

1. Suppose $\langle x \rangle_{\alpha \in A}$ is a net in X that does not converge to $x \in X$. Show that there is an open set U containing x and a subnet $\langle x_{\alpha_\beta} \rangle_{\beta \in B}$ such that $x_{\alpha_\beta} \notin U$ for all $\beta \in B$. Hint: For a particular ‘bad’ U , take B to be the entire subset of A such that $x_\beta \notin U$ and show that B is directed. Then show that there is a natural increasing cofinal map from B to A .
2. Crossley 6.1 Show that the spaces $[0, 1]$ and $(0, 1)$ are homotopy equivalent by finding an explicit homotopy equivalence and its inverse between the two spaces.
3. Crossley 6.4
Suppose $f : X \rightarrow S^n$ is continuous and not surjective. Show that it is homotopic to a constant map.
4. Crossley 6.5
Show by means of an explicit homotopy that the map $f : S^1 \rightarrow S^1$ given by $f(x, y) = (-x, -y)$ is homotopic to the identity.
5. Show that a space X is contractible if and only if $[X, X]$ consists of a single element.
6. Lee 7-1