

FTC Part I

$$\frac{d}{dx} \int_a^x f(s) ds = f(x)$$

FTC Part II

If $F'(x) = f(x),$

$$\int_a^b f(x) dx = F(b) - F(a)$$

Indefinite integral

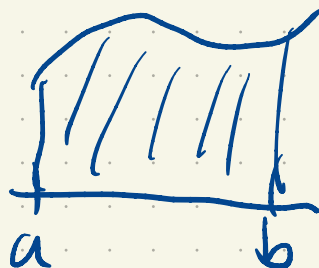
$$\int f(x) dx = F(x)$$

$$\hookrightarrow F'(x) = f(x)$$

$$\underbrace{\frac{d}{dx} f(x)}_{f'(x)}$$

$$\int_a^b f(x) dx$$

↓
number



$$\int f(x) dx$$

↪ function

$$\int \cos(x) dx = \sin(x) + C$$

→ family of functions

$$\int_0^{\pi/2} \cos(x) dx$$

$$\int \cos(x) dx = \sin(x) + C$$

→ $= \sin(\pi/2) - \sin(0) = 1 - 0 = 1$

Another perspective on FTC II

If $F'(x) = f(x)$ then

Net Change
Theorem

$$\int_a^b f(x) dx = F(b) - F(a)$$

$$\int_a^b \underbrace{F'(x)}_{\text{rate of change}} dx = \underbrace{F(b) - F(a)}_{\text{net change in } F}$$

rate of change

net change in F

If the height of a ball has a
rate of change $h'(t)$

then

$$\int_1^3 \underbrace{h'(t)}_{\substack{\text{rate of change} \\ \text{of height} \\ \text{of the ball}}} dt = \underbrace{h(3) - h(1)}_{\substack{\text{net change in height} \\ \text{of the ball}}}$$

rate of change
of height
of the ball

net change in height
of the ball