



Algebraic Fractions

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The aim of this document is to provide a short, self assessment programme for students who wish to acquire a basic competence in the use of algebraic fractions.

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1. Algebraic Fractions(Introduction)

Algebraic fractions have properties which are the same as those for numerical fractions, the only difference being that the the **numerator** (top) and **denominator** (bottom) are both algebraic expressions.

Example 1 Simplify each of the following fractions.

$$(a) \quad \frac{2b}{7b^2}, \quad (b) \quad \frac{3x+x^2}{6x^2}.$$

Solution

$$(a) \quad \frac{2b}{7b^2} = \frac{2 \times \cancel{b}}{7 \times b \times \cancel{b}} = \frac{2}{7b}$$

$$(b) \quad \begin{aligned} \frac{3x+x^2}{6x^2} &= \frac{x \times (3+x)}{x \times 6x} \\ &= \frac{\cancel{x} \times (3+x)}{\cancel{x} \times 6x} = \frac{3+x}{6x} \end{aligned}$$

N.B. The cancellation in **(b)** is allowed since x is a common factor of the **numerator** and the **denominator**.

Sometimes a little more work is necessary before an algebraic fraction can be reduced to a simpler form.

Example 2 Simplify the algebraic fraction

$$\frac{x^2 - 2x + 1}{x^2 + 2x - 3}$$

Solution

In this case the numerator and denominator can be factored into two terms, thus

$$x^2 - 2x + 1 = (x - 1)^2, \quad \text{and} \quad x^2 + 2x - 3 = (x - 1)(x + 3).$$

(See the package on **factorising expressions**). With this established the simplification proceeds as follows:

$$\begin{aligned} \frac{x^2 - 2x + 1}{x^2 + 2x - 3} &= \frac{(x - 1) \times (x - 1)}{(x + 3) \times (x - 1)} \\ &= \frac{x - 1}{x + 3} \quad (\text{cancelling } (x - 1)) \end{aligned}$$

EXERCISE 1. Simplify each of the following algebraic fractions. (Click on the **green** letters for solution.)

(a) $\frac{8y}{2y^3}$

(b) $\frac{2y}{4x}$

(c) $\frac{7a^6b^3}{14a^5b^4}$

(d) $\frac{(2x)^2}{-4x}$

(e) $\frac{5y + 2y^2}{-7y}$

(f) $\frac{5ax}{15a + 10a^2}$

(g) $\frac{2z^2 - 4z}{2z^2 - 10z}$

(h) $\frac{y^2 + 7y + 10}{y^2 - 25}$

(i) $\frac{w^2 - 5w - 14}{w^2 - 4w - 21}$

Now try this short quiz.

Quiz Which of the following is a simplified version of

$$\frac{t^2 + 3t - 4}{t^2 - 3t + 2}?$$

(a) $\frac{t - 4}{t - 2}$

(b) $\frac{t - 4}{t + 2}$

(c) $\frac{t + 4}{t - 2}$

(d) $\frac{t + 4}{t + 2}$

So far, simplification has been achieved by cancelling common factors from the **numerator** and **denominator**. There are fractions which can be simplified by *multiplying* the **numerator** and **denominator** by an appropriate common factor, thus obtaining an equivalent, simpler expression.

Example 3 Simplify the following fractions.

$$(a) \quad \frac{\frac{1}{4} + y}{\frac{1}{2}} \qquad (b) \quad \frac{3}{2} + \frac{1}{x}$$

Solution

(a) In this case, multiplying both the **numerator** and the **denominator** by 4 gives:

$$\frac{\frac{1}{4} + y}{\frac{1}{2}} = \frac{4 \cdot \frac{1}{4} + y}{4 \cdot \frac{1}{2}} = \frac{1 + 4y}{2}$$

(b) To simplify this expression, multiply the **numerator** and **denominator** by x . Thus

$$\frac{3x + \frac{1}{x}}{2} = \frac{x \cdot 3x + x \cdot \frac{1}{x}}{2x} = \frac{3x^2 + 1}{2x}$$

Now try this exercise on similar examples.

EXERCISE 2. Simplify each of the following algebraic fractions. (Click on the **green** letters for solution.)

$$(a) \quad \frac{4y - \frac{3}{2}}{2}$$

$$(b) \quad \frac{2 - \frac{1}{2}}{x + \frac{1}{4}}$$

$$(c) \quad \frac{\frac{z}{3}}{\frac{z}{\frac{1}{2}}}$$

$$(d) \quad \frac{2 - \frac{1}{x}}{2}$$

$$(e) \quad \frac{3t - \frac{2}{t}}{\frac{1}{2}}$$

$$(f) \quad \frac{\frac{z}{2z}}{z - \frac{1}{3z}}$$

For the last part of this section, try the following short quiz.

Quiz Which of the following is a simplified version of

$$\frac{x - \frac{1}{x+1}}{x - 1} ?$$

$$(a) \quad \frac{x^2 - x + 1}{x^2 + x + 1}$$

$$(b) \quad \frac{x^2 - x + 1}{x^2 - 1}$$

$$(c) \quad \frac{x^2 - 1}{x^2 - x - 1}$$

$$(d) \quad \frac{x^2 + x - 1}{x^2 - 1}$$

2. Addition of Algebraic Fractions

Addition (and subtraction) of algebraic fractions proceeds in exactly the same manner as for numerical fractions.

Example 4 Write the following sum as a single fraction in its simplest form.

$$\frac{2}{x+1} + \frac{1}{x+2}$$

Solution The *least common multiple* of the denominators (see the package on **fractions**) is $(x+1)(x+2)$. Thus

$$\begin{aligned}\frac{2}{x+1} + \frac{1}{x+2} &= \frac{2 \times (x+2)}{(x+1) \times (x+2)} + \frac{1 \times (x+1)}{(x+2) \times (x+1)} \\ &= \frac{2x+4}{(x+1)(x+2)} + \frac{x+1}{(x+1)(x+2)} \\ &= \frac{(2x+4) + (x+1)}{(x+1)(x+2)} = \frac{3x+5}{(x+1)(x+2)}\end{aligned}$$

EXERCISE 3. Evaluate each of the following fractions. (Click on the green letters for solution.)

(a) $\frac{2}{y} + \frac{3}{z}$

(b) $\frac{1}{3y} - \frac{2}{5y}$

(c) $\frac{3z+1}{3} - \frac{2z+1}{2}$

(d) $\frac{3t+1}{2} + \frac{1}{t}$

(e) $\frac{x+1}{2} + \frac{1}{x-1}$

(f) $\frac{2}{w+3} - \frac{5}{w-1}$

Quiz Which of the following values of **a** is needed if

$$\frac{\mathbf{a}}{2x+1} + \frac{1}{x+2} = \frac{4x+5}{(2x+1)(x+2)}?$$

(a) **a** = 3

(b) **a** = -3

(c) **a** = 2

(d) **a** = -2

3. Simple Partial Fractions

The last quiz was an example of *partial fractions*, i.e. the technique of decomposing a fraction as a sum of simpler fractions. This section will consider the simpler forms of this technique.

Example 5 Find the partial fraction decomposition of $4/(x^2 - 4)$.

Solution The denominator factorises as $x^2 - 4 = (x - 2)(x + 2)$. (See the package on **quadratics**.) The partial fractions will, therefore, be of the form $a/(x - 2)$ and $b/(x + 2)$. Thus

$$\begin{aligned}\frac{a}{x-2} + \frac{b}{x+2} &= \frac{4}{(x-2)(x+2)} \\ \frac{a(x+2)}{(x-2)(x+2)} + \frac{b(x-2)}{(x+2)(x-2)} &= \frac{4}{(x-2)(x+2)} \\ \frac{(a+b)x + 2(a-b)}{(x-2)(x+2)} &= \frac{4}{(x-2)(x+2)} \\ \text{so that } (a+b)x + 2(a-b) &= 4\end{aligned}$$

The last line is

$$(a + b)x + 2(a - b) = 4,$$

and this enables a and b to be found. For the equation to be true *for all* values of x the coefficients must match, i.e.

$$a + b = 0 \quad (\text{coefficients of } x)$$

$$2a - 2b = 4 \quad (\text{constant terms})$$

where the first equation holds since there is no x term in $4/(x^2 - 4)$. This set of simultaneous equations may be solved to give $a = 1$ and $b = -1$. (See the package on **simultaneous equations** for a method of finding these solutions.)

Thus

$$\frac{4}{(x-2)(x+2)} = \frac{1}{x-2} - \frac{1}{(x+2)}$$

EXERCISE 4. For each of the following, find a and b . (Click on the green letters for solution.)

$$(a) \quad \frac{a}{x-2} + \frac{b}{x+2} = \frac{4x}{(x-2)(x+2)}$$

$$(b) \quad \frac{a}{z-3} + \frac{b}{z+2} = \frac{z+7}{(z-3)(z+2)}$$

$$(c) \quad \frac{a}{w-4} + \frac{b}{w+1} = \frac{3w-2}{(w-4)(w+1)}$$

Now try this final, short quiz.

Quiz If $\frac{a}{2x-3} + \frac{b}{3x+4} = \frac{x+7}{(2x-3)(3x+4)}$, which of the following is the solution to the equation?

(a) $a = 1, b = -1$

(b) $a = -1, b = 1$

(c) $a = 1, b = 1$

(d) $a = -1, b = -1$

4. Quiz on Algebraic Fractions

Begin Quiz In each of the following, choose:

1. the simplified form of $(z^2 + 4z - 5)/(z^2 - 4z + 3)$

(a) $(z + 5)/(z + 3)$

(b) $(z + 5)/(z - 3)$

(c) $(z - 5)/(z + 3)$

(d) $(z - 5)/(z - 3)$

2. the sum $[1/(w - 2)] - [1/(w + 7)]$

(a) $5/(w^2 + 5w - 14)$

(b) $5/(w^2 - 5w + 14)$

(c) $9/(w^2 - 5w + 14)$

(d) $9/(w^2 + 5w - 14)$

3. a and b if

$$[a/(3x + 2)] + [b/(4x - 3)] = (x - 5)/[(3x + 2)(4x - 3)]$$

(a) $a = 1, b = 1$

(b) $a = -1, b = 1$

(c) $a = -1, b = -1$

(d) $a = 1, b = -1$

End Quiz

Score:

Correct

Solutions to Exercises

Exercise 1(a)

$$\begin{aligned}\frac{8y}{2y^3} &= \frac{4 \times 2y}{y^2 \times 2y} \\ &= \frac{4 \times \cancel{2} \times \cancel{y}}{y^2 \times \cancel{2} \times \cancel{y}} \\ &= \frac{4}{y^2}\end{aligned}$$

Click on the green square to return

Exercise 1(b)

$$\begin{aligned}\frac{2y}{4x} &= \frac{2 \times y}{2 \times 2x} \\ &= \frac{\cancel{2} \times y}{\cancel{2} \times 2x} \\ &= \frac{y}{2x}\end{aligned}$$

Click on the green square to return

Exercise 1(c) The fraction is $\frac{7a^6b^3}{14a^5b^4}$. This time, instead of expanding the factors, it is easier to use the rule for powers

$$\frac{a^m}{a^n} = a^{m-n}.$$

(See the package on **powers**.) Thus

$$\begin{aligned}\frac{7a^6b^3}{14a^5b^4} &= \frac{7}{14} \times \frac{a^6}{a^5} \times \frac{b^3}{b^4} \\ &= \frac{1}{2} \times a^{6-5} \times b^{3-4} \\ &= \frac{1}{2} \times a^1 \times b^{-1} = \frac{a}{2b}\end{aligned}$$

Click on the green square to return

Exercise 1(d)

$$\begin{aligned}\frac{(2x)^2}{4x} &= \frac{2 \times x \times 2 \times x}{2 \times 2 \times x} \\ &= \frac{\cancel{2} \times \cancel{x} \times \cancel{2} \times x}{\cancel{2} \times \cancel{2} \times \cancel{x}} \\ &= x.\end{aligned}$$

Click on the green square to return

Exercise 1(e)

$$\begin{aligned}\frac{5y + 2y^2}{7y} &= \frac{y \times (5 + 2y)}{7 \times y} \\ &= \frac{/y \times (5 + 2y)}{7 \times /y} \\ &= \frac{5 + 2y}{7}.\end{aligned}$$

Click on the green square to return

Exercise 1(f)

$$\begin{aligned}\frac{5ax}{15a + 10a^2} &= \frac{5 \times a \times x}{5 \times a \times (3 + 2a)} \\ &= \frac{\cancel{5} \times \cancel{a} \times x}{\cancel{5} \times \cancel{a} \times (3 + 2a)} \\ &= \frac{x}{3 + 2a}\end{aligned}$$

Click on the green square to return

Exercise 1(g)

$$\begin{aligned}\frac{2z^2 - 4z}{2z^2 - 10z} &= \frac{2 \times z \times (z - 2)}{2 \times z \times (z - 5)} \\ &= \frac{/2 \times /z \times (z - 2)}{/2 \times /z \times (z - 5)} \\ &= \frac{z - 2}{z - 5}.\end{aligned}$$

Click on the green square to return

Exercise 1(h) In this case, some initial factorisation is needed (see the package on **factorising expressions**).

$$y^2 + 7y + 10 = (y + 5)(y + 2) \quad \text{and} \quad y^2 - 25 = (y + 5)(y - 5)$$

Thus

$$\begin{aligned} \frac{y^2 + 7y + 10}{y^2 - 25} &= \frac{(y + 5)(y + 2)}{(y + 5)(y - 5)} \\ &= \frac{y + 2}{y - 5} \end{aligned}$$

where the factor $(y + 5)$ has been cancelled.

[Click on the green square to return](#)

Exercise 1(i) Again, in this case, some initial factorisation is needed (see the package on **factorising expressions**).

$$w^2 - 5w - 14 = (w - 7)(w + 2) \quad \text{and} \quad w^2 - 4w - 21 = (w - 7)(w + 3)$$

Thus

$$\begin{aligned} \frac{w^2 - 5w - 14}{w^2 - 4w - 21} &= \frac{(w - 7)(w + 2)}{(w - 7)(w + 3)} \\ &= \frac{w + 2}{w + 3}, \end{aligned}$$

where the factor $(w - 7)$ has been cancelled.

[Click on the green square to return](#)

Exercise 2(a) The fraction is simplified by multiplying both the **numerator** and the **denominator** by 2.

$$\frac{4y - \frac{3}{2}}{2} = \frac{2}{2 \times 2} \frac{4y - \frac{3}{2}}{2} = \frac{8y - 3}{4}$$

Click on the green square to return

Exercise 2(b) This fraction is simplified by multiplying both the **numerator** and the **denominator** by 4. Thus

$$\frac{2x + \frac{1}{2}}{x + \frac{1}{4}} = \frac{4 \cdot 2x + \frac{4}{2}}{4 \cdot x + \frac{4}{4}} = \frac{8x + 2}{4x + 1}$$

Click on the green square to return

Exercise 2(c) In this case, since the **numerator** contains the fraction $1/3$ and the **denominator** contains the fraction $1/2$, the common factor needed is $2 \times 3 = 6$. Thus

$$\frac{z - \frac{1}{3}}{z - \frac{1}{2}} = \frac{6 \ z - \frac{1}{3}}{6 \ z - \frac{1}{2}} = \frac{6z - 2}{6z - 3}$$

Click on the green square to return

Exercise 2(d) For this fraction the required multiplier is x .

$$\frac{2 - \frac{1}{x}}{2} = x \cdot \frac{2 - \frac{1}{x}}{2x} = \frac{2 - \frac{1}{x}}{2x} \cdot x$$

Click on the green square to return

Exercise 2(e) Here the **numerator** includes the fraction $2/t$ and the **denominator** is the fraction $1/2$, so the required multiplier is $2t$.

$$\frac{3t - \frac{2}{t}}{\frac{1}{2}} = \frac{2t \cdot 3t - \frac{2}{t}}{2t \cdot \frac{1}{2}} = \frac{6t^2 - 4}{t}$$

Click on the green square to return

Exercise 2(f) For the last part of this exercise, since the **numerator** includes the fraction $1/2z$ and the **denominator** includes the fraction $1/3z$, the common multiplier is $6z$.

$$\frac{z - \frac{1}{2z}}{z - \frac{1}{3z}} = \frac{6z \cdot z - \frac{1}{2z} \cdot 6z}{6z \cdot z - \frac{1}{3z} \cdot 6z} = \frac{6z^2 - 3}{6z^2 - 2}$$

Click on the green square to return

Exercise 3(a) The *least common multiple* of the denominators is yz .
Thus

$$\begin{aligned}\frac{2}{y} + \frac{3}{z} &= \frac{2 \times z}{y \times z} + \frac{3 \times y}{z \times y} \\ &= \frac{2z}{yz} + \frac{3y}{yz} \\ &= \frac{3y + 2z}{yz}\end{aligned}$$

Click on the green square to return

Exercise 3(b) Here the *least common multiple* of the denominators is $15y$, so

$$\begin{aligned}\frac{1}{3y} - \frac{2}{5y} &= \frac{5 \times 1}{5 \times 3y} - \frac{3 \times 2}{3 \times 5y} \\ &= \frac{5}{15y} - \frac{6}{15y} \\ &= \frac{5-6}{15y} = -\frac{1}{15y}\end{aligned}$$

Click on the green square to return

Exercise 3(c) The *least common multiple* of the denominators of the two fractions in this case is 6. Thus

$$\begin{aligned}\frac{3z+1}{3} - \frac{2z+1}{2} &= \frac{2 \times (3z+1)}{2 \times 3} - \frac{3 \times (2z+1)}{3 \times 2} \\ &= \frac{6z+2}{6} - \frac{6z+3}{6} \\ &= \frac{(6z+2) - (6z+3)}{6} = -\frac{1}{6}\end{aligned}$$

Simplification in this case has shown that the difference of these two fractions is independent of z .

Click on the green square to return

Exercise 3(d) The *least common multiple* of the denominators of the two fractions in this case is $2t$. The sum simplifies as follows.

$$\begin{aligned}\frac{3t+1}{2} + \frac{1}{t} &= \frac{t \times (3t+1)}{t \times 2} + \frac{2 \times 1}{2 \times t} \\ &= \frac{3t^2+t}{2t} + \frac{2}{2t} = \frac{3t^2+t+2}{2t}\end{aligned}$$

Click on the green square to return

Exercise 3(e)

Here the required *least common multiple* of the denominators is the factor $2(x-1)$. Proceeding as before:

$$\begin{aligned}\frac{x+1}{2} + \frac{1}{x-1} &= \frac{(x-1) \times (x+1)}{(x-1) \times 2} + \frac{2 \times 1}{2 \times (x-1)} \\ &= \frac{(x^2-1)}{2(x-1)} + \frac{2}{2(x-1)} \\ &= \frac{(x^2-1)+2}{2(x-1)} = \frac{x^2+1}{2(x-1)}\end{aligned}$$

Click on the green square to return

Exercise 3(f)

Here the required *least common multiple* of the denominators is

$$(w + 3)(w - 1) = w^2 + 2w - 3.$$

With this in mind,

$$\begin{aligned}\frac{2}{w+3} - \frac{5}{w-1} &= \frac{(w-1) \times 2}{(w-1) \times (w+3)} - \frac{(w+3) \times 5}{(w+3) \times (w-1)} \\ &= \frac{2w-2}{(w+3)(w-1)} - \frac{5w+15}{(w+3)(w-1)} \\ &= \frac{(2w-2) - (5w+15)}{(w+3)(w-1)} \\ &= \frac{-3w-17}{(w+3)(w-1)} = - \frac{3w+17}{(w+3)(w-1)}\end{aligned}$$

Click on the green square to return

Exercise 4(a) Taking common denominators:

$$\begin{aligned}\frac{a}{x-2} + \frac{b}{x+2} &= \frac{4x}{(x-2)(x+2)} \\ \frac{a(x+2)}{(x-2)(x+2)} + \frac{b(x-2)}{(x-2)(x+2)} &= \frac{4x}{(x-2)(x+2)} \\ \frac{(a+b)x + (2a-2b)}{(x-2)(x+2)} &= \frac{4x}{(x-2)(x+2)}\end{aligned}$$

so that $(a+b)x + (2a-2b) = 4x$. Equating coefficients in this case gives

$$\begin{aligned}a+b &= 4 && \text{(coefficients of } x) \\ 2a-2b &= 0 && \text{(constant terms)}\end{aligned}$$

Solving this set of equations gives $a=2$, $b=2$. Hence

$$\frac{2}{x-2} + \frac{2}{x+2} = \frac{4x}{(x-2)(x+2)}$$

Click on the green square to return

Exercise 4(b) Taking common denominators:

$$\begin{aligned}\frac{a}{z-3} + \frac{b}{z+2} &= \frac{z+7}{(z-3)(z+2)} \\ \frac{a(z+2)}{(z-3)(z+2)} + \frac{b(z-3)}{(z-3)(z+2)} &= \frac{z+7}{(z-3)(z+2)} \\ \frac{(a+b)z + (2a-3b)}{(z-3)(z+2)} &= \frac{z+7}{(z-3)(z+2)}\end{aligned}$$

so that $(a+b)z + (2a-3b) = z+7$. Equating coefficients in this case gives

$$\begin{aligned}a+b &= 1 && \text{(coefficients of } z) \\ 2a-3b &= 7 && \text{(constant terms)}\end{aligned}$$

Solving this set of equations gives $a=2$, $b=-1$. Hence

$$\frac{2}{z-3} - \frac{1}{z+2} = \frac{z+7}{(z-3)(z+2)}$$

Click on the green square to return

Exercise 4(c) Taking common denominators:

$$\begin{aligned}\frac{a}{w-4} + \frac{b}{w+1} &= \frac{3w-2}{(w-4)(w+1)} \\ \frac{a(w+1)}{(w-4)(w+1)} + \frac{b(w-4)}{(w-4)(w+1)} &= \frac{3w-2}{(w-4)(w+1)} \\ \frac{(a+b)w + (a-4b)}{(w-4)(w+1)} &= \frac{3w-2}{(w-4)(w+1)}\end{aligned}$$

so that $(a+b)w + (a-4b) = 3w-2$. Equating coefficients in this case gives

$$\begin{aligned}a+b &= 3 && \text{(coefficients of } w) \\ a-4b &= -2 && \text{(constant terms)}\end{aligned}$$

Solving this set of equations gives $a=2$, $b=1$. Hence

$$\frac{2}{w-4} + \frac{1}{w+1} = \frac{3w-2}{(w-4)(w+1)}$$

[Click on the green square to return](#)

Solutions to Quizzes

Solution to Quiz: The numerator and denominator respectively factorise as

$$t^2 + 3t - 4 = (t - 1)(t + 4) \quad \text{and} \quad t^2 - 3t + 2 = (t - 1)(t - 2)$$

so that

$$\begin{aligned} \frac{t^2 + 3t - 4}{t^2 - 3t + 2} &= \frac{(t - 1)(t + 4)}{(t - 1)(t - 2)} \\ &= \frac{t + 4}{t - 2} \end{aligned}$$

where the factor $(t - 1)$ has been cancelled from the first equation.

End Quiz

Solution to Quiz:

For $\frac{x - \frac{1}{x+1}}{x-1}$, the common multiplier is $(x+1)$. Multiplying the **numerator** and the **denominator** by this gives:

$$\begin{aligned}\frac{x - \frac{1}{x+1}}{x-1} &= \frac{(x+1) \left(x - \frac{1}{x+1} \right)}{(x+1)(x-1)} \\ &= \frac{(x+1)x - (x+1) \frac{1}{(x+1)}}{(x^2-1)} \\ &= \frac{x^2 + x - 1}{x^2 - 1}\end{aligned}$$

End Quiz

Solution to Quiz: Writing all the fractions with a common denominator

$$\begin{aligned}\frac{4x+5}{(2x+1)(x+2)} &= \frac{\mathbf{a}}{2x+1} + \frac{1}{x+2} \\&= \frac{\mathbf{a}(x+2)}{(2x+1)(x+2)} + \frac{(2x+1)}{(2x+1)(x+2)} \\&= \frac{\mathbf{a}x + 2\mathbf{a} + 2x + 1}{(2x+1)(x+2)} \\&= \frac{(\mathbf{a}+2)x + (2\mathbf{a}+1)}{(2x+1)(x+2)}\end{aligned}$$

so that $(\mathbf{a}+2)x + (2\mathbf{a}+1) = 4x+5$. This gives two equations

$$\begin{array}{rcl}\mathbf{a} + 2 & = & 4 \quad \text{coefficients of } x \\2\mathbf{a} + 1 & = & 5 \quad \text{constant coefficients}\end{array}$$

The solution is $\mathbf{a} = 2$.

End Quiz

Solution to Quiz: Writing all the fractions with a common denominator

$$\begin{aligned}\frac{a}{2x-3} + \frac{b}{3x+4} &= \frac{x+7}{(2x-3)(3x+4)} \\ \frac{a(3x+4)}{(2x-3)(3x+4)} + \frac{b(2x-3)}{(2x-3)(3x+4)} &= \frac{x+7}{(2x-3)(3x+4)} \\ \frac{(3a+2b)x + (4a-3b)}{(2x-3)(3x+4)} &= \frac{x+7}{(2x-3)(3x+4)}\end{aligned}$$

so that $(3a+2b)x + (4a-3b) = x+7$. This gives two equations

$$3a + 2b = 1 \quad (\text{coefficients of } x)$$

$$4a - 3b = 7 \quad (\text{constant terms})$$

Solving these gives $a = 1$, $b = -1$.

End Quiz