

Tarefa Básica - Sistemas Lineares, regra de Cramer

01)
$$\begin{cases} 2x - y = 2 \\ -x + 3y = -3 \end{cases}$$

$$D = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} \quad \begin{array}{l} 6 - 1 = 5 \\ D = 5 \end{array}$$

$$D_y = \begin{vmatrix} 2 & 2 \\ -1 & -3 \end{vmatrix} \quad \begin{array}{l} -6 - (-2) = -4 \\ D_y = -8 \end{array}$$

$$D_x = \begin{vmatrix} 2 & -1 \\ -3 & 3 \end{vmatrix} \quad \begin{array}{l} 6 - 3 = 3 \\ D_x = 3 \end{array}$$

$$x = \frac{D_x}{D} = \frac{3}{5}$$

$$y = \frac{D_y}{D} = \frac{-4}{5}$$

$$S = \left\{ \left(\frac{3}{5}, -\frac{4}{5} \right) \right\}$$

02)
$$\begin{cases} 3x - y + z = 1 \\ 2x + 3y = -1 \\ 4x + y - 2z = 7 \end{cases}$$

$$D = \begin{vmatrix} 3 & -1 & 1 & 3 & -1 \\ 2 & 0 & 3 & 2 & 0 \\ 4 & 1 & -2 & 4 & 1 \end{vmatrix} \quad \begin{array}{l} 0 + 9 + 4 = 13 \\ -10 - 13 = -23 \\ D = -23 \end{array}$$

$$0 + (-12) + 2 = -10$$

$$D_x = \begin{vmatrix} 1 & -1 & 1 & 1 & -1 \\ -1 & 0 & 3 & -1 & 0 \\ 7 & 1 & 2 & 7 & 1 \end{vmatrix} \quad \begin{array}{l} 0 + 3 + 2 = 5 \\ -22 - 5 = -27 \\ D_x = -27 \end{array}$$

$$0 + (-21) + (-1) = -22$$

$$D_y = \begin{vmatrix} 3 & 1 & 1 & | & 3 & 1 \\ 2 & -1 & 3 & | & 2 & -1 \\ 4 & 7 & -2 & | & 4 & 7 \end{vmatrix}$$

$$-4 + (6 + (-4)) = 55$$

$$32 - 55 = -23$$

$$D_y = -23$$

$$D_z = \begin{vmatrix} 3 & -1 & 1 & | & 3 & -1 \\ 2 & 0 & -1 & | & 2 & 0 \\ 4 & 1 & 7 & | & 4 & 1 \end{vmatrix}$$

$$6 + 12 + 14 = 32$$

$$0 + (-3) + (-14) = -17$$

$$6 - (-17) = 23$$

$$D_z = 23$$

$$0 + 4 + 2 = 6$$

$$x = \frac{D_x}{D} = \frac{-27}{-23} = 1 \quad y = \frac{D_y}{D} = \frac{-23}{-23} = 1 \quad z = \frac{D_z}{D} = \frac{23}{-23} = -1$$

$$S = \{(1, 1, -1)\}$$

$$\textcircled{22} \begin{cases} 3x + 4y - z = 1 \\ -1x + 4x + 5y + 2z = 12 \\ x - 2y + 3z = 8 \end{cases}$$

$$\textcircled{+} \begin{cases} -4x - 5y - 2z = -12 \\ 4x + 2y + 2z = 9 \\ -3y = -3 \end{cases}$$

$$y = \frac{-3}{-3} = 1$$

$$\begin{cases} 3x + 4y - z = 1 \\ x - 2y + 3z = 8 \\ 4x + 2y + 2z = 9 \end{cases}$$

03
$$\begin{cases} x + 2y + z = 1 \\ 3x + y - 11z = -2 \\ 2x + 3y - z = 1 \end{cases}$$

$$D = \begin{vmatrix} 1 & 2 & 1 & 1 & 2 \\ 3 & 1 & -11 & 3 & 1 \\ 2 & 3 & -1 & 2 & 3 \end{vmatrix}$$

$2 + (-33) + (-6) = -37$
 $-34 - (-37) = 3$
 $D = 3$

$$D_x = \begin{vmatrix} 1 & 2 & 1 & 1 & 2 \\ -2 & 1 & -11 & -2 & 1 \\ 1 & 3 & -1 & 1 & 3 \end{vmatrix}$$

$1 + (-33) + 4 = -28$
 $-29 - (-28) = -1$
 $D_x = -1$

$$D_y = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 3 & -2 & -11 & 3 & -2 \\ 2 & 1 & -1 & 2 & 1 \end{vmatrix}$$

$-1 + (-22) + (-6) = -29$
 $-17 - (-18) = 1$
 $D_y = 1$

$-4 + (-11) + (-3) = -18$ $2 + (-22) + 3 = -17$

$$D_z = \begin{vmatrix} 1 & 2 & 1 & 1 & 2 \\ 3 & 1 & -2 & 3 & 1 \\ 2 & 3 & 1 & 2 & 3 \end{vmatrix}$$

$2 + (-6) + 6 = 2$
 $2 - 2 = 0$
 $D_z = 0$

$1 + (-8) + 9 = 2$

$1a + b + c$
 $-1 + 1 + 0 = 0$

(04)

$$\begin{cases} x+2y-3z=29 \\ x+3y+2z=4 \\ x-y-2z=8 \end{cases}$$

$$D = \begin{vmatrix} 1 & 2 & -3 \\ 1 & 3 & 2 \\ 1 & -1 & -2 \end{vmatrix}$$

$$-9 + (-2) + (-4) = -15$$

$$1 - (-15) = 16$$

$$D = 16$$

$$D_x = \begin{vmatrix} 29 & 2 & -3 \\ 4 & 3 & 2 \\ 8 & -1 & -2 \end{vmatrix}$$

$$-72 + (-58) + (-16) = -146$$

$$-6 + 4 + 3 = 1$$

$$-130 - (-146) = 16$$

$$D_x = 16$$

$$-174 + 32 + 12 = -130$$

$$D_y = \begin{vmatrix} 1 & 29 & -3 \\ 1 & 4 & 2 \\ 1 & 8 & -2 \end{vmatrix}$$

$$1 = 26 - (-54) = 80$$

$$-12 + 16 + (-58) = -54$$

$$-8 + 58 + (-24) = 26$$

$$87 + (-4) + 16 = 99$$

$$D_z = \begin{vmatrix} 1 & 2 & 29 \\ 1 & 3 & 4 \\ 1 & -1 & 8 \end{vmatrix}$$

$$3 - 99 = -96$$

$$24 + 8 + (-29) = 3$$

$$(x + y + z) = 16 + 80 + (-96) = 0 //$$

(05)

$$\begin{cases} 2x + y = 5 \\ 2y + z = 3 \\ 3x + 2y + z = 7 \end{cases}$$

$$D = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 3 & 2 & 1 \end{vmatrix}$$

$$0 + 4 + 0 = 4$$

$$7 - 4 = 3$$

$$D = 3$$

$$4 + 3 + 0 = 7$$

$$D_x = \begin{vmatrix} 5 & 1 & 0 \\ 3 & 2 & 1 \\ 7 & 2 & 1 \end{vmatrix}$$

$$0 + 10 + 3 = 13$$

$$17 - 13 = 4$$

$$D_x = 4$$

$$10 + 7 + 0 = 17$$

$$D_y = \begin{vmatrix} 2 & 5 & 0 \\ 0 & 3 & 1 \\ 3 & 7 & 1 \end{vmatrix}$$

$$0 + 14 + 0 = 14$$

$$21 - 14 = 7$$

$$D_y = 7$$

$$6 + 15 + 0 = 21$$

$$D_z = \begin{vmatrix} 2 & 1 & 5 \\ 0 & 2 & 3 \\ 3 & 2 & 7 \end{vmatrix}$$

$$30 + 12 + 0 = 42$$

$$37 - 42 = -5$$

$$D_z = -5$$

$$28 + 9 + 0 = 37$$

$$x = \frac{4}{3}$$

$$y = \frac{7}{3}$$

$$z = -\frac{5}{3}$$

$$S = \left\{ \frac{4}{3}, \frac{7}{3}, -\frac{5}{3} \right\}$$

$$\textcircled{Q6} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ -1 \end{bmatrix}$$

$$\begin{cases} x = 3 \\ 2x + y = 7 \\ -x + 2y + 2z = -1 \end{cases}$$

$$D = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 2 & 1 \\ -1 & 2 & 2 & -1 & 2 \end{bmatrix}$$

$0+0+0=0$

$2-0=2$

$2+0+0=2$

$$D_x = \begin{bmatrix} 3 & 0 & 0 & 3 & 0 \\ 7 & 1 & 0 & 7 & 1 \\ -1 & 2 & 2 & -1 & 2 \end{bmatrix}$$

$0+0+0=0$

$6-0=6$

$6+0+0=6$

$$D_y = \begin{bmatrix} 1 & 3 & 0 & 1 & 3 \\ 2 & 7 & 0 & 2 & 7 \\ -1 & -1 & 2 & -1 & -1 \end{bmatrix}$$

$0+0+12=12$

$14-12=2$

$D_y=2$

$21-0+8=14+0+0=14$

$$D_z = \begin{bmatrix} 1 & 0 & 3 & 1 & 0 \\ 2 & 1 & 7 & 2 & 1 \\ -1 & 2 & -1 & -1 & 2 \end{bmatrix}$$

$-3+14+0=11$

$11-11=0$

$D_z=0$

$-1+0+12=11$

$$x = \frac{6}{2} = 3$$

$$y = \frac{2}{2} = 1$$

$$z = \frac{0}{2} = 0$$

$$V = \{3, 1, 0\}$$

Escalonamento - Tarifa Básica

$$01 \quad S = \begin{cases} 2x - y - 3z = -5 \\ x + 3y - z = 11 \\ x - 5z = 3 \end{cases}$$

$$\begin{array}{l} x=2 \quad \begin{bmatrix} 1 & 0 & -5 & : & 3 \\ 2 & -1 & -3 & : & -5 \\ 1 & -3 & -1 & : & 11 \end{bmatrix} \quad \begin{array}{l} \times 3 \\ \downarrow \end{array} \quad \begin{bmatrix} 0 & -1 & 7 & : & 11 \\ 0 & 3 & 4 & : & 8 \end{bmatrix} \quad \begin{array}{l} \rightarrow -y - 7 \cdot -1 = 11 \\ y = 4 \end{array} \\ \begin{bmatrix} 1 & 0 & -5 & : & 3 \\ 0 & 0 & 25 & : & -25 \end{bmatrix} \end{array}$$

$$\begin{aligned} x - 5 \cdot (-1) &= 3 \\ x &= -2 \end{aligned}$$

$$\begin{aligned} 25z &= -25 \\ z &= -1 \end{aligned}$$

$$02 \quad \begin{cases} x = 2y \\ 2y = 3z \\ x + y + z = 11 \end{cases}$$

$$z = 2 \cdot \left(\frac{x}{2} \right) = z = \frac{x}{3}$$

$$y = \frac{x}{2}$$

$$x + \left(\frac{x}{2} \right) + \left(\frac{x}{3} \right) = 11$$

$$x \cdot 6 + \frac{x}{2} \cdot 6 + \frac{x}{3} \cdot 6 = 11 \cdot 6$$

$$x \cdot 6 + 3x + 2x = 66$$

$$11x = 66$$

$$x = \frac{66}{11} = 6$$

$$6 \cdot 2 \cdot \left(\frac{x}{2} \right) + 2 \cdot \left(\frac{x}{3} \right)$$

$$6 + x + x =$$

$$6 + 6 + 6 = 18$$

$$x = 18$$

$$\textcircled{03} \begin{cases} x + y + z = 0 \\ 2x - y - 2z = 1 \\ 6y + 3z = -12 \end{cases}$$

$$x + (-9) + 2 = 0$$

$$x = 7$$

$$\times 2 \begin{bmatrix} 1 & 1 & 1 & : & 0 \\ 2 & -1 & -2 & : & 1 \\ 0 & 6 & 3 & : & -12 \end{bmatrix}$$

$$\times 2 \begin{bmatrix} 0 & -3 & -4 & : & 1 \\ 0 & 6 & 3 & : & -12 \end{bmatrix}$$

$$\hookrightarrow y + (-4) \cdot 2 = 1$$

$$y = -9$$

$$\begin{bmatrix} 0 & 0 & -5 & : & -10 \end{bmatrix}$$

$$\hookrightarrow -5z = -10$$

$$z = 2$$

$$\textcircled{04} A + B + C = \text{R\$} 68,00$$

$$B + \frac{20}{100} \cdot C = A \rightarrow B + \frac{C}{5} = A \rightarrow 5B + C = 5A \rightarrow 5A - 5B - C = 0$$

$$C + \frac{20}{100} \cdot A = 3B \rightarrow C + \frac{A}{5} = 3B \rightarrow 5C + A = 15B \rightarrow A - 15B - 5C = 0$$

$$\begin{cases} A + B + C = 68 \\ 5A - 5B - C = 0 \\ A - 15B + 5C = 0 \end{cases}$$

$$D = \begin{bmatrix} 1 & 1 & 1 & | & 68 \\ 5 & -5 & -1 & | & 0 \\ 1 & -15 & 5 & | & 0 \end{bmatrix} \begin{array}{l} -5 + 15 + 25 = 35 \\ -101 - 35 = -136 \\ D = 136 \end{array}$$

$$-25 + (-1) + (-15) = -41$$

$$D_A = \begin{bmatrix} 68 & 1 & 1 & | & 68 & 1 \\ 0 & -5 & -1 & | & 0 & -5 \\ 0 & -15 & 5 & | & 0 & -15 \end{bmatrix} \begin{array}{l} 0 + 1020 + 0 = 1020 \\ -1700 - 1020 = -2720 \\ D_A = -2720 \end{array}$$

$$-1700 + 0 + 0 = -1700$$

$$0+0+1700=1700$$

$$DB = \begin{array}{ccc|ccc} 1 & 68 & 1 & 1 & 68 & \\ 5 & 0 & -1 & 5 & 0 & \\ 1 & 0 & 5 & 1 & 0 & \end{array}$$

$$-68 - 1700 = -1768$$

$$DB = -1768$$

$$0 + (-68) + 0 = -68$$

$$-340 + 0 + 0 = -340$$

$$Dc = \begin{array}{ccc|ccc} 1 & 1 & 68 & 1 & 1 & \\ 5 & -5 & 0 & 5 & -5 & \\ 1 & -15 & 0 & 1 & -15 & \end{array}$$

$$-5100 - (-340) = -4760$$

$$Dc = -4760$$

$$0 + 0 + (-5100) = -5100$$

$$A = \frac{-2700}{-136} = 20$$

$$B = \frac{-1768}{-136} = 13$$

$$C = \frac{-4760}{-136} = 35$$

Ali: R\$20, Bia: R\$13,00 u Cao: R\$35,00

$$C - A = 35 - 20 = R\$15,00$$

$$(05) \quad A = \begin{bmatrix} 0 & 3 & 4 \\ 1 & 0 & 5 \\ 2 & 1 & 0 \end{bmatrix} \text{ u } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{cases} 0x & 3y & 4z = 134 \\ 1x & 0y & 5z = 115 \\ 2x & 1y & 0z = 48 \end{cases}$$

$$0+0+0=0$$

$$D = \begin{array}{ccc|ccc} 0 & 3 & 4 & 0 & 3 & \\ 1 & 0 & 5 & 1 & 0 & \\ 2 & 1 & 0 & 2 & 1 & \end{array}$$

$$34 - 0 = 34$$

$$D = 34$$

$$0 + 30 + 4 = 34$$

$$D_x = \begin{vmatrix} 134 & 3 & 4 & 134 & 3 \\ 115 & 0 & 5 & 115 & 0 \\ 48 & 1 & 0 & 48 & 1 \end{vmatrix}$$

$$0 + 670 + 0 = 670$$

$$1180 - 670 = 510$$

$$D_x = 510$$

$$0 + 720 + 460 = 1180$$

$$D_y = \begin{vmatrix} 0 & 134 & 4 & 0 & 134 \\ 1 & 115 & 5 & 1 & 115 \\ 2 & 48 & 0 & 2 & 48 \end{vmatrix}$$

$$920 + 0 + 0 = 920$$

$$1532 - 920 = 612$$

$$D_y = 612$$

$$0 + 1340 + 192 = 1532$$

$$D_z = \begin{vmatrix} 0 & 3 & 134 & 0 & 3 \\ 1 & 0 & 115 & 1 & 0 \\ 2 & 1 & 48 & 2 & 1 \end{vmatrix}$$

$$0 + 0 + 144 = 144$$

$$824 - 144 = 680$$

$$D_z = 680$$

$$0 + 690 + 134 = 824$$

$$x = \frac{510}{34} = 15$$

$$y = \frac{612}{34} = 18$$

$$z = \frac{680}{34} = 20$$

$$x + y + z = 15 + 18 + 20 = R\$53,00$$