

## Tarefa Básica - Matrizes Inversas

01)  $A = \begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix}$   $B = \begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix}$

$$\begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} x \cdot 3 + 1 \cdot y & x \cdot (-1) + 1 \cdot 2 \\ 5 \cdot 3 + 3 \cdot y & 5 \cdot (-1) + 3 \cdot 2 \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{cases} 3x + y = 1 \\ 3x = 1 - y \\ 3x = 6 \\ x = \frac{6}{3} = 2 \end{cases} \quad \begin{cases} -x + 2 = 0 \\ -x = -2 \\ x = \frac{-2}{-1} = 2 \end{cases} \quad \begin{cases} 15 + 3y = 0 \\ 3y + 15 = 0 \\ 3y = -15 \\ y = \frac{-15}{3} = -5 \end{cases} \quad \begin{cases} -5 + 6 = 1 \\ 1 = 1 \\ 1 = 1 \end{cases} \quad \left\{ A^{-1} = \begin{pmatrix} 2 & -2 \\ -5 & 1 \end{pmatrix} \right.$$

$$(x+y) = 2 + (-5) = -3$$

02)  $\begin{bmatrix} 1 & 0 & 1 \\ K & 1 & 3 \\ 1 & K & 3 \end{bmatrix}$

$$\begin{array}{ccc|cc} 1 & 0 & 1 & 1 & 0 \\ K & 1 & 3 & K & 1 \\ 1 & K & 3 & 1 & K \end{array}$$

$$1 + 3K + 0 = 1 + 3K$$

$$3 + K^2 - 1 - 3K = K^2 - 3K + 2$$

$$\Delta = b^2 - 4 \cdot a \cdot c \quad \left\{ \begin{array}{l} x = \frac{-b \pm \sqrt{\Delta}}{2 \cdot a} \end{array} \right.$$

$$\Delta = 3^2 - 4 \cdot 1 \cdot 2$$

$$3 + 0 + K^2 = 3 + K^2$$

$$\Delta = 9 - 8$$

$$\Delta = 1$$

$$x' = \frac{3+1}{2} = \frac{4}{2} = 2$$

$$x'' = \frac{3-1}{2} = \frac{2}{2} = 1$$

03

$$A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$$

10      12

$$12 - 10 = 2 \rightarrow \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} \div 2 = \begin{bmatrix} 3/2 & 5/2 \\ 1 & 2 \end{bmatrix}$$

det = 2

destacando a diagonal  $\rightarrow \begin{bmatrix} 2 & -5/2 \\ -1 & 3/2 \end{bmatrix}$   
e trocando sinal

04

$$\begin{bmatrix} x & 1 & 2 \\ 3 & 1 & 2 \\ 10 & 1 & x \end{bmatrix}$$

$$\begin{bmatrix} x & 1 & 2 & x & 1 \\ 3 & 1 & 2 & 3 & 1 \\ 10 & 1 & x & 10 & 1 \end{bmatrix}$$

$20 + 2x + 3x = 20 + 5x$   
 $x^2 + 26 - 20 - 5x = x^2 + 6 - 5x$   
 $x^2 - 5x + 6 = 0$

$$x^2 + 20 + 6 = x^2 + 26$$

$$\Delta = b^2 - 4 \cdot a \cdot c$$

$$\Delta = -5^2 - 4 \cdot 1 \cdot 6$$

$$\Delta = 25 - 24$$

$$\Delta = 1$$

$$x = \frac{-(-5) \pm \sqrt{1}}{2 \cdot 1}$$

$$x' = \frac{5+1}{2} = \frac{6}{2} = 3$$

$$x'' = \frac{5-1}{2} = \frac{4}{2} = 2$$

05

$$\begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix}$$

$A^{-1} = ?$

$$\begin{bmatrix} -1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & 2 & 1 \\ 1 & 1 & -1 & 1 & 1 \end{bmatrix}$$

$2 + 2 + 2 = 6$   
 $7 - 6 = 1$   
det = 1

$$1 + 2 + 4 = 7$$

Calculando matriz adjunta

$$\begin{bmatrix} -1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & 2 & 1 \\ 1 & 1 & -1 & 1 & 1 \\ -1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & 2 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{pmatrix}$$

Continuação 5

$$A = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix} + A^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -2 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

$$A + A^{-1} = \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \end{bmatrix} B_{//}$$

06  $(X \cdot A)^T = B$

$$X \cdot A \cdot A^T = B^T \cdot A^{-1}$$

$$X \cdot I = B^T \cdot A^{-1}$$

$$X = B^T \cdot A^{-1} \quad \textcircled{B}$$

07  $B = \begin{bmatrix} x \\ y \end{bmatrix}$  e  $C = \begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix}$

$$A = \begin{bmatrix} 4 & 5 \\ 5 & 6 \end{bmatrix} \cdot B = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix}$$

$$\det A = -1$$

$$A^{-1} = \begin{bmatrix} 6 & -5 \\ 5 & 4 \end{bmatrix} \div (-1) = A^{-1} = \begin{bmatrix} -6 & 5 \\ 5 & -4 \end{bmatrix} \quad \textcircled{D}$$



08

$$A = \begin{bmatrix} 2 & k \\ -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & k \\ -2 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & k \\ 0 & 1-2k \end{bmatrix}$$

$-2k \quad 2$

$$2 \cdot (-2k) = 2k + 2$$

$$2k_1 + 2 = 1$$

$$2k_1 = -1$$

$$k_1 = -\frac{1}{2}$$

$$\begin{cases} 2k_2 + 2 = -1 \end{cases}$$

$$\begin{cases} 2k_2 = -3 \end{cases}$$

$$\begin{cases} k_2 = -\frac{3}{2} \end{cases}$$

$$\frac{-1}{2} + \left( \frac{-3}{2} \right) = \frac{-4}{2} = -2 //$$

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