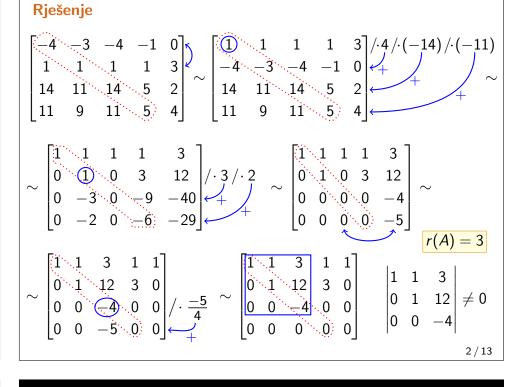
# Sustavi linearnih jednadžbi. Kronecker-Capellijev teorem

MATEMATIKA ZA EKONOMISTE 1

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## Zadatak 1

Odredite rang matrice

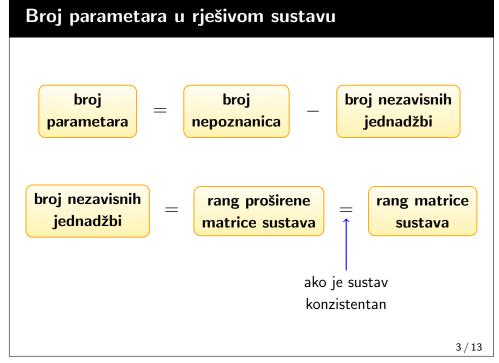
subdeterminanta reda 2

subdeterminanta reda 3

$$\begin{vmatrix} -4 & -4 & -1 \\ 1 & 1 & 1 \\ 14 & 14 & 5 \end{vmatrix}$$

$$(1.) (3.) (4.)$$

subdeterminanta reda 4



#### Zadatak 2

Zadan je sustav linearnih jednadžbi

$$2x_1 + 3x_2 + 2x_3 + 6x_4 = 1$$
$$-2x_1 + 3x_2 - 6x_3 + 12x_4 = -19.$$
$$2x_1 + 6x_2 + 15x_4 = -8$$

- a) Pomoću Kronecker-Capellijevog teorema ispitajte koliko rješenja ima zadani sustav.
- b) Riješite zadani sustav Gaussovim postupkom.

 $r(A) = r(A_p) = 2 < \text{broj nepoznanica}$ zadani sustav zadani sustav ima ima rješenje beskonačno mnogo rješenja

b) broj parametara u općem rješenju

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broj parametara = broj nepoznanica - r(A)broj parametara = 4 - 2broj parametara = 2

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Rješenje

a) 
$$A = \begin{bmatrix} 2 & 3 & 2 & 6 \\ -2 & 3 & -6 & 12 \\ 2 & 6 & 0 & 15 \end{bmatrix} \qquad A_p = \begin{bmatrix} 2 & 3 & 2 & 6 & 1 \\ -2 & 3 & -6 & 12 & -19 \\ 2 & 6 & 0 & 15 & -8 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & 2 & 6 & 1 \\ -2 & 3 & -6 & 12 & -19 \\ 2 & 6 & 0 & 15 & -8 \end{bmatrix} \xrightarrow{/ \cdot 1/\cdot (-1)} \sim \begin{bmatrix} 2 & 3 & 2 & 6 & 1 \\ 0 & 6 & -4 & 18 & -18 \\ 0 & 3 & -2 & 9 & -9 \end{bmatrix} \xrightarrow{\sim}$$

$$\sim \begin{bmatrix} 2 & 3 & 2 & 6 & 1 \\ 0 & 3 & -2 & 9 & -9 \\ 0 & 6 & -4 & 18 & -18 \end{bmatrix} / \cdot (-2) \sim \begin{bmatrix} 2 & 3 & 2 & 6 & 1 \\ 0 & 3 & -2 & 9 & -9 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$r(A)=2 \qquad r(A_p)=2$$

ešenje  $A = \begin{bmatrix} 2 & 3 & 2 & 6 \\ -2 & 3 & -6 & 12 \\ 2 & 6 & 0 & 15 \end{bmatrix} \qquad A_p = \begin{bmatrix} 2 & 3 & 2 & 6 & 1 \\ -2 & 3 & -6 & 12 & -19 \\ 2 & 6 & 0 & 15 & -8 \end{bmatrix} \qquad \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ \hline 2 & 3 & 2 & 6 & 1 \\ \hline -2 & 3 & -6 & 12 & -19 \\ \hline 2 & 6 & 0 & 15 & -8 \\ \hline 0 & -3 & 2 & -9 & 9 & / \cdot 3 \end{bmatrix} \qquad \begin{bmatrix} -3x_2 + 2x_3 - 9x_4 = 9 \\ 2x_3 = 9 + 3x_2 + 9x_4 \\ \hline x_3 = \frac{9}{2} + \frac{3}{2}x_2 + \frac{9}{2}x_4 \\ \hline 0 & -3 & 2 & -9 & 9 & / \cdot 3 \end{bmatrix}$  $2x_1 + 6x_2 + 15x_4 = -8$ Opće rješenje sustava  $x_1 = -4 - 3u - \frac{15}{2}v$  $u, v \in \mathbb{R}$  $\begin{array}{c}
-3x_2 + 2x_3 - 9x_4 = 9 \\
2x_1 + 6x_2 + 15x_4 = -8
\end{array}
\qquad x_3 = \frac{9}{2} + \frac{3}{2}u + \frac{9}{2}v$   $x_4 = v$ 7 / 13

### Neka posebna rješenja

• bazično rješenje: u = 0, v = 0

$$x_1 = -4, \ x_2 = 0, \ x_3 = \frac{9}{2}, \ x_4 = 0$$

$$x_{1} = -4 - 3u - \frac{15}{2}v$$

$$x_{2} = u$$

$$x_{3} = \frac{9}{2} + \frac{3}{2}u + \frac{9}{2}v$$

$$x_{4} = v$$

• 
$$u = 0, v = 1$$

$$x_1 = -\frac{23}{2}, \ x_2 = 0, \ x_3 = 9, \ x_4 = 1$$

•  $u = \sqrt{2}, v = \pi$ 

$$x_1 = -4 - 3\sqrt{2} - \frac{15}{2}\pi$$
,  $x_2 = \sqrt{2}$ ,  $x_3 = \frac{9}{2} + \frac{3}{2}\sqrt{2} + \frac{9}{2}\pi$ ,  $x_4 = \pi$ 

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$$\begin{bmatrix} 1 & -4 & 5 & 6 \\ 0 & -3 & 2 & -12 \\ 2 & 7 & 0 & 35 \end{bmatrix} / \cdot (-2) \sim \begin{bmatrix} 1 & -4 & 5 & 6 \\ 0 & 3 & 2 & -12 \\ 0 & 15 & -10 & 23 \end{bmatrix} / \cdot \underbrace{5}_{+} \sim$$

$$\sim \begin{bmatrix}
1 & -4 & 5 & 6 \\
0 & -3 & 2 & -12 \\
0 & 0 & 0 & -37
\end{bmatrix}
\sim \begin{bmatrix}
1 & -4 & 6 & 5 \\
0 & -3 & -12 & 2 \\
0 & 0 & -37 & 0
\end{bmatrix}$$

$$r(A) = 2$$

$$r(A_p) = 3$$

$$r(A) \neq r(A_p) \longrightarrow$$
 zadani sustav je kontradiktoran

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## Zadatak 3

Pomoću Kronecker-Capellijevog teorema ispitajte koliko rješenja ima sustav linearnih jednadžbi

$$x_1 - 4x_2 + 5x_3 = 6$$
  
 $-3x_2 + 2x_3 = -12$ .  
 $2x_1 + 7x_2 = 35$ 

# Rješenje

$$A = \begin{bmatrix} 1 & -4 & 5 \\ 0 & -3 & 2 \\ 2 & 7 & 0 \end{bmatrix} \qquad A_p = \begin{bmatrix} 1 & -4 & 5 & 6 \\ 0 & -3 & 2 & -12 \\ 2 & 7 & 0 & 35 \end{bmatrix}$$

#### Zadatak 4

Zadan je homogeni sustav linearnih jednadžbi

$$6x_1 - 4x_2 + x_3 = 0$$
$$-x_1 + x_2 + 4x_3 = 0.$$
$$4x_1 - 2x_2 + ax_3 = 0$$

- a) Odredite sve vrijednosti parametra  $a \in \mathbb{R}$  za koje sustav ima i netrivijalnih rješenja.
- b) Za sve pronađene vrijednosti parametra  $a \in \mathbb{R}$  iz a) dijela zadatka riješite pripadni sustav jednadžbi.

## Rješenje

a) Roucheov teorem

$$\begin{vmatrix} 6 & -4 & 1 & 6 & -4 \\ -1 & 1 & 4 & -1 & 1 \\ 4 & -2 & 3 & 4 & -2 \end{vmatrix} =$$

$$6x_1 - 4x_2 + x_3 = 0$$

$$-x_1 + x_2 + 4x_3 = 0$$

$$4x_1 - 2x_2 + ax_3 = 0$$

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$$= 6 \cdot 1 \cdot a + (-4) \cdot 4 \cdot 4 + 1 \cdot (-1) \cdot (-2) -$$

$$- 4 \cdot 1 \cdot 1 - (-2) \cdot 4 \cdot 6 - a \cdot (-1) \cdot (-4) =$$

$$= 6a - 64 + 2 - 4 + 48 - 4a = 2a - 18$$

$$2a - 18 = 0$$

$$a = 9$$

- Za a = 9 pripadni homogeni sustav ima i netrivijalnih rješenja.
- Za  $a \in \mathbb{R} \setminus \{9\}$  pripadni homogeni sustav ima samo trivijalno rješenje.

a = 9 $6x_1 - 4x_2 + x_3 = 0$  $-x_1 + x_2 + 4x_3 = 0$  $4x_1 - 2x_2 + ax_3 = 0$  $-2x_2 + 25x_3 = 0$  $-x_1 - \frac{17}{2}x_3 = 0$  $x_1 = -\frac{17}{2}x_3$ Opće rješenje sustava  $\left(-\frac{17}{2}t, -\frac{25}{2}t, t\right), \quad t \in \mathbb{R}$ 13 / 13