Globally convergent decomposition algorithm for risk parity problem in portfolio selection

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1 Introduction

2 Preliminary background

Let us consider the following optimization problem:

$$\min_{x,y} \quad f(x,y) \tag{1a}$$
$$\text{s.t.} \quad l \le x \le u \tag{1b}$$
$$\mathbf{1}^T x = 1 \tag{1c}$$
$$x \ge 0 \tag{1d}$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}$, $f : \mathbb{R}^{n+1} \to \mathbb{R}$ is a *which are the hypothesis on f?*, $l, u \in \mathbb{R}^n$ with $l < u$ and $\mathbf{1} \in \mathbb{R}^n$ is the identity vector. We indicate by $\mathcal{F}$ the feasible set of Problem (1), namely

$$\mathcal{F} = \{x \in \mathbb{R}^n : \mathbf{1}^T x = 1, l \le x \le u, x \ge 0\}. \tag{2}$$

Since the constraints of Problem (1) are linear, we have that a feasible point (x, y) is a stationary point of Problem (1) if and only if the Karush-Kuhn-Tucker (KKT) conditions are satisfied.

Proposition 2.1 (Optimality conditions (Necessary)) *Let $(x^*, y^*) \in \mathbb{R}^{n+1}$, with $x^* \in \mathcal{F}$, a local optimum for Problem (1). Then there are two multipliers $\lambda^* \in \mathbb{R}$, $\mu^* \in \mathbb{R}^n$ satisfying*

$$\frac{\partial f(x^*, y^*)}{\partial y} = 0 \tag{3a}$$
$$\frac{\partial f(x^*, y^*)}{\partial x_i} + \lambda^* - \mu_i^* = 0 \tag{3b}$$
$$\mu_i^* x_i^* = 0 \tag{3c}$$
$$\mu_i^* \ge 0 \tag{3d}$$

From Proposition (2.1) it follows that:

Corollary 2.2 *If* $(x^*, y^*) \in \mathbb{R}^{n+1}$ *is a local optimum for Problem (1), then*

$$x_j^* > 0 \quad \Rightarrow \quad \frac{\partial f(x^*, y^*)}{\partial x_j} \leq \frac{\partial f(x^*, y^*)}{\partial x_i} \quad \forall i \in \{1, .., n\} \tag{4}$$

3 A decomposition framework

4 Convergence analysis

5 Computational experiments