See **Rules** on following page.

1. A subset A of a topological space X is said to be nowhere dense if $\operatorname{Int} \overline{A} = \emptyset$.

a) Let U be an open subset of a topological space. Prove that ∂U is closed and nowhere dense.

b) Let V be a closed and nowhere dense set. Show that V is the boundary of an open set.

2. Let f and g be continuous maps from a topological space X to a Hausdorff space Y. Suppose $f = g$ on a dense subset of X. Prove that $f = g$.

3. Suppose A and B are disjoint compact subsets of a Hausdorff space. Show that there are disjoint open sets U_A and U_B with $A \subset U_A$ and $B \subset U_B$.

4. Suppose X and Y are spaces and Y is compact. Show that the projection $X \times Y \to Y$ is a closed map.

5. Let G be an algebraic group. We say that G is a **topological group** if in addition G is a topological space such that that the multiplication map $m : G \times G \to G$ and the inversion map $i : G \to G$ defined by $m(g, h) = g \cdot h$ and $i(g) = g^{-1}$ are continuous.

a) Suppose G is an algebraic group and a T_1 topological space. Show that G is a topological group if and only if the map $f : G \times G \to G$ defined by $f(g, h) = gh^{-1}$ is continuous.

b) Let G be a topological group and let H be a subgroup. Show that $\overline{H}$ is a subgroup. Hint: that map f from the previous part is continuous.

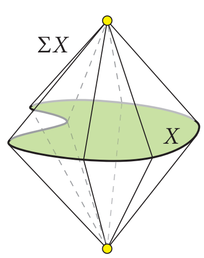
6. Let $\{x_n\}_n$ be a sequence in an arbitrary product $\prod X_\alpha$. Show that $x_n \to x$ if and only if $\pi_\alpha(x_n) \to \pi_\alpha(x)$ for every α. Then show that this result is false if we assume instead that $\prod X_\alpha$ is given the box topology.

7. Lee Problem 4-4

8. Lee Problem 4-5

9. Lee Problem 4-11

10. Let X be a topological space. The **suspension** of X, denoted by ΣX, is the quotient of $X \times [-1, 1]$ where all points of the form $(x, 1)$ are identified, and all points of the form $(x, -1)$ are identified. Determine, with proof, a familiar space that is homeomorphic to ΣS^n.



Rules and format:

- You are welcome to discuss this exam with me (David Maxwell) to ask for hints and so forth.
- If you find a suspected error or mispring, please contact me as soon as possible and I will communicate it to the class if needed.
- You my not discuss the exam with anyone else until after the due date/time.
- You are permitted to reference Lee or any other topology text you like. If you use another text, you must cite it when you use it.
- You may not consult any internet resources, including search engines.
- Each problem is weighted equally.
- The due date/time is absolutely firm.
- The problem session on March 7 will be a hints session.