

## *... Teoria Deciziilor*

- *Metode multicriteriale de analiză a deciziilor*

...

➤ Metoda *ELECTRE*

...

## ELECTRE methods

ELECTRE is a family of multi-criteria decision analysis methods that originated in Europe in the mid-1960s. The acronym ELECTRE stands for: ELimination Et Choix Traduisant la REalité (ELimination and Choice Expressing REality).



The method was first proposed by Bernard Roy and his colleagues at SEMA consultancy company. A team at SEMA was working on the concrete, multiple criteria, real-world problem of how firms could decide on new activities and had encountered problems using a weighted sum technique. Bernard Roy was called in as a consultant and the group devised the ELECTRE method. As it was first applied in 1965, the ELECTRE method was to choose the best action(s) from a given set of actions, but it was soon applied to three main problems: choosing, ranking and sorting. The method became more widely known when a paper by B. Roy appeared in a French operations research journal.[1] It evolved into ELECTRE I (*electre one*) and the evolutions have continued with ELECTRE II, ELECTRE III, ELECTRE IV, ELECTRE IS and ELECTRE TRI (*electre tree*), to mention a few.[2]

Bernard Roy is widely recognized as the father of the ELECTRE method, which was one of the earliest approaches in what is sometimes known as the French School of decision making. It is usually classified as an "outranking method" of decision making.

There are two main parts to an ELECTRE application: first, the construction of one or several outranking relations, which aims at comparing in a comprehensive way each pair of actions; second, an exploitation procedure that elaborates on the recommendations obtained in the first phase. The nature of the recommendation depends on the problem being addressed: choosing, ranking or sorting.

Criteria in ELECTRE methods have two distinct sets of parameters: the importance coefficients and the veto thresholds.

## References

1.  Roy, Bernard (1968). "Classement et choix en présence de points de vue multiples (la méthode ELECTRE)". *la Revue d'Informatique et de Recherche Opérationnelle (RIRO)* (8): 57–75.
2.  Figueira, José; Salvatore Greco, Matthias Ehrgott (2005). *Multiple Criteria Decision Analysis: State of the Art Surveys*. New York,: Springer Science + Business Media, Inc.. ISBN ISBN 0-387-23081-5.

## **Introduction to Decision Making Methods, János Fülöp**

Laboratory of Operations Research and Decision Systems,

Computer and Automation Institute, Hungarian Academy of Sciences

<http://academic.evergreen.edu/projects/bdei/documents/decisionmakingmethods.pdf>

## **Outranking methods**

The principal outranking methods assume data availability broadly similar to that required for the MAUT methods. That is, they require alternatives and criteria to be specified, and use the same data of the decision table, namely the  $a_{ij}$ s and  $w_i$ s.

Vincke (1992) provides an introduction to the best known outranking methods; see also Figueira et al. (2004) for state-of-art surveys. Here, the two most popular families of the outranking methods, the **ELECTRE** and the **PROMETHEE** methods will be briefly outlined.

<http://academic.evergreen.edu/projects/bdei/documents/decisionmakingmethods.pdf>

## The ELECTRE methods

The simplest method of the ELECTRE family is ELECTRE I.

The ELECTRE methodology is based on the concordance and discordance indices defined as follows. We start from the data of the decision matrix, and assume here that the sum of the weights of all criteria equals to 1. For an ordered pair of alternatives ( $\mathbf{A}_j, \mathbf{A}_k$ ), the concordance index  $c_{jk}$  is the sum of all the weights for those criteria where the performance score of  $\mathbf{A}_j$  is least as high as that of  $\mathbf{A}_k$ , i.e.

$$c_{jk} = \sum_{i: a_{ij} \geq a_{ik}} w_i, \quad j, k = 1, \dots, n, \quad j \neq k.$$

Clearly, the concordance index lies between 0 and 1.

The computation of the discordance index  $d_{jk}$  is a bit more complicated:  $d_{jk}=0$  if  $a_{ij} > a_{ik}$ ,  $i=1, \dots, m$ , i.e. the discordance index is zero if  $\mathbf{A}_j$  performs better than  $\mathbf{A}_k$  on all criteria. Otherwise,

$$d_{jk} = \max_{i=1, \dots, m} \frac{a_{ik} - a_{ij}}{\max_{j=1, \dots, n} a_{ij} - \min_{j=1, \dots, n} a_{ij}}, \quad j, k = 1, \dots, n, \quad j \neq k,$$

i.e. for each criterion where  $\mathbf{A}_k$  outperforms  $\mathbf{A}_j$ , the ratio is calculated between the difference in performance level between  $\mathbf{A}_k$  and  $\mathbf{A}_j$  and the maximum difference in score on the criterion concerned between any pair of alternatives. The maximum of these ratios (which must lie between 0 and 1) is the discordance index.

A concordance threshold  $c^*$  and discordance threshold  $d^*$  are then defined such that  $0 < d^* < c^* < 1$ .

Then,  $\mathbf{A}_j$  outranks  $\mathbf{A}_k$  if the  $c_{jk} > c^*$  and  $d_{jk} < d^*$ , i.e. the concordance index is above and the discordance index is below its threshold, respectively.



This outranking defines a partial ranking on the set of alternatives. Consider the set of all alternatives that outrank at least one other alternative and are themselves not outranked. This set contains the promising alternatives for this decision problem. Interactively changing the level thresholds, we also can change the size of this set.

The ELECTRE I method is used to construct a partial ranking and choose a set of promising alternatives. ELECTRE II is used for ranking the alternatives. In ELECTRE III an outranking degree is established, representing an outranking creditability between two alternatives which makes this method more sophisticated (and, of course, more complicated and difficult to interpret).

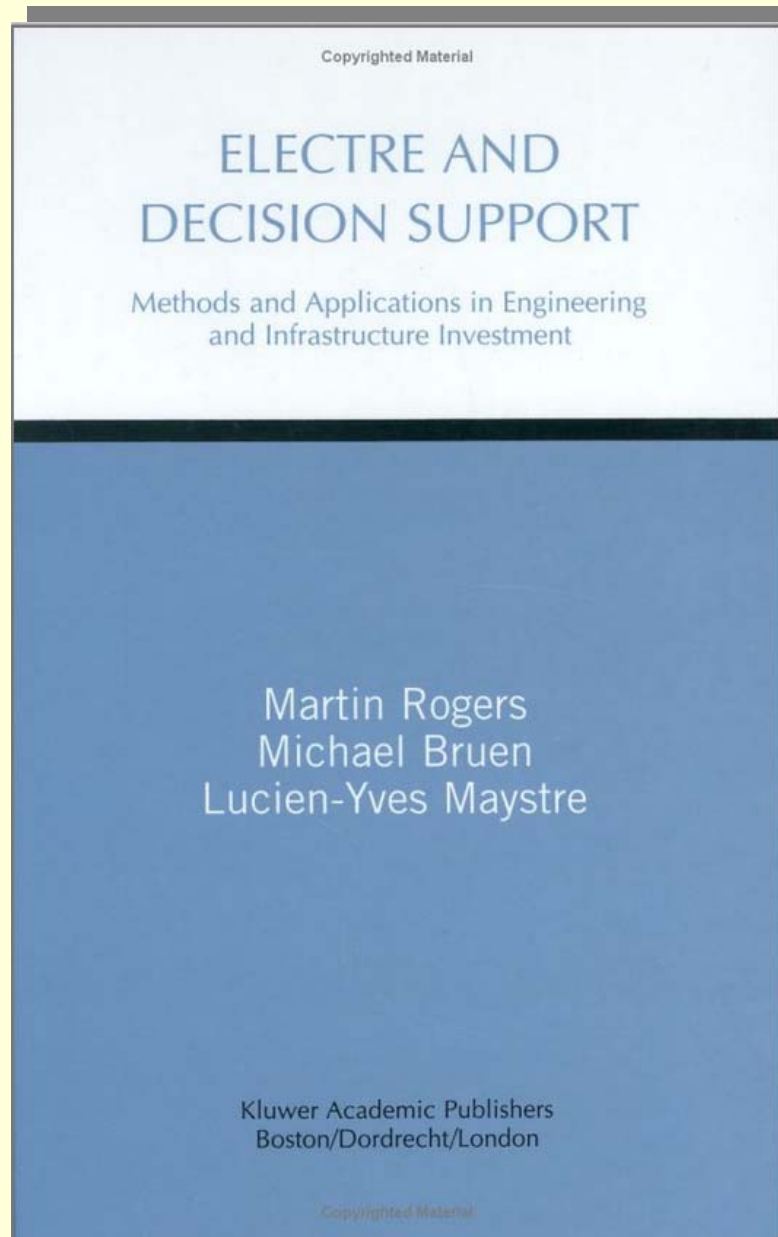
See Figueira et al (2004) for more details and further members of the ELECTRE family.

## References

- Aczél, J. and Saaty, T.L. (1983) .Procedures for synthesizing ratio judgements., *Journal of Mathematical Psychology*, 27, 93-102.
- Baker, D., Bridges, D., Hunter, R., Johnson, G., Krupa, J., Murphy, J. and Sorenson, K. (2002) *Guidebook to Decision-Making Methods*, WSRC-IM-2002-00002, Department of Energy, USA. [http://emi-web.inel.gov/Nissmg/Guidebook\\_2002.pdf](http://emi-web.inel.gov/Nissmg/Guidebook_2002.pdf)
- Barron, F.H. and Barrett, B.E. (1996) .The efficacy of SMARTER . Simple Multi-Attribute Rating Technique Extended to Ranking., *Acta Psychologica*, 93, 23-36.
- Bose, U., Davey, A.M. and Olson, D.L. (1997) .Multi-attribute utility methods in group decision making: Past applications and potential for inclusion in GDSS., *Omega*, 25, 691-706.
- Brans, J.P. and Vincke, Ph. (1985) "A preference ranking organization method", *Management Science*, 31, 647-656.
- Brans, J.P., Vincke, Ph. and Marechal, B. (1986) "How to select and how to rank projects: The PROMETHEE method", *European Journal of Operational Research*, 24, 228- 238.
- Brans, J.-P. and Mareschal, B. (1994) .The PROMCALC & GAIA decision support system for multicriteria decision aid., *Decision Support Systems*, 12, 297-310.
- Csáki, P., Rapcsák, T., Turchányi, P. and Vermes, M. (1995) .Research and development for group decision aid in Hungary by WINGDSS, a Microsoft Windows based group decision support system., *Decision Support Systems* 14, 205-21.
- Dyer, R.F. and Forman, E.H. (1992) .Group decision support with the Analytic Hierarchy Process., *Decision Support Systems*, 8, 99-124.
- Edwards, W. (1977) .How to use multiattribute utility measurement for social decisionmaking., *IEEE Transactions on Systems, Man, and Cybernetics*, SMC-7, 326-340.
- Edwards, W. and Barron, F.H. (1994) .SMARTS and SMARTER: Improved simple methods for multiattribute utility measurements., *Organizational Behavior and Human Decision Processes*, 60, 306-325.
- Ekárt, A. and Németh, S.Z. (2005) .Stability analysis of tree structured decision functions., *European Journal of Operational Research*, 160, 676-695.
- Figueira, J., Greco, S. and Ehrgott, M. (Eds.) (2004) *Multiple Criteria Decision Analysis: State of the Art Surveys*, Springer, New York.
- Forman , E. and Selly, M.A. (2001) *Decision by Objectives*, World Scientific.
- Gass, S. I. and Rapcsák, T. (1998) .A note on synthesizing group decisions., *Decision Support Systems*, 22, 59-63.
- Gass, S. I. and Rapcsák, T. (2004) .Singular value decomposition in AHP., *European Journal of Operational Research*, 154, 573-584.
- Harris, R. (1998) *Introduction to Decision Making*, VirtualSalt. <http://www.virtualsalt.com/crebook5.htm>
- Keeney, R.L. and Raiffa, H. (1976) *Decisions with Multiple Objectives: Performances and Value Trade-Offs*, Wiley, New York.
- Keeney, R.L. (1976) .A group preference axiomatization with cardinal utility., *Management Science*, 23, 140 .145.



- Lai, V.S., Bo K.W. and Cheung, W. (2002) .Group decision making in a multiple criteria environment: A case using the AHP in software selection., *European Journal of Operational Research*, 137, 134-144.
- Leyva-López, J-C. and Fernández-González, E. (2003) .A new method for group decision support based on ELECTRE III methodology., *European Journal of Operational Research*, 148, 14-27.
- Linkov, I., Varghese, A., Jamil, S., Seager, T.P., Kiker, G. and Bridges, T. (2004) .Multi-criteria decision analysis: A framework for structuring remedial decisions at the contaminated sites., In: Linkov, I. and Ramadan, A.B. (Eds.) *Comparative Risk Assessment and Environmental Decision Making*, Springer, New York, pp. 15-54.
- Macharis, C., Brans, J.P. and Mareschal, B. (1998). .The GDSS PROMETHEE Procedure., *Journal of Decision Systems*, 7, 283-307.
- Macharis, C., Springael, J., De Brucker, K. and Verbeke, A. (2004) .PROMETHEE and AHP: The design of operational synergies in multicriteria analysis.: Strengthening PROMETHEE with ideas of AHP., *European Journal of Operational Research*, 153, 307-317.
- Mareschal, B. (1988) .Weight stability intervals in multicriteria decision aid., *European Journal of Operational Research*, 33, 54-64.
- Mészáros, Cs. and Rapcsák, T. (1996) .On sensitivity analysis for a class of decision systems., *Decision Support Systems* 16, 231-240.
- Munda, G. (1996) .Cost-benefit analysis in integrated environmental assessment: some methodological issues., *Ecological Economics*, 19, 157-168.
- Nemhauser, G.L., Rinnoy Kan, A.H.G. and Todd, M.J. (1989) *Handbooks in Operations Research and Management Science: Volume 1 Optimization*, North-Holland, Amsterdam.
- Rapcsák, T. (2004) *Multiattribute Decision Making*, Lecture notes, Department of Decisions in Economics, Corvinus University, Budapest. (in Hungarian)
- Roy, B. (1968) "Classement et choix en présence de points de vue multiple (la méthode electre).", *RAIRO*, 2, 57-75.
- Saaty, T.L. (1980) *The Analytic Hierarchy Process*, McGraw Hill.
- Saaty, T.L. and Vargas, L.G. (1984) .Comparison of eigenvalue, logarithmic least squares and least squares methods in estimating ratios., *Mathematical Modelling*, 5, 309-324.
- Steuer, R. E. (1986) *Multiple Criteria Optimization: Theory, Computation and Application*, Wiley, New York.
- Triantaphyllou, E. and Sanchez, A. (1997) "A sensitivity analysis approach for some deterministic multi-criteria decision making methods", *Decision Sciences*, 28, 151-194.
- Triantaphyllou, E. (2000) *Multi-Criteria Decision Making Methods: A Comparative Study*, Kluwer Academic Publishers, Dordrecht.
- UK DTLR (2001) *Multi Criteria Analysis: A Manual*, Department for Transport, Local Government and the Regions, UK. [http://www.odpm.gov.uk/stellent/groups/odpm\\_about/documents/page/odpm\\_about\\_608524.hcsp](http://www.odpm.gov.uk/stellent/groups/odpm_about/documents/page/odpm_about_608524.hcsp)
- US EPA (2000) *Guidelines for Preparing Economic Analysis*, United States Environmental Protection Agency, EPA 240-R-00-003. <http://yosemite.epa.gov/ee/epa/eed.nsf/webpages/Guidelines.html>
- Vincke, P. (1992) *Multi-criteria Decision-Aid*, John Wiley, Chichester.
- Wolters, W.T.M. and Mareschal, B. (1995) .Novel types of sensitivity analysis for additive MCDM methods., *European Journal of Operational Research*, 81, 281-290.



Springer's  
International Series



ADVANCING THE  
STATE-OF-THE-ART

Volume Contributors:

Carlos Henggeler Antunes  
Denis Bouyssou  
Jean-Pierre Brans  
João Climaco  
José Craveirinha  
Jean-Marie De Corte  
Danae Diakoulaki  
James S. Dyer  
Carlos Bana e Costa  
Matthias Ehrgott  
José Figueira  
Michel Grabisch  
Salvatore Greco  
Evangelos Grigoroudis  
Masahiro Inuiguchi  
Pekka Korhonen  
Christophe Labreuche  
Bertrand Mareschal  
Jean Marc Martel  
António Martins  
Benedetto Matarazzo  
Nikolaos F. Matsatsinis  
Alexander Mechtlov  
Patrick Meyer  
Helen Moshkovich  
Vincent Mousseau  
Giuseppe Munda  
Subhass Narula  
Stefan Nickel  
David L. Olson  
Meltem Öztürk  
Marc Pirlot  
Justo Puerto  
Antonio Rodríguez-Chía  
Marc Roubens  
Bernard Roy  
Thomas Saaty  
Yannis Siskos  
Roman Slowinski  
Charles Smith  
Jaap Sprenk  
Ralph L. Stewart  
Thao Stewart  
Alexis Tsoulfas  
Jean-Claude Vansnick  
Philippe Vincke  
Roland Weistroffer  
Margaret M. Wiecek  
Constantin Zopounidis

# MULTIPLE CRITERIA DECISION ANALYSIS

## State of the Art Surveys

edited by  
**José Figueira**  
**Salvatore Greco**  
**Matthias Ehrgott**

### Chapter 4

#### ELECTRE METHODS

José Figueira

*Faculdade de Economia and INESC-Coimbra*  
*Universidade de Coimbra*  
*Av. Dias da Silva, 165, 3004-512 Coimbra*  
*Portugal*  
[figueira@fe.uc.pt](mailto:figueira@fe.uc.pt)

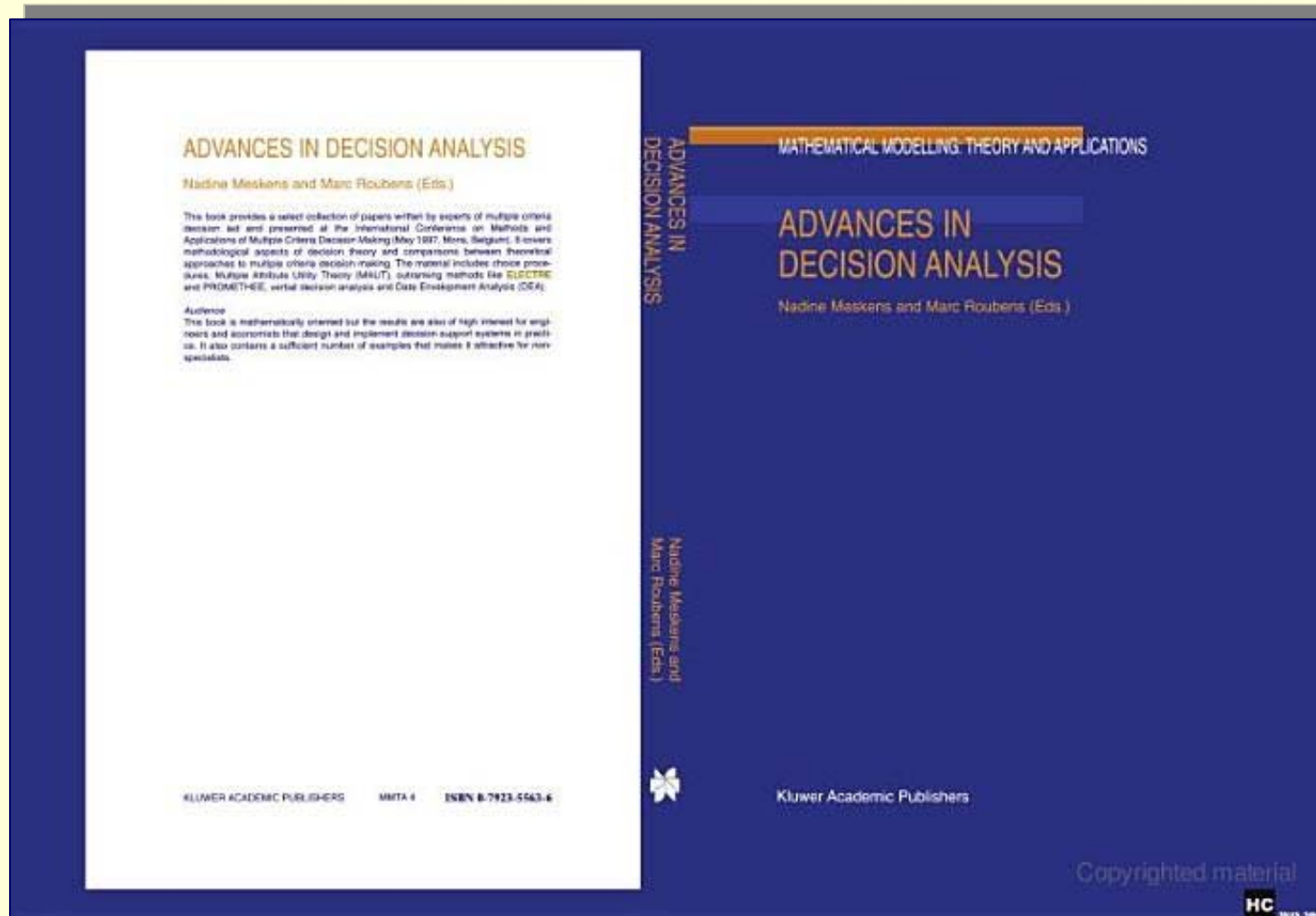
Vincent Mousseau, Bernard Roy

*LAMSADE*  
*Université Paris-Dauphine*  
*Place du Maréchal De Lattre de Tassigny, 75775 Paris Cedex 16*  
*France*  
[{mousseau,roy}@lamsade.dauphine.fr](mailto:{mousseau,roy}@lamsade.dauphine.fr)

**Abstract** Over the last three decades a large body of research in the field of ELECTRE family methods appeared. This research has been conducted by several researchers mainly in Europe. The purpose of this chapter is to present a survey of the ELECTRE methods since their first appearance in mid-sixties, when ELECTRE I was proposed by Bernard Roy and his colleagues at SEMA consultancy company. The chapter is organized in five sections. The first section presents a brief history of ELECTRE methods. The second section is devoted to the main features of ELECTRE methods. The third section describes the different ELECTRE methods existing in the literature according to the three main problematics: choosing, ranking and sorting. The fourth section presents the recent developments and future issues on ELECTRE methods. Finally, the fifth section is devoted to the software and applications. An extensive and up-to-date bibliography is also provided in the end of this chapter.

**Keywords:** Multiple criteria decision aiding, Outranking approaches, ELECTRE methods.

[http://books.google.com/books?id=YM1X0jHrzEMC&printsec=frontcover&dq=ELECTRE&source=gbs\\_similarbooks\\_s&cad=1#v=onepage&q=ELECTRE&f=false](http://books.google.com/books?id=YM1X0jHrzEMC&printsec=frontcover&dq=ELECTRE&source=gbs_similarbooks_s&cad=1#v=onepage&q=ELECTRE&f=false)





# The PROMETHEE methods





## St. Cloud State University & LEO (Literacy Education Online)

**LEO** provides online handouts about a variety of writing topics. Although LEO is affiliated with the Write Place (the writing center at St. Cloud State University), LEO does not offer online tutoring, answer questions about grammar or punctuation, or give feedback about your writing or papers.

If you are interested in information about tutoring or other Write Place services, please head to the writing center homepage at <http://www.stcloudstate.edu/writeplace>.

**LEO** is also the online site for the early issues (1990-1998) of *Kaleidoscope*, SCSU's multicultural literary magazine. For current issues, please visit *Kaleidoscope's* new home on the Write Place web site: <http://web.stcloudstate.edu/kaleidoscope/>.

<http://leo.stcloudstate.edu/index.html>



## Strategies for Writing a Conclusion

Conclusions are often the most difficult part of an essay to write, and many writers feel that they have nothing left to say after having written the paper. A writer needs to keep in mind that the conclusion is often what a reader remembers best. Your conclusion should be the best part of your paper.

### A conclusion should

- stress the importance of the thesis statement,
- give the essay a sense of completeness, and
- leave a final impression on the reader.

### Suggestions

- **Answer the question "So What?"** Show your readers why this paper was important. Show them that your paper was meaningful and useful.
- **Synthesize, don't summarize**
  - Don't simply repeat things that were in your paper. They have read it. Show them how the points you made and the support and examples you used were not random, but fit together.
- **Redirect your readers**
  - Give your reader something to think about, perhaps a way to use your paper in the "real" world. If your introduction went from general to specific, make your conclusion go from specific to general. Think globally.
- **Create a new meaning**
  - You don't have to give new information to create a new meaning. By demonstrating how your ideas work together, you can create a new picture. Often the sum of the paper is worth more than its parts.

## Strategies

- **Echoing the introduction:** Echoing your introduction can be a good strategy if it is meant to bring the reader full-circle. If you begin by describing a scenario, you can end with the same scenario as proof that your essay was helpful in creating a new understanding.

### Example

#### *Introduction*

From the parking lot, I could see the towers of the castle of the Magic Kingdom standing stately against the blue sky. To the right, the tall peak of The Matterhorn rose even higher. From the left, I could hear the jungle sounds of Adventureland. As I entered the gate, Main Street stretched before me with its quaint shops evoking an old-fashioned small town so charming it could never have existed. I was entranced. Disneyland may have been built for children, but it brings out the child in adults.

#### *Conclusion*

I thought I would spend a few hours at Disneyland, but here I was at 1:00 A.M., closing time, leaving the front gates with the now dark towers of the Magic Kingdom behind me. I could see tired children, toddling along and struggling to keep their eyes open as best they could. Others slept in their parents' arms as we waited for the parking lot tram that would take us to our cars. My forty-year-old feet ached, and I felt a bit sad to think that in a couple of days I would be leaving California, my vacation over, to go back to my desk. But then I smiled to think that for at least a day I felt ten years old again.

- **Challenging the reader:** By issuing a challenge to your readers, you are helping them to redirect the information in the paper, and they may apply it to their own lives.

#### **Example**

Though serving on a jury is not only a civic responsibility but also an interesting experience, many people still view jury duty as a chore that interrupts their jobs and the routine of their daily lives. However, juries are part of America's attempt to be a free and just society. Thus, jury duty challenges us to be interested and responsible citizens.

- **Looking to the future:** Looking to the future can emphasize the importance of your paper or redirect the readers' thought process. It may help them apply the new information to their lives or see things more globally.

#### **Example**

Without well-qualified teachers, schools are little more than buildings and equipment. If higher-paying careers continue to attract the best and the brightest students, there will not only be a shortage of teachers, but the teachers available may not have the best qualifications. Our youth will suffer. And when youth suffers, the future suffers.

- **Posing questions:** Posing questions, either to your readers or in general, may help your readers gain a new perspective on the topic, which they may not have held before reading your conclusion. It may also bring your main ideas together to create a new meaning.

#### **Example**

Campaign advertisements should help us understand the candidate's qualifications and positions on the issues. Instead, most tell us what a boob or knave the opposing candidate is, or they present general images of the candidate as a family person or God-fearing American. Do such advertisements contribute to creating an informed electorate or a people who choose political leaders the same way they choose soft drinks and soap?

## ***Metoda ELECTRE***

Metoda ELECTRE (*ELimination Et Choix Traduisant la REalité*), dezvoltată în Franța și aplicată în special în țările europene [Roy, 1968; Roy și Vanderpooten, 1996], se bazează pe **conceptul de surclasare** pentru eliminarea alternativelor care sunt într-un anumit sens "dominate". Noțiunea de "dominanță" în cadrul oferit de surclasare este o generalizare a dominanței clasice și folosește ponderi pentru a ierarhiza criteriile (unele dintre ele au influență mai mare decât altele asupra deciziei).

La fel ca și celelalte metode de analiză multicriterială, metoda ELECTRE are aceleași **etape**:

- specificarea alternativelor și criteriilor,
- evaluarea performanțelor în funcție de criterii și
- stabilirea ponderilor asociate criteriilor, care stabilesc importanța relativă a acestora.

## ... Metoda ELECTRE

Conceptul de *surclasare* a fost definit de Roy astfel: alternativa  $A_i$  *surclasează* alternativa  $A_k$  dacă, fiind date preferințele decidentului, calitatea evaluării alternativelor și contextul problemei, există suficiente argumente să se decidă că  $A_i$  **este cel puțin la fel de bună ca**  $A_k$  și nu există nici un motiv evident care să contrazică afirmația.

Pe baza acestei idei, au fost dezvoltate o serie de proceduri pentru concretizarea conceptului de *surclasare* ca modalitate de asistare a deciziilor multicriteriale.

În general este nevoie de **două etape**:

- a) specificarea unei modalități precise de *stabilire a existenței surclasării* între două alternative și
- b) combinarea evaluărilor de surclasare a alternativelor pentru **a le ierarhiza**.

Esența metodei ELECTRE constă în *identificarea relațiilor de dominanță*.

Ea determină o submulțime de alternative  $E \subseteq A$  astfel încât orice alternativă  $A_k \notin E$  este surclasată de cel puțin o alternativă  $A_i \in E$ .

Obiectivul este ca submulțimea  $E$  să aibă cât mai puține elemente, care vor reprezenta alternative candidat pentru decizia finală.

## Pasul 1. Determinarea indicilor de concordanță și discordanță

Pentru determinarea gradului de concordanță și discordanță ale alternativelor se folosesc **indicii de concordanță și discordanță**. Indicele de concordanță  $c_{ik}$  se calculează pentru fiecare pereche de alternative  $(A_i, A_k)$  ca sumă a ponderilor acelor criterii de decizie  $C_j$  pentru care performanța alternativei  $A_i$  este mai mare decât performanța alternativei  $A_k$ . Indicele de discordanță  $d_{ik}$  se determină ca elementul maxim al mulțimii rapoartelor dintre diferența performanțelor alternativelor  $A_k$  și  $A_i$  și diferența maximă de performanță în raport cu fiecare dintre criteriile  $j$  pentru care performanța alternativei  $A_k$  este cel puțin egală performanței alternativei  $A_i$ :

$$c_{ik} = \sum_{\substack{j=1 \\ r_{ij} > r_{kj}}}^n p_j \quad d_{ik} = \max_{\substack{1 \leq j \leq n \\ r_{ij} \leq r_{kj}}} \left\{ \frac{r_{kj} - r_{ij}}{\max_{1 \leq l \leq m} r_{lj} - \min_{1 \leq l \leq m} r_{lj}} \right\} \quad (1 \leq i, k \leq m).$$

În acest fel, **indicele de discordanță** este 0 dacă performanța alternativei  $A_i$  este mai bună decât performanța alternativei  $A_k$ , respectiv un număr subunitar în caz contrar.

**Indicii de concordanță**  $c_{ik}$  și **discordanță**  $d_{ik}$  definesc **matricile de concordanță MC** și **discordanță MD**, matrici pătratice de ordinul  $m =$  numărul de alternative decizionale.



## Pasul 2. Combinarea indicilor de concordanță și discordanță

Pentru combinarea indicilor de concordanță și discordanță se introduce un prag (relativ mare) de concordanță  $c^*$  și un prag (relativ mic) de discordanță  $d^*$ .

Cu ajutorul acestora se poate defini concret conceptul de *surclasare*:

alternativa  $A_i$  *surclasează* alternativa  $A_k$  dacă  $c_{ik} > c^*$  și  $d_{ik} < d^*$ .

Mulțimea  $E$  a *alternativelor promițătoare* este formată din acele alternative decizionale care surclasează cel puțin o altă alternativă și nu sunt la rândul lor surclasate.

Dacă mulțimea  $E$  conține **prea multe** elemente, ea se poate restrânge prin **majorarea pragului de concordanță și micșorarea pragului de discordanță**.

Dacă  $E$  este **prea mică** (de exemplu mulțimea vidă), atunci se **micșorează pragul de concordanță și se majorează pragul de discordanță**.

**Exemplu:** [Dodgson, 2000]

Alternative decizionale:  $A = \{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}\}$  ( $m=10$ ),

Criterii de decizie:  $C = \{C_1, C_2, C_3, C_4, C_5, C_6\}$  ( $n=6$ ),

Ponderile asociate criteriilor:  $P = \{0.25, 0.10, 0.15, 0.25, 0.05, 0.20\}$ .

**Matricea performanțelor:**

Alternative decizionale	Criterii de decizie					
	$p_1 = 0.25$	$p_2 = 0.1$	$p_3 = 0.15$	$p_4 = 0.25$	$p_5 = 0.05$	$p_6 = 0.2$
	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$
$A_1$	6	300	27	18	570	12
$A_2$	2	450	21	19	400	23
$A_3$	16	350	27	12	420	18
$A_4$	10	500	20	12	450	20
$A_5$	11	380	23	20	400	16
$A_6$	5	250	31	10	430	18
$A_7$	16	390	24	18	510	21
$A_8$	17	400	22	26	380	23
$A_9$	10	410	16	23	410	20
$A_{10}$	5	250	18	21	400	22
diferențe max – min	17-2= 15	500-250= 250	31-16= 15	26-10= 16	570-380= 190	23-12= 11

Pasul 1. Determinarea indicilor de concordanță și discordanță

În această fază se calculează matricile indicilor de concordanță și de discordanță. Pentru exemplificare, se consideră alternativele  $A_3$  și  $A_4$ . Criteriile de decizie pentru care se calculează indicele de concordanță sunt  $\{C_1, C_3\}$  (acele criterii pentru care performanța alternativei  $A_3$  este mai bună decât performanța alternativei  $A_4$ :  $r_{31} > r_{41}$ , respectiv  $r_{33} > r_{43}$ ), iar cele pentru care se calculează indicele de discordanță sunt  $\{C_2, C_4, C_5, C_6\}$  (acele criterii pentru care performanța alternativei  $A_3$  nu este mai bună decât performanța alternativei  $A_4$ :  $r_{32} < r_{42}$ ,  $r_{34} = r_{44}$ ,  $r_{35} < r_{45}$ , respectiv  $r_{36} < r_{46}$ ). Conform definiției, indicele de concordanță  $c_{34} = p_1 + p_3 = 0.25 + 0.15 = 0.4$ . Pentru determinarea indicelui de discordanță  $d_{34}$  se determină elementul maxim al mulțimii:

$$\left\{ \frac{r_{42} - r_{32}}{250}, \frac{r_{44} - r_{34}}{16}, \frac{r_{45} - r_{35}}{190}, \frac{r_{46} - r_{36}}{11} \right\} = \left\{ \frac{500 - 350}{250}, \frac{12 - 12}{16}, \frac{450 - 420}{190}, \frac{20 - 18}{11} \right\} =$$

$$= \left\{ \frac{150}{250}, \frac{0}{16}, \frac{30}{190}, \frac{2}{11} \right\} = \left\{ \frac{3}{5}, 0, \frac{3}{19}, \frac{2}{11} \right\},$$

prin urmare  $d_{34} = 0.6$ . Elementele mulțimii din care s-a calculat  $d_{34}$  sunt rapoartele dintre diferența performanțelor alternativelor  $A_3$  și  $A_4$  și diferențele maxime de performanță, determinate pentru criteriile  $\{C_2, C_4, C_5, C_6\}$  în raport cu care  $A_3$  nu are performanță mai bună decât  $A_4$ .

... Pasul 1. Determinarea indicilor de concordanță și discordanță

Matricea indicilor de *concordanță* este următoarea:

<i>Matricea indicilor de concordanță <math>c_{ik}</math></i>										
	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$A_7$	$A_8$	$A_9$	$A_{10}$
$A_1$	-	0.45	0.30	0.45	0.20	0.65	0.20	0.20	0.20	0.55
$A_2$	0.55	-	0.55	0.60	0.30	0.55	0.55	0.15	0.45	0.45
$A_3$	0.55	0.45	-	0.40	0.65	0.60	0.15	0.20	0.45	0.55
$A_4$	0.55	0.40	0.35	-	0.35	0.85	0.10	0.15	0.30	0.55
$A_5$	0.80	0.65	0.35	0.65	-	0.60	0.25	0.20	0.40	0.50
$A_6$	0.35	0.45	0.02	0.15	0.40	-	0.15	0.20	0.20	0.20
$A_7$	0.55	0.45	0.60	0.90	0.75	0.85	-	0.20	0.65	0.55
$A_8$	0.80	0.65	0.80	0.85	0.80	0.80	0.80	-	0.85	0.95
$A_9$	0.80	0.55	0.55	0.25	0.60	0.80	0.35	0.15	-	0.65
$A_{10}$	0.45	0.50	0.45	0.45	0.45	0.45	0.45	0.05	0.35	-

... Pasul 1. Determinarea indicilor de concordanță și discordanță

Matricea indicilor de *discordanță* este :

<i>Matricea indicilor de discordanță <math>d_{ik}</math></i>										
	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$A_7$	$A_8$	$A_9$	$A_{10}$
$A_1$	-	1.00	0.67	0.80	0.36	0.55	0.82	1.00	0.73	0.91
$A_2$	0.89	-	0.93	0.53	0.60	0.67	0.93	1.00	0.53	0.20
$A_3$	0.79	0.45	-	0.60	0.50	0.27	0.47	0.88	0.69	0.56
$A_4$	0.63	0.44	0.47	-	0.50	0.73	0.40	0.88	0.69	0.56
$A_5$	0.89	0.64	0.33	0.48	-	0.53	0.58	0.64	0.36	0.55
$A_6$	0.74	0.80	0.73	1.00	0.63	-	0.95	1.00	0.81	0.69
$A_7$	0.32	0.24	0.20	0.44	0.13	0.47	-	0.50	0.31	0.19
$A_8$	1.00	0.20	0.33	0.40	0.10	0.60	0.68	-	0.16	0.11
$A_9$	0.84	0.33	0.73	0.76	0.47	1.00	0.53	0.47	-	0.18
$A_{10}$	0.89	0.80	0.73	1.00	0.52	0.87	0.73	0.80	0.64	-

Pasul 2. Combinarea indicilor de concordanță și discordanță

Indicii de concordanță și discordanță se combină pentru a defini conceptul (relativ) de surclasare. Pentru aceasta, mai este nevoie de stabilirea pragurilor de concordanță  $c^*$  și discordanță  $d^*$ . De obicei, valorile de start ale acestora se setează la media elementelor din matricile indicatorilor de concordanță și discordanță:

$$c^* = \frac{1}{m(m-1)} \sum_{\substack{i,k=1 \\ i \neq k}}^m c_{ik}, \quad d^* = \frac{1}{m(m-1)} \sum_{\substack{i,k=1 \\ i \neq k}}^m d_{ik}.$$

Pentru exemplul curent, valorile acestora sunt  $c^* = 0.48$  and  $d^* = 0.61$ . Prin definiție, alternativa  $A_i$  surclasează alternativa  $A_k$  dacă  $c_{ik} > c^*$  și  $d_{ik} < d^*$ .



... Pasul 2. Combinarea indicilor de concordanță și discordanță

Examinând matricile indicatorilor de concordanță și discordanță, rezultă că:

- $A_1$  surclasează  $A_6$ ;
- $A_2$  surclasează  $A_4$ ;
- $A_3$  surclasează  $A_5, A_6, A_{10}$ ;
- $A_4$  surclasează  $A_{10}$ ;
- $A_5$  surclasează  $A_4, A_6, A_{10}$ ;
- $A_7$  surclasează  $A_1, A_3, A_4, A_5, A_6, A_9, A_{10}$ ;
- $A_8$  surclasează  $A_2, A_3, A_4, A_5, A_6, A_9, A_{10}$ ;
- $A_9$  surclasează  $A_2, A_5, A_{10}$ .

Pentru că o alternativă este *dominantă* dacă surclasează cel puțin o altă alternativă și nu este surclasată de nici o altă alternativă, alternativele dominante sunt doar  $A_7$  și  $A_8$  (relația de surclasare nu este tranzitivă ).

Pentru a determina o mulțime mai redusă (de preferat cu un singur element) de alternative dominante, procedeul de mai sus se poate repeta, folosind praguri mai stricte.

Uzual,  $c^*$  se mărește cu 10%, iar  $d^*$  se micșorează cu 10%, dar se pot folosi și alte scheme.

Tabelul alăturat  
ilustrează modificarea  
relațiilor de dominanță  
dacă  $c^*$  se mărește, iar  
 $d^*$  se micșorează.

Rezultatele  
precedente reprezintă  
**Iterația 1).**

<b>Modificarea relațiilor de dominanță dacă <math>c^*</math> se mărește, iar <math>d^*</math> se micșorează</b>				
<b>Iterația</b>	<b><math>c^*</math></b>	<b><math>d^*</math></b>	<b>Alternativa</b>	<b>surclasează alternativele</b>
<b>2</b>	<b>0.53</b>	<b>0.54</b>	$A_2$	$A_4$
			$A_3$	$A_5, A_6$
			$A_5$	$A_4, A_6$
			$A_7$	$A_1, A_3, A_4, A_5, A_6, A_9, A_{10}$
			$A_8$	$A_2, A_3, A_4, A_5, A_9, A_{10}$
			$A_9$	$A_2, A_5, A_{10}$
<b>3</b>	<b>0.58</b>	<b>0.49</b>	$A_3$	$A_6$
			$A_5$	$A_4$
			$A_7$	$A_3, A_4, A_5, A_6, A_9$
			$A_8$	$A_2, A_3, A_4, A_5, A_9, A_{10}$
			$A_9$	$A_5, A_{10}$
<b>4</b>	<b>0.64</b>	<b>0.44</b>	$A_7$	$A_5, A_9$
			$A_8$	$A_2, A_3, A_4, A_5, A_9, A_{10}$
			$A_9$	$A_{10}$
<b>5</b>	<b>0.70</b>	<b>0.40</b>	$A_7$	$A_5$
			$A_8$	$A_3, A_5, A_9, A_{10}$
<b>6</b>	<b>0.75</b>	<b>0.38</b>	$A_8$	$A_3, A_5, A_9, A_{10}$

În primele cinci iterații, mulțimea alternativelor dominante nu are mai puțin de două elemente,  $A_7$  și  $A_8$ . Se observă că doar la **iterația 6** s-a obținut alternativa dominantă  $A_8$ .

Dacă mulțimea alternativelor dominante **nu se reduce la un singur element și dacă se cere recomandarea unei singure alternative**, atunci trebuie să se recurgă la **alte tehnici**.

Dacă însă se cere **ierarhizarea tuturor alternativelor**, se poate proceda în două moduri:

1. Prima variantă, iterativă, elimină la fiecare pas din mulțimea alternativelor pe cea considerată "optimă". Pasul inițial folosește mulțimea tuturor alternativelor, iar după  $m-1$  iterații se obține ierarhizarea dorită: alternativa eliminată la pasul  $k$  ocupă poziția  $k$  în clasament.
2. A doua variantă ia în considerare *meritele relative* ale alternativelor, folosind analiza efectuată anterior. Un clasament *brut* al alternativelor este:  
 $A_8, A_7, A_9, A_3, A_5, A_2, A_1, A_4, A_6, A_{10}$ .

# Decision Theory

