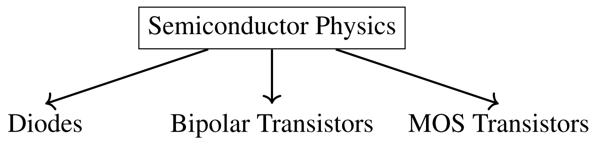
Contents

1. use this article to add to ch1 about doping
2. practice problems for ch1
3. practice problems for ch2
4. finish up ch3 i burned out while trying to be thorough. p117 in reader, p152 in sedra
5. dead week planning
   (a) tuesday: finish pn junction, diodes, mosfets
   (b) Wedesnday: moscap, amplifiers maybe small signal model

1.1 Charge Carriers and Doping

1.1.1 Introduction

We start learning about resistors, capacitors, and inductors from earlier courses such as EECS 16A and 16B. With more components like transistors, diodes, and op-amps (which are all based on semiconductors), we are able to expand upon circuit design. We need to understand semiconductor physics in order to understand how these components operate.



We can also redefine Ohm's Law which we know as ($V = IR$) as $J = \sigma\mathbf{E}$. We can also write resistance(R) and conductance(G) in terms of other variables

- J: current density, A/m^2 (Amperes per meter squared)
- σ: conductivity, S/m (Siemens per meter)
- $\mathbf{E}$: electric field, V/m (Volts per meter)

$R = \frac{R}{I} = \rho\frac{l}{A}$ and $G = R^{-1} = \sigma\frac{A}{l}$

- σ: conductivity
- ρ: resistivity

For collisions in gas, we focus on the idea that initial velocity and direction is lost/randomized after a few collisions. So, when we sum over the random velocities of the particles and average it, it comes out to zero. Average momentum gain is:

$$\bar{\mu} = \frac{\mathbf{E}q\tau}{M} = \mu\mathbf{E}, \quad \mu := \frac{q\tau}{M} = \frac{\bar{v}}{\mathbf{E}}$$

- μ: mobility, $m^2/(V \cdot s)$
- q: electric charge, 1.60×10^{-19}, Coulombs = Amperes/second
- τ: mean free time
- M: mass
- $\bar{v}$: average velocity

Different elements have a different number of outer shell electrons. For semiconductors like silicon, we can increase the temperature to increase its conductivity. Silicon atoms are arranged in a diamond structure and in general, the energy levels that an atom can occupy are discrete. The **valence band** electrons are at a lower energy state (bound to host atoms) while **conduction band** electrons are at a higher energy state and are "free" electrons. These electrons are free to move around the crystal and take part in conduction.

1.1.2 Conduction and Fermi Dirac Distribution

Thermal energy is on average about $\sim 26\ eV$ at room temperature How large the **band-gap**, the gap between the conduction and valence band, determines how conductive a material is:

- Insulators: band gap $\sim 15\ eV$
    - Glass, rubber, oil, plastic, diamond

- Semiconductors: band gap $\sim 1eV$
  - Silicon = 1.12 eV
- Conductors: Not applicable due to overlapping conduction/valence bands

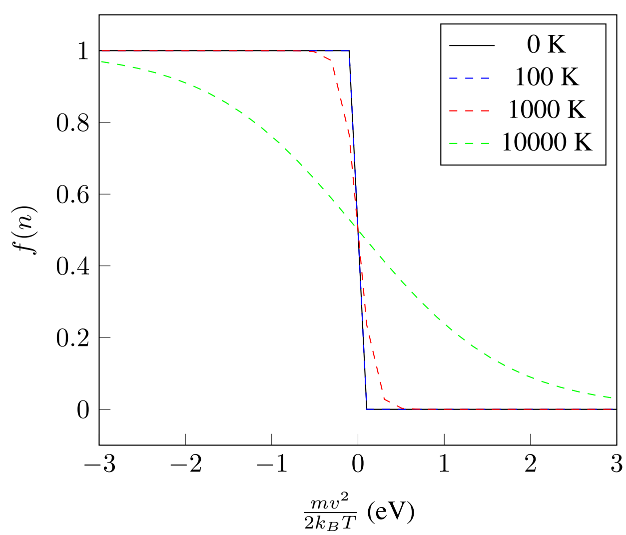
This band gap energy is the minumum energy required to break a covalent bond and generate an electron-hole pair.

Because electrons are a type of particle called a **fermion**, we can say that

$$f(E) = \frac{1}{e^{\frac{E-E_F}{k_B T}} + 1}$$

- $f(E)$: occupational probability of a state energy ϵ
- E: energy of the particle
- E_F: fermi energy, eV
- k_B: Boltzmann's constant, 1.380649×10^{-23} J/K (Joules per Kelvin), 8.62×10^{-5} eV/K
- T: temperature in Kelvin

The graph below is a plot of the Fermi-Dirac Distribution at different temperatures. We can use this plot to find the probability of the concentration of electrons ad holes at certain temperatures.



Ok so why is this graph significant. To repeat, we can relate the concentration of holes or electrons to the

Thermal generation occurs when sufficient thermal energy exist to break some covalent bonds, which results in a **free electron**. This free electron is now free to conduct electric current if an electric field is applied to the crystal. However, this leaves behind a net positive charge. **Holes** are positively charged carrers, which can also move around the crystal and are available to conduct electric current. As temperature increases, so do electron-hole pairs and the conductivity of the crystal.

Recombination is the process of free electrons and holes moving through the silicon crystal structure and electrons filling in some of the holes. The recombination rate is proportional to the number of free electrons and holes, which also determines the **thermal generation** rate.

At thermal equilibrium, $n = p = n_i$. We can alternatively express as

$$pn = n_i^2$$

- n: concentration of free electrons, cm^{-3}
- p: concentration of holes, cm^{-3}
- n_i: number of free electrons nd holes in a unit volume (cm^3) of intrinsic silicon at a given temperature

We can also write n_i as a function of temperature

$$n_i = BT^{3/2}e^{-E_G/2kT}$$

- B: material-dependent parameter, $7.3 \times 10^{15}\text{cm}^{-3}\text{K}^{-3/2}$ for Silicon
- T: temperature in K
- E_G: bandgap energy, 1.12 eV for silicon
- k, Boltzmann's constant

1.1.3 Doping

In a non-doped crystal like in the above texts, there will be equal concentrations of free electrons and holes from thermal generation. **Doping** is defined as introducing impurities inside a silicon crystal to adjust the carrier concentration. The following materials are commonly used:

- Group III elements: boron, aluminum, gallium $\rightarrow$ acceptors; increase holes
- Group IV elements: germanium and silicon
- Group V elements: phosphorus, arsenic, antimony $\rightarrow$ donors; increase free electrons

Doping with donors $\rightarrow$ **n type**.

Doping with acceptors $\rightarrow$ **p type**

Sometimes we assume that the number of electrons we add is much greater than the original free electrons that pure silicon had (10^{10} per cubic centimeter). This leads to the simplification that the number of free electrons in our silicon crystal is N_D, where N_D is the number of donor atoms that we add per cubic centimeter.

Analysis : Doping concentrations

Suppose we dope silicon with phosphorus, an acceptor. This means that four electrons from its outer shell will form a covalent bond with silicon. This results in one free electron, meaning that each phosphorus atom donates one free electron to the silicon. No holes are generated during this process. Usually, the concentration of donor atoms, N_D, is much greater htan n_i, so

$$n_n \simeq N_D$$

- n_n: for n-type silicon, concentration of free electrons
- p_n: for n-type silicon, concentratation of holes
- N_D: donor doping concentration
- N_A: acceptor doping concentration

n_n is determined by doping concentration, and independent of temperature, while p_n is determined by:

$$p_n n_n = n_i^2$$
$$p_n \simeq \frac{n_i^2}{N_D}$$

An analysis with boron, an acceptor, will yield similar conclusions with a change in variables.

1.1.4 Practice Problems

1. Calculate the intrinsic carrier density n_i for silicon at $T = 50$ K and 350 K.

$$\begin{aligned} n_{i,1} &= BT^{3/2}e^{-E_G/2kT} \\ &= (7.3 \times 10^{15}\text{cm}^{-3}\text{K}^{-3/2})(50K)^{3/2}e^{-1.12eV/2(8.62\times10^{-5}eV/K)(50K)} \\ &= 9.63217875 \times 10^{-39}/\text{cm}^3 \\ n_{i,2} &= (7.3 \times 10^{15}\text{cm}^{-3}\text{K}^{-3/2})(350K)^{3/2}e^{-1.12eV/2(8.62\times10^{-5}eV/K)(350K)} \\ &= 4.15216354 \times 10^{11}/\text{cm}^3 \end{aligned}$$

2. Consider an n-type silicon for which the dopant concentration $N_D = 10^{17}/\text{cm}^3$. Find the electron and hole concentrations at 350 K. Refer to previous example for the value of n_i at $T = 350$ K.
   Since this is an n-type silicon,
$$n_n \simeq N_D = 10^{17}/\text{cm}^3$$
   From this we can find the concentration of minority holes as
$$p_n \simeq \frac{n_i^2}{N_D} = \frac{(4.15 \times 10^{11})^2}{10^17} = 1.72 \times 10^6/\text{cm}^3$$

1.1.5 Sources

- Razavi Electronics 1, Lec 1, Intro., Charge Carriers, Doping
- EE105 Reader

- Sedra, Adel S., et al. Microelectronic Circuits. Oxford University Press, 2021
- Engineering LibreTexts: The Fermi-Dirac Distribution
- Fermi-dirac distribution graph

1.2 Drift and Diffusion Current

Total current flow is made up of drift current and diffusion current. If we apply an electric field E to a semiconductor crystal, then holes accelerate in the direction of E and free electrons accelerate in the opposite direction of E. While **drift current** is movement caused by electric fields, **diffusion current** is movement caused by variation in the carrier concentration.

1.2.1 Drift Current

$$v_{p-drift} = \mu_p E, \quad v_{n-drift} = -\mu_n E$$

- E: electric field, V/cm
- $v_{p-drift}$: drift velocity of holes, cm/s
- $v_{n-drift}$: drift velocity of electrons, cm/s
- μ_p: hole mobility, cm^2/V · s, 480 cm^2/V · s for intrinsic silicon
- μ_n: electron mobility, cm^2/V · s, 1350 cm^2/V · s for intrinsic silicon

Current density is the current per unit cross-sectional area

$$J_p = qn\mu_p E, \quad J_n = qn\mu_n E$$

Total drift current density is

$$J = J_p + J_n = q(p\mu_p + n\mu_n)E = \sigma E$$

Via pattern matching, we can see that conductivity σ is given by $\sigma = q(p\mu_p + n\mu_n)$ and resistivity is the inverse of this.

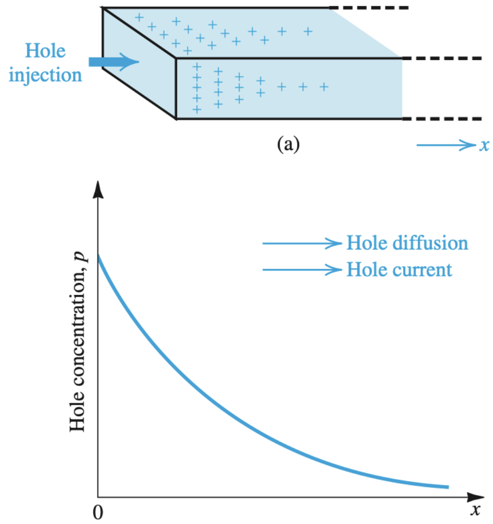
1.2.2 Diffusion Current

Diffusion current is the diffusion of charge carriers that give rose to a net flow of charge. We can write the current density as a function of the **concentration gradient** (slope of the concentration profile) at any point such that:

$$J_p = -qD_p \frac{dp(x)}{dx}, \quad J_n = qD_n \frac{dn(X)}{dx}$$

- J_p, J_n: hole/electron-current density, A/cm^2
- q: magnitude of the electron charge
- D_p, D_n: diffusion constant or diffusivity of holes/electrons. For intrinsic silicon, $D_p = 12$ cm^2/s and $D_n = 35$ cm^2/s
- $p(x), n(x)$: hole/electron concentration at point x. Note that if $\frac{dp(x)}{dx} < 0$, J_p ends up being positive.

The image below shows a bar of silicon and an injection of holes on the left side, which will result in hole diffusion current in the same direct (positive direction of x)



Suppose we fill a gas chamber that is divided into two sections with a gases of temperature T on one side. If we remove this divider, the gas will fill the entire volume of the new chamber. This occurs due to the concentration gradient. If the gas molecules here were charged, there would be a net current flow.

Analysis : Einstein Relationship

The following equation is known as the **Einstein relationship**:

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T = \frac{kT}{q}$$

- V_T: thermal voltage; at $T \simeq 300$ K, $V_T = 25.9$ mV

We see that the diffusion constant is related to the mobility.

Total current will be given by the sum of drift and diffusion currents. In resistors, since the carrier is approximately uniform, diffusion current is nearly zero.

$$J = J_{drift} + J_{diff} = q\mu_n nE + qD_n\frac{dn}{dx}$$

1.2.3 Practice Problems

1. A uniform bar of n-type silicon of 2-μm length has a voltage of 1 V applied across it. If $N_D = 10^{16}/\text{cm}^3$ and $\mu_n = 1350$ cm^2/V $\cdot$ s, find (a) the electron drift velocity, (b) the time it takes an electron to cross the 2-μm length, (c) the drift-current density, and the (d) drift current in the case that the silicon bar has a cross-sectional area of 0.25 μm^2.

   (a) TODO: finish out this question

2.

1.2.4 Sources

- Sedra, Adel S., et al. Microelectronic Circuits. Oxford University Press, 2021
- EE105 Reader

1.3 PN Junctions

A **PN junction** is the junction between an N-type semiconductor and P-type semiconductor. Understanding the PN junction will set up us for understanding diodes, BJTs, and MOSFETs later. It seems like we draw it as two separate silicon crystals, but in actual practice the p and n regions are part of the same silicon crystal, accomplished by creating regions of different doping.

Plus ("+") signs represent majority holes while minus ("-") signs represent majority el ectrons. The following diagram is from Seda and Adel's *Microelectronic Circuits*.

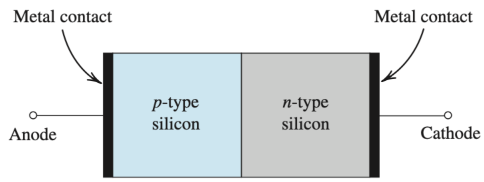


Figure 1.1: Simplified physical structure of the PN junction

1.3.1 Diffusion Current

Although it doesn't show it in the diagram, there minority holes generated by thermal ionization in the n-type material and there are minority electrons generated in the p-type material. Due to concentration difference of holes in the p region and the n region, holes diffuse across the junction from the p side to the n side. This results in **diffusion current,** I_D, whose direction is from the p to n side.

So current Damanic is wondering right now "if this stuff is diffusing then won't this entire block be the same mush at the end." Here we introduce the depletion region. Holes that diffuse across the junction into the n region recombine with majority electrons there. A charge is said to be **uncovered** when some of the bound positive charge is no longer neutralized by free electrons. This introduces the idea that at a region close to the junction, it is depleted of free electrons and contains unbound positive charge for the n region.

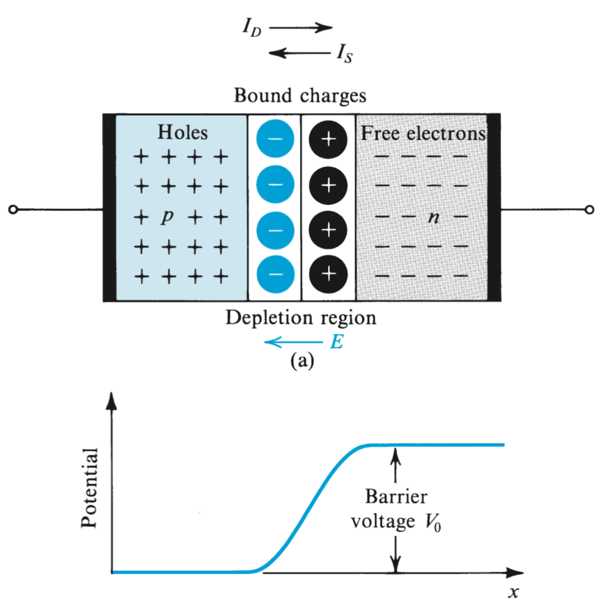


Figure 1.2: Top image shows PN-junction with bound charges and bottom image is potential along an axis perpendicular to the junction

The left side of the PN junction (p region) will be negatively charged while the right side (n region) will be positively charged. To sum it up, this is because at some point, electrons that diffuse across the junction into the p region will recombine with holes, and those holes will disappear leaving uncovered bound negative charge. Vice versa for holes diffusing into the n region.

From the figure above we see that the n region will be positively charged and the p side if negatively charged. This is the **depletion region**, or the **space-charge region** or **depletion layer**. There are no *mobile* charge carriers present here. Charges on both sides of the depletion region results in an electric field E. Here we introduce the idea that a larger barrier voltage results in a small number of carriers that can overcome this barrier. This leads to a decrease in magnitude of diffusion current since it is more difficult for holes to diffuse into the n region and electrons to diffuse into the p region. Referring again to figure 1.2, we see that V_0 is the barrier voltage. Therefore the diffusion current I_D has a strong relationship with V_0, the voltage drop across the depletion region.

1.3.2 Drift Current and Equilibrium

Recall that drift current is caused by electric fields and I_S is independent of the value of the depletion-layer voltage V_0. Under open-circuit conditions, there is no external current, so

$$I_D = I_S$$

This condition is maintained by V_0.

Analysis : I_S and I_D at Equilibrium

- V_O: barrier voltage
- I_S: drift current whose direction is from the n side to the p side of the junction
- I_D: diffusion current whose direction is from the p side to the n side of the junction

1. $I_D > I_S$: more bound charge is uncovered on both sides $\rightarrow$ the depletion layer widens (vertically)$\rightarrow$ V_0 increases $\rightarrow$ I_D decreases until $I_D = I_S$ (equilibrium)
2. $I_D < I_S$: uncovered charge decreases $\rightarrow$ depletion layer narrows (vertically) $\rightarrow$ V_0 decreases $\rightarrow$ I_D increases until $I_D = I_S$ (equilibrium)

Under the zero bias equilibrium condition (no external voltage is applied to the PN junction), does the diffusion and drift current "cancel" out here, meaning that current density is zero. Their individual components are also equal here, i.e. hole/electron drift current is equal to hole/electron diffusion current, respectively.

$$J_n = 0 = qn_0\mu_n E_0 + qD_n\frac{dn_0}{dx}$$

V_0 has been referred to so far as barrier voltage boltage, but it's also called **junction built-in voltage**.

$$\phi_{bi} = V_{th}\ln\left(\frac{N_A N_D}{n_i^2}\right) = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Remember here that $V_t h$ is thermal voltage which is $\approx$ 26 mV at room temperature. ϕ_{bi} is typically 0.6 V to 0.9V for room temperature silicon. In the EE105 reader, ϕ_{bi} and V_{th} has the same meaning as V_0 and V_T, respectively, in the *Microelectronic Circuits* textbook. I'm writing down the EE105 reader notation here for clarity.

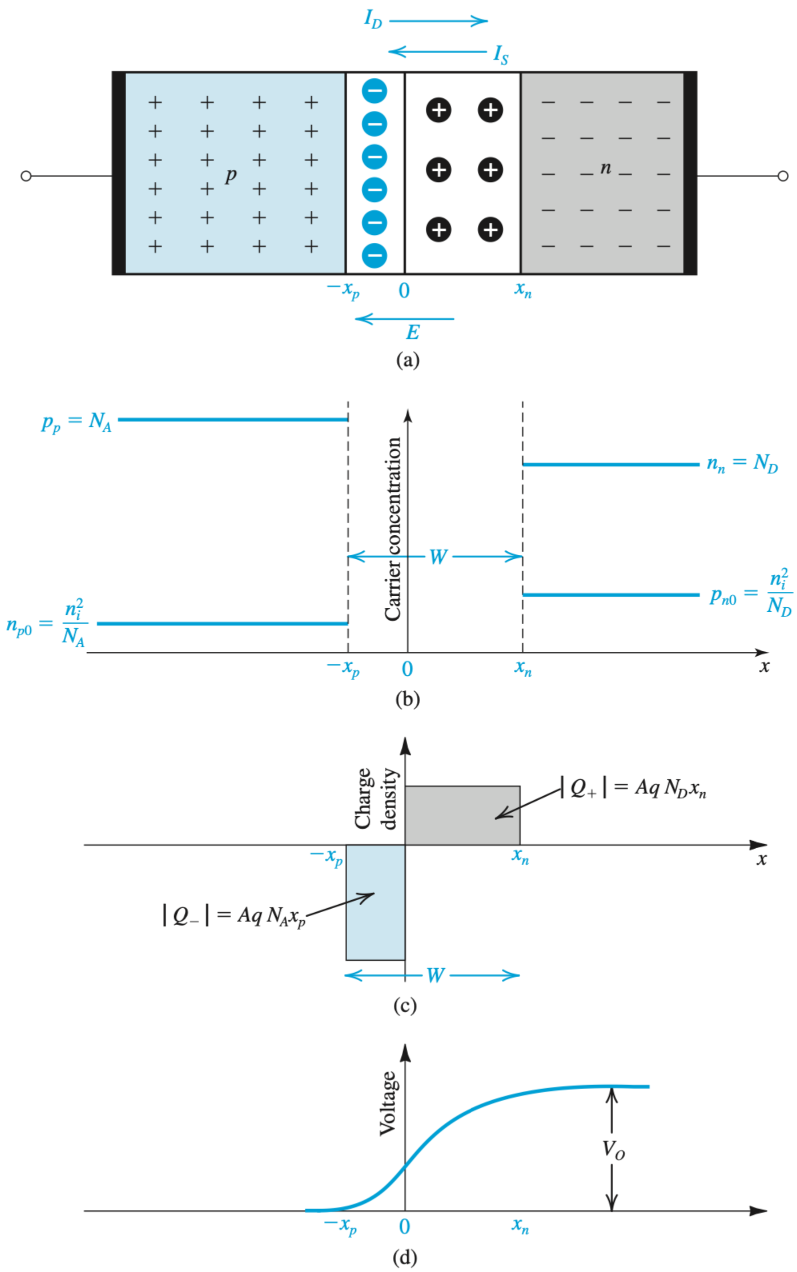


Figure 1.3: Graphs of (a) a PN junction (b) carrier concentrations (c) charge density (d) built in voltage V_0

How we got from each graph.

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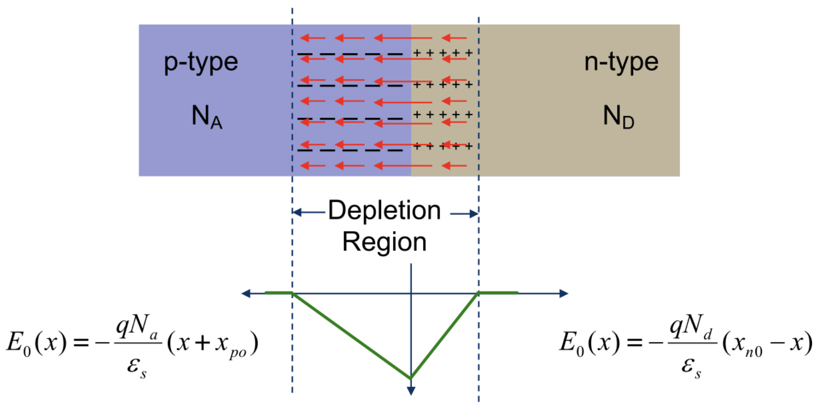

Figure 1.4: Graph of electric field of PN junction

Here in figure 1.4, the electric fields in the depletion region are negative because positive charges are on the right and the negative charges are on the left, assuming zero fields in neutral P-region and N-regions. Key takeaways:

- Width of the depletion region is not symmetric
- The negative peak of the electric field always occurs at the junction

1.3.3 Reverse Bias

1.3.4 Forward Bias

1.3.5 PN Junction with External Voltage Applied

The above section was discussing the PN junction at equilibrium.

1.3.6 Practice Problems

1. Show that
$$V_0 = \frac{1}{2}(\frac{q}{\epsilon_s})(\frac{N_A N_D}{N_A + N_D})W^2$$
2. Show that for a PN junction in which the p side is much more heavily dped than the n side (i.e. $N_A \gg N_D$) referred to as a p^+n diode. The following can be written as follows:
$$W \simeq \sqrt{\frac{2\epsilon_s}{qN_D}V_0}, \quad x_n \simeq W, \quad x_p \simeq \frac{W}{N_A/N_D}$$
$$Q_j \simeq AqN_D W, \quad Q_j \simeq A\sqrt{2\epsilon_s q N_D V_0}$$
3.

1.3.7 Sources

- Sedra, Adel S., et al. Microelectronic Circuits. Oxford University Press, 2021: Specifically screenshots of the graphs I need to redo this when I learn how to use graphing/tikzpicture better in LaTex
- EE105 Reader