


Applied Statistical Analysis I

Hypothesis testing, experiments, difference in means

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Hypothesis Testing

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- What is a hypothesis?

Hypothesis Testing

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- What are the five steps of null-hypothesis significance testing?

- A **hypothesis** is a statement about a population. It is a prediction that a parameter takes a particular numerical value or falls in a certain range of values. Agresti and Finlay, 2009, p.143

Hypothesis Testing

- Usually, we want to learn about population parameters based on sample statistics → **statistical inference**.
- For example: is a population parameter different from zero? If you find a value different from zero in your sample, can you be sure it's not just random variation?
- Formulate competing hypotheses:
 H_0 : The population parameter equals zero (**null hypothesis**).
 H_A : The population parameter differs from zero (**alternative hypothesis**).

The 5 Steps of Hypothesis Testing

TABLE 6.1: The Five Parts of a Statistical Significance Test

-
- | | |
|----|---|
| 1. | Assumptions Type of data, randomization, population distribution, sample size condition |
| 2. | Hypotheses Null hypothesis, H_0 (parameter value for “no effect”) Alternative hypothesis, H_a (alternative parameter values) |
| 3. | Test statistic Compares point estimate to H_0 parameter value |
| 4. | P-value Weight of evidence against H_0 ; smaller P is stronger evidence |
| 5. | Conclusion Report P -value Formal decision (optional; see Section 6.4) |
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Figure 1: Source: Agresti and Finlay, 2009, p.147

Hypothesis Testing: Step 1

Assumptions Agresti and Finlay, 2009, p.144

- Type of data: continuous, categorical, etc.
- Sampling method: randomly obtained (e.g., random sample)
- Population distribution: assume a certain distribution (e.g., normal)
- Sample size: validity improves with larger samples

Hypothesis Testing: Step 2

State the Hypotheses

- Example: Someone claims the mean income of a country is 1400.
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- In other words: H_0 = the population mean **is equal** to the sample mean and H_A = the population mean **is not equal** to the sample mean.

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- In other words: H_0 = the population mean **is equal** to the sample mean and H_A = the population mean **is not equal** to the sample mean.
- The hypothesis can be one-sided ($<$, $>$, \geq , \leq) or two-sided ($=$, \neq).

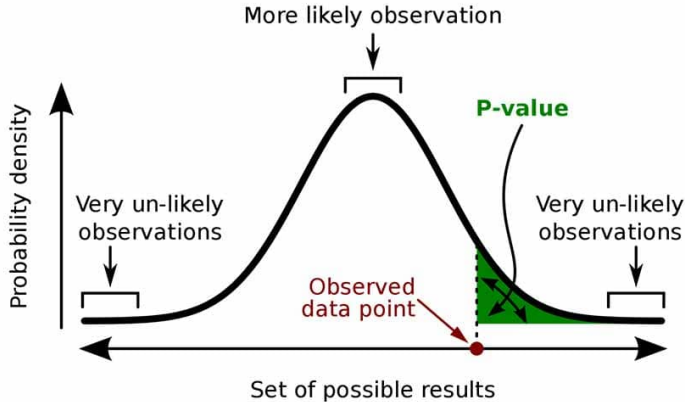
Test Statistic

- “The test statistic summarizes how far the estimate falls from the parameter value in H_0 . Often this is expressed by the number of standard errors between the estimate and the H_0 value.” Agresti and Finlay, 2009, p.145
- Depending on the distribution of the test statistic, it is often referred to as a z-statistic (when the population standard deviation is known and n is large) or a t-statistic (when the n is small and/or the population variance is unknown).

Step 4: p-value

- **p-value**: The p-value is the probability that the test statistic equals the observed value or a value even more extreme in the direction predicted by H_A . It is calculated by presuming that H_0 is true. The smaller the p-value, the stronger the evidence is against H_0 (Agresti and Finlay 2009, 145).

Step 4: p-value



A **p-value** (shaded green area) is the probability of an observed (or more extreme) result assuming that the null hypothesis is true.

Hypothesis Testing: Step 5

Conclusion

- Validate whether the obtained test statistic is unlikely to occur, under the assumption that the null hypothesis (H_0) is true \rightarrow p-value, if probability is low, we reject H_0 .
- Select an α -level, which indicates acceptable probability of Type 1 error (usually 0.05, 0.01).
- If $\text{p-value} < \alpha$, we reject $H_0 \rightarrow$ proof by contradiction.
- Be careful when you are drawing a conclusion - there is still a probability that we falsely reject the null, even if $\text{p-value} < \alpha$.

Hypothesis Testing: Step 5

| | | Null hypothesis (H_0) is | |
|--|--------------|--|---|
| | | True | False |
| Decision about Null hypothesis (H_0) | Don't reject | Correct inference (true negative) (probability = $1-\alpha$) | Type II Error (false negative) (probability = β) |
| | Reject | Type I Error (false positive) (probability = α) | Correct inference (true positive) (probability = $1-\beta$) |

We are most concerned about Type I error: the probability of obtaining a false positive result.

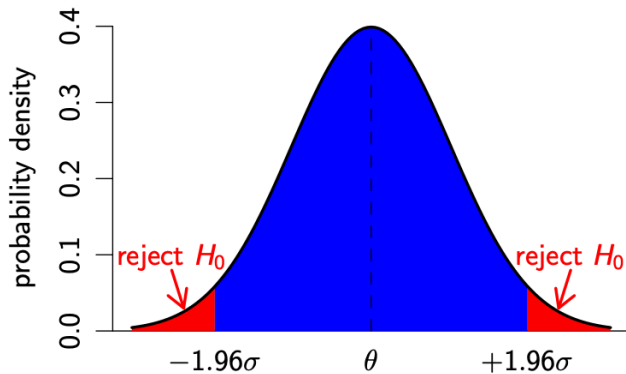
Calculating a Test Statistic

- Let's take our previous example: We take a random sample of size $n = 1000$ and obtain a mean of 1350 with a standard deviation of 750.
- At a level of 95% confidence, the critical z-values on a standard normal distribution are ± 1.96
- How much is the sample mean (i.e., 1350) away from the hypothesized population mean (i.e., 1400) in repeated samples? Calculate the z-score of your realized sample:

$$z = \frac{\bar{x} - \mu_{pop}}{\sigma_{pop}/\sqrt{n}} = \frac{1350 - 1400}{750/\sqrt{1000}} = -2.1$$

- This is smaller than the critical value and you would reject H_0 .

Calculating a Test Statistic



Decision Based on Confidence Intervals

- A $100(1 - \alpha)\%$ **confidence interval (CI)** for a population mean is an interval that, in repeated random samples, would contain the true mean about $100(1 - \alpha)\%$ of the time.
- For a two-sided test with significance level $\alpha = 0.05$, the CI is a **95% CI**.
- Our example:

$$\bar{x} \pm z_{1-\alpha/2} \cdot \frac{s}{\sqrt{n}} = 1350 \pm 1.96 \cdot \frac{750}{\sqrt{1000}} = [1303, 1396].$$

- The hypothesized mean 1400 is **not in this 95% CI** \Rightarrow reject H_0 at $\alpha = 0.05$.
- **Key idea:** For a two-sided test at level α ,

$$\mu_0 \notin 100(1 - \alpha)\% \text{ CI} \iff p < \alpha.$$

Example

Significance test for a mean (t-test)

TABLE 6.2: Responses of Subjects on a Scale of Political Ideology

| Response | Race | | |
|-----------------------------|-----------|------------|----------|
| | Black | White | Other |
| 1. Extremely liberal | 10 | 36 | 1 |
| 2. Liberal | 21 | 109 | 13 |
| 3. Slightly liberal | 22 | 124 | 13 |
| 4. Moderate, middle of road | 74 | 421 | 27 |
| 5. Slightly conservative | 21 | 179 | 9 |
| 6. Conservative | 27 | 176 | 7 |
| 7. Extremely conservative | 11 | 28 | 2 |
| | $n = 186$ | $n = 1073$ | $n = 72$ |

H_0 : Mean political ideology is 'moderate' $\rightarrow \mu = 4$

H_0 : Mean political ideology falls in liberal or conservative direction $\rightarrow \mu \neq 4$
($\mu < 4$ or $\mu > 4$)

Significance test for a mean (t-test)

- With $\bar{x} = 4.075$, $s = 1.512$, we can calculate the SE:

$$SE = \frac{s}{\sqrt{n}} = \frac{1.512}{\sqrt{186}} = 0.111$$

- And then the test statistic:

$$t = \frac{\bar{x} - \mu_0}{SE} = \frac{4.075 - 4}{0.111} = 0.68$$

- Degrees of freedom: $df = n - 1 = 185$.
- How to interpret this value?

Significance test for a mean (t-test)

- For a **two-sided** $\alpha = 0.05$ and $df = 185$, the critical value is : $t_{0.975,185} \approx 1.97$
- Our $t = 0.68$ is far inside $[-1.97, 1.97] \Rightarrow$ fail to reject H_0 .
- p-value: $P(|T| > 0.68) \approx 0.50$ (from the t distribution).

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Checking the CIs too:

$$4.075 \pm 1.97 \cdot 0.111 = [3.86, 4.29]$$

Since 4 is inside the 95% CI, result agrees: fail to reject H_0 .

Significance test for a mean (t-test)

What is the conclusion?

p-value $\approx 0.50 > 0.05$, thus **we cannot reject the null (H_0)**. It is plausible that the population mean is 4, and therefore moderate.

Introduction to causality

What is a causal effect?

What is the fundamental problem of causal inference?



Key idea

A **causal effect** is the change in an outcome Y that would occur if we switched a unit from one condition to another.

Variables

- Treatment T :
 - $T = 1$ = treated
 - $T = 0$ = control
- Outcome Y : what we measure

Potential outcomes

$Y_i(1)$ = outcome if treated

$Y_i(0)$ = outcome if control

Individual causal effect

$$\tau_i = Y_i(1) - Y_i(0)$$

The difference between what *did* happen and what *would have happened* in the counterfactual world.

The fundamental problem with causal inference: “we can only observe, at most, one of the two quantities, for any individual at a particular point in time”
→ **The causal effect is unobservable.**

(Bueno de Mesquita & Fowler, 2021, p.164)

What is the sample average treatment effect (SATE)?
What can we actually observe?

$$\frac{1}{n} \sum_{i=1}^n \{Y_i(1) - Y_i(0)\}$$

- It is the average of individual-level treatment effects in the sample.
- SATE is unobservable due to the fundamental problem of causal inference → we only observe **sample difference in means**.

SATE: Difference in Means

Naïve estimator (difference in means)

$$\hat{\Delta} = \bar{Y}_{T=1} - \bar{Y}_{T=0}$$

Difference in average outcome between treated and control groups.

Key point

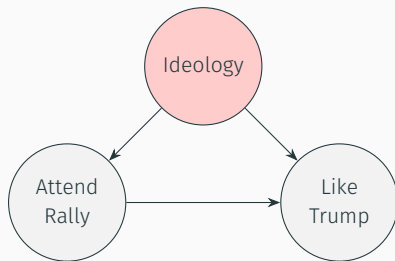
- $\hat{\Delta}$ is an **unbiased estimate of SATE** only if treatment is **as-if random**.
- Otherwise it may be **biased by selection** (treated and control differ in unobserved ways).
- **Correlation does not imply causation.**

What are the sources of bias? And how can we overcome bias?

- **Baseline differences:** “...difference in the average potential outcome between two groups (e.g., the treated and untreated groups), even when those two groups have the same treatment status”
- **Confounders** may cause baseline differences, which may cause bias (omitted variable bias).

Confounders

- It has an effect on the treatment status.
- It also affects the potential outcome *beyond* its effect through treatment.



Other sources of bias

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- Reverse causality
- Unobserved unit heterogeneity (special type of OVB)
- Post-treatment bias

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Randomly assigning units to treatment and control groups eliminates baseline differences.

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3. Causal inference methods: Difference-in-Differences design, Instrumental Variables etc.