

(3, 2)

(  
↑ ,      ↑      →      ↑  
kg      liters      kg  
of      of oil      of sugar

## Basic Operations

N

$$a = (1, 3, 7, 5)$$

$a_1 \ a_2 \ a_3 \ a_4$

indexing       $a_3 = 7$

subsetting       $a_{2:3} = (3, 7)$

$$a_{2:4} = (3, 7, 5)$$

concatenation       $b = (2, -3, 9)$

$$(a, b) = (1, 3, 7, 5, 2, -3, 9)$$

$$O_n = \underbrace{(0, 0, \dots, 0)}_n$$

zero vector  $(0, 0, 1, 0, 0, 0)$

$O$

$$\vec{1}_n = \underbrace{(1, 1, \dots, 1)}_n$$

$\vec{1}$  which vector  $(e_j)_k$  entry  
 $e_1, e_2, e_3, \dots, e_n$

standard basis vectors  
 (unit vectors)

$$e_k = (0, 0, \dots, 0, \underset{\uparrow}{1}, 0, \dots, 0)$$

slot  $k$   
 $n$  (suppressed)

# Fundamental Vector Operations

- 1) Vector addition
- 2) Scalar multiplication

$$a = \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix} \quad b = \begin{bmatrix} 4 \\ -2 \\ -2 \end{bmatrix}$$

$$a + b = \begin{bmatrix} 6 \\ 3 \\ 4 \end{bmatrix} \quad \begin{aligned} 2 + 4 &= 6 \\ 1 + 2 &= 3 \\ 6 + (-2) &= 4 \end{aligned}$$

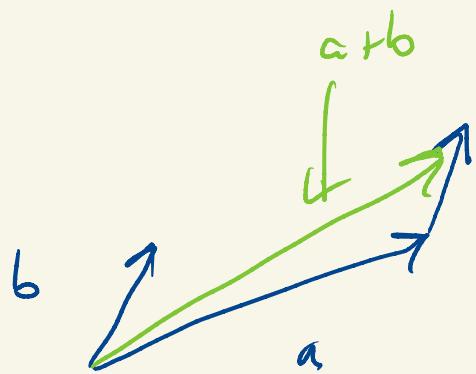
Portfolio assets ( $, , , ,$ )  
↑ ↓ . . . -

# of  
shares of AAPL

new  
portfolios

Time series audio signal

combining audio signals



Scalar Multiplication

$$7 \cdot \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 14 \\ 7 \\ 28 \end{bmatrix}$$

audio signals louder or quieter  
portfolio



These operations get along with each other

$a, b, c$  vectors of same length

$\alpha, \beta$  numbers

$$a + b = b + a$$

$$(a+b) + c = a + (b+c)$$

$$(a+b+c)$$

$$a + 0 = a = 0 + a$$

$$a + (-a) = 0$$

$$-a = (-a_1, -a_2, \dots, -a_n)$$

$$\alpha(\beta a) = (\alpha\beta)a$$

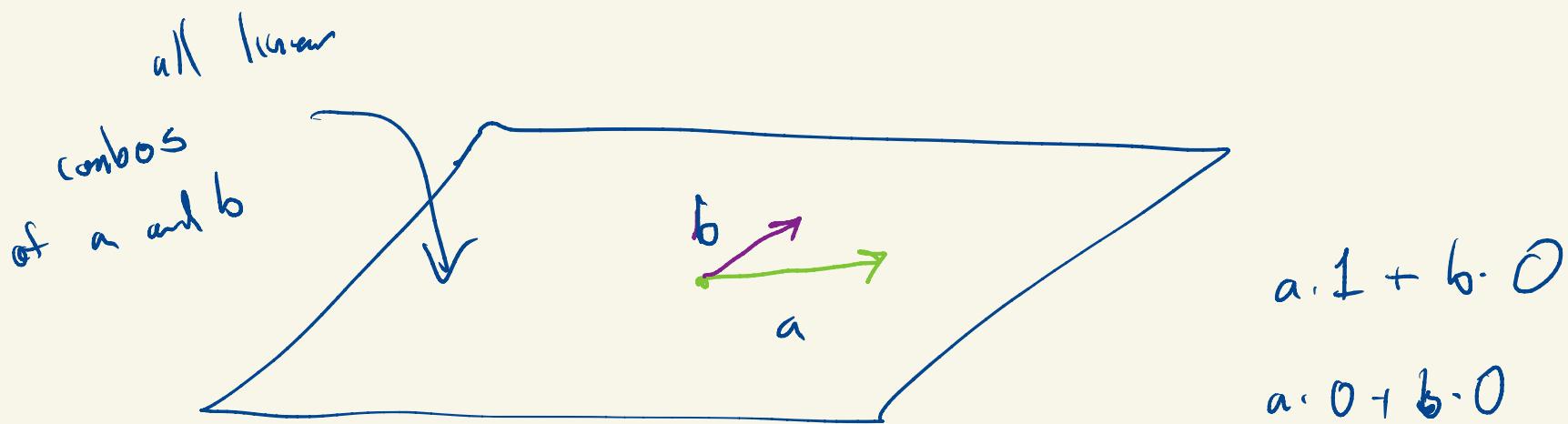
$$\alpha(a+b) = \alpha a + \alpha b$$

$$(\alpha\beta)a = \alpha a + \beta a$$

$a, b$  vectors       $\alpha, \beta$  numbers

$$\alpha a + \beta b$$

→ a linear combination of  $a$  and  $b$



Inner Product (duality) (dot product)

$$a = (a_1, a_2, a_3, a_4)$$

$$b = (b_1, b_2, b_3, b_4)$$

$$a^T b = a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4$$

we are adding up the entries of  $b$  with weights  
coming from  $a$ .

$b$ : portfolio assets

(# of shares, MSFT, AMZN)  
at AAPL

$a$ : price per share of  
the same stocks.

$$a^T b = a_1 b_1 + a_2 b_2 + a_3 b_3$$

total value of portfolio

$$a = \vec{1}_4 \quad b = (b_1, b_2, b_3, b_4)$$

$$a^T b = b_1 + b_2 + b_3 + b_4$$

$$a = \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right), \quad a^T b = \frac{1}{4}b_1 + \frac{1}{4}b_2 + \frac{1}{4}b_3 + \frac{1}{4}b_4 \\ = \frac{1}{4}(b_1 + \dots + b_4)$$

$$a = e_3 \\ = (0, 0, 1, 0) \quad a^T \cdot b = 0 \cdot b_1 + 0 \cdot b_2 + 1 \cdot b_3 + 0 \cdot b_4 \\ = b_3$$

$$a^T \cdot b = a_1 \cdot b_1 + a_2 \cdot b_2 + \dots + a_n \cdot b_n$$

$$= b_1 \cdot a_1 + b_2 \cdot a_2 + \dots + b_n \cdot a_n$$

$$= b^T a$$