Compute the derivatives of the following functions.

$$\frac{d}{dx}\sqrt{x} = \frac{1}{2}\frac{1}{\sqrt{x}}$$

1. $f(x) = \sqrt{1+x^2}$

$$\frac{d}{dx}\sqrt{1+x^2} = \frac{1}{2}\frac{1}{\sqrt{1+x^2}} \cdot \frac{d}{dx}(1+x^2)$$

$$= \frac{1}{2}\frac{1}{\sqrt{1+x^2}} \cdot 2x$$

$$= \frac{x}{\sqrt{1+x^2}}$$

2. $f(\theta) = \tan(4\theta + 9)$

$$\frac{d}{dx}\tan(\theta) = \sec^2(\theta)$$

$$f'(\theta) = \sec^2(4\theta+9) \cdot \frac{d}{d\theta}(4\theta+9)$$

$$= \sec^2(4\theta+9) \cdot 4$$

$$= 4\sec^2(4\theta+9)$$

3. $f(t) = e^{t^2}(1 + \cos(t))$

$$\frac{d}{dt} e^{t^2} \cdot (1+\cos(t)) = \left(\frac{d}{dt} e^{t^2}\right) \cdot (1+\cos(t)) + e^{t^2} \cdot \frac{d}{dt}(1+\cos(t))$$

inside: t^2 inside': $2t$

outside: e^t outside': e^t

$$\frac{d}{dt} e^{t^2} = e^{t^2} \cdot 2t$$

$$= \frac{d}{dt} e^{t^2} \cdot (1+\cos(t)) - e^{t^2} \sin(t)$$

$$= 2t e^{t^2} \cdot (1+\cos(t)) - e^{t^2} \sin(t)$$

$$= e^{t^2}\left[2t(1+\cos(t)) - \sin(t)\right]$$

4. $f(v) = \sec\left(\dfrac{1}{1+v^2}\right)$

$$\frac{d}{dx} \sec(x) = \sec(x)\tan(x)$$

$$\frac{d}{dv} \sec\left(\frac{1}{1+v^2}\right) = \sec\left(\frac{1}{1+v^2}\right)\tan\left(\frac{1}{1+v^2}\right) \cdot \frac{d}{dv}\left(\frac{1}{1+v^2}\right)$$

$$= \sec\left(\frac{1}{1+v^2}\right)\tan\left(\frac{1}{1+v^2}\right) \cdot (-1)\frac{\frac{d}{dv}(1+v^2)}{(1+v^2)^2}$$

$$= -\sec\left(\frac{1}{1+v^2}\right)\tan\left(\frac{1}{1+v^2}\right)\frac{2v}{(1+v^2)^2}$$

5. The cost of building wooden pencils is given by a function $C(n)$ where C is the cost in dollars and n is the number of pencils, measured in thousands.

a) Explain what $C'(50) = 37.5$ means in language your parents could understand.

After 50000 pencils have been built, it costs \$37.5 / thousand pencils to build more.

b) Suppose it costs \$20000 to build 50000 pencils and $C'(50) = 37.5$. Estimate the cost of building 51000 pencils.

one thousand

$$\$20000 + \$37.5 \cdot 1$$

$$= \$20037.5$$

c) Under the same assumptions, estimate the cost of building 50100 pencils.

$$\$20000 + \$37.5 \cdot 0.1$$

$$= \$20003.75$$

6. $f(x) = \cos(x^{1/3}e^x)$

inside: $x^{1/3}e^x$

outside: $\cos(x)$

$$\frac{d}{dx}\cos(x^{1/3}e^x) = -\sin(x^{1/3}e^x)\cdot\frac{d}{dx}(x^{1/3}e^x)$$

$$= -\sin(x^{1/3}e^x)\cdot\left(\left(\frac{d}{dx}x^{1/3}\right)e^x + x^{1/3}\frac{d}{dx}e^x\right)$$

$$= -\sin(x^{1/3}e^x)\cdot\left(\frac{1}{3}x^{-2/3}e^x + x^{1/3}e^x\right)$$

$$= -x^{-2/3}e^x\sin(x^{1/3}e^x)\left(\frac{1}{3} + x\right)$$

7. $f(x) = \sqrt{x + e^{x^2}}$ $\sin(\sqrt{1+x^2})$

inside: $x + e^{x^2}$

outside: $\sqrt{x}$ $\sqrt{☺}$

outside': $\frac{1}{2}☺^{-1/2}$

$\frac{1}{2\sqrt{☺}}$

$$f'(x) = \frac{1}{2}(x + e^{x^2})^{-\frac{1}{2}}\cdot\frac{d}{dx}\left(x + e^{x^2}\right)$$

$$= \frac{1}{2}(x + e^{x^2})^{-\frac{1}{2}}\cdot\left(1 + 2xe^{x^2}\right)$$

$$\frac{d}{dx}e^{x^2} = e^{x^2}\cdot 2x$$

$\frac{d}{dx}x^2$

$$\frac{d}{dx} \sin\left(\sqrt{1+x^2}\right) = \cos\left(\sqrt{1+x^2}\right) \cdot \frac{d}{dx}\sqrt{1+x^2}$$

$$= \cos\left(\sqrt{1+x^2}\right) \frac{1}{2}\left(1+x^2\right)^{-1/2} \cdot \frac{d}{dx}\left(1+x^2\right)$$

$$= \cos\left(\sqrt{1+x^2}\right) \frac{1}{2}\left(1+x^2\right)^{-1/2} 2x$$

$$= x\left(1+x^2\right)^{-1/2} \cos\left(\sqrt{1+x^2}\right)$$