MULTICRITERIA DECISION MAKING

Contents

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Literature:

1. *Multiple Attribute Decision Making, Methods and Applications.* G-H Tzeng and J-J Huang. CRC Press. 2011

2. *Multiple Attribute Decision Making, Methods and Applications, A State-of-the-Art Survey*, C-L. Hwang and K.Yoon, Springer-Verlag, 1981.

3. "How to Select and How to Rank Projects: The Promethee Method", J.P.Brans, Ph. Vincke, B.Mareschal. *European Journal of Operational Research*, 24, 1986, pages 228-238.

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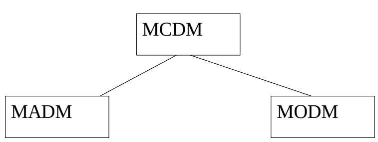
1. Basic Concepts and Foundations

Overview:

Multicriteria decision making (MCDM) refers to making decisions in the presence of multiple, usually conflicting criteria.

Multiattribute decision making (MADM)

Multiobjective decision making (MODM)



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Decision Matrix

$\boldsymbol{A} = \{A_1, A_2, \ldots, A_n\}$ Alternatives to choose between

$\boldsymbol{C} = \{C_1, C_2, \ldots, C_k\}$ Criteria

$$D = \begin{bmatrix} f_{11} & \cdots & f_{1k} \\ \vdots & \ddots & \vdots \\ f_{n1} & \cdots & f_{nk} \end{bmatrix}$$

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Numerical example.
Fighter aircraft selection.

Attributes:

C_1 : maximum speed (Mach)
C_2 : ferry range (NM)
C_3 : maximum payload (pounds)
C_4 : purchasing cost (£ × 10^6)
C_5 : reliability (high - low)
C_6 : maneuverability (high - low)

$$D = \begin{bmatrix} 2.0 & 1500 & 20000 & 5.5 & average & very\ high \\ 2.5 & 2700 & 18000 & 6.5 & low & average \\ 1.8 & 2000 & 21000 & 4.5 & high & high \\ 2.2 & 1800 & 20000 & 5.0 & average & average \end{bmatrix}$$

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Transformation of Attributes

Scaling problems and issues
(this example is for a maximisation attribute)

0	
1.0	*very low*
3.0	*low*
5.0	*average*
7.0	*high*
9.0	*very high*

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Transformation of Attributes

Normalisation

Once you have numerical values for all attributes, you should normalise them.

1. Linear Scale
   Exercise
2. Mapping to [0,1]
3. Vector normalisation

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1. Linear scale transformation

$$
\begin{array}{cccccc}
\text{max} & \text{max} & \text{max} & \text{min} & \text{max} & \text{max}
\end{array}
$$

$$
D = \begin{bmatrix}
2.0 & 1500 & 20000 & 5.5 & 5 & 9 \\
2.5 & 2700 & 18000 & 6.5 & 3 & 5 \\
1.8 & 2000 & 21000 & 4.5 & 7 & 7 \\
2.2 & 1800 & 20000 & 5.0 & 5 & 5
\end{bmatrix}
$$

$$
R = \begin{bmatrix}
0.8\,(=2.0/2.5) & 0.56 & 0.95 & 0.82\,(=4.5/5.5) & 0.71 & 1.0 \\
1.0\,(=2.5/2.5) & 1.0 & 0.86 & 0.69\,(=4.5/6.5) & 0.43 & 0.56 \\
0.72\,(=1.8/2.5) & 0.74 & 1.0 & 1.0\,(=4.5/4.5) & 1.0 & 0.78 \\
0.88\,(=2.2/2.5) & 0.67 & 0.95 & 0.90\,(=4.5/5.0) & 0.71 & 0.56
\end{bmatrix}
$$

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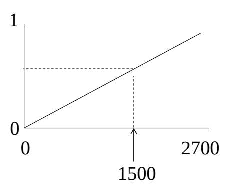
1. Linear scale transformation

$$
\begin{array}{cccccc}
\text{max} & \text{max} & \text{max} & \text{min} & \text{max} & \text{max}
\end{array}
$$

$$
D = \begin{bmatrix}
2.0 & 1500 & 20000 & 5.5 & 5 & 9 \\
2.5 & 2700 & 18000 & 6.5 & 3 & 5 \\
1.8 & 2000 & 21000 & 4.5 & 7 & 7 \\
2.2 & 1800 & 20000 & 5.0 & 5 & 5
\end{bmatrix}
$$



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2. Transformation which maps criteria values to interval [0,1]

• maximisation criterion

$$r_{nk} = \frac{f_{nk} - f_k^{\min}}{f_k^{\max} - f_k^{\min}}, \qquad 0 \le r_{nk} \le 1$$

• minimisation criterion

$$r_{nk} = \frac{f_k^{\max} - f_{nk}}{f_k^{\max} - f_k^{\min}}, \qquad 0 \le r_{nk} \le 1$$

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2. Transformation which maps criteria values to interval [0,1]

$$\begin{array}{cc} & \begin{array}{cccccc} \max & \max & \max & \min & \max & \max \end{array} \\ D = & \begin{bmatrix} 2.0 & 1500 & 20000 & 5.5 & 5 & 9 \\ 2.5 & 2700 & 18000 & 6.5 & 3 & 5 \\ 1.8 & 2000 & 21000 & 4.5 & 7 & 7 \\ 2.2 & 1800 & 20000 & 5.0 & 5 & 5 \end{bmatrix} \end{array}$$

$$R = \begin{bmatrix} 0.29\ (=\frac{2.0-1.8}{2.5-1.8}) & 0.0 & 0.67 & 0.5\ (=\frac{6.5-5.5}{6.5-4.5}) & 0.5 & 1.0 \\ 1.0\ (=\frac{2.5-1.8}{2.5-1.8}) & 1.0 & 0.0 & 0.0\ (=\frac{6.5-6.5}{6.5-4.5}) & 0.0 & 0.0 \\ 0.0\ (=\frac{1.8-1.8}{2.5-1.8}) & 0.42 & 1.0 & 1.0\ (=\frac{6.5-4.5}{6.5-4.5}) & 1.0 & 0.5 \\ 0.57\ (=\frac{2.2-1.8}{2.5-1.8}) & 0.25 & 0.67 & 0.75\ (=\frac{6.5-5.0}{6.5-4.5}) & 0.5 & 0.0 \end{bmatrix}$$

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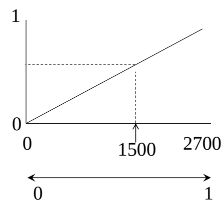
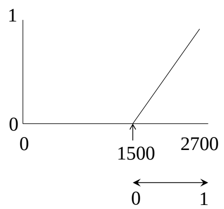
2. Transformation which maps criteria values to interval [0,1]

$$\begin{array}{cc} & \begin{array}{cccccc} \max & \max & \max & \min & \max & \max \end{array} \\ D = & \begin{bmatrix} 2.0 & 1500 & 20000 & 5.5 & 5 & 9 \\ 2.5 & 2700 & 18000 & 6.5 & 3 & 5 \\ 1.8 & 2000 & 21000 & 4.5 & 7 & 7 \\ 2.2 & 1800 & 20000 & 5.0 & 5 & 5 \end{bmatrix} \end{array}$$



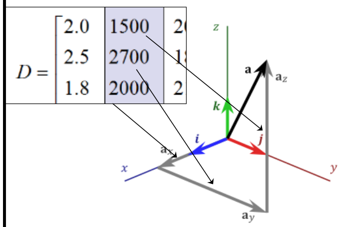


3. Vector normalisation

$$r_{nk} = \frac{f_{nk}}{\sqrt{\sum_{n=1}^{N} f_{nk}^{2}}}$$

The value. Can be seen as a 'contribution' to the length

Length of the vector



(image is for 3 alternatives, but the concept is extended when dealing with more dimensions)

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3. Vector normalisation

$$D = \begin{array}{cccccc} \text{max} & \text{max} & \text{max} & \text{min} & \text{max} & \text{max} \\ \end{array}$$
$$D = \begin{bmatrix} 2.0 & 1500 & 20000 & 5.5 & 5 & 9 \\ 2.5 & 2700 & 18000 & 6.5 & 3 & 5 \\ 1.8 & 2000 & 21000 & 4.5 & 7 & 7 \\ 2.2 & 1800 & 20000 & 5.0 & 5 & 5 \end{bmatrix}$$

$$R = \begin{bmatrix} 0.4671\,(=\frac{2}{\sqrt{2^2+2.5^2+1.8^2+2.2^2}}) & 0.3662 & 0.5056 & 0.5063 & 0.4811 & 0.6708 \\ 0.5839\,(=\frac{2.5}{\sqrt{2^2+2.5^2+1.8^2+2.2^2}}) & 0.6591 & 0.4550 & 0.5983 & 0.2887 & 0.3727 \\ 0.4204\,(=\frac{1.8}{\sqrt{2^2+2.5^2+1.8^2+2.2^2}}) & 0.4882 & 1.0 & 0.4143 & 0.6736 & 0.5217 \\ 0.5139(=\frac{2.3}{\sqrt{2^2+2.5^2+1.8^2+2.2^2}}) & 0.4392 & 0.5056 & 0.4603 & 0.4811 & 0.3727 \end{bmatrix}$$

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2. Simple Additive Weighting Method

Construct the normalised decision matrix.
For each alternative, calculate the weighted sum of values.

$$S_n = \frac{\sum_{k=1}^{K} w_k \cdot r_{nk}}{\sum_{k=1}^{K} w_k}$$

weights are normalised : $\sum_{k=1}^{K} w_k = 1$

$$\begin{aligned} S_1 &= w_1 \cdot r_{11} + w_2 \cdot r_{12} + \cdots + w_K \cdot r_{1K} \\ S_2 &= w_1 \cdot r_{21} + w_2 \cdot r_{22} + \cdots + w_K \cdot r_{2K} \\ &\dots \\ S_N &= w_1 \cdot r_{N1} + w_2 \cdot r_{N2} + \cdots + w_K \cdot r_{NK} \end{aligned}$$

Select alternative(s) A^*:

$$A^* \in \left\{ A_{n*} \mid S_{n*} = \max_n \frac{\sum_{k=1}^{K} w_k r_{nk}}{\sum_{k=1}^{K} w_k} \right\}$$

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Example
Fighter aircraft selection problem.

$$\begin{array}{cc} & \begin{array}{cccccc} \max & \max & \max & \min & \max & \max \end{array} \\ D = & \begin{bmatrix} 2.0 & 1500 & 20000 & 5.5 & 5 & 9 \\ 2.5 & 2700 & 18000 & 6.5 & 3 & 5 \\ 1.8 & 2000 & 21000 & 4.5 & 7 & 7 \\ 2.2 & 1800 & 20000 & 5.0 & 5 & 5 \end{bmatrix} \end{array}$$

$\boldsymbol{W} = \{0.2, 0.1, 0.1, 0.1, 0.2, 0.3\}$

Linear scale transformation

$$R = \begin{bmatrix} 0.8\,(=2.0/2.5) & 0.56 & 0.95 & 0.82\,(=4.5/5.5) & 0.71 & 1.0 \\ 1.0\,(=2.5/2.5) & 1.0 & 0.86 & 0.69\,(=4.5/6.5) & 0.43 & 0.56 \\ 0.72\,(=1.8/2.5) & 0.74 & 1.0 & 1.0\,(=4.5/4.5) & 1.0 & 0.78 \\ 0.88\,(=2.2/2.5) & 0.67 & 0.95 & 0.90\,(=4.5/5.0) & 0.71 & 0.56 \end{bmatrix}$$

Example
Fighter aircraft selection problem.

$\boldsymbol{W} = \{0.2, 0.1, 0.1, 0.1, 0.2, 0.3\}$

$$R = \begin{bmatrix} 0.8\,(=2.0/2.5) & 0.56 & 0.95 & 0.82\,(=4.5/5.5) & 0.71 & 1.0 \\ 1.0\,(=2.5/2.5) & 1.0 & 0.86 & 0.69\,(=4.5/6.5) & 0.43 & 0.56 \\ 0.72\,(=1.8/2.5) & 0.74 & 1.0 & 1.0\,(=4.5/4.5) & 1.0 & 0.78 \\ 0.88\,(=2.2/2.5) & 0.67 & 0.95 & 0.90\,(=4.5/5.0) & 0.71 & 0.56 \end{bmatrix}$$

						Sum
0.16 = (0.8*0.2)	0.056	0.095	0.082	0.142	0.3	0.835
0.2 = (1.0*0.2)	0.1	0.086	0.069	0.086	0.168	0.709
0.144 = (0.72*0.2)	0.074	0.1	0.1	0.2	0.234	0.852
0.176 = (0.88*0.2)	0.067	0.095	0.09	0.142	0.168	0.738

3. TOPSIS Method

- ideal point A^+
- nadir $A_\#$

☞ select the solution which has the shortest distance from the ideal solution and the farthest from the nadir.

Nadir is the ‘worst’ point.

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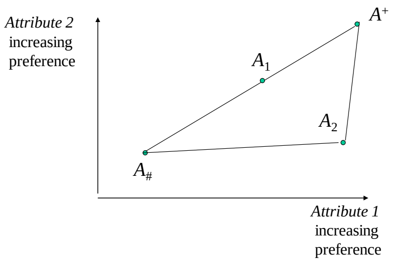

☞ Euclidean distance: The point closest to the ideal point is not necessarily farthest from the nadir.

Example: A_1 is closest to A^+, but not farthest from $A_\#$.

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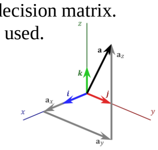
Step 1. Construct the normalised decision matrix.
Vector normalisation is most often used.

$$r_{nk} = \frac{f_{nk}}{\sqrt{\sum_{n=1}^{N} f_{nk}^2}}$$



Step 2. Construct the **weighted** normalised decision matrix.

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Example   Fighter aircraft selection problem.

Decision matrix

$$\begin{array}{cc} & \begin{array}{cccccc} \text{max} & \text{max} & \text{max} & \text{min} & \text{max} & \text{max} \end{array} \\ D = & \begin{bmatrix} 2.0 & 1500 & 20000 & 5.5 & 5 & 9 \\ 2.5 & 2700 & 18000 & 6.5 & 3 & 5 \\ 1.8 & 2000 & 21000 & 4.5 & 7 & 7 \\ 2.2 & 1800 & 20000 & 5.0 & 5 & 5 \end{bmatrix} \end{array}$$

$\boldsymbol{W}$ = {0.2, 0.1, 0.1, 0.1, 0.2, 0.3}

Normalised decision matrix

$$R = \begin{bmatrix} 0.4671\,(=\frac{2}{\sqrt{2^2+2.5^2+1.8^2+2.2^2}}) & 0.3662 & 0.5056 & 0.5063 & 0.4811 & 0.6708 \\ 0.5839\,(=\frac{2.5}{\sqrt{2^2+2.5^2+1.8^2+2.2^2}}) & 0.6591 & 0.4550 & 0.5983 & 0.2887 & 0.3727 \\ 0.4204\,(=\frac{1.8}{\sqrt{2^2+2.5^2+1.8^2+2.2^2}}) & 0.4882 & 1.0 & 0.4143 & 0.6736 & 0.5217 \\ 0.5139(=\frac{2.3}{\sqrt{2^2+2.5^2+1.8^2+2.2^2}}) & 0.4392 & 0.5056 & 0.4603 & 0.4811 & 0.3727 \end{bmatrix}$$

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Example Fighter aircraft selection problem.

Calculate weighted normalised decision matrix

$$\boldsymbol{W} = \{0.2, 0.1, 0.1, 0.1, 0.2, 0.3\}$$

$$V = \begin{array}{c} \begin{array}{cccccc} \text{max} & \text{max} & \text{max} & \text{min} & \text{max} & \text{max} \end{array} \\ \begin{bmatrix} 0.0934 = 0.2 \cdot 0.4671 & 0.0366 & 0.0506 & 0.0506 & 0.0962 & 0.2012 \\ 0.1168 = 0.2 \cdot 0.5839 & 0.0659 & 0.0455 & 0.0598 & 0.0577 & 0.1118 \\ 0.0841 = 0.2 \cdot 0.4204 & 0.0488 & 0.0531 & 0.0414 & 0.1347 & 0.1565 \\ 0.1028 = 0.2 \cdot 0.5139 & 0.0439 & 0.0506 & 0.0460 & 0.0962 & 0.1118 \end{bmatrix} \end{array}$$

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Example Fighter aircraft selection problem.

Calculate Ideal point and Nadir point

$$V = \begin{array}{c} \begin{array}{cccccc} \text{max} & \text{max} & \text{max} & \text{min} & \text{max} & \text{max} \end{array} \\ \begin{bmatrix} 0.0934 = 0.2 \cdot 0.4671 & 0.0366 & 0.0506 & 0.0506 & 0.0962 & 0.2012 \\ 0.1168 = 0.2 \cdot 0.5839 & 0.0659 & 0.0455 & 0.0598 & 0.0577 & 0.1118 \\ 0.0841 = 0.2 \cdot 0.4204 & 0.0488 & 0.0531 & 0.0414 & 0.1347 & 0.1565 \\ 0.1028 = 0.2 \cdot 0.5139 & 0.0439 & 0.0506 & 0.0460 & 0.0962 & 0.1118 \end{bmatrix} \end{array}$$

Ideal point

$\mathbf{A}^+ = \{0.1168, 0.0659, 0.0531, 0.0414, 0.1347, 0.2012\}$

Nadir point

$\mathbf{A}_\# = \{0.0841, 0.0366, 0.0455, 0.0598, 0.0577, 0.1118\}$

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Step 4. Calculate the distance between each alternative and ideal point A^+ and nadir $A_\#$.

Euclidean distance

$$S_{n+} = \sqrt{\sum_{k=1}^{K} (v_{nk} - v_k^{\,+})^2} \quad , \quad n = 1,\ldots,N$$

$$S_{n\#} = \sqrt{\sum_{k=1}^{K} (v_{nk} - v_{k\#})^2} \quad , \quad n = 1,\ldots,N$$

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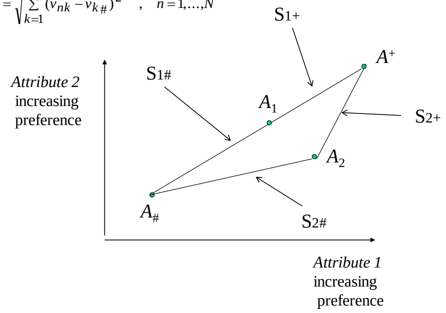
$$S_{n+} = \sqrt{\sum_{k=1}^{K} (v_{nk} - v_k^{+})^2} \quad , \quad n = 1,\ldots,N$$

$$S_{n\#} = \sqrt{\sum_{k=1}^{K} (v_{nk} - v_{k\#})^2} \quad , \quad n = 1,\ldots,N$$



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<u>Example</u> Fighter aircraft selection problem.

Distances between the alternatives and ideal point A^+ and nadir $A_\#$.

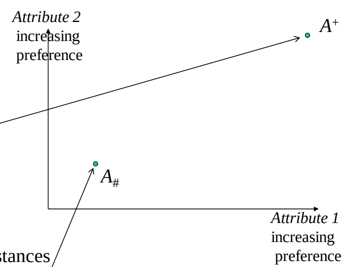
$$S_{1^+} = \sqrt{(0.0934-0.1168)^2 + (0.0366-0.0659)^2 + (0.0506-0.0531)^2 + (0.0506-0.0414)^2 + (0.0962-0.1347)^2 + (0.2012-0.2012)^2}$$
$$= 0.0545$$

S_{2+}=0.1197
S_{3+}=0.0580
S_{4+}=0.1009

Distances from here

$S_{1\#}$=0.0983
$S_{2\#}$=0.439
$S_{3\#}$=0.0920
$S_{4\#}$=0.0458

Distances from here



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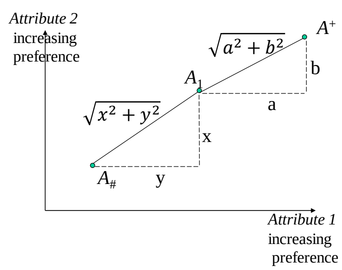
<u>Example</u> Fighter aircraft selection problem.

Distances between alternatives and ideal point A^+ and nadir $A_\#$.

$$S_{1^+} = \sqrt{(0.0934-0.1168)^2 + (0.0366-0.0659)^2 + (0.0506-0.0531)^2 + (0.0506-0.0414)^2 + (0.0962-0.1347)^2 + (0.2012-0.2012)^2}$$
$$= 0.0545$$



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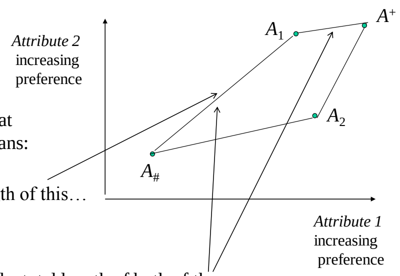
Step 5. Calculate the **relative closeness** to the ideal point A^+



Before we look at the maths, it means:

What is the length of this…

…compared to the total length of both of these

If it is a large proportion, then A_1 is closer to the Ideal Point than to the Nadir

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Step 5. Calculate the relative closeness to the ideal point A^+ .

$$P_n = \frac{S_{n\#}}{S_{n+} + S_{n\#}}, \quad P_n \in [0,1]$$

IF $A_n = A^+$ THEN $P_n = 1$

IF $A_n = A_\#$ THEN $P_n = 0$

Step 6. Rank the alternatives by descending P_n

$$A^* \in \{A_{n^*} \mid P_{n^*} = \max_n P_n\}$$

The best alternative is the highest value of P_n

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<u>Example</u> Fighter aircraft selection problem.

Relative closeness to the ideal point:

$P_1 = S_{1\#} / (S_{1+} + S_{1\#}) = 0.0983/(0.0545+0.0983) = 0.643$
$P_2 = 0.268$
$P_3 = 0.613$
$P_4 = 0.312$

- Aircraft A_1 is selected
- It is closer to the Ideal Point in the normalised space

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4. Promethee Methods

alternatives: $\boldsymbol{A} = \{a,b,c,d,\ldots\}$
attributes: $\boldsymbol{C} = \{f_1, f_2,\ldots,f_K\}$
weights: $\boldsymbol{W} = (w_1, w_2,\ldots,w_K)$

	f_1	f_2	…	f_k		f_K
a	$f_1(a)$	$f_2(a)$		$f_k(a)$		$f_K(a)$
b	$f_1(b)$	$f_2(b)$		$f_k(b)$		$f_K(b)$
c	$f_1(c)$	$f_2(c)$		$f_k(c)$		$f_K(c)$
d	$f_1(d)$	$f_2(d)$		$f_k(d)$		$f_K(d)$
…						
W	w_1	w_2		w_k		w_K

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Dominance Relation:

(all criteria are maximisation)

alternative a ***dominates*** alternative b iff

$f_k(a) \geq f_k(b)$, for all $k=1,\ldots,K$ $\longleftarrow$ It is better or equal in all attributes

and

$(\exists j)\ j \in \{1,\ldots,K\}\ \ f_j(a) > f_j(b)$ $\longleftarrow$ It is strictly better in at least one attribute

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	ex. I		ex. II		ex. III		ex. IV		ex.V	
	f_1 max	f_2 max	f_1 max	f_2 max	f_1 max	f_2 max	f_1 max	f_2 max	f_1 max	f_2 max
a	100	100	100	20	100	99	100	99	100	100
b	30	20	30	100	20	100	99	100	99	99

- ex. I: a should be recommended
- ex. II: a and b are incomparable
- ex. III: a should be recommended
- ex. IV: a and b are **indifferent**
- ex. V: a and b are indifferent

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Motivation

- The magnitude of the deviations between the criteria values should be considered.
- The scaling effects should be eliminated.
- Incomparability should **not** be excluded in case of pairwise comparisons.
- An appropriate methods should be simple to understand by the decision maker.

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Promethee methods

Step 1. Design the "Generalised Preference Criteria"

f is to be maximised

- Usually, this is the dominance relation for each criteria:
  f(a) > f(b) → a is preferred to b
  $f(a) = f(b)$ → a is indifferent to b

•This is inadequate in most applications. The Promethee method "enriches" the dominance relation, replacing it with a *preference* relation.
•We *prefer* one "to a certain degree" [0-1].

•$P(a,b)$ preference function
- ***a*** is not better than ***b*** with respect to criterion → $P(a, b) = 0$
- ***a*** is "slightly" better than ***b*** with respect to criterion → $P(a, b) \sim 0$
- ***a*** is "strongly" better than ***b*** with respect to criterion → $P(a, b) \sim 1$
- ***a*** is "strictly" better than ***b*** with respect to criterion → $P(a, b) = 1$

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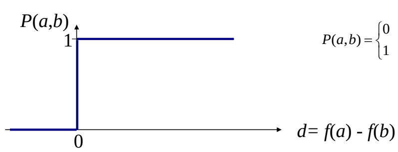
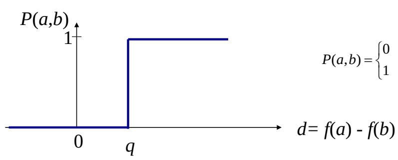
Type I: Usual criterion (for a maximising criterion)



$$P(a,b)=\begin{cases}0 & d\le 0\\ 1 & d>0\end{cases}$$

Type II: Quasi criterion (for a maximising criterion)

- q - is a parameter defining an indifference area



$$P(a,b)=\begin{cases}0 & d\le q\\ 1 & d>q\end{cases}$$

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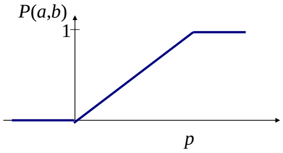
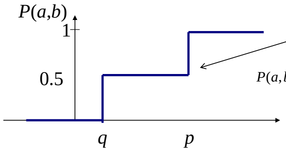
<u>Type III</u>: Criterion with linear preference (for a maximising criterion)

- p - is a parameter defining a strict preference area



$$P(a,b)=\begin{cases} 0 & 0 \le d \\ \dfrac{d}{p} & 0 \le d \le p \\ 1 & d > p \end{cases}$$

<u>Type IV</u>: Level criterion (for a maximising criterion)



$$P(a,b)=\begin{cases} 0 & d \le q \\ 0.5 & q < d \le p \\ 1 & d > p \end{cases}$$

"mid-way" between indifference and full preference

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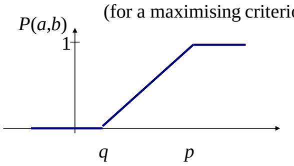
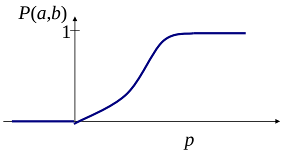
<u>Type V</u>: Criterion with linear preference and indifference interval (for a maximising criterion)



$$P(a,b)=\begin{cases} 0 & d \le q \\ \dfrac{d-q}{p-q} & q < d \le p \\ 1 & d > p \end{cases}$$

<u>Type VI</u>: Gaussian criterion (for a maximising criterion)

- s - is a parameter of Gaussian distribution $(q < s < p)$



$$P(a,b) = 1 - e^{-\frac{d^2}{2s^2}}$$

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<u>Example</u>.
Location of an electric power plant.

<u>Alternatives</u>:
a_1: Italy
a_2: Belgium
a_3: Germany
a_4: UK
a_5: Portugal
a_6: France

<u>Attributes</u>:
C_1: Manpower for running the plant
C_2: Power (in Megawatt)
C_3: Construction costs (in million \$)
C_4: Annual maintenance costs (in million \$)
C_5: Ecology; number of villages to evacuate
C_6: Safety level

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Step 2. Outranking Graph

- for each pair of alternatives *a* and *b* and for each criterion *k* calculate: $d_k = f_k(a) - f_k(b)$, and from that you get the preference index: $P_k(a,b)$

Aggregated preference index (with weights):

$$P(a,b) = w_1 P_1(a,b) + w_2 P_2(a,b) + \dots w_K P_K(a,b)$$

$$P(a,b) = \sum_{k=1}^{K} w_k P_k(a,b), \quad \sum_{k=1}^{K} w_k = 1$$

- Aggregated preference index if weights of all criteria are the same:

$$P(a,b) = \frac{1}{K} P_1(a,b) + \frac{1}{K} P_2(a,b) + \dots + \frac{1}{K} P_K(a,b)$$

$$= \frac{1}{K} \left(P_1(a,b) + P_2(a,b) + \dots + P_K(a,b) \right)$$

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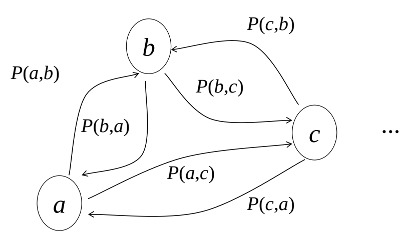

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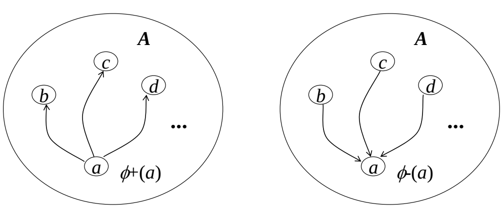
Step 3. Calculate two 'flows'

How much does it outrank others? → 1. Positive outranking flow

$$\phi^{+}(a) = (P(a,b) + P(a,c) + P(a,d) + \dots)$$

How much is it outranked by others? → 2. Negative outranking flow

$$\phi^{-}(a) = (P(b,a) + P(c,a) + P(d,a) + \dots)$$



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- **Promethee I** gives partial rank
(preference $P^{(I)}$, indifference $I^{(I)}$, and incomparability $R^{(I)}$)

$$\begin{cases} a\,P^{(I)}\,b & \text{if } \phi^+(a) \geq \phi^+(b) \text{ and } \phi^-(a) \leq \phi^-(b) \\ & \text{(only one inequality being strict)} \\ a\,I^{(I)}\,b & \text{if } \phi^+(a) = \phi^+(b) \text{ and } \phi^-(a) = \phi^-(b) \\ a\,R^{(I)}\,b & \text{otherwise} \end{cases}$$

Informally (and less accurately):
If it outranks more alternatives, and it is outranked by less alternatives, then it is preferred.

- **Promethee II** gives complete rank.
No pairs are incomparable, but you lose some information.

Uses the net outranking flow:
$\phi(a) = \phi+(a) - \phi-(a)$

$\phi(a) > \phi(b)$ → a outranks b
$\phi(a) = \phi(b)$ → a indifferent b

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Example.
Location of an electric power plant.

Alternatives:
a_1: Italy
a_2: Belgium
a_3: Germany
a_4: UK
a_5: Portugal
a_6: France

Attributes:
C_1: Manpower for running the plant
C_2: Power (in Megawatt)
C_3: Construction costs (in million $)
C_4: Annual maintenance costs (in million $)
C_5: Ecology; number of villages to evacuate
C_6: Safety level

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	C_1	C_2	C_3	C_4	C_5	C_6
	min	max	min	min	min	max
A_1	80	90	600	54	8	5
A_2	65	58	**200**	97	**1**	1
A_3	83	60	400	72	4	7
A_4	**40**	80	1000	75	7	**10**
A_5	52	72	600	**20**	3	8
A_6	94	**96**	700	36	5	6
Weights	1	1	1	1	1	1
Gen.Cri.	II	III	V	IV	I	VI
Parameters	q=10	p=30	q=50 p=500	q=10 p=60	I	s=5

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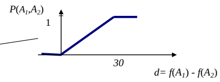
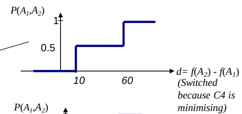
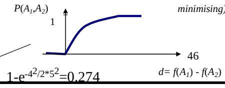
	C_1	C_2	C_3	C_4	C_5	C_6
	min	max	min	min	min	max
A_1	80	90	600	54	8	5
A_2	65	58	**200**	97	**1**	1

$P(A_1, A_2) =$ 0
\+
1 ⟶ d=32
\+
0
\+
0.5 ⟶ d=43
\+
0
\+
0.274 ⟶ d=4







$1-e^{-4^2/2*5^2}=0.274$

	A_1	A_2	A_3	A_4	A_5	A_6	$\phi^+(a)$	$\phi^-(a)$	$\phi(a)$
A_1	0	0.296	0.250	0.268	0.1	0.185	**1.1**	**1.827**	**-0.727**
A_2	0.462	0	0.389	0.333	0.296	0.500	**1.98**	**1.895**	**0.085**
A_3	0.236	0.180	0	0.333	0.056	0.429	**1.23**	**1.681**	**-0.451**
A_4	0.399	0.505	0.305	0	0.223	0.212	**1.64**	**1.746**	**-0.106**
A_5	0.444	0.515	0.487	0.380	0	0.448	**2.27**	**0.808**	**1.462**
A_6	0.286	0.399	0.250	0.432	0.133	0	**1.5**	**1.774**	**-0.274**
$\phi^-(a)$	**1.827**	**1.895**	**1.681**	**1.746**	**0.808**	**1.774**			

☞ A_5 is selected. Largest net outranking flow

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Summary

- Multicriteria decision making refers to making decisions in the presence of multiple, usually conflicting criteria, expressed in different measurement units.
- A large number of MCDM methods exist. They can produce different results!

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