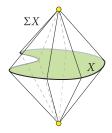
See **Rules** on following page.

- **1.** A subset A of a topological space X is said to be nowhere dense if Int $\overline{A} = \emptyset$.
 - a) Let U be an open subset of a topological space. Prove that ∂U is closed and nowhere dense.
 - b) Let V be a closed and nowhere dense set. Show that V is the boundary of an open set.
- **2.** Let f and g be continuous maps from a topological space X to a Hausdorff space Y. Suppose f = g on a dense subset of X. Prove that f = g.
- **3.** Exercise 4.38
- **4.** Suppose *X* and *Y* are spaces and *Y* is compact. Show that the projection $X \times Y \to Y$ is a closed map.
- **5.** Let *G* be an algebraic group. We say that *G* is a **topological group** if in addition *G* is a topological space such that the multiplication map $m: G \times G \to G$ and the inversion map $i: G \to G$ defined by $m(g,h) = g \cdot h$ and $i(g) = g^{-1}$ are continuous.
 - a) Suppose G is an algebraic group and a T_1 topological space. Show that G is a topological group if and only if the map $f: G \times G \to G$ defined by $f(g,h) = gh^{-1}$ is continuous.
 - b) Let G be a topological group and let H be a subgroup. Show that \overline{H} is a subgroup. Hint: that map f from the previous part is continuous.
- **6.** Let $\{x_n\}_n$ be a sequence in an arbitrary product $\prod X_\alpha$. Show that $x_n \to x$ if and only if $\pi_\alpha(x_n) \to \pi_\alpha(x)$ for every α . Then show that this result is false if we assume instead that $\prod X_\alpha$ is given the box topology.
- 7. Lee Problem 4-4
- **8.** Lee Problem 4-5
- 9. Lee Problem 4-11
- 10. Let X be a topological space. The **suspension** of X, denoted by ΣX , is the quotient of $X \times [-1, 1]$ where all points of the form (x, 1) are identified, and all points of the form (x, -1) are identified. Determine, with proof, a familiar space that is homeomorphic to ΣS^n .



Rules and format:

- You are welcome to discuss this exam with me (David Maxwell) to ask for hints and so forth.
- If you find a suspected error or mispring, please contact me as soon as possible and I will communicate it to the class if needed.
- You my not discuss the exam with anyone else until after the due date/time.
- You are permitted to reference Lee or any other topology text you like. If you use another text, you must cite it when you use it.
- You may not consult any internet resources, including search engines.
- Each problem is weighted equally.
- The due date/time is absolutely firm.
- The problem session on March 7 will be a hints session.