1. Problem 10.5

2. Problem 10.7

3. Continuing with the theme that some sample points are better than others, recall that polynomial interpolation with high-order polynomials is prone to making large oscillation errors, but that this can be minimized using Chebyshev polynomials, which are the Lagrange polynomials associated with the sample points

$$x_j = \cos(\pi + (\pi j/n)), \qquad j = 0, \ldots, n$$

on the interval $[-1, 1]$. Clenshaw-Curtis integration is integration using polynomial interpolation at these sample points.

Use the MATLAB `polyfit` function to perform polynomial interpolation at these sample points for $n = 4, 6, 10$ and then use the resulting polynomials to approximate

$$\int_{-1}^{1} x \sin(x) \, dx.$$

Compare your approximations to the exact answer (which you should compute by hand). Integration by parts!

4. Recall that 5 point Gauss-Legendre integration uses sample points $[-\beta, -\alpha, 0, \alpha, \beta]$ where

$$\alpha = \frac{1}{3}\sqrt{5 - 2\sqrt{\frac{10}{7}}}$$

$$\beta = \frac{1}{3}\sqrt{5 + 2\sqrt{\frac{10}{7}}}.$$

Write a code that performs composite Gauss-Legendre integration with these sample points. Your code should have the signature

```
function q=glquad(f,a,b,N)
...
end
```

were $f$ is the function to integrate, $a$ and $b$ are the endpoints of integration, and $N$ is the number of subintervals. Your code should perform Gauss-Legendre integration on each subinterval and add them up. Then apply your function to compute

$$\int_{-1}^{1} x \sin(x) \, dx$$

using $N = 1, 2, 4, 10$. Compare the results of Gauss-Legendre integration to the results you saw using Clenshaw-Curtis integration.