Math F302 Midterm 2 Spring 2022

Name: _____

Student Id: _____

Rules:

You have 60 minutes to complete the exam.

Partial credit will be awarded, but you must show your work.

One notecard of notes are allowed.

No calculators, books or other aids are permitted.

Turn off anything that might go beep during the exam.

If you need extra space, you can use the back sides of the pages. Please make it obvious when you have done so.

Good luck!

Problem	Possible	Score
1	10	
2	10	
3	10	
4	10	
5	10	
EC	5	
Total	50	

1. (10 points)

Consider the points $A = i - d$ and $B = i + d$ in the upper half plane where d is a positive real number.

a. (5 points) Use the cross ratio to compute the hyperbolic distance between A and B.

b. (5 points) Consider the Euclidean line segment between A and B. Compute its hyperbolic length.

2. (10 points)

The hyperbolic plane can be tiled with "squares" (four sided figures where each side has equal length and each vertex has the same interior angle) such that five squares meet at each vertex. What is the area of these squares? You may assume that the area of an asymptotic triangle is π.

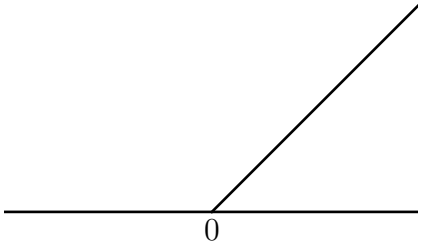
3. (10 points)

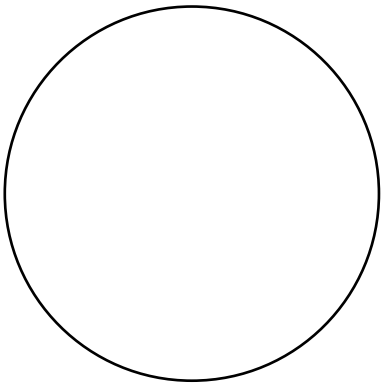
Recall that our standard transformation from the ball model to the half plane model of hyperbolic geometry is

$$T(z) = i\frac{1-z}{1+z}$$

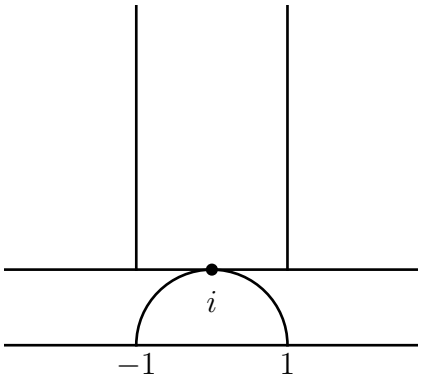
In the diagrams below I have drawn a figure in one of the two models. Draw the corresponding figure in the other model.

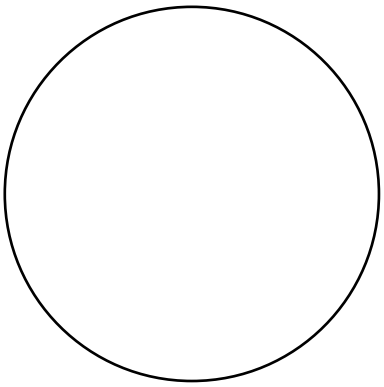
a. (2 points)



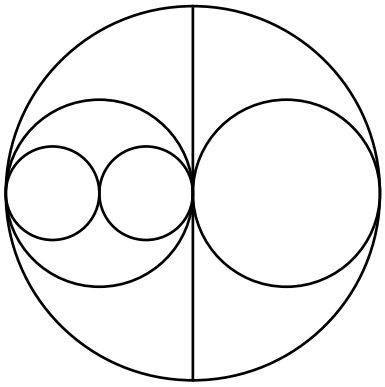


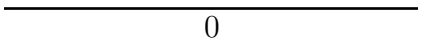
b. (4 points)





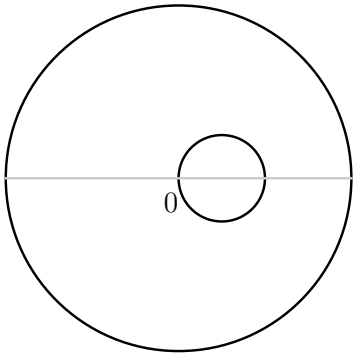
c. (4 points)





4. (15 points)

In the ball model of hyperbolic geometry, consider the Euclidean circle with center $z = 1/4$ and Euclidean radius $1/4$.



a. (7 points) What is the hyperbolic radius of this circle? Hint: you might find it easier to compute the diameter of the circle. Maybe think about ideal points...

b. (3 points) What are the coordinates of the hyperbolic center of this circle?

5. (10 points)

Consider the Euclidean triangular region in the upper half plane bounded by the points i, $i+1$, and $2i+1$. What is the area of this region?

Extra Credit: (5 points)

Find, with full justification, a formula for all hyperbolic transformations in the half plane model that have 1 and -1 as fixed points.

Length and Area Formulas

Ball model:

$$L = \int_a^b \frac{2|z'|}{1-r^2}\, dt$$

$$A = \int_\Omega \frac{4r}{(1-r^2)^2}\, dr\, d\theta$$

Upper half plane model:

$$L = \int_a^b \frac{|z'|}{y}\, dt$$

$$A = \int_\Omega \frac{1}{y^2}\, dx\, dy$$

$$L = \int_a^b \frac{2|z'|}{1-r^2}\, dt$$