Limits: $\frac{0}{0} \longrightarrow$ instantaneous rate of change

$\frac{1}{0} \longrightarrow \pm\infty$

"Limits at ∞"

↳ long term behavior

$$f(x) = \frac{1}{x-1} + 2$$

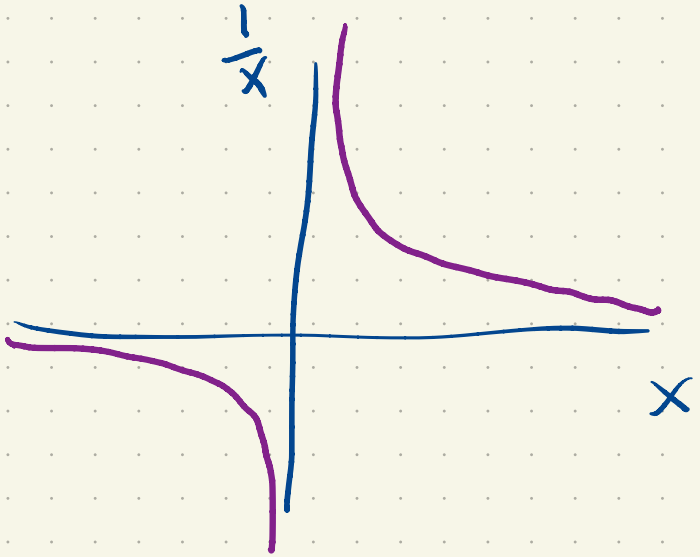
$f(\infty) = \ldots$

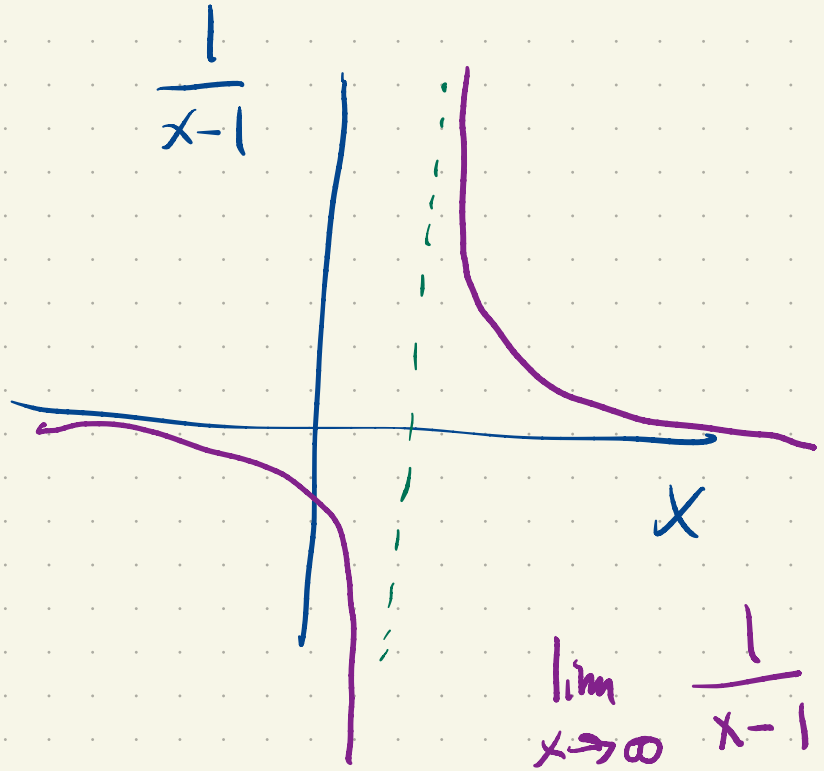
↑

you can't go there.

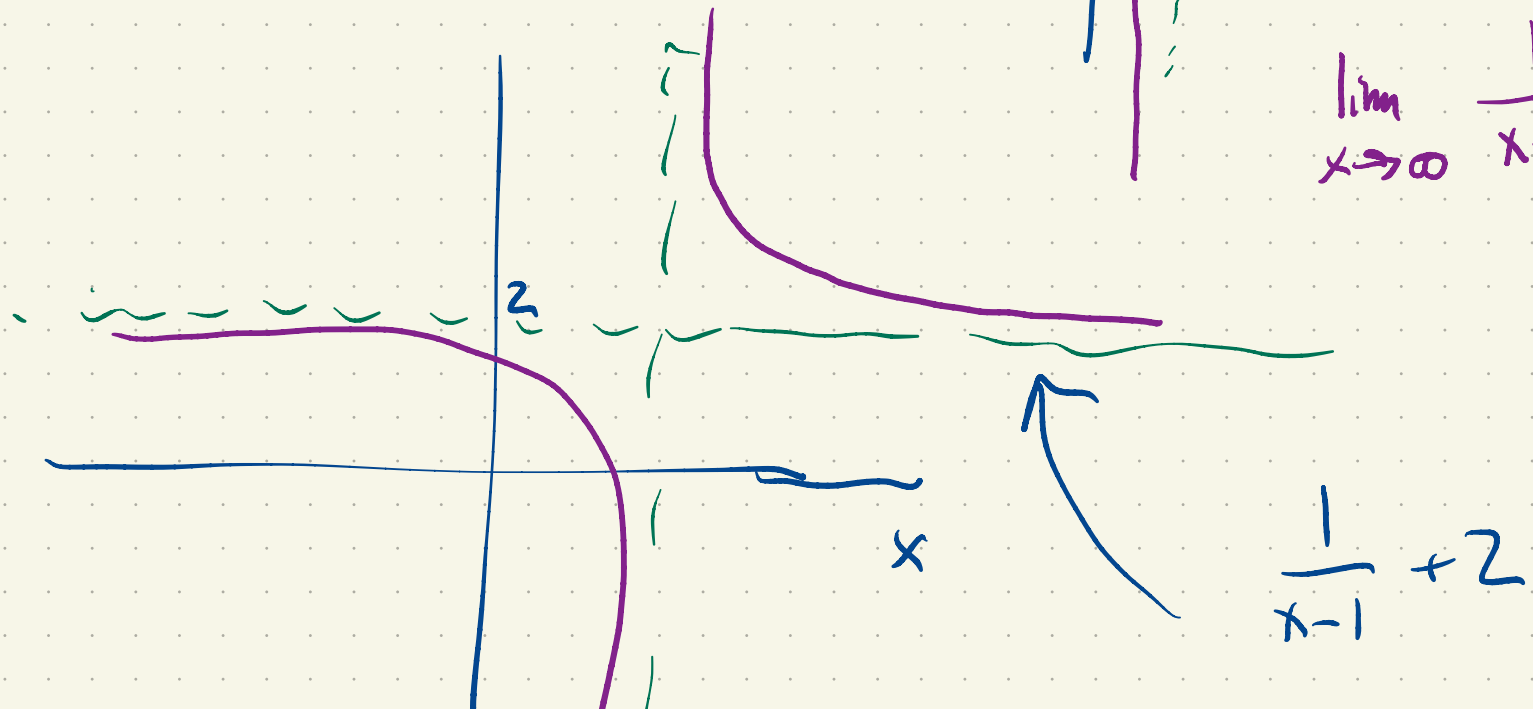
$$\lim_{x \to \infty} f(x) = ?$$

$$\lim_{x \to \infty} \left[\frac{1}{x-1} + 2 \right] = ?$$

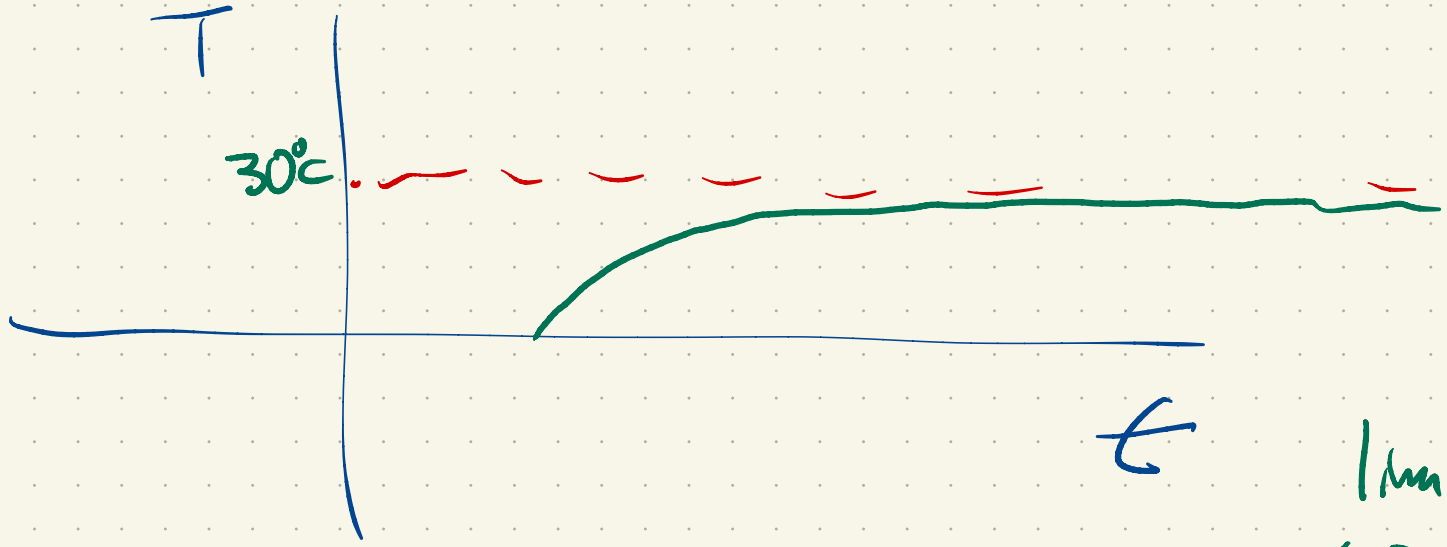




$$\lim_{x \to \infty} \frac{1}{x-1} = 0$$



$$\lim_{x \to \infty} \left(\frac{1}{x-1} + 2 \right) = 2$$



$$\lim_{t \to \infty} T(t) = 30$$

$$\frac{1}{x-1} + 2 = \frac{1}{x-1} + \frac{2(x-1)}{x-1} = \frac{1}{x-1} + \frac{2x-2}{x-1}$$

$$= \frac{2x-1}{x-1}$$

$$\lim_{x\to\infty} \frac{2x-1}{x-1} \longrightarrow \frac{\infty}{\infty} \qquad \frac{0}{0}$$

$$\frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{2x-1}{x-1} = \lim_{x\to\infty} \frac{1/x}{1/x} \frac{2x-1}{x-1}$$

$$\lim_{x\to\infty} \frac{1}{x} = 0$$

$$= \lim_{x\to\infty} \frac{2-\frac{1}{x}}{1-1/x}$$

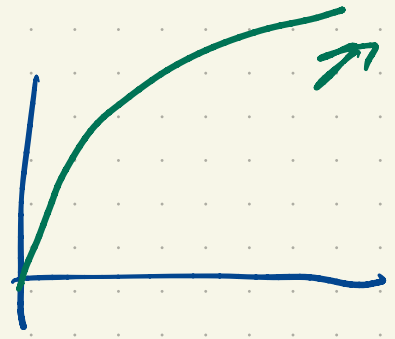
$$= \frac{\lim_{x\to\infty} 2-\frac{1}{x}}{\lim_{x\to\infty} 1-1/x} = \frac{2-0}{1-0} = 2$$

Basic limits:

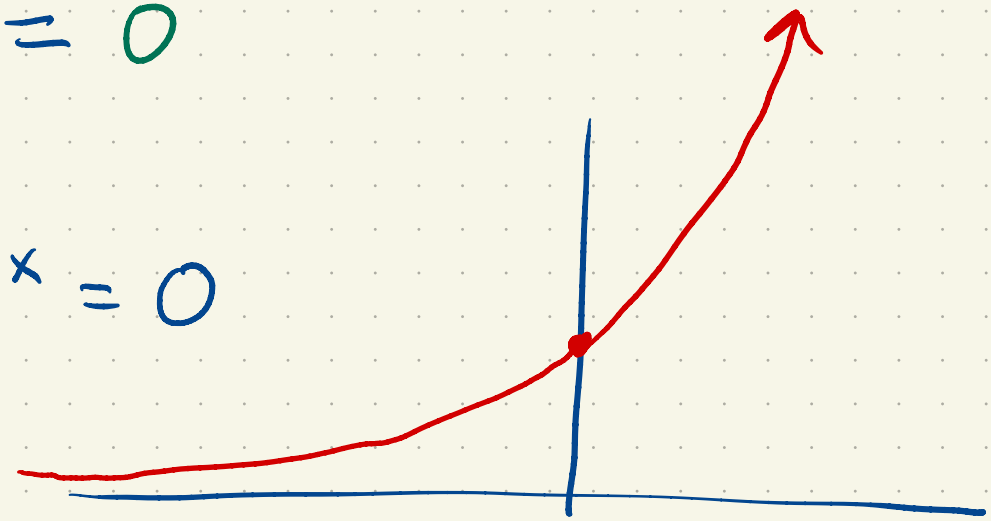
$$\lim_{x \to \infty} \frac{1}{x} = 0$$

$$\lim_{x \to \infty} \frac{1}{x^2} = 0$$

$$\lim_{x \to \infty} \frac{1}{\sqrt{x}} = 0$$



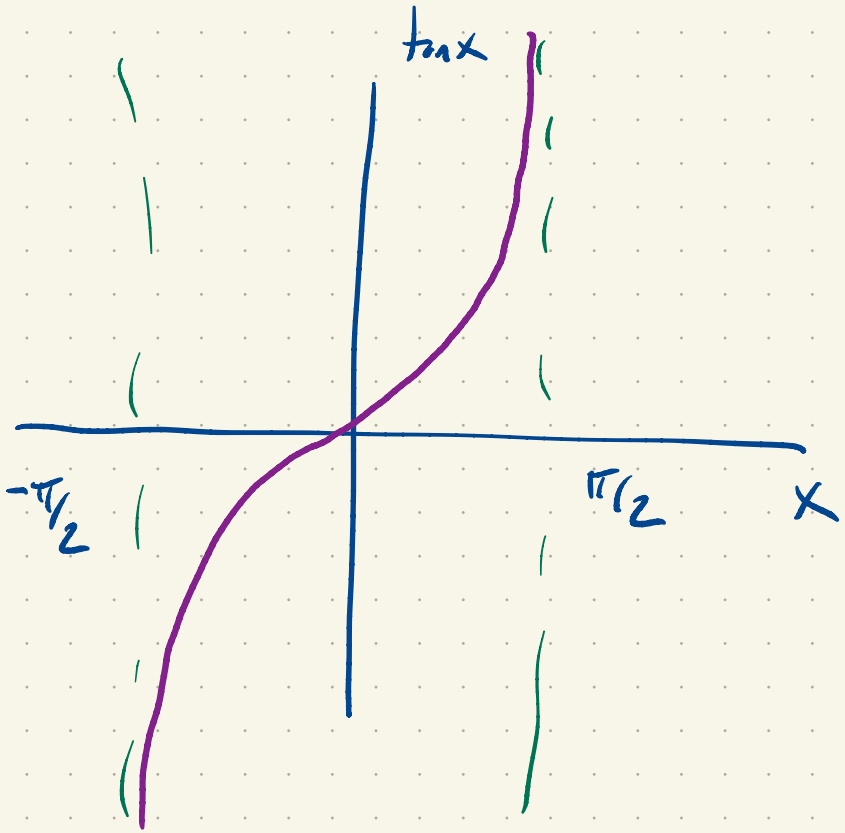
$$\lim_{x \to -\infty} 10^x = 0$$

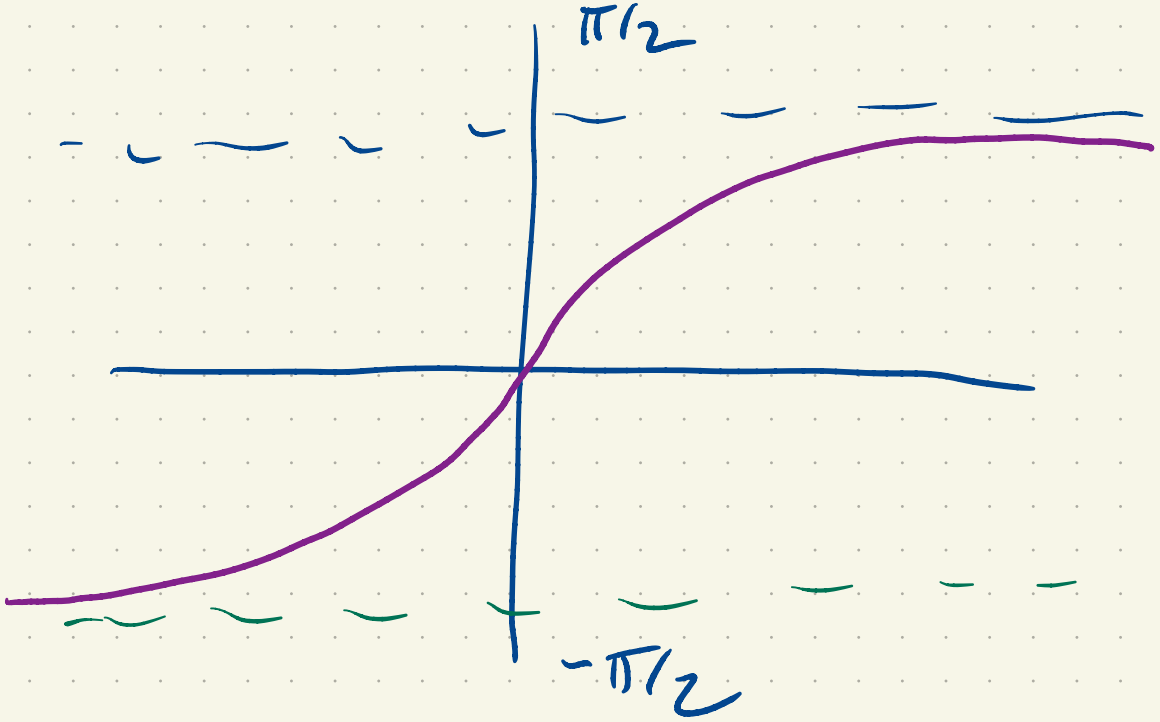


$$\lim_{x \to \infty} 10^x = \infty$$

$\arctan(x) = \tan^{-1}(x)$

$\frac{1}{\tan(x)}$





$$\lim_{x \to \infty} \arctan(x) = \frac{\pi}{2}$$

$$\lim_{x \to -\infty} \arctan(x) = -\pi/2$$