Math F651: Take Home Midterm Due: 10:00pm March 7, 2025

See **Rules** on following page.

1. Suppose $q : X \to Y$ is a quotient map and that each fiber of q is connected. Show that if Y is connected, then so is X.

2. A subset A of a topological space X is said to be nowhere dense if $\text{Int}\,\overline{A} = \emptyset$.

a) Let U be an open subset of a topological space. Prove that ∂U is closed and nowhere dense.

b) Let V be a closed and nowhere dense set. Show that V is the boundary of an open set.

3. Let f and g be continuous maps from a topological space X to a Hausdorff space Y. Suppose $f = g$ on a dense subset of X. Prove that $f = g$.

4. Exercise 4.38

5. Let G be an algebraic group. We say that G is a **topological group** if in addition G is a topological space such that that the multiplication map $m : G \times G \to G$ and the inversion map $i : G \to G$ defined by $m(g, h) = g \cdot h$ and $i(g) = g^{-1}$ are continuous.

a) Suppose G is an algebraic group and a topological space. Show that G is a topological group if and only if the map $f : G \times G \to G$ defined by $f(g, h) = gh^{-1}$ is continuous.

b) Let G be a topological group and let H be a subgroup. Show that $\overline{H}$ is a subgroup. Hint: that map f from the previous part is continuous.

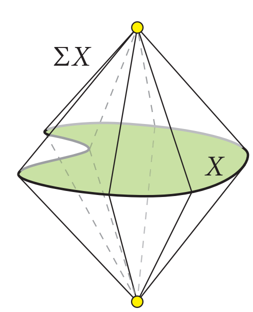
6. Suppose the spaces X_α, αinI are all connected and nonempty, and let a be a point in $X = \prod_{\alpha inI} X_\alpha$.

a) Given any finite set $K \subset I$, let X_K denote the subspace of X where $x_\alpha = a_\alpha$ for all $\alpha \ni nK$. Show that each X_K is connected.

b) Show that $Y = \cup_K X_K$ is connected.

c) Show that $\overline{Y} = X$ and conclude that X is connected.

7. Let X be a topological space. The **suspension** of X, denoted by ΣX, is the quotient of $X \times [-1, 1]$ where all points of the form $(x, 1)$ are identified, and all points of the form $(x, -1)$ are identified. Determine, with proof, a familiar space that is homeomorphic to ΣS^n.



8. Lee Problem 4-4

9. Lee Problem 4-5

10. Lee Problem 4-11

Rules and format:

- You are welcome to discuss this exam with me (David Maxwell) to ask for hints and so forth.
- You are not permitted to use any form of AI to assist you in any part of this exam.
- If you find a suspected error or misprint, please contact me as soon as possible and I will communicate it to the class if needed.
- You my not discuss the exam with anyone else until after the due date/time.
- You are permitted to reference Lee or any other topology text you like. If you use another text, you must cite it when you use it.
- You may not consult any internet resources, including search engines.
- Each problem is weighted equally.
- The due date/time is absolutely firm.
- The problem session on March 5 will be a hints session.