Teorema do Binômio

Tarefa Básica

1- $(1+2x^2)^6$ $\quad x^8 = ?$

$$\binom{6}{k} 1^{6-k} \cdot (2x^2)^k = \binom{6}{k} 2^k \cdot x^{2k} \rightarrow k = 0, 1, 2, \ldots, 6$$

$2k = 8$

$k = \frac{8}{2} = 4$

$$\binom{6}{4} 2^4 \cdot x^8 = \frac{6!}{4!\,2!} \cdot 16 \cdot x^8 = 240x^8 \quad \text{(C)}$$

2- $(14x-13y)^{237}$

$x = 1$

$y = 1$

$$\rightarrow (14x-13y)^{237} = (14.1-13.1)^{237} = (14-13)^{237} = 1^{237} = 1 \quad \text{(B)}$$

3- $(x+a)^{11}$ igual a $1386x^5$

$$T_{k+1} = \binom{11}{x} x^{11-k} a^k = 1386x^5$$

$11 - k = 5$

$11 - 5 = k$

$k = 6$

$$\rightarrow T_{6+1} = \binom{11}{6} x^{11-6} a^6 = 1386x^5$$

$$T1 = \frac{11!}{6!\,5!} a^6 = 1386$$

$$T1 = \frac{11.10.9.8.7.\cancel{6!}}{\cancel{6!}\,5!} a^6 = 1368$$

$462a^6 = 1386$

$a^6 = \frac{1386}{462}$

$a^6 = 3$

$a = \sqrt[6]{3}$ (A)

④ - $\left[x+\frac{1}{x^2}\right]^9$ $\quad T_{p+1}=\binom{9}{p}\cdot(x)^{9-p}\cdot\left(\frac{1}{x^2}\right)^p$

$T_{p+1}=\binom{9}{p}\cdot(x)^{9-p}\cdot(x^{2\cdot-1})^p=\binom{9}{p}x^{\frac{9-p}{2}}\cdot x^{-p}=\binom{9}{p}x^{\frac{9-p}{2}-p}$

$\frac{9-p}{2}-p \rightsquigarrow \frac{9-p}{2}-\frac{2p}{2}=\frac{9-3p}{2}$

$T_{p+1}=\binom{9}{p}x^{\frac{9-3p}{2}}$

$x^0=1$

$\frac{9-3p}{2}=0 \rightsquigarrow 9-3p=0 \rightsquigarrow 9=3p \rightsquigarrow p=\left(\frac{9}{3}\right)$ // (D)

⑤ - $\left[x+\frac{1}{x^2}\right]^n$ $\quad T_{p+1}=\binom{n}{p}\cdot(x)^{n-p}\cdot\left(\frac{1}{x^2}\right)^n$

$T_{p+1}=\binom{n}{p}\cdot(x)^{n-p}\cdot(x^{2\cdot-1})^n=\binom{n}{p}x^{\frac{n-p}{2}}\cdot x^{-n}=\binom{n}{p}x^{\frac{n-p}{2}-n}$

$\frac{n-p}{2}-n \rightsquigarrow \frac{n-p}{2}-\frac{2n}{2}=\frac{n-3p}{2}$

$T_{p+1}=\binom{n}{p}x^{\frac{n-3p}{2}}$

$x^0=1$

$\frac{n-3p}{2}=0 \rightsquigarrow n-3p=0 \rightsquigarrow p=\frac{n}{3}$ // (C)

(06) - Seja $K = \left(3x^2 + \frac{2}{x^2}\right)^5 - \left[243x^{15} + 810x^{10} + 1080x^5 + \frac{240}{x^5} + \frac{32}{x^{10}}\right]$

$x = 1$

$\left(3.1^3 + \frac{2}{1^2}\right)^5 - \left(234.1^{15} + 810.1^{10} + 1080^5 + \frac{240}{1^5} + \frac{32}{1^{10}}\right)$

$\left(3.1 + \frac{2}{1}\right)^5 - \left(234 + 810 + 1080 + 240 + 32\right)$

$\left(3.1 + \frac{2}{1}\right)^5 - 2405$

$(3+2)^5 - 2405$

$5^5 - 2405 = 3125 - 2405 = 720$

(07) $(2x+y)^5$

$(2x+y)^5 = \binom{5}{0}(2x)^5y^0 + \binom{5}{1}(2x)^4y^1 + \ldots + \binom{5}{4}(2x)^1y^4 + \binom{5}{5}(2x)^0y^5$

$\binom{5}{0}2^5 + \binom{5}{1}2^4 + \binom{5}{2}2^3 + \binom{5}{3}2^2 + \binom{5}{4}2^1 + \binom{5}{5}2^0 = 2^5 + 5.2^4 + 10.2^3 + 10.2^2 + 5.2 + 1 =$

$32 + 80 + 80 + 40 + 10 + 1 = 243$

(C)