Compute the derivatives of the following functions.

$$\frac{d}{dx}\sqrt{x} = \frac{1}{2}\frac{1}{\sqrt{x}}$$

1. $f(x) = \sqrt{1+x^2}$

$$\frac{d}{dx}\sqrt{1+x^2} = \frac{1}{2}\frac{1}{\sqrt{1+x^2}}\cdot\frac{d}{dx}(1+x^2)$$

$$= \frac{1}{2}\frac{1}{\sqrt{1+x^2}}\cdot 2x$$

$$= \frac{x}{\sqrt{1+x^2}}$$

2. $f(\theta) = \tan(4\theta + 9)$

$$\frac{d}{dx}\tan(\theta) = \sec^2(\theta)$$

$$f'(\theta) = \sec^2(4\theta+9)\cdot\frac{d}{d\theta}(4\theta+9)$$

$$= \sec^2(4\theta+9)\cdot 4$$

$$= 4\sec^2(4\theta+9)$$

3. $f(t) = e^{t^2}(1+\cos(t))$

$$\frac{d}{dt} e^{t^2} \cdot (1+\cos(t)) = \left(\frac{d}{dt} e^{t^2}\right) \cdot (1+\cos(t)) + e^{t^2} \cdot \frac{d}{dt}(1+\cos(t))$$

inside: t^2 inside': $2t$

outside: e^t outside': e^t

$$\frac{d}{dt} e^{t^2} = e^{t^2} \cdot 2t$$

$$= \frac{d}{dt} e^{t^2} \cdot (1+\cos(t)) - e^{t^2} \sin(t)$$

$$= 2t e^{t^2} \cdot (1+\cos(t)) - e^{t^2} \sin(t)$$

$$= e^{t^2}\left[2t(1+\cos(t)) - \sin(t)\right]$$

4. $f(v) = \sec\left(\frac{1}{1+v^2}\right)$

$$\frac{d}{dx} \sec(x) = \sec(x)\tan(x)$$

$$\frac{d}{dv} \sec\left(\frac{1}{1+v^2}\right) = \sec\left(\frac{1}{1+v^2}\right)\tan\left(\frac{1}{1+v^2}\right) \cdot \frac{d}{dv} \frac{1}{1+v^2}$$

$$= \sec\left(\frac{1}{1+v^2}\right)\tan\left(\frac{1}{1+v^2}\right) \frac{-2v}{(1+v^2)^2}$$

reciprocal rule!

5. The cost of building wooden pencils is given by a function $C(n)$ where C is the cost in dollars and n is the number of pencils, measured in thousands.

a) Explain what $C'(50) = 37.5$ means in language your parents could understand.

b) Suppose it costs \$20000 to build 50000 pencils and $C'(50) = 37.5$. Estimate the cost of building 51000 pencils.

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c) Under the same assumptions, estimate the cost of building 50100 pencils.

6. $f(x) = \cos(x^{1/3}e^x)$

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7. $f(x) = \sqrt{x + e^{x^2}}$

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