Seminari 7

Matematika za ekonomiste 2

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Sadržaj

prvi zadatak

drugi zadatak

treći zadatak

četvrti zadatak

Križna elastičnost

peti zadatak

šesti zadatak

prvi zadatak

Zadatak 1

Dane su cijene dvaju dobara u ovisnosti o količinama proizvodnje $p_1=15-Q_1$ i $p_2=10-Q_2$ te funkcija troškova

$$T(Q_1, Q_2) = 5Q_1 + 4Q_2 + 5.$$

Pronađite optimalnu kombinaciju proizvodnje tako da dobit bude maksimalna. Koliko iznosi maksimalna dobit?

DOBIT (ili PROFIT) = PRIHOD - TROŠKOVI

$$p_2 = 10 - Q_2$$

$$P(Q_1, Q_2) =$$

$$P(Q_1, Q_2) = p_1 Q_1$$

$$p_2 = 10 - Q_2$$

$$P(Q_1, Q_2) = p_1 Q_1 +$$

$$P(Q_1, Q_2) = p_1 Q_1 + p_2 Q_2$$

$$P(Q_1, Q_2) = p_1 Q_1 + p_2 Q_2 = (15 - Q_1) Q_1$$

$$-Q$$

$$P(Q_1, Q_2) = p_1 Q_1 + p_2 Q_2 = (15 - Q_1)Q_1 +$$

$$-Q_1$$

$$P(Q_1, Q_2) = p_1Q_1 + p_2Q_2 = (15 - Q_1)Q_1 + (10 - Q_2)Q_2$$

$$p_1 = 15 - Q_1$$
 $p_2 = 10 - Q_2$

$$P(Q_1, Q_2) = p_1 Q_1 + p_2 Q_2 = (15 - Q_1) Q_1 + (10 - Q_2) Q_2$$

 $P(Q_1, Q_2) =$

$$P(Q_1, Q_2) = p_1 Q_1 + p_2 Q_2 = (15 - Q_1) Q_1 + (10 - Q_2) Q_2$$

 $P(Q_1, Q_2) = -Q_1^2 - Q_2^2 + 15 Q_1 + 10 Q_2$

Riešenie

Prihod kao funkcija količine proizvodnje

$$egin{aligned} P(Q_1,Q_2) &= p_1 Q_1 + p_2 Q_2 = (15-Q_1) Q_1 + (10-Q_2) Q_2 \ P(Q_1,Q_2) &= -Q_1^2 - Q_2^2 + 15 Q_1 + 10 Q_2 \end{aligned}$$

$$D(Q_1,Q_2) =$$

Prihod kao funkcija količine proizvodnje

$$egin{aligned} P(Q_1,\,Q_2) &= p_1 Q_1 + p_2 Q_2 = (15 - Q_1) Q_1 + (10 - Q_2) Q_2 \ P(Q_1,\,Q_2) &= -Q_1^2 - Q_2^2 + 15 Q_1 + 10 Q_2 \end{aligned}$$

$$D(Q_1, Q_2) = P(Q_1, Q_2) - T(Q_1, Q_2)$$

• Prihod kao funkcija količine proizvodnje

$$egin{aligned} P(Q_1,\,Q_2) &= p_1 Q_1 + p_2 Q_2 = (15 - Q_1) Q_1 + (10 - Q_2) Q_2 \ P(Q_1,\,Q_2) &= -Q_1^2 - Q_2^2 + 15 Q_1 + 10 Q_2 \end{aligned}$$

$$egin{aligned} D(Q_1,\,Q_2) &= P(Q_1,\,Q_2) - \, \mathcal{T}(Q_1,\,Q_2) \ D(Q_1,\,Q_2) &= \end{aligned}$$

DOBIT (ili PROFIT) = PRIHOD - TROŠKOVI

Prihod kao funkcija količine proizvodnje

$$egin{aligned} P(Q_1,Q_2) &= p_1 Q_1 + p_2 Q_2 = (15-Q_1)Q_1 + (10-Q_2)Q_2 \ P(Q_1,Q_2) &= -Q_1^2 - Q_2^2 + 15Q_1 + 10Q_2 \end{aligned}$$

$$egin{aligned} D(Q_1,\,Q_2) &= P(Q_1,\,Q_2) - T(Q_1,\,Q_2) \ D(Q_1,\,Q_2) &= \left(-\,Q_1^2 - \,Q_2^2 + 15\,Q_1 + 10\,Q_2
ight) \end{aligned}$$

$$n_1 - 15 - Q_1$$
 $n_2 - 10 - Q_2$

Prihod kao funkcija količine proizvodnje

$$egin{aligned} P(Q_1,Q_2) &= p_1 Q_1 + p_2 Q_2 = (15-Q_1)Q_1 + (10-Q_2)Q_2 \ P(Q_1,Q_2) &= -Q_1^2 - Q_2^2 + 15Q_1 + 10Q_2 \end{aligned}$$

$$egin{aligned} D(Q_1,\,Q_2) &= P(Q_1,\,Q_2) - T(Q_1,\,Q_2) \ D(Q_1,\,Q_2) &= \left(-\,Q_1^2 - \,Q_2^2 + 15\,Q_1 + 10\,Q_2
ight) - \end{aligned}$$

$$p_1 = 15 - Q_1$$
 $p_2 = 10 - Q_2$

Prihod kao funkcija količine proizvodnje

$$P(Q_1, Q_2) = p_1 Q_1 + p_2 Q_2 = (15 - Q_1) Q_1 + (10 - Q_2) Q_2$$

 $P(Q_1, Q_2) = -Q_1^2 - Q_2^2 + 15 Q_1 + 10 Q_2$

$$egin{aligned} D(Q_1,\,Q_2) &= P(Q_1,\,Q_2) - \mathcal{T}(Q_1,\,Q_2) \ D(Q_1,\,Q_2) &= \left(-\,Q_1^2 - \,Q_2^2 + 15\,Q_1 + 10\,Q_2
ight) - \left(5\,Q_1 + 4\,Q_2 + 5
ight) \end{aligned}$$

$$p_1 = 15 - Q_1$$
 $p_2 = 10 - Q_2$ $T(Q_1, Q_2) = 5Q_1 + 4Q_2 + 5$

DOBIT (ili PROFIT) =
$$PRIHOD - TROŠKOVI$$

Prihod kao funkcija količine proizvodnje

$$egin{align} P(Q_1,\,Q_2) &= p_1 Q_1 + p_2 Q_2 = (15-Q_1) Q_1 + (10-Q_2) Q_2 \ P(Q_1,\,Q_2) &= -Q_1^2 - Q_2^2 + 15 Q_1 + 10 Q_2 \ \end{matrix}$$

$$egin{align} D(Q_1,Q_2)&=P(Q_1,Q_2)-\mathcal{T}(Q_1,Q_2)\ D(Q_1,Q_2)&=\left(-Q_1^2-Q_2^2+15Q_1+10Q_2
ight)-\left(5Q_1+4Q_2+5
ight)\ D(Q_1,Q_2)&= \end{array}$$

$$D(\mathit{Q}_{1},\mathit{Q}_{2}) =$$

DOBIT (ili PROFIT) =
$$PRIHOD - TROŠKOVI$$

Prihod kao funkcija količine proizvodnje

$$egin{align} P(Q_1,\,Q_2) &= p_1 Q_1 + p_2 Q_2 = (15-Q_1) Q_1 + (10-Q_2) Q_2 \ P(Q_1,\,Q_2) &= -Q_1^2 - Q_2^2 + 15 Q_1 + 10 Q_2 \ \end{pmatrix}$$

Dobit kao funkcija količine proizvodnje

$$egin{aligned} D(Q_1,Q_2) &= P(Q_1,Q_2) - \mathcal{T}(Q_1,Q_2) \ D(Q_1,Q_2) &= \left(-Q_1^2 - Q_2^2 + 15Q_1 + 10Q_2
ight) - \left(5Q_1 + 4Q_2 + 5
ight) \ D(Q_1,Q_2) &= -Q_1^2 - Q_2^2 + 10Q_1 + 6Q_2 - 5 \end{aligned}$$

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$$D(Q_1, Q_2) = -Q_1^2 - Q_2^2 + 10Q_1 + 6Q_2 - 5$$

 $D_{Q_1} =$

$$D(Q_1, Q_2) = -Q_1^2 - Q_2^2 + 10Q_1 + 6Q_2 - 5$$

$$D_{Q_1} = -2Q_1 + 10$$

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$$D(Q_1, Q_2) = -Q_1^2 - Q_2^2 + 10Q_1 + 6Q_2 - 5$$

$$D_{Q_1} = -2Q_1 + 10$$

$$D_{Q_2} = -2Q_2 + 6$$

$$D(Q_1, Q_2) = -Q_1^2 - Q_2^2 + 10Q_1 + 6Q_2 - 5$$

$$D_{Q_1} = -2Q_1 + 10$$
 $-2Q_1 + 10 = 0$ $-2Q_2 + 6 = 0$

$$D(Q_1, Q_2) = -Q_1^2 - Q_2^2 + 10Q_1 + 6Q_2 - 5$$

$$D_{Q_1} = -2Q_1 + 10$$
 $-2Q_1 + 10 = 0 \longrightarrow Q_1 = 5$ $D_{Q_2} = -2Q_2 + 6 = 0$

$$D(Q_1, Q_2) = -Q_1^2 - Q_2^2 + 10Q_1 + 6Q_2 - 5$$

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$$D_{Q_1Q_1} =$$

$$D(Q_1, Q_2) = -Q_1^2 - Q_2^2 + 10Q_1 + 6Q_2 - 5$$

$$D_{Q_1} = -2Q_1 + 10$$
 $-2Q_1 + 10 = 0 \longrightarrow Q_1 = 5$
 $D_{Q_2} = -2Q_2 + 6$ $-2Q_2 + 6 = 0 \longrightarrow Q_2 = 3$

$$D_{Q_1Q_1}=-2$$

$$D(Q_1, Q_2) = -Q_1^2 - Q_2^2 + 10Q_1 + 6Q_2 - 5$$

$$D_{Q_1} = -2Q_1 + 10$$
 $-2Q_1 + 10 = 0 \longrightarrow Q_1 = 5$
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$$egin{aligned} D_{Q_1Q_1} &= -2 \ D_{Q_1Q_2} &= \end{aligned}$$

$$D(Q_1, Q_2) = -Q_1^2 - Q_2^2 + 10Q_1 + 6Q_2 - 5$$

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$$egin{align} D_{Q_1Q_1} &= -2 \ D_{Q_1Q_2} &= 0 \ D_{Q_2Q_2} &= \end{array}$$

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$$egin{align} D_{Q_1Q_1} &= -2 \ D_{Q_1Q_2} &= 0 \ D_{Q_2Q_2} &= -2 \ \end{pmatrix} egin{align} H(Q_1,\,Q_2) &= egin{align} D_{Q_1Q_1} & D_{Q_1Q_2} \ D_{Q_1Q_2} & D_{Q_2Q_2} \ \end{pmatrix} \ \end{array}$$

$$D(Q_1, Q_2) = -Q_1^2 - Q_2^2 + 10Q_1 + 6Q_2 - 5$$

$$D_{Q_1} = -2Q_1 + 10$$
 $-2Q_1 + 10 = 0 \longrightarrow Q_1 = 5$ $-2Q_2 + 6 = 0 \longrightarrow Q_2 = 3$

Stacionarna točka:
$$(5,3)$$

$$D_{Q_1Q_1} = -2$$

$$D_{Q_1Q_2} = 0$$

$$D_{Q_2Q_2} = -2$$

$$H(Q_1, Q_2) = \begin{vmatrix} D_{Q_1Q_1} & D_{Q_1Q_2} \\ D_{Q_1Q_2} & D_{Q_2Q_2} \end{vmatrix} = \begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix}$$

$$D(Q_1, Q_2) = -Q_1^2 - Q_2^2 + 10Q_1 + 6Q_2 - 5$$

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Stacionarna točka:
$$(5,3)$$

$$D_{Q_1Q_1} = -2$$

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$$Q_1 Q_2 H(5,3) =$$

$$D(Q_1, Q_2) = -Q_1^2 - Q_2^2 + 10Q_1 + 6Q_2 - 5$$

$$D_{Q_1} = -2Q_1 + 10$$
 $-2Q_1 + 10 = 0 \longrightarrow Q_1 = 5$ $D_{Q_2} = -2Q_2 + 6$ $-2Q_2 + 6 = 0 \longrightarrow Q_2 = 3$

Stacionarna točka:
$$(5,3)$$

$$D_{Q_1Q_1} = -2$$

$$D_{Q_1Q_2} = 0 H(Q_1,Q_2) = \begin{vmatrix} D_{Q_1Q_1} & D_{Q_1Q_2} \\ D_{Q_1Q_2} & D_{Q_2Q_2} \end{vmatrix} = \begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix}$$

$$D_{Q_1Q_2} = 0$$
 $D_{Q_2Q_2} = -2$

$$= -2$$

$$H(5,3) = \begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix}$$

$$D(Q_1, Q_2) = -Q_1^2 - Q_2^2 + 10Q_1 + 6Q_2 - 5$$

$$D_{Q_1} = -2Q_1 + 10$$
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$$egin{aligned} D_{Q_1Q_1} &= -2 \ D_{Q_1Q_2} &= 0 \ D_{Q_2Q_2} &= -2 \end{aligned} \qquad egin{aligned} H(Q_1,Q_2) &= egin{aligned} D_{Q_1Q_1} & D_{Q_1Q_2} \ D_{Q_1Q_2} & D_{Q_2Q_2} \end{aligned} = egin{aligned} -2 & 0 \ 0 & -2 \end{aligned}$$

$$H(5,3) = \begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix} = 4$$

$$D(Q_1, Q_2) = -Q_1^2 - Q_2^2 + 10Q_1 + 6Q_2 - 5$$

$$D_{Q_1} = -2Q_1 + 10$$
 $-2Q_1 + 10 = 0 \longrightarrow Q_1 = 5$
 $D_{Q_2} = -2Q_2 + 6$ $-2Q_2 + 6 = 0 \longrightarrow Q_2 = 3$

 $D_{Q_1Q_1} = -2$

 $D_{Q_1Q_2} = 0$

$$egin{align*} D_{Q_1Q_2} &= 0 & H(Q_1,Q_2) = egin{align*} D_{Q_1Q_1} & D_{Q_1Q_2} \ D_{Q_2Q_2} &= -2 & D_{Q_2Q_2} \ D_{Q_2Q$$

$$D(Q_1, Q_2) = -Q_1^2 - Q_2^2 + 10Q_1 + 6Q_2 - 5$$

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 $D_{Q_1Q_1} = -2$

$$D_{Q_1Q_2} = 0$$
 $D_{Q_1Q_2} = -2$ $D_{Q_1Q_2} = \begin{vmatrix} D_{Q_1Q_1} & D_{Q_1Q_2} \\ D_{Q_1Q_2} & D_{Q_2Q_2} \end{vmatrix} = \begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix}$

$$H(5,3) = \begin{vmatrix} Q_1 & Q_2 \\ 0 & -2 \end{vmatrix} = 4 > 0$$

$$D(Q_1, Q_2) = -Q_1^2 - Q_2^2 + 10Q_1 + 6Q_2 - 5$$

$$D_{Q_1} = -2Q_1 + 10$$
 $-2Q_1 + 10 = 0 \longrightarrow Q_1 = 5$
 $D_{Q_2} = -2Q_2 + 6$ $-2Q_2 + 6 = 0 \longrightarrow Q_2 = 3$

$$egin{aligned} D_{Q_1Q_1} &= -2 \ D_{Q_1Q_2} &= 0 \ D_{Q_2Q_2} &= -2 \end{aligned} \qquad egin{aligned} \mathcal{H}(Q_1,\,Q_2) &= egin{aligned} D_{Q_1Q_1} & D_{Q_1Q_2} \ D_{Q_2Q_2} & D_{Q_2Q_2} \end{aligned} = egin{aligned} -2 & 0 \ 0 & -2 \end{aligned}$$

$$H(5,3) = \begin{vmatrix} Q_1 & Q_2 \\ 0 & -2 \end{vmatrix} = 4 > 0 \longrightarrow \text{točka lokalnog maksimuma}$$

$$D(Q_1, Q_2) = -Q_1^2 - Q_2^2 + 10Q_1 + 6Q_2 - 5$$

$$D_{Q_1} = -2Q_1 + 10$$
 $-2Q_1 + 10 = 0 \longrightarrow Q_1 = 5$ $-2Q_2 + 6 = 0 \longrightarrow Q_2 = 3$

Stacionarna točka:
$$Q_1 Q_2 \ (5,3)$$

$$D_{Q_1Q_1} = -2$$

$$\begin{array}{ccc} -2 & & & & \\ 0 & & H(Q_1,Q_2) = \begin{vmatrix} D_{Q_1Q_1} & D_{Q_1Q_2} \\ D_{Q_1Q_2} & D_{Q_2Q_2} \end{vmatrix} = \begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix}$$

 $D_{Q_1Q_2} = 0$

$$D_{Q_{1}Q_{2}} = 0 H(Q_{1}, Q_{2}) = \begin{vmatrix} D_{Q_{1}Q_{1}} & D_{Q_{1}Q_{2}} \\ D_{Q_{1}Q_{2}} & D_{Q_{2}Q_{2}} \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 0 & -2 \end{vmatrix}$$

$$H(5, 3) = \begin{vmatrix} Q_{1} & Q_{2} \\ 0 & -2 \end{vmatrix} = 4 > 0 \longrightarrow \text{točka lokalnog maksimuma}$$

 $D_{O_2O_2} = -2$

D(5,3) =

$$\mathcal{A}(Q_1,Q_2) = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$$

$$_{1}Q_{1}$$
 $_{1}Q_{2}$

$$D_{Q_1}$$

$$Q_1 Q_2$$

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$$D(Q_1, Q_2) = -Q_1^2 - Q_2^2 + 10Q_1 + 6Q_2 - 5$$

$$D_{Q_1} = -2Q_1 + 10$$
 $-2Q_1 + 10 = 0 \longrightarrow Q_1 = 5$ $-2Q_2 + 6 = 0 \longrightarrow Q_2 = 3$

Stacionarna točka:
$$(5,3)$$

$$D_{Q_1Q_1} = -2$$

$$D_{Q_1Q_2} = 0$$

$$H(Q_1, Q_2) = \begin{vmatrix} D_{Q_1Q_1} & D_{Q_1Q_2} \\ D_{Q_1Q_2} & D_{Q_2Q_2} \end{vmatrix} = \begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix}$$

 $D(5.3) = -5^2 - 3^2 + 10 \cdot 5 + 6 \cdot 3 - 5$

 $D_{Q_1Q_2} = 0$

$$H(Q_1, \dots, Q_n)$$

$$D_{Q_1Q_2}=0$$

$$D_{Q_1Q_2} = 0$$
 $D_{Q_2Q_2} = -2$

$$D_{Q_2Q_2} = -2$$

$$D_{Q_2Q_2} = -2$$

$$\begin{vmatrix} D_{Q_1Q_2} & D_{Q_2Q_2} \\ D_{Q_2Q_2} & D_{Q_2Q_2} \end{vmatrix} \quad 0 = 2$$

$$H(5,3) = \begin{vmatrix} 0 & 0 \\ 0 & -2 \end{vmatrix} = 4 > 0 \longrightarrow \text{točka lokalnog maksimuma}$$

$$D_{Q_1Q_2}$$

$$D_1 = L$$

$$D_{Q_1Q_2}$$

$$Q_1 Q_2$$

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$$D(Q_1, Q_2) = -Q_1^2 - Q_2^2 + 10Q_1 + 6Q_2 - 5$$

$$D_{Q_1} = -2Q_1 + 10$$
 $-2Q_1 + 10 = 0 \longrightarrow Q_1 = 5$ $-2Q_2 + 6 = 0 \longrightarrow Q_2 = 3$

Stacionarna točka:
$$(5,3)$$

$$D_{Q_1Q_1} = -2$$

$$D_{Q_1Q_2} = 0$$

$$D_{Q_1Q_1} = -2$$

$$D_{Q_1Q_1} = 0$$

$$D_{Q_1Q_2} = 0$$

$$D_{Q_1Q_1} = 0$$

 $D(5.3) = -5^2 - 3^2 + 10 \cdot 5 + 6 \cdot 3 - 5 = 29$

 $D_{Q_1Q_2} = 0$

 $D_{Q_2Q_2} = -2$

$$D_{Q_2Q_2} = -2$$

$$\begin{vmatrix} Q_1 Q_2 \\ H(5,3) = \begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix} = 4 > 0 \longrightarrow \text{točka lokalnog maksimuma}$$

$$\begin{pmatrix} Q_2 \\ Q_3 \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & 0 \end{pmatrix} = 4 > 0$$

$$H(Q_1,Q_2) = egin{bmatrix} D_{Q_1Q_1} & D_{Q_1Q_2} \ D_{Q_1Q_2} & D_{Q_2Q_2} \end{bmatrix} = egin{bmatrix} -2 & 0 \ 0 & -2 \end{bmatrix}$$

$$ig| \mathcal{D}_{Q_1Q_2} \ \ \mathcal{D}_{Q_2Q_2} ig| \ \ \ ig| \ 0$$

$$D(Q_1, Q_2) = -Q_1^2 - Q_2^2 + 10Q_1 + 6Q_2 - 5$$

 $D_{Q_1} = -2Q_1 + 10$

$$D_{Q_2}=-2Q_2+6$$
 $-2Q_2+6=0\longrightarrow Q_2=3$ Stacionarna točka: $\binom{Q_1}{5}, \binom{Q_2}{3}$ Maksimalna dobit iznosi 29 novčanih

jedinica, a postiže se za $Q_1 = 5$ i $Q_2 = 3$. $D_{Q_1Q_1} = -2$

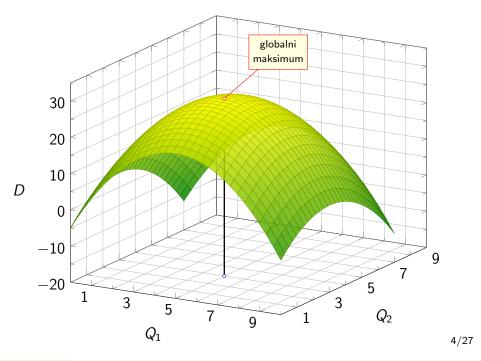
 $-2Q_1 + 10 = 0 \longrightarrow Q_1 = 5$

$$D_{Q_1Q_1}=-2$$
 $D_{Q_1Q_2}=0$ $D_{Q_2Q_2}=-2$ $D_{Q_2Q_2}=-2$ $D_{Q_1Q_2}=0$ $D_{Q_1Q_2}=0$ $D_{Q_2Q_2}=0$ $D_{Q_2Q_2}=0$ $D_{Q_2Q_2}=0$ $D_{Q_2Q_2}=0$ $D_{Q_2Q_2}=0$ $D_{Q_2Q_2}=0$

$$D_{Q_1Q_2} = 0 H(Q_1, Q_2) = \begin{vmatrix} D_{Q_1Q_1} & D_{Q_1Q_2} \\ D_{Q_1Q_2} & D_{Q_2Q_2} \end{vmatrix} = \begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix}$$

$$D_{Q_2Q_2} = -2 H(5, 3) = \begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix} = 4 > 0 \longrightarrow \text{točka lokalnog maksimuma}$$

$$H(5,3) = \begin{vmatrix} 0 & -2 \\ 0 & -2 \end{vmatrix} = 4 > 0 \longrightarrow \text{točka lokalnog mak}$$
 $D(5,3) = -5^2 - 3^2 + 10 \cdot 5 + 6 \cdot 3 - 5 = 29$



drugi zadatak

Zadatak 2

Zadana je funkcija troškova

$$T(Q_1, Q_2) = 2Q_1^2 + Q_1Q_2 + Q_2^2$$

u ovisnosti o količinama proizvodnje dva proizvoda. Odredite uz koju kombinaciju proizvodnje su troškovi minimalni ako je ukupna proizvodnja jednaka 20 proizvoda.

$$T(Q_1, Q_2) = 2Q_1^2 + Q_1Q_2 + Q_2^2$$

$$T(Q_1, Q_2) = 2Q_1^2 + Q_1Q_2 + Q_2^2 \leftarrow funkcija$$

$$\mathcal{T}(\mathit{Q}_1,\mathit{Q}_2) = 2\mathit{Q}_1^2 + \mathit{Q}_1\mathit{Q}_2 + \mathit{Q}_2^2 \longleftarrow$$
 funkcija

$$Q_1+Q_2=20$$

 $Q_1 + Q_2 = 20 \leftarrow$ uvjet

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$$\mathcal{T}(\mathit{Q}_1,\mathit{Q}_2) = 2\mathit{Q}_1^2 + \mathit{Q}_1\mathit{Q}_2 + \mathit{Q}_2^2 \longleftarrow$$
 funkcija

$$Q_1+Q_2=20$$
 — uvjet $Q_1+Q_2=20$ — $Q_2=20-Q_1$

$$\mathcal{T}(\mathit{Q}_1,\mathit{Q}_2) = 2\mathit{Q}_1^2 + \mathit{Q}_1\mathit{Q}_2 + \mathit{Q}_2^2 \longleftarrow$$
 funkcija

$$Q_1+Q_2=20$$
 — uvjet $Q_1+Q_2=20$ — $Q_2=20-Q_1$

$$T(Q_1, Q_2) = 2Q_1^2 + Q_1Q_2 + Q_2^2 \longleftarrow$$
 funkcija

$$Q_1+Q_2=20 \longleftarrow ext{uvjet} \qquad \qquad Q_1+Q_2=20 \longrightarrow \boxed{Q_2=20-Q_1}$$

 $T(Q_1,$

$$\mathcal{T}(\mathit{Q}_1,\mathit{Q}_2) = 2\mathit{Q}_1^2 + \mathit{Q}_1\mathit{Q}_2 + \mathit{Q}_2^2 \longleftarrow$$
 funkcija

$$Q_1+Q_2=20$$
 — uvjet $Q_1+Q_2=20$ — $Q_2=20-Q_1$

$$T(Q_1,20-Q_1)$$

$$T(Q_1, Q_2) = 2Q_1^2 + Q_1Q_2 + Q_2^2 \longleftarrow$$
 funkcija

$$Q_1 + Q_2 = 20 \longleftarrow \text{uvjet}$$

$$T(Q_1,20-Q_1)=$$

$$\mathcal{T}(\mathit{Q}_1,\mathit{Q}_2) = 2\mathit{Q}_1^2 + \mathit{Q}_1\mathit{Q}_2 + \mathit{Q}_2^2 \longleftarrow$$
 funkcija

$$T(Q_1, 20 - Q_1) = 2Q_1^2$$

$$\mathcal{T}(\mathit{Q}_1,\mathit{Q}_2) = 2\mathit{Q}_1^2 + \mathit{Q}_1\mathit{Q}_2 + \mathit{Q}_2^2 \longleftarrow$$
 funkcija

$$Q_1+Q_2=20$$
 — uvjet $Q_1+Q_2=20$ — $Q_2=20-Q_1$

$$T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1$$

$$\mathcal{T}(\mathit{Q}_1,\mathit{Q}_2) = 2\mathit{Q}_1^2 + \mathit{Q}_1\mathit{Q}_2 + \mathit{Q}_2^2 \longleftarrow$$
 funkcija

$$T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1)$$

$$T(Q_1, Q_2) = 2Q_1^2 + Q_1Q_2 + Q_2^2 \longleftarrow$$
 funkcija

$$Q_1+Q_2=20 \longleftarrow ext{uvjet} \qquad \qquad Q_1+Q_2=20 \longrightarrow \boxed{Q_2=20-Q_1}$$

$$T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1) + (20 - Q_1)^2$$

$$T(Q_1, Q_2) = 2Q_1^2 + Q_1Q_2 + Q_2^2 \longleftarrow$$
 funkcija

$$Q_1+Q_2=20 \longleftarrow ext{uvjet} \qquad \qquad Q_1+Q_2=20 \longrightarrow \boxed{Q_2=20-Q_1}$$

$$T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1) + (20 - Q_1)^2 =$$

$$T(Q_1, Q_2) = 2Q_1^2 + Q_1Q_2 + Q_2^2 \longleftarrow$$
 funkcija

$$Q_1+Q_2=20$$
 — uvjet $Q_1+Q_2=20$ — $Q_2=20-Q_1$

$$T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1) + (20 - Q_1)^2 =$$

= $2Q_1^2$

$$\mathcal{T}(Q_1,Q_2) = 2Q_1^2 + Q_1Q_2 + Q_2^2 \longleftarrow$$
 funkcija

$$Q_1 + Q_2 = 20$$
 — uvjet $Q_1 + Q_2 = 20$ — $Q_2 = 20 - Q_1$

$$T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1) + (20 - Q_1)^2 =$$

= $2Q_1^2 + 20Q_1$

$$\mathcal{T}(\mathit{Q}_1,\mathit{Q}_2) = 2\mathit{Q}_1^2 + \mathit{Q}_1\mathit{Q}_2 + \mathit{Q}_2^2 \longleftarrow$$
 funkcija

$$Q_1+Q_2=20$$
 — uvjet $Q_1+Q_2=20$ — $Q_2=20-Q_1$

$$T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1) + (20 - Q_1)^2 =$$

= $2Q_1^2 + 20Q_1 - Q_1^2$

$$T(Q_1, Q_2) = 2Q_1^2 + Q_1Q_2 + Q_2^2 \longleftarrow$$
 funkcija

$$T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1) + (20 - Q_1)^2 =$$

= $2Q_1^2 + 20Q_1 - Q_1^2 + 400$

$$\mathcal{T}(\mathit{Q}_1,\mathit{Q}_2) = 2\mathit{Q}_1^2 + \mathit{Q}_1\mathit{Q}_2 + \mathit{Q}_2^2 \longleftarrow$$
 funkcija

$$Q_1+Q_2=20$$
 — uvjet $Q_1+Q_2=20$ — $Q_2=20-Q_1$

$$T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1) + (20 - Q_1)^2 =$$

= $2Q_1^2 + 20Q_1 - Q_1^2 + 400 - 40Q_1$

$$\mathcal{T}(\mathit{Q}_1,\mathit{Q}_2) = 2\mathit{Q}_1^2 + \mathit{Q}_1\mathit{Q}_2 + \mathit{Q}_2^2 \longleftarrow$$
 funkcija

$$Q_1+Q_2=20 \longleftarrow ext{uvjet} \qquad \qquad Q_1+Q_2=20 \longrightarrow \boxed{Q_2=20-Q_1}$$

$$T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1) + (20 - Q_1)^2 =$$

= $2Q_1^2 + 20Q_1 - Q_1^2 + 400 - 40Q_1 + Q_1^2$

$$\mathcal{T}(\mathit{Q}_1,\mathit{Q}_2) = 2\mathit{Q}_1^2 + \mathit{Q}_1\mathit{Q}_2 + \mathit{Q}_2^2 \longleftarrow$$
 funkcija

$$Q_1+Q_2=20 \longleftarrow ext{uvjet} \qquad \qquad Q_1+Q_2=20 \longrightarrow \boxed{Q_2=20-Q_1}$$

$$T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1) + (20 - Q_1)^2 =$$

= $2Q_1^2 + 20Q_1 - Q_1^2 + 400 - 40Q_1 + Q_1^2 =$

$$\mathcal{T}(\mathit{Q}_1,\mathit{Q}_2) = 2\mathit{Q}_1^2 + \mathit{Q}_1\mathit{Q}_2 + \mathit{Q}_2^2 \longleftarrow$$
 funkcija

$$Q_1+Q_2=20 \longleftarrow ext{uvjet} \qquad \qquad Q_1+Q_2=20 \longrightarrow \boxed{Q_2=20-Q_1}$$

$$T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1) + (20 - Q_1)^2 =$$

= $2Q_1^2 + 20Q_1 - Q_1^2 + 400 - 40Q_1 + Q_1^2 = 2Q_1^2$

$$\mathcal{T}(\mathit{Q}_1,\mathit{Q}_2) = 2\mathit{Q}_1^2 + \mathit{Q}_1\mathit{Q}_2 + \mathit{Q}_2^2 \longleftarrow$$
 funkcija

$$Q_1+Q_2=20 \longleftarrow ext{uvjet} \qquad \qquad Q_1+Q_2=20 \longrightarrow \boxed{Q_2=20-Q_1}$$

$$T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1) + (20 - Q_1)^2 =$$

= $2Q_1^2 + 20Q_1 - Q_1^2 + 400 - 40Q_1 + Q_1^2 = 2Q_1^2 - 20Q_1$

$$T(Q_1, Q_2) = 2Q_1^2 + Q_1Q_2 + Q_2^2$$
 funkcija $Q_1 + Q_2 = 20$ wvjet $Q_1 + Q_2 = 20$ $Q_2 = 20 - Q_1$ $Q_2 = 20 - Q_1$ $Q_1 + Q_2 = 20$ funkcija $Q_1 + Q_2 = 20$ $Q_2 = 20 - Q_1$

$$I(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1) + (20 - Q_1)^2 =$$

$$= 2Q_1^2 + 20Q_1 - Q_1^2 + 400 - 40Q_1 + Q_1^2 = 2Q_1^2 - 20Q_1 + 400$$

$$T(Q_1, Q_2) = 2Q_1^2 + Q_1Q_2 + Q_2^2 \longleftarrow$$
 funkcija

$$Q_1+Q_2=20$$
 — uvjet $Q_1+Q_2=20$ — $Q_2=20-Q_1$

$$T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1) + (20 - Q_1)^2 =$$

= $2Q_1^2 + 20Q_1 - Q_1^2 + 400 - 40Q_1 + Q_1^2 = 2Q_1^2 - 20Q_1 + 400$

$$f(Q_1) = 2Q_1^2 - 20Q_1 + 400$$

Riešenie

$$T(Q_1, Q_2) = 2Q_1^2 + Q_1Q_2 + Q_2^2 \longleftarrow$$
 funkcija

$$Q_1+Q_2=20 \longleftarrow ext{uvjet} \qquad \qquad Q_1+Q_2=20 \longrightarrow \boxed{Q_2=20-Q_1}$$

$$T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1) + (20 - Q_1)^2 =$$

= $2Q_1^2 + 20Q_1 - Q_1^2 + 400 - 40Q_1 + Q_1^2 = 2Q_1^2 - 20Q_1 + 400$

$$f(Q_1) = 2Q_1^2 - 20Q_1 + 400$$

$$f'(Q_1) = 2Q_1 - 20Q_1 + 400$$
 $f'(Q_1) =$

$$T(Q_1, Q_2) = 2Q_1^2 + Q_1Q_2 + Q_2^2 \longleftarrow$$
 funkcija

$$Q_1+Q_2=20 \longleftarrow ext{uvjet}$$
 $Q_1+Q_2=20 \longrightarrow Q_2=20-Q_1$

$$T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1) + (20 - Q_1)^2 =$$

= $2Q_1^2 + 20Q_1 - Q_1^2 + 400 - 40Q_1 + Q_1^2 = 2Q_1^2 - 20Q_1 + 400$

$$f(Q_1) = 2Q_1^2 - 20Q_1 + 400$$

$$f'(Q_1) = 4Q_1 - 20$$

$$T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1) + (20 - Q_1)^2 =$$

$$= 2Q_1^2 + 20Q_1 - Q_1^2 + 400 - 40Q_1 + Q_1^2 = 2Q_1^2 - 20Q_1 + 400$$

$$f(Q_1) = 2Q_1^2 - 20Q_1 + 400$$

 $f'(Q_1) = 4Q_1 - 20$

$$= 4Q_1 - 2$$

$$4Q_1-20=0$$

$$\mathcal{T}(\mathit{Q}_1,\mathit{Q}_2) = 2\mathit{Q}_1^2 + \mathit{Q}_1\mathit{Q}_2 + \mathit{Q}_2^2 \longleftarrow$$
 funkcija

$$Q_1+Q_2=20$$
 — uvjet $Q_1+Q_2=20$ — $Q_2=20-Q_1$

$$egin{aligned} Tig(Q_1,20-Q_1ig) &= 2Q_1^2 + Q_1\cdot (20-Q_1) + (20-Q_1)^2 = \ &= 2Q_1^2 + 20Q_1 - Q_1^2 + 400 - 40Q_1 + Q_1^2 = 2Q_1^2 - 20Q_1 + 400 \end{aligned}$$

$$f(Q_1) = 2Q_1^2 - 20Q_1 + 400$$

 $f'(Q_1) = 4Q_1 - 20$
 $4Q_1 - 20 = 0$

$$-20 = 0$$

 $Q_1 = 5$

$$\mathcal{T}(\mathit{Q}_1,\mathit{Q}_2) = 2\mathit{Q}_1^2 + \mathit{Q}_1\mathit{Q}_2 + \mathit{Q}_2^2 \longleftarrow$$
 funkcija

$$Q_1+Q_2=20$$
 — uvjet $Q_1+Q_2=20$ — $Q_2=20-Q_1$

$$T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1) + (20 - Q_1)^2 =$$

= $2Q_1^2 + 20Q_1 - Q_1^2 + 400 - 40Q_1 + Q_1^2 = 2Q_1^2 - 20Q_1 + 400$

$$f(Q_1) = 2Q_1^2 - 20Q_1 + 400$$

 $f'(Q_1) = 4Q_1 - 20$

$$4Q_1 - 20 = 0$$



 $4Q_1 - 20 = 0$

$$T(Q_1,Q_2)=2Q_1^2+Q_1Q_2+Q_2^2 \longleftarrow \text{funkcija}$$
 $Q_1+Q_2=20 \longleftarrow \text{uvjet}$ $Q_1+Q_2=20 \longrightarrow \boxed{Q_2=20-Q_1}$
 $T(Q_1,20-Q_1)=2Q_1^2+Q_1\cdot(20-Q_1)+(20-Q_1)^2=$
 $=2Q_1^2+20Q_1-Q_1^2+400-40Q_1+Q_1^2=2Q_1^2-20Q_1+400$
 $f(Q_1)=2Q_1^2-20Q_1+400$ $f''(Q_1)=$
 $f'(Q_1)=4Q_1-20$

$$T(Q_1, Q_2) = 2Q_1^2 + Q_1Q_2 + Q_2^2 \longleftarrow$$
 funkcija

$$Q_1+Q_2=20 \longleftarrow ext{uvjet} \qquad \qquad Q_1+Q_2=20 \longrightarrow \overline{Q_2=20-Q_1}$$

$$T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1) + (20 - Q_1)^2 =$$

= $2Q_1^2 + 20Q_1 - Q_1^2 + 400 - 40Q_1 + Q_1^2 = 2Q_1^2 - 20Q_1 + 400$

$$f(Q_1) = 2Q_1^2 - 20Q_1 + 400$$
 $f''(Q_1) = 4$
 $f'(Q_1) = 4Q_1 - 20$
 $4Q_1 - 20 = 0$

 $Q_1 = 5$

Riešenje

$$T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1) + (20 - Q_1)^2 =$$

= $2Q_1^2 + 20Q_1 - Q_1^2 + 400 - 40Q_1 + Q_1^2 = 2Q_1^2 - 20Q_1 + 400$

$$f(Q_1) = 2Q_1^2 - 20Q_1 + 400$$
 $f''(Q_1) = 4$
 $f'(Q_1) = 4Q_1 - 20$ $f''(5) = 4$
 $4Q_1 - 20 = 0$

$$20 = 0$$

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Riešenje

$$T(Q_1, Q_2) = 2Q_1^2 + Q_1Q_2 + Q_2^2$$
 funkcija $Q_1 + Q_2 = 20$ wvjet $Q_1 + Q_2 = 20$ $Q_2 = 20 - Q_1$ $Q_2 = 20 - Q_1$ $Q_1 + Q_2 = 20$ funkcija $Q_1 + Q_2 = 20$ $Q_2 = 20 - Q_1$

$$egin{align} \mathcal{T}(Q_1,20-Q_1) &= 2Q_1^2 + Q_1 \cdot (20-Q_1) + (20-Q_1)^2 = \ &= 2Q_1^2 + 20Q_1 - Q_1^2 + 400 - 40Q_1 + Q_1^2 = 2Q_1^2 - 20Q_1 + 400 \ &= Q_1(Q_1) = 2Q_1^2 - 20Q_1 + 400 & f''(Q_1) = 4 \ \end{pmatrix}$$

$$f(Q_1) = 2Q_1^2 - 20Q_1 + 400$$
 $f''(Q_1) = 4$ $f'(Q_1) = 4 > 0$ $f''(5) = 4 > 0$ $f''(5) = 4 > 0$

 $Q_1 = 5$

$$T(Q_1,Q_2)=2Q_1^2+Q_1Q_2+Q_2^2$$
 funkcija $Q_1+Q_2=20$ wyjet $Q_1+Q_2=20$ $Q_2=20-Q_1$ $T(Q_1,20-Q_1)=2Q_1^2+Q_1\cdot(20-Q_1)+(20-Q_1)^2=$ $=2Q_1^2+20Q_1-Q_1^2+400-40Q_1+Q_1^2=2Q_1^2-20Q_1+400$ $f(Q_1)=2Q_1^2-20Q_1+400$ $f''(Q_1)=4$ $f''(Q_1)=4Q_1-20$ $f'''(5)=4>0$ minimum $4Q_1-20=0$

$$T(Q_1,Q_2)=2Q_1^2+Q_1Q_2+Q_2^2 \longleftarrow \text{funkcija}$$
 $Q_1+Q_2=20 \longleftarrow \text{uvjet}$
 $Q_1+Q_2=20 \longrightarrow \boxed{Q_2=20-Q_1}$
 $T(Q_1,20-Q_1)=2Q_1^2+Q_1\cdot(20-Q_1)+(20-Q_1)^2=$
 $=2Q_1^2+20Q_1-Q_1^2+400-40Q_1+Q_1^2=2Q_1^2-20Q_1+400$
 $f(Q_1)=2Q_1^2-20Q_1+400$
 $f''(Q_1)=4$
 $f''(Q_1)=4Q_1-20$
 $f'''(S)=4>0 \longrightarrow \text{minimum}$
 $4Q_1-20=0$
 $Q_2=$

$$T(Q_1,Q_2)=2Q_1^2+Q_1Q_2+Q_2^2 \longleftarrow \text{funkcija}$$
 $Q_1+Q_2=20 \longleftarrow \text{uvjet}$
 $Q_1+Q_2=20 \longrightarrow Q_2=20-Q_1$
 $T(Q_1,20-Q_1)=2Q_1^2+Q_1\cdot(20-Q_1)+(20-Q_1)^2=$
 $=2Q_1^2+20Q_1-Q_1^2+400-40Q_1+Q_1^2=2Q_1^2-20Q_1+400$
 $f(Q_1)=2Q_1^2-20Q_1+400$
 $f''(Q_1)=4$
 $f''(Q_1)=4Q_1-20$
 $f'''(Q_1)=4>0$ minimum
 $4Q_1-20=0$ $Q_2=20-5$

$$T(Q_1,Q_2)=2Q_1^2+Q_1Q_2+Q_2^2$$
 funkcija $Q_1+Q_2=20$ uvjet $Q_1+Q_2=20$ $Q_2=20-Q_1$ $T(Q_1,20-Q_1)=2Q_1^2+Q_1\cdot(20-Q_1)+(20-Q_1)^2=$ $=2Q_1^2+20Q_1-Q_1^2+400-40Q_1+Q_1^2=2Q_1^2-20Q_1+400$ $f(Q_1)=2Q_1^2-20Q_1+400$ $f''(Q_1)=4$ $f''(Q_1)=4Q_1-20$ $f'''(5)=4>0$ minimum $4Q_1-20=0$ $Q_2=20-5$ $Q_2=15$

$$T(Q_1, Q_2) = 2Q_1^2 + Q_1Q_2 + Q_2^2 \qquad \text{funkcija}$$

$$Q_1 + Q_2 = 20 \qquad \text{uvjet} \qquad Q_1 + Q_2 = 20 \longrightarrow Q_2 = 20 - Q_1$$

$$T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1) + (20 - Q_1)^2 =$$

$$= 2Q_1^2 + 20Q_1 - Q_1^2 + 400 - 40Q_1 + Q_1^2 = 2Q_1^2 - 20Q_1 + 400$$

$$f(Q_1) = 2Q_1^2 - 20Q_1 + 400 \qquad f''(Q_1) = 4$$

$$f'(Q_1) = 4Q_1 - 20 \qquad f''(5) = 4 > 0 \longrightarrow \text{minimum}$$

$$4Q_1 - 20 = 0 \qquad Q_2 = 20 - 5 \checkmark \qquad Q_2 = 15$$

$$T(Q_1, Q_2) = 2Q_1^2 + Q_1Q_2 + Q_2^2$$
 funkcija
 $Q_1 + Q_2 = 20$ wyjet $Q_1 + Q_2 = 20$ $Q_2 = 20 - Q_1$
 $T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1) + (20 - Q_1)^2 =$
 $= 2Q_1^2 + 20Q_1 - Q_1^2 + 400 - 40Q_1 + Q_1^2 = 2Q_1^2 - 20Q_1 + 400$
 $f(Q_1) = 2Q_1^2 - 20Q_1 + 400$ $f''(Q_1) = 4$
 $f'(Q_1) = 4Q_1 - 20$ $f''(5) = 4 > 0$ minimum
 $4Q_1 - 20 = 0$ $Q_2 = 20 - 5$ $Q_2 = 15$
Stacionarna točka:

$$T(Q_1, Q_2) = 2Q_1^2 + Q_1Q_2 + Q_2^2$$
 funkcija
 $Q_1 + Q_2 = 20$ wyjet $Q_1 + Q_2 = 20$ $Q_2 = 20 - Q_1$
 $T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1) + (20 - Q_1)^2 =$
 $= 2Q_1^2 + 20Q_1 - Q_1^2 + 400 - 40Q_1 + Q_1^2 = 2Q_1^2 - 20Q_1 + 400$
 $f(Q_1) = 2Q_1^2 - 20Q_1 + 400$ $f''(Q_1) = 4$
 $f'(Q_1) = 4Q_1 - 20$ $f''(S) = 4 > 0$ minimum
 $4Q_1 - 20 = 0$ $Q_2 = 20 - 5$ $Q_2 = 15$
Stacionarna točka: $(5, 15)$

$$T(Q_1,Q_2)=2Q_1^2+Q_1Q_2+Q_2^2 \leftarrow \qquad \text{funkcija}$$
 $Q_1+Q_2=20 \leftarrow \text{uvjet} \qquad Q_1+Q_2=20 \longrightarrow Q_2=20-Q_1$
 $T(Q_1,20-Q_1)=2Q_1^2+Q_1\cdot(20-Q_1)+(20-Q_1)^2=$
 $=2Q_1^2+20Q_1-Q_1^2+400-40Q_1+Q_1^2=2Q_1^2-20Q_1+400$
 $f(Q_1)=2Q_1^2-20Q_1+400 \qquad f''(Q_1)=4$
 $f'(Q_1)=4Q_1-20 \qquad f''(5)=4>0 \longrightarrow \text{minimum}$
 $4Q_1-20=0 \qquad Q_2=20-5 \qquad Q_2=15$
 $Q_1=5 \qquad \text{Stacionarna točka: } (5,15)$
 $C(0,1)=1$

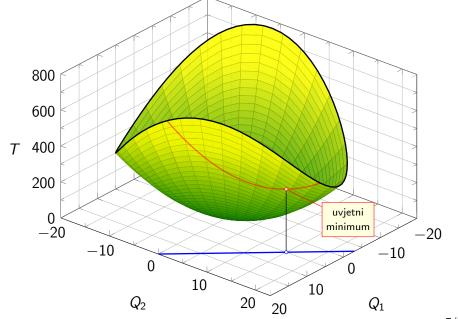
$$T(Q_1, Q_2) = 2Q_1^2 + Q_1Q_2 + Q_2^2$$
 funkcija
 $Q_1 + Q_2 = 20$ wyjet $Q_1 + Q_2 = 20$ $Q_2 = 20 - Q_1$
 $T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1) + (20 - Q_1)^2 =$
 $= 2Q_1^2 + 20Q_1 - Q_1^2 + 400 - 40Q_1 + Q_1^2 = 2Q_1^2 - 20Q_1 + 400$
 $f(Q_1) = 2Q_1^2 - 20Q_1 + 400$ $f''(Q_1) = 4$
 $f'(Q_1) = 4Q_1 - 20$ $f''(5) = 4 > 0$ minimum
 $4Q_1 - 20 = 0$ $Q_2 = 20 - 5$ $Q_2 = 15$
 $Q_1 = 5$ Stacionarna točka: $(5, 15)$

$$T(Q_1, Q_2) = 2Q_1^2 + Q_1Q_2 + Q_2^2$$
 funkcija
 $Q_1 + Q_2 = 20$ uvjet $Q_1 + Q_2 = 20$ $Q_2 = 20 - Q_1$
 $T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1) + (20 - Q_1)^2 =$
 $= 2Q_1^2 + 20Q_1 - Q_1^2 + 400 - 40Q_1 + Q_1^2 = 2Q_1^2 - 20Q_1 + 400$
 $f(Q_1) = 2Q_1^2 - 20Q_1 + 400$ $f''(Q_1) = 4$
 $f'(Q_1) = 4Q_1 - 20$ $f''(5) = 4 > 0$ minimum
 $4Q_1 - 20 = 0$ $Q_2 = 20 - 5$ $Q_2 = 15$
 $Q_1 = 5$ Stacionarna točka: $(5, 15)$

Rješenje $T(Q_1, Q_2) = 2Q_1^2 + Q_1Q_2 + Q_2^2 \leftarrow \text{funkcija}$

$$Q_1 + Q_2 = 20$$
 — uvjet $Q_1 + Q_2 = 20$ — $Q_2 = 20 - Q_1$ $T(Q_1, 20 - Q_1) = 2Q_1^2 + Q_1 \cdot (20 - Q_1) + (20 - Q_1)^2 = 2Q_1^2 + 20Q_1 - Q_1^2 + 400 - 40Q_1 + Q_1^2 = 2Q_1^2 - 20Q_1 + 400$ $f'(Q_1) = 2Q_1^2 - 20Q_1 + 400$ $f''(Q_1) = 4$ $f''(Q_1) = 4Q_1 - 20$ $f'''(5) = 4 > 0$ — minimum $Q_2 = 20 - 5$ $Q_2 = 15$ $Q_1 = 5$ Stacionarna točka: $Q_1 = 20$ Stacionarna točka: $Q_1 = 20$ $Q_2 = 20$ $Q_2 = 20$

Minimalni troškovi za 20 proizvoda iznose 350 novčanih jedinica, a postižu se za $Q_1=5$ i $Q_2=15$.



treći zadatak

Zadatak 3

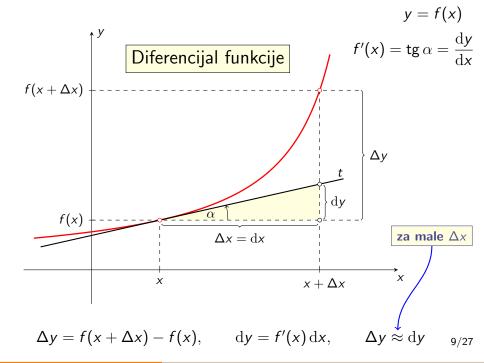
Zadana je funkcija ponude dobra D₁

$$s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$$

- u ovisnosti o cijenama p_1 i p_2 dobara D_1 i D_2 .
 - a) Za koliko se približno promijeni ponuda s_1 kada se cijena p_1 na nivou $p_1 = 1$, $p_2 = 2$ poveća za 0.02?
 - b) Za koliko se približno promijeni ponuda s_1 kada se cijena p_2 na nivou $p_1=1,\ p_2=2$ smanji za 0.01?
 - c) Za koliko se približno promijeni ponuda s₁ kada istovremeno napravimo promjene iz a) i b) dijela zadatka?

U svakom slučaju također izračunajte stvarne promjene ponude s_1 i usporedite ih s približnim promjenama koje su aproksimirane pomoću diferencijala.

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Rješenje
$$p_1 = 1, p_2 = 2,$$

a) Stvarna promjena ponude:

$$\Delta s_1 =$$

 $s_1(p_1,p_2) = 10\sqrt{p_1} - 2p_2^2$

Rješenje
$$p_1 = 1, p_2 = 2,$$

 $s_1(p_1,p_2) = 10\sqrt{p_1} - 2p_2^2$

$$\Delta s_1 =$$

Rješenje
$$p_1 = 1, p_2 = 2,$$

 $\Delta s_1 =$

 $s_1(p_1,p_2) = 10\sqrt{p_1} - 2p_2^2$

a) Stvarna promjena ponude: nova ponuda – stara ponuda

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Rješenje
$$p_1 = 1, p_2 = 2,$$

a) Stvarna promjena ponude: nova ponuda – stara ponuda

 $s_1(p_1,p_2) = 10\sqrt{p_1} - 2p_2^2$

$$\Delta s_1 = -s_1(1,2)$$

Rješenje
$$p_1 = 1, p_2 = 2, dp_1 = 0.02$$

 $s_1(p_1,p_2) = 10\sqrt{p_1} - 2p_2^2$

$$\Delta s_1 = -s_1(1,2)$$

Rješenje
$$p_1 = 1, p_2 = 2, dp_1 = 0.02$$

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

$$\Delta s_1 = s_1(1.02,2) - s_1(1,2)$$

nje $p_1 = 1, p_2 = 2, dp_1 = 0.02$

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

$$\Delta s_1 = s_1(1.02,2) - s_1(1,2) \approx$$

Rješenje
$$p_1 = 1, p_2 = 2, dp_1 = 0.02$$

$$\Delta s_1 = s_1(1.02, 2) - s_1(1, 2) \approx 2.0995$$

Rješenje
$$p_1 = 1, p_2 = 2, dp_1 = 0.02$$

$$\Delta s_1 = s_1(1.02, 2) - s_1(1, 2) \approx 2.0995 -$$

Rješenje
$$p_1 = 1, p_2 = 2, dp_1 = 0.02$$

$$\Delta s_1 = s_1(1.02, 2) - s_1(1, 2) \approx 2.0995 - 2$$

Rješenje
$$p_1 = 1, p_2 = 2, dp_1 = 0.02$$

$$\Delta s_1 = s_1(1.02, 2) - s_1(1, 2) \approx 2.0995 - 2 = 0.0995$$

$$\Delta s_1 = s_1(1.02,2) - s_1(1,2) \approx 2.0995 - 2 = 0.0995$$

Rješenje

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

Približna promjena ponude

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$

$$y = y(x), \quad dy = y'dx$$

$$\Delta s_1 = s_1(1.02,2) - s_1(1,2) pprox 2.0995 - 2 = 0.0995$$
Približna promjena ponude

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$

a) Stvarna promjena ponude: nova ponuda – stara ponuda

$$\Delta s_1 pprox rac{\partial s_1}{\partial p_1}(1,2) \cdot \mathrm{d}p_1$$

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

$$y = y(x), \quad dy = y'dx$$

$$\Delta y \approx \mathrm{d}y$$
, za male pomake $\mathrm{d}x$

$$\Delta s_1 = s_1(1)$$

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

$$\Delta s_1 = s_1(1.02, 2) - s_1(1, 2) \approx 2.0995 - 2 = 0.0995$$

Približna promjena ponude

$$\Delta s_1 pprox rac{\partial s_1}{\partial p_1} (1,2) \cdot \mathrm{d} p_1 \ rac{\partial s_1}{\partial p_1} =$$

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$

$$y = y(x), \quad dy = y'dx$$

$$\Delta s_1 = s_1(1)$$

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

$$\Delta s_1 = s_1(1.02, 2) - s_1(1, 2) \approx 2.0995 - 2 = 0.0995$$

$$rac{\partial s_1}{\partial
ho_1} = 10 \ \cdot$$

 $\Delta s_1 \approx \frac{\partial s_1}{\partial p_1} (1,2) \cdot \mathrm{d} p_1$

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$

$$y = y(x), \quad dy = y'dx$$

$$\Delta s_1 = s_1($$

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

$$\Delta s_1 = s_1(1.02, 2) - s_1(1, 2) \approx 2.0995 - 2 = 0.0995$$

$$egin{align} \Delta s_1 &pprox rac{\partial s_1}{\partial
ho_1}(1,2) \cdot \mathrm{d}
ho_1 \ & \ rac{\partial s_1}{\partial
ho_1} = 10 \cdot rac{1}{2\sqrt{
ho_1}} \ \end{aligned}$$

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$

$$y = y(x), \quad dy = y'dx$$

$$\Delta s_1 = s_1(1.02, 2) - s_1(1, 2) \approx 2.0995 - 2 = 0.0995$$

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$

a) Stvarna promjena ponude: nova ponuda – stara ponuda

$$\Delta s_1 pprox rac{\partial s_1}{\partial p_1} (1,2) \cdot \mathrm{d} p_1$$

$$y = y(x), \quad dy = y'dx$$

$$dx = \Delta y \approx dy$$
, za male pomake dx

 $\frac{\partial s_1}{\partial p_1} = 10 \cdot \frac{1}{2\sqrt{p_1}} = \frac{5}{\sqrt{p_1}}$

$$\Delta s_1 = s_1($$

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

$$\Delta s_1 = s_1(1.02, 2) - s_1(1, 2) \approx 2.0995 - 2 = 0.0995$$

Približna promjena ponude

$$egin{align} \Delta s_1 &pprox rac{\partial s_1}{\partial
ho_1} inom{
ho_1}{(1,2)} \cdot \mathrm{d}
ho_1 = \ & rac{\partial s_1}{\partial
ho_1} = 10 \cdot rac{1}{2\sqrt{
ho_1}} = rac{5}{\sqrt{
ho_1}} \end{split}$$

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$

$$y = y' d$$

$$\Delta s_1 = s_1$$

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

$$\Delta s_1 = s_1(1.02, 2) - s_1(1, 2) \approx 2.0995 - 2 = 0.0995$$

$$\Delta s_1 pprox rac{\partial s_1}{\partial p_1} rac{p_1}{(1,2)} \cdot \mathrm{d} p_1 = rac{5}{\sqrt{1}}$$

$$\frac{\partial s_1}{\partial p_1} = 10 \cdot \frac{1}{2\sqrt{p_1}} = \frac{5}{\sqrt{p_1}}$$

a) Stvarna promjena ponude: nova ponuda – stara ponuda

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$

$$y = y(x), dy = y'dx$$

$$\Delta s_1 = s_1(1.02, 2) - s_1(1, 2) \approx 2.0995 - 2 = 0.0995$$

Približna promjena ponude

Riešenie

$$\Delta s_1 pprox rac{\partial s_1}{\partial p_1} \stackrel{p_1}{(1,2)} \cdot \mathrm{d}p_1 = rac{5}{\sqrt{1}} \cdot 0.02$$

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

$$rac{\partial s_1}{\partial p_1} = 10 \cdot rac{1}{2\sqrt{p_1}} = rac{5}{\sqrt{p_1}}$$

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$

$$y = y(x)$$
, $dy = y'dx$ $\Delta y \approx dy$, za male pomake dx

$$\Delta s_1 = s_1(1.02, 2) - s_1(1, 2) \approx 2.0995 - 2 = 0.0995$$

Približna promjena ponude
$$\Delta s_1 \approx \frac{\partial s_1}{\partial p_1} {p_1 \choose 1, 2} \cdot \mathrm{d} p_1 = \frac{5}{\sqrt{1}} \cdot 0.02 = 0.1$$

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

$$\frac{\partial s_1}{\partial p_1} = 10 \cdot \frac{1}{2\sqrt{p_1}} = \frac{5}{\sqrt{p_1}}$$

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$

$$y = y(x)$$
, $dy = y'dx$ $\Delta y \approx dy$, za male pomake dx

Približna promjena ponude

Riešenie

 $\frac{\partial s_1}{\partial p_1} = 10 \cdot \frac{1}{2\sqrt{p_1}} = \frac{5}{\sqrt{p_1}}$

Ponuda dobra D_1 se poveća približno za 0.1.

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$

a) Stvarna promjena ponude: nova ponuda – stara ponuda

$$y = y(x), \quad dy = y'dx$$

$$y' dx$$
 $\Delta y \approx dy$, za male pomake dx

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

$$p_1 = 1, p_2 = 2,$$

b) Stvarna promjena ponude:

$$\Delta s_1 =$$

$$p_1 = 1, p_2 = 2,$$

$$s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$$

$$\Delta s_1 =$$

$$p_1 = 1, p_2 = 2,$$

 $\Delta s_1 =$

$$s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$$

$$p_1 = 1, p_2 = 2,$$

$$\Delta s_1 = -s_1(1,2)$$

$$p_1 = 1, \ p_2 = 2, \ \mathrm{d}p_2 = -0.01$$

$$\Delta s_1 = -s_1(1,2)$$

$$p_1 = 1, \ p_2 = 2, \ \mathrm{d}p_2 = -0.01$$

$$\Delta s_1 = s_1(1, 1.99) - s_1(1, 2)$$

$$p_1 = 1, \ p_2 = 2, \ \mathrm{d}p_2 = -0.01$$

$$\Delta s_1 = s_1(1, 1.99) - s_1(1, 2) =$$

$$p_1 = 1, \ p_2 = 2, \ \mathrm{d}p_2 = -0.01$$

$$\Delta s_1 = s_1(1, 1.99) - s_1(1, 2) = 2.0798$$

$$p_1 = 1, \ p_2 = 2, \ \mathrm{d}p_2 = -0.01$$

$$\Delta s_1 = s_1(1, 1.99) - s_1(1, 2) = 2.0798 -$$

$$p_1 = 1, \ p_2 = 2, \ \mathrm{d}p_2 = -0.01$$

$$s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$$

$$\Delta s_1 = s_1(1, 1.99) - s_1(1, 2) = 2.0798 - 2$$

$$p_1 = 1, \ p_2 = 2, \ \mathrm{d}p_2 = -0.01$$

$$s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$$

$$\Delta s_1 = s_1(1, 1.99) - s_1(1, 2) = 2.0798 - 2 = 0.0798$$

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

b) Stvarna promjena ponude: nova ponuda – stara ponuda

$$\Delta s_1 = s_1(1, 1.99) - s_1(1, 2) = 2.0798 - 2 = 0.0798$$

Približna promjena ponude

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

$$\Delta s_1 = s_1(1, 1.99) - s_1(1, 2) = 2.0798 - 2 = 0.0798$$

Približna promjena ponude

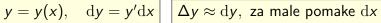
$$\Delta s_1 pprox rac{\partial s_1}{\partial p_2} (1,2) \cdot \mathrm{d} p_2$$

$$y = y(x), \quad dy = y'dx$$

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

$$\Delta s_1 pprox rac{\partial s_1}{\partial p_2} (1,2) \cdot \mathrm{d} p_2 \ \partial s_1$$

$$y = y'd$$



 $\Delta s_1 = s_1(1, 1.99) - s_1(1, 2) = 2.0798 - 2 = 0.0798$

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

 $\Delta s_1 = s_1(1, 1.99) - s_1(1, 2) = 2.0798 - 2 = 0.0798$

$$\Delta s_1 = s_1(1, 1.99) - s_1(1, 2) = 2.0798 - 2 = 0.0798$$

Približna promjena ponude

$$egin{align} \Delta s_1 &pprox rac{\partial s_1}{\partial
ho_2} (1,2) \cdot \mathrm{d}
ho_2 \ & \ rac{\partial s_1}{\partial
ho_2} = 0 - \ \end{align*}$$

$$y = y(x)$$
, $dy = y'dx$ $\Delta y \approx dy$, za male pomake dx

 $\Delta s_1 \approx \frac{\partial s_1}{\partial p_2} (1,2) \cdot \mathrm{d} p_2$

 $p_1 = 1$, $p_2 = 2$, $dp_2 = -0.01$

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

$$\frac{\partial s_1}{\partial p_2} = 0 - 4p_2$$

y = y(x), dy = y'dx

 $\Delta y \approx \mathrm{d}y$, za male pomake $\mathrm{d}x$

$$\Delta s_1 = s_1(1, 1.99) - s_1(1, 2) = 2.0798 - 2 = 0.0798$$

Približna promjena ponude

$$\Delta s_1 pprox rac{\partial s_1}{\partial p_2} (1,2) \cdot \mathrm{d} p_2$$

 $p_1 = 1$, $p_2 = 2$, $dp_2 = -0.01$

$$\frac{\partial s_1}{\partial p_2} = 0 - 4p_2 = -4p_2$$

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

b) Stvarna promjena ponude: nova ponuda – stara ponuda

$$\Delta s_1 = s_1(1, 1.99) - s_1(1, 2) = 2.0798 - 2 = 0.0798$$

Približna promjena ponude

$$\Delta s_1 pprox rac{\partial s_1}{\partial
ho_2} \stackrel{oldsymbol{
ho_1}}{(1,2)} \cdot \mathrm{d}
ho_2 =$$

$$\frac{\partial s_1}{\partial p_2} = 0 - 4p_2 = -4p_2$$

$$y = y(x), dy = y'dx$$

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

b) Stvarna promjena ponude: nova ponuda — stara ponuda

$$\Delta s_1 = s_1(1, 1.99) - s_1(1, 2) = 2.0798 - 2 = 0.0798$$

Približna promjena ponude

$$\Delta s_1 pprox \frac{\partial s_1}{\partial p_2} (1,2) \cdot \mathrm{d}p_2 = -4 \cdot 2$$

$$\frac{\partial s_1}{\partial p_2} = 0 - 4p_2 = -4p_2$$

$$y = y(x), \quad dy = y'dx$$

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

$$egin{align} \Delta s_1 &pprox rac{\partial s_1}{\partial p_2} (1,2) \cdot \mathrm{d} p_2 = -4 \cdot 2 \cdot (-0.01) \ & rac{\partial s_1}{\partial p_2} = 0 - 4 p_2 = -4 p_2 \end{gathered}$$

$$= y' dx$$

 $\Delta s_1 = s_1(1, 1.99) - s_1(1, 2) = 2.0798 - 2 = 0.0798$

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

,

Priblizna promjena ponud

$$\Delta s_1 pprox rac{\partial s_1}{\partial p_2} \stackrel{oldsymbol{p}_1}{(1,2)} \cdot \mathrm{d} p_2 = -4 \cdot 2 \cdot (-0.01) = 0.08$$
 $rac{\partial s_1}{\partial p_2} = 0 - 4p_2 = -4p_2$

$$y = y(x), \quad dy = y'dx$$

 $\Delta s_1 = s_1(1, 1.99) - s_1(1, 2) = 2.0798 - 2 = 0.0798$

 $p_1 = 1$, $p_2 = 2$, $dp_2 = -0.01$

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

Približna promjena ponude

$$\Delta s_1 pprox rac{\partial s_1}{\partial p_2} \stackrel{p_1}{(1,2)} \cdot \mathrm{d}p_2 = -4 \cdot 2 \cdot (-0.01) = 0.08$$

$$\frac{\partial s_1}{\partial p_2} = 0 - 4p_2 = -4p_2$$

Ponuda dobra D_1 se poveća približno za 0.08.

$$y = y(x)$$
, $dy = y'dx$ $\Delta y \approx dy$, za male pomake dx

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$, $dp_2 = -0.01$

c) Stvarna promjena ponude:

 $\Delta s_1 =$

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 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$, $dp_2 = -0.01$

 $\Delta s_1 =$

c) Stvarna promjena ponude: nova ponuda – stara ponuda

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

c) Stvarna promjena ponude: nova ponuda – stara ponuda

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$, $dp_2 = -0.01$

 $\Delta s_1 =$

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

 $-s_1(1,2)$

c) Stvarna promjena ponude: nova ponuda – stara ponuda

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$, $dp_2 = -0.01$

 $\Delta s_1 =$

 $|s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2|$

c) Stvarna promjena ponude: nova ponuda – stara ponuda

 $\Delta s_1 = s_1(1.02, 1.99) - s_1(1, 2)$

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$, $dp_2 = -0.01$

 $|s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2|$

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$, $dp_2 = -0.01$

c) Stvarna promjena ponude: nova ponuda – stara ponuda

 $\Delta s_1 = s_1(1.02, 1.99) - s_1(1, 2) \approx$

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 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$, $dp_2 = -0.01$

c) Stvarna promjena ponude: nova ponuda – stara ponuda

 $\Delta s_1 = s_1(1.02, 1.99) - s_1(1, 2) \approx 2.1793$

 $|s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2|$

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 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$, $dp_2 = -0.01$

c) Stvarna promjena ponude: nova ponuda – stara ponuda

 $\Delta s_1 = s_1(1.02, 1.99) - s_1(1, 2) \approx 2.1793 -$

 $|s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2|$

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$, $dp_2 = -0.01$

c) Stvarna promjena ponude: nova ponuda – stara ponuda

 $\Delta s_1 = s_1(1.02, 1.99) - s_1(1, 2) \approx 2.1793 - 2$

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 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

 $\Delta s_1 = s_1(1.02, 1.99) - s_1(1, 2) \approx 2.1793 - 2 = 0.1793$

c) Stvarna promjena ponude: nova ponuda – stara ponuda

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$, $dp_2 = -0.01$

 $s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2$

 $\Delta s_1 = s_1(1.02, 1.99) - s_1(1, 2) \approx 2.1793 - 2 = 0.1793$

c) Stvarna promjena ponude: nova ponuda – stara ponuda

$$\Delta z = z(x + \Delta x, y + \Delta y) - z(x, y)$$

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$, $dp_2 = -0.01$

Približna promjena ponude

$$z = z(x, y), \quad dz = z_x dx + z_y dy$$
 $\Delta z \approx dz$

 $|s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2|$

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$, $dp_2 = -0.01$

$$\Delta z = z(x + \Delta x, y + \Delta y) - z(x, y)$$

c) Stvarna promjena ponude: nova ponuda – stara ponuda

 $\Delta s_1 \approx \frac{\partial s_1}{\partial p_1}(1,2) \cdot \mathrm{d}p_1 + \frac{\partial s_1}{\partial p_2}(1,2) \cdot \mathrm{d}p_2$

$$z = z(x, y), \quad dz = z_x dx + z_y dy$$

$$\Delta z \approx dz$$

c) Stvarna promjena ponude: nova ponuda – stara ponuda

 $\Delta s_1 \approx \frac{\partial s_1}{\partial p_1}(1,2) \cdot \mathrm{d}p_1 + \frac{\partial s_1}{\partial p_2}(1,2) \cdot \mathrm{d}p_2$

 $\Delta s_1 \approx$

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$, $dp_2 = -0.01$

$$\Delta z = z(x + \Delta x, y + \Delta y) - z(x, y)$$

$$\Delta z = z(x + \Delta x, y + \Delta y) - z(x, y)$$

$$z = z(x, y), \quad dz = z_x dx + z_y dy$$
 $\Delta z \approx dz$

c) Stvarna promjena ponude: nova ponuda – stara ponuda

 $\Delta s_1 \approx \frac{\partial s_1}{\partial p_1}(1,2) \cdot \mathrm{d}p_1 + \frac{\partial s_1}{\partial p_2}(1,2) \cdot \mathrm{d}p_2$

 $\Delta s_1 \approx 5$

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$, $dp_2 = -0.01$

$$\Delta z = z(x + \Delta x, y + \Delta y) - z(x, y)$$

$$z = z(x + \Delta x, y + \Delta y) - z(x, y)$$

$$z = z(x, y), \quad dz = z_x dx + z_y dy$$
 $\Delta z \approx dz$

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c) Stvarna promjena ponude: nova ponuda – stara ponuda

 $\Delta s_1 \approx \frac{\partial s_1}{\partial p_1}(1,2) \cdot \mathrm{d}p_1 + \frac{\partial s_1}{\partial p_2}(1,2) \cdot \mathrm{d}p_2$

 $z = z(x, y), \quad dz = z_x dx + z_y dy \quad \Delta z \approx dz$

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$, $dp_2 = -0.01$

$$\Delta z = z(x + \Delta x, y + \Delta y) - z(x, y)$$

 $\Delta s_1 \approx 5$.

$$\frac{12-2(\lambda+\Delta\lambda,y+\Delta y)-2(\lambda,y)}{2}$$

c) Stvarna promjena ponude: nova ponuda – stara ponuda

 $\Delta s_1 pprox \frac{\partial s_1}{\partial p_1}(1,2) \cdot \mathrm{d}p_1 + \frac{\partial s_1}{\partial p_2}(1,2) \cdot \mathrm{d}p_2$

 $\Delta s_1 \approx 5 \cdot 0.02$

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$, $dp_2 = -0.01$

$$\Delta z = z(x + \Delta x, y + \Delta y) - z(x, y)$$

$$z = z(x, y), \quad dz = z_x dx + z_y dy$$
 $\Delta z \approx dz$

c) Stvarna promjena ponude: nova ponuda – stara ponuda

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$, $dp_2 = -0.01$

$$\Delta s_1 \approx \frac{\partial s_1}{\partial p_1} (1, 2) \cdot dp_1 + \frac{\partial s_1}{\partial p_2} (1, 2) \cdot dp_2$$

$$\Delta s_1 \approx 5 \cdot 0.02 +$$

$$\Delta z = z(x + \Delta x, y + \Delta y) - z(x, y)$$

$$\Delta z = z(x + \Delta x, y + \Delta y) - z(x, y)$$

$$z = z(x, y), \quad dz = z_x dx + z_y dy$$
 $\Delta z \approx dz$

c) Stvarna promjena ponude: nova ponuda – stara ponuda

 $\Delta s_1 \approx \frac{\partial s_1}{\partial p_1}(1,2) \cdot \mathrm{d}p_1 + \frac{\partial s_1}{\partial p_2}(1,2) \cdot \mathrm{d}p_2$

 $\Delta s_1 \approx 5 \cdot 0.02 + (-8)$

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$, $dp_2 = -0.01$

$$\Delta z = z(x + \Delta x, y + \Delta y) - z(x, y)$$

$$z = z(x, y), \quad dz = z_x dx + z_y dy$$
 $\Delta z \approx dz$

$$\Delta s_1 = s_1(1.02, 1.99) - s_1(1, 2) pprox 2.1793 - 2 = 0.1793$$

Približna promjena ponude

c) Stvarna promjena ponude: nova ponuda – stara ponuda

 $\Delta s_1 \approx \frac{\partial s_1}{\partial p_1}(1,2) \cdot \mathrm{d}p_1 + \frac{\partial s_1}{\partial p_2}(1,2) \cdot \mathrm{d}p_2$

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$, $dp_2 = -0.01$

$$\Delta z = z(x + \Delta x, y + \Delta y) - z(x, y)$$

 $\Delta s_1 \approx 5 \cdot 0.02 + (-8)$

$$z = z(x, y), \quad dz = z_x dx + z_y dy$$
 $\Delta z \approx dz$

$$\Delta s_1 = s_1(1.02, 1.99) - s_1(1, 2) \approx 2.1793 - 2 = 0.1793$$
Približna promjena ponude

c) Stvarna promjena ponude: nova ponuda – stara ponuda

 $\Delta s_1 \approx \frac{\partial s_1}{\partial p_1}(1,2) \cdot \mathrm{d}p_1 + \frac{\partial s_1}{\partial p_2}(1,2) \cdot \mathrm{d}p_2$

 $\Delta s_1 \approx 5 \cdot 0.02 + (-8) \cdot (-0.01)$

 $|s_1(p_1, p_2) = 10\sqrt{p_1} - 2p_2^2|$

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$, $dp_2 = -0.01$

$$\Delta z = z(x + \Delta x, y + \Delta y) - z(x, y)$$

 $z = z(x, y), \quad dz = z_x dx + z_y dy$ $\Delta z \approx dz$

$$\Delta s_1 = s_1(1.02, 1.99) - s_1(1, 2) \approx 2.1793 - 2 = 0.1793$$
Približna promjena ponude

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$, $dp_2 = -0.01$

 $\Delta s_1 \approx \frac{\partial s_1}{\partial p_1}(1,2) \cdot \mathrm{d}p_1 + \frac{\partial s_1}{\partial p_2}(1,2) \cdot \mathrm{d}p_2$

 $\Delta s_1 \approx 5 \cdot 0.02 + (-8) \cdot (-0.01) = 0.1 + 0.08$

c) Stvarna promjena ponude: nova ponuda – stara ponuda

$$\Delta z = z(x + \Delta x, y + \Delta y) - z(x, y)$$

$$z = z(x, y), \quad dz = z_x dx + z_y dy \quad \Delta z \approx dz$$

$$\Delta s_1 = s_1(1.02, 1.99) - s_1(1, 2) \approx 2.1793 - 2 = 0.1793$$
Približna promjena ponude

 $\Delta s_1 \approx 5 \cdot 0.02 + (-8) \cdot (-0.01) = 0.1 + 0.08 = 0.18$

c) Stvarna promjena ponude: nova ponuda – stara ponuda

 $\Delta s_1 \approx \frac{\partial s_1}{\partial p_1}(1,2) \cdot \mathrm{d}p_1 + \frac{\partial s_1}{\partial p_2}(1,2) \cdot \mathrm{d}p_2$

 $\Delta z = z(x + \Delta x, y + \Delta y) - z(x, y)$

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$, $dp_2 = -0.01$

$$z = z(x, y), \quad dz = z_x dx + z_y dy$$
 $\Delta z \approx dz$

c) Stvarna promjena ponude: nova ponuda – stara ponuda

 $\Delta s_1 \approx \frac{\partial s_1}{\partial p_1}(1,2) \cdot \mathrm{d}p_1 + \frac{\partial s_1}{\partial p_2}(1,2) \cdot \mathrm{d}p_2$

Približna promjena ponude

$$\Delta s_1 pprox 5 \cdot 0.02 + (-8) \cdot (-0.01) = 0.1 + 0.08 = 0.18$$

Ponuda dobra D_1 se poveća približno za 0.18

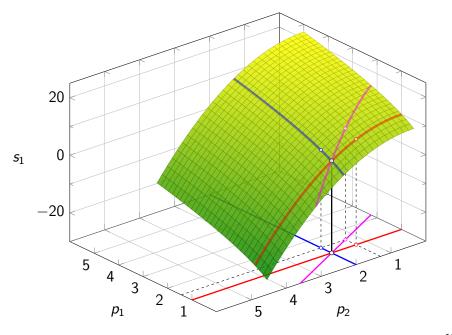
Ponuda dobra D_1 se poveća približno za 0.18.

 $p_1 = 1$, $p_2 = 2$, $dp_1 = 0.02$, $dp_2 = -0.01$

$$\Delta z = z(x + \Delta x, y + \Delta y) - z(x, y)$$

 $z = z(x, y), \quad dz = z_x dx + z_y dy \quad \Delta z \approx dz$

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četvrti zadatak

Izračunajte koeficijente parcijalnih elastičnosti funkcije

$$f(x,y) = \sqrt{x - y^2}$$

na nivou x = 25, y = 3 i interpretirajte dobivene rezultate.

Izračunajte koeficijente parcijalnih elastičnosti funkcije

$$f(x,y) = \sqrt{x - y^2}$$

na nivou x = 25, y = 3 i interpretirajte dobivene rezultate.

$$f_x =$$

Izračunajte koeficijente parcijalnih elastičnosti funkcije

$$f(x,y) = \sqrt{x - y^2}$$

na nivou x = 25, y = 3 i interpretirajte dobivene rezultate.

$$f_x = \frac{1}{2\sqrt{x - y^2}}$$

Izračunajte koeficijente parcijalnih elastičnosti funkcije

$$f(x,y) = \sqrt{x - y^2}$$

na nivou x = 25, y = 3 i interpretirajte dobivene rezultate.

$$f_{x} = \frac{1}{2\sqrt{x - y^2}} \, \cdot$$

Izračunajte koeficijente parcijalnih elastičnosti funkcije

$$f(x,y) = \sqrt{x - y^2}$$

na nivou x = 25, y = 3 i interpretirajte dobivene rezultate.

$$f_{x} = \frac{1}{2\sqrt{x - v^2}} \cdot 1$$

Izračunajte koeficijente parcijalnih elastičnosti funkcije

$$f(x,y) = \sqrt{x - y^2}$$

na nivou x = 25, y = 3 i interpretirajte dobivene rezultate.

$$f_{x} = \frac{1}{2\sqrt{x - y^{2}}} \cdot 1 = \frac{1}{2\sqrt{x - y^{2}}}$$

Izračunajte koeficijente parcijalnih elastičnosti funkcije

$$f(x,y) = \sqrt{x - y^2}$$

na nivou x = 25, y = 3 i interpretirajte dobivene rezultate.

$$f_x = \frac{1}{2\sqrt{x - y^2}} \cdot 1 = \frac{1}{2\sqrt{x - y^2}}$$

$$f_y =$$

Izračunajte koeficijente parcijalnih elastičnosti funkcije

$$f(x,y) = \sqrt{x - y^2}$$

na nivou x = 25, y = 3 i interpretirajte dobivene rezultate.

$$f_x = \frac{1}{2\sqrt{x - y^2}} \cdot 1 = \frac{1}{2\sqrt{x - y^2}}$$

$$f_y = \frac{1}{2\sqrt{x - y^2}}$$

Izračunajte koeficijente parcijalnih elastičnosti funkcije

$$f(x,y) = \sqrt{x - y^2}$$

na nivou x = 25, y = 3 i interpretirajte dobivene rezultate.

$$f_x = \frac{1}{2\sqrt{x - y^2}} \cdot 1 = \frac{1}{2\sqrt{x - y^2}}$$
 $f_y = \frac{1}{2\sqrt{x - y^2}}$

Izračunajte koeficijente parcijalnih elastičnosti funkcije

$$f(x,y) = \sqrt{x - y^2}$$

na nivou x = 25, y = 3 i interpretirajte dobivene rezultate.

$$f_x = \frac{1}{2\sqrt{x - y^2}} \cdot 1 = \frac{1}{2\sqrt{x - y^2}}$$
 $f_y = \frac{1}{2\sqrt{x - y^2}} \cdot (-2y)$

Zadatak 4

Izračunajte koeficijente parcijalnih elastičnosti funkcije

$$f(x,y) = \sqrt{x - y^2}$$

na nivou x = 25, y = 3 i interpretirajte dobivene rezultate.

Rješenje

$$f_x = \frac{1}{2\sqrt{x - y^2}} \cdot 1 = \frac{1}{2\sqrt{x - y^2}}$$

$$f_y = \frac{1}{2\sqrt{x - y^2}} \cdot (-2y) = \frac{-y}{\sqrt{x - y^2}}$$

 $f(x,y) = \sqrt{x - y^2}$

 $E_{f,x} = \frac{x}{f} \cdot f_x$

 $E_{f,x}(25,3) =$

 $E_{f,x}(25,3) = -$

 $f(x,y) = \sqrt{x-y^2}$ $f_x = \frac{1}{2\sqrt{x-y^2}}$

 $E_{f,x} = \frac{x}{f} \cdot f_x$

 $E_{f,x}(25,3) = \frac{25}{}$

 $f(x,y) = \sqrt{x - y^2}$ $f_x = \frac{1}{2\sqrt{x - y^2}}$

 $E_{f,x} = \frac{x}{f} \cdot f_x$

 $E_{f,x}(25,3) = \frac{25}{f(25,3)}$

 $f(x,y) = \sqrt{x - y^2}$ $f_x = \frac{1}{2\sqrt{x - y^2}}$

 $E_{f,x} = \frac{x}{f} \cdot f_x$

 $E_{f,x}(25,3) = \frac{25}{f(25,3)}$.

 $f(x,y) = \sqrt{x-y^2}$ $f_x = \frac{1}{2\sqrt{x-y^2}}$

 $E_{f,x} = \frac{x}{f} \cdot f_x$

 $E_{f,x}(25,3) = \frac{25}{f(25,3)} \cdot f_x(25,3)$

 $f(x,y) = \sqrt{x-y^2}$ $f_x = \frac{1}{2\sqrt{x-y^2}}$

$$f(25,3) =$$

 $f(x,y) = \sqrt{x-y^2} \qquad f_x = \frac{1}{2\sqrt{x-y^2}}$

$$f(25,3) = \sqrt{25 - 3^2}$$

 $f(x,y) = \sqrt{x-y^2} \qquad f_x = \frac{1}{2\sqrt{x-y^2}}$

$$f(25,3) = \sqrt{25 - 3^2} = \sqrt{16}$$

 $f(x,y) = \sqrt{x-y^2}$ $f_x = \frac{1}{2\sqrt{x-y^2}}$

$$f(25,3) = \sqrt{25-3^2} = \sqrt{16} = 4$$

 $f(x,y) = \sqrt{x-y^2}$ $f_x = \frac{1}{2\sqrt{x-y^2}}$

$$E_{f,x} = \frac{x}{f} \cdot f_x$$

 $f_{x}(25,3) =$

$$E_{f,x}(25,3) = \frac{25}{f(25,3)} \cdot f_x(25,3) =$$

$$f(25,3) = \sqrt{25-3^2} = \sqrt{16} = 4$$

$$f_x(25,3) =$$

$$E_{f,x} = \frac{x}{f} \cdot f_x$$

- 3²

$$E_{f,x}(25,3) = \frac{25}{f(25,3)} \cdot f_x(25,3) =$$

$$f(25,3) = \sqrt{25 - 3^2} = \sqrt{16} = 4$$

$$f_x(25,3) = \frac{1}{2\sqrt{25 - 3^2}}$$

$$E_{f,x} = \frac{x}{f} \cdot f_x$$

 $E_{f,x}(\overset{x}{25},\overset{y}{3}) = \frac{25}{f(25.3)} \cdot f_x(25,3) =$

 $f(25,3) = \sqrt{25-3^2} = \sqrt{16} = 4$

 $f_{x}(25,3) = \frac{1}{2\sqrt{25-3^2}} = \frac{1}{2\sqrt{16}}$

$$E_{f,x} = \frac{x}{f} \cdot f_x$$

 $f(25,3) = \sqrt{25-3^2} = \sqrt{16} = 4$

 $f_{x}(25,3) = \frac{1}{2\sqrt{25-3^2}} = \frac{1}{2\sqrt{16}} = \frac{1}{8}$

$$E_{f,x} = \frac{x}{f} \cdot f_x$$

 $f_{x}(25,3) = \frac{1}{2\sqrt{25-3^2}} = \frac{1}{2\sqrt{16}} = \frac{1}{8}$

$$E_{f,x}(25,3) = \frac{25}{f(25,3)} \cdot f_x(25,3) = \frac{25}{f(25,3)}$$
$$f(25,3) = \sqrt{25-3^2} = \sqrt{16} = 4$$

$$E_{f,x} = \frac{x}{f} \cdot f_x$$

- 5 2 10 0

$$E_{f,x}(25,3) = \frac{25}{f(25,3)} \cdot f_x(25,3) = \frac{25}{4}$$

$$f(25,3) = \sqrt{25 - 3^2} = \sqrt{16} = 4$$

$$f_x(25,3) = \frac{1}{2\sqrt{25 - 3^2}} = \frac{1}{2\sqrt{16}} = \frac{1}{8}$$

$$E_{f,x} = \frac{x}{f} \cdot f_x$$

 $E_{f,x}(\overset{x}{25},\overset{y}{3})=\frac{25}{f(25.3)}\cdot f_x(25,3)=\frac{25}{4}$

 $f(25,3) = \sqrt{25-3^2} = \sqrt{16} = 4$

 $f_x(25,3) = \frac{1}{2\sqrt{25-3^2}} = \frac{1}{2\sqrt{16}} = \frac{1}{8}$

$$E_{f,x} = \frac{x}{f} \cdot f_x$$

 $f_{x}(25,3) = \frac{1}{2\sqrt{25-3^2}} = \frac{1}{2\sqrt{16}} = \frac{1}{8}$

 $f(x,y) = \sqrt{x-y^2}$ $f_x = \frac{1}{2\sqrt{x-y^2}}$

$$E_{f,x}(25,3) = \frac{25}{f(25,3)} \cdot f_x(25,3) = \frac{25}{4} \cdot \frac{1}{8}$$
$$f(25,3) = \sqrt{25 - 3^2} = \sqrt{16} = 4$$

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$$E_{f,x} = \frac{x}{f} \cdot f_x$$

 $f(x,y) = \sqrt{x-y^2}$ $f_x = \frac{1}{2\sqrt{x-y^2}}$

$$f_x(25,3) = \frac{1}{2\sqrt{25-3^2}} = \frac{1}{2\sqrt{16}} = \frac{1}{8}$$

 $E_{f,x}(25,3) = \frac{25}{f(25,3)} \cdot f_x(25,3) = \frac{25}{4} \cdot \frac{1}{8} = \frac{25}{32}$

 $f(25,3) = \sqrt{25-3^2} = \sqrt{16} = 4$

$$E_{f,x} = \frac{x}{f} \cdot f_x$$

 $f_x(25,3) = \frac{1}{2\sqrt{25-3^2}} = \frac{1}{2\sqrt{16}} = \frac{1}{8}$

 $E_{f,x}(25,3) = \frac{25}{f(25,3)} \cdot f_x(25,3) = \frac{25}{4} \cdot \frac{1}{8} = \frac{25}{32} = 0.78125$

 $f(25,3) = \sqrt{25-3^2} = \sqrt{16} = 4$

$$f_x(25,3) = \frac{1}{2\sqrt{25-3^2}} = \frac{1}{2\sqrt{16}} = \frac{1}{8}$$

 $f(x,y) = \sqrt{x-y^2}$ $f_x = \frac{1}{2\sqrt{x-y^2}}$

Ako na nivou (25,3) varijablu x povećamo za 1%, funkcija f će se povećati za 0.78125%.

 $f(25,3) = \sqrt{25-3^2} = \sqrt{16} = 4$

 $E_{f,x}(25,3) = \frac{25}{f(25,3)} \cdot f_x(25,3) = \frac{25}{4} \cdot \frac{1}{8} = \frac{25}{32} = 0.78125$

 $f(x,y) = \sqrt{x - y^2}$

 $E_{f,y} = \frac{y}{f} \cdot f_y$

 $E_{f,y}(25,3) =$

 $f(x,y) = \sqrt{x - y^2}$

 $E_{f,y} = \frac{y}{f} \cdot f_y$

 $E_{f,y}(25,3) = -$

 $E_{f,y}(25,3) = \frac{3}{}$

 $f(x,y) = \sqrt{x - y^2} \qquad f_y = \frac{-y}{\sqrt{x - y^2}}$

 $E_{f,y}(25,3) = \frac{3}{f(25,3)}$

 $f(x,y) = \sqrt{x - y^2} \qquad f_y = \frac{-y}{\sqrt{x - y^2}}$

$$E_{f,y}(25,3) = \frac{3}{f(25,3)}$$
.

 $f(x,y) = \sqrt{x - y^2} \qquad f_y = \frac{-y}{\sqrt{x - y^2}}$

 $f(x,y) = \sqrt{x - y^2} \quad \left| f_y = \frac{-y}{\sqrt{x - y^2}} \right|$

 $E_{f,y} = \frac{y}{f} \cdot f_y$

$$f(25,3) =$$

 $f(x,y) = \sqrt{x - y^2}$ $f_y = \frac{-y}{\sqrt{x - y^2}}$

$$f(25,3) = \sqrt{25-3^2}$$

 $f(x,y) = \sqrt{x - y^2} \qquad f_y = \frac{-y}{\sqrt{x - y^2}}$

$$E_{f,y}(25,3) = \frac{3}{f(25,3)} \cdot f_y(25,3) = f(25,3) = \sqrt{25-3^2} = \sqrt{16}$$

$$f(25,3) = \sqrt{25-3^2} = \sqrt{16} = 4$$

 $E_{f,y}(\overset{X}{25},\overset{Y}{3}) = \frac{3}{f(25,3)} \cdot f_y(25,3) =$

 $E_{f,y} = \frac{y}{f} \cdot f_y$

 $f(x,y) = \sqrt{x-y^2}$ $f_y = \frac{-y}{\sqrt{x-y^2}}$

$$f(25,3) = \sqrt{25-3^2} = \sqrt{16} = 4$$

 $f_y(25,3) =$

 $E_{f,y}(25,3) = \frac{3}{f(25,3)} \cdot f_y(25,3) =$

$$E_{f,y} = \frac{y}{f} \cdot f_y$$

$$E_{f,y}(25,3) = \frac{3}{f(25,3)} \cdot f_y(25,3) =$$

$$f(25,3) = \sqrt{25 - 3^2} = \sqrt{16} = 4$$

$$f_y(25,3) = \frac{-3}{\sqrt{25 - 3^2}}$$

$$E_{f,y} = \frac{y}{f} \cdot f_y$$

$$\sqrt{16}$$

 $f(x,y) = \sqrt{x-y^2}$ $f_y = \frac{-y}{\sqrt{x-y^2}}$

$$E_{f,y}(25,3) = \frac{3}{f(25,3)} \cdot f_y(25,3) =$$

$$f(25,3) = \sqrt{25 - 3^2} = \sqrt{16} = 4$$

$$f_y(25,3) = \frac{-3}{\sqrt{25 - 3^2}} = \frac{-3}{\sqrt{16}}$$

$$E_{f,y} = \frac{y}{f} \cdot f_y$$

$$\frac{-3}{\sqrt{16}} = \frac{-3}{4}$$

 $f(x,y) = \sqrt{x-y^2}$ $f_y = \frac{-y}{\sqrt{x-y^2}}$

$$E_{f,y} = \frac{y}{f} \cdot f_y$$

$$\frac{-3}{\sqrt{16}} = \frac{-3}{4}$$

 $f(x,y) = \sqrt{x - y^2}$ $f_y = \frac{-y}{\sqrt{x - y^2}}$

$$E_{f,y}(25,3) = \frac{3}{f(25,3)} \cdot f_y(25,3) = \frac{3}{f(25,3)} \cdot f_y(25,3) = \frac{3}{f(25,3)} = \frac{3}$$

$$E_{f,y} = \frac{y}{f} \cdot f_y$$

 $f(x,y) = \sqrt{x - y^2} \qquad f_y = \frac{-y}{\sqrt{x - y^2}}$

$$E_{f,y}(25,3) = \frac{3}{f(25,3)} \cdot f_y(25,3) = \frac{3}{4}$$

$$f(25,3) = \sqrt{25 - 3^2} = \sqrt{16} = 4$$

$$f_y(25,3) = \frac{-3}{\sqrt{25 - 3^2}} = \frac{-3}{\sqrt{16}} = \frac{-3}{4}$$

$$E_{f,y} = \frac{y}{f} \cdot f_y$$

$$f_y(25,3) = \frac{-3}{\sqrt{25-3^2}} = \frac{-3}{\sqrt{16}} = \frac{-3}{4}$$

 $f(x,y) = \sqrt{x - y^2}$ $f_y = \frac{-y}{\sqrt{x - y^2}}$

$$E_{f,y}(25,3) = \frac{3}{f(25,3)} \cdot f_y(25,3) = \frac{3}{4} \cdot f_y(25,3) = \frac{3}{4} \cdot f_y(25,3) = \sqrt{25-3^2} = \sqrt{16} = 4$$

$$E_{f,y} = \frac{y}{f} \cdot f_y$$

$$f_y(25,3) = \frac{-3}{\sqrt{25-3^2}} = \frac{-3}{\sqrt{16}} = \frac{-3}{4}$$

 $f(x,y) = \sqrt{x - y^2}$ $f_y = \frac{-y}{\sqrt{x - v^2}}$

$$E_{f,y}(25,3) = \frac{3}{f(25,3)} \cdot f_y(25,3) = \frac{3}{4} \cdot \frac{-3}{4}$$
$$f(25,3) = \sqrt{25-3^2} = \sqrt{16} = 4$$

$$E_{f,y} = \frac{y}{f} \cdot f_y$$

$$f_y(25,3) = \frac{-3}{\sqrt{25-3^2}} = \frac{-3}{\sqrt{16}} = \frac{-3}{4}$$

 $f(x,y) = \sqrt{x - y^2} \qquad f_y = \frac{-y}{\sqrt{x - y^2}}$

$$E_{f,y}(25,3) = \frac{3}{f(25,3)} \cdot f_y(25,3) = \frac{3}{4} \cdot \frac{-3}{4} = \frac{-9}{16}$$
$$f(25,3) = \sqrt{25-3^2} = \sqrt{16} = 4$$

$$E_{f,y} = \frac{y}{f} \cdot f_y$$

 $f(x,y) = \sqrt{x - y^2}$ $f_y = \frac{-y}{\sqrt{x - y^2}}$

 $E_{f,y}(25,3) = \frac{3}{f(25,3)} \cdot f_y(25,3) = \frac{3}{4} \cdot \frac{-3}{4} = \frac{-9}{16} = -0.5625$

 $f(25,3) = \sqrt{25-3^2} = \sqrt{16} = 4$

 $f_y(25,3) = \frac{-3}{\sqrt{25-3^2}} = \frac{-3}{\sqrt{16}} = \frac{-3}{4}$

 $E_{f,y} = \frac{y}{f} \cdot f_y$

$$f_y(25,3) = \frac{-3}{\sqrt{25-3^2}} = \frac{-3}{\sqrt{16}} = \frac{-3}{4}$$

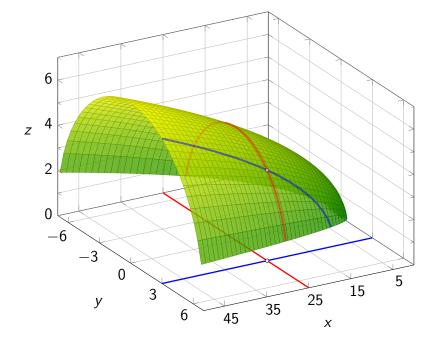
 $f(x,y) = \sqrt{x - y^2}$ $f_y = \frac{-y}{\sqrt{x - y^2}}$

Ako na nivou
$$(25,3)$$
 varijablu y povećamo za 1% , funkcija f će se

 $E_{f,y}(25,3) = \frac{3}{f(25,3)} \cdot f_y(25,3) = \frac{3}{4} \cdot \frac{-3}{4} = \frac{-9}{16} = -0.5625$

 $f(25,3) = \sqrt{25-3^2} = \sqrt{16} = 4$

smanjiti za 0.5625%.



Križna elastičnost

- Funkcija potražnje prvog proizvoda: $q_1=q_1(p_1,p_2)$ Funkcija potražnje drugog proizvoda: $q_2=q_2(p_1,p_2)$
- Koeficijent parcijalne elastičnosti opisuje ponašanje funkcije potražnje jednog proizvoda u slučaju kada se mijenja cijena tog proizvoda: E_{q_1,p_1} , E_{q_2,p_2}
- Proizvod je normalno dobro ukoliko povećanje cijene tog proizvoda uzrokuje pad njegove potražnje. U tom slučaju koeficijent parcijalne elastičnosti je negativni broj.
- Proizvod nije normalno dobro ukoliko povećanje cijene tog proizvoda uzrokuje rast njegove potražnje. U tom slučaju koeficijent parcijalne elastičnosti je pozitivni broj.

- Funkcija potražnje prvog proizvoda: $q_1 = q_1(p_1, p_2)$ Funkcija potražnje drugog proizvoda: $q_2 = q_2(p_1, p_2)$
- Koeficijent križne elastičnosti opisuje ponašanje funkcije potražnje jednog proizvoda u slučaju kada se mijenja cijena drugog proizvoda: E_{q_1,p_2} , E_{q_2,p_1}
- Proizvodi su supstituti ukoliko rast cijene jednog od njih uzrokuje rast potražnje za drugim. U tom slučaju koeficijent križne elastičnosti je pozitivni broj.
- Proizvodi su komplementi ukoliko rast cijene jednog od njih uzrokuje pad potražnje za drugim. U tom slučaju koeficijent križne elastičnosti je negativni broj.

peti zadatak

Dana je funkcija potražnje proizvoda D₁

$$q_1(p_1,p_2)=rac{1}{2}p_1^2+rac{6}{p_2}$$

u ovisnosti o cijenama p_1 i p_2 proizvoda D_1 i D_2 . Izračunajte i interpretirajte koeficijente parcijalne i križne elastičnosti na nivou cijena $p_1 = 1$, $p_2 = 2$. Jesu li ti proizvodi komplementi ili supstituti? Je li proizvod D_1 normalno dobro?

Dana je funkcija potražnje proizvoda D₁

$$q_1 = \frac{1}{2}p_1^2 + 6p_2^{-1}$$

$$q_1(p_1,p_2)=rac{1}{2}p_1^2+rac{6}{p_2}$$

u ovisnosti o cijenama p_1 i p_2 proizvoda D_1 i D_2 . Izračunajte i interpretirajte koeficijente parcijalne i križne elastičnosti na nivou cijena $p_1 = 1$, $p_2 = 2$. Jesu li ti proizvodi komplementi ili supstituti? Je li proizvod D_1 normalno dobro?

$$\frac{\partial q_1}{\partial p_1} =$$

Dana je funkcija potražnje proizvoda D₁

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$$\frac{\partial q_1}{\partial p_1} = \frac{1}{2} \cdot$$

Dana je funkcija potražnje proizvoda D₁

$$q_1 = \frac{1}{2}p_1^2 + 6p_2^{-1}$$

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$$\frac{\partial q_1}{\partial p_1} = \frac{1}{2} \cdot 2p_1$$

Dana je funkcija potražnje proizvoda D₁

$$q_1 = \frac{1}{2}p_1^2 + 6p_2^{-1}$$

$$q_1(p_1,p_2)=rac{1}{2}p_1^2+rac{6}{p_2}$$

u ovisnosti o cijenama p_1 i p_2 proizvoda D_1 i D_2 . Izračunajte i interpretirajte koeficijente parcijalne i križne elastičnosti na nivou cijena $p_1 = 1$, $p_2 = 2$. Jesu li ti proizvodi komplementi ili supstituti? Je li proizvod D_1 normalno dobro?

$$\frac{\partial q_1}{\partial p_1} = \frac{1}{2} \cdot 2p_1 + 0$$

Dana je funkcija potražnje proizvoda D₁

$$q_1 = \frac{1}{2}p_1^2 + 6p_2^{-1}$$

$$q_1(p_1,p_2)=rac{1}{2}p_1^2+rac{6}{p_2}$$

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$$\frac{\partial q_1}{\partial p_1} = \frac{1}{2} \cdot 2p_1 + 0 = p_1$$

Dana je funkcija potražnje proizvoda D_1

$$q_1 = \frac{1}{2}p_1^2 + 6p_2^{-1}$$

$$q_1(p_1,p_2)=rac{1}{2}p_1^2+rac{6}{p_2}$$

u ovisnosti o cijenama p_1 i p_2 proizvoda D_1 i D_2 . Izračunajte i interpretirajte koeficijente parcijalne i križne elastičnosti na nivou cijena $p_1=1,\ p_2=2$. Jesu li ti proizvodi komplementi ili supstituti? Je li proizvod D_1 normalno dobro?

$$\begin{aligned} \frac{\partial q_1}{\partial p_1} &= \frac{1}{2} \cdot 2p_1 + 0 = p_1 \\ \frac{\partial q_1}{\partial p_2} &= \end{aligned}$$

Dana je funkcija potražnje proizvoda D_1

$$q_1 = \frac{1}{2}p_1^2 + 6p_2^{-1}$$

$$q_1(p_1,p_2)=rac{1}{2}p_1^2+rac{6}{p_2}$$

u ovisnosti o cijenama p_1 i p_2 proizvoda D_1 i D_2 . Izračunajte i interpretirajte koeficijente parcijalne i križne elastičnosti na nivou cijena $p_1=1$, $p_2=2$. Jesu li ti proizvodi komplementi ili supstituti? Je li proizvod D_1 normalno dobro?

$$\frac{\partial q_1}{\partial p_1} = \frac{1}{2} \cdot 2p_1 + 0 = p_1$$
$$\frac{\partial q_1}{\partial p_2} = 0$$

Dana je funkcija potražnje proizvoda D_1

$$q_1 = \frac{1}{2}p_1^2 + 6p_2^{-1}$$

$$q_1(p_1,p_2)=rac{1}{2}p_1^2+rac{6}{p_2}$$

u ovisnosti o cijenama p_1 i p_2 proizvoda D_1 i D_2 . Izračunajte i interpretirajte koeficijente parcijalne i križne elastičnosti na nivou cijena $p_1=1$, $p_2=2$. Jesu li ti proizvodi komplementi ili supstituti? Je li proizvod D_1 normalno dobro?

$$\frac{\partial q_1}{\partial p_1} = \frac{1}{2} \cdot 2p_1 + 0 = p_1$$
$$\frac{\partial q_1}{\partial p_2} = 0 + 6$$

Dana je funkcija potražnje proizvoda D_1

$$q_1 = \frac{1}{2}p_1^2 + 6p_2^{-1}$$

$$q_1(p_1,p_2)=rac{1}{2}p_1^2+rac{6}{p_2}$$

u ovisnosti o cijenama p_1 i p_2 proizvoda D_1 i D_2 . Izračunajte i interpretirajte koeficijente parcijalne i križne elastičnosti na nivou cijena $p_1=1$, $p_2=2$. Jesu li ti proizvodi komplementi ili supstituti? Je li proizvod D_1 normalno dobro?

$$\frac{\partial q_1}{\partial p_1} = \frac{1}{2} \cdot 2p_1 + 0 = p_1$$
$$\frac{\partial q_1}{\partial p_2} = 0 + 6 \cdot (-p_2^{-2})$$

Dana je funkcija potražnje proizvoda D₁

$$q_1 = \frac{1}{2}p_1^2 + 6p_2^{-1}$$

$$q_1(p_1,p_2)=rac{1}{2}p_1^2+rac{6}{p_2}$$

u ovisnosti o cijenama p_1 i p_2 proizvoda D_1 i D_2 . Izračunajte i interpretirajte koeficijente parcijalne i križne elastičnosti na nivou cijena $p_1=1$, $p_2=2$. Jesu li ti proizvodi komplementi ili supstituti? Je li proizvod D_1 normalno dobro?

$$\frac{\partial q_1}{\partial p_1} = \frac{1}{2} \cdot 2p_1 + 0 = p_1$$

$$\frac{\partial q_1}{\partial p_2} = 0 + 6 \cdot (-p_2^{-2}) = \frac{-6}{p_2^2}$$

$$\frac{\partial q_1}{\partial p_1}$$

• Koeficijent parcijalne elastičnosti

 $E_{q_1,p_1}(1,2) =$

 $q_1(p_1,p_2) = \frac{1}{2}p_1^2 + \frac{6}{p_2}$

$$\frac{\partial q_1}{\partial p_1}$$

 $q_1(p_1,p_2) = \frac{1}{2}p_1^2 + \frac{6}{p_2}$

Koeficijent parcijalne elastičnosti

$$E_{q_1,p_1}(1,2) = ----$$

$$\frac{\partial q_1}{\partial p_1}$$

 $q_1(p_1,p_2)=rac{1}{2}p_1^2+rac{6}{p_2}\left|
ight. \left|rac{\partial q_1}{\partial p_1}=p_1
ight.$

$$E_{q_1,p_1}(1,2) = -1$$

$$\frac{\partial q_1}{\partial p_1}$$

 $q_1(p_1,p_2) = \frac{1}{2}p_1^2 + \frac{6}{p_2}$

$$p_1 p_2$$
 1

Koeficijent parcijalne elastičnosti

$$E_{q_1,p_1}(1,2) = \frac{1}{q_1(1,2)}$$

$$\frac{\partial q_1}{\partial p_1}$$

 $q_1(p_1,p_2) = \frac{1}{2}p_1^2 + \frac{6}{p_2}$

$$E_{q_1,p_1} \stackrel{m{p_1}}{(1,2)} = rac{1}{q_1(1,2)} \cdot$$

$$\frac{\partial q_1}{\partial p_1}$$

 $p_1 p_2 \qquad 1 \qquad \partial q_1$

 $q_1(p_1, p_2) = \frac{1}{2}p_1^2 + \frac{6}{p_2} \left| \begin{array}{c} \frac{\partial q_1}{\partial p_1} = p_1 \end{array} \right|$

$$E_{q_1,p_1}(1,2) = \frac{1}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_1}(1,2)$$

• Koeficijent parcijalne elastičnosti

$$\frac{\partial q_1}{\partial p_1}$$

 $q_1(p_1, p_2) = \frac{1}{2}p_1^2 + \frac{6}{p_2} \left| \begin{array}{c} \frac{\partial q_1}{\partial p_1} = p_1 \end{array} \right|$

$$E_{q_1,p_1}(1,2) = \frac{1}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_1}(1,2) =$$

$$\frac{\partial q_1}{\partial p_1}$$

Koeficijent parcijalne elastičnosti

$$q_1(1,2) =$$

$$\frac{\partial q_1}{\partial p_1}$$

$$egin{aligned} E_{q_1,p_1}(1,2) &= rac{1}{q_1(1,2)} \cdot rac{\partial q_1}{\partial p_1}(1,2) = \ q_1(1,2) &= rac{p_1}{2} \cdot 1^2 + rac{6}{2} \end{aligned}$$

$$\frac{\partial q_1}{\partial p_1}$$

Koeficijent parcijalne elastičnosti

$$E_{q_1,p_1}(1,2) = rac{1}{q_1(1,2)} \cdot rac{\partial q_1}{\partial p_1}(1,2) = \ q_1(1,2) = rac{p_1}{2} \cdot 1^2 + rac{6}{2} = rac{7}{2}$$

$$\frac{\partial q_1}{\partial p_1}$$

Koeficijent parcijalne elastičnosti

$$E_{q_1,p_1}(1,2) = rac{1}{q_1(1,2)} \cdot rac{\partial q_1}{\partial p_1}(1,2) = \ q_1(1,2) = rac{p_1}{2} \cdot 1^2 + rac{6}{2} = rac{7}{2} = 3.5$$

$$\frac{\partial q_1}{\partial p_1}$$

$$E_{q_1,p_1}(1,2) = rac{1}{q_1(1,2)} \cdot rac{\partial q_1}{\partial p_1}(1,2) = \ q_1(1,2) = rac{p_1}{2} \cdot 1^2 + rac{6}{2} = rac{7}{2} = 3.5 \ rac{\partial q_1}{\partial p_1}(1,2) = \ \frac{\partial q_1}{\partial p_$$

$$\frac{\partial q_1}{\partial p_1}$$

$$E_{q_1,p_1}(1,2) = \frac{1}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_1}(1,2) =$$

$$q_1(1,2) = \frac{1}{2} \cdot 1^2 + \frac{6}{2} = \frac{7}{2} = 3.5$$

$$\frac{\partial q_1}{\partial p_1}(1,2) = 1$$

$$\frac{\partial q_1}{\partial p_1}$$

• Koeficijent parcijalne elastičnosti

$$E_{q_1,p_1}(1,2) = \frac{1}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_1}(1,2) = \frac{1}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_1}(1,2) = \frac{1}{q_1(1,2)}$$

$$q_1(1,2) = \frac{1}{2} \cdot 1^2 + \frac{6}{2} = \frac{7}{2} = 3.5$$
 $\frac{\partial q_1}{\partial p_1}(1,2) = 1$

$$\frac{\partial q_1}{\partial p_1}$$

Koeficijent parcijalne elastičnosti

$$E_{q_1,p_1}(1,2) = \frac{1}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_1}(1,2) = \frac{1}{3.5}$$

$$q_1(1,2) = \frac{1}{2} \cdot 1^2 + \frac{6}{2} = \frac{7}{2} = 3.5$$

$$\frac{\partial q_1}{\partial p_1}(1,2) = 1$$

$$\frac{\partial q_1}{\partial p_1}$$

Koeficijent parcijalne elastičnosti

$$E_{q_1,p_1}(1,2) = \frac{1}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_1}(1,2) = \frac{1}{3.5}$$

$$q_1(1,2) = \frac{p_1}{2} \cdot 1^2 + \frac{6}{2} = \frac{7}{2} = 3.5$$

$$rac{\partial q_1}{\partial p_1} \stackrel{\mathbf{p}_1}{(1,2)} = 1$$

$$\frac{\partial q_1}{\partial p_1}$$

 $\frac{\partial q_1}{\partial p_2} \stackrel{p_1}{(1,2)} = 1$

 $q_1(p_1, p_2) = \frac{1}{2}p_1^2 + \frac{6}{p_2} \left| \frac{\partial q_1}{\partial p_1} = p_1 \right|$

Koeficijent parcijalne elastičnosti

 $q_1(1,2) = \frac{1}{2} \cdot 1^2 + \frac{6}{2} = \frac{7}{2} = 3.5$

$$E_{q_1,p_1}(1,2) = \frac{1}{(1,2)} \cdot \frac{\partial q_1}{\partial q_1}$$

$$E_{q_1,p_1}(1,2) = \frac{1}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_1}(1,2) = \frac{1}{3.5} \cdot 1$$

$$\frac{\partial q_1}{\partial p_1}$$

 $\frac{\partial q_1}{\partial p_1} \stackrel{p_1}{(1,2)} = 1$

• Koeficijent parcijalne elastičnosti

$$E_{q_1,p_1}(1,2) = \frac{1}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_1}(1,2) = \frac{1}{3.5} \cdot 1 = \frac{2}{7}$$

$$q_1(1,2) = \frac{1}{2} \cdot 1^2 + \frac{6}{2} = \frac{7}{2} = 3.5$$

 $q_1(p_1, p_2) = \frac{1}{2}p_1^2 + \frac{6}{p_2} \left| \frac{\partial q_1}{\partial p_1} = p_1 \right|$

$$\frac{\partial q_1}{\partial p_1}$$

 $\frac{\partial q_1}{\partial p_2} \stackrel{p_1}{(1,2)} = 1$

$$q_1(1,2) = \frac{1}{2} \cdot 1^2 + \frac{6}{2} = \frac{7}{2} = 3.5$$
 $q_1 \stackrel{p_1}{(1,2)} \stackrel{p_2}{(1,2)} = \frac{1}{2} \cdot 1^2 + \frac{6}{2} = \frac{7}{2} = 3.5$

 $q_1(p_1, p_2) = \frac{1}{2}p_1^2 + \frac{6}{p_2} \left| \frac{\partial q_1}{\partial p_1} = p_1 \right|$

$$E_{q_1,p_1}(1,2) = \frac{1}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_1}(1,2) = \frac{1}{3.5} \cdot 1 = \frac{2}{7} \approx 0.29$$

$$p_1$$

$$=\frac{7}{2}=3.5$$

$$q_1(1,2) = \frac{1}{2} \cdot 1^2 + \frac{6}{2} = \frac{7}{2} = 3.5$$

$$\frac{\partial q_1}{\partial p_1}(1,2) = 1$$

Ako na nivou cijena $p_1 = 1$, $p_2 = 2$ cijenu p_1 proizvoda D_1 povećamo za 1%, potražnja za proizvodom D_1 se poveća za 0.29%.

 $E_{q_1,p_1}(1,2) = \frac{1}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_1}(1,2) = \frac{1}{3.5} \cdot 1 = \frac{2}{7} \approx 0.29$

$$\frac{q_1}{p_1}$$

$$E_{q_1,p_1}(1,2) = \frac{1}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_1}(1,2) = \frac{1}{3.5} \cdot 1 = \frac{2}{7} \approx 0.29$$

Koeficijent parcijalne elastičnosti

$$rac{\partial q_1}{\partial p_1} \stackrel{oldsymbol{p}_1}{(1,2)} = 1$$

Ako na nivou cijena $p_1 = 1$, $p_2 = 2$ cijenu p_1 proizvoda D_1 povećamo za 1%, potražnja za proizvodom D_1 se poveća za 0.29%.

Proizvod D_1 nije normalno dobro.

 $q_1(1,2) = \frac{1}{2} \cdot 1^2 + \frac{6}{2} = \frac{7}{2} = 3.5$

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$$\frac{\partial q_1}{\partial p_2}$$

 $q_1(p_1,p_2) = \frac{1}{2}p_1^2 + \frac{6}{p_2}$

 $E_{q_1,p_2}(1,2) =$

$$\frac{\partial q_1}{\partial p_2}$$

 $q_1(p_1,p_2)=rac{1}{2}p_1^2+rac{6}{p_2}$

$$E_{q_1,p_2}(1,2) = ----$$

$$\frac{\partial q_1}{\partial p_2}$$

 $q_1(p_1,p_2)=rac{1}{2}p_1^2+rac{6}{p_2}$

$$E_{q_1,p_2}(1,2) = \frac{2}{2}$$

$$\frac{\partial q_1}{\partial p_2}$$

 $q_1(p_1, p_2) = \frac{1}{2}p_1^2 + \frac{6}{p_2}$

$$E_{q_1,p_2}(1,2) = \frac{2}{q_1(1,2)}$$

$$\frac{\partial q_1}{\partial p_2}$$

 $q_1(p_1, p_2) = \frac{1}{2}p_1^2 + \frac{6}{p_2}$

$$E_{q_1,p_2}(1,2) = \frac{2}{q_1(1,2)}$$

$$\frac{\partial q_1}{\partial p_2}$$

 $q_1(p_1,p_2)=rac{1}{2}p_1^2+rac{6}{p_2}$

$$E_{q_1,p_2}(1,2) = \frac{2}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_2}(1,2)$$

$$\frac{\partial q_1}{\partial p_2}$$

 $q_1(p_1,p_2) = \frac{1}{2}p_1^2 + \frac{6}{p_2}$

$$E_{q_1,p_2}(1,2) = \frac{2}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_2}(1,2) =$$

$$\frac{\partial q_1}{\partial p_2}$$

 $q_1(p_1,p_2) = \frac{1}{2}p_1^2 + \frac{6}{p_2}$

$$E_{q_1,p_2}(1,2) = \frac{2}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_2}(1,2) =$$
 $q_1(1,2) =$

$$\frac{\partial q_1}{\partial p_2}$$

 $q_1(p_1,p_2) = \frac{1}{2}p_1^2 + \frac{6}{p_2}$

Koeficijent križne elastičnosti

$$E_{q_1,p_2}(1,2) = rac{2}{q_1(1,2)} \cdot rac{\partial q_1}{\partial p_2}(1,2) = \ q_1(1,2) = rac{p_1}{2} \cdot 1^2 + rac{6}{2}$$

$$\frac{\partial q_1}{\partial p_2}$$

$$E_{q_1,p_2}(1,2) = rac{2}{q_1(1,2)} \cdot rac{\partial q_1}{\partial p_2}(1,2) = \ q_1(1,2) = rac{p_1}{2} \cdot 1^2 + rac{6}{2} = rac{7}{2}$$

 $q_1(p_1,p_2) = \frac{1}{2}p_1^2 + \frac{6}{p_2}$

$$\frac{\partial q_1}{\partial p_2}$$

Koeficijent križne elastičnosti

$$E_{q_1,p_2}(1,2) = \frac{2}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_2}(1,2) =$$
$$q_1(1,2) = \frac{1}{2} \cdot 1^2 + \frac{6}{2} = \frac{7}{2} = 3.5$$

$$\frac{\partial q_1}{\partial p_2}$$

Koeficijent križne elastičnosti

$$E_{q_1,p_2}(1,2) = \frac{2}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_2}(1,2) =$$

$$q_1(1,2) = \frac{1}{2} \cdot 1^2 + \frac{6}{2} = \frac{7}{2} = 3.5$$

 $\frac{\partial q_1}{\partial p_2}(1,2) =$

$$\frac{\partial q_1}{\partial p_2}$$

$$E_{q_1,p_2}(1,2) = \frac{2}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_2}(1,2) =$$

$$q_1(1,2) \quad \partial p_2(1,2)$$

$$q_1(1,2) = \frac{1}{2} \cdot 1^2 + \frac{6}{2} = \frac{7}{2} = 3.5$$

 $\frac{\partial q_1}{\partial p_2} (1,2) = \frac{-6}{2^2}$

 $q_1(p_1, p_2) = \frac{1}{2}p_1^2 + \frac{6}{p_2} \left| \frac{\partial q_1}{\partial p_2} = \right|$

$$\frac{\partial q_1}{\partial p_2}$$

 $\frac{\partial q_1}{\partial p_2} \stackrel{p_1}{(1,2)} = \frac{-6}{2^2} = -\frac{3}{2}$

Koeficijent križne elastičnosti

$$E_{q_1,p_2}(1,2) = \frac{2}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_2}(1,2) =$$

$$q_1(1,2) = \frac{1}{2} \cdot 1^2 + \frac{6}{2} = \frac{7}{2} = 3.5$$

 $q_1(p_1, p_2) = \frac{1}{2}p_1^2 + \frac{6}{p_2} \left| \frac{\partial q_1}{\partial p_2} \right| = 0$

$$\frac{\partial q_1}{\partial p_2}$$

$$E_{q_1,p_2}(1,2) = \frac{2}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_2}(1,2) =$$

$$q_1(1,2) = \frac{1}{2} \cdot 1^2 + \frac{6}{2} = \frac{7}{2} = 3.5$$

$$\frac{\partial q_1}{\partial p_2}(1,2) = \frac{-6}{2^2} = -\frac{3}{2} = -1.5$$

$$\frac{\partial q_1}{\partial p_2}$$

$$E_{q_1,p_2}(1,2) = \frac{2}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_2}(1,2) = \frac{2}{q_1(1,2)}$$
$$q_1(1,2) = \frac{1}{2} \cdot 1^2 + \frac{6}{2} = \frac{7}{2} = 3.5$$

$$q_1(1,2) = \frac{1}{2} \cdot 1^2 + \frac{1}{2} = \frac{1}{2} = 3.3$$

$$\frac{\partial q_1}{\partial p_2}(1,2) = \frac{-6}{2^2} = -\frac{3}{2} = -1.5$$

$$\frac{\partial q_1}{\partial p_2}$$

$$E_{q_1,p_2}(1,2) = \frac{2}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_2}(1,2) = \frac{2}{3.5}$$

$$q_1 \stackrel{p_1}{(1,2)} = \frac{1}{2} \cdot 1^2 + \frac{6}{2} = \frac{7}{2} = 3.5$$

$$\frac{\partial q_1}{\partial p_2} \stackrel{p_1}{(1,2)} = \frac{-6}{2^2} = -\frac{3}{2} = -1.5$$

 $q_1(p_1, p_2) = \frac{1}{2}p_1^2 + \frac{6}{p_2} \left| \frac{\partial q_1}{\partial p_2} \right|$

$$\frac{\partial q_1}{\partial p_2}$$

$$F \qquad \stackrel{p_1}{(1,2)} = \frac{2}{} \cdot \frac{\partial q_1}{\partial q_2} (1)$$

$$E_{q_1,p_2} \stackrel{p_1}{(1,2)} = rac{2}{q_1(1,2)} \cdot rac{\partial q_1}{\partial p_2} (1,2) = rac{2}{3.5} \cdot$$

 $\frac{\partial q_1}{\partial p_2} \stackrel{p_1}{(1,2)} = \frac{-6}{2^2} = -\frac{3}{2} = -1.5$

$$q_1(1,2) = 6p_2$$

$$q_1(1,2) = \frac{1}{2} \cdot 1^2 + \frac{6}{2} = \frac{7}{2} = 3.5$$

 $q_1(p_1,p_2) = \frac{1}{2}p_1^2 + \frac{6}{p_2} \left| \begin{array}{c} \frac{\partial q_1}{\partial p_2} = \end{array} \right|$

$$\frac{\partial q_1}{\partial p_2}$$

 $q_1(p_1,p_2) = \frac{1}{2}p_1^2 + \frac{6}{p_2} \left| \frac{\partial q_1}{\partial p_2} \right| = \frac{1}{2}p_2^2$

Koeficijent križne elastičnosti

$$E_{q_1,p_2}(1,2) = \frac{2}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_2}(1,2) = \frac{2}{3.5} \cdot (-1.5)$$

$$\frac{\partial q_1}{\partial p_2} \stackrel{p_1}{(1,2)} = \frac{-6}{2^2} = -\frac{3}{2} = -1.5$$

 $q_1(1,2) = \frac{1}{2} \cdot 1^2 + \frac{6}{2} = \frac{7}{2} = 3.5$

$$\frac{\partial q_1}{\partial p_2}$$

$$q_1(1,2) = \frac{1}{2} \cdot 1^2 + \frac{6}{2} = \frac{7}{2} = 3.5$$

$$E_{q_1,p_2}(1,2) = \frac{2}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_2}(1,2) = \frac{2}{3.5} \cdot (-1.5) = -\frac{6}{7}$$

$$\frac{\partial q_1}{\partial p_2} \stackrel{p_1}{(1,2)} = \frac{-6}{2^2} = -\frac{3}{2} = -1.5$$

$$\frac{\partial q_1}{\partial p_2}$$

$$E_{q_1,p_2}(1,2) = \frac{2}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_2}(1,2) = \frac{2}{3.5} \cdot (-1.5) = -\frac{6}{7} \approx -0.86$$

$$q_1(1,2) = \frac{1}{2} \cdot 1^2 + \frac{6}{2} = \frac{7}{2} = 3.5$$

$$\partial q_1 \stackrel{p_1}{(1,2)} \stackrel{p_2}{(1,2)} = -6 \qquad 3$$

$$\frac{\partial q_1}{\partial p_2} (1,2) = \frac{-6}{2^2} = -\frac{3}{2} = -1.5$$

$$q_1(1,2) = \frac{1}{2} \cdot 1^2 + \frac{6}{2} = \frac{7}{2} = 3.5$$

$$\frac{\partial q_1}{\partial p_2} {1,2} = \frac{-6}{2^2} = -\frac{3}{2} = -1.5$$

1%, potražnja za proizvodom D_1 se smanji za 0.86%.

Ako na nivou cijena $p_1 = 1$, $p_2 = 2$ cijenu p_2 proizvoda D_2 povećamo za

 $E_{q_1,p_2}(1,2) = \frac{2}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_2}(1,2) = \frac{2}{3.5} \cdot (-1.5) = -\frac{6}{7} \approx -0.86$

$$\frac{q_1,p_2}{q_1} = \frac{r_2}{q_1} \cdot \frac{r_1}{\partial p_2}$$
• Koeficijent križne elastičnosti

 $E_{q_1,p_2}(1,2) = \frac{2}{q_1(1,2)} \cdot \frac{\partial q_1}{\partial p_2}(1,2) = \frac{2}{3.5} \cdot (-1.5) = -\frac{6}{7} \approx -0.86$ $q_1(1,2) = \frac{1}{2} \cdot 1^2 + \frac{6}{2} = \frac{7}{2} = 3.5$

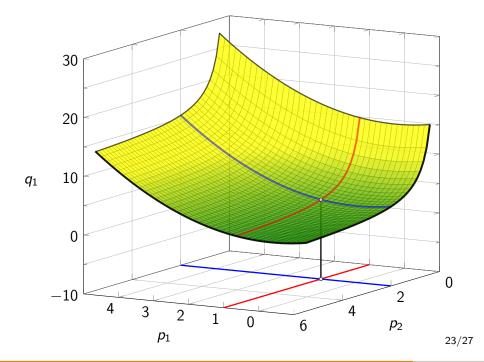
 $q_1(p_1, p_2) = \frac{1}{2}p_1^2 + \frac{6}{p_2} \left| \frac{\partial q_1}{\partial p_2} = \frac{-6}{p_2^2} \right|$

 $\frac{\partial q_1}{\partial p_2} \stackrel{p_1}{(1,2)} = \frac{-6}{2^2} = -\frac{3}{2} = -1.5$

Ako na nivou cijena $p_1 = 1$, $p_2 = 2$ cijenu p_2 proizvoda D_2 povećamo za 1%, potražnja za proizvodom D_1 se smanji za 0.86%.

Proizvodi D_1 i D_2 su komplementi.

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Zadatak 6

Dana je funkcija potražnje proizvoda D₂

$$q_2(p_1,p_2)=2p_1\sqrt{10-p_2}$$

u ovisnosti o cijenama p_1 i p_2 proizvoda D_1 i D_2 . Izračunajte i interpretirajte koeficijente parcijalne i križne elastičnosti na nivou cijena $p_1 = 5$, $p_2 = 1$. Jesu li ti proizvodi komplementi ili supstituti? Je li proizvod D_2 normalno dobro?

Dana je funkcija potražnje proizvoda D₂

$$q_2(p_1, p_2) = 2p_1\sqrt{10-p_2}$$

u ovisnosti o cijenama p_1 i p_2 proizvoda D_1 i D_2 . Izračunajte i interpretirajte koeficijente parcijalne i križne elastičnosti na nivou cijena $p_1 = 5$, $p_2 = 1$. Jesu li ti proizvodi komplementi ili supstituti? Je li proizvod D_2 normalno dobro?

$$\frac{\partial q_2}{\partial p_1} =$$

Dana je funkcija potražnje proizvoda D₂

$$q_2(p_1, p_2) = 2p_1\sqrt{10-p_2}$$

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$$\frac{\partial q_2}{\partial p_1} = 2\sqrt{10 - p_2} \cdot$$

Dana je funkcija potražnje proizvoda D₂

$$q_2(p_1, p_2) = 2p_1\sqrt{10-p_2}$$

u ovisnosti o cijenama p_1 i p_2 proizvoda D_1 i D_2 . Izračunajte i interpretirajte koeficijente parcijalne i križne elastičnosti na nivou cijena $p_1 = 5$, $p_2 = 1$. Jesu li ti proizvodi komplementi ili supstituti? Je li proizvod D_2 normalno dobro?

$$\frac{\partial q_2}{\partial p_1} = 2\sqrt{10 - p_2} \cdot 1$$

Dana je funkcija potražnje proizvoda D₂

$$q_2(p_1, p_2) = 2p_1\sqrt{10-p_2}$$

u ovisnosti o cijenama p_1 i p_2 proizvoda D_1 i D_2 . Izračunajte i interpretirajte koeficijente parcijalne i križne elastičnosti na nivou cijena $p_1 = 5$, $p_2 = 1$. Jesu li ti proizvodi komplementi ili supstituti? Je li proizvod D_2 normalno dobro?

$$\frac{\partial q_2}{\partial p_1} = 2\sqrt{10 - p_2} \cdot 1 = 2\sqrt{10 - p_2}$$

Dana je funkcija potražnje proizvoda D₂

$$q_2(p_1, p_2) = 2p_1\sqrt{10-p_2}$$

u ovisnosti o cijenama p_1 i p_2 proizvoda D_1 i D_2 . Izračunajte i interpretirajte koeficijente parcijalne i križne elastičnosti na nivou cijena $p_1 = 5$, $p_2 = 1$. Jesu li ti proizvodi komplementi ili supstituti? Je li proizvod D_2 normalno dobro?

$$\frac{\partial q_2}{\partial p_1} = 2\sqrt{10 - p_2} \cdot 1 = 2\sqrt{10 - p_2}$$

$$\frac{\partial q_2}{\partial p_2} =$$

Dana je funkcija potražnje proizvoda D₂

$$q_2(p_1,p_2)=2p_1\sqrt{10-p_2}$$

u ovisnosti o cijenama p_1 i p_2 proizvoda D_1 i D_2 . Izračunajte i interpretirajte koeficijente parcijalne i križne elastičnosti na nivou cijena $p_1 = 5$, $p_2 = 1$. Jesu li ti proizvodi komplementi ili supstituti? Je li proizvod D_2 normalno dobro?

$$\frac{\partial q_2}{\partial p_1} = 2\sqrt{10 - p_2} \cdot 1 = 2\sqrt{10 - p_2}$$

$$\frac{\partial q_2}{\partial p_2} = 2p_1$$

Dana je funkcija potražnje proizvoda D₂

$$q_2(p_1, p_2) = 2p_1\sqrt{10-p_2}$$

u ovisnosti o cijenama p_1 i p_2 proizvoda D_1 i D_2 . Izračunajte i interpretirajte koeficijente parcijalne i križne elastičnosti na nivou cijena $p_1 = 5$, $p_2 = 1$. Jesu li ti proizvodi komplementi ili supstituti? Je li proizvod D_2 normalno dobro?

$$\frac{\partial q_2}{\partial p_1} = 2\sqrt{10 - p_2} \cdot 1 = 2\sqrt{10 - p_2}$$
$$\frac{\partial q_2}{\partial p_2} = 2p_1 \cdot \frac{1}{2\sqrt{10 - p_2}}$$

Dana je funkcija potražnje proizvoda D₂

$$q_2(p_1, p_2) = 2p_1\sqrt{10-p_2}$$

u ovisnosti o cijenama p_1 i p_2 proizvoda D_1 i D_2 . Izračunajte i interpretirajte koeficijente parcijalne i križne elastičnosti na nivou cijena $p_1 = 5$, $p_2 = 1$. Jesu li ti proizvodi komplementi ili supstituti? Je li proizvod D_2 normalno dobro?

$$\begin{aligned} \frac{\partial q_2}{\partial p_1} &= 2\sqrt{10 - p_2} \cdot 1 = 2\sqrt{10 - p_2} \\ \frac{\partial q_2}{\partial p_2} &= 2p_1 \cdot \frac{1}{2\sqrt{10 - p_2}} \cdot (-1) \end{aligned}$$

Dana je funkcija potražnje proizvoda D₂

$$q_2(p_1,p_2)=2p_1\sqrt{10-p_2}$$

u ovisnosti o cijenama p_1 i p_2 proizvoda D_1 i D_2 . Izračunajte i interpretirajte koeficijente parcijalne i križne elastičnosti na nivou cijena $p_1 = 5$, $p_2 = 1$. Jesu li ti proizvodi komplementi ili supstituti? Je li proizvod D_2 normalno dobro?

$$\begin{aligned} \frac{\partial q_2}{\partial p_1} &= 2\sqrt{10 - p_2} \cdot 1 = 2\sqrt{10 - p_2} \\ \frac{\partial q_2}{\partial p_2} &= 2p_1 \cdot \frac{1}{2\sqrt{10 - p_2}} \cdot (-1) = \frac{-p_1}{\sqrt{10 - p_2}} \end{aligned}$$

$$E_{q_2,p_2}(5,1) =$$

 $q_2(p_1, p_2) = 2p_1\sqrt{10-p_2}$

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$$E_{q_2,p_2}(5,1) = ----$$

 $q_2(p_1, p_2) = 2p_1\sqrt{10-p_2}$

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$$E_{q_2,p_2}(5,1) = \frac{1}{2}$$

$$\frac{p_1}{p_2} \frac{p_2}{p_2} = 1$$

Koeficijent parcijalne elastičnosti

$$E_{q_2,p_2}(f 5,1)=rac{1}{q_2(f 5,1)}$$

 $q_2(p_1,p_2)=2p_1\sqrt{10-p_2}$

$$E_{q_2,p_2}(f 5,f 1)=rac{1}{q_2(f 5,f 1)}\, \cdot$$

 $q_2(p_1,p_2)=2p_1\sqrt{10-p_2}$

$$E_{q_2,
ho_2}(f 5,1) = rac{1}{q_2(f 5,1)} \cdot rac{\partial q_2}{\partial
ho_2}(f 5,1)$$

 $q_2(p_1, p_2) = 2p_1\sqrt{10-p_2}$

$$E_{q_2,
ho_2}(5,1) = rac{1}{q_2(5,1)} \cdot rac{\partial q_2}{\partial
ho_2}(5,1) =$$

 $q_2(p_1, p_2) = 2p_1\sqrt{10-p_2}$

$$E_{q_2,p_2}\!\!\left(5,1
ight) = rac{1}{q_2(5,1)} \cdot rac{\partial q_2}{\partial p_2}\!\!\left(5,1
ight) =$$

 $q_2(5,1) =$

 $q_2(p_1,p_2)=2p_1\sqrt{10-p_2}$

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$$E_{q_2,p_2}(5,1) = rac{1}{q_2(5,1)} \cdot rac{\partial q_2}{\partial p_2}(5,1) = rac{p_1}{p_2} rac{p_2}{p_2}$$

Koeficijent parcijalne elastičnosti

$$\mathcal{L}_{q_2,p_2}(5,1) = \frac{1}{q_2(5,1)} \cdot \frac{1}{\partial p_2}(5,1)$$
 $q_2(5,1) = 2 \cdot 5 \cdot \sqrt{10-1}$

$$E_{q_2,p_2}(5,1) = \frac{1}{q_2(5,1)} \cdot \frac{\partial q_2}{\partial p_2}(5,1) = \frac{p_1}{p_2}$$

$$E_{q_2,p_2}(5,1) = rac{1}{q_2(5,1)} \cdot rac{\partial q_2}{\partial p_2}(5,1) = rac{p_1}{q_2(5,1)} = 2 \cdot 5 \cdot \sqrt{10-1} = 30$$

Koeficijent parcijalne elastičnosti

$$\frac{p_2}{-1} = 30$$

$$E_{q_2,p_2}(5,1) = rac{1}{q_2(5,1)} \cdot rac{\partial q_2}{\partial p_2}(5,1) = \ q_2(5,1) = 2 \cdot 5 \cdot \sqrt{10-1} = 30 \ rac{\partial q_2}{\partial p_2}(5,1) =$$

 $E_{q_2,p_2}(5,1) = \frac{1}{q_2(5,1)} \cdot \frac{\partial q_2}{\partial p_2}(5,1) =$

$$q_{2}(5,1) = 2 \cdot 5 \cdot \sqrt{10 - 1} = 30$$

$$\frac{\partial q_{2}}{\partial p_{2}} {p_{1} \choose 5, 1} = \frac{-5}{\sqrt{10 - 1}}$$

$$E_{q_2,p_2}(5,1) = \frac{1}{q_2(5,1)} \cdot \frac{\partial q_2}{\partial p_2}(5,1) =$$

$$q_2(5,1) = 2 \cdot 5 \cdot \sqrt{10-1} = 30$$

$$\frac{\partial q_2}{\partial p_2}(5,1) = \frac{-5}{\sqrt{10-1}} = \frac{-5}{3}$$

$$E_{q_2,p_2}(5,1) = \frac{1}{q_2(5,1)} \cdot \frac{\partial q_2}{\partial p_2}(5,1) = \frac{1}{q_2(5,1)}$$

$$q_2(5,1) = 2 \cdot 5 \cdot \sqrt{10-1} = 30$$

$$\frac{\partial q_2}{\partial p_2}(5,1) = \frac{-5}{\sqrt{10-1}} = \frac{-5}{3}$$

$$E_{q_2,p_2}(5,1) = \frac{1}{q_2(5,1)} \cdot \frac{\partial q_2}{\partial p_2}(5,1) = \frac{1}{30}$$

$$q_2(5,1) = 2 \cdot 5 \cdot \sqrt{10-1} = 30$$

$$\frac{\partial q_2}{\partial p_2}(5,1) = \frac{-5}{\sqrt{10-1}} = \frac{-5}{3}$$

• Koeficijent parcijalne elastičnosti
$$E_{q_2,p_2}(5,1) = \frac{1}{q_2(5,1)} \cdot \frac{\partial q_2}{\partial p_2}(5,1) = \frac{1}{30} \cdot$$

$$q_{2}(5,1) = 2 \cdot 5 \cdot \sqrt{10 - 1} = 30$$

$$\frac{\partial q_{2}}{\partial p_{2}}(5,1) = \frac{-5}{\sqrt{10 - 1}} = \frac{-5}{3}$$

 $q_2(p_1, p_2) = 2p_1\sqrt{10 - p_2}$ $\frac{\partial q_2}{\partial p_2} = \frac{-p_1}{\sqrt{10 - p_2}}$

$$E_{q_2,p_2}(5,1) = \frac{1}{q_2(5,1)} \cdot \frac{\partial q_2}{\partial p_2}(5,1) = \frac{1}{30} \cdot \frac{-5}{3}$$

$$q_2(5,1) = 2 \cdot 5 \cdot \sqrt{10-1} = 30$$

$$\frac{\partial q_2}{\partial p_2}(5,1) = \frac{-5}{\sqrt{10-1}} = \frac{-5}{3}$$

 $q_2(p_1, p_2) = 2p_1\sqrt{10 - p_2}$ $\frac{\partial q_2}{\partial p_2} = \frac{-p_1}{\sqrt{10 - p_2}}$

$$E_{q_2,p_2}(5,1) = \frac{1}{q_2(5,1)} \cdot \frac{\partial q_2}{\partial p_2}(5,1) = \frac{1}{30} \cdot \frac{-5}{3} = \frac{-1}{18}$$

$$q_2(5,1) = 2 \cdot 5 \cdot \sqrt{10-1} = 30$$

$$\frac{\partial q_2 \stackrel{p_1}{(5,1)}}{(5,1)} = \frac{-5}{30} = \frac{-5}{30}$$

$$\frac{\partial q_2}{\partial p_2}(5,1) = \frac{-5}{\sqrt{10-1}} = \frac{-5}{3}$$

$$E_{q_2,p_2}(5,1) = \frac{1}{q_2(5,1)} \cdot \frac{\partial q_2}{\partial p_2}(5,1) = \frac{1}{30} \cdot \frac{-5}{3} = \frac{-1}{18} \approx -0.06$$

$$\frac{p_1}{q_2(5,1)} = 2 \cdot 5 \cdot \sqrt{10-1} = 30$$

 $q_2(p_1, p_2) = 2p_1\sqrt{10 - p_2}$ $\frac{\partial q_2}{\partial p_2} = \frac{-p_1}{\sqrt{10 - p_2}}$

$$q_2(5,1) = 2 \cdot 5 \cdot \sqrt{10 - 1} = \frac{\partial q_2}{\partial p_2}(5,1) = \frac{-5}{\sqrt{10 - 1}} = \frac{-5}{3}$$

Koeficijent parcijalne elastičnosti

$$q_2(5,1) = 2 \cdot 5 \cdot \sqrt{10-1} = 30$$

$$\frac{\partial q_2}{\partial p_2}(5,1) = \frac{-5}{\sqrt{10-1}} = \frac{-5}{3}$$

Ako na nivou cijena $p_1 = 5$, $p_2 = 1$ cijenu p_2 proizvoda D_2 povećamo za

1%, potražnja za proizvodom D_2 se smanji za 0.06%.

 $E_{q_2,p_2}(5,1) = \frac{1}{a_2(5,1)} \cdot \frac{\partial q_2}{\partial p_2}(5,1) = \frac{1}{30} \cdot \frac{-5}{3} = \frac{-1}{18} \approx -0.06$

Koeficijent parcijalne elastičnosti

 $q_2(p_1, p_2) = 2p_1\sqrt{10 - p_2}$ $\frac{\partial q_2}{\partial p_2} = \frac{-p_1}{\sqrt{10 - p_2}}$

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$$q_2(5,1) = 2 \cdot 5 \cdot \sqrt{10-1} = 30$$

 $E_{q_2,p_2}(5,1) = \frac{1}{a_2(5,1)} \cdot \frac{\partial q_2}{\partial p_2}(5,1) = \frac{1}{30} \cdot \frac{-5}{3} = \frac{-1}{18} \approx -0.06$

Koeficijent parcijalne elastičnosti

 $\frac{\partial q_2}{\partial p_2}(5,1) = \frac{-5}{\sqrt{10-1}} = \frac{-5}{3}$

 $q_2(p_1, p_2) = 2p_1\sqrt{10 - p_2}$ $\frac{\partial q_2}{\partial p_2} = \frac{-p_1}{\sqrt{10 - p_2}}$

Ako na nivou cijena $p_1 = 5$, $p_2 = 1$ cijenu p_2 proizvoda D_2 povećamo za 1%, potražnja za proizvodom D_2 se smanji za 0.06%.

Proizvod D_2 je normalno dobro.

$$E_{q_2,p_1}(5,1)=$$

Koeficijent križne elastičnosti

 $q_2(p_1, p_2) = 2p_1\sqrt{10-p_2}$

 $q_2(p_1, p_2) = 2p_1\sqrt{10 - p_2}$ $\frac{\partial q_2}{\partial p_1} = 2\sqrt{10 - p_2}$

Koeficijent križne elastičnosti

 $q_2(p_1, p_2) = 2p_1\sqrt{10 - p_2}$ $\frac{\partial q_2}{\partial p_1} = 2\sqrt{10 - p_2}$

$$q_2(5,1)$$

Koeficijent križne elastičnosti

$$(5,1)=\frac{1}{q_2(5,1)}$$

 $\frac{\partial q_2}{\partial p_1} = 2\sqrt{10 - p_2}$

$$q_2(5,1)$$

$$E_{q_2,p_1}\!\!\left(5,1
ight) = rac{p_1}{q_2(5,1)} \cdot$$

 $\frac{\partial q_2}{\partial p_1} = 2\sqrt{10 - p_2}$

$$E_{q_2,
ho_1}(5,1) = rac{5}{q_2(5,1)} \cdot rac{\partial q_2}{\partial
ho_1}(5,1)$$

Koeficijent križne elastičnosti

 $q_2(p_1, p_2) = 2p_1\sqrt{10 - p_2}$ $\frac{\partial q_2}{\partial p_1} = 2\sqrt{10 - p_2}$

$$E_{q_2,p_1}(5,1) = rac{5}{q_2(5,1)} \cdot rac{\partial q_2}{\partial p_1}(5,1) =$$

$$q_2(5,1)=$$

 $E_{q_2,p_1}(5,1) = \frac{5}{q_2(5,1)} \cdot \frac{\partial q_2}{\partial p_1}(5,1) =$

Koeficijent križne elastičnosti

$$egin{aligned} E_{q_2,
ho_1}(5,1) &= rac{5}{q_2(5,1)} \cdot rac{\partial q_2}{\partial
ho_1}(5,1) = \ q_2(5,1) &= 2 \cdot 5 \cdot \sqrt{10-1} \end{aligned}$$

$$E_{q_2,p_1}(5,1) = rac{5}{q_2(5,1)} \cdot rac{\partial q_2}{\partial p_1}(5,1) = \ q_2(5,1) = 2 \cdot 5 \cdot \sqrt{10-1} = 30$$

$$E_{q_2,
ho_1}(5,1) = rac{5}{q_2(5,1)} \cdot rac{\partial q_2}{\partial
ho_1}(5,1) =$$

 $q_2(p_1, p_2) = 2p_1\sqrt{10 - p_2}$ $\frac{\partial q_2}{\partial p_1} = 2\sqrt{10 - p_2}$

$$q_{2}(5,1) \quad \partial p_{1}$$
 $q_{2}(5,1) \quad \partial p_{1}$
 $q_{2}(5,1) = 2 \cdot 5 \cdot \sqrt{10-1} = 30$
 $\frac{\partial q_{2}}{\partial p_{1}}(5,1) = 0$

Koeficijent križne elastičnosti

$$egin{align} E_{q_2,
ho_1}(f 5,1) &= rac{5}{q_2(5,1)} \cdot rac{\partial q_2}{\partial
ho_1}(5,1) = \ q_2(5,1) &= 2 \cdot 5 \cdot \sqrt{10-1} = 30 \end{array}$$

 $\frac{\partial q_2}{\partial p_1}(5,1) = 2\sqrt{10-1}$

$$E_{q_2,p_1}(5,1) = \frac{1}{q_2(5,1)} \cdot \frac{1}{\partial p_1}(5,1) = \frac{p_1}{(5,1)} \cdot \frac{p_2}{(5,1)} = \frac{1}{(5,1)} \cdot \frac{1}{(5,1)} = \frac{1}{(5,1)} = \frac{1}{(5,1)} \cdot \frac{1}{(5,1)} = \frac{1}{(5,1)} \cdot \frac{1}{(5,1)} = \frac{1}{(5,1)}$$

 $q_2(p_1, p_2) = 2p_1\sqrt{10 - p_2}$ $\frac{\partial q_2}{\partial p_1} = 2\sqrt{10 - p_2}$

$$E_{q_2,p_1}(5,1) = \frac{5}{q_2(5,1)} \cdot \frac{\partial q_2}{\partial p_1}(5,1) =$$

$$q_2(5,1) = 2 \cdot 5 \cdot \sqrt{10-1} = 30$$

$$\frac{\partial q_2}{\partial p_1}(5,1) = 2\sqrt{10-1} = 6$$

Koeficijent križne elastičnosti

$$q_2(5,1) = 2 \cdot 5 \cdot \sqrt{10-1} = 30$$

$$\frac{\partial q_2}{\partial p_1} {p_1 \choose 5, 1} = 2\sqrt{10-1} = 6$$

 $E_{q_2,p_1}(5,1) = \frac{5}{q_2(5,1)} \cdot \frac{\partial q_2}{\partial p_1}(5,1) = \frac{5}{q_2(5,1)}$

Koeficijent križne elastičnosti

$$q_2(5,1) = 2 \cdot 5 \cdot \sqrt{10 - 1} = 30$$

$$\frac{\partial q_2}{\partial p_1} {p_1 \choose 5, 1} = 2\sqrt{10 - 1} = 6$$

 $E_{q_2,p_1}(5,1) = \frac{5}{q_2(5,1)} \cdot \frac{\partial q_2}{\partial p_1}(5,1) = \frac{5}{30}$

Koeficijent križne elastičnosti

$$q_2(5,1) = 2 \cdot 5 \cdot \sqrt{10 - 1} = 30$$

$$\frac{\partial q_2}{\partial p_1}(5,1) = 2\sqrt{10 - 1} = 6$$

 $E_{q_2,p_1}(5,1) = \frac{5}{q_2(5,1)} \cdot \frac{\partial q_2}{\partial p_1}(5,1) = \frac{5}{30}$

Koeficijent križne elastičnosti

$$q_2(5,1) = 2 \cdot 5 \cdot \sqrt{10-1} = 30$$

$$\frac{\partial q_2}{\partial p_1}(5,1) = 2\sqrt{10-1} = 6$$

 $E_{q_2,p_1}(5,1) = \frac{5}{q_2(5,1)} \cdot \frac{\partial q_2}{\partial p_1}(5,1) = \frac{5}{30} \cdot 6$

Koeficijent križne elastičnosti

$$q_2(5,1) = 2 \cdot 5 \cdot \sqrt{10-1} = 30$$

$$\frac{\partial q_2}{\partial p_1}(5,1) = 2\sqrt{10-1} = 6$$

 $E_{q_2,p_1}(5,1) = \frac{5}{q_2(5,1)} \cdot \frac{\partial q_2}{\partial p_1}(5,1) = \frac{5}{30} \cdot 6 = 1$

Koeficijent križne elastičnosti

$$rac{\partial q_2}{\partial p_1} inom{
ho_1}{(5,1)} = 2\sqrt{10-1} = 6$$

 $a_2(5,1) = 2 \cdot 5 \cdot \sqrt{10-1} = 30$

 $E_{q_2,p_1}(5,1) = \frac{5}{a_2(5,1)} \cdot \frac{\partial q_2}{\partial p_1}(5,1) = \frac{5}{30} \cdot 6 = 1$

Koeficijent križne elastičnosti

 $q_2(p_1, p_2) = 2p_1\sqrt{10-p_2}$ $\frac{\partial q_2}{\partial p_1} = 2\sqrt{10-p_2}$

Ako na nivou cijena
$$p_1=5,\ p_2=1$$
 cijenu p_1 proizvoda D_1 povećamo za

1%, potražnja za proizvodom D_2 se poveća za 1%.

$$\frac{\partial q_2}{\partial x} (5,1) = 2\sqrt{2}$$

 $q_2(p_1, p_2) = 2p_1\sqrt{10-p_2}$ $\frac{\partial q_2}{\partial p_1} = 2\sqrt{10-p_2}$

$$rac{\partial q_2}{\partial p_1} \stackrel{p_1}{(5,1)} = 2\sqrt{10-1} = 6$$

Ako na nivou cijena $p_1=5,\ p_2=1$ cijenu p_1 proizvoda D_1 povećamo za 1% , potražnja za proizvodom D_2 se poveća za 1% .

 $E_{q_2,p_1}(5,1) = \frac{5}{q_2(5,1)} \cdot \frac{\partial q_2}{\partial p_1}(5,1) = \frac{5}{30} \cdot 6 = 1$

 $a_2(5,1) = 2 \cdot 5 \cdot \sqrt{10-1} = 30$

Proizvodi D_1 i D_2 su supstituti.

