Quiz 6

Math 202 F01

Name: Solutions

3/4/15

1. Wheat production, W, in a given year depends on the average yearly temperature, T, and the annual rainfall, R. It is estimated that for current production in a certain farming district

$$\frac{\partial W}{\partial T} = -2 \ \frac{\text{tons}}{\text{degree}} \quad \text{and} \quad \frac{\partial W}{\partial R} = 8 \ \frac{\text{tons}}{\text{inch}}.$$

In this district, climate change is causing temperature to rise at a rate of 0.05 degree/year, and rainfall to increase at a rate of 0.1 inches/year.

Use the multivariable chain rule to determine the rate at which wheat production is changing. Indicate appropriate units.

$$\frac{dW}{dt} = \frac{\partial W}{\partial T}\frac{dT}{dt} + \frac{\partial W}{\partial R}\frac{dR}{dt}$$

$$= (-2)(.05) + (8)(.1)$$

$$= -.1 + .8 = \boxed{.7 \ \frac{\text{tons}}{\text{year}}}$$

2. The volume of a cone with a circular base of radius r and height h is

$$V(r, h) = \frac{\pi}{3}r^2 h.$$

Find the total differential of V, and use it to estimate the error in the volume if (r, h) are measured to be $(5, 10)$ cm, with error at most $\pm.05$ cm in each. Indicate appropriate units.

$$dV = \frac{\partial V}{\partial r}dr + \frac{\partial V}{\partial h}dh$$

$$= \left(\frac{2\pi}{3}rh\right)\Big|_{(5,10)} dr + \left(\frac{\pi}{3}r^2\right)\Big|_{(5,10)} dh$$

$$= \frac{100\pi}{3}dr + \frac{25\pi}{3}dh$$

If dr, dh are at most $.05$ cm

dV is at most $\frac{100\pi}{3}(.05) + \frac{25\pi}{3}(.05) = \frac{125\pi}{3}(.05) = \boxed{\frac{25\pi}{12} \ \text{cm}^3}$