First Derivative Test

Suppose f is a function with a derivative on (a, b), and if c is a point in the interval with f'(c) = 0.

- If f'(x) > 0 for x just to the left of c and f'(x) < 0 for x just to the right of c, then f has a local maximum at c.
- If f'(x) < 0 for x just to the left of c and f'(x) > 0 for x just to the right of c, then f has a local minimum at c.
- If f'(c) = 0 and f'(x) < 0 on both sides of c or f'(x) > 0 on both sides of c, then there is neither a local min nor a local max at c.

Second Derivative Test

Suppose f is a function with a continuous second derivative on (a, b), and that c is a point in the interval with f'(c) = 0.

- If f''(c) > 0 then f has a local minimum at c.
- If f''(c) < 0 then f has a local maximum at c.

Concave Up: f'(x) increasing; f''(x) > 0

Concave Down: f'(x) decreasing; f''(x) < 0

Point of Inflection: Value x where concavity changes; often f''(x) = 0

This worksheet considers the function

$$g(x) = x^2 e^x$$

1. Find all critical points of *g*.

2. Determine the intervals where g is increasing and where g is decreasing.

3. Determine the intervals where g is concave up and where f is concave down.

4. Find all points of inflection if *g*.

5. Use the First Derivative Test to classify each critical point as a local min/local max.

6. Use the Second Derivative Test to classify each critical point as a local min/local max (if possible).

7. Determine the value of g at each of its critical points.

8. Use the information determined thus far to sketch the graph of g(x). You may used the fact, which we will justify next class, that $\lim_{x\to-\infty} f(x) = 0$.