

**Exercise 1.2.9:** Show that the sequence  $(x_1, x_2, x_3, ...)$  defined in Example 1.2.7 is bounded above by 2. That is, show that for every  $i \in \mathbb{N}$ ,  $x_i \le 2$ .

Proof.

**Exercise 1.3.4:** Assume that *A* and *B* are nonempty, bounded above, and satisfy  $B \subseteq A$ . Show that  $\sup B \leq \sup A$ .

Proof. **Exercise 1.3.5:** Let A be bounded above and let  $c \in \mathbb{R}$ . Define the sets  $c + A = \{a + c : a \in \mathbb{R} \}$  $a \in A$  and  $cA = \{ca : a \in A\}$ . (a) Show that  $\sup(c + A) = c + \sup(A)$ . (b) If  $c \ge 0$ , show that  $\sup(cA) = c \sup(A)$ . (c) Postulate a similar statuent for  $\sup(cA)$  when c < 0. Proof(a). Proof (b). Statement for part (c): **Exercise 1.3.6:** Compute, without proof, the suprema and infima of the following sets. (a)  $\{n \in \mathbb{N} : n^2 < 10\}$ . (b)  $\{n/(n+m) : n, m \in \mathbb{N}\}.$ (c)  $\{n/(2n+1) : n \in \mathbb{N}\}.$ (d)  $\{n/m : m, n \in \mathbb{N} \text{ with } m + n \le 10\}.$ **Solution:** (a) (b) (c) (d) Exercise 1.3.7: Prove that if a is an upper bound for A and if a is also an element of A, then  $a = \sup A$ . Proof. 

Exercise 1.3.8: If  $\sup A < \sup B$  then show that there exists an element  $b \in B$  that is an upper bound for A.

*Proof.*  $\Box$