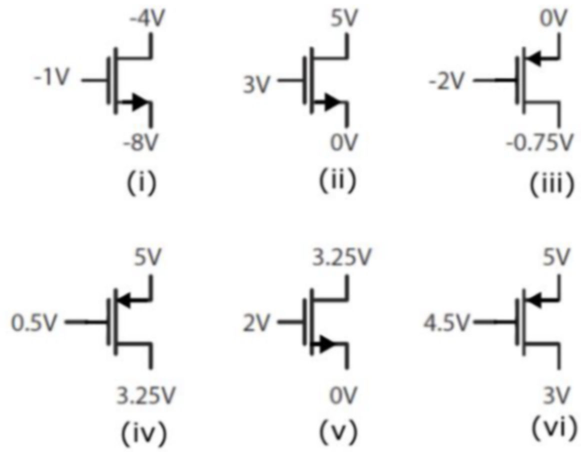
0.1 Practice Problems

1. Find the region of operation for the following transistors. You may use $V_{tn} = 0.9$V and $| V_{tp} = 1$V. The operation condition of NMOS is also shown. PMOS is conducted by holes which is opposite to NMOS so the voltage polarity is different.



Region of operation	Condsitions	i_{DS}
Cut-off	$v_{GS} < V_T$	$i_{DS} \sim 0$A
Linear/Triode	$V_{GS} > V_T, V_{DS} < V_{GS} - V_T$	$i_{DS} = \frac{W}{L}\mu_n C_{ox}(V_{GS} - V_T - \frac{V_{DS}}{2})V_{DS}$
Saturation:	$V_{GS} > V_T, V_{DS} > V_{GS} - V_T$	$i_{DS} = \frac{W}{L}\frac{\mu_n C_{ox}}{2}(V_{GS} - V_T)^2(1 + \lambda V_{DS})$

(a) We identify this transistor as a NMOS transistor due to the direction of the arrow. In a NMOS transistor, current flows from drain to source due to electrons being the charge carriers in the NMOS transistor. $v_{GS} = v_G - v_S = 7$V and $v_{DS} = v_D - v_S = -4 - (-8) - 4$V. This transistor is in the linear/triode region.

(b) $v_{GS} = 3$V and $v_{DS} = 5$V. This transistor is in the saturation region.

(c) This is a PMOS transistor. In a PMOS transistor, current flows from source to drain. $|v_{GS}| = 2$V and $|v_{DS}| = 0.75$V. This is in the linear/triode region.

(d) $|v_{GS}| = 4.5$V and $|v_{DS}| = |3.25 - 5| = 1.75$V. This is in the linear/triode region.

(e) $v_{GS} = 2$V and $v_{DS} = 3.25 - 0 = 3.25$V. This is in the saturation region.

(f) $|v_{GS}| = |4.5 - 5| = 0.5$V. This is in the cutoff region.

2. An ideal N-channel MOSFET has the following parameters: $W = 100$ μm, $L = 1$ μm, $t_{ox} = 15$ nm, the oxide relative permittivity is 4, the silicon relative permittivity is 12, $N_A = 10^{15}$ cm^{-3}, $n_i = 10^{10}$ cm^{-3}, $V_{FB} = -0.2$ V, $\mu_n = 300$ cm^2/V $\cdot$ sat 300K. $\lambda = 0$.

(a) Find the threshold voltage.

Threshold voltage is given by the following formula:

$$\phi_B = \frac{kT}{q}\ln\frac{N_A}{n_i} = 0.026V\ln\left(\frac{10^{15}\text{cm}^{-3}}{10^{10}\text{cm}^{-3}}\right) = 0.2993\ldots V$$
$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{4(8.854\times10^{-14}\text{F/cm})}{15\text{nm}} = 3.54\times10^{-7}\text{F/cm}^2$$
$$V_t = V_{FB} + 2\phi_B + \frac{\sqrt{2q\epsilon_s N_A 2\phi_B}}{C_{ox}}$$
$$= -0.2 + 2(0.299) + \frac{2(12)(8.854\times10^{14}\text{F/cm})(1.602\times10^{-19}C)(10^{15}\text{cm}^{-3})(2)(0.2993V)}{3.54\times10^{-7}\text{F/cm}^2}$$
$$= 0.44\text{V}$$

(b) What is the minumum v_{DS} value for the MOSFET to be at saturation region at v_{GS} = 2V.
For the MOSFET to remain in saturation, v_{DS} = v_{GS} - $V_t n$, so

$$V_{DS} - V_{DS} - V_{tn} = 2 - 0.44 = 1.56\text{V}$$

(c) Find the channel resistance at v_{GS} = 2V and v_{DS} = 0.01 V.
Channel resistance is given by calculating what i_{DS} is given that we know v_{DS} is.

$$R_{ch} = \frac{V_{DS}}{I_{DS}} = \frac{0.01V}{\frac{W}{L}\mu_n C_{ox}(V_{GS}-V_T)V_{DS}}$$
$$= \frac{1}{300\times100\times3.54\times10^{-7}\times1.56}$$
$$= 60.36\Omega$$

(d) Find I_D at v_{GS} = 2V and v_{DS} = 1V.
At these values, the transistor is in the linear/triode region, so

$$I_D = \frac{\mu_n W}{L}C_{ox}((v_{GS}-V_{tn})v_{DS} - \frac{v_{DS}^2}{2})$$
$$= 300\times100\times3.54\times10^{-7}(2-0.44)(1-\frac{1^2}{2})$$
$$= 0.011A$$

(e) Find I_D at v_{GS} = 2V and v_{DS} = 2V.
At these values, the transistor is in the saturation region.

$$I_D = \frac{\mu_n W}{2L}C_{ox}(v_{gs}-v_t)^2$$
$$= \frac{300}{2}\times100\times3.54\times10^{-7}\times(2-0.44)^2$$
$$= 0.013A$$

3. A circuit designer intending to operate a MOSFET in saturation is considering the effect of changing the device dimensions and operating voltages on the drain current I_D. Specifically, by what factor does I_D change in each of the following cases?

(a) The channel length is doubled.
The problem says that the MOSFET is in saturation. I_D in saturation is

$$I_D = \frac{1}{2}\mu_n C_{ox}(\frac{W}{L})(v_{GS} - V_{tn})^2$$

From the above equation, we see that I_D is inversely proportional to channel length. So, if channel length is doubled then I_D will be halved.

(b) The channel width is doubled.
I_D will be doubled (refer to equation for I_D in saturation above).

(c) The overdrive voltage is doubled.
Overdrive voltage is equal to v_{GS} - V_t, so I_D will quadruple if the overdrive voltage is doubled.

(d) The drain to source voltage is doubled.
If we doubled v_{DS} , there will be no effect on the drain current since v_{DS} already reaches overdrive voltage drain current saturates and remains constant.

(e) Changes (a), (b), (c), and (d) are made simultaneously.
Simultaneously doing change (a) and change (b) results in the current remaining constant. Double the overdrive voltage results in drain current quadrupling while (d) will not change drain current so the drainc current will stay

(f) Which of these changes might cause the MOSFET to leave the saturation region?
Decreasing v_{DS} may cause this since this may change the channel underneath.

4. An N-channel MOSFET with the following parameters: $W = 10\ \mu$m, $L = 1\ \mu$m, $V_t = 0.5$V, $\mu_n = 400\ $ cm^2/V $\cdot$ s, $C_{ox} = 4 \times 10^{-7}$ F/cm^2, $\lambda = 0.01$V^{-1}.

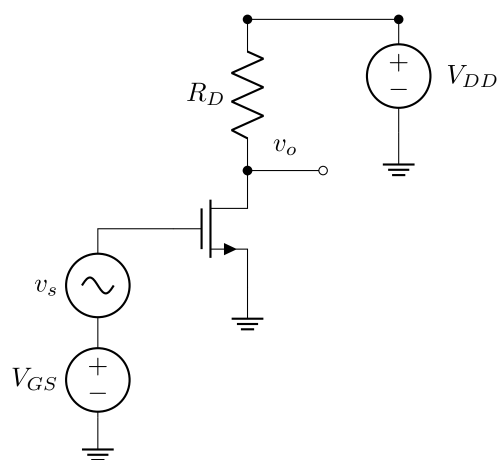


Figure 1: Circuit for this problem

(a) Calculate g_m at v_{GS} = 1V and v_{DS} = 1.2V.
The formula is

$$\begin{aligned}
g_m &= \mu_n C_{ox}\frac{W}{L}(v_{GS} - V_t)(1 + \lambda V_{DS}) \\
&= (400\frac{cm^2}{V \cdot s})(4 \times 10^{-7}\frac{F}{cm^2})(\frac{10\mu\text{m}}{1\mu\text{m}})(1V - 0.5V)(1 + 0.01V^{-1}(1.2V)) \\
&= 8.096 \times 10^{-4}\Omega^{-1} \\
&= 8.096 \times 10^{-4}S
\end{aligned}$$

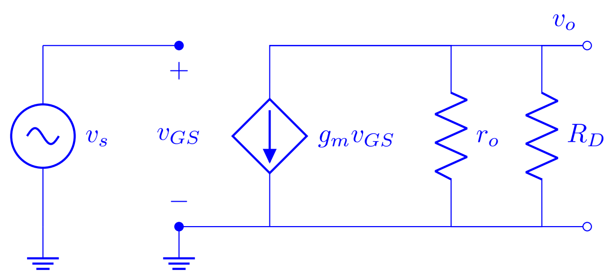
(b) Calculate r_o at v_{GS} = 1V and v_{DS} =1.2V.

Formula for r_o (output resistance) is

$$\begin{aligned} r_o &= (\frac{\partial I_{DS}}{\partial v_{DS}})^{-1} \\ &= \frac{1}{\mu_n C_{ox} \frac{W}{2L}(v_{GS} - V_t)^2 \lambda} \\ &= \left((\frac{400}{2} \frac{cm^2}{V \cdot s})(4 \times 10^{-7} \frac{F}{cm^2})(\frac{10\mu\text{m}}{1\mu\text{m}})(1V - 0.5V)(0.01V^{-1}) \right)^{-1} \\ &= 500k\Omega \end{aligned}$$

(c) Find the gain $\frac{v_o}{v_s}$ of this common source amplifier using this NMOS with v_{GS} = 1V, R_D = 1 kΩ and V_{DD} = 1.4V. You can ignore the λ when finding the DC bias point for simplicity. Don't need to consider capacitances and g_{mb}.

Draw the small signal model.



We see that v_{GS} = v_s. Find v_o by Ohm's Law and equivalent resistance.

$$\begin{aligned} v_o &= -g_m v_{GS}(r_o || R_D) \\ &= -g_m v_s (r_o || R_D) \\ \frac{v_o}{v_s} &= -g_m (r_o || R_D) \\ &= -8.096 \times 10^{-4}\Omega^{-1}(500k\Omega || 1k\Omega) \\ &= -0.808 \end{aligned}$$

5. For a PMOS common source amplifier:

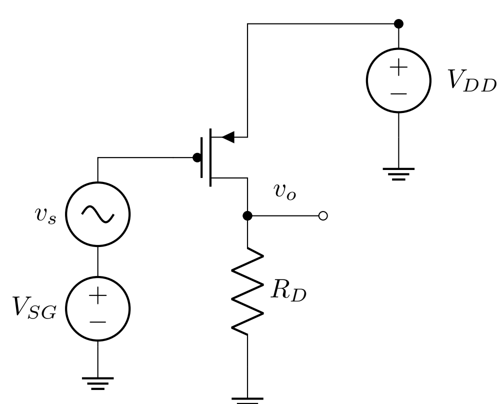


Figure 2: Circuit for this problem

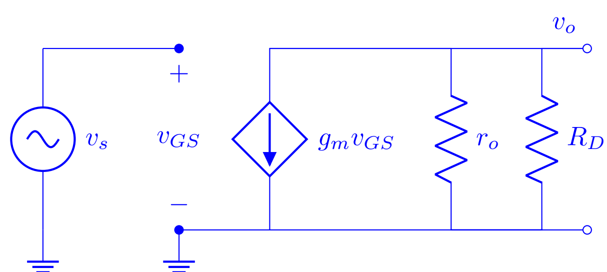
(a) If $v_{SG} - |V_{tp}| = V_0$, and $k = \mu_p C_{ox} \frac{W}{L}$, find the maximum R_D symbolically to make PMOS operation in the saturation region. Assume $\lambda = 0$.

For a PMOS transistor, it will be in saturation if $V_{SD} \geq V_{SD} - V_{tp}$

$$
\begin{aligned}
I_{DS} &= \frac{\mu C_{ox}}{2} \frac{W}{L} (V_{GS} - V_t)^2 \\
&= \frac{k}{2} (V_{GS} - V_t)^2 \\
&= \frac{k}{2} V_0^2 \\
V_{SD} &= V_{DD} - R_D I_{DS} \geq V_0
\end{aligned}
$$

$$
\begin{aligned}
V_{DD} - V_0 &\geq R_D I_{DS} \\
V_{DD} - V_0 &\geq R_D \frac{k V_0^2}{2} \\
R_D &\leq \frac{2(V_{DD} - V_0)}{k V_0^2}
\end{aligned}
$$

(b) Draw its small-signal equivalent circuit and find out the $A_v = \frac{v_o}{v_s}$ symbolically (R_D, g_m, r_o, g_{mb}, and no capacitances). Please consider a finite PMOS output resistance (r_o) and assume it is in the saturation region. Body is grounded.



Notice that the small signal model for a PMOS and NMOS transistor are the same here.

$$
\begin{aligned}
v_o &= -g m v_s (r_o || R_D) \\
A_v &= \frac{v_o}{v_s} = -g_m (r_o || R_D) \\
A_v &= \frac{v_o}{v_s} = -g_m \frac{r_o R_D}{r_o + R_D}
\end{aligned}
$$

(c) For the same circuit in part (b) but the body is connected to v_o, find A_v.

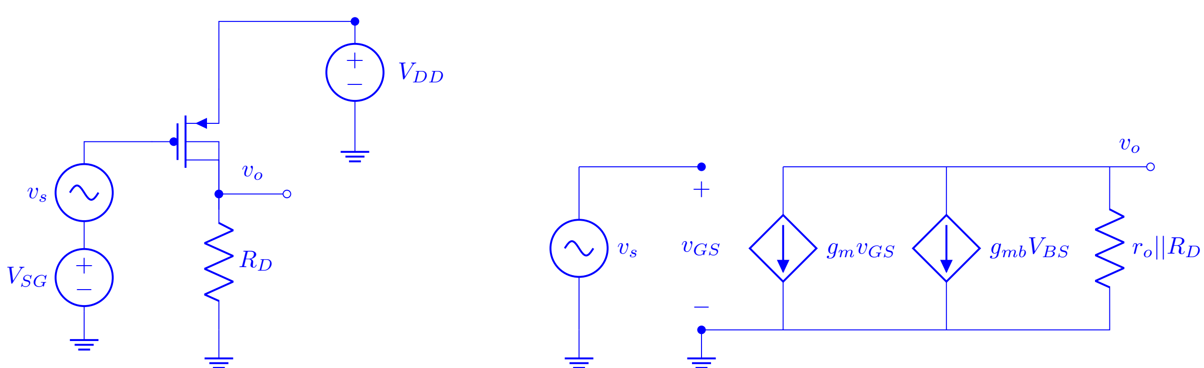


Figure 3: Circuit for this problem