

**Exercise 1.2.5:** Use the triangle inequality to establish the following inequalities:

(a)  $|a - b| \leq |a| + |b|$ ;

(b)  $||a| - |b|| \leq |a - b|$ .

*Proof.*

□

**Exercise 1.2.6(b), (d):** Given a function  $f$  and a subset  $A$  of its domain, let  $f(A)$  represent the range of  $f$  over the set  $A$ ; that is,  $f(A) = \{f(x) : x \in A\}$ .

(b) Find two sets  $A$  and  $B$  for which  $f(A \cap B) \neq f(A) \cap f(B)$ .

(d) Form and prove a conjecture concerning  $f(A \cup B)$  and  $f(A) \cup f(B)$ .

*Proof (b).*

□

*Proof (d).*

□

**Exercise 1.2.8:** Form the logical negation of each claim. Do not use the easy way out: "It is not the case that. . ." is not permitted

(a) For all real numbers satisfying  $a < b$ , there exists  $n \in \mathbb{N}$  such that  $a + (1/n) < b$ .

(b) Between every two distinct real numbers there is a rational number.

(c) For all natural numbers  $n \in \mathbb{N}$ ,  $\sqrt{n}$  is either a natural number or is an irrational number.

(d) Given any real number  $x \in \mathbb{R}$  there exists  $n \in \mathbb{N}$  satisfying  $n > x$ .

**Solution:**

(a)

(b)

(c)

(d)

**Exercise 1.2.9:** Show that the sequence  $(x_1, x_2, x_3, \dots)$  defined in Example 1.2.7 is bounded above by 2. That is, show that for every  $i \in \mathbb{N}$ ,  $x_i \leq 2$ .

*Proof.*

□

**Exercise 1.3.4:** Assume that  $A$  and  $B$  are nonempty, bounded above, and satisfy  $B \subseteq A$ . Show that  $\sup B \leq \sup A$ .

*Proof.* □

**Exercise 1.3.5:** Let  $A$  be bounded above and let  $c \in \mathbb{R}$ . Define the sets  $c + A = \{a + c : a \in A\}$  and  $cA = \{ca : a \in A\}$ .

- (a) Show that  $\sup(c + A) = c + \sup(A)$ .
- (b) If  $c \geq 0$ , show that  $\sup(cA) = c \sup(A)$ .
- (c) Postulate a similar statment for  $\sup(cA)$  when  $c < 0$ .

*Proof (a).* □

*Proof (b).* □

Statement for part (c):

**Exercise 1.3.6:** Compute, without proof, the suprema and infima of the following sets.

- (a)  $\{n \in \mathbb{N} : n^2 < 10\}$ .
- (b)  $\{n/(n + m) : n, m \in \mathbb{N}\}$ .
- (c)  $\{n/(2n + 1) : n \in \mathbb{N}\}$ .
- (d)  $\{n/m : m, n \in \mathbb{N} \text{ with } m + n \leq 10\}$ .

**Solution:**

- (a)
- (b)
- (c)
- (d)

**Exercise 1.3.7:** Prove that if  $a$  is an upper bound for  $A$  and  $a \in A$ , then  $a = \sup A$ .

*Proof.* □

**Exercise 1.3.8:** If  $\sup A < \sup B$  then show that there exists an element  $b \in B$  that is an upper bound for  $A$ .

*Proof.* □