Preamble: There is a total of **70** points on this exam, plus **4** points of extra credit. Please write out your proofs and your statements of theorems and definitions as clearly as you can. In grading, however, some consideration will be made for the exam's time constraints. Good luck!

1. [20 points] Define the following

a) The sequence (x_n) of real numbers converges to L.

b) The series $\sum_{k=1}^{\infty} a_k$ converges to L.

c) A countable set.

d) The supremum of a set $A \subseteq \mathbb{R}$. (You must define any new technical words you introduce).

e) The sequence of functions $f_n : A \to \mathbb{R}$ converges uniformly to f.

f) A limit point of a set $A \subseteq \mathbb{R}$.

g) The function $f : A \to \mathbb{R}$ has a limit L at a limit point $c \in A$.

h) The function $f : A \to \mathbb{R}$ is continuous at $c \in A$.

i) The function $f : A \to \mathbb{R}$ is differentiable at a limit point $c \in A$.

j) The function $f : A \to \mathbb{R}$ is uniformly continuous.

2. [18 points] State the following theorems.

a) The Mean Value Theorem

b) The Interior Extremum Theorem

c) The Fundamental Theorem of Calculus (Part I or Part II)

d) The Fundamental Theorem of Calculus (the other Part)

e) The Intermediate Value Theorem

f) The Alternating Series Test

g) The Squeeze Theorem

h) A theorem that ensures $f : A \to \mathbb{R}$ is uniformly continuous.

i) The Sequential Criterion For Continuity

3. **[8 points]** State the Axiom of Completeness. Then state any three other results we proved that are equivalent to the Axiom of Completeness. You must state each theorem, not just give a name. (You may assume the Archimedean Property if needed).

4. **[4 points]** Suppose (a_n) and (b_n) are sequences that converge to A and B respectively. Show that
$$\lim_{n\to\infty} a_n + b_n = A + B.$$

5. **[4 points]** Prove that every convergent sequence is Cauchy.

6. **[4 points]** Suppose $f : \mathbb{R} \to \mathbb{R}$ is continuous, $c \in \mathbb{R}$, and $f(c) > 0$. Show that there is an $\delta > 0$ such that if $|x - c| < \delta$, then $f(x) > 0$.

7. **[4 points]** From the definition of the Riemann integral, show that if k is a constant, then $\int_a^b k = k(b - a)$.

8. **[4 points]** Carefully show that $f(x) = |x|$ is not differentiable at $x = 0$.

9. **[4 points]** Suppose $f : [0,1] \to \mathbb{R}$ is differentiable and f' is bounded. Show that f is uniformly continuous.

10. **[Extra Credit: 4 points]** Suppose $f : [a,b] \to \mathbb{R}$ is continuous. Show there is a $c \in [a,b]$ such that $f(c) = \frac{1}{b-a} \int_a^b f(x)\, dx$.