1. Carothers 18.1
2. Carothers 18.3
3. Carothers 18.4
4. Carothers 18.6
5. Carothers 18.9
6. Carothers 18.11
7. Let $f \geq 0$ be Riemann integrable. In this exercise you will show that f is measurable and that its Riemann integral $(R)\int_a^b f$ equals its Lebesgue integral $(L)\int_a^b f$. In your work, you are welcome to use the obvious fact that the Riemann integral and the Lebesgue integral agree for step functions.

   a) Show that there exists a monotone increasing sequence of step functions φ_n and a monotone decreasing sequence of step functions ψ_n such that $\varphi_n \leq f \leq \psi_n$ for each n and such that

   $$(R)\int_a^b (\psi_n - \varphi_n) \to 0.$$

   b) Let $\Phi = \sup \varphi_n$ and $\Psi = \inf \varphi_n$. Show that $\Psi - \Phi = 0$ almost everywhere.

   c) Conclude that f is measurable.

   d) Conclude that $(R)\int_a^b f = (L)\int_a^b f$.
8. Carothers 18.16
9. Carothers 18.17