$$\frac{\sin(\pi/3+h) - \sin(\pi/3)}{h} - \cos(\frac{\pi}{3})$$

$$\text{error} \approx C h^{\alpha}$$

$O(h^2)$

!

slope 1

rise: 7

run: 7

$\frac{7}{7} = 1$

$$g(h) = c_1 h + \frac{c_2 \varepsilon}{h}$$

10^{-16}

$$g(h) = C_1 h + \frac{C_2 \varepsilon}{h}$$

Minimize: $g'(h) = 0$

$$g'(h) = C_1 - \frac{C_2 \varepsilon}{h^2}$$

$$C_1 - \frac{C_2 \varepsilon}{h^2} = 0 \qquad h^2 = \frac{C_2 \varepsilon}{C_1}$$

Error is $O(h^2)$

$$\text{error} = Ch^q$$

10^{-16}

$g(h) = C_1 h^2 + \frac{C_2}{h}\varepsilon$ $\qquad g'(h) = 0$ $\qquad \approx 10^{-5}$

$h = C_3 \varepsilon^{1/3}$

$g'(h) = 2C_1 h - \frac{C_2 \varepsilon}{h^2}$

$2C_1 h^3 = C_2 \varepsilon$ $\qquad h^3 = \frac{C_2}{2C_1}\varepsilon$