

1. Prove that every ball  $B_r(x)$  in a metric space  $(X, d)$  is an open set.
2. Let  $V$  be a subset of a metric space  $(X, d)$ . The set of limit points of  $V$  are those points  $x$  that can be written as the limit of a sequence of points in  $V$ . Show that a set  $V \subseteq X$  is closed if and only if it contains its limit points.
3. Let  $d_1$  and  $d_2$  be two metrics on a set  $X$ . Show that the following conditions are equivalent.
  - a) For every sequence  $\{p_i\}_{i=1}^{\infty}$ , if  $p_i \xrightarrow{d_2} p$  then  $p_i \xrightarrow{d_1} p$ .
  - b) For every function  $f : X \rightarrow \mathbb{R}$ , if  $f$  is continuous with respect to  $d_1$  then  $f$  is continuous with respect to  $d_2$ .
  - c) For every set  $V$ , if  $V$  is closed with respect to  $d_1$  then  $V$  is closed with respect to  $d_2$ .
  - d) For every set  $U$ , if  $U$  is open with respect to  $d_1$  then  $U$  is open with respect to  $d_2$ .

*Hint:* You might want to show  $a) \iff b)$  and  $a) \implies c) \implies d) \implies a)$ .
4. Lee, Problem 2-1
5. Lee, Exercise (Not Problem) 2.6