1. Prove that every ball $B_r(x)$ in a metric space (X, d) is an open set.

2. Let V be a subset of a metric space (X, d). The set of limit points of V are those points x that can be written as a the limit of a sequence of points in V. Show that a set $V \subseteq X$ is closed if and only if it contains its limit points.

3. Let d_1 and d_2 be two metrics on a set X. Show that the following conditions are equivalent.

a) For every sequence $\{p_i\}_{i=1}^{\infty}$, if $p_i \underset{d_2}{\longrightarrow} p$ then $p_i \underset{d_1}{\longrightarrow} p$.

b) For every function $f : X \to \mathbb{R}$, if f is continuous with respect to d_1 then f is continuous with respect to d_2.

c) For every set V, if V is closed with respect to d_1 then V is closed with respect to d_2.

d) For every set U, if U is open with respect to d_1 then U is open with respect to d_2.

Hint: You might want to show a) $\iff$ b) and a) $\implies$ c) $\implies$ d) $\implies$ a).

4. Lee, Problem 2-1

5. Lee, Exercise (Not Problem) 2.6