

$$\vec{r}(t) = \langle \cos(2t), \sin(2t), t \rangle \quad 0 \leq t \leq 2\pi$$

$\circlearrowleft t=2\pi \quad (z=2\pi)$   
 $\circlearrowleft t=0 \quad (z=0)$

$$\vec{q}(s) = \langle \cos(s), \sin(s), s/2 \rangle \quad 0 \leq s \leq 4\pi$$

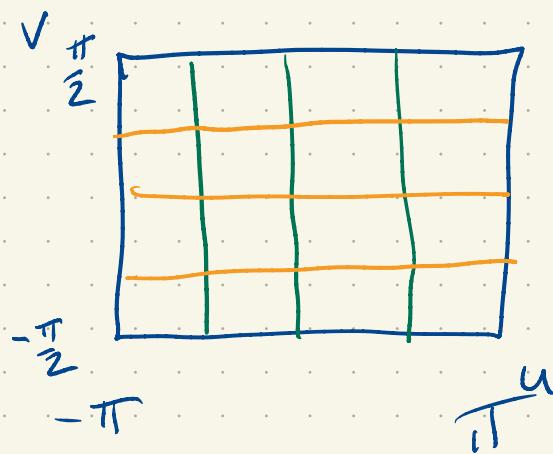
$$\vec{r}(u, v) = \langle x(u, v), y(u, v), z(u, v) \rangle$$

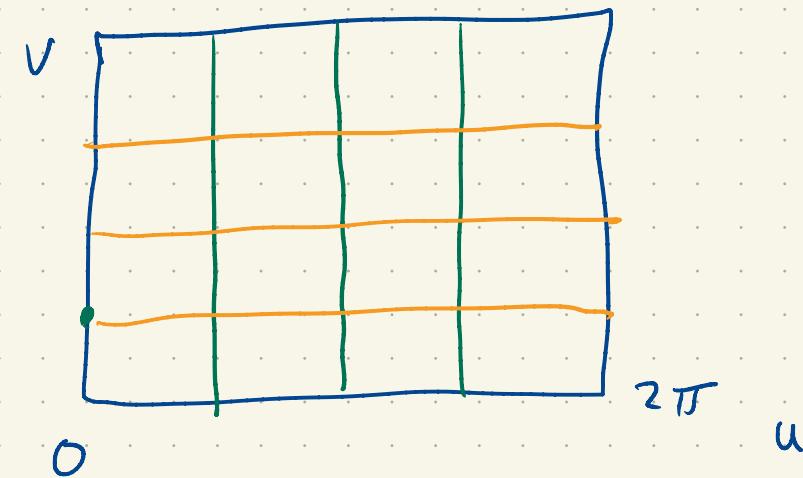
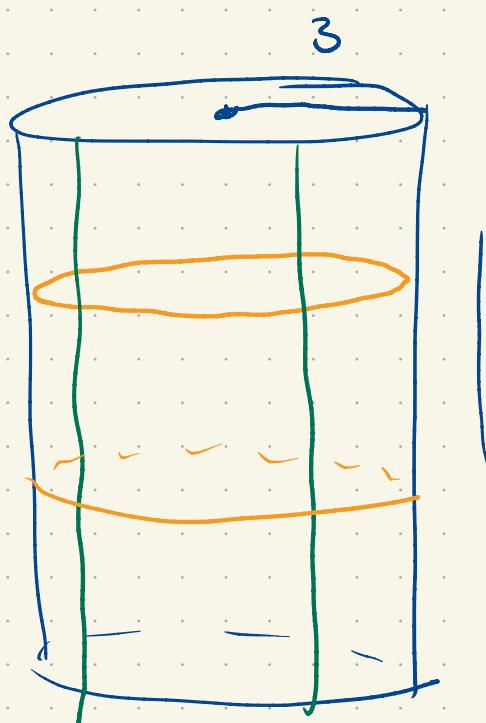
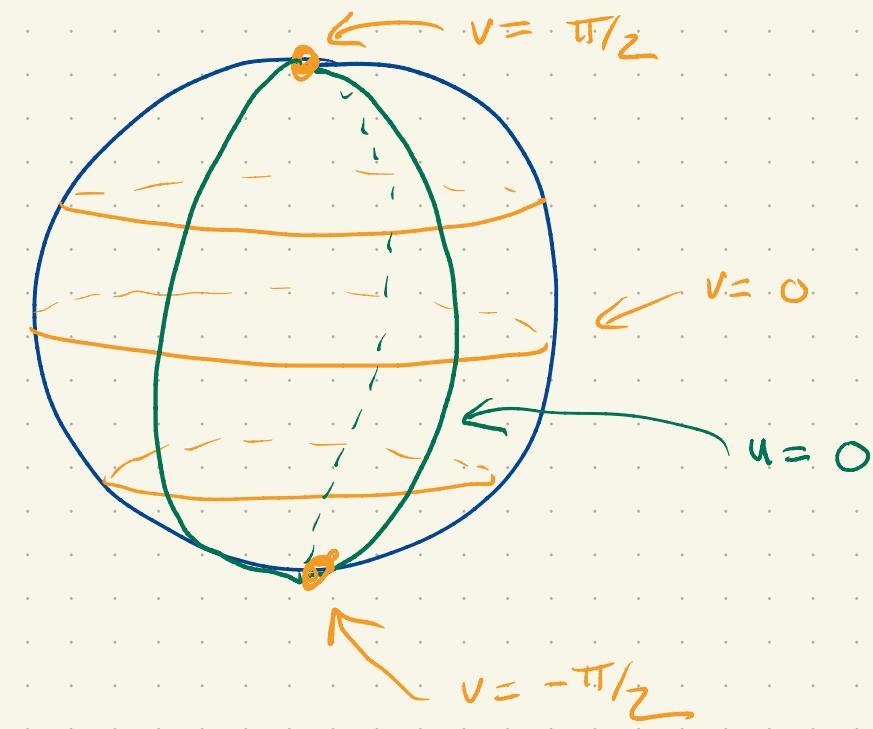
"parameterized surface"

$$\vec{r}(u, v) = \langle \underbrace{\cos u \cos v, \sin u \cos v, \sin v}_{x(u, v)}, \underbrace{\langle \cos v, 0, \sin v \rangle}_{y(u, v)} \rangle$$

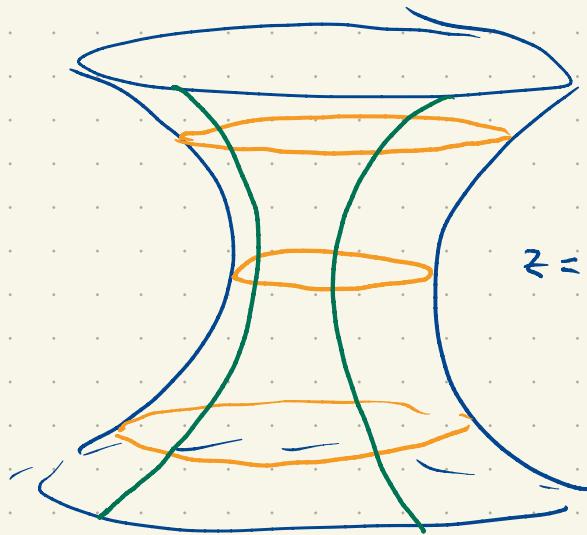
$$\begin{aligned} x^2 + y^2 &= \cos^2 u \cos^2 v + \sin^2 u \cos^2 v \\ &= \cos^2 v \end{aligned}$$

$$x^2 + y^2 + z^2 = \cos^2 v + \sin^2 v = 1$$





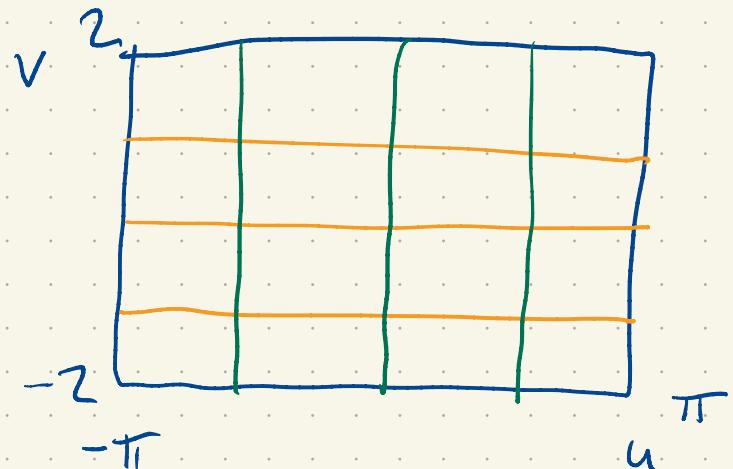
$$\vec{r}(u, v) = \langle 3\cos(u), 3\sin(u), v \rangle$$



$$z = 2$$

$$x^2 + y^2 = z^2 + 1$$

$$r = \sqrt{z^2 + 1}$$



$$\vec{r}(u, v) = \langle r \cos u, r \sin u, v \rangle$$

$$= \left\langle \sqrt{v^2+1} \cos u, \sqrt{v^2+1} \sin u, v \right\rangle$$

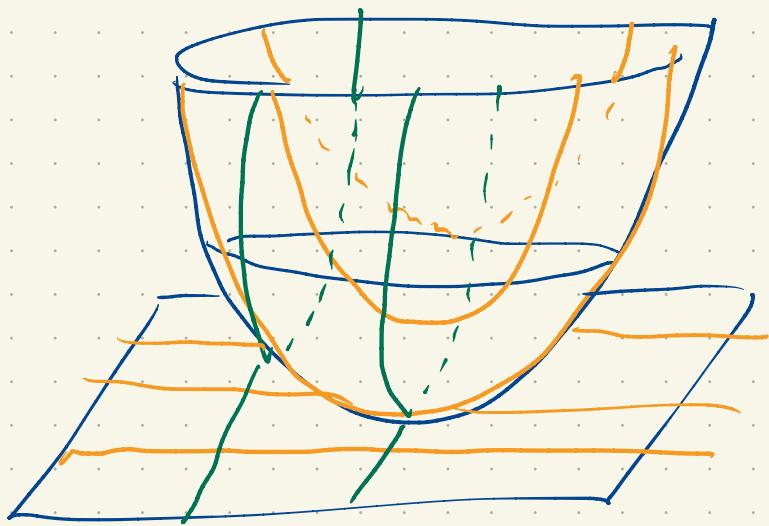
$$\iint_{-2}^2 \iint_{-\pi}^{\pi} \text{d}u \text{d}v$$

$$x^2 + y^2 = (v^2 + 1) \cos^2 u + (v^2 + 1) \sin^2 u = v^2 + 1$$

$$z^2 + 1 = v^2 + 1 \quad \begin{matrix} \nearrow \\ = \checkmark \end{matrix}$$

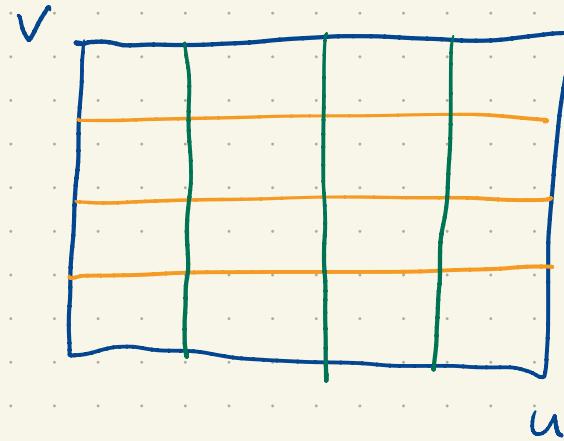
$$z = f(x, y)$$

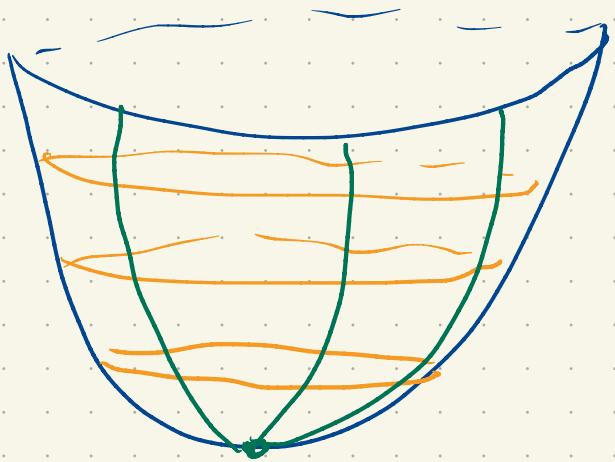
$$z = x^2 + y^2$$



$$\vec{r}(u, v) = \langle u, v, f(u, v) \rangle$$

$$\vec{r}(u, v) = \langle u, v, u^2 + v^2 \rangle$$

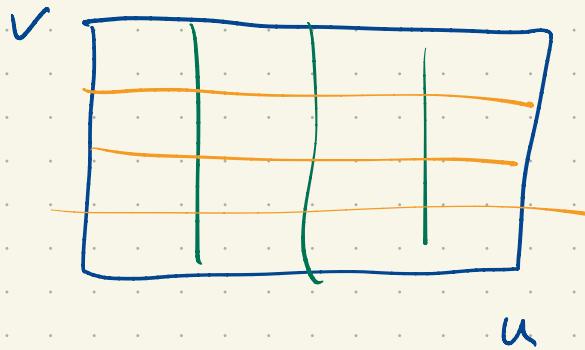




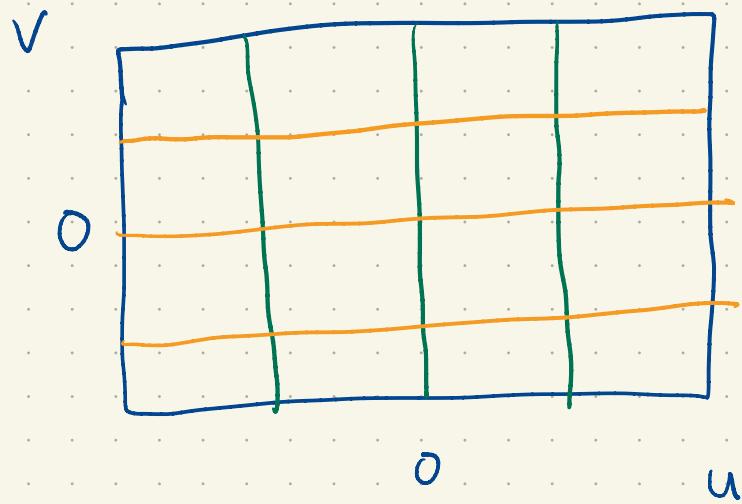
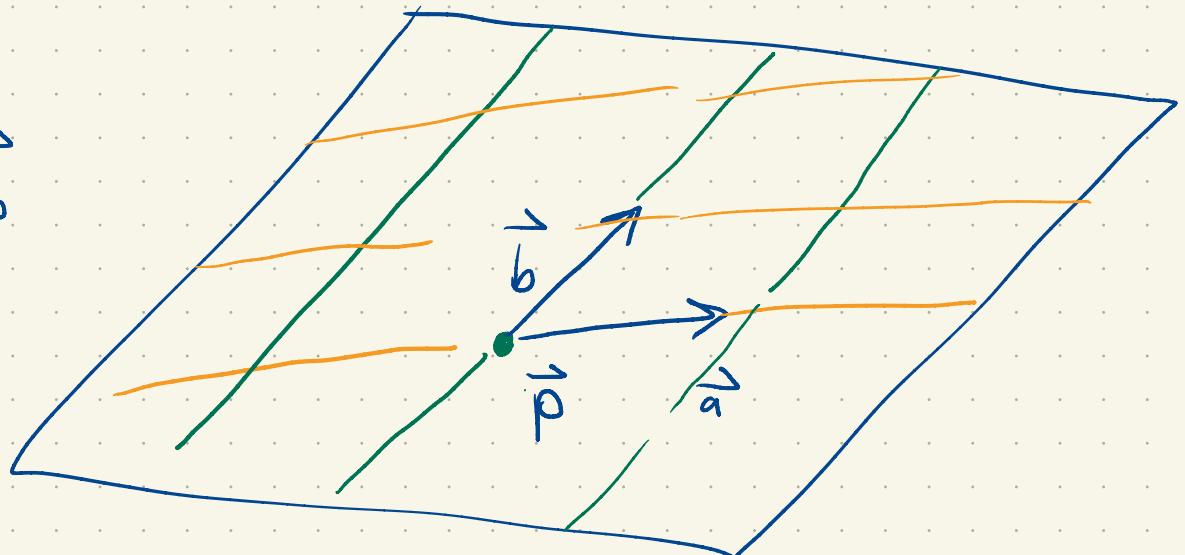
$$z = x^2 + y^2$$

$$\vec{r}(u,v) = (\sqrt{v} \cos u, \sqrt{v} \sin u, v)$$

$$x^2 + y^2 = v \cos^2 u + v \sin^2 u = v = z \checkmark$$

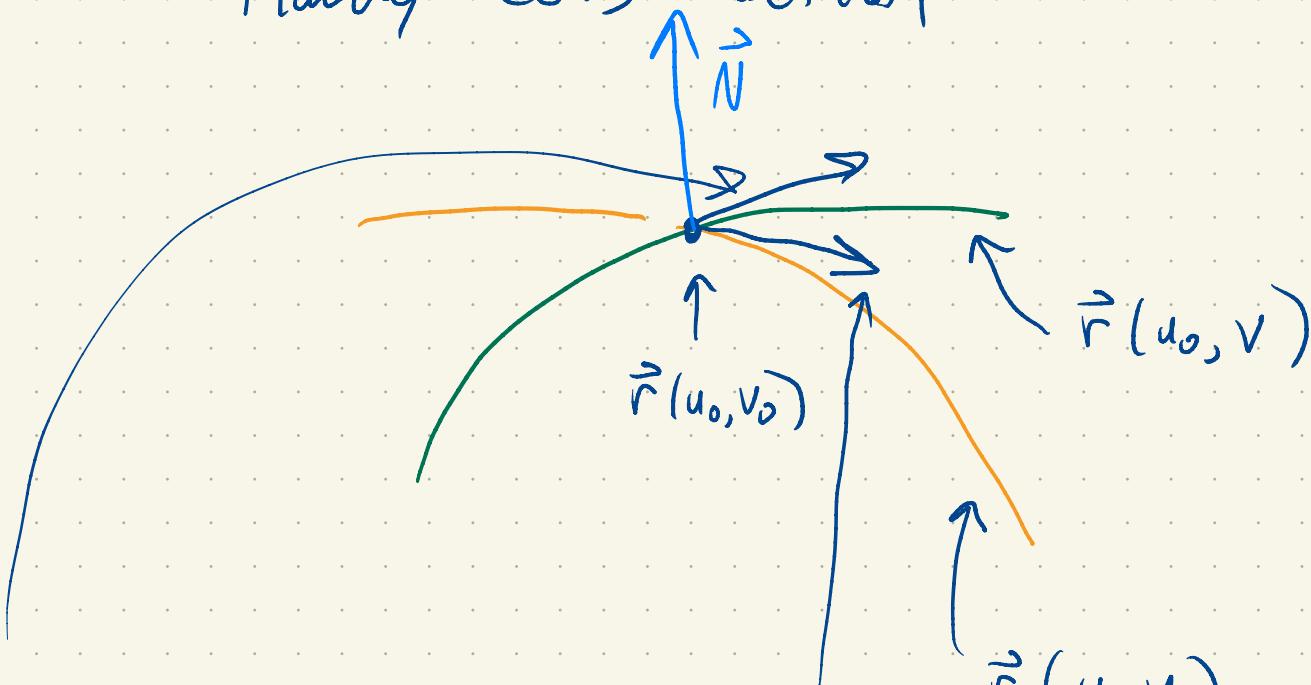


$$\vec{r}(u,v) = \vec{p} + u\vec{a} + v\vec{b}$$



$$\vec{r}(t) = \vec{p} + t\vec{v}$$

# Hardy Construction



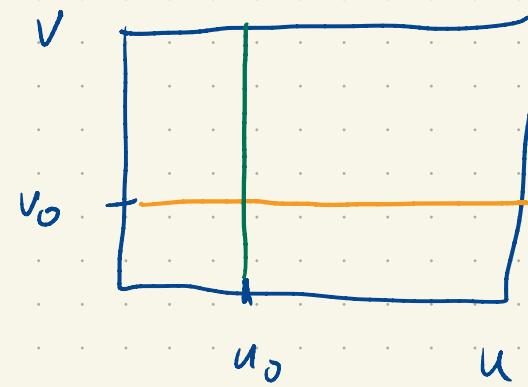
$$\frac{\partial \vec{r}}{\partial v}(u_0, v_0)$$

$$\frac{\partial \vec{r}}{\partial u}(u_0, v_0)$$

$$\frac{\partial \vec{x}}{\partial u}(u_0, v_0)$$

$$\left\langle \frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial z}{\partial u} \right\rangle$$

$$\vec{N} = \frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v}$$



$$\vec{r}(u,v) = \langle \cos u, \sin u, v \rangle$$

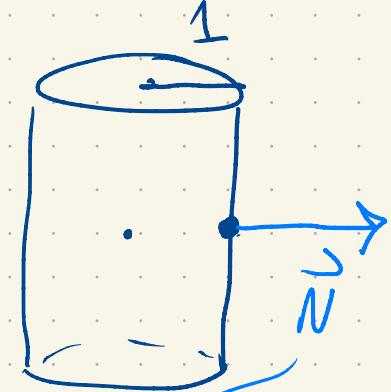
$$\frac{\partial \vec{r}}{\partial u} = \langle -\sin u, \cos u, 0 \rangle$$

$$\frac{\partial \vec{r}}{\partial v} = \langle 0, 0, 1 \rangle$$

$$\vec{N} = \frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v} = \begin{vmatrix} \uparrow & \uparrow & \uparrow \\ -\sin u & \cos u & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \langle \cos u, \sin u, 0 \rangle$$

$$\vec{N}(0,0) = \langle 1, 0, 0 \rangle$$



$$u=0, v=0$$

$$\langle 1, 0, 0 \rangle$$

parallel to

$$\langle 1, 0, 0 \rangle$$