |t#1| (|GcdDomain|)))))
(QUOTE
 (((|OrderedSet|) (|has| |t#1| (|OrderedSet|)))
  ((|ConvertibleTo| (|InputForm|))
   (AND (|has| |t#3| (|ConvertibleTo| (|InputForm|)))
        (|has| |t#1| (|ConvertibleTo| (|InputForm|)))))
  ((|ConvertibleTo| (|Pattern| (|Integer|)))
   (AND (|has| |t#3| (|ConvertibleTo| (|Pattern| (|Integer|))))
        (|has| |t#1| (|ConvertibleTo| (|Pattern| (|Integer|))))))
  ((|ConvertibleTo| (|Pattern| (|Float|)))
   (AND (|has| |t#3| (|ConvertibleTo| (|Pattern| (|Float|))))
        (|has| |t#1| (|ConvertibleTo| (|Pattern| (|Float|))))))
  ((|PatternMatchable| (|Integer|))
   (AND
    (|has| |t#3| (|PatternMatchable| (|Integer|)))
    (|has| |t#1| (|PatternMatchable| (|Integer|)))))
  ((|PatternMatchable| (|Float|))
   (AND
    (|has| |t#3| (|PatternMatchable| (|Float|)))
    (|has| |t#1| (|PatternMatchable| (|Float|)))))
  ((|GcdDomain|) (|has| |t#1| (|GcdDomain|)))
  (|canonicalUnitNormal|
    (|has| |t#1| (ATTRIBUTE |canonicalUnitNormal|)))
  ((|PolynomialFactorizationExplicit|)
    (|has| |t#1| (|PolynomialFactorizationExplicit|)))))
(QUOTE
 ((|Factored| $)
  (|List| $)
  (|List| |t#3|)
  (|NonNegativeInteger|)
  (|SparseUnivariatePolynomial| $)
```

```
             (|SparseUnivariatePolynomial| |t#1|)
             (|List| (|NonNegativeInteger|))))
            NIL))
          . #1=(|PolynomialCategory|)))))
      . #1#)
    (SETELT #0# 0
     (LIST (QUOTE |PolynomialCategory|)
      (|devaluate| |t#1|) (|devaluate| |t#2|) (|devaluate| |t#3|)))))))
```

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21.44 POLYCAT-.lsp BOOTSTRAP

POLYCAT- depends on **POLYCAT**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **POLYCAT-** category which we can write into the **MID** directory. We compile the lisp code and copy the **POLYCAT-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— POLYCAT-.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)


(/VERSIONCHECK 2)

(DEFUN |POLYCAT-;eval;SLS;1| (|p| |l| $)
 (PROG (#0=#:G1444 #1=#:G1438 #2=#:G1445 #3=#:G1446 |lvar| #4=#:G1447
        |e| #5=#:G1448)
  (RETURN
   (SEQ
    (COND
     ((NULL |l|) |p|)
     ((QUOTE T)
      (SEQ
       (SEQ
        (EXIT
         (SEQ
          (LETT |e| NIL |POLYCAT-;eval;SLS;1|)
          (LETT #0# |l| |POLYCAT-;eval;SLS;1|)
          G190
          (COND
           ((OR (ATOM #0#)
                (PROGN (LETT |e| (CAR #0#) |POLYCAT-;eval;SLS;1|) NIL))
            (GO G191)))
          (SEQ
           (EXIT
            (COND
             ((QEQCAR
               (SPADCALL (SPADCALL |e| (QREFELT $ 11)) (QREFELT $ 13)) 1)
              (PROGN
```

```
              (LETT #1#
               (|error| "cannot find a variable to evaluate")
               |POLYCAT-;eval;SLS;1|)
              (GO #1#))))))
         (LETT #0# (CDR #0#) |POLYCAT-;eval;SLS;1|)
         (GO G190)
         G191
         (EXIT NIL)))
       #1# (EXIT #1#))
      (LETT |lvar|
       (PROGN
        (LETT #2# NIL |POLYCAT-;eval;SLS;1|)
        (SEQ
         (LETT |e| NIL |POLYCAT-;eval;SLS;1|)
         (LETT #3# |l| |POLYCAT-;eval;SLS;1|)
         G190
         (COND
          ((OR (ATOM #3#)
               (PROGN (LETT |e| (CAR #3#) |POLYCAT-;eval;SLS;1|) NIL))
           (GO G191)))
         (SEQ
          (EXIT
           (LETT #2#
            (CONS (SPADCALL (SPADCALL |e| (QREFELT $ 11)) (QREFELT $ 14))
                  #2#)
            |POLYCAT-;eval;SLS;1|)))
         (LETT #3# (CDR #3#) |POLYCAT-;eval;SLS;1|)
         (GO G190)
         G191
         (EXIT (NREVERSE0 #2#))))
        |POLYCAT-;eval;SLS;1|)
      (EXIT
       (SPADCALL |p| |lvar|
        (PROGN
         (LETT #4# NIL |POLYCAT-;eval;SLS;1|)
         (SEQ
          (LETT |e| NIL |POLYCAT-;eval;SLS;1|)
          (LETT #5# |l| |POLYCAT-;eval;SLS;1|)
          G190
          (COND
           ((OR (ATOM #5#)
                (PROGN (LETT |e| (CAR #5#) |POLYCAT-;eval;SLS;1|) NIL))
             (GO G191)))
          (SEQ
           (EXIT
            (LETT #4# (CONS (SPADCALL |e| (QREFELT $ 15)) #4#)
             |POLYCAT-;eval;SLS;1|)))
          (LETT #5# (CDR #5#) |POLYCAT-;eval;SLS;1|)
          (GO G190)
          G191
          (EXIT (NREVERSE0 #4#))))
        (QREFELT $ 18)))))))))

(DEFUN |POLYCAT-;monomials;SL;2| (|p| $)
```

```
 (PROG (|ml|)
  (RETURN
   (SEQ
    (LETT |ml| NIL |POLYCAT-;monomials;SL;2|)
    (SEQ G190
     (COND
      ((NULL
        (COND
         ((SPADCALL |p| (|spadConstant| $ 21) (QREFELT $ 24)) (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (LETT |ml|
       (CONS (SPADCALL |p| (QREFELT $ 25)) |ml|)
       |POLYCAT-;monomials;SL;2|)
      (EXIT
       (LETT |p| (SPADCALL |p| (QREFELT $ 26)) |POLYCAT-;monomials;SL;2|)))
      NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT (REVERSE |ml|))))))

(DEFUN |POLYCAT-;isPlus;SU;3| (|p| $)
 (PROG (|l|)
  (RETURN
   (COND
    ((NULL
       (CDR (LETT |l| (SPADCALL |p| (QREFELT $ 28)) |POLYCAT-;isPlus;SU;3|)))
      (CONS 1 "failed"))
    ((QUOTE T) (CONS 0 |l|))))))

(DEFUN |POLYCAT-;isTimes;SU;4| (|p| $)
 (PROG (|lv| #0=#:G1470 |v| #1=#:G1471 |l| |r|)
  (RETURN
   (SEQ
    (COND
     ((OR (NULL
           (LETT |lv| (SPADCALL |p| (QREFELT $ 31)) |POLYCAT-;isTimes;SU;4|))
          (NULL (SPADCALL |p| (QREFELT $ 32))))
       (CONS 1 "failed"))
     ((QUOTE T)
      (SEQ
       (LETT |l|
        (PROGN
         (LETT #0# NIL |POLYCAT-;isTimes;SU;4|)
         (SEQ
          (LETT |v| NIL |POLYCAT-;isTimes;SU;4|)
          (LETT #1# |lv| |POLYCAT-;isTimes;SU;4|)
          G190
          (COND
           ((OR (ATOM #1#)
                (PROGN (LETT |v| (CAR #1#) |POLYCAT-;isTimes;SU;4|) NIL))
            (GO G191)))
```

```
          (SEQ
           (EXIT
            (LETT #0#
             (CONS
              (SPADCALL (|spadConstant| $ 33) |v|
               (SPADCALL |p| |v| (QREFELT $ 36)) (QREFELT $ 37))
              #0#)
              |POLYCAT-;isTimes;SU;4|)))
          (LETT #1# (CDR #1#) |POLYCAT-;isTimes;SU;4|)
          (GO G190)
          G191
         (EXIT (NREVERSE0 #0#))))
        |POLYCAT-;isTimes;SU;4|)
       (LETT |r| (SPADCALL |p| (QREFELT $ 38)) |POLYCAT-;isTimes;SU;4|)
       (EXIT
        (COND
         ((SPADCALL |r| (|spadConstant| $ 34) (QREFELT $ 39))
           (COND
            ((NULL (CDR |lv|)) (CONS 1 "failed"))
            ((QUOTE T) (CONS 0 |l|))))
         ((QUOTE T)
           (CONS 0 (CONS (SPADCALL |r| (QREFELT $ 40)) |l|))))))))))))

(DEFUN |POLYCAT-;isExpt;SU;5| (|p| $)
 (PROG (|u| |d|)
  (RETURN
   (SEQ
    (LETT |u| (SPADCALL |p| (QREFELT $ 42)) |POLYCAT-;isExpt;SU;5|)
    (EXIT
     (COND
      ((OR (QEQCAR |u| 1)
           (NULL
            (SPADCALL |p|
             (SPADCALL (|spadConstant| $ 33)
              (QCDR |u|)
              (LETT |d| (SPADCALL |p| (QCDR |u|) (QREFELT $ 36))
               |POLYCAT-;isExpt;SU;5|)
              (QREFELT $ 37))
             (QREFELT $ 24))))
       (CONS 1 "failed"))
      ((QUOTE T) (CONS 0 (CONS (QCDR |u|) |d|)))))))))

(DEFUN |POLYCAT-;coefficient;SVarSetNniS;6| (|p| |v| |n| $)
 (SPADCALL (SPADCALL |p| |v| (QREFELT $ 47)) |n| (QREFELT $ 49)))

(DEFUN |POLYCAT-;coefficient;SLLS;7| (|p| |lv| |ln| $)
 (COND
  ((NULL |lv|)
   (COND
    ((NULL |ln|) |p|)
    ((QUOTE T) (|error| "mismatched lists in coefficient"))))
  ((NULL |ln|) (|error| "mismatched lists in coefficient"))
  ((QUOTE T)
   (SPADCALL
```

```
    (SPADCALL
     (SPADCALL |p| (|SPADfirst| |lv|) (QREFELT $ 47))
     (|SPADfirst| |ln|)
     (QREFELT $ 49))
    (CDR |lv|)
    (CDR |ln|)
    (QREFELT $ 52)))))

(DEFUN |POLYCAT-;monomial;SLLS;8| (|p| |lv| |ln| $)
 (COND
  ((NULL |lv|)
   (COND
    ((NULL |ln|) |p|)
    ((QUOTE T) (|error| "mismatched lists in monomial"))))
  ((NULL |ln|) (|error| "mismatched lists in monomial"))
  ((QUOTE T)
   (SPADCALL
    (SPADCALL |p| (|SPADfirst| |lv|) (|SPADfirst| |ln|) (QREFELT $ 37))
    (CDR |lv|)
    (CDR |ln|)
    (QREFELT $ 54)))))

(DEFUN |POLYCAT-;retract;SVarSet;9| (|p| $)
 (PROG (#0=#:G1496 |q|)
  (RETURN
   (SEQ
    (LETT |q|
     (PROG2
      (LETT #0# (SPADCALL |p| (QREFELT $ 42)) |POLYCAT-;retract;SVarSet;9|)
      (QCDR #0#)
      (|check-union| (QEQCAR #0# 0) (QREFELT $ 9) #0#))
     |POLYCAT-;retract;SVarSet;9|)
    (EXIT
     (COND
      ((SPADCALL (SPADCALL |q| (QREFELT $ 56)) |p| (QREFELT $ 24)) |q|)
      ((QUOTE T) (|error| "Polynomial is not a single variable"))))))))

(DEFUN |POLYCAT-;retractIfCan;SU;10| (|p| $)
 (PROG (|q| #0=#:G1504)
  (RETURN
   (SEQ
    (EXIT
     (SEQ
      (SEQ
       (LETT |q| (SPADCALL |p| (QREFELT $ 42)) |POLYCAT-;retractIfCan;SU;10|)
       (EXIT
        (COND
         ((QEQCAR |q| 0)
          (COND
           ((SPADCALL (SPADCALL (QCDR |q|) (QREFELT $ 56)) |p| (QREFELT $ 24))
             (PROGN
              (LETT #0# |q| |POLYCAT-;retractIfCan;SU;10|)
              (GO #0#))))))))
      (EXIT (CONS 1 "failed"))))
```

```
     #0#
     (EXIT #0#)))))

(DEFUN |POLYCAT-;mkPrim| (|p| $)
 (SPADCALL
  (|spadConstant| $ 34)
  (SPADCALL |p| (QREFELT $ 59))
  (QREFELT $ 60)))

(DEFUN |POLYCAT-;primitiveMonomials;SL;12| (|p| $)
 (PROG (#0=#:G1509 |q| #1=#:G1510)
  (RETURN
   (SEQ
    (PROGN
     (LETT #0# NIL |POLYCAT-;primitiveMonomials;SL;12|)
     (SEQ
      (LETT |q| NIL |POLYCAT-;primitiveMonomials;SL;12|)
      (LETT #1# (SPADCALL |p| (QREFELT $ 28)) |POLYCAT-;primitiveMonomials;SL;12|)
      G190
      (COND
       ((OR (ATOM #1#)
            (PROGN
             (LETT |q| (CAR #1#) |POLYCAT-;primitiveMonomials;SL;12|)
             NIL))
        (GO G191)))
      (SEQ
       (EXIT
        (LETT #0# (CONS (|POLYCAT-;mkPrim| |q| $) #0#)
         |POLYCAT-;primitiveMonomials;SL;12|)))
      (LETT #1# (CDR #1#) |POLYCAT-;primitiveMonomials;SL;12|)
      (GO G190)
      G191
      (EXIT (NREVERSE0 #0#))))))))

(DEFUN |POLYCAT-;totalDegree;SNni;13| (|p| $)
 (PROG (#0=#:G1512 |d| |u|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |p| (QREFELT $ 62)) 0)
     ((QUOTE T)
      (SEQ
       (LETT |u|
        (SPADCALL |p|
         (PROG2
          (LETT #0#
           (SPADCALL |p| (QREFELT $ 42))
           |POLYCAT-;totalDegree;SNni;13|)
          (QCDR #0#)
          (|check-union| (QEQCAR #0# 0) (QREFELT $ 9) #0#))
         (QREFELT $ 47))
        |POLYCAT-;totalDegree;SNni;13|)
       (LETT |d| 0 |POLYCAT-;totalDegree;SNni;13|)
       (SEQ G190
```

```
        (COND
         ((NULL
           (COND
            ((SPADCALL |u| (|spadConstant| $ 63) (QREFELT $ 64)) (QUOTE NIL))
            ((QUOTE T) (QUOTE T)))) (GO G191)))
        (SEQ
         (LETT |d|
          (MAX |d|
           (+
            (SPADCALL |u| (QREFELT $ 65))
            (SPADCALL (SPADCALL |u| (QREFELT $ 66)) (QREFELT $ 67))))
          |POLYCAT-;totalDegree;SNni;13|)
         (EXIT
          (LETT |u|
           (SPADCALL |u| (QREFELT $ 68)) |POLYCAT-;totalDegree;SNni;13|)))
        NIL
        (GO G190)
        G191
        (EXIT NIL))
       (EXIT |d|))))))))

(DEFUN |POLYCAT-;totalDegree;SLNni;14| (|p| |lv| $)
 (PROG (#0=#:G1520 |v| |w| |d| |u|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |p| (QREFELT $ 62)) 0)
     ((QUOTE T)
      (SEQ
       (LETT |u|
        (SPADCALL |p|
         (LETT |v|
          (PROG2
           (LETT #0#
            (SPADCALL |p| (QREFELT $ 42))
            |POLYCAT-;totalDegree;SLNni;14|)
           (QCDR #0#)
           (|check-union| (QEQCAR #0# 0) (QREFELT $ 9) #0#))
          |POLYCAT-;totalDegree;SLNni;14|)
         (QREFELT $ 47))
        |POLYCAT-;totalDegree;SLNni;14|)
       (LETT |d| 0 |POLYCAT-;totalDegree;SLNni;14|)
       (LETT |w| 0 |POLYCAT-;totalDegree;SLNni;14|)
       (COND
        ((SPADCALL |v| |lv| (QREFELT $ 70))
          (LETT |w| 1 |POLYCAT-;totalDegree;SLNni;14|)))
       (SEQ G190
        (COND
         ((NULL
           (COND
            ((SPADCALL |u| (|spadConstant| $ 63) (QREFELT $ 64)) (QUOTE NIL))
            ((QUOTE T) (QUOTE T)))) (GO G191)))
        (SEQ
         (LETT |d|
```

```
          (MAX |d|
           (+
            (* |w| (SPADCALL |u| (QREFELT $ 65)))
            (SPADCALL (SPADCALL |u| (QREFELT $ 66)) |lv| (QREFELT $ 71))))
          |POLYCAT-;totalDegree;SLNni;14|)
         (EXIT
          (LETT |u|
           (SPADCALL |u| (QREFELT $ 68))
           |POLYCAT-;totalDegree;SLNni;14|)))
         NIL
         (GO G190)
         G191
         (EXIT NIL))
       (EXIT |d|))))))))

(DEFUN |POLYCAT-;resultant;2SVarSetS;15| (|p1| |p2| |mvar| $)
 (SPADCALL
  (SPADCALL |p1| |mvar| (QREFELT $ 47))
  (SPADCALL |p2| |mvar| (QREFELT $ 47))
  (QREFELT $ 73)))

(DEFUN |POLYCAT-;discriminant;SVarSetS;16| (|p| |var| $)
 (SPADCALL (SPADCALL |p| |var| (QREFELT $ 47)) (QREFELT $ 75)))

(DEFUN |POLYCAT-;allMonoms| (|l| $)
 (PROG (#0=#:G1532 |p| #1=#:G1533)
  (RETURN
   (SEQ
    (SPADCALL
     (SPADCALL
      (PROGN
       (LETT #0# NIL |POLYCAT-;allMonoms|)
       (SEQ
        (LETT |p| NIL |POLYCAT-;allMonoms|)
        (LETT #1# |l| |POLYCAT-;allMonoms|)
        G190
        (COND
         ((OR (ATOM #1#) (PROGN (LETT |p| (CAR #1#) |POLYCAT-;allMonoms|) NIL))
          (GO G191)))
        (SEQ
         (EXIT
          (LETT #0#
           (CONS (SPADCALL |p| (QREFELT $ 77)) #0#)
           |POLYCAT-;allMonoms|)))
        (LETT #1# (CDR #1#) |POLYCAT-;allMonoms|)
        (GO G190)
        G191
        (EXIT (NREVERSE0 #0#))))
      (QREFELT $ 79))
     (QREFELT $ 80))))))

(DEFUN |POLYCAT-;P2R| (|p| |b| |n| $)
 (PROG (|w| |bj| #0=#:G1538 |i| #1=#:G1537)
  (RETURN
```

```
   (SEQ
    (LETT |w|
     (SPADCALL |n| (|spadConstant| $ 22) (QREFELT $ 82))
     |POLYCAT-;P2R|)
    (SEQ
     (LETT |bj| NIL |POLYCAT-;P2R|)
     (LETT #0# |b| |POLYCAT-;P2R|)
     (LETT |i| (SPADCALL |w| (QREFELT $ 84)) |POLYCAT-;P2R|)
     (LETT #1# (QVSIZE |w|) |POLYCAT-;P2R|)
     G190
     (COND
      ((OR (> |i| #1#)
           (ATOM #0#)
           (PROGN (LETT |bj| (CAR #0#) |POLYCAT-;P2R|) NIL))
       (GO G191)))
     (SEQ
      (EXIT
       (SPADCALL |w| |i| (SPADCALL |p| |bj| (QREFELT $ 85)) (QREFELT $ 86))))
     (LETT |i|
      (PROG1 (+ |i| 1) (LETT #0# (CDR #0#) |POLYCAT-;P2R|)) |POLYCAT-;P2R|)
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |w|)))))

(DEFUN |POLYCAT-;eq2R| (|l| |b| $)
 (PROG (#0=#:G1542 |bj| #1=#:G1543 #2=#:G1544 |p| #3=#:G1545)
  (RETURN
   (SEQ
    (SPADCALL
     (PROGN
      (LETT #0# NIL |POLYCAT-;eq2R|)
      (SEQ
       (LETT |bj| NIL |POLYCAT-;eq2R|)
       (LETT #1# |b| |POLYCAT-;eq2R|)
       G190
       (COND
        ((OR (ATOM #1#)
             (PROGN (LETT |bj| (CAR #1#) |POLYCAT-;eq2R|) NIL)) (GO G191)))
       (SEQ
        (EXIT
         (LETT #0#
          (CONS
           (PROGN
            (LETT #2# NIL |POLYCAT-;eq2R|)
            (SEQ
             (LETT |p| NIL |POLYCAT-;eq2R|)
             (LETT #3# |l| |POLYCAT-;eq2R|)
             G190
             (COND
              ((OR (ATOM #3#) (PROGN (LETT |p| (CAR #3#) |POLYCAT-;eq2R|) NIL))
               (GO G191)))
             (SEQ
              (EXIT
```

```
              (LETT #2#
               (CONS (SPADCALL |p| |bj| (QREFELT $ 85)) #2#)
               |POLYCAT-;eq2R|)))
             (LETT #3# (CDR #3#) |POLYCAT-;eq2R|)
             (GO G190)
             G191
             (EXIT (NREVERSEO #2#))))
           #0#)
          |POLYCAT-;eq2R|)))
       (LETT #1# (CDR #1#) |POLYCAT-;eq2R|)
       (GO G190)
       G191
       (EXIT (NREVERSEO #0#))))
     (QREFELT $ 89))))))

(DEFUN |POLYCAT-;reducedSystem;MM;20| (|m| $)
 (PROG (#0=#:G1555 |r| #1=#:G1556 |b| #2=#:G1557 |bj| #3=#:G1558 |d| |mm| |l|)
  (RETURN
   (SEQ
    (LETT |l| (SPADCALL |m| (QREFELT $ 92)) |POLYCAT-;reducedSystem;MM;20|)
    (LETT |b|
     (SPADCALL
      (SPADCALL
       (PROGN
        (LETT #0# NIL |POLYCAT-;reducedSystem;MM;20|)
        (SEQ
         (LETT |r| NIL |POLYCAT-;reducedSystem;MM;20|)
         (LETT #1# |l| |POLYCAT-;reducedSystem;MM;20|)
         G190
        (COND
         ((OR (ATOM #1#)
              (PROGN (LETT |r| (CAR #1#) |POLYCAT-;reducedSystem;MM;20|) NIL))
          (GO G191)))
        (SEQ
         (EXIT
          (LETT #0#
           (CONS (|POLYCAT-;allMonoms| |r| $) #0#)
           |POLYCAT-;reducedSystem;MM;20|)))
        (LETT #1# (CDR #1#) |POLYCAT-;reducedSystem;MM;20|)
        (GO G190)
        G191
        (EXIT (NREVERSEO #0#))))
       (QREFELT $ 79))
      (QREFELT $ 80))
     |POLYCAT-;reducedSystem;MM;20|)
    (LETT |d|
     (PROGN
      (LETT #2# NIL |POLYCAT-;reducedSystem;MM;20|)
      (SEQ
       (LETT |bj| NIL |POLYCAT-;reducedSystem;MM;20|)
       (LETT #3# |b| |POLYCAT-;reducedSystem;MM;20|)
       G190
       (COND
        ((OR (ATOM #3#)
```

```
            (PROGN (LETT |bj| (CAR #3#) |POLYCAT-;reducedSystem;MM;20|) NIL))
          (GO G191)))
        (SEQ
         (EXIT
          (LETT #2#
           (CONS (SPADCALL |bj| (QREFELT $ 59)) #2#)
           |POLYCAT-;reducedSystem;MM;20|)))
        (LETT #3# (CDR #3#) |POLYCAT-;reducedSystem;MM;20|)
        (GO G190)
        G191
        (EXIT (NREVERSE0 #2#))))
       |POLYCAT-;reducedSystem;MM;20|)
     (LETT |mm|
      (|POLYCAT-;eq2R| (|SPADfirst| |l|) |d| $) |POLYCAT-;reducedSystem;MM;20|)
     (LETT |l| (CDR |l|) |POLYCAT-;reducedSystem;MM;20|)
     (SEQ G190
      (COND
       ((NULL (COND ((NULL |l|) (QUOTE NIL)) ((QUOTE T) (QUOTE T)))) (GO G191)))
      (SEQ
       (LETT |mm|
        (SPADCALL |mm| (|POLYCAT-;eq2R| (|SPADfirst| |l|) |d| $) (QREFELT $ 93))
        |POLYCAT-;reducedSystem;MM;20|)
       (EXIT (LETT |l| (CDR |l|) |POLYCAT-;reducedSystem;MM;20|)))
      NIL
      (GO G190)
      G191
      (EXIT NIL))
     (EXIT |mm|)))))

(DEFUN |POLYCAT-;reducedSystem;MVR;21| (|m| |v| $)
 (PROG (#0=#:G1570 |s| #1=#:G1571 |b| #2=#:G1572 |bj| #3=#:G1573 |d| |n|
        |mm| |w| |l| |r|)
  (RETURN
   (SEQ
    (LETT |l| (SPADCALL |m| (QREFELT $ 92)) |POLYCAT-;reducedSystem;MVR;21|)
    (LETT |r| (SPADCALL |v| (QREFELT $ 97)) |POLYCAT-;reducedSystem;MVR;21|)
    (LETT |b|
     (SPADCALL
      (SPADCALL
       (|POLYCAT-;allMonoms| |r| $)
       (SPADCALL
        (PROGN
         (LETT #0# NIL |POLYCAT-;reducedSystem;MVR;21|)
         (SEQ
          (LETT |s| NIL |POLYCAT-;reducedSystem;MVR;21|)
          (LETT #1# |l| |POLYCAT-;reducedSystem;MVR;21|)
          G190
          (COND
           ((OR (ATOM #1#)
                (PROGN
                 (LETT |s| (CAR #1#) |POLYCAT-;reducedSystem;MVR;21|)
                  NIL))
             (GO G191)))
          (SEQ
```

```
       (EXIT
        (LETT #0#
         (CONS (|POLYCAT-;allMonoms| |s| $) #0#)
         |POLYCAT-;reducedSystem;MVR;21|)))
      (LETT #1# (CDR #1#) |POLYCAT-;reducedSystem;MVR;21|)
      (GO G190)
      G191
      (EXIT (NREVERSE0 #0#))))
    (QREFELT $ 79))
   (QREFELT $ 98))
  (QREFELT $ 80))
 |POLYCAT-;reducedSystem;MVR;21|)
(LETT |d|
 (PROGN
  (LETT #2# NIL |POLYCAT-;reducedSystem;MVR;21|)
  (SEQ
   (LETT |bj| NIL |POLYCAT-;reducedSystem;MVR;21|)
   (LETT #3# |b| |POLYCAT-;reducedSystem;MVR;21|)
   G190
   (COND
    ((OR (ATOM #3#)
         (PROGN (LETT |bj| (CAR #3#) |POLYCAT-;reducedSystem;MVR;21|) NIL))
     (GO G191)))
   (SEQ
    (EXIT
     (LETT #2#
      (CONS (SPADCALL |bj| (QREFELT $ 59)) #2#)
      |POLYCAT-;reducedSystem;MVR;21|)))
   (LETT #3# (CDR #3#) |POLYCAT-;reducedSystem;MVR;21|)
   (GO G190)
   G191
   (EXIT (NREVERSE0 #2#))))
  |POLYCAT-;reducedSystem;MVR;21|)
(LETT |n| (LENGTH |d|) |POLYCAT-;reducedSystem;MVR;21|)
(LETT |mm|
 (|POLYCAT-;eq2R| (|SPADfirst| |l|) |d| $)
 |POLYCAT-;reducedSystem;MVR;21|)
(LETT |w|
 (|POLYCAT-;P2R| (|SPADfirst| |r|) |d| |n| $)
 |POLYCAT-;reducedSystem;MVR;21|)
(LETT |l| (CDR |l|) |POLYCAT-;reducedSystem;MVR;21|)
(LETT |r| (CDR |r|) |POLYCAT-;reducedSystem;MVR;21|)
(SEQ G190
 (COND
  ((NULL (COND ((NULL |l|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
   (GO G191)))
 (SEQ
  (LETT |mm|
   (SPADCALL |mm| (|POLYCAT-;eq2R| (|SPADfirst| |l|) |d| $) (QREFELT $ 93))
   |POLYCAT-;reducedSystem;MVR;21|)
  (LETT |w|
   (SPADCALL |w|
    (|POLYCAT-;P2R| (|SPADfirst| |r|) |d| |n| $)
    (QREFELT $ 99))
```

```
      |POLYCAT-;reducedSystem;MVR;21|)
     (LETT |l| (CDR |l|) |POLYCAT-;reducedSystem;MVR;21|)
     (EXIT (LETT |r| (CDR |r|) |POLYCAT-;reducedSystem;MVR;21|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
   (EXIT (CONS |mm| |w|))))))

(DEFUN |POLYCAT-;gcdPolynomial;3Sup;22| (|pp| |qq| $)
 (SPADCALL |pp| |qq| (QREFELT $ 104)))

(DEFUN |POLYCAT-;solveLinearPolynomialEquation;LSupU;23| (|lpp| |pp| $)
 (SPADCALL |lpp| |pp| (QREFELT $ 109)))

(DEFUN |POLYCAT-;factorPolynomial;SupF;24| (|pp| $)
 (SPADCALL |pp| (QREFELT $ 114)))

(DEFUN |POLYCAT-;factorSquareFreePolynomial;SupF;25| (|pp| $)
 (SPADCALL |pp| (QREFELT $ 117)))

(DEFUN |POLYCAT-;factor;SF;26| (|p| $)
 (PROG (|v| |ansR| #0=#:G1615 |w| #1=#:G1616 |up| |ansSUP| #2=#:G1617
       |ww| #3=#:G1618)
  (RETURN
   (SEQ
    (LETT |v| (SPADCALL |p| (QREFELT $ 42)) |POLYCAT-;factor;SF;26|)
    (EXIT
     (COND
      ((QEQCAR |v| 1)
       (SEQ
        (LETT |ansR|
         (SPADCALL (SPADCALL |p| (QREFELT $ 38)) (QREFELT $ 120))
         |POLYCAT-;factor;SF;26|)
        (EXIT
         (SPADCALL
          (SPADCALL (SPADCALL |ansR| (QREFELT $ 122)) (QREFELT $ 40))
          (PROGN
           (LETT #0# NIL |POLYCAT-;factor;SF;26|)
           (SEQ
            (LETT |w| NIL |POLYCAT-;factor;SF;26|)
            (LETT #1#
             (SPADCALL |ansR| (QREFELT $ 126))
             |POLYCAT-;factor;SF;26|)
            G190
            (COND
             ((OR (ATOM #1#)
                  (PROGN (LETT |w| (CAR #1#) |POLYCAT-;factor;SF;26|) NIL))
              (GO G191)))
            (SEQ
             (EXIT
              (LETT #0#
               (CONS
                (VECTOR (QVELT |w| 0)
```

```
                (SPADCALL (QVELT |w| 1) (QREFELT $ 40)) (QVELT |w| 2))
              #0#)
             |POLYCAT-;factor;SF;26|)))
          (LETT #1# (CDR #1#) |POLYCAT-;factor;SF;26|)
          (GO G190)
          G191
          (EXIT (NREVERSE0 #0#))))
        (QREFELT $ 130)))))
    ((QUOTE T)
     (SEQ
      (LETT |up|
       (SPADCALL |p| (QCDR |v|) (QREFELT $ 47)) |POLYCAT-;factor;SF;26|)
      (LETT |ansSUP| (SPADCALL |up| (QREFELT $ 114)) |POLYCAT-;factor;SF;26|)
      (EXIT
       (SPADCALL
        (SPADCALL
         (SPADCALL |ansSUP| (QREFELT $ 131)) (QCDR |v|) (QREFELT $ 132))
        (PROGN
         (LETT #2# NIL |POLYCAT-;factor;SF;26|)
         (SEQ
          (LETT |ww| NIL |POLYCAT-;factor;SF;26|)
          (LETT #3#
           (SPADCALL |ansSUP| (QREFELT $ 135))
           |POLYCAT-;factor;SF;26|)
          G190
          (COND
           ((OR (ATOM #3#)
                (PROGN (LETT |ww| (CAR #3#) |POLYCAT-;factor;SF;26|) NIL))
            (GO G191)))
          (SEQ
           (EXIT
            (LETT #2#
             (CONS
              (VECTOR (QVELT |ww| 0) (SPADCALL (QVELT |ww| 1) (QCDR |v|)
                      (QREFELT $ 132)) (QVELT |ww| 2))
              #2#)
             |POLYCAT-;factor;SF;26|)))
          (LETT #3# (CDR #3#) |POLYCAT-;factor;SF;26|)
          (GO G190)
          G191
          (EXIT (NREVERSE0 #2#))))
        (QREFELT $ 130))))))))))

(DEFUN |POLYCAT-;conditionP;MU;27| (|mat| $)
 (PROG (|ll| #0=#:G1653 |z| #1=#:G1654 |ch| |l| #2=#:G1655 #3=#:G1656
       #4=#:G1625 #5=#:G1623 #6=#:G1624 #7=#:G1657 |vars| |degs|
       #8=#:G1658 |d| #9=#:G1659 |nd| #10=#:G1652 #11=#:G1632 |deg1|
       |redmons| #12=#:G1660 |v| #13=#:G1662 |u| #14=#:G1661 |llR|
       |monslist| |ans| #15=#:G1663 #16=#:G1664 |mons| #17=#:G1665 |m|
       #18=#:G1666 |i| #19=#:G1648 #20=#:G1646 #21=#:G1647)
  (RETURN
   (SEQ
    (EXIT
     (SEQ
```

```
(LETT |ll|
 (SPADCALL (SPADCALL |mat| (QREFELT $ 137)) (QREFELT $ 92))
 |POLYCAT-;conditionP;MU;27|)
(LETT |llR|
 (PROGN
  (LETT #0# NIL |POLYCAT-;conditionP;MU;27|)
  (SEQ
   (LETT |z| NIL |POLYCAT-;conditionP;MU;27|)
   (LETT #1# (|SPADfirst| |ll|) |POLYCAT-;conditionP;MU;27|)
   G190
   (COND
    ((OR (ATOM #1#)
         (PROGN (LETT |z| (CAR #1#) |POLYCAT-;conditionP;MU;27|) NIL))
     (GO G191)))
   (SEQ (EXIT (LETT #0# (CONS NIL #0#) |POLYCAT-;conditionP;MU;27|)))
   (LETT #1# (CDR #1#) |POLYCAT-;conditionP;MU;27|)
   (GO G190)
   G191
   (EXIT (NREVERSE0 #0#))))
 |POLYCAT-;conditionP;MU;27|)
(LETT |monslist| NIL |POLYCAT-;conditionP;MU;27|)
(LETT |ch| (SPADCALL (QREFELT $ 138)) |POLYCAT-;conditionP;MU;27|)
(SEQ
 (LETT |l| NIL |POLYCAT-;conditionP;MU;27|)
 (LETT #2# |ll| |POLYCAT-;conditionP;MU;27|)
 G190
 (COND
  ((OR (ATOM #2#)
       (PROGN (LETT |l| (CAR #2#) |POLYCAT-;conditionP;MU;27|) NIL))
   (GO G191)))
 (SEQ
  (LETT |mons|
   (PROGN
    (LETT #6# NIL |POLYCAT-;conditionP;MU;27|)
    (SEQ
     (LETT |u| NIL |POLYCAT-;conditionP;MU;27|)
     (LETT #3# |l| |POLYCAT-;conditionP;MU;27|)
     G190
     (COND
      ((OR (ATOM #3#)
           (PROGN (LETT |u| (CAR #3#) |POLYCAT-;conditionP;MU;27|) NIL))
       (GO G191)))
     (SEQ
      (EXIT
       (PROGN
        (LETT #4#
         (SPADCALL |u| (QREFELT $ 77))
         |POLYCAT-;conditionP;MU;27|)
        (COND
         (#6#
          (LETT #5#
           (SPADCALL #5# #4# (QREFELT $ 139))
           |POLYCAT-;conditionP;MU;27|))
         ((QUOTE T)
```

```
          (PROGN
           (LETT #5# #4# |POLYCAT-;conditionP;MU;27|)
           (LETT #6# (QUOTE T) |POLYCAT-;conditionP;MU;27|)))))))
     (LETT #3# (CDR #3#) |POLYCAT-;conditionP;MU;27|)
     (GO G190)
     G191
     (EXIT NIL))
    (COND (#6# #5#) ((QUOTE T) (|IdentityError| (QUOTE |setUnion|)))))
   |POLYCAT-;conditionP;MU;27|)
  (LETT |redmons| NIL |POLYCAT-;conditionP;MU;27|)
  (SEQ
   (LETT |m| NIL |POLYCAT-;conditionP;MU;27|)
   (LETT #7# |mons| |POLYCAT-;conditionP;MU;27|)
   G190
   (COND
    ((OR (ATOM #7#)
         (PROGN (LETT |m| (CAR #7#) |POLYCAT-;conditionP;MU;27|) NIL))
      (GO G191)))
   (SEQ
    (LETT |vars|
     (SPADCALL |m| (QREFELT $ 31))
     |POLYCAT-;conditionP;MU;27|)
    (LETT |degs|
     (SPADCALL |m| |vars| (QREFELT $ 140))
     |POLYCAT-;conditionP;MU;27|)
    (LETT |deg1|
     (PROGN
      (LETT #8# NIL |POLYCAT-;conditionP;MU;27|)
      (SEQ
       (LETT |d| NIL |POLYCAT-;conditionP;MU;27|)
       (LETT #9# |degs| |POLYCAT-;conditionP;MU;27|)
       G190
       (COND
        ((OR (ATOM #9#)
             (PROGN
              (LETT |d| (CAR #9#) |POLYCAT-;conditionP;MU;27|)
              NIL))
          (GO G191)))
       (SEQ
        (EXIT
         (LETT #8#
          (CONS
           (SEQ
            (LETT |nd|
             (SPADCALL |d| |ch| (QREFELT $ 142))
             |POLYCAT-;conditionP;MU;27|)
            (EXIT
             (COND
              ((QEQCAR |nd| 1)
                (PROGN
                  (LETT #10#
                   (CONS 1 "failed") |POLYCAT-;conditionP;MU;27|)
                  (GO #10#)))
              ((QUOTE T)
```

```
                (PROG1
                 (LETT #11# (QCDR |nd|) |POLYCAT-;conditionP;MU;27|)
                 (|check-subtype|
                  (>= #11# 0) (QUOTE (|NonNegativeInteger|)) #11#))))))
          #8#)
         |POLYCAT-;conditionP;MU;27|)))
   (LETT #9# (CDR #9#) |POLYCAT-;conditionP;MU;27|)
   (GO G190)
   G191
   (EXIT (NREVERSE0 #8#))))
  |POLYCAT-;conditionP;MU;27|)
 (LETT |redmons|
  (CONS
   (SPADCALL (|spadConstant| $ 33) |vars| |deg1| (QREFELT $ 54))
   |redmons|)
  |POLYCAT-;conditionP;MU;27|)
 (EXIT
  (LETT |llR|
   (PROGN
    (LETT #12# NIL |POLYCAT-;conditionP;MU;27|)
    (SEQ
     (LETT |v| NIL |POLYCAT-;conditionP;MU;27|)
     (LETT #13# |llR| |POLYCAT-;conditionP;MU;27|)
     (LETT |u| NIL |POLYCAT-;conditionP;MU;27|)
     (LETT #14# |l| |POLYCAT-;conditionP;MU;27|)
     G190
     (COND
      ((OR (ATOM #14#)
        (PROGN
         (LETT |u| (CAR #14#) |POLYCAT-;conditionP;MU;27|)
          NIL)
        (ATOM #13#)
        (PROGN (LETT |v| (CAR #13#) |POLYCAT-;conditionP;MU;27|) NIL))
       (GO G191)))
     (SEQ
      (EXIT
       (LETT #12#
        (CONS
         (CONS
          (SPADCALL
           (SPADCALL |u| |vars| |degs| (QREFELT $ 52))
           (QREFELT $ 143))
          |v|)
         #12#)
        |POLYCAT-;conditionP;MU;27|)))
     (LETT #14#
      (PROG1
       (CDR #14#)
       (LETT #13# (CDR #13#) |POLYCAT-;conditionP;MU;27|))
      |POLYCAT-;conditionP;MU;27|)
     (GO G190)
     G191
     (EXIT (NREVERSE0 #12#))))
   |POLYCAT-;conditionP;MU;27|)))
```

```
    (LETT #7# (CDR #7#) |POLYCAT-;conditionP;MU;27|)
    (GO G190)
    G191
    (EXIT NIL))
   (EXIT
    (LETT |monslist|
     (CONS |redmons| |monslist|)
     |POLYCAT-;conditionP;MU;27|)))
  (LETT #2# (CDR #2#) |POLYCAT-;conditionP;MU;27|)
  (GO G190)
  G191
  (EXIT NIL))
 (LETT |ans|
  (SPADCALL (SPADCALL (SPADCALL |llR| (QREFELT $ 89)) (QREFELT $ 144))
            (QREFELT $ 146))
  |POLYCAT-;conditionP;MU;27|)
 (EXIT
  (COND
   ((QEQCAR |ans| 1) (CONS 1 "failed"))
   ((QUOTE T)
    (SEQ
     (LETT |i| 0 |POLYCAT-;conditionP;MU;27|)
     (EXIT
      (CONS 0
       (PRIMVEC2ARR
        (PROGN
         (LETT #15# (make-array (SIZE |monslist|))
          |POLYCAT-;conditionP;MU;27|)
         (SEQ
          (LETT #16# 0 |POLYCAT-;conditionP;MU;27|)
          (LETT |mons| NIL |POLYCAT-;conditionP;MU;27|)
          (LETT #17# |monslist| |POLYCAT-;conditionP;MU;27|)
          G190
          (COND
           ((OR (ATOM #17#)
                (PROGN
                 (LETT |mons| (CAR #17#) |POLYCAT-;conditionP;MU;27|)
                 NIL))
             (GO G191)))
          (SEQ
           (EXIT
            (SETELT #15# #16#
             (PROGN
              (LETT #21# NIL |POLYCAT-;conditionP;MU;27|)
              (SEQ
               (LETT |m| NIL |POLYCAT-;conditionP;MU;27|)
               (LETT #18# |mons| |POLYCAT-;conditionP;MU;27|)
               G190
               (COND
                ((OR (ATOM #18#)
                   (PROGN
                    (LETT |m| (CAR #18#) |POLYCAT-;conditionP;MU;27|)
                     NIL))
                  (GO G191)))
```

```
                    (SEQ
                     (EXIT
                      (PROGN
                       (LETT #19#
                        (SPADCALL |m|
                         (SPADCALL
                          (SPADCALL
                           (QCDR |ans|)
                           (LETT |i| (+ |i| 1) |POLYCAT-;conditionP;MU;27|)
                           (QREFELT $ 147))
                          (QREFELT $ 40))
                         (QREFELT $ 148))
                        |POLYCAT-;conditionP;MU;27|)
                       (COND
                        (#21#
                         (LETT #20#
                          (SPADCALL #20# #19# (QREFELT $ 149))
                          |POLYCAT-;conditionP;MU;27|))
                        ((QUOTE T)
                          (PROGN
                           (LETT #20# #19# |POLYCAT-;conditionP;MU;27|)
                           (LETT #21#
                            (QUOTE T)
                            |POLYCAT-;conditionP;MU;27|)))))))
                    (LETT #18# (CDR #18#) |POLYCAT-;conditionP;MU;27|)
                    (GO G190)
                    G191
                    (EXIT NIL))
                   (COND (#21# #20#) ((QUOTE T) (|spadConstant| $ 21)))))))
               (LETT #17#
                (PROG1
                 (CDR #17#)
                 (LETT #16# (QSADD1 #16#) |POLYCAT-;conditionP;MU;27|))
                |POLYCAT-;conditionP;MU;27|)
               (GO G190)
               G191
               (EXIT NIL))
              #15#))))))))))
   #10#
   (EXIT #10#)))))

(DEFUN |POLYCAT-;charthRoot;SU;28| (|p| $)
 (PROG (|vars| |ans| |ch|)
  (RETURN
   (SEQ
    (LETT |vars| (SPADCALL |p| (QREFELT $ 31)) |POLYCAT-;charthRoot;SU;28|)
    (EXIT
     (COND
      ((NULL |vars|)
        (SEQ
         (LETT |ans|
          (SPADCALL (SPADCALL |p| (QREFELT $ 143)) (QREFELT $ 151))
          |POLYCAT-;charthRoot;SU;28|)
         (EXIT
```

```
          (COND
           ((QEQCAR |ans| 1) (CONS 1 "failed"))
           ((QUOTE T) (CONS 0 (SPADCALL (QCDR |ans|) (QREFELT $ 40))))))))
      ((QUOTE T)
       (SEQ
        (LETT |ch| (SPADCALL (QREFELT $ 138)) |POLYCAT-;charthRoot;SU;28|)
        (EXIT (|POLYCAT-;charthRootlv| |p| |vars| |ch| $))))))))))

(DEFUN |POLYCAT-;charthRootlv| (|p| |vars| |ch| $)
 (PROG (|v| |dd| |cp| |d| #0=#:G1687 |ans| |ansx| #1=#:G1694)
  (RETURN
   (SEQ
    (EXIT
     (COND
      ((NULL |vars|)
       (SEQ
        (LETT |ans|
         (SPADCALL (SPADCALL |p| (QREFELT $ 143)) (QREFELT $ 151))
         |POLYCAT-;charthRootlv|)
        (EXIT
         (COND
          ((QEQCAR |ans| 1) (CONS 1 "failed"))
          ((QUOTE T) (CONS 0 (SPADCALL (QCDR |ans|) (QREFELT $ 40))))))))
      ((QUOTE T)
       (SEQ
        (LETT |v| (|SPADfirst| |vars|) |POLYCAT-;charthRootlv|)
        (LETT |vars| (CDR |vars|) |POLYCAT-;charthRootlv|)
        (LETT |d| (SPADCALL |p| |v| (QREFELT $ 36)) |POLYCAT-;charthRootlv|)
        (LETT |ans| (|spadConstant| $ 21) |POLYCAT-;charthRootlv|)
        (SEQ G190
         (COND ((NULL (< 0 |d|)) (GO G191)))
         (SEQ
          (LETT |dd|
           (SPADCALL |d| |ch| (QREFELT $ 142))
           |POLYCAT-;charthRootlv|)
          (EXIT
           (COND
            ((QEQCAR |dd| 1)
              (PROGN
               (LETT #1# (CONS 1 "failed") |POLYCAT-;charthRootlv|)
               (GO #1#)))
            ((QUOTE T)
             (SEQ
              (LETT |cp|
               (SPADCALL |p| |v| |d| (QREFELT $ 154))
               |POLYCAT-;charthRootlv|)
              (LETT |p|
               (SPADCALL |p|
                (SPADCALL |cp| |v| |d| (QREFELT $ 37))
                (QREFELT $ 155))
               |POLYCAT-;charthRootlv|)
              (LETT |ansx|
               (|POLYCAT-;charthRootlv| |cp| |vars| |ch| $)
               |POLYCAT-;charthRootlv|)
```

```
              (EXIT
               (COND
                ((QEQCAR |ansx| 1)
                  (PROGN
                   (LETT #1# (CONS 1 "failed") |POLYCAT-;charthRootlv|)
                   (GO #1#)))
                ((QUOTE T)
                 (SEQ
                  (LETT |d|
                   (SPADCALL |p| |v| (QREFELT $ 36))
                   |POLYCAT-;charthRootlv|)
                  (EXIT
                   (LETT |ans|
                    (SPADCALL |ans|
                     (SPADCALL (QCDR |ansx|) |v|
                      (PROG1
                       (LETT #0# (QCDR |dd|) |POLYCAT-;charthRootlv|)
                       (|check-subtype| (>= #0# 0)
                        (QUOTE (|NonNegativeInteger|)) #0#))
                      (QREFELT $ 37))
                     (QREFELT $ 149))
                    |POLYCAT-;charthRootlv|)))))))))
         NIL
         (GO G190)
         G191
         (EXIT NIL))
        (LETT |ansx|
         (|POLYCAT-;charthRootlv| |p| |vars| |ch| $)
         |POLYCAT-;charthRootlv|)
        (EXIT
         (COND
          ((QEQCAR |ansx| 1)
           (PROGN
            (LETT #1# (CONS 1 "failed") |POLYCAT-;charthRootlv|)
            (GO #1#)))
          ((QUOTE T)
           (PROGN
            (LETT #1#
             (CONS 0 (SPADCALL |ans| (QCDR |ansx|) (QREFELT $ 149)))
             |POLYCAT-;charthRootlv|)
            (GO #1#))))))))
    #1#
    (EXIT #1#)))))

(DEFUN |POLYCAT-;monicDivide;2SVarSetR;30| (|p1| |p2| |mvar| $)
 (PROG (|result|)
  (RETURN
   (SEQ
    (LETT |result|
     (SPADCALL
      (SPADCALL |p1| |mvar| (QREFELT $ 47))
      (SPADCALL |p2| |mvar| (QREFELT $ 47))
      (QREFELT $ 157))
     |POLYCAT-;monicDivide;2SVarSetR;30|)
```

```
    (EXIT
     (CONS
      (SPADCALL (QCAR |result|) |mvar| (QREFELT $ 132))
      (SPADCALL (QCDR |result|) |mvar| (QREFELT $ 132))))))))

(DEFUN |POLYCAT-;squareFree;SF;31| (|p| $)
 (SPADCALL |p| (QREFELT $ 160)))

(DEFUN |POLYCAT-;squareFree;SF;32| (|p| $)
 (SPADCALL |p| (QREFELT $ 163)))

(DEFUN |POLYCAT-;squareFree;SF;33| (|p| $)
 (SPADCALL |p| (QREFELT $ 163)))

(DEFUN |POLYCAT-;squareFreePart;2S;34| (|p| $)
 (PROG (|s| |f| #0=#:G1710 #1=#:G1708 #2=#:G1706 #3=#:G1707)
  (RETURN
   (SEQ
    (SPADCALL
     (SPADCALL
      (LETT |s| (SPADCALL |p| (QREFELT $ 164)) |POLYCAT-;squareFreePart;2S;34|)
      (QREFELT $ 165))
     (PROGN
      (LETT #3# NIL |POLYCAT-;squareFreePart;2S;34|)
      (SEQ
       (LETT |f| NIL |POLYCAT-;squareFreePart;2S;34|)
       (LETT #0# (SPADCALL |s| (QREFELT $ 168)) |POLYCAT-;squareFreePart;2S;34|)
       G190
       (COND
        ((OR (ATOM #0#)
             (PROGN (LETT |f| (CAR #0#) |POLYCAT-;squareFreePart;2S;34|) NIL))
         (GO G191)))
       (SEQ
        (EXIT
         (PROGN
          (LETT #1# (QCAR |f|) |POLYCAT-;squareFreePart;2S;34|)
          (COND
           (#3#
            (LETT #2#
             (SPADCALL #2# #1# (QREFELT $ 148))
             |POLYCAT-;squareFreePart;2S;34|))
           ((QUOTE T)
            (PROGN
             (LETT #2# #1# |POLYCAT-;squareFreePart;2S;34|)
             (LETT #3# (QUOTE T) |POLYCAT-;squareFreePart;2S;34|)))))))
       (LETT #0# (CDR #0#) |POLYCAT-;squareFreePart;2S;34|)
       (GO G190)
       G191
       (EXIT NIL))
      (COND (#3# #2#) ((QUOTE T) (|spadConstant| $ 33))))
     (QREFELT $ 148))))))

(DEFUN |POLYCAT-;content;SVarSetS;35| (|p| |v| $)
 (SPADCALL (SPADCALL |p| |v| (QREFELT $ 47)) (QREFELT $ 170)))
```

```
(DEFUN |POLYCAT-;primitivePart;2S;36| (|p| $)
 (PROG (#0=#:G1713)
  (RETURN
   (COND
    ((SPADCALL |p| (QREFELT $ 172)) |p|)
    ((QUOTE T)
     (QVELT
      (SPADCALL
       (PROG2
        (LETT #0#
         (SPADCALL |p| (SPADCALL |p| (QREFELT $ 173)) (QREFELT $ 174))
         |POLYCAT-;primitivePart;2S;36|)
        (QCDR #0#)
        (|check-union| (QEQCAR #0# 0) (QREFELT $ 6) #0#))
       (QREFELT $ 176))
      1))))))

(DEFUN |POLYCAT-;primitivePart;SVarSetS;37| (|p| |v| $)
 (PROG (#0=#:G1720)
  (RETURN
   (COND
    ((SPADCALL |p| (QREFELT $ 172)) |p|)
    ((QUOTE T)
     (QVELT
      (SPADCALL
       (PROG2
        (LETT #0#
         (SPADCALL |p| (SPADCALL |p| |v| (QREFELT $ 178)) (QREFELT $ 179))
         |POLYCAT-;primitivePart;SVarSetS;37|)
        (QCDR #0#)
        (|check-union| (QEQCAR #0# 0) (QREFELT $ 6) #0#))
       (QREFELT $ 176))
      1))))))

(DEFUN |POLYCAT-;<;2SB;38| (|p| |q| $)
 (PROG (|dp| |dq|)
  (RETURN
   (SEQ
    (LETT |dp| (SPADCALL |p| (QREFELT $ 59)) |POLYCAT-;<;2SB;38|)
    (LETT |dq| (SPADCALL |q| (QREFELT $ 59)) |POLYCAT-;<;2SB;38|)
    (EXIT
     (COND
      ((SPADCALL |dp| |dq| (QREFELT $ 181))
        (SPADCALL
         (|spadConstant| $ 22)
         (SPADCALL |q| (QREFELT $ 38))
         (QREFELT $ 182)))
      ((SPADCALL |dq| |dp| (QREFELT $ 181))
        (SPADCALL
         (SPADCALL |p| (QREFELT $ 38))
         (|spadConstant| $ 22)
         (QREFELT $ 182)))
      ((QUOTE T)
```

```
      (SPADCALL
       (SPADCALL (SPADCALL |p| |q| (QREFELT $ 155)) (QREFELT $ 38))
       (|spadConstant| $ 22)
       (QREFELT $ 182))))))))

(DEFUN |POLYCAT-;patternMatch;SP2Pmr;39| (|p| |pat| |l| $)
 (SPADCALL |p| |pat| |l| (QREFELT $ 187)))

(DEFUN |POLYCAT-;patternMatch;SP2Pmr;40| (|p| |pat| |l| $)
 (SPADCALL |p| |pat| |l| (QREFELT $ 193)))

(DEFUN |POLYCAT-;convert;SP;41| (|x| $)
 (SPADCALL (ELT $ 196) (ELT $ 197) |x| (QREFELT $ 201)))

(DEFUN |POLYCAT-;convert;SP;42| (|x| $)
 (SPADCALL (ELT $ 203) (ELT $ 204) |x| (QREFELT $ 208)))

(DEFUN |POLYCAT-;convert;SIf;43| (|p| $)
 (SPADCALL (ELT $ 211) (ELT $ 212) |p| (QREFELT $ 216)))

(DEFUN |PolynomialCategory&| (|#1| |#2| |#3| |#4|)
 (PROG (DV$1 DV$2 DV$3 DV$4 |dv$| $ |pv$|)
  (RETURN
   (PROGN
    (LETT DV$1 (|devaluate| |#1|) . #0=(|PolynomialCategory&|))
    (LETT DV$2 (|devaluate| |#2|) . #0#)
    (LETT DV$3 (|devaluate| |#3|) . #0#)
    (LETT DV$4 (|devaluate| |#4|) . #0#)
    (LETT |dv$| (LIST (QUOTE |PolynomialCategory&|) DV$1 DV$2 DV$3 DV$4) . #0#)
    (LETT $ (make-array 226) . #0#)
    (QSETREFV $ 0 |dv$|)
    (QSETREFV $ 3
     (LETT |pv$|
      (|buildPredVector| 0 0
       (LIST
        (|HasCategory| |#2| (QUOTE (|PolynomialFactorizationExplicit|)))
        (|HasAttribute| |#2| (QUOTE |canonicalUnitNormal|))
        (|HasCategory| |#2| (QUOTE (|GcdDomain|)))
        (|HasCategory| |#2| (QUOTE (|CommutativeRing|)))
        (|HasCategory| |#4| (QUOTE (|PatternMatchable| (|Float|))))
        (|HasCategory| |#2| (QUOTE (|PatternMatchable| (|Float|))))
        (|HasCategory| |#4| (QUOTE (|PatternMatchable| (|Integer|))))
        (|HasCategory| |#2| (QUOTE (|PatternMatchable| (|Integer|))))
        (|HasCategory| |#4| (QUOTE (|ConvertibleTo| (|Pattern| (|Float|)))))
        (|HasCategory| |#2| (QUOTE (|ConvertibleTo| (|Pattern| (|Float|)))))
        (|HasCategory| |#4| (QUOTE (|ConvertibleTo| (|Pattern| (|Integer|)))))
        (|HasCategory| |#2| (QUOTE (|ConvertibleTo| (|Pattern| (|Integer|)))))
        (|HasCategory| |#4| (QUOTE (|ConvertibleTo| (|InputForm|))))
        (|HasCategory| |#2| (QUOTE (|ConvertibleTo| (|InputForm|))))
        (|HasCategory| |#2| (QUOTE (|OrderedSet|)))))
     . #0#))
    (|stuffDomainSlots| $)
    (QSETREFV $ 6 |#1|)
    (QSETREFV $ 7 |#2|)
```

```
(QSETREFV $ 8 |#3|)
(QSETREFV $ 9 |#4|)
(COND
 ((|testBitVector| |pv$| 4)
  (PROGN
   (QSETREFV $ 74
    (CONS (|dispatchFunction| |POLYCAT-;resultant;2SVarSetS;15|) $))
   (QSETREFV $ 76
    (CONS (|dispatchFunction| |POLYCAT-;discriminant;SVarSetS;16|) $)))))
(COND
 ((|HasCategory| |#2| (QUOTE (|IntegralDomain|)))
  (PROGN
   (QSETREFV $ 95
    (CONS (|dispatchFunction| |POLYCAT-;reducedSystem;MM;20|) $))
   (QSETREFV $ 102
    (CONS (|dispatchFunction| |POLYCAT-;reducedSystem;MVR;21|) $)))))
(COND
 ((|testBitVector| |pv$| 1)
  (PROGN
   (QSETREFV $ 105
    (CONS (|dispatchFunction| |POLYCAT-;gcdPolynomial;3Sup;22|) $))
   (QSETREFV $ 112
    (CONS
     (|dispatchFunction|
       |POLYCAT-;solveLinearPolynomialEquation;LSupU;23|)
     $))
   (QSETREFV $ 116
    (CONS (|dispatchFunction| |POLYCAT-;factorPolynomial;SupF;24|) $))
   (QSETREFV $ 118
    (CONS
     (|dispatchFunction| |POLYCAT-;factorSquareFreePolynomial;SupF;25|)
     $))
   (QSETREFV $ 136 (CONS (|dispatchFunction| |POLYCAT-;factor;SF;26|) $))
   (COND
    ((|HasCategory| |#2| (QUOTE (|CharacteristicNonZero|)))
     (PROGN
      (QSETREFV $ 150
       (CONS (|dispatchFunction| |POLYCAT-;conditionP;MU;27|) $))))))))
(COND
 ((|HasCategory| |#2| (QUOTE (|CharacteristicNonZero|)))
  (PROGN
   (QSETREFV $ 152
    (CONS (|dispatchFunction| |POLYCAT-;charthRoot;SU;28|) $)))))
(COND
 ((|testBitVector| |pv$| 3)
  (PROGN
   (COND
    ((|HasCategory| |#2| (QUOTE (|EuclideanDomain|)))
     (COND
      ((|HasCategory| |#2| (QUOTE (|CharacteristicZero|)))
       (QSETREFV $ 161
        (CONS (|dispatchFunction| |POLYCAT-;squareFree;SF;31|) $)))
      ((QUOTE T)
       (QSETREFV $ 161
```

```
            (CONS (|dispatchFunction| |POLYCAT-;squareFree;SF;32|) $)))))
        ((QUOTE T)
         (QSETREFV $ 161
          (CONS (|dispatchFunction| |POLYCAT-;squareFree;SF;33|) $))))
      (QSETREFV $ 169
       (CONS (|dispatchFunction| |POLYCAT-;squareFreePart;2S;34|) $))
      (QSETREFV $ 171
       (CONS (|dispatchFunction| |POLYCAT-;content;SVarSetS;35|) $))
      (QSETREFV $ 177
       (CONS (|dispatchFunction| |POLYCAT-;primitivePart;2S;36|) $))
      (QSETREFV $ 180
       (CONS (|dispatchFunction| |POLYCAT-;primitivePart;SVarSetS;37|) $)))))
    (COND
     ((|testBitVector| |pv$| 15)
      (PROGN
       (QSETREFV $ 183 (CONS (|dispatchFunction| |POLYCAT-;<;2SB;38|) $))
       (COND
        ((|testBitVector| |pv$| 8)
         (COND
          ((|testBitVector| |pv$| 7)
           (QSETREFV $ 189
            (CONS
             (|dispatchFunction| |POLYCAT-;patternMatch;SP2Pmr;39|)
             $))))))
       (COND
        ((|testBitVector| |pv$| 6)
         (COND
         ((|testBitVector| |pv$| 5)
          (QSETREFV $ 195
           (CONS
            (|dispatchFunction| |POLYCAT-;patternMatch;SP2Pmr;40|)
            $)))))))))
    (COND
     ((|testBitVector| |pv$| 12)
      (COND
       ((|testBitVector| |pv$| 11)
        (QSETREFV $ 202
         (CONS (|dispatchFunction| |POLYCAT-;convert;SP;41|) $))))))
    (COND
     ((|testBitVector| |pv$| 10)
      (COND
       ((|testBitVector| |pv$| 9)
        (QSETREFV $ 209
         (CONS (|dispatchFunction| |POLYCAT-;convert;SP;42|) $))))))
    (COND
     ((|testBitVector| |pv$| 14)
      (COND
       ((|testBitVector| |pv$| 13)
         (QSETREFV $ 217
          (CONS (|dispatchFunction| |POLYCAT-;convert;SIf;43|) $))))))
    $))))

(setf (get
 (QUOTE |PolynomialCategory&|)
```

```
(QUOTE |infovec|))
(LIST (QUOTE
 #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|) (|local| |#3|)
   (|local| |#4|) (|Equation| 6) (0 . |lhs|) (|Union| 9 (QUOTE "failed"))
   (5 . |retractIfCan|) (10 . |retract|) (15 . |rhs|) (|List| 9) (|List| $)
   (20 . |eval|) (|List| 221) |POLYCAT-;eval;SLS;1| (27 . |Zero|)
   (31 . |Zero|) (|Boolean|) (35 . =) (41 . |leadingMonomial|)
   (46 . |reductum|) |POLYCAT-;monomials;SL;2| (51 . |monomials|)
   (|Union| 17 (QUOTE "failed")) |POLYCAT-;isPlus;SU;3| (56 . |variables|)
   (61 . |monomial?|) (66 . |One|) (70 . |One|) (|NonNegativeInteger|)
   (74 . |degree|) (80 . |monomial|) (87 . |leadingCoefficient|) (92 . =)
   (98 . |coerce|) |POLYCAT-;isTimes;SU;4| (103 . |mainVariable|)
   (|Record| (|:| |var| 9) (|:| |exponent| 35))
   (|Union| 43 (QUOTE "failed")) |POLYCAT-;isExpt;SU;5|
   (|SparseUnivariatePolynomial| $) (108 . |univariate|)
   (|SparseUnivariatePolynomial| 6) (114 . |coefficient|)
   |POLYCAT-;coefficient;SVarSetNniS;6| (|List| 35) (120 . |coefficient|)
   |POLYCAT-;coefficient;SLLS;7| (127 . |monomial|)
   |POLYCAT-;monomial;SLLS;8| (134 . |coerce|)
   |POLYCAT-;retract;SVarSet;9| |POLYCAT-;retractIfCan;SU;10|
   (139 . |degree|) (144 . |monomial|) |POLYCAT-;primitiveMonomials;SL;12|
   (150 . |ground?|) (155 . |Zero|) (159 . =) (165 . |degree|)
   (170 . |leadingCoefficient|) (175 . |totalDegree|) (180 . |reductum|)
   |POLYCAT-;totalDegree;SNni;13| (185 . |member?|) (191 . |totalDegree|)
   |POLYCAT-;totalDegree;SLNni;14| (197 . |resultant|) (203 . |resultant|)
   (210 . |discriminant|) (215 . |discriminant|) (221 . |primitiveMonomials|)
   (|List| 6) (226 . |concat|) (231 . |removeDuplicates!|) (|Vector| 7)
   (236 . |new|) (|Integer|) (242 . |minIndex|) (247 . |coefficient|)
   (253 . |qsetelt!|) (|List| 220) (|Matrix| 7) (260 . |matrix|)
   (|List| 78) (|Matrix| 6) (265 . |listOfLists|) (270 . |vertConcat|)
   (|Matrix| $) (276 . |reducedSystem|) (|Vector| 6) (281 . |entries|)
   (286 . |concat|) (292 . |concat|)
   (|Record| (|:| |mat| 88) (|:| |vec| 81)) (|Vector| $)
   (298 . |reducedSystem|) (|GeneralPolynomialGcdPackage| 8 9 7 6)
   (304 . |gcdPolynomial|) (310 . |gcdPolynomial|)
   (|Union| 107 (QUOTE "failed")) (|List| 48)
   (|PolynomialFactorizationByRecursion| 7 8 9 6)
   (316 . |solveLinearPolynomialEquationByRecursion|)
   (|Union| 111 (QUOTE "failed")) (|List| 46)
   (322 . |solveLinearPolynomialEquation|) (|Factored| 48)
   (328 . |factorByRecursion|) (|Factored| 46) (333 . |factorPolynomial|)
   (338 . |factorSquareFreeByRecursion|)
   (343 . |factorSquareFreePolynomial|) (|Factored| $) (348 . |factor|)
   (|Factored| 7) (353 . |unit|)
   (|Union| (QUOTE "nil") (QUOTE "sqfr") (QUOTE "irred") (QUOTE "prime"))
   (|Record| (|:| |flg| 123) (|:| |fctr| 7) (|:| |xpnt| 83))
   (|List| 124) (358 . |factorList|)
   (|Record| (|:| |flg| 123) (|:| |fctr| 6) (|:| |xpnt| 83))
   (|List| 127) (|Factored| 6) (363 . |makeFR|) (369 . |unit|)
   (374 . |multivariate|)
   (|Record| (|:| |flg| 123) (|:| |fctr| 48) (|:| |xpnt| 83))
   (|List| 133) (380 . |factorList|) (385 . |factor|) (390 . |transpose|)
   (395 . |characteristic|) (399 . |setUnion|) (405 . |degree|)
   (|Union| $ (QUOTE "failed")) (411 . |exquo|) (417 . |ground|)
```

```
  (422 . |transpose|) (|Union| 101 (QUOTE "failed")) (427 . |conditionP|)
  (432 . |elt|) (438 . *) (444 . +) (450 . |conditionP|)
  (455 . |charthRoot|) (460 . |charthRoot|) (465 . |Zero|)
  (469 . |coefficient|) (476 . -)
  (|Record| (|:| |quotient| $) (|:| |remainder| $))
  (482 . |monicDivide|) |POLYCAT-;monicDivide;2SVarSetR;30|
  (|MultivariateSquareFree| 8 9 7 6) (488 . |squareFree|)
  (493 . |squareFree|) (|PolynomialSquareFree| 9 8 7 6)
  (498 . |squareFree|) (503 . |squareFree|) (508 . |unit|)
  (|Record| (|:| |factor| 6) (|:| |exponent| 83)) (|List| 166)
  (513 . |factors|) (518 . |squareFreePart|) (523 . |content|)
  (528 . |content|) (534 . |zero?|) (539 . |content|) (544 . |exquo|)
  (|Record| (|:| |unit| $) (|:| |canonical| $) (|:| |associate| $))
  (550 . |unitNormal|) (555 . |primitivePart|) (560 . |content|)
  (566 . |exquo|) (572 . |primitivePart|) (578 . <) (584 . <) (590 . <)
  (|PatternMatchResult| 83 6) (|Pattern| 83)
  (|PatternMatchPolynomialCategory| 83 8 9 7 6) (596 . |patternMatch|)
  (|PatternMatchResult| 83 $) (603 . |patternMatch|)
  (|PatternMatchResult| (|Float|) 6) (|Pattern| (|Float|))
  (|PatternMatchPolynomialCategory| (|Float|) 8 9 7 6)
  (610 . |patternMatch|) (|PatternMatchResult| (|Float|) $)
  (617 . |patternMatch|) (624 . |convert|) (629 . |convert|)
  (|Mapping| 185 9) (|Mapping| 185 7)
  (|PolynomialCategoryLifting| 8 9 7 6 185) (634 . |map|)
  (641 . |convert|) (646 . |convert|) (651 . |convert|) (|Mapping| 191 9)
  (|Mapping| 191 7) (|PolynomialCategoryLifting| 8 9 7 6 191)
  (656 . |map|) (663 . |convert|) (|InputForm|) (668 . |convert|)
  (673 . |convert|) (|Mapping| 210 9) (|Mapping| 210 7)
  (|PolynomialCategoryLifting| 8 9 7 6 210) (678 . |map|)
  (685 . |convert|) (|Record| (|:| |mat| 219) (|:| |vec| (|Vector| 83)))
  (|Matrix| 83) (|List| 7) (|Equation| $) (|Union| 83 (QUOTE "failed"))
  (|Union| 224 (QUOTE "failed")) (|Fraction| 83)
  (|Union| 7 (QUOTE "failed"))))
(QUOTE #(|totalDegree| 690 |squareFreePart| 701 |squareFree| 706
 |solveLinearPolynomialEquation| 711 |retractIfCan| 717 |retract| 722
 |resultant| 727 |reducedSystem| 734 |primitivePart| 745
 |primitiveMonomials| 756 |patternMatch| 761 |monomials| 775
 |monomial| 780 |monicDivide| 787 |isTimes| 794 |isPlus| 799
 |isExpt| 804 |gcdPolynomial| 809 |factorSquareFreePolynomial| 815
 |factorPolynomial| 820 |factor| 825 |eval| 830 |discriminant| 836
 |convert| 842 |content| 857 |conditionP| 863 |coefficient| 868
 |charthRoot| 882 < 887))
(QUOTE NIL)
(CONS (|makeByteWordVec2| 1 (QUOTE NIL))
 (CONS (QUOTE #())
  (CONS (QUOTE #())
   (|makeByteWordVec2| 217 (QUOTE
    (1 10 6 0 11 1 6 12 0 13 1 6 9 0 14 1 10 6 0 15 3 6 0 0 16 17 18 0 6 0
     21 0 7 0 22 2 6 23 0 0 24 1 6 0 0 25 1 6 0 0 26 1 6 17 0 28 1 6 16 0
     31 1 6 23 0 32 0 6 0 33 0 7 0 34 2 6 35 0 9 36 3 6 0 0 9 35 37 1 6 7
     0 38 2 7 23 0 0 39 1 6 0 7 40 1 6 12 0 42 2 6 46 0 9 47 2 48 6 0 35
     49 3 6 0 0 16 51 52 3 6 0 0 16 51 54 1 6 0 9 56 1 6 8 0 59 2 6 0 7 8
     60 1 6 23 0 62 0 48 0 63 2 48 23 0 0 64 1 48 35 0 65 1 48 6 0 66 1 6
     35 0 67 1 48 0 0 68 2 16 23 9 0 70 2 6 35 0 16 71 2 48 6 0 0 73 3 0
```

```
      0 0 0 9 74 1 48 6 0 75 2 0 0 0 9 76 1 6 17 0 77 1 78 0 17 79 1 78 0
      0 80 2 81 0 35 7 82 1 81 83 0 84 2 6 7 0 8 85 3 81 7 0 83 7 86 1 88
      0 87 89 1 91 90 0 92 2 88 0 0 0 93 1 0 88 94 95 1 96 78 0 97 2 78 0
      0 0 98 2 81 0 0 0 99 2 0 100 94 101 102 2 103 48 48 48 104 2 0 46 46
      46 105 2 108 106 107 48 109 2 0 110 111 46 112 1 108 113 48 114 1 0
      115 46 116 1 108 113 48 117 1 0 115 46 118 1 7 119 0 120 1 121 7 0
      122 1 121 125 0 126 2 129 0 6 128 130 1 113 48 0 131 2 6 0 46 9 132
      1 113 134 0 135 1 0 119 0 136 1 91 0 0 137 0 6 35 138 2 78 0 0 0 139
      2 6 51 0 16 140 2 83 141 0 0 142 1 6 7 0 143 1 88 0 0 144 1 7 145 94
      146 2 81 7 0 83 147 2 6 0 0 0 148 2 6 0 0 0 149 1 0 145 94 150 1 7
      141 0 151 1 0 141 0 152 0 8 0 153 3 6 0 0 9 35 154 2 6 0 0 0 155 2
      48 156 0 0 157 1 159 129 6 160 1 0 119 0 161 1 162 129 6 163 1 6 119
      0 164 1 129 6 0 165 1 129 167 0 168 1 0 0 0 169 1 48 6 0 170 2 0 0 0
      9 171 1 6 23 0 172 1 6 7 0 173 2 6 141 0 7 174 1 6 175 0 176 1 0 0 0
      177 2 6 0 0 9 178 2 6 141 0 0 179 2 0 0 0 9 180 2 8 23 0 0 181 2 7 23
      0 0 182 2 0 23 0 0 183 3 186 184 6 185 184 187 3 0 188 0 185 188 189
      3 192 190 6 191 190 193 3 0 194 0 191 194 195 1 9 185 0 196 1 7 185
      0 197 3 200 185 198 199 6 201 1 0 185 0 202 1 9 191 0 203 1 7 191 0
      204 3 207 191 205 206 6 208 1 0 191 0 209 1 9 210 0 211 1 7 210 0
      212 3 215 210 213 214 6 216 1 0 210 0 217 2 0 35 0 16 72 1 0 35 0 69
      1 0 0 0 169 1 0 119 0 161 2 0 110 111 46 112 1 0 12 0 58 1 0 9 0 57
      3 0 0 0 0 9 74 1 0 88 94 95 2 0 100 94 101 102 2 0 0 0 9 180 1 0 0 0
      177 1 0 17 0 61 3 0 188 0 185 188 189 3 0 194 0 191 194 195 1 0 17 0
      27 3 0 0 0 16 51 55 3 0 156 0 0 9 158 1 0 29 0 41 1 0 29 0 30 1 0 44
      0 45 2 0 46 46 46 105 1 0 115 46 118 1 0 115 46 116 1 0 119 0 136 2
      0 0 0 19 20 2 0 0 0 9 76 1 0 210 0 217 1 0 185 0 202 1 0 191 0 209 2
      0 0 0 9 171 1 0 145 94 150 3 0 0 0 16 51 53 3 0 0 0 9 35 50 1 0 141
      0 152 2 0 23 0 0 183))))))
  (QUOTE |lookupComplete|)))
```

21.45 PSETCAT.lsp BOOTSTRAP

PSETCAT depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **PSETCAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **PSETCAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— **PSETCAT.lsp BOOTSTRAP** —

```
(|/VERSIONCHECK| 2)

(SETQ |PolynomialSetCategory;CAT| (QUOTE NIL))

(SETQ |PolynomialSetCategory;AL| (QUOTE NIL))

(DEFUN |PolynomialSetCategory| (|&REST| #1=#:G82375 |&AUX| #2=#:G82373)
 (declare (special |PolynomialSetCategory;AL|))
 (DSETQ #2# #1#)
```

```
 (LET (#3=#:G82374)
  (COND
   ((SETQ #3# (|assoc| (|devaluateList| #2#) |PolynomialSetCategory;AL|))
     (CDR #3#))
   (T
    (SETQ |PolynomialSetCategory;AL|
     (|cons5|
      (CONS
       (|devaluateList| #2#)
       (SETQ #3# (APPLY (FUNCTION |PolynomialSetCategory;|) #2#)))
      |PolynomialSetCategory;AL|))
    #3#))))

(DEFUN |PolynomialSetCategory;| (|t#1| |t#2| |t#3| |t#4|)
 (declare (special |PolynomialSetCategory;CAT|))
 (PROG (#1=#:G82372)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR
       (QUOTE (|t#1| |t#2| |t#3| |t#4|))
       (LIST
        (|devaluate| |t#1|)
        (|devaluate| |t#2|)
        (|devaluate| |t#3|)
        (|devaluate| |t#4|)))
      (|sublisV|
       (PAIR (QUOTE (#2=#:G82371)) (LIST (QUOTE (|List| |t#4|))))
       (COND
        (|PolynomialSetCategory;CAT|)
        ((QUOTE T)
         (LETT |PolynomialSetCategory;CAT|
          (|Join|
           (|SetCategory|)
           (|Collection| (QUOTE |t#4|))
           (|CoercibleTo| (QUOTE #2#))
           (|mkCategory|
            (QUOTE |domain|)
            (QUOTE (
             ((|retractIfCan| ((|Union| |$| "failed") (|List| |t#4|))) T)
             ((|retract| (|$| (|List| |t#4|))) T)
             ((|mvar| (|t#3| |$|)) T)
             ((|variables| ((|List| |t#3|) |$|)) T)
             ((|mainVariables| ((|List| |t#3|) |$|)) T)
             ((|mainVariable?| ((|Boolean|) |t#3| |$|)) T)
             ((|collectUnder| (|$| |$| |t#3|)) T)
             ((|collect| (|$| |$| |t#3|)) T)
             ((|collectUpper| (|$| |$| |t#3|)) T)
             ((|sort|
              ((|Record|
                (|:| |under| |$|)
                (|:| |floor| |$|)
                (|:| |upper| |$|)) |$| |t#3|))
```

```
            T)
           ((|trivialIdeal?| ((|Boolean|) |$|)) T)
           ((|roughBase?| ((|Boolean|) |$|))
             (|has| |t#1| (|IntegralDomain|)))
           ((|roughSubIdeal?| ((|Boolean|) |$| |$|))
             (|has| |t#1| (|IntegralDomain|)))
           ((|roughEqualIdeals?| ((|Boolean|) |$| |$|))
             (|has| |t#1| (|IntegralDomain|)))
           ((|roughUnitIdeal?| ((|Boolean|) |$|))
             (|has| |t#1| (|IntegralDomain|)))
           ((|headRemainder|
             ((|Record|
               (|:| |num| |t#4|)
               (|:| |den| |t#1|)) |t#4| |$|))
              (|has| |t#1| (|IntegralDomain|)))
           ((|remainder|
             ((|Record|
               (|:| |rnum| |t#1|)
               (|:| |polnum| |t#4|)
               (|:| |den| |t#1|)) |t#4| |$|))
              (|has| |t#1| (|IntegralDomain|)))
           ((|rewriteIdealWithHeadRemainder|
            ((|List| |t#4|) (|List| |t#4|) |$|))
              (|has| |t#1| (|IntegralDomain|)))
           ((|rewriteIdealWithRemainder| ((|List| |t#4|) (|List| |t#4|) |$|))
             (|has| |t#1| (|IntegralDomain|)))
           ((|triangular?| ((|Boolean|) |$|))
             (|has| |t#1| (|IntegralDomain|)))))
          (QUOTE ((|finiteAggregate| T)))
          (QUOTE ((|Boolean|) (|List| |t#4|) (|List| |t#3|)))
          NIL))
     . #3=(|PolynomialSetCategory|))))))
   . #3#)
  (SETELT #1# 0
   (LIST
    (QUOTE |PolynomialSetCategory|)
    (|devaluate| |t#1|)
    (|devaluate| |t#2|)
    (|devaluate| |t#3|)
    (|devaluate| |t#4|)))))))
```

21.46 PSETCAT-.lsp BOOTSTRAP

PSETCAT- depends on **PSETCAT**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **PSETCAT-** category which we can write into the **MID** directory. We compile the lisp code and copy the **PSETCAT-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— **PSETCAT-.lsp BOOTSTRAP** —

```
(|/VERSIONCHECK| 2)

(DEFUN |PSETCAT-;elements| (|ps| |$|)
 (PROG (|lp|)
  (RETURN
   (LETT |lp| (SPADCALL |ps| (QREFELT |$| 12)) |PSETCAT-;elements|))))

(DEFUN |PSETCAT-;variables1| (|lp| |$|)
 (PROG (#1=#:G82392 |p| #2=#:G82393 |lvars|)
  (RETURN
   (SEQ
    (LETT |lvars|
     (PROGN
      (LETT #1# NIL |PSETCAT-;variables1|)
      (SEQ
       (LETT |p| NIL |PSETCAT-;variables1|)
       (LETT #2# |lp| |PSETCAT-;variables1|)
       G190
       (COND
        ((OR (ATOM #2#) (PROGN (LETT |p| (CAR #2#) |PSETCAT-;variables1|) NIL))
          (GO G191)))
       (SEQ
        (EXIT
         (LETT #1#
          (CONS (SPADCALL |p| (QREFELT |$| 14)) #1#) |PSETCAT-;variables1|)))
       (LETT #2# (CDR #2#) |PSETCAT-;variables1|)
       (GO G190)
       G191
       (EXIT (NREVERSE0 #1#))))
     |PSETCAT-;variables1|)
    (EXIT
     (SPADCALL
      (CONS (FUNCTION |PSETCAT-;variables1!0|) |$|)
      (SPADCALL (SPADCALL |lvars| (QREFELT |$| 18)) (QREFELT |$| 19))
      (QREFELT |$| 21)))))))

(DEFUN |PSETCAT-;variables1!0| (|#1| |#2| |$|)
 (SPADCALL |#2| |#1| (QREFELT |$| 16)))

(DEFUN |PSETCAT-;variables2| (|lp| |$|)
 (PROG (#1=#:G82397 |p| #2=#:G82398 |lvars|)
  (RETURN
   (SEQ
    (LETT |lvars|
     (PROGN
      (LETT #1# NIL |PSETCAT-;variables2|)
      (SEQ
       (LETT |p| NIL |PSETCAT-;variables2|)
       (LETT #2# |lp| |PSETCAT-;variables2|)
       G190
       (COND
        ((OR (ATOM #2#) (PROGN (LETT |p| (CAR #2#) |PSETCAT-;variables2|) NIL))
          (GO G191)))
```

```
       (SEQ
        (EXIT
         (LETT #1#
          (CONS (SPADCALL |p| (QREFELT |$| 22)) #1#) |PSETCAT-;variables2|)))
       (LETT #2# (CDR #2#) |PSETCAT-;variables2|)
       (GO G190)
       G191
       (EXIT (NREVERSE0 #1#))))
     |PSETCAT-;variables2|)
    (EXIT
     (SPADCALL
      (CONS (FUNCTION |PSETCAT-;variables2!0|) |$|)
      (SPADCALL |lvars| (QREFELT |$| 19))
      (QREFELT |$| 21)))))))

(DEFUN |PSETCAT-;variables2!0| (|#1| |#2| |$|)
 (SPADCALL |#2| |#1| (QREFELT |$| 16)))

(DEFUN |PSETCAT-;variables;SL;4| (|ps| |$|)
 (|PSETCAT-;variables1| (|PSETCAT-;elements| |ps| |$|) |$|))

(DEFUN |PSETCAT-;mainVariables;SL;5| (|ps| |$|)
 (|PSETCAT-;variables2|
  (SPADCALL (ELT |$| 24) (|PSETCAT-;elements| |ps| |$|) (QREFELT |$| 26))
  |$|))

(DEFUN |PSETCAT-;mainVariable?;VarSetSB;6| (|v| |ps| |$|)
 (PROG (|lp|)
  (RETURN
   (SEQ
    (LETT |lp|
     (SPADCALL (ELT |$| 24) (|PSETCAT-;elements| |ps| |$|) (QREFELT |$| 26))
     |PSETCAT-;mainVariable?;VarSetSB;6|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((OR
           (NULL |lp|)
           (SPADCALL
            (SPADCALL (|SPADfirst| |lp|) (QREFELT |$| 22))
            |v|
            (QREFELT |$| 28)))
          (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ (EXIT (LETT |lp| (CDR |lp|) |PSETCAT-;mainVariable?;VarSetSB;6|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))))))
```

```
(DEFUN |PSETCAT-;collectUnder;SVarSetS;7| (|ps| |v| |$|)
 (PROG (|p| |lp| |lq|)
  (RETURN
   (SEQ
    (LETT |lp|
     (|PSETCAT-;elements| |ps| |$|)
     |PSETCAT-;collectUnder;SVarSetS;7|)
    (LETT |lq| NIL |PSETCAT-;collectUnder;SVarSetS;7|)
    (SEQ
     G190
     (COND
      ((NULL (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
        (GO G191)))
     (SEQ
      (LETT |p| (|SPADfirst| |lp|) |PSETCAT-;collectUnder;SVarSetS;7|)
      (LETT |lp| (CDR |lp|) |PSETCAT-;collectUnder;SVarSetS;7|)
      (EXIT
       (COND
        ((OR
          (SPADCALL |p| (QREFELT |$| 24))
          (SPADCALL (SPADCALL |p| (QREFELT |$| 22)) |v| (QREFELT |$| 16)))
         (LETT |lq| (CONS |p| |lq|) |PSETCAT-;collectUnder;SVarSetS;7|)))))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT (SPADCALL |lq| (QREFELT |$| 30)))))))

(DEFUN |PSETCAT-;collectUpper;SVarSetS;8| (|ps| |v| |$|)
 (PROG (|p| |lp| |lq|)
  (RETURN
   (SEQ
    (LETT |lp|
     (|PSETCAT-;elements| |ps| |$|)
     |PSETCAT-;collectUpper;SVarSetS;8|)
    (LETT |lq| NIL |PSETCAT-;collectUpper;SVarSetS;8|)
    (SEQ
     G190
     (COND
      ((NULL (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
        (GO G191)))
     (SEQ
      (LETT |p| (|SPADfirst| |lp|) |PSETCAT-;collectUpper;SVarSetS;8|)
      (LETT |lp| (CDR |lp|) |PSETCAT-;collectUpper;SVarSetS;8|)
      (EXIT
       (COND
        ((NULL (SPADCALL |p| (QREFELT |$| 24)))
         (COND
          ((SPADCALL |v| (SPADCALL |p| (QREFELT |$| 22)) (QREFELT |$| 16))
            (LETT |lq| (CONS |p| |lq|) |PSETCAT-;collectUpper;SVarSetS;8|)))))))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
```

```
    (EXIT (SPADCALL |lq| (QREFELT |$| 30)))))))

(DEFUN |PSETCAT-;collect;SVarSetS;9| (|ps| |v| |$|)
 (PROG (|p| |lp| |lq|)
  (RETURN
   (SEQ
    (LETT |lp| (|PSETCAT-;elements| |ps| |$|) |PSETCAT-;collect;SVarSetS;9|)
    (LETT |lq| NIL |PSETCAT-;collect;SVarSetS;9|)
    (SEQ
     G190
     (COND
      ((NULL (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
        (GO G191)))
     (SEQ
      (LETT |p| (|SPADfirst| |lp|) |PSETCAT-;collect;SVarSetS;9|)
      (LETT |lp| (CDR |lp|) |PSETCAT-;collect;SVarSetS;9|)
      (EXIT
       (COND
        ((NULL (SPADCALL |p| (QREFELT |$| 24)))
         (COND
          ((SPADCALL (SPADCALL |p| (QREFELT |$| 22)) |v| (QREFELT |$| 28))
            (LETT |lq| (CONS |p| |lq|) |PSETCAT-;collect;SVarSetS;9|)))))))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT (SPADCALL |lq| (QREFELT |$| 30)))))))

(DEFUN |PSETCAT-;sort;SVarSetR;10| (|ps| |v| |$|)
 (PROG (|p| |lp| |us| |vs| |ws|)
  (RETURN
   (SEQ
    (LETT |lp| (|PSETCAT-;elements| |ps| |$|) |PSETCAT-;sort;SVarSetR;10|)
    (LETT |us| NIL |PSETCAT-;sort;SVarSetR;10|)
    (LETT |vs| NIL |PSETCAT-;sort;SVarSetR;10|)
    (LETT |ws| NIL |PSETCAT-;sort;SVarSetR;10|)
    (SEQ
     G190
     (COND
      ((NULL (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (LETT |p| (|SPADfirst| |lp|) |PSETCAT-;sort;SVarSetR;10|)
      (LETT |lp| (CDR |lp|) |PSETCAT-;sort;SVarSetR;10|)
      (EXIT
       (COND
        ((OR
           (SPADCALL |p| (QREFELT |$| 24))
           (SPADCALL (SPADCALL |p| (QREFELT |$| 22)) |v| (QREFELT |$| 16)))
          (LETT |us| (CONS |p| |us|) |PSETCAT-;sort;SVarSetR;10|))
        ((SPADCALL (SPADCALL |p| (QREFELT |$| 22)) |v| (QREFELT |$| 28))
          (LETT |vs| (CONS |p| |vs|) |PSETCAT-;sort;SVarSetR;10|))
        ((QUOTE T) (LETT |ws| (CONS |p| |ws|) |PSETCAT-;sort;SVarSetR;10|)))))
     NIL
```

```
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT
     (VECTOR
      (SPADCALL |us| (QREFELT |$| 30))
      (SPADCALL |vs| (QREFELT |$| 30))
      (SPADCALL |ws| (QREFELT |$| 30))))))))

(DEFUN |PSETCAT-;=;2SB;11| (|ps1| |ps2| |$|)
 (PROG (#1=#:G82439 #2=#:G82440 #3=#:G82437 |p| #4=#:G82438)
  (RETURN
   (SEQ
    (SPADCALL
     (SPADCALL
      (PROGN
       (LETT #1# NIL |PSETCAT-;=;2SB;11|)
       (SEQ
        (LETT |p| NIL |PSETCAT-;=;2SB;11|)
        (LETT #2# (|PSETCAT-;elements| |ps1| |$|) |PSETCAT-;=;2SB;11|)
        G190
        (COND
         ((OR (ATOM #2#) (PROGN (LETT |p| (CAR #2#) |PSETCAT-;=;2SB;11|) NIL))
           (GO G191)))
        (SEQ (EXIT (LETT #1# (CONS |p| #1#) |PSETCAT-;=;2SB;11|)))
        (LETT #2# (CDR #2#) |PSETCAT-;=;2SB;11|)
        (GO G190)
        G191
        (EXIT (NREVERSE0 #1#))))
      (QREFELT |$| 37))
     (SPADCALL
      (PROGN
       (LETT #3# NIL |PSETCAT-;=;2SB;11|)
       (SEQ
        (LETT |p| NIL |PSETCAT-;=;2SB;11|)
        (LETT #4# (|PSETCAT-;elements| |ps2| |$|) |PSETCAT-;=;2SB;11|)
        G190
        (COND
         ((OR (ATOM #4#) (PROGN (LETT |p| (CAR #4#) |PSETCAT-;=;2SB;11|) NIL))
           (GO G191)))
        (SEQ (EXIT (LETT #3# (CONS |p| #3#) |PSETCAT-;=;2SB;11|)))
        (LETT #4# (CDR #4#) |PSETCAT-;=;2SB;11|)
        (GO G190)
        G191
        (EXIT (NREVERSE0 #3#))))
      (QREFELT |$| 37))
     (QREFELT |$| 38))))))

(DEFUN |PSETCAT-;localInf?| (|p| |q| |$|)
 (SPADCALL
  (SPADCALL |p| (QREFELT |$| 40))
  (SPADCALL |q| (QREFELT |$| 40))
  (QREFELT |$| 41)))
```

```
(DEFUN |PSETCAT-;localTriangular?| (|lp| |$|)
 (PROG (|q| |p|)
  (RETURN
   (SEQ
    (LETT |lp|
     (SPADCALL (ELT |$| 42) |lp| (QREFELT |$| 26)) |PSETCAT-;localTriangular?|)
    (EXIT
     (COND
      ((NULL |lp|) (QUOTE T))
      ((SPADCALL (ELT |$| 24) |lp| (QREFELT |$| 43)) (QUOTE NIL))
      ((QUOTE T)
       (SEQ
        (LETT |lp|
         (SPADCALL
          (CONS (FUNCTION |PSETCAT-;localTriangular?!0|) |$|)
          |lp|
          (QREFELT |$| 45))
         |PSETCAT-;localTriangular?|)
        (LETT |p| (|SPADfirst| |lp|) |PSETCAT-;localTriangular?|)
        (LETT |lp| (CDR |lp|) |PSETCAT-;localTriangular?|)
        (SEQ
         G190
         (COND
          ((NULL
            (COND
             ((NULL |lp|) (QUOTE NIL))
             ((QUOTE T)
               (SPADCALL
                (SPADCALL
                 (LETT |q|
                  (|SPADfirst| |lp|)
                  |PSETCAT-;localTriangular?|)
                 (QREFELT |$| 22))
                (SPADCALL |p| (QREFELT |$| 22)) (QREFELT |$| 16)))))
           (GO G191)))
         (SEQ
          (LETT |p| |q| |PSETCAT-;localTriangular?|)
          (EXIT (LETT |lp| (CDR |lp|) |PSETCAT-;localTriangular?|)))
         NIL
         (GO G190)
         G191
         (EXIT NIL))
        (EXIT (NULL |lp|))))))))))

(DEFUN |PSETCAT-;localTriangular?!0| (|#1| |#2| |$|)
 (SPADCALL
  (SPADCALL |#2| (QREFELT |$| 22))
  (SPADCALL |#1| (QREFELT |$| 22))
  (QREFELT |$| 16)))

(DEFUN |PSETCAT-;triangular?;SB;14| (|ps| |$|)
 (|PSETCAT-;localTriangular?| (|PSETCAT-;elements| |ps| |$|) |$|))

(DEFUN |PSETCAT-;trivialIdeal?;SB;15| (|ps| |$|)
```

```
 (NULL
  (SPADCALL (ELT |$| 42) (|PSETCAT-;elements| |ps| |$|) (QREFELT |$| 26))))

(DEFUN |PSETCAT-;roughUnitIdeal?;SB;16| (|ps| |$|)
 (SPADCALL
  (ELT |$| 24)
  (SPADCALL (ELT |$| 42) (|PSETCAT-;elements| |ps| |$|) (QREFELT |$| 26))
  (QREFELT |$| 43)))

(DEFUN |PSETCAT-;relativelyPrimeLeadingMonomials?| (|p| |q| |$|)
 (PROG (|dp| |dq|)
  (RETURN
   (SEQ
    (LETT |dp|
     (SPADCALL |p| (QREFELT |$| 40))
     |PSETCAT-;relativelyPrimeLeadingMonomials?|)
    (LETT |dq|
     (SPADCALL |q| (QREFELT |$| 40))
     |PSETCAT-;relativelyPrimeLeadingMonomials?|)
    (EXIT
     (SPADCALL
      (SPADCALL |dp| |dq| (QREFELT |$| 49))
      (SPADCALL |dp| |dq| (QREFELT |$| 50))
      (QREFELT |$| 51)))))))

(DEFUN |PSETCAT-;roughBase?;SB;18| (|ps| |$|)
 (PROG (|p| |lp| |rB?| |copylp|)
  (RETURN
   (SEQ
    (LETT |lp|
     (SPADCALL (ELT |$| 42) (|PSETCAT-;elements| |ps| |$|) (QREFELT |$| 26))
     |PSETCAT-;roughBase?;SB;18|)
    (EXIT
     (COND
      ((NULL |lp|) (QUOTE T))
      ((QUOTE T)
       (SEQ
        (LETT |rB?| (QUOTE T) |PSETCAT-;roughBase?;SB;18|)
        (SEQ
         G190
         (COND
          ((NULL (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) |rB?|)))
           (GO G191)))
         (SEQ
          (LETT |p| (|SPADfirst| |lp|) |PSETCAT-;roughBase?;SB;18|)
          (LETT |lp| (CDR |lp|) |PSETCAT-;roughBase?;SB;18|)
          (LETT |copylp| |lp| |PSETCAT-;roughBase?;SB;18|)
          (EXIT
           (SEQ
            G190
            (COND
             ((NULL (COND ((NULL |copylp|) (QUOTE NIL)) ((QUOTE T) |rB?|)))
              (GO G191)))
            (SEQ
```

```
             (LETT |rB?|
              (|PSETCAT-;relativelyPrimeLeadingMonomials?| |p|
               (|SPADfirst| |copylp|) |$|)
              |PSETCAT-;roughBase?;SB;18|)
             (EXIT (LETT |copylp| (CDR |copylp|) |PSETCAT-;roughBase?;SB;18|)))
            NIL
            (GO G190)
            G191
            (EXIT NIL))))
         NIL
         (GO G190)
         G191
         (EXIT NIL))
        (EXIT |rB?|)))))))))

(DEFUN |PSETCAT-;roughSubIdeal?;2SB;19| (|ps1| |ps2| |$|)
 (PROG (|lp|)
  (RETURN
   (SEQ
    (LETT |lp|
     (SPADCALL (|PSETCAT-;elements| |ps1| |$|) |ps2| (QREFELT |$| 53))
     |PSETCAT-;roughSubIdeal?;2SB;19|)
    (EXIT (NULL (SPADCALL (ELT |$| 42) |lp| (QREFELT |$| 26))))))))

(DEFUN |PSETCAT-;roughEqualIdeals?;2SB;20| (|ps1| |ps2| |$|)
 (COND
  ((SPADCALL |ps1| |ps2| (QREFELT |$| 55)) (QUOTE T))
  ((SPADCALL |ps1| |ps2| (QREFELT |$| 56))
    (SPADCALL |ps2| |ps1| (QREFELT |$| 56)))
  ((QUOTE T) (QUOTE NIL))))

(DEFUN |PSETCAT-;exactQuo| (|r| |s| |$|)
 (SPADCALL |r| |s| (QREFELT |$| 58)))

(DEFUN |PSETCAT-;exactQuo| (|r| |s| |$|)
 (PROG (#1=#:G82473)
  (RETURN
   (PROG2
    (LETT #1# (SPADCALL |r| |s| (QREFELT |$| 60)) |PSETCAT-;exactQuo|)
    (QCDR #1#)
    (|check-union| (QEQCAR #1# 0) (QREFELT |$| 7) #1#)))))

(DEFUN |PSETCAT-;headRemainder;PSR;23| (|a| |ps| |$|)
 (PROG (|lp1| |p| |e| |g| |#G47| |#G48| |lca| |lcp| |r| |lp2|)
  (RETURN
   (SEQ
    (LETT |lp1|
     (SPADCALL (ELT |$| 42) (|PSETCAT-;elements| |ps| |$|) (QREFELT |$| 26))
     |PSETCAT-;headRemainder;PSR;23|)
    (EXIT
     (COND
      ((NULL |lp1|) (CONS |a| (|spadConstant| |$| 61)))
      ((SPADCALL (ELT |$| 24) |lp1| (QREFELT |$| 43))
        (CONS (SPADCALL |a| (QREFELT |$| 62)) (|spadConstant| |$| 61)))
```

```
((QUOTE T)
 (SEQ
  (LETT |r| (|spadConstant| |$| 61) |PSETCAT-;headRemainder;PSR;23|)
  (LETT |lp1|
   (SPADCALL
    (CONS #'|PSETCAT-;localInf?| |$|)
    (REVERSE (|PSETCAT-;elements| |ps| |$|))
    (QREFELT |$| 45))
   |PSETCAT-;headRemainder;PSR;23|)
  (LETT |lp2| |lp1| |PSETCAT-;headRemainder;PSR;23|)
  (SEQ
   G190
   (COND
    ((NULL
      (COND
       ((OR (SPADCALL |a| (QREFELT |$| 42)) (NULL |lp2|)) (QUOTE NIL))
       ((QUOTE T) (QUOTE T))))
     (GO G191)))
   (SEQ
    (LETT |p| (|SPADfirst| |lp2|) |PSETCAT-;headRemainder;PSR;23|)
    (LETT |e|
     (SPADCALL
      (SPADCALL |a| (QREFELT |$| 40))
      (SPADCALL |p| (QREFELT |$| 40))
      (QREFELT |$| 63))
     |PSETCAT-;headRemainder;PSR;23|)
    (EXIT
     (COND
      ((QEQCAR |e| 0)
       (SEQ
        (LETT |g|
         (SPADCALL
          (LETT |lca|
           (SPADCALL |a| (QREFELT |$| 64))
           |PSETCAT-;headRemainder;PSR;23|)
          (LETT |lcp|
           (SPADCALL |p| (QREFELT |$| 64))
           |PSETCAT-;headRemainder;PSR;23|)
          (QREFELT |$| 65))
         |PSETCAT-;headRemainder;PSR;23|)
        (PROGN
         (LETT |#G47|
          (|PSETCAT-;exactQuo| |lca| |g| |$|)
          |PSETCAT-;headRemainder;PSR;23|)
         (LETT |#G48|
          (|PSETCAT-;exactQuo| |lcp| |g| |$|)
          |PSETCAT-;headRemainder;PSR;23|)
         (LETT |lca| |#G47| |PSETCAT-;headRemainder;PSR;23|)
         (LETT |lcp| |#G48| |PSETCAT-;headRemainder;PSR;23|))
        (LETT |a|
         (SPADCALL
          (SPADCALL |lcp|
           (SPADCALL |a| (QREFELT |$| 62))
           (QREFELT |$| 66))
```

```
                (SPADCALL
                 (SPADCALL |lca| (QCDR |e|) (QREFELT |$| 67))
                 (SPADCALL |p| (QREFELT |$| 62)) (QREFELT |$| 68))
                (QREFELT |$| 69))
               |PSETCAT-;headRemainder;PSR;23|)
              (LETT |r|
               (SPADCALL |r| |lcp| (QREFELT |$| 70))
               |PSETCAT-;headRemainder;PSR;23|)
              (EXIT (LETT |lp2| |lp1| |PSETCAT-;headRemainder;PSR;23|))))
            ((QUOTE T)
             (LETT |lp2| (CDR |lp2|) |PSETCAT-;headRemainder;PSR;23|)))))
         NIL
         (GO G190)
         G191
         (EXIT NIL))
        (EXIT (CONS |a| |r|)))))))))

(DEFUN |PSETCAT-;makeIrreducible!| (|frac| |$|)
 (PROG (|g|)
  (RETURN
   (SEQ
    (LETT |g|
     (SPADCALL (QCDR |frac|) (QCAR |frac|) (QREFELT |$| 73))
     |PSETCAT-;makeIrreducible!|)
    (EXIT
     (COND
      ((SPADCALL |g| (QREFELT |$| 74)) |frac|)
      ((QUOTE T)
       (SEQ
        (PROGN
         (RPLACA |frac| (SPADCALL (QCAR |frac|) |g| (QREFELT |$| 75)))
         (QCAR |frac|))
        (PROGN
         (RPLACD |frac| (|PSETCAT-;exactQuo| (QCDR |frac|) |g| |$|))
         (QCDR |frac|))
        (EXIT |frac|))))))))

(DEFUN |PSETCAT-;remainder;PSR;25| (|a| |ps| |$|)
 (PROG (|hRa| |r| |lca| |g| |b| |c|)
  (RETURN
   (SEQ
    (LETT |hRa|
     (|PSETCAT-;makeIrreducible!| (SPADCALL |a| |ps| (QREFELT |$| 76)) |$|)
     |PSETCAT-;remainder;PSR;25|)
    (LETT |a| (QCAR |hRa|) |PSETCAT-;remainder;PSR;25|)
    (LETT |r| (QCDR |hRa|) |PSETCAT-;remainder;PSR;25|)
    (EXIT
     (COND
      ((SPADCALL |a| (QREFELT |$| 42))
        (VECTOR (|spadConstant| |$| 61) |a| |r|))
      ((QUOTE T)
       (SEQ
        (LETT |b|
         (SPADCALL
```

```
  (|spadConstant| |$| 61)
  (SPADCALL |a| (QREFELT |$| 40))
  (QREFELT |$| 67))
 |PSETCAT-;remainder;PSR;25|)
(LETT |c| (SPADCALL |a| (QREFELT |$| 64)) |PSETCAT-;remainder;PSR;25|)
(SEQ
 G190
 (COND
  ((NULL
    (COND
     ((SPADCALL
        (LETT |a|
         (SPADCALL |a| (QREFELT |$| 62))
         |PSETCAT-;remainder;PSR;25|)
        (QREFELT |$| 42))
       (QUOTE NIL))
     ((QUOTE T) (QUOTE T))))
   (GO G191)))
 (SEQ
  (LETT |hRa|
   (|PSETCAT-;makeIrreducible!|
    (SPADCALL |a| |ps| (QREFELT |$| 76))
    |$|)
   |PSETCAT-;remainder;PSR;25|)
  (LETT |a| (QCAR |hRa|) |PSETCAT-;remainder;PSR;25|)
  (LETT |r|
   (SPADCALL |r| (QCDR |hRa|) (QREFELT |$| 70))
   |PSETCAT-;remainder;PSR;25|)
  (LETT |g|
   (SPADCALL |c|
    (LETT |lca|
     (SPADCALL |a| (QREFELT |$| 64))
     |PSETCAT-;remainder;PSR;25|)
    (QREFELT |$| 65))
   |PSETCAT-;remainder;PSR;25|)
  (LETT |b|
   (SPADCALL
    (SPADCALL
     (SPADCALL
      (QCDR |hRa|)
      (|PSETCAT-;exactQuo| |c| |g| |$|)
      (QREFELT |$| 70))
     |b|
     (QREFELT |$| 66))
    (SPADCALL
     (|PSETCAT-;exactQuo| |lca| |g| |$|)
     (SPADCALL |a| (QREFELT |$| 40))
     (QREFELT |$| 67))
    (QREFELT |$| 77))
   |PSETCAT-;remainder;PSR;25|)
  (EXIT (LETT |c| |g| |PSETCAT-;remainder;PSR;25|)))
 NIL
 (GO G190)
 G191
```

```
         (EXIT NIL))
        (EXIT (VECTOR |c| |b| |r|))))))))))

(DEFUN |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26| (|ps| |cs| |$|)
 (PROG (|p| |rs|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |cs| (QREFELT |$| 80)) |ps|)
     ((SPADCALL |cs| (QREFELT |$| 81)) (LIST (|spadConstant| |$| 82)))
     ((QUOTE T)
      (SEQ
       (LETT |ps|
        (SPADCALL (ELT |$| 42) |ps| (QREFELT |$| 26))
        |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|)
       (EXIT
        (COND
         ((NULL |ps|) |ps|)
         ((SPADCALL (ELT |$| 24) |ps| (QREFELT |$| 43))
           (LIST (|spadConstant| |$| 83)))
         ((QUOTE T)
           (SEQ
            (LETT |rs| NIL |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|)
            (SEQ
             G190
             (COND
              ((NULL (COND ((NULL |ps|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
                (GO G191)))
             (SEQ
              (LETT |p|
               (|SPADfirst| |ps|)
               |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|)
              (LETT |ps|
               (CDR |ps|)
               |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|)
              (LETT |p|
               (QCAR (SPADCALL |p| |cs| (QREFELT |$| 76)))
               |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|)
              (EXIT
               (COND
                ((NULL (SPADCALL |p| (QREFELT |$| 42)))
                  (COND
                   ((SPADCALL |p| (QREFELT |$| 24))
                    (SEQ
                     (LETT |ps| NIL
                      |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|)
                     (EXIT
                      (LETT |rs|
                       (LIST (|spadConstant| |$| 83))
                       |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|))))
                   ((QUOTE T)
                    (SEQ
                     (SPADCALL |p| (QREFELT |$| 84))
                     (EXIT
```

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                    (LETT |rs|
                     (CONS |p| |rs|)
                     |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|)))))))))
             NIL
             (GO G190)
             G191
             (EXIT NIL))
            (EXIT (SPADCALL |rs| (QREFELT |$| 85)))))))))))))

(DEFUN |PSETCAT-;rewriteIdealWithRemainder;LSL;27| (|ps| |cs| |$|)
 (PROG (|p| |rs|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |cs| (QREFELT |$| 80)) |ps|)
     ((SPADCALL |cs| (QREFELT |$| 81)) (LIST (|spadConstant| |$| 82)))
     ((QUOTE T)
      (SEQ
       (LETT |ps|
        (SPADCALL (ELT |$| 42) |ps| (QREFELT |$| 26))
        |PSETCAT-;rewriteIdealWithRemainder;LSL;27|)
       (EXIT
        (COND
         ((NULL |ps|) |ps|)
         ((SPADCALL (ELT |$| 24) |ps| (QREFELT |$| 43))
           (LIST (|spadConstant| |$| 83)))
         ((QUOTE T)
          (SEQ
           (LETT |rs| NIL |PSETCAT-;rewriteIdealWithRemainder;LSL;27|)
           (SEQ
            G190
            (COND
             ((NULL (COND ((NULL |ps|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
               (GO G191)))
            (SEQ
             (LETT |p|
              (|SPADfirst| |ps|)
              |PSETCAT-;rewriteIdealWithRemainder;LSL;27|)
             (LETT |ps| (CDR |ps|) |PSETCAT-;rewriteIdealWithRemainder;LSL;27|)
             (LETT |p|
              (QVELT (SPADCALL |p| |cs| (QREFELT |$| 87)) 1)
              |PSETCAT-;rewriteIdealWithRemainder;LSL;27|)
             (EXIT
              (COND
               ((NULL (SPADCALL |p| (QREFELT |$| 42)))
                (COND
                 ((SPADCALL |p| (QREFELT |$| 24))
                  (SEQ
                   (LETT |ps| NIL |PSETCAT-;rewriteIdealWithRemainder;LSL;27|)
                   (EXIT
                    (LETT |rs|
                     (LIST (|spadConstant| |$| 83))
                     |PSETCAT-;rewriteIdealWithRemainder;LSL;27|))))
                 ((QUOTE T)
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                 (LETT |rs|
                  (CONS (SPADCALL |p| (QREFELT |$| 88)) |rs|)
                  |PSETCAT-;rewriteIdealWithRemainder;LSL;27|)))))))
           NIL
           (GO G190)
           G191
           (EXIT NIL))
          (EXIT (SPADCALL |rs| (QREFELT |$| 85)))))))))))))

(DEFUN |PolynomialSetCategory&| (|#1| |#2| |#3| |#4| |#5|)
 (PROG (|DV$1| |DV$2| |DV$3| |DV$4| |DV$5| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|PolynomialSetCategory&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |DV$3| (|devaluate| |#3|) . #1#)
    (LETT |DV$4| (|devaluate| |#4|) . #1#)
    (LETT |DV$5| (|devaluate| |#5|) . #1#)
    (LETT |dv$|
     (LIST
      (QUOTE |PolynomialSetCategory&|)
      |DV$1| |DV$2| |DV$3| |DV$4| |DV$5|) . #1#)
    (LETT |$| (make-array 90) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3
     (LETT |pv$|
      (|buildPredVector| 0 0
       (LIST (|HasCategory| |#2| (QUOTE (|IntegralDomain|))))) . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
    (QSETREFV |$| 8 |#3|)
    (QSETREFV |$| 9 |#4|)
    (QSETREFV |$| 10 |#5|)
    (COND
     ((|testBitVector| |pv$| 1)
      (PROGN
       (QSETREFV |$| 48
        (CONS (|dispatchFunction| |PSETCAT-;roughUnitIdeal?;SB;16|) |$|))
       (QSETREFV |$| 52
        (CONS (|dispatchFunction| |PSETCAT-;roughBase?;SB;18|) |$|))
       (QSETREFV |$| 54
        (CONS (|dispatchFunction| |PSETCAT-;roughSubIdeal?;2SB;19|) |$|))
       (QSETREFV |$| 57
        (CONS (|dispatchFunction| |PSETCAT-;roughEqualIdeals?;2SB;20|) |$|)))))
    (COND
     ((|HasCategory| |#2| (QUOTE (|GcdDomain|)))
     (COND
      ((|HasCategory| |#4| (QUOTE (|ConvertibleTo| (|Symbol|))))
       (PROGN
        (QSETREFV |$| 72
         (CONS (|dispatchFunction| |PSETCAT-;headRemainder;PSR;23|) |$|))
        (QSETREFV |$| 79
         (CONS (|dispatchFunction| |PSETCAT-;remainder;PSR;25|) |$|))
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        (QSETREFV |$| 86
         (CONS
          (|dispatchFunction| |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|)
          |$|))
        (QSETREFV |$| 89
         (CONS
          (|dispatchFunction| |PSETCAT-;rewriteIdealWithRemainder;LSL;27|)
          |$|)))))))
    |$|))))

(setf (get
 (QUOTE |PolynomialSetCategory&|)
 (QUOTE |infovec|))
 (LIST
  (QUOTE
   #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|) (|local| |#3|)
     (|local| |#4|) (|local| |#5|) (|List| 10) (0 . |members|) (|List| 9)
     (5 . |variables|) (|Boolean|) (10 . |<|) (|List| |$|) (16 . |concat|)
     (21 . |removeDuplicates|) (|Mapping| 15 9 9) (26 . |sort|)
     (32 . |mvar|) |PSETCAT-;variables;SL;4| (37 . |ground?|)
     (|Mapping| 15 10) (42 . |remove|) |PSETCAT-;mainVariables;SL;5|
     (48 . |=|) |PSETCAT-;mainVariable?;VarSetSB;6| (54 . |construct|)
     |PSETCAT-;collectUnder;SVarSetS;7| |PSETCAT-;collectUpper;SVarSetS;8|
     |PSETCAT-;collect;SVarSetS;9| (|Record| (|:| |under| |$|)
     (|:| |floor| |$|) (|:| |upper| |$|)) |PSETCAT-;sort;SVarSetR;10|
     (|Set| 10) (59 . |brace|) (64 . |=|) |PSETCAT-;=;2SB;11|
     (70 . |degree|) (75 . |<|) (81 . |zero?|) (86 . |any?|)
     (|Mapping| 15 10 10) (92 . |sort|) |PSETCAT-;triangular?;SB;14|
     |PSETCAT-;trivialIdeal?;SB;15| (98 . |roughUnitIdeal?|)
     (103 . |sup|) (109 . |+|) (115 . |=|) (121 . |roughBase?|)
     (126 . |rewriteIdealWithRemainder|) (132 . |roughSubIdeal?|)
     (138 . |=|) (144 . |roughSubIdeal?|) (150 . |roughEqualIdeals?|)
     (156 . |quo|) (|Union| |$| (QUOTE "failed")) (162 . |exquo|)
     (168 . |One|) (172 . |reductum|) (177 . |subtractIfCan|)
     (183 . |leadingCoefficient|) (188 . |gcd|) (194 . |*|)
     (200 . |monomial|) (206 . |*|) (212 . |-|) (218 . |*|)
     (|Record| (|:| |num| 10) (|:| |den| 7)) (224 . |headRemainder|)
     (230 . |gcd|) (236 . |one?|) (241 . |exactQuotient!|)
     (247 . |headRemainder|) (253 . |+|) (|Record| (|:| |rnum| 7)
     (|:| |polnum| 10) (|:| |den| 7)) (259 . |remainder|)
     (265 . |trivialIdeal?|) (270 . |roughUnitIdeal?|)
     (275 . |Zero|) (279 . |One|) (283 . |primitivePart!|)
     (288 . |removeDuplicates|) (293 . |rewriteIdealWithHeadRemainder|)
     (299 . |remainder|) (305 . |unitCanonical|)
     (310 . |rewriteIdealWithRemainder|)))
  (QUOTE #(|variables| 316 |trivialIdeal?| 321 |triangular?| 326
     |sort| 331 |roughUnitIdeal?| 337 |roughSubIdeal?| 342
     |roughEqualIdeals?| 348 |roughBase?| 354 |rewriteIdealWithRemainder|
     359 |rewriteIdealWithHeadRemainder| 365 |remainder| 371 |mainVariables|
     377 |mainVariable?| 382 |headRemainder| 388 |collectUpper| 394
     |collectUnder| 400 |collect| 406 |=| 412))
  (QUOTE NIL)
  (CONS
   (|makeByteWordVec2| 1 (QUOTE NIL))
```

```
  (CONS
   (QUOTE #())
   (CONS
    (QUOTE #())
    (|makeByteWordVec2| 89 (QUOTE (1 6 11 0 12 1 10 13 0 14 2 9 15 0
     0 16 1 13 0 17 18 1 13 0 0 19 2 13 0 20 0 21 1 10 9 0 22 1 10 15 0 24
     2 11 0 25 0 26 2 9 15 0 0 28 1 6 0 11 30 1 36 0 11 37 2 36 15 0 0 38 1
     10 8 0 40 2 8 15 0 0 41 1 10 15 0 42 2 11 15 25 0 43 2 11 0 44 0 45 1
     0 15 0 48 2 8 0 0 0 49 2 8 0 0 0 50 2 8 15 0 0 51 1 0 15 0 52 2 6 11 11
     0 53 2 0 15 0 0 54 2 6 15 0 0 55 2 6 15 0 0 56 2 0 15 0 0 57 2 7 0 0 0
     58 2 7 59 0 0 60 0 7 0 61 1 10 0 0 62 2 8 59 0 0 63 1 10 7 0 64 2 7 0
     0 0 65 2 10 0 7 0 66 2 10 0 7 8 67 2 10 0 0 0 68 2 10 0 0 0 69 2 7 0
     0 0 70 2 0 71 10 0 72 2 10 7 7 0 73 1 7 15 0 74 2 10 0 0 7 75 2 6 71
     10 0 76 2 10 0 0 0 77 2 0 78 10 0 79 1 6 15 0 80 1 6 15 0 81 0 10 0
     82 0 10 0 83 1 10 0 0 84 1 11 0 0 85 2 0 11 11 0 86 2 6 78 10 0 87 1
     10 0 0 88 2 0 11 11 0 89 1 0 13 0 23 1 0 15 0 47 1 0 15 0 46 2 0 34
     0 9 35 1 0 15 0 48 2 0 15 0 0 54 2 0 15 0 0 57 1 0 15 0 52 2 0 11 11
     0 89 2 0 11 11 0 86 2 0 78 10 0 79 1 0 13 0 27 2 0 15 9 0 29 2 0 71
     10 0 72 2 0 0 0 9 32 2 0 0 0 9 31 2 0 0 0 9 33 2 0 15 0 0 39))))))
 (QUOTE |lookupComplete|)))
```

21.47 QFCAT.lsp BOOTSTRAP

QFCAT depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **QFCAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **QFCAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— QFCAT.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |QuotientFieldCategory;CAT| (QUOTE NIL))

(SETQ |QuotientFieldCategory;AL| (QUOTE NIL))

(DEFUN |QuotientFieldCategory| (#1=#:G103631)
 (declare (special |QuotientFieldCategory;AL|))
 (LET (#2=#:G103632)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |QuotientFieldCategory;AL|))
     (CDR #2#))
   (T
    (SETQ |QuotientFieldCategory;AL|
     (|cons5|
      (CONS (|devaluate| #1#) (SETQ #2# (|QuotientFieldCategory;| #1#)))
      |QuotientFieldCategory;AL|))
    #2#))))
```

```
(DEFUN |QuotientFieldCategory;| (|t#1|)
 (declare (special |QuotientFieldCategory;CAT|))
 (PROG (#1=#:G103630)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (COND
       (|QuotientFieldCategory;CAT|)
       ((QUOTE T)
        (LETT |QuotientFieldCategory;CAT|
         (|Join|
          (|Field|)
          (|Algebra| (QUOTE |t#1|))
          (|RetractableTo| (QUOTE |t#1|))
          (|FullyEvalableOver| (QUOTE |t#1|))
          (|DifferentialExtension| (QUOTE |t#1|))
          (|FullyLinearlyExplicitRingOver| (QUOTE |t#1|))
          (|Patternable| (QUOTE |t#1|))
          (|FullyPatternMatchable| (QUOTE |t#1|))
          (|mkCategory|
           (QUOTE |domain|)
           (QUOTE (
            ((|/| (|$| |t#1| |t#1|)) T)
            ((|numer| (|t#1| |$|)) T)
            ((|denom| (|t#1| |$|)) T)
            ((|numerator| (|$| |$|)) T)
            ((|denominator| (|$| |$|)) T)
            ((|wholePart| (|t#1| |$|)) (|has| |t#1| (|EuclideanDomain|)))
            ((|fractionPart| (|$| |$|)) (|has| |t#1| (|EuclideanDomain|)))
            ((|random| (|$|)) (|has| |t#1| (|IntegerNumberSystem|)))
            ((|ceiling| (|t#1| |$|)) (|has| |t#1| (|IntegerNumberSystem|)))
            ((|floor| (|t#1| |$|)) (|has| |t#1| (|IntegerNumberSystem|)))))
          (QUOTE (
            ((|StepThrough|) (|has| |t#1| (|StepThrough|)))
            ((|RetractableTo| (|Integer|))
               (|has| |t#1| (|RetractableTo| (|Integer|))))
            ((|RetractableTo| (|Fraction| (|Integer|)))
               (|has| |t#1| (|RetractableTo| (|Integer|))))
            ((|OrderedSet|) (|has| |t#1| (|OrderedSet|)))
            ((|OrderedIntegralDomain|) (|has| |t#1| (|OrderedIntegralDomain|)))
            ((|RealConstant|) (|has| |t#1| (|RealConstant|)))
            ((|ConvertibleTo| (|InputForm|))
               (|has| |t#1| (|ConvertibleTo| (|InputForm|))))
            ((|CharacteristicZero|) (|has| |t#1| (|CharacteristicZero|)))
            ((|CharacteristicNonZero|) (|has| |t#1| (|CharacteristicNonZero|)))
            ((|RetractableTo| (|Symbol|))
               (|has| |t#1| (|RetractableTo| (|Symbol|))))
            ((|PolynomialFactorizationExplicit|)
               (|has| |t#1| (|PolynomialFactorizationExplicit|)))))
          (QUOTE NIL) NIL)) . #2=(|QuotientFieldCategory|))))) . #2#)
    (SETELT #1# 0
```

```
    (LIST (QUOTE |QuotientFieldCategory|) (|devaluate| |t#1|)))))))
```

———-

21.48 QFCAT-.lsp BOOTSTRAP

QFCAT- depends on **QFCAT**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **QFCAT-** category which we can write into the **MID** directory. We compile the lisp code and copy the **QFCAT-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— QFCAT-.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(DEFUN |QFCAT-;numerator;2A;1| (|x| |$|)
 (SPADCALL (SPADCALL |x| (QREFELT |$| 8)) (QREFELT |$| 9)))

(DEFUN |QFCAT-;denominator;2A;2| (|x| |$|)
 (SPADCALL (SPADCALL |x| (QREFELT |$| 11)) (QREFELT |$| 9)))

(DEFUN |QFCAT-;init;A;3| (|$|)
 (SPADCALL (|spadConstant| |$| 13) (|spadConstant| |$| 14) (QREFELT |$| 15)))

(DEFUN |QFCAT-;nextItem;AU;4| (|n| |$|)
 (PROG (|m|)
  (RETURN
   (SEQ
    (LETT |m|
     (SPADCALL
      (SPADCALL |n| (QREFELT |$| 8))
      (QREFELT |$| 18))
     |QFCAT-;nextItem;AU;4|)
    (EXIT
     (COND
      ((QEQCAR |m| 1)
       (|error| "We seem to have a Fraction of a finite object"))
      ((QUOTE T)
       (CONS 0
        (SPADCALL (QCDR |m|) (|spadConstant| |$| 14) (QREFELT |$| 15))))))))))

(DEFUN |QFCAT-;map;M2A;5| (|fn| |x| |$|)
 (SPADCALL
  (SPADCALL (SPADCALL |x| (QREFELT |$| 8)) |fn|)
  (SPADCALL (SPADCALL |x| (QREFELT |$| 11)) |fn|)
  (QREFELT |$| 15)))

(DEFUN |QFCAT-;reducedSystem;MM;6| (|m| |$|)
 (SPADCALL |m| (QREFELT |$| 26)))

(DEFUN |QFCAT-;characteristic;Nni;7| (|$|)
```

```
 (SPADCALL (QREFELT |$| 30)))

(DEFUN |QFCAT-;differentiate;AMA;8| (|x| |deriv| |$|)
 (PROG (|n| |d|)
  (RETURN
   (SEQ
    (LETT |n| (SPADCALL |x| (QREFELT |$| 8)) |QFCAT-;differentiate;AMA;8|)
    (LETT |d| (SPADCALL |x| (QREFELT |$| 11)) |QFCAT-;differentiate;AMA;8|)
    (EXIT
     (SPADCALL
      (SPADCALL
       (SPADCALL (SPADCALL |n| |deriv|) |d| (QREFELT |$| 32))
       (SPADCALL |n| (SPADCALL |d| |deriv|) (QREFELT |$| 32))
       (QREFELT |$| 33))
      (SPADCALL |d| 2 (QREFELT |$| 35)) (QREFELT |$| 15)))))))

(DEFUN |QFCAT-;convert;AIf;9| (|x| |$|)
 (SPADCALL
  (SPADCALL (SPADCALL |x| (QREFELT |$| 8)) (QREFELT |$| 38))
  (SPADCALL (SPADCALL |x| (QREFELT |$| 11)) (QREFELT |$| 38))
  (QREFELT |$| 39)))

(DEFUN |QFCAT-;convert;AF;10| (|x| |$|)
 (SPADCALL
  (SPADCALL (SPADCALL |x| (QREFELT |$| 8)) (QREFELT |$| 42))
  (SPADCALL (SPADCALL |x| (QREFELT |$| 11)) (QREFELT |$| 42))
  (QREFELT |$| 43)))

(DEFUN |QFCAT-;convert;ADf;11| (|x| |$|)
 (|/|
  (SPADCALL (SPADCALL |x| (QREFELT |$| 8)) (QREFELT |$| 46))
  (SPADCALL (SPADCALL |x| (QREFELT |$| 11)) (QREFELT |$| 46))))

(DEFUN |QFCAT-;<;2AB;12| (|x| |y| |$|)
 (SPADCALL
  (SPADCALL
   (SPADCALL |x| (QREFELT |$| 8))
   (SPADCALL |y| (QREFELT |$| 11))
   (QREFELT |$| 32))
  (SPADCALL
   (SPADCALL |y| (QREFELT |$| 8))
   (SPADCALL |x| (QREFELT |$| 11))
   (QREFELT |$| 32))
  (QREFELT |$| 49)))

(DEFUN |QFCAT-;<;2AB;13| (|x| |y| |$|)
 (PROG (|#G19| |#G20| |#G21| |#G22|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL
       (SPADCALL |x| (QREFELT |$| 11))
       (|spadConstant| |$| 51)
       (QREFELT |$| 49))
```

```
      (PROGN
       (LETT |#G19| |y| |QFCAT-;<;2AB;13|)
       (LETT |#G20| |x| |QFCAT-;<;2AB;13|)
       (LETT |x| |#G19| |QFCAT-;<;2AB;13|)
       (LETT |y| |#G20| |QFCAT-;<;2AB;13|))))
    (COND
     ((SPADCALL
        (SPADCALL |y| (QREFELT |$| 11))
        (|spadConstant| |$| 51)
        (QREFELT |$| 49))
       (PROGN
        (LETT |#G21| |y| |QFCAT-;<;2AB;13|)
        (LETT |#G22| |x| |QFCAT-;<;2AB;13|)
        (LETT |x| |#G21| |QFCAT-;<;2AB;13|)
        (LETT |y| |#G22| |QFCAT-;<;2AB;13|))))
    (EXIT
     (SPADCALL
      (SPADCALL
       (SPADCALL |x| (QREFELT |$| 8))
       (SPADCALL |y| (QREFELT |$| 11))
       (QREFELT |$| 32))
      (SPADCALL
       (SPADCALL |y| (QREFELT |$| 8))
       (SPADCALL |x| (QREFELT |$| 11))
       (QREFELT |$| 32))
      (QREFELT |$| 49)))))))

(DEFUN |QFCAT-;<;2AB;14| (|x| |y| |$|)
 (SPADCALL
  (SPADCALL
   (SPADCALL |x| (QREFELT |$| 8))
   (SPADCALL |y| (QREFELT |$| 11))
   (QREFELT |$| 32))
  (SPADCALL
   (SPADCALL |y| (QREFELT |$| 8))
   (SPADCALL |x| (QREFELT |$| 11))
   (QREFELT |$| 32))
  (QREFELT |$| 49)))

(DEFUN |QFCAT-;fractionPart;2A;15| (|x| |$|)
 (SPADCALL |x|
  (SPADCALL (SPADCALL |x| (QREFELT |$| 52)) (QREFELT |$| 9))
  (QREFELT |$| 53)))

(DEFUN |QFCAT-;coerce;SA;16| (|s| |$|)
 (SPADCALL (SPADCALL |s| (QREFELT |$| 56)) (QREFELT |$| 9)))

(DEFUN |QFCAT-;retract;AS;17| (|x| |$|)
 (SPADCALL (SPADCALL |x| (QREFELT |$| 58)) (QREFELT |$| 59)))

(DEFUN |QFCAT-;retractIfCan;AU;18| (|x| |$|)
 (PROG (|r|)
  (RETURN
   (SEQ
```

```
    (LETT |r| (SPADCALL |x| (QREFELT |$| 62)) |QFCAT-;retractIfCan;AU;18|)
    (EXIT
     (COND
      ((QEQCAR |r| 1) (CONS 1 "failed"))
      ((QUOTE T) (SPADCALL (QCDR |r|) (QREFELT |$| 64))))))))

(DEFUN |QFCAT-;convert;AP;19| (|x| |$|)
 (SPADCALL
  (SPADCALL (SPADCALL |x| (QREFELT |$| 8)) (QREFELT |$| 67))
  (SPADCALL (SPADCALL |x| (QREFELT |$| 11)) (QREFELT |$| 67))
  (QREFELT |$| 68)))

(DEFUN |QFCAT-;patternMatch;AP2Pmr;20| (|x| |p| |l| |$|)
 (SPADCALL |x| |p| |l| (QREFELT |$| 72)))

(DEFUN |QFCAT-;convert;AP;21| (|x| |$|)
 (SPADCALL
  (SPADCALL (SPADCALL |x| (QREFELT |$| 8)) (QREFELT |$| 76))
  (SPADCALL (SPADCALL |x| (QREFELT |$| 11)) (QREFELT |$| 76))
  (QREFELT |$| 77)))

(DEFUN |QFCAT-;patternMatch;AP2Pmr;22| (|x| |p| |l| |$|)
 (SPADCALL |x| |p| |l| (QREFELT |$| 81)))

(DEFUN |QFCAT-;coerce;FA;23| (|x| |$|)
 (SPADCALL
  (SPADCALL (SPADCALL |x| (QREFELT |$| 86)) (QREFELT |$| 87))
  (SPADCALL (SPADCALL |x| (QREFELT |$| 88)) (QREFELT |$| 87))
  (QREFELT |$| 89)))

(DEFUN |QFCAT-;retract;AI;24| (|x| |$|)
 (SPADCALL (SPADCALL |x| (QREFELT |$| 58)) (QREFELT |$| 91)))

(DEFUN |QFCAT-;retractIfCan;AU;25| (|x| |$|)
 (PROG (|u|)
  (RETURN
   (SEQ
    (LETT |u| (SPADCALL |x| (QREFELT |$| 62)) |QFCAT-;retractIfCan;AU;25|)
    (EXIT
     (COND
      ((QEQCAR |u| 1) (CONS 1 "failed"))
      ((QUOTE T) (SPADCALL (QCDR |u|) (QREFELT |$| 94))))))))

(DEFUN |QFCAT-;random;A;26| (|$|)
 (PROG (|d|)
  (RETURN
   (SEQ
    (SEQ
      G190
      (COND
        ((NULL
          (SPADCALL
           (LETT |d| (SPADCALL (QREFELT |$| 96)) |QFCAT-;random;A;26|)
           (QREFELT |$| 97)))
```

```
         (GO G191)))
      (SEQ (EXIT |d|))
      NIL
      (GO G190)
      G191
     (EXIT NIL))
    (EXIT (SPADCALL (SPADCALL (QREFELT |$| 96)) |d| (QREFELT |$| 15)))))))

(DEFUN |QFCAT-;reducedSystem;MVR;27| (|m| |v| |$|)
 (PROG (|n|)
  (RETURN
   (SEQ
    (LETT |n|
     (SPADCALL
      (SPADCALL (SPADCALL |v| (QREFELT |$| 100)) |m| (QREFELT |$| 101))
      (QREFELT |$| 102))
     |QFCAT-;reducedSystem;MVR;27|)
    (EXIT
     (CONS
      (SPADCALL |n|
       (SPADCALL |n| (QREFELT |$| 103))
       (SPADCALL |n| (QREFELT |$| 104))
       (|+| 1 (SPADCALL |n| (QREFELT |$| 105)))
       (SPADCALL |n| (QREFELT |$| 106))
       (QREFELT |$| 107))
      (SPADCALL |n| (SPADCALL |n| (QREFELT |$| 105)) (QREFELT |$| 109))))))))

(DEFUN |QuotientFieldCategory&| (|#1| |#2|)
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|QuotientFieldCategory&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |QuotientFieldCategory&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (make-array 119) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3
     (LETT |pv$|
      (|buildPredVector| 0 0 (LIST
       (|HasCategory| |#2| (QUOTE (|PolynomialFactorizationExplicit|)))
       (|HasCategory| |#2| (QUOTE (|IntegerNumberSystem|)))
       (|HasCategory| |#2| (QUOTE (|EuclideanDomain|)))
       (|HasCategory| |#2| (QUOTE (|RetractableTo| (|Symbol|))))
       (|HasCategory| |#2| (QUOTE (|CharacteristicNonZero|)))
       (|HasCategory| |#2| (QUOTE (|CharacteristicZero|)))
       (|HasCategory| |#2| (QUOTE (|ConvertibleTo| (|InputForm|))))
       (|HasCategory| |#2| (QUOTE (|RealConstant|)))
       (|HasCategory| |#2| (QUOTE (|OrderedIntegralDomain|)))
       (|HasCategory| |#2| (QUOTE (|OrderedSet|)))
       (|HasCategory| |#2| (QUOTE (|RetractableTo| (|Integer|))))
       (|HasCategory| |#2| (QUOTE (|StepThrough|))))) . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
```

```
(COND
 ((|testBitVector| |pv$| 12)
   (PROGN
    (QSETREFV |$| 16 (CONS (|dispatchFunction| |QFCAT-;init;A;3|) |$|))
    (QSETREFV |$| 20
     (CONS (|dispatchFunction| |QFCAT-;nextItem;AU;4|) |$|)))))
(COND
 ((|testBitVector| |pv$| 7)
   (QSETREFV |$| 40
    (CONS (|dispatchFunction| |QFCAT-;convert;AIf;9|) |$|))))
(COND
 ((|testBitVector| |pv$| 8)
   (PROGN
    (QSETREFV |$| 44
     (CONS (|dispatchFunction| |QFCAT-;convert;AF;10|) |$|))
    (QSETREFV |$| 47
     (CONS (|dispatchFunction| |QFCAT-;convert;ADf;11|) |$|)))))
(COND
 ((|testBitVector| |pv$| 9)
  (COND
   ((|HasAttribute| |#2| (QUOTE |canonicalUnitNormal|))
     (QSETREFV |$| 50 (CONS (|dispatchFunction| |QFCAT-;<;2AB;12|) |$|)))
   ((QUOTE T)
     (QSETREFV |$| 50 (CONS (|dispatchFunction| |QFCAT-;<;2AB;13|) |$|)))))
 ((|testBitVector| |pv$| 10)
  (QSETREFV |$| 50 (CONS (|dispatchFunction| |QFCAT-;<;2AB;14|) |$|))))
(COND
 ((|testBitVector| |pv$| 3)
   (QSETREFV |$| 54
    (CONS (|dispatchFunction| |QFCAT-;fractionPart;2A;15|) |$|))))
(COND
 ((|testBitVector| |pv$| 4)
  (PROGN
   (QSETREFV |$| 57 (CONS (|dispatchFunction| |QFCAT-;coerce;SA;16|) |$|))
   (QSETREFV |$| 60 (CONS (|dispatchFunction| |QFCAT-;retract;AS;17|) |$|))
   (QSETREFV |$| 65
    (CONS (|dispatchFunction| |QFCAT-;retractIfCan;AU;18|) |$|)))))
(COND
 ((|HasCategory| |#2| (QUOTE (|ConvertibleTo| (|Pattern| (|Integer|)))))
  (PROGN
   (QSETREFV |$| 69 (CONS (|dispatchFunction| |QFCAT-;convert;AP;19|) |$|))
   (COND
    ((|HasCategory| |#2| (QUOTE (|PatternMatchable| (|Integer|))))
      (QSETREFV |$| 74
       (CONS
        (|dispatchFunction| |QFCAT-;patternMatch;AP2Pmr;20|) |$|)))))))
(COND
 ((|HasCategory| |#2| (QUOTE (|ConvertibleTo| (|Pattern| (|Float|)))))
  (PROGN
   (QSETREFV |$| 78 (CONS (|dispatchFunction| |QFCAT-;convert;AP;21|) |$|))
   (COND
    ((|HasCategory| |#2| (QUOTE (|PatternMatchable| (|Float|))))
     (QSETREFV |$| 83
      (CONS (|dispatchFunction| |QFCAT-;patternMatch;AP2Pmr;22|) |$|)))))))
```

```
    (COND
     ((|testBitVector| |pv$| 11)
      (PROGN
       (QSETREFV |$| 90 (CONS (|dispatchFunction| |QFCAT-;coerce;FA;23|) |$|))
       (COND
        ((|domainEqual| |#2| (|Integer|)))
        ((QUOTE T)
         (PROGN
          (QSETREFV |$| 92
           (CONS (|dispatchFunction| |QFCAT-;retract;AI;24|) |$|))
          (QSETREFV |$| 95
           (CONS (|dispatchFunction| |QFCAT-;retractIfCan;AU;25|) |$|))))))))
    (COND
     ((|testBitVector| |pv$| 2)
       (QSETREFV |$| 98 (CONS (|dispatchFunction| |QFCAT-;random;A;26|) |$|))))
    |$|))))

(setf (get
 (QUOTE |QuotientFieldCategory&|)
 (QUOTE |infovec|))
 (LIST
  (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|)
   (0 . |numer|) (5 . |coerce|) |QFCAT-;numerator;2A;1| (10 . |denom|)
   |QFCAT-;denominator;2A;2| (15 . |init|) (19 . |One|) (23 . |/|)
   (29 . |init|) (|Union| |$| (QUOTE "failed")) (33 . |nextItem|)
   (38 . |One|) (42 . |nextItem|) (|Mapping| 7 7) |QFCAT-;map;M2A;5|
   (|Matrix| 7) (|Matrix| 6) (|MatrixCommonDenominator| 7 6)
   (47 . |clearDenominator|) (|Matrix| |$|) |QFCAT-;reducedSystem;MM;6|
   (|NonNegativeInteger|) (52 . |characteristic|)
   |QFCAT-;characteristic;Nni;7| (56 . |*|) (62 . |-|)
   (|PositiveInteger|) (68 . |**|) |QFCAT-;differentiate;AMA;8|
   (|InputForm|) (74 . |convert|) (79 . |/|) (85 . |convert|)
   (|Float|) (90 . |convert|) (95 . |/|) (101 . |convert|)
   (|DoubleFloat|) (106 . |convert|) (111 . |convert|) (|Boolean|)
   (116 . |<|) (122 . |<|) (128 . |Zero|) (132 . |wholePart|)
   (137 . |-|) (143 . |fractionPart|) (|Symbol|) (148 . |coerce|)
   (153 . |coerce|) (158 . |retract|) (163 . |retract|)
   (168 . |retract|) (|Union| 7 (QUOTE "failed")) (173 . |retractIfCan|)
   (|Union| 55 (QUOTE "failed")) (178 . |retractIfCan|)
   (183 . |retractIfCan|) (|Pattern| 84) (188 . |convert|)
   (193 . |/|) (199 . |convert|) (|PatternMatchResult| 84 6)
   (|PatternMatchQuotientFieldCategory| 84 7 6) (204 . |patternMatch|)
   (|PatternMatchResult| 84 |$|) (211 . |patternMatch|) (|Pattern| 41)
   (218 . |convert|) (223 . |/|) (229 . |convert|)
   (|PatternMatchResult| 41 6)
   (|PatternMatchQuotientFieldCategory| 41 7 6) (234 . |patternMatch|)
   (|PatternMatchResult| 41 |$|) (241 . |patternMatch|) (|Integer|)
   (|Fraction| 84) (248 . |numer|) (253 . |coerce|) (258 . |denom|)
   (263 . |/|) (269 . |coerce|) (274 . |retract|) (279 . |retract|)
   (|Union| 84 (QUOTE "failed")) (284 . |retractIfCan|)
   (289 . |retractIfCan|) (294 . |random|) (298 . |zero?|)
   (303 . |random|) (|Vector| 6) (307 . |coerce|) (312 . |horizConcat|)
   (318 . |reducedSystem|) (323 . |minRowIndex|) (328 . |maxRowIndex|)
   (333 . |minColIndex|) (338 . |maxColIndex|) (343 . |subMatrix|)
```

```
 (|Vector| 7) (352 . |column|) (|Record| (|:| |mat| 23)
 (|:| |vec| 108)) (|Vector| |$|) |QFCAT-;reducedSystem;MVR;27|
 (|Union| 85 (QUOTE "failed")) (|Record| (|:| |mat| 115)
 (|:| |vec| (|Vector| 84))) (|Matrix| 84) (|List| 55) (|List| 29)
 (|OutputForm|)))
(QUOTE #(|retractIfCan| 358 |retract| 368 |reducedSystem| 378
 |random| 389 |patternMatch| 393 |numerator| 407 |nextItem| 412
 |map| 417 |init| 423 |fractionPart| 427 |differentiate| 432
 |denominator| 438 |convert| 443 |coerce| 468 |characteristic| 478
 |<| 482))
(QUOTE NIL)
(CONS
 (|makeByteWordVec2| 1 (QUOTE NIL))
 (CONS
  (QUOTE #())
  (CONS (QUOTE #())
   (|makeByteWordVec2| 112 (QUOTE
    (1 6 7 0 8 1 6 0 7 9 1 6 7 0 11 0 7 0 13 0 7 0 14 2 6 0 7 7 15 0
     0 0 16 1 7 17 0 18 0 6 0 19 1 0 17 0 20 1 25 23 24 26 0 7 29 30 2
     7 0 0 0 32 2 7 0 0 0 33 2 7 0 0 34 35 1 7 37 0 38 2 37 0 0 0 39 1
     0 37 0 40 1 7 41 0 42 2 41 0 0 0 43 1 0 41 0 44 1 7 45 0 46 1 0 45
     0 47 2 7 48 0 0 49 2 0 48 0 0 50 0 7 0 51 1 6 7 0 52 2 6 0 0 0 53 1
     0 0 0 54 1 7 0 55 56 1 0 0 55 57 1 6 7 0 58 1 7 55 0 59 1 0 55 0 60
     1 6 61 0 62 1 7 63 0 64 1 0 63 0 65 1 7 66 0 67 2 66 0 0 0 68 1 0
     66 0 69 3 71 70 6 66 70 72 3 0 73 0 66 73 74 1 7 75 0 76 2 75 0 0
     0 77 1 0 75 0 78 3 80 79 6 75 79 81 3 0 82 0 75 82 83 1 85 84 0 86
     1 6 0 84 87 1 85 84 0 88 2 6 0 0 0 89 1 0 0 85 90 1 7 84 0 91 1 0
     84 0 92 1 7 93 0 94 1 0 93 0 95 0 7 0 96 1 7 48 0 97 0 0 0 98 1 24
     0 99 100 2 24 0 0 0 101 1 6 23 27 102 1 23 84 0 103 1 23 84 0 104
     1 23 84 0 105 1 23 84 0 106 5 23 0 0 84 84 84 84 107 2 23 108 0 84
     109 1 0 93 0 95 1 0 63 0 65 1 0 84 0 92 1 0 55 0 60 2 0 110 27 111
     112 1 0 23 27 28 0 0 0 98 3 0 82 0 75 82 83 3 0 73 0 66 73 74 1 0
     0 0 10 1 0 17 0 20 2 0 0 21 0 22 0 0 0 16 1 0 0 0 54 2 0 0 0 21 36
     1 0 0 0 12 1 0 45 0 47 1 0 37 0 40 1 0 41 0 44 1 0 66 0 69 1 0 75 0
     78 1 0 0 55 57 1 0 0 85 90 0 0 29 31 2 0 48 0 0 50))))))
(QUOTE |lookupComplete|)))
```

21.49 RCAGG.lsp BOOTSTRAP

RCAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **RCAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **RCAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— RCAGG.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)
```

```
(SETQ |RecursiveAggregate;CAT| (QUOTE NIL))

(SETQ |RecursiveAggregate;AL| (QUOTE NIL))

(DEFUN |RecursiveAggregate| (#1=#:G84501)
 (declare (special |RecursiveAggregate;AL|))
 (LET (#2=#:G84502)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |RecursiveAggregate;AL|)) (CDR #2#))
   (T
    (SETQ |RecursiveAggregate;AL|
     (|cons5|
      (CONS (|devaluate| #1#) (SETQ #2# (|RecursiveAggregate;| #1#)))
      |RecursiveAggregate;AL|))
    #2#))))

(DEFUN |RecursiveAggregate;| (|t#1|)
 (declare (special |RecursiveAggregate;CAT|))
 (PROG (#1=#:G84500)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (COND
       (|RecursiveAggregate;CAT|)
       ((QUOTE T)
        (LETT |RecursiveAggregate;CAT|
         (|Join|
          (|HomogeneousAggregate| (QUOTE |t#1|))
          (|mkCategory|
           (QUOTE |domain|)
           (QUOTE (
            ((|children| ((|List| |$|) |$|)) T)
            ((|nodes| ((|List| |$|) |$|)) T)
            ((|leaf?| ((|Boolean|) |$|)) T)
            ((|value| (|t#1| |$|)) T)
            ((|elt| (|t#1| |$| "value")) T)
            ((|cyclic?| ((|Boolean|) |$|)) T)
            ((|leaves| ((|List| |t#1|) |$|)) T)
            ((|distance| ((|Integer|) |$| |$|)) T)
            ((|child?| ((|Boolean|) |$| |$|)) (|has| |t#1| (|SetCategory|)))
            ((|node?| ((|Boolean|) |$| |$|)) (|has| |t#1| (|SetCategory|)))
            ((|setchildren!| (|$| |$| (|List| |$|)))
              (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
            ((|setelt| (|t#1| |$| "value" |t#1|))
              (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
            ((|setvalue!| (|t#1| |$| |t#1|))
              (|has| |$| (ATTRIBUTE |shallowlyMutable|)))))
           NIL
           (QUOTE ((|List| |$|) (|Boolean|) (|Integer|) (|List| |t#1|)))
           NIL))
         . #2=(|RecursiveAggregate|)))))
      . #2#)
```

```
(SETELT #1# 0 (LIST (QUOTE |RecursiveAggregate|) (|devaluate| |t#1|)))))))
```

———-

21.50 RCAGG-.lsp BOOTSTRAP

RCAGG- depends on **RCAGG**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **RCAGG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **RCAGG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— RCAGG-.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(DEFUN |RCAGG-;elt;AvalueS;1| (|x| G84515 |$|)
 (declare (ignore G84515))
 (SPADCALL |x| (QREFELT |$| 8)))

(DEFUN |RCAGG-;setelt;Avalue2S;2| (|x| G84517 |y| |$|)
 (declare (ignore G84517))
 (SPADCALL |x| |y| (QREFELT |$| 11)))

(DEFUN |RCAGG-;child?;2AB;3| (|x| |l| |$|)
 (SPADCALL |x| (SPADCALL |l| (QREFELT |$| 14)) (QREFELT |$| 17)))

(DEFUN |RecursiveAggregate&| (|#1| |#2|)
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|RecursiveAggregate&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |RecursiveAggregate&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (make-array 19) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3
     (LETT |pv$|
      (|buildPredVector| 0 0
       (LIST
        (|HasAttribute| |#1| (QUOTE |shallowlyMutable|))
        (|HasCategory| |#2| (QUOTE (|SetCategory|)))))
      . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
    (COND
     ((|testBitVector| |pv$| 1)
      (QSETREFV |$| 12
       (CONS (|dispatchFunction| |RCAGG-;setelt;Avalue2S;2|) |$|))))
    (COND
     ((|testBitVector| |pv$| 2)
```

```
      (QSETREFV |$| 18 (CONS (|dispatchFunction| |RCAGG-;child?;2AB;3|) |$|))))
    |$|))))

(setf (get
 (QUOTE |RecursiveAggregate&|)
 (QUOTE |infovec|))
 (LIST
  (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|)
   (0 . |value|) (QUOTE "value") |RCAGG-;elt;AvalueS;1| (5 . |setvalue!|)
   (11 . |setelt|) (|List| |$|) (18 . |children|) (|Boolean|) (|List| 6)
   (23 . |member?|) (29 . |child?|)))
  (QUOTE #(|setelt| 35 |elt| 42 |child?| 48))
  (QUOTE NIL)
  (CONS (|makeByteWordVec2| 1 (QUOTE NIL))
 (CONS
  (QUOTE #())
  (CONS
   (QUOTE #())
   (|makeByteWordVec2| 18 (QUOTE (1 6 7 0 8 2 6 7 0 7 11 3 0 7 0 9 7 12
     1 6 13 0 14 2 16 15 6 0 17 2 0 15 0 0 18 3 0 7 0 9 7 12 2 0 7 0 9
     10 2 0 15 0 0 18))))))
  (QUOTE |lookupComplete|)))
```

21.51 RING.lsp BOOTSTRAP

RING depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **RING** category which we can write into the **MID** directory. We compile the lisp code and copy the **RING.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— RING.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |Ring;AL| (QUOTE NIL))

(DEFUN |Ring| NIL
  (declare (special |Ring;AL|))
    (COND
      (|Ring;AL|)
      (T (SETQ |Ring;AL| (|Ring;|)))))

(DEFUN |Ring;| NIL
  (PROG (#1=#:G82787)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|Rng|)
```

```
            (|Monoid|)
            (|LeftModule| (QUOTE |$|))
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|characteristic| ((|NonNegativeInteger|))) T)
                ((|coerce| (|$| (|Integer|))) T)))
              (QUOTE ((|unitsKnown| T)))
              (QUOTE ((|Integer|) (|NonNegativeInteger|)))
              NIL))
           |Ring|)
        (SETELT #1# 0 (QUOTE (|Ring|)))))))

(setf (get (QUOTE |Ring|) (QUOTE NILADIC)) T)
```

21.52 RING-.lsp BOOTSTRAP

RING- depends on **RING**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **RING-** category which we can write into the **MID** directory. We compile the lisp code and copy the **RING-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— RING-.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(DEFUN |RING-;coerce;IS;1| (|n| |$|)
  (SPADCALL |n| (|spadConstant| |$| 7) (QREFELT |$| 9)))

(DEFUN |Ring&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|Ring&|))
        (LETT |dv$| (LIST (QUOTE |Ring&|) |DV$1|) . #1#)
        (LETT |$| (make-array 12) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        |$|))))

(setf (get
  (QUOTE |Ring&|)
  (QUOTE |infovec|))
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
```

```
        (|local| |#1|)
        (0 . |One|)
        (|Integer|)
        (4 . |*|)
        |RING-;coerce;IS;1|
        (|OutputForm|)))
  (QUOTE #(|coerce| 10))
  (QUOTE NIL)
  (CONS
    (|makeByteWordVec2| 1 (QUOTE NIL))
    (CONS
      (QUOTE #())
      (CONS
        (QUOTE #())
        (|makeByteWordVec2| 10 (QUOTE (0 6 0 7 2 6 0 8 0 9 1 0 0 8 10))))))
   (QUOTE |lookupComplete|)))
```

21.53 RNG.lsp BOOTSTRAP

RNG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **RNG** category which we can write into the **MID** directory. We compile the lisp code and copy the **RNG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— RNG.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |Rng;AL| (QUOTE NIL))

(DEFUN |Rng| NIL
  (declare (special |Rng;AL|))
    (COND
      (|Rng;AL|)
      (T (SETQ |Rng;AL| (|Rng;|)))))

(DEFUN |Rng;| NIL
  (PROG (#1=#:G82720)
    (RETURN
      (PROG1
        (LETT #1# (|Join| (|AbelianGroup|) (|SemiGroup|)) |Rng|)
        (SETELT #1# 0 (QUOTE (|Rng|)))))))

(setf (get (QUOTE |Rng|) (QUOTE NILADIC)) T)
```

21.54 RNS.lsp BOOTSTRAP

RNS depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **RNS** category which we can write into the **MID** directory. We compile the lisp code and copy the **RNS.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— RNS.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |RealNumberSystem;AL| (QUOTE NIL))

(DEFUN |RealNumberSystem| NIL
  (declare (special |RealNumberSystem;AL|))
    (COND
      (|RealNumberSystem;AL|)
      (T (SETQ |RealNumberSystem;AL| (|RealNumberSystem;|)))))

(DEFUN |RealNumberSystem;| NIL
  (PROG (#1=#:G105476)
    (RETURN
      (PROG1
        (LETT #1#
          (|sublisV|
            (PAIR
              (QUOTE (#2=#:G105472 #3=#:G105473 #4=#:G105474 #5=#:G105475))
              (LIST
                (QUOTE (|Integer|))
                (QUOTE (|Fraction| (|Integer|)))
                (QUOTE (|Pattern| (|Float|)))
                (QUOTE (|Float|))))
            (|Join|
              (|Field|)
              (|OrderedRing|)
              (|RealConstant|)
              (|RetractableTo| (QUOTE #2#))
              (|RetractableTo| (QUOTE #3#))
              (|RadicalCategory|)
              (|ConvertibleTo| (QUOTE #4#))
              (|PatternMatchable| (QUOTE #5#))
              (|CharacteristicZero|)
              (|mkCategory|
                (QUOTE |domain|)
                (QUOTE (
                  ((|norm| (|$| |$|)) T)
                  ((|ceiling| (|$| |$|)) T)
                  ((|floor| (|$| |$|)) T)
                  ((|wholePart| ((|Integer|) |$|)) T)
                  ((|fractionPart| (|$| |$|)) T)
                  ((|truncate| (|$| |$|)) T)
                  ((|round| (|$| |$|)) T)
```

```
                ((|abs| (|$| |$|)) T)))
               NIL
               (QUOTE ((|Integer|)))
               NIL)))
         |RealNumberSystem|)
       (SETELT #1# 0 (QUOTE (|RealNumberSystem|)))))))

(setf (get (QUOTE |RealNumberSystem|) (QUOTE NILADIC)) T)
```

———-

21.55 RNS-.lsp BOOTSTRAP

RNS- depends **RNS**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **RNS-** category which we can write into the **MID** directory. We compile the lisp code and copy the **RNS.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— RNS-.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(PUT
  (QUOTE |RNS-;characteristic;Nni;1|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL 0)))

(DEFUN |RNS-;characteristic;Nni;1| (|$|)
 (declare (ignore |$|))
 0)

(DEFUN |RNS-;fractionPart;2S;2| (|x| |$|)
  (SPADCALL |x| (SPADCALL |x| (QREFELT |$| 9)) (QREFELT |$| 10)))

(DEFUN |RNS-;truncate;2S;3| (|x| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 13))
      (SPADCALL
        (SPADCALL
          (SPADCALL |x| (QREFELT |$| 14))
          (QREFELT |$| 15))
        (QREFELT |$| 14)))
    ((QUOTE T) (SPADCALL |x| (QREFELT |$| 15)))))

(DEFUN |RNS-;round;2S;4| (|x| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 13))
      (SPADCALL
        (SPADCALL |x|
          (SPADCALL
```

```
            (|spadConstant| |$| 17)
            (SPADCALL 2 (QREFELT |$| 19))
            (QREFELT |$| 20))
          (QREFELT |$| 10))
        (QREFELT |$| 9)))
    ((QUOTE T)
      (SPADCALL
        (SPADCALL |x|
          (SPADCALL
            (|spadConstant| |$| 17)
            (SPADCALL 2 (QREFELT |$| 19))
            (QREFELT |$| 20))
          (QREFELT |$| 21))
        (QREFELT |$| 9)))))

(DEFUN |RNS-;norm;2S;5| (|x| |$|)
  (SPADCALL |x| (QREFELT |$| 23)))

(DEFUN |RNS-;coerce;FS;6| (|x| |$|)
  (SPADCALL
    (SPADCALL
      (SPADCALL |x| (QREFELT |$| 26))
      (QREFELT |$| 19))
    (SPADCALL
      (SPADCALL |x| (QREFELT |$| 27))
      (QREFELT |$| 19))
    (QREFELT |$| 20)))

(DEFUN |RNS-;convert;SP;7| (|x| |$|)
  (SPADCALL (SPADCALL |x| (QREFELT |$| 30)) (QREFELT |$| 32)))

(DEFUN |RNS-;floor;2S;8| (|x| |$|)
  (PROG (|x1|)
    (RETURN
      (SEQ
        (LETT |x1|
          (SPADCALL (SPADCALL |x| (QREFELT |$| 34)) (QREFELT |$| 19))
          |RNS-;floor;2S;8|)
        (EXIT
          (COND
            ((SPADCALL |x| |x1| (QREFELT |$| 35)) |x|)
            ((SPADCALL |x| (|spadConstant| |$| 36) (QREFELT |$| 37))
              (SPADCALL |x1| (|spadConstant| |$| 17) (QREFELT |$| 10)))
            ((QUOTE T) |x1|)))))))

(DEFUN |RNS-;ceiling;2S;9| (|x| |$|)
  (PROG (|x1|)
    (RETURN
      (SEQ
        (LETT |x1|
          (SPADCALL (SPADCALL |x| (QREFELT |$| 34)) (QREFELT |$| 19))
          |RNS-;ceiling;2S;9|)
        (EXIT
          (COND
```

```
            ((SPADCALL |x| |x1| (QREFELT |$| 35)) |x|)
            ((SPADCALL |x| (|spadConstant| |$| 36) (QREFELT |$| 37)) |x1|)
            ((QUOTE T)
              (SPADCALL |x1| (|spadConstant| |$| 17) (QREFELT |$| 21)))))))))

(DEFUN |RNS-;patternMatch;SP2Pmr;10| (|x| |p| |l| |$|)
  (PROG (|r|)
    (RETURN
      (SEQ
        (COND
          ((SPADCALL |p| (QREFELT |$| 40))
            (SPADCALL |p| |x| |l| (QREFELT |$| 42)))
          ((SPADCALL |p| (QREFELT |$| 43))
            (SEQ
              (LETT |r|
                (SPADCALL |p| (QREFELT |$| 45))
                |RNS-;patternMatch;SP2Pmr;10|)
              (EXIT
                (COND
                  ((QEQCAR |r| 0)
                    (COND
                      ((SPADCALL
                         (SPADCALL |x| (QREFELT |$| 30))
                         (QCDR |r|)
                         (QREFELT |$| 46))
                        |l|)
                      ((QUOTE T) (SPADCALL (QREFELT |$| 47)))))
                  ((QUOTE T) (SPADCALL (QREFELT |$| 47)))))))
          ((QUOTE T) (SPADCALL (QREFELT |$| 47))))))))

(DEFUN |RealNumberSystem&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|RealNumberSystem&|))
        (LETT |dv$| (LIST (QUOTE |RealNumberSystem&|) |DV$1|) . #1#)
        (LETT |$| (make-array 52) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        |$|))))

(setf (get
  (QUOTE |RealNumberSystem&|)
  (QUOTE |infovec|))
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (|NonNegativeInteger|)
        |RNS-;characteristic;Nni;1|
        (0 . |truncate|)
        (5 . |-|)
```

```
    |RNS-;fractionPart;2S;2|
    (|Boolean|)
    (11 . |negative?|)
    (16 . |-|)
    (21 . |floor|)
    |RNS-;truncate;2S;3|
    (26 . |One|)
    (|Integer|)
    (30 . |coerce|)
    (35 . |/|)
    (41 . |+|)
    |RNS-;round;2S;4|
    (47 . |abs|)
    |RNS-;norm;2S;5|
    (|Fraction| 18)
    (52 . |numer|)
    (57 . |denom|)
    |RNS-;coerce;FS;6|
    (|Float|)
    (62 . |convert|)
    (|Pattern| 29)
    (67 . |coerce|)
    |RNS-;convert;SP;7|
    (72 . |wholePart|)
    (77 . |=|)
    (83 . |Zero|)
    (87 . |<|)
    |RNS-;floor;2S;8|
    |RNS-;ceiling;2S;9|
    (93 . |generic?|)
    (|PatternMatchResult| 29 6)
    (98 . |addMatch|)
    (105 . |constant?|)
    (|Union| 29 (QUOTE "failed"))
    (110 . |retractIfCan|)
    (115 . |=|)
    (121 . |failed|)
    (|PatternMatchResult| 29 |$|)
    |RNS-;patternMatch;SP2Pmr;10|
    (|DoubleFloat|)
    (|OutputForm|)))
(QUOTE
  #(|truncate| 125 |round| 130 |patternMatch| 135 |norm| 142
    |fractionPart| 147 |floor| 152 |convert| 157 |coerce| 162
    |characteristic| 172 |ceiling| 176))
(QUOTE NIL)
(CONS
  (|makeByteWordVec2| 1 (QUOTE NIL))
  (CONS
    (QUOTE #())
    (CONS
      (QUOTE #())
      (|makeByteWordVec2| 49
        (QUOTE
```

```
            (1 6 0 0 9 2 6 0 0 0 10 1 6 12 0 13 1 6 0 0 14 1 6 0 0 15 0 6 0
             17 1 6 0 18 19 2 6 0 0 0 20 2 6 0 0 0 21 1 6 0 0 23 1 25 18 0
             26 1 25 18 0 27 1 6 29 0 30 1 31 0 29 32 1 6 18 0 34 2 6 12 0
             0 35 0 6 0 36 2 6 12 0 0 37 1 31 12 0 40 3 41 0 31 6 0 42 1 31
             12 0 43 1 31 44 0 45 2 29 12 0 0 46 0 41 0 47 1 0 0 0 16 1 0 0
             0 22 3 0 48 0 31 48 49 1 0 0 0 24 1 0 0 0 11 1 0 0 0 38 1 0 31
             0 33 1 0 0 25 28 1 0 0 25 28 0 0 7 8 1 0 0 0 39))))))
    (QUOTE |lookupComplete|)))
```

———-

21.56 SETAGG.lsp BOOTSTRAP

SETAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **SETAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **SETAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— SETAGG.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |SetAggregate;CAT| (QUOTE NIL))

(SETQ |SetAggregate;AL| (QUOTE NIL))

(DEFUN |SetAggregate| (#1=#:G83200)
 (declare (special |SetAggregate;AL|))
 (LET (#2=#:G83201)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |SetAggregate;AL|)) (CDR #2#))
   (T
    (SETQ |SetAggregate;AL|
     (|cons5|
      (CONS (|devaluate| #1#) (SETQ #2# (|SetAggregate;| #1#)))
      |SetAggregate;AL|))
    #2#))))

(DEFUN |SetAggregate;| (|t#1|)
 (declare (special |SetAggregate;CAT|))
 (PROG (#1=#:G83199)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (COND
       (|SetAggregate;CAT|)
       ((QUOTE T)
        (LETT |SetAggregate;CAT|
```

```
        (|Join|
         (|SetCategory|)
         (|Collection| (QUOTE |t#1|))
         (|mkCategory|
          (QUOTE |domain|)
          (QUOTE (
           ((|<| ((|Boolean|) |$| |$|)) T)
           ((|brace| (|$|)) T)
           ((|brace| (|$| (|List| |t#1|))) T)
           ((|set| (|$|)) T)
           ((|set| (|$| (|List| |t#1|))) T)
           ((|intersect| (|$| |$| |$|)) T)
           ((|difference| (|$| |$| |$|)) T)
           ((|difference| (|$| |$| |t#1|)) T)
           ((|symmetricDifference| (|$| |$| |$|)) T)
           ((|subset?| ((|Boolean|) |$| |$|)) T)
           ((|union| (|$| |$| |$|)) T)
           ((|union| (|$| |$| |t#1|)) T)
           ((|union| (|$| |t#1| |$|)) T)))
          (QUOTE ((|partiallyOrderedSet| T)))
          (QUOTE ((|Boolean|) (|List| |t#1|)))
          NIL))
       . #2=(|SetAggregate|)))))
    . #2#)
  (SETELT #1# 0 (LIST (QUOTE |SetAggregate|) (|devaluate| |t#1|)))))))
```

21.57 SETAGG-.lsp BOOTSTRAP

SETAGG- depends on **SETAGG**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **SETAGG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **SETAGG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— SETAGG-.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(DEFUN |SETAGG-;symmetricDifference;3A;1| (|x| |y| |$|)
 (SPADCALL
  (SPADCALL |x| |y| (QREFELT |$| 8))
  (SPADCALL |y| |x| (QREFELT |$| 8))
  (QREFELT |$| 9)))

(DEFUN |SETAGG-;union;ASA;2| (|s| |x| |$|)
 (SPADCALL |s| (SPADCALL (LIST |x|) (QREFELT |$| 12)) (QREFELT |$| 9)))

(DEFUN |SETAGG-;union;S2A;3| (|x| |s| |$|)
 (SPADCALL |s| (SPADCALL (LIST |x|) (QREFELT |$| 12)) (QREFELT |$| 9)))
```

```
(DEFUN |SETAGG-;difference;ASA;4| (|s| |x| |$|)
 (SPADCALL |s| (SPADCALL (LIST |x|) (QREFELT |$| 12)) (QREFELT |$| 8)))

(DEFUN |SetAggregate&| (|#1| |#2|)
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|SetAggregate&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |SetAggregate&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (make-array 16) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
    |$|))))

(setf (get
 (QUOTE |SetAggregate&|)
 (QUOTE |infovec|))
 (LIST
  (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|)
    (0 . |difference|) (6 . |union|) |SETAGG-;symmetricDifference;3A;1|
    (|List| 7) (12 . |brace|) |SETAGG-;union;ASA;2| |SETAGG-;union;S2A;3|
    |SETAGG-;difference;ASA;4|))
  (QUOTE #(|union| 17 |symmetricDifference| 29 |difference| 35))
  (QUOTE NIL)
  (CONS
   (|makeByteWordVec2| 1 (QUOTE NIL))
   (CONS
    (QUOTE #())
    (CONS
     (QUOTE #())
     (|makeByteWordVec2| 15 (QUOTE (2 6 0 0 0 8 2 6 0 0 0 9 1 6 0 11 12 2
       0 0 7 0 14 2 0 0 0 7 13 2 0 0 0 0 10 2 0 0 0 7 15))))))
  (QUOTE |lookupComplete|)))
```

21.58 SETCAT.lsp BOOTSTRAP

SETCAT needs **SINT** which needs **UFD** which needs **GCDDOM** which needs **COMRING** which needs **RING** which needs **RNG** which needs **ABELGRP** which needs **CABMON** which needs **ABELMON** which needs **ABELSG** which needs **SETCAT**. We break this chain with **SETCAT.lsp** which we cache here. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **SETCAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **SETCAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— SETCAT.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |SetCategory;AL| (QUOTE NIL))

(DEFUN |SetCategory| NIL
  (declare (special |SetCategory;AL|))
    (COND
      (|SetCategory;AL|)
      (T (SETQ |SetCategory;AL| (|SetCategory;|)))))

(DEFUN |SetCategory;| NIL
  (PROG (#1=#:G82357)
    (RETURN
      (PROG1
        (LETT #1#
          (|sublisV|
            (PAIR
              (QUOTE (#2=#:G82356))
              (LIST (QUOTE (|OutputForm|))))
            (|Join|
              (|BasicType|)
              (|CoercibleTo| (QUOTE #2#))
              (|mkCategory|
                (QUOTE |domain|)
                (QUOTE (
                  ((|hash| ((|SingleInteger|) |$|)) T)
                  ((|latex| ((|String|) |$|)) T)))
                NIL
                (QUOTE ((|String|) (|SingleInteger|)))
                NIL)))
          |SetCategory|)
        (SETELT #1# 0 (QUOTE (|SetCategory|)))))))

(setf (get (QUOTE |SetCategory|) (QUOTE NILADIC)) T)
```

————-

21.59 SETCAT-.lsp BOOTSTRAP

SETCAT- is the implementation of the operations exported by **SETCAT**. It comes into existance whenever **SETCAT** gets compiled by Axiom. However this will not happen at the lisp level so we also cache this information here. See the explanation under the **SETCAT.lsp** section for more details.

Note that this code is not included in the generated catdef.spad file.

— SETCAT-.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(PUT
```

```
  (QUOTE |SETCAT-;hash;SSi;1|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|s|) 0)))

(DEFUN |SETCAT-;hash;SSi;1| (|s| |$|)
 (declare (ignore |s|))
 (declare (ignore |$|))
 0)

(PUT
  (QUOTE |SETCAT-;latex;SS;2|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|s|) "\\mbox{\\bf Unimplemented}")))

(DEFUN |SETCAT-;latex;SS;2| (|s| |$|)
 (declare (ignore |s|))
 (declare (ignore |$|))
  "\\mbox{\\bf Unimplemented}")

(DEFUN |SetCategory&| (|#1|)
 (PROG (|DV$1| |dv$| |$| |pv$|)
   (RETURN
     (PROGN
       (LETT |DV$1| (|devaluate| |#1|) . #1=(|SetCategory&|))
       (LETT |dv$| (LIST (QUOTE |SetCategory&|) |DV$1|) . #1#)
       (LETT |$| (make-array 11) . #1#)
       (QSETREFV |$| 0 |dv$|)
       (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
       (|stuffDomainSlots| |$|)
       (QSETREFV |$| 6 |#1|)
       |$|))))

(setf (get
  (QUOTE |SetCategory&|)
  (QUOTE |infovec|))
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (|SingleInteger|)
        |SETCAT-;hash;SSi;1|
        (|String|)
        |SETCAT-;latex;SS;2|))
    (QUOTE
      #(|latex| 0 |hash| 5))
    (QUOTE NIL)
    (CONS
      (|makeByteWordVec2| 1 (QUOTE NIL))
      (CONS
        (QUOTE #())
        (CONS
          (QUOTE #())
          (|makeByteWordVec2|
            10
```

```
        (QUOTE (1 0 9 0 10 1 0 7 0 8))))))
    (QUOTE |lookupComplete|)))
```

———-

21.60 STAGG.lsp BOOTSTRAP

STAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **STAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **STAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— **STAGG.lsp BOOTSTRAP** —

```
(|/VERSIONCHECK| 2)

(SETQ |StreamAggregate;CAT| (QUOTE NIL))

(SETQ |StreamAggregate;AL| (QUOTE NIL))

(DEFUN |StreamAggregate| (#1=#:G87035)
 (declare (special |StreamAggregate;AL|))
 (LET (#2=#:G87036)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |StreamAggregate;AL|)) (CDR #2#))
   (T
    (SETQ |StreamAggregate;AL|
     (|cons5|
      (CONS (|devaluate| #1#) (SETQ #2# (|StreamAggregate;| #1#)))
      |StreamAggregate;AL|))
    #2#))))

(DEFUN |StreamAggregate;| (|t#1|)
 (declare (special |StreamAggregate;CAT|))
 (PROG (#1=#:G87034)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (COND
       (|StreamAggregate;CAT|)
       ((QUOTE T)
        (LETT |StreamAggregate;CAT|
         (|Join|
          (|UnaryRecursiveAggregate| (QUOTE |t#1|))
          (|LinearAggregate| (QUOTE |t#1|))
          (|mkCategory|
           (QUOTE |domain|)
           (QUOTE (
```

```
           ((|explicitlyFinite?| ((|Boolean|) |$|)) T)
           ((|possiblyInfinite?| ((|Boolean|) |$|)) T)))
          NIL
          (QUOTE ((|Boolean|)))
          NIL))
        . #2=(|StreamAggregate|)))))
      . #2#)
    (SETELT #1# 0 (LIST (QUOTE |StreamAggregate|) (|devaluate| |t#1|)))))))
```

———-

21.61 STAGG-.lsp BOOTSTRAP

STAGG- depends on **STAGG**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **STAGG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **STAGG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— STAGG-.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(DEFUN |STAGG-;explicitlyFinite?;AB;1| (|x| |$|)
 (COND ((SPADCALL |x| (QREFELT |$| 9)) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))

(DEFUN |STAGG-;possiblyInfinite?;AB;2| (|x| |$|)
 (SPADCALL |x| (QREFELT |$| 9)))

(DEFUN |STAGG-;first;ANniA;3| (|x| |n| |$|)
 (PROG (#1=#:G87053 |i|)
  (RETURN
   (SEQ
    (SPADCALL
     (PROGN
      (LETT #1# NIL |STAGG-;first;ANniA;3|)
      (SEQ
       (LETT |i| 1 |STAGG-;first;ANniA;3|)
       G190
       (COND ((QSGREATERP |i| |n|) (GO G191)))
       (SEQ
        (EXIT
         (LETT #1#
          (CONS
           (|STAGG-;c2| |x|
            (LETT |x| (SPADCALL |x| (QREFELT |$| 12)) |STAGG-;first;ANniA;3|)
            |$|)
            #1#)
          |STAGG-;first;ANniA;3|)))
       (LETT |i| (QSADD1 |i|) |STAGG-;first;ANniA;3|)
       (GO G190)
       G191
```

```
      (EXIT (NREVERSE0 #1#))))
    (QREFELT |$| 14))))))

(DEFUN |STAGG-;c2| (|x| |r| |$|)
 (declare (ignore |r|))
 (COND
   ((SPADCALL |x| (QREFELT |$| 17)) (|error| "Index out of range"))
   ((QUOTE T) (SPADCALL |x| (QREFELT |$| 18)))))

(DEFUN |STAGG-;elt;AIS;5| (|x| |i| |$|)
 (PROG (#1=#:G87056)
  (RETURN
   (SEQ
    (LETT |i| (|-| |i| (SPADCALL |x| (QREFELT |$| 20))) |STAGG-;elt;AIS;5|)
    (COND
     ((OR
       (|<| |i| 0)
       (SPADCALL
        (LETT |x|
         (SPADCALL |x|
          (PROG1
           (LETT #1# |i| |STAGG-;elt;AIS;5|)
           (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
          (QREFELT |$| 21))
         |STAGG-;elt;AIS;5|)
        (QREFELT |$| 17)))
      (EXIT (|error| "index out of range"))))
    (EXIT (SPADCALL |x| (QREFELT |$| 18)))))))

(DEFUN |STAGG-;elt;AUsA;6| (|x| |i| |$|)
 (PROG (|l| #1=#:G87060 |h| #2=#:G87062 #3=#:G87063)
  (RETURN
   (SEQ
    (LETT |l|
     (|-| (SPADCALL |i| (QREFELT |$| 24)) (SPADCALL |x| (QREFELT |$| 20)))
     |STAGG-;elt;AUsA;6|)
    (EXIT
     (COND
      ((|<| |l| 0) (|error| "index out of range"))
      ((NULL (SPADCALL |i| (QREFELT |$| 25)))
        (SPADCALL
         (SPADCALL |x|
          (PROG1
           (LETT #1# |l| |STAGG-;elt;AUsA;6|)
           (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
          (QREFELT |$| 21))
         (QREFELT |$| 26)))
      ((QUOTE T)
       (SEQ
        (LETT |h|
         (|-| (SPADCALL |i| (QREFELT |$| 27)) (SPADCALL |x| (QREFELT |$| 20)))
         |STAGG-;elt;AUsA;6|)
        (EXIT
         (COND
```

```
          ((|<| |h| |l|) (SPADCALL (QREFELT |$| 28)))
          ((QUOTE T)
           (SPADCALL
            (SPADCALL |x|
             (PROG1
              (LETT #2# |l| |STAGG-;elt;AUsA;6|)
              (|check-subtype|
                (|>=| #2# 0) (QUOTE (|NonNegativeInteger|)) #2#))
             (QREFELT |$| 21))
            (PROG1
             (LETT #3# (|+| (|-| |h| |l|) 1) |STAGG-;elt;AUsA;6|)
             (|check-subtype| (|>=| #3# 0) (QUOTE (|NonNegativeInteger|)) #3#))
            (QREFELT |$| 29)))))))))))))

(DEFUN |STAGG-;concat;3A;7| (|x| |y| |$|)
 (SPADCALL (SPADCALL |x| (QREFELT |$| 26)) |y| (QREFELT |$| 31)))

(DEFUN |STAGG-;concat;LA;8| (|l| |$|)
 (COND
  ((NULL |l|) (SPADCALL (QREFELT |$| 28)))
  ((QUOTE T)
   (SPADCALL
    (SPADCALL (|SPADfirst| |l|) (QREFELT |$| 26))
    (SPADCALL (CDR |l|) (QREFELT |$| 34))
    (QREFELT |$| 31)))))

(DEFUN |STAGG-;map!;M2A;9| (|f| |l| |$|)
 (PROG (|y|)
  (RETURN
   (SEQ
    (LETT |y| |l| |STAGG-;map!;M2A;9|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((SPADCALL |l| (QREFELT |$| 17)) (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (SPADCALL |l|
       (SPADCALL (SPADCALL |l| (QREFELT |$| 18)) |f|) (QREFELT |$| 36))
      (EXIT (LETT |l| (SPADCALL |l| (QREFELT |$| 12)) |STAGG-;map!;M2A;9|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |y|)))))

(DEFUN |STAGG-;fill!;ASA;10| (|x| |s| |$|)
 (PROG (|y|)
  (RETURN
   (SEQ
    (LETT |y| |x| |STAGG-;fill!;ASA;10|)
```

```
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((SPADCALL |y| (QREFELT |$| 17)) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (SPADCALL |y| |s| (QREFELT |$| 36))
      (EXIT (LETT |y| (SPADCALL |y| (QREFELT |$| 12)) |STAGG-;fill!;ASA;10|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |x|)))))

(DEFUN |STAGG-;setelt;AI2S;11| (|x| |i| |s| |$|)
 (PROG (#1=#:G87081)
  (RETURN
   (SEQ
    (LETT |i|
     (|-| |i| (SPADCALL |x| (QREFELT |$| 20))) |STAGG-;setelt;AI2S;11|)
    (COND
     ((OR
       (|<| |i| 0)
       (SPADCALL
        (LETT |x|
         (SPADCALL |x|
          (PROG1
           (LETT #1# |i| |STAGG-;setelt;AI2S;11|)
           (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
          (QREFELT |$| 21))
         |STAGG-;setelt;AI2S;11|)
        (QREFELT |$| 17)))
      (EXIT (|error| "index out of range"))))
    (EXIT (SPADCALL |x| |s| (QREFELT |$| 36)))))))

(DEFUN |STAGG-;setelt;AUs2S;12| (|x| |i| |s| |$|)
 (PROG (|l| |h| #1=#:G87086 #2=#:G87087 |z| |y|)
  (RETURN
   (SEQ
    (LETT |l|
     (|-| (SPADCALL |i| (QREFELT |$| 24)) (SPADCALL |x| (QREFELT |$| 20)))
      |STAGG-;setelt;AUs2S;12|)
    (EXIT
     (COND
      ((|<| |l| 0) (|error| "index out of range"))
      ((QUOTE T)
       (SEQ
        (LETT |h|
         (COND
          ((SPADCALL |i| (QREFELT |$| 25))
            (|-|
             (SPADCALL |i| (QREFELT |$| 27))
```

```
            (SPADCALL |x| (QREFELT |$| 20))))
         ((QUOTE T) (SPADCALL |x| (QREFELT |$| 41))))
        |STAGG-;setelt;AUs2S;12|)
       (EXIT
        (COND
         ((|<| |h| |l|) |s|)
         ((QUOTE T)
           (SEQ
            (LETT |y|
             (SPADCALL |x|
              (PROG1
               (LETT #1# |l| |STAGG-;setelt;AUs2S;12|)
               (|check-subtype|
                (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
              (QREFELT |$| 21))
             |STAGG-;setelt;AUs2S;12|)
            (LETT |z|
             (SPADCALL |y|
              (PROG1
               (LETT #2# (|+| (|-| |h| |l|) 1) |STAGG-;setelt;AUs2S;12|)
               (|check-subtype|
                 (|>=| #2# 0) (QUOTE (|NonNegativeInteger|)) #2#))
              (QREFELT |$| 21))
             |STAGG-;setelt;AUs2S;12|)
            (SEQ
              G190
              (COND
               ((NULL
                 (COND
                  ((SPADCALL |y| |z| (QREFELT |$| 42)) (QUOTE NIL))
                  ((QUOTE T) (QUOTE T))))
                (GO G191)))
              (SEQ
               (SPADCALL |y| |s| (QREFELT |$| 36))
               (EXIT
                (LETT |y|
                 (SPADCALL |y| (QREFELT |$| 12))
                 |STAGG-;setelt;AUs2S;12|)))
              NIL
              (GO G190)
              G191
              (EXIT NIL))
            (EXIT |s|)))))))))))))

(DEFUN |STAGG-;concat!;3A;13| (|x| |y| |$|)
 (SEQ
  (COND
   ((SPADCALL |x| (QREFELT |$| 17)) |y|)
   ((QUOTE T)
     (SEQ
      (SPADCALL (SPADCALL |x| (QREFELT |$| 44)) |y| (QREFELT |$| 45))
      (EXIT |x|))))))

(DEFUN |StreamAggregate&| (|#1| |#2|)
```

```
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|StreamAggregate&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |StreamAggregate&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (make-array 51) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
    (COND
     ((|HasAttribute| |#1| (QUOTE |shallowlyMutable|))
      (PROGN
       (QSETREFV |$| 32 (CONS (|dispatchFunction| |STAGG-;concat;3A;7|) |$|))
       (QSETREFV |$| 35 (CONS (|dispatchFunction| |STAGG-;concat;LA;8|) |$|))
       (QSETREFV |$| 38 (CONS (|dispatchFunction| |STAGG-;map!;M2A;9|) |$|))
       (QSETREFV |$| 39 (CONS (|dispatchFunction| |STAGG-;fill!;ASA;10|) |$|))
       (QSETREFV |$| 40
         (CONS (|dispatchFunction| |STAGG-;setelt;AI2S;11|) |$|))
       (QSETREFV |$| 43
         (CONS (|dispatchFunction| |STAGG-;setelt;AUs2S;12|) |$|))
       (QSETREFV |$| 46
         (CONS (|dispatchFunction| |STAGG-;concat!;3A;13|) |$|)))))
    |$|))))

(setf (get
 (QUOTE |StreamAggregate&|)
 (QUOTE |infovec|))
 (LIST
  (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|)
   (|Boolean|) (0 . |cyclic?|) |STAGG-;explicitlyFinite?;AB;1|
   |STAGG-;possiblyInfinite?;AB;2| (5 . |rest|) (|List| 7)
   (10 . |construct|) (|NonNegativeInteger|) |STAGG-;first;ANniA;3|
   (15 . |empty?|) (20 . |first|) (|Integer|) (25 . |minIndex|)
   (30 . |rest|) |STAGG-;elt;AIS;5| (|UniversalSegment| 19) (36 . |lo|)
   (41 . |hasHi|) (46 . |copy|) (51 . |hi|) (56 . |empty|) (60 . |first|)
   |STAGG-;elt;AUsA;6| (66 . |concat!|) (72 . |concat|) (|List| |$|)
   (78 . |concat|) (83 . |concat|) (88 . |setfirst!|) (|Mapping| 7 7)
   (94 . |map!|) (100 . |fill!|) (106 . |setelt|) (113 . |maxIndex|)
   (118 . |eq?|) (124 . |setelt|) (131 . |tail|) (136 . |setrest!|)
   (142 . |concat!|) (QUOTE "rest") (QUOTE "last") (QUOTE "first")
   (QUOTE "value")))
  (QUOTE #(|setelt| 148 |possiblyInfinite?| 162 |map!| 167 |first| 173
    |fill!| 179 |explicitlyFinite?| 185 |elt| 190 |concat!| 202 |concat| 208))
  (QUOTE NIL)
  (CONS
   (|makeByteWordVec2| 1 (QUOTE NIL))
   (CONS
    (QUOTE #())
    (CONS
     (QUOTE #())
     (|makeByteWordVec2| 46 (QUOTE (1 6 8 0 9 1 6 0 0 12 1 6 0 13 14 1 6
```

```
      8 0 17 1 6 7 0 18 1 6 19 0 20 2 6 0 0 15 21 1 23 19 0 24 1 23 8
      0 25 1 6 0 0 26 1 23 19 0 27 0 6 0 28 2 6 0 0 15 29 2 6 0 0 0 31
      2 0 0 0 0 32 1 6 0 33 34 1 0 0 33 35 2 6 7 0 7 36 2 0 0 37 0 38 2
      0 0 0 7 39 3 0 7 0 19 7 40 1 6 19 0 41 2 6 8 0 0 42 3 0 7 0 23 7 43
      1 6 0 0 44 2 6 0 0 0 45 2 0 0 0 0 46 3 0 7 0 19 7 40 3 0 7 0 23 7 43
      1 0 8 0 11 2 0 0 37 0 38 2 0 0 0 15 16 2 0 0 0 7 39 1 0 8 0 10 2 0 7
      0 19 22 2 0 0 0 23 30 2 0 0 0 0 46 1 0 0 33 35 2 0 0 0 0 32))))))
  (QUOTE |lookupComplete|)))
```

———-

21.62 TSETCAT.lsp BOOTSTRAP

TSETCAT depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **TSETCAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **TSETCAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— **TSETCAT.lsp BOOTSTRAP** —

```
(|/VERSIONCHECK| 2)

(SETQ |TriangularSetCategory;CAT| (QUOTE NIL))

(SETQ |TriangularSetCategory;AL| (QUOTE NIL))

(DEFUN |TriangularSetCategory| (|&REST| #1=#:G82394 |&AUX| #2=#:G82392)
 (declare (special |TriangularSetCategory;AL|))
 (DSETQ #2# #1#)
 (LET (#3=#:G82393)
  (COND
   ((SETQ #3# (|assoc| (|devaluateList| #2#) |TriangularSetCategory;AL|))
     (CDR #3#))
   (T
    (SETQ |TriangularSetCategory;AL|
     (|cons5|
      (CONS
       (|devaluateList| #2#)
       (SETQ #3# (APPLY (FUNCTION |TriangularSetCategory;|) #2#)))
      |TriangularSetCategory;AL|))
     #3#))))

(DEFUN |TriangularSetCategory;| (|t#1| |t#2| |t#3| |t#4|)
 (declare (special |TriangularSetCategory;CAT|))
 (PROG (#1=#:G82391)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR
       (QUOTE (|t#1| |t#2| |t#3| |t#4|))
```

```
 (LIST
  (|devaluate| |t#1|)
  (|devaluate| |t#2|)
  (|devaluate| |t#3|)
  (|devaluate| |t#4|)))
(COND
 (|TriangularSetCategory;CAT|)
 ((QUOTE T)
  (LETT |TriangularSetCategory;CAT|
   (|Join|
    (|PolynomialSetCategory|
     (QUOTE |t#1|)
     (QUOTE |t#2|)
     (QUOTE |t#3|)
     (QUOTE |t#4|))
    (|mkCategory|
     (QUOTE |domain|)
     (QUOTE (
      ((|infRittWu?| ((|Boolean|) |$| |$|)) T)
      ((|basicSet| (
         (|Union|
          (|Record| (|:| |bas| |$|) (|:| |top| (|List| |t#4|))) "failed")
         (|List| |t#4|)
         (|Mapping| (|Boolean|) |t#4| |t#4|)))
        T)
      ((|basicSet| (
         (|Union|
          (|Record| (|:| |bas| |$|) (|:| |top| (|List| |t#4|))) "failed")
         (|List| |t#4|)
         (|Mapping| (|Boolean|) |t#4|)
         (|Mapping| (|Boolean|) |t#4| |t#4|)))
        T)
      ((|initials| ((|List| |t#4|) |$|)) T)
      ((|degree| ((|NonNegativeInteger|) |$|)) T)
      ((|quasiComponent| (
         (|Record|
          (|:| |close| (|List| |t#4|))
          (|:| |open| (|List| |t#4|)))
         |$|))
        T)
      ((|normalized?| ((|Boolean|) |t#4| |$|)) T)
      ((|normalized?| ((|Boolean|) |$|)) T)
      ((|reduced?| (
         (|Boolean|)
         |t#4|
         |$|
         (|Mapping| (|Boolean|) |t#4| |t#4|)))
        T)
      ((|stronglyReduced?| ((|Boolean|) |t#4| |$|)) T)
      ((|headReduced?| ((|Boolean|) |t#4| |$|)) T)
      ((|initiallyReduced?| ((|Boolean|) |t#4| |$|)) T)
      ((|autoReduced?| (
        (|Boolean|)
        |$|
```

```
            (|Mapping| (|Boolean|) |t#4| (|List| |t#4|))))
            T)
          ((|stronglyReduced?| ((|Boolean|) |$|)) T)
          ((|headReduced?| ((|Boolean|) |$|)) T)
          ((|initiallyReduced?| ((|Boolean|) |$|)) T)
          ((|reduce| (
             |t#4|
             |t#4|
             |$|
             (|Mapping| |t#4| |t#4| |t#4|)
             (|Mapping| (|Boolean|) |t#4| |t#4|)))
            T)
          ((|rewriteSetWithReduction| (
            (|List| |t#4|)
            (|List| |t#4|)
            |$|
            (|Mapping| |t#4| |t#4| |t#4|)
            (|Mapping| (|Boolean|) |t#4| |t#4|)))
            T)
          ((|stronglyReduce| (|t#4| |t#4| |$|)) T)
          ((|headReduce| (|t#4| |t#4| |$|)) T)
          ((|initiallyReduce| (|t#4| |t#4| |$|)) T)
          ((|removeZero| (|t#4| |t#4| |$|)) T)
          ((|collectQuasiMonic| (|$| |$|)) T)
          ((|reduceByQuasiMonic| (|t#4| |t#4| |$|)) T)
          ((|zeroSetSplit| ((|List| |$|) (|List| |t#4|))) T)
          ((|zeroSetSplitIntoTriangularSystems|
            ((|List|
              (|Record| (|:| |close| |$|) (|:| |open| (|List| |t#4|))))
             (|List| |t#4|)))
            T)
          ((|first| ((|Union| |t#4| "failed") |$|)) T)
          ((|last| ((|Union| |t#4| "failed") |$|)) T)
          ((|rest| ((|Union| |$| "failed") |$|)) T)
          ((|algebraicVariables| ((|List| |t#3|) |$|)) T)
          ((|algebraic?| ((|Boolean|) |t#3| |$|)) T)
          ((|select| ((|Union| |t#4| "failed") |$| |t#3|)) T)
          ((|extendIfCan| ((|Union| |$| "failed") |$| |t#4|)) T)
          ((|extend| (|$| |$| |t#4|)) T)
          ((|coHeight| ((|NonNegativeInteger|) |$|))
            (|has| |t#3| (|Finite|)))))
         (QUOTE (
          (|finiteAggregate| T)
          (|shallowlyMutable| T)))
         (QUOTE ((|NonNegativeInteger|)
          (|Boolean|)
          (|List| |t#3|)
          (|List| (|Record| (|:| |close| |$|) (|:| |open| (|List| |t#4|))))
          (|List| |t#4|)
          (|List| |$|)))
         NIL))
   . #2=(|TriangularSetCategory|)))))
  . #2#)
 (SETELT #1# 0
```

```
    (LIST
     (QUOTE |TriangularSetCategory|)
     (|devaluate| |t#1|)
     (|devaluate| |t#2|)
     (|devaluate| |t#3|)
     (|devaluate| |t#4|)))))))
```

———-

21.63 TSETCAT-.lsp BOOTSTRAP

TSETCAT depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **TSETCAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **TSETCAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— TSETCAT-.lsp BOOTSTRAP —

```
(/VERSIONCHECK 2)

(DEFUN |TSETCAT-;=;2SB;1| (|ts| |us| $)
 (PROG (#0=#:G1475 #1=#:G1481)
  (RETURN
   (COND
    ((SPADCALL |ts| (QREFELT $ 12)) (SPADCALL |us| (QREFELT $ 12)))
    ((OR
      (SPADCALL |us| (QREFELT $ 12))
      (NULL
       (SPADCALL
        (PROG2
         (LETT #0# (SPADCALL |ts| (QREFELT $ 14)) |TSETCAT-;=;2SB;1|)
         (QCDR #0#)
         (|check-union| (QEQCAR #0# 0) (QREFELT $ 10) #0#))
        (PROG2
         (LETT #0# (SPADCALL |us| (QREFELT $ 14)) |TSETCAT-;=;2SB;1|)
         (QCDR #0#)
         (|check-union| (QEQCAR #0# 0) (QREFELT $ 10) #0#))
        (QREFELT $ 15))))
      (QUOTE NIL))
    ((QUOTE T)
     (SPADCALL
      (PROG2
       (LETT #1# (SPADCALL |ts| (QREFELT $ 17)) |TSETCAT-;=;2SB;1|)
       (QCDR #1#)
       (|check-union| (QEQCAR #1# 0) (QREFELT $ 6) #1#))
      (PROG2
       (LETT #1# (SPADCALL |us| (QREFELT $ 17)) |TSETCAT-;=;2SB;1|)
       (QCDR #1#)
       (|check-union| (QEQCAR #1# 0) (QREFELT $ 6) #1#))
      (QREFELT $ 18)))))))
```

```
(DEFUN |TSETCAT-;infRittWu?;2SB;2| (|ts| |us| $)
 (PROG (|p| #0=#:G1489 |q| |v|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |us| (QREFELT $ 12))
       (COND
        ((SPADCALL |ts| (QREFELT $ 12)) (QUOTE NIL))
        ((QUOTE T) (QUOTE T))))
     ((SPADCALL |ts| (QREFELT $ 12)) (QUOTE NIL))
     ((QUOTE T)
      (SEQ
       (LETT |p|
        (PROG2
         (LETT #0# (SPADCALL |ts| (QREFELT $ 20)) |TSETCAT-;infRittWu?;2SB;2|)
         (QCDR #0#)
         (|check-union| (QEQCAR #0# 0) (QREFELT $ 10) #0#))
        |TSETCAT-;infRittWu?;2SB;2|)
       (LETT |q|
        (PROG2
         (LETT #0# (SPADCALL |us| (QREFELT $ 20)) |TSETCAT-;infRittWu?;2SB;2|)
         (QCDR #0#)
         (|check-union| (QEQCAR #0# 0) (QREFELT $ 10) #0#))
        |TSETCAT-;infRittWu?;2SB;2|)
       (EXIT
        (COND
         ((SPADCALL |p| |q| (QREFELT $ 21)) (QUOTE T))
         ((SPADCALL |p| |q| (QREFELT $ 22)) (QUOTE NIL))
         ((QUOTE T)
          (SEQ
           (LETT |v| (SPADCALL |p| (QREFELT $ 23)) |TSETCAT-;infRittWu?;2SB;2|)
           (EXIT
            (SPADCALL
             (SPADCALL |ts| |v| (QREFELT $ 24))
             (SPADCALL |us| |v| (QREFELT $ 24)) (QREFELT $ 25)))))))))))))

(DEFUN |TSETCAT-;reduced?;PSMB;3| (|p| |ts| |redOp?| $)
 (PROG (|lp|)
  (RETURN
   (SEQ
    (LETT |lp| (SPADCALL |ts| (QREFELT $ 28)) |TSETCAT-;reduced?;PSMB;3|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((NULL |lp|) (QUOTE NIL))
         ((QUOTE T) (SPADCALL |p| (|SPADfirst| |lp|) |redOp?|))))
       (GO G191)))
     (SEQ (EXIT (LETT |lp| (CDR |lp|) |TSETCAT-;reduced?;PSMB;3|)))
     NIL
     (GO G190)
     G191
```

```
     (EXIT NIL))
    (EXIT (NULL |lp|))))))

(DEFUN |TSETCAT-;basicSet;LMU;4| (|ps| |redOp?| $)
 (PROG (|b| |bs| |p| |ts|)
  (RETURN
   (SEQ
    (LETT |ps|
     (SPADCALL (ELT $ 31) |ps| (QREFELT $ 33))
     |TSETCAT-;basicSet;LMU;4|)
    (EXIT
     (COND
      ((SPADCALL (ELT $ 34) |ps| (QREFELT $ 35)) (CONS 1 "failed"))
      ((QUOTE T)
       (SEQ
        (LETT |ps|
         (SPADCALL (ELT $ 21) |ps| (QREFELT $ 36))
         |TSETCAT-;basicSet;LMU;4|)
        (LETT |bs| (SPADCALL (QREFELT $ 37)) |TSETCAT-;basicSet;LMU;4|)
        (LETT |ts| NIL |TSETCAT-;basicSet;LMU;4|)
        (SEQ
         G190
         (COND
          ((NULL (COND ((NULL |ps|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
            (GO G191)))
         (SEQ
          (LETT |b| (|SPADfirst| |ps|) |TSETCAT-;basicSet;LMU;4|)
          (LETT |bs|
           (SPADCALL |bs| |b| (QREFELT $ 38))
           |TSETCAT-;basicSet;LMU;4|)
          (LETT |ps| (CDR |ps|) |TSETCAT-;basicSet;LMU;4|)
          (EXIT
           (SEQ
            G190
            (COND
             ((NULL
              (COND
               ((OR
                  (NULL |ps|)
                  (SPADCALL
                   (LETT |p| (|SPADfirst| |ps|) |TSETCAT-;basicSet;LMU;4|)
                   |bs| |redOp?| (QREFELT $ 39)))
                  (QUOTE NIL))
               ((QUOTE T) (QUOTE T))))
              (GO G191)))
            (SEQ
             (LETT |ts| (CONS |p| |ts|) |TSETCAT-;basicSet;LMU;4|)
             (EXIT (LETT |ps| (CDR |ps|) |TSETCAT-;basicSet;LMU;4|)))
            NIL
            (GO G190)
            G191
            (EXIT NIL))))
          NIL
          (GO G190)
```

```
          G191
          (EXIT NIL))
        (EXIT (CONS 0 (CONS |bs| |ts|)))))))))))

(DEFUN |TSETCAT-;basicSet;LMMU;5| (|ps| |pred?| |redOp?| $)
 (PROG (|bps| |b| |bs| |p| |gps| |ts|)
  (RETURN
   (SEQ
    (LETT |ps|
     (SPADCALL (ELT $ 31) |ps| (QREFELT $ 33))
     |TSETCAT-;basicSet;LMMU;5|)
    (EXIT
     (COND
      ((SPADCALL (ELT $ 34) |ps| (QREFELT $ 35)) (CONS 1 "failed"))
      ((QUOTE T)
       (SEQ
        (LETT |gps| NIL |TSETCAT-;basicSet;LMMU;5|)
        (LETT |bps| NIL |TSETCAT-;basicSet;LMMU;5|)
        (SEQ
         G190
         (COND
          ((NULL (COND ((NULL |ps|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
            (GO G191)))
         (SEQ
          (LETT |p| (|SPADfirst| |ps|) |TSETCAT-;basicSet;LMMU;5|)
          (LETT |ps| (CDR |ps|) |TSETCAT-;basicSet;LMMU;5|)
          (EXIT
           (COND
            ((SPADCALL |p| |pred?|)
             (LETT |gps| (CONS |p| |gps|) |TSETCAT-;basicSet;LMMU;5|))
            ((QUOTE T)
             (LETT |bps| (CONS |p| |bps|) |TSETCAT-;basicSet;LMMU;5|)))))
          NIL
          (GO G190)
          G191
          (EXIT NIL))
        (LETT |gps|
         (SPADCALL (ELT $ 21) |gps| (QREFELT $ 36)) |TSETCAT-;basicSet;LMMU;5|)
        (LETT |bs| (SPADCALL (QREFELT $ 37)) |TSETCAT-;basicSet;LMMU;5|)
        (LETT |ts| NIL |TSETCAT-;basicSet;LMMU;5|)
        (SEQ
         G190
         (COND
          ((NULL (COND ((NULL |gps|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
           (GO G191)))
         (SEQ
          (LETT |b| (|SPADfirst| |gps|) |TSETCAT-;basicSet;LMMU;5|)
          (LETT |bs|
           (SPADCALL |bs| |b| (QREFELT $ 38)) |TSETCAT-;basicSet;LMMU;5|)
          (LETT |gps| (CDR |gps|) |TSETCAT-;basicSet;LMMU;5|)
          (EXIT
           (SEQ
            G190
            (COND
```

```
             ((NULL
              (COND
               ((OR
                 (NULL |gps|)
                 (SPADCALL
                  (LETT |p| (|SPADfirst| |gps|) |TSETCAT-;basicSet;LMMU;5|)
                  |bs| |redOp?| (QREFELT $ 39)))
                (QUOTE NIL))
               ((QUOTE T) (QUOTE T))))
              (GO G191)))
            (SEQ
             (LETT |ts| (CONS |p| |ts|) |TSETCAT-;basicSet;LMMU;5|)
             (EXIT (LETT |gps| (CDR |gps|) |TSETCAT-;basicSet;LMMU;5|)))
            NIL
            (GO G190)
            G191
            (EXIT NIL))))
         NIL
         (GO G190)
         G191
         (EXIT NIL))
        (LETT |ts|
         (SPADCALL
          (ELT $ 21)
          (SPADCALL |ts| |bps| (QREFELT $ 43))
          (QREFELT $ 36))
         |TSETCAT-;basicSet;LMMU;5|)
        (EXIT (CONS 0 (CONS |bs| |ts|)))))))))))

(DEFUN |TSETCAT-;initials;SL;6| (|ts| $)
 (PROG (|p| |ip| |lip| |lp|)
  (RETURN
   (SEQ
    (LETT |lip| NIL |TSETCAT-;initials;SL;6|)
    (EXIT
     (COND
      ((SPADCALL |ts| (QREFELT $ 12)) |lip|)
      ((QUOTE T)
       (SEQ
        (LETT |lp| (SPADCALL |ts| (QREFELT $ 28)) |TSETCAT-;initials;SL;6|)
        (SEQ
         G190
         (COND
          ((NULL (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
            (GO G191)))
         (SEQ
          (LETT |p| (|SPADfirst| |lp|) |TSETCAT-;initials;SL;6|)
          (COND
           ((NULL
             (SPADCALL
              (LETT |ip|
               (SPADCALL |p| (QREFELT $ 45))
               |TSETCAT-;initials;SL;6|)
              (QREFELT $ 34)))
```

```
            (LETT |lip|
             (CONS (SPADCALL |ip| (QREFELT $ 46)) |lip|)
             |TSETCAT-;initials;SL;6|)))
           (EXIT (LETT |lp| (CDR |lp|) |TSETCAT-;initials;SL;6|)))
          NIL
          (GO G190)
          G191
          (EXIT NIL))
         (EXIT (SPADCALL |lip| (QREFELT $ 47))))))))))

(DEFUN |TSETCAT-;degree;SNni;7| (|ts| $)
 (PROG (|lp| |d|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |ts| (QREFELT $ 12)) 0)
     ((QUOTE T)
      (SEQ
       (LETT |lp| (SPADCALL |ts| (QREFELT $ 28)) |TSETCAT-;degree;SNni;7|)
       (LETT |d|
        (SPADCALL (|SPADfirst| |lp|) (QREFELT $ 50))
        |TSETCAT-;degree;SNni;7|)
       (SEQ
        G190
        (COND
         ((NULL
           (COND
            ((NULL
              (LETT |lp| (CDR |lp|) |TSETCAT-;degree;SNni;7|)) (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
           (GO G191)))
        (SEQ
         (EXIT
          (LETT |d|
           (* |d| (SPADCALL (|SPADfirst| |lp|) (QREFELT $ 50)))
           |TSETCAT-;degree;SNni;7|)))
        NIL
        (GO G190)
        G191
        (EXIT NIL))
       (EXIT |d|))))))))

(DEFUN |TSETCAT-;quasiComponent;SR;8| (|ts| $)
 (CONS (SPADCALL |ts| (QREFELT $ 28)) (SPADCALL |ts| (QREFELT $ 52))))

(DEFUN |TSETCAT-;normalized?;PSB;9| (|p| |ts| $)
 (SPADCALL |p| (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 56)))

(DEFUN |TSETCAT-;stronglyReduced?;PSB;10| (|p| |ts| $)
 (SPADCALL |p| (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 58)))

(DEFUN |TSETCAT-;headReduced?;PSB;11| (|p| |ts| $)
 (SPADCALL (SPADCALL |p| (QREFELT $ 60)) |ts| (QREFELT $ 61)))
```

```
(DEFUN |TSETCAT-;initiallyReduced?;PSB;12| (|p| |ts| $)
 (PROG (|lp| |red|)
  (RETURN
   (SEQ
    (LETT |lp|
     (SPADCALL |ts| (QREFELT $ 28)) |TSETCAT-;initiallyReduced?;PSB;12|)
    (LETT |red| (QUOTE T) |TSETCAT-;initiallyReduced?;PSB;12|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((OR (NULL |lp|) (SPADCALL |p| (QREFELT $ 34))) (QUOTE NIL))
         ((QUOTE T) |red|)))
       (GO G191)))
     (SEQ
      (SEQ
       G190
       (COND
        ((NULL
          (COND
           ((NULL |lp|) (QUOTE NIL))
           ((QUOTE T)
            (SPADCALL
             (SPADCALL |p| (QREFELT $ 23))
             (SPADCALL (|SPADfirst| |lp|) (QREFELT $ 23))
             (QREFELT $ 63)))))
          (GO G191)))
       (SEQ (EXIT (LETT |lp| (CDR |lp|) |TSETCAT-;initiallyReduced?;PSB;12|)))
       NIL
       (GO G190)
       G191
       (EXIT NIL))
      (EXIT
       (COND
        ((NULL (NULL |lp|))
         (COND
          ((SPADCALL
            (SPADCALL (|SPADfirst| |lp|) (QREFELT $ 23))
            (SPADCALL |p| (QREFELT $ 23))
            (QREFELT $ 64))
           (COND
            ((SPADCALL |p| (|SPADfirst| |lp|) (QREFELT $ 65))
             (SEQ
              (LETT |lp| (CDR |lp|) |TSETCAT-;initiallyReduced?;PSB;12|)
              (EXIT
               (LETT |p|
                (SPADCALL |p| (QREFELT $ 45))
                |TSETCAT-;initiallyReduced?;PSB;12|))))
            ((QUOTE T)
             (LETT |red| (QUOTE NIL) |TSETCAT-;initiallyReduced?;PSB;12|))))
          ((QUOTE T)
           (LETT |p|
            (SPADCALL |p| (QREFELT $ 45))
```

```
          |TSETCAT-;initiallyReduced?;PSB;12|)))))))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |red|)))))

(DEFUN |TSETCAT-;reduce;PSMMP;13| (|p| |ts| |redOp| |redOp?| $)
 (PROG (|ts0| #0=#:G1572 |reductor| #1=#:G1575)
  (RETURN
   (SEQ
    (COND
     ((OR (SPADCALL |ts| (QREFELT $ 12)) (SPADCALL |p| (QREFELT $ 34))) |p|)
     ((QUOTE T)
      (SEQ
       (LETT |ts0| |ts| |TSETCAT-;reduce;PSMMP;13|)
       (SEQ
        G190
        (COND
         ((NULL
          (COND
           ((OR
             (SPADCALL |ts| (QREFELT $ 12))
             (SPADCALL |p| (QREFELT $ 34)))
            (QUOTE NIL))
           ((QUOTE T) (QUOTE T))))
          (GO G191)))
        (SEQ
         (LETT |reductor|
          (PROG2
           (LETT #0# (SPADCALL |ts| (QREFELT $ 14)) |TSETCAT-;reduce;PSMMP;13|)
           (QCDR #0#)
           (|check-union| (QEQCAR #0# 0) (QREFELT $ 10) #0#))
          |TSETCAT-;reduce;PSMMP;13|)
         (LETT |ts|
          (PROG2
           (LETT #1# (SPADCALL |ts| (QREFELT $ 17)) |TSETCAT-;reduce;PSMMP;13|)
           (QCDR #1#)
           (|check-union| (QEQCAR #1# 0) (QREFELT $ 6) #1#))
          |TSETCAT-;reduce;PSMMP;13|)
         (EXIT
          (COND
           ((NULL (SPADCALL |p| |reductor| |redOp?|))
            (SEQ
             (LETT |p|
              (SPADCALL |p| |reductor| |redOp|) |TSETCAT-;reduce;PSMMP;13|)
             (EXIT (LETT |ts| |ts0| |TSETCAT-;reduce;PSMMP;13|)))))))
        NIL
        (GO G190)
        G191
        (EXIT NIL))
       (EXIT |p|))))))))

(DEFUN |TSETCAT-;rewriteSetWithReduction;LSMML;14|
```

```
    (|lp| |ts| |redOp| |redOp?| $)
 (PROG (|p| |rs|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |ts| (QREFELT $ 69)) |lp|)
     ((QUOTE T)
      (SEQ
       (LETT |lp|
        (SPADCALL (ELT $ 31) |lp| (QREFELT $ 33))
        |TSETCAT-;rewriteSetWithReduction;LSMML;14|)
       (EXIT
        (COND
         ((NULL |lp|) |lp|)
         ((SPADCALL (ELT $ 34) |lp| (QREFELT $ 35))
           (LIST (|spadConstant| $ 70)))
         ((QUOTE T)
          (SEQ
           (LETT |rs| NIL |TSETCAT-;rewriteSetWithReduction;LSMML;14|)
           (SEQ
            G190
            (COND
             ((NULL (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
              (GO G191)))
            (SEQ
             (LETT |p|
              (|SPADfirst| |lp|)
              |TSETCAT-;rewriteSetWithReduction;LSMML;14|)
             (LETT |lp| (CDR |lp|) |TSETCAT-;rewriteSetWithReduction;LSMML;14|)
             (LETT |p|
              (SPADCALL
               (SPADCALL |p| |ts| |redOp| |redOp?| (QREFELT $ 71))
               (QREFELT $ 46))
              |TSETCAT-;rewriteSetWithReduction;LSMML;14|)
             (EXIT
              (COND
               ((NULL (SPADCALL |p| (QREFELT $ 31)))
                 (COND
                  ((SPADCALL |p| (QREFELT $ 34))
                   (SEQ
                    (LETT |lp| NIL |TSETCAT-;rewriteSetWithReduction;LSMML;14|)
                    (EXIT
                     (LETT |rs|
                      (LIST (|spadConstant| $ 70))
                      |TSETCAT-;rewriteSetWithReduction;LSMML;14|))))
                  ((QUOTE T)
                   (LETT |rs|
                    (CONS |p| |rs|)
                    |TSETCAT-;rewriteSetWithReduction;LSMML;14|)))))))
            NIL
            (GO G190)
            G191
            (EXIT NIL))
           (EXIT (SPADCALL |rs| (QREFELT $ 47)))))))))))))
```

```
(DEFUN |TSETCAT-;stronglyReduce;PSP;15| (|p| |ts| $)
 (SPADCALL |p| |ts| (ELT $ 73) (ELT $ 65) (QREFELT $ 71)))

(DEFUN |TSETCAT-;headReduce;PSP;16| (|p| |ts| $)
 (SPADCALL |p| |ts| (ELT $ 75) (ELT $ 76) (QREFELT $ 71)))

(DEFUN |TSETCAT-;initiallyReduce;PSP;17| (|p| |ts| $)
 (SPADCALL |p| |ts| (ELT $ 78) (ELT $ 79) (QREFELT $ 71)))

(DEFUN |TSETCAT-;removeZero;PSP;18| (|p| |ts| $)
 (PROG (|v| |tsv-| #0=#:G1599 #1=#:G1608 |q|)
  (RETURN
   (SEQ
    (EXIT
     (COND
      ((OR (SPADCALL |p| (QREFELT $ 34)) (SPADCALL |ts| (QREFELT $ 12))) |p|)
      ((QUOTE T)
       (SEQ
        (LETT |v| (SPADCALL |p| (QREFELT $ 23)) |TSETCAT-;removeZero;PSP;18|)
        (LETT |tsv-|
         (SPADCALL |ts| |v| (QREFELT $ 81))
         |TSETCAT-;removeZero;PSP;18|)
        (COND
         ((SPADCALL |v| |ts| (QREFELT $ 82))
          (SEQ
           (LETT |q|
            (SPADCALL |p|
             (PROG2
              (LETT #0#
               (SPADCALL |ts| |v| (QREFELT $ 83))
               |TSETCAT-;removeZero;PSP;18|)
              (QCDR #0#)
              (|check-union| (QEQCAR #0# 0) (QREFELT $ 10) #0#))
             (QREFELT $ 73))
            |TSETCAT-;removeZero;PSP;18|)
           (EXIT
            (COND
             ((SPADCALL |q| (QREFELT $ 31))
               (PROGN (LETT #1# |q| |TSETCAT-;removeZero;PSP;18|) (GO #1#)))
              ((SPADCALL (SPADCALL |q| |tsv-| (QREFELT $ 84)) (QREFELT $ 31))
                (PROGN
                 (LETT #1#
                  (|spadConstant| $ 85)
                  |TSETCAT-;removeZero;PSP;18|) (GO #1#))))))))
        (EXIT
         (COND
          ((SPADCALL |tsv-| (QREFELT $ 12)) |p|)
          ((QUOTE T)
           (SEQ
            (LETT |q| (|spadConstant| $ 85) |TSETCAT-;removeZero;PSP;18|)
            (SEQ
             G190
             (COND
```

```
              ((NULL
                (SPADCALL (SPADCALL |p| |v| (QREFELT $ 86)) (QREFELT $ 88)))
               (GO G191)))
             (SEQ
              (LETT |q|
               (SPADCALL
                (SPADCALL
                 (SPADCALL
                  (SPADCALL |p| (QREFELT $ 45))
                   |tsv-| (QREFELT $ 84))
                  (SPADCALL |p| (QREFELT $ 89))
                 (QREFELT $ 90))
                |q|
                (QREFELT $ 91))
               |TSETCAT-;removeZero;PSP;18|)
              (EXIT
               (LETT |p|
                (SPADCALL |p| (QREFELT $ 92))
                |TSETCAT-;removeZero;PSP;18|)))
             NIL
             (GO G190)
             G191
             (EXIT NIL))
            (EXIT
             (SPADCALL |q|
              (SPADCALL |p| |tsv-| (QREFELT $ 84))
              (QREFELT $ 91)))))))))))
    #1#
    (EXIT #1#)))))

(DEFUN |TSETCAT-;reduceByQuasiMonic;PSP;19| (|p| |ts| $)
 (COND
  ((OR (SPADCALL |p| (QREFELT $ 34)) (SPADCALL |ts| (QREFELT $ 12))) |p|)
  ((QUOTE T)
    (QVELT (SPADCALL |p| (SPADCALL |ts| (QREFELT $ 94)) (QREFELT $ 96)) 1))))

(DEFUN |TSETCAT-;autoReduced?;SMB;20| (|ts| |redOp?| $)
 (PROG (|p| |lp|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |ts| (QREFELT $ 12)) (QUOTE T))
     ((QUOTE T)
      (SEQ
       (LETT |lp|
        (SPADCALL |ts| (QREFELT $ 28)) |TSETCAT-;autoReduced?;SMB;20|)
       (LETT |p| (|SPADfirst| |lp|) |TSETCAT-;autoReduced?;SMB;20|)
       (LETT |lp| (CDR |lp|) |TSETCAT-;autoReduced?;SMB;20|)
       (SEQ
        G190
        (COND
         ((NULL
           (COND
            ((NULL |lp|) (QUOTE NIL))
```

```
           ((QUOTE T) (SPADCALL |p| |lp| |redOp?|))))
          (GO G191)))
        (SEQ
         (LETT |p| (|SPADfirst| |lp|) |TSETCAT-;autoReduced?;SMB;20|)
         (EXIT (LETT |lp| (CDR |lp|) |TSETCAT-;autoReduced?;SMB;20|)))
        NIL
        (GO G190)
        G191
        (EXIT NIL))
       (EXIT (NULL |lp|)))))))))

(DEFUN |TSETCAT-;stronglyReduced?;SB;21| (|ts| $)
 (SPADCALL |ts| (ELT $ 58) (QREFELT $ 100)))

(DEFUN |TSETCAT-;normalized?;SB;22| (|ts| $)
 (SPADCALL |ts| (ELT $ 56) (QREFELT $ 100)))

(DEFUN |TSETCAT-;headReduced?;SB;23| (|ts| $)
 (SPADCALL |ts| (ELT $ 103) (QREFELT $ 100)))

(DEFUN |TSETCAT-;initiallyReduced?;SB;24| (|ts| $)
 (SPADCALL |ts| (ELT $ 105) (QREFELT $ 100)))

(DEFUN |TSETCAT-;mvar;SV;25| (|ts| $)
 (PROG (#0=#:G1627)
  (RETURN
   (COND
    ((SPADCALL |ts| (QREFELT $ 12))
      (|error| "Error from TSETCAT in mvar : #1 is empty"))
    ((QUOTE T)
      (SPADCALL
       (PROG2
        (LETT #0# (SPADCALL |ts| (QREFELT $ 14)) |TSETCAT-;mvar;SV;25|)
        (QCDR #0#)
        (|check-union| (QEQCAR #0# 0) (QREFELT $ 10) #0#))
       (QREFELT $ 23)))))))

(DEFUN |TSETCAT-;first;SU;26| (|ts| $)
 (PROG (|lp|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |ts| (QREFELT $ 12)) (CONS 1 "failed"))
     ((QUOTE T)
      (SEQ
       (LETT |lp|
        (SPADCALL (ELT $ 22) (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 36))
        |TSETCAT-;first;SU;26|)
       (EXIT (CONS 0 (|SPADfirst| |lp|))))))))))

(DEFUN |TSETCAT-;last;SU;27| (|ts| $)
 (PROG (|lp|)
  (RETURN
   (SEQ
```

```
    (COND
     ((SPADCALL |ts| (QREFELT $ 12)) (CONS 1 "failed"))
     ((QUOTE T)
      (SEQ
       (LETT |lp|
        (SPADCALL (ELT $ 21) (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 36))
        |TSETCAT-;last;SU;27|)
       (EXIT (CONS 0 (|SPADfirst| |lp|))))))))))

(DEFUN |TSETCAT-;rest;SU;28| (|ts| $)
 (PROG (|lp|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |ts| (QREFELT $ 12)) (CONS 1 "failed"))
     ((QUOTE T)
      (SEQ
       (LETT |lp|
        (SPADCALL (ELT $ 22) (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 36))
        |TSETCAT-;rest;SU;28|)
       (EXIT (CONS 0 (SPADCALL (CDR |lp|) (QREFELT $ 110))))))))))

(DEFUN |TSETCAT-;coerce;SL;29| (|ts| $)
 (SPADCALL (ELT $ 22) (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 36)))

(DEFUN |TSETCAT-;algebraicVariables;SL;30| (|ts| $)
 (PROG (#0=#:G1652 |p| #1=#:G1653)
  (RETURN
   (SEQ
    (PROGN
     (LETT #0# NIL |TSETCAT-;algebraicVariables;SL;30|)
     (SEQ
      (LETT |p| NIL |TSETCAT-;algebraicVariables;SL;30|)
      (LETT #1#
       (SPADCALL |ts| (QREFELT $ 28)) |TSETCAT-;algebraicVariables;SL;30|)
      G190
      (COND
       ((OR
         (ATOM #1#)
         (PROGN (LETT |p| (CAR #1#) |TSETCAT-;algebraicVariables;SL;30|) NIL))
        (GO G191)))
      (SEQ
       (EXIT
        (LETT #0#
         (CONS (SPADCALL |p| (QREFELT $ 23)) #0#)
         |TSETCAT-;algebraicVariables;SL;30|)))
      (LETT #1# (CDR #1#) |TSETCAT-;algebraicVariables;SL;30|)
      (GO G190)
      G191
      (EXIT (NREVERSE0 #0#))))))))

(DEFUN |TSETCAT-;algebraic?;VSB;31| (|v| |ts| $)
 (SPADCALL |v| (SPADCALL |ts| (QREFELT $ 115)) (QREFELT $ 116)))
```

```
(DEFUN |TSETCAT-;select;SVU;32| (|ts| |v| $)
 (PROG (|lp|)
  (RETURN
   (SEQ
    (LETT |lp|
     (SPADCALL (ELT $ 22) (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 36))
     |TSETCAT-;select;SVU;32|)
    (SEQ
     G190
     (COND
      ((NULL
       (COND
        ((OR
          (NULL |lp|)
          (SPADCALL |v|
           (SPADCALL (|SPADfirst| |lp|) (QREFELT $ 23))
           (QREFELT $ 64)))
          (QUOTE NIL))
        ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ (EXIT (LETT |lp| (CDR |lp|) |TSETCAT-;select;SVU;32|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT
     (COND
      ((NULL |lp|) (CONS 1 "failed"))
      ((QUOTE T) (CONS 0 (|SPADfirst| |lp|)))))))))

(DEFUN |TSETCAT-;collectQuasiMonic;2S;33| (|ts| $)
 (PROG (|newlp| |lp|)
  (RETURN
   (SEQ
    (LETT |lp|
     (SPADCALL |ts| (QREFELT $ 28))
     |TSETCAT-;collectQuasiMonic;2S;33|)
    (LETT |newlp| NIL |TSETCAT-;collectQuasiMonic;2S;33|)
    (SEQ
     G190
     (COND
      ((NULL (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
        (GO G191)))
     (SEQ
      (COND
       ((SPADCALL (SPADCALL (|SPADfirst| |lp|) (QREFELT $ 45)) (QREFELT $ 34))
        (LETT |newlp|
         (CONS (|SPADfirst| |lp|) |newlp|)
         |TSETCAT-;collectQuasiMonic;2S;33|)))
      (EXIT (LETT |lp| (CDR |lp|) |TSETCAT-;collectQuasiMonic;2S;33|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
```

```
    (EXIT (SPADCALL |newlp| (QREFELT $ 110)))))))

(DEFUN |TSETCAT-;collectUnder;SVS;34| (|ts| |v| $)
 (PROG (|lp|)
  (RETURN
   (SEQ
    (LETT |lp|
     (SPADCALL (ELT $ 22) (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 36))
     |TSETCAT-;collectUnder;SVS;34|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((OR
           (NULL |lp|)
           (SPADCALL
            (SPADCALL (|SPADfirst| |lp|) (QREFELT $ 23))
            |v|
            (QREFELT $ 63)))
          (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ (EXIT (LETT |lp| (CDR |lp|) |TSETCAT-;collectUnder;SVS;34|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT (SPADCALL |lp| (QREFELT $ 110)))))))

(DEFUN |TSETCAT-;collectUpper;SVS;35| (|ts| |v| $)
 (PROG (|lp2| |lp1|)
  (RETURN
   (SEQ
    (LETT |lp1|
     (SPADCALL (ELT $ 22) (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 36))
     |TSETCAT-;collectUpper;SVS;35|)
    (LETT |lp2| NIL |TSETCAT-;collectUpper;SVS;35|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((NULL |lp1|) (QUOTE NIL))
         ((QUOTE T)
          (SPADCALL |v|
           (SPADCALL (|SPADfirst| |lp1|) (QREFELT $ 23))
           (QREFELT $ 63)))))
       (GO G191)))
     (SEQ
      (LETT |lp2|
       (CONS (|SPADfirst| |lp1|) |lp2|)
       |TSETCAT-;collectUpper;SVS;35|)
      (EXIT (LETT |lp1| (CDR |lp1|) |TSETCAT-;collectUpper;SVS;35|)))
```

```
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT (SPADCALL (REVERSE |lp2|) (QREFELT $ 110)))))))

(DEFUN |TSETCAT-;construct;LS;36| (|lp| $)
 (PROG (|rif|)
  (RETURN
   (SEQ
    (LETT |rif| (SPADCALL |lp| (QREFELT $ 122)) |TSETCAT-;construct;LS;36|)
    (EXIT
     (COND
      ((QEQCAR |rif| 0) (QCDR |rif|))
      ((QUOTE T)
       (|error| "in construct : LP -> $ from TSETCAT : bad arg"))))))))

(DEFUN |TSETCAT-;retractIfCan;LU;37| (|lp| $)
 (PROG (|rif|)
  (RETURN
   (SEQ
    (COND
     ((NULL |lp|) (CONS 0 (SPADCALL (QREFELT $ 37))))
     ((QUOTE T)
      (SEQ
       (LETT |lp|
        (SPADCALL (ELT $ 22) |lp| (QREFELT $ 36))
        |TSETCAT-;retractIfCan;LU;37|)
       (LETT |rif|
        (SPADCALL (CDR |lp|) (QREFELT $ 122))
        |TSETCAT-;retractIfCan;LU;37|)
       (EXIT
        (COND
         ((QEQCAR |rif| 0)
           (SPADCALL (QCDR |rif|) (|SPADfirst| |lp|) (QREFELT $ 124)))
         ((QUOTE T)
           (|error| "in retractIfCan : LP -> ... from TSETCAT : bad arg")
  ))))))))))

(DEFUN |TSETCAT-;extend;SPS;38| (|ts| |p| $)
 (PROG (|eif|)
  (RETURN
   (SEQ
    (LETT |eif| (SPADCALL |ts| |p| (QREFELT $ 124)) |TSETCAT-;extend;SPS;38|)
    (EXIT
     (COND
      ((QEQCAR |eif| 0) (QCDR |eif|))
      ((QUOTE T)
       (|error| "in extend : ($,P) -> $ from TSETCAT : bad ars"))))))))

(DEFUN |TSETCAT-;coHeight;SNni;39| (|ts| $)
 (PROG (|n| |m| #0=#:G1696)
  (RETURN
   (SEQ
```

```
    (LETT |n| (SPADCALL (QREFELT $ 127)) |TSETCAT-;coHeight;SNni;39|)
    (LETT |m|
     (LENGTH (SPADCALL |ts| (QREFELT $ 28)))
     |TSETCAT-;coHeight;SNni;39|)
    (EXIT
     (PROG2
      (LETT #0# (SPADCALL |n| |m| (QREFELT $ 128)) |TSETCAT-;coHeight;SNni;39|)
      (QCDR #0#)
      (|check-union| (QEQCAR #0# 0) (|NonNegativeInteger|) #0#)))))))

(DEFUN |TriangularSetCategory&| (|#1| |#2| |#3| |#4| |#5|)
 (PROG (DV$1 DV$2 DV$3 DV$4 DV$5 |dv$| $ |pv$|)
  (RETURN
   (PROGN
    (LETT DV$1 (|devaluate| |#1|) . #0=(|TriangularSetCategory&|))
    (LETT DV$2 (|devaluate| |#2|) . #0#)
    (LETT DV$3 (|devaluate| |#3|) . #0#)
    (LETT DV$4 (|devaluate| |#4|) . #0#)
    (LETT DV$5 (|devaluate| |#5|) . #0#)
    (LETT |dv$|
     (LIST (QUOTE |TriangularSetCategory&|) DV$1 DV$2 DV$3 DV$4 DV$5) . #0#)
    (LETT $ (make-array 131) . #0#)
    (QSETREFV $ 0 |dv$|)
    (QSETREFV $ 3
     (LETT |pv$|
      (|buildPredVector| 0 0 (LIST (|HasCategory| |#4| (QUOTE (|Finite|)))))
      . #0#))
    (|stuffDomainSlots| $)
    (QSETREFV $ 6 |#1|)
    (QSETREFV $ 7 |#2|)
    (QSETREFV $ 8 |#3|)
    (QSETREFV $ 9 |#4|)
    (QSETREFV $ 10 |#5|)
    (COND
     ((|testBitVector| |pv$| 1)
       (QSETREFV $ 129
        (CONS (|dispatchFunction| |TSETCAT-;coHeight;SNni;39|) $))))
     $))))

(setf (get
 (QUOTE |TriangularSetCategory&|)
 (QUOTE |infovec|))
 (LIST (QUOTE
  #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|) (|local| |#3|)
    (|local| |#4|) (|local| |#5|) (|Boolean|) (0 . |empty?|)
    (|Union| 10 (QUOTE "failed")) (5 . |first|) (10 . =)
    (|Union| $ (QUOTE "failed")) (16 . |rest|) (21 . =) |TSETCAT-;=;2SB;1|
    (27 . |last|) (32 . |infRittWu?|) (38 . |supRittWu?|) (44 . |mvar|)
    (49 . |collectUpper|) (55 . |infRittWu?|) |TSETCAT-;infRittWu?;2SB;2|
    (|List| 10) (61 . |members|) (|Mapping| 11 10 10)
    |TSETCAT-;reduced?;PSMB;3| (66 . |zero?|) (|Mapping| 11 10)
    (71 . |remove|) (77 . |ground?|) (82 . |any?|) (88 . |sort|)
    (94 . |empty|) (98 . |extend|) (104 . |reduced?|)
    (|Record| (|:| |bas| $) (|:| |top| 27))
```

```
   (|Union| 40 (QUOTE "failed")) |TSETCAT-;basicSet;LMU;4|
   (111 . |concat|) |TSETCAT-;basicSet;LMMU;5| (117 . |init|)
   (122 . |primPartElseUnitCanonical|) (127 . |removeDuplicates|)
   |TSETCAT-;initials;SL;6| (|NonNegativeInteger|) (132 . |mdeg|)
   |TSETCAT-;degree;SNni;7| (137 . |initials|)
   (|Record| (|:| |close| 27) (|:| |open| 27))
   |TSETCAT-;quasiComponent;SR;8| (|List| $) (142 . |normalized?|)
   |TSETCAT-;normalized?;PSB;9| (148 . |reduced?|)
   |TSETCAT-;stronglyReduced?;PSB;10| (154 . |head|)
   (159 . |stronglyReduced?|) |TSETCAT-;headReduced?;PSB;11|
   (165 . <) (171 . =) (177 . |reduced?|)
   |TSETCAT-;initiallyReduced?;PSB;12| (|Mapping| 10 10 10)
   |TSETCAT-;reduce;PSMMP;13| (183 . |trivialIdeal?|) (188 . |One|)
   (192 . |reduce|) |TSETCAT-;rewriteSetWithReduction;LSMML;14|
   (200 . |lazyPrem|) |TSETCAT-;stronglyReduce;PSP;15|
   (206 . |headReduce|) (212 . |headReduced?|)
   |TSETCAT-;headReduce;PSP;16| (218 . |initiallyReduce|)
   (224 . |initiallyReduced?|) |TSETCAT-;initiallyReduce;PSP;17|
   (230 . |collectUnder|) (236 . |algebraic?|) (242 . |select|)
   (248 . |removeZero|) (254 . |Zero|) (258 . |degree|) (|Integer|)
   (264 . |positive?|) (269 . |mainMonomial|) (274 . *) (280 . +)
   (286 . |tail|) |TSETCAT-;removeZero;PSP;18| (291 . |collectQuasiMonic|)
   (|Record| (|:| |rnum| 7) (|:| |polnum| 10) (|:| |den| 7))
   (296 . |remainder|) |TSETCAT-;reduceByQuasiMonic;PSP;19|
   (|Mapping| 11 10 27) |TSETCAT-;autoReduced?;SMB;20|
   (302 . |autoReduced?|) |TSETCAT-;stronglyReduced?;SB;21|
   |TSETCAT-;normalized?;SB;22| (308 . |headReduced?|)
   |TSETCAT-;headReduced?;SB;23| (314 . |initiallyReduced?|)
   |TSETCAT-;initiallyReduced?;SB;24| |TSETCAT-;mvar;SV;25|
   |TSETCAT-;first;SU;26| |TSETCAT-;last;SU;27| (320 . |construct|)
   |TSETCAT-;rest;SU;28| |TSETCAT-;coerce;SL;29| (|List| 9)
   |TSETCAT-;algebraicVariables;SL;30| (325 . |algebraicVariables|)
   (330 . |member?|) |TSETCAT-;algebraic?;VSB;31|
   |TSETCAT-;select;SVU;32| |TSETCAT-;collectQuasiMonic;2S;33|
   |TSETCAT-;collectUnder;SVS;34| |TSETCAT-;collectUpper;SVS;35|
   (336 . |retractIfCan|) |TSETCAT-;construct;LS;36|
   (341 . |extendIfCan|) |TSETCAT-;retractIfCan;LU;37|
   |TSETCAT-;extend;SPS;38| (347 . |size|) (351 . |subtractIfCan|)
   (357 . |coHeight|) (|OutputForm|)))
(QUOTE #(|stronglyReduced?| 362 |stronglyReduce| 373 |select| 379
   |rewriteSetWithReduction| 385 |retractIfCan| 393 |rest| 398 |removeZero|
   403 |reduced?| 409 |reduceByQuasiMonic| 416 |reduce| 422
   |quasiComponent| 430 |normalized?| 435 |mvar| 446 |last| 451
   |initials| 456 |initiallyReduced?| 461 |initiallyReduce| 472
   |infRittWu?| 478 |headReduced?| 484 |headReduce| 495 |first| 501
   |extend| 506 |degree| 512 |construct| 517 |collectUpper| 522
   |collectUnder| 528 |collectQuasiMonic| 534 |coerce| 539 |coHeight|
   544 |basicSet| 549 |autoReduced?| 562 |algebraicVariables| 568
   |algebraic?| 573 = 579))
(QUOTE NIL)
(CONS
 (|makeByteWordVec2| 1 (QUOTE NIL))
 (CONS (QUOTE #()) (CONS (QUOTE #())
  (|makeByteWordVec2| 129 (QUOTE (1 6 11 0 12 1 6 13 0 14 2 10 11 0 0 15
```

```
     1 6 16 0 17 2 6 11 0 0 18 1 6 13 0 20 2 10 11 0 0 21 2 10 11 0 0 22
     1 10 9 0 23 2 6 0 0 9 24 2 6 11 0 0 25 1 6 27 0 28 1 10 11 0 31 2 27
     0 32 0 33 1 10 11 0 34 2 27 11 32 0 35 2 27 0 29 0 36 0 6 0 37 2 6 0
     0 10 38 3 6 11 10 0 29 39 2 27 0 0 0 43 1 10 0 0 45 1 10 0 0 46 1 27
     0 0 47 1 10 49 0 50 1 6 27 0 52 2 10 11 0 55 56 2 10 11 0 55 58 1 10
     0 0 60 2 6 11 10 0 61 2 9 11 0 0 63 2 9 11 0 0 64 2 10 11 0 0 65 1 6
     11 0 69 0 10 0 70 4 6 10 10 0 67 29 71 2 10 0 0 0 73 2 10 0 0 0 75 2
     10 11 0 0 76 2 10 0 0 0 78 2 10 11 0 0 79 2 6 0 0 9 81 2 6 11 9 0 82
     2 6 13 0 9 83 2 6 10 10 0 84 0 10 0 85 2 10 49 0 9 86 1 87 11 0 88 1
     10 0 0 89 2 10 0 0 0 90 2 10 0 0 0 91 1 10 0 0 92 1 6 0 0 94 2 6 95
     10 0 96 2 6 11 0 98 100 2 10 11 0 55 103 2 10 11 0 55 105 1 6 0 27
     110 1 6 113 0 115 2 113 11 9 0 116 1 6 16 27 122 2 6 16 0 10 124 0
     9 49 127 2 49 16 0 0 128 1 0 49 0 129 1 0 11 0 101 2 0 11 10 0 59 2
     0 10 10 0 74 2 0 13 0 9 118 4 0 27 27 0 67 29 72 1 0 16 27 125 1 0
     16 0 111 2 0 10 10 0 93 3 0 11 10 0 29 30 2 0 10 10 0 97 4 0 10 10 0
     67 29 68 1 0 53 0 54 1 0 11 0 102 2 0 11 10 0 57 1 0 9 0 107 1 0 13
     0 109 1 0 27 0 48 1 0 11 0 106 2 0 11 10 0 66 2 0 10 10 0 80 2 0 11
     0 0 26 1 0 11 0 104 2 0 11 10 0 62 2 0 10 10 0 77 1 0 13 0 108 2 0 0
     0 10 126 1 0 49 0 51 1 0 0 27 123 2 0 0 0 9 121 2 0 0 0 9 120 1 0 0
     0 119 1 0 27 0 112 1 0 49 0 129 3 0 41 27 32 29 44 2 0 41 27 29 42 2
     0 11 0 98 99 1 0 113 0 114 2 0 11 9 0 117 2 0 11 0 0 19))))))
  (QUOTE |lookupComplete|)))
```

———-

21.64 UFD.lsp BOOTSTRAP

UFD needs **GCDDOM** which needs **COMRING** which needs **RING** which needs **RNG** which needs **ABELGRP** which needs **CABMON** which needs **ABELMON** which needs **ABELSG** which needs **SETCAT** which needs **SINT** which needs **UFD**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **UFD** category which we can write into the **MID** directory. We compile the lisp code and copy the **UFD.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— UFD.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |UniqueFactorizationDomain;AL| (QUOTE NIL))

(DEFUN |UniqueFactorizationDomain| NIL
  (declare (special |UniqueFactorizationDomain;AL|))
    (COND
      (|UniqueFactorizationDomain;AL|)
      (T
        (SETQ
          |UniqueFactorizationDomain;AL|
          (|UniqueFactorizationDomain;|)))))

(DEFUN |UniqueFactorizationDomain;| NIL
```

```
  (PROG (#1=#:G83332)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|GcdDomain|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|prime?| ((|Boolean|) |$|)) T)
                ((|squareFree| ((|Factored| |$|) |$|)) T)
                ((|squareFreePart| (|$| |$|)) T)
                ((|factor| ((|Factored| |$|) |$|)) T)))
              NIL
              (QUOTE ((|Factored| |$|) (|Boolean|)))
              NIL))
          |UniqueFactorizationDomain|)
        (SETELT #1# 0 (QUOTE (|UniqueFactorizationDomain|)))))))

(setf (get (QUOTE |UniqueFactorizationDomain|) (QUOTE NILADIC)) T)
```

————-

21.65 UFD-.lsp BOOTSTRAP

UFD- needs **UFD**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **UFD-** category which we can write into the **MID** directory. We compile the lisp code and copy the **UFD-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— UFD-.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(DEFUN |UFD-;squareFreePart;2S;1| (|x| |$|)
  (PROG (|s| |f| #1=#:G83349 #2=#:G83347 #3=#:G83345 #4=#:G83346)
    (RETURN
      (SEQ
        (SPADCALL
          (SPADCALL
            (LETT |s|
              (SPADCALL |x| (QREFELT |$| 8))
              |UFD-;squareFreePart;2S;1|)
            (QREFELT |$| 10))
          (PROGN
            (LETT #4# NIL |UFD-;squareFreePart;2S;1|)
            (SEQ
              (LETT |f| NIL |UFD-;squareFreePart;2S;1|)
              (LETT #1#
                (SPADCALL |s| (QREFELT |$| 13))
```

```
                |UFD-;squareFreePart;2S;1|)
              G190
              (COND
                ((OR
                   (ATOM #1#)
                   (PROGN
                     (LETT |f| (CAR #1#) |UFD-;squareFreePart;2S;1|)
                     NIL))
                 (GO G191)))
              (SEQ
                (EXIT
                  (PROGN
                    (LETT #2# (QCAR |f|) |UFD-;squareFreePart;2S;1|)
                    (COND
                      (#4#
                        (LETT #3#
                          (SPADCALL #3# #2# (QREFELT |$| 14))
                          |UFD-;squareFreePart;2S;1|))
                      ((QUOTE T)
                        (PROGN
                          (LETT #3# #2# |UFD-;squareFreePart;2S;1|)
                          (LETT #4# (QUOTE T)
                            |UFD-;squareFreePart;2S;1|)))))))
              (LETT #1# (CDR #1#) |UFD-;squareFreePart;2S;1|)
              (GO G190)
              G191
              (EXIT NIL))
            (COND
              (#4# #3#)
              ((QUOTE T) (|spadConstant| |$| 15))))
       (QREFELT |$| 14))))))

(DEFUN |UFD-;prime?;SB;2| (|x| |$|)
  (EQL
    (LENGTH (SPADCALL (SPADCALL |x| (QREFELT |$| 17)) (QREFELT |$| 21))) 1))

(DEFUN |UniqueFactorizationDomain&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|UniqueFactorizationDomain&|))
        (LETT |dv$| (LIST (QUOTE |UniqueFactorizationDomain&|) |DV$1|) . #1#)
        (LETT |$| (make-array 24) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        |$|))))

(setf (get
  (QUOTE |UniqueFactorizationDomain&|)
  (QUOTE |infovec|))
  (LIST
    (QUOTE
```

```
   #(NIL NIL NIL NIL NIL NIL
     (|local| |#1|)
     (|Factored| |$|)
     (0 . |squareFree|)
     (|Factored| 6)
     (5 . |unit|)
     (|Record| (|:| |factor| 6) (|:| |exponent| (|Integer|)))
     (|List| 11)
     (10 . |factors|)
     (15 . |*|)
     (21 . |One|)
     |UFD-;squareFreePart;2S;1|
     (25 . |factor|)
     (|Union|
       (QUOTE "nil") (QUOTE "sqfr") (QUOTE "irred") (QUOTE "prime"))
     (|Record| (|:| |flg| 18) (|:| |fctr| 6) (|:| |xpnt| (|Integer|)))
     (|List| 19)
     (30 . |factorList|)
     (|Boolean|)
     |UFD-;prime?;SB;2|))
  (QUOTE #(|squareFreePart| 35 |prime?| 40))
  (QUOTE NIL)
  (CONS
    (|makeByteWordVec2| 1 (QUOTE NIL))
    (CONS
      (QUOTE #())
      (CONS
        (QUOTE #())
        (|makeByteWordVec2| 23
          (QUOTE
            (1 6 7 0 8 1 9 6 0 10 1 9 12 0 13 2 6 0 0 0 14 0 6 0 15 1 6 7
             0 17 1 9 20 0 21 1 0 0 0 16 1 0 22 0 23))))))
   (QUOTE |lookupComplete|)))
```

21.66 ULSCAT.lsp BOOTSTRAP

ULSCAT depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ULSCAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **ULSCAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— ULSCAT.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |UnivariateLaurentSeriesCategory;CAT| (QUOTE NIL))

(SETQ |UnivariateLaurentSeriesCategory;AL| (QUOTE NIL))
```

```
(DEFUN |UnivariateLaurentSeriesCategory| (#1=#:G83278)
 (declare (special |UnivariateLaurentSeriesCategory;AL|))
 (LET (#2=#:G83279)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |UnivariateLaurentSeriesCategory;AL|))
     (CDR #2#))
   (T
    (SETQ |UnivariateLaurentSeriesCategory;AL|
     (|cons5|
      (CONS
       (|devaluate| #1#)
       (SETQ #2# (|UnivariateLaurentSeriesCategory;| #1#)))
      |UnivariateLaurentSeriesCategory;AL|))
     #2#))))

(DEFUN |UnivariateLaurentSeriesCategory;| (|t#1|)
 (declare (special |UnivariateLaurentSeriesCategory;CAT|))
 (PROG (#1=#:G83277)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (|sublisV|
       (PAIR (QUOTE (#2=#:G83276)) (LIST (QUOTE (|Integer|))))
       (COND
        (|UnivariateLaurentSeriesCategory;CAT|)
        ((QUOTE T)
         (LETT |UnivariateLaurentSeriesCategory;CAT|
          (|Join|
           (|UnivariatePowerSeriesCategory| (QUOTE |t#1|) (QUOTE #2#))
           (|mkCategory| (QUOTE |domain|)
            (QUOTE (
             ((|series|
               (|$|
                (|Stream| (|Record| (|:| |k| (|Integer|)) (|:| |c| |t#1|)))))
                T)
             ((|multiplyCoefficients| (|$| (|Mapping| |t#1| (|Integer|)) |$|))
                T)
             ((|rationalFunction|
               ((|Fraction| (|Polynomial| |t#1|)) |$| (|Integer|)))
                (|has| |t#1| (|IntegralDomain|)))
             ((|rationalFunction|
               ((|Fraction| (|Polynomial| |t#1|)) |$| (|Integer|) (|Integer|)))
                (|has| |t#1| (|IntegralDomain|)))
             ((|integrate| (|$| |$|))
               (|has| |t#1| (|Algebra| (|Fraction| (|Integer|)))))
             ((|integrate| (|$| |$| (|Symbol|)))
               (AND
                (|has| |t#1|
                  (SIGNATURE |variables| ((|List| (|Symbol|)) |t#1|)))
                (|has| |t#1| (SIGNATURE |integrate| (|t#1| |t#1| (|Symbol|))))
                (|has| |t#1| (|Algebra| (|Fraction| (|Integer|))))))
```

```
              ((|integrate| (|$| |$| (|Symbol|)))
                (AND
                 (|has| |t#1| (|AlgebraicallyClosedFunctionSpace| (|Integer|)))
                 (|has| |t#1| (|PrimitiveFunctionCategory|))
                 (|has| |t#1| (|TranscendentalFunctionCategory|))
                 (|has| |t#1| (|Algebra| (|Fraction| (|Integer|))))))))
            (QUOTE (
             ((|RadicalCategory|)
               (|has| |t#1| (|Algebra| (|Fraction| (|Integer|)))))
             ((|TranscendentalFunctionCategory|)
               (|has| |t#1| (|Algebra| (|Fraction| (|Integer|)))))
             ((|Field|) (|has| |t#1| (|Field|)))))
            (QUOTE (
             (|Symbol|)
             (|Fraction| (|Polynomial| |t#1|))
             (|Integer|)
             (|Stream| (|Record| (|:| |k| (|Integer|)) (|:| |c| |t#1|))))
             NIL))
            . #3=(|UnivariateLaurentSeriesCategory|))))))
           . #3#)
    (SETELT #1# 0
     (LIST (QUOTE |UnivariateLaurentSeriesCategory|) (|devaluate| |t#1|)))))))
```

21.67 UPOLYC.lsp BOOTSTRAP

UPOLYC depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **UPOLYC** category which we can write into the **MID** directory. We compile the lisp code and copy the **UPOLYC.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— UPOLYC.lsp BOOTSTRAP —

```
(/VERSIONCHECK 2)

(SETQ |UnivariatePolynomialCategory;CAT| (QUOTE NIL))

(SETQ |UnivariatePolynomialCategory;AL| (QUOTE NIL))

(DEFUN |UnivariatePolynomialCategory| (#0=#:G1438)
 (declare (special |UnivariatePolynomialCategory;AL|))
 (LET (#1=#:G1439)
  (COND
   ((SETQ #1# (|assoc| (|devaluate| #0#) |UnivariatePolynomialCategory;AL|))
    (CDR #1#))
   (T
    (SETQ |UnivariatePolynomialCategory;AL|
     (|cons5|
      (CONS (|devaluate| #0#) (SETQ #1# (|UnivariatePolynomialCategory;| #0#)))
      |UnivariatePolynomialCategory;AL|))
```

```
    #1#))))

(DEFUN |UnivariatePolynomialCategory;| (|t#1|)
 (declare (special |UnivariatePolynomialCategory;CAT|))
 (PROG (#0=#:G1437)
  (RETURN
   (PROG1
    (LETT #0#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (|sublisV|
       (PAIR
        (QUOTE (#1=#:G1435 #2=#:G1436))
        (LIST
         (QUOTE (|NonNegativeInteger|))
         (QUOTE (|SingletonAsOrderedSet|))))
       (COND
        (|UnivariatePolynomialCategory;CAT|)
        ((QUOTE T)
         (LETT |UnivariatePolynomialCategory;CAT|
          (|Join|
           (|PolynomialCategory| (QUOTE |t#1|) (QUOTE #1#) (QUOTE #2#))
           (|Eltable| (QUOTE |t#1|) (QUOTE |t#1|))
           (|Eltable| (QUOTE $) (QUOTE $))
           (|DifferentialRing|)
           (|DifferentialExtension| (QUOTE |t#1|))
           (|mkCategory|
            (QUOTE |domain|)
            (QUOTE
             (((|vectorise| ((|Vector| |t#1|) $ (|NonNegativeInteger|))) T)
              ((|unvectorise| ($ (|Vector| |t#1|))) T)
              ((|makeSUP| ((|SparseUnivariatePolynomial| |t#1|) $)) T)
              ((|unmakeSUP| ($ (|SparseUnivariatePolynomial| |t#1|))) T)
              ((|multiplyExponents| ($ $ (|NonNegativeInteger|))) T)
              ((|divideExponents|
                ((|Union| $ "failed") $ (|NonNegativeInteger|))) T)
              ((|monicDivide|
                ((|Record| (|:| |quotient| $) (|:| |remainder| $)) $ $)) T)
              ((|karatsubaDivide|
                ((|Record| (|:| |quotient| $) (|:| |remainder| $)) $
                 (|NonNegativeInteger|))) T)
              ((|shiftRight| ($ $ (|NonNegativeInteger|))) T)
              ((|shiftLeft| ($ $ (|NonNegativeInteger|))) T)
              ((|pseudoRemainder| ($ $ $)) T)
              ((|differentiate| ($ $ (|Mapping| |t#1| |t#1|) $)) T)
              ((|discriminant| (|t#1| $)) (|has| |t#1| (|CommutativeRing|)))
              ((|resultant| (|t#1| $ $)) (|has| |t#1| (|CommutativeRing|)))
              ((|elt| ((|Fraction| $) (|Fraction| $) (|Fraction| $)))
                (|has| |t#1| (|IntegralDomain|)))
              ((|order| ((|NonNegativeInteger|) $ $))
                (|has| |t#1| (|IntegralDomain|)))
              ((|subResultantGcd| ($ $ $))
                (|has| |t#1| (|IntegralDomain|)))
              ((|composite| ((|Union| $ "failed") $ $))
```

```
              (|has| |t#1| (|IntegralDomain|)))
            ((|composite|
               ((|Union| (|Fraction| $) "failed") (|Fraction| $) $))
              (|has| |t#1| (|IntegralDomain|)))
            ((|pseudoQuotient| ($ $ $)) (|has| |t#1| (|IntegralDomain|)))
            ((|pseudoDivide|
              ((|Record| (|:| |coef| |t#1|) (|:| |quotient| $)
                         (|:| |remainder| $)) $ $))
              (|has| |t#1| (|IntegralDomain|)))
            ((|separate|
              ((|Record| (|:| |primePart| $) (|:| |commonPart| $)) $ $))
              (|has| |t#1| (|GcdDomain|)))
            ((|elt| (|t#1| (|Fraction| $) |t#1|)) (|has| |t#1| (|Field|)))
            ((|integrate| ($ $))
              (|has| |t#1| (|Algebra| (|Fraction| (|Integer|)))))))
          (QUOTE (
           ((|StepThrough|) (|has| |t#1| (|StepThrough|)))
           ((|Eltable| (|Fraction| $) (|Fraction| $))
             (|has| |t#1| (|IntegralDomain|)))
           ((|EuclideanDomain|) (|has| |t#1| (|Field|)))
           (|additiveValuation| (|has| |t#1| (|Field|)))))
          (QUOTE (
           (|Fraction| $)
           (|NonNegativeInteger|)
           (|SparseUnivariatePolynomial| |t#1|)
           (|Vector| |t#1|)))
          NIL))
      . #3=(|UnivariatePolynomialCategory|))))))
    . #3#)
  (SETELT #0# 0
   (LIST (QUOTE |UnivariatePolynomialCategory|) (|devaluate| |t#1|)))))))
```

———-

21.68 UPOLYC-.lsp BOOTSTRAP

UPOLYC- depends on **UPOLYC**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **UPOLYC-** category which we can write into the **MID** directory. We compile the lisp code and copy the **UPOLYC-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— UPOLYC-.lsp BOOTSTRAP —

```
(/VERSIONCHECK 2)

(DEFUN |UPOLYC-;variables;SL;1| (|p| $)
 (COND
  ((OR (SPADCALL |p| (QREFELT $ 9)) (ZEROP (SPADCALL |p| (QREFELT $ 11))))
     NIL)
  ((QUOTE T) (LIST (SPADCALL (QREFELT $ 13))))))
```

```
(DEFUN |UPOLYC-;degree;SSaosNni;2| (|p| |v| $)
 (declare (ignore |v|))
 (SPADCALL |p| (QREFELT $ 11)))

(DEFUN |UPOLYC-;totalDegree;SLNni;3| (|p| |lv| $)
 (COND ((NULL |lv|) 0) ((QUOTE T) (SPADCALL |p| (QREFELT $ 17)))))

(DEFUN |UPOLYC-;degree;SLL;4| (|p| |lv| $)
 (COND ((NULL |lv|) NIL) ((QUOTE T) (LIST (SPADCALL |p| (QREFELT $ 11))))))

(DEFUN |UPOLYC-;eval;SLLS;5| (|p| |lv| |lq| $)
 (COND
  ((NULL |lv|) |p|)
  ((NULL (NULL (CDR |lv|)))
    (|error| "can only eval a univariate polynomial once"))
   ((QUOTE T)
    (SPADCALL |p| (|SPADfirst| |lv|) (|SPADfirst| |lq|) (QREFELT $ 21)))))

(DEFUN |UPOLYC-;eval;SSaos2S;6| (|p| |v| |q| $)
 (declare (ignore |v|))
 (SPADCALL |p| |q| (QREFELT $ 24)))

(DEFUN |UPOLYC-;eval;SLLS;7| (|p| |lv| |lr| $)
 (COND
  ((NULL |lv|) |p|)
  ((NULL (NULL (CDR |lv|)))
    (|error| "can only eval a univariate polynomial once"))
   ((QUOTE T)
    (SPADCALL |p| (|SPADfirst| |lv|) (|SPADfirst| |lr|) (QREFELT $ 26)))))

(DEFUN |UPOLYC-;eval;SSaosRS;8| (|p| |v| |r| $)
 (declare (ignore |v|))
 (SPADCALL (SPADCALL |p| |r| (QREFELT $ 29)) (QREFELT $ 30)))

(DEFUN |UPOLYC-;eval;SLS;9| (|p| |le| $)
 (COND
  ((NULL |le|) |p|)
  ((NULL (NULL (CDR |le|)))
    (|error| "can only eval a univariate polynomial once"))
  ((QUOTE T)
   (COND
    ((QEQCAR
      (SPADCALL
       (SPADCALL (|SPADfirst| |le|) (QREFELT $ 33))
       (QREFELT $ 35)) 1) |p|)
    ((QUOTE T)
     (SPADCALL |p|
      (SPADCALL (|SPADfirst| |le|) (QREFELT $ 36)) (QREFELT $ 24))))))))

(DEFUN |UPOLYC-;mainVariable;SU;10| (|p| $)
 (COND
  ((ZEROP (SPADCALL |p| (QREFELT $ 11))) (CONS 1 "failed"))
  ((QUOTE T) (CONS 0 (SPADCALL (QREFELT $ 13))))))
```

```
(DEFUN |UPOLYC-;minimumDegree;SSaosNni;11| (|p| |v| $)
 (declare (ignore |v|))
 (SPADCALL |p| (QREFELT $ 40)))

(DEFUN |UPOLYC-;minimumDegree;SLL;12| (|p| |lv| $)
 (COND ((NULL |lv|) NIL) ((QUOTE T) (LIST (SPADCALL |p| (QREFELT $ 40))))))

(DEFUN |UPOLYC-;monomial;SSaosNniS;13| (|p| |v| |n| $)
 (declare (ignore |v|))
 (PROG NIL
  (RETURN
   (SPADCALL
    (CONS (FUNCTION |UPOLYC-;monomial;SSaosNniS;13!0|) (VECTOR $ |n|))
    |p| (QREFELT $ 45)))))

(DEFUN |UPOLYC-;monomial;SSaosNniS;13!0| (|x1| $$)
 (PROG (|n| $)
  (LETT |n| (QREFELT $$ 1) . #0=(|UPOLYC-;monomial;SSaosNniS;13|))
  (LETT $ (QREFELT $$ 0) . #0#)
  (RETURN (PROGN (SPADCALL |x1| |n| (QREFELT $ 43))))))

(DEFUN |UPOLYC-;coerce;SaosS;14| (|v| $)
 (declare (ignore |v|))
 (SPADCALL (|spadConstant| $ 48) 1 (QREFELT $ 49)))

(DEFUN |UPOLYC-;makeSUP;SSup;15| (|p| $)
 (COND
  ((SPADCALL |p| (QREFELT $ 9)) (|spadConstant| $ 52))
  ((QUOTE T)
   (SPADCALL
    (SPADCALL
     (SPADCALL |p| (QREFELT $ 53))
     (SPADCALL |p| (QREFELT $ 11))
     (QREFELT $ 54))
    (SPADCALL
     (SPADCALL |p| (QREFELT $ 55))
     (QREFELT $ 56))
    (QREFELT $ 57)))))

(DEFUN |UPOLYC-;unmakeSUP;SupS;16| (|sp| $)
 (COND
  ((SPADCALL |sp| (QREFELT $ 59)) (|spadConstant| $ 60))
  ((QUOTE T)
   (SPADCALL
    (SPADCALL
     (SPADCALL |sp| (QREFELT $ 61))
     (SPADCALL |sp| (QREFELT $ 62))
     (QREFELT $ 49))
    (SPADCALL (SPADCALL |sp| (QREFELT $ 63)) (QREFELT $ 64))
    (QREFELT $ 65)))))

(DEFUN |UPOLYC-;karatsubaDivide;SNniR;17| (|p| |n| $)
 (SPADCALL |p|
```

```
  (SPADCALL (|spadConstant| $ 48) |n| (QREFELT $ 49))
  (QREFELT $ 68)))

(DEFUN |UPOLYC-;shiftRight;SNniS;18| (|p| |n| $)
 (QCAR
  (SPADCALL |p|
   (SPADCALL (|spadConstant| $ 48) |n| (QREFELT $ 49))
   (QREFELT $ 68))))

(DEFUN |UPOLYC-;shiftLeft;SNniS;19| (|p| |n| $)
 (SPADCALL |p|
  (SPADCALL (|spadConstant| $ 48) |n| (QREFELT $ 49))
  (QREFELT $ 71)))

(DEFUN |UPOLYC-;solveLinearPolynomialEquation;LSupU;20| (|lpp| |pp| $)
 (SPADCALL |lpp| |pp| (QREFELT $ 77)))

(DEFUN |UPOLYC-;factorPolynomial;SupF;21| (|pp| $)
 (SPADCALL |pp| (QREFELT $ 83)))

(DEFUN |UPOLYC-;factorSquareFreePolynomial;SupF;22| (|pp| $)
 (SPADCALL |pp| (QREFELT $ 86)))

(DEFUN |UPOLYC-;factor;SF;23| (|p| $)
 (PROG (|ansR| #0=#:G1523 |w| #1=#:G1524)
  (RETURN
   (SEQ
    (COND
     ((ZEROP (SPADCALL |p| (QREFELT $ 11)))
      (SEQ
       (LETT |ansR|
        (SPADCALL (SPADCALL |p| (QREFELT $ 53)) (QREFELT $ 89))
        |UPOLYC-;factor;SF;23|)
       (EXIT
        (SPADCALL
         (SPADCALL (SPADCALL |ansR| (QREFELT $ 91)) (QREFELT $ 30))
         (PROGN
          (LETT #0# NIL |UPOLYC-;factor;SF;23|)
          (SEQ
           (LETT |w| NIL |UPOLYC-;factor;SF;23|)
           (LETT #1# (SPADCALL |ansR| (QREFELT $ 95)) |UPOLYC-;factor;SF;23|)
          G190
           (COND
            ((OR (ATOM #1#)
                 (PROGN (LETT |w| (CAR #1#) |UPOLYC-;factor;SF;23|) NIL))
              (GO G191)))
           (SEQ
            (EXIT
             (LETT #0#
              (CONS
               (VECTOR
                (QVELT |w| 0)
                (SPADCALL (QVELT |w| 1) (QREFELT $ 30))
                (QVELT |w| 2))
```

```
                #0#)
               |UPOLYC-;factor;SF;23|)))
            (LETT #1# (CDR #1#) |UPOLYC-;factor;SF;23|)
            (GO G190)
           G191
            (EXIT (NREVERSE0 #0#))))
          (QREFELT $ 99)))))
      ((QUOTE T)
       (SPADCALL (ELT $ 64)
        (SPADCALL (SPADCALL |p| (QREFELT $ 56)) (QREFELT $ 100))
        (QREFELT $ 104))))))))

(DEFUN |UPOLYC-;vectorise;SNniV;24| (|p| |n| $)
 (PROG (|v| |m| |i| #0=#:G1529 #1=#:G1525)
  (RETURN
   (SEQ
    (LETT |m|
     (SPADCALL
      (LETT |v|
       (SPADCALL |n| (|spadConstant| $ 106) (QREFELT $ 108))
       |UPOLYC-;vectorise;SNniV;24|)
      (QREFELT $ 110))
     |UPOLYC-;vectorise;SNniV;24|)
    (SEQ (LETT |i| (SPADCALL |v| (QREFELT $ 110)) |UPOLYC-;vectorise;SNniV;24|) (LETT #0#
  (QVSIZE |v|)
  |UPOLYC-;vectorise;SNniV;24|)
  G190
  (COND ((> |i| #0#) (GO G191)))
  (SEQ
   (EXIT
    (SPADCALL |v| |i|
     (SPADCALL |p|
      (PROG1
       (LETT #1# (- |i| |m|) |UPOLYC-;vectorise;SNniV;24|)
       (|check-subtype| (>= #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
      (QREFELT $ 111))
     (QREFELT $ 112))))
  (LETT |i| (+ |i| 1) |UPOLYC-;vectorise;SNniV;24|) (GO G190) G191 (EXIT NIL))
    (EXIT |v|)))))

(DEFUN |UPOLYC-;unvectorise;VS;25| (|v| $)
 (PROG (|i| #0=#:G1534 #1=#:G1530 |p|)
  (RETURN
   (SEQ
    (LETT |p| (|spadConstant| $ 60) |UPOLYC-;unvectorise;VS;25|)
    (SEQ
     (LETT |i| 1 |UPOLYC-;unvectorise;VS;25|)
     (LETT #0# (QVSIZE |v|) |UPOLYC-;unvectorise;VS;25|)
     G190
     (COND ((QSGREATERP |i| #0#) (GO G191)))
     (SEQ
      (EXIT
       (LETT |p|
        (SPADCALL |p|
```

```
         (SPADCALL
          (SPADCALL |v| |i| (QREFELT $ 114))
          (PROG1
           (LETT #1# (- |i| 1) |UPOLYC-;unvectorise;VS;25|)
           (|check-subtype| (>= #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
          (QREFELT $ 49))
         (QREFELT $ 65))
        |UPOLYC-;unvectorise;VS;25|)))
     (LETT |i| (QSADD1 |i|) |UPOLYC-;unvectorise;VS;25|)
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |p|)))))

(DEFUN |UPOLYC-;retract;SR;26| (|p| $)
 (COND
  ((SPADCALL |p| (QREFELT $ 9)) (|spadConstant| $ 106))
  ((ZEROP (SPADCALL |p| (QREFELT $ 11))) (SPADCALL |p| (QREFELT $ 53)))
  ((QUOTE T) (|error| "Polynomial is not of degree 0"))))

(DEFUN |UPOLYC-;retractIfCan;SU;27| (|p| $)
 (COND
  ((SPADCALL |p| (QREFELT $ 9)) (CONS 0 (|spadConstant| $ 106)))
  ((ZEROP (SPADCALL |p| (QREFELT $ 11)))
   (CONS 0 (SPADCALL |p| (QREFELT $ 53))))
  ((QUOTE T) (CONS 1 "failed"))))

(DEFUN |UPOLYC-;init;S;28| ($)
 (SPADCALL (|spadConstant| $ 119) (QREFELT $ 30)))

(DEFUN |UPOLYC-;nextItemInner| (|n| $)
 (PROG (|nn| |n1| |n2| #0=#:G1555 |n3|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |n| (QREFELT $ 9))
      (CONS 0
       (SPADCALL
        (PROG2
         (LETT #0#
          (SPADCALL (|spadConstant| $ 106) (QREFELT $ 122))
          |UPOLYC-;nextItemInner|)
         (QCDR #0#)
         (|check-union| (QEQCAR #0# 0) (QREFELT $ 7) #0#))
        (QREFELT $ 30))))
     ((ZEROP (SPADCALL |n| (QREFELT $ 11)))
       (SEQ
        (LETT |nn|
         (SPADCALL (SPADCALL |n| (QREFELT $ 53)) (QREFELT $ 122))
         |UPOLYC-;nextItemInner|)
        (EXIT
        (COND
         ((QEQCAR |nn| 1) (CONS 1 "failed"))
         ((QUOTE T) (CONS 0 (SPADCALL (QCDR |nn|) (QREFELT $ 30)))))))
```

```
      ((QUOTE T)
       (SEQ
        (LETT |n1| (SPADCALL |n| (QREFELT $ 55)) |UPOLYC-;nextItemInner|)
        (LETT |n2| (|UPOLYC-;nextItemInner| |n1| $) |UPOLYC-;nextItemInner|)
        (EXIT
         (COND
          ((QEQCAR |n2| 0)
            (CONS 0
             (SPADCALL
              (SPADCALL
               (SPADCALL |n| (QREFELT $ 53))
               (SPADCALL |n| (QREFELT $ 11))
               (QREFELT $ 49))
             (QCDR |n2|)
             (QREFELT $ 65))))
          ((< (+ 1 (SPADCALL |n1| (QREFELT $ 11)))
              (SPADCALL |n| (QREFELT $ 11)))
            (CONS 0
             (SPADCALL
              (SPADCALL
               (SPADCALL |n| (QREFELT $ 53))
               (SPADCALL |n| (QREFELT $ 11))
               (QREFELT $ 49))
              (SPADCALL
               (PROG2
                (LETT #0#
                 (SPADCALL (|spadConstant| $ 119) (QREFELT $ 122))
                 |UPOLYC-;nextItemInner|)
                (QCDR #0#)
                (|check-union| (QEQCAR #0# 0) (QREFELT $ 7) #0#))
               (+ 1 (SPADCALL |n1| (QREFELT $ 11)))
               (QREFELT $ 49))
              (QREFELT $ 65))))
          ((QUOTE T)
           (SEQ
            (LETT |n3|
             (SPADCALL (SPADCALL |n| (QREFELT $ 53)) (QREFELT $ 122))
             |UPOLYC-;nextItemInner|)
            (EXIT
             (COND
              ((QEQCAR |n3| 1) (CONS 1 "failed"))
              ((QUOTE T)
               (CONS 0
                (SPADCALL (QCDR |n3|)
                 (SPADCALL |n| (QREFELT $ 11)) (QREFELT $ 49))))))))))))))))

(DEFUN |UPOLYC-;nextItem;SU;30| (|n| $)
 (PROG (|n1| #0=#:G1568)
  (RETURN
   (SEQ
    (LETT |n1| (|UPOLYC-;nextItemInner| |n| $) |UPOLYC-;nextItem;SU;30|)
    (EXIT
     (COND
      ((QEQCAR |n1| 1)
```

```
        (CONS 0
         (SPADCALL
          (PROG2
           (LETT #0#
            (SPADCALL (|spadConstant| $ 119) (QREFELT $ 122))
            |UPOLYC-;nextItem;SU;30|)
           (QCDR #0#)
           (|check-union| (QEQCAR #0# 0) (QREFELT $ 7) #0#))
          (+ 1 (SPADCALL |n| (QREFELT $ 11))) (QREFELT $ 49))))
      ((QUOTE T) |n1|)))))))

(DEFUN |UPOLYC-;content;SSaosS;31| (|p| |v| $)
 (declare (ignore |v|))
 (SPADCALL (SPADCALL |p| (QREFELT $ 125)) (QREFELT $ 30)))

(DEFUN |UPOLYC-;primeFactor| (|p| |q| $)
 (PROG (#0=#:G1574 |p1|)
  (RETURN
   (SEQ
    (LETT |p1|
     (PROG2
      (LETT #0# (SPADCALL |p| (SPADCALL |p| |q| (QREFELT $ 127)) (QREFELT $ 128)) |UPOLYC-;primeFactor|)
      (QCDR #0#)
      (|check-union| (QEQCAR #0# 0) (QREFELT $ 6) #0#))
     |UPOLYC-;primeFactor|)
    (EXIT
     (COND
      ((SPADCALL |p1| |p| (QREFELT $ 129)) |p|)
      ((QUOTE T) (|UPOLYC-;primeFactor| |p1| |q| $))))))))

(DEFUN |UPOLYC-;separate;2SR;33| (|p| |q| $)
 (PROG (|a| #0=#:G1580)
  (RETURN
   (SEQ
    (LETT |a| (|UPOLYC-;primeFactor| |p| |q| $) |UPOLYC-;separate;2SR;33|)
    (EXIT
     (CONS |a|
      (PROG2
       (LETT #0# (SPADCALL |p| |a| (QREFELT $ 128)) |UPOLYC-;separate;2SR;33|)
       (QCDR #0#)
       (|check-union| (QEQCAR #0# 0) (QREFELT $ 6) #0#))))))))

(DEFUN |UPOLYC-;differentiate;SM2S;34| (|x| |deriv| |x'| $)
 (PROG (|dg| |lc| #0=#:G1585 |d|)
  (RETURN
   (SEQ
    (LETT |d| (|spadConstant| $ 60) |UPOLYC-;differentiate;SM2S;34|)
    (SEQ G190
     (COND
      ((NULL (< 0
         (LETT |dg|
          (SPADCALL |x| (QREFELT $ 11)) |UPOLYC-;differentiate;SM2S;34|)))
       (GO G191)))
     (SEQ
```

```
      (LETT |lc| (SPADCALL |x| (QREFELT $ 53)) |UPOLYC-;differentiate;SM2S;34|)
      (LETT |d|
       (SPADCALL
        (SPADCALL |d|
         (SPADCALL |x'|
          (SPADCALL
           (SPADCALL |dg| |lc| (QREFELT $ 133))
            (PROG1
             (LETT #0# (- |dg| 1) |UPOLYC-;differentiate;SM2S;34|)
              (|check-subtype| (>= #0# 0) (QUOTE (|NonNegativeInteger|)) #0#))
            (QREFELT $ 49))
           (QREFELT $ 71))
         (QREFELT $ 65))
        (SPADCALL (SPADCALL |lc| |deriv|) |dg| (QREFELT $ 49)) (QREFELT $ 65))
       |UPOLYC-;differentiate;SM2S;34|)
      (EXIT
       (LETT |x|
        (SPADCALL |x| (QREFELT $ 55)) |UPOLYC-;differentiate;SM2S;34|)))
      NIL
      (GO G190)
      G191
      (EXIT NIL))
    (EXIT
     (SPADCALL |d|
      (SPADCALL
       (SPADCALL (SPADCALL |x| (QREFELT $ 53)) |deriv|)
       (QREFELT $ 30))
      (QREFELT $ 65)))))))

(DEFUN |UPOLYC-;ncdiff| (|n| |x'| $)
 (PROG (#0=#:G1603 |n1|)
  (RETURN
   (COND
    ((ZEROP |n|) (|spadConstant| $ 60))
    ((ZEROP
      (LETT |n1|
       (PROG1
        (LETT #0# (- |n| 1) |UPOLYC-;ncdiff|)
        (|check-subtype| (>= #0# 0) (QUOTE (|NonNegativeInteger|)) #0#))
       |UPOLYC-;ncdiff|))
      |x'|)
    ((QUOTE T)
     (SPADCALL
      (SPADCALL |x'|
       (SPADCALL (|spadConstant| $ 48) |n1| (QREFELT $ 49)) (QREFELT $ 71))
      (SPADCALL
       (SPADCALL (|spadConstant| $ 48) 1 (QREFELT $ 49))
       (|UPOLYC-;ncdiff| |n1| |x'| $)
       (QREFELT $ 71))
      (QREFELT $ 65)))))))

(DEFUN |UPOLYC-;differentiate;SM2S;36| (|x| |deriv| |x'| $)
 (PROG (|dg| |lc| |d|)
  (RETURN
```

```
   (SEQ
    (LETT |d| (|spadConstant| $ 60) |UPOLYC-;differentiate;SM2S;36|)
    (SEQ
      G190
      (COND
       ((NULL (< 0
         (LETT |dg|
           (SPADCALL |x| (QREFELT $ 11)) |UPOLYC-;differentiate;SM2S;36|)))
        (GO G191)))
      (SEQ
       (LETT |lc|
        (SPADCALL |x| (QREFELT $ 53)) |UPOLYC-;differentiate;SM2S;36|)
       (LETT |d|
        (SPADCALL
         (SPADCALL |d|
          (SPADCALL (SPADCALL |lc| |deriv|) |dg| (QREFELT $ 49))
          (QREFELT $ 65))
         (SPADCALL |lc| (|UPOLYC-;ncdiff| |dg| |x'| $) (QREFELT $ 136))
         (QREFELT $ 65))
        |UPOLYC-;differentiate;SM2S;36|)
       (EXIT
        (LETT |x|
         (SPADCALL |x| (QREFELT $ 55)) |UPOLYC-;differentiate;SM2S;36|)))
      NIL
      (GO G190)
      G191
      (EXIT NIL))
    (EXIT
     (SPADCALL |d|
      (SPADCALL
       (SPADCALL (SPADCALL |x| (QREFELT $ 53)) |deriv|) (QREFELT $ 30))
      (QREFELT $ 65)))))))

(DEFUN |UPOLYC-;differentiate;SMS;37| (|x| |deriv| $)
 (SPADCALL |x| |deriv| (|spadConstant| $ 47) (QREFELT $ 137)))

(DEFUN |UPOLYC-;differentiate;2S;38| (|x| $)
 (PROG (|dg| #0=#:G1612 |d|)
  (RETURN
   (SEQ
    (LETT |d| (|spadConstant| $ 60) |UPOLYC-;differentiate;2S;38|)
    (SEQ
     G190
     (COND
      ((NULL (< 0
        (LETT |dg|
         (SPADCALL |x| (QREFELT $ 11)) |UPOLYC-;differentiate;2S;38|)))
       (GO G191)))
     (SEQ
      (LETT |d|
       (SPADCALL |d|
        (SPADCALL
         (SPADCALL |dg| (SPADCALL |x| (QREFELT $ 53)) (QREFELT $ 133))
         (PROG1
```

```
          (LETT #0# (- |dg| 1) |UPOLYC-;differentiate;2S;38|)
          (|check-subtype| (>= #0# 0) (QUOTE (|NonNegativeInteger|)) #0#))
         (QREFELT $ 49))
        (QREFELT $ 65))
       |UPOLYC-;differentiate;2S;38|)
      (EXIT
       (LETT |x| (SPADCALL |x| (QREFELT $ 55)) |UPOLYC-;differentiate;2S;38|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |d|)))))

(DEFUN |UPOLYC-;differentiate;SSaosS;39| (|x| |v| $)
 (declare (ignore |v|))
 (SPADCALL |x| (QREFELT $ 140)))

(DEFUN |UPOLYC-;elt;3F;40| (|g| |f| $)
 (SPADCALL
  (SPADCALL (SPADCALL |g| (QREFELT $ 143)) |f| (QREFELT $ 145))
  (SPADCALL (SPADCALL |g| (QREFELT $ 146)) |f| (QREFELT $ 145))
  (QREFELT $ 147)))

(DEFUN |UPOLYC-;pseudoQuotient;3S;41| (|p| |q| $)
 (PROG (|n| #0=#:G1658 #1=#:G1660)
  (RETURN
   (SEQ
    (LETT |n|
     (+ (- (SPADCALL |p| (QREFELT $ 11)) (SPADCALL |q| (QREFELT $ 11))) 1)
     |UPOLYC-;pseudoQuotient;3S;41|)
    (EXIT
     (COND
      ((< |n| 1) (|spadConstant| $ 60))
      ((QUOTE T)
       (PROG2
        (LETT #1#
         (SPADCALL
          (SPADCALL
           (SPADCALL
            (SPADCALL
             (SPADCALL |q| (QREFELT $ 53))
              (PROG1
               (LETT #0# |n| |UPOLYC-;pseudoQuotient;3S;41|)
               (|check-subtype| (>= #0# 0) (QUOTE (|NonNegativeInteger|)) #0#))
              (QREFELT $ 149))
            |p| (QREFELT $ 136))
           (SPADCALL |p| |q| (QREFELT $ 150)) (QREFELT $ 151))
          |q| (QREFELT $ 128))
         |UPOLYC-;pseudoQuotient;3S;41|)
        (QCDR #1#)
        (|check-union| (QEQCAR #1# 0) (QREFELT $ 6) #1#)))))))))

(DEFUN |UPOLYC-;pseudoDivide;2SR;42| (|p| |q| $)
 (PROG (|n| |prem| #0=#:G1666 |lc| #1=#:G1668)
```

```
  (RETURN
   (SEQ
    (LETT |n|
     (+ (- (SPADCALL |p| (QREFELT $ 11)) (SPADCALL |q| (QREFELT $ 11))) 1)
     |UPOLYC-;pseudoDivide;2SR;42|)
    (EXIT
     (COND
      ((< |n| 1) (VECTOR (|spadConstant| $ 48) (|spadConstant| $ 60) |p|))
      ((QUOTE T)
       (SEQ
        (LETT |prem|
         (SPADCALL |p| |q| (QREFELT $ 150)) |UPOLYC-;pseudoDivide;2SR;42|)
        (LETT |lc|
         (SPADCALL
          (SPADCALL |q| (QREFELT $ 53))
          (PROG1
           (LETT #0# |n| |UPOLYC-;pseudoDivide;2SR;42|)
            (|check-subtype| (>= #0# 0) (QUOTE (|NonNegativeInteger|)) #0#))
          (QREFELT $ 149))
         |UPOLYC-;pseudoDivide;2SR;42|)
        (EXIT
         (VECTOR |lc|
          (PROG2
           (LETT #1#
            (SPADCALL
             (SPADCALL
              (SPADCALL |lc| |p| (QREFELT $ 136)) |prem| (QREFELT $ 151))
             |q| (QREFELT $ 128))
            |UPOLYC-;pseudoDivide;2SR;42|)
           (QCDR #1#)
           (|check-union| (QEQCAR #1# 0) (QREFELT $ 6) #1#))
          |prem|))))))))))

(DEFUN |UPOLYC-;composite;FSU;43| (|f| |q| $)
 (PROG (|n| |d|)
  (RETURN
   (SEQ
    (LETT |n|
     (SPADCALL (SPADCALL |f| (QREFELT $ 143)) |q| (QREFELT $ 155))
     |UPOLYC-;composite;FSU;43|)
    (EXIT
     (COND
      ((QEQCAR |n| 1) (CONS 1 "failed"))
      ((QUOTE T)
       (SEQ
        (LETT |d|
         (SPADCALL (SPADCALL |f| (QREFELT $ 146)) |q| (QREFELT $ 155))
         |UPOLYC-;composite;FSU;43|)
        (EXIT
         (COND
          ((QEQCAR |d| 1) (CONS 1 "failed"))
          ((QUOTE T)
           (CONS 0
            (SPADCALL (QCDR |n|) (QCDR |d|) (QREFELT $ 156)))))))))))))))
```

```
(DEFUN |UPOLYC-;composite;2SU;44| (|p| |q| $)
 (PROG (|cqr| |v| |u| |w| #0=#:G1694)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |p| (QREFELT $ 159)) (CONS 0 |p|))
     ((QUOTE T)
      (SEQ
       (EXIT
        (SEQ
         (LETT |cqr|
          (SPADCALL |p| |q| (QREFELT $ 160)) |UPOLYC-;composite;2SU;44|)
         (COND
          ((SPADCALL (QVELT |cqr| 2) (QREFELT $ 159))
           (SEQ
            (LETT |v|
             (SPADCALL (QVELT |cqr| 2) (QVELT |cqr| 0) (QREFELT $ 161))
             |UPOLYC-;composite;2SU;44|)
            (EXIT
             (COND
              ((QEQCAR |v| 0)
               (SEQ
                (LETT |u|
                 (SPADCALL (QVELT |cqr| 1) |q| (QREFELT $ 155))
                 |UPOLYC-;composite;2SU;44|)
                (EXIT
                 (COND
                  ((QEQCAR |u| 0)
                   (SEQ
                    (LETT |w|
                     (SPADCALL (QCDR |u|) (QVELT |cqr| 0) (QREFELT $ 161))
                     |UPOLYC-;composite;2SU;44|)
                    (EXIT
                     (COND
                      ((QEQCAR |w| 0)
                       (PROGN
                        (LETT #0#
                         (CONS 0
                          (SPADCALL (QCDR |v|)
                           (SPADCALL
                            (SPADCALL (|spadConstant| $ 48) 1 (QREFELT $ 49))
                            (QCDR |w|)
                            (QREFELT $ 71))
                           (QREFELT $ 65)))
                         |UPOLYC-;composite;2SU;44|)
                        (GO #0#))))))))))))))))
         (EXIT (CONS 1 "failed")))) #0# (EXIT #0#)))))))

(DEFUN |UPOLYC-;elt;S2F;45| (|p| |f| $)
 (PROG (|n| #0=#:G1701 |ans|)
  (RETURN
   (SEQ
    (COND
```

```
     ((SPADCALL |p| (QREFELT $ 9)) (|spadConstant| $ 163))
     ((QUOTE T)
      (SEQ
       (LETT |ans|
        (SPADCALL
         (SPADCALL (SPADCALL |p| (QREFELT $ 53)) (QREFELT $ 30))
         (QREFELT $ 164))
         |UPOLYC-;elt;S2F;45|)
       (LETT |n| (SPADCALL |p| (QREFELT $ 11)) |UPOLYC-;elt;S2F;45|)
       (SEQ
        G190
        (COND
         ((NULL
          (COND
           ((SPADCALL
            (LETT |p|
             (SPADCALL |p| (QREFELT $ 55)) |UPOLYC-;elt;S2F;45|) (QREFELT $ 9))
            (QUOTE NIL))
           ((QUOTE T) (QUOTE T))))
          (GO G191)))
        (SEQ
         (EXIT
          (LETT |ans|
           (SPADCALL
            (SPADCALL |ans|
             (SPADCALL |f|
              (PROG1
               (LETT #0#
                (- |n|
                 (LETT |n|
                  (SPADCALL |p| (QREFELT $ 11))
                  |UPOLYC-;elt;S2F;45|))
                |UPOLYC-;elt;S2F;45|)
               (|check-subtype| (>= #0# 0) (QUOTE (|NonNegativeInteger|)) #0#))
              (QREFELT $ 165))
             (QREFELT $ 166))
            (SPADCALL
             (SPADCALL (SPADCALL |p| (QREFELT $ 53)) (QREFELT $ 30))
             (QREFELT $ 164))
            (QREFELT $ 167))
           |UPOLYC-;elt;S2F;45|)))
        NIL
        (GO G190)
        G191
        (EXIT NIL))
       (EXIT
        (COND
         ((ZEROP |n|) |ans|)
         ((QUOTE T)
          (SPADCALL |ans|
           (SPADCALL |f| |n| (QREFELT $ 168)) (QREFELT $ 166)))))))))))

(DEFUN |UPOLYC-;order;2SNni;46| (|p| |q| $)
 (PROG (|u| #0=#:G1715 |ans|)
```

```
  (RETURN
   (SEQ
    (EXIT
     (COND
      ((SPADCALL |p| (QREFELT $ 9))
       (|error| "order: arguments must be nonzero"))
      ((< (SPADCALL |q| (QREFELT $ 11)) 1)
       (|error| "order: place must be non-trivial"))
      ((QUOTE T)
       (SEQ
        (LETT |ans| 0 |UPOLYC-;order;2SNni;46|)
        (EXIT
         (SEQ
         G190
         NIL
         (SEQ
          (LETT |u|
           (SPADCALL |p| |q| (QREFELT $ 128)) |UPOLYC-;order;2SNni;46|)
          (EXIT
           (COND
            ((QEQCAR |u| 1)
             (PROGN (LETT #0# |ans| |UPOLYC-;order;2SNni;46|) (GO #0#)))
            ((QUOTE T)
             (SEQ
              (LETT |p| (QCDR |u|) |UPOLYC-;order;2SNni;46|)
              (EXIT (LETT |ans| (+ |ans| 1) |UPOLYC-;order;2SNni;46|)))))))
         NIL
         (GO G190)
         G191
         (EXIT NIL)))))))
     #0# (EXIT #0#)))))

(DEFUN |UPOLYC-;squareFree;SF;47| (|p| $) (SPADCALL |p| (QREFELT $ 172)))

(DEFUN |UPOLYC-;squareFreePart;2S;48| (|p| $) (SPADCALL |p| (QREFELT $ 174)))

(DEFUN |UPOLYC-;gcdPolynomial;3Sup;49| (|pp| |qq| $)
 (COND
  ((SPADCALL |pp| (QREFELT $ 176)) (SPADCALL |qq| (QREFELT $ 177)))
  ((SPADCALL |qq| (QREFELT $ 176)) (SPADCALL |pp| (QREFELT $ 177)))
  ((QUOTE T)
   (SPADCALL
    (SPADCALL
     (SPADCALL
      (SPADCALL |pp| (QREFELT $ 178))
      (SPADCALL |qq| (QREFELT $ 178))
      (QREFELT $ 127))
     (SPADCALL
      (SPADCALL
       (SPADCALL |pp| (QREFELT $ 179))
       (SPADCALL |qq| (QREFELT $ 179))
       (QREFELT $ 180))
      (QREFELT $ 179))
     (QREFELT $ 181))
```

```
    (QREFELT $ 177)))))

(DEFUN |UPOLYC-;squareFreePolynomial;SupF;50| (|pp| $)
 (SPADCALL |pp| (QREFELT $ 184)))

(DEFUN |UPOLYC-;elt;F2R;51| (|f| |r| $)
 (SPADCALL
  (SPADCALL (SPADCALL |f| (QREFELT $ 143)) |r| (QREFELT $ 29))
  (SPADCALL (SPADCALL |f| (QREFELT $ 146)) |r| (QREFELT $ 29))
  (QREFELT $ 186)))

(DEFUN |UPOLYC-;euclideanSize;SNni;52| (|x| $)
 (COND
  ((SPADCALL |x| (QREFELT $ 9))
   (|error| "euclideanSize called on 0 in Univariate Polynomial"))
  ((QUOTE T)
   (SPADCALL |x| (QREFELT $ 11)))))

(DEFUN |UPOLYC-;divide;2SR;53| (|x| |y| $)
 (PROG (|lc| |f| #0=#:G1728 |n| |quot|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |y| (QREFELT $ 9))
      (|error| "division by 0 in Univariate Polynomials"))
     ((QUOTE T)
      (SEQ
       (LETT |quot| (|spadConstant| $ 60) |UPOLYC-;divide;2SR;53|)
       (LETT |lc|
        (SPADCALL (SPADCALL |y| (QREFELT $ 53)) (QREFELT $ 189))
        |UPOLYC-;divide;2SR;53|)
       (SEQ
        G190
        (COND
         ((NULL
           (COND
            ((OR (SPADCALL |x| (QREFELT $ 9))
                 (< (SPADCALL |x| (QREFELT $ 11))
                    (SPADCALL |y| (QREFELT $ 11))))
              (QUOTE NIL))
            ((QUOTE T) (QUOTE T))))
          (GO G191)))
        (SEQ
         (LETT |f|
          (SPADCALL |lc| (SPADCALL |x| (QREFELT $ 53)) (QREFELT $ 190))
          |UPOLYC-;divide;2SR;53|)
         (LETT |n|
          (PROG1
           (LETT #0#
            (- (SPADCALL |x| (QREFELT $ 11)) (SPADCALL |y| (QREFELT $ 11)))
            |UPOLYC-;divide;2SR;53|)
           (|check-subtype| (>= #0# 0) (QUOTE (|NonNegativeInteger|)) #0#))
          |UPOLYC-;divide;2SR;53|)
         (LETT |quot|
```

```
          (SPADCALL |quot| (SPADCALL |f| |n| (QREFELT $ 49)) (QREFELT $ 65))
          |UPOLYC-;divide;2SR;53|)
         (EXIT
          (LETT |x|
           (SPADCALL |x|
            (SPADCALL (SPADCALL |f| |n| (QREFELT $ 49)) |y| (QREFELT $ 71))
            (QREFELT $ 151))
           |UPOLYC-;divide;2SR;53|)))
        NIL
        (GO G190)
        G191
        (EXIT NIL))
       (EXIT (CONS |quot| |x|)))))))))

(DEFUN |UPOLYC-;integrate;2S;54| (|p| $)
 (PROG (|l| |d| |ans|)
  (RETURN
   (SEQ
    (LETT |ans| (|spadConstant| $ 60) |UPOLYC-;integrate;2S;54|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((SPADCALL |p| (|spadConstant| $ 60) (QREFELT $ 129)) (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (LETT |l| (SPADCALL |p| (QREFELT $ 53)) |UPOLYC-;integrate;2S;54|)
      (LETT |d| (+ 1 (SPADCALL |p| (QREFELT $ 11))) |UPOLYC-;integrate;2S;54|)
      (LETT |ans|
       (SPADCALL |ans|
        (SPADCALL
         (SPADCALL (SPADCALL |d| (QREFELT $ 193)) (QREFELT $ 194))
         (SPADCALL |l| |d| (QREFELT $ 49))
         (QREFELT $ 195))
        (QREFELT $ 65))
       |UPOLYC-;integrate;2S;54|)
      (EXIT
       (LETT |p| (SPADCALL |p| (QREFELT $ 55)) |UPOLYC-;integrate;2S;54|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |ans|)))))

(DEFUN |UnivariatePolynomialCategory&| (|#1| |#2|)
 (PROG (DV$1 DV$2 |dv$| $ |pv$|)
  (RETURN
   (PROGN
    (LETT DV$1 (|devaluate| |#1|) . #0=(|UnivariatePolynomialCategory&|))
    (LETT DV$2 (|devaluate| |#2|) . #0#)
    (LETT |dv$| (LIST (QUOTE |UnivariatePolynomialCategory&|) DV$1 DV$2) . #0#)
    (LETT $ (MAKE-ARRAY 203) . #0#)
```

```
(QSETREFV $ 0 |dv$|)
(QSETREFV $ 3
 (LETT |pv$|
  (|buildPredVector| 0 0
   (LIST
    (|HasCategory| |#2| (QUOTE (|Algebra| (|Fraction| (|Integer|)))))
    (|HasCategory| |#2| (QUOTE (|Field|)))
    (|HasCategory| |#2| (QUOTE (|GcdDomain|)))
    (|HasCategory| |#2| (QUOTE (|IntegralDomain|)))
    (|HasCategory| |#2| (QUOTE (|CommutativeRing|)))
    (|HasCategory| |#2| (QUOTE (|StepThrough|)))))
   . #0#))
(|stuffDomainSlots| $)
(QSETREFV $ 6 |#1|)
(QSETREFV $ 7 |#2|)
(COND
 ((|HasCategory| |#2| (QUOTE (|PolynomialFactorizationExplicit|)))
  (PROGN
   (QSETREFV $ 81
    (CONS
     (|dispatchFunction| |UPOLYC-;solveLinearPolynomialEquation;LSupU;20|)
     $))
   (QSETREFV $ 85
    (CONS (|dispatchFunction| |UPOLYC-;factorPolynomial;SupF;21|) $))
   (QSETREFV $ 87
    (CONS
     (|dispatchFunction| |UPOLYC-;factorSquareFreePolynomial;SupF;22|)
     $))
   (QSETREFV $ 105 (CONS (|dispatchFunction| |UPOLYC-;factor;SF;23|) $)))))
 (COND
  ((|testBitVector| |pv$| 6)
   (PROGN
    (QSETREFV $ 120 (CONS (|dispatchFunction| |UPOLYC-;init;S;28|) $))
    NIL
    (QSETREFV $ 124
     (CONS (|dispatchFunction| |UPOLYC-;nextItem;SU;30|) $)))))
 (COND
  ((|testBitVector| |pv$| 3)
   (PROGN
    (QSETREFV $ 126
     (CONS (|dispatchFunction| |UPOLYC-;content;SSaosS;31|) $))
    NIL
    (QSETREFV $ 131
     (CONS (|dispatchFunction| |UPOLYC-;separate;2SR;33|) $)))))
 (COND
  ((|testBitVector| |pv$| 5)
   (QSETREFV $ 135
    (CONS (|dispatchFunction| |UPOLYC-;differentiate;SM2S;34|) $)))
  ((QUOTE T)
   (PROGN
    (QSETREFV $ 135
     (CONS (|dispatchFunction| |UPOLYC-;differentiate;SM2S;36|) $)))))
 (COND
  ((|testBitVector| |pv$| 4)
```

```
      (PROGN
       (QSETREFV $ 148 (CONS (|dispatchFunction| |UPOLYC-;elt;3F;40|) $))
       (QSETREFV $ 152
         (CONS (|dispatchFunction| |UPOLYC-;pseudoQuotient;3S;41|) $))
       (QSETREFV $ 154
         (CONS (|dispatchFunction| |UPOLYC-;pseudoDivide;2SR;42|) $))
       (QSETREFV $ 158
         (CONS (|dispatchFunction| |UPOLYC-;composite;FSU;43|) $))
       (QSETREFV $ 162
         (CONS (|dispatchFunction| |UPOLYC-;composite;2SU;44|) $))
       (QSETREFV $ 169 (CONS (|dispatchFunction| |UPOLYC-;elt;S2F;45|) $))
       (QSETREFV $ 170
         (CONS (|dispatchFunction| |UPOLYC-;order;2SNni;46|) $)))))
    (COND
     ((|testBitVector| |pv$| 3)
      (PROGN
       (QSETREFV $ 173
        (CONS (|dispatchFunction| |UPOLYC-;squareFree;SF;47|) $))
       (QSETREFV $ 175
        (CONS (|dispatchFunction| |UPOLYC-;squareFreePart;2S;48|) $)))))
    (COND
     ((|HasCategory| |#2| (QUOTE (|PolynomialFactorizationExplicit|)))
      (PROGN
       (QSETREFV $ 182
        (CONS (|dispatchFunction| |UPOLYC-;gcdPolynomial;3Sup;49|) $))
       (QSETREFV $ 185
        (CONS
         (|dispatchFunction| |UPOLYC-;squareFreePolynomial;SupF;50|)
         $)))))
    (COND
     ((|testBitVector| |pv$| 2)
      (PROGN
       (QSETREFV $ 187 (CONS (|dispatchFunction| |UPOLYC-;elt;F2R;51|) $))
       (QSETREFV $ 188
        (CONS (|dispatchFunction| |UPOLYC-;euclideanSize;SNni;52|) $))
       (QSETREFV $ 191
        (CONS (|dispatchFunction| |UPOLYC-;divide;2SR;53|) $)))))
    (COND
     ((|testBitVector| |pv$| 1)
      (QSETREFV $ 196
       (CONS (|dispatchFunction| |UPOLYC-;integrate;2S;54|) $))))
    $))))

(SETF
 (GET (QUOTE |UnivariatePolynomialCategory&|) (QUOTE |infovec|))
 (LIST
  (QUOTE
    #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|) (|Boolean|)
      (0 . |zero?|) (|NonNegativeInteger|) (5 . |degree|)
      (|SingletonAsOrderedSet|) (10 . |create|) (|List| 12)
      |UPOLYC-;variables;SL;1| |UPOLYC-;degree;SSaosNni;2|
      (14 . |totalDegree|) |UPOLYC-;totalDegree;SLNni;3| (|List| 10)
      |UPOLYC-;degree;SLL;4| (19 . |eval|) (|List| $) |UPOLYC-;eval;SLLS;5|
      (26 . |elt|) |UPOLYC-;eval;SSaos2S;6| (32 . |eval|) (|List| 7)
```

```
|UPOLYC-;eval;SLLS;7| (39 . |elt|) (45 . |coerce|)
|UPOLYC-;eval;SSaosRS;8| (|Equation| 6) (50 . |lhs|)
(|Union| 12 (QUOTE "failed")) (55 . |mainVariable|) (60 . |rhs|)
(|List| 199) |UPOLYC-;eval;SLS;9| |UPOLYC-;mainVariable;SU;10|
(65 . |minimumDegree|) |UPOLYC-;minimumDegree;SSaosNni;11|
|UPOLYC-;minimumDegree;SLL;12| (70 . +) (|Mapping| 10 10)
(76 . |mapExponents|) |UPOLYC-;monomial;SSaosNniS;13| (82 . |One|)
(86 . |One|) (90 . |monomial|) |UPOLYC-;coerce;SaosS;14|
(|SparseUnivariatePolynomial| 7) (96 . |Zero|)
(100 . |leadingCoefficient|) (105 . |monomial|) (111 . |reductum|)
(116 . |makeSUP|) (121 . +) |UPOLYC-;makeSUP;SSup;15| (127 . |zero?|)
(132 . |Zero|) (136 . |leadingCoefficient|) (141 . |degree|)
(146 . |reductum|) (151 . |unmakeSUP|) (156 . +)
|UPOLYC-;unmakeSUP;SupS;16| (|Record| (|:| |quotient| $)
(|:| |remainder| $)) (162 . |monicDivide|)
|UPOLYC-;karatsubaDivide;SNniR;17| |UPOLYC-;shiftRight;SNniS;18|
(168 . *) |UPOLYC-;shiftLeft;SNniS;19| (|Union| 74 (QUOTE "failed"))
(|List| 75) (|SparseUnivariatePolynomial| 6)
(|PolynomialFactorizationByRecursionUnivariate| 7 6)
(174 . |solveLinearPolynomialEquationByRecursion|)
(|Union| 79 (QUOTE "failed")) (|List| 80)
(|SparseUnivariatePolynomial| $)
(180 . |solveLinearPolynomialEquation|) (|Factored| 75)
(186 . |factorByRecursion|) (|Factored| 80) (191 . |factorPolynomial|)
(196 . |factorSquareFreeByRecursion|)
(201 . |factorSquareFreePolynomial|) (|Factored| $) (206 . |factor|)
(|Factored| 7) (211 . |unit|) (|Union| (QUOTE "nil") (QUOTE "sqfr")
(QUOTE "irred") (QUOTE "prime")) (|Record| (|:| |flg| 92)
(|:| |fctr| 7) (|:| |xpnt| 109)) (|List| 93) (216 . |factorList|)
(|Record| (|:| |flg| 92) (|:| |fctr| 6) (|:| |xpnt| 109)) (|List| 96)
(|Factored| 6) (221 . |makeFR|) (227 . |factorPolynomial|)
(|Mapping| 6 51) (|Factored| 51) (|FactoredFunctions2| 51 6)
(232 . |map|) (238 . |factor|) (243 . |Zero|) (|Vector| 7)
(247 . |new|) (|Integer|) (253 . |minIndex|) (258 . |coefficient|)
(264 . |qsetelt!|) |UPOLYC-;vectorise;SNniV;24| (271 . |elt|)
|UPOLYC-;unvectorise;VS;25| |UPOLYC-;retract;SR;26|
(|Union| 7 (QUOTE "failed")) |UPOLYC-;retractIfCan;SU;27|
(277 . |init|) (281 . |init|) (|Union| $ (QUOTE "failed"))
(285 . |nextItem|) (290 . |One|) (294 . |nextItem|) (299 . |content|)
(304 . |content|) (310 . |gcd|) (316 . |exquo|) (322 . =)
(|Record| (|:| |primePart| $) (|:| |commonPart| $)) (328 . |separate|)
(334 . |Zero|) (338 . *) (|Mapping| 7 7) (344 . |differentiate|)
(351 . *) (357 . |differentiate|) |UPOLYC-;differentiate;SMS;37|
|UPOLYC-;differentiate;2S;38| (364 . |differentiate|)
|UPOLYC-;differentiate;SSaosS;39| (|Fraction| 6) (369 . |numer|)
(|Fraction| $) (374 . |elt|) (380 . |denom|) (385 . /) (391 . |elt|)
(397 . **) (403 . |pseudoRemainder|) (409 . -) (415 . |pseudoQuotient|)
(|Record| (|:| |coef| 7) (|:| |quotient| $) (|:| |remainder| $))
(421 . |pseudoDivide|) (427 . |composite|) (433 . /)
(|Union| 144 (QUOTE "failed")) (439 . |composite|) (445 . |ground?|)
(450 . |pseudoDivide|) (456 . |exquo|) (462 . |composite|)
(468 . |Zero|) (472 . |coerce|) (477 . **) (483 . *) (489 . +)
(495 . **) (501 . |elt|) (507 . |order|)
(|UnivariatePolynomialSquareFree| 7 6) (513 . |squareFree|)
```

```
    (518 . |squareFree|) (523 . |squareFreePart|) (528 . |squareFreePart|)
    (533 . |zero?|) (538 . |unitCanonical|) (543 . |content|)
    (548 . |primitivePart|) (553 . |subResultantGcd|) (559 . *)
    (565 . |gcdPolynomial|) (|UnivariatePolynomialSquareFree| 6 75)
    (571 . |squareFree|) (576 . |squareFreePolynomial|) (581 . /)
    (587 . |elt|) (593 . |euclideanSize|) (598 . |inv|) (603 . *)
    (609 . |divide|) (|Fraction| 109) (615 . |coerce|) (620 . |inv|)
    (625 . *) (631 . |integrate|) (|List| 198) (|Symbol|) (|Equation| $)
    (|Union| 109 (QUOTE "failed")) (|Union| 192 (QUOTE "failed"))
    (|OutputForm|)))
 (QUOTE #(|vectorise| 636 |variables| 642 |unvectorise| 647
          |unmakeSUP| 652 |totalDegree| 657 |squareFreePolynomial| 663
          |squareFreePart| 668 |squareFree| 673
          |solveLinearPolynomialEquation| 678 |shiftRight| 684
          |shiftLeft| 690 |separate| 696 |retractIfCan| 702
          |retract| 707 |pseudoQuotient| 712 |pseudoDivide| 718
          |order| 724 |nextItem| 730 |monomial| 735 |minimumDegree| 742
          |makeSUP| 754 |mainVariable| 759 |karatsubaDivide| 764
          |integrate| 770 |init| 775 |gcdPolynomial| 779
          |factorSquareFreePolynomial| 785 |factorPolynomial| 790
          |factor| 795 |eval| 800 |euclideanSize| 834 |elt| 839
          |divide| 857 |differentiate| 863 |degree| 887 |content| 899
          |composite| 905 |coerce| 917))
 (QUOTE NIL)
 (CONS (|makeByteWordVec2| 1 (QUOTE NIL))
  (CONS
   (QUOTE #())
   (CONS
    (QUOTE #())
    (|makeByteWordVec2| 196 (QUOTE (1 6 8 0 9 1 6 10 0 11 0 12 0 13 1 6
      10 0 17 3 6 0 0 12 0 21 2 6 0 0 0 24 3 6 0 0 12 7 26 2 6 7 0 7 29
      1 6 0 7 30 1 32 6 0 33 1 6 34 0 35 1 32 6 0 36 1 6 10 0 40 2 10 0
      0 0 43 2 6 0 44 0 45 0 6 0 47 0 7 0 48 2 6 0 7 10 49 0 51 0 52 1
      6 7 0 53 2 51 0 7 10 54 1 6 0 0 55 1 6 51 0 56 2 51 0 0 0 57 1 51
      8 0 59 0 6 0 60 1 51 7 0 61 1 51 10 0 62 1 51 0 0 63 1 6 0 51 64
      2 6 0 0 0 65 2 6 67 0 0 68 2 6 0 0 0 71 2 76 73 74 75 77 2 0 78
      79 80 81 1 76 82 75 83 1 0 84 80 85 1 76 82 75 86 1 0 84 80 87
      1 7 88 0 89 1 90 7 0 91 1 90 94 0 95 2 98 0 6 97 99 1 7 84 80
      100 2 103 98 101 102 104 1 0 88 0 105 0 7 0 106 2 107 0 10 7 108
      1 107 109 0 110 2 6 7 0 10 111 3 107 7 0 109 7 112 2 107 7 0 109
      114 0 7 0 119 0 0 0 120 1 7 121 0 122 0 75 0 123 1 0 121 0 124 1
      6 7 0 125 2 0 0 0 12 126 2 6 0 0 0 127 2 6 121 0 0 128 2 6 8 0 0
      129 2 0 130 0 0 131 0 75 0 132 2 7 0 10 0 133 3 0 0 0 134 0 135
      2 6 0 7 0 136 3 6 0 0 134 0 137 1 6 0 0 140 1 142 6 0 143 2 6 144
      0 144 145 1 142 6 0 146 2 142 0 0 0 147 2 0 144 144 144 148 2 7 0
      0 10 149 2 6 0 0 0 150 2 6 0 0 0 151 2 0 0 0 0 152 2 0 153 0 0 154
      2 6 121 0 0 155 2 142 0 6 6 156 2 0 157 144 0 158 1 6 8 0 159 2 6
      153 0 0 160 2 6 121 0 7 161 2 0 121 0 0 162 0 142 0 163 1 142 0 6
      164 2 142 0 0 109 165 2 142 0 0 0 166 2 142 0 0 0 167 2 142 0 0 10
      168 2 0 144 0 144 169 2 0 10 0 0 170 1 171 98 6 172 1 0 88 0 173 1
      171 6 6 174 1 0 0 0 175 1 75 8 0 176 1 75 0 0 177 1 75 6 0 178 1
      75 0 0 179 2 75 0 0 0 180 2 75 0 6 0 181 2 0 80 80 80 182 1 183 82
      75 184 1 0 84 80 185 2 7 0 0 0 186 2 0 7 144 7 187 1 0 10 0 188 1
      7 0 0 189 2 7 0 0 0 190 2 0 67 0 0 191 1 192 0 109 193 1 192 0 0
```

```
        194 2 6 0 192 0 195 1 0 0 0 196 2 0 107 0 10 113 1 0 14 0 15 1 0
        0 107 115 1 0 0 51 66 2 0 10 0 14 18 1 0 84 80 185 1 0 0 0 175 1
        0 88 0 173 2 0 78 79 80 81 2 0 0 0 10 70 2 0 0 0 10 72 2 0 130 0
        0 131 1 0 117 0 118 1 0 7 0 116 2 0 0 0 0 152 2 0 153 0 0 154 2
        0 10 0 0 170 1 0 121 0 124 3 0 0 0 12 10 46 2 0 10 0 12 41 2 0
        19 0 14 42 1 0 51 0 58 1 0 34 0 39 2 0 67 0 10 69 1 0 0 0 196
        0 0 0 120 2 0 80 80 80 182 1 0 84 80 87 1 0 84 80 85 1 0 88 0
        105 3 0 0 0 14 22 23 3 0 0 0 12 0 25 3 0 0 0 12 7 31 3 0 0 0 14
        27 28 2 0 0 0 37 38 1 0 10 0 188 2 0 144 0 144 169 2 0 7 144 7
        187 2 0 144 144 144 148 2 0 67 0 0 191 3 0 0 0 134 0 135 2 0 0
        0 134 138 1 0 0 0 139 2 0 0 0 12 141 2 0 19 0 14 20 2 0 10 0 12
        16 2 0 0 0 12 126 2 0 121 0 0 162 2 0 157 144 0 158 1 0 0 12 50))))))
   (QUOTE |lookupComplete|)))
```

21.69 URAGG.lsp BOOTSTRAP

URAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **URAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **URAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— URAGG.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(SETQ |UnaryRecursiveAggregate;CAT| (QUOTE NIL))

(SETQ |UnaryRecursiveAggregate;AL| (QUOTE NIL))

(DEFUN |UnaryRecursiveAggregate| (#1=#:G84596)
 (declare (special |UnaryRecursiveAggregate;AL|))
 (LET (#2=#:G84597)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |UnaryRecursiveAggregate;AL|))
     (CDR #2#))
   (T
    (SETQ |UnaryRecursiveAggregate;AL|
     (|cons5|
      (CONS (|devaluate| #1#) (SETQ #2# (|UnaryRecursiveAggregate;| #1#)))
      |UnaryRecursiveAggregate;AL|))
    #2#))))

(DEFUN |UnaryRecursiveAggregate;| (|t#1|)
 (declare (special |UnaryRecursiveAggregate;CAT|))
 (PROG (#1=#:G84595)
  (RETURN
   (PROG1
    (LETT #1#
```

```
 (|sublisV|
  (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
  (COND
   (|UnaryRecursiveAggregate;CAT|)
   ((QUOTE T)
    (LETT |UnaryRecursiveAggregate;CAT|
     (|Join|
      (|RecursiveAggregate| (QUOTE |t#1|))
      (|mkCategory|
       (QUOTE |domain|)
       (QUOTE (
        ((|concat| (|$| |$| |$|)) T)
        ((|concat| (|$| |t#1| |$|)) T)
        ((|first| (|t#1| |$|)) T)
        ((|elt| (|t#1| |$| "first")) T)
        ((|first| (|$| |$| (|NonNegativeInteger|))) T)
        ((|rest| (|$| |$|)) T)
        ((|elt| (|$| |$| "rest")) T)
        ((|rest| (|$| |$| (|NonNegativeInteger|))) T)
        ((|last| (|t#1| |$|)) T)
        ((|elt| (|t#1| |$| "last")) T)
        ((|last| (|$| |$| (|NonNegativeInteger|))) T)
        ((|tail| (|$| |$|)) T)
        ((|second| (|t#1| |$|)) T)
        ((|third| (|t#1| |$|)) T)
        ((|cycleEntry| (|$| |$|)) T)
        ((|cycleLength| ((|NonNegativeInteger|) |$|)) T)
        ((|cycleTail| (|$| |$|)) T)
        ((|concat!| (|$| |$| |$|))
          (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
        ((|concat!| (|$| |$| |t#1|))
          (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
        ((|cycleSplit!| (|$| |$|))
          (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
        ((|setfirst!| (|t#1| |$| |t#1|))
          (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
        ((|setelt| (|t#1| |$| "first" |t#1|))
          (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
        ((|setrest!| (|$| |$| |$|))
          (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
        ((|setelt| (|$| |$| "rest" |$|))
          (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
        ((|setlast!| (|t#1| |$| |t#1|))
          (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
        ((|setelt| (|t#1| |$| "last" |t#1|))
          (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
        ((|split!| (|$| |$| (|Integer|)))
          (|has| |$| (ATTRIBUTE |shallowlyMutable|)))))
       NIL
       (QUOTE ((|Integer|) (|NonNegativeInteger|)))
       NIL))
     . #2=(|UnaryRecursiveAggregate|)))))
 . #2#)
(SETELT #1# 0
```

```
(LIST (QUOTE |UnaryRecursiveAggregate|) (|devaluate| |t#1|)))))))
```

——————-

21.70 URAGG-.lsp BOOTSTRAP

URAGG- depends on **URAGG**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **URAGG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **URAGG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

— URAGG-.lsp BOOTSTRAP —

```
(|/VERSIONCHECK| 2)

(DEFUN |URAGG-;elt;AfirstS;1| (|x| G84610 |$|)
 (declare (ignore G84610))
 (SPADCALL |x| (QREFELT |$| 8)))

(DEFUN |URAGG-;elt;AlastS;2| (|x| G84612 |$|)
 (declare (ignore G84612))
 (SPADCALL |x| (QREFELT |$| 11)))

(DEFUN |URAGG-;elt;ArestA;3| (|x| G84614 |$|)
 (declare (ignore G84614))
 (SPADCALL |x| (QREFELT |$| 14)))

(DEFUN |URAGG-;second;AS;4| (|x| |$|)
 (SPADCALL (SPADCALL |x| (QREFELT |$| 14)) (QREFELT |$| 8)))

(DEFUN |URAGG-;third;AS;5| (|x| |$|)
 (SPADCALL
  (SPADCALL (SPADCALL |x| (QREFELT |$| 14)) (QREFELT |$| 14))
  (QREFELT |$| 8)))

(DEFUN |URAGG-;cyclic?;AB;6| (|x| |$|)
 (COND
  ((OR
    (SPADCALL |x| (QREFELT |$| 20))
    (SPADCALL (|URAGG-;findCycle| |x| |$|) (QREFELT |$| 20)))
     (QUOTE NIL))
   ((QUOTE T) (QUOTE T))))

(DEFUN |URAGG-;last;AS;7| (|x| |$|)
 (SPADCALL (SPADCALL |x| (QREFELT |$| 22)) (QREFELT |$| 8)))

(DEFUN |URAGG-;nodes;AL;8| (|x| |$|)
 (PROG (|l|)
  (RETURN
   (SEQ
    (LETT |l| NIL |URAGG-;nodes;AL;8|)
```

```
    (SEQ
     G190
     (COND
      ((NULL
        (COND
          ((SPADCALL |x| (QREFELT |$| 20)) (QUOTE NIL))
          ((QUOTE T) (QUOTE T))))
        (GO G191)))
     (SEQ
      (LETT |l| (CONS |x| |l|) |URAGG-;nodes;AL;8|)
      (EXIT (LETT |x| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;nodes;AL;8|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT (NREVERSE |l|))))))

(DEFUN |URAGG-;children;AL;9| (|x| |$|)
 (PROG (|l|)
  (RETURN
   (SEQ
    (LETT |l| NIL |URAGG-;children;AL;9|)
    (EXIT
     (COND
      ((SPADCALL |x| (QREFELT |$| 20)) |l|)
      ((QUOTE T) (CONS (SPADCALL |x| (QREFELT |$| 14)) |l|))))))))

(DEFUN |URAGG-;leaf?;AB;10| (|x| |$|)
 (SPADCALL |x| (QREFELT |$| 20)))

(DEFUN |URAGG-;value;AS;11| (|x| |$|)
 (COND
  ((SPADCALL |x| (QREFELT |$| 20)) (|error| "value of empty object"))
  ((QUOTE T) (SPADCALL |x| (QREFELT |$| 8)))))

(DEFUN |URAGG-;less?;ANniB;12| (|l| |n| |$|)
 (PROG (|i|)
  (RETURN
   (SEQ
    (LETT |i| |n| |URAGG-;less?;ANniB;12|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((|<| 0 |i|)
          (COND
           ((SPADCALL |l| (QREFELT |$| 20)) (QUOTE NIL))
           ((QUOTE T) (QUOTE T))))
         ((QUOTE T) (QUOTE NIL))))
       (GO G191)))
     (SEQ
      (LETT |l| (SPADCALL |l| (QREFELT |$| 14)) |URAGG-;less?;ANniB;12|)
      (EXIT (LETT |i| (|-| |i| 1) |URAGG-;less?;ANniB;12|)))
```

```
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT (|<| 0 |i|))))))

(DEFUN |URAGG-;more?;ANniB;13| (|l| |n| |$|)
 (PROG (|i|)
  (RETURN
   (SEQ
    (LETT |i| |n| |URAGG-;more?;ANniB;13|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((|<| 0 |i|)
          (COND
           ((SPADCALL |l| (QREFELT |$| 20)) (QUOTE NIL))
           ((QUOTE T) (QUOTE T))))
         ((QUOTE T) (QUOTE NIL))))
       (GO G191)))
     (SEQ
      (LETT |l| (SPADCALL |l| (QREFELT |$| 14)) |URAGG-;more?;ANniB;13|)
      (EXIT (LETT |i| (|-| |i| 1) |URAGG-;more?;ANniB;13|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT
     (COND
      ((ZEROP |i|)
       (COND
        ((SPADCALL |l| (QREFELT |$| 20)) (QUOTE NIL))
        ((QUOTE T) (QUOTE T))))
      ((QUOTE T) (QUOTE NIL))))))))

(DEFUN |URAGG-;size?;ANniB;14| (|l| |n| |$|)
 (PROG (|i|)
  (RETURN
   (SEQ
    (LETT |i| |n| |URAGG-;size?;ANniB;14|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((SPADCALL |l| (QREFELT |$| 20)) (QUOTE NIL))
         ((QUOTE T) (|<| 0 |i|))))
       (GO G191)))
     (SEQ
      (LETT |l| (SPADCALL |l| (QREFELT |$| 14)) |URAGG-;size?;ANniB;14|)
      (EXIT (LETT |i| (|-| |i| 1) |URAGG-;size?;ANniB;14|)))
     NIL
```

```
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT
     (COND
      ((SPADCALL |l| (QREFELT |$| 20)) (ZEROP |i|))
      ((QUOTE T) (QUOTE NIL))))))))

(DEFUN |URAGG-;#;ANni;15| (|x| |$|)
 (PROG (|k|)
  (RETURN
   (SEQ
    (SEQ
     (LETT |k| 0 |URAGG-;#;ANni;15|)
     G190
     (COND
      ((NULL
        (COND
         ((SPADCALL |x| (QREFELT |$| 20)) (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (COND
       ((EQL |k| 1000)
         (COND
          ((SPADCALL |x| (QREFELT |$| 33)) (EXIT (|error| "cyclic list"))))))
       (EXIT (LETT |x| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;#;ANni;15|)))
     (LETT |k| (QSADD1 |k|) |URAGG-;#;ANni;15|)
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |k|)))))

(DEFUN |URAGG-;tail;2A;16| (|x| |$|)
 (PROG (|k| |y|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |x| (QREFELT |$| 20)) (|error| "empty list"))
     ((QUOTE T)
      (SEQ
       (LETT |y| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;tail;2A;16|)
       (SEQ
        (LETT |k| 0 |URAGG-;tail;2A;16|)
        G190
        (COND
         ((NULL
           (COND
            ((SPADCALL |y| (QREFELT |$| 20)) (QUOTE NIL))
            ((QUOTE T) (QUOTE T))))
          (GO G191)))
        (SEQ
         (COND
          ((EQL |k| 1000)
```

```
            (COND
             ((SPADCALL |x| (QREFELT |$| 33))
               (EXIT (|error| "cyclic list"))))))
         (EXIT
          (LETT |y|
           (SPADCALL (LETT |x| |y| |URAGG-;tail;2A;16|) (QREFELT |$| 14))
           |URAGG-;tail;2A;16|)))
        (LETT |k| (QSADD1 |k|) |URAGG-;tail;2A;16|)
        (GO G190)
        G191
        (EXIT NIL))
       (EXIT |x|))))))))

(DEFUN |URAGG-;findCycle| (|x| |$|)
 (PROG (#1=#:G84667 |y|)
  (RETURN
   (SEQ
    (EXIT
     (SEQ
      (LETT |y| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;findCycle|)
      (SEQ
       G190
       (COND
        ((NULL
          (COND
           ((SPADCALL |y| (QREFELT |$| 20)) (QUOTE NIL))
           ((QUOTE T) (QUOTE T))))
         (GO G191)))
       (SEQ
        (COND
         ((SPADCALL |x| |y| (QREFELT |$| 36))
           (PROGN (LETT #1# |x| |URAGG-;findCycle|) (GO #1#))))
        (LETT |x| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;findCycle|)
        (LETT |y| (SPADCALL |y| (QREFELT |$| 14)) |URAGG-;findCycle|)
        (COND
         ((SPADCALL |y| (QREFELT |$| 20))
           (PROGN (LETT #1# |y| |URAGG-;findCycle|) (GO #1#))))
        (COND
         ((SPADCALL |x| |y| (QREFELT |$| 36))
           (PROGN (LETT #1# |y| |URAGG-;findCycle|) (GO #1#))))
        (EXIT (LETT |y| (SPADCALL |y| (QREFELT |$| 14)) |URAGG-;findCycle|)))
       NIL
       (GO G190)
       G191
       (EXIT NIL))
      (EXIT |y|)))
    #1#
    (EXIT #1#)))))

(DEFUN |URAGG-;cycleTail;2A;18| (|x| |$|)
 (PROG (|y| |z|)
  (RETURN
   (SEQ
    (COND
```

```
     ((SPADCALL
       (LETT |y|
        (LETT |x| (SPADCALL |x| (QREFELT |$| 37)) |URAGG-;cycleTail;2A;18|)
        |URAGG-;cycleTail;2A;18|)
       (QREFELT |$| 20))
      |x|)
     ((QUOTE T)
       (SEQ
        (LETT |z| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;cycleTail;2A;18|)
        (SEQ
         G190
         (COND
          ((NULL
            (COND
             ((SPADCALL |x| |z| (QREFELT |$| 36)) (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
           (GO G191)))
         (SEQ
          (LETT |y| |z| |URAGG-;cycleTail;2A;18|)
          (EXIT
           (LETT |z|
            (SPADCALL |z| (QREFELT |$| 14)) |URAGG-;cycleTail;2A;18|)))
         NIL
         (GO G190)
         G191
         (EXIT NIL))
        (EXIT |y|))))))))

(DEFUN |URAGG-;cycleEntry;2A;19| (|x| |$|)
 (PROG (|l| |z| |k| |y|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |x| (QREFELT |$| 20)) |x|)
     ((SPADCALL
       (LETT |y| (|URAGG-;findCycle| |x| |$|) |URAGG-;cycleEntry;2A;19|)
       (QREFELT |$| 20))
      |y|)
     ((QUOTE T)
      (SEQ
       (LETT |z| (SPADCALL |y| (QREFELT |$| 14)) |URAGG-;cycleEntry;2A;19|)
       (SEQ
        (LETT |l| 1 |URAGG-;cycleEntry;2A;19|)
        G190
        (COND
         ((NULL
           (COND
            ((SPADCALL |y| |z| (QREFELT |$| 36)) (QUOTE NIL))
            ((QUOTE T) (QUOTE T))))
           (GO G191)))
        (SEQ
         (EXIT
          (LETT |z|
           (SPADCALL |z| (QREFELT |$| 14)) |URAGG-;cycleEntry;2A;19|)))
```

```
        (LETT |l|
         (QSADD1 |l|) |URAGG-;cycleEntry;2A;19|) (GO G190) G191 (EXIT NIL))
       (LETT |y| |x| |URAGG-;cycleEntry;2A;19|)
       (SEQ
        (LETT |k| 1 |URAGG-;cycleEntry;2A;19|)
        G190
        (COND ((QSGREATERP |k| |l|) (GO G191)))
        (SEQ
         (EXIT
          (LETT |y|
           (SPADCALL |y| (QREFELT |$| 14)) |URAGG-;cycleEntry;2A;19|)))
        (LETT |k| (QSADD1 |k|) |URAGG-;cycleEntry;2A;19|)
        (GO G190)
        G191
        (EXIT NIL))
       (SEQ
        G190
        (COND
          ((NULL
            (COND
             ((SPADCALL |x| |y| (QREFELT |$| 36)) (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
           (GO G191)))
        (SEQ
         (LETT |x| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;cycleEntry;2A;19|)
         (EXIT
          (LETT |y|
           (SPADCALL |y| (QREFELT |$| 14)) |URAGG-;cycleEntry;2A;19|)))
        NIL
        (GO G190)
        G191
        (EXIT NIL))
       (EXIT |x|))))))))

(DEFUN |URAGG-;cycleLength;ANni;20| (|x| |$|)
 (PROG (|k| |y|)
  (RETURN
   (SEQ
    (COND
     ((OR
       (SPADCALL |x| (QREFELT |$| 20))
       (SPADCALL
        (LETT |x| (|URAGG-;findCycle| |x| |$|) |URAGG-;cycleLength;ANni;20|)
        (QREFELT |$| 20)))
       0)
     ((QUOTE T)
      (SEQ
       (LETT |y| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;cycleLength;ANni;20|)
       (SEQ
        (LETT |k| 1 |URAGG-;cycleLength;ANni;20|)
        G190
        (COND
         ((NULL
           (COND
```

```
             ((SPADCALL |x| |y| (QREFELT |$| 36)) (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
          (GO G191)))
        (SEQ
         (EXIT
          (LETT |y|
           (SPADCALL |y| (QREFELT |$| 14)) |URAGG-;cycleLength;ANni;20|)))
        (LETT |k| (QSADD1 |k|) |URAGG-;cycleLength;ANni;20|)
        (GO G190)
         G191
         (EXIT NIL))
       (EXIT |k|))))))))

(DEFUN |URAGG-;rest;ANniA;21| (|x| |n| |$|)
 (PROG (|i|)
  (RETURN
   (SEQ
    (SEQ
     (LETT |i| 1 |URAGG-;rest;ANniA;21|)
     G190
     (COND ((QSGREATERP |i| |n|) (GO G191)))
     (SEQ
      (EXIT
       (COND
        ((SPADCALL |x| (QREFELT |$| 20)) (|error| "Index out of range"))
        ((QUOTE T)
          (LETT |x| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;rest;ANniA;21|)))))
     (LETT |i| (QSADD1 |i|) |URAGG-;rest;ANniA;21|)
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |x|)))))

(DEFUN |URAGG-;last;ANniA;22| (|x| |n| |$|)
 (PROG (|m| #1=#:G84694)
  (RETURN
   (SEQ
    (LETT |m| (SPADCALL |x| (QREFELT |$| 42)) |URAGG-;last;ANniA;22|)
    (EXIT
     (COND
      ((|<| |m| |n|) (|error| "index out of range"))
      ((QUOTE T)
       (SPADCALL
        (SPADCALL |x|
         (PROG1
          (LETT #1# (|-| |m| |n|) |URAGG-;last;ANniA;22|)
          (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
         (QREFELT |$| 43))
        (QREFELT |$| 44))))))))))

(DEFUN |URAGG-;=;2AB;23| (|x| |y| |$|)
 (PROG (|k| #1=#:G84705)
  (RETURN
   (SEQ
```

```
    (EXIT
     (COND
      ((SPADCALL |x| |y| (QREFELT |$| 36)) (QUOTE T))
      ((QUOTE T)
       (SEQ
        (SEQ
         (LETT |k| 0 |URAGG-;=;2AB;23|)
         G190
         (COND
          ((NULL
            (COND
             ((OR
               (SPADCALL |x| (QREFELT |$| 20))
               (SPADCALL |y| (QREFELT |$| 20)))
               (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
           (GO G191)))
         (SEQ
          (COND
           ((EQL |k| 1000)
            (COND
             ((SPADCALL |x| (QREFELT |$| 33))
               (EXIT (|error| "cyclic list"))))))
          (COND
           ((NULL
             (SPADCALL
              (SPADCALL |x| (QREFELT |$| 8))
              (SPADCALL |y| (QREFELT |$| 8))
              (QREFELT |$| 46)))
            (EXIT (PROGN (LETT #1# (QUOTE NIL) |URAGG-;=;2AB;23|) (GO #1#)))))
          (LETT |x| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;=;2AB;23|)
          (EXIT (LETT |y| (SPADCALL |y| (QREFELT |$| 14)) |URAGG-;=;2AB;23|)))
         (LETT |k| (QSADD1 |k|) |URAGG-;=;2AB;23|)
         (GO G190)
         G191
         (EXIT NIL))
        (EXIT
         (COND
          ((SPADCALL |x| (QREFELT |$| 20)) (SPADCALL |y| (QREFELT |$| 20)))
          ((QUOTE T) (QUOTE NIL))))))))
    #1#
    (EXIT #1#)))))

(DEFUN |URAGG-;node?;2AB;24| (|u| |v| |$|)
 (PROG (|k| #1=#:G84711)
  (RETURN
   (SEQ
    (EXIT
     (SEQ
      (SEQ
       (LETT |k| 0 |URAGG-;node?;2AB;24|)
       G190
       (COND
         ((NULL
```

```
         (COND
          ((SPADCALL |v| (QREFELT |$| 20)) (QUOTE NIL))
          ((QUOTE T) (QUOTE T))))
        (GO G191)))
       (SEQ
        (EXIT
         (COND
          ((SPADCALL |u| |v| (QREFELT |$| 48))
            (PROGN (LETT #1# (QUOTE T) |URAGG-;node?;2AB;24|) (GO #1#)))
          ((QUOTE T)
           (SEQ
            (COND
             ((EQL |k| 1000)
              (COND
               ((SPADCALL |v| (QREFELT |$| 33))
                 (EXIT (|error| "cyclic list"))))))
            (EXIT
             (LETT |v|
              (SPADCALL |v| (QREFELT |$| 14))
              |URAGG-;node?;2AB;24|)))))))
       (LETT |k| (QSADD1 |k|) |URAGG-;node?;2AB;24|) (GO G190) G191 (EXIT NIL))
      (EXIT (SPADCALL |u| |v| (QREFELT |$| 48)))))
    #1# (EXIT #1#)))))

(DEFUN |URAGG-;setelt;Afirst2S;25| (|x| G84713 |a| |$|)
 (declare (ignore G84713))
 (SPADCALL |x| |a| (QREFELT |$| 50)))

(DEFUN |URAGG-;setelt;Alast2S;26| (|x| G84715 |a| |$|)
 (declare (ignore G84715))
 (SPADCALL |x| |a| (QREFELT |$| 52)))

(DEFUN |URAGG-;setelt;Arest2A;27| (|x| G84717 |a| |$|)
 (declare (ignore G84717))
 (SPADCALL |x| |a| (QREFELT |$| 54)))

(DEFUN |URAGG-;concat;3A;28| (|x| |y| |$|)
 (SPADCALL (SPADCALL |x| (QREFELT |$| 44)) |y| (QREFELT |$| 56)))

(DEFUN |URAGG-;setlast!;A2S;29| (|x| |s| |$|)
 (SEQ
  (COND
   ((SPADCALL |x| (QREFELT |$| 20)) (|error| "setlast: empty list"))
   ((QUOTE T)
    (SEQ
     (SPADCALL (SPADCALL |x| (QREFELT |$| 22)) |s| (QREFELT |$| 50))
     (EXIT |s|))))))

(DEFUN |URAGG-;setchildren!;ALA;30| (|u| |lv| |$|)
 (COND
  ((EQL (LENGTH |lv|) 1) (SPADCALL |u| (|SPADfirst| |lv|) (QREFELT |$| 54)))
  ((QUOTE T) (|error| "wrong number of children specified"))))

(DEFUN |URAGG-;setvalue!;A2S;31| (|u| |s| |$|)
```

```
 (SPADCALL |u| |s| (QREFELT |$| 50)))

(DEFUN |URAGG-;split!;AIA;32| (|p| |n| |$|)
 (PROG (#1=#:G84725 |q|)
  (RETURN
   (SEQ
    (COND
     ((|<| |n| 1) (|error| "index out of range"))
     ((QUOTE T)
      (SEQ
       (LETT |p|
        (SPADCALL |p|
         (PROG1
          (LETT #1# (|-| |n| 1) |URAGG-;split!;AIA;32|)
          (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
         (QREFELT |$| 43))
        |URAGG-;split!;AIA;32|)
       (LETT |q| (SPADCALL |p| (QREFELT |$| 14)) |URAGG-;split!;AIA;32|)
       (SPADCALL |p| (SPADCALL (QREFELT |$| 61)) (QREFELT |$| 54))
       (EXIT |q|))))))))

(DEFUN |URAGG-;cycleSplit!;2A;33| (|x| |$|)
 (PROG (|y| |z|)
  (RETURN
   (SEQ
    (COND
     ((OR
       (SPADCALL
        (LETT |y| (SPADCALL |x| (QREFELT |$| 37)) |URAGG-;cycleSplit!;2A;33|)
        (QREFELT |$| 20))
       (SPADCALL |x| |y| (QREFELT |$| 36))) |y|)
     ((QUOTE T)
      (SEQ
       (LETT |z| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;cycleSplit!;2A;33|)
       (SEQ G190
        (COND
         ((NULL
           (COND
            ((SPADCALL |z| |y| (QREFELT |$| 36)) (QUOTE NIL))
            ((QUOTE T) (QUOTE T))))
          (GO G191)))
        (SEQ
         (LETT |x| |z| |URAGG-;cycleSplit!;2A;33|)
         (EXIT
          (LETT |z|
           (SPADCALL |z| (QREFELT |$| 14)) |URAGG-;cycleSplit!;2A;33|)))
        NIL
        (GO G190)
        G191
        (EXIT NIL))
       (SPADCALL |x|
        (SPADCALL (QREFELT |$| 61)) (QREFELT |$| 54)) (EXIT |y|))))))))

(DEFUN |UnaryRecursiveAggregate&| (|#1| |#2|)
```

```
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|UnaryRecursiveAggregate&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |UnaryRecursiveAggregate&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (make-array 66) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3
     (LETT |pv$|
      (|buildPredVector| 0 0
       (LIST (|HasAttribute| |#1| (QUOTE |shallowlyMutable|))))
      . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
    (COND
      ((|HasAttribute| |#1| (QUOTE |finiteAggregate|))
        (QSETREFV |$| 45
          (CONS (|dispatchFunction| |URAGG-;last;ANniA;22|) |$|))))
    (COND
     ((|HasCategory| |#2| (QUOTE (|SetCategory|)))
      (PROGN
       (QSETREFV |$| 47 (CONS (|dispatchFunction| |URAGG-;=;2AB;23|) |$|))
       (QSETREFV |$| 49
        (CONS (|dispatchFunction| |URAGG-;node?;2AB;24|) |$|)))))
    (COND
     ((|testBitVector| |pv$| 1)
      (PROGN
       (QSETREFV |$| 51
        (CONS (|dispatchFunction| |URAGG-;setelt;Afirst2S;25|) |$|))
       (QSETREFV |$| 53
        (CONS (|dispatchFunction| |URAGG-;setelt;Alast2S;26|) |$|))
       (QSETREFV |$| 55
        (CONS (|dispatchFunction| |URAGG-;setelt;Arest2A;27|) |$|))
       (QSETREFV |$| 57
        (CONS (|dispatchFunction| |URAGG-;concat;3A;28|) |$|))
       (QSETREFV |$| 58
        (CONS (|dispatchFunction| |URAGG-;setlast!;A2S;29|) |$|))
       (QSETREFV |$| 59
        (CONS (|dispatchFunction| |URAGG-;setchildren!;ALA;30|) |$|))
       (QSETREFV |$| 60
        (CONS (|dispatchFunction| |URAGG-;setvalue!;A2S;31|) |$|))
       (QSETREFV |$| 63
        (CONS (|dispatchFunction| |URAGG-;split!;AIA;32|) |$|))
       (QSETREFV |$| 64
        (CONS (|dispatchFunction| |URAGG-;cycleSplit!;2A;33|) |$|)))))
    |$|))))

(setf (get
 (QUOTE |UnaryRecursiveAggregate&|)
 (QUOTE |infovec|))
 (LIST
  (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|)
```

```
  (0 . |first|) (QUOTE "first") |URAGG-;elt;AfirstS;1| (5 . |last|)
  (QUOTE "last") |URAGG-;elt;AlastS;2| (10 . |rest|) (QUOTE "rest")
  |URAGG-;elt;ArestA;3| |URAGG-;second;AS;4| |URAGG-;third;AS;5|
  (|Boolean|) (15 . |empty?|) |URAGG-;cyclic?;AB;6| (20 . |tail|)
  |URAGG-;last;AS;7| (|List| |$|) |URAGG-;nodes;AL;8|
  |URAGG-;children;AL;9| |URAGG-;leaf?;AB;10| |URAGG-;value;AS;11|
  (|NonNegativeInteger|) |URAGG-;less?;ANniB;12| |URAGG-;more?;ANniB;13|
  |URAGG-;size?;ANniB;14| (25 . |cyclic?|) |URAGG-;#;ANni;15|
  |URAGG-;tail;2A;16| (30 . |eq?|) (36 . |cycleEntry|)
  |URAGG-;cycleTail;2A;18| |URAGG-;cycleEntry;2A;19|
  |URAGG-;cycleLength;ANni;20| |URAGG-;rest;ANniA;21| (41 . |#|)
  (46 . |rest|) (52 . |copy|) (57 . |last|) (63 . |=|) (69 . |=|)
  (75 . |=|) (81 . |node?|) (87 . |setfirst!|) (93 . |setelt|)
  (100 . |setlast!|) (106 . |setelt|) (113 . |setrest!|)
  (119 . |setelt|) (126 . |concat!|) (132 . |concat|) (138 . |setlast!|)
  (144 . |setchildren!|) (150 . |setvalue!|) (156 . |empty|) (|Integer|)
  (160 . |split!|) (166 . |cycleSplit!|) (QUOTE "value")))
 (QUOTE #(|value| 171 |third| 176 |tail| 181 |split!| 186 |size?| 192
  |setvalue!| 198 |setlast!| 204 |setelt| 210 |setchildren!| 231 |second|
  237 |rest| 242 |nodes| 248 |node?| 253 |more?| 259 |less?| 265 |leaf?|
  271 |last| 276 |elt| 287 |cyclic?| 305 |cycleTail| 310 |cycleSplit!|
  315 |cycleLength| 320 |cycleEntry| 325 |concat| 330 |children| 336 |=|
  341 |#| 347))
 (QUOTE NIL)
 (CONS
  (|makeByteWordVec2| 1 (QUOTE NIL))
  (CONS
   (QUOTE #())
   (CONS
    (QUOTE #())
    (|makeByteWordVec2| 64 (QUOTE (1 6 7 0 8 1 6 7 0 11 1 6 0 0 14 1 6
      19 0 20 1 6 0 0 22 1 6 19 0 33 2 6 19 0 0 36 1 6 0 0 37 1 6 29 0
      42 2 6 0 0 29 43 1 6 0 0 44 2 0 0 0 29 45 2 7 19 0 0 46 2 0 19 0
      0 47 2 6 19 0 0 48 2 0 19 0 0 49 2 6 7 0 7 50 3 0 7 0 9 7 51 2 6
      7 0 7 52 3 0 7 0 12 7 53 2 6 0 0 0 54 3 0 0 0 15 0 55 2 6 0 0 0 56
      2 0 0 0 0 57 2 0 7 0 7 58 2 0 0 0 24 59 2 0 7 0 7 60 0 6 0 61 2 0
      0 0 62 63 1 0 0 0 64 1 0 7 0 28 1 0 7 0 18 1 0 0 0 35 2 0 0 0 62 63
      2 0 19 0 29 32 2 0 7 0 7 60 2 0 7 0 7 58 3 0 7 0 12 7 53 3 0 0 0 15
      0 55 3 0 7 0 9 7 51 2 0 0 0 24 59 1 0 7 0 17 2 0 0 0 29 41 1 0 24 0
      25 2 0 19 0 0 49 2 0 19 0 29 31 2 0 19 0 29 30 1 0 19 0 27 2 0 0 0
      29 45 1 0 7 0 23 2 0 7 0 12 13 2 0 0 0 15 16 2 0 7 0 9 10 1 0 19 0
      21 1 0 0 0 38 1 0 0 0 64 1 0 29 0 40 1 0 0 0 39 2 0 0 0 0 57 1 0 24
      0 26 2 0 19 0 0 47 1 0 29 0 34))))))
 (QUOTE |lookupComplete|)))
```

Chapter 22

The Proofs

— coq —

```
Module Categories
\getchunk{COQ BASTYPE}
\getchunk{COQ ELEMFUN}
\getchunk{COQ HYPCAT}
\getchunk{COQ IEVALAB}
\getchunk{COQ ATJACID}
\getchunk{COQ ATLUNIT}
\getchunk{COQ ATMULVA}
\getchunk{COQ ATNZDIV}
\getchunk{COQ ATNULSQ}
\getchunk{COQ ATPOSET}
\getchunk{COQ RADCAT}
\getchunk{COQ RETRACT}
\getchunk{COQ ATRUNIT}
\getchunk{COQ TRIGCAT}
\getchunk{COQ AGG}
\getchunk{COQ ELTAGG}
\getchunk{COQ EVALAB}
\getchunk{COQ FRETRCT}
\getchunk{COQ LOGIC}
\getchunk{COQ SETCAT}
\getchunk{COQ TRANFUN}
\getchunk{COQ ABELSG}
\getchunk{COQ FINITE}
\getchunk{COQ GRMOD}
\getchunk{COQ HOAGG}
\getchunk{COQ MONAD}
\getchunk{COQ ORDSET}
\getchunk{COQ RRCC}
\getchunk{COQ SGROUP}
\getchunk{COQ ABELMON}
\getchunk{COQ BGAGG}
\getchunk{COQ CLAGG}
\getchunk{COQ DVARCAT}
```

```
\getchunk{COQ ES}
\getchunk{COQ GRALG}
\getchunk{COQ IXAGG}
\getchunk{COQ MONADWU}
\getchunk{COQ MONOID}
\getchunk{COQ RCAGG}
\getchunk{COQ ARR2CAT}
\getchunk{COQ BRAGG}
\getchunk{COQ DIOPS}
\getchunk{COQ GROUP}
\getchunk{COQ LNAGG}
\getchunk{COQ MATCAT}
\getchunk{COQ OASGP}
\getchunk{COQ ORDMON}
\getchunk{COQ PSETCAT}
\getchunk{COQ SETAGG}
\getchunk{COQ URAGG}
\getchunk{COQ ABELGRP}
\getchunk{COQ BTCAT}
\getchunk{COQ DIAGG}
\getchunk{COQ ELAGG}
\getchunk{COQ FLAGG}
\getchunk{COQ STAGG}
\getchunk{COQ TSETCAT}
\getchunk{COQ FDIVCAT}
\getchunk{COQ FSAGG}
\getchunk{COQ KDAGG}
\getchunk{COQ LZSTAGG}
\getchunk{COQ LSAGG}
\getchunk{COQ NARNG}
\getchunk{COQ A1AGG}
\getchunk{COQ RSETCAT}
\getchunk{COQ RMODULE}
\getchunk{COQ RNG}
\getchunk{COQ BMODULE}
\getchunk{COQ BTAGG}
\getchunk{COQ NASRING}
\getchunk{COQ OAMONS}
\getchunk{COQ RING}
\getchunk{COQ SRAGG}
\getchunk{COQ TBAGG}
\getchunk{COQ VECTCAT}
\getchunk{COQ DIFRING}
\getchunk{COQ ENTIRER}
\getchunk{COQ LALG}
\getchunk{COQ MODULE}
\getchunk{COQ ORDRING}
\getchunk{COQ PDRING}
\getchunk{COQ RMATCAT}
\getchunk{COQ OREPCAT}
\getchunk{COQ ALGEBRA}
\getchunk{COQ DIFEXT}
\getchunk{COQ LIECAT}
\getchunk{COQ LODOCAT}
```

```
\getchunk{COQ NAALG}
\getchunk{COQ VSPACE}
\getchunk{COQ DIRPCAT}
\getchunk{COQ DIVRING}
\getchunk{COQ FINAALG}
\getchunk{COQ INTDOM}
\getchunk{COQ OC}
\getchunk{COQ QUATCAT}
\getchunk{COQ SMATCAT}
\getchunk{COQ AMR}
\getchunk{COQ FRNAALG}
\getchunk{COQ GCDDOM}
\getchunk{COQ FAMR}
\getchunk{COQ PSCAT}
\getchunk{COQ UFD}
\getchunk{COQ EUCDOM}
\getchunk{COQ PFECAT}
\getchunk{COQ UPSCAT}
\getchunk{COQ FIELD}
\getchunk{COQ INS}
\getchunk{COQ POLYCAT}
\getchunk{COQ UTSCAT}
\getchunk{COQ ACF}
\getchunk{COQ DPOLCAT}
\getchunk{COQ FPC}
\getchunk{COQ FINRALG}
\getchunk{COQ FS}
\getchunk{COQ QFCAT}
\getchunk{COQ RCFIELD}
\getchunk{COQ RNS}
\getchunk{COQ RPOLCAT}
\getchunk{COQ UPOLYC}
\getchunk{COQ ACFS}
\getchunk{COQ XF}
\getchunk{COQ FFIELDC}
\getchunk{COQ FPS}
\getchunk{COQ FRAMALG}
\getchunk{COQ ULSCCAT}
\getchunk{COQ UPXSCCA}
\getchunk{COQ FAXF}
\getchunk{COQ MONOGEN}
\getchunk{COQ COMPCAT}
\getchunk{COQ FFCAT}
End Categories
```

Chapter 23

Chunk collections

— algebra —

```
\getchunk{category ABELGRP AbelianGroup}
\getchunk{category ABELMON AbelianMonoid}
\getchunk{category AMR AbelianMonoidRing}
\getchunk{category ABELSG AbelianSemiGroup}
\getchunk{category AGG Aggregate}
\getchunk{category ALGEBRA Algebra}
\getchunk{category ACF AlgebraicallyClosedField}
\getchunk{category ACFS AlgebraicallyClosedFunctionSpace}
\getchunk{category ATADDVA AdditiveValuationAttribute}
\getchunk{category ATAPPRO ApproximateAttribute}
\getchunk{category ATARBEX ArbitraryExponentAttribute}
\getchunk{category ATARBPR ArbitraryPrecisionAttribute}
\getchunk{category AHYP ArcHyperbolicFunctionCategory}
\getchunk{category ATRIG ArcTrigonometricFunctionCategory}
\getchunk{category ALAGG AssociationListAggregate}
\getchunk{category ATTREG AttributeRegistry}

\getchunk{category BGAGG BagAggregate}
\getchunk{category BASTYPE BasicType}
\getchunk{category BLMETCT BlowUpMethodCategory}
\getchunk{category BMODULE BiModule}
\getchunk{category BRAGG BinaryRecursiveAggregate}
\getchunk{category BTCAT BinaryTreeCategory}
\getchunk{category BTAGG BitAggregate}

\getchunk{category CACHSET CachableSet}
\getchunk{category CABMON CancellationAbelianMonoid}
\getchunk{category ATCANON CanonicalAttribute}
\getchunk{category ATCANCL CanonicalClosedAttribute}
\getchunk{category ATCUNOR CanonicalUnitNormalAttribute}
\getchunk{category ATCENRL CentralAttribute}
\getchunk{category CHARNZ CharacteristicNonZero}
\getchunk{category CHARZ CharacteristicZero}
\getchunk{category KOERCE CoercibleTo}
```

```
\getchunk{category CLAGG Collection}
\getchunk{category CFCAT CombinatorialFunctionCategory}
\getchunk{category COMBOPC CombinatorialOpsCategory}
\getchunk{category ATCS CommutativeStarAttribute}
\getchunk{category COMPAR Comparable}
\getchunk{category COMRING CommutativeRing}
\getchunk{category COMPCAT ComplexCategory}
\getchunk{category KONVERT ConvertibleTo}

\getchunk{category DQAGG DequeueAggregate}
\getchunk{category DSTRCAT DesingTreeCategory}
\getchunk{category DIAGG Dictionary}
\getchunk{category DIOPS DictionaryOperations}
\getchunk{category DIFEXT DifferentialExtension}
\getchunk{category DPOLCAT DifferentialPolynomialCategory}
\getchunk{category DIFRING DifferentialRing}
\getchunk{category DIVCAT DivisorCategory}
\getchunk{category DVARCAT DifferentialVariableCategory}
\getchunk{category DIRPCAT DirectProductCategory}
\getchunk{category DIVRING DivisionRing}
\getchunk{category DLAGG DoublyLinkedAggregate}

\getchunk{category ELEMFUN ElementaryFunctionCategory}
\getchunk{category ELTAB Eltable}
\getchunk{category ELTAGG EltableAggregate}
\getchunk{category ENTIRER EntireRing}
\getchunk{category EUCDOM EuclideanDomain}
\getchunk{category EVALAB Evalable}
\getchunk{category ES ExpressionSpace}
\getchunk{category ELAGG ExtensibleLinearAggregate}
\getchunk{category XF ExtensionField}

\getchunk{category FIELD Field}
\getchunk{category FPC FieldOfPrimeCharacteristic}
\getchunk{category FILECAT FileCategory}
\getchunk{category FNCAT FileNameCategory}
\getchunk{category FINITE Finite}
\getchunk{category FAMR FiniteAbelianMonoidRing}
\getchunk{category ATFINAG FiniteAggregateAttribute}
\getchunk{category FAXF FiniteAlgebraicExtensionField}
\getchunk{category FDIVCAT FiniteDivisorCategory}
\getchunk{category FFIELDC FiniteFieldCategory}
\getchunk{category FLAGG FiniteLinearAggregate}
\getchunk{category FINRALG FiniteRankAlgebra}
\getchunk{category FINAALG FiniteRankNonAssociativeAlgebra}
\getchunk{category FSAGG FiniteSetAggregate}
\getchunk{category FPS FloatingPointSystem}
\getchunk{category FORTFN FortranFunctionCategory}
\getchunk{category FMTC FortranMachineTypeCategory}
\getchunk{category FMC FortranMatrixCategory}
\getchunk{category FMFUN FortranMatrixFunctionCategory}
\getchunk{category FORTCAT FortranProgramCategory}
\getchunk{category FVC FortranVectorCategory}
\getchunk{category FVFUN FortranVectorFunctionCategory}
```

```
\getchunk{category FRAMALG FramedAlgebra}
\getchunk{category FRNAALG FramedNonAssociativeAlgebra}
\getchunk{category FAMONC FreeAbelianMonoidCategory}
\getchunk{category FLALG FreeLieAlgebra}
\getchunk{category FMCAT FreeModuleCat}
\getchunk{category FEVALAB FullyEvalableOver}
\getchunk{category FLINEXP FullyLinearlyExplicitRingOver}
\getchunk{category FPATMAB FullyPatternMatchable}
\getchunk{category FRETRCT FullyRetractableTo}
\getchunk{category FFCAT FunctionFieldCategory}
\getchunk{category FS FunctionSpace}

\getchunk{category GCDDOM GcdDomain}
\getchunk{category GRALG GradedAlgebra}
\getchunk{category GRMOD GradedModule}
\getchunk{category GROUP Group}

\getchunk{category HOAGG HomogeneousAggregate}
\getchunk{category HYPCAT HyperbolicFunctionCategory}

\getchunk{category IXAGG IndexedAggregate}
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\getchunk{category INTDOM IntegralDomain}
\getchunk{category INTCAT IntervalCategory}

\getchunk{category ATJACID JacobiIdentityAttribute}

\getchunk{category KDAGG KeyedDictionary}

\getchunk{category ATLR LazyRepresentationAttribute}
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\getchunk{category MONADWU MonadWithUnit}
\getchunk{category MONOGEN MonogenicAlgebra}
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\getchunk{category MDAGG MultiDictionary}
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\getchunk{category NAALG NonAssociativeAlgebra}
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\getchunk{category NARNG NonAssociativeRng}
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\getchunk{category ATNZDIV NoZeroDivisorsAttribute}
\getchunk{category ATNULSQ NullSquareAttribute}
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\getchunk{category OPTCAT NumericalOptimizationCategory}

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\getchunk{category A1AGG OneDimensionalArrayAggregate}
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\getchunk{category OAMONS OrderedAbelianMonoidSup}
\getchunk{category OASGP OrderedAbelianSemiGroup}
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\getchunk{category PID PrincipalIdealDomain}
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\getchunk{category REAL RealConstant}
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\getchunk{category TBAGG TableAggregate}
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\getchunk{category TRIGCAT TrigonometricFunctionCategory}
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[Abra01] Sergei Abramov and Manuel Bronstein. On Solutions of Linear Functional Systems, 2001.

Abstract: We describe a new direct algorithm for transforming a linear system of recurrences into an equivalent one with nonsingular leading or trailing matrix. Our algorithm, which is an improvement to the EG elimination method, uses only elementary linear algebra operations (ranks, kernels, and determinants) to produce an equation satisfied by the degress of the solutions with finite support. As a consequence, we can boudn and compute the polynomial and rational solutions of very general linear functional systems such as systems of differential or (q)-difference equations.

Link: http://www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mb_papers.html

Algebra:
(p1105) category OREPCAT UnivariateSkewPolynomialCategory
(p1149) category LODOCAT LinearOrdinaryDifferentialOperatorCategory
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(p**??**) domain OREUP UnivariateSkewPolynomial
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(p**??**) domain LODO2 LinearOrdinaryDifferentialOperator2
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(p**??**) package OREPCTO UnivariateSkewPolynomialCategoryOps
(p**??**) package LODOF LinearOrdinaryDifferentialOperatorFactorizer
(p**??**) package LODOOPS LinearOrdinaryDifferentialOperatorsOps

[Aubr96] Phillippe Aubry and Marc Moreno Maza. Triangular Sets for Solving Polynomial Systems: a Comparison of Four Methods, 1996.

Abstract: Four methods for solving polynomial systems by means of triangular sets are presented and implemented in a unified way. These methods are those of Wu, Lazard, Kalkbrener, and Wang. They are compared on various examples with emphasis on efficiency, conciseness and legibility of the outputs.

Link: http://www.lip6.fr/lip6/reports/1997/lip6.1997.009.ps.gz

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(p**??**) package RSDCMPK RegularSetDecompositionPackage

[Aubr99] Phillippe Aubry, Daniel Lazard, and Marc Moreno Maza. On the Theories of Triangular Sets. *Journal of Symbolic Computation*, 28:105–124, 1999.

Abstract: Different notions of triangular sets are presented. The relationship between these notions are studied. The main result is that four different existing notions of *good* triangular sets are equivalent.

Link: http://www.csd.uwo.ca/~moreno/Publications/Aubry-Lazard-MorenoMaza-1999-JSC.pdf

Algebra:
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(p**??**) package LEXTRIPK LexTriangularPackage
(p**??**) package RSDCMPK RegularSetDecompositionPackage

[Aubr99a] Phillippe Aubry and Marc Moreno Maza. Triangular Sets for Solving Polynomial Systems: a Comparison of Four Methods. *J. Symb. Comput.*, 28(1-2):125–154, 1999.

Abstract: Four methods for solving polynomial systems by means of triangular sets are presented and implemented in a unified way. These methods are those of Wu, Lazard, Kalkbrener, and Wang. They are compared on various examples with emphasis on efficiency, conciseness and legibility of the outputs.

Link: http://www.csd.uwo.ca/~moreno/Publications/Aubry-MorenoMaza-1999-JSC.pdf

Algebra:
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(p929) category NTSCAT NormalizedTriangularSetCategory
(p958) category SFRTCAT SquareFreeRegularTriangularSetCategory
(p**??**) package RSDCMPK RegularSetDecompositionPackage

[Brem08] Murray R. Bremner, Lucia I. Murakami, and Ivan P. Shestakov. Nonassociative Algebras, 2008.

Abstract: One of the earliest surveys on nonassociative algebras is the article by Shirshov which introduced the phrase “rings that are nearly associative”. The first book in the English language devoted to a systematic study of nonassociative algebras is Schafer (Scha66). A comprehensive exposition of the work of the Russian School is Zhevlakov, Slinko, Shestakov and Shirshov. A collection of open research problems in algebra, including many problems on nonassociative algebra, is the *Dniester Notebook*; the survey article by Kuzmin and Shetakov is from the same period. Three books on Jordan algebras which contain substantial material on general nonassociative algebras are Braun and Koecher, Jacobson, and McCrimmon. Recent research appears in the Proceedings of the International Conferences on Nonassociative Algebras and its Applications. The present section provides very limited information on Lie algebras, since they are the subject of Section 16.4. The last part (section 9) of the present section presents three applications of computational linear algebra to the study of polynomial identiies for nonassociative algebras: pseudorandom vectors in a nonassociative algebra, the expansion matrix for a nonassociative operation, and the representation theory of the symmetric group.

Link: http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.146.2665

Algebra:
(p851) category NARNG NonAssociativeRng

[Bron95] Manuel Bronstein. On radical solutions of linear ordinary differential equations, 1995.

Link: ftp://ftp.inf.ethz.ch/org/cathode/workshops/jan95/abstracts/bronstein.ps

Algebra:
(p1105) category OREPCAT UnivariateSkewPolynomialCategory
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(p**??**) domain ORESUP SparseUnivariateSkewPolynomial
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(p**??**) domain LODO1 LinearOrdinaryDifferentialOperator1
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(p**??**) package OREPCTO UnivariateSkewPolynomialCategoryOps
(p**??**) package LODOF LinearOrdinaryDifferentialOperatorFactorizer
(p**??**) package LODOOPS LinearOrdinaryDifferentialOperatorsOps

[Bron96a] Manuel Bronstein and Marko Petkovsek. An introduction to pseudo-linear algebra. *Theoretical Computer Science*, 157:3–33, 1966.

Abstract: Pseudo-linear algebra is the study of common properties of linear differential and difference operators. We introduce in this paper its basic objects (pseudo-derivations, skew polynomials, and pseudo-linear operators) and describe several recent algorithms on them, which, when applied in the differential and difference cases, yield algorithms for uncoupling and solving systems of linear differential and difference equations in closed form.

Link: http://www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mb_papers.html

Algebra:
(p276) category LORER LeftOreRing

[Corl00] Robert M. Corless, David J. Jeffrey, Stephen M. Watt, and James H. Davenport. "According to Abramowitz and Stegun" or arccoth needn't be Uncouth. *SIGSAM Bulletin - Special Issue on OpenMath*, 34(2):58–65, 2000.

Abstract: This paper addresses the definitions in OpenMath of the elementary functions. The original OpenMath definitions, like most other sources, simply cite [2] as the definition. We show that this is not adequate, and propose precise definitions, and explore the relationships between these definitions.In particular, we introduce the concept of a couth pair of definitions, e.g. of arcsin and arcsinh, and show that the pair arccot and *arccoth* can be couth.

Algebra:
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(p**??**) domain FLOAT Float
(p**??**) domain FRAC Fraction
(p**??**) domain INT Integer
(p**??**) domain LIST List
(p**??**) domain SINT SingleInteger
(p**??**) domain STRING String
(p**??**) domain SYMBOL Symbol
(p**??**) package OMEXPR ExpressionToOpenMath
(p**??**) package OMSERVER OpenMathServerPackage

[Dele06] Jonathon Delenclos and André Leroy. Noncommutative Symmetric functions and W-polynomials.

Abstract: Let K, S, D be a division ring an endomorphism and a S-derivation of K, respectively. In this setting we introduce generalized noncommutative symmetric functions and obtain Viéte formula and decompositions of different operators. W-polynomials show up naturally, their connetions with P-independency. Vandermonde and Wronskian matrices are briefly studied. The different linear factorizations of W-polynomials are analysed. Connections between the existence of LLCM (least left common multiples) of monic linear polynomials with coefficients in a ring and the left duo property are established at the end of the paper.

Link: http://arxiv.org/pdf/math/0606614.pdf

Algebra:
(p276) category LORER LeftOreRing

[Fate01a] Richard J. Fateman. A Critique of OpenMath and Thoughts on Encoding Mathematics, 2001.

Abstract: The OpenMath project, as portrayed in the Special Issue of the SIGSAM Bulletin (volume 34 no. 2), seems to have a number of problems to face. One of them is the (apparently implicit) assumption that OpenMath designers, through dint of mathematical thought and the advice of the members of the Open Math Society, have solved, in their domain, one of the most pressing problems of software engineering today, namely software re-use. After six years there is insufficient evidence on which to base any claims of success and it appears that most substantive practical issues of mathematical representation and communication have yet to be addressed. We also raise questions about related computational mathematical goals and mathematical encodings.

Link: http://www.cs.berkeley.edu/~/fateman/papers/openmathcrit.pdf

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(p**??**) domain STRING String
(p**??**) domain SYMBOL Symbol
(p**??**) package OMEXPR ExpressionToOpenMath
(p**??**) package OMSERVER OpenMathServerPackage

[Grab92] Johannes Grabmeier and Alfred Scheerhorn. Finite fields in Axiom. technical report AXIOM Technical Report TR7/92 (ATR/5)(NP2522), Numerical Algorithms Group, Inc., Downer's Grove, IL, USA and Oxford, UK, 1992.

Abstract: Finite fields play an important role for many applications (e.g. coding theory, cryptograpy). There are different ways to construct a finite field for a given prime power. The paper describes the different constructions implemented in AXIOM. These are *polynomial basis representation*, *cyclic group representation*, and *normal basis representation*. Furthermore, the concept of the implementation, the used algorithms and the various datatype coercions between these representations are discussed.

Link: http://www.nag.co.uk/doc/TechRep/axiomtr.html

Algebra:
(p1012) category CHARNZ CharacteristicNonZero
(p1535) category FPC FieldOfPrimeCharacteristic
(p1759) category XF ExtensionField
(p1767) category FFIELDC FiniteFieldCategory
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(p**??**) domain IPF InnerPrimeField
(p**??**) domain PF PrimeField
(p**??**) domain FFP FiniteFieldExtensionByPolynomial
(p**??**) domain FFCGP FiniteFieldCyclicGroupExtensionByPolynomial
(p**??**) domain FFNBP FiniteFieldNormalBasisExtensionByPolynomial
(p**??**) domain FFX FiniteFieldExtension
(p**??**) domain FFCGX FiniteFieldCyclicGroupExtension
(p**??**) domain FFNBX FiniteFieldNormalBasisExtension

(p**??**) domain IFF InnerFiniteField
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(p**??**) domain FFCG FiniteFieldCyclicGroup
(p**??**) domain FFNB FiniteFieldNormalBasis
(p**??**) package DLP DiscreteLogarithmPackage
(p**??**) package FFF FiniteFieldFunctions
(p**??**) package INBFF InnerNormalBasisFieldFunctions
(p**??**) package FFPOLY FiniteFieldPolynomialPackage
(p**??**) package FFPOLY2 FiniteFieldPolynomialPackage2
(p**??**) package FFHOM FiniteFieldHomomorphisms
(p**??**) package FFFACTSE FiniteFieldFactorizationWithSizeParseBySideEffect

[Hold11] Tom Hohold, Jacobus H. van Lint, and Ruud Pellikaan. *Algebraic Geometry Codes*, volume I. V.S Pless, W.C. Huffman and R.A. Brualdi (eds), 2011, 9780444814722.

Link: http://www.win.tue.nl/~ruudp/paper/31.pdf

Algebra:
(p471) category PRSPCAT ProjectiveSpaceCategory

[Hubb] John H. Hubbard and Benjamin Lundell. A First Look at Differential Algebra.

Abstract: The object of the paper is to prove that the differential equation

$$u^{'}(t) = t - [u(t)]^2$$

has no solutions which can be written using elementary functions, or anti-derivatives of elementary functions, or exponentials of such anti-derivatives, or anti-derivative of those, etc. We should note that Equation 1 can be solved using power series, integrals which depend on a parameter, or Bessel functions of order 1/3. However, as we will see, none of these methods of solution are “algebraic” in nature. We aim to give a precise definition of “algebraic” by developing the theory of *differential algebra*, which is largely the work of Ritt. Other contributors are Liouville, Picard, Vessoit, Kolchin, Rosenlicht, ... The part of differential Galois theory which leads to a proof of Abel’s celebrated result that a general polynomial equation of degree five or higher cannot be solved by radicals. In effort to derive these two areas in parallel, we will also explain why the polynomial equation

$$x^5 - 4x^2 - 2 = 0$$

has no solutions which can be written as radicals of solutions to lower degree polyomial equations.

Link: http://www.math.cornell.edu/~hubbard/diffalg1.pdf

Algebra:
(p414) category DVARCAT DifferentialVariableCategory

[Iyan60] Shokichi Iyanaga and Yukiyosi Kawada Iyanaga. *Encyclopedia Dictionary of Mathematics*. Mathematical Society of Japan, 1960, 978-0262090261. **Algebra:** (p438) category GRALG GradedAlgebra

[Jaco51] Nathan Jacobson. General Representation Theory of Jordan Algebras. *Trans. of the American Mathematical Society*, 70(3):509–530, 1951.

Link: http://www.math.uci.edu/~brusso/jacobson1951.pdf

Algebra:
(p305) category MONAD Monad

[Jaco68] Nathan Jacobson. Structure and Representations of Jordan Algebras. *Bull. Amer. Math. Soc*, 79(3):509–514, 1973.

Link: http://projecteuclid.org/euclid.bams/1183534656

Algebra:
(p305) category MONAD Monad,
(p451) category MONADWU MonadWithUnit

[Kalk91] M. Kalkbrener. *Three contributions to elimination theory.* PhD thesis, University of Linz, Austria, 1991. **Algebra:**
(p756) category TSETCAT TriangularSetCategory
(p880) category RSETCAT RegularTriangularSetCategory
(p929) category NTSCAT NormalizedTriangularSetCategory
(p958) category SFRTCAT SquareFreeRegularTriangularSetCategory
(p**??**) package RSDCMPK RegularSetDecompositionPackage

[Kalk98] M. Kalkbrener. Algorithmic properties of polynomial rings. *Journal of Symbolic Computation*, 1998.

Abstract: In this paper we investigate how algorithms for computing heights, radicals, unmixed and primary decompositions of ideals can be lifted from a Noetherian commutative ring R to polynomial rings over R.

Algebra:
(p756) category TSETCAT TriangularSetCategory
(p880) category RSETCAT RegularTriangularSetCategory
(p929) category NTSCAT NormalizedTriangularSetCategory
(p958) category SFRTCAT SquareFreeRegularTriangularSetCategory
(p**??**) package RSDCMPK RegularSetDecompositionPackage

[Lamb92] Larry Lambe. Next Generation Computer Algebra Systems AXIOM and the Scratchpad Concept: Applications to Research in Algebra. *unknown*, 1992.

Abstract: One way in which mathematicians deal with infinite amounts of data is symbolic representation. A simple example is the quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a formula which uses symbolic representation to describe the solutions to an infinite class of equations. Most computer algebra systems can deal with polynomials with symbolic coefficients, but what if symbolic exponents are called for (e.g. $1 + t^i$)? What if symbolic limits on summations are also called for, for example

$$1 + t + \ldots + t^i = \sum_j t^j$$

The “Scratchpad Concept” is a theoretical ideal which allows the implementation of objects at this level of abstraction and beyond in a mathematically consistent way. The Axiom computer algebra system is an implementation of a major part of the Scratchpad Concept. Axiom (formerly called Scratchpad) is a language with extensible parameterized types and generic operators which is based on the notions of domains and categories. By examining some aspects of the Axiom system, the Scratchpad Concept will be illustrated. It will be shown how some complex problems in homologicial algebra were solved through the use of this system.

Link: http://axiom-wiki.newsynthesis.org/public/refs/axiom-21cong.pdf

Algebra:
(p1036) category FMCAT FreeModuleCat

[Laza91] Daniel Lazard. A New Method for Solving Algebraic Systems of Positive Dimension. *Discrete. Applied. Mathematics*, 33:147–160, 1991.

Abstract: A new algorithm is presented for solving algebraic systems of equations, which is designed from the structure which is wanted for the result. This algorithm is not yet implemented; thus technical details and proofs are omitted, for emphasising on the relation between the algorithm design and a good representation of the result. The algorithm is based on a new theorem of decomposition for algebraic varieties.

Algebra:
(p756) category TSETCAT TriangularSetCategory
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(p958) category SFRTCAT SquareFreeRegularTriangularSetCategory
(p**??**) package RSDCMPK RegularSetDecompositionPackage

[Lips81] John D. Lipson. *Elements of Algebra and Algebraic Computing.* Addison-Wesley Educational Publishers, 1981, 978-0201041156. **Algebra:**
(p1767) category FFIELDC FiniteFieldCategory

[Maza00] Marc Moreno Maza. On Triangular Decompositions of Algebraic Varieties. technical report TR 4/99, Numerical Algorithms Group, June 2000.

Abstract: Different kinds of triangular decompositions of algebraic varieties are presented. The main result is an efficient method for obtaining them. Our strategy is based on a lifting theorem for polynomial computations module regular chains.

Link: http://www.csd.uwo.ca/~moreno//Publications

Algebra:
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(p929) category NTSCAT NormalizedTriangularSetCategory
(p958) category SFRTCAT SquareFreeRegularTriangularSetCategory
(p**??**) package RSDCMPK RegularSetDecompositionPackage

[Maza95] Marc Moreno Maza and Renaud Rioboo. Polynomial Gcd Computations over Towers of Algebraic Extensions. *Proceedings of AAECC11*, 1995.

Abstract: Some methods for polynomial system solving require efficient techniques for computing univariate polynomial gcd over algebraic extensions of a field. Currently used techniques compute *generic* univariate polynomial gcd before *specializing* the result using algebraic relations in the ring of coefficients. This strategy generates very big intermediate data and fails for many problems. We present here a new approach which takes permanently into account those algebraic relations. It is based on a property of subresultant remainder sequences and leads to a great increase of the speed of computations and thus the size of accessible systems.

Algebra:
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(p**??**) package RSDCMPK RegularSetDecompositionPackage

[Maza97] Marc Moreno Maza. *Calculs de pgcd au-dessus des tours d'extensions simples et resolution des systemes d'equations algebriques.* PhD thesis, Universite P.etM. Curie, 1997.

Abstract: This thesis is dedicated to polynomial system solving by means of triangular sets. A first prt presents two algorithms to compute polynomial gcds over tower of simple extensions. The first one was designed by Renaud Rioboo and applies to algebraic towers. The second one is a generalization of the previous one to the most general case of seperable towers. These algorithms lead to an efficient implementation of two methods suggested by Daniel Lazard to solve polynomial systems by means of triangular sets. These programs solved problems that were previously unreachable. The second method was only sketched by its author. So, a second part of this thesis presents the necessary developements to describe a right implementation. Moreover, a theorecal and unified presentation, together with an experimental comparison with similar methods due to Wu Wen-Tsun, Dongming Wang and Michael Kalkbrener were realized by Philippe Aubry and are reported in a third part of this document.

Keyword: axiomref

Link: http://www.csd.uwo.ca/~moreno//Publications/MorenoMaza-Thesis-1997.ps.gz

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(p**??**) package RSDCMPK RegularSetDecompositionPackage

[Maza98] M. Moreno Maza. A new algorithm for computing triangular decomposition of algebraic varieties. Technical report, Numerical Algorithms Group (NAG), 1998.
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(p756) category TSETCAT TriangularSetCategory
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[Muld95] Thom Mulders. Primitives: Orepoly and Lodo, 1995.

Link: ftp://ftp.inf.ethz.ch/org/cathode/workshops/jan95/abstracts/mulders.ps

Algebra:
(p1105) category OREPCAT UnivariateSkewPolynomialCategory
(p1149) category LODOCAT LinearOrdinaryDifferentialOperatorCategory
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(p??) package OREPCTO UnivariateSkewPolynomialCategoryOps
(p??) package LODOF LinearOrdinaryDifferentialOperatorFactorizer
(p??) package LODOOPS LinearOrdinaryDifferentialOperatorsOps

[Ritt50] Joseph Fels Ritt. Differential Algebra. *AMS Colloquium Publications*, 33, 1950, 978-0-8218-4638-4. **Algebra:**
(p414) category DVARCAT DifferentialVariableCategory

[SALSA] SALSA. Solvers for Algebraic Systems and Applications.

Link: http://www.ens-lyon.fr/LIP/Arenaire/SYMB/teams/salsa/proposal-salsa.pdf

Algebra:
(p756) category TSETCAT TriangularSetCategory
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(p958) category SFRTCAT SquareFreeRegularTriangularSetCategory
(p??) package RSDCMPK RegularSetDecompositionPackage

[Saun79] Saunders MacLane and Garrett Birkhoff. *Algebra, Second Edition*. MacMillan, 1979. **Algebra:**
(p271) category GRMOD GradedModule

[Scha10] R.D. Schafer. *An Introduction to Nonassociative Algebras*. Benediction Classics, 2010, 978-1849025904.

Abstract: Concise study presents in a short space some of the important ideas and results in the theory of non-associative algebras, with particular emphasis on alternative and (commutative) Jordan algebras. Written as an introduction for graduate students and other mathematicians meeting the subject for the first time.

Algebra:
(p851) category NARNG NonAssociativeRng

[Scha61] R.D. Schafer. An Introduction to Nonassociative Algebras. *Advanced Subject Matter Institute*, 1961.

Abstract: These are notes for my lectures in July, 1961, at the Advanced Subject Matter Institute in Algebra which was held at Oklahoma State University in the summer of 1961. Students at the Institute were provided with reprints of my paper, *Structure and representation of nonassociate algebras* (Bulletin of the American Mathematical Society, vol. 61 (1955), pp469-484), together with copies of a selective bibliography of more recent papers on non-associative algebras. These notes supplement the 1955 Bulletin article, bringing the statements there up to date and providing detailed proofs of a selected group of theorems. The proofs illustrate a number of important techniques used in the study of nonassociative algebras.

Link: `http://www.gutenberg.org/ebooks/25156`

Algebra:
(p851) category NARNG NonAssociativeRng

[Scha66] R.D. Schafer. *An Introduction to Nonassociative Algebras.* Academic Press, New York, 1966. **Algebra:**
(p851) category NARNG NonAssociativeRng
(p919) category NASRING NonAssociativeRing
(p**??**) domain ALGSC AlgebraGivenByStructuralConstants

[lidl83] Rudolf Lidl and Harald Niederreiter. *Finite Field, Encyclopedia of Mathematics and Its Applications*, volume 20. Cambridge Univ. Press, 1983, 0-521-30240-4. **Algebra:**
(p1836) category FAXF FiniteAlgebraicExtensionField
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(p**??**) domain FFNBX FiniteFieldNormalBasisExtension
(p**??**) domain FFNBP FiniteFieldNormalBasisExtensionByPolynomial
(p**??**) domain IFF InnerFiniteField
(p**??**) domain IPF InnerPrimeField
(p**??**) domain PF PrimeField
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(p**??**) package FFPOLY2 FiniteFieldPolynomialPackage2
(p**??**) package FFPOLY FiniteFieldPolynomialPackage
(p**??**) package FFHOM FiniteFieldHomomorphisms
(p**??**) package FFF FiniteFieldFunctions

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