**First Derivative Test**
Suppose $f$ is a function with a derivative on $(a, b)$, and if $c$ is a point in the interval with $f'(c) = 0$.

- If $f'(x) > 0$ for $x$ just to the left of $c$ and $f'(x) < 0$ for $x$ just to the right of $c$, then $f$ has a local maximum at $c$.
- If $f'(x) < 0$ for $x$ just to the left of $c$ and $f'(x) > 0$ for $x$ just to the right of $c$, then $f$ has a local minimum at $c$.
- If $f'(c) = 0$ and $f'(x) < 0$ on both sides of $c$ or $f'(x) > 0$ on both sides of $c$, then there is neither a local min nor a local max at $c$.

**Second Derivative Test**
Suppose $f$ is a function with a continuous second derivative on $(a, b)$, and that $c$ is a point in the interval with $f'(c) = 0$.

- If $f''(c) > 0$ then $f$ has a local minimum at $c$.
- If $f''(c) < 0$ then $f$ has a local maximum at $c$.

**Concave Up:** $f'(x)$ increasing; $f''(x) > 0$

**Concave Down:** $f'(x)$ decreasing; $f''(x) < 0$

**Point of Inflection:** Value $x$ where concavity changes; often $f''(x) = 0$

This worksheet considers the function

$$g(x) = x^2 e^x$$

number

**1.** Find all critical points of $g$.

Look for $g'(x) = 0$ or DNE

$g'(x) = 2x e^x + x^2 e^x$

$= (2x + x^2) e^x$

$= x(2 + x) e^x$

$x = 0, \quad x = -2$

$-1\ (2+(-1))\,e^{-1}$



**2.** Determine the intervals where $g$ is increasing and where $g$ is decreasing.

$g'(x) > 0$ $\quad$ $g'(x) < 0$

$g'(x)$: $+$ $\;|\;$ $-$ $\;|\;$ $+$

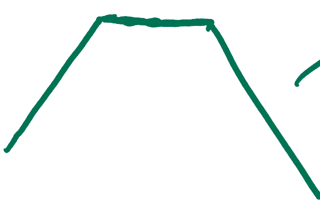
$-2$ $\quad$ $0$

incr. $\quad$ dec. $\quad$ inc.

**3.** Use the First Derivative Test to classify each critical point as a local min/local max.

$+$ $\;|\;$ $-$

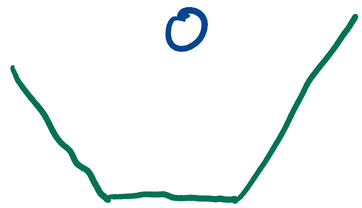
$-2$



local max

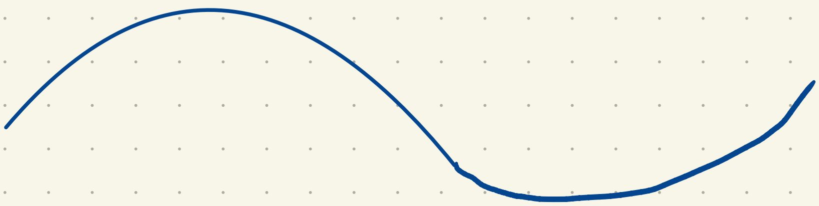
$-$ $\;|\;$ $+$

$0$



local min


+
−
+
−2
0



**4.** Determine the intervals where $g$ is concave $\underbrace{\text{up}}$ and where $\underbrace{\cancel{f}\ g \text{ is concave down.}}$

$\cup$

$g''(x) < 0$

$g''(x) > 0$

$g'(x) = (2x + x^2)e^x$

$g''(x) = (2+2x)e^x + (2x+x^2)e^x = \underbrace{(2+4x+x^2)e^x}$

**5.** Find all points of inflection of $g$.

analyze this!

**6.** Use the Second Derivative Test to classify each critical point as a local min/local max (if possible).

**7.** Determine the value of $g$ at each of its critical points.

**8.** Use the information determined thus far to sketch the graph of $g(x)$. You may used the fact, which we will justify next class, that $\lim_{x \to -\infty} f(x) = 0$.

**j**. $f(x) = \dfrac{2x+5}{2\ln x + \ln 5}$

**k**. $g(x) = \arctan(e^x)$

**l**. Compute $\dfrac{dy}{dx}$ if $e^{x+y} = xy + 3\cos y$. You must solve for $\dfrac{dy}{dx}$.