Section 6.5 Additional Problems

1. There is a large number of identities combining vector derivative operators, the dot and cross product, and scalar multiplication. See, e.g., the wikipedia page for "Vector calculus identities". The point of this exercise is to verify one of these identities to give you a sense of the kind of thing that is possible. Let f be a function and let $\mathbf{V}$ be a vector field. Show that the following "product rule" holds:

$$\nabla \times (f\mathbf{V}) = (\nabla f) \times \mathbf{V} + f(\nabla \times \mathbf{V}).$$

2. Let $\mathbf{F} = \langle e^y, xe^y, 1\rangle$. First show that $\nabla \times F = 0$. Since $\mathbb{R}^3$ is simply connected, this implies $\mathbf{F}$ is conservative. Now find a potential for $\mathbf{F}$. That is, find a function f such that $\mathbf{F} = \nabla f$.