Name:

1. Consider the vector-valued function

$$\mathbf{r}(t) = t^3\mathbf{i} + e^{2t}\mathbf{j} + \cos(2t)\mathbf{k}$$

Compute $\mathbf{r}'(t)$

$$r'(t) = 3t^2\hat{i} + 2e^{2t}\hat{j} - 2\sin(2t)\hat{k}$$

2. The function in the problem 1 desribes the position of a particle as a function of time. The $\mathbf{i}$ and $\mathbf{j}$ directions are horizontal and the $\mathbf{k}$ direction is vertical. List 3 different times when the particle is moving only in a horizontal direction.

vertical component of velocity: $-2\sin(2t)$

This vanishes at $t = 0, \pi/2, \pi, \ldots$

3. A vector-valued function has **derivative**

$$\mathbf{r}'(t) = te^{t^2}\mathbf{i} + \sin(3t)\mathbf{j}.$$

We are given the additional data $\mathbf{r}(0) = 2\mathbf{j}$. Determine $\mathbf{r}(t)$.

$$\vec{r}(t) = \int te^{t^2}\hat{\imath} + \int \sin(3t)\hat{\jmath} + \vec{C}$$

$$= \frac{1}{2}e^{t^2}\hat{\imath} - \frac{1}{3}\cos(3t)\hat{\jmath} + \vec{C}$$

$$\hat{\jmath} = \frac{1}{2}\hat{\imath} - \frac{1}{3}\hat{\jmath} + \vec{C} \Rightarrow \vec{C} = -\frac{1}{2}\hat{\imath} + \frac{4}{3}\hat{\jmath}$$

$$\vec{r}(t) = \frac{1}{2}\left(e^{t^2} - 1\right)\hat{\imath} + \left(\frac{4}{3} - \frac{1}{3}\cos(3t)\right)\hat{\jmath}$$

4. Find an equation for the tangent line of the curve $\mathbf{r}(t) = e^{2t}\mathbf{i} + e^{t}\mathbf{j}$. at $t = 0$.

$$\vec{r}(0) = \hat{\imath} + \hat{\jmath}$$

$$\vec{r}\,'(t) = 2te^{2t}\hat{\imath} + e^{t}\hat{\jmath}$$

$$\vec{r}\,'(0) = 2\hat{\imath} + \hat{\jmath}$$

tangent line: $(\hat{\imath} + \hat{\jmath}) + t(2\hat{\imath} + \hat{\jmath})$, a.k.a

$$\langle 1+2t, 1+t, 0 \rangle$$