

closed bounded domain.
(includes boundary)

fits in a box.

A continuous function on such a domain will
attain a max/min.

This happens either at

- 1) an interior critical point
- 2) on the boundary.

E.g. Maximize $V = xyz$ subject to $x, y, z \geq 0$

$$x+y+z \leq 96$$

$$z \leq 96-x-y$$

(slipping negs!)

$$z = 96 - x - y$$

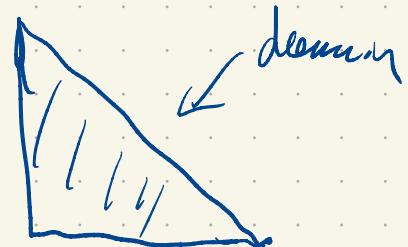
$$x \geq 0$$

$$y \geq 0$$

$$x+y \leq 96$$

$$V_x = y(96-x-y) - xy$$

$$= y(96-y) - 2xy = y[96-y-2x]$$



$$V_y = x(96-x) - 2xy = x[96-x-2y]$$

$$V_x = 0 : y=0 \text{ or } 96-y-2x=0$$

$$V_y = 0 : x=0 \text{ or } 96-x-2y=0$$

$$(0,0), \quad (0,96) \quad (96,0)$$

$$-y+x -2x+2y = 0$$

$$-x+y = 0 \quad \boxed{x=y}$$

$$96-x-2x = 96-3x \cancel{-2x}$$

$$x=32$$

$$y=32$$

$$V_{xx} = -2y \quad V_{yy} = -2x \quad V_{xy} = 96 - 2y - 2x$$

$$D = V_{xx} V_{yy} - (V_{xy})^2 = 4x^2 - (96 - 2x - 2y)^2$$

$$D(32,32) = 3072 > 0$$

$V_{xy} < 0 \Rightarrow$ local max

(when)

$$z = 96 - 32 - 32 = 32 (!)$$

Cube: 32^3

Last class:

critical point: $\nabla f = 0$ or DNE.

At a local min/max in interior of domain,
we have a crit point.

So if looking for max/min, in interior
need only look at critical points.

for $f(x,y)$ (2-d) we have a 2nd der test

$$H = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix}$$

$$D = |H| = f_{xx}f_{yy} - (f_{xy})^2$$

If $D > 0 \Rightarrow$ local min/max

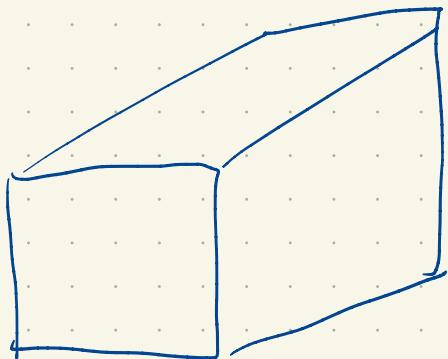
$D < 0 \Rightarrow$ saddle

$D = 0 \Rightarrow$ inconclusive

$f_{xx} > 0 \Rightarrow$ local min (Sxy also)

$f_{xx} < 0 \Rightarrow$ local max

Section Lagrange Multipliers



$$V = xyz$$

$$\text{girth + length} \leq 108$$

$$2x+2y+z \leq 108$$

clearly an increase, so

Maximize $V = xyz$ subject to

$$2x+2y+z = 108,$$

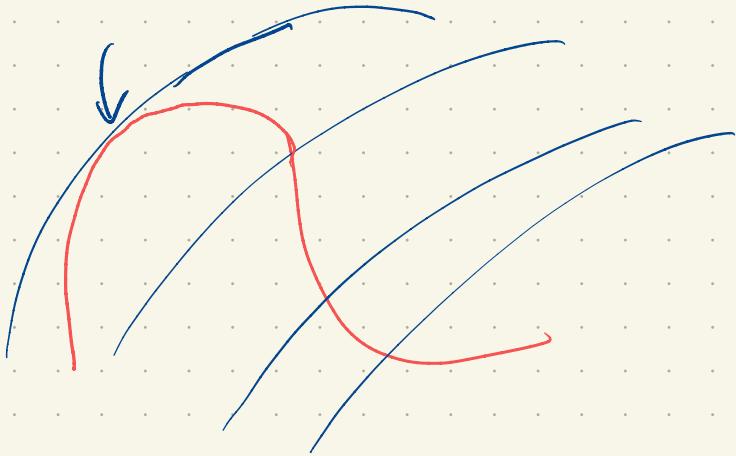
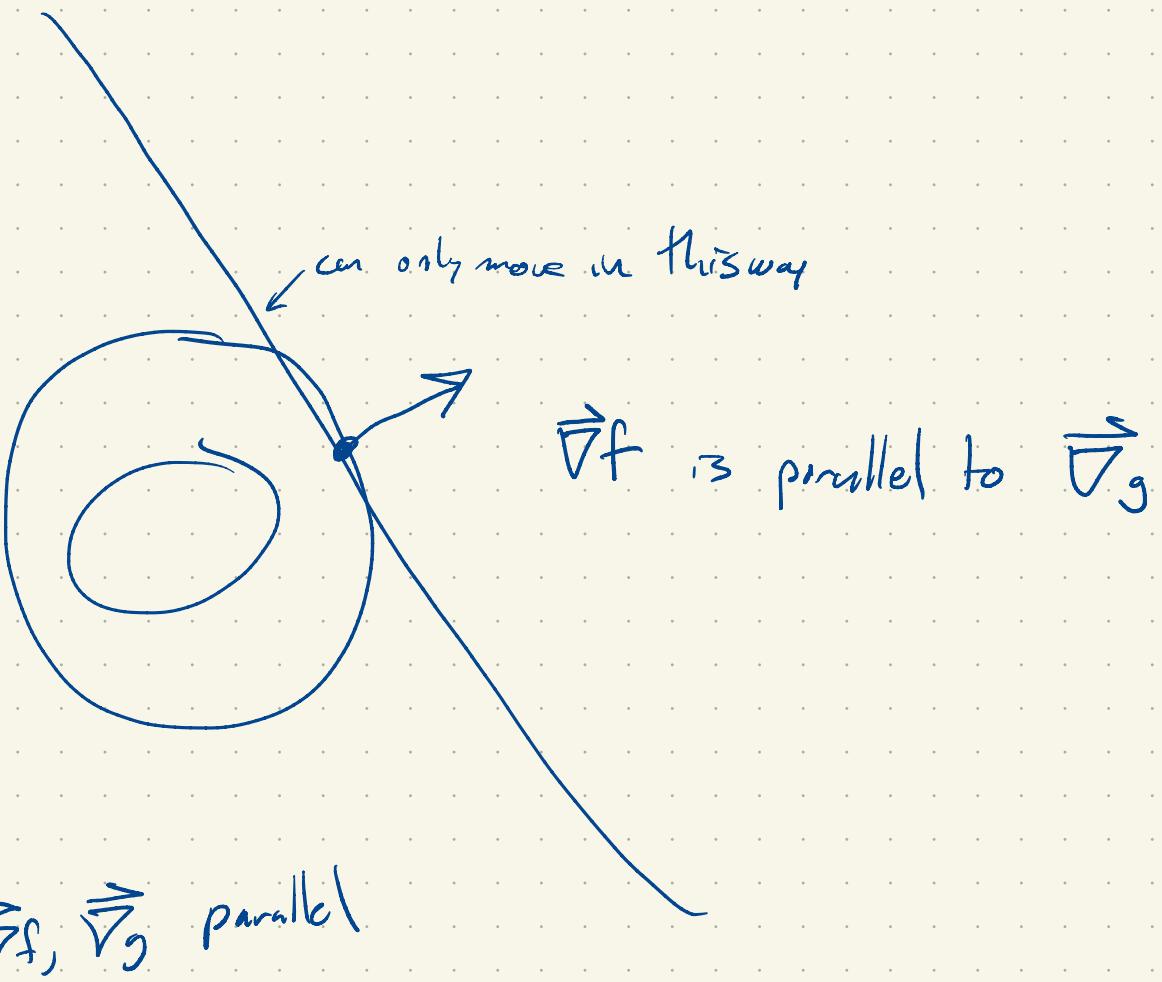
↳ constraint.

We'll come back to this.

Let us instead minimize

$$f(x,y) = x^2 + y^2 \quad \text{subject to } x+y = 9$$

$$g(x,y)$$



At a maximizer

$$\vec{\nabla}f(x_0, y_0) = \lambda \vec{\nabla}g(x_0, y_0)$$

$$g(x_0, y_0) = c \quad (c \text{ in general})$$

3 eq's for

3 unknowns

$$f_x(x_0, y_0) = \lambda g_x(x_0, y_0)$$

$$f_y(x_0, y_0) = \lambda g_y(x_0, y_0)$$