

HW2

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Section 7.8

7.1: Books contains the daily sales of paperback and hardcover books at the same store. Forecast the next four days; sales for paperback and hardcover books

(a) Plot the series and discuss the main features of the data

```
library(fpp2)

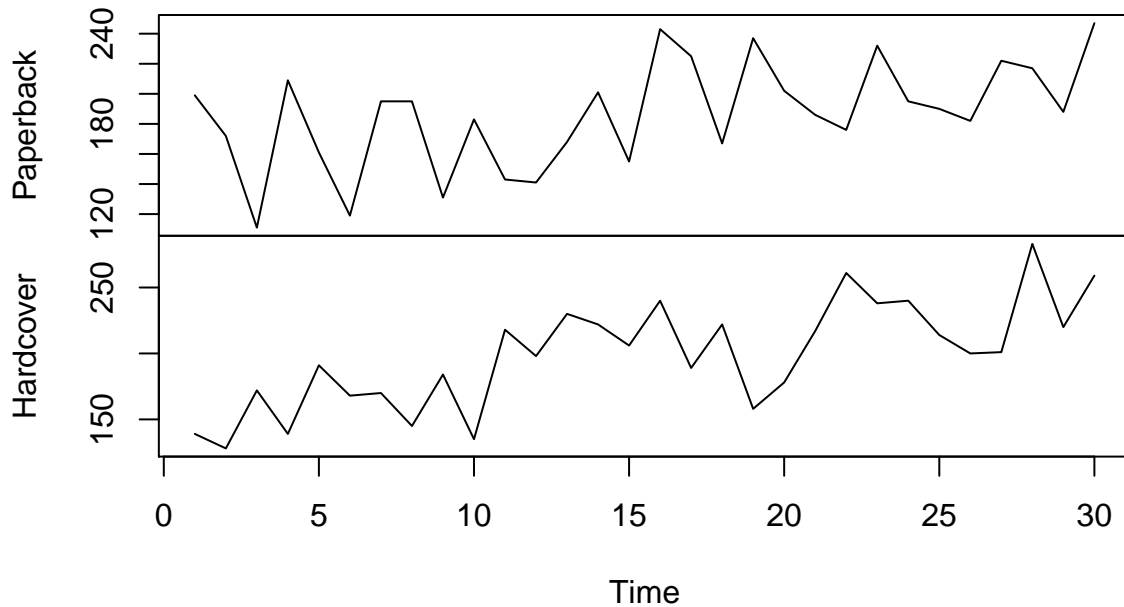
## Loading required package: ggplot2
## Loading required package: forecast
## Warning in as.POSIXlt.POSIXct(Sys.time()): unknown timezone 'zone/tz/2018e.
## 1.0/zoneinfo/America/New_York'
##
## Attaching package: 'forecast'
## The following object is masked from 'package:ggplot2':
##
##     autolayer
## Loading required package: fma
## Loading required package: expsmooth
## Warning: replacing previous import 'forecast::autolayer' by
## 'ggplot2::autolayer' when loading 'fpp2'
data(books)
colnames(books)

## [1] "Paperback" "Hardcover"
```

The seasonal trend in both hardcover and paperback sales is visible. There is a general upward trend in hardcover sales, but not as present in paperback sales.

```
plot(books, main="Daily Sales of Paperback and Hardcover", ylab="Daily Sales", xlab="Time")
```

Daily Sales of Paperback and Hardcover



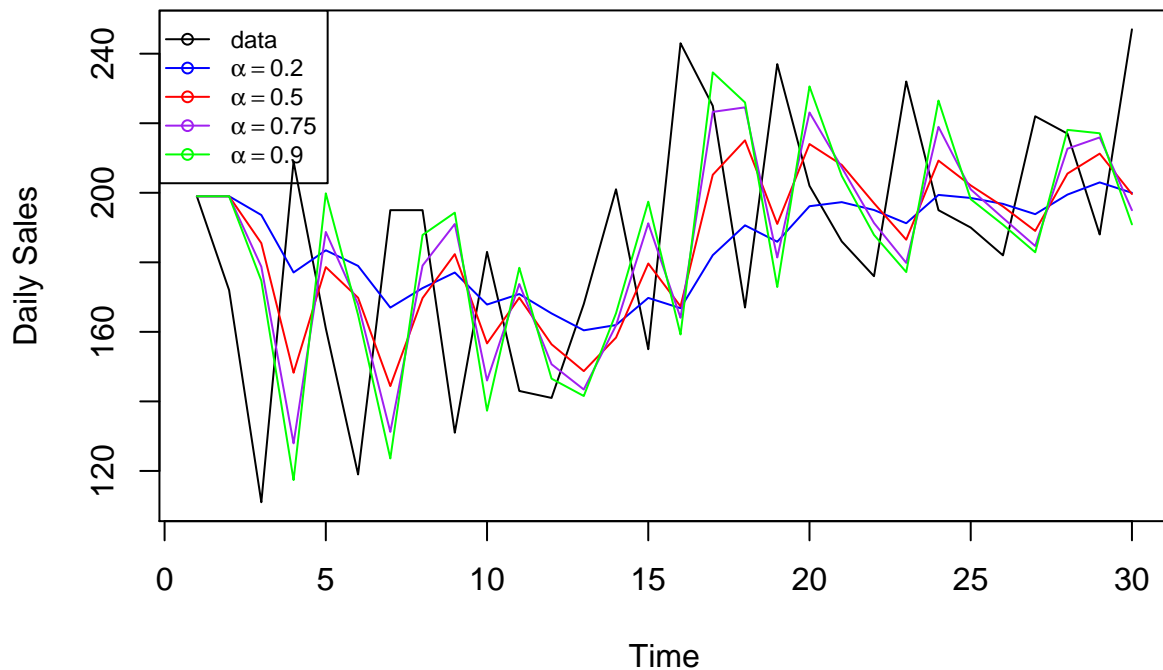
(b) Use SES and explore different values for alpha.

Paperback Sales

```
fit1<-ses(books[,1], alpha = 0.2, initial = "simple", h=1)
fit2<- ses(books[,1], alpha = 0.5, initial = "simple", h=1)
fit3<-ses(books[,1], alpha = 0.75, initial = "simple", h=1)
fit4<-ses(books[,1], alpha = 0.90, initial = "simple", h=1)

plot(books[,1], main="Daily Sales of Paperback", ylab="Daily Sales", xlab="Time")
lines(fitted(fit1), col="blue")
lines(fitted(fit2), col="red")
lines(fitted(fit3), col="purple")
lines(fitted(fit4), col="green")
legend("topleft", lty = 1, col = c(1,"blue", "red", "purple", "green"),
      c("data", expression(alpha == 0.2), expression(alpha == 0.5),
        expression(alpha == 0.75), expression(alpha == 0.9)), pch = 1, cex = 0.75)
```

Daily Sales of Paperback

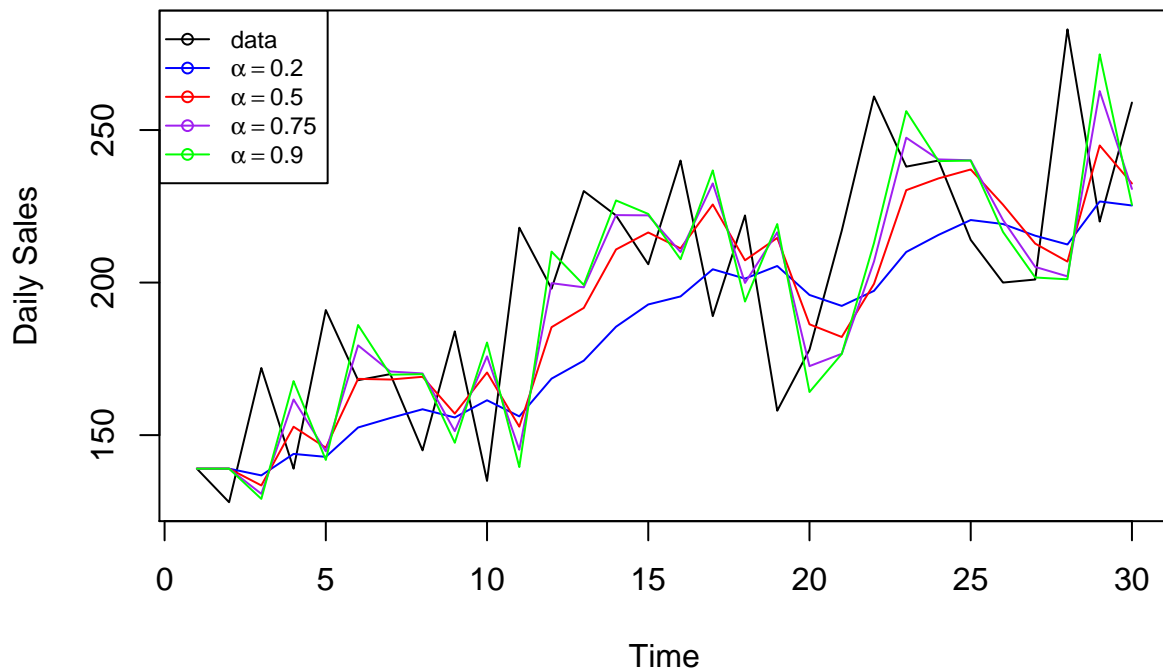


Hardcover Sales

```
fit5<-ses(books[,2], alpha = 0.2, initial = "simple", h=1)
fit6<- ses(books[,2], alpha = 0.5, initial = "simple", h=1)
fit7<-ses(books[,2], alpha = 0.75, initial = "simple", h=1)
fit8<-ses(books[,2], alpha = 0.90, initial = "simple", h=1)

plot(books[,2], main="Daily Sales of Hardback", ylab="Daily Sales", xlab="Time")
lines(fitted(fit5), col="blue")
lines(fitted(fit6), col="red")
lines(fitted(fit7), col="purple")
lines(fitted(fit8), col="green")
legend("topleft", lty = 1, col = c(1,"blue", "red", "purple", "green"),
      c("data", expression(alpha == 0.2), expression(alpha == 0.5),
        expression(alpha == 0.75), expression(alpha == 0.9)), pch = 1, cex = 0.75)
```

Daily Sales of Hardback



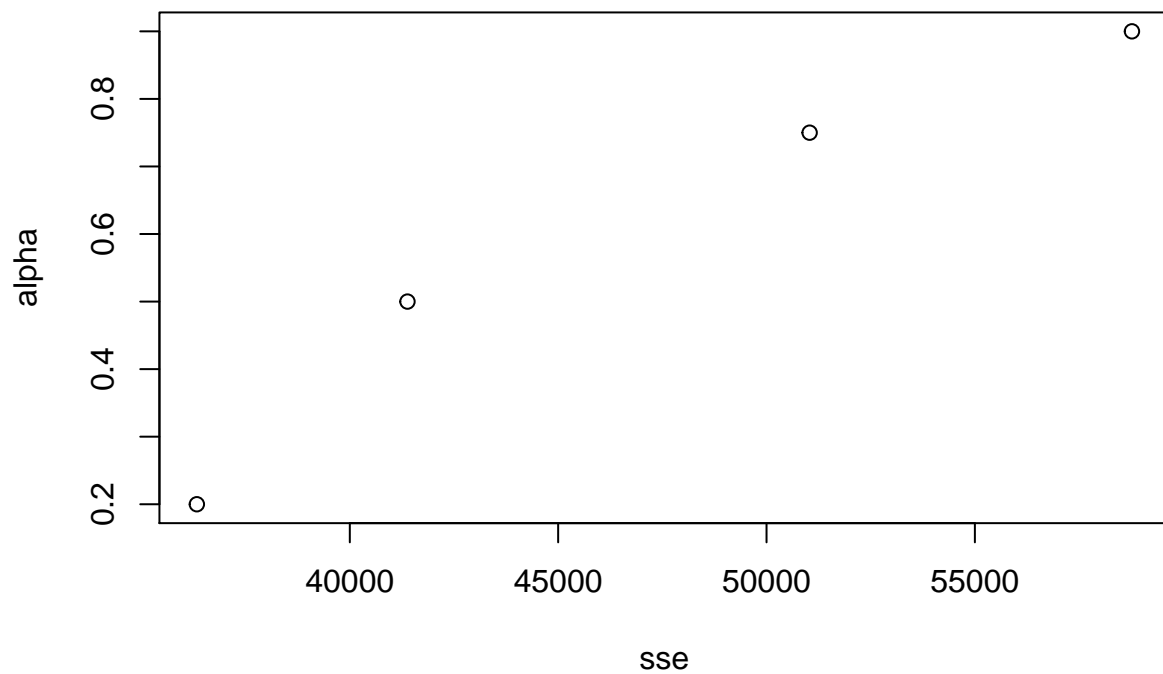
Record the within sample SSE for the one-step forecasts. Plot SSE against the alpha and find which value of alpha works best.

Paperback Sales

```
fit1.sse<-sum(residuals(fit1)^2)
fit2.sse<-sum(residuals(fit2)^2)
fit3.sse<-sum(residuals(fit3)^2)
fit4.sse<-sum(residuals(fit4)^2)

sse<-c(fit1.sse, fit2.sse, fit3.sse, fit4.sse)
alpha<-c(0.2, 0.5, 0.75, 0.9)

plot(sse, alpha)
```

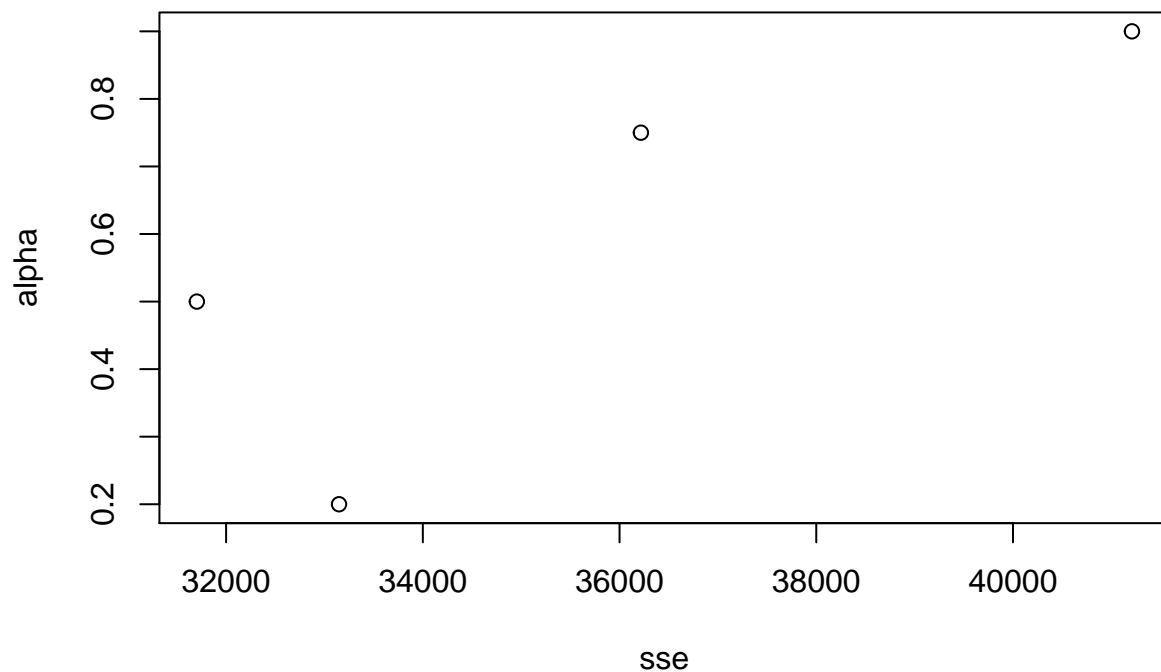


```
####Hardcover
```

```
fit5.sse<-sum(residuals(fit5)^2)
fit6.sse<-sum(residuals(fit6)^2)
fit7.sse<-sum(residuals(fit7)^2)
fit8.sse<-sum(residuals(fit8)^2)
```

```
sse<-c(fit5.sse, fit6.sse, fit7.sse, fit8.sse)
alpha<-c(0.2, 0.5, 0.75, 0.9)
```

```
plot(sse, alpha)
```



###Paperback

accuracy(fit1)

```
##           ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 1.731374 34.79911 28.51298 -2.805469 16.51268 0.719023
##           ACF1
## Training set -0.1128428
```

accuracy(fit2)

```
##           ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 1.620964 37.14104 31.95714 -2.67478 18.23098 0.8058757
##           ACF1
## Training set -0.3274686
```

accuracy(fit3)

```
##           ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 1.555328 41.24503 34.68559 -2.781873 19.65236 0.87468
##           ACF1
## Training set -0.4172491
```

accuracy(fit4)

```
##           ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 1.57004 44.26038 36.8182 -2.801562 20.6701 0.928459
##           ACF1
## Training set -0.4567634
```

Hardcover

```
accuracy(fit5)
```

```
##              ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 15.50292 33.24062 27.71224 6.01686 13.48065 0.8268056
##              ACF1
## Training set -0.08268141
```

```
accuracy(fit6)
```

```
##              ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 7.115701 32.50774 25.92563 1.776891 12.93637 0.7735014
##              ACF1
## Training set -0.2620811
```

```
accuracy(fit7)
```

```
##              ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 5.018773 34.74542 27.36976 0.6923409 13.805 0.8165876
##              ACF1
## Training set -0.4438901
```

```
accuracy(fit8)
```

```
##              ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 4.320299 37.0628 30.11231 0.2818677 15.24951 0.8984127
##              ACF1
## Training set -0.5483903
```

What is the effect of alpha on the forecasts?

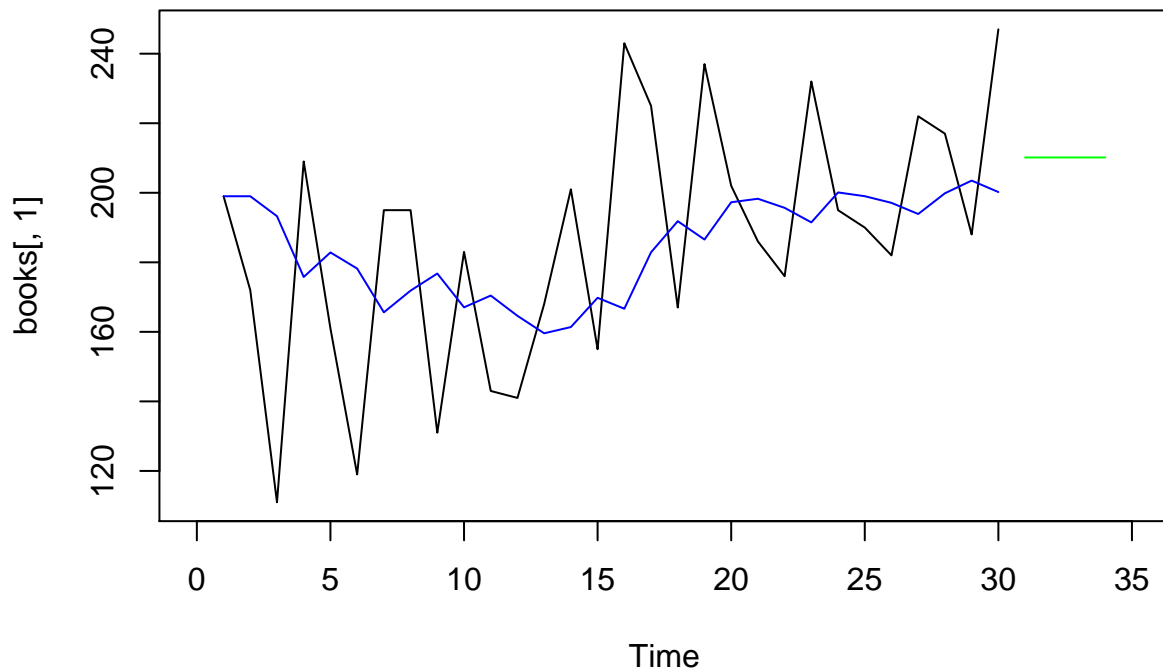
The effect of alpha is different for the paperback sales versus the hardback sales. In reference to paperback sales, as the alpha increases, the SSE increases. This is shown in the graph, but also in the Mean Average Squared Error (MASE) presented in the summary plot.

In contrast, the hardcover sales has a smaller SSE and MASE at alpha equals 0.5. This is visible in the graph and the accuracy stats.

(c) Let SES select the optimal value of alpha. Use this value to generate forecasts for the next four days. Compare the results with (b)

Paperback

```
fit10<-ses(books[,1], initial= "simple", h=4)
plot(books[,1], xlim=c(0,35))
lines(fitted(fit10), col="blue")
lines(fit10$mean, col="green")
```



```
summary(fit10)
```

```
##
## Forecast method: Simple exponential smoothing
##
## Model Information:
## Simple exponential smoothing
##
## Call:
## ses(y = books[, 1], h = 4, initial = "simple")
##
## Smoothing parameters:
##   alpha = 0.2125
##
## Initial states:
##   l = 199
##
## sigma: 34.7918
## Error measures:
##           ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 1.749509 34.79175 28.64424 -2.770157 16.56938 0.7223331
##           ACF1
## Training set -0.1268119
##
## Forecasts:
##   Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
```



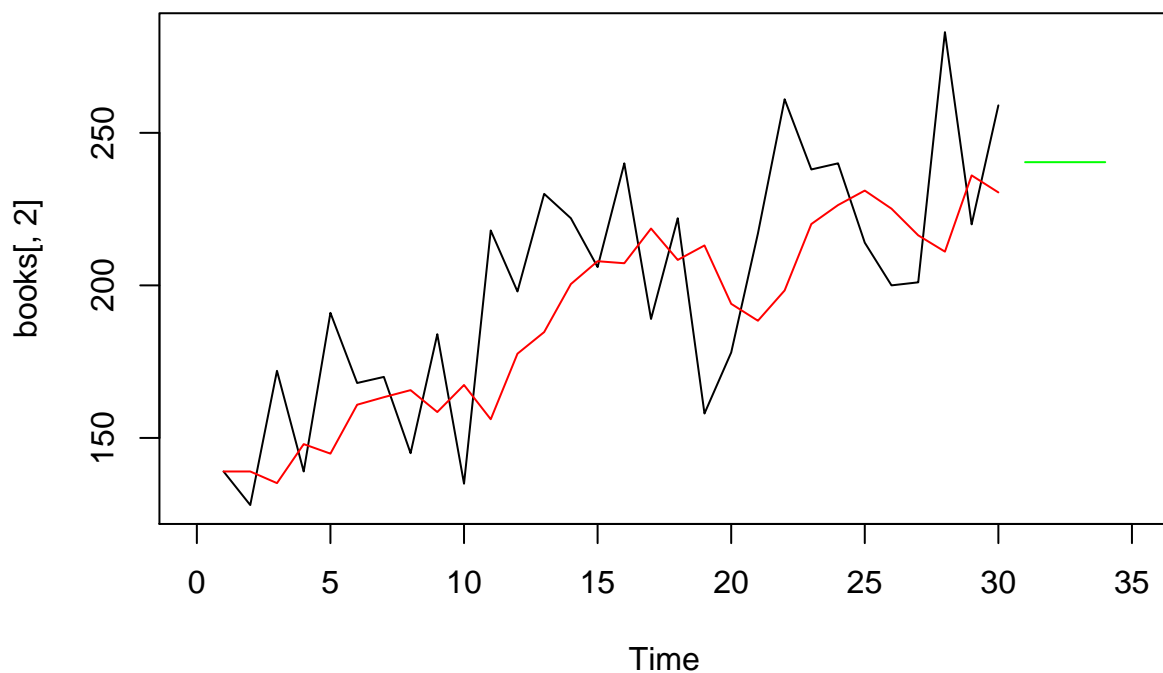
```
## 31      210.1537 165.5663 254.7411 141.9631 278.3443
## 32      210.1537 164.5706 255.7368 140.4404 279.8671
## 33      210.1537 163.5962 256.7112 138.9501 281.3573
## 34      210.1537 162.6418 257.6657 137.4905 282.8170
```

```
fit10.sse<-sum(residuals(fit10)^2)
fit10.sse
```

```
## [1] 36313.98
```

Hardcover

```
fit11<-ses(books[,2],initial= "simple", h=4)
plot(books[,2], xlim=c(0,35))
lines(fitted(fit11), col="red")
lines(fit11$mean, col="green")
```



```
summary(fit11)
```

```
##
## Forecast method: Simple exponential smoothing
##
## Model Information:
## Simple exponential smoothing
##
## Call:
```

```
## ses(y = books[, 2], h = 4, initial = "simple")
##
## Smoothing parameters:
##   alpha = 0.3473
##
## Initial states:
##   l = 139
##
## sigma: 32.0198
## Error measures:
##           ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 9.72952 32.01982 26.34467 3.104211 13.05063 0.7860035
##           ACF1
## Training set -0.1629042
##
## Forecasts:
##   Point Forecast   Lo 80   Hi 80   Lo 95   Hi 95
## 31      240.3808 199.3457 281.4158 177.6231 303.1385
## 32      240.3808 196.9410 283.8206 173.9453 306.8162
## 33      240.3808 194.6625 286.0990 170.4608 310.3008
## 34      240.3808 192.4924 288.2691 167.1418 313.6197
fit11.sse<-sum(residuals(fit11)^2)
fit11.sse

## [1] 30758.07
```

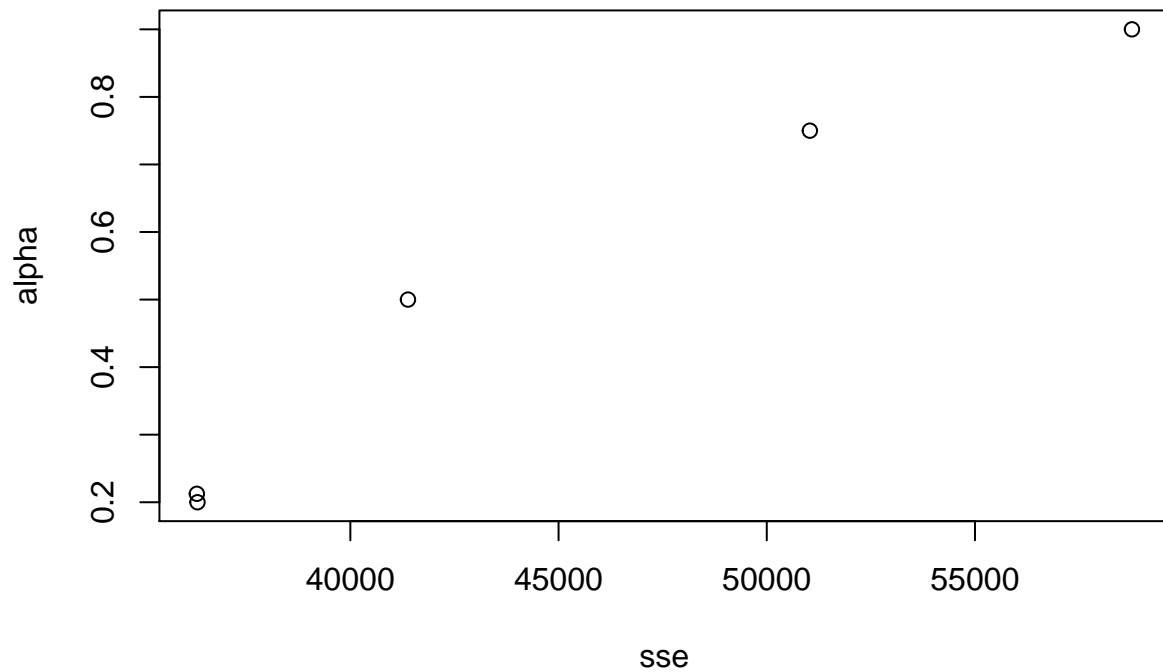
Lets see where these point lie in relation to the others

Paperback

As presented in the summary stats above, the alpha is 0.1685. This alpha value corresponds with the sse value of 33944.82, which is the lowest SSE value, or the most accurate forecast.

```
sse<-c(fit1.sse, fit2.sse, fit3.sse, fit4.sse, fit10.sse)
alpha<-c(0.2, 0.5, 0.75, 0.9, 0.2125)

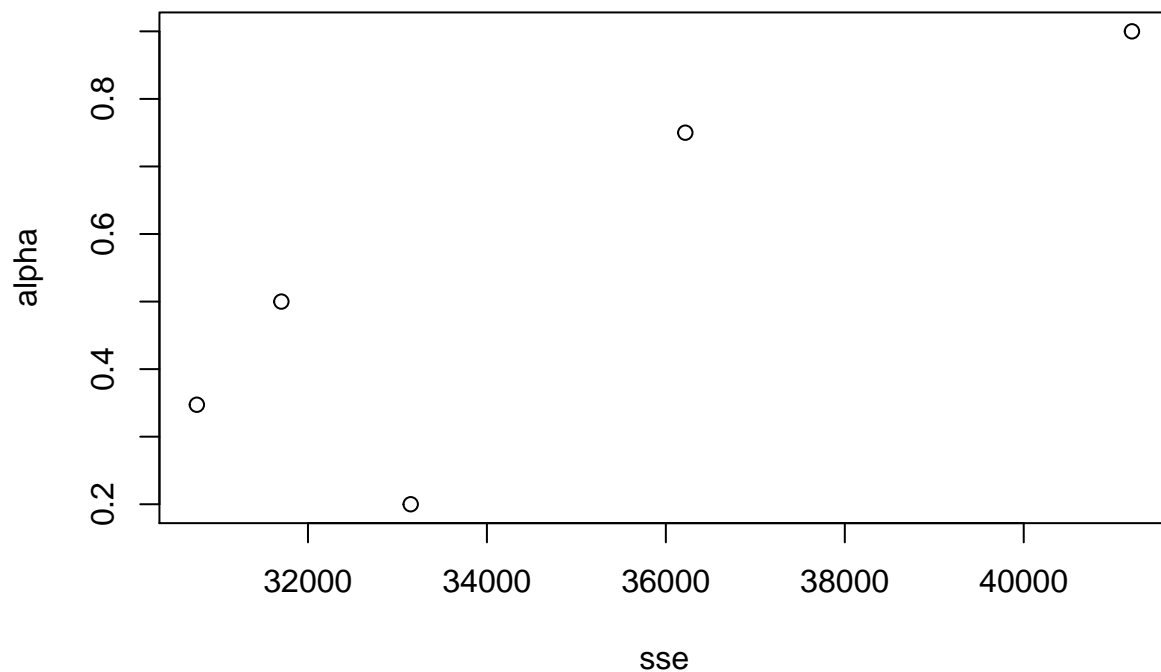
plot(sse, alpha)
```



Hardcover

As for Hardcover, the alpha value is selected as 0.3283 and the SSE is 30587.69, this combination has the lowest SSE, as shown in the graph below. This point is the one on the bottom left of the scatter plot.

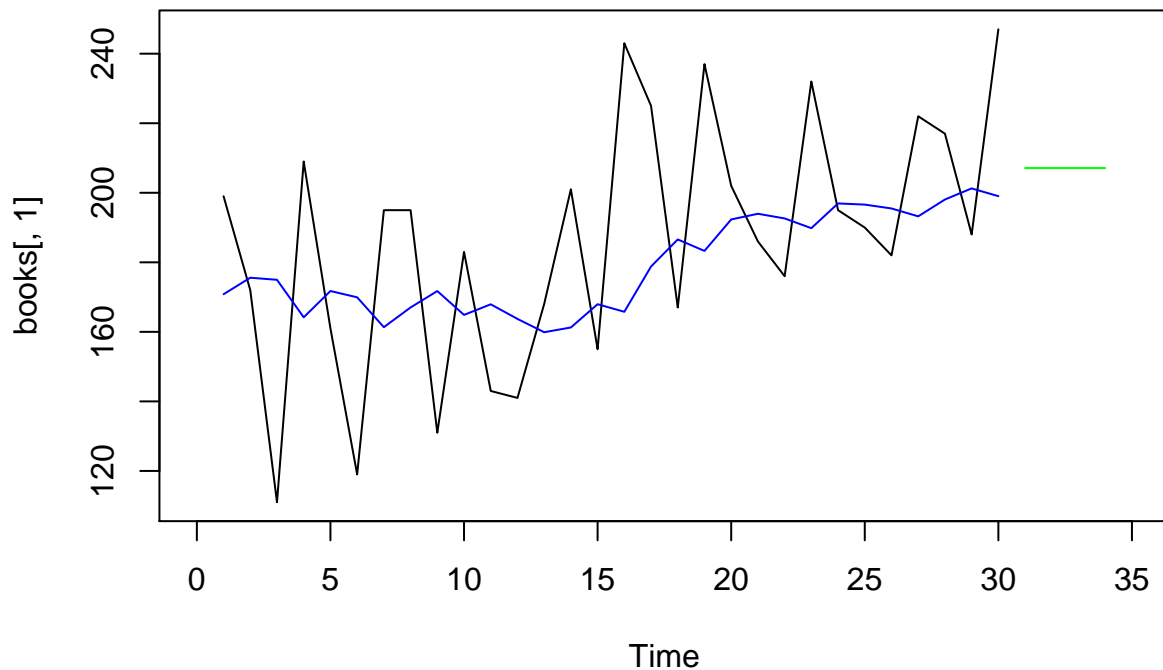
```
sse<-c(fit5.sse, fit6.sse, fit7.sse, fit8.sse, fit11.sse)
alpha<-c(0.2, 0.5, 0.75, 0.9, 0.3473)
plot(sse, alpha)
```



(d) Repeat but with initial =“optimal”. How much difference does an optimal initial level make?

Paperback

```
fit1.opt<-ses(books[,1],initial = "optimal", h=4)
plot(books[,1], xlim=c(0,35))
lines(fitted(fit1.opt), col="blue")
lines(fit1.opt$mean, col="green")
```



```
summary(fit1.opt)
```

```
##
## Forecast method: Simple exponential smoothing
##
## Model Information:
## Simple exponential smoothing
##
## Call:
## ses(y = books[, 1], h = 4, initial = "optimal")
##
## Smoothing parameters:
##   alpha = 0.1685
##
## Initial states:
##   l = 170.8257
##
## sigma: 33.6377
##
##      AIC      AICc      BIC
## 318.9747 319.8978 323.1783
##
## Error measures:
##           ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 7.176212 33.63769 27.8431 0.4737524 15.57782 0.7021303
##           ACF1
```

```
## Training set -0.2117579
##
## Forecasts:
##      Point Forecast      Lo 80      Hi 80      Lo 95      Hi 95
## 31      207.1098 164.0013 250.2182 141.1811 273.0384
## 32      207.1098 163.3934 250.8261 140.2513 273.9682
## 33      207.1098 162.7937 251.4258 139.3342 274.8853
## 34      207.1098 162.2021 252.0174 138.4294 275.7901
```

Paperback SSE

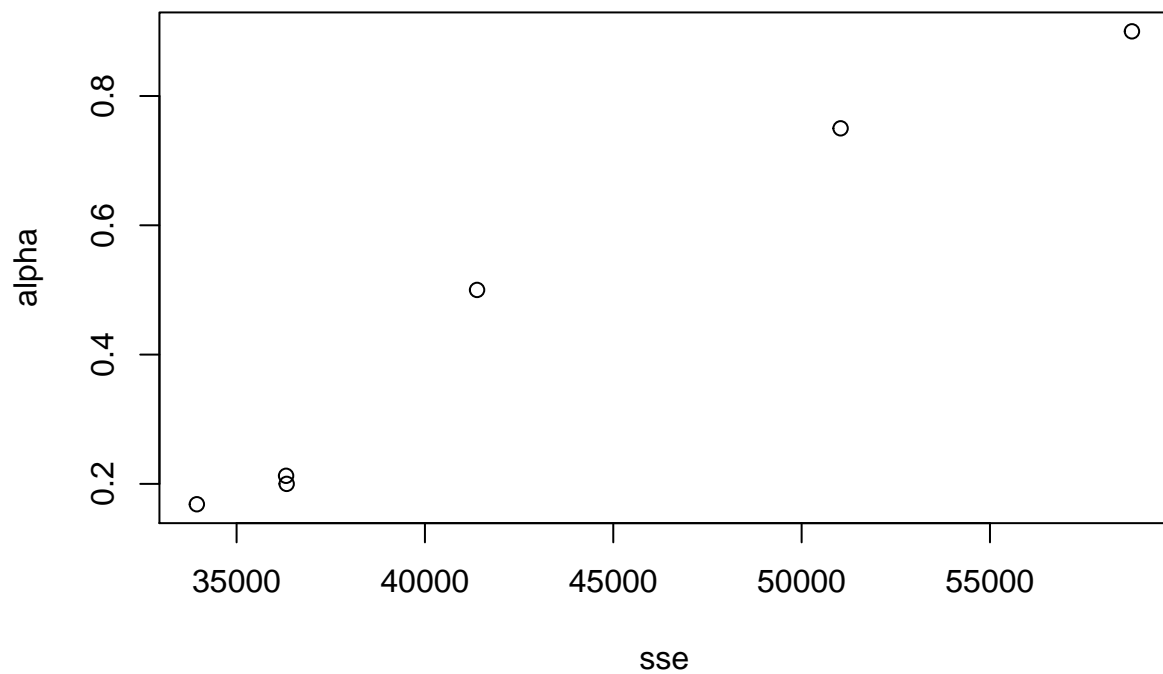
```
fit1.opt.sse<-sum(residuals(fit1.opt)^2)
fit1.opt.sse
```

```
## [1] 33944.82
```

Identifying the optimal start point, decreases the SSEs. The SSE with the optimal start point and optimal alpha is plotted below. This point is located on the bottom left. It is obvious that this is the best model, in regards to SSE.

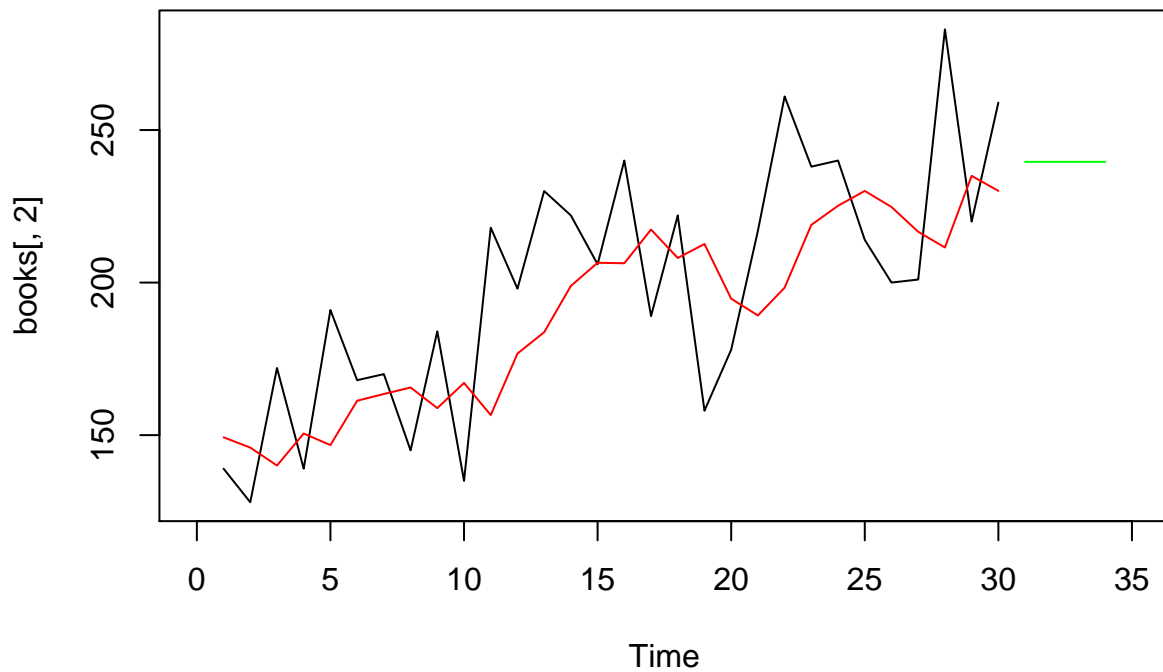
```
sse<-c(fit1.sse, fit2.sse, fit3.sse, fit4.sse, fit10.sse, fit1.opt.sse)
alpha<-c(0.2, 0.5, 0.75, 0.9, 0.2125, 0.1685)

plot(sse, alpha)
```



Hardcover

```
fit2.opt<-ses(books[,2],initial = "optimal", h=4)
plot(books[,2], xlim=c(0,35))
lines(fitted(fit2.opt), col="red")
lines(fit2.opt$mean, col="green")
```



```
summary(fit2.opt)
```

```
##
## Forecast method: Simple exponential smoothing
##
## Model Information:
## Simple exponential smoothing
##
## Call:
## ses(y = books[, 2], h = 4, initial = "optimal")
##
## Smoothing parameters:
##   alpha = 0.3283
##
## Initial states:
##   l = 149.2836
##
## sigma: 31.931
##
##      AIC      AICc      BIC
## 315.8506 316.7737 320.0542
##
## Error measures:
##           ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 9.166918 31.93101 26.7731 2.636328 13.39479 0.7987858
##           ACF1
```



```
## Training set -0.1417817
##
## Forecasts:
##      Point Forecast      Lo 80      Hi 80      Lo 95      Hi 95
## 31      239.5602 198.6390 280.4815 176.9766 302.1439
## 32      239.5602 196.4905 282.6299 173.6908 305.4297
## 33      239.5602 194.4443 284.6762 170.5613 308.5591
## 34      239.5602 192.4869 286.6336 167.5677 311.5527
```

SSE

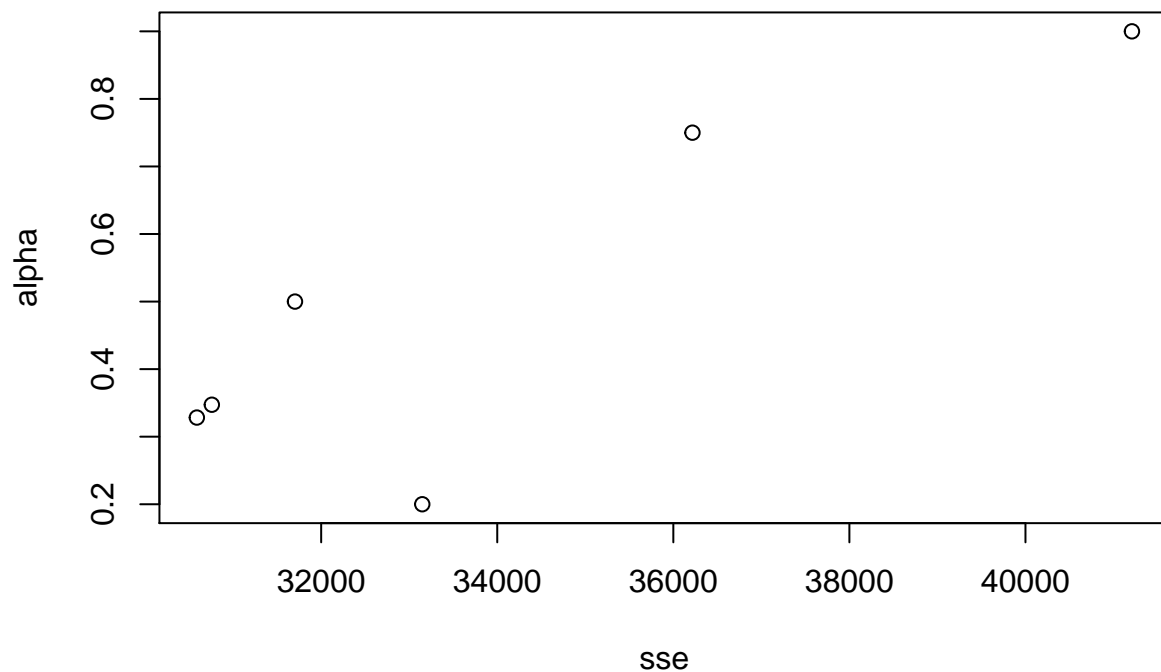
```
fit2.opt.sse<-sum(residuals(fit2.opt)^2)
fit2.opt.sse
```

```
## [1] 30587.69
```

The improvement in hardcover is not as dramatic as with paperback books. It is still an improvement, as shown in the scatterplot below. The optimal start point and alpha is the point on the bottom left, at alpha .32 with a SSE of 30587

```
sse<-c(fit5.sse, fit6.sse, fit7.sse, fit8.sse, fit11.sse, fit2.opt.sse)
alpha<-c(0.2, 0.5, 0.75, 0.9, 0.3473, 0.3283)

plot(sse, alpha)
```



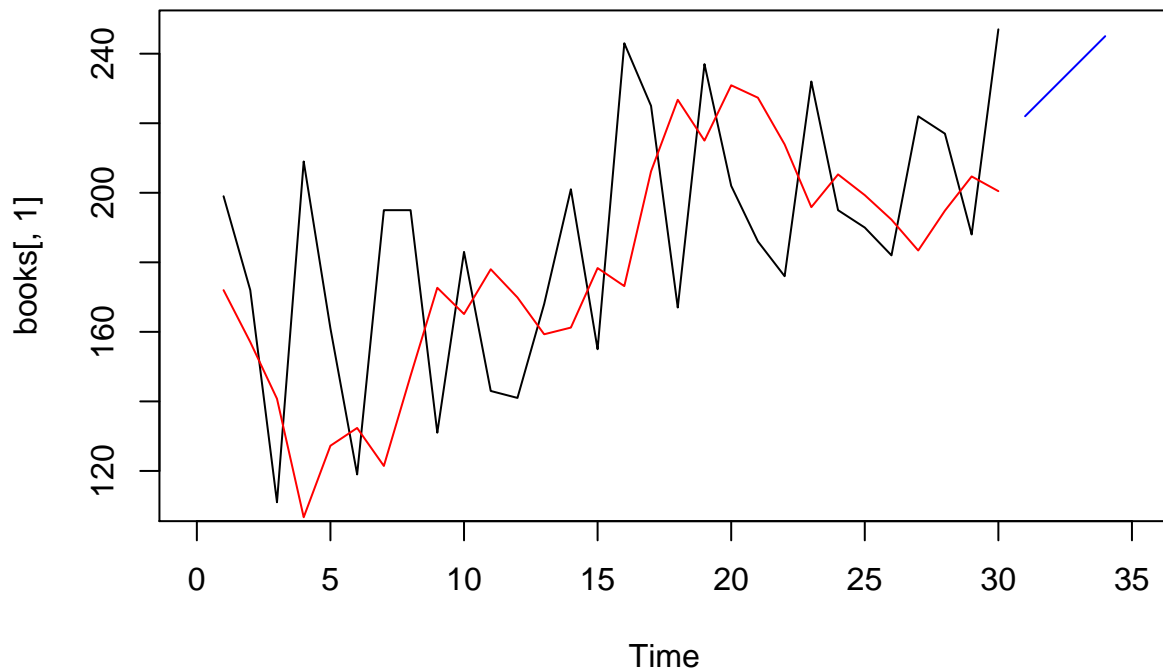
7.2

(a) Apply Holt's linear method to the paperback and hardback series and compute four-day forecasts in each case

Paperback

```
fit.7.2<-holt(books[,1], initial = "simple", h=4)

plot(books[,1], xlim=c(0,35))
lines(fitted(fit.7.2), col="red")
lines(fit.7.2$mean, col="blue")
```



```
summary(fit.7.2)
```

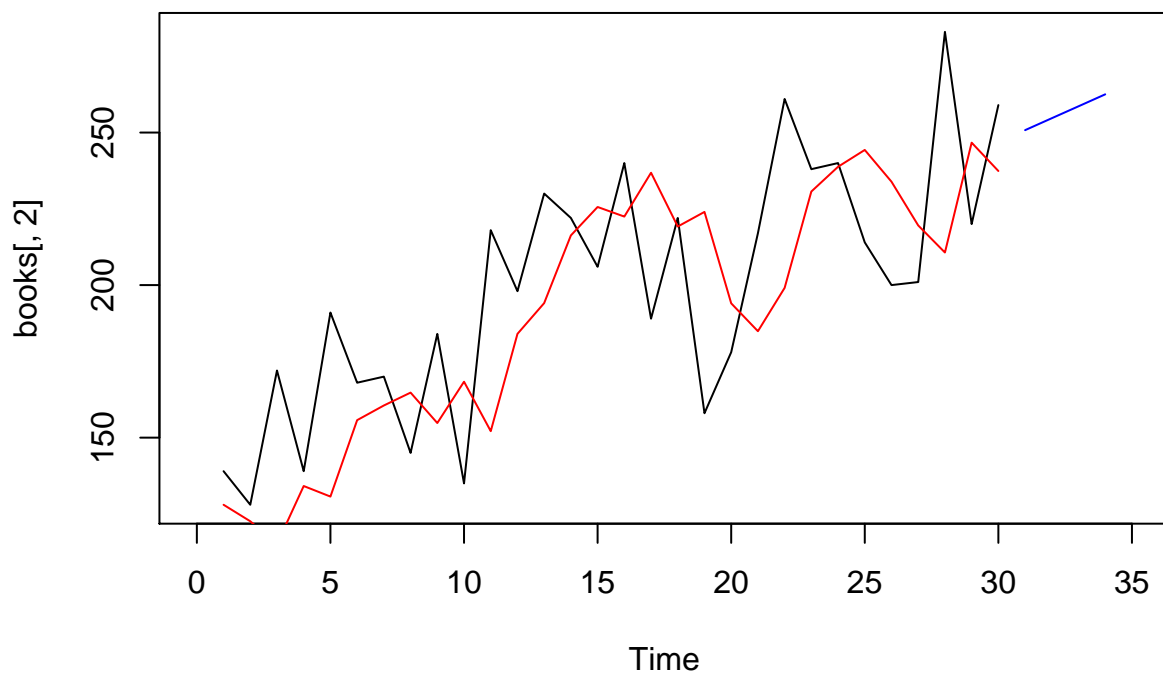
```
##
## Forecast method: Holt's method
##
## Model Information:
## Holt's method
##
## Call:
## holt(y = books[, 1], h = 4, initial = "simple")
##
## Smoothing parameters:
##   alpha = 0.2984
##   beta  = 0.4984
##
## Initial states:
##   l = 199
##   b = -27
##
## sigma: 39.5463
## Error measures:
##           ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 7.769844 39.54634 33.5377 1.633306 18.19621 0.8457332
##           ACF1
## Training set -0.1088681
##
```

```
## Forecasts:
##      Point Forecast      Lo 80      Hi 80      Lo 95      Hi 95
## 31      222.0201 171.3394 272.7007 144.51068 299.5295
## 32      229.6904 164.8872 294.4935 130.58245 328.7983
## 33      237.3606 145.1175 329.6038  96.28696 378.4343
## 34      245.0309 115.5211 374.5407  46.96280 443.0991
```

Hardback

```
fit.7.2.2<-holt(books[,2], initial = "simple", h=4)

plot(books[,2], xlim=c(0,35))
lines(fitted(fit.7.2.2), col="red")
lines(fit.7.2.2$mean, col="blue")
```



```
summary(fit.7.2.2)
```

```
##
## Forecast method: Holt's method
##
## Model Information:
## Holt's method
##
## Call:
## holt(y = books[, 2], h = 4, initial = "simple")
```

```
##
## Smoothing parameters:
##   alpha = 0.439
##   beta  = 0.1574
##
## Initial states:
##   l = 139
##   b = -11
##
## sigma: 35.0438
## Error measures:
##           ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 7.193267 35.04383 27.99174 2.423793 14.18241 0.8351445
##           ACF1
## Training set -0.07743714
##
## Forecasts:
##   Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 31      250.7889 205.8784 295.6993 182.1042 319.4735
## 32      254.7003 202.4087 306.9918 174.7273 334.6733
## 33      258.6117 196.3181 320.9052 163.3419 353.8815
## 34      262.5231 187.9903 337.0558 148.5350 376.5111
```

(b) Compare the SSE measures of Holt's method for the two series to those of simple exponential smoothing in the previous questions. Discuss the merits of the two forecasting methods for these data sets.

Paperback

```
fit.7.2.sse<-sum(residuals(fit.7.2)^2)
fit.7.2.sse
```

```
## [1] 46917.39
```

All Paperback SSEs

```
fit1.sse
```

```
## [1] 36329.34
```

```
fit2.sse
```

```
## [1] 41383.7
```

```
fit3.sse
```

```
## [1] 51034.58
```

```
fit4.sse
```

```
## [1] 58769.45
```

```
fit10.sse
```

```
## [1] 36313.98
```

```
fit1.opt.sse
```

```
## [1] 33944.82
```

Hardback

```
fit.7.2.2.sse<-sum(residuals(fit.7.2.2)^2)  
fit.7.2.2.sse
```

```
## [1] 36842.1
```

All Hardback SSEs

```
fit5.sse
```

```
## [1] 33148.16
```

```
fit6.sse
```

```
## [1] 31702.6
```

```
fit7.sse
```

```
## [1] 36217.34
```

```
fit8.sse
```

```
## [1] 41209.53
```

```
fit11.sse
```

```
## [1] 30758.07
```

```
fit2.opt.sse
```

```
## [1] 30587.69
```

Discuss the merits of the two forecasting methods for these data sets.

Judging by the SSEs, Holt's method does not perform better than the optimized SES model, with SSE values of 33944.82 for paperback and 30587.69 for hardback. The Holt values for these, respectively, are 46917.39 and 36842.1.

(c) Compare the forecasts for the two series using both methods. Which do you think is best?

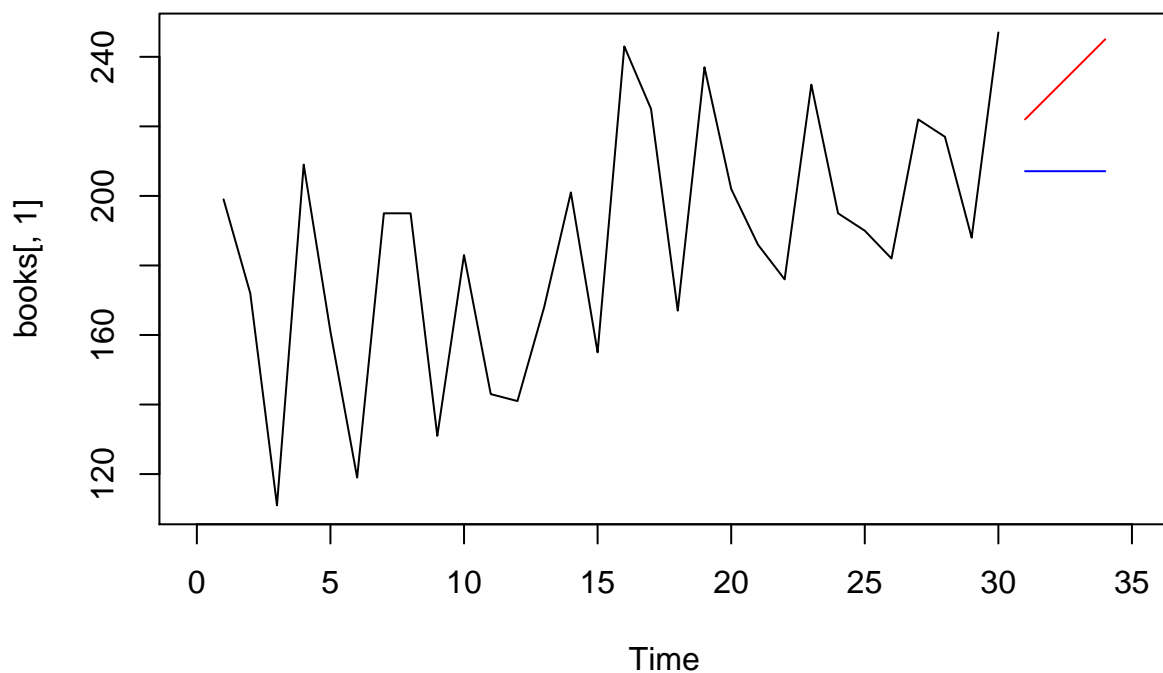
In looking at the graphs below, the holt forecast seems better, in that it captures the trend of the paperback and hardback sales. This is different from the conclusion drawn when viewing the SSE.

Paperback

Blue is Optimal SES forecast

Red is Holts forecast

```
plot(books[,1], xlim=c(0,35))  
lines(fit1.opt$mean, col="blue")  
lines(fit.7.2$mean, col="red")
```

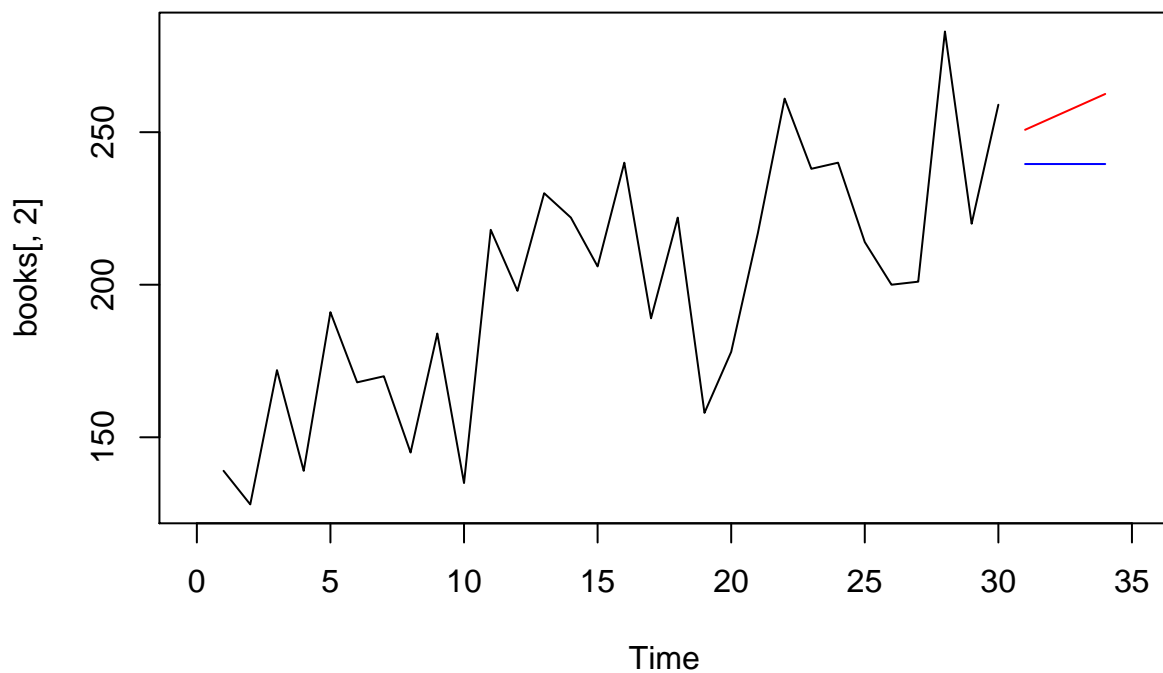


Hardback

Blue is Optimal SES forecast

Red is Holts forecast

```
plot(books[,2], xlim=c(0,35))  
lines(fit2.opt$mean, col="blue")  
lines(fit.7.2.2$mean, col="red")
```

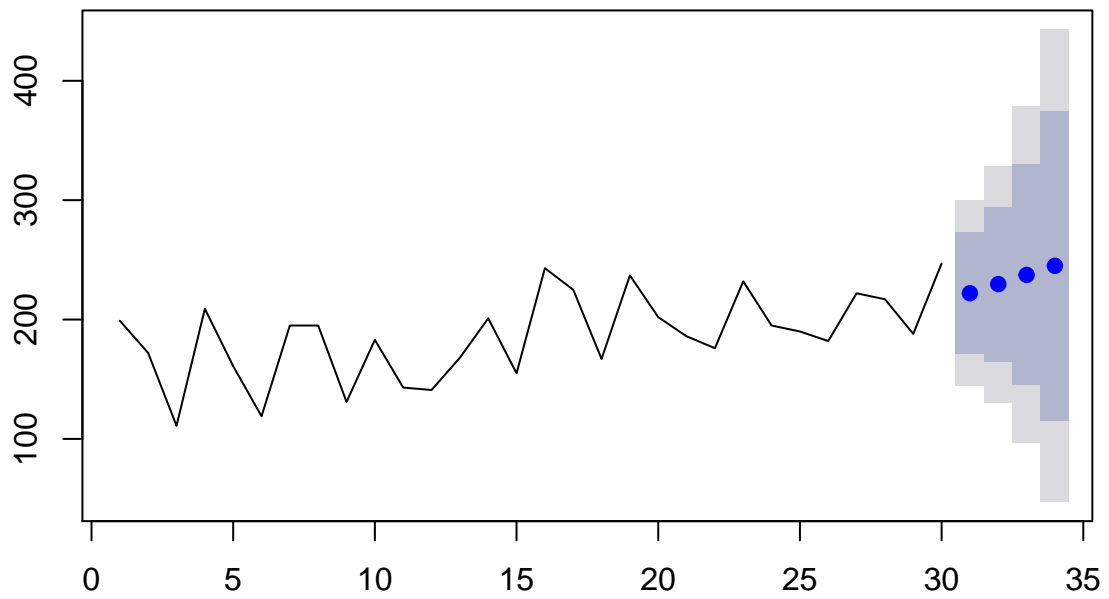


(d) Calculate the 95% prediction interval for the first forecast for each series using both methods, assuming normal errors. Compare your forecasts with those produced by R.

Paperback

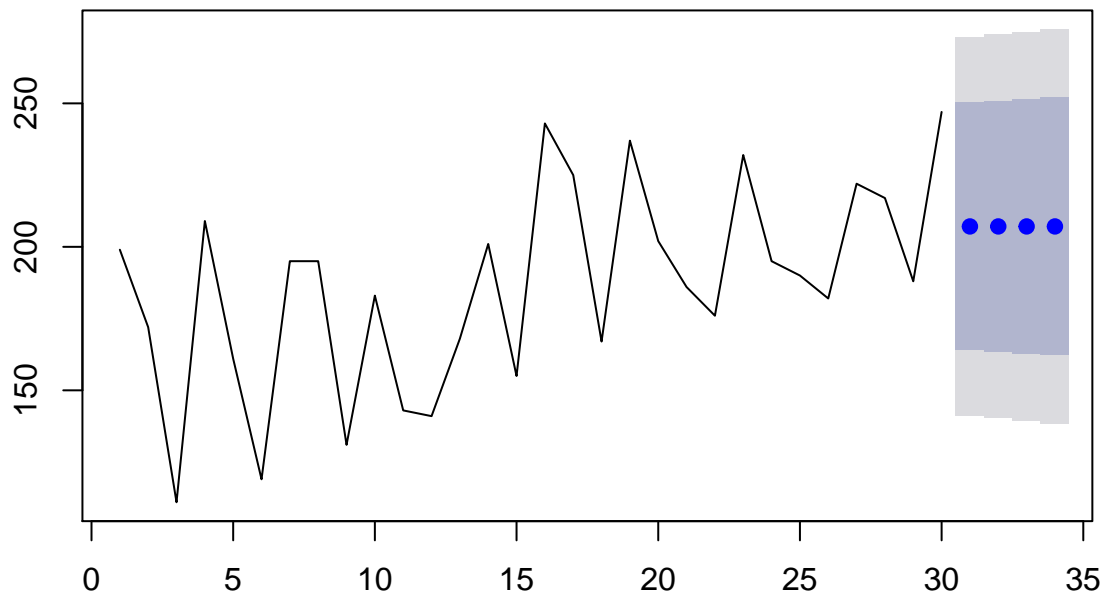
```
plot(fit.7.2)
```


Forecasts from Holt's method



```
plot(fit1.opt)
```

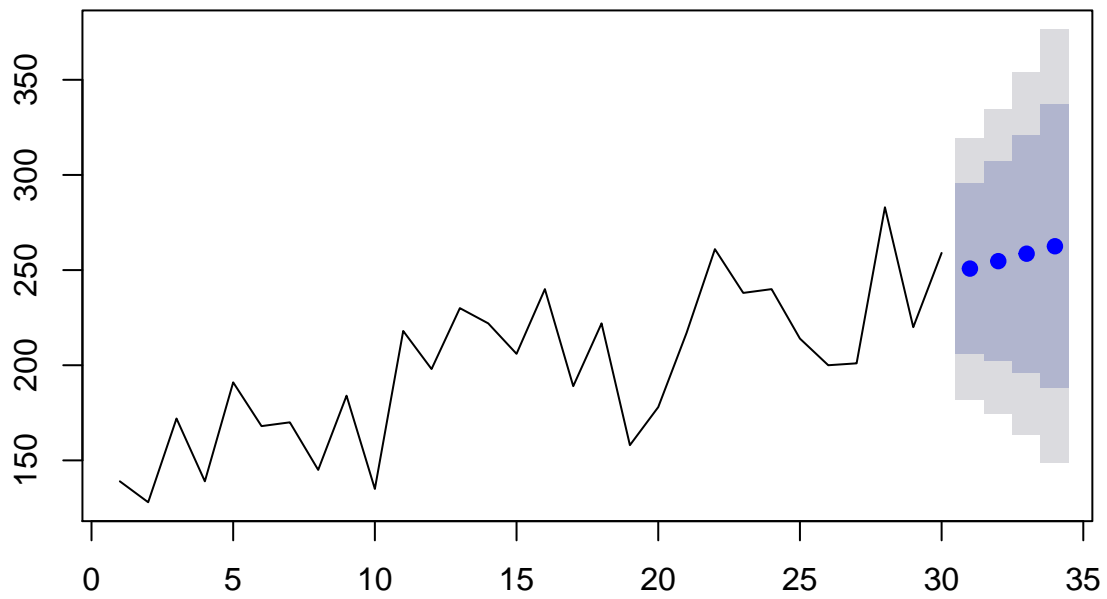
Forecasts from Simple exponential smoothing



Hardcover

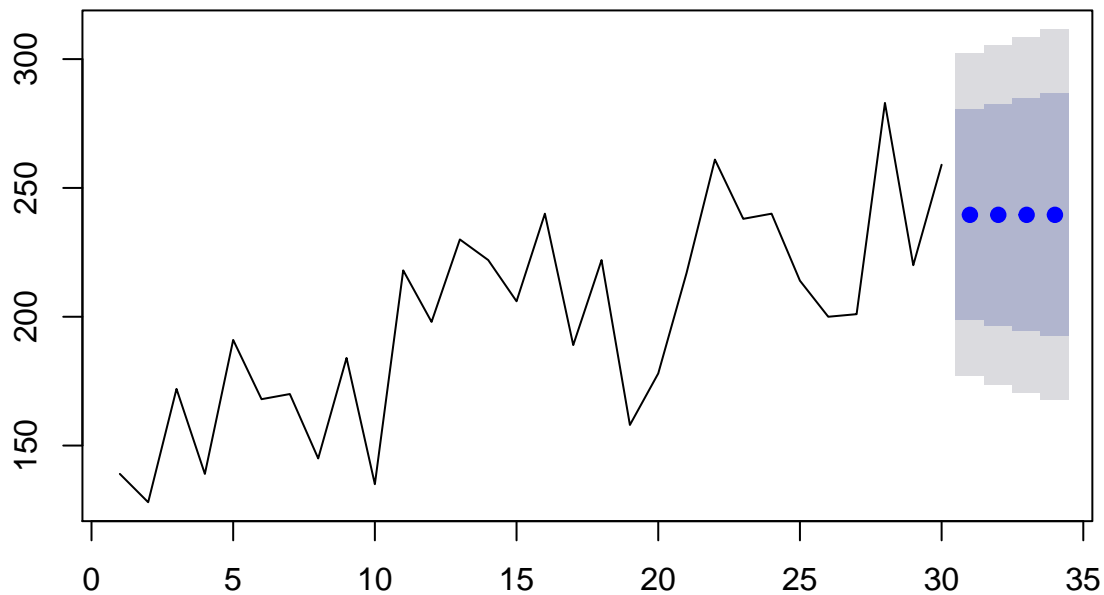
```
plot(fit.7.2.2)
```

Forecasts from Holt's method



```
plot(fit2.opt)
```

Forecasts from Simple exponential smoothing



7.3 Use data set eggs. Experiment with various options in the `holt()` function to see how much the forecasts change with damped or exponential trend. Also, try changing the parameter values for `alpha` and `beta` to see how they affect the forecasts

Hint: Use `h=100` when calling `holt()` so you can clearly see the difference between the various options when plotting the forecasts.

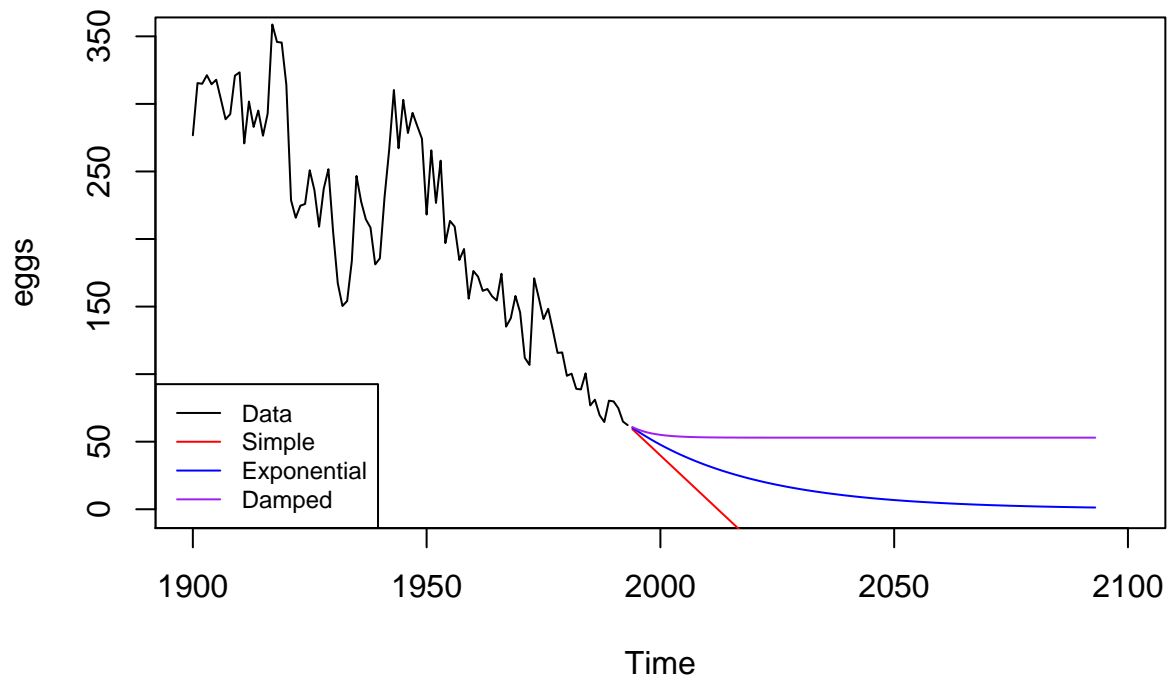
Which model gives the best RMSE?

```
data(eggs)

holt1<-holt(eggs, alpha=0.8, beta=0.2, initial="simple", h=100)
holt2<-holt(eggs, alpha=0.8, beta=0.2, initial="simple", exponential=TRUE, h=100)
holt3<-holt(eggs, alpha=0.8, beta=0.2, damped=TRUE, initial="optimal", h=100)

plot(eggs, xlim = c(1900,2100), ylim=c(0,350), main= "Holt Trend with Alpha=0.8 and Beta=0.2")
lines(holt1$mean, col="red")
lines(holt2$mean, col="blue")
lines(holt3$mean, col="purple")
legend("bottomleft", lty=1, col=c("black", "red", "blue", "purple"),
      c("Data", "Simple", "Exponential", "Damped"), cex = 0.75)
```

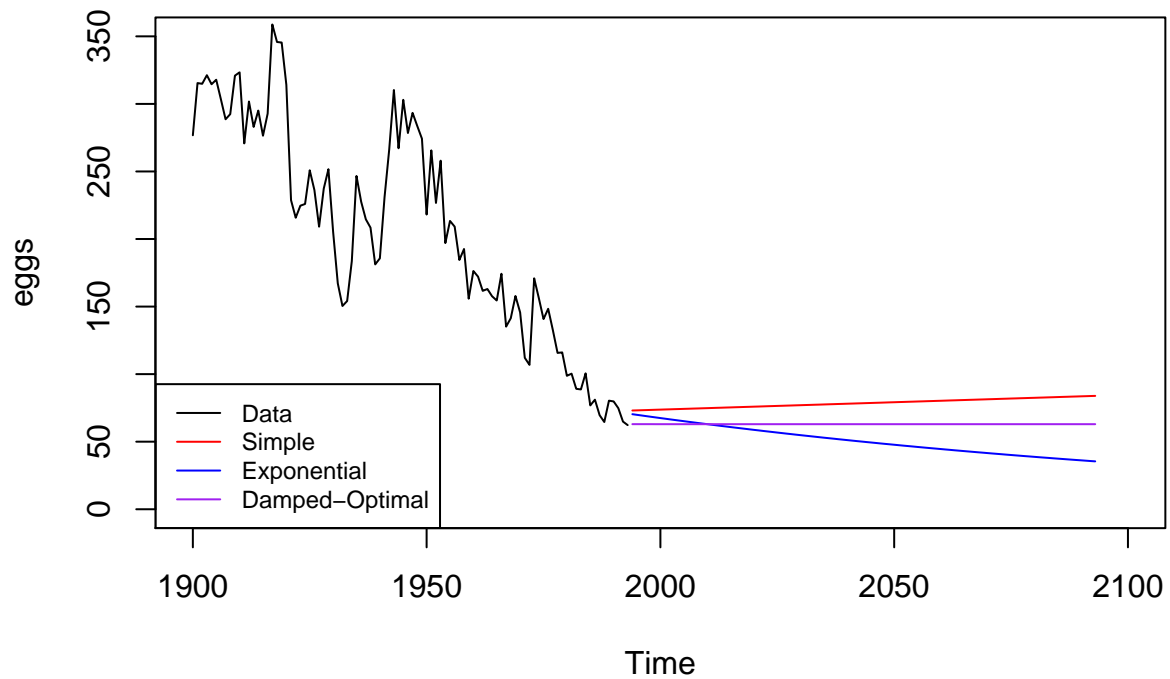
Holt Trend with Alpha=0.8 and Beta=0.2



```
holt4<-holt(eggs, alpha=0.2, beta=0.8, initial="simple", h=100)
holt5<-holt(eggs, alpha=0.2, beta=0.8, initial="simple", exponential=TRUE, h=100)
holt6<-holt(eggs, damped=TRUE, initial="optimal", h=100)

plot(eggs, xlim = c(1900,2100), ylim=c(0,350),main= "Holt Trend with Alpha=0.2 and Beta=0.8")
lines(holt4$mean, col="red")
lines(holt5$mean, col="blue")
lines(holt6$mean, col="purple")
legend("bottomleft", lty=1, col=c("black", "red", "blue", "purple"),
      c("Data", "Simple", "Exponential", "Damped-Optimal"), cex = 0.75)
```

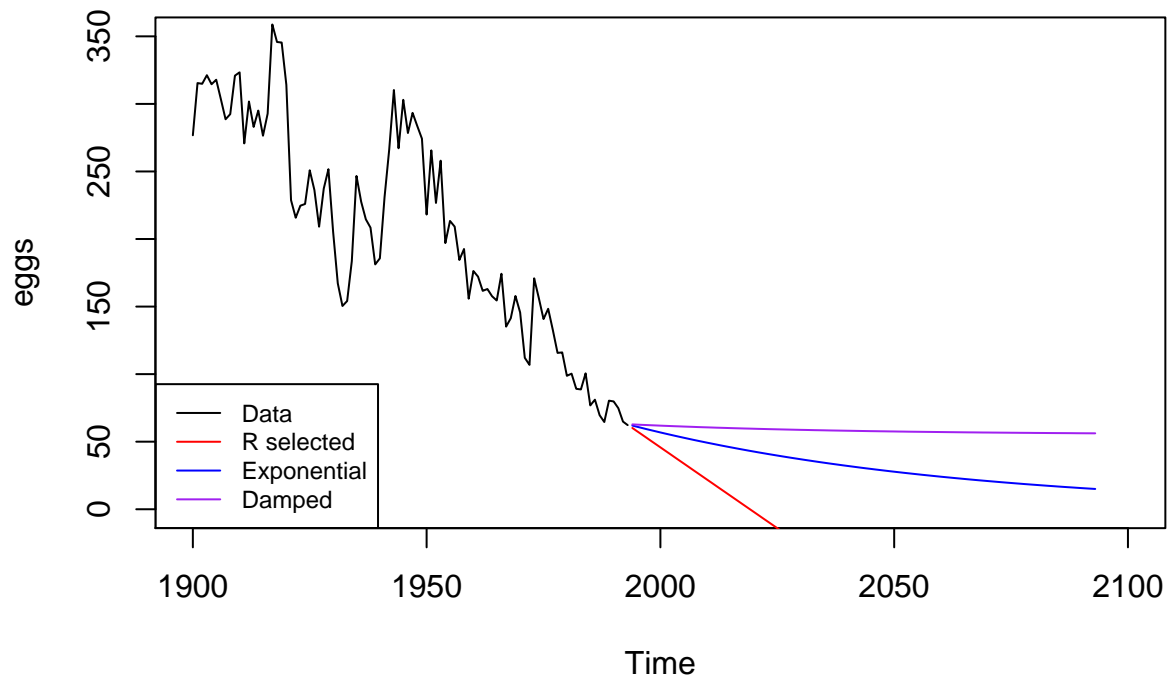
Holt Trend with Alpha=0.2 and Beta=0.8



```
holt7<-holt(eggs, h=100)
holt8<-holt(eggs, exponential=TRUE, h=100)
holt10<-holt(eggs, exponential=TRUE, damped=TRUE, h=100)

plot(eggs, xlim = c(1900,2100), ylim=c(0,350), main= "Holt Trend with Optimal Alpha and Beta")
lines(holt7$mean, col="red")
lines(holt8$mean, col="blue")
lines(holt10$mean, col="purple")
legend("bottomleft", lty=1, col=c("black", "red", "blue", "purple"),
      c("Data", "R selected", "Exponential", "Damped"), cex = 0.75)
```

Holt Trend with Optimal Alpha and Beta



Holt8 has the lowest RMSE. It is the model with exponential smoothing and R choosen the optimal alpha and beta values and no dampening. In the final graph, we can see that Holt8 is the blue line. It maintains the downward trend, but levels to a constant value.

```
accuracy(holt1)
```

```
##              ME      RMSE      MAE      MPE      MAPE      MASE
## Training set -2.784254 29.23521 22.3058 -1.55109 11.02945 1.100312
##              ACF1
## Training set 0.06902877
```

```
accuracy(holt2)
```

```
##              ME      RMSE      MAE      MPE      MAPE      MASE
## Training set -5.415946 29.6436 22.68924 -2.986354 11.22238 1.119226
##              ACF1
## Training set 0.06999484
```

```
accuracy(holt3)
```

```
##              ME      RMSE      MAE      MPE      MAPE      MASE
## Training set -1.478575 27.71097 20.83628 -1.81307 10.60224 1.027823
##              ACF1
## Training set -0.02909399
```

```
accuracy(holt4)
```

```
##              ME      RMSE      MAE      MPE      MAPE      MASE
## Training set -2.561214 39.44921 30.14474 -1.604226 15.04488 1.486995
##              ACF1
```

```
## Training set 0.6227753
```

```
accuracy(holt5)
```

```
##           ME      RMSE      MAE      MPE      MAPE      MASE
## Training set -6.985339 40.36501 30.16092 -3.784936 14.5846 1.487793
##           ACF1
## Training set 0.6334021
```

```
accuracy(holt6)
```

```
##           ME      RMSE      MAE      MPE      MAPE      MASE
## Training set -3.092257 26.66198 19.51158 -3.023266 10.11049 0.9624771
##           ACF1
## Training set -0.005832122
```

```
accuracy(holt7)
```

```
##           ME      RMSE      MAE      MPE      MAPE      MASE
## Training set -0.2517656 26.50294 19.16922 -1.334809 9.684965 0.9455886
##           ACF1
## Training set 0.007078776
```

```
accuracy(holt8)
```

```
##           ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 0.4755243 26.38645 19.22237 -1.27959 9.754134 0.9482108
##           ACF1
## Training set 0.00705702
```

```
accuracy(holt10)
```

```
##           ME      RMSE      MAE      MPE      MAPE      MASE
## Training set -0.8824596 26.52585 19.51446 -2.100753 10.01524 0.9626188
##           ACF1
## Training set 0.005081461
```

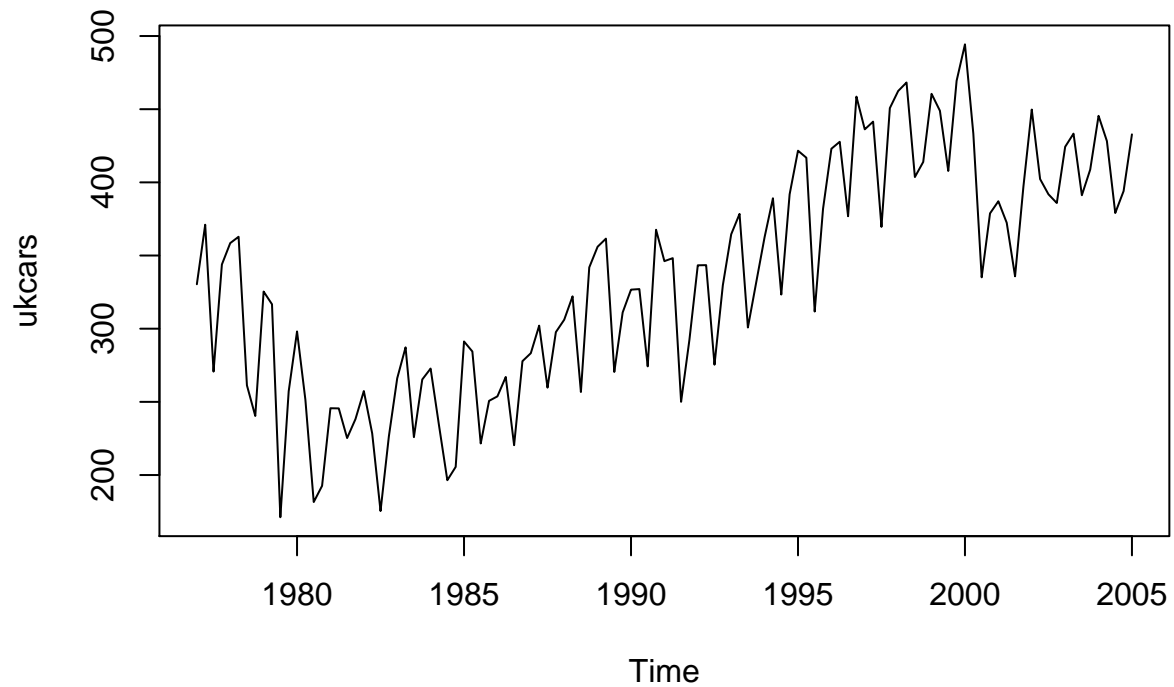
7.4 Use data set ukcars

(a) Plot the data and describe the main features of the series

There is a strong seasonal effect in this dataset, and an inconsistent trend. In the beginning of the dataset, we see a downward trend, but it becomes an upward trend which levels out towards the end of the dataset. The most prevalent feature is the seasonal variation present in the data.

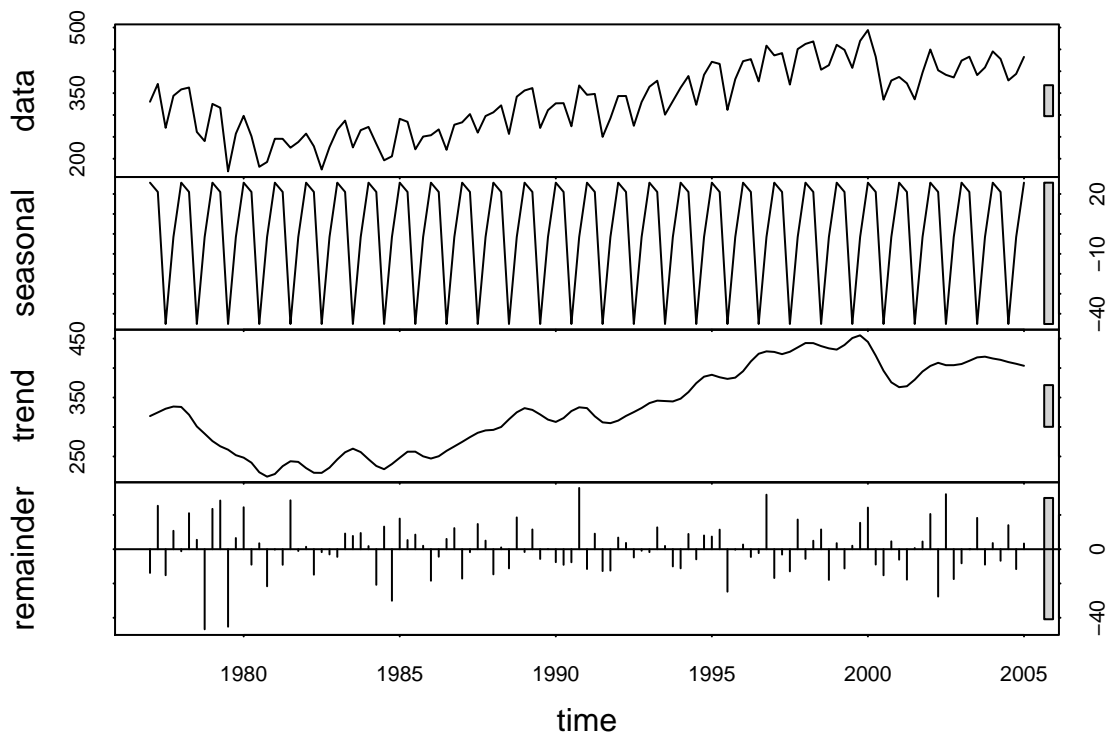
```
data(ukcars)
plot(ukcars, main="UK Passenger Vehicle Production Data")
```


UK Passenger Vehicle Production Data



(b) Decompose the series using STL and obtain the seasonally adjusted data

```
seas<-stl(ukcars, s.window = "periodic")  
plot(seas)
```



```
seas.adj<-seasadj(seas)
```

(c) Forecast the next 2 years using an additive dampened trend method applied to seasonally adjusted data. Then reseasonalize the forecasts. Record the parameters of the methods and report the RMSE of the one-step forecasts from your method.

```
fcast<-holt(seas.adj, damped = TRUE, seasonal="additive", h=24, robust=TRUE)
#View(seas.adj)
accuracy(fcast)
```

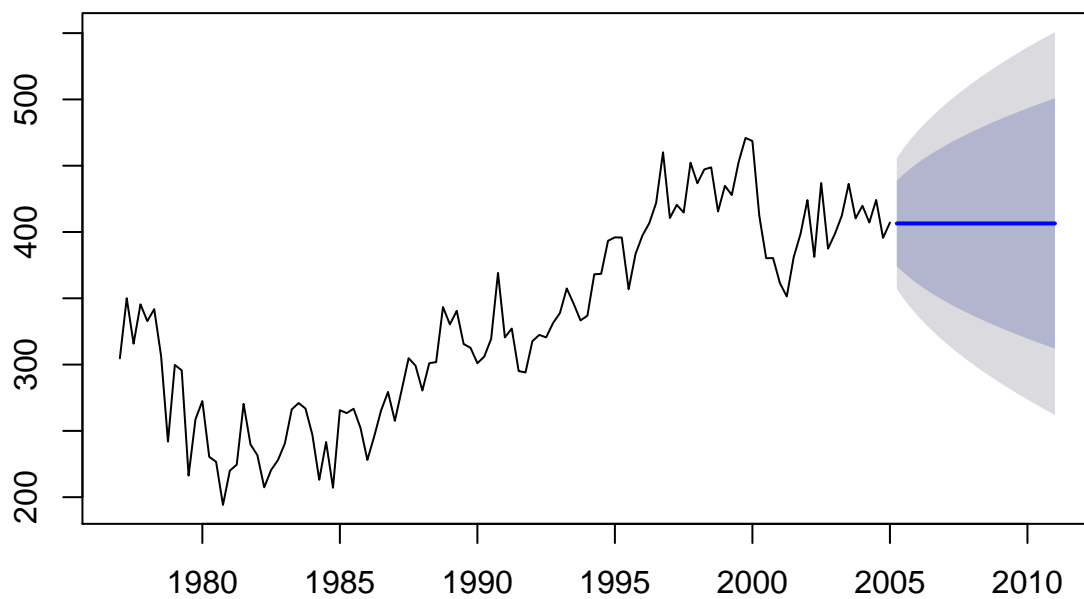
```
##               ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 2.542419 25.13965 20.50991 0.319648 6.568478 0.6684081
##               ACF1
## Training set 0.03469485
```

```
fcast$model
```

```
## Damped Holt's method
##
## Call:
## holt(y = seas.adj, h = 24, damped = TRUE, seasonal = "additive",
##
## Call:
##      robust = TRUE)
```

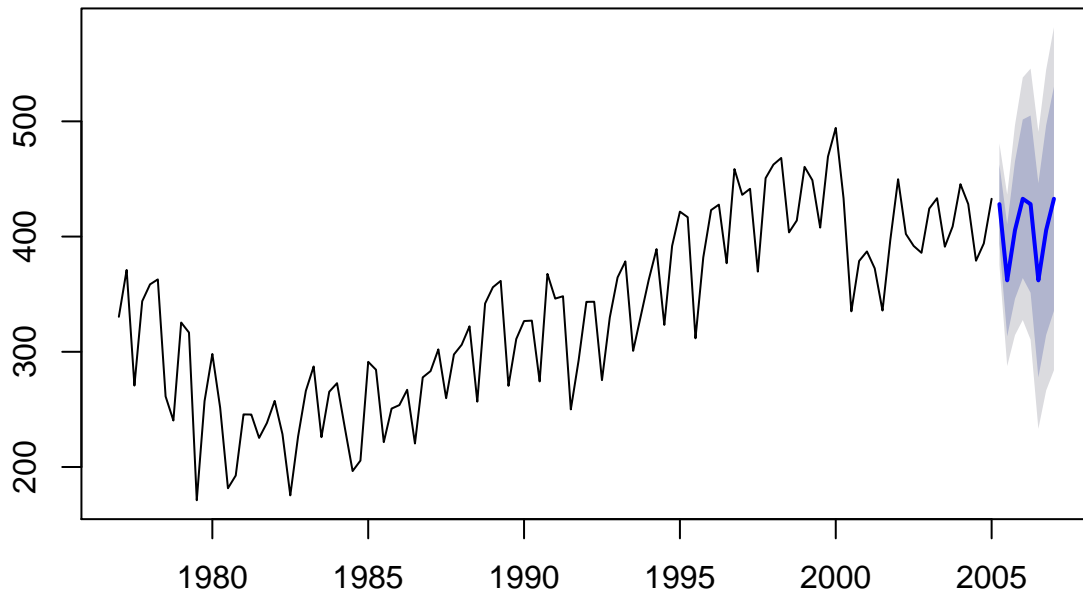
```
##
## Smoothing parameters:
##   alpha = 0.5737
##   beta  = 1e-04
##   phi   = 0.9106
##
## Initial states:
##   l = 342.6908
##   b = -9.9556
##
## sigma: 25.1396
##
##      AIC      AICc      BIC
## 1274.920 1275.712 1291.284
plot(forecast(fcast, h=24))
```

Forecasts from Damped Holt's method



```
fcast2 <- forecast(seas, method="naive")
plot(fcast2)
```

Forecasts from STL + Random walk



(d) Forecast the next 2 years using Holt's linear method applied to seasonally adjusted data. Then reseasonalize the forecasts. Record the parameters of the methods and report the RMSE of the one-step forecasts from your method.

```
fcast3<-holt(seas.adj, initial = "simple", h=24, robust=TRUE)
#View(seas.adj)
accuracy(fcast3)
```

```
##              ME      RMSE      MAE      MPE      MAPE      MASE
## Training set -2.669845 28.70238 22.50912 -1.16915 7.324064 0.7335612
##              ACF1
## Training set -0.008396094
```

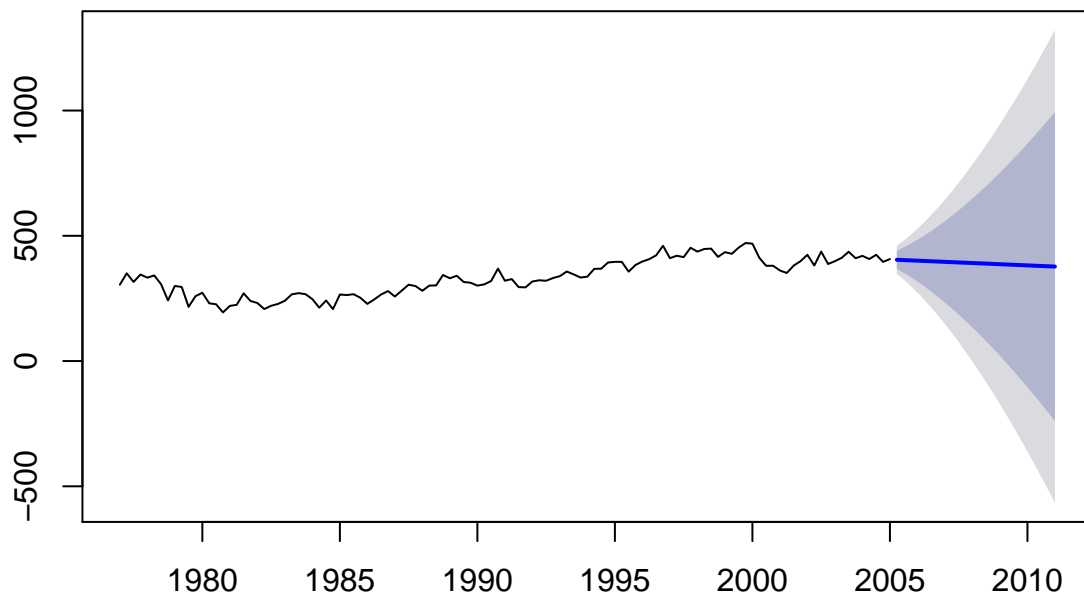
```
fcast3$model
```

```
## Holt's method
##
## Call:
## holt(y = seas.adj, h = 24, initial = "simple", robust = TRUE)
##
## Smoothing parameters:
##   alpha = 0.7505
##   beta  = 0.2054
##
## Initial states:
```

```
##      l = 304.7042
##      b = 45.3325
##
##      sigma: 28.7024
```

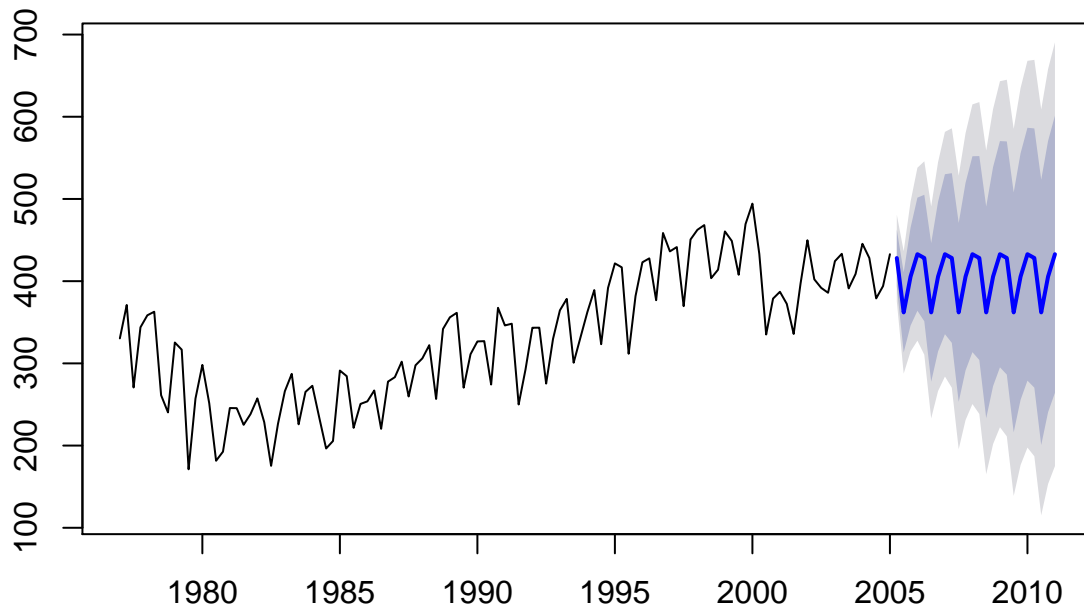
```
linpred<-forecast(fcast3, h=24)
plot(forecast(fcast3, h=24))
```

Forecasts from Holt's method



```
fcast4 <- forecast(seas, method= "naive", h=24)
plot(fcast4)
```

Forecasts from STL + Random walk



```
summary(fcast4)
```

```
##
## Forecast method: STL + Random walk
##
## Model Information:
## Call: rwf(y = x, h = h, drift = FALSE, level = level)
##
## Residual sd: 26.945
##
## Error measures:
##           ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 0.9145089 26.84003 21.2837 -0.1849312 6.965207 0.6936254
##           ACF1
## Training set -0.3048612
##
## Forecasts:
##      Point Forecast      Lo 80      Hi 80      Lo 95      Hi 95
## 2005 Q2      428.1435 393.7467 462.5404 375.5381 480.7490
## 2005 Q3      362.0045 313.3599 410.6490 287.6091 436.3999
## 2005 Q4      405.5729 345.9958 465.1500 314.4575 496.6883
## 2006 Q1      432.7960 364.0022 501.5898 327.5850 538.0070
## 2006 Q2      428.1435 351.2298 505.0573 310.5141 545.7730
## 2006 Q3      362.0045 277.7497 446.2593 233.1479 490.8611
## 2006 Q4      405.5729 314.5673 496.5785 266.3919 544.7539
## 2007 Q1      432.7960 335.5069 530.0851 284.0052 581.5868
```

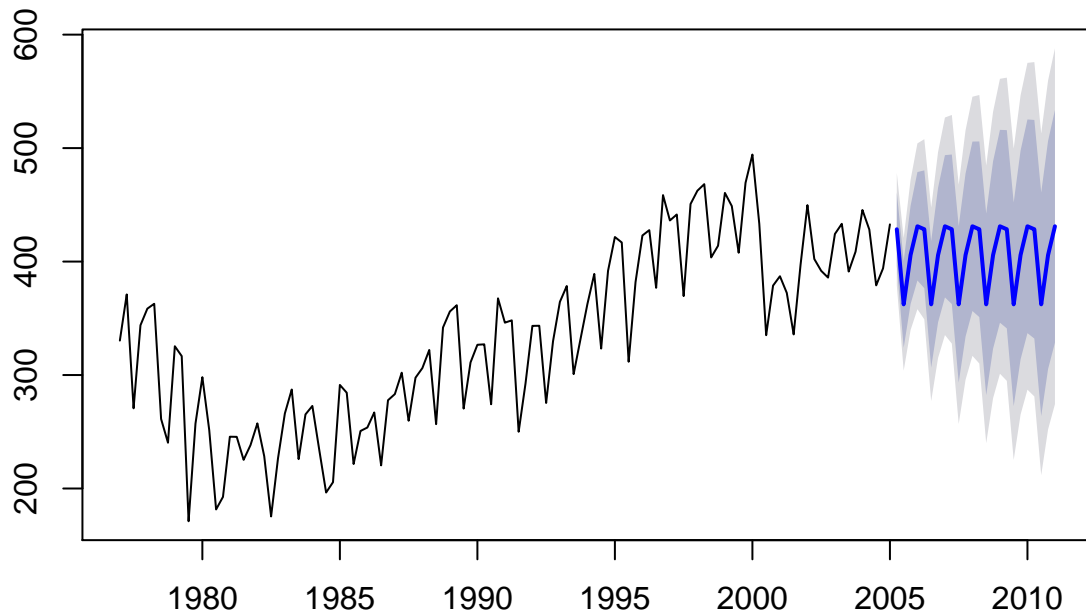
```
## 2007 Q2      428.1435 324.9529 531.3342 270.3271 585.9600
## 2007 Q3      362.0045 253.2320 470.7769 195.6513 528.3576
## 2007 Q4      405.5729 291.4914 519.6544 231.1002 580.0456
## 2008 Q1      432.7960 313.6417 551.9503 250.5652 615.0268
## 2008 Q2      428.1435 304.1238 552.1633 238.4718 617.8153
## 2008 Q3      362.0045 233.3031 490.7058 165.1728 558.8362
## 2008 Q4      405.5729 272.3544 538.7914 201.8327 609.3131
## 2009 Q1      432.7960 295.2085 570.3835 222.3741 643.2179
## 2009 Q2      428.1435 286.3216 569.9655 211.2456 645.0415
## 2009 Q3      362.0045 216.0709 507.9381 138.8183 585.1906
## 2009 Q4      405.5729 255.6404 555.5054 176.2709 634.8749
## 2010 Q1      432.7960 278.9685 586.6235 197.5371 668.0549
## 2010 Q2      428.1435 270.5172 585.7698 187.0749 669.2122
## 2010 Q3      362.0045 200.6688 523.3401 115.2629 608.7461
## 2010 Q4      405.5729 240.6113 570.5345 153.2859 657.8600
## 2011 Q1      432.7960 264.2864 601.3056 175.0828 690.5092
```

(e) Now use the `ets()` to choose a seasonal model for the data.

```
fets<- ets(ukcars)
summary(fets)

## ETS(A,N,A)
##
## Call:
## ets(y = ukcars)
##
## Smoothing parameters:
##   alpha = 0.6267
##   gamma = 1e-04
##
## Initial states:
##   l = 313.0916
##   s=-1.1271 -44.606 21.5553 24.1778
##
## sigma: 25.2579
##
##      AIC      AICc      BIC
## 1277.980 1279.047 1297.072
##
## Training set error measures:
##           ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 1.324962 25.25792 20.12508 -0.1634983 6.609629 0.6558666
##           ACF1
## Training set 0.01909295
plot(forecast(fets, h=24))
```

Forecasts from ETS(A,N,A)



(f) Compare the RMSE of the fitted model with the RMSE of the model you obtained using STL Decomposition with Holt's Method. Which gives the better in-sample fits?

The RMSE for the Holt method on the seasonally adjusted data is better than the naive forecast on the seasonal data. The ETS model selected by the `ets()` fit has a competitive RMSE to the Holt method on the seasonally adjusted data.

```
accuracy(fets)
```

```
##               ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 1.324962 25.25792 20.12508 -0.1634983 6.609629 0.6558666
##               ACF1
## Training set 0.01909295
```

```
accuracy(fcast)
```

```
##               ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 2.542419 25.13965 20.50991 0.319648 6.568478 0.6684081
##               ACF1
## Training set 0.03469485
```

```
accuracy(fcast4)
```

```
##               ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 0.9145089 26.84003 21.2837 -0.1849312 6.965207 0.6936254
```



```
##                               ACF1
## Training set -0.3048612
```

(g) Compare the forecasts from the two approaches? Which seems most reasonable?

In viewing the forecasted results, the ETS model has a smaller prediction interval and is slightly more accurate than the reseasonalized prediction (fcast4) in all accuracy measures, except ME.

Section 8.11

8.5 Use R to simulate and plot some data from simple ARIMA models

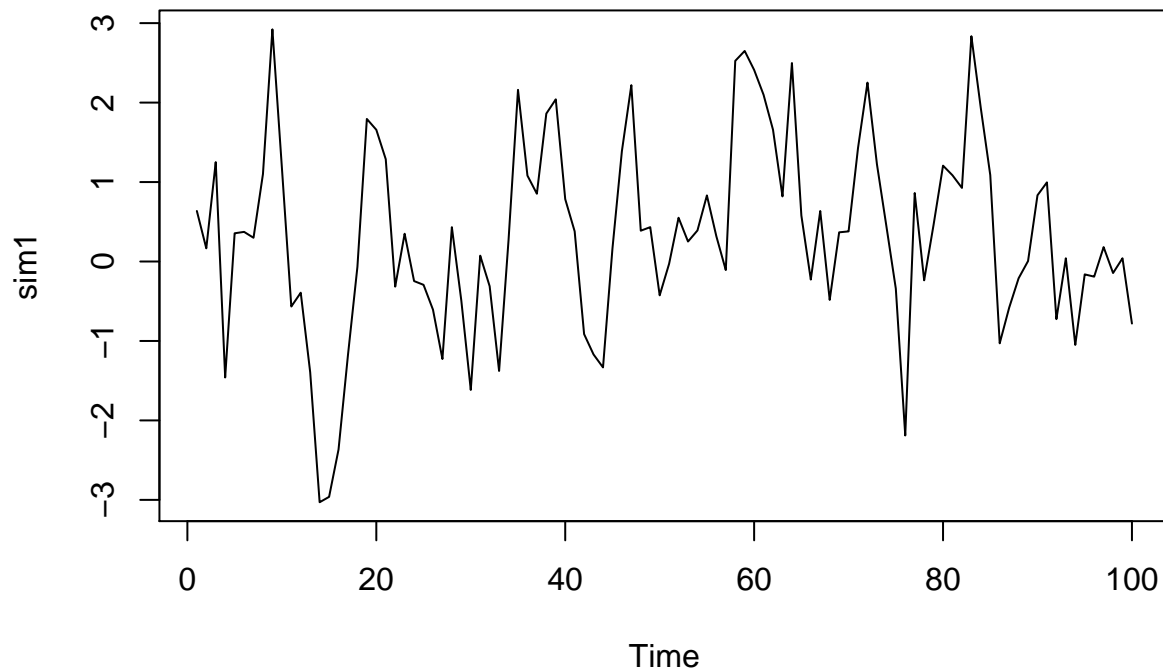
(a) Generate data from an AR(1) model with $\theta=0.6$ and $\sigma^2=1$. Start with $y_0=0$

$\theta = \text{ar}$, $Y_0 = c$, $\sqrt{\sigma^2} = \text{sd}$

```
sim1<-arima.sim(list( ar=0.6, sd=sqrt(1), c=0), n=100)
```

(b) Produce a time plot for the series. How does the plot change as you change θ ?

```
plot(sim1)
```



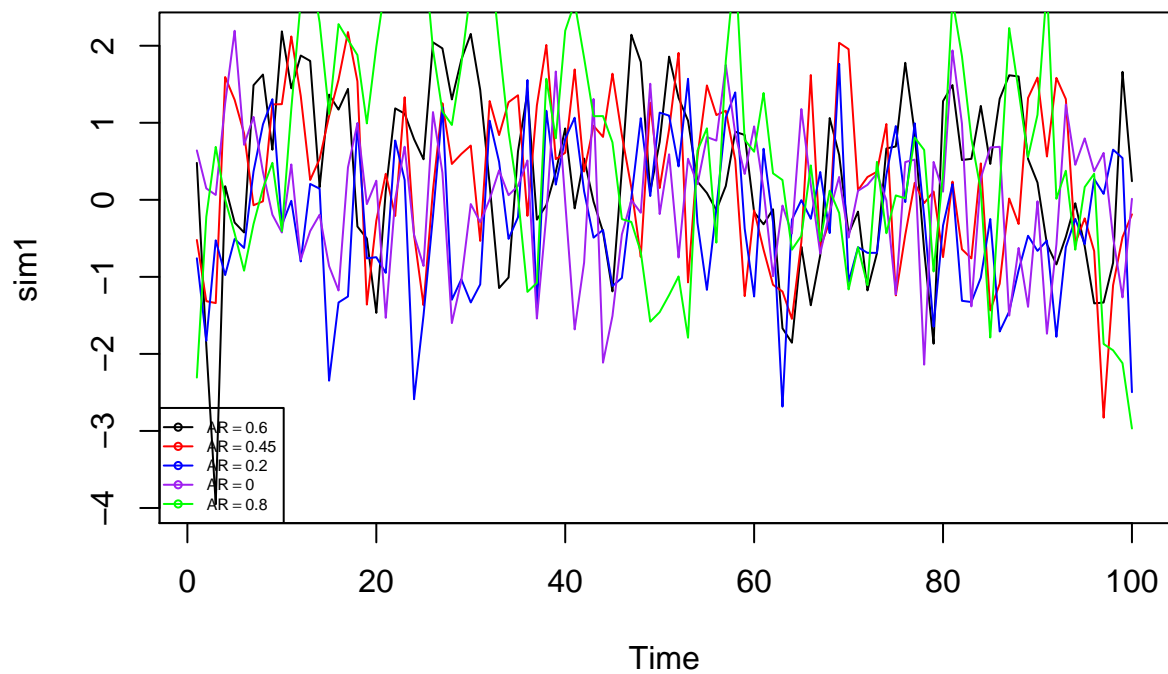
```
sim1<-arima.sim(list( ar=0.6, sd=sqrt(1), c=0), n=100)
sim1.1 <-arima.sim(list( ar=.45, sd=sqrt(1), c=0), n=100)
sim1.2 <-arima.sim(list( ar=0.2, sd=sqrt(1), c=0), n=100)
sim1.3 <-arima.sim(list( ar=0, sd=sqrt(1), c=0), n=100)
```

```
## Warning in min(Mod(polyroot(c(1, -model$ar)))): no non-missing arguments to
## min; returning Inf
```

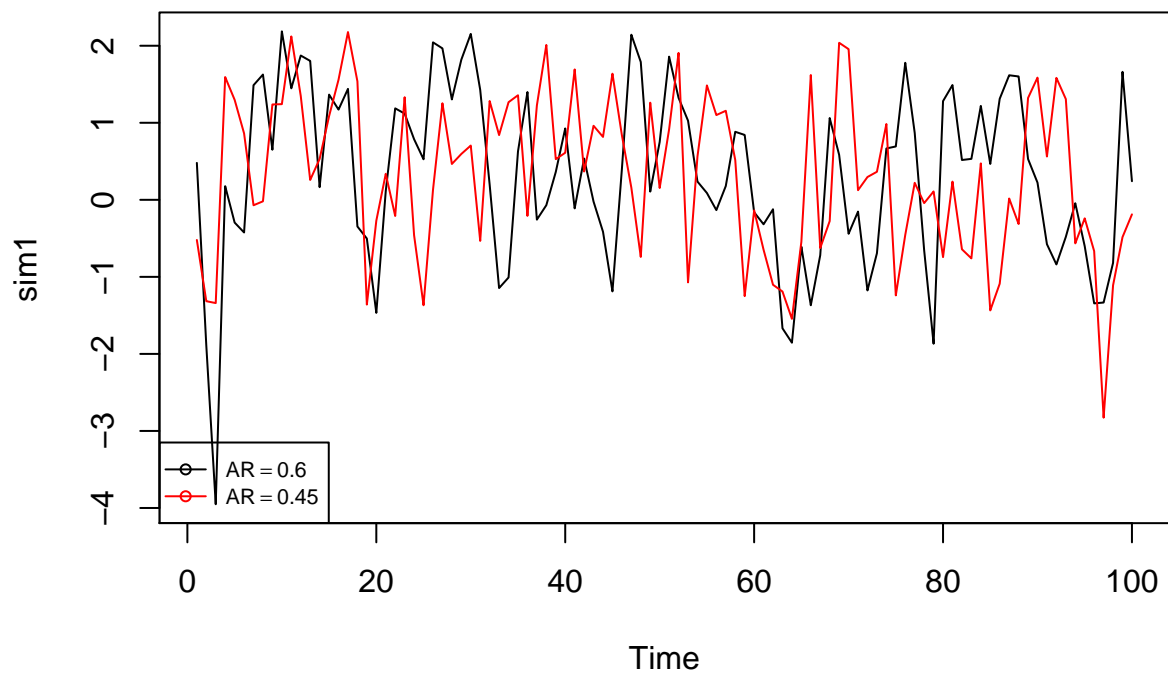
```
sim1.4 <-arima.sim(list( ar=.80, sd=sqrt(1), c=0), n=100)
```

Well, that looks like abstract art

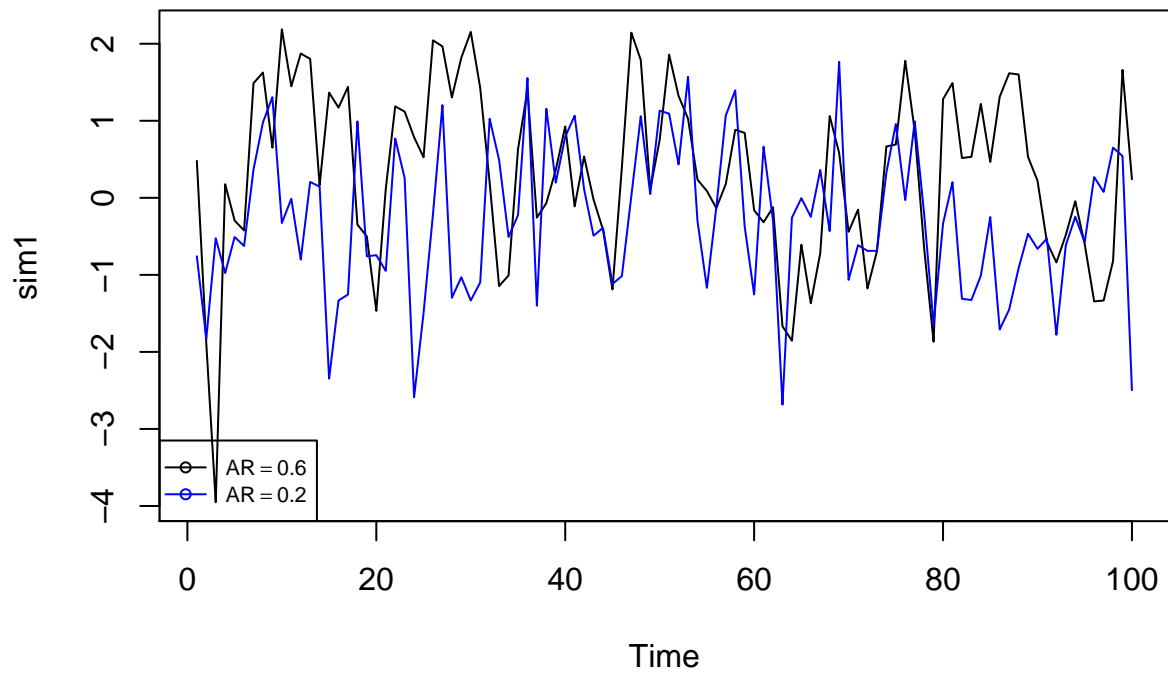
```
plot(sim1)
lines(sim1.1, col="red")
lines(sim1.2, col= "blue")
lines(sim1.3, col="purple")
lines(sim1.4, col= "green")
legend("bottomleft", lty = 1, col = c(1, "red","blue", "purple", "green"),
      c(expression(AR==0.6), expression(AR==0.45), expression(AR==0.20),
        expression(AR==0.0), expression(AR==0.80)), pch = 1, cex = 0.50)
```



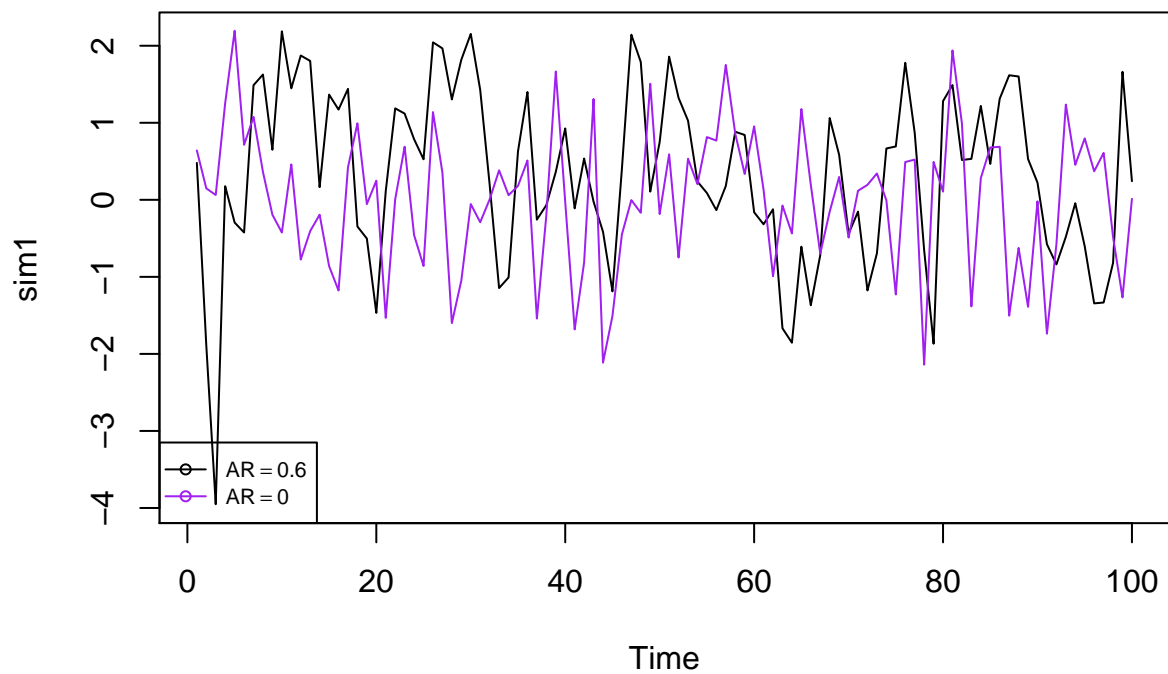
```
plot(sim1)
lines(sim1.1, col="red")
legend("bottomleft", lty = 1, col = c(1, "red"), c(expression(AR==0.6),
                                                    expression(AR==0.45)),
      pch = 1, cex = 0.70)
```



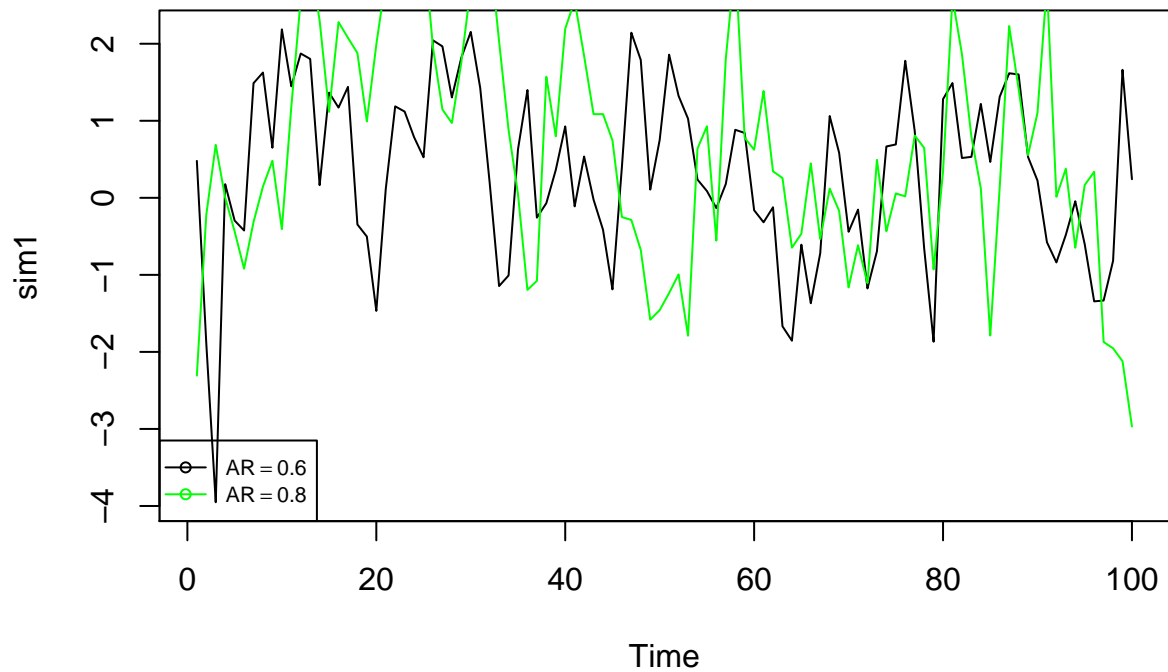
```
plot(sim1)
lines(sim1.2, col= "blue")
legend("bottomleft", lty = 1, col = c(1, "blue"),
      c(expression(AR==0.6), expression(AR==0.20)), pch = 1, cex = 0.70)
```



```
plot(sim1)
lines(sim1.3, col="purple")
legend("bottomleft", lty = 1, col = c(1, "purple"),
      c(expression(AR==0.6), expression(AR==0.0)), pch = 1, cex = 0.70)
```



```
plot(sim1)
lines(sim2, col= "green")
legend("bottomleft", lty = 1, col = c(1,"green"),
      c(expression(AR==0.6), expression(AR==0.80)), pch = 1, cex = 0.70)
```

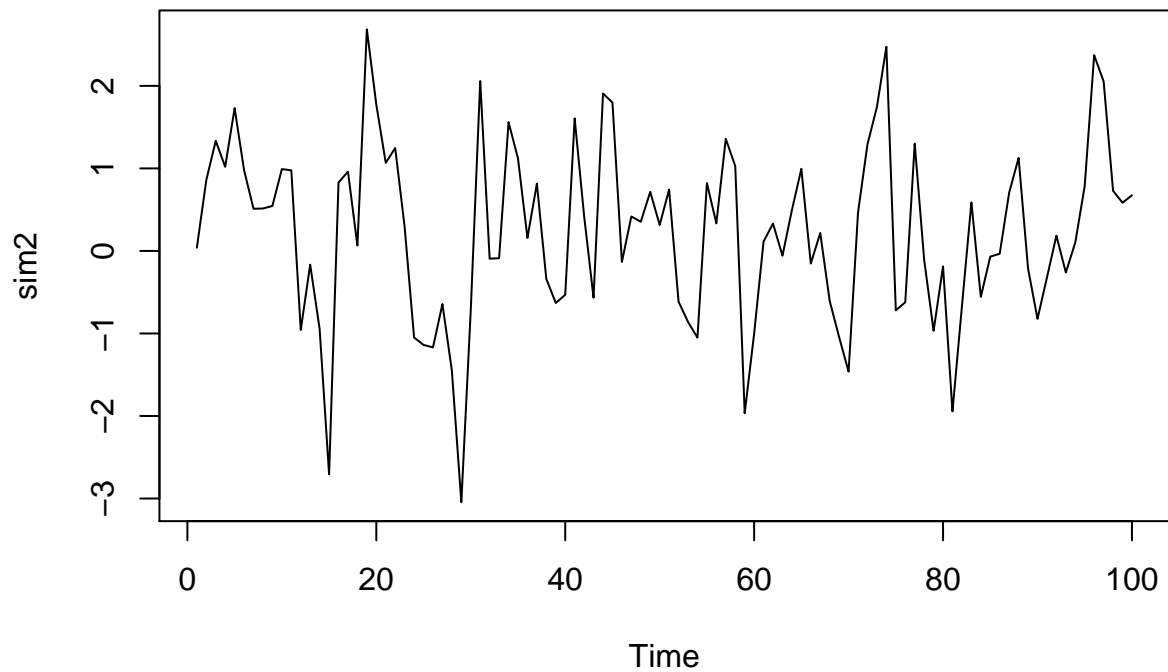


(c) Generate data from an MA(1) model with $\theta = 0.6$, and $\sigma^2 = 1$, start with $e_0 = 0$.

```
sim2<-arima.sim(list( ma=0.6, sd=sqrt(1), c=0), n=100)
```

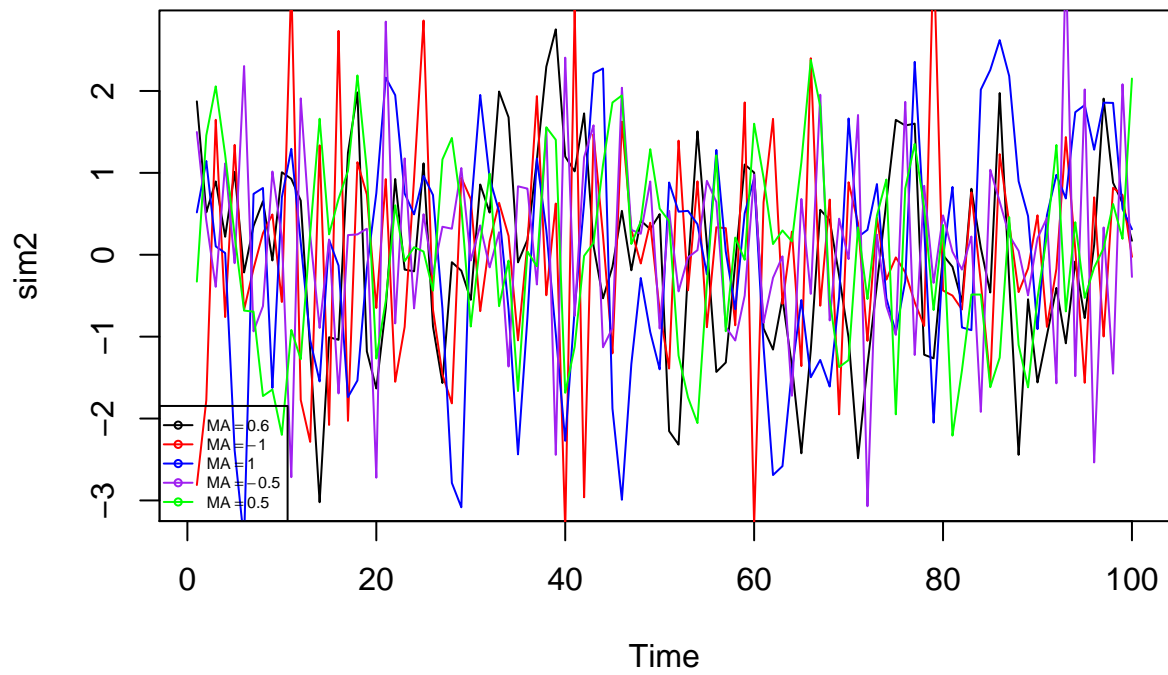
(d) Produce a time plot for the series. How does the plot change as you change the θ

```
plot(sim2)
```

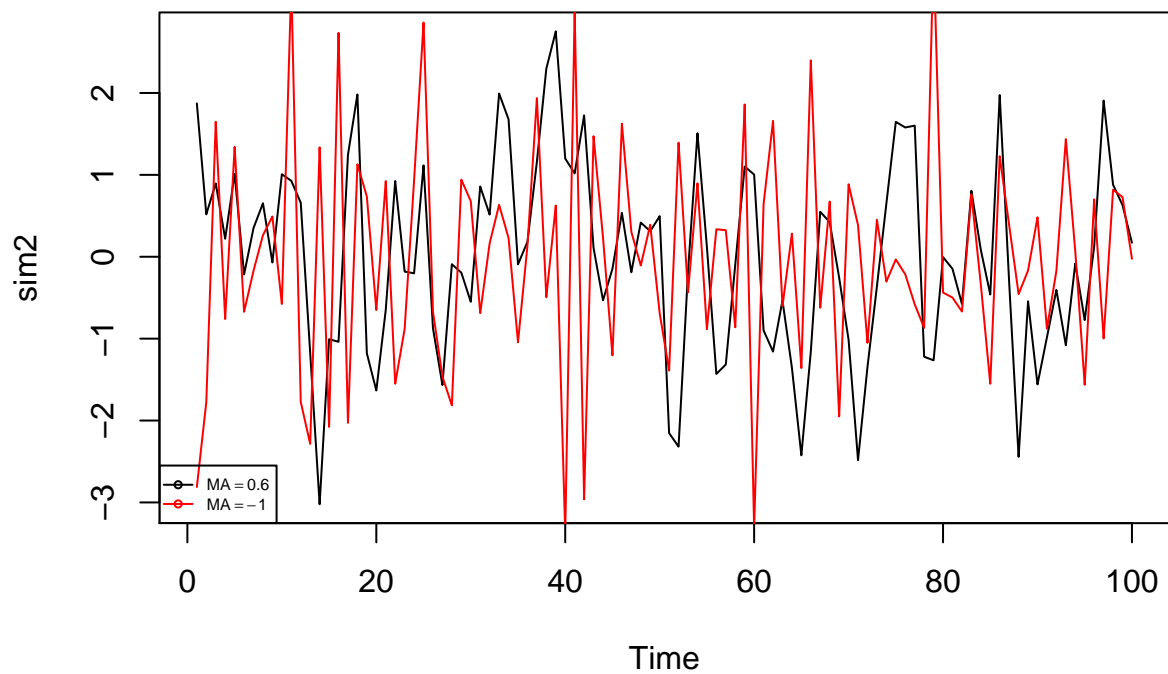


```
sim2<-arima.sim(list( ma=0.6, sd=sqrt(1), c=0), n=100)
sim2.1<-arima.sim(list( ma=-1.0, sd=sqrt(1), c=0), n=100)
sim2.2<-arima.sim(list( ma=1.0, sd=sqrt(1), c=0), n=100)
sim2.3<-arima.sim(list( ma=-0.50, sd=sqrt(1), c=0), n=100)
sim2.4<-arima.sim(list( ma=0.50, sd=sqrt(1), c=0), n=100)
```

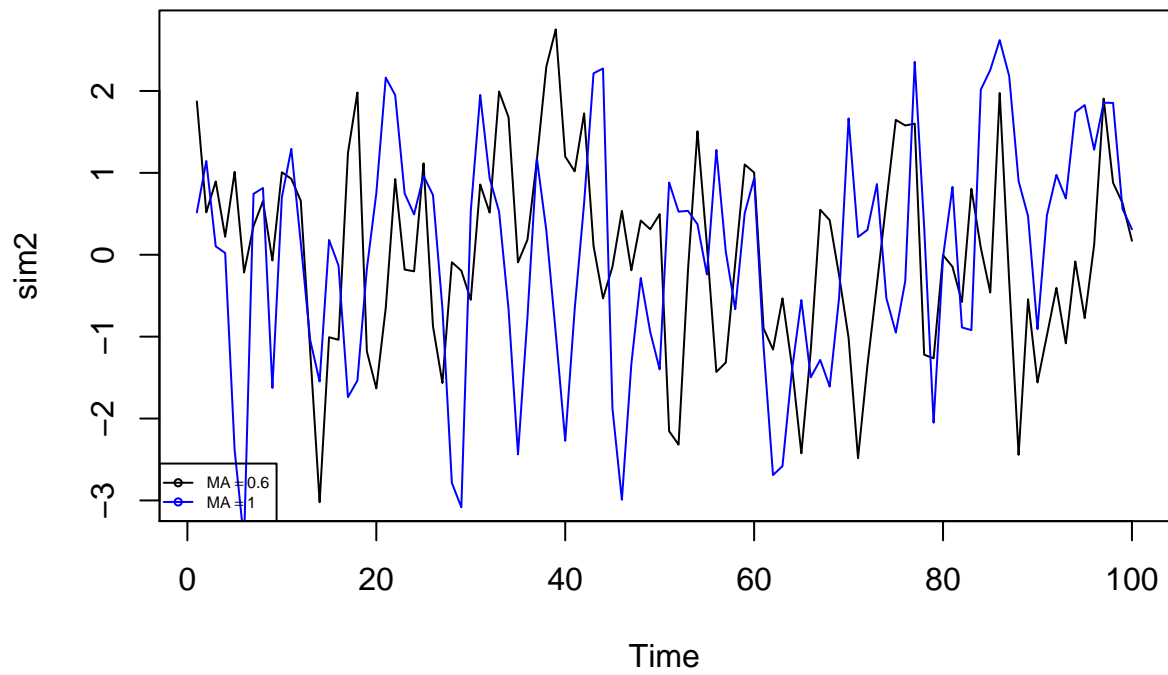
```
plot(sim2)
lines(sim2.1, col="red")
lines(sim2.2, col= "blue")
lines(sim2.3, col="purple")
lines(sim2.4, col= "green")
legend("bottomleft", lty = 1, col = c(1, "red","blue", "purple", "green"),
      c(expression(MA==0.6), expression(MA==1.0),
        expression(MA==1.00), expression(MA==0.50), expression(MA==0.50)),
      pch = 1, cex = 0.50)
```

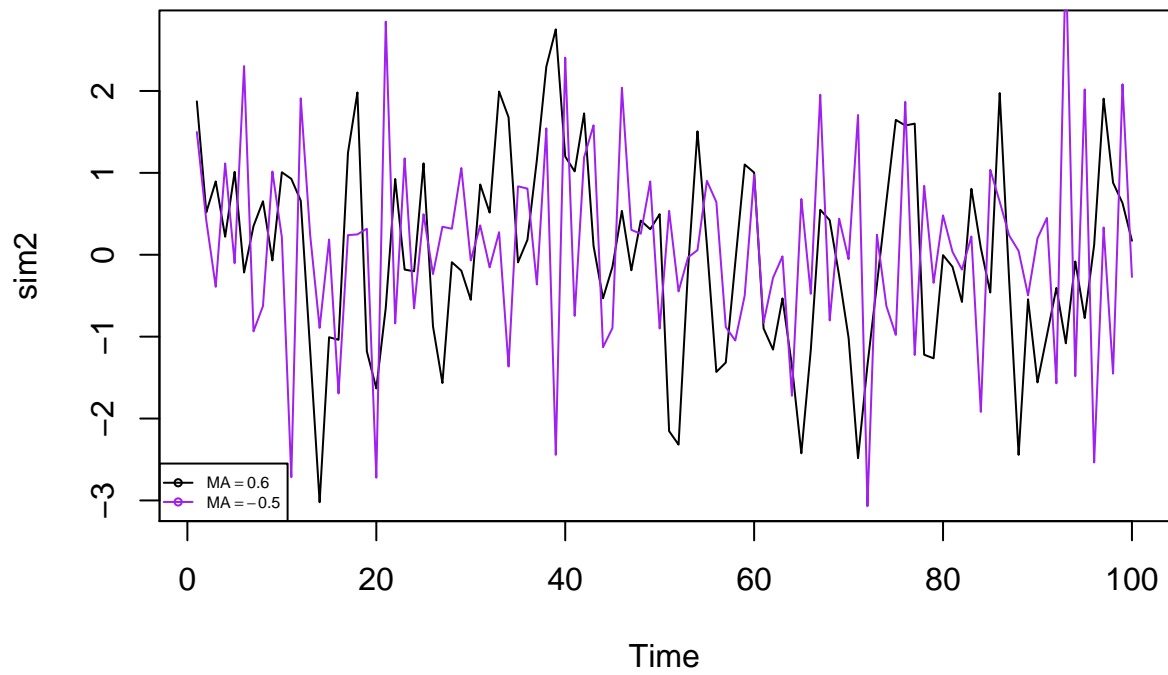
```
plot(sim2)
lines(sim2.1, col="red")
legend("bottomleft", lty = 1, col = c(1, "red"),
      c(expression(MA==0.6), expression(MA==1.0)),
      pch = 1, cex = 0.50)
```



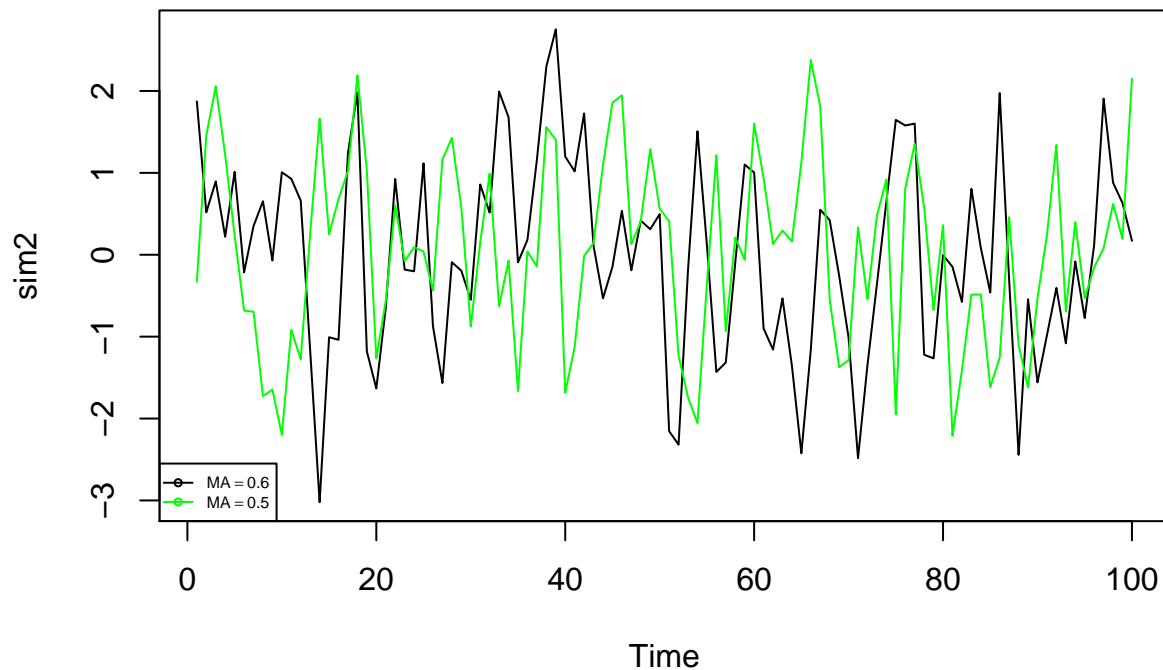
```
plot(sim2)
lines(sim2.2, col= "blue")
legend("bottomleft", lty = 1, col = c(1, "blue"),
      c(expression(MA==0.6), expression(MA==1.00)), pch = 1, cex = 0.50)
```



```
plot(sim2)
lines(sim2.3, col="purple")
legend("bottomleft", lty = 1, col = c(1, "purple"),
      c(expression(MA==0.6), expression(MA==0.50)), pch = 1, cex = 0.50)
```



```
plot(sim2)
lines(sim2.4, col= "green")
legend("bottomleft", lty = 1, col = c(1, "green"),
      c(expression(MA==0.6), expression(MA==0.50)), pch = 1, cex = 0.50)
```



(e) Generate data from an ARMA (1,1) model with AR=0.6 and MA=0.6 and $sd=\sqrt{1}$. Start with $y_0=y_{-1}=0$

```
sim3<-arima.sim(list( ar=0.6, ma=0.6, sd=sqrt(1), c=0), n=100)
```

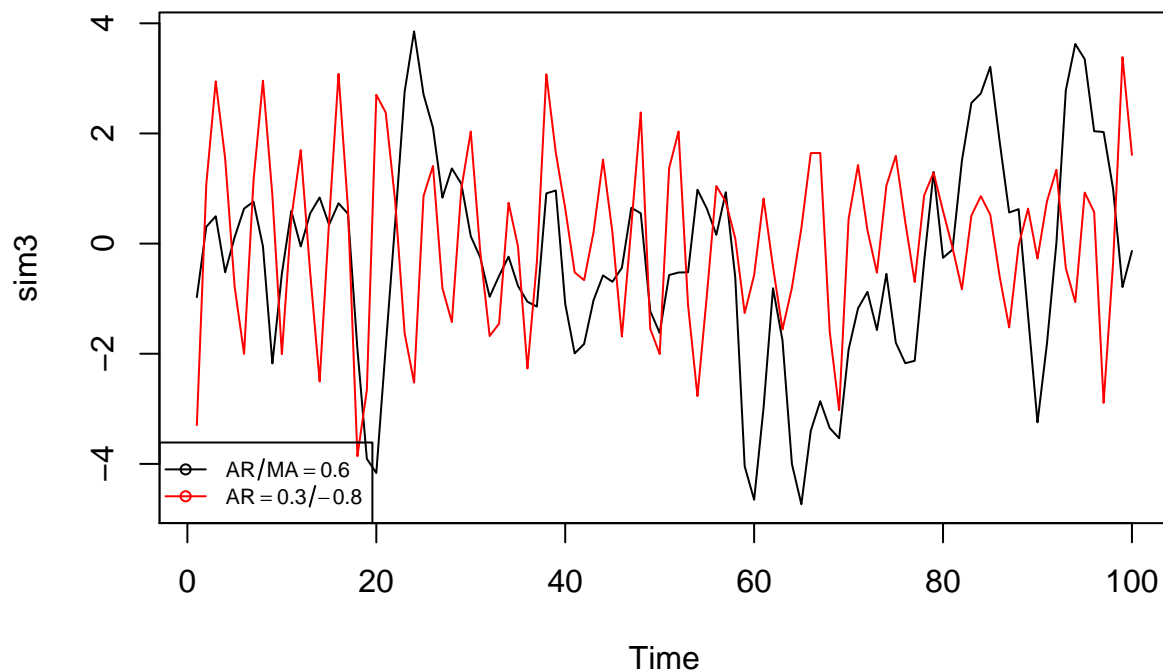
(f) Generate data from an AR(2) model with AR=-0.8 and 0.3 and $sd=\sqrt{1}$. Start with $y_0=y_{-1}=0$

```
sim4<-arima.sim(list( ar=c( 0.3,-0.8), sd=sqrt(1), c=0), n=100)
```

(g) Graph the latter two series and compare them.

The ARMA with values (-.8,.3) primarily stays centered around 0. It does show ebbs and flows, but is lacking the peaks seen in the ARMA with values (.6,.6). With this ARMA model, the model is centered around 0, but with extreme peaks and dips. TO stay centered around 0, these extreme peaks are followed by extreme dips.

```
plot(sim3)
lines(sim4, col="red")
legend("bottomleft", lty = 1, col = c(1, "red"),
      c(expression(AR/MA==0.6), expression(AR==0.3/-0.8)), pch = 1, cex = 0.70)
```



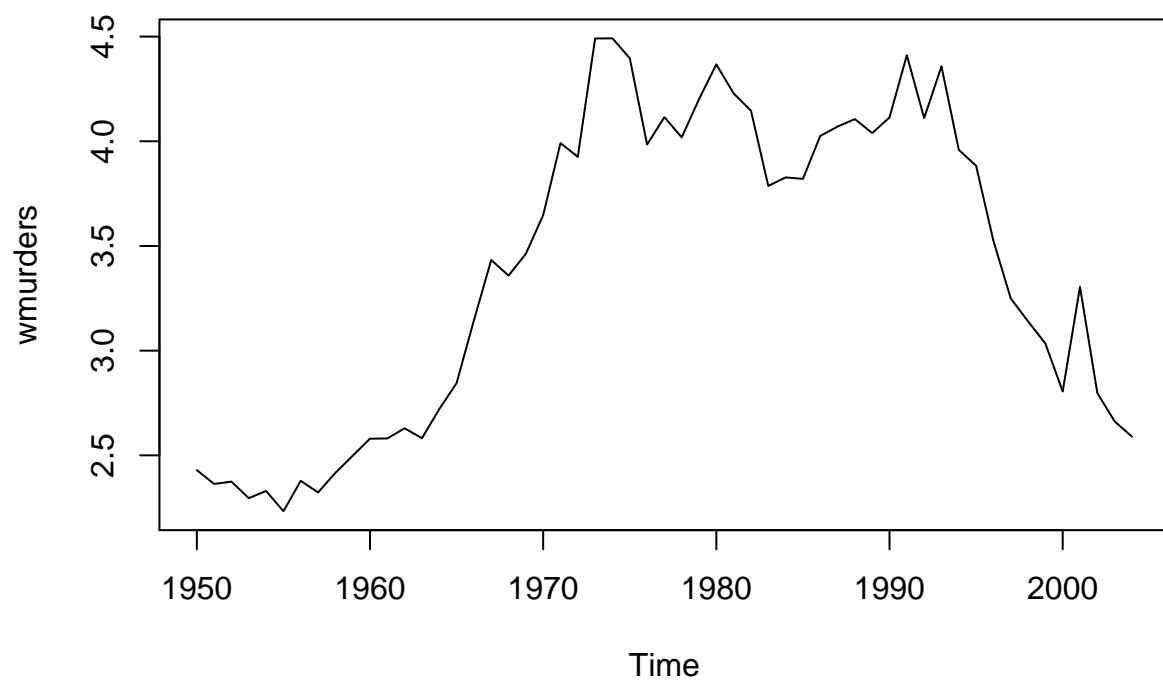
8.6 Data set wmurders

```
data(wmurders)
```

(a) By studying appropriate graphs of the series in R, find an appropriate ARIMA(p,d,q) model for these data

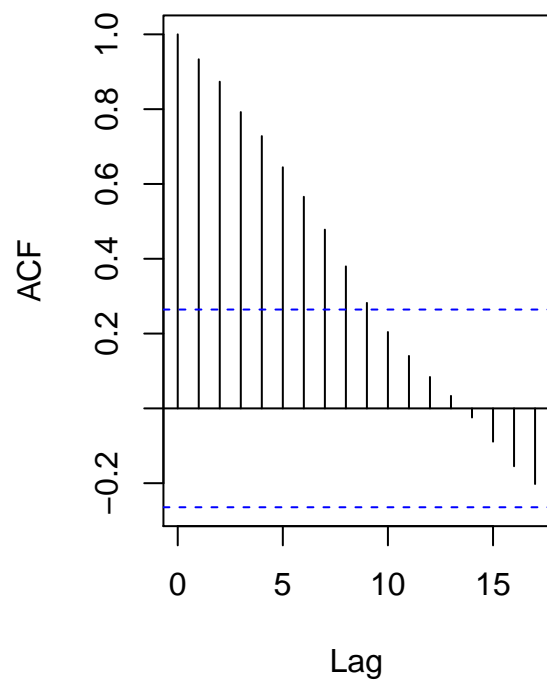
In looking at the plot, we first notice the model is not stationary. We'll take the first difference to try and make it stationary.

```
plot(wmurders)
```

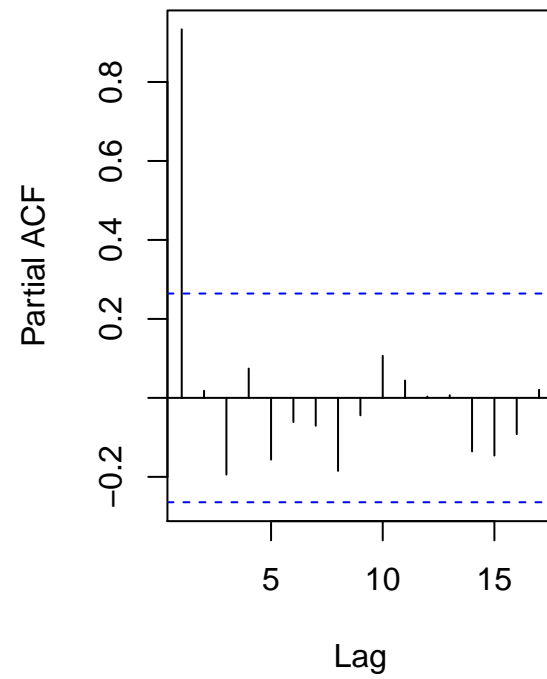


```
par(mfrow=c(1,2))  
acf(wmurders)  
pacf(wmurders)
```

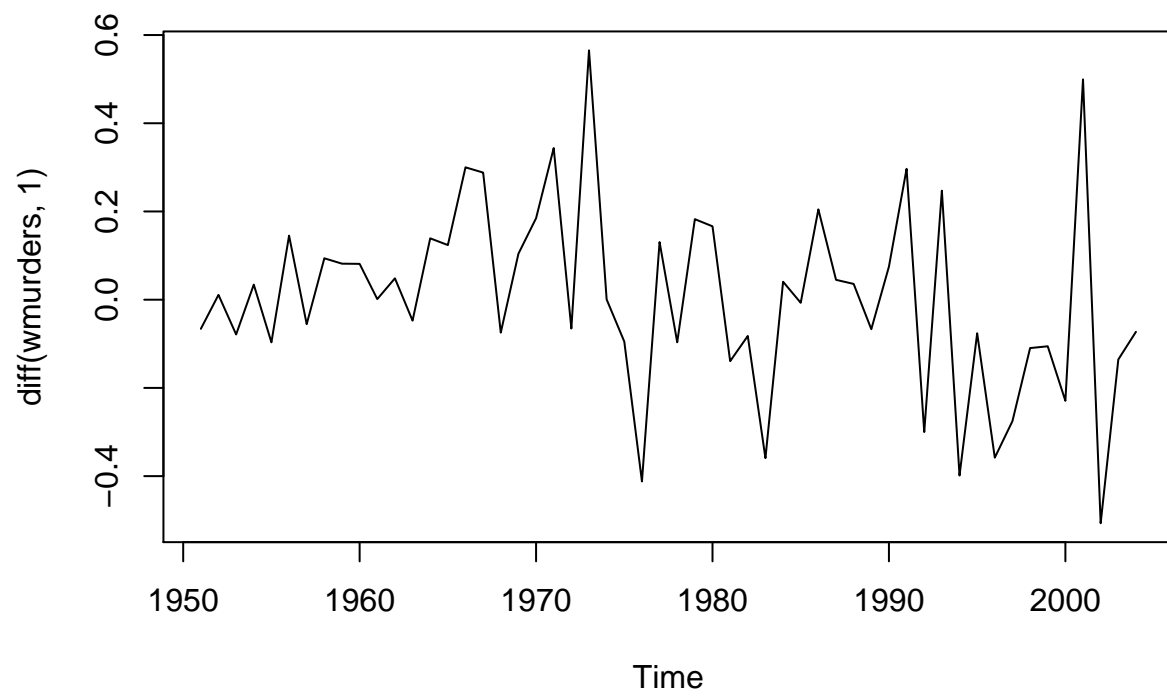
Series wmurders



Series wmurders

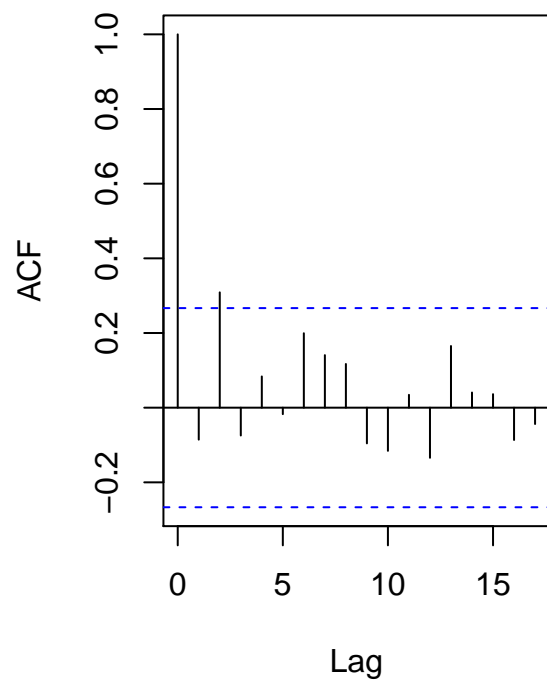


```
plot(diff(wmurders, 1))
```

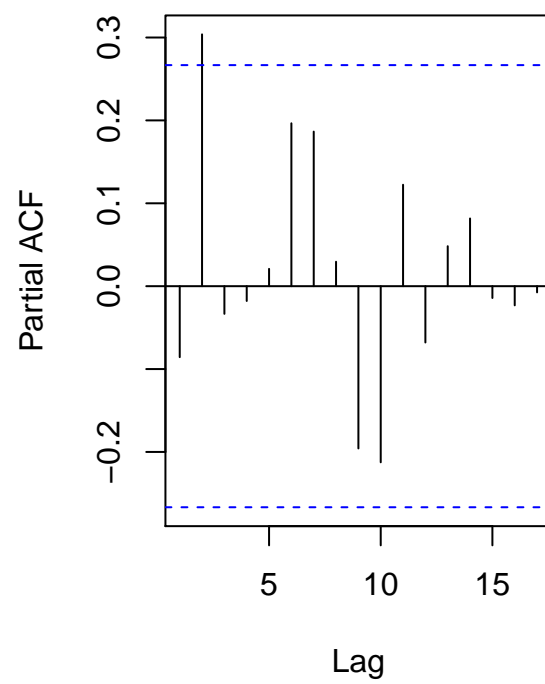



```
par(mfrow=c(1,2))  
acf(diff(wmurders, 1))  
pacf(diff(wmurders, 1))
```

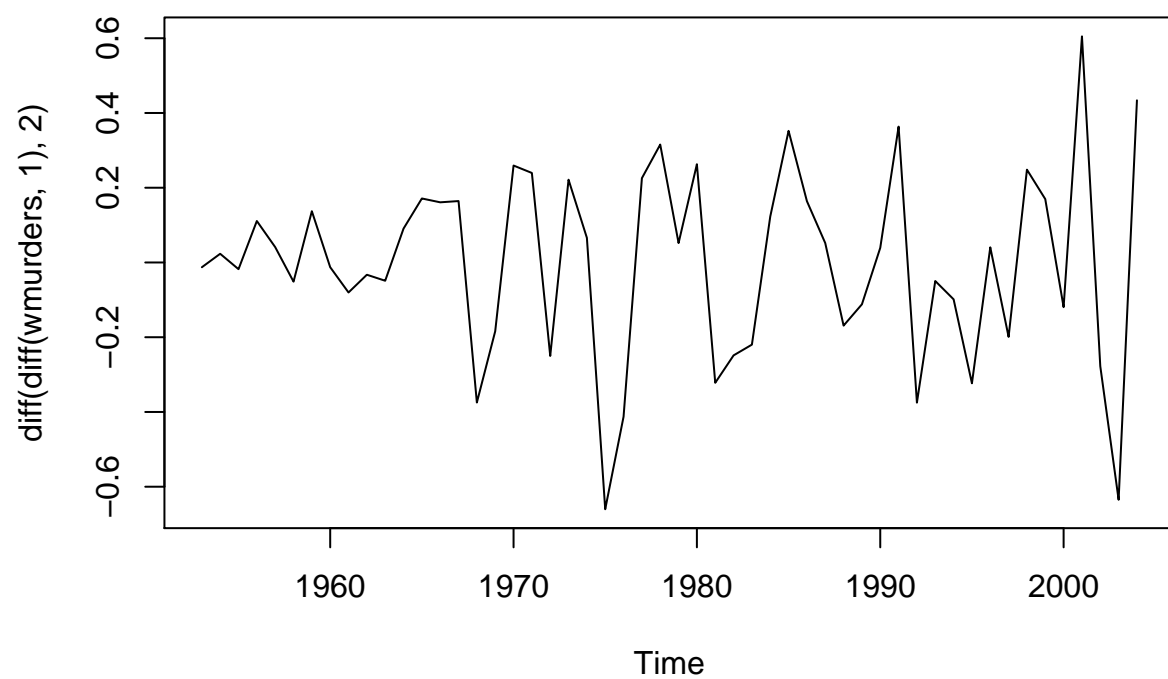
Series diff(wmurders, 1)



Series diff(wmurders, 1)



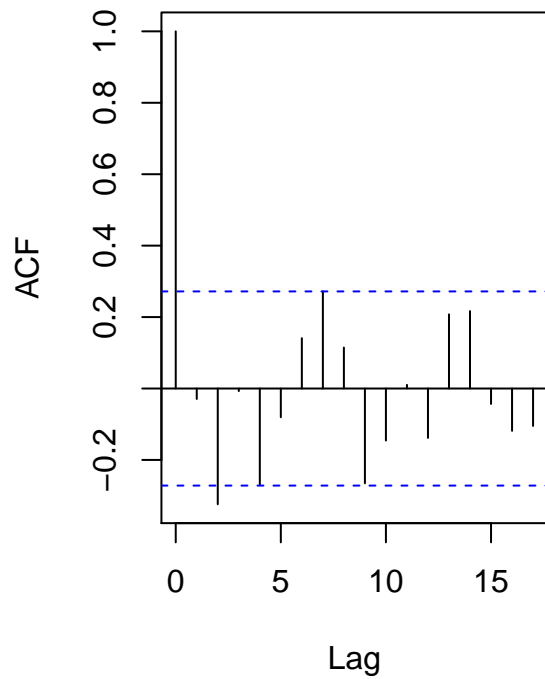
```
plot(diff(diff(wmurders, 1),2))
```



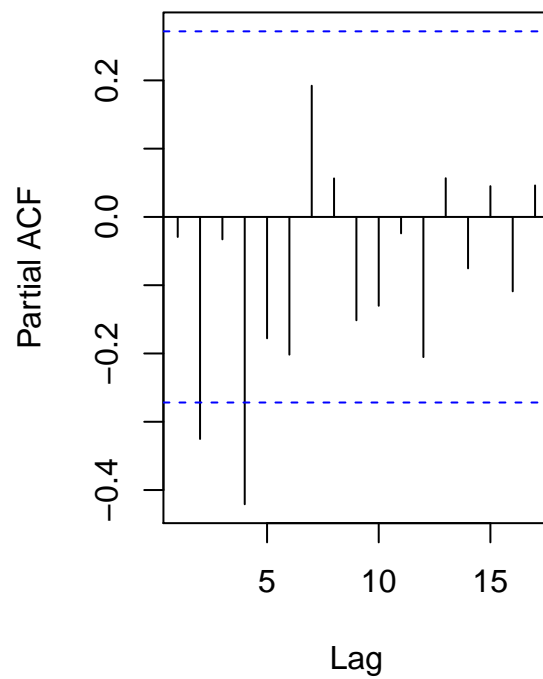
```
##AR
```

```
par(mfrow=c(1,2))  
acf(diff(diff(wmurders, 1),2))  
pacf(diff(diff(wmurders, 1),2))
```

Series diff(diff(wmurders, 1), 2)



Series diff(diff(wmurders, 1), 2)



```
## Yeah! I got it right!!
```

```
ndiffs(wmurders)
```

```
## [1] 2
```

Now lets try and build this model

```
dif2<-diff(diff(wmurders, 1),2)
```

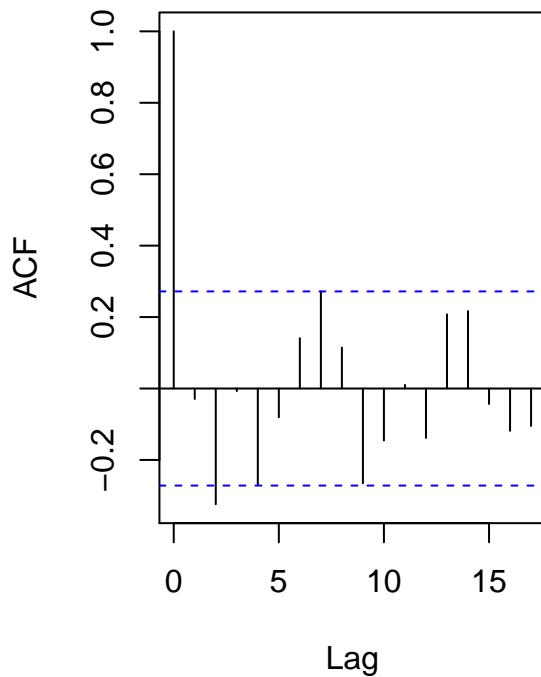
```
ARIMA(1,2,2)?
```

```
par(mfrow=c(1,2))
```

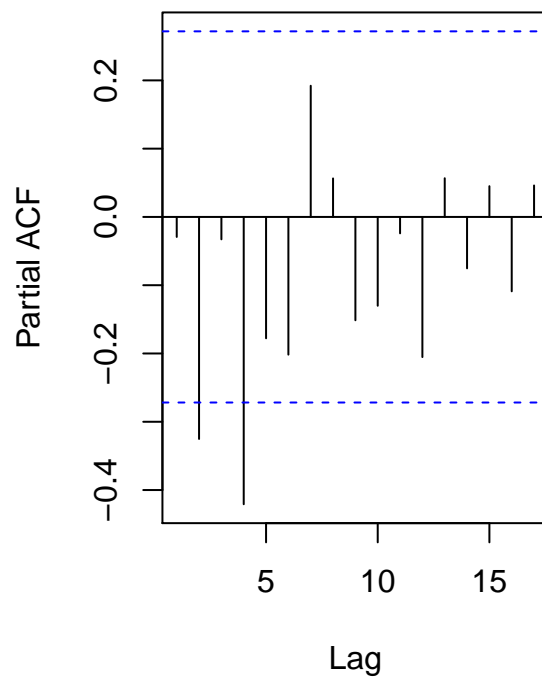
```
acf(dif2)
```

```
pacf(dif2)
```

Series dif2



Series dif2



```
fitA1<-Arima(wmurders, order = c(1,2,3))
summary(fitA1)
```

```
## Series: wmurders
## ARIMA(1,2,3)
##
## Coefficients:
##      ar1      ma1      ma2      ma3
##    -0.0931 -0.9317  0.3287 -0.2880
## s.e.   0.5181   0.4973  0.6336  0.2482
##
## sigma^2 estimated as 0.04565:  log likelihood=7.79
## AIC=-5.58   AICc=-4.3   BIC=4.27
##
## Training set error measures:
##              ME      RMSE      MAE      MPE      MAPE      MASE
## Training set -0.0131169 0.2016676 0.1521779 -0.3198772 4.361731 0.9358229
##              ACF1
## Training set -0.02486969
```

```
fitA<-Arima(wmurders, order = c(2,2,3))
summary(fitA)
```

```
## Series: wmurders
## ARIMA(2,2,3)
##
## Coefficients:
```

```
##          ar1          ar2          ma1          ma2          ma3
##          0.3500   -0.5222   -1.3508   1.3963   -0.9200
## s.e.    0.1449    0.1360    0.0882   0.1258    0.1069
##
## sigma^2 estimated as 0.03965:  log likelihood=10.03
## AIC=-8.05   AICc=-6.23   BIC=3.77
##
## Training set error measures:
##              ME          RMSE          MAE          MPE          MAPE          MASE
## Training set -0.01353027  0.1860191  0.1378711  -0.3377097  3.997824  0.8478428
##              ACF1
## Training set -0.1087861
```

(b) Should you include a constant in the model? Explain.

The model that i've selected above has a $d > 1$, so adding a constant is particularly dangerous when forecasting.

(c) Write this model in terms of the backshift operator. ARIMA(1,2,3)

$$(1 - \text{Sigma}(B))(1-B)^2 y_t = (1 + \text{Theta}(B) + \text{Theta}(B)^2 + \text{Theta}(B)^3) \epsilon_t$$

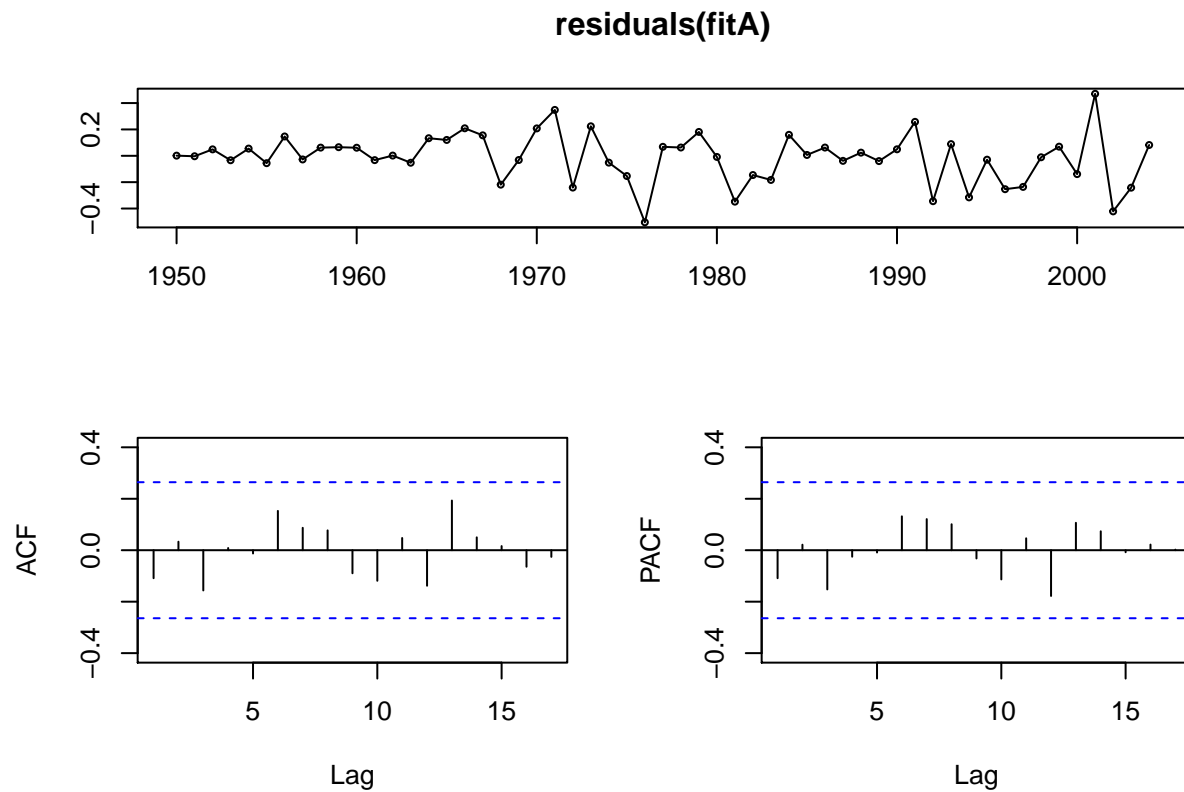
ARIMA(2,2,3)

$$(1 - \text{Sigma}(B) - \text{Sigma}(B)^2)(1-B)^2 y_t = (1 + \text{Theta}(B) + \text{Theta}(B)^2 + \text{Theta}(B)^3) \epsilon_t$$

(d) Fit the model using R and examine the residuals. Is the model satisfactory?

The model appears to be satisfactory. There are no significant spikes in the ACF or PACF and the residuals appear to be white noise.

```
tsdisplay(residuals(fitA))
```



(e) Forecast three times ahead.

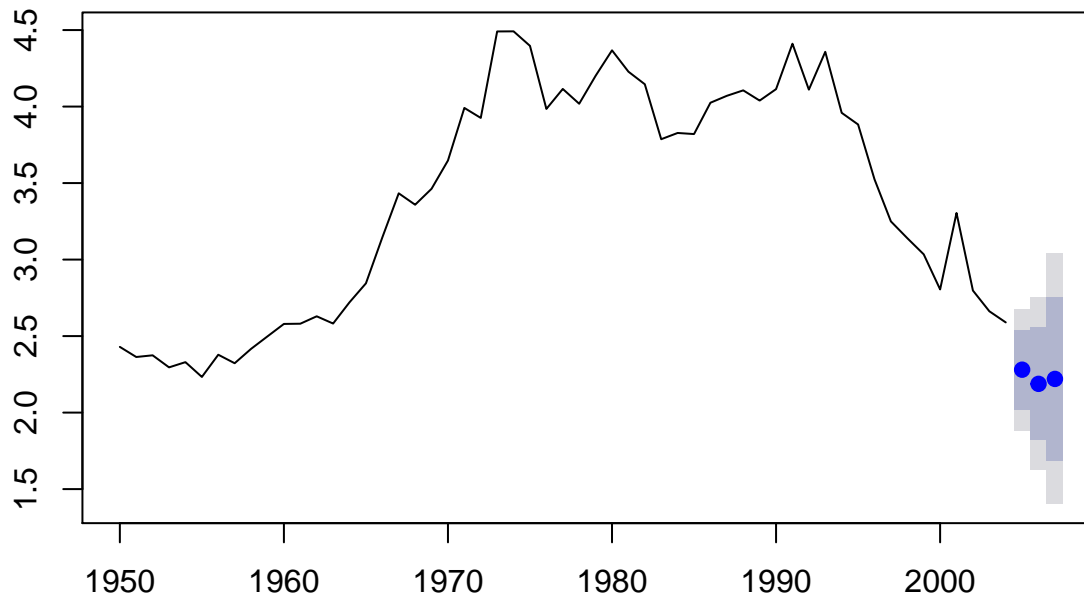
```
forecast(fitA, h=3)
```

##	Point Forecast	Lo 80	Hi 80	Lo 95	Hi 95
## 2005	2.280465	2.021042	2.539888	1.883712	2.677218
## 2006	2.187847	1.819820	2.555873	1.624999	2.750694
## 2007	2.220101	1.684602	2.755600	1.401126	3.039076

(f) Create a plot of the series with forecasts and prediction intervals for the next three periods shown.

```
plot(forecast(fitA, h=3))
```

Forecasts from ARIMA(2,2,3)



(g) Does `auto.arima` give the same model you have chosen? If not, which model do you think is better?

The `Auto.Arima` model did not choose the same model. If we base our selection on the smallest AICc, we choose the `auto.arima` model. If we go based off of the RMSE, we choose the `ARIMA(2,2,3)`. Since the models are different, we will use the RMSE and go with the handcrafted `ARIMA(2,2,3)` model.

```
aafit<-auto.arima(wmurders)
summary(aafit)
```

```
## Series: wmurders
## ARIMA(1,2,1)
##
## Coefficients:
##      ar1      ma1
##    -0.2434 -0.8261
## s.e.   0.1553   0.1143
##
## sigma^2 estimated as 0.04632: log likelihood=6.44
## AIC=-6.88 AICc=-6.39 BIC=-0.97
##
## Training set error measures:
##              ME      RMSE      MAE      MPE      MAPE      MASE
```



```
## Training set -0.01065956 0.2072523 0.1528734 -0.2149476 4.335214 0.9400996
##               ACF1
## Training set 0.02176343
```

(h) Find the latest data and compare with your forecasts.

```
forecast(fitA, h=3)
```

```
##      Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 2005      2.280465 2.021042 2.539888 1.883712 2.677218
## 2006      2.187847 1.819820 2.555873 1.624999 2.750694
## 2007      2.220101 1.684602 2.755600 1.401126 3.039076
```

The latest data isn't present on the website.

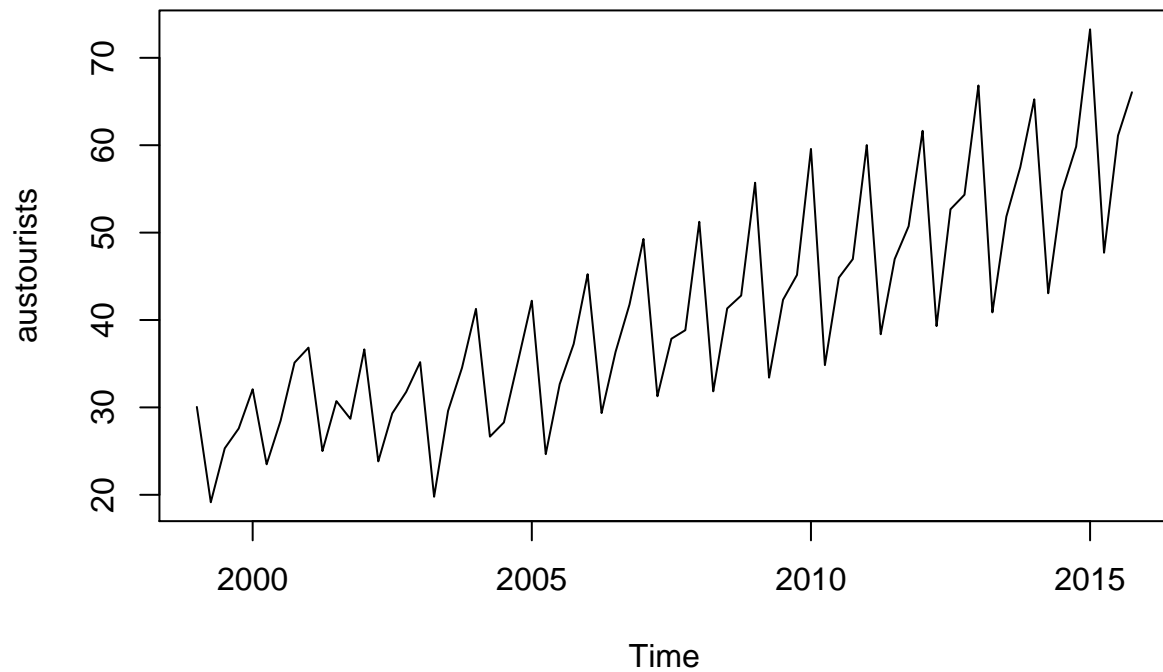
8.7 Dataset austourists

```
data("austourists")
```

(a) Describe the time plot.

This dataset has strong seasonal trend, and a general upward trend.

```
plot(austourists)
```

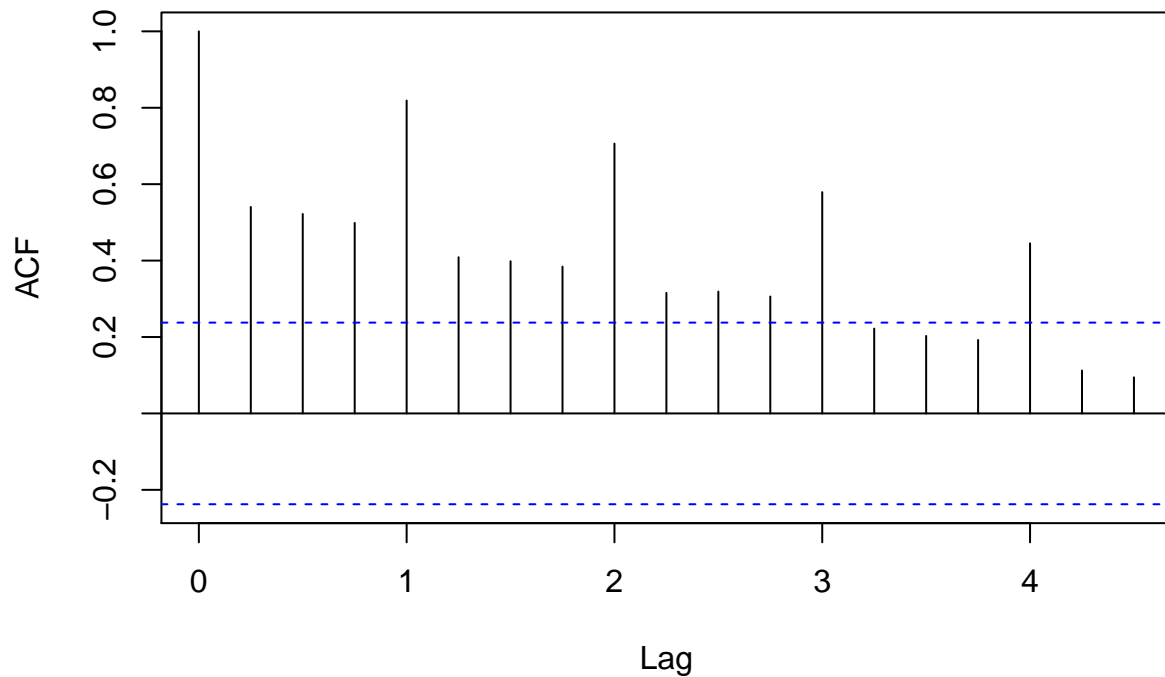


(b) What can you learn from the ACF graph?

The ACF Plot shows the relationship between y_t and y_{t-k} . So this ACF shows us that the value of y_t is impacted by previous values, specifically, 1 year prior. The seasonal impact is likely yearly in this quarterly dataset. We also can learn that the dataset is not stationary, which we noticed in the plot of the data.

```
acf(austourists)
```

Series austourists

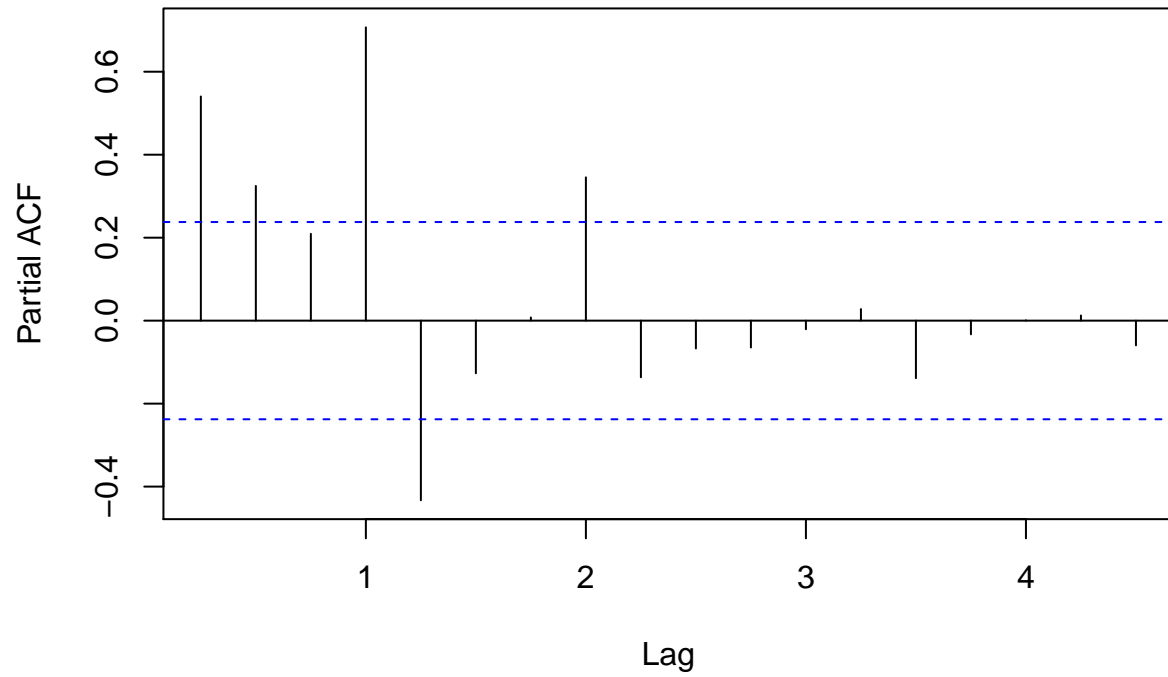


(c) What can you learn from the PACF graph?

The PACF shows us the relationship of y_t to $y(t-k)$ after removing the effects of other time lags prior to $y(t-k)$. We notice the same yearly spikes in the PACF and the non stationary aspect of the dataset.

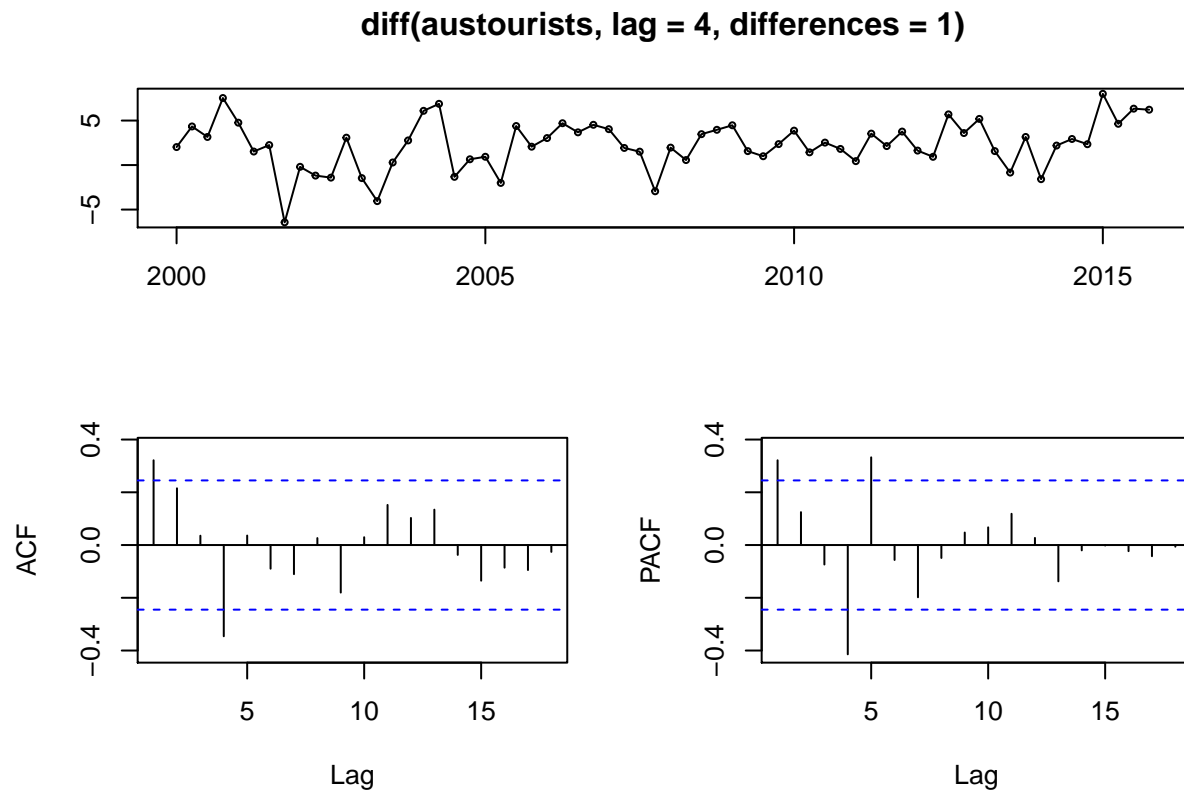
```
pacf(austourists)
```

Series austourists



(d) Produce plots of the seasonally differenced data $(1-B^4)Y_t$.

```
#seasd<-seasadj(stl(austourists, s.window = "periodic"))
tsdisplay(diff(austourists, lag = 4, differences = 1))
```



##What model do these graphs suggest? ### The significant lag at 1 in the ACF suggests a non-seasonal MA(1), the significant spike at 4 in the ACF suggests a seasonal MA(1) component. So we begin with a ARIMA (0,1,1)(0,1,1)[4] model. The PACF shows a significant spike at 1,4 and 5. The spike at 4 is consistent with the seasonal nature of the data. We end with a ARIMA (1,1,1)(1,1,1)[4] model.

(e) Does `auto.arima` give the same model that you chose? If not, which model do you think is better?

The `auto.arima` model chose a different model than the one I selected. In running the model I selected, the model selected by the `auto.arima` is better, as measured by the smaller AICc and smaller RMSE

```
aa8.7<-auto.arima(austourists)
summary(aa8.7)

## Series: austourists
## ARIMA(1,0,0)(1,1,0)[4] with drift
##
## Coefficients:
##          ar1      sar1    drift
##          0.4705  -0.5305  0.5489
## s.e.    0.1154   0.1122  0.0864
##
## sigma^2 estimated as 5.15:  log likelihood=-142.48
## AIC=292.97   AICc=293.65   BIC=301.6
```

```
##
## Training set error measures:
##           ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 0.02200144 2.149384 1.620917 -0.7072593 4.388288 0.5378929
##           ACF1
## Training set -0.06393238

aamy<-Arima(austourists, order = c(1,1,1), seasonal = c(1,1,1))
summary(aamy)

## Series: austourists
## ARIMA(1,1,1)(1,1,1)[4]
##
## Coefficients:
##           ar1      ma1      sar1      sma1
##           0.4023 -0.9202 -0.4463 -0.1153
## s.e.    0.1548   0.0785   0.2258   0.2442
##
## sigma^2 estimated as 5.403:  log likelihood=-142.05
## AIC=294.1   AICc=295.15   BIC=304.82
##
## Training set error measures:
##           ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 0.1596318 2.165125 1.699392 -0.305583 4.588511 0.5639345
##           ACF1
## Training set -0.07172687
```

(f) Write the model in terms of the backshift operator, and then without using the backshift operator.

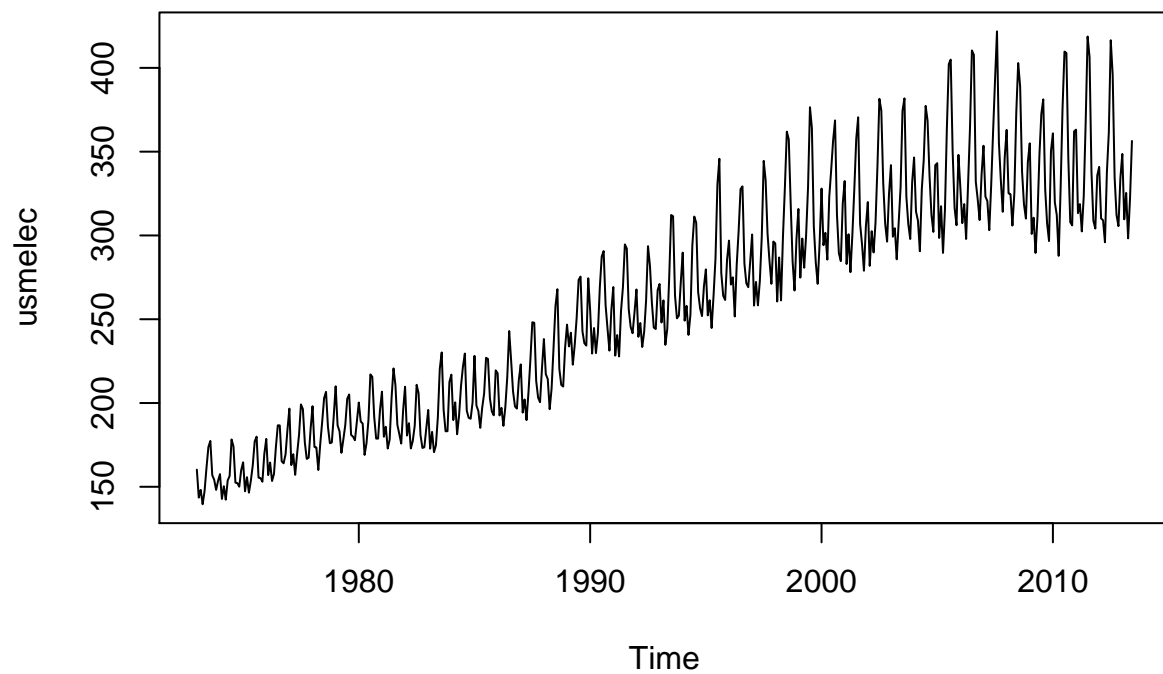
$$(1 - \phi(B))(1 - \Phi(B))(1 - B)(1 - B) y_t = (1 + \theta(B))(1 + \Theta(B)) \epsilon_t$$

8.8 Dataset usmelec. In general there are two peaks per year: mid-summer and mid-winter.

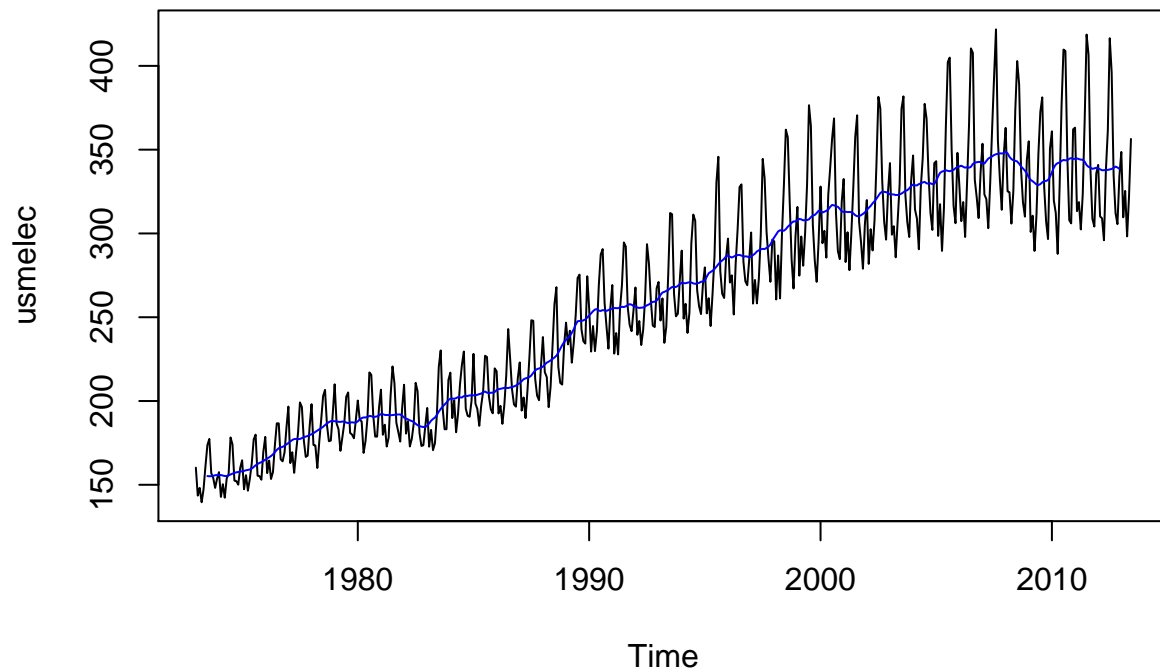
```
data("usmelec")
```

(a) Examine the 12 month moving average of this series to see what kind of trend is involved.

```
plot(usmelec)
```



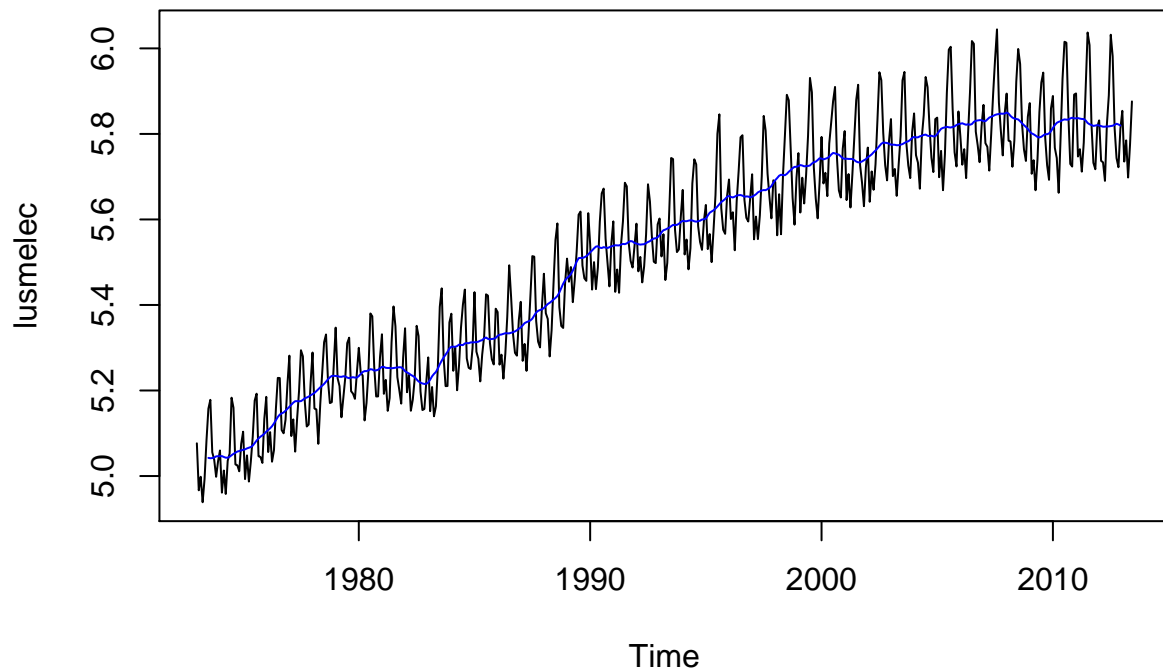
```
ma12<-ma(usmelec, 12)
plot(usmelec)
lines(ma12, col="blue")
```



(b) Do the data need transforming? If so, find a suitable transformation.

The seasonal effect of the data increases as time goes on. This dataset will likely benefit from a log transformation, which will remove the increased variance of the later observations.

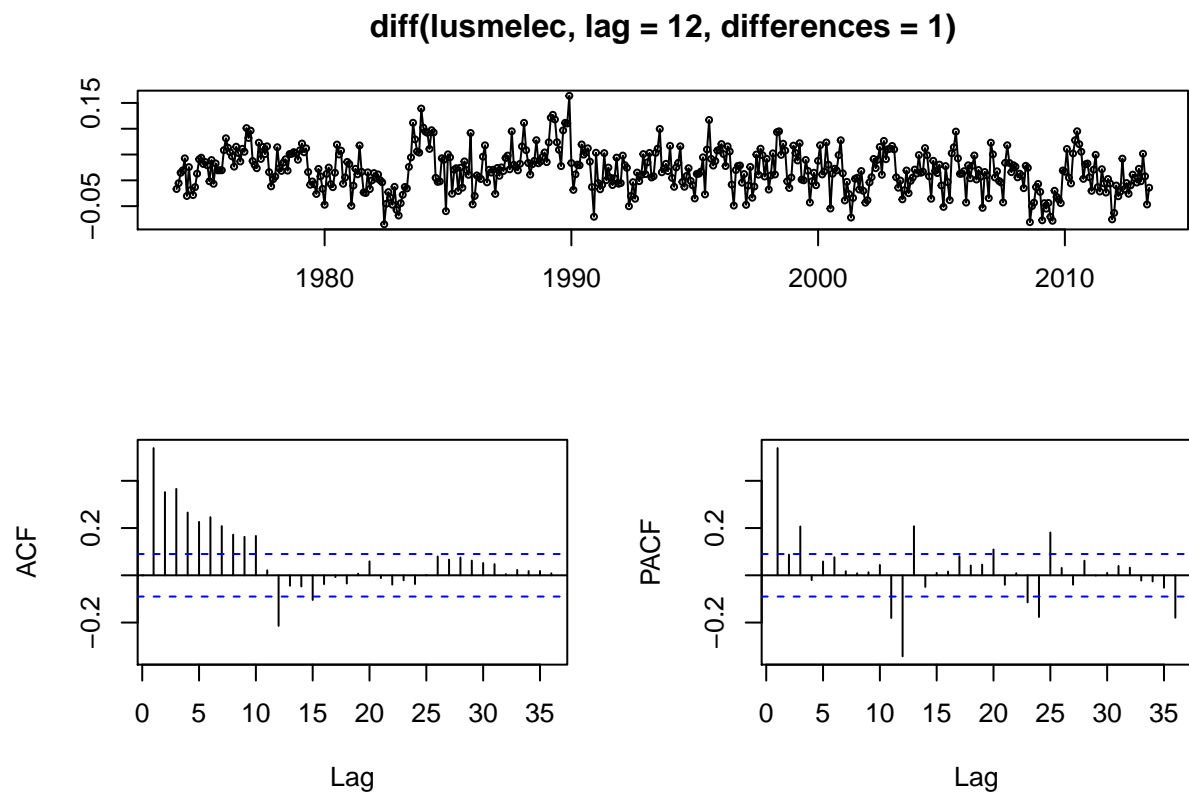
```
lusmelec<-log(usmelec)
lma12<-ma(lusmelec, 12)
plot(lusmelec)
lines(lma12, col="blue")
```

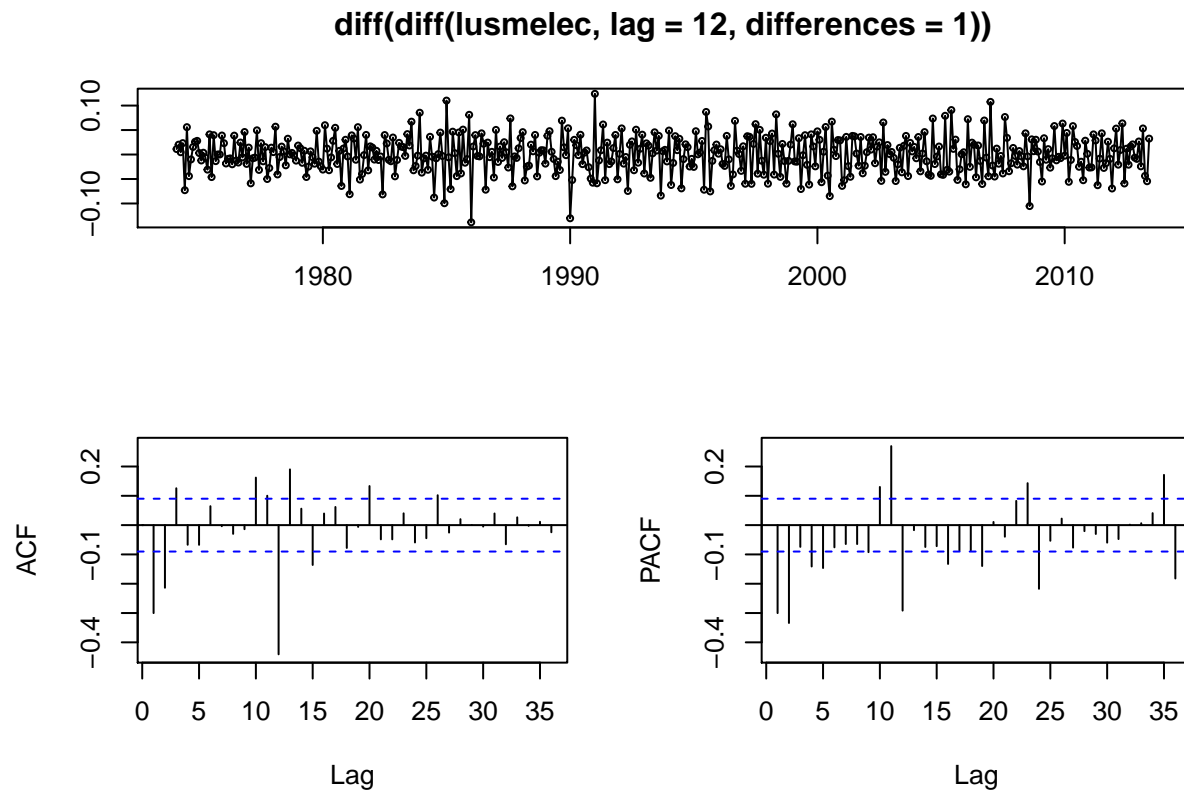
(c) Are the data stationary? If not, find an appropriate differencing which yields stationary data.

The data is not stationary, but a single difference appears to make the dataset stationary.

```
tsdisplay(diff(lusmelec, lag = 12, differences = 1))
```



```
tsdisplay(diff(diff(lusmelec, lag = 12, differences = 1)))
```



(d) Identify a couple of ARIMA models that might be useful in describing the time series. Which of your models is the best according to the AIC values.

In looking at the ACF and PACF, we see exponential decay in the ACF, with a significant spike at 12. In the PACF, There is a significant lag at 1,3, 11,12,13,23,24,25,36.

PACF is AR – ARIMA(AR,D,MA)

ARIMA(2,0,0)(3,1,1)[12] is the best model of the ones tested below, based off of the AIC, AICc and BIC.

```
arima1<-Arima(lusmelec, order = c(2,0,0), seasonal = c(3,1,1))
summary(arima1)
```

```
## Series: lusmelec
## ARIMA(2,0,0)(3,1,1)[12]
##
## Coefficients:
##          ar1      ar2      sar1      sar2      sar3      sma1
##          0.7316  0.2502 -0.0635 -0.1544 -0.1081 -0.7714
## s.e.      0.0455  0.0446  0.0615  0.0548  0.0535  0.0488
##
## sigma^2 estimated as 0.0007709: log likelihood=1021.09
## AIC=-2028.19  AICc=-2027.95  BIC=-1999.06
```

```
##
## Training set error measures:
##           ME           RMSE           MAE           MPE           MAPE
## Training set 0.00211664 0.02724639 0.02078942 0.03755393 0.3755733
##           MASE           ACF1
## Training set 0.5928503 -0.06465986

arima2<-Arima(lusmelec, order = c(2,1,3), seasonal = c(3,1,1))
summary(arima2)

## Series: lusmelec
## ARIMA(2,1,3)(3,1,1)[12]
##
## Coefficients:
##           ar1           ar2           ma1           ma2           ma3           sar1           sar2           sar3
##           0.0685    0.1786   -0.4894   -0.3782    0.0649   -0.0360   -0.1315   -0.1049
## s.e.    0.4525    0.1605    0.4516    0.3066    0.1077    0.0663    0.0576    0.0559
##           sma1
##           -0.7744
## s.e.    0.0506
##
## sigma^2 estimated as 0.0006827:  log likelihood=1049.54
## AIC=-2079.07  AICc=-2078.6  BIC=-2037.48
##
## Training set error measures:
##           ME           RMSE           MAE           MPE           MAPE
## Training set -0.0009605182 0.02553068 0.01961125 -0.01836626 0.354044
##           MASE           ACF1
## Training set 0.5592525 -0.003915312

arima2<-Arima(lusmelec, order = c(2,1,2), seasonal = c(3,1,1))
summary(arima2)

## Series: lusmelec
## ARIMA(2,1,2)(3,1,1)[12]
##
## Coefficients:
##           ar1           ar2           ma1           ma2           sar1           sar2           sar3
##           -0.1367    0.1720   -0.2815   -0.4610   -0.0401   -0.1330   -0.1048
## s.e.    0.3037    0.1474    0.2967    0.2547    0.0657    0.0575    0.0559
##           sma1
##           -0.7742
## s.e.    0.0504
##
## sigma^2 estimated as 0.0006816:  log likelihood=1049.38
## AIC=-2080.76  AICc=-2080.38  BIC=-2043.33
##
## Training set error measures:
##           ME           RMSE           MAE           MPE           MAPE
## Training set -0.0009413485 0.0255378 0.01961076 -0.01802902 0.354006
##           MASE           ACF1
## Training set 0.5592386 -0.006891814
```

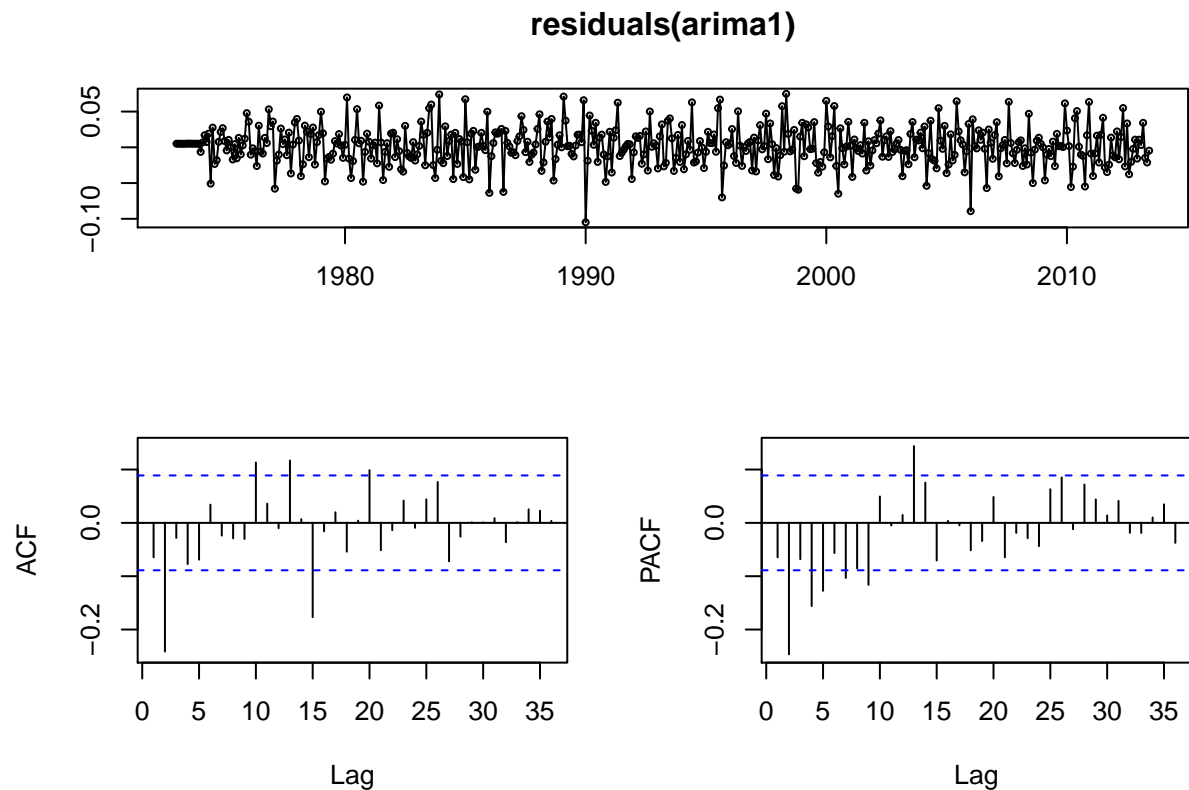
(e) Estimate the parameters of your best model and do diagnostic testing on the residuals. Do the residuals resemble white noise? If not, try to find another ARIMA model which fits better.

```
summary(arima1)
```

```
## Series: lusmelec
## ARIMA(2,0,0)(3,1,1)[12]
##
## Coefficients:
##          ar1      ar2      sar1      sar2      sar3      sma1
##          0.7316  0.2502 -0.0635 -0.1544 -0.1081 -0.7714
## s.e.    0.0455  0.0446   0.0615   0.0548   0.0535   0.0488
##
## sigma^2 estimated as 0.0007709:  log likelihood=1021.09
## AIC=-2028.19   AICc=-2027.95   BIC=-1999.06
##
## Training set error measures:
##              ME      RMSE      MAE      MPE      MAPE
## Training set 0.00211664 0.02724639 0.02078942 0.03755393 0.3755733
##              MASE      ACF1
## Training set 0.5928503 -0.06465986
```

The ACF and PACF show that there is still correlation left in the residuals. A better model will be estimated. The ACF and PACF both show significant spikes at 2 and 10.

```
tsdisplay(residuals(arima1))
```



##P-values are extremely small, so the residuals are distinguishable from white noise. Prediction can be used by not the forecast intervals.

```
Box.test(residuals(arima1), lag = 12, fitdf = 6, type = "Ljung")
```

```
##
## Box-Ljung test
##
## data: residuals(arima1)
## X-squared = 45.041, df = 6, p-value = 4.594e-08
```

ARIMA (3,0,0)(1,1,1)[12]

```
arima4<-Arima(lusmelec, order = c(3,0,0), seasonal = c(1,1,1))
summary(arima4)
```

```
## Series: lusmelec
## ARIMA(3,0,0)(1,1,1)[12]
##
## Coefficients:
##      ar1      ar2      ar3      sar1      sma1
##    0.6613  0.0748  0.2491  0.0600 -0.8394
## s.e.  0.0458  0.0537  0.0451  0.0547   0.0308
##
## sigma^2 estimated as 0.0007402:  log likelihood=1031.55
## AIC=-2051.09   AICc=-2050.91   BIC=-2026.13
##
```

```
## Training set error measures:
##           ME           RMSE           MAE           MPE           MAPE
## Training set 0.001762829 0.02672695 0.0204812 0.03102697 0.3699123
##           MASE           ACF1
## Training set 0.5840607 -0.02444075
```

ARIMA (2,0,1)(1,1,1)[12]

```
arima5<-Arima(lusmelec, order = c(2,0,1), seasonal = c(3,1,1))
summary(arima5)
```

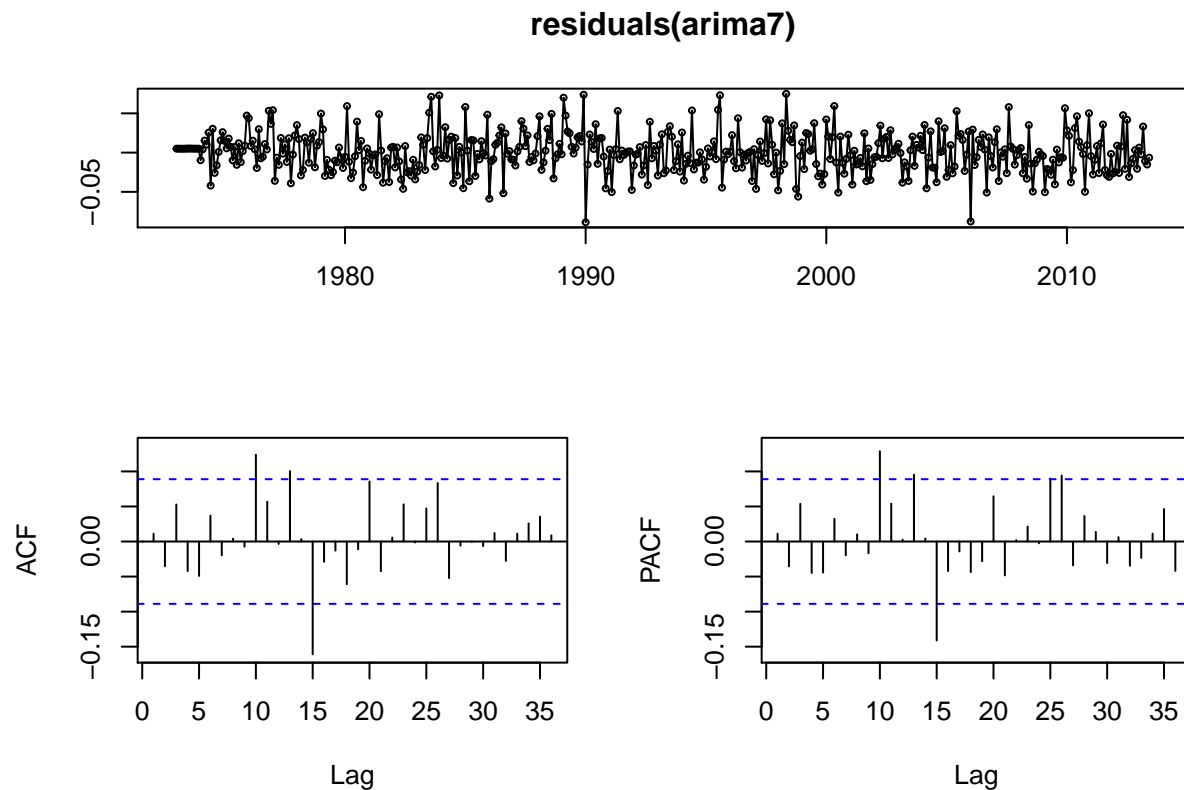
```
## Series: lusmelec
## ARIMA(2,0,1)(3,1,1)[12]
##
## Coefficients:
##           ar1           ar2           ma1           sar1           sar2           sar3           sma1
##           1.4095 -0.4104 -0.8454 -0.0427 -0.1366 -0.1010 -0.7755
## s.e. 0.0621 0.0618 0.0352 0.0651 0.0570 0.0555 0.0497
##
## sigma^2 estimated as 0.0006811: log likelihood=1051.32
## AIC=-2086.64 AICc=-2086.33 BIC=-2053.35
##
## Training set error measures:
##           ME           RMSE           MAE           MPE           MAPE
## Training set 0.0001133247 0.02558203 0.01968217 0.001622631 0.3554641
##           MASE           ACF1
## Training set 0.561275 0.01110217
```

```
arima7<-Arima(lusmelec, order = c(2,0,1), seasonal = c(3,1,1))
summary(arima7)
```

```
## Series: lusmelec
## ARIMA(2,0,1)(3,1,1)[12]
##
## Coefficients:
##           ar1           ar2           ma1           sar1           sar2           sar3           sma1
##           1.4095 -0.4104 -0.8454 -0.0427 -0.1366 -0.1010 -0.7755
## s.e. 0.0621 0.0618 0.0352 0.0651 0.0570 0.0555 0.0497
##
## sigma^2 estimated as 0.0006811: log likelihood=1051.32
## AIC=-2086.64 AICc=-2086.33 BIC=-2053.35
##
## Training set error measures:
##           ME           RMSE           MAE           MPE           MAPE
## Training set 0.0001133247 0.02558203 0.01968217 0.001622631 0.3554641
##           MASE           ACF1
## Training set 0.561275 0.01110217
```

significant spikes at 9 and 15, missing some non seasonal

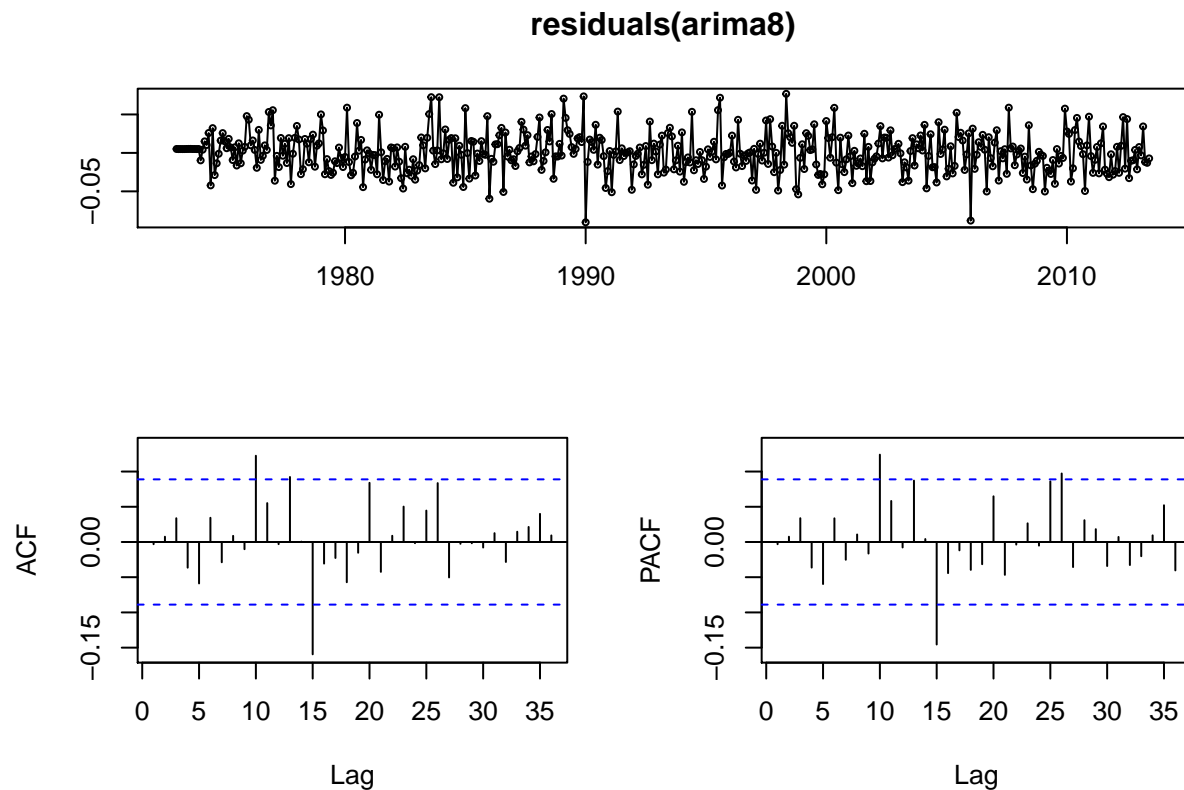
```
tsdisplay(residuals(arima7))
```



```
arima8<-Arima(lusmelec, order = c(3,0,2), seasonal = c(3,1,1))
summary(arima8)
```

```
## Series: lusmelec
## ARIMA(3,0,2)(3,1,1)[12]
##
## Coefficients:
##      ar1      ar2      ar3      ma1      ma2      sar1      sar2      sar3
##      0.8566  0.3135 -0.1718 -0.2763 -0.4628 -0.0402 -0.1327 -0.1048
## s.e.  0.2399  0.3531  0.1301  0.2356  0.2100  0.0661  0.0577  0.0586
##      sma1
##      -0.7730
## s.e.   0.0513
##
## sigma^2 estimated as 0.0006824:  log likelihood=1051.98
## AIC=-2083.96  AICc=-2083.49  BIC=-2042.35
##
## Training set error measures:
##              ME          RMSE          MAE          MPE          MAPE
## Training set 0.0001822759 0.02555216 0.0196574 0.002866167 0.3549809
##              MASE          ACF1
## Training set 0.5605684 -0.003147958
```

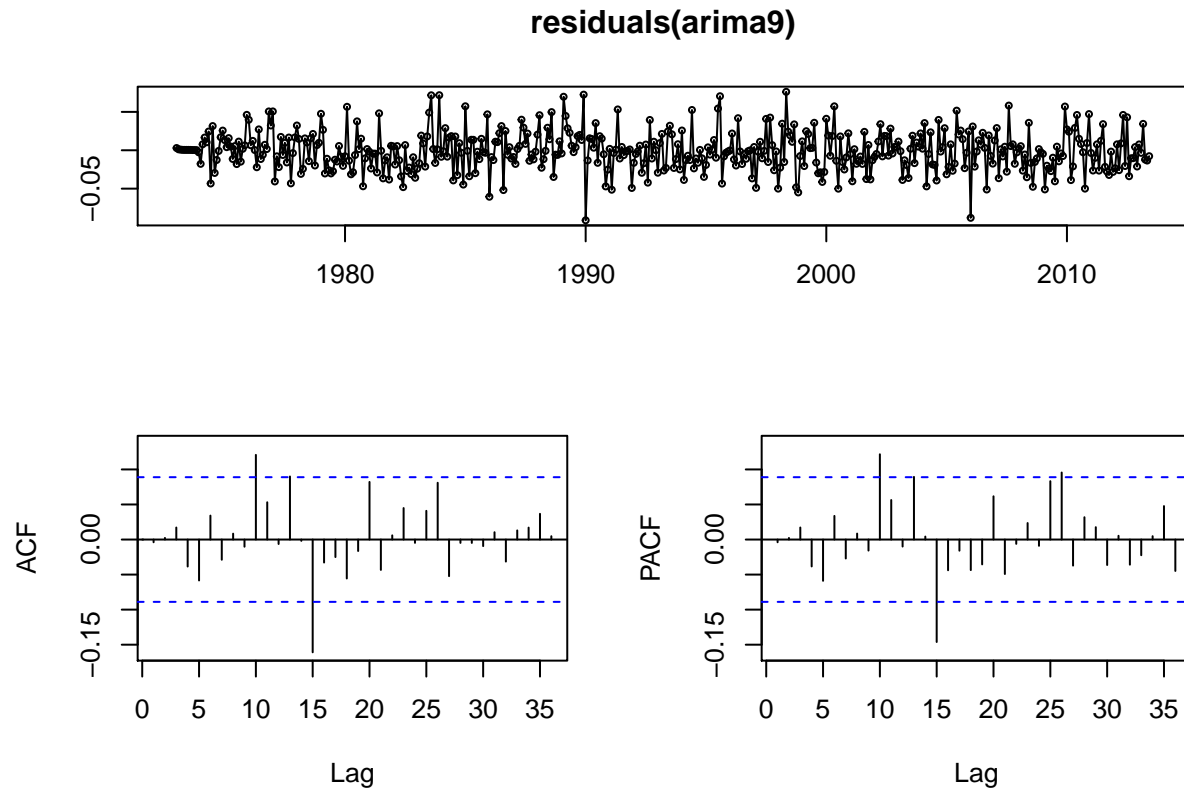
```
tsdisplay(residuals(arima8))
```

```
arima9<-Arima(lusmelec, order = c(2,1,3), seasonal = c(3,1,1))
summary(arima9)
```

```
## Series: lusmelec
## ARIMA(2,1,3)(3,1,1)[12]
##
## Coefficients:
##          ar1      ar2      ma1      ma2      ma3      sar1      sar2      sar3
##          0.0685  0.1786 -0.4894 -0.3782  0.0649 -0.0360 -0.1315 -0.1049
## s.e.      0.4525  0.1605   0.4516   0.3066  0.1077   0.0663   0.0576   0.0559
##          sma1
##          -0.7744
## s.e.      0.0506
##
## sigma^2 estimated as 0.0006827:  log likelihood=1049.54
## AIC=-2079.07   AICc=-2078.6   BIC=-2037.48
##
## Training set error measures:
##              ME          RMSE          MAE          MPE          MAPE
## Training set -0.0009605182  0.02553068  0.01961125 -0.01836626  0.354044
##              MASE          ACF1
## Training set 0.5592525 -0.003915312
```

```
tsdisplay(residuals(arima9))
```

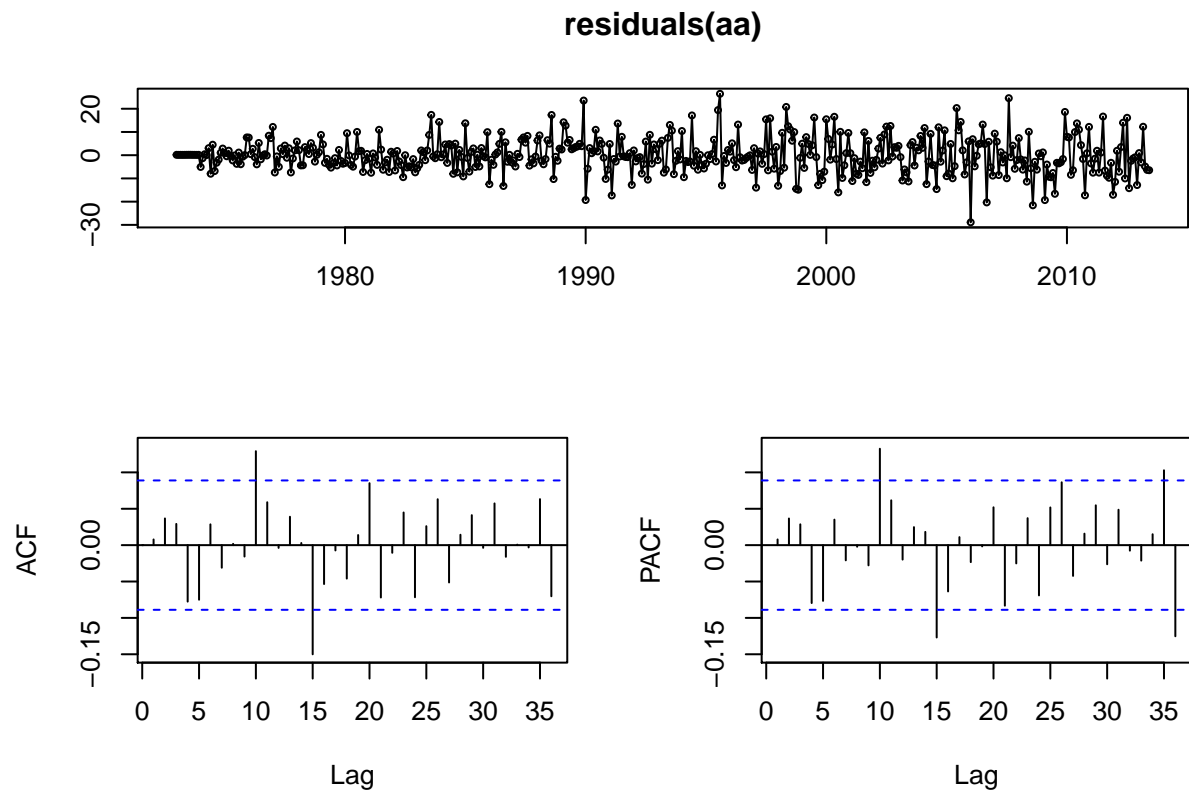


###What does the auto arima say it should be?

```
aa<-auto.arima(usmelec)
summary(aa)
```

```
## Series: usmelec
## ARIMA(1,0,2)(0,1,1)[12] with drift
##
## Coefficients:
##      ar1      ma1      ma2      sma1  drift
##      0.9717 -0.4374 -0.2774 -0.7061  0.3834
## s.e.  0.0163  0.0483  0.0493  0.0310  0.0868
##
## sigma^2 estimated as 57.67:  log likelihood=-1635.13
## AIC=3282.26  AICc=3282.44  BIC=3307.22
##
## Training set error measures:
##              ME      RMSE      MAE      MPE      MAPE      MASE
## Training set -6.778489e-05 7.460306 5.564703 -0.0698123 2.074949 0.6179747
##              ACF1
## Training set 0.007817298
```

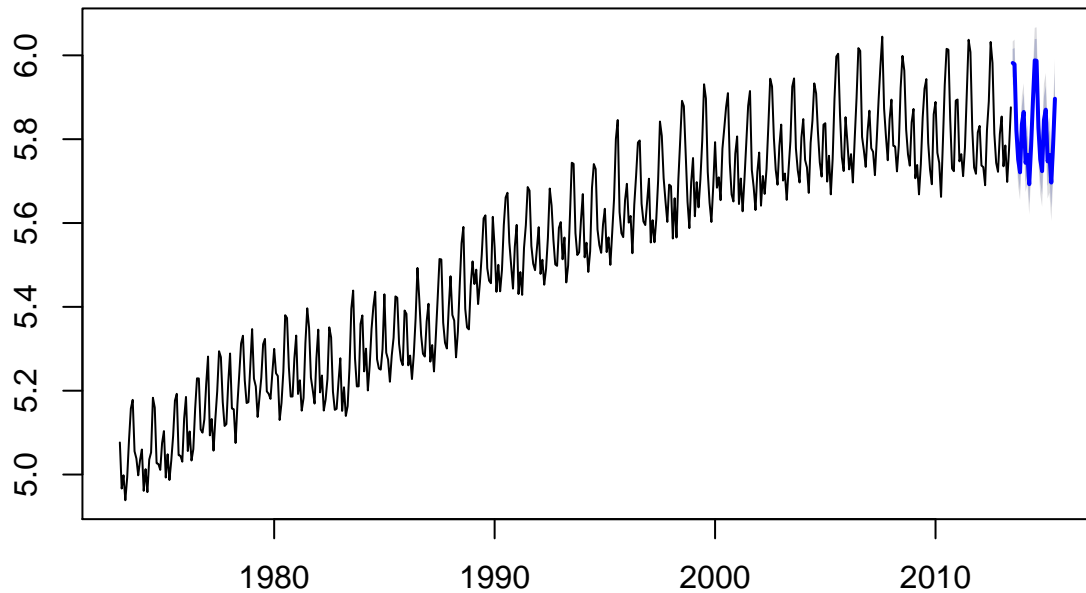
```
tsdisplay(residuals(aa))
```



(f) Forecast the next 15 years.

```
plot(forecast(arima7), n=180)
```

Forecasts from ARIMA(2,0,1)(3,1,1)[12]



(g) How many years of forecasts do you think are sufficiently accurate to be usable?

The further from the last observation we get, the less accurate and reliable the forecast is likely to be. Since we are forecasting 15 years into the future, the forecasts from year 10 on, may be highly suspect. Another issue to be wary about is the prediction intervals. Since I was unable to account for all significant spikes in building a seasonal ARIMA model, the prediction intervals should not be relied upon. The forecasts can be relied upon.