Heat equation

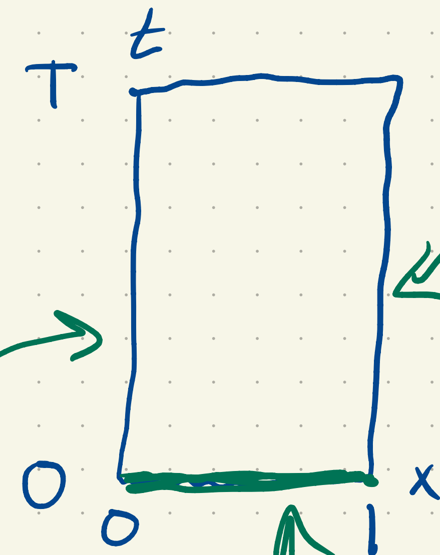
$u(x, t)$

space, time

$$u_t = u_{xx}$$

$$0 \le x \le 1$$

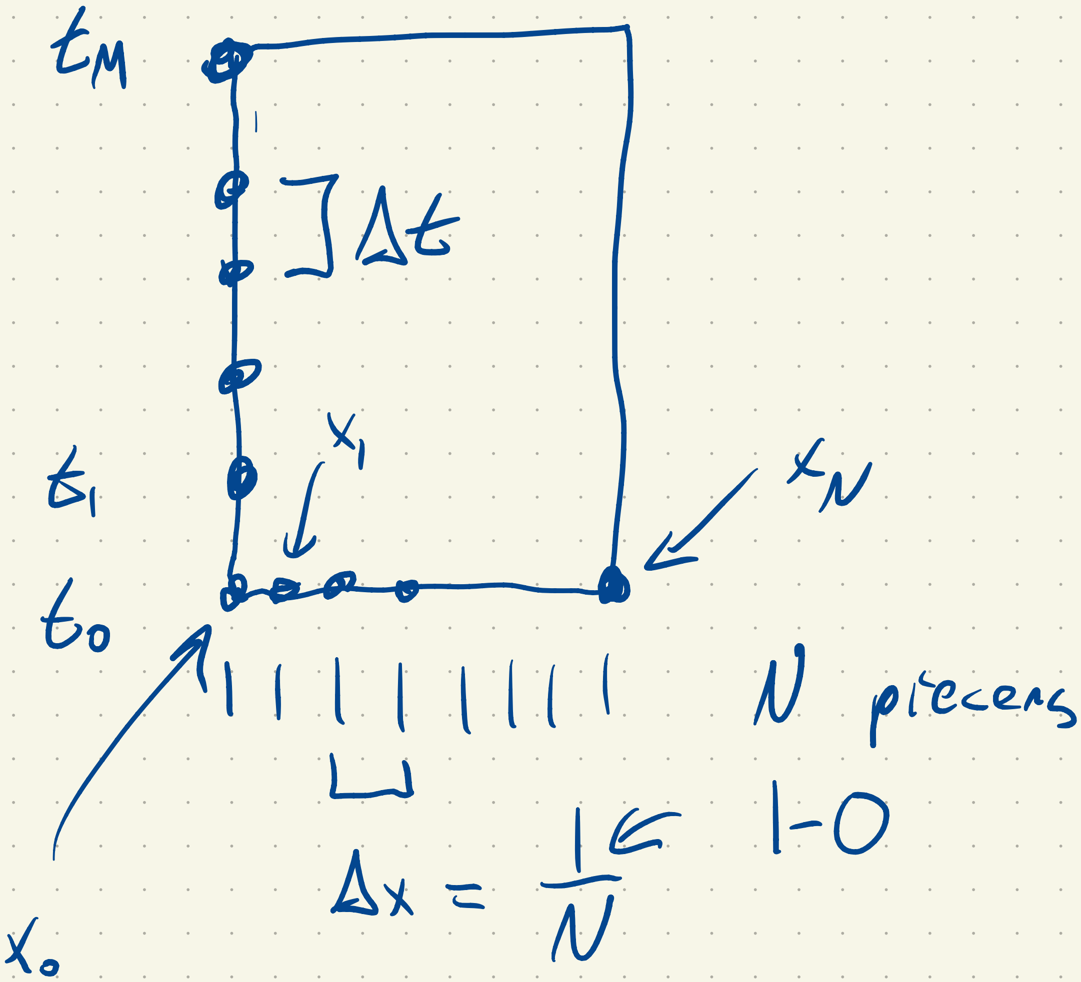
$$0 \le t \le T$$

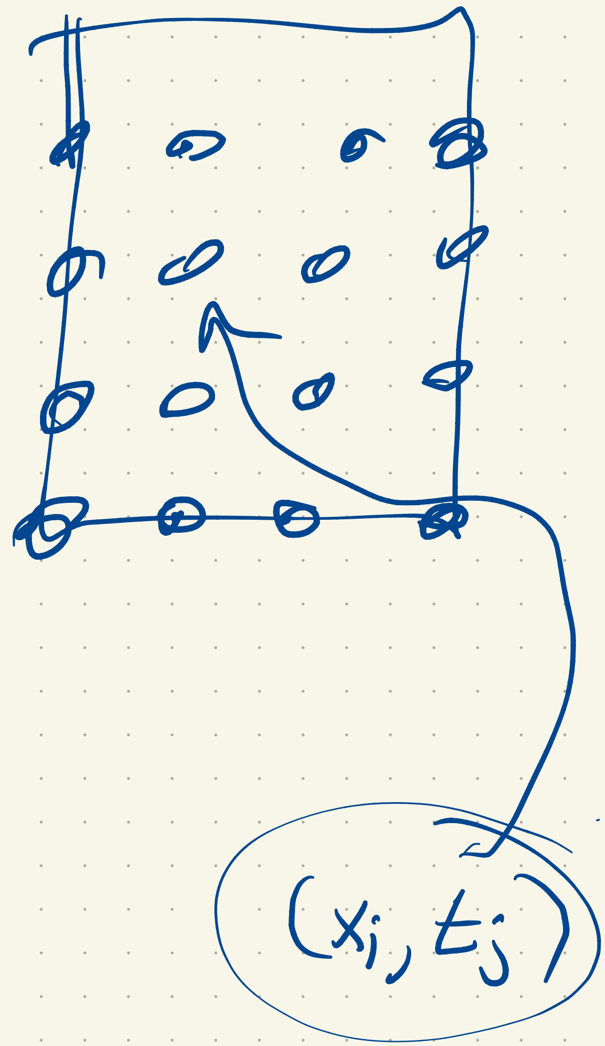


$u(1, t) = 0$

$u(0, t) = 0$

$u(x, 0) = u_0(x)$





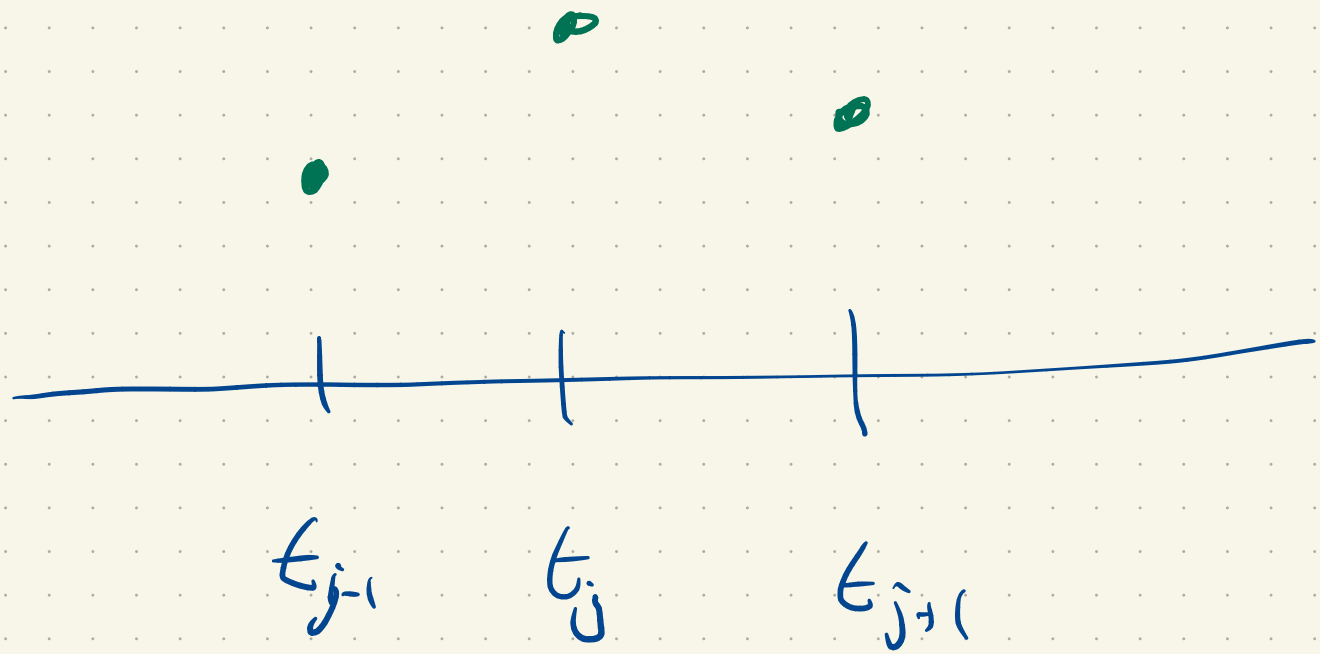
$1 - 0$

$$\Delta x = \frac{1}{N}$$

$$\Delta t = \frac{T - 0}{M} = \frac{T}{M}$$

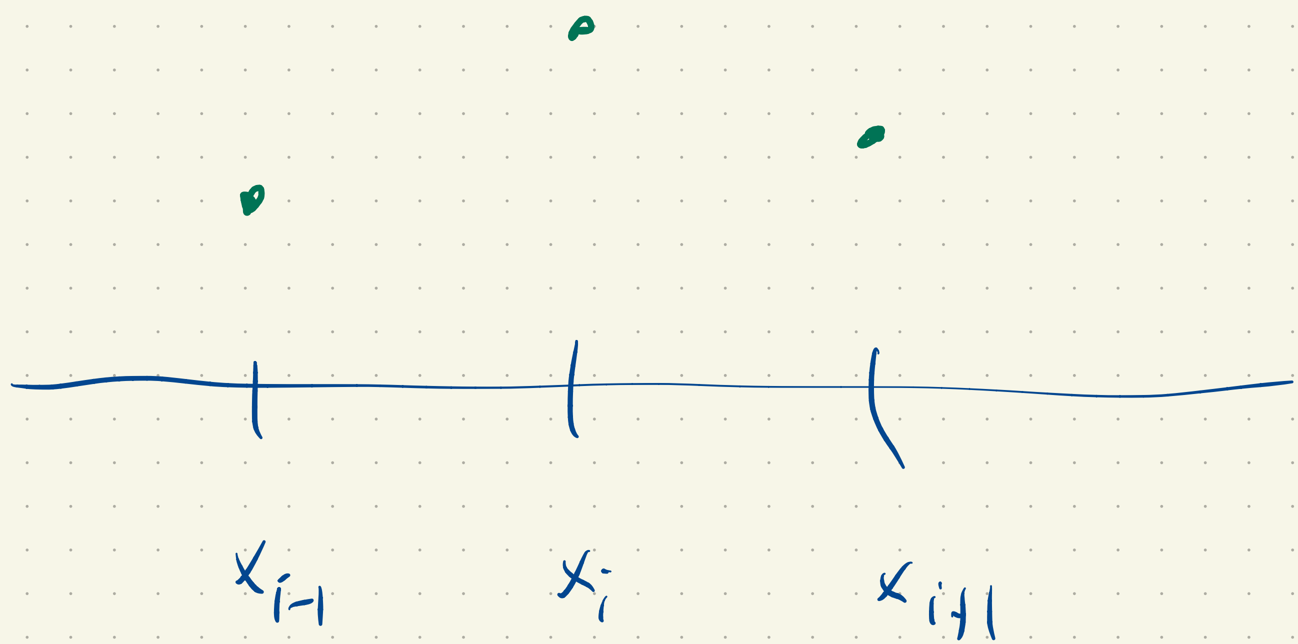
$$u(x_i, t_j) = u_{i,j}$$

$$u_t(x_i, t_j) \approx \frac{u(x_i, t_{j+1}) - u(x_i, t_j)}{\Delta t}$$



$$u_x(x_i, t_j) \approx \frac{u(x_{i+1}, t_j) - u(x_i, t_j)}{\Delta x}$$

$$\approx \frac{u(x_i, t_j) - u(x_{i-1}, t_j)}{\Delta x}$$

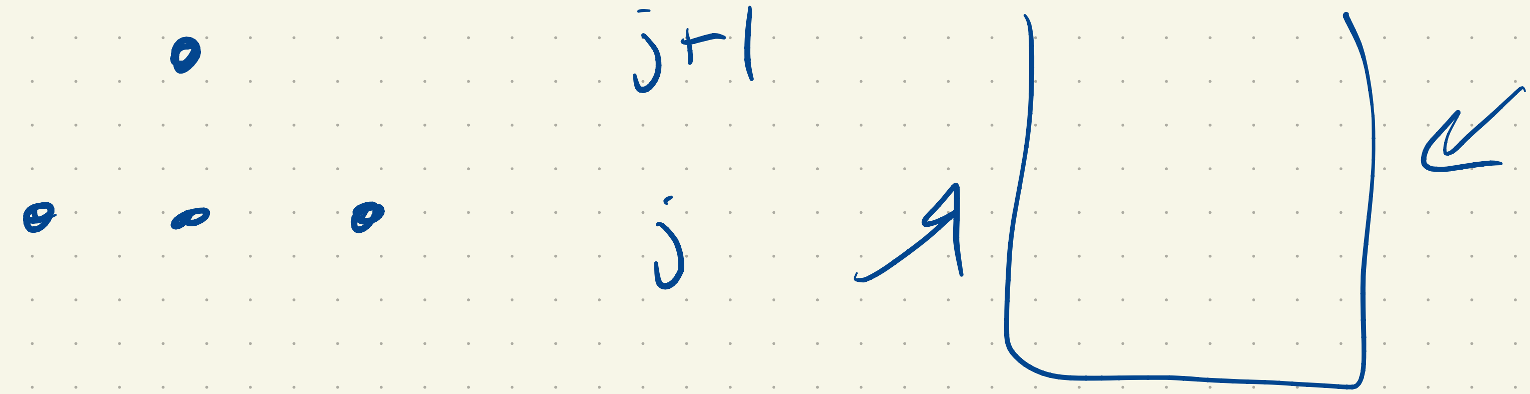


$$u_{xx} \approx \frac{(u_{i+1,j} - u_{i,j}) - (u_{i,j} - u_{i-1,j})}{\Delta x^2}$$

$$= \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2}$$

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} =$$

$$u_{i,j+1} = u_{i,j} + \frac{\Delta t}{\Delta x^2}\left[u_{i+1,j} - 2u_{i,j} + u_{i-1,j}\right]$$



$u_{0,j} = 0$ $\quad$ $u_{N,j} = 0$

$\underbrace{u_{i,0}}$ give me this.

u_0 $\qquad$ $[u_0, u_1, u_2, \dots, u_M]$

Exact solutions: $u(x,t) = \sin(k\pi x)\, e^{-k^2\pi^2 t}$

Exercise: $u_t = u_{xx}$ ✓

$u(0,t) = 0$

$u(1,t) = 0$

$u(x,0) = \sin(k\pi x)$

$k \in \mathbb{Z}$