Parallel Computing Assignment 1
Shared Memory

November 16, 2016

Contents

List of Figures

1 Approach to Parallelisation

1.1 Introduction

The assignment was to implement matrix relaxation for a shared memory architecture using pthreads, on a matrix $\mathcal{M}$ of variable square dimension d, using t threads, working to a floating point precision of p.

My implementation uses two barriers and no other synchronisation primitives, taking advantage of the case of redundant-write data races discussed in [4] to avoid the overheads of locking a shared global variable.

1.2 Preventing Data Races

Data races are one of the most difficult problems arising from multithreaded code to debug, due to the seemingly random nature of how threads are scheduled [2]. They occur when concurrent threads attempt to access a shared variable (which is not protected by an explicit synchronisation mechanism) and at least one of the accesses is a write [7].

I order to preserve the consistency of data being read by performing the *relax* operation on a different cell, no cell can be overwritten with its new value until the new value every other cell has been calculated. This is the case regardless of whether the relaxation is performed sequentially or in parallel, and necessitates using a second matrix of the same dimensions, into which the relaxed values are written. After each complete relaxation, the two matrices can then be swapped so that the result of the most recent iteration can be relaxed, overwriting the result of the second most recent. There is of course additional space complexity associated with allocating space for $2n$ elements, but this is still linear ($\mathcal{O}(2n)$ generalises to $\mathcal{O}(n)$) and is therefore a reasonable trade-off against the overheads of alternative solutions.

Although during the process of relaxation any cell $a_{i,j}$ will be read as part of performing the *relax* operation on at most four of its neighbours ($a_{i-1,j}, a_{i,j-1}, a_{i+1,j}$ and $a_{i,j+1}$), it will only ever be written to when $a_{i,j}$ itself is relaxed. This means that no synchronisation is required when writing values to the second array, as write-access to every element is guaranteed to only happen once, during which time there are no reads. Similarly, checking the precision of a cell requires one read operation in each array but no writes, so again avoids potential races.

1.3 Division of Work

Relaxing a square matrix of size $n \times n$ with t threads requires $(n-2)^2$ cells to be relaxed each iteration, as the array boundary values remain fixed. Dividing the work as evenly as possible can be done with simple arithmetic.

All threads relax a minimum of $(n-2)^2$ div t cells, where div is the integer division operation. This leaves $(n-2)^2$ mod t cells remaining, and as there is no way of predicting how each thread will be scheduled and therefore which ones will finish first, for the sake of simplicity this leaves the first $((n-2)^2$ mod $t)$ threads assigned to do $((n-2)^2$ div $t) + 1$ cells each.

Calculating the precision for a given cell is not dependent on the values of any of its adjacent neighbours, so threads both relax and compute the precision for every cell they are assigned, which avoids the cost of needlessly looping again.

Since array access in C is row-major, indexing into a row's columns is a faster operation than indexing into a column's rows, so I assign cells to threads by varying the second index in the inner loop.

1.4 Thread Work

My approach to parallelising the solution was to reduce contention for global variables as much as possible. There are two global condition variables, `global_continue` and `global_done`. `global_continue` gets set to 1 by any thread, if any of its cells did not reach the required precision, meaning another iteration is required. `global_done` is a binary flag set only by the main thread (to 1 if the threads can return or 0 if they need to continue.)

I used two barriers to synchronise threads, as illustrated in Figures 1 and 2. The first is hit once all the worker threads have relaxed their assigned cells and either set `global_continue` or not set it. Once the main thread hits also this barrier, the worker threads hit the second barrier and wait. The main thread resets `global_continue` if it has been set, ready for the next iteration. If `global_continue` was zero, all threads have finished, so the main thread sets `global_done` which will cause all the threads to return.

There is obvious contention for `global_continue` between all threads which may want to increment it. However, I avoided having to use a lock by treating any non-zero value as `true`. Either an arbitrary number of threads assign `global_continue = 1` in succession (which will be read by the main thread as `true`) or two or more of the assignments happen simultaneously and the value is still a non-zero one (and will therefore still be read by the main thread as `true`.) This particular case of race has been found to be acceptably safe [4].

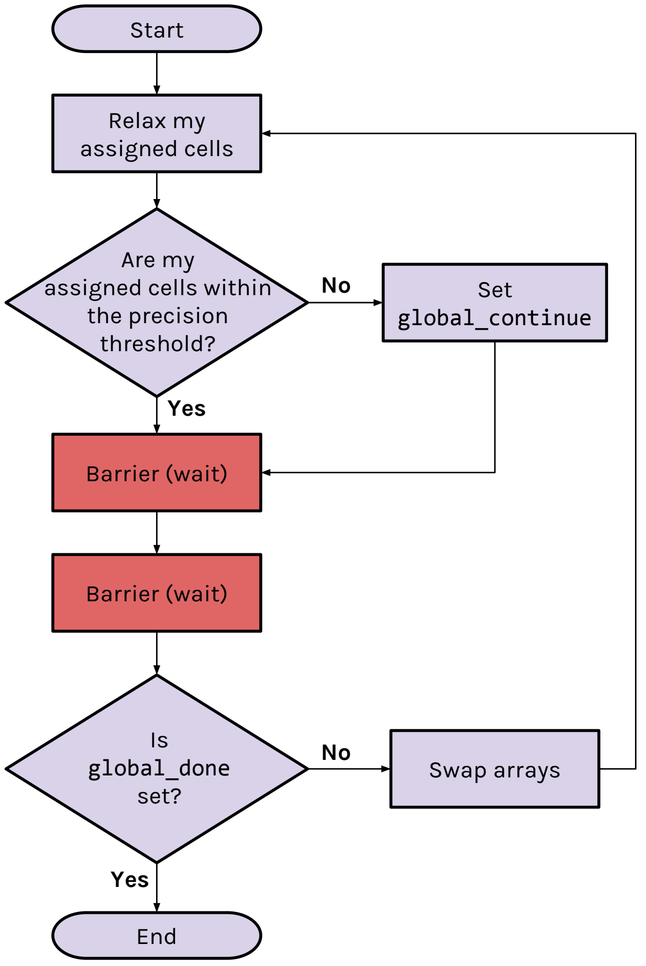


Figure 1: Worker thread control flow

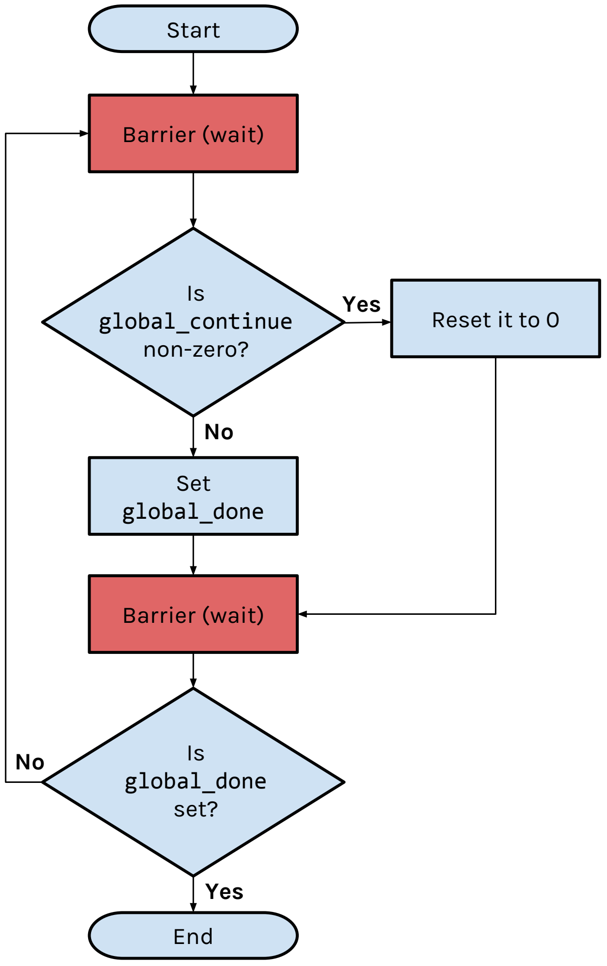


Figure 2: Main thread control flow

None of my worker threads allocate thread-local variables on the heap, as this is unnecessary and also requires allocator and deallocator synchronisation across every thread [3].

2 Correctness Testing

To verify that my implementation of relaxation was correct, I manually calculated every step in relaxing a matrix and verified not only that both solutions terminated after the same number of iterations, but also that after each iteration, my matrix was identical to the computed one. Both my hand-computed relaxation (below) and my single-threaded implementation required four iterations to relax the 5×5 matrix to a precision of 0.75.

Initial Matrix

1.000000	1.000000	1.000000	1.000000	1.000000
1.000000	3.000000	7.000000	2.000000	1.000000
1.000000	8.000000	6.000000	5.000000	1.000000
1.000000	9.000000	0.000000	4.000000	1.000000
1.000000	1.000000	1.000000	1.000000	1.000000

Max Δ: abs(9-2.5) = 6.5
6.5 ≥ 0.75 so continue.

1.000000	1.000000	1.000000	1.000000	1.000000
1.000000	4.250000	3.000000	3.500000	1.000000
1.000000	4.750000	5.000000	3.250000	1.000000
1.000000	2.500000	5.000000	1.750000	1.000000
1.000000	1.000000	1.000000	1.000000	1.000000

Max Δ: abs(5-2.5625) = 2.4375
2.4375 ≥ 0.75 so continue.

1.000000	1.000000	1.000000	1.000000	1.000000
1.000000	2.437500	3.437500	2.062500	1.000000
1.000000	3.187500	4.000000	2.812500	1.000000
1.000000	2.937500	2.562500	2.562500	1.000000
1.000000	1.000000	1.000000	1.000000	1.000000

Max Δ: abs(3.4375-2.375) = 1.0625
1.0625 ≥ 0.75 so continue.

1.000000	1.000000	1.000000	1.000000	1.000000
1.000000	2.156250	2.375000	2.062500	1.000000
1.000000	2.593750	3.000000	2.406250	1.000000
1.000000	1.937500	2.625000	1.843750	1.000000
1.000000	1.000000	1.000000	1.000000	1.000000

Max Δ: abs(2.625-1.9453125) = 0.6796875
0.6796875 < 0.75 (done).

1.000000	1.000000	1.000000	1.000000	1.000000
1.000000	1.742188	2.054688	1.695312	1.000000
1.000000	2.023438	2.500000	1.976562	1.000000
1.000000	1.804688	1.945312	1.757812	1.000000
1.000000	1.000000	1.000000	1.000000	1.000000

Relaxed matrix (after 4 iterations)

The hand-computed results show my checking of the maximum delta for each array after relaxing it to determine whether another iteration is required. Once I verified that my implementation was correct with one thread, I relaxed under the same conditions with 2-9 threads, printing all intermediate values and the iteration count at termination. I then used the Unix `diff` utility to check all 9 outputs were identical. These files have been included in the `correctness` directory, for verification.

3 Scalability Investigation

3.1 Fixed Problem Size

Once I had verified the correctness of my relaxation implementation, I chose a fixed array size and precision, and ran my program relaxing the same array with 1-16 threads. The time, speedup and efficiency (speedup achieved by n processors divided by n [5]) for this can be seen in Figures 3 and 4.

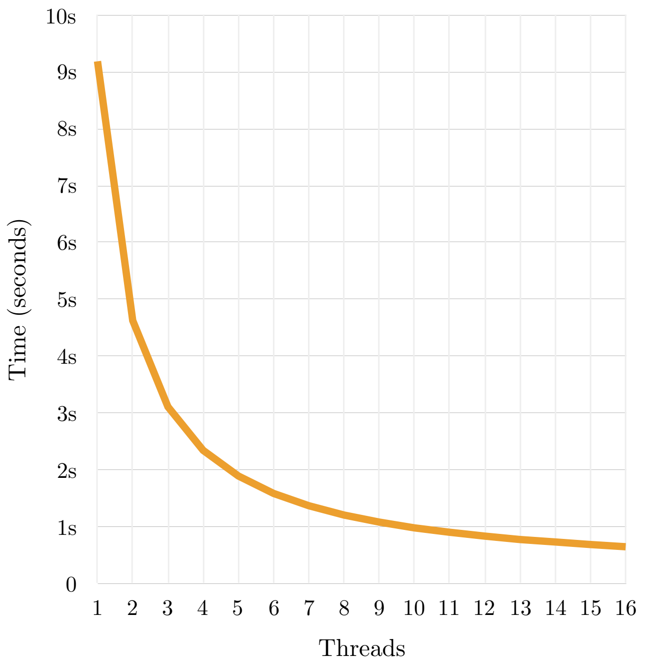


Figure 3: Time for 1-16 threads to relax the same matrix, $d = 1000$, $p = 0.5$

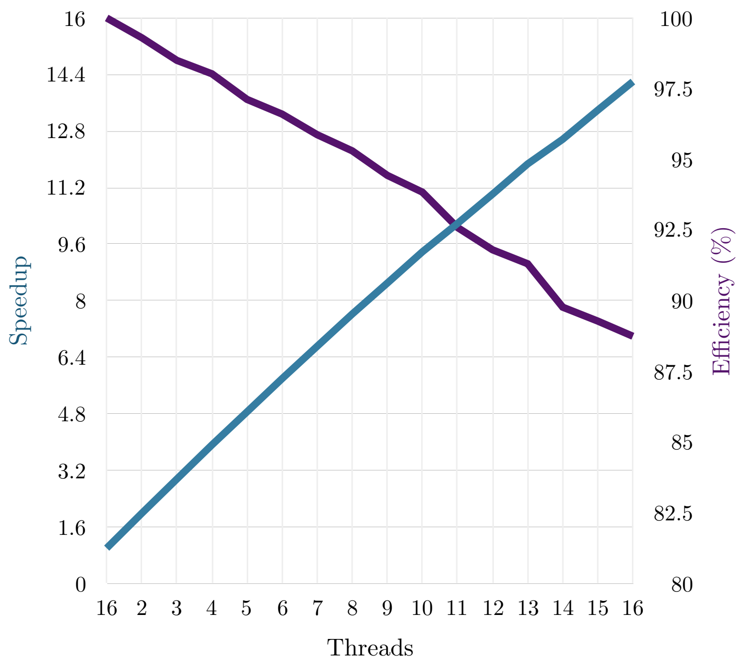


Figure 4: Speedup and efficiency achieved by 1-16 threads relaxing the matrix, $d = 1000$, $p = 0.5$

Each thread[1] saw efficiency reduce, as the speedup had a progressively smaller impact on the time reduction (diminishing returns; [1]). This is consistent with Amdahl's law, a corollary of which states that for P processors and a computation whose sequential proportion is denoted f, predicted speedup is bound by $\frac{1}{f}$ [6]. Working backwards from this upper bound and assuming a maximum speedup of 14.5, this would imply that the sequential proportion of my solution is ≤ 0.069, or 6.9% of the total computation for this fixed problem size. It is important to note that the sequential proportion is not fixed, as changing the number of threads, precision or dimensions would alter the overhead needed to allocate, assign thread work, and wait at barriers.

The speedup I achieved for this problem size was sub-linear, but close to linear. The sequential proportion of the computation appears to become the limiting factor above 16 threads, preventing further speedup through parallelisation.[2] In order to verify this, I repeated the experiment with even numbers of threads between 2 and 32 on the same matrix. The resulting time, speedup and efficiency values can be seen in Figures 5 and 6, with the values for 1 thread (i.e. the sequential computation) left in for comparison.[3]

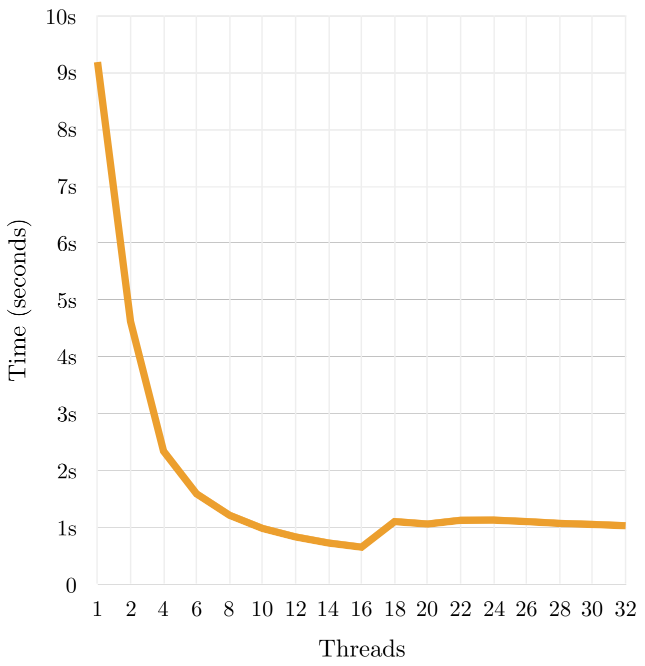


Figure 5: Time for 2-32 threads to relax the same matrix, $d = 1000$, $p = 0.5$

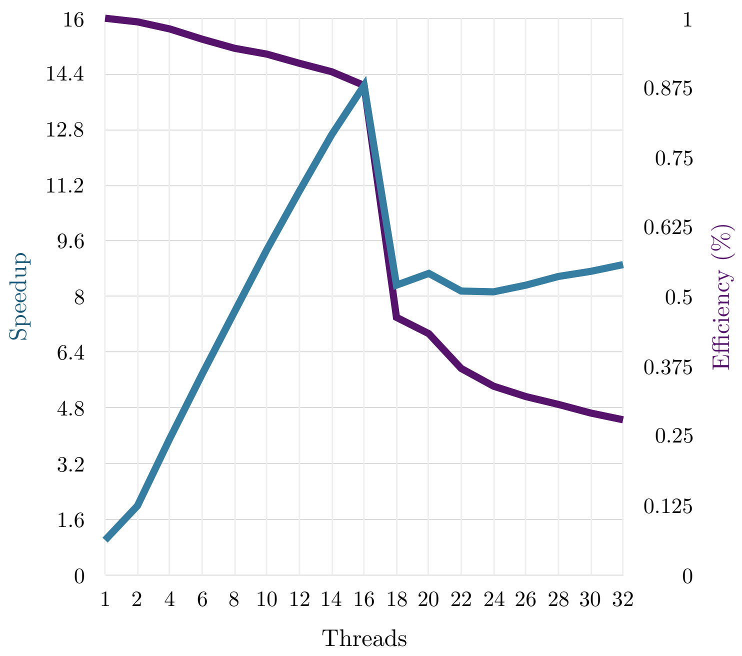


Figure 6: Speedup achieved by 2-32 threads relaxing the matrix, $d = 1000$, $p = 0.5$

[1]The thread counts specified in this section refer to worker threads, i.e. the main thread is not included.
[2]Full data in Appendix A.1.
[3]Full data in Appendix A.2.

3.2 Variable Problem Size

The second factor I tested was the effect of varying the matrix dimensions on the time, for various thread counts. The results of this can be seen in Figure 7[4]. The x-axis in the graph is scaled with respect to dimensions but not size, since the actual computation required for each dimension is proportional to the square of the dimension, not the dimension itself.

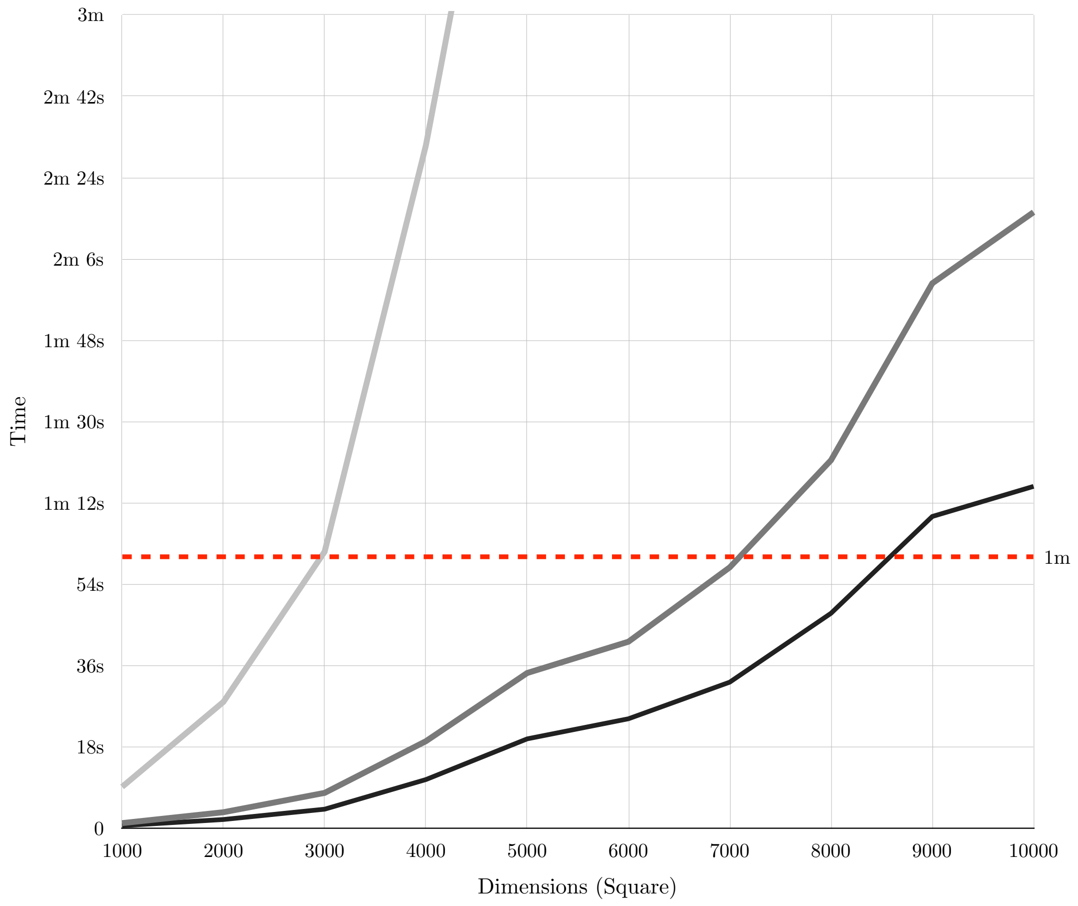


Figure 7: Matrix dimensions against time for 1 (light grey), 8 (medium grey) and 16 (dark grey) threads.

Consider the values which intersect the dotted line in Figure 7. In one minute, one thread can relax a 3000^2 matrix, with 9,000,000 elements. In the same amount of time, eight threads can relax a 7000^2 matrix (49,000,000 elements, and a speedup of 7) and sixteen threads can relax approximately 8500^2 (72,250,000 elements, and a speedup of 8.03). The relative drop in speedup from 1-8 and 8-16 threads again demonstrates the effect of diminishing returns and efficiency; the efficiency of eight threads is $7 \div 8 = 87.5\%$; for sixteen threads this

[4]Full data in Appendix A.3

becomes $8.03 \div 16 = 50.1\%$. It is important to note however that the most efficient number of processors is not the number which will yield the greatest absolute speedup. Mathematically, peak efficiency is reached by maximising $\frac{\text{speedup}}{n}$, which is equivalent to $\frac{T_{\text{seq}}}{n \times T_n}$. T_{seq} is fixed, so maximum efficiency is found minimising $n \times T_n$; the solution to which will be $n = 1$ unless speedup is superlinear.

3.3 Precision

I conducted a single experiment into the effects of lowering the precision threshold on the time and iterations required for my solution to terminate. Below are the results for a fixed array with $d = 500$ and 16 threads[5], showing iterations and time increasing at the same rate as the precision threshold is lowered.

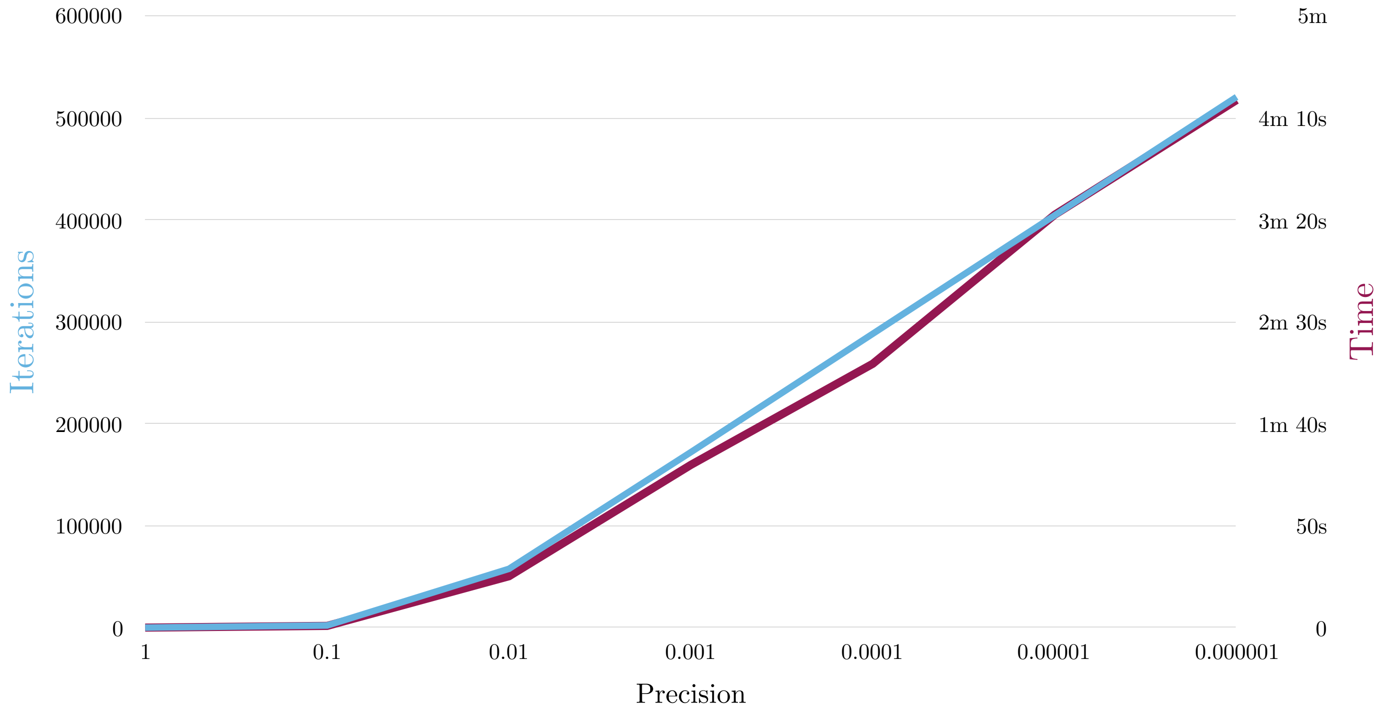


Figure 8: Precision values against iterations and time

I tried to run my solution on a precision of 0.0000001, but it timed out on Balena, due to either exceeding the 15 minute limit while relaxing, or reaching the point at which applying the relax operation left every cell unchanged. Not every matrix when relaxed to under a

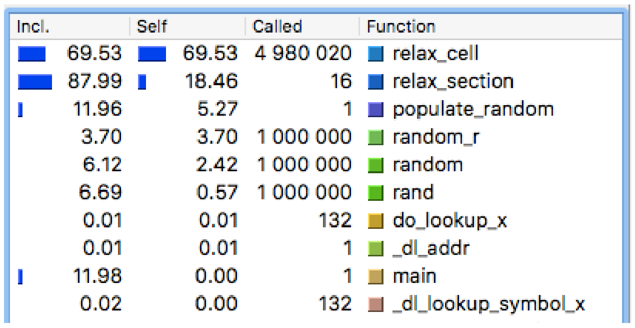
[5]Full data in Appendix A.4

given precision will terminate. My implementation does not attempt to identify these cases and would therefore loop infinitely for precision values which aren't reachable for a given matrix.

4 Cycle Accounting

To determine whether there were any simple optimisations I could make to my implementation to improve speeds, I ran it on Balena under Valgrind, using the `callgrind` and `cachegrind` tools. I then exported the output file and analysed it with qcachegrind[6].

In `callgrind`, I ordered functions by cycle count (ignoring cycle counts of their callees). Although cycle count is not perfectly analogous to latency, it is the closest metric callgrind provides. This allowed me to see the functions which were dominating execution cycles, as shown in Figure 9. At the lower end of the list are functions which can be expected such as



Incl.	Self	Called	Function
69.53	69.53	4 980 020	relax_cell
87.99	18.46	16	relax_section
11.96	5.27	1	populate_random
3.70	3.70	1 000 000	random_r
6.12	2.42	1 000 000	random
6.69	0.57	1 000 000	rand
0.01	0.01	132	do_lookup_x
0.01	0.01	1	_dl_addr
11.98	0.00	1	main
0.02	0.00	132	_dl_lookup_symbol_x

Figure 9: The top ten functions by self-cycles

`main`, functions prefixed `dl_` which are the dynamic linker loading pthread library functions, and functions responsible for populating the array with random numbers. None of these are associated with the actual relaxation logic.

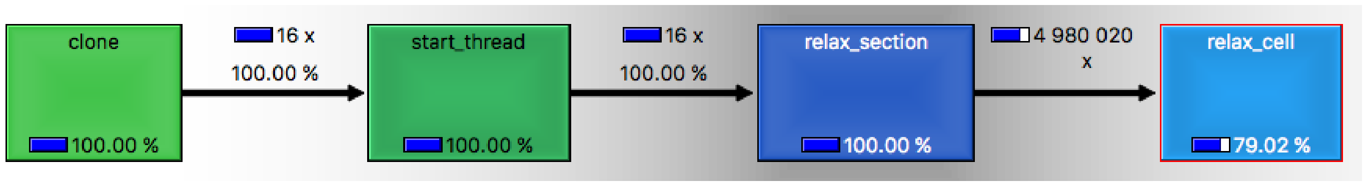


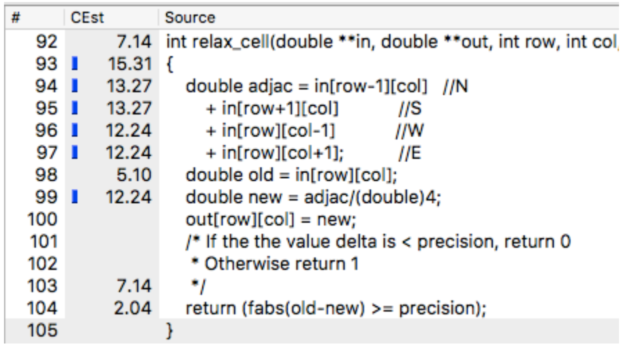
Figure 10: Call graph showing `relax_cell` and `relax_section`

[6]https://kcachegrind.github.io/html/Home.html

The top two functions; `relax_cell` and its caller, my thread function `relax_section` account for the vast majority of cycles. Figure 10 shows that the 4,980,020 calls (over 5 iterations) to `relax_cell` account for 79.02% of the cycles accross the 16 (one for each thread) calls to `relax_section`.

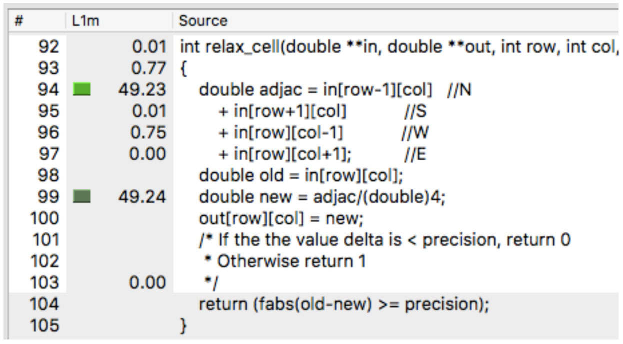
There was no obvious improvement which could be made to the logic in `relax_cell`, as Figure 11 shows the average cycles to be fairly similar for all source lines which need to access the array (implying the cycles are due to load latency of the data), and for the assignment of the variable `new` (where the cycles would be due to latency of the store).

To confirm this, I used the `cachegrind` tool on the same input and looked at the L1 misses for reads and writes in `relax_cell` (See Figure 12.)



#		CEst	Source
92		7.14	int relax_cell(double **in, double **out, int row, int col
93	▮	15.31	{
94	▮	13.27	double adjac = in[row-1][col] //N
95	▮	13.27	+ in[row+1][col] //S
96	▮	12.24	+ in[row][col-1] //W
97	▮	12.24	+ in[row][col+1]; //E
98		5.10	double old = in[row][col];
99	▮	12.24	double new = adjac/(double)4;
100			out[row][col] = new;
101			/* If the the value delta is < precision, return 0
102			* Otherwise return 1
103		7.14	*/
104		2.04	return (fabs(old-new) >= precision);
105			}

Figure 11: Cycles detection averages in `relax_cell`



#		L1m	Source
92		0.01	int relax_cell(double **in, double **out, int row, int col,
93		0.77	{
94	🟩	49.23	double adjac = in[row-1][col] //N
95		0.01	+ in[row+1][col] //S
96		0.75	+ in[row][col-1] //W
97		0.00	+ in[row][col+1]; //E
98			double old = in[row][col];
99	🟩	49.24	double new = adjac/(double)4;
100			out[row][col] = new;
101			/* If the the value delta is < precision, return 0
102			* Otherwise return 1
103		0.00	*/
104			return (fabs(old-new) >= precision);
105			}

Figure 12: L1 Cache Read (light green) and Write (dark green) misses in `relax_cell`

There is no easy fix for cache misses, as this is clearly a physical constraint and therefore machine-specific. Consequentially, as I had identified `relax_cell` as the single critical performance hotspot in my application, I did not do any further performance tuning as it would have yielded inconsequential improvements.

I have included the output from valgrind in both `callgrind` and `cachegrind` mode in the `valgrind` directory, from which all of the figures in this section can be reconstructed and explored. The options used for this run of my program were $d = 1000$, $p = 5.0$ and $t = 16$.

References

[1] Gene M. Amdahl. Validity of the single processor approach to achieving large scale computing capabilities. In *Proceedings of the April 18-20, 1967, Spring Joint Computer Conference*, AFIPS '67 (Spring), pages 483–485, New York, NY, USA, 1967. ACM. doi: 10.1145/1465482.1465560. URL `http://doi.acm.org/10.1145/1465482.1465560`.

[2] Cyrille Artho, Klaus Havelund, and Armin Biere. High-level data races. *Software Testing, Verification and Reliability*, 13(4):207–227, 2003.

[3] Andrew Binstock. Memory constraints on thread performance. `http://www.drdobbs.com/tools/memory-constraints-on-thread-performance/231300494`, 2011. Accessed: 10/11/2016.

[4] Hans-J. Boehm. How to miscompile programs with "benign" data races. In *Proceedings of the 3rd USENIX Conference on Hot Topic in Parallelism*, HotPar'11, pages 3–3, Berkeley, CA, USA, 2011. USENIX Association. URL `http://dl.acm.org/citation.cfm?id=2001252.2001255`.

[5] Derek L Eager, John Zahorjan, and Edward D Lazowska. Speedup versus efficiency in parallel systems. *IEEE Transactions on Computers*, 38(3):408–423, 1989.

[6] John L. Gustafson. *Amdahl's Law*, pages 53–60. Springer US, Boston, MA, 2011. ISBN 978-0-387-09766-4. doi: 10.1007/978-0-387-09766-4_77. URL `http://dx.doi.org/10.1007/978-0-387-09766-4_77`.

[7] Stefan Savage, Michael Burrows, Greg Nelson, Patrick Sobalvarro, and Thomas Anderson. Eraser: A dynamic data race detector for multi-threaded programs. *SIGOPS Oper. Syst. Rev.*, 31(5):27–37, October 1997. ISSN 0163-5980. doi: 10.1145/269005.266641. URL `http://doi.acm.org/10.1145/269005.266641`.

A Full Testing Results

A.1 Time and Speedup for 1-16 threads, $d = 1000$, $p = 0.5$

Threads	Time	Speedup
1	9s 187ms	1
2	4s 626ms	1.986
3	3s 109ms	2.955
4	2s 343ms	3.921
5	1s 892ms	4.856
6	1s 585ms	5.796
7	1s 369ms	6.711
8	1s 205ms	7.624
9	1s 81ms	8.499
10	979ms	9.384
11	902ms	10.185
12	834ms	11.016
13	774ms	11.87
14	731ms	12.568
15	686ms	13.392
16	647ms	14.199

A.2 Time and Speedup for 2-32 threads, $d = 1000$, $p = 0.5$

Threads	Time	Speedup
2	4s 624ms	1.987
4	2s 342ms	3.923
6	1s 591ms	5.774
8	1s 214ms	7.568
10	982ms	9.355
12	833ms	11.029
14	726ms	12.654
16	653ms	14.069
18	1s 103ms	8.329
20	1s 60ms	8.667
22	1s 126ms	8.159
24	1s 129ms	8.137
26	1s 103ms	8.329
28	1s 71ms	8.578
30	1s 53ms	8.725
32	1s 30ms	8.919

A.3 Dimensions against time, $p = 0.5$, $t = \{1, 8, 16\}$

Dimension	1 Thread	8 Threads	16 Threads
1000	9s 231ms	1s 207ms	646ms
2000	28s 13ms	3s 616ms	2s 7ms
3000	1m 1s 257ms	7s 896ms	4s 291ms
4000	2m 31s 143ms	19s 331ms	10s 843ms
5000	4m 29s 167ms	34s 413ms	19s 845ms
6000	5m 23s 100ms	41s 339ms	24s 290ms
7000	7m 29s 558ms	57s 778ms	32s 389ms
8000	(timed out)	1m 21s 480ms	47s 624ms
9000	(timed out)	2m 0s 608ms	1m 9s 36ms
10000	(timed out)	2m 16s 340ms	1m 15s 692ms

A.4 Precision against iterations and time, $d = 500$, $t = 16$

Precision	Time	Iterations
1	38ms	78
0.1	872ms	2160
0.01	25s 239ms	57755
0.001	1m 19s 879ms	172235
0.0001	2m 9s 480ms	288416
0.00001	3m 22s 586ms	404599
0.000001	4m 18s 886ms	520783
0.0000001	(timed out)	(timed out)