

derivative of a vector field?

$$\vec{V} = \langle P, Q, R \rangle$$

$$\operatorname{div} \vec{V}$$

The divergence of \vec{V} , written $\vec{\nabla} \cdot \vec{V} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$

$$\left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \langle P, Q, R \rangle =$$

$$\vec{V} = \langle P, Q, R \\ x\gamma, x\sin(\gamma), z \rangle$$

$$\vec{\nabla} \cdot \vec{V} = \gamma + x\cos(\gamma) + 1$$

$$P = x\gamma \quad \frac{\partial P}{\partial x} = \gamma$$

$$Q = x\sin\gamma \quad \frac{\partial Q}{\partial y} = x\cos\gamma$$

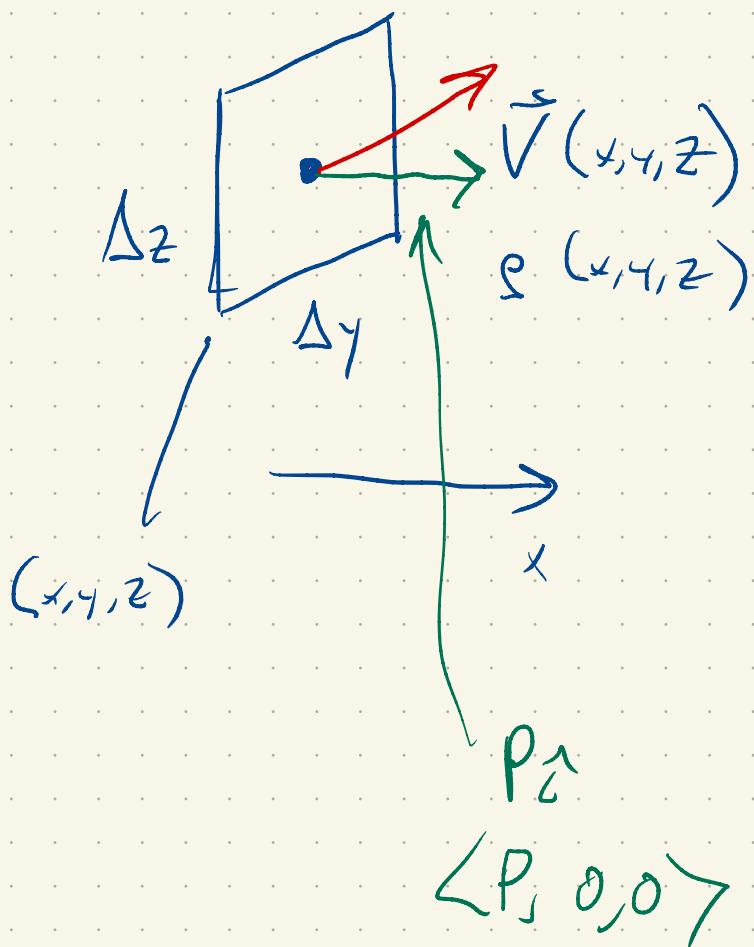
$$R = z \quad \frac{\partial R}{\partial z} = 1$$

What is the rate

at which smoke

is passing through the window?

(mg/s)



ρ smoke density mg/cm³

\vec{V} smoke velocity cm/s

$$\vec{V} = \langle P, Q, R \rangle$$

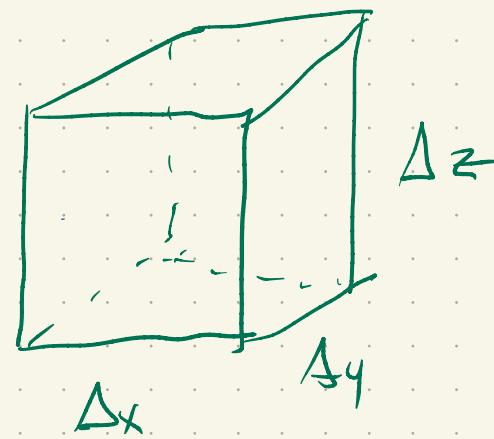
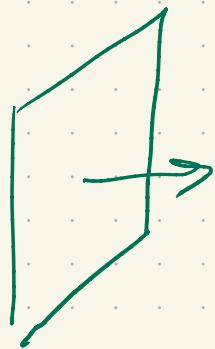
$$[\rho \ P] = [\rho] [P]$$

$$\frac{\text{mg}}{\text{cm}^3} \cdot \frac{\text{cm}}{\text{s}} = \frac{\text{mg}}{\text{cm}^2 \text{s}} = \frac{\text{mg}}{\text{s}} \cdot \frac{1}{\text{cm}^2}$$

mass / time ← mass flux

$$\text{mass flux} = \rho P \Delta y \Delta z$$

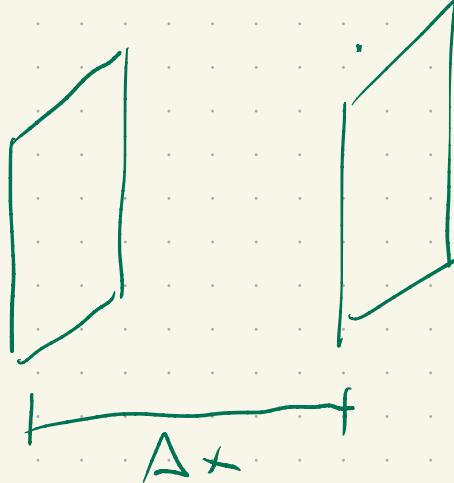
At what rate is
smoke entering the
box?



$$\rho P \Delta y \Delta z$$

at what rate is smoke entering the

box?



$$\int P(x, y, z) \Delta y \Delta z$$

$$\int P(x + \Delta x, y, z) \Delta y \Delta z$$

$$\int P(x, y, z) \Delta y \Delta z - \int P(x + \Delta x, y, z) \Delta y \Delta z$$

$$\int [P(x, y, z) - P(x + \Delta x, y, z)] \Delta y \Delta z$$

$$- \int [P(x + \Delta x, y, z) - P(x, y, z)] \Delta y \Delta z$$

$$- \int \left[\frac{\partial P}{\partial x} \Delta x \right] \Delta y \Delta z$$

$$- \int \frac{\partial P}{\partial x} \Delta x \Delta y \Delta z \quad \begin{matrix} \leftarrow \text{contribution sum} \\ \text{two parallel windows} \end{matrix}$$

Diagram illustrating two adjacent rectangular volumes in a 3D space. The top volume has dimensions Δx , Δy , and Δz . The bottom volume is similar. A vertical vector labeled $\vec{S} \frac{\partial R}{\partial z} \Delta x \Delta y \Delta z$ points downwards from the top volume, representing the mass flux.

$$Q$$

$$\downarrow$$

$$-\int \frac{\partial Q}{\partial y} \Delta x \Delta y \Delta z$$

$$-\int \vec{S} \frac{\partial R}{\partial z} \Delta x \Delta y \Delta z$$

Total mass flux into the box is

$$-\int \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \right) \Delta x \Delta y \Delta z$$

$\vec{F} \cdot \vec{V}$

\uparrow density of source

\uparrow little volume

$$f(5 + 0.01) \approx f(5) + f'(5) \cdot 0.01$$

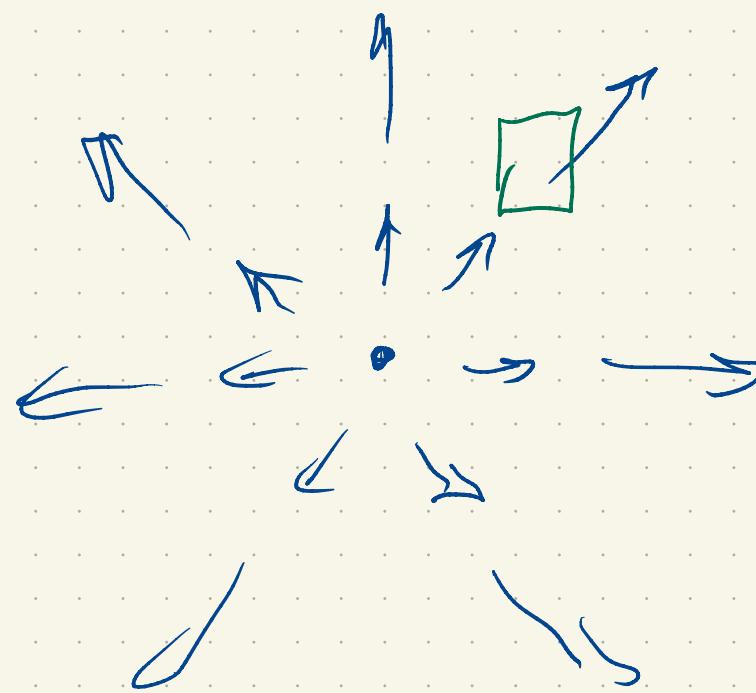
$$f(5 + 0.01) - f(5) \approx f'(5) \cdot 0.01$$

$$\vec{J} \cdot \vec{V} = P_x + Q_y + R_z$$

$\vec{J} \cdot \vec{V} > 0$ smoke is locally dispersing

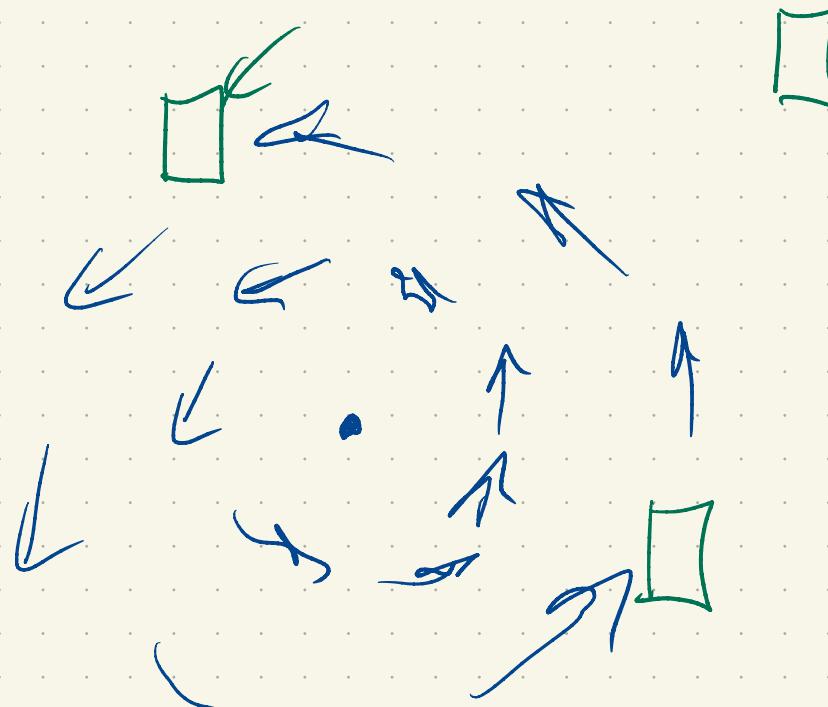
$\vec{J} \cdot \vec{V} < 0$ smoke is locally concentratory

$$\vec{V} = \langle x, -y \rangle$$



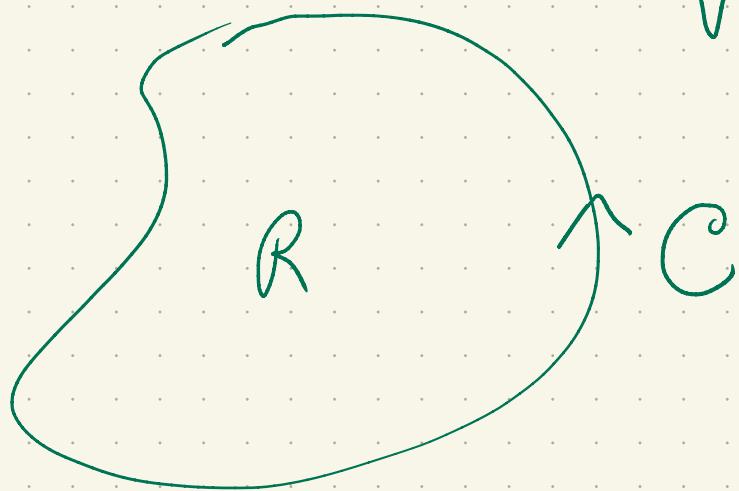
$$\vec{V} \cdot \vec{V} = \frac{\partial x}{\partial x} + \frac{\partial (-y)}{\partial y} = 1 + 1 = 2 > 0$$

$$\vec{V} = \langle -y, x \rangle$$



$$\vec{\nabla} \cdot \vec{V} = \frac{\partial (-y)}{\partial x} + \frac{\partial x}{\partial y} = 0 + 0 = 0$$

(no
concentration)
dispersing)

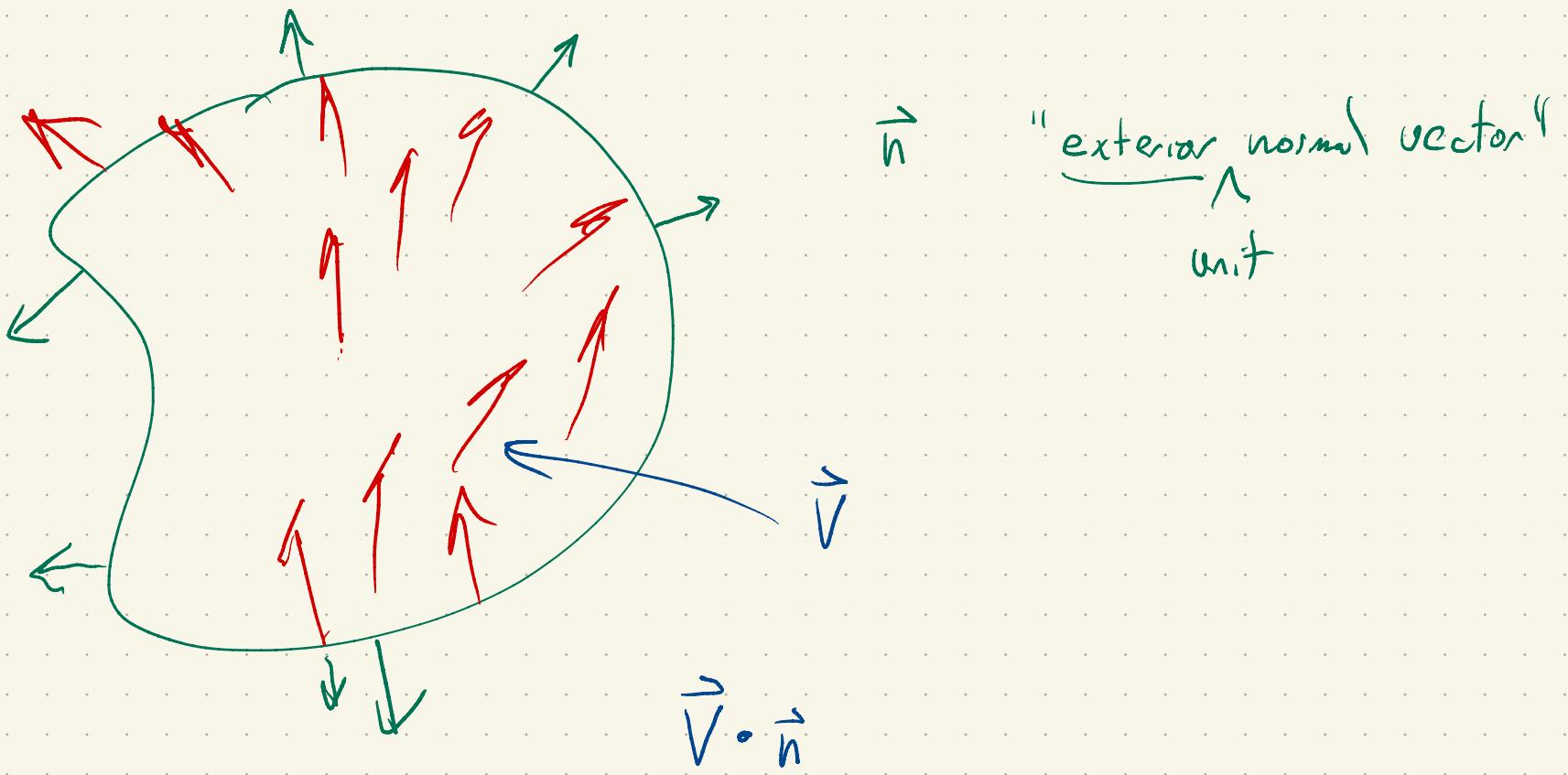


$$\vec{V} = \langle P, Q \rangle$$

$$P_x \quad Q_y$$

$$\oint_C P dx + Q dy = \iint_R -P_y + Q_x dxdy$$





$\int_C \vec{V} \cdot \vec{n} ds \rightarrow$ total rate of dispersion
 in the region.

$$\int_C \vec{V} \cdot \vec{n} ds = \iint_R \vec{\nabla} \cdot \vec{V} dx dy$$

