MATH 253
Quiz 5

Name: SOLUTIONS

February 19, 2021

Instructions: Five points total. Show all work for credit. **GS:** Scan ONE page for your solutions.

1. (a) (1 pt.) Prove that the following limit does not exist: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2+y^8}$

$\boxed{\text{Since } 0 \neq 1/2, \text{ this Limit D.N.E.}}$

One possible answer:

Approach along x-axis ($y=0$) $\displaystyle\lim_{x\to 0} \frac{x(0)^4}{x^2} = \lim_{x\to 0} \frac{0}{x} = 0$

Approach along $x = y^4$ $\displaystyle\lim_{y\to 0} \frac{(y^4)(y^4)}{(y^4)^2+y^8} = \lim_{y\to 0} \frac{y^8}{2y^8} = \lim_{y\to 0} \frac{1}{2} = \frac{1}{2}$

(b) ()1 pt.) Find the value of $\displaystyle\lim_{(x,y)\to(2,2)} \frac{xy}{9+e^{y-x}}$ and give a brief mathematical justification that this limit exists.

$\frac{xy}{9+e^{-(y-x)}}$ is continuous at $(2,2)$ so the limit exists

$$\lim_{(x,y)\to(2,2)} \frac{xy}{9+e^{y-x}} = \frac{(2)(2)}{9+e^{(2-2)}} = \frac{4}{10} = \boxed{\frac{2}{5}}$$

2. (3 pts.) Consider the function $g(x,y) = y\tan(xy)$.

(a) (1 pt.) Is the function $g(x,y)$ increasing, decreasing, or stable in the x-direction at the point in its domain $(\frac{\pi}{6}, 2)$? Briefly justify your answer.

$g_x(x,y) = y^2 \sec^2(xy)$ and at $(\pi/6, 2)$, $g_x(\frac{\pi}{6}, 2) = 2^2 \sec^2(\frac{\pi}{3}) = 16 > 0$

Since the partial derivative is positive at $(\frac{\pi}{6}, 2)$, $g(x,y)$ is $\boxed{\text{increasing}}$

(b) (2 pts.) Find the equation of the tangent plane to $g(x,y)$ at the point $\left(\frac{\pi}{6}, 2, g\left(\frac{\pi}{6}, 2\right)\right)$.

The function value is $g(\pi/6, 2) = 2\tan(\pi/3) = 2\sqrt{3}$.

$$z - g(\pi/6, 2) = g_x(\tfrac{\pi}{6}, 2)(x - \tfrac{\pi}{6}) + g_y(\tfrac{\pi}{6}, 2)(y-2)$$

$$z = 2\sqrt{3} + 16(x - \tfrac{\pi}{6}) + g_y(\tfrac{\pi}{6}, 2)(y-2)$$

$$z = 2\sqrt{3} + 16(x - \pi/6) + (\tfrac{4\pi}{3} + \sqrt{3})y - 2$$

$$z = 16x + \sqrt{3}y + (\cancel{2\sqrt{3}} - \tfrac{8\pi}{3} - \tfrac{8\pi}{3} - \cancel{2\sqrt{3}})$$

$$\boxed{z = x + (\tfrac{4\pi}{3} + \sqrt{3})y - \tfrac{16\pi}{3}}$$

$g_y(x,y) = xy\sec^2(xy) + \tan(x,y)$

$g_y(\frac{\pi}{6}, 2) = \frac{\pi}{3}\sec^2(\frac{\pi}{3}) + \tan(\frac{\pi}{3})$

$= \frac{\pi}{3} \frac{1}{(\frac{1}{2})^2} + \sqrt{3}$

$= \frac{4\pi}{3} + \sqrt{3}$