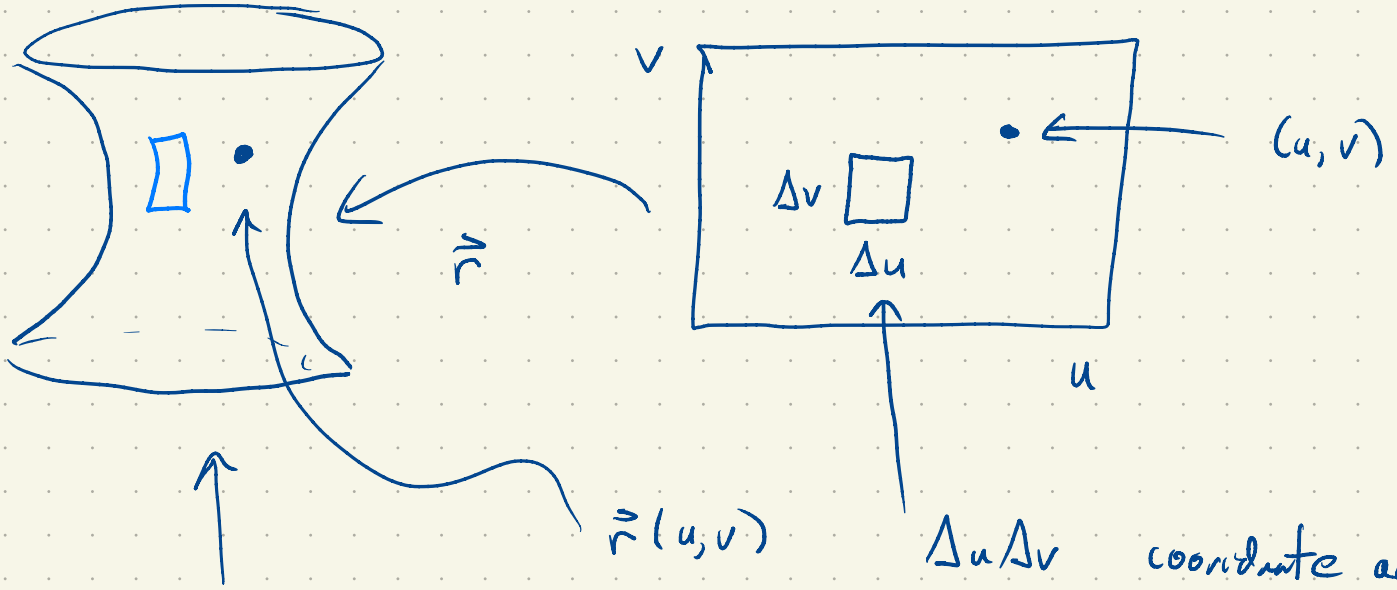
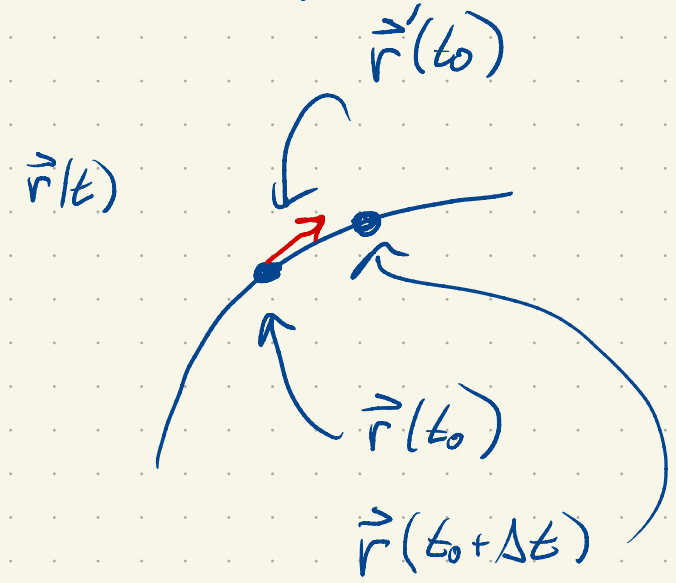
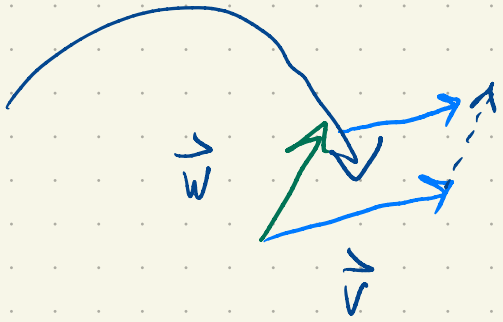


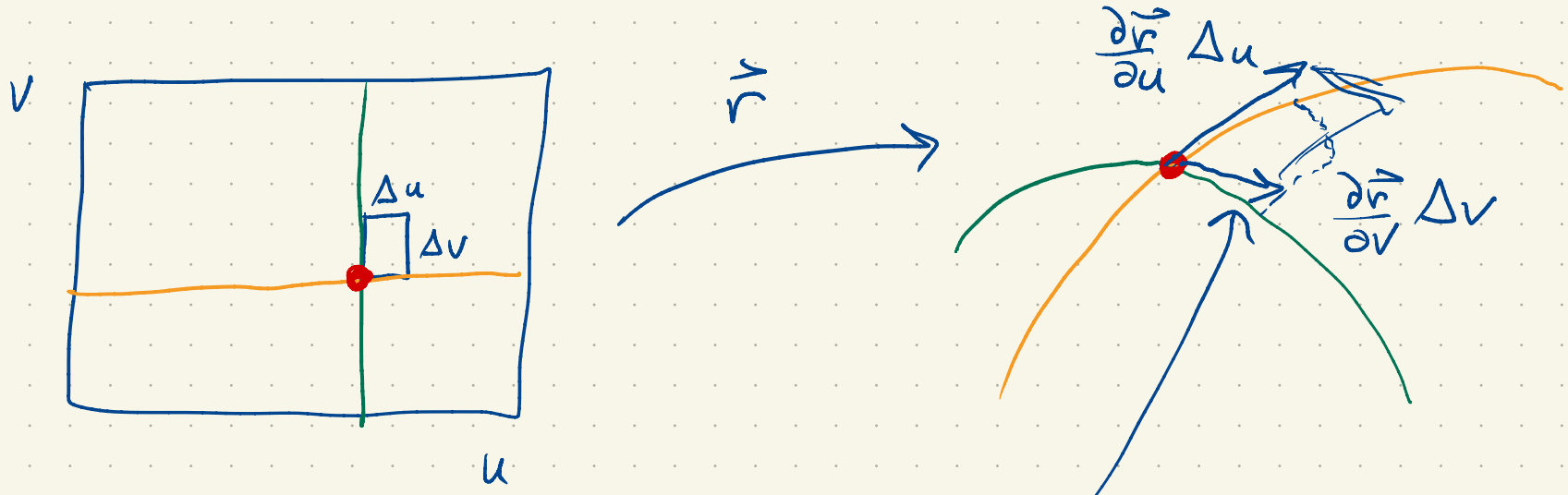


$$\vec{v} \cdot \vec{w} = \|\vec{v}\| \, \|\vec{w}\| \cos\theta$$

$\| \vec{v} \times \vec{w} \|$ = area



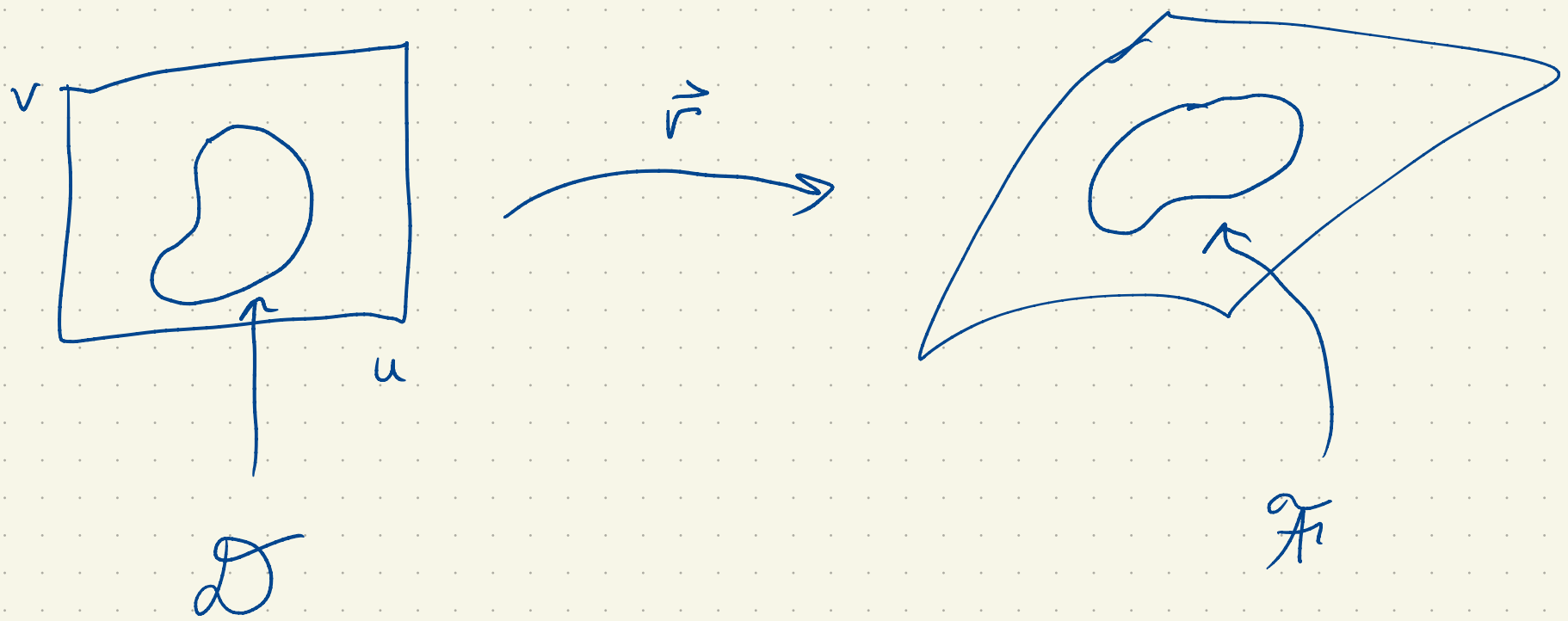
$$\vec{r}(t_0+\Delta t) \approx \vec{r}(t_0) + \vec{r}'(t_0)\Delta t$$



The area of

is approximately $\| \vec{r}_u \times \vec{r}_v \| \Delta u \Delta v$

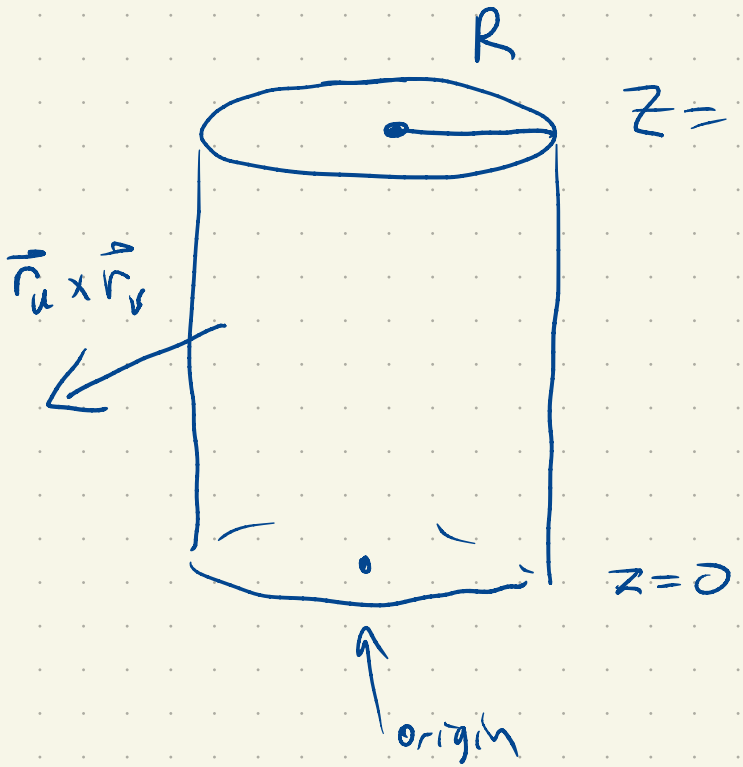
$\vec{r}_v \rightarrow \frac{\partial \vec{r}}{\partial v}$



$$\text{area of } \mathcal{T} = \iint_{\mathcal{T}} 1 \, dS$$

dS → "bit of surface area"

$$\iint_{\mathcal{T}} 1 \, dS = \iint_{\mathcal{D}} \|\vec{r}_u \times \vec{r}_v\| \, du\,dv$$



area $= 2\pi Rh$

$\vec{r}(u,v) = \langle R\cos(u), R\sin(u), v \rangle$

$-\pi \le u \le \pi \qquad 0 \le v \le h$

$\vec{r}_u = \langle -R\sin u, R\cos u, 0 \rangle$

$\vec{r}_v = \langle 0, 0, 1 \rangle$

$\vec{r}_u \times \vec{r}_v = \langle R\cos u, R\sin u, 0 \rangle$

$\vec{N}$

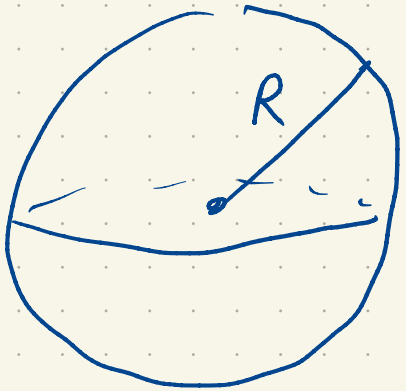
$\| \vec{r}_u \times \vec{r}_v \| = \left(R^2\cos^2 u + R^2 \sin^2 u \right)^{1/2}$

$= R$

surface area: $\int_0^h \int_{-\pi}^{\pi} R \, du \, dv = \int_0^h R \, 2\pi \, dv$

$= R 2\pi h$

$= 2\pi R h$ ☺



$\frac{4}{3}\pi r^3$

$4\pi r^2 \leftarrow$

area $4\pi R^2$

$\vec{r}(u,v) = \langle R\cos u \cos v, R \sin u \cos v, R \sin v \rangle$

$-\pi \leq u \leq \pi$

$-\frac{\pi}{2} \leq v \leq \pi/2$

$\|\vec{r}_u \times \vec{r}_v\|$

$$\vec{r}_u = \langle -R\sin u \cos v,\ R\cos u\cos v,\ 0 \rangle$$

$$\vec{r}_v = \langle -R\cos u \sin v,\ -R\sin u \sin v,\ R\cos v \rangle$$

$$\vec{r}_u \times \vec{r}_v = \langle R^2 \cos u \cos^2 v,\ R^2 \sin u \cos^2 v,\ R^2 \sin^2 u \cos v \sin v + R^2 \cos^2 u \cos v \sin v \rangle$$

$$= \langle R^2 \cos u \cos^2 v,\ R^2 \sin u \cos^2 v,\ R^2 \cos v \sin v \rangle$$

$$\|\vec{r}_u \times \vec{r}_v\|^2 = R^4 \cos^2 u \cos^4 v + R^4 \sin^2 u \cos^4 v + R^4 \cos^2 v \sin^2 v$$

$$= R^4 \cos^4 v + R^4 \cos^2 v \sin^2 v$$

$$= R^4 \cos^2 v (\cos^2 v + \sin^2 v)$$

$$= R^4 \cos^2 v$$

$$\|\vec{r}_u \times \vec{r}_v\| = R^2 \cos v \qquad \left(-\tfrac{\pi}{2} \le v \le \pi/2 \Rightarrow \cos v \ge 0\right)$$

surface area $$\iint_{\mathcal{A}} 1 \, dS = \int_{-\frac{\pi}{2}}^{\pi/2} \int_{-\pi}^{\pi} R^2 \cos v \, du \, dv$$

$$= 2\pi \int_{-\frac{\pi}{2}}^{\pi/2} R^2 \cos v \, dv$$

$$= 2\pi R^2 \int_{-\pi/2}^{\pi/2} \cos v \, dv$$

$$= 2\pi R^2 \cdot 2$$

$$= 4\pi R^2$$