Tarefa Básica - Matriz Inversa

01 $A = \begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix}$

$$\begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} x.3+1.y & x.(-1)+1.2 \\ 5.3+3.y & 5.(-1)+3.2 \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$3x+(-5)=1$
$3x=1+5$
$3x=6$
$x=\frac{6}{3}=2$

$-x+2=0$
$-x=-2$
$x=\frac{-2}{-1}=2$

$15+3y=0$
$3y+15=0$
$3y=-15$
$y=\frac{-15}{3}=-5$

$-5+6=1$
$1=1$
$\frac{1}{1}=1$

$A^{-1}=\begin{pmatrix} 2 & 2 \\ -5 & 1 \end{pmatrix}$

$(x+y)=2+(-5)=-3$

02 $\begin{bmatrix} 1 & 0 & 1 \\ K & 1 & 3 \\ 1 & K & 3 \end{bmatrix}$

$1+3K+0=1+3K$

$$\begin{vmatrix} 1 & 0 & 1 \\ K & 1 & 3 \\ 1 & K & 3 \end{vmatrix} \begin{matrix} 1 & 0 \\ K & 1 \\ 1 & K \end{matrix}$$

$3+0+K^2=3+K^2$

$3+K^2-1-3K=K^2-3K+2$

$\Delta=b^2-4.a.c$
$\Delta=-3^2-4.1.2$
$\Delta=9-8$
$\Delta=1$

$x=\frac{-b\pm\sqrt{\Delta}}{2.a}$

$x'=\frac{3+1}{2}=\frac{4}{2}=2$

$x''=\frac{3-1}{2}=\frac{2}{2}=1$

(03)

$A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$ 10 12

$12 - 10 = 2$
det $= 2$

$\rightarrow \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} \div 2 = \begin{bmatrix} 3/2 & 5/2 \\ 1 & 2 \end{bmatrix}$

destacando a diagonal e trocando sinal $\rightarrow \begin{bmatrix} 2 & -5/2 \\ -1 & 3/2 \end{bmatrix}$ c//

(04) $\begin{bmatrix} x & 1 & 2 \\ 3 & 1 & 2 \\ 10 & 1 & x \end{bmatrix}$

$\left[\begin{array}{ccc|cc} x & 1 & 2 & x & 1 \\ 3 & 1 & 2 & 3 & 1 \\ 10 & 1 & x & 10 & 1 \end{array}\right]$

$20 + 2x + 3x = 20 + 5x$

$x^2 + 26 - 20 - 5x = x^2 + 6 - 5x$

$x^2 - 5x + 6 = 0$

$x^2 + 20 + 6 = x^2 + 26$

$\Delta = b^2 - 4 . a . c$
$\Delta = -5^2 - 4 . 1 . 6$
$\Delta = 25 - 24$
$\Delta = 1$

$x = \dfrac{-(-5) \pm \sqrt{1}}{2 . 1}$

$x' = \dfrac{5+1}{2} = \dfrac{6}{2} = 3$//

$x'' = \dfrac{5-1}{2} = \dfrac{4}{2} = 2$//

(05) $\begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix}$ $A^{-1} = ?$

$2 + 2 + 2 = 6$

$\left[\begin{array}{ccc|cc} -1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & 2 & 1 \\ 1 & 1 & -1 & 1 & 1 \end{array}\right]$

$7 - 6 = 1$
det $= 1$

$1 + 2 + 4 = 7$

Calculando matriz adjunta

$\left[\begin{array}{ccc|cc} \cancel{-1} & \cancel{-1} & \cancel{2} & \cancel{-1} & \cancel{-1} \\ 2 & 1 & -2 & 2 & 1 \\ 1 & 1 & -1 & 1 & 1 \\ -1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & 2 & 1 \end{array}\right]$

$A^{-1} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{pmatrix}$

Credeal

Continuação 5

$A = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix} + A^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -2 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

$A + A^{-1} = \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \end{bmatrix}$ B//

06 $(x.A)^T = B$

$XA.A = B^T.A^{-1}$

$XI = B^T.A^{-1}$

$X = B^T.A^{-1}$ // B

07 $B = \begin{bmatrix} x \\ y \end{bmatrix}$ e $C = \begin{bmatrix} 4x+5y \\ 5x+6y \end{bmatrix}$

$A = \begin{bmatrix} 4 & 5 \\ 5 & 6 \end{bmatrix} . B = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix}$

25 24

$\det A = -1$

$A^{-1} = \begin{bmatrix} 6 & -5 \\ 5 & 4 \end{bmatrix} \div (-1) = A^{-1} = \begin{bmatrix} -6 & 5 \\ -5 & -4 \end{bmatrix}$ D

08

$A = \begin{bmatrix} 2 & k \\ -2 & 1 \end{bmatrix}$ $\begin{bmatrix} 2 & k \\ -2 & 1 \end{bmatrix} = 2-(-2k) = 2k+2$

$-2k \quad 2$

$2k_1 + 2 = 1$
$2k_1 = -1$
$k_1 = -\frac{1}{2}$

$2k_2 + 2 = -1$
$2k_2 = -3$
$k_2 = -\frac{3}{2}$

$-\frac{1}{2} + \left(-\frac{3}{2}\right) = -\frac{4}{2} = -2$ // B

09 A e B matrizes quadradas de ordem 2 ($\det(A) \neq 0$ e $\det(B) \neq 0$)

a) $(A+B).(A-B) = A^2 - AB + BA - B^2 \quad \neq 0$

b) $AB \neq BA \rightarrow (A+B)^2 = A^2 + 2AB + B^2$ se $AB = BA$

c) $\det A = \begin{vmatrix} a & b \end{vmatrix} = ad - bc$

$\det -A = \begin{vmatrix} -a & -b \end{vmatrix} = ad - bc$

$\frac{\det A}{\det -A} = \frac{ad-bc}{ad-bc} = 1$ $\quad \begin{vmatrix} c & d \end{vmatrix} \begin{vmatrix} c & -d \end{vmatrix}$

d) $\det A^{-1} = \frac{1}{\det A}$ $\quad B = A^{-1}$

$\det B = \frac{1}{\det A}$