Section 14.1   Multivariate functions

$$PV = nRT$$

of
particles — *# of particles*

gas constant

temp
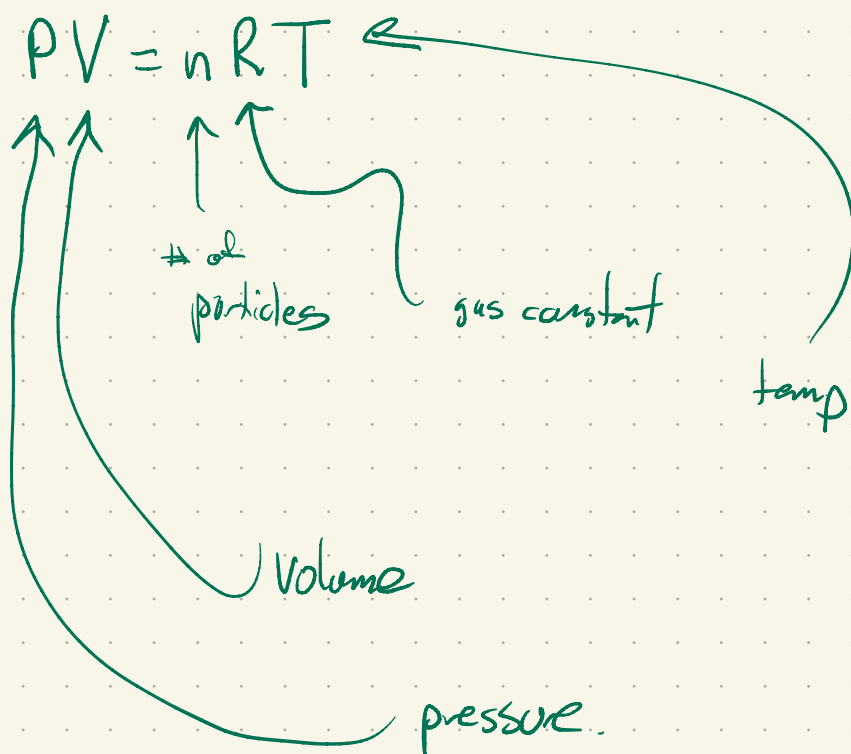
Volume

pressure.

Let us suppose n is fixed but _____ are not.

$$P = (nR) \, T/V$$ determines pressure
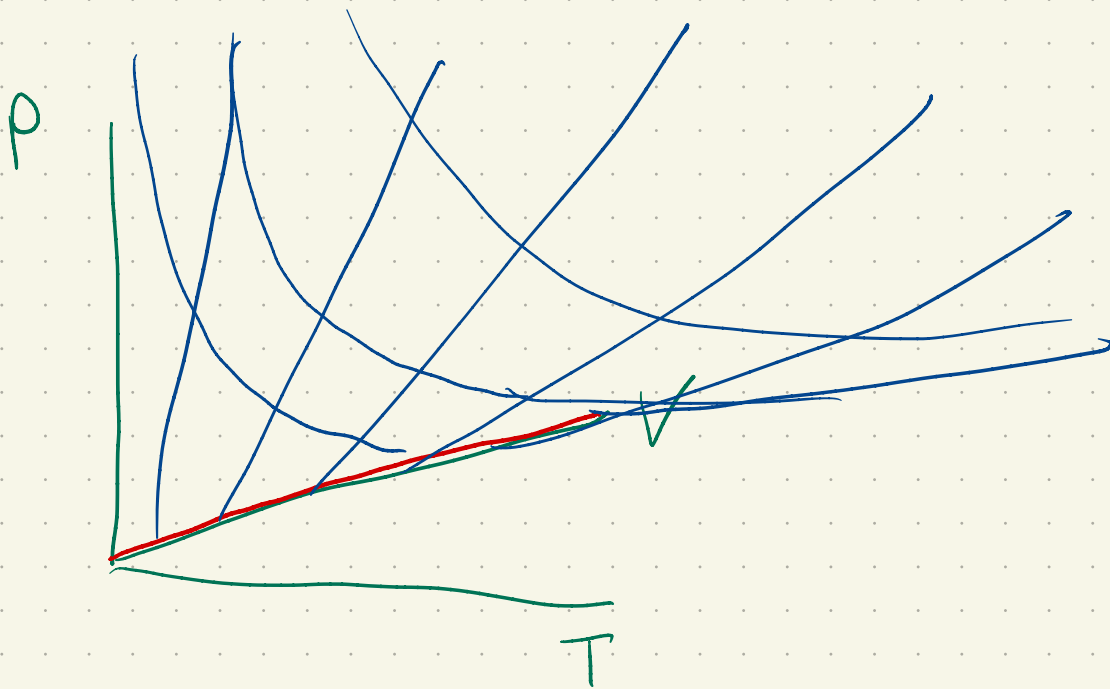
as a function of temp
and volume

you controll T and V.

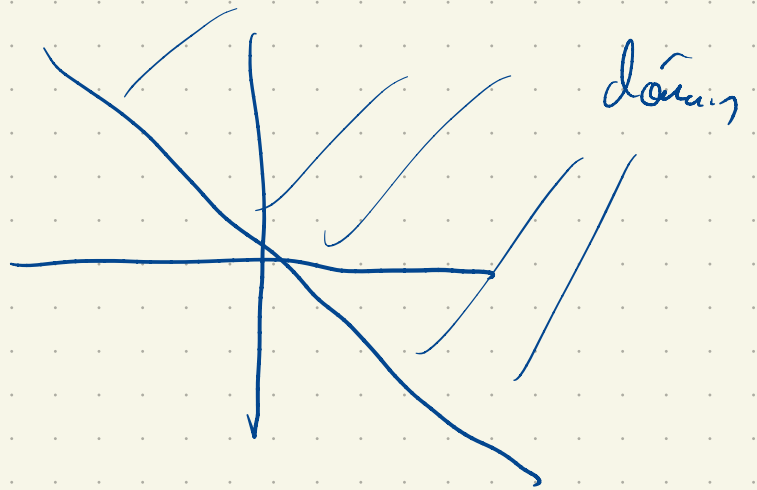P is told back to you

V goes up, P goes down

T goes up, P goes up

Vocab: $f(x,y)$ domain: allowable input

range all outputs

e.g. $f(x,y) = \left(\ln(x+y) \right)^2$

$x+y > 0 \qquad y > -x$

domain

range: $\quad f(x,y) \geq 0$ for sue

$\qquad > 0$ in fact
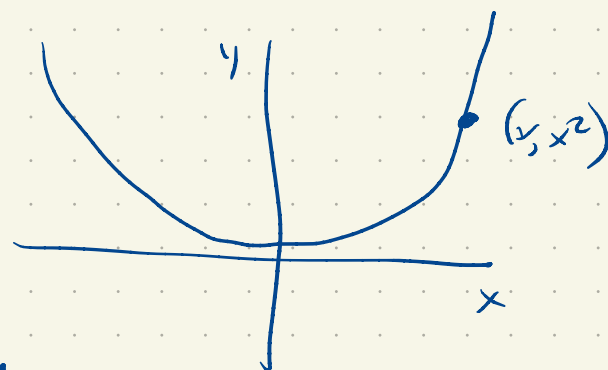
do we get all $z > 0$?

$\sqrt{z} = \ln(x) \qquad x = e^{\sqrt{z}}$

$f(e^{\sqrt{z}}, 0) = \left[\log(e^{\sqrt{z}} + 0) \right]^2$

$\qquad = (\sqrt{z})^2 = z.$

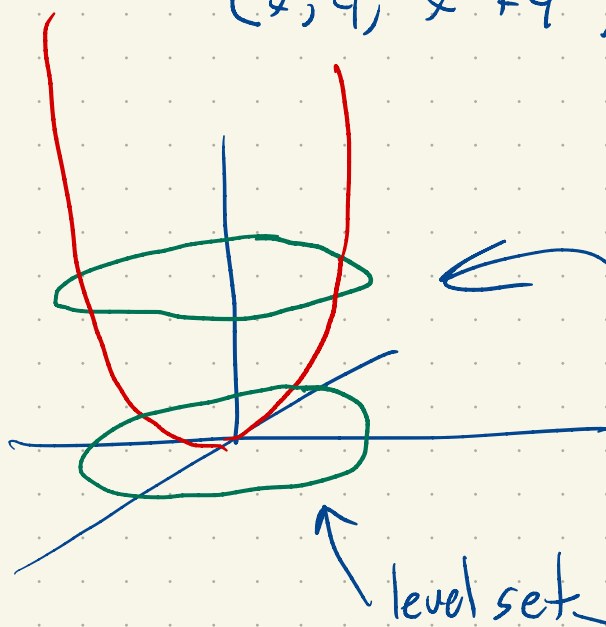Let's visualize some functions of x, y.

$f(x,y) = x^2 + y^2$

Graph: $(x, y, z = f(x,y))$

$(x, y = f(x))$ in old days

$(x, y, x^2 + y^2)$

$z = x^2$ if $y = 0$

$z = y^2$ if $x = 0$
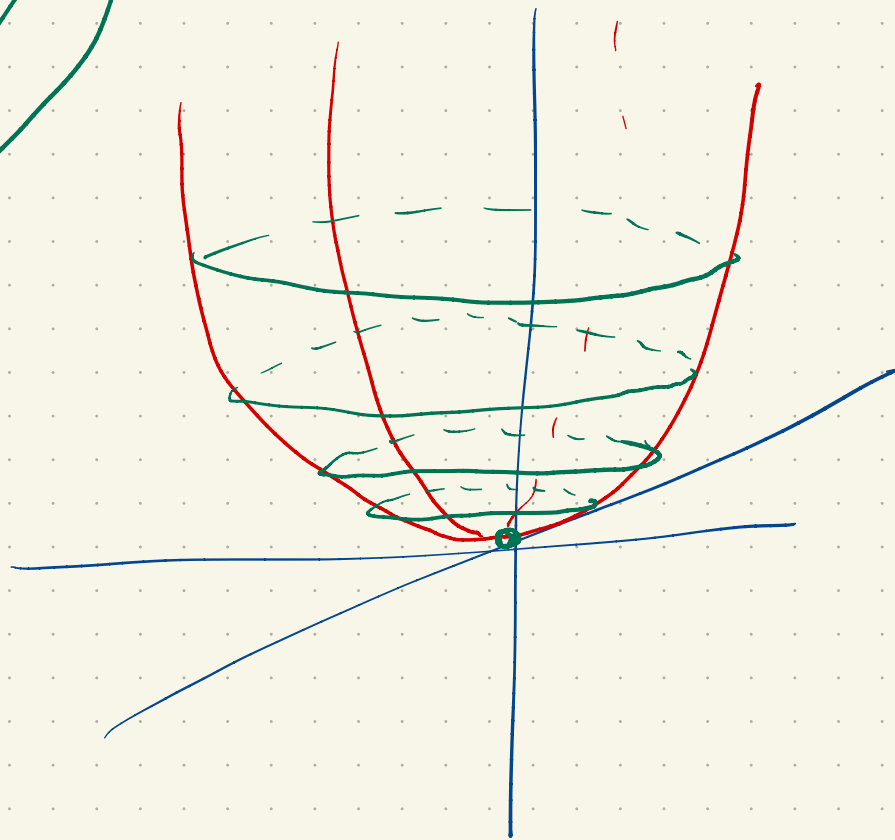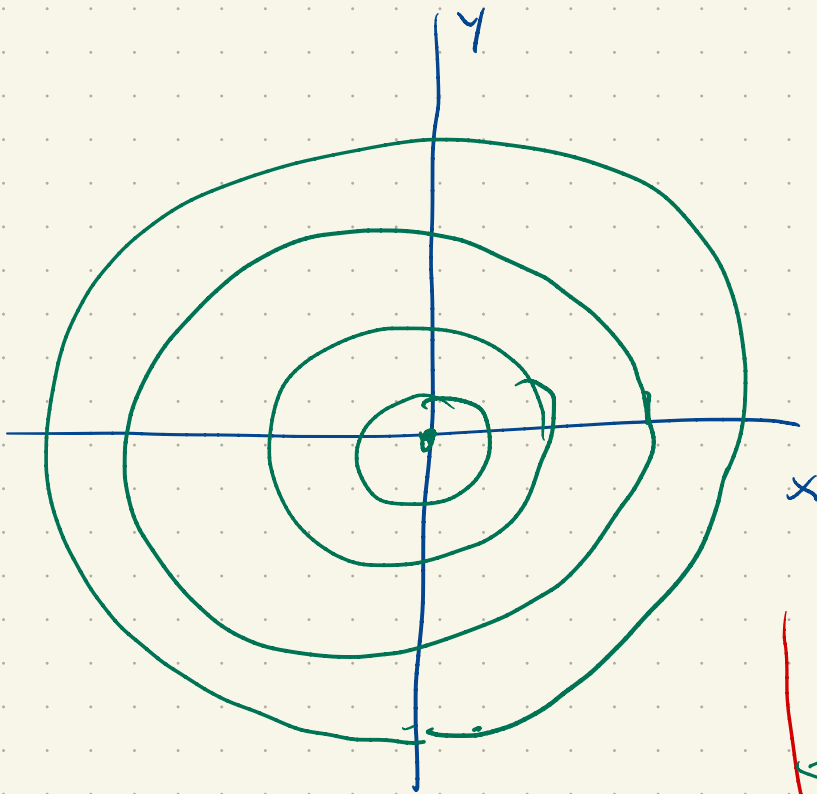
$\{(x,y): x^2 + y^2 = c\}$
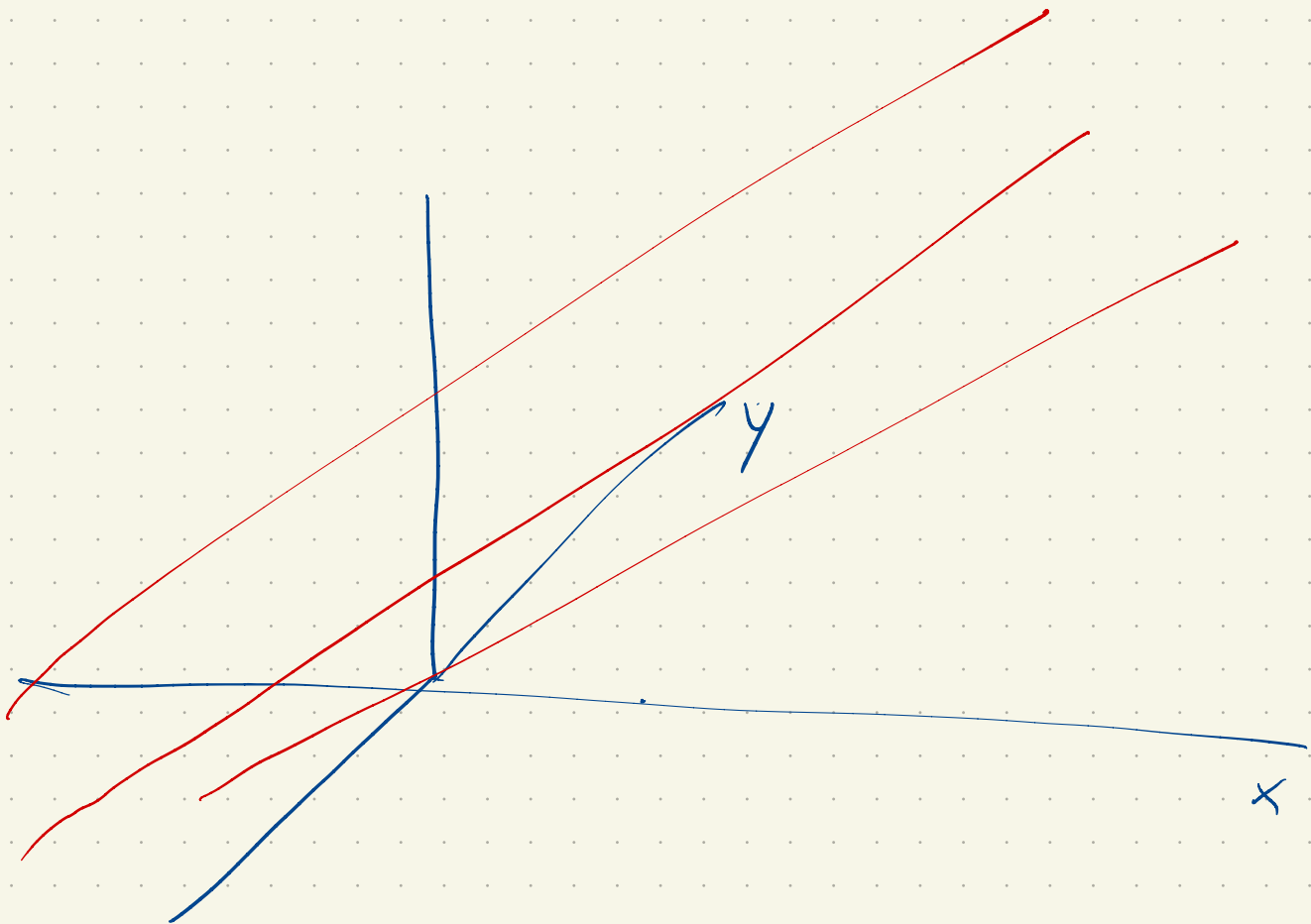
$\hookrightarrow$ circle

level set

Contour plot

Eg. $\underbrace{f(x,y)}_{z} = 2x - 3y + 5$

$z = 2x - 3y + 5$

$y = 0 \qquad z = 2x + 5$

$y = 1 \qquad z = 2x + 2$
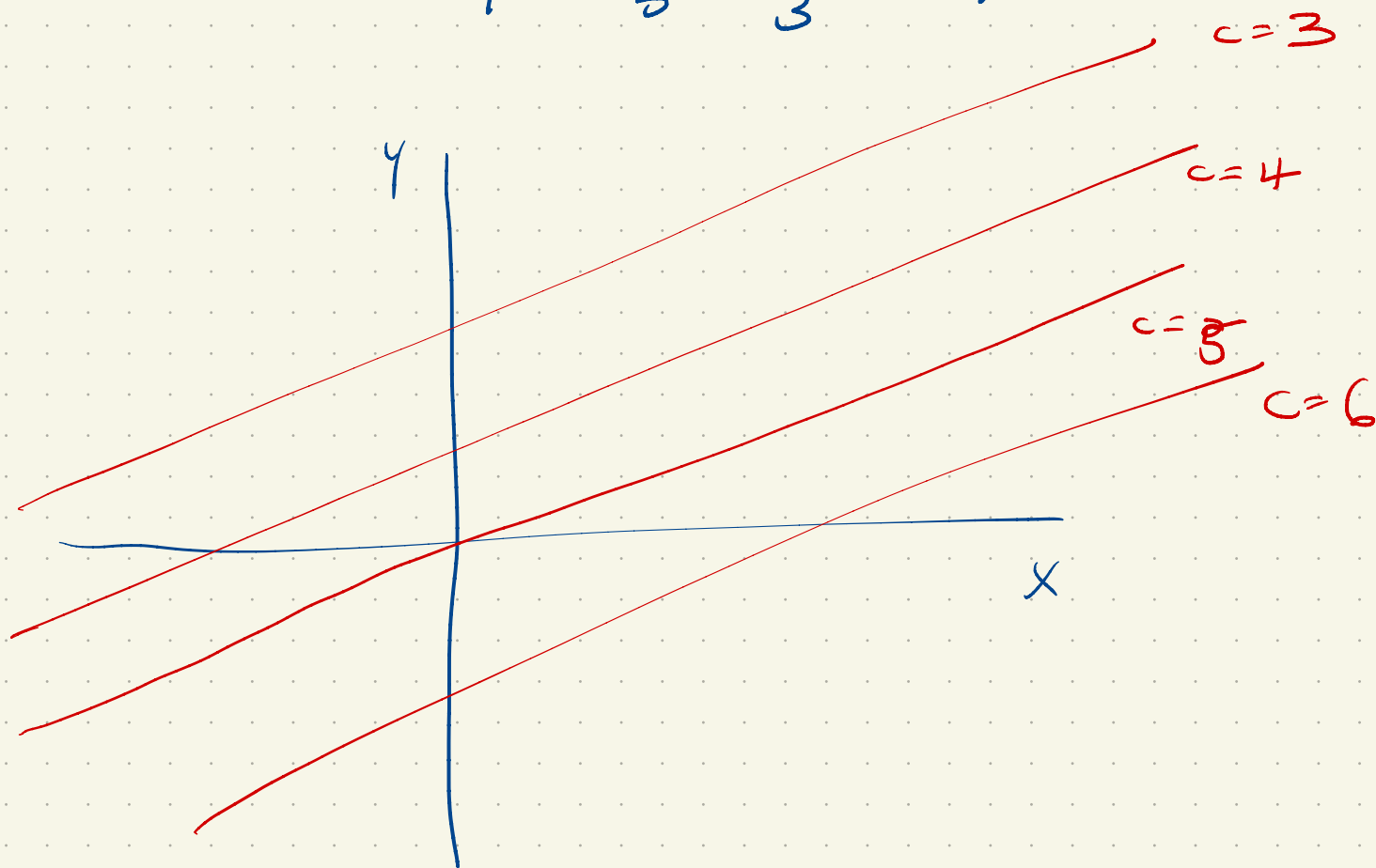
$y = -1 \qquad z = 2x + 8$
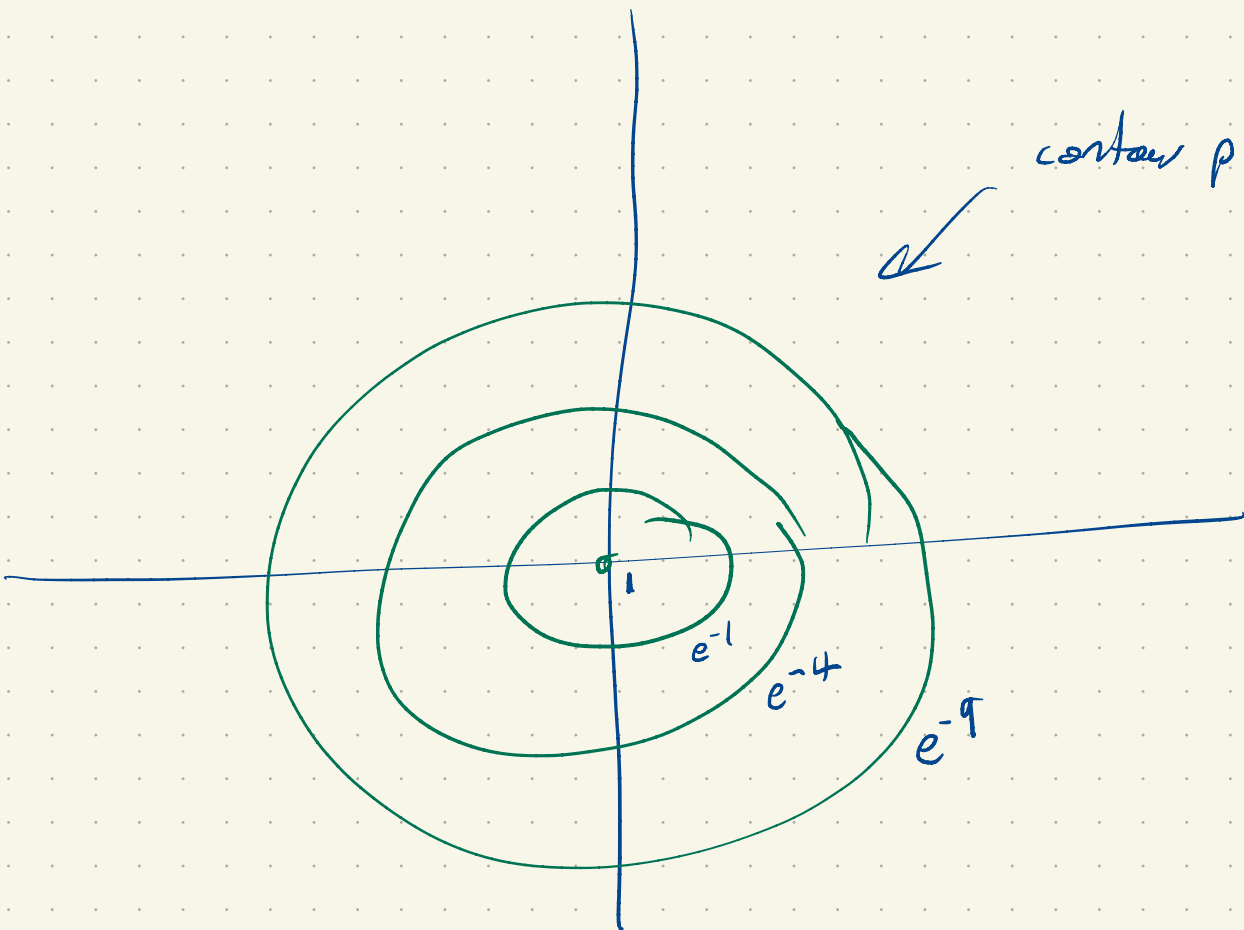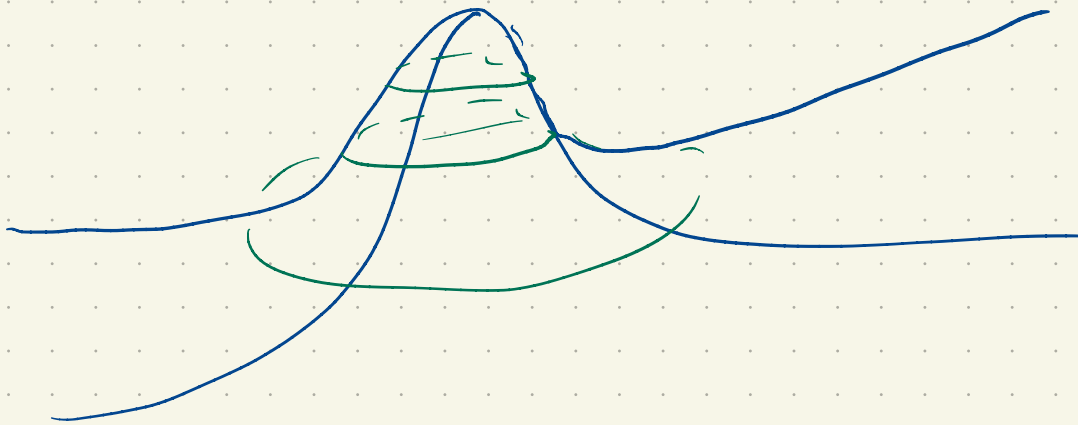
$$2x - 3y - z = 5$$

This is a plane!

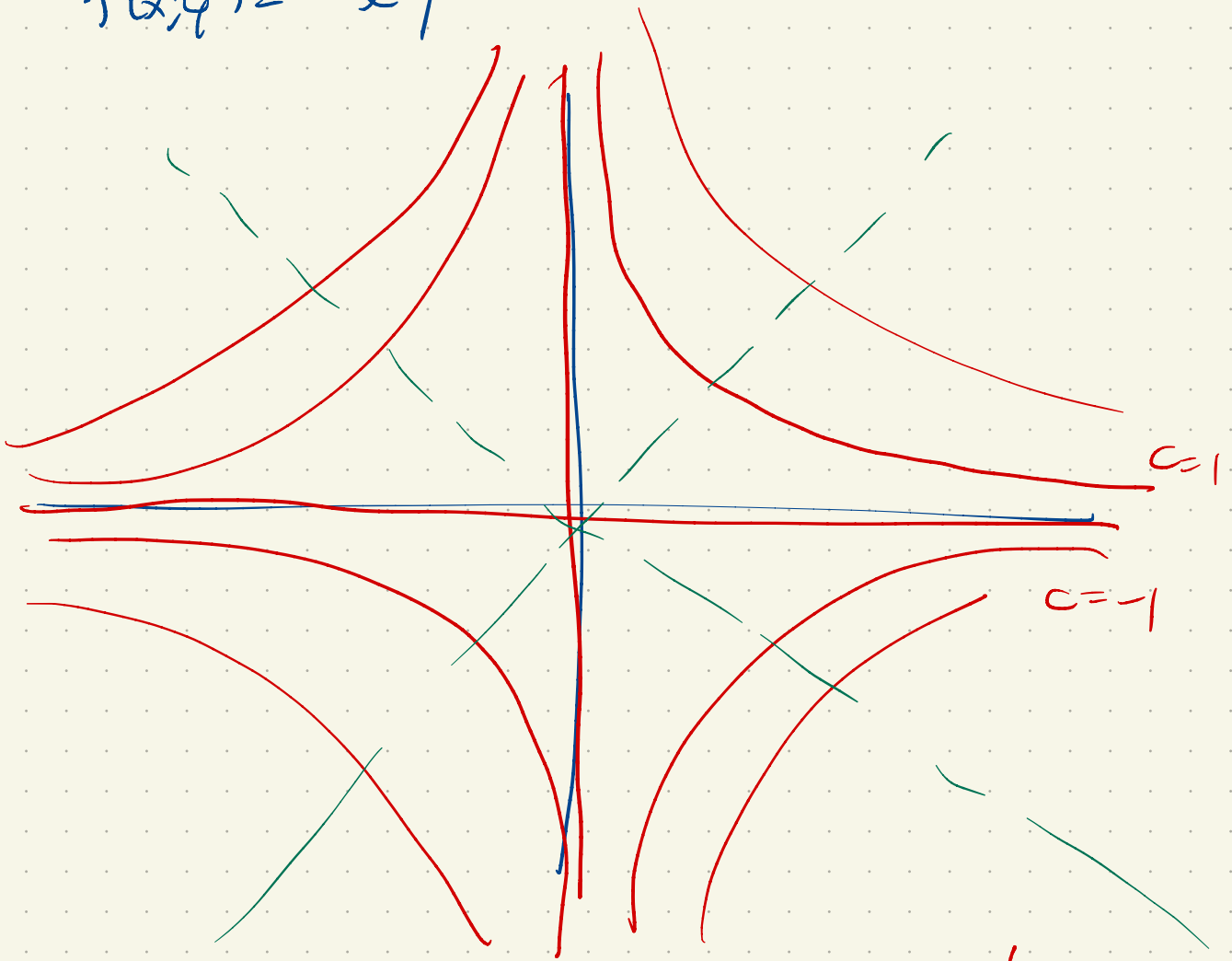Level sets: $2x - 3y + 5 = c$

$$2x - 3y = (c - 5)$$

$$y = \frac{2}{3}x - \frac{1}{3}(c - 5)$$

c = 3
c = 4
c = 5
c = 6

e.g. $\qquad f(x,y) = \exp(-x^2 - y^2)$

contour plot.

0

1

e^{-1}

e^{-4}

e^{-9}

$f(x,y) = xy$

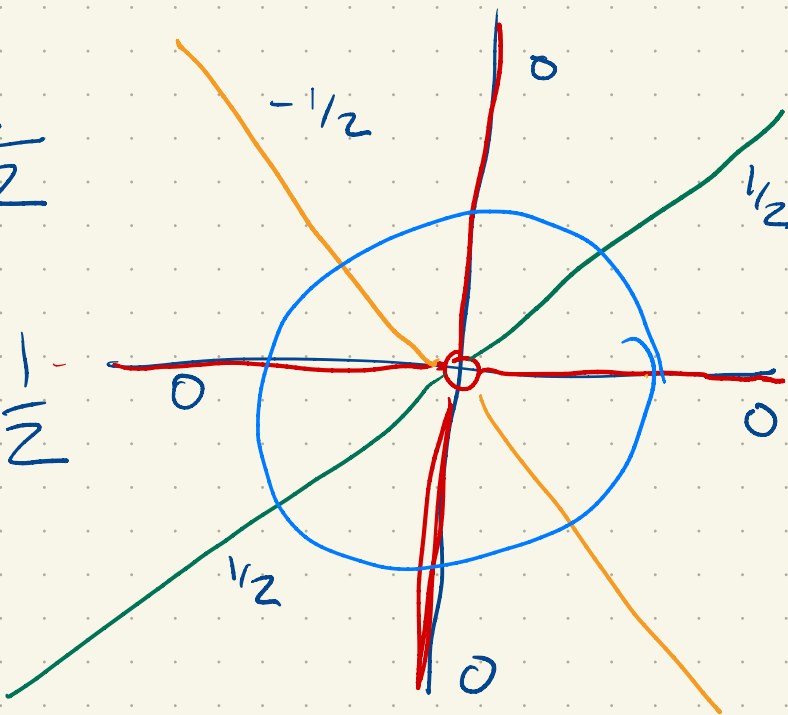$c=1$

$c=-1$

z

$z = xy$

y

x
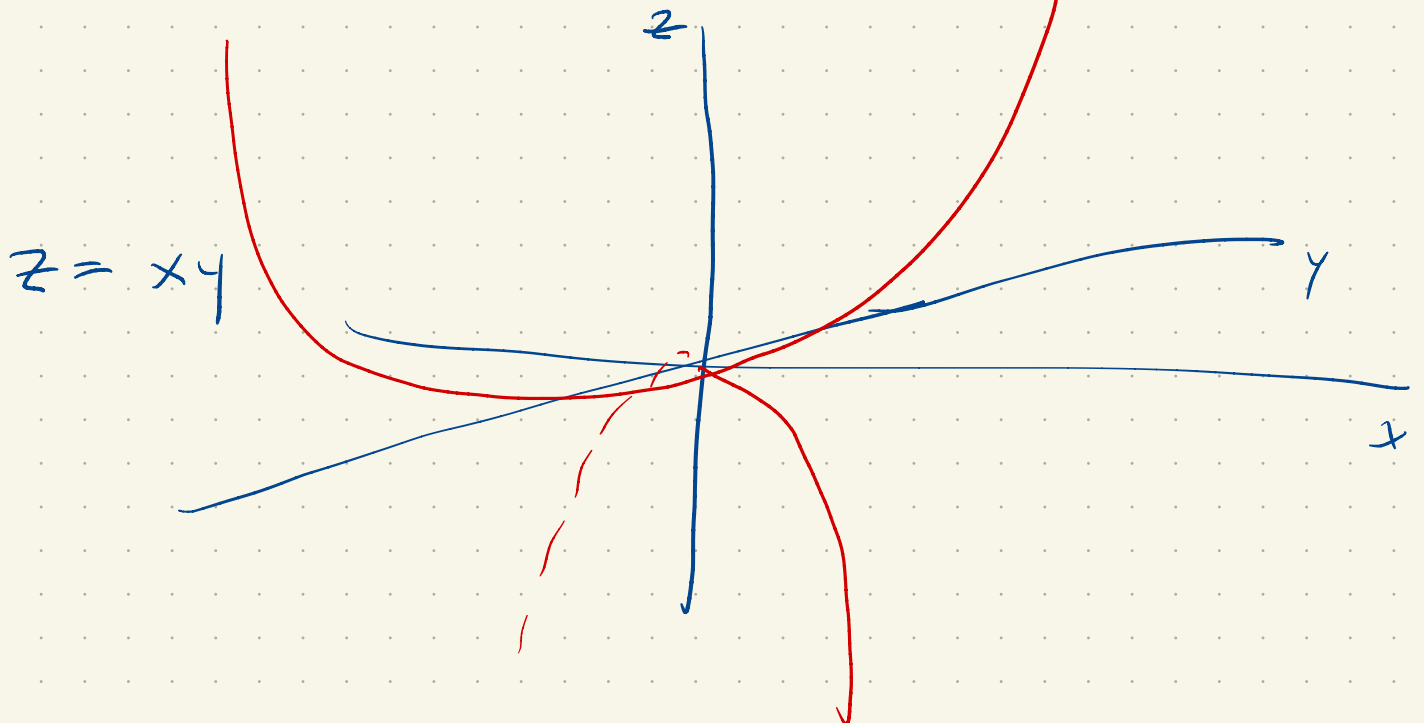
$$f(x,y) = \frac{xy}{x^2+y^2} \qquad (x,y) \neq (0,0)$$
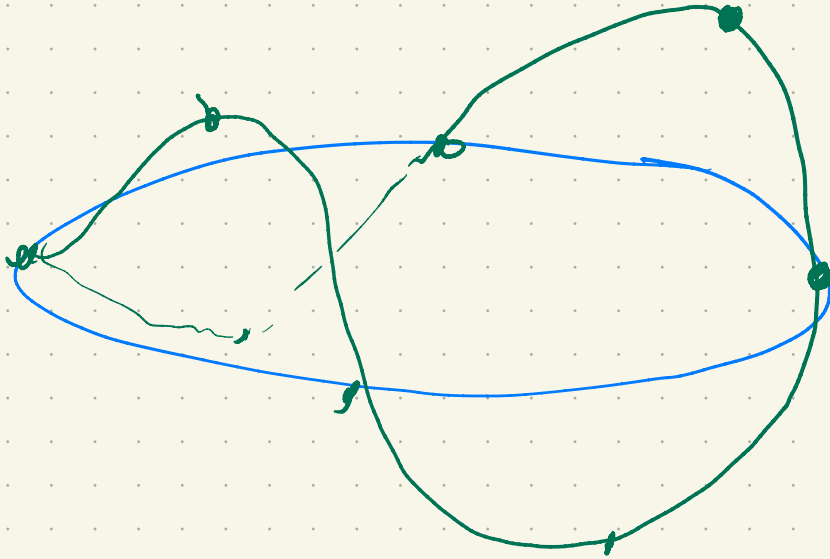
$$x=0 \;,\; y=0 \;\Rightarrow\; f(x,y)=0$$

$$y=x \qquad \frac{x^2}{x^2+x^2} = \frac{1}{2}$$

$$y=-x \qquad \frac{-x^2}{x^2+x^2} = -\frac{1}{2}$$

$$x=\cos\theta \qquad y=\sin\theta$$

$$\cos\theta \sin\theta = \frac{1}{2}\sin(2\theta)$$

Matlab:

```
x = linspace (0, 1, 5)

x = linspace (0, 2*pi, 10)

plot (x, sin(x))
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