A diagonal matrix:

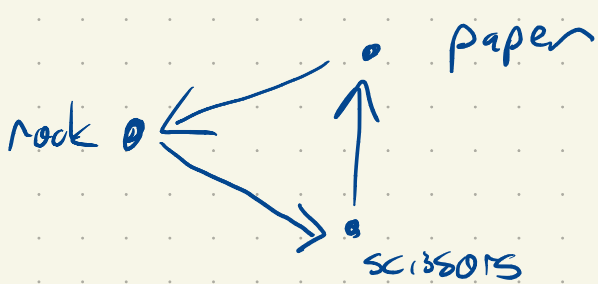
$$\text{diag}(3,7,-1,6) = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

All entries are zero, except on the diagonals

$$I = \text{diag}(1,1,1,1).$$

Linear Algebra I

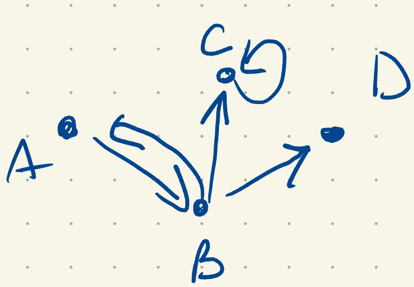
Directed Graphs



$$\begin{array}{cc} & \begin{array}{ccc} \text{rock} & \text{s} & \text{p} \end{array} \\ \begin{array}{c} \text{rock} \\ \text{scissors} \\ \text{paper} \end{array} & \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

rock's arrows

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$



$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ \\ 1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Operations With matrices:

1) Transpose

Finally the T in $x^T y$

$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad x^T = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

it swaps columns + rows

$$(A^T)_{ij} = A_{ji}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 4 & 1 & 0 \\ 5 & 6 & 3 \end{bmatrix}^T = \begin{bmatrix} 2 & 4 & 5 \\ 0 & 1 & 6 \\ 0 & 0 & 3 \end{bmatrix}$$

lower triangular

upper triangular.

$$\begin{bmatrix} a_1 & \cdots & a_k \end{bmatrix}^T = \begin{bmatrix} a_1^T \\ \vdots \\ a_k^T \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ \vdots \\ b_k \end{bmatrix}^T = \begin{bmatrix} b_1^T & \cdots & b_k^T \end{bmatrix}$$

$$A = \begin{bmatrix} a' \\ A' \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \qquad \begin{matrix} \boxed{a} & \boxed{b} \\ \boxed{A'} & \boxed{B'} \\ \boxed{C} & \boxed{D} \end{matrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^T = \begin{bmatrix} a^T & (A')^T & C^T \\ b^T & (B')^T & D^T \end{bmatrix}$$

$$= \begin{bmatrix} A^T & C^T \\ B^T & D^T \end{bmatrix}$$

cool.

What does it mean?

graph = arrows go the other way.

contingency: swap categories

image: reflect

$I^T = I$

$D^T = D$

A matrix is symmetric if $A^T = A$ (symmetry about the diagonal)

skew-symmetric if $A^T = -A$

Adding matrices:

must be same dimensions

A $m \times n$, B $m \times n$

$$(A+B)_{ij} = A_{ij} + B_{ij}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 7 \\ 2 & 0 & 6 \end{bmatrix} = \begin{bmatrix} 10 & 2 & 10 \\ 6 & 5 & 12 \end{bmatrix}$$

Does this mean anything?

images get combined,
contingency props get merged
column vectors get added.

Scalar multiplication

$$(cA)_{ij} = c A_{ij}$$

$$2 \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \end{bmatrix}$$

$$\left.\begin{aligned} c(A+B) &= cA + cB \\ (c+d)A &= cA + dA \end{aligned}\right]$$

matrices obey the rules for vectors!

$$c(dA) = cdA$$

Just like vectors, matrices have norms.

$$\left\| \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \right\| = \sqrt{1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2}$$

(essentially treat the matrix like a vector!

$$\|A + B\| \leq \|A\| + \|B\| \quad !$$

Now the big moment: matrix, vector multiplication

$$2a - 3b + 5c = 3$$
$$-a + b + 2c = 5$$

$$a\begin{bmatrix} 2 \\ -1 \end{bmatrix} + b\begin{bmatrix} -3 \\ 1 \end{bmatrix} + c\begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 5 \\ -1 & 1 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$A x = b$$