

# Contents

<b>1</b>	<b>Charge Carriers and Doping</b>	<b>3</b>
1.1	Introduction . . . . .	3
1.2	Conduction and Fermi Dirac Distribution . . . . .	4
1.3	Doping . . . . .	5
1.4	Compensated Semiconductor Material . . . . .	6
1.5	Practice Problems . . . . .	6
1.6	Sources . . . . .	8
<b>2</b>	<b>Drift and Diffusion Current</b>	<b>9</b>
2.1	Drift Current . . . . .	9
2.2	Diffusion Current . . . . .	9
2.3	Practice Problems . . . . .	10
2.4	Sources . . . . .	11
<b>3</b>	<b>PN Junctions</b>	<b>12</b>
3.1	Diffusion Current . . . . .	12
3.2	Drift Current and Equilibrium . . . . .	13
3.3	Reverse Bias . . . . .	15
3.4	Forward Bias . . . . .	16
3.5	Practice Problems . . . . .	16
3.6	Sources . . . . .	17
<b>4</b>	<b>MOS Capacitors</b>	<b>19</b>
4.1	MOSCAP at Equilibrium . . . . .	19
4.2	Regions of Operation . . . . .	19
4.3	Practice Problems . . . . .	19
4.4	Sources . . . . .	19

1. use this article to add to ch1 about doping
2. practice problems for ch1
3. practice problems for ch2
4. practice problems for ch3
5. finish up ch3 i burned out while trying to be thorough. p117 in reader, p152 in sedra
6. dead week planning
  - (a) tuesday: finish pn junction, diodes, mosfets
  - (b) Wednesday: moscap, amplifiers maybe small signal model

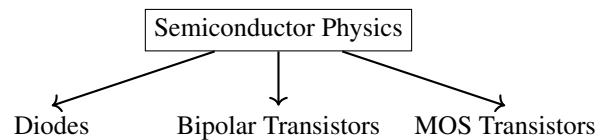
x

# Chapter 1

## Charge Carriers and Doping

### 1.1 Introduction

We start learning about resistors, capacitors, and inductors from earlier courses such as EECS 16A and 16B. With more components like transistors, diodes, and op-amps (which are all based on semiconductors), we are able to expand upon circuit design. We need to understand semiconductor physics in order to understand how these components operate.



We can also redefine Ohm's Law which we know as ( $V = IR$ ) as  $J = \sigma \mathbf{E}$ . We can also write resistance( $R$ ) and conductance( $G$ ) in terms of other variables

- $J$ : current density,  $A/m^2$  (Amperes per meter squared)
- $\sigma$ : conductivity,  $S/m$  (Siemens per meter)
- $\mathbf{E}$ : electric field,  $V/m$  (Volts per meter)

$$R = \frac{R}{l} = \rho \frac{l}{A} \text{ and } G = R^{-1} = \sigma \frac{A}{l}$$

- $\sigma$ : conductivity
- $\rho$ : resistivity

For collisions in gas, we focus on the idea that initial velocity and direction is lost/randomized after a few collisions. So, when we sum over the random velocities of the particles and average it, it comes out to zero. Average momentum gain is:

$$\bar{\mu} = \frac{\mathbf{E}q\tau}{M} = \mu \mathbf{E}, \quad \mu := \frac{q\tau}{M} = \frac{\bar{v}}{\mathbf{E}}$$

- $\mu$ : mobility,  $m^2/(V \cdot s)$
- $q$ : electric charge,  $1.60 \times 10^{-19}$ , Coulombs = Amperes/second
- $\tau$ : mean free time
- $M$ : mass
- $\bar{v}$ : average velocity

Different elements have a different number of outer shell electrons. For semiconductors like silicon, we can increase the temperature to increase its conductivity. Silicon atoms are arranged in a diamond structure and in general, the energy levels that an atom can occupy are discrete. The **valence band** electrons are at a lower energy state (bound to host atoms) while **conduction band** electrons are at a higher energy state and are "free" electrons. These electrons are free to move around the crystal and take part in conduction.

## 1.2 Conduction and Fermi Dirac Distribution

Thermal energy is on average about  $\sim 26 \text{ eV}$  at room temperature. How large the **band-gap**, the gap between the conduction and valence band, determines how conductive a material is:

- Insulators: band gap  $\sim 15 \text{ eV}$ 
  - Glass, rubber, oil, plastic, diamond
- Semiconductors: band gap  $\sim 1 \text{ eV}$ 
  - Silicon =  $1.12 \text{ eV}$
- Conductors: Not applicable due to overlapping conduction/valence bands

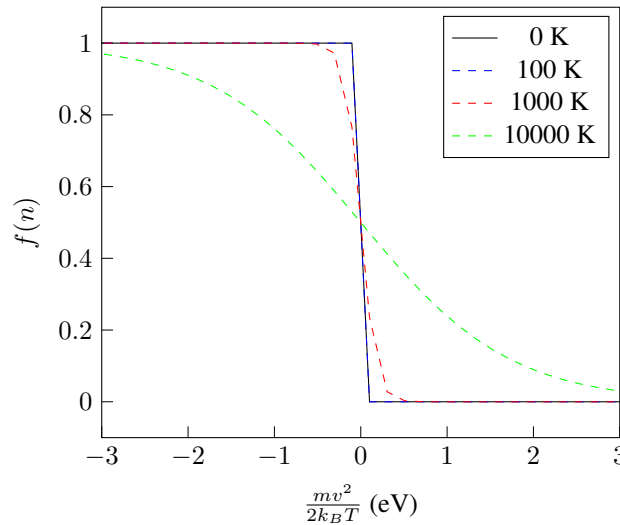
This band gap energy is the minimum energy required to break a covalent bond and generate an electron-hole pair.

Because electrons are a type of particle called a **fermion**, we can say that

$$f(E) = \frac{1}{e^{\frac{E-E_F}{k_B T}} + 1}$$

- $f(E)$ : occupational probability of a state energy  $\epsilon$
- $E$ : energy of the particle
- $E_F$ : fermi energy, eV
- $k_B$ : Boltzmann's constant,  $1.380649 \times 10^{-23} \text{ J/K}$  (Joules per Kelvin),  $8.62 \times 10^{-5} \text{ eV/K}$
- $T$ : temperature in Kelvin

The graph below is a plot of the Fermi-Dirac Distribution at different temperatures. We can use this plot to find the probability of the concentration of electrons and holes at certain temperatures.



Ok so why is this graph significant. To repeat, we can relate the concentration of holes or electrons to the temperature of its surroundings. We see that for the plot for a high temperature, there is a greater probability of charge carriers having enough energy to cross into the conduction band.

**Thermal generation** occurs when sufficient thermal energy exist to break some covalent bonds, which results in a **free electron**. This free electron is now free to conduct electric current if an electric field is applied to the crystal. However, this leaves behind a net positive charge. **Holes** are positively charged carrers, which can also move around the crystal and are available to conduct electric current. As temperature increases, so do electron-hole pairs and the conductivity of the crystal.

**Recombination** is the process of free electrons and holes moving through the silicon crystal structure and electrons filling in some of the holes. The recombination rate is proportional to the number of free electrons and holes, which also determines the **thermal generation** rate.

At thermal equilibrium,  $n = p = n_i$ . We can alternatively express as

$$pn = n_i^2$$

- $n$ : concentration of free electrons,  $cm^{-3}$
- $p$ : concentration of holes,  $cm^{-3}$
- $n_i$ : number of free electrons nd holes in a unit volume ( $cm^3$ ) of intrinsic silicon at a given temperature

We can also write  $n_i$  as a function of temperature

$$n_i = BT^{3/2}e^{-E_G/2kT}$$

- $B$ : material-dependent parameter,  $7.3 \times 10^{15} cm^{-3} K^{-3/2}$  for Silicon
- $T$ : temperature in  $K$
- $E_G$ : bandgap energy, 1.12 eV for silicon
- $k$ , Boltzmann's constant

## 1.3 Doping

In a non-doped crystal like in the above texts, there will be equal concentrations of free electrons and holes from thermal generation. **Doping** is defined as introducing impurities inside a silicon crystal to adjust the carrier concentration. The following materials are commonly used:

- Group III elements: boron, aluminum, gallium  $\rightarrow$  acceptors; increase holes
- Group IV elements: germanium and silicon
- Group V elements: phosphorus, arsenic, antimony  $\rightarrow$  donors; increase free electrons

Doping with donors  $\rightarrow$  **n type**.

Doping with acceptors  $\rightarrow$  **p type**

Sometimes we assume that the number of electrons we add is much greater than the original free electrons that pure silicon had ( $10^{10}$  per cubic centimeter). This leads to the simplification that the number of free electrons in our silicon crystal is  $N_D$ , where  $N_D$  is the number of donor atoms that we add per cubic centimeter.

**Analysis : Doping concentrations**

Suppose we dope silicon with phosphorus, an acceptor. This means that four electrons from its outer shell will form a covalent bond with silicon. This results in one free electron, meaning that each phosphorus atom donates one free electron to the silicon. No holes are generated during this process. Usually, the concentration of donor atoms,  $N_D$ , is much greater than  $n_i$ , so

$$n_n \simeq N_D$$

- $n_n$ : for  $n$ -type silicon, concentration of free electrons
- $p_n$ : for  $n$ -type silicon, concentration of holes
- $N_D$ : donor doping concentration
- $N_A$ : acceptor doping concentration

$n_n$  is determined by doping concentration, and independent of temperature, while  $p_n$  is determined by:

$$\begin{aligned} p_n n_n &= n_i^2 \\ p_n &\simeq \frac{n_i^2}{N_D} \end{aligned}$$

An analysis with boron, an acceptor, will yield similar conclusions with a change in variables.

If the concentration of donors/acceptors is greater than the other by a factor of at least 100, then we can say that  $n/p = N_D/N_A$ , respectively.

## 1.4 Compensated Semiconductor Material

We can dope a semiconductor with both acceptors and donors. A **compensated semiconductor material** has both acceptors and donors.

**N-type material:**  $N_D > N_A$

$$n \approx N_D - N_A, \quad p \approx \frac{n_i^2}{N_D - N_A}$$

**P-type material:**  $N_D < N_A$

$$p \approx N_A - N_D, \quad n \approx \frac{n_i^2}{N_A - N_D}$$

## 1.5 Practice Problems

1. Calculate the intrinsic carrier density  $n_i$  for silicon at  $T = 50$  K and 350 K.

$$\begin{aligned} n_{i,1} &= BT^{3/2}e^{-E_G/2kT} \\ &= (7.3 \times 10^{15} \text{cm}^{-3} \text{K}^{-3/2})(50 \text{K})^{3/2}e^{-1.12 \text{eV}/2(8.62 \times 10^{-5} \text{eV/K})(50 \text{K})} \\ &= 9.63217875 \times 10^{-39} / \text{cm}^3 \\ n_{i,2} &= (7.3 \times 10^{15} \text{cm}^{-3} \text{K}^{-3/2})(350 \text{K})^{3/2}e^{-1.12 \text{eV}/2(8.62 \times 10^{-5} \text{eV/K})(350 \text{K})} \\ &= 4.15216354 \times 10^{11} / \text{cm}^3 \end{aligned}$$

2. Consider an  $n$ -type silicon for which the dopant concentration  $N_D = 10^{17} \text{ cm}^{-3}$ . Find the electron and hole concentrations at 350 K. Refer to previous example for the value of  $n_i$  at  $T = 350 \text{ K}$ .

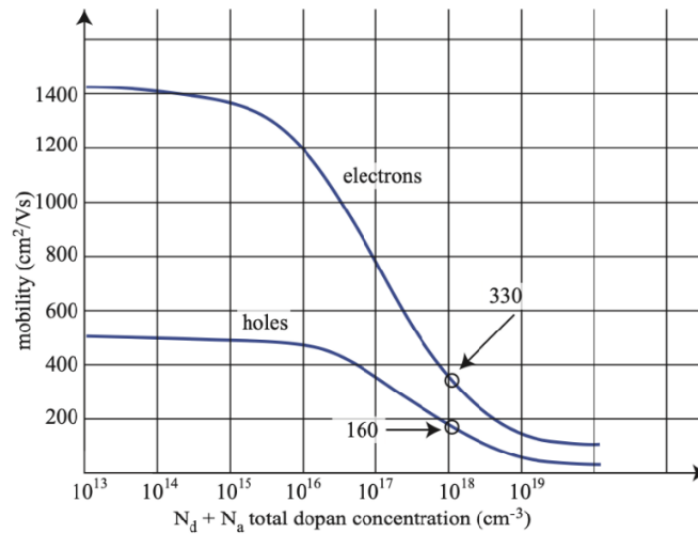
Since this is an  $n$ -type silicon,

$$n_n \simeq N_D = 10^{17} / \text{cm}^3$$

From this we can find the concentration of minority holes as

$$p_n \simeq \frac{n_i^2}{N_D} = \frac{(4.15 \times 10^{11})^2}{10^{17}} = 1.72 \times 10^6 / \text{cm}^3$$

3. Here is a  $1 \mu\text{m}$  long Si bar doped with  $6 \times 10^{15} \text{ cm}^{-3}$  of As and  $4 \times 10^{15} \text{ cm}^{-3}$  at 300 K. Assume  $n_i = 10^{10} \text{ cm}^{-3}$ . Answer the following questions (show the unit). Assume complete ionization. Use the given figure to find the mobility.



- (a) Find the concentration of  $n$  and  $p$ .

Arsenic is a donor, meaning that it has 5 valence electrons in its outermost shell.  $n \approx 6 \times 10^{15} - 4 \times 10^{15} = 2 \times 10^{15}$

$$p = \frac{n_i^2}{n} = 5 \times 10^4 \text{ cm}^{-3}$$

- (b) Calculate its resistivity.

$$\rho = \frac{1}{q(\mu_n n + \mu_p p)}$$

Looking at the graph we see that at  $N_A + N_D = 10^{16}$ ,  $\mu_n \approx 1200 \text{ cm}^2/\text{V} \cdot \text{s}$  and  $\mu_p \approx 450 \text{ cm}^2/\text{V} \cdot \text{s}$ . We can also simplify our equation above as

$$\rho \approx \frac{1}{q\mu_n n} = 2.6 \Omega \text{ cm}$$

- (c) Find the current density when 1 V is applied across the  $1 \mu\text{m}$  length direction.

$$J = \sigma E = \frac{E}{\rho} = \frac{1}{2.6 \times 10^{-4} \text{ cm}} \approx 3840 \text{ A/cm}^2$$

- (d) If we keep increasing the voltage, will the current keep increasing?

No, due to velocity saturation.

## 1.6 Sources

- [Razavi Electronics 1, Lec 1, Intro., Charge Carriers, Doping](#)
- EE105 Reader
- Sedra, Adel S., et al. Microelectronic Circuits. Oxford University Press, 2021
- Engineering LibreTexts: The Fermi-Dirac Distribution
- Fermi-dirac distribution graph
- Q3 appeared as Q1 as EE105 HW6



## Chapter 2

# Drift and Diffusion Current

Total current flow is made up of drift current and diffusion current. If we apply an electric field  $E$  to a semiconductor crystal, then holes accelerate in the direction of  $E$  and free electrons accelerate in the opposite direction of  $E$ . While **drift current** is movement caused by electric fields, **diffusion current** is movement caused by variation in the carrier concentration.

### 2.1 Drift Current

$$v_{p-drift} = \mu_p E, \quad v_{n-drift} = -\mu_n E$$

- $E$ : electric field, V/cm
- $v_{p-drift}$ : drift velocity of holes, cm/s
- $v_{n-drift}$ : drift velocity of electrons, cm/s
- $\mu_p$ : hole mobility,  $\text{cm}^2/\text{V} \cdot \text{s}$ ,  $480 \text{ cm}^2/\text{V} \cdot \text{s}$  for intrinsic silicon
- $\mu_n$ : electron mobility,  $\text{cm}^2/\text{V} \cdot \text{s}$ ,  $1350 \text{ cm}^2/\text{V} \cdot \text{s}$  for intrinsic silicon

Current density is the current per unit cross-sectional area

$$J_p = qn\mu_p E, \quad J_n = qn\mu_n E$$

Total drift current density is

$$J = J_p + J_n = q(p\mu_p + n\mu_n)E = \sigma E$$

Via pattern matching, we can see that conductivity  $\sigma$  is given by  $\sigma = q(p\mu_p + n\mu_n)$  and resistivity is the inverse of this.

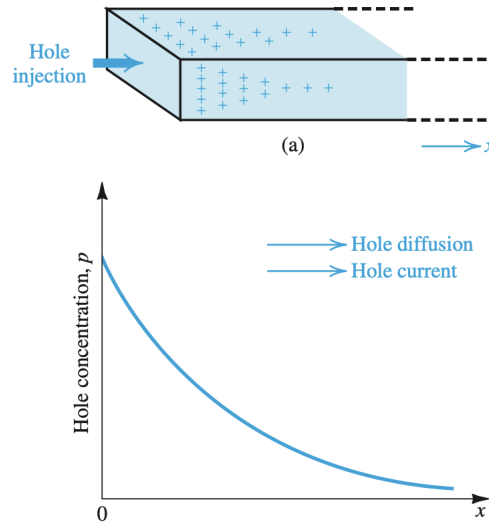
### 2.2 Diffusion Current

**Diffusion current** is the diffusion of charge carriers that give rise to a net flow of charge from a region of high concentration to a region of low concentration. This implies that we can have current without voltage. We can write the current density as a function of the **concentration gradient** (slope of the concentration profile) at any point such that:

$$J_p = -qD_p \frac{dp(x)}{dx}, \quad J_n = qD_n \frac{dn(X)}{dx}$$

- $J_p, J_n$ : hole/electron-current density, A/cm<sup>2</sup>
- $q$ : magnitude of the electron charge
- $D_p, D_n$ : diffusion constant or diffusivity of holes/electrons. For intrinsic silicon,  $D_p = 12$  cm<sup>2</sup>/s and  $D_n = 35$  cm<sup>2</sup>/s
- $p(x), n(x)$ : hole/electron concentration at point  $x$ . Note that if  $\frac{dp(x)}{dx} < 0$ ,  $J_p$  ends up being positive.

This equation shows that current density is proportional to the slope of the concentration. The image below shows a bar of silicon and an injection of holes on the left side, which will result in hole diffusion current in the same direct (positive direction of  $x$ )



Suppose we fill a gas chamber that is divided into two sections with a gases of temperature  $T$  on one side. If we remove this divider, the gas will fill the entire volume of the new chamber. This occurs due to the concentration gradient. If the gas molecules here were charged, there would be a net current flow.

#### Analysis : Einstein Relationship

The following equation is known as the **Einstein relationship**:

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T = \frac{kT}{q}$$

- $V_T$ : thermal voltage; at  $T \simeq 300$  K,  $V_T = 25.9$  mV

We see that the diffusion constant is related to the mobility.

Total current will be given by the sum of drift and diffusion currents. In resistors, since the carrier is approximately uniform, diffusion current is nearly zero.

$$J^n = J_{drift}^n + J_{diff}^n = q\mu_n nE + qD_n \frac{dn}{dx}$$

## 2.3 Practice Problems

1. A uniform bar of  $n$ -type silicon of  $2\text{-}\mu\text{m}$  length has a voltage of 1 V applied across it. If  $N_D = 10^{16}$  cm<sup>-3</sup> and  $\mu_n = 1350$  cm<sup>2</sup>/V · s, find (a) the electron drift velocity, (b) the time it takes an electron to cross the  $2\text{-}\mu\text{m}$

length, (c) the drift-current density, and the (d) drift current in the case that the silicon bar has a cross-sectional area of  $0.25 \mu\text{m}^2$ .

**I** (a) TODO: finish out this question

2. A general relationship for the current density carried out by holes of density  $p$  is  $J = qpv$ , where  $q$  is the electronic charge and  $v$  is the hole velocity.

- (a) Find the velocity of holes,  $v(x)$ , that are moving only by diffusion if they have a density distribution of  $p(x) = p_0 e^{-x/l}$ . The electric field is zero.
- (b) What would be the electric field that would lead to a hole drift velocity equal to that of the diffusion velocity in part(a)? Use Einstein's relation to answer this question.
- (c) At 300 K, what is the value of  $l$  to make the electric field in part (b) be 1000 V/cm?

## 2.4 Sources

- Sedra, Adel S., et al. Microelectronic Circuits. Oxford University Press, 2021
- EE105 Reader
- Q2 from here is Q2 from EE105 HW6

## Chapter 3

# PN Junctions

A **PN junction** is the junction between an *N*-type semiconductor and *P*-type semiconductor. Understanding the PN junction will set up us for understanding diodes, BJTs, and MOSFETs later. It seems like we draw it as two separate silicon crystals, but in actual practice the *p* and *n* regions are part of the same silicon crystal, accomplished by creating regions of different doping.

Plus ("+") signs represent majority holes while minus ("-") signs represent majority electrons. The following diagram is from Seda and Adel's *Microelectronic Circuits*.

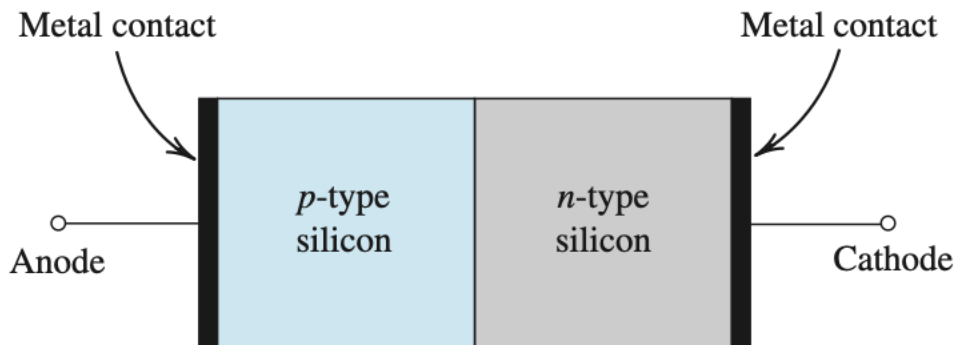


Figure 3.1: Simplified physical structure of the PN junction

### 3.1 Diffusion Current

Although it doesn't show it in the diagram, there are minority holes generated by thermal ionization in the *n*-type material and there are minority electrons generated in the *p*-type material. Due to concentration difference of holes in the *p* region and the *n* region, holes diffuse across the junction from the *p* side to the *n* side. This results in **diffusion current**,  $I_D$ , whose direction is from the *p* to *n* side.

So current Damanic is wondering right now "if this stuff is diffusing then won't this entire block be the same mush at the end." Here we introduce the depletion region. Holes that diffuse across the junction into the *n* region recombine with majority electrons there. A charge is said to be **uncovered** when some of the bound positive charge is no longer neutralized by free electrons. This introduces the idea that at a region close to the junction, it is depleted of free electrons and contains unbound positive charge for the *n* region.

The left side of the PN junction (*p* region) will be negatively charged while the right side (*n* region) will be positively

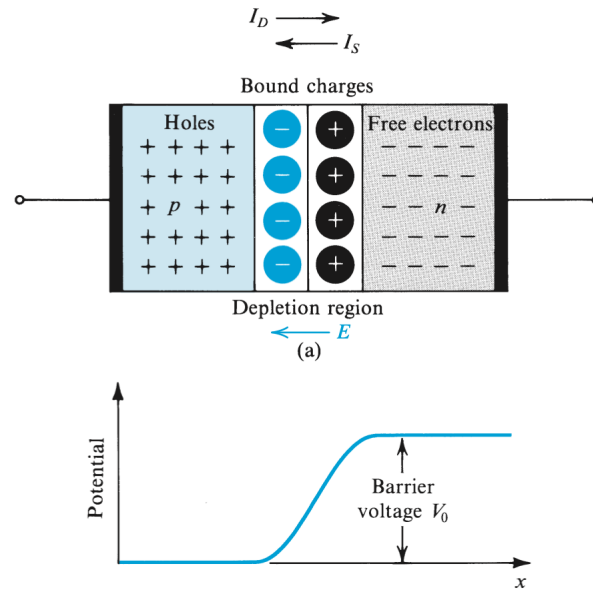


Figure 3.2: Top image shows PN-junction with bound charges and bottom image is potential along an axis perpendicular to the junction

charged. To sum it up, this is because at some point, electrons that diffuse across the junction into the  $p$  region will recombine with holes, and those holes will disappear leaving uncovered bound negative charge. Vice versa for holes diffusing into the  $n$  region.

From the figure above we see that the  $n$  region will be positively charged and the  $p$  side if negatively charged. This is the **depletion region**, or the **space-charge region** or **depletion layer**. There are no *mobile* charge carriers present here. Charges on both sides of the depletion region results in an electric field  $E$ . Here we introduce the idea that a larger barrier voltage results in a small number of carriers that can overcome this barrier. This leads to a decrease in magnitude of diffusion current since it is more difficult for holes to diffuse into the  $n$  region and electrons to diffuse into the  $p$  region. Referring again to figure 3.2, we see that  $V_0$  is the barrier voltage. Therefore the diffusion current  $I_D$  has a strong relationship with  $V_0$ , the voltage drop across the depletion region.

## 3.2 Drift Current and Equilibrium

Recall that drift current is caused by electric fields and  $I_S$  is independent of the value of the depletion-layer voltage  $V_0$ . Under open-circuit conditions, there is no external current, so

$$I_D = I_S$$

This condition is maintained by  $V_0$ .

**Analysis :  $I_S$  and  $I_D$  at Equilibrium**

- $V_O$ : barrier voltage
  - $I_S$ : drift current whose direction is from the  $n$  side to the  $p$  side of the junction
  - $I_D$ : diffusion current whose direction is from the  $p$  side to the  $n$  side of the junction
1.  $I_D > I_S$ : more bound charge is uncovered on both sides  $\rightarrow$  the depletion layer widens (vertically)  $\rightarrow V_0$  increases  $\rightarrow I_D$  decreases until  $I_D = I_S$  (equilibrium)
  2.  $I_D < I_S$ : uncovered charge decreases  $\rightarrow$  depletion layer narrows (vertically)  $\rightarrow V_0$  decreases  $\rightarrow I_D$  increases until  $I_D = I_S$  (equilibrium)

Under the zero bias equilibrium condition (no external voltage is applied to the PN junction), does the diffusion and drift current "cancel" out here, meaning that current density is nearly zero. Their individual components are also equal here, i.e. hole/electron drift current is equal to hole/electron diffusion current, respectively.

$$J_n = 0 = qn_0\mu_n E_0 + qD_n \frac{dn_0}{dx}$$

$V_0$  has been referred to so far as barrier voltage, but it's also called **junction built-in voltage**.

$$\phi_{bi} = V_{th} \ln \left( \frac{N_A N_D}{n_i^2} \right) = \frac{kT}{q} \ln \left( \frac{N_A N_D}{n_i^2} \right)$$

Remember here that  $V_{th}$  is thermal voltage which is  $\approx 26$  mV at room temperature.  $\phi_{bi}$  is typically 0.6 V to 0.9V for room temperature silicon. In the EE105 reader,  $\phi_{bi}$  and  $V_{th}$  has the same meaning as  $V_0$  and  $V_T$ , respectively, in the *Microelectronic Circuits* textbook. I'm writing down the EE105 reader notation here for clarity.

1. How we got from each graph

This graph shows a junction where  $N_A > N_D$ . If we heavily dope one side, the depletion region will exist almost entirely on the lightly doped side.

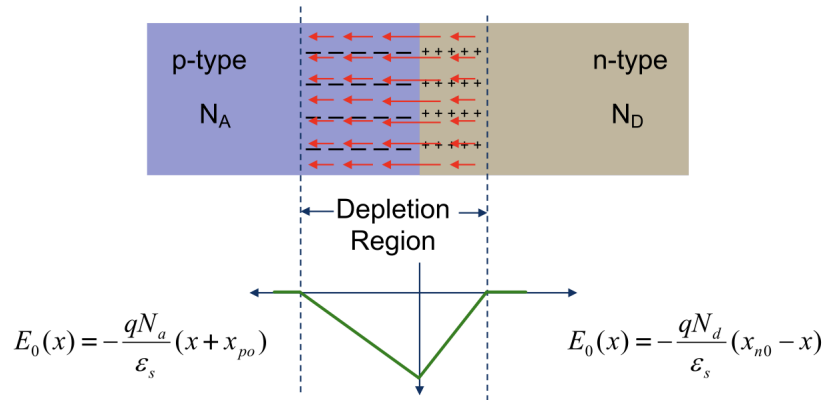


Figure 3.4: Graph of electric field of PN junction

Here in figure 3.4, the electric fields in the depletion region are negative because positive charges are on the right and the negative charges are on the left, assuming zero fields in neutral  $P$ -region and  $N$ -regions. Key takeaways due to difference in doping levels in the  $N$ -type and  $P$ -type sides:

- Width of the depletion region is not symmetric
- Slope of the electric field is larger in one region than the other
- The negative peak of the electric field always occurs at the junction

#### Analysis : Gauss's Law

Gauss's Law states that the total electric flux out of a closed surface is equal to the charge enclosed by the permittivity. Electric flux is the electric field multiplied by the area of the surface projected in a plane and perpendicular to the field.

$$\oint_S \vec{E}_n \cdot d\vec{s} = \frac{1}{\epsilon_0} Q_{inside}$$

1. finish this

By Gauss's Law, the total fixed charged in the  $N$ -region is the same as the total fixed charge in the  $P$ -region.

$$qN_A x_{p0} = qN_D x_{n0}$$

Here we'll derive the equations for depletion width, which are dependent on externally applied voltage, which are given by the following. The zero is for the zero bias case.

$$\text{Depletion width on N-side: } x_{n0} = \sqrt{\frac{2\epsilon_s \phi_{bi}}{qN_D} \left( \frac{N_A}{N_A + N_D} \right)}$$

$$\text{Depletion width on P-side: } x_{p0} = \sqrt{\frac{2\epsilon_s \phi_{bi}}{qN_A} \left( \frac{N_D}{N_A + N_D} \right)}$$

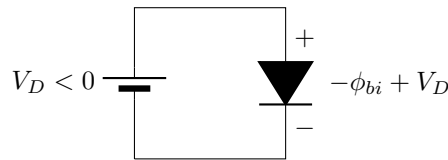
$$\text{Built in potential: } \phi_{bi} \equiv \phi_n = \phi_p > 0$$

$$\text{Total depletion width } X_{dep0} = x_{n0} + x_{p0} = \sqrt{\frac{2\epsilon_s \phi_{bi}}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right)}$$

1. derivation here

For the zero bias case (under thermal equilibrium), net current is zero. Diffusion current is very small since few carriers have enough energy to penetrate the barrier. Drift current is small since minority carriers are few.

### 3.3 Reverse Bias



The above section was discussing the PN junction at equilibrium. At equilibrium, the  $PN$ -junction doesn't draw any current. Under reverse bias like in figure above, meaning that a negative voltage is applied, charge will increase and the depletion region widens horizontally. We can rewrite the depletion width formulas from above as a function of  $V_D$ .

$$x_n(V_D) = \sqrt{\frac{2\epsilon_s(\phi_{bi} - V_D)}{qN_D} \left( \frac{N_A}{N_A + N_D} \right)} = x_{n0} \sqrt{1 - \frac{V_D}{\phi_{bi}}}$$

$$x_p(V_D) = \sqrt{\frac{2\epsilon_s(\phi_{bi} - V_D)}{qN_A} \left( \frac{N_D}{N_A + N_D} \right)} = x_{p0} \sqrt{1 - \frac{V_D}{\phi_{bi}}}$$

$$X_{dep0} = x_n(V_D) + x_p(V_D) = \sqrt{\frac{2\epsilon_s(\phi_{bi}-V_D)}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)} = X_{dep0} \sqrt{1 - \frac{V_D}{\phi_{bi}}}$$

Keep these in mind:

- Minority drift current is independent of the barrier height ( $\phi_{bi}$ )
- Diffusion current is a strong exponential function of the barrier height

### 3.4 Forward Bias

This occurs when  $V_D$  is nonnegative. A forward bias results in an exponential increase in the number of carriers, which have enough energy to break the barrier. This results in an exponential increase in diffusion current. Drift current doesn't change with forward bias.

1. skipped small signal model for this
2. skipped a lot of practice problems for this too

### 3.5 Practice Problems

1. Show that

$$V_0 = \frac{1}{2} \left( \frac{q}{\epsilon_s} \right) \left( \frac{N_A N_D}{N_A + N_D} \right) W^2$$

2. Show that for a PN junction in which the  $p$  side is much more heavily doped than the  $n$  side (i.e.  $N_A \gg N_D$ ) referred to as a  $p^+n$  diode. The following can be written as follows:

$$W \simeq \sqrt{\frac{2\epsilon_s}{qN_D}} V_0, \quad x_n \simeq W, \quad x_p \simeq \frac{W}{N_A/N_D}$$

$$Q_j \simeq AqN_D W, \quad Q_j \simeq A\sqrt{2\epsilon_s q N_D V_0}$$

3. For a Si PN junction that looks like this, where  $N_A = 1 \times 10^{18} \text{ cm}^{-3}$  and  $N_D = 1 \times 10^{17} \text{ cm}^{-3}$ . Given the built-in potential is 0. V and the relative permittivity of Si is 12. No voltage is applied.



- (a) Draw the charge along the horizontal axis.
  - (b) Draw the electric field along the horizontal axis. Label the critical points such as charge density, depletion width, and maximum electric field in your graphs.
4. Considering a PN junction where the  $n$  side is doped with  $N_D = 10^{17} \text{ cm}^{-3}$  and  $p$  side is doped with  $N_A = 10^{20} \text{ cm}^{-3}$ ,  $n_i = 10^{10} \text{ cm}^{-3}$ ,  $kT/q = 26 \text{ mV}$ ,  $\mu_n = 100 \text{ cm}^2/\text{V} \cdot \text{sat}$   $p$  side,  $\mu_p = 400 \text{ cm}^2/\text{V} \cdot \text{sat}$   $n$  side,  $L_n = 2.3 \mu\text{m}$  at  $p$  side, and  $L_p = 3.2 \mu\text{m}$  at  $n$  side, vacuum permittivity is  $\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)$ , the permittivity of silicon is  $11.7\epsilon_0$ . Area of the PN junction is  $1 \mu\text{m}^2$ 
    - (a) Calculate the built-in potential.
    - (b) Calculate the total depletion width at zero bias.
    - (c) Draw the electric field profile at zero bias (You need to calculate the maximum electric field). The left-hand side is N and the right-hand side is P.
    - (d) Calculate the depletion capacitance when the bias -1V.
    - (e) Calculate the current when the bias is 1V.
    - (f) Find out the small-signal resistance and diffusion capacitance at 1V Given  $\tau = 1\mu\text{s}$ .



## 3.6 Sources

- Sedra, Adel S., et al. Microelectronic Circuits. Oxford University Press, 2021: Specifically screenshots of the graphs I need to redo this when I learn how to use graphing/tikzpicture better in LaTeX
- EE105 Reader
- Q1 and Q2 are excercises from Sedra, Adel chapter3
- Q3 from here is Q4 from EE105 HW6
- Q4 from here is Q1 from EE105 Hw7
- Explanation of Gauss's Law

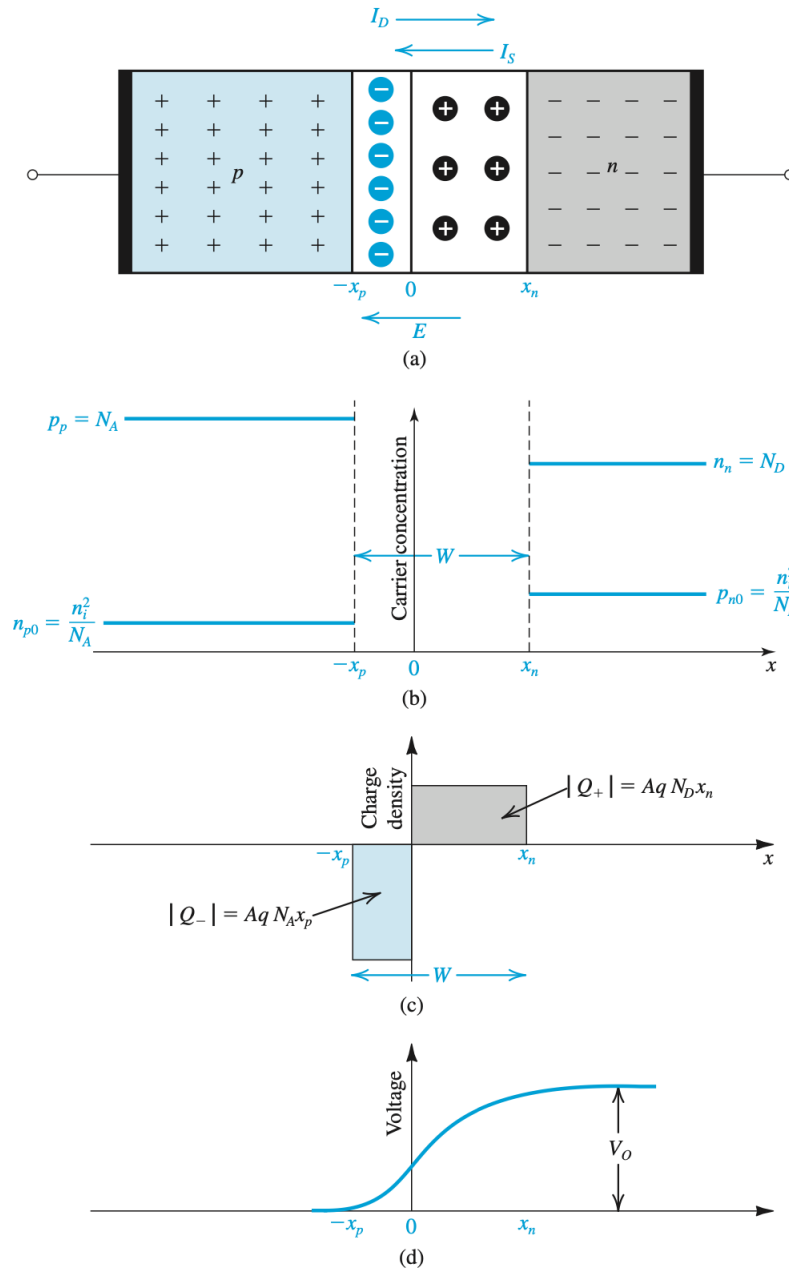


Figure 3.3: Graphs of (a) a PN junction (b) carrier concentrations (c) charge density (d) built in voltage  $V_0$

## Chapter 4

# MOS Capacitors

The MOS in MOS capacitor stands for Metal-Oxide-Silicon. Sometimes the S is also written as substrate in other places. Understanding threshold voltage here for a MOS-CAP is useful for also understanding MOS transistors in a later chapter.

- Metal: usually this is a heavily doped polysilicon (Poly-Si) layer doped with  $n^+$  or  $p^+$ , due to high temperature processing for aluminum
- Oxide:  $\text{SiO}_2$  where  $\epsilon_{ox} = 3.9\epsilon_0$ , can be any insulator actually
- Substrate:  $\epsilon_s = 11.7\epsilon_0$ ; NMOS capacitor  $\rightarrow P$ -type substrate, PMOS capacitor  $\rightarrow N$ -type substrate

### 4.1 MOSCAP at Equilibrium

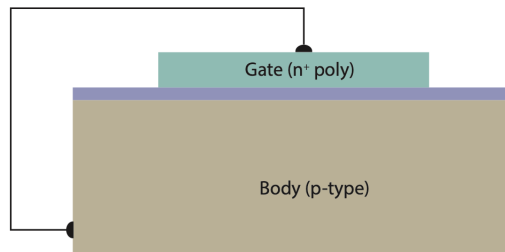


Figure 4.1: MOS CAP with gate and body shorted in equilibrium

In equilibrium, the gate and body, which are the terminals where voltage is applied, are shorted like in fig ???. The previous chapter on PN junctions is relevant here. Under thermal equilibrium, there is no current flow, so an  $n^+$  type polysilicon gate

### 4.2 Regions of Operation

### 4.3 Practice Problems

### 4.4 Sources

•