

**First Derivative Test**

Suppose  $f$  is a function with a derivative on  $(a, b)$ , and if  $c$  is a point in the interval with  $f'(c) = 0$ .

- If  $f'(x) > 0$  for  $x$  just to the left of  $c$  and  $f'(x) < 0$  for  $x$  just to the right of  $c$ , then  $f$  has a local maximum at  $c$ .
- If  $f'(x) < 0$  for  $x$  just to the left of  $c$  and  $f'(x) > 0$  for  $x$  just to the right of  $c$ , then  $f$  has a local minimum at  $c$ .
- If  $f'(c) = 0$  and  $f'(x) < 0$  on both sides of  $c$  or  $f'(x) > 0$  on both sides of  $c$ , then there is neither a local min nor a local max at  $c$ .

**Second Derivative Test**

Suppose  $f$  is a function with a continuous second derivative on  $(a, b)$ , and that  $c$  is a point in the interval with  $f'(c) = 0$ .

- If  $f''(c) > 0$  then  $f$  has a local minimum at  $c$ .
- If  $f''(c) < 0$  then  $f$  has a local maximum at  $c$ .

**Concave Up:**  $f'(x)$  increasing;  $f''(x) > 0$

**Concave Down:**  $f'(x)$  decreasing;  $f''(x) < 0$

**Point of Inflection:** Value  $x$  where concavity changes; often  $f''(x) = 0$

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This worksheet considers the function

$$g(x) = x^2 e^x$$

1. Find all critical points of  $g$ .
2. Determine the intervals where  $g$  is increasing and where  $g$  is decreasing.

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