In [1]:
```python
import numpy as np
```

Floating point arithmetic

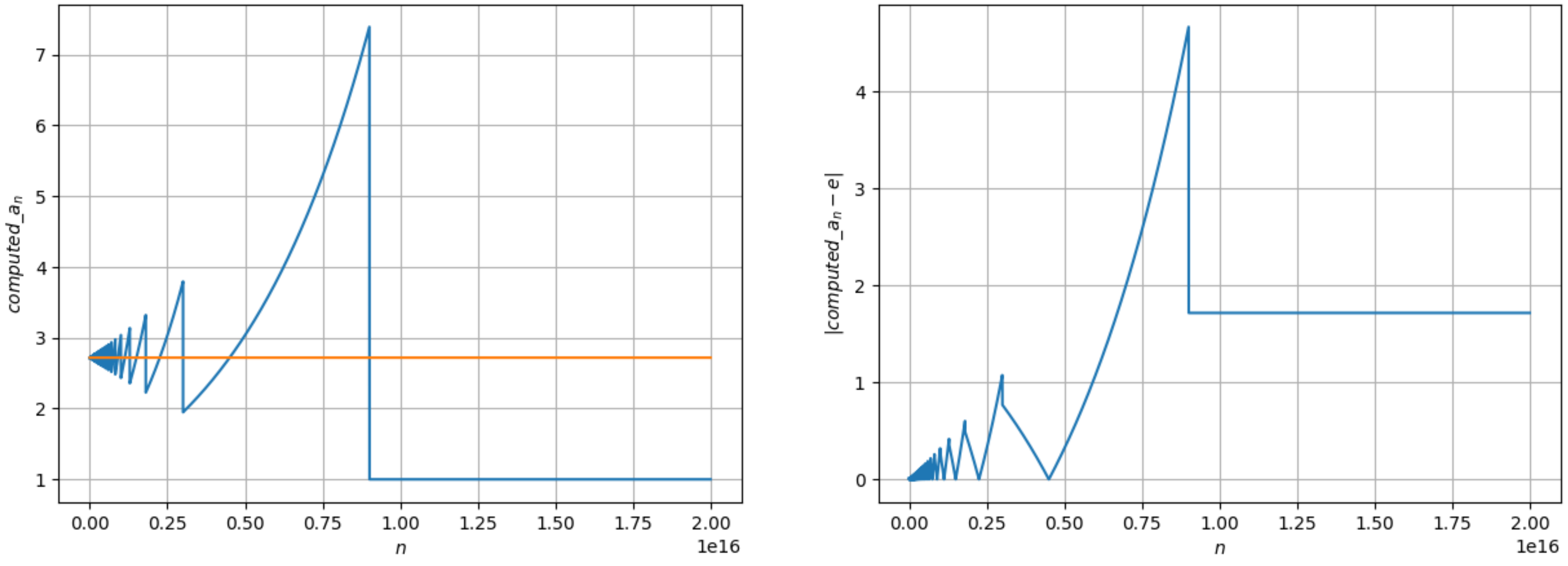
The Machine epsilon is defined as the smallest floating point number such that it holds: fl(1+eps) > 1. Compute eps. Tips: use a while structure.

In [2]:
```python
eps = 1.0 # initialize epsilon
while 1 + eps > 1:
    eps /= 2
eps *= 2
print(eps)
```

```
2.220446049250313e-16
```

Let's consider the sequence $a_n = (1 + \frac{1}{n})^n$. It is well known that: $\lim_{x\to\infty} a_n = e$, where is the Nepero number. Choose different values for n, compute a_n and compare it to the real value of the Nepero number. What happens if you choose a large value of n?

In [4]:
```python
from matplotlib.pyplot import subplots

#n_values = np.arange(1, 100, 1)
#n_values = np.arange(10, 1e14, 1e8)
#n_values = np.arange(100, 5e15, 1e10)
n_values = np.arange(100, 2e16, 1e10)

a_n = (1 + (1/n_values))**n_values
abs_err = abs(a_n - np.e)


fig, axs = subplots(figsize=(15,5), nrows=1, ncols=2)

ax = axs[0]
x = range(1,len(n_values)+1)
ax.plot(n_values, a_n)
ax.plot(n_values,  np.full(len(a_n), np.e))
ax.set_xlabel(r"$n$")
ax.set_ylabel(r"$computed\_a_n$")
#add legend
ax.grid()

ax = axs[1]
ax.plot(n_values, abs_err)
ax.set_xlabel(r"$n$")
ax.set_ylabel(r"$|computed\_a_n - e|$")
ax.grid()
```



Compute the rank of A and B and their eigenvalues. Are A and B full-rank matrices? Can you infer some relationship between the values of the eigenvalues and the full-rank condition? Please, corroborate your deduction with other examples.

In [5]:
```python
A = np.array([[4, 2], [1, 3]])
B = np.array([[4, 2], [2, 1]])

rk_A = np.linalg.matrix_rank(A) #np.linalg.matrix_rank uses SVD decomposition to compute the rank
rk_B = np.linalg.matrix_rank(B)
print('Rk(A) = ' + str(rk_A) + ', so A is full-rank')
print('Rk(B) = ' + str(rk_B) + ', so A is not full-rank'+ '\n')

eigenvals_A = np.linalg.eigvals(A)
eigenvals_B = np.linalg.eigvals(B)
print("Eigenvalus of A: " + str(eigenvals_A))
print("Eigenvalus of B: " + str(eigenvals_B))
```

```
Rk(A) = 2, so A is full-rank
Rk(B) = 1, so A is not full-rank

Eigenvalus of A: [5. 2.]
Eigenvalus of B: [5. 0.]
```

We can observe that a square matrix has full rank if and only if all its eigenvalues are non-zero. For example the matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ is full-rank with eigenvalues 1 and 3. The matrix

$$B = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \tag{2}$$

is not full-rank with eigenvalues 1 and 0.

In [ ]:
```python

```