Exercise 1.2.6 [Modified]: Use the triangle inequality to establish the following inequalities:

(a) $|a - b| \leq |a| + |b|$;

(b) $||a| - |b|| \leq |a - b|$.

Proof. □

Exercise 1.2.7(b), (d): Given a function f and a subset A of its domain, let $f(A)$ represent the range of f over the set A; that is, $f(a) = \{f(x) : x \in A\}$.

(b) Find two sets A and B for which $f(A \cap B) \neq f(A) \cap f(B)$.

(d) Form and prove a conjecture concerning $f(A \cup B)$ and $f(A) \cup f(B)$.

Proof (b). □

Proof (d). □

Exercise 1.2.11: Form the logical negation of each claim. Do not use the easy way out: "It is not the case that..." is not permitted

(a) For all real numbers satisfying $a < b$, there exists $n \in \mathbb{N}$ such that $a + (1/n) < b$.

(b) There exist a real number $x > 0$ such that $x < 1/n$ for all $n \in \mathbb{N}$.

(c) Between every two distinct real numbers there is a rational number.

Solution:

(a)

(b)

(c)

Exercise [1.2 Supplement]: Show that the sequence $(x_1, x_2, x_3, \ldots)$ defined in Example 1.2.7 is bounded above by 2. That is, show that for every $i \in \mathbb{N}$, $x_i \leq 2$.

Proof. □

Exercise 1.3.5: Let A be bounded above and let $c \in \mathbb{R}$. Define $cA = \{ca : a \in A\}$.

(a) If $c \geq 0$, show that $\sup(cA) = c \sup(A)$.

(b) Postulate a similar statment for $\sup(cA)$ when $c < 0$.

Proof (a). □

Statement for part (b):

Exercise 1.3.7: Prove that if a is an upper bound for A and if a is also an element of A, then $a = \sup A$.

Proof. □

Exercise 1.3.8: Compute, without proof, the suprema and infima of the following sets.

(a) $\{m/n : m, n \in \mathbb{N} \text{ with } m < n\}$.

(b) $\{(-1)^m/n : n, m \in \mathbb{N}\}$.

(c) $\{n/(3n+1) : n \in \mathbb{N}\}$.

(d) $\{m/(m+n) : m, n \in \mathbb{N}\}$.

Solution:

(a)

(b)

(c)

(d)