1. Let

$$R_\theta = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}.$$

Let $H = \{R_\theta : \theta \in \mathbb{R}\}$. That is, H is $SO(2)$.

1. Show that H is group isomorphic to S^1. You must exhibit the isomorphism, show that it is a homomorphism, and show that it is bejective. You may find it easiest to make the isomorphism go from S^1 to H.
2. Suppose A is a 2×2 real matrix in $O(2)$ and $AR_\theta = R_\theta A$ for all θ. Show that there exists θ' with $A = R_{\theta'}$.
3. Conclude that $SO(2)$ is a maximal torus in $O(2)$.