

$$= \lim_{h \rightarrow 0} \frac{\sin(x) \cos(h) + \sin(h) \cos(x) - \sin(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sin(x) [\cos(h) - 1] + \sin(h) \cos(x)}{h}$$

$$= \sin(x) \lim_{h \rightarrow 0} \frac{\cos(h) - 1}{h} + \cos(x) \lim_{h \rightarrow 0} \frac{\sin(h)}{h}$$

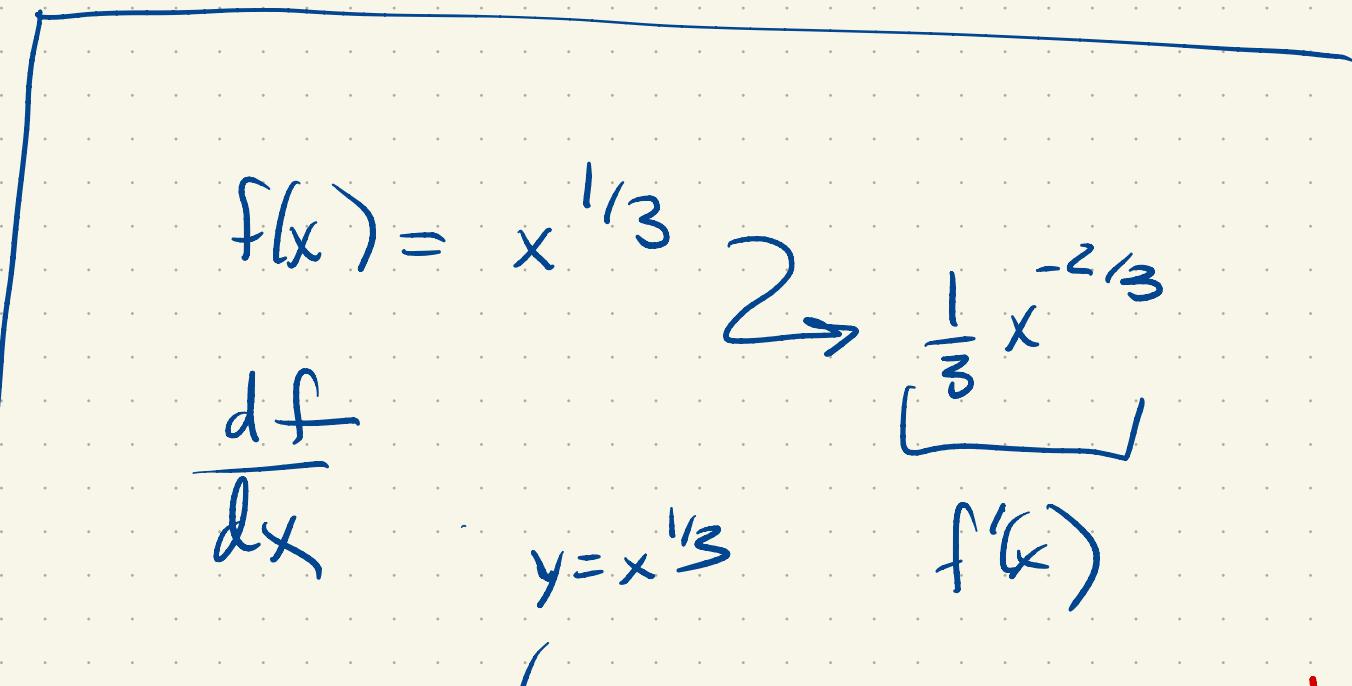
$$= \sin(x) \cdot 0 + \cos(x) \cdot 1$$

$$= \cos(x) \boxed{\frac{d}{dx} \cos(x)}$$


Section 3.5 (Implicit Differentiation)

$$\frac{d}{dx} \ln(x)$$

$$\frac{d}{dx} \arctan(x)$$

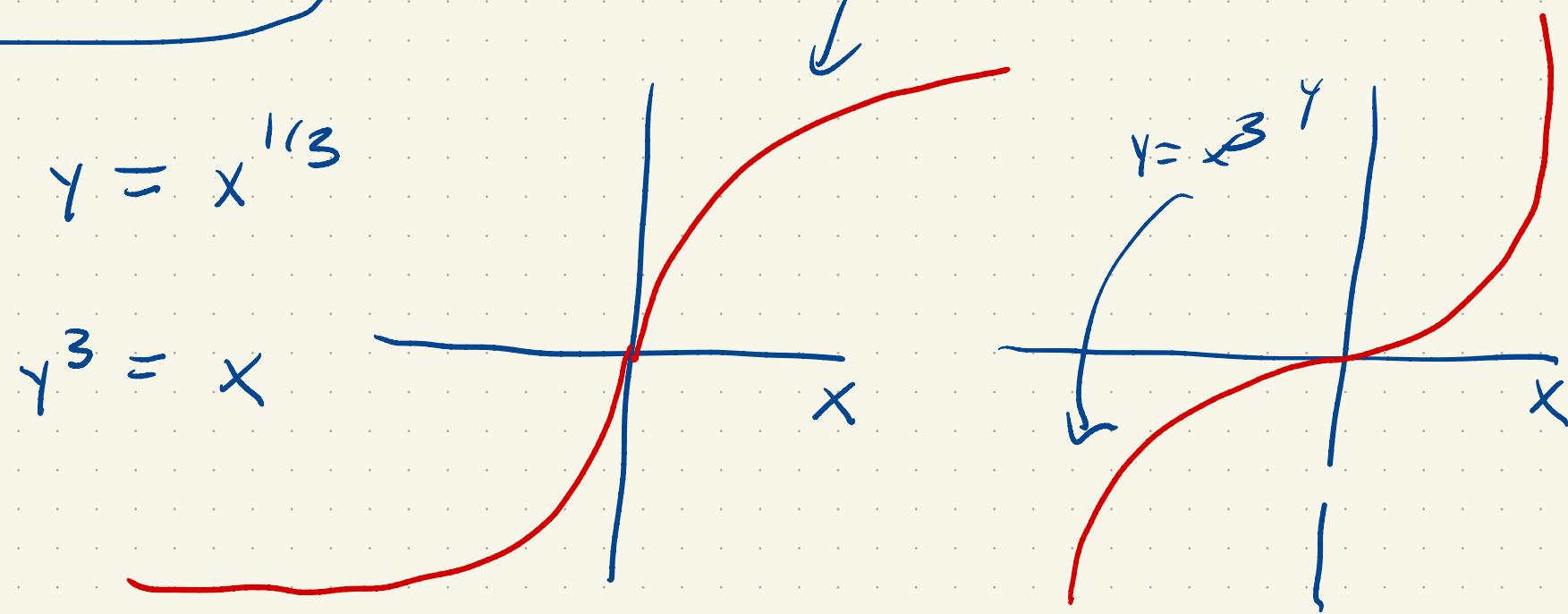


A graph of the function $f(x) = x^{1/3}$ on a Cartesian coordinate system. The curve passes through the origin (0,0) and is increasing for all x . It has vertical asymptotes at $x = -1$ and $x = 1$.

$$f(x) = x^{1/3}$$
$$\frac{df}{dx}$$
$$y = x^{1/3}$$
$$f'(x)$$

$$y = x^{1/3}$$

$$y^3 = x$$



Two graphs illustrating implicit differentiation. The left graph shows the curve $y = x^{1/3}$ (blue) and the curve $y^3 = x$ (red). The right graph shows the curve $y = x^{1/3}$ (blue) and the curve $y = x^{3/2}$ (red). Arrows point from the derivative terms in the top diagram to the corresponding curves in the bottom graphs.

$$y = x^{3/2}$$
$$y = x^{1/3}$$

$$\frac{dy}{dx} \quad \text{d}y \quad y = x^{1/3}$$

$$y^3 = x$$

$$\rightarrow (y(x))^3 = x$$



$$\frac{d}{dx} (y(x))^3 = \frac{d}{dx} x = 1$$

||

$$3(y(x))^2 \cdot \frac{d}{dx} y(x)$$

$$3(y(x))^2 \frac{dy}{dx} = 1$$

$$\begin{aligned} & \frac{d}{dt} (\sin(t))^3 \\ &= 3 \sin^2(t) \cdot \cos(t) \end{aligned}$$

$$y(x) = x^{1/3}$$

$$\frac{dy}{dx} = \frac{1}{3(y(x))^2}$$

$$\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3(x^{1/3})^2} = \frac{1}{3} \cdot \frac{1}{x^{2/3}} = \frac{1}{3} x^{-2/3}$$

$$\frac{dy}{dx} = \frac{1}{3} x^{-2/3}$$

Find $\frac{dy}{dx}$ if $y = x^{1/3}$.

Start with $y^3 = x$

$$\frac{d}{dx} y^3 = \frac{d}{dx} x$$

$$3y^2 \cdot \frac{dy}{dx} = 1$$

$$y = x^{4/3}$$

$$y^2 = x^{2/3}$$

$$\frac{dy}{dx} = \frac{1}{3y^2}$$

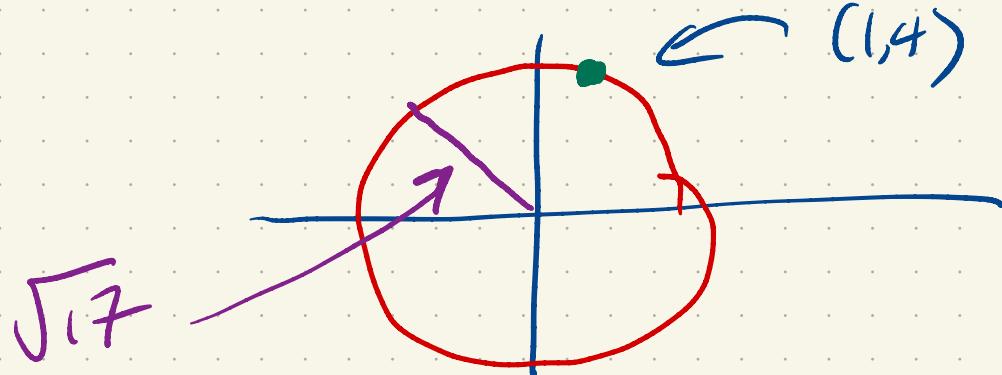


$$\frac{dy}{dx} = \frac{1}{3} \cdot \frac{1}{x^{2/3}} = \frac{1}{3} x^{-2/3}$$

Silly geometric fun:

$$y = \text{circle}$$

Consider the curve $x^2 + y^2 = 17$



$$(x,y) = (1,4)$$

is on the curve.

We want to compute the equation of the tangent line to the curve at $(1, 4)$

We need the slope of the tangent line.

We want $\frac{dy}{dx}$ at $x=1$ ($y=4$)

$$x^2 + y^2 = 1$$

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx} 1$$

$$\frac{d}{dx}x^2 + \frac{d}{dx}y^2 = 0$$

$\downarrow (y(x))^2$

$$\frac{d}{dx}(y(x))^2 = 2y(x)\frac{dy}{dx}$$

$$2x + 2y \cdot \frac{dy}{dx} = 0$$

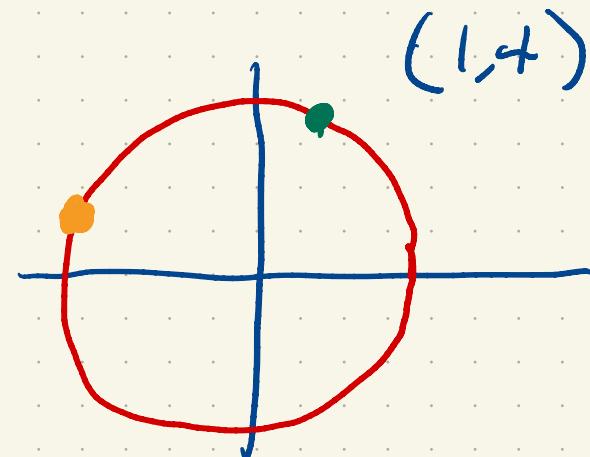
$$2y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}$$

$$x^2 + y^2 = 17$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\frac{dy}{dx} = -\frac{1}{4} \text{ at } (1,4)$$



$$(-4, 1) = (x, y) \quad x^2 + y^2 = 17$$

$$\frac{dy}{dx} = -\frac{-4}{1} = +4$$

$$x^3 = y^4 + x^2 \sin(y) + 1 \quad \text{curve}$$

$$1^3 = 0^4 + 1^2 \sin(0) + 1 \quad \text{point on the curve}$$

$$(x, y) = (1, 0)$$

What is $\frac{dy}{dx}$ at this point?

$$(x, y) = (1, 0)$$

$$x^3 = y^4 + x^2 \sin(y) + 1$$

$$\frac{d}{dx} x^3 = \frac{d}{dx} [y^4 + x^2 \sin(y) + 1]$$

$$3x^2 = \frac{d}{dx} y^4 + \frac{d}{dx} (x^2 \sin(y)) + \frac{d}{dx} 1$$

$$= 4y^3 \frac{dy}{dx} + \frac{d}{dx} (x^2) \cdot \sin(y) + x^2 \frac{d}{dx} \sin(y)$$

$$= 4y^3 \frac{dy}{dx} + 2x \sin(y) + x^2 \cos(y) \frac{dy}{dx}$$

$$3x^2 = 4y^3 \frac{dy}{dx} + 2x \sin(y) + x^2 \cos(y) \frac{dy}{dx}$$

$$3x^2 - 2x \sin(y) = \left[4y^3 + x^2 \cos(y) \right] \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{3x^2 - 2x \sin(y)}{4y^3 + x^2 \cos(y)}$$

$$x^3 = y^4 + x^2 \sin(y) + 1 \quad (1, 0)$$

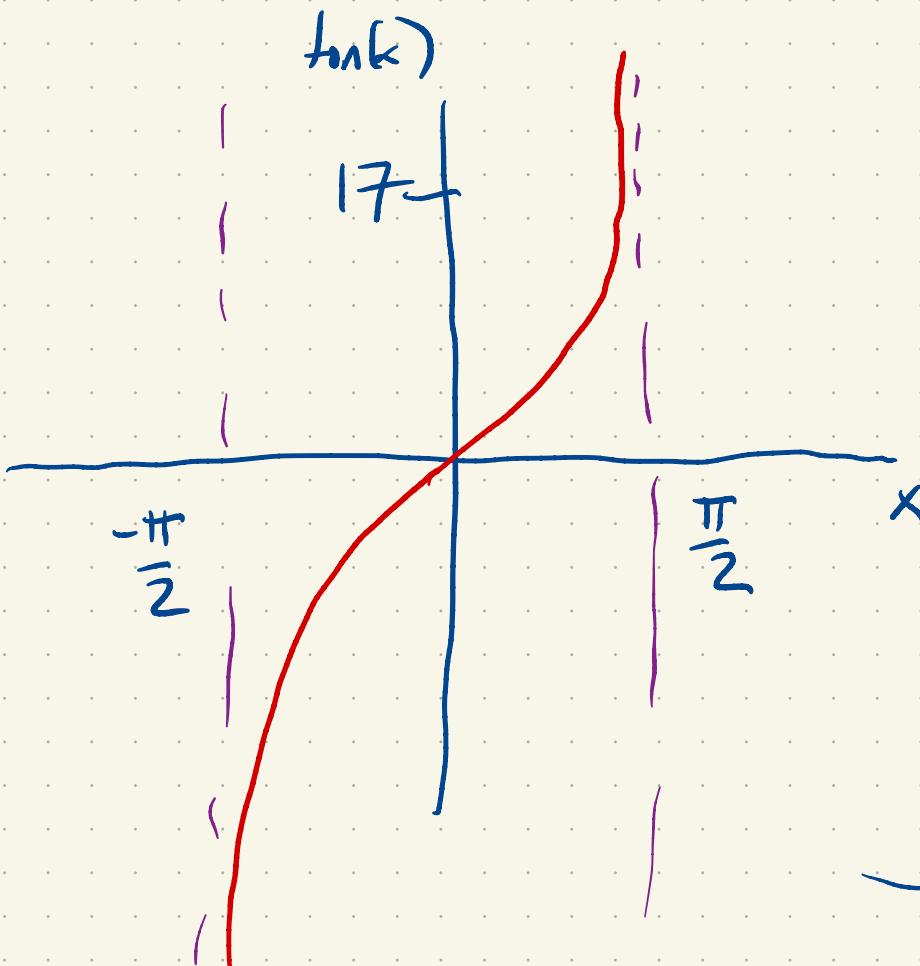
At $(x, y) = (1, 0)$

$$\frac{dy}{dx} = \frac{3 \cdot 1^2 - 2 \cdot 1 \sin(0)}{4 \cdot 0^3 + 1^2 \cos(0)} = \frac{3 - 2 \cdot 0}{0 + 1}$$

$$= \frac{3}{1}$$

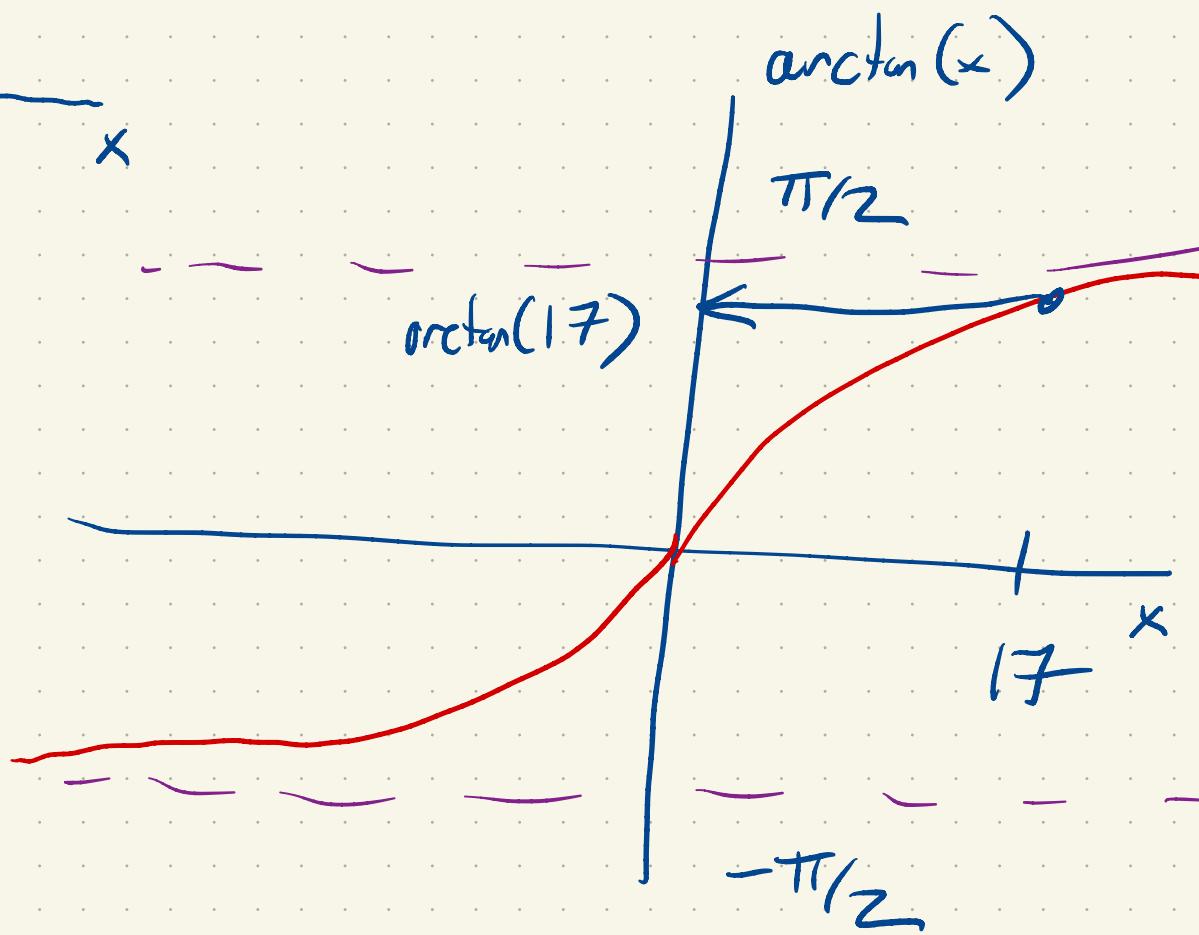
$$= 3$$

Inverse Trig Functions



$$\frac{\sin(\theta)}{\cos(\theta)}$$

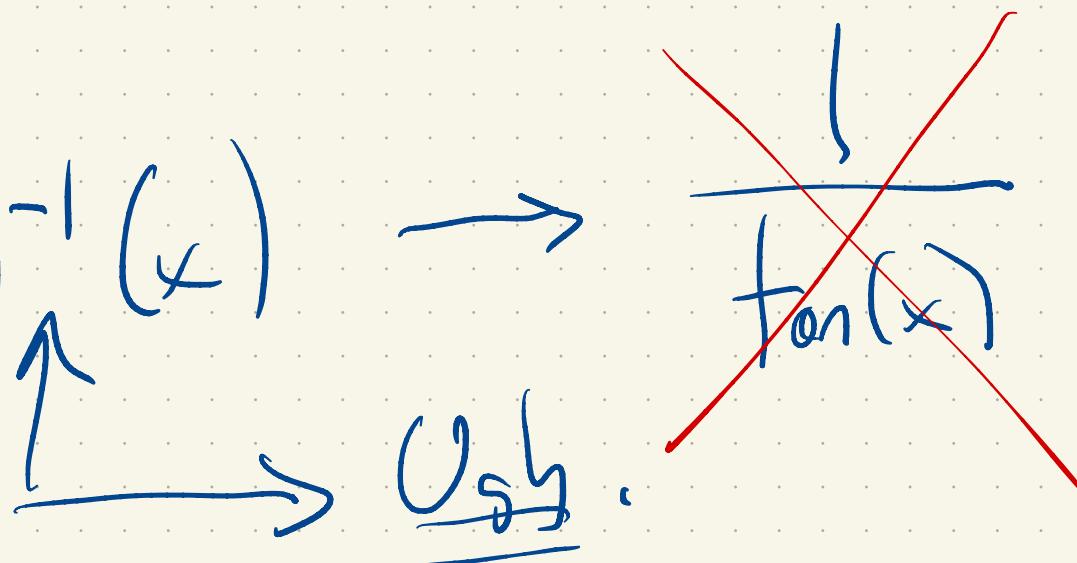
$$\tan(\theta) = \sqrt{3}$$



$$\tan(\arctan(z)) = z \quad \text{always}$$

$$\arctan(\tan(\theta)) = \theta \quad \text{only if}$$

$$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

Notation $\tan^{-1}(x) \rightarrow$ 
The diagram shows a blue arrow pointing from the expression $\tan^{-1}(x)$ to a horizontal blue line segment representing the range of the function. A large red 'X' is drawn over this blue line segment, indicating that the inverse tangent function does not have a single, continuous range.