**Exercise 1.2.5:** Use the triangle inequality to establish the following inequalities:

(a)
$$|a - b| \leq |a| + |b|,$$

(b)
$$||a| - |b|| \leq |a - b|.$$

*Part (a).* From the triangle inequality,

$$|a - b| \leq |a| + | - b| = |a| + |b|.$$

□

*Part (b).* First, we apply the triange inequality to observe

$$|a| = |a - b + b| \leq |a - b| + |b|.$$

Hence
$$|a| - |b| \leq |a - b|. \tag{1}$$
On the other hand another application of the triangle inequality implies

$$|b| = |b - a + a| \leq |b - a| + |a| = |a - b| + |a|.$$

Hence
$$|b| - |a| \leq |a - b|. \tag{2}$$
Combining equations (**??**) and (**??**) we find

$$-|a - b| \leq |a| - |b| \leq |a - b|$$

which is equivalent to
$$||a| - |b|| \leq |a - b|$$

□