FCA based Constant and Coherent Signed Bicluster Identification and its Application in Biodiversity Study

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Abstract. The data mining task of finding coherent signed bicluster is not new in the field of gene expression data. It could also be applied in the area of computation-oriented biodiversity study with a significant impact for exposing domain-specific coherency. The present study considers a symbolic table filled with signs having meaning imposed by the users and proposes a novel signed biclustering methodology using formal concept analysis. The present work has the ability to identify both the constant and coherent signed biclusters. Moreover, aiming at revealing the usefulness of the proposed approach, we prepare a signed dataset corresponding to the spatio-temporal changes of abundance data of Sundarban mangroves, the vulnerable mangrove ecosystem. In this article, we explain our methodology theoretically with the help of a related but smaller synthetic dataset.

Keywords: Bicluster · Biodiversity · Ecosystem · Data mining · Constant and coherent clusters · Formal Concept Analysis

1 Introduction

Background and motivation: The mangrove forest is identified as one of the most threatened ecosystems in the world [27]. The major reasons lying behind the world-wide mangrove loss are anthropogenic activities and the rapid rate of climate change [10]. Furthermore, along with the declining ecosystem, coastal livelihood is also substantially affected. This situation is turned into a crucial challenge where the identification of spatial and temporal changes of mangrove cover is deadly needed [12,21] as it can lead towards the planning for long-term conservation of the mangrove forest [6,7].

From the designing point of view, biclustering [18] is related to standard clustering in a matrix along both the row and column dimensions. To form a bicluster, a subset of objects along the rows get clustered down based upon a subset of attributes along with the columns. Whereas, formal concept analysis (FCA) [4] in mathematical theory, is becoming popular [17,24], derives a concept hierarchy based upon the objects and their properties. Here the objects form a

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group by their common attributes and create a formal concept. Thus, a formal concept corresponds to a bicluster. In this aspect, it can be stated that FCA is homogeneous to biclustering as it brings out all the maximal rectangles from a binary matrix and arranges them in a hierarchical concept lattice [15].

Problem Statement: In this paper, we would like to propose a signed biclustering algorithm to derive a specific type of bicluster based upon the constant symbol and coherent symbolic changing data. Related to our addressed problem, this kind of bicluster would efficiently extract all the clustered regions having constant or coherent changes in mangrove cover considering all the aspects of the biodiversity.

Contribution:

- Here, our intention is to show the use of signed bicluster in analyzing data in the domain of biodiversity.
- For this, we have curated a dataset on the mangrove cover changes over the years for demonstrating the employment of signed bicluster.
- We have provided a formalization based upon the FCA and partition-based pattern structure by taking into account the direction of the symbolic changes and no-changes as well.
- The addressed constant and coherent sign changing bicluster is novel as we are not restricted to binary symbols. We consider the change in symbolic direction instead of the magnitude of the attribute-value. This kind of multi-symbolic sign-changing bicluster identification and its domain specific study is new in literature.
- We present a theoretical illustration of the hierarchical structure of all the biclusters and frame an interpretation of the derived clusters from the viewpoint of an expert.

Related work: The term biclustering is familiar to the researchers since the former studies in the field of gene expression data [9,3,20,19]. Conceptually, in the gene expression dataset, any bicluster seems to have a subset of genes expressing similar behavior under a subset of conditions [26]. One important variation of such bicluster is identified in [26] and named as coherent-sign-changes bicluster [18] where the gene expression values are either increasing or decreasing based upon the specific conditions under the submatrix forming the biclusters. Coming to the FCA based bicluster formation, [11,8], are the significant research articles where biclusters are discovered from the dense binary matrix. Instead of exact biclusters, here, the authors have shown a way to form approximate biclusters considering some empty cells as well. Bicluster formation from the numerical matrix is addressed in [16]. Triadic concept analysis, an extension to the formal concept analysis, is studied here. In addition to this, formal concept analysis and pattern structure-based biclustering methodology are studied in [14] where the symbolic matrix is used. Another variation, order-preserving bicluster is discussed in [15].

Regarding the study on the mangrove ecosystem, alpha diversity (special map of species richness), beta diversity (temporal changes in species composition), gamma diversity (overall diversity) of Sundarban mangrove, are studied in

[22]. These kinds of studies help in strategic planning for the conversion [25] as spatial and temporal changes in species map can identify the regions that need proper protection policies. As per [23], Bangladesh Sundarban is divided into three ecological zones, hypo-saline, meso-saline, and hyper-saline (from lower to higher salinity). But the Farakka barrage in India (1975), causes a great reduction in freshwater supply [1]. Therefore, a major transformation in the salinity level is found in different zones [6]. Along with the zonal transformation due to the salinity, vegetation pattern is also changing. [22] reveals an explicit representation of the changing pattern of the geographic range and mangrove species abundance in the interval of 1980 to 2014. Our approach for extracting signed biclusters is appropriate to study this kind of dataset for deriving knowledge in conservation policy. Using the symbolic table, it is possible to cluster down whether a species is increasing in count or decreasing, whether a species is newly appeared to a site or completely disappeared, etc.

2 Background

2.1 Related concepts

In this section, based upon the basic background of biclustering [18], formal concept, and pattern structure [5] we would discuss a few related terms [13] for the purpose of explanation of our proposed methodology. Let us consider a 2-dimensional matrix where the rows are represented by the set of objects M, and the columns are represented by the set of attributes N. So, we would work upon a dataset of (M, N) where M = {m1, m2, m3. . . ..mi}, and N = {n1, n2, n3...nj} where i is the number of rows and j is the number of columns.

1. Formal context and formal concept: For every binary relation a complete lattice can be formed [2] and this establishes the basis for the formal concept analysis. Now, considering our dataset (M, N), let R $\subseteq$ MxN, where R is a set of symbols, then, for any pair (P, Q), satisfying P $\subseteq$ M and Q $\subseteq$ N, P' = Q and Q' = P, is called a formal concept with respect to the formal context (M, N, R), i.e. a formal context is a triplet where there is a set of objects, attributes, and a relation. They form a complete lattice named as concept lattice of (M, N, R). A formal concept basically reflects that an object m $\in$ M encodes an attribute value r $\in$ R for an attribute n $\in$ N.
2. Pattern structure: It is a combination of a set of objects with their descriptions, where there is a semi-lattice among the descriptions with a similarity operation, and a mapping from the objects to the descriptions. Below, we describe a pattern structure in definition 12 denoted by (M, (V,$\sqcap$), δ).
3. Constant signed bicluster: An extracted constant-signed bicluster represents a subset of objects exhibiting similar signed values for a subset of attributes. If a subset of objects is denoted as A $\in$ M, for any attribute n *in* N, n(A) denotes the column sub-matrix. In case of a constant-signed bicluster, for i^{th} attribute and j^{th} attribute, $n_i(A) = n_j(A)$, and they would form a bicluster having an identical sign for all the elements.
4. Coherent signed bicluster: Considering the dataset (M, N), a coherent signed bicluster can be represented by (P, Q) where (P $\subseteq$ M) and (Q $\subseteq$ N). Now, if

the bicluster is column coherent, then $\forall\ q_i, q_j \in$ Q, $q_i(P)$ would be column coherent to $q_j(P)$. If the bicluster is row coherent, then $\forall\ p_i, p_j \in$ P, $p_i(Q)$ would be row coherent to $p_j(Q)$.
We define the coherency based upon the signs. As we said that coherency would be decided by the individuals, specific to our case, the coherency is discussed later.
5. Signed attribute: Let the set of attributes is denoted by N. n $\in$ N, be an attribute, and $* \in \{-1, 0, \tilde{}, +, +1\}$, be a sign. So, n^* would be called a signed attribute having sign $*$. For example, $n1^+$ could be denoted as a signed attribute where + sign is assigned to n1.
6. Signed partition component: Let s be a signed partition component and s is a subset of N. Each attribute in s posses the corresponding sign $*$. Therefore, the signed partition component or sp-component s can be represented as, s = $(n1^*, n3^*, \ldots, nn^*)$.
7. Constant sp-component: All the attributes within s exhibit the same symbol. For example, s1= $(n1^+, n3^+, n5^+)$ is a constant-sp component.
8. Coherent sp-component: It contains signed attributes where the signs exhibit coherency among themselves.
9. Equality of two sp-component: A sp-component contains attributes along with signs. Therefore, the equality of two sp-components can be recognized if they have the identical set of attributes with the same sign associated with them (constant signed cluster) or the same set of attributes coherently signed (coherent signed cluster). If s1= $(n1^+, n3^+, n5^+)$ and s2= $(n1^+, n3^+, n5^+)$, then only we could say that s1 and s2 are equal and they are constant signed bicluster. Types of coherency are discussed later. As coherency would be domain-specific, we illustrate it using an example in our case.
10. Signed partition: A signed partition P is formed by a collection of sp-components, i.e. P = $\sum_{i=1}^{n} s$. Thus P is the set of signed partitions where every attribute in N must be covered and should be present in exactly one component. Say, an object mj = $(n1^+, n2^+, n3^+, n4^-, n5^+)$, a valid signed partition could be $\{\{n1^+, n2^+, n3^+\}$,$\{n4^-, n5^+\}\}$.
Let V represents the set of all signed partitions. Now, we have to create a signed partition mapping, $\delta : M \to V$, for assigning an object to a signed partition over N. Let us say, for any object m, $\delta(m)$ represents a signed partition. It may have only one sp-component. This sp-component should cover all attributes in N for that particular object m, where m $\in$ M. For example, $\delta(m) = (n1^+, n2^+, n3^+, n4^-, n5^+)$; if in our addressed dataset (MxN), a specific row holds $(n1^+, n2^+, n3^+, n4^-, n5^+)$ entry for object m.
11. Signed partition space: The relation between any two sp-components can extract the biclusters. To illustrate the concept of extracting bicluster, we need the notation n(s) that denotes the sign for an attribute n in a sp-component s. For, s = $(n1^+, n2^+, n3^+, n4^-, n5^+)$; n5(s) = +. With this notion, the similarity between two sp-components, can be specified with $\cap$.
s1 $\cap^*$ s2 = $\{n_j^* \in s1 | n_j(s1) = n_j(s2)\}$ representing constant signed bicluster; and s1 $\cap^{*\diamond}$ s2 = $\{n_j^{*\diamond}$ where $n_j^* \in s1$ and $n_j^\diamond \in s2$ and $n_j(s1)$ coherent to $n_j(s2)\}$ representing coherent signed bicluster;

12. Signed partition pattern structure: For our considered dataset (MxN), the lattice of signed partitions of N is $(V, \sqcap)$, where $\delta : M \rightarrow V$ corresponds to an object mapped to a signed partition. A signed partition pattern structure can be denoted by $(M, (V, \sqcap), \delta)$ where (A, B) is a signed partition pattern concept. Here, A $\subseteq$ M, and B $\in$ V; a signed bicluster can be formed from a signed partition pattern concept. For the pattern concept (A, B), a signed bicluster would be (A, b) where b $\in$ B.

2.2 Constant and coherent signed biclusters: Illustration with examples with respect to our addressed biodiversity domain:

Table 1: Symbolic representation for the changes in Sundarban Mangrove Species count data in between 1986 to 2014 [22]

Sl No.	**Species**	**Hyposaline**	**Mesosaline**	**Hypersaline**
1	*Excoecaria agallocha*	$\sim$	+	$\sim$
2	*Heritiera fomes*	$\sim$	-	-
3	*Avicennia officinalis*	-	-1	-
4	*Sonneratia apetala*	-	0	-
5	*Amoora cucullata*	$\sim$	-	-
6	*Bruguiera sexangula*	+	$\sim$	$\sim$
7	*Xylocarpus moluccensis*	+	$\sim$	-
8	*Cynometra ramiflora*	-	-	0
9	*Cerbera manghas*	-	0	0
10	*Talipariti tiliaceum*	-	0	0
11	*Aegiceras corniculatum*	-	0	0
12	*Excoecaria indica*	-	0	0
13	*Tamarix dioica*	-	0	0
14	*Barringtonia racemosa*	-1	0	0
15	*Ceriops decandra*	1	1	+
16	*Sonneratia caseolaris*	-1	0	0
17	*Intsia bijuga*	+	0	0
18	*Lannea coromandelica*	-1	0	0
19	*Xylocarpus granatum*	+	-	-
20	*Pongamia pinnata*	+	0	0
21	*Syzygium fruticosum*	+	0	0
22	*Hypobathrum racemosum*	1	1	0
23	*Salacia chinensis*	0	-1	0
24	*Rhizophora mucronata*	0	1	0
25	*Lumnitzera racemosa*	0	-	0

Table 1 is derived from the data presented in an article [22] where Sundarban mangrove abundance data for 25 species are highlighted in hyposaline, mesosaline, and hypersaline zones. We represent the data symbolically to express the changes in species count data that is given in between 1986 to 2014. Each symbol is conveying a specific meaning. We have taken five such symbols, viz; 0, -, $\sim$, +, and 1; where,

0: Historical absence (A species is absent in both 1986 and 2014)

-: Range contraction (Decrease in species count)

$\sim$: Unchanged (Same abundance data in both 1984 and 2014)

+: Range expansion(Increase in species count)

1: Introduced (Absent in 1986 but present in 2014)

-1: Disappeared (Present in 1986 but absent in 2014)

Our approach for finding out both the constant and coherent signed biclusters would be appropriate for this kind of dataset. As we are elaborating the theoretical framework here, for the convenience of the illustration, and to cover all the possible aspects, we would consider a smaller synthetic symbolic dataset.

Table 2: Synthetic example dataset for illustration

	n1	**n2**	**n3**	**n4**	**n5**
m1	+	-	+	-	-
m2	+	-	+	-	~
m3	+	0	+	0	~
m4	+	0	+	0	~
m5	+	1	+	1	~

Our example dataset MxN is given in Table 2. Say, M represents a list of species and N represents a list of zones. Each cell is representing the changes in total species count between an interval in years. As specified before, for Table 2, we have considered all the five symbols: 0, -, ∼, +, and 1; representing their respective meanings. All the left-hand-side symbols including ∼, i.e. 0, -, and ∼ are forming the negative domain whereas all the right-hand-side symbols including ∼, i.e. ∼, +, 1 are forming the positive domain. The types of the biclusters are illustrated with examples below:

1. In Table 3, a positive constant bicluster is indicating an increasing species count (for species m1, m2). Or, in other words, the diversity increases in terms of species count in the regions numbered n1, n2, and n3.

Table 3: Constant bicluster

Condition 1: Constant bicluster i.e. bicluster with the same sign				
m1	$n1^{+}$	$n2^{+}$	$n3^{+}$	
m2	$n1^{+}$	$n2^{+}$	$n3^{+}$	

2. Table 4 is showing a few scenarios for condition 2. It extracts biclusters where a particular species diversity behaves homogeneously along with varying sites. At the same time, it also extracts a list of species that behave oppositely with respect to a particular site. The inversely signed elements for a particular site can be identified through this kind of biclusters. Thus the overall scenario for a site can be noted.

Table 4: Example for row constant and inverse column coherent biclusters

Condition2: Row constant + inverse column coherent values, i.e. row-wise same valued symbol and column wise symbol from the opposite domains except considering ∼													
m1	$n1^{-}$	$n2^{-}$	$n3^{-}$	or	m1	$n1^{-}$	$n2^{-}$	$n3^{-}$	or	m1	$n1^{+}$	$n2^{+}$	$n3^{+}$
m2	$n1^{+}$	$n2^{+}$	$n3^{+}$		m2	$n1^{1}$	$n2^{1}$	$n3^{1}$		m2	$n1^{0}$	$n2^{0}$	$n3^{0}$

3. Table 5 inferences that the sites (n1, n2, n3) are exhibiting diminishing biodiversity for the species m1 and m2. More specifically, m2 has been disappearing, and m1 is shrinking (denoted by the left-hand-side bicluster).

Table 5: Example for row constant and negative column coherent biclusters

Condition 3: Row constant + negative column coherent values i.e. row wise same valued symbol and column wise symbol from the negative domain								
m1	$n1^{-}$	$n2^{-}$	$n3^{-}$	or	m1	$n1^{-}$	$n2^{-}$	$n3^{-}$
m2	$n1^{0}$	$n2^{0}$	$n3^{0}$		m2	$n1^{\sim}$	$n2^{\sim}$	$n3^{\sim}$

4. Table 6 is indicating a positive response to biodiversity. As per the example shown here, m1 is expanding in nature at the sites n1, n2, and n3, whereas, m2 is newly introduced (for the left-sided bicluster) or keeping its count consistent (for the right-sided bicluster).

Table 6: Example for row constant and positive column coherent biclusters

Condition 4: Row constant + positive column coherent values, i.e. row wise same valued symbol and column wise symbol from the positive domain								
m1	$n1^{+}$	$n2^{+}$	$n3^{+}$	or	m1	$n1^{+}$	$n2^{+}$	$n3^{+}$
m2	$n1^{1}$	$n2^{1}$	$n3^{1}$		m2	$n1^{\sim}$	$n2^{\sim}$	$n3^{\sim}$

Table 7: Example for column constant and inverse row coherent biclusters

Condition 5: Column Constant + inverse row coherent values, i.e. column-wise same valued symbol and the row wise symbols are from the opposite domain except considering $\sim$										
m1	$n1^-$	$n2^+$	or	m1	$n1^-$	$n2^1$	or	m1	$n1^+$	$n2^0$
m2	$n1^-$	$n2^+$		m2	$n1^-$	$n2^1$		m2	$n1^+$	$n2^0$

5. The kind of biclusters showing in Table 7 reveals the site-wise similar changes towards species-biodiversity. This may lead to the identification of vulnerable sites where multiple species(m1, m2) counts are dropping, or they are gradually disappearing. Similarly, the positive scenario for any sites can also be identified.
6. Table 8 is showing a bicluster where site-wise, multiple species (m1, m2) are clustered down based upon the common pessimistic scenario. As per the table, both m1 and m2 are losing their biodiversity across multiple sites.

Table 8: Example for column constant and negative row coherent bicluster

Condition 6: Column constant + negative row coherent values, i.e. column wise same valued symbol and all the symbols are in negative domain.			
m1	$n1^-$	$n2^0$	$n3^\sim$
m2	$n1^-$	$n2^0$	$n3^\sim$

Table 9: Example for column constant and positive row coherent bicluster

Condition 7: Column constant + positive row coherent values, i.e. column wise same valued symbol and all symbols are from positive domain			
m1	$n1^1$	$n2^+$	$n3^1$
m2	$n1^1$	$n2^+$	$n3^1$

7. The example shown in Table 9 identifies the optimistic scenario related to biodiversity. It highlights the positive circumstances of specific sites for some specific species. As per Table 9, site n2 has a list of species with increasing species count, whereas site n1 and n3 have a newly introduced species list.
8. Table 10 identifies a bicluster with coherent evolution on the positive domain. It tends towards an overall optimistic evolution.

Table 10: Example for bicluster with coherent evolution on positive domain

Condition 8: Coherent evolution in positive domain		
m1	$n1^+$	$n2^+$
m2	$n1^+$	$n2^1$

Table 11: Example for bicluster with coherent evolution on negative domain

Condition 9: Coherent evolution on negative domain		
m1	$n1^\sim$	$n2^-$
m2	$n1^-$	$n2^0$

9. Table 11 identifies a bicluster with coherent evolution on the negative domain. It highlights the overall alarmist scenario.

3 Pattern structure for the signed partition & bicluster generation

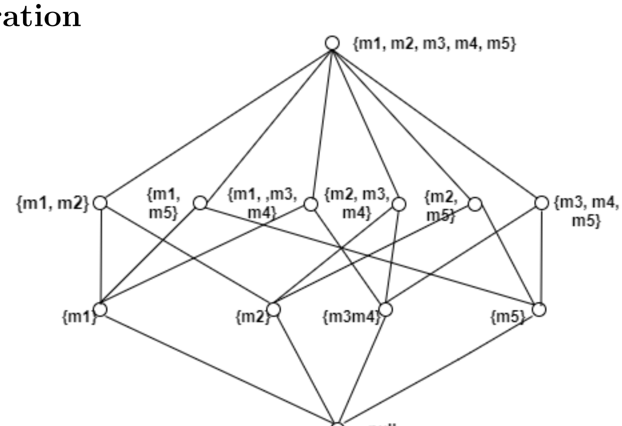

Fig. 1: Signed partition pattern lattice for the pattern concept shown in Table 12: Only the Extents are shown within the diagram.

Table 12 has listed all the signed partition patterns and the corresponding biclusters that could be derived from Table 2. The extents for the concepts of Table 12 are hierarchically shown as a lattice in Figure 1. The corresponding intents can be found in Table 12

4 Conclusions

This paper presents a novel approach to mine a multiple-signed dataset to identify both the constant and coherent signed biclusters. Bicluster has its importance in the recommendation system, along with the study related to commonly known bioinformatics. Here we have revealed another direction of using biclusters, i.e. in biodiversity study. We can conclude that this kind of signed bicluster retrieval from the spatio-temporal dataset of species vs region may help in identifying the vulnerable regions along with the unprotected or endangered species for biodiversity conservation. Not only that, it would help the conservationists for conserving or restoring a declining species, a community, or even an ecosystem. In the future, we would like to experiment with our proposed methodology to find its efficacy with respect to computation time. Another variation is required to address where the minimum number of attributes will be specified as a threshold to reduce the number of generated biclusters. Understanding the importance of fuzzy logic in dealing with uncertain data, fuzzy-based formal concept analysis in manipulating biodiversity data could be studied.

Acknowledgements

The authors are grateful to the Department of Science & Technology, Government of India, New Delhi, for financial assistance under the scheme of WOS-A (Women Scientist Scheme A) to carry out this Ph.D. research project.

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Table 12: Signed partition concepts generated from Table 2 and the respective biclusters along with their types

Concept		Bicluster		Type
Extent	Intent	Objects	Attributes	
{m1}	$\{n1^+, n2^-, n3^+, n4^-, n5^-\}$	{m1}	$\{n1^+, n2^-, n3^+, n4^-, n5^-\}$	
{m2}	$\{n1^+, n2^-, n3^+, n4^-, n5^\sim\}$	{m2}	$\{n1^+, n2^-, n3^+, n4^-, n5^\sim\}$	
{m3m4}	$\{n1^+, n2^0, n3^+, n4^0, n5^\sim\}$	{m3m4}	$\{n1^+, n2^0, n3^+, n4^0, n5^\sim\}$	
{m5}	$\{n1^+, n2^1, n3^+, n4^1, n5^\sim\}$	{m5}	$\{n1^+, n2^1, n3^+, n4^1, n5^\sim\}$	
{m1, m2}	$\{\{n1^+, n2^-, n3^+, n4^-\}\{n5^{-\sim}\}\}$	{m1, m2}	$\{n1^+, n2^-, n3^+, n4^-\}$	Column constant & inverse row coherent;
		{m1, m2}	$\{n5^{-\sim}\}$	Negative coherent evolution;
{m1, m3, m4}	$\{\{n1^+,n3^+\},\{n2^{-0}, n4^{-0}, n5^{-\sim}\}\}$	{m1, m3, m4}	$\{n1^+,n3^+\}$	Constant;
		{m1, m3, m4}	$\{n2^{-0}, n4^{-0}, n5^{-\sim}\}$	Negative coherent evolution;
{m1, m5}	$\{\{n1^+, n3^+\}\ \{n2^{-1}, n4^{-1}\},\{n5^{-\sim}\}\ \}$	{m1, m5}	$\{n1^+, n3^+\}$	Constant;
		{m1, m5}	$\{n2^{-1}, n4^{-1}\}$	Row constant & inverse column coherent;
		{m1, m5}	$\{n5^{-\sim}\}$	Negative coherent evolution;
{m2, m3, m4}	$\{\{n1^+, n3^+, n5^\sim\},\{n2^{-,0}, n4^{-,0}\}\}$	{m2, m3, m4}	$\{\ n1^+, n3^+, n5^\sim\ \}$	Positive coherent evolution;
		{m2, m3, m4}	$\{n^{-,0}, n4^{-,0}\ \}$	Row constant & negative column coherent;
{m2, m5}	$\{\{n1^+, n3^+, n5^\sim\}, \{n2^{-1},n4^{-1}\}\}$	{m2, m5}	$\{n1^+, n3^+, n5^\sim\}$	Positive coherent evolution;
		{m2, m5}	$\{n2^{-1},n4^{-1}\}\}$	Row constant & inverse column coherent evolution;
{m3, m4, m5}	$\{\{n1^+, n3^+, n5^\sim\ \}\{n2^{01},n4^{01}\}\}$	{m3, m4, m5}	$\{n1^+, n3^+, n5^\sim\ \}$	Positive coherent evolution;
		{m3, m4, m5}	$\{n2^{01},n4^{01}\}$	Row constant & inverse column coherent;
{m1,m2, m3, m4}	$\{\{n1^+, n3^+\}\ \{\ n2^{-0},n4^{-0},n5^{-\sim}\ \}\}$	{m1,m2, m3, m4}	$\{n1^+, n3^+\}$	Constant;
		{m1,m2, m3, m4}	$\{\ n2^{-0},n4^{-0},n5^{-\sim}\ \}$	Negative coherent evolution;
{m1, m2, m5}	$\{\{n1^+,\ n3^+\}\ \{\ n2^{-1},\ n4^{-1}\}\{\ n5^{-\sim}\}\}$	{m1, m2, m5}	$\{n1^+, n3^+\}$	Constant;
		{m1, m2, m5}	$\{\ n2^{-1}, n4^{-1}\}$	Row constant & inverse column coherent;
		{m1, m2, m5}	$\{\ n5^{-\sim}\}$	Negative coherent evolution;
{m1,m2, m3, m4, m5}	$\{\ \{n1^+, n3^+\}\ \{\ n2^{-01},\ n4{-}01\}\ \{\ n5^{-\sim}\}\ \}$	{m1,m2, m3, m4, m5}	$\{n1^+, n3^+\}$	Constant;
		{m1,m2, m3, m4, m5}	$\{\ n2^{-01},\ n4{-}01\}$	Row constant & inverse column coherent;
		{m1,m2, m3, m4, m5}	$\{\ n5^{-\sim}\}$	Negative coherent evolution;
{m1, m3, m4, m5}	$\{\ \{n1^+, n3^+\}\ \{\ n2^{-01},\ n4{-}01\}\ \{\ n5^{-\sim}\}\ \}$	{m1, m3, m4, m5}	$\{n1^+,n3^+\}$	Constant;
		{m1, m3, m4, m5}	$\{\ n2^{-01},n4{-}01\}$	Row constant & inverse column coherent;
		{m1, m3, m4, m5}	$\{\ n5^{-\sim}\}$	Negative coherent evolution;
{m2, m3, m4, m5}	$\{\ \{n1^+, n3^+\}\ \{\ n2^{-01},\ n4{-}01\}\ \{\ n5^{-\sim}\}\ \}$	{m2, m3, m4, m5}	$\{n1^+, n3^+\}$	Constant;
		{m2, m3, m4, m5}	$\{\ n2^{-01},n4{-}01\}$	Row constant & negative column coherent;
		{m2, m3, m4, m5}	$\{\ n5^{-\sim}\}$	Negative coherent evolution;
{m1,m2, m3, m4, m5}	$\{\ \{n1^+, n3^+\}\ \{\ n2^{-01},n4{-}01\}\ \{\ n5^{-\sim}\}\ \}$	{m1,m2, m3, m4, m5}	$\{n1^+, n3^+\}$	Constant;
		{m1,m2, m3, m4, m5}	$\{\ n2^{-01},n4{-}01\}$	Row constant & inverse column coherent;
		{m1,m2, m3, m4, m5}	$\{\ n5^{-\sim}\}$	Negative coherent evolution;

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