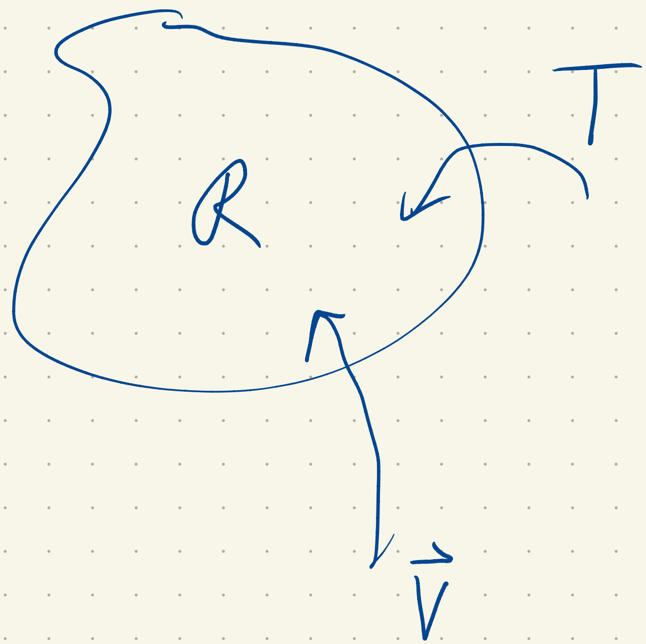


f'

$\vec{\nabla} T$

derivative of a vector field?

$\vec{V} = \langle P, Q, R \rangle$

$\operatorname{div} \vec{V}$

The divergence of $\vec{V}$, written $\vec{\nabla} \cdot \vec{V} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$

$\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle \cdot \langle P, Q, R \rangle =$

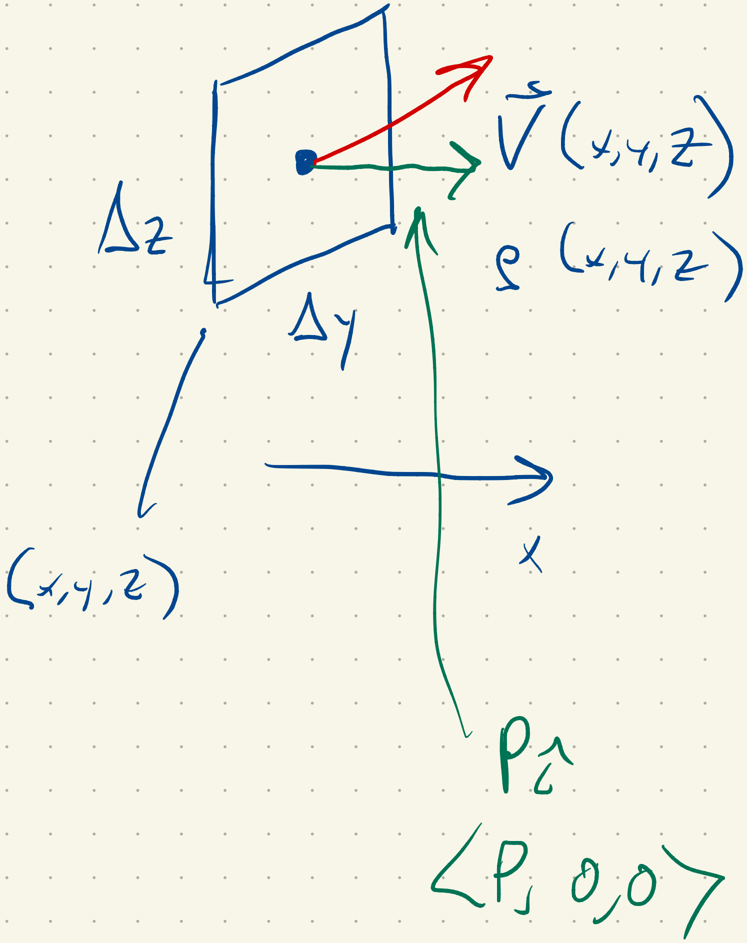
$$\vec{V} = \langle \overset{P}{xy}, \overset{Q}{x\sin(y)}, \overset{R}{z} \rangle$$

$$\vec{\nabla} \cdot \vec{V} = y + x\cos(y) + 1$$

$$P = xy \qquad \frac{\partial P}{\partial x} = y$$

$$Q = x\sin y \qquad \frac{\partial Q}{\partial y} = x\cos y$$

$$R = z \qquad \frac{\partial R}{\partial z} = 1$$



What is the rate at which smoke is passing through the window?

($\mu g/s$)

ρ smoke density $\mu g/cm^3$

$\vec{V}$ smoke velocity cm/s

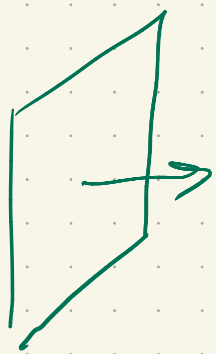
$$\vec{V} = \langle P, Q, R \rangle$$

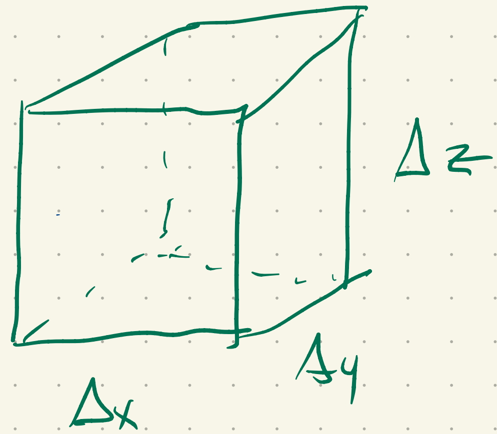
$$[\rho P] = [\rho][P] \qquad \frac{\mu g}{cm^3} \cdot \frac{cm}{s} = \frac{\mu g}{cm^2\, s} = \frac{\mu g}{s} \cdot \frac{1}{cm^2}$$

mass / time ← mass flux

$$\text{mass flux} = \rho P \Delta y \Delta z$$

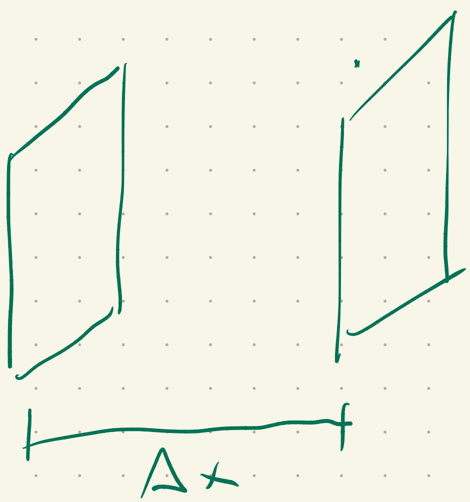
At what rate is smoke entering the box?





$$\rho P \Delta y \Delta z$$

at what rate is smoke entering the box?



$$\rho P(x,y,z)\,\Delta y\,\Delta z \qquad\qquad \rho P(x+\Delta x,y,z)\,\Delta y\,\Delta z$$

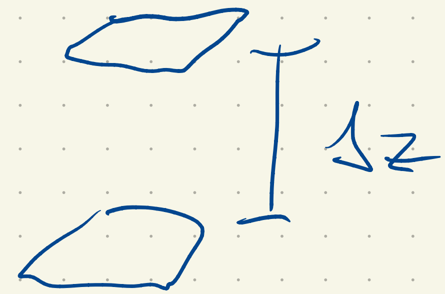
$$\rho P(x,y,z)\Delta y\,\Delta z - \rho P(x+\Delta x,y,z)\,\Delta y\,\Delta z$$

$$\rho\left[P(x,y,z) - P(x+\Delta x,y,z)\right]\Delta y\,\Delta z$$

$$-\rho\left[P(x+\Delta x,y,z) - P(x,y,z)\right]\Delta y\,\Delta z$$

$$-\rho\left[\frac{\partial P}{\partial x}\Delta x\right]\Delta y\,\Delta z$$

$$-\rho\frac{\partial P}{\partial x}\Delta x\,\Delta y\,\Delta z$$ ← contribution from two parallel windows



$$-\rho \frac{\partial Q}{\partial y} \Delta x \Delta y \Delta z$$

$$-\rho \frac{\partial R}{\partial z} \Delta x \Delta y \Delta z$$

total mass flux into the box is

$$\underbrace{-\rho}_{\text{density of smoke}} \underbrace{\left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \right)}_{\vec{\nabla} \cdot \vec{V}} \underbrace{\Delta x \Delta y \Delta z}_{\text{little volume}}$$

$$f(5+0.01) \approx f(5) + f'(5) \cdot 0.01$$

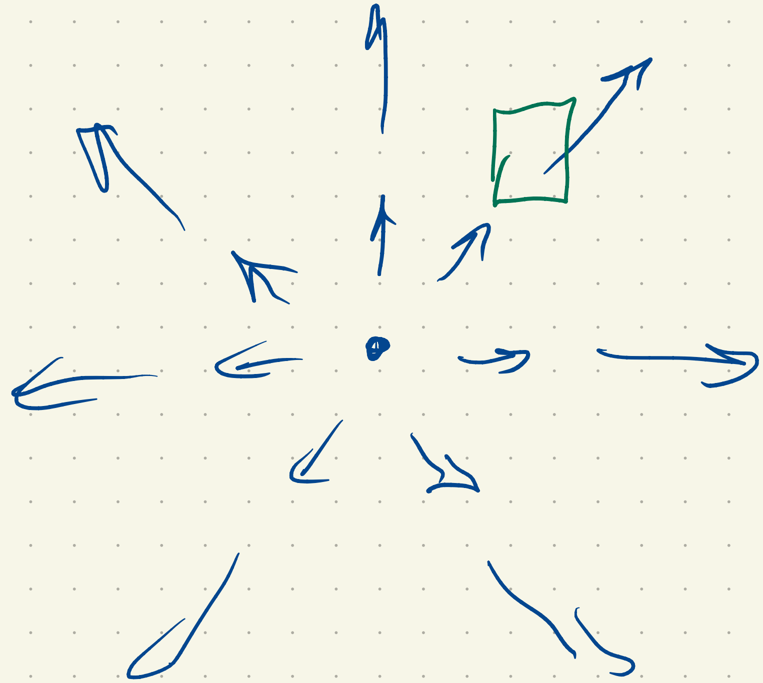
$$f(5+0.01) - f(5) \approx f'(5) \cdot 0.01$$

$$\vec{\nabla} \cdot \vec{V} = P_x + Q_y + R_z$$

$\vec{\nabla} \cdot \vec{V} > 0$ smoke is locally dispersing

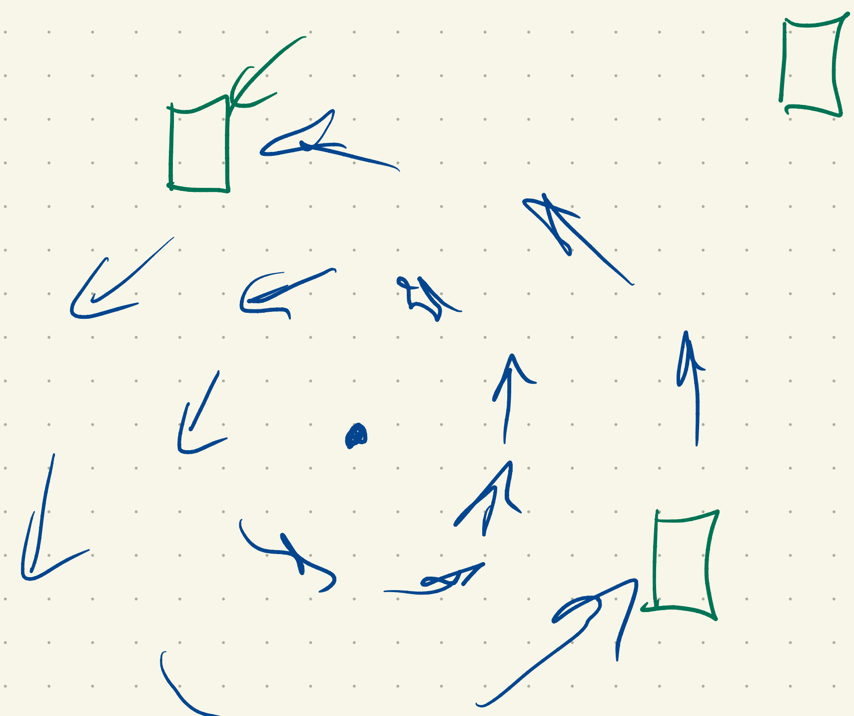
$\vec{\nabla} \cdot \vec{V} < 0$ smoke is locally concentrating

$\vec{V} = \langle x, y \rangle$



$\vec{\nabla} \cdot \vec{V} = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} = 1 + 1 = 2 > 0$

$\vec{V} = \langle -y, x \rangle$

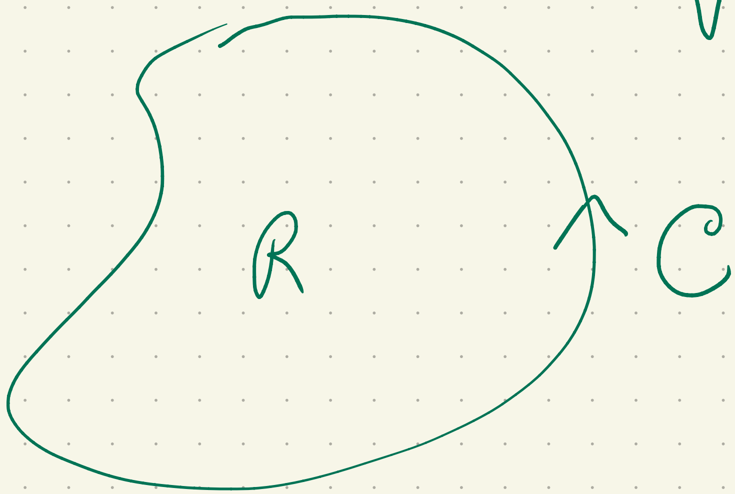


$$\vec{\nabla} \cdot \vec{V} = \frac{\partial(-y)}{\partial x} + \frac{\partial x}{\partial y} = 0 + 0 = 0$$

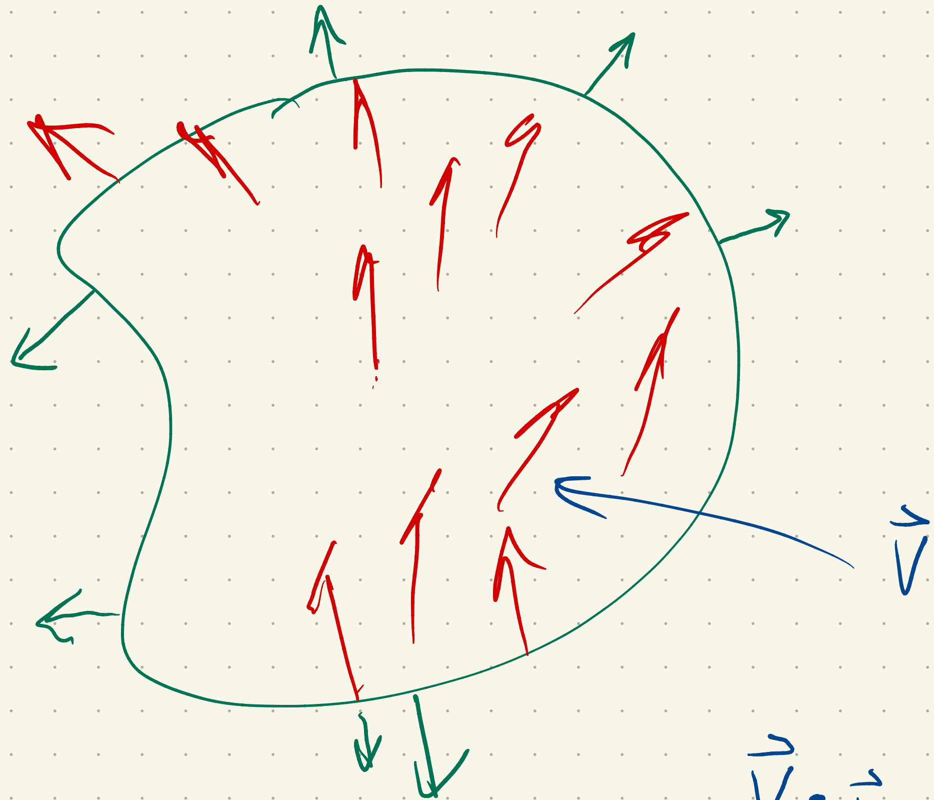
(no concentrating/dispersing)

$P_x \quad Q_y$

$$\vec{V} = \langle P, Q \rangle$$



$$\oint_C P\,dx + Q\,dy = \iint_R -P_y + Q_x \, dx\,dy$$



$\vec{n}$ "exterior normal vector"

unit

$\vec{V} \cdot \vec{n}$

$$\int_C \vec{V} \cdot \vec{n} \, ds \rightarrow \text{total rate of dispersion in the region.}$$

$$\int_C \vec{V} \cdot \vec{n} \, ds = \iint_R \vec{\nabla} \cdot \vec{V} \, dx \, dy$$