Seminari 1

Matematičke metode za informatičare

Damir Horvat

FOI, Varaždin

Sadržaj

Oznake

prvi zadatak

drugi zadatak

treći zadatak

četvrti zadatak

peti zadatak

Dijeljenje dužine u zadanom omjeru

AB



AB ← pravac kroz točke A i B



\(\overline{AB} \)

AB ← pravac kroz točke A i B



AB ← dužina čiji su krajevi točke A i B

AB ← pravac kroz točke A i B



• \overline{AB} «www-dužina čiji su krajevi točke A i B



AB ← pravac kroz točke A i B





|AB|

AB ← pravac kroz točke A i B



• \overline{AB} «www-dužina čiji su krajevi točke A i B



• |AB| \leftarrow duljina dužine \overline{AB}

AB ← pravac kroz točke A i B



• \overline{AB} «www-dužina čiji su krajevi točke A i B



• |AB| «www-duljina dužine \overline{AB} (nenegativni realni broj)

 \bullet \overrightarrow{AB}

ullet \overrightarrow{AB} common orijentirana dužina čiji početak je točka A, a kraj točka B

ullet \overrightarrow{AB} common orijentirana dužina čiji početak je točka A, a kraj točka B



orijentirana dužina \overrightarrow{AB}

- ullet \overrightarrow{AB} «www-orijentirana dužina čiji početak je točka A, a kraj točka B
- [\overrightarrow{AB}]



orijentirana dužina \overrightarrow{AB}

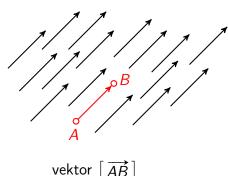
- ullet \overrightarrow{AB} \ullet orijentirana dužina čiji početak je točka A, a kraj točka B
- $[\overrightarrow{AB}]$ *** vektor čiji reprezentant je orijentirana dužina \overrightarrow{AB}

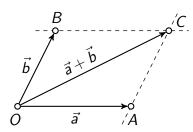


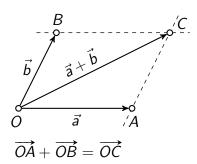
orijentirana dužina \overrightarrow{AB}

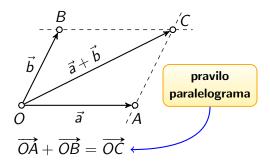
- \overrightarrow{AB} comordijentirana dužina čiji početak je točka A, a kraj točka B

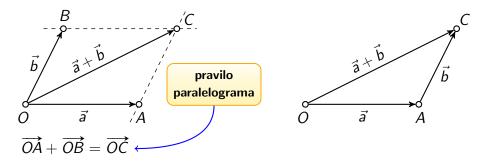


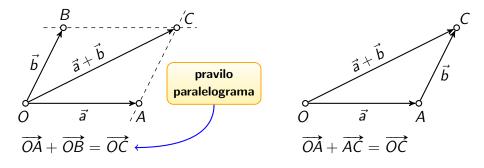


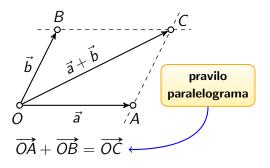


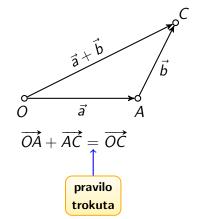


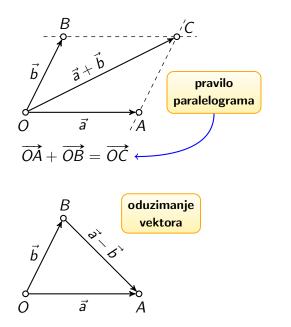


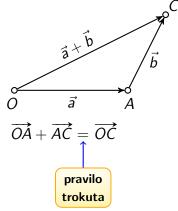


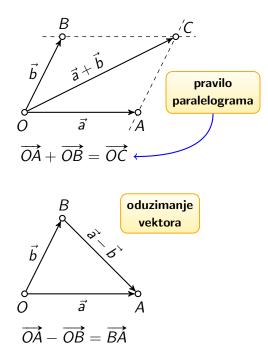


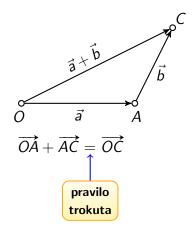


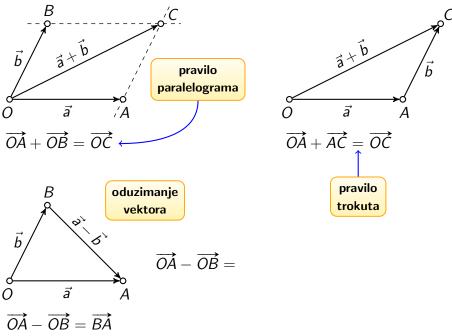


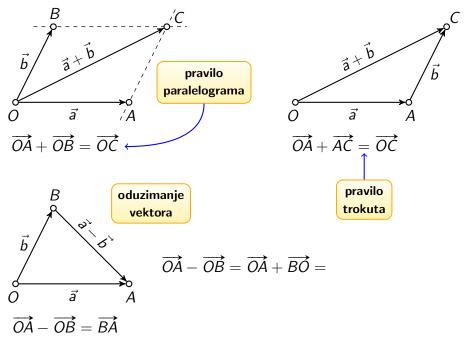


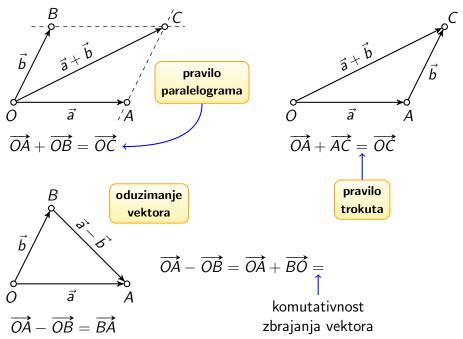


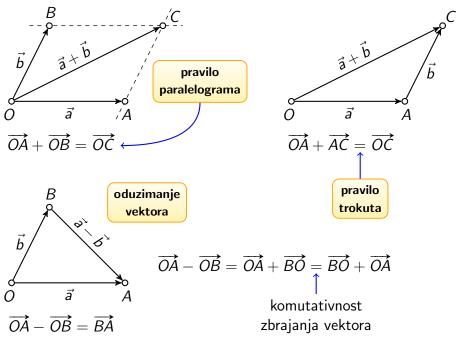


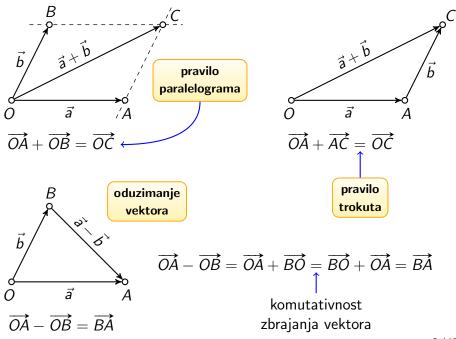












prvi zadatak

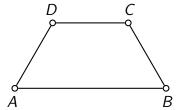
Zadatak 1

Točke A, B, C i D su redom vrhovi jednakokračnog trapeza kojemu je duljina kraka jednaka d i kojemu su duljine osnovica |AB| = 2d i |CD| = d.

- a) Nacrtajte vektor $\vec{v} = \frac{1}{2} \overrightarrow{AB} \overrightarrow{CB}$.
- b) Kolika je norma (duljina) vektora \vec{v} ?

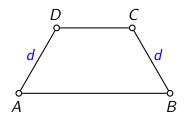
Rješenje

a)



Rješenje

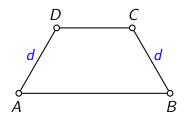
a)



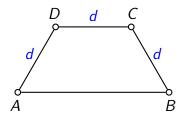
$$|AD| = |BC| = d$$

Rješenje

a)

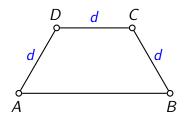


$$|AD| = |BC| = d$$
$$|AB| = 2d$$



$$|AD| = |BC| = d$$

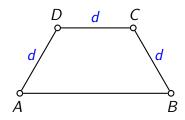
 $|AB| = 2d, \quad |CD| = d$



$$\vec{v} = \frac{1}{2} \overrightarrow{AB} - \overrightarrow{CB}$$

$$|AD| = |BC| = d$$

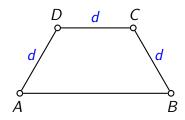
 $|AB| = 2d, \quad |CD| = d$



$$\vec{v} = \frac{1}{2} \overrightarrow{AB} - \overrightarrow{CB} =$$

$$|AD| = |BC| = d$$

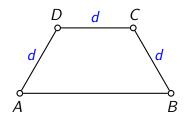
 $|AB| = 2d, \quad |CD| = d$



$$\vec{v} = \frac{1}{2} \overrightarrow{AB} - \overrightarrow{CB} = \frac{1}{2} \overrightarrow{AB}$$

$$|AD| = |BC| = d$$

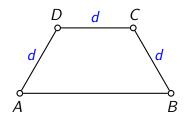
 $|AB| = 2d, \quad |CD| = d$



$$\vec{v} = \frac{1}{2}\overrightarrow{AB} - \overrightarrow{CB} = \frac{1}{2}\overrightarrow{AB} + \overrightarrow{BC}$$

$$|AD| = |BC| = d$$

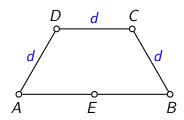
 $|AB| = 2d, \quad |CD| = d$



$$\vec{v} = \frac{1}{2}\overrightarrow{AB} - \overrightarrow{CB} = \frac{1}{2}\overrightarrow{AB} + \overrightarrow{BC} =$$

$$|AD| = |BC| = d$$

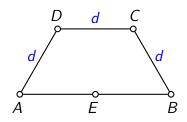
 $|AB| = 2d, \quad |CD| = d$



$$\vec{v} = \frac{1}{2}\overrightarrow{AB} - \overrightarrow{CB} = \frac{1}{2}\overrightarrow{AB} + \overrightarrow{BC} =$$

$$|AD| = |BC| = d$$

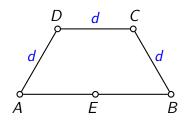
 $|AB| = 2d, \quad |CD| = d$



$$\vec{v} = \frac{1}{2}\overrightarrow{AB} - \overrightarrow{CB} = \frac{1}{2}\overrightarrow{AB} + \overrightarrow{BC} =$$

$$|AD| = |BC| = d$$

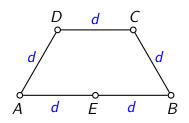
 $|AB| = 2d, \quad |CD| = d$
 $|AE| = |BE|$



$$\vec{v} = \frac{1}{2} \overrightarrow{AB} - \overrightarrow{CB} = \frac{1}{2} \overrightarrow{AB} + \overrightarrow{BC} =$$

$$|AD| = |BC| = d$$

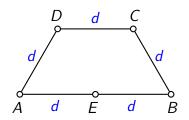
 $|AB| = 2d, \quad |CD| = d$
 $|AE| = |BE| = d$



$$\vec{v} = \frac{1}{2}\overrightarrow{AB} - \overrightarrow{CB} = \frac{1}{2}\overrightarrow{AB} + \overrightarrow{BC} =$$

$$|AD| = |BC| = d$$

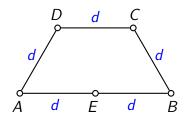
 $|AB| = 2d, \quad |CD| = d$
 $|AE| = |BE| = d$



$$|AD| = |BC| = d$$

 $|AB| = 2d, \quad |CD| = d$
 $|AE| = |BE| = d$

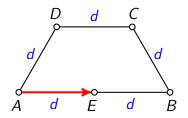
$$\vec{v} = \frac{1}{2}\overrightarrow{AB} - \overrightarrow{CB} = \frac{1}{2}\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AE}$$



$$|AD| = |BC| = d$$

 $|AB| = 2d, \quad |CD| = d$
 $|AE| = |BE| = d$

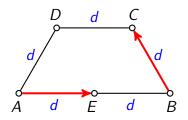
$$\vec{v} = \frac{1}{2}\overrightarrow{AB} - \overrightarrow{CB} = \frac{1}{2}\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AE} + \overrightarrow{BC}$$



$$|AD| = |BC| = d$$

 $|AB| = 2d, \quad |CD| = d$
 $|AE| = |BE| = d$

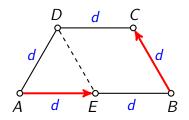
$$\vec{v} = \frac{1}{2}\overrightarrow{AB} - \overrightarrow{CB} = \frac{1}{2}\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AE} + \overrightarrow{BC}$$



$$|AD| = |BC| = d$$

 $|AB| = 2d, \quad |CD| = d$
 $|AE| = |BE| = d$

$$\vec{v} = \frac{1}{2}\overrightarrow{AB} - \overrightarrow{CB} = \frac{1}{2}\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AE} + \overrightarrow{BC}$$

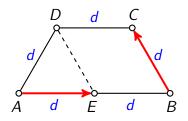


$$|AD| = |BC| = d$$

 $|AB| = 2d, \quad |CD| = d$
 $|AE| = |BE| = d$

$$\vec{v} = \frac{1}{2}\overrightarrow{AB} - \overrightarrow{CB} = \frac{1}{2}\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AE} + \overrightarrow{BC}$$

 $EB \parallel DC$,

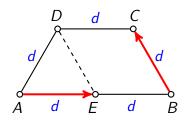


$$|AD| = |BC| = d$$

 $|AB| = 2d, \quad |CD| = d$
 $|AE| = |BE| = d$

$$\vec{v} = \frac{1}{2}\overrightarrow{AB} - \overrightarrow{CB} = \frac{1}{2}\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AE} + \overrightarrow{BC}$$

$EB \parallel DC, \mid EB \mid = \mid DC \mid$

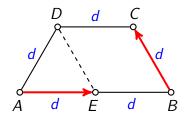


$$|AD| = |BC| = d$$

 $|AB| = 2d, \quad |CD| = d$
 $|AE| = |BE| = d$

$$\vec{v} = \frac{1}{2}\overrightarrow{AB} - \overrightarrow{CB} = \frac{1}{2}\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AE} + \overrightarrow{BC}$$

$$EB \parallel DC, \mid EB \mid = \mid DC \mid \implies EBCD$$
 je paralelogram

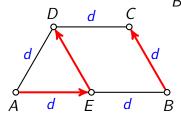


$$|AD| = |BC| = d$$

 $|AB| = 2d, \quad |CD| = d$
 $|AE| = |BE| = d$

$$\vec{v} = \frac{1}{2}\overrightarrow{AB} - \overrightarrow{CB} = \frac{1}{2}\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AE} + \overrightarrow{BC}$$

 $EB \parallel DC, \mid EB \mid = \mid DC \mid \implies EBCD$ je paralelogram



$$\overrightarrow{BC} = \overrightarrow{ED}$$

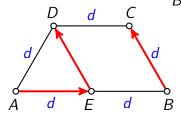
$$|AD| = |BC| = d$$

$$|AB| = 2d, \quad |CD| = d$$

 $|AE| = |BE| = d$

$$\vec{v} = \frac{1}{2}\overrightarrow{AB} - \overrightarrow{CB} = \frac{1}{2}\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AE} + \overrightarrow{BC}$$

 $EB \parallel DC, \mid EB \mid = \mid DC \mid \implies EBCD$ je paralelogram



$$\overrightarrow{BC} = \overrightarrow{ED}$$

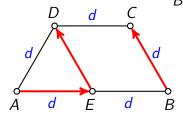
$$|AD| = |BC| = d$$

$$|AB|=2d, \quad |CD|=d$$

$$|AE| = |BE| = d$$

$$\vec{v} = \frac{1}{2} \overrightarrow{AB} - \overrightarrow{CB} = \frac{1}{2} \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AE} + \overrightarrow{BC} =$$

 $EB \parallel DC, \mid EB \mid = \mid DC \mid \implies EBCD$ je paralelogram



$$\overrightarrow{BC} = \overrightarrow{ED}$$

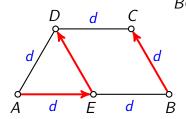
$$|AD| = |BC| = d$$

 $|AB| = 2d, \quad |CD| = d$
 $|AE| = |BE| = d$

$$\vec{v} = \frac{1}{2} \overrightarrow{AB} - \overrightarrow{CB} = \frac{1}{2} \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AE} + \overrightarrow{BC} = \overrightarrow{AE}$$

 $EB \parallel DC, \mid EB \mid = \mid DC \mid \implies EBCD$ je paralelogram

a)



$$\overrightarrow{BC} = \overrightarrow{ED}$$

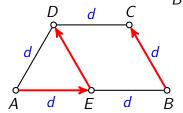
$$|AD| = |BC| = d$$

$$|AB| = 2d, \quad |CD| = d$$

|AE| = |BE| = d

$$\vec{v} = \frac{1}{2} \overrightarrow{AB} - \overrightarrow{CB} = \frac{1}{2} \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AE} + \overrightarrow{BC} = \overrightarrow{AE} + \overrightarrow{ED}$$

$$EB \parallel DC, \mid EB \mid = \mid DC \mid \implies EBCD$$
 je paralelogram



$$\overrightarrow{BC} = \overrightarrow{ED}$$

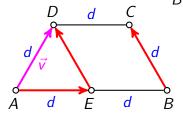
$$|AD| = |BC| = d$$

$$|AB| = 2d, \quad |CD| = d$$

$$|AE| = |BE| = d$$

$$\vec{v} = \frac{1}{2} \overrightarrow{AB} - \overrightarrow{CB} = \frac{1}{2} \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AE} + \overrightarrow{BC} = \overrightarrow{AE} + \overrightarrow{ED} = \overrightarrow{AD}$$

 $EB \parallel DC, \mid EB \mid = \mid DC \mid \implies EBCD$ je paralelogram



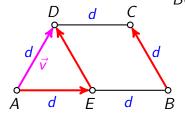
$$\overrightarrow{BC} = \overrightarrow{ED}$$

$$|AD| = |BC| = d$$

 $|AB| = 2d, \quad |CD| = d$
 $|AE| = |BE| = d$

$$\vec{v} = \frac{1}{2} \overrightarrow{AB} - \overrightarrow{CB} = \frac{1}{2} \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AE} + \overrightarrow{BC} = \overrightarrow{AE} + \overrightarrow{ED} = \overrightarrow{AD}$$

$$EB \parallel DC, \mid EB \mid = \mid DC \mid \implies EBCD$$
 je paralelogram



$$\overrightarrow{BC} = \overrightarrow{ED}$$

$$|AD| = |BC| = d$$

 $|AB| = 2d, \quad |CD| = d$
 $|AE| = |BE| = d$

$$\vec{v} = \frac{1}{2} \overrightarrow{AB} - \overrightarrow{CB} = \frac{1}{2} \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AE} + \overrightarrow{BC} = \overrightarrow{AE} + \overrightarrow{ED} = \overrightarrow{AD}$$

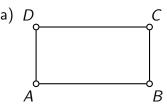
b)
$$|\vec{v}| = d$$

drugi zadatak

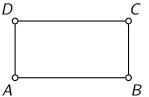
Zadatak 2

Točke A, B, C i D su redom vrhovi pravokutnika kojemu su duljine susjednih stranica |AB| = a i |AD| = b.

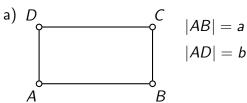
- a) Nacrtajte vektor $\overrightarrow{AE} = \overrightarrow{AB} \overrightarrow{BC}$.
- b) Kolika je norma vektora \overrightarrow{AE} ?
- c) Nacrtajte vektor $\overrightarrow{AF} = \overrightarrow{AB} \overrightarrow{AC}$.
- d) Kolika je norma vektora \overrightarrow{AF} ?



a) *D*



|AB| = a



a)
$$D$$

$$|AB| = a$$

$$|AD| = B$$

$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC}$$

(a)
$$D$$
 C $|AB| = a$ $|AD| = B$

$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB}$$

(a)
$$D$$
 C $|AB| = a$ $|AD| = B$

$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{CB}$$

(a)
$$D$$
 C $|AB| = a$ $|AD| = B$

$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{CB}$$

(a) D
$$|AB| = a$$

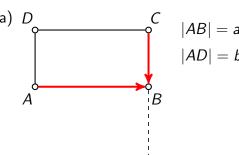
$$|AD| = b$$

$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{CB}$$

(a)
$$D$$
 C $|AB| = a$ $|AD| = b$

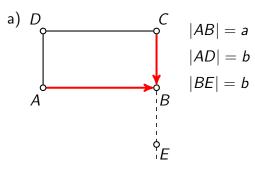
$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{CB} =$$

$$= \overrightarrow{AB} +$$



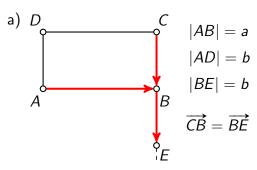
$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{CB} =$$

$$= \overrightarrow{AB} +$$



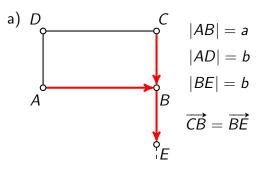
$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{CB} =$$

$$= \overrightarrow{AB} +$$



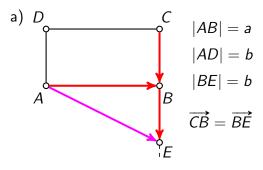
$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{CB} =$$

$$= \overrightarrow{AB} +$$



$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{CB} =$$

$$= \overrightarrow{AB} + \overrightarrow{BE}$$



$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{CB} =$$

$$= \overrightarrow{AB} + \overrightarrow{BE}$$

a) D A = A = A |AB| = A = A |AB| = B = A |BE| = B = B = A $\overrightarrow{CB} = \overrightarrow{BE}$

$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{CB} =$$

$$= \overrightarrow{AB} + \overrightarrow{BE}$$

b)
$$|\overrightarrow{AE}| =$$

a)
$$D$$

$$|AB| = a$$

$$|AD| = b$$

$$|BE| = b$$

$$\overrightarrow{CB} = \overrightarrow{BE}$$

$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{CB} =$$

$$= \overrightarrow{AB} + \overrightarrow{BE}$$

b)
$$|\overrightarrow{AE}| =$$

(a)
$$D$$

$$A = A$$

$$A$$

$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{CB} =$$

$$= \overrightarrow{AB} + \overrightarrow{BE}$$

b)
$$|\overrightarrow{AE}| =$$

(a) D
$$|AB| = a$$

$$|AD| = b$$

$$|BE| = b$$

$$\overrightarrow{CB} = \overrightarrow{BE}$$

$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{CB} =$$

$$= \overrightarrow{AB} + \overrightarrow{BE}$$

b)
$$|\overrightarrow{AE}| = \sqrt{a^2 + b^2}$$

a)
$$D$$

$$A = A$$

$$A =$$

$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{CB} =$$

$$= \overrightarrow{AB} + \overrightarrow{BE}$$

b)
$$|\overrightarrow{AE}| = \sqrt{a^2 + b^2}$$

$$|AB| = a$$

$$|AD| = b$$

$$|BE| = b$$

$$\overrightarrow{CB} = \overrightarrow{BE}$$

$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{CB} =$$

$$= \overrightarrow{AB} + \overrightarrow{BE}$$

b)
$$|\overrightarrow{AE}| = \sqrt{a^2 + b^2}$$

$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} =$$

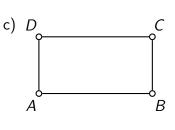
$$\begin{vmatrix} AB | = a \\ |AD| = b \\ |BE| = b \end{vmatrix}$$

$$\overrightarrow{CB} = \overrightarrow{BE}$$

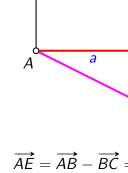
$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{CB} =$$

$$= \overrightarrow{AB} + \overrightarrow{BE}$$

b)
$$\left|\overrightarrow{AE}\right| = \sqrt{a^2 + b^2}$$



$$\overrightarrow{AF} = \overrightarrow{AB} - \overrightarrow{AC}$$



$$|AB| = a$$

$$|AD| = b$$

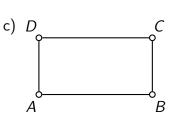
$$|BE| = b$$

$$\overrightarrow{CB} = \overrightarrow{BE}$$

$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{CB} =$$

$$= \overrightarrow{AB} + \overrightarrow{BE}$$

b)
$$|\overrightarrow{AE}| = \sqrt{a^2 + b^2}$$



$$\overrightarrow{AF} = \overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CB}$$

$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{A}$$

$$|AB| = a$$

$$|AD| = b$$

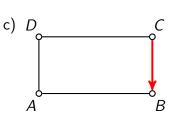
$$|BE| = b$$

$$\overrightarrow{CB} = \overrightarrow{BE}$$

$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{CB} =$$

$$= \overrightarrow{AB} + \overrightarrow{BE}$$

b)
$$|\overrightarrow{AE}| = \sqrt{a^2 + b^2}$$



$$\overrightarrow{AF} = \overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CB}$$

$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB}$$

$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{CB} =$$

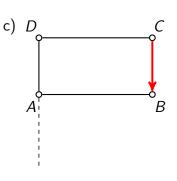
$$= \overrightarrow{AB} + \overrightarrow{BE}$$

|AB| = a

|BE| = b

 $\overrightarrow{CB} = \overrightarrow{BE}$

b)
$$\left|\overrightarrow{AE}\right| = \sqrt{a^2 + b^2}$$



$$\overrightarrow{AF} = \overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CB}$$

A

$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{CB} =$$

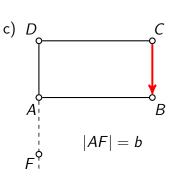
$$= \overrightarrow{AB} + \overrightarrow{BE}$$

|AB| = a

|BE| = b

 $\overrightarrow{CB} = \overrightarrow{BE}$

b)
$$|\overrightarrow{AE}| = \sqrt{a^2 + b^2}$$



$$\overrightarrow{AF} = \overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CB}$$

$$|AB| = a$$

$$|AD| = b$$

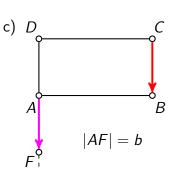
$$|BE| = b$$

$$\overrightarrow{CB} = \overrightarrow{BE}$$

$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{CB} =$$

$$= \overrightarrow{AB} + \overrightarrow{BE}$$

b)
$$|\overrightarrow{AE}| = \sqrt{a^2 + b^2}$$



$$\overrightarrow{AF} = \overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CB}$$

$$|BE| = b$$
 $\overrightarrow{CB} = \overrightarrow{BE}$

|AB| = a

$$\overrightarrow{AE} = \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{CB} =$$

$$= \overrightarrow{AB} + \overrightarrow{BE}$$

b)
$$|\overrightarrow{AE}| = \sqrt{a^2 + b^2}$$

c)
$$D$$
 C $AF = b$

 $\overrightarrow{AF} = \overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CB}$

d)
$$\left|\overrightarrow{AF}\right| = b$$

treći zadatak

Definicija kolinearnih vektora

Za dva vektora iz V^3 kažemo da su kolinearni ako imaju isti smjer. Po dogovoru je nulvektor kolinearan sa svakim vektorom iz V^3 .

Definicija kolinearnih vektora

Za dva vektora iz V^3 kažemo da su kolinearni ako imaju isti smjer. Po dogovoru je nulvektor kolinearan sa svakim vektorom iz V^3 .

Iznimno važna propozicija

Neka su $\vec{a}, \vec{b} \in V^3$ različiti od nulvektora. Tada vrijedi:

 \vec{a} i \vec{b} imaju isti smjer $\Leftrightarrow \exists \lambda \in \mathbb{R}, \ \vec{b} = \lambda \vec{a}$.

U tom je slučaju $\lambda \neq 0$ i vrijedi $\vec{a} = \frac{1}{\lambda} \vec{b}$. Nadalje, takav λ je jedinstven.

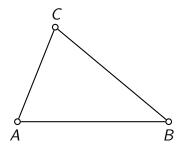
Dokažite da je srednjica trokuta paralelna s nasuprotnom stranicom trokuta i da je njezina duljina jednaka polovici duljine te stranice.

Dokažite da je srednjica trokuta paralelna s nasuprotnom stranicom trokuta i da je njezina duljina jednaka polovici duljine te stranice.

Rješenje

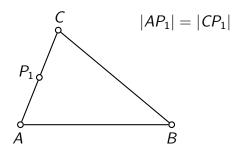
Dokažite da je srednjica trokuta paralelna s nasuprotnom stranicom trokuta i da je njezina duljina jednaka polovici duljine te stranice.

Rješenje



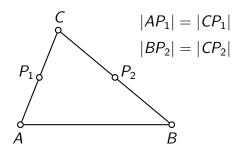
Dokažite da je srednjica trokuta paralelna s nasuprotnom stranicom trokuta i da je njezina duljina jednaka polovici duljine te stranice.

Rješenje



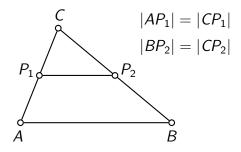
Dokažite da je srednjica trokuta paralelna s nasuprotnom stranicom trokuta i da je njezina duljina jednaka polovici duljine te stranice.

Rješenje



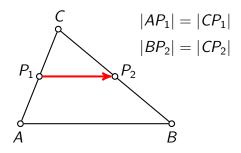
Dokažite da je srednjica trokuta paralelna s nasuprotnom stranicom trokuta i da je njezina duljina jednaka polovici duljine te stranice.

Rješenje



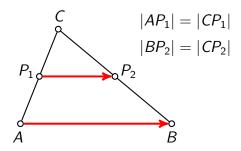
Dokažite da je srednjica trokuta paralelna s nasuprotnom stranicom trokuta i da je njezina duljina jednaka polovici duljine te stranice.

Rješenje



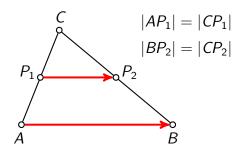
Dokažite da je srednjica trokuta paralelna s nasuprotnom stranicom trokuta i da je njezina duljina jednaka polovici duljine te stranice.

Rješenje



Dokažite da je srednjica trokuta paralelna s nasuprotnom stranicom trokuta i da je njezina duljina jednaka polovici duljine te stranice.

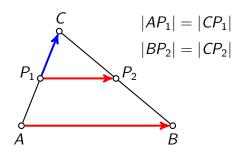
Rješenje



$$\overrightarrow{P_1P_2} =$$

Dokažite da je srednjica trokuta paralelna s nasuprotnom stranicom trokuta i da je njezina duljina jednaka polovici duljine te stranice.

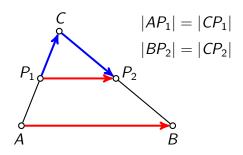
Rješenje



$$\overrightarrow{P_1P_2} = \overrightarrow{P_1C} +$$

Dokažite da je srednjica trokuta paralelna s nasuprotnom stranicom trokuta i da je njezina duljina jednaka polovici duljine te stranice.

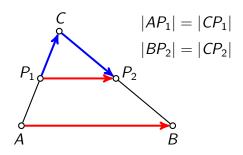
Rješenje



$$\overrightarrow{P_1P_2} = \overrightarrow{P_1C} + \overrightarrow{CP_2}$$

Dokažite da je srednjica trokuta paralelna s nasuprotnom stranicom trokuta i da je njezina duljina jednaka polovici duljine te stranice.

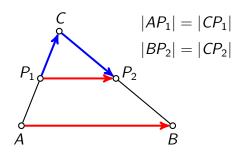
Rješenje



$$\overrightarrow{P_1P_2} = \overrightarrow{P_1C} + \overrightarrow{CP_2} = \frac{1}{2}\overrightarrow{AC}$$

Dokažite da je srednjica trokuta paralelna s nasuprotnom stranicom trokuta i da je njezina duljina jednaka polovici duljine te stranice.

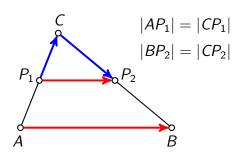
Rješenje



$$\overrightarrow{P_1P_2} = \overrightarrow{P_1C} + \overrightarrow{CP_2} = \frac{1}{2}\overrightarrow{AC} + \overrightarrow{CP_2}$$

Dokažite da je srednjica trokuta paralelna s nasuprotnom stranicom trokuta i da je njezina duljina jednaka polovici duljine te stranice.

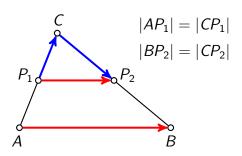
Rješenje



$$\overrightarrow{P_1P_2} = \overrightarrow{P_1C} + \overrightarrow{CP_2} = \frac{1}{2}\overrightarrow{AC} + \frac{1}{2}\overrightarrow{CB}$$

Dokažite da je srednjica trokuta paralelna s nasuprotnom stranicom trokuta i da je njezina duljina jednaka polovici duljine te stranice.

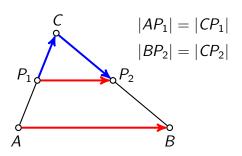
Rješenje



$$\overrightarrow{P_1P_2} = \overrightarrow{P_1C} + \overrightarrow{CP_2} = \frac{1}{2}\overrightarrow{AC} + \frac{1}{2}\overrightarrow{CB} = \frac{1}{2}\left(\overrightarrow{AC} + \overrightarrow{CB}\right)$$

Dokažite da je srednjica trokuta paralelna s nasuprotnom stranicom trokuta i da je njezina duljina jednaka polovici duljine te stranice.

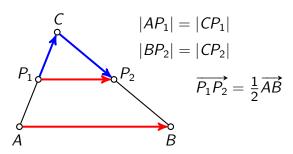
Rješenje



$$\overrightarrow{P_1P_2} = \overrightarrow{P_1C} + \overrightarrow{CP_2} = \frac{1}{2}\overrightarrow{AC} + \frac{1}{2}\overrightarrow{CB} = \frac{1}{2}\left(\overrightarrow{AC} + \overrightarrow{CB}\right) = \frac{1}{2}\overrightarrow{AB}$$

Dokažite da je srednjica trokuta paralelna s nasuprotnom stranicom trokuta i da je njezina duljina jednaka polovici duljine te stranice.

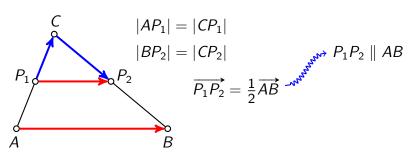
Rješenje



$$\overrightarrow{P_1P_2} = \overrightarrow{P_1C} + \overrightarrow{CP_2} = \frac{1}{2}\overrightarrow{AC} + \frac{1}{2}\overrightarrow{CB} = \frac{1}{2}\left(\overrightarrow{AC} + \overrightarrow{CB}\right) = \frac{1}{2}\overrightarrow{AB}$$

Dokažite da je srednjica trokuta paralelna s nasuprotnom stranicom trokuta i da je njezina duljina jednaka polovici duljine te stranice.

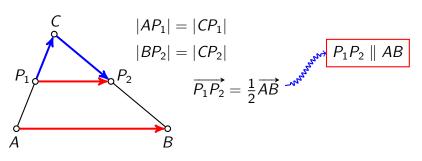
Rješenje



$$\overrightarrow{P_1P_2} = \overrightarrow{P_1C} + \overrightarrow{CP_2} = \frac{1}{2}\overrightarrow{AC} + \frac{1}{2}\overrightarrow{CB} = \frac{1}{2}\left(\overrightarrow{AC} + \overrightarrow{CB}\right) = \frac{1}{2}\overrightarrow{AB}$$

Dokažite da je srednjica trokuta paralelna s nasuprotnom stranicom trokuta i da je njezina duljina jednaka polovici duljine te stranice.

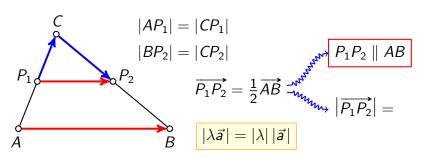
Rješenje



$$\overrightarrow{P_1P_2} = \overrightarrow{P_1C} + \overrightarrow{CP_2} = \frac{1}{2}\overrightarrow{AC} + \frac{1}{2}\overrightarrow{CB} = \frac{1}{2}\left(\overrightarrow{AC} + \overrightarrow{CB}\right) = \frac{1}{2}\overrightarrow{AB}$$

Dokažite da je srednjica trokuta paralelna s nasuprotnom stranicom trokuta i da je njezina duljina jednaka polovici duljine te stranice.

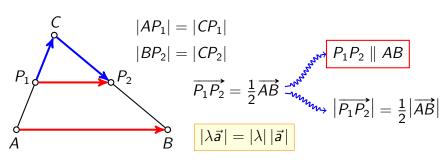
Rješenje



$$\overrightarrow{P_1P_2} = \overrightarrow{P_1C} + \overrightarrow{CP_2} = \frac{1}{2}\overrightarrow{AC} + \frac{1}{2}\overrightarrow{CB} = \frac{1}{2}\left(\overrightarrow{AC} + \overrightarrow{CB}\right) = \frac{1}{2}\overrightarrow{AB}$$

Dokažite da je srednjica trokuta paralelna s nasuprotnom stranicom trokuta i da je njezina duljina jednaka polovici duljine te stranice.

Rješenje



$$\overrightarrow{P_1P_2} = \overrightarrow{P_1C} + \overrightarrow{CP_2} = \frac{1}{2}\overrightarrow{AC} + \frac{1}{2}\overrightarrow{CB} = \frac{1}{2}\left(\overrightarrow{AC} + \overrightarrow{CB}\right) = \frac{1}{2}\overrightarrow{AB}$$

Dokažite da je srednjica trokuta paralelna s nasuprotnom stranicom trokuta i da je njezina duljina jednaka polovici duljine te stranice.

Rješenje

$$|AP_1| = |CP_1|$$

$$|BP_2| = |CP_2|$$

$$|P_1 \rightarrow P_2| = \frac{1}{2} \overrightarrow{AB} \xrightarrow{\mathcal{A} \rightarrow \mathcal{A} \rightarrow \mathcal{A}} |P_1 P_2| = \frac{1}{2} |\overrightarrow{AB}|$$

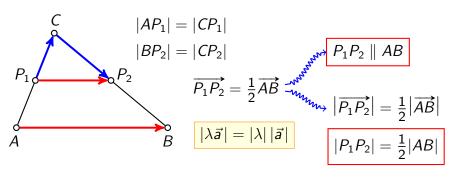
$$|\lambda \overrightarrow{a}| = |\lambda| |\overrightarrow{a}|$$

$$|P_1 P_2| = \frac{1}{2} |AB|$$

$$\overrightarrow{P_1P_2} = \overrightarrow{P_1C} + \overrightarrow{CP_2} = \frac{1}{2}\overrightarrow{AC} + \frac{1}{2}\overrightarrow{CB} = \frac{1}{2}\left(\overrightarrow{AC} + \overrightarrow{CB}\right) = \frac{1}{2}\overrightarrow{AB}$$

Dokažite da je srednjica trokuta paralelna s nasuprotnom stranicom trokuta i da je njezina duljina jednaka polovici duljine te stranice.

Rješenje

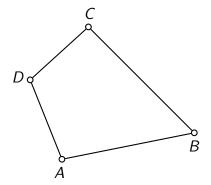


$$\overrightarrow{P_1P_2} = \overrightarrow{P_1C} + \overrightarrow{CP_2} = \frac{1}{2}\overrightarrow{AC} + \frac{1}{2}\overrightarrow{CB} = \frac{1}{2}\left(\overrightarrow{AC} + \overrightarrow{CB}\right) = \frac{1}{2}\overrightarrow{AB}$$

četvrti zadatak

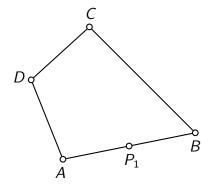
Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.



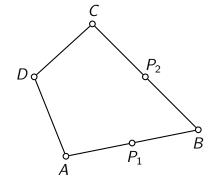
Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

$$|AP_1| = |BP_1|$$



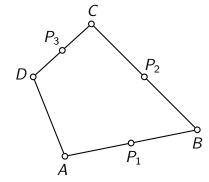
Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

$$|AP_1| = |BP_1|$$
$$|BP_2| = |CP_2|$$



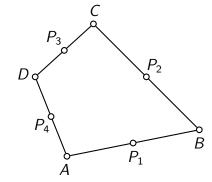
Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

$$|AP_1| = |BP_1|$$
 $|CP_3| = |DP_3|$
 $|BP_2| = |CP_2|$



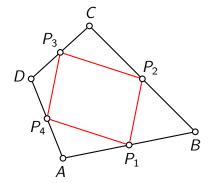
Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

$$|AP_1| = |BP_1|$$
 $|CP_3| = |DP_3|$
 $|BP_2| = |CP_2|$ $|AP_4| = |DP_4|$



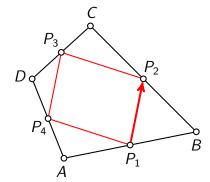
Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

$$|AP_1| = |BP_1|$$
 $|CP_3| = |DP_3|$
 $|BP_2| = |CP_2|$ $|AP_4| = |DP_4|$



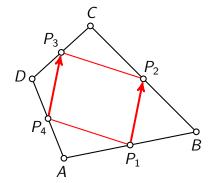
Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

$$|AP_1| = |BP_1|$$
 $|CP_3| = |DP_3|$
 $|BP_2| = |CP_2|$ $|AP_4| = |DP_4|$



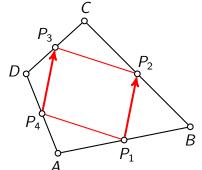
Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

$$|AP_1| = |BP_1|$$
 $|CP_3| = |DP_3|$
 $|BP_2| = |CP_2|$ $|AP_4| = |DP_4|$



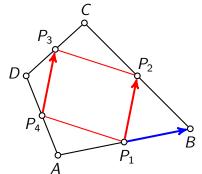
Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

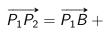
$$|AP_1| = |BP_1|$$
 $|CP_3| = |DP_3|$ $\overrightarrow{P_1P_2} = |BP_2| = |CP_2|$ $|AP_4| = |DP_4|$



Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

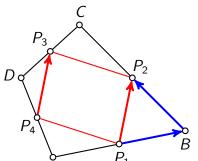
$$|AP_1| = |BP_1|$$
 $|CP_3| = |DP_3|$ $\overrightarrow{P_1P_2} = \overrightarrow{P_1B} + |BP_2| = |CP_2|$ $|AP_4| = |DP_4|$

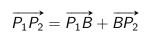




Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

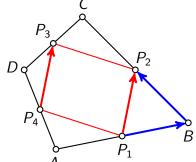
$$|AP_1| = |BP_1|$$
 $|CP_3| = |DP_3|$ $\overrightarrow{P_1P_2} = \overrightarrow{P_1B} + \overrightarrow{BP_2}$
 $|BP_2| = |CP_2|$ $|AP_4| = |DP_4|$

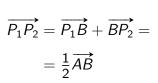




Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

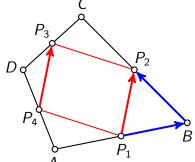
$$|AP_1| = |BP_1|$$
 $|CP_3| = |DP_3|$ $\overrightarrow{P_1P_2} = \overrightarrow{P_1B} + \overrightarrow{BP_2} = |BP_2| = |CP_2|$ $|AP_4| = |DP_4|$ $= \frac{1}{2}\overrightarrow{AB}$

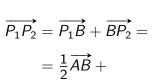




Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

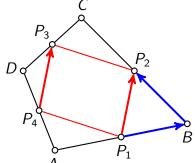
$$|AP_1| = |BP_1|$$
 $|CP_3| = |DP_3|$ $\overrightarrow{P_1P_2} = \overrightarrow{P_1B} + \overrightarrow{BP_2} = |BP_2| = |CP_2|$ $|AP_4| = |DP_4|$ $= \frac{1}{2}\overrightarrow{AB} + |CP_2|$





Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

$$|AP_1| = |BP_1|$$
 $|CP_3| = |DP_3|$ $\overrightarrow{P_1P_2} = \overrightarrow{P_1B} + \overrightarrow{BP_2} = |BP_2| = |CP_2|$ $|AP_4| = |DP_4|$ $= \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC}$

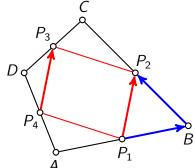


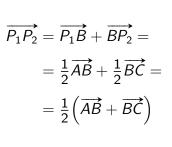
$$\overrightarrow{P_1P_2} = \overrightarrow{P_1B} + \overrightarrow{BP_2} =$$

$$= \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC}$$

Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

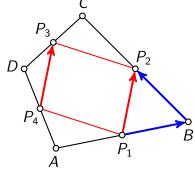
$$|AP_1| = |BP_1|$$
 $|CP_3| = |DP_3|$
 $|BP_2| = |CP_2|$ $|AP_4| = |DP_4|$





Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

$$|AP_1| = |BP_1|$$
 $|CP_3| = |DP_3|$
 $|BP_2| = |CP_2|$ $|AP_4| = |DP_4|$



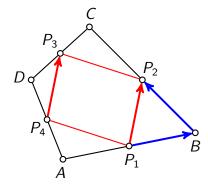
$$\overrightarrow{P_1P_2} = \overrightarrow{P_1B} + \overrightarrow{BP_2} =$$

$$= \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} =$$

$$= \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{BC}) = \frac{1}{2}\overrightarrow{AC}$$

Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

$$|AP_1| = |BP_1|$$
 $|CP_3| = |DP_3|$
 $|BP_2| = |CP_2|$ $|AP_4| = |DP_4|$



$$\overrightarrow{P_1P_2} = \overrightarrow{P_1B} + \overrightarrow{BP_2} =$$

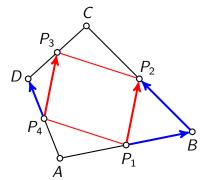
$$= \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} =$$

$$= \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{BC}) = \frac{1}{2}\overrightarrow{AC}$$

$$\overrightarrow{P_4P_3} =$$

Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

$$|AP_1| = |BP_1|$$
 $|CP_3| = |DP_3|$
 $|BP_2| = |CP_2|$ $|AP_4| = |DP_4|$



$$\overrightarrow{P_1P_2} = \overrightarrow{P_1B} + \overrightarrow{BP_2} =$$

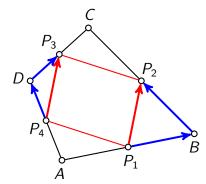
$$= \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} =$$

$$= \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{BC}) = \frac{1}{2}\overrightarrow{AC}$$

$$\overrightarrow{P_4P_3} = \overrightarrow{P_4D} +$$

Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

$$|AP_1| = |BP_1|$$
 $|CP_3| = |DP_3|$
 $|BP_2| = |CP_2|$ $|AP_4| = |DP_4|$



$$\overrightarrow{P_1P_2} = \overrightarrow{P_1B} + \overrightarrow{BP_2} =$$

$$= \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} =$$

$$= \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{BC}) = \frac{1}{2}\overrightarrow{AC}$$

$$\overrightarrow{P_4P_3} = \overrightarrow{P_4D} + \overrightarrow{DP_3}$$

Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

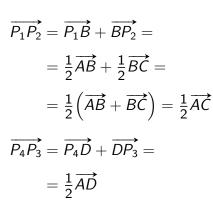
Rješenje

$$|BP_2| = |CP_2| \qquad |AP_4| = |DP_4|$$

$$C$$

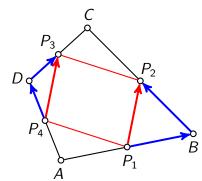
$$P_3$$

 $|AP_1| = |BP_1|$ $|CP_3| = |DP_3|$



Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

$$|AP_1| = |BP_1|$$
 $|CP_3| = |DP_3|$
 $|BP_2| = |CP_2|$ $|AP_4| = |DP_4|$



$$\overrightarrow{P_1P_2} = \overrightarrow{P_1B} + \overrightarrow{BP_2} =$$

$$= \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} =$$

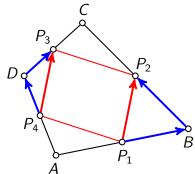
$$= \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{BC}) = \frac{1}{2}\overrightarrow{AC}$$

$$\overrightarrow{P_4P_3} = \overrightarrow{P_4D} + \overrightarrow{DP_3} =$$

$$= \frac{1}{2}\overrightarrow{AD} +$$

Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

$$|AP_1| = |BP_1|$$
 $|CP_3| = |DP_3|$
 $|BP_2| = |CP_2|$ $|AP_4| = |DP_4|$



$$\overrightarrow{P_1P_2} = \overrightarrow{P_1B} + \overrightarrow{BP_2} =$$

$$= \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} =$$

$$= \frac{1}{2}\left(\overrightarrow{AB} + \overrightarrow{BC}\right) = \frac{1}{2}\overrightarrow{AC}$$

$$\overrightarrow{P_4P_3} = \overrightarrow{P_4D} + \overrightarrow{DP_3} =$$

$$= \frac{1}{2}\overrightarrow{AD} + \frac{1}{2}\overrightarrow{DC}$$

Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

Rješenje

$$|BP_2| = |CP_2| \qquad |AP_4| = |DP_4|$$

$$C$$

$$P_3$$

$$P_4$$

$$P_1$$

$$B$$

 $|AP_1| = |BP_1|$ $|CP_3| = |DP_3|$

$$\overrightarrow{P_1P_2} = \overrightarrow{P_1B} + \overrightarrow{BP_2} =$$

$$= \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} =$$

$$= \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{BC}) = \frac{1}{2}\overrightarrow{AC}$$

$$\overrightarrow{P_4P_3} = \overrightarrow{P_4D} + \overrightarrow{DP_3} =$$

$$= \frac{1}{2}\overrightarrow{AD} + \frac{1}{2}\overrightarrow{DC} =$$

$$= \frac{1}{2}(\overrightarrow{AD} + \overrightarrow{DC})$$

Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

Rješenje

$$|AP_{1}| = |BP_{1}|$$
 $|CP_{3}| = |DP_{3}|$
 $|BP_{2}| = |CP_{2}|$ $|AP_{4}| = |DP_{4}|$
 P_{3}
 P_{4}
 P_{4}
 P_{4}
 P_{4}
 P_{4}
 P_{4}

$$\overrightarrow{P_1P_2} = \overrightarrow{P_1B} + \overrightarrow{BP_2} =$$

$$= \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} =$$

$$= \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{BC}) = \frac{1}{2}\overrightarrow{AC}$$

$$\overrightarrow{P_4P_3} = \overrightarrow{P_4D} + \overrightarrow{DP_3} =$$

$$= \frac{1}{2}\overrightarrow{AD} + \frac{1}{2}\overrightarrow{DC} =$$

 $=\frac{1}{2}(\overrightarrow{AD}+\overrightarrow{DC})=\frac{1}{2}\overrightarrow{AC}$

Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

Rješenje

$$\implies \overrightarrow{P_1P_2} = \overrightarrow{P_4P_3}$$
 ešenje

 $|AP_1| = |BP_1|$ $|CP_3| = |DP_3|$

$$|BP_2| = |CP_2| \qquad |AP_4| = |DP_4|$$

$$C$$

$$P_3$$

$$P_4$$

$$\overrightarrow{P_1P_2} = \overrightarrow{P_1B} + \overrightarrow{BP_2} =$$

$$= \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} =$$

$$= \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{BC}) = \frac{1}{2}\overrightarrow{AC}$$

 $\overrightarrow{P_AP_3} = \overrightarrow{P_AD} + \overrightarrow{DP_3} =$

$$= \frac{1}{2}\overrightarrow{AD} + \frac{1}{2}\overrightarrow{DC} =$$

$$= \frac{1}{2}\left(\overrightarrow{AD} + \overrightarrow{DC}\right) = \frac{1}{2}\overrightarrow{AC}$$
10/17

Dokažite da su polovišta stranica bilo kojeg četverokuta vrhovi paralelograma.

 $\implies \overrightarrow{P_1P_2} = \overrightarrow{P_4P_3} \implies P_1P_2P_3P_4$ je paralelogram

 $|BP_2| = |CP_2|$

$$|AP_1| = |BP_1| \qquad |CP_3| = |DP_3|$$

 $|AP_4| = |DP_4|$

$$P_3$$
 P_2
 P_4
 P_1
 P_2
 P_3
 P_4

$$\overrightarrow{P_1P_2} = \overrightarrow{P_1B} + \overrightarrow{BP_2} =$$

$$= \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} =$$

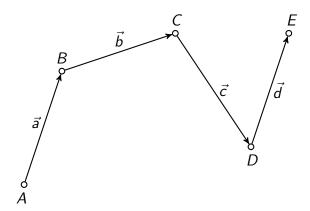
$$= \frac{1}{2} \left(\overrightarrow{AB} + \overrightarrow{BC} \right) = \frac{1}{2} \overrightarrow{AC}$$

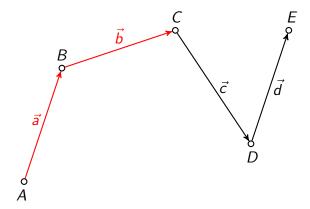
$$\overrightarrow{P_4P_3} = \overrightarrow{P_4D} + \overrightarrow{DP_3} =$$

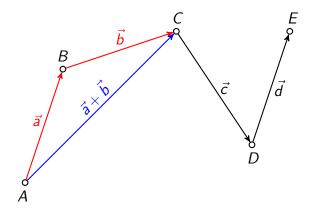
$$= \frac{1}{2}\overrightarrow{AD} + \frac{1}{2}\overrightarrow{DC} =$$

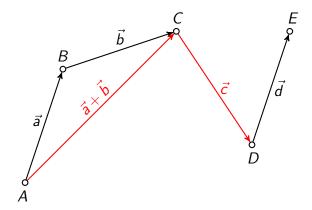
$$= \frac{1}{2} \left(\overrightarrow{AD} + \overrightarrow{DC} \right) = \frac{1}{2}\overrightarrow{AC}$$

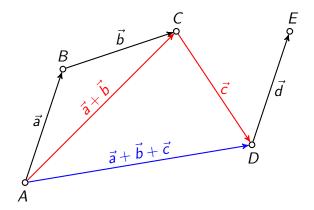
peti zadatak

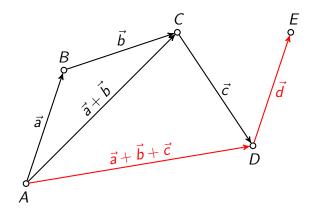


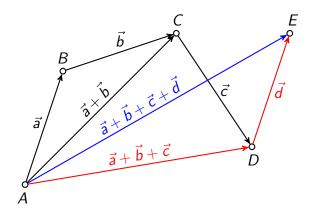


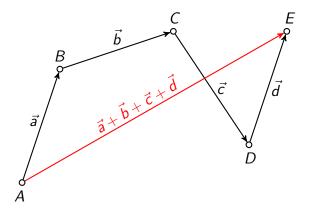












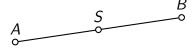
Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.

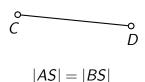
Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.



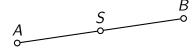


Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.





Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.

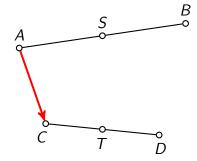


$$C$$
 T D

$$|AS| = |BS|$$

$$|CT| = |DT|$$

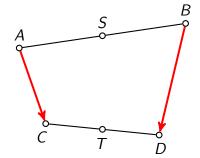
Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.



$$|AS| = |BS|$$

$$|CT| = |DT|$$

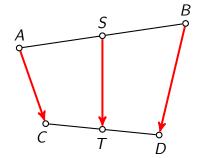
Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.



$$|AS| = |BS|$$

$$|CT| = |DT|$$

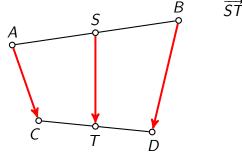
Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.



$$|AS| = |BS|$$

$$|CT| = |DT|$$

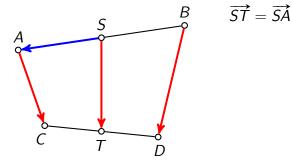
Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.



$$\overrightarrow{ST} =$$

$$|AS| = |BS|$$
$$|CT| = |DT|$$

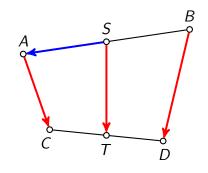
Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.



$$|AS| = |BS|$$

$$|CT| = |DT|$$

Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.

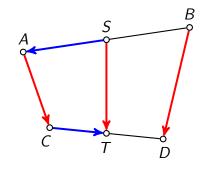


$$\overrightarrow{ST} = \overrightarrow{SA} + \overrightarrow{AC}$$

$$|AS| = |BS|$$

$$|CT| = |DT|$$

Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.

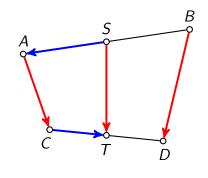


$$\overrightarrow{ST} = \overrightarrow{SA} + \overrightarrow{AC} + \overrightarrow{CT}$$

$$|AS| = |BS|$$

$$|CT| = |DT|$$

Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.

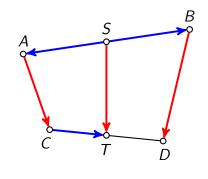


$$\overrightarrow{ST} = \overrightarrow{SA} + \overrightarrow{AC} + \overrightarrow{CT}$$

$$\overrightarrow{ST} =$$

$$|AS| = |BS|$$
$$|CT| = |DT|$$

Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.

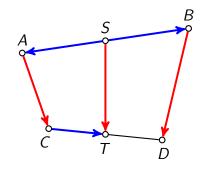


$$\overrightarrow{ST} = \overrightarrow{SA} + \overrightarrow{AC} + \overrightarrow{CT}$$

$$\overrightarrow{ST} = \overrightarrow{SB}$$

$$|AS| = |BS|$$
$$|CT| = |DT|$$

Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.

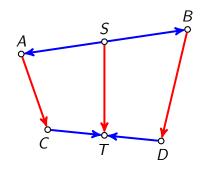


$$\overrightarrow{ST} = \overrightarrow{SA} + \overrightarrow{AC} + \overrightarrow{CT}$$

$$\overrightarrow{ST} = \overrightarrow{SB} + \overrightarrow{BD}$$

$$|AS| = |BS|$$
$$|CT| = |DT|$$

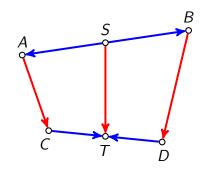
Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.



$$\overrightarrow{ST} = \overrightarrow{SA} + \overrightarrow{AC} + \overrightarrow{CT}$$
$$\overrightarrow{ST} = \overrightarrow{SB} + \overrightarrow{BD} + \overrightarrow{DT}$$

$$|AS| = |BS|$$
$$|CT| = |DT|$$

Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.



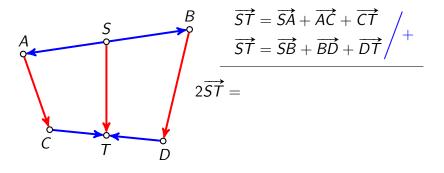
$$\overrightarrow{ST} = \overrightarrow{SA} + \overrightarrow{AC} + \overrightarrow{CT} / +$$

$$\overrightarrow{ST} = \overrightarrow{SB} + \overrightarrow{BD} + \overrightarrow{DT} / +$$

$$|AS| = |BS|$$

$$|CT| = |DT|$$

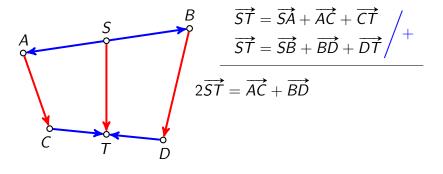
Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.



$$|AS| = |BS|$$

$$|CT| = |DT|$$

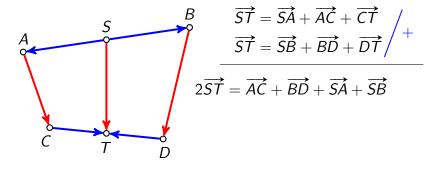
Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.



$$|AS| = |BS|$$

 $|CT| = |DT|$

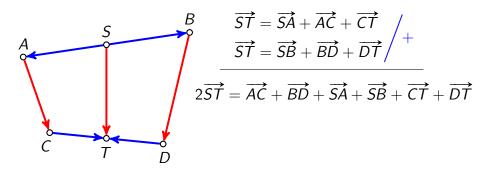
Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.



$$|AS| = |BS|$$

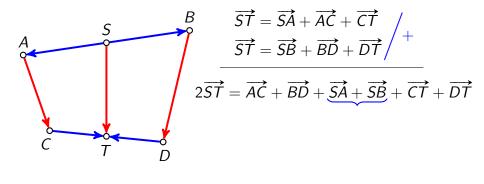
$$|CT| = |DT|$$

Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.



$$|AS| = |BS|$$
$$|CT| = |DT|$$

Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.

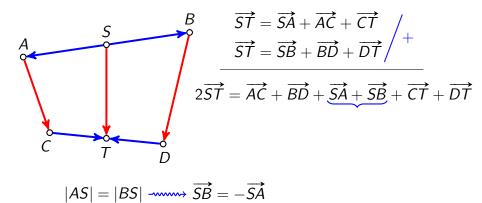


$$|AS| = |BS|$$
$$|CT| = |DT|$$

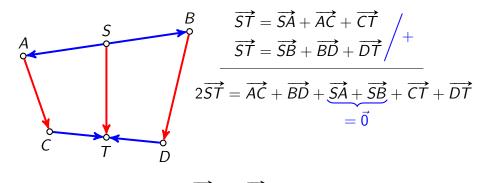
Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.

Rješenje

|CT| = |DT|



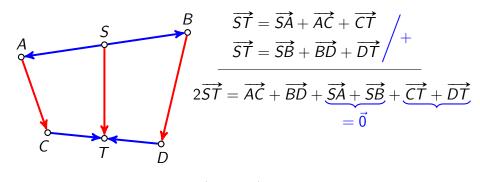
Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.



$$|AS| = |BS| \longrightarrow \overrightarrow{SB} = -\overrightarrow{SA}$$

 $|CT| = |DT|$

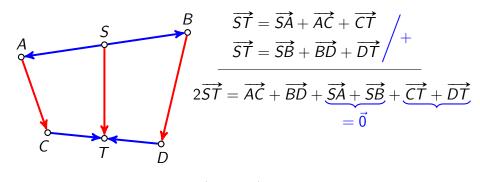
Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.



$$|AS| = |BS| \longrightarrow \overrightarrow{SB} = -\overrightarrow{SA}$$

 $|CT| = |DT|$

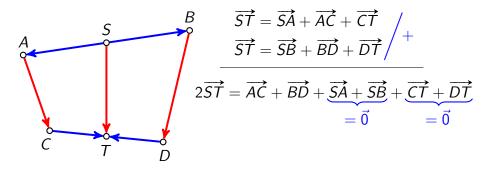
Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.



$$|AS| = |BS| \longrightarrow \overrightarrow{SB} = -\overrightarrow{SA}$$

 $|CT| = |DT| \longrightarrow \overrightarrow{DT} = -\overrightarrow{CT}$

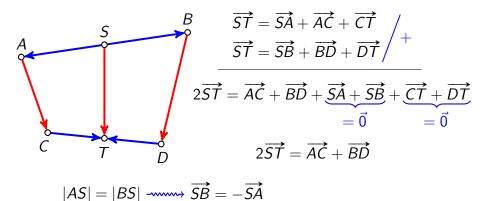
Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.



$$|AS| = |BS| \longrightarrow \overrightarrow{SB} = -\overrightarrow{SA}$$

 $|CT| = |DT| \longrightarrow \overrightarrow{DT} = -\overrightarrow{CT}$

Neka su \overline{AB} i \overline{CD} bilo koje dužine, a točke S i T redom polovišta tih dužina. Dokažite da je $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{ST}$.



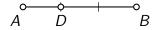
$$|CT| = |DT| \longrightarrow \overrightarrow{DT} = -\overrightarrow{CT}$$

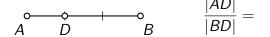
omjeru

Dijeljenje dužine u zadanom

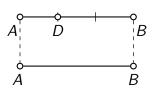




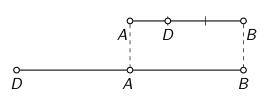




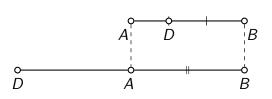
$$\frac{|AD|}{|BD|} = \frac{1}{2}$$



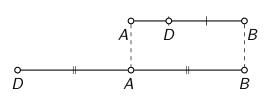
$$\frac{|AD|}{|BD|} = \frac{1}{2}$$



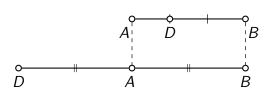
$$\frac{|AD|}{|BD|} = \frac{1}{2}$$



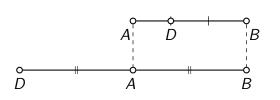
$$\frac{|AD|}{|BD|} = \frac{1}{2}$$



$$\frac{|AD|}{|BD|} = \frac{1}{2}$$

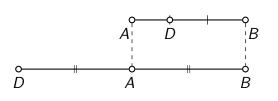


$$\frac{|AD|}{|BD|} = \frac{1}{2}$$
$$\frac{|AD|}{|BD|} =$$

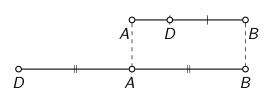


$$\frac{|AD|}{|BD|} = \frac{1}{2}$$

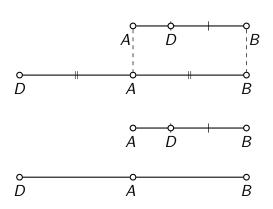
$$\frac{AD|}{BD|} = \frac{1}{2}$$



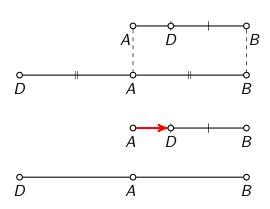
$$\frac{|AD|}{|BD|} = \frac{1}{2}$$
 iznutra
$$\frac{|AD|}{|BD|} = \frac{1}{2}$$



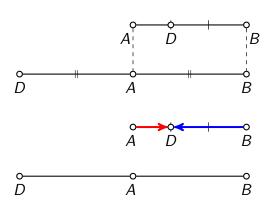
$$\frac{|AD|}{|BD|} = \frac{1}{2}$$
 iznutra $\frac{|AD|}{|BD|} = \frac{1}{2}$ izvana



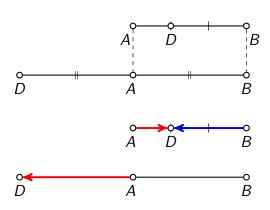
$$\frac{|AD|}{|BD|} = \frac{1}{2}$$
 iznutra $|AD|$ 1 .



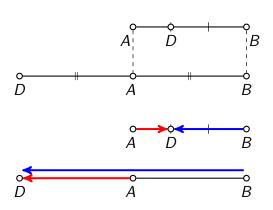
$$\frac{|AD|}{|BD|} = \frac{1}{2}$$
 iznutra $\frac{|AD|}{|BD|} = \frac{1}{2}$ izvana



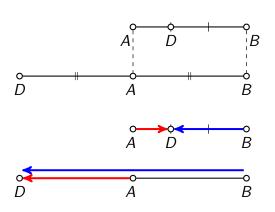
$$rac{|AD|}{|BD|} = rac{1}{2}$$
 iznutra $rac{|AD|}{|BD|} = rac{1}{2}$ izvana



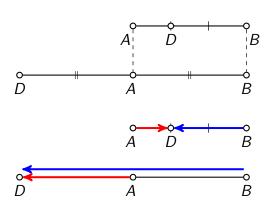
$$rac{|AD|}{|BD|} = rac{1}{2}$$
 iznutra $rac{|AD|}{|BD|} = rac{1}{2}$ izvana



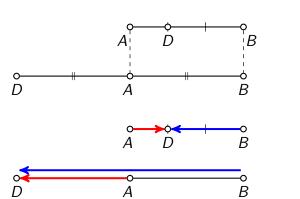
$$\frac{|AD|}{|BD|} = \frac{1}{2}$$
 iznutra $\frac{|AD|}{|BD|} = \frac{1}{2}$ izvana



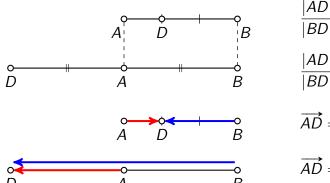
$$\frac{|AD|}{|BD|} = \frac{1}{2}$$
 iznutra $\frac{|AD|}{|BD|} = \frac{1}{2}$ izvana $\overrightarrow{AD} =$



$$\frac{|AD|}{|BD|} = \frac{1}{2}$$
 iznutra $\frac{|AD|}{|BD|} = \frac{1}{2}$ izvana $\overrightarrow{AD} = -\frac{1}{2}\overrightarrow{BD}$



$$\frac{|AD|}{|BD|} = \frac{1}{2}$$
 iznutra $\frac{|AD|}{|BD|} = \frac{1}{2}$ izvana $\overrightarrow{AD} = -\frac{1}{2}\overrightarrow{BD}$

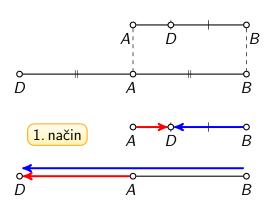


$$\frac{|AD|}{|BD|} = \frac{1}{2} \quad \text{iznutra}$$

$$\frac{|AD|}{|BD|} = \frac{1}{2} \quad \text{izvana}$$

$$\overrightarrow{AD} = -\frac{1}{2}\overrightarrow{BD}$$

$$\overrightarrow{AD} = \frac{1}{2}\overrightarrow{BD}$$



$$\frac{|AD|}{|BD|} = \frac{1}{2} \quad \text{iznutra}$$

$$\frac{|AD|}{|BD|} = \frac{1}{2} \quad \text{izvana}$$

$$\overrightarrow{AD} = -\frac{1}{2}\overrightarrow{BD}$$

$$\overrightarrow{AD} = \frac{1}{2}\overrightarrow{BD}$$

$$\begin{array}{c|cccc}
A & D & B \\
\hline
D & A & B \\
\hline
1. način & A & D & B \\
\hline
D & A & B \\
\hline
2. način & A & D & B \\
\hline
D & A & B \\
\hline
B & B & B \\
\hline
D & A & B & B \\
\hline$$

$$\frac{|AD|}{|BD|} = \frac{1}{2}$$
 iznutra $\frac{|AD|}{|BD|} = \frac{1}{2}$ izvana $\overrightarrow{AD} = -\frac{1}{2}\overrightarrow{BD}$

$$\overrightarrow{AD} = \frac{1}{2}\overrightarrow{BD}$$

$$\begin{array}{c|cccc}
A & D & B \\
\hline
D & A & B \\
\hline
1. način & A & D & B \\
\hline
D & A & B \\
\hline
2. način & A & D & B \\
\hline
D & A & B \\
\hline
B & B & B \\
\hline
D & A & B & B \\
\hline$$

$$\frac{|AD|}{|BD|} = \frac{1}{2}$$
 iznutra $\frac{|AD|}{|BD|} = \frac{1}{2}$ izvana $\overrightarrow{AD} = -\frac{1}{2}\overrightarrow{BD}$

$$\overrightarrow{AD} = \frac{1}{2}\overrightarrow{BD}$$

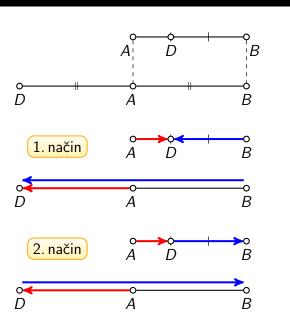
$$\frac{|AD|}{|BD|} = \frac{1}{2}$$
 iznutra $\frac{|AD|}{|BD|} = \frac{1}{2}$ izvana

$$\overrightarrow{AD} = \frac{1}{2}\overrightarrow{BD}$$

 $\overrightarrow{AD} = -\frac{1}{2}\overrightarrow{BD}$

$$\frac{|AD|}{|BD|} = \frac{1}{2}$$
 iznutra $\frac{|AD|}{|BD|} = \frac{1}{2}$ izvana $\overrightarrow{AD} = -\frac{1}{2}\overrightarrow{BD}$

$$\overrightarrow{AD} = \frac{1}{2}\overrightarrow{BD}$$



$$\frac{|AD|}{|BD|} = \frac{1}{2}$$
 iznutra $\frac{|AD|}{|BD|} = \frac{1}{2}$ izvana

$$\overrightarrow{AD} = -\frac{1}{2}\overrightarrow{BD}$$

$$\overrightarrow{AD} = \frac{1}{2}\overrightarrow{BD}$$

$$\frac{|AD|}{|BD|} = \frac{1}{2} \quad \text{iznutra}$$

$$\frac{|AD|}{|BD|} = \frac{1}{2} \quad \text{izvana}$$

$$\overrightarrow{AD} = -\frac{1}{2}\overrightarrow{BD}$$

$$\overrightarrow{AD} = \frac{1}{2}\overrightarrow{BD}$$

$$\overrightarrow{AD} =$$

$$\frac{|AD|}{|BD|} = \frac{1}{2} \quad \text{iznutra}$$

$$\frac{|AD|}{|BD|} = \frac{1}{2} \quad \text{izvana}$$

$$\overrightarrow{AD} = -\frac{1}{2}\overrightarrow{BD}$$

$$\overrightarrow{AD} = \frac{1}{2}\overrightarrow{BD}$$

$$\overrightarrow{AD} = \frac{1}{2}\overrightarrow{DB}$$

$$|AD| = \frac{1}{2} \text{ iznutra}$$

$$|AD| = \frac{1}{2} \text{ izvana}$$

$$|AD| = \frac{1}{2} \overrightarrow{BD}$$

$$|AD| = \frac{1}{2} \overrightarrow{BD}$$

$$|AD| = \frac{1}{2} \overrightarrow{DB}$$

$$|AD| = \frac{1}{2} \overrightarrow{DB}$$

$$\overrightarrow{A} \qquad \overrightarrow{D} \qquad \overrightarrow{B} \qquad \overrightarrow{|BD|} = \frac{1}{2} \quad \text{iznutra}$$

$$\overrightarrow{AD} = \frac{1}{2} \overrightarrow{BD}$$

$$\overrightarrow{AD} = \frac{1}{2} \overrightarrow{BD}$$

$$\overrightarrow{AD} = \frac{1}{2} \overrightarrow{BD}$$

$$\overrightarrow{AD} = \frac{1}{2} \overrightarrow{BD}$$

$$\overrightarrow{AD} = \frac{1}{2} \overrightarrow{DB}$$

$$\overrightarrow{AD} = -\frac{1}{2} \overrightarrow{DB}$$

$$\overrightarrow{AD} = -\frac{1}{2} \overrightarrow{DB}$$

13 / 17

1. način

Kažemo da točka D dijeli dužinu \overline{AB} u omjeru λ ako je $\overrightarrow{AD}=\lambda \overrightarrow{BD}.$

1. način

Kažemo da točka D dijeli dužinu \overline{AB} u omjeru λ ako je $\overrightarrow{AD}=\lambda\overrightarrow{BD}$.

Ako je $\lambda < 0$, točka D se nalazi unutar dužine \overline{AB} .

1. način

Kažemo da točka D dijeli dužinu \overline{AB} u omjeru λ ako je $\overrightarrow{AD} = \lambda \overrightarrow{BD}$.

Ako je $\lambda < 0$, točka D se nalazi unutar dužine \overline{AB} .

Ako je $\lambda > 0$, točka D se nalazi izvan dužine \overline{AB} .

Kažemo da točka D dijeli dužinu \overline{AB} u omjeru λ ako je $\overrightarrow{AD}=\lambda\overrightarrow{BD}$.

Ako je $\lambda < 0$, točka D se nalazi unutar dužine \overline{AB} .

Ako je $\lambda > 0$, točka D se nalazi izvan dužine \overline{AB} .

2. način

Kažemo da točka D dijeli dužinu \overline{AB} u omjeru λ ako je $\overrightarrow{AD} = \lambda \overrightarrow{BD}$.

Ako je $\lambda < 0$, točka D se nalazi unutar dužine \overline{AB} .

Ako je $\lambda > 0$, točka D se nalazi izvan dužine \overline{AB} .

2. način

Kažemo da točka D dijeli dužinu \overline{AB} u omjeru λ ako je $\overrightarrow{AD} = \lambda \overrightarrow{DB}$.

Kažemo da točka D dijeli dužinu \overline{AB} u omjeru λ ako je $\overrightarrow{AD} = \lambda \overrightarrow{BD}$.

Ako je $\lambda < 0$, točka D se nalazi unutar dužine \overline{AB} .

Ako je $\lambda > 0$, točka D se nalazi izvan dužine \overline{AB} .

2. način

Kažemo da točka D dijeli dužinu \overline{AB} u omjeru λ ako je $\overrightarrow{AD} = \lambda \overrightarrow{DB}$.

Ako je $\lambda > 0$, točka D se nalazi unutar dužine \overline{AB} .

Kažemo da točka D dijeli dužinu \overline{AB} u omjeru λ ako je $\overrightarrow{AD} = \lambda \overrightarrow{BD}$.

Ako je $\lambda < 0$, točka D se nalazi unutar dužine \overline{AB} .

Ako je $\lambda > 0$, točka D se nalazi izvan dužine \overline{AB} .

2. način

Kažemo da točka D dijeli dužinu \overline{AB} u omjeru λ ako je $\overrightarrow{AD} = \lambda \overrightarrow{DB}$.

Ako je $\lambda > 0$, točka D se nalazi unutar dužine \overline{AB} .

Ako je $\lambda < 0$, točka D se nalazi izvan dužine \overline{AB} .

1. način www- koristit ćemo ovaj pristup

Kažemo da točka D dijeli dužinu \overline{AB} u omjeru λ ako je $\overrightarrow{AD} = \lambda \overrightarrow{BD}$.

Ako je $\lambda < 0$, točka D se nalazi unutar dužine \overline{AB} .

Ako je $\lambda > 0$, točka D se nalazi izvan dužine \overline{AB} .

2. način

Kažemo da točka D dijeli dužinu \overline{AB} u omjeru λ ako je $\overrightarrow{AD} = \lambda \overrightarrow{DB}$.

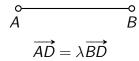
Ako je $\lambda > 0$, točka D se nalazi unutar dužine \overline{AB} .

Ako je $\lambda < 0$, točka D se nalazi izvan dužine \overline{AB} .

 $\lambda = 0$



$$\lambda = 0$$



$$\lambda = \mathbf{0}$$

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0 \cdot \overrightarrow{BD}$$

$$\lambda = 0$$

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0 \cdot \overrightarrow{BD}$$

$$\overrightarrow{AD} = \vec{0}$$

$$\lambda = \mathbf{0}$$

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0 \cdot \overrightarrow{BD}$$

$$\overrightarrow{AD} = \overrightarrow{0}$$

$$\overrightarrow{AD} = \overrightarrow{0}$$

$$D = A$$

$$\lambda = 0$$

$$D$$

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0 \cdot \overrightarrow{BD}$$

$$\overrightarrow{AD} = \vec{0}$$

$$D = A$$

$$\lambda = 0$$

$$\overrightarrow{A} \qquad \overrightarrow{B}$$

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0 \cdot \overrightarrow{BD}$$

$$\overrightarrow{AD} = \vec{0}$$

$$D = A$$

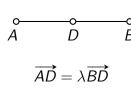
polovište



$$\begin{array}{ccc}
\lambda = 0 \\
D \\
A \\
\overrightarrow{AD} = \lambda \overrightarrow{BD} \\
\overrightarrow{AD} = 0 \cdot \overrightarrow{BD} \\
\overrightarrow{AD} = \overrightarrow{0} \\
D = A
\end{array}$$

$$\begin{array}{ccc}
\lambda = 0 \\
D \\
\overrightarrow{A} & B
\end{array}$$

$$\overrightarrow{AD} = \lambda \overrightarrow{BD} \\
\overrightarrow{AD} = 0 \cdot \overrightarrow{BD} \\
\overrightarrow{AD} = \vec{0} \\
D = A$$



$$\lambda = 0$$

$$\overrightarrow{A} \qquad \overrightarrow{BD}$$

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$

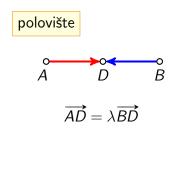
$$\overrightarrow{AD} = 0 \cdot \overrightarrow{BD}$$

$$\overrightarrow{AD} = \overrightarrow{0}$$

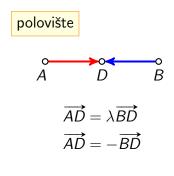
$$D = A$$

polovište
$$\overrightarrow{A} \overrightarrow{D} = \lambda \overrightarrow{BD}$$

$$\begin{array}{c}
\lambda = 0 \\
\overrightarrow{A} \\
\overrightarrow{AD} = \lambda \overrightarrow{BD} \\
\overrightarrow{AD} = 0 \cdot \overrightarrow{BD} \\
\overrightarrow{AD} = \overrightarrow{0} \\
D = A
\end{array}$$



$$\begin{array}{ccc}
\lambda = 0 \\
D \\
A \\
\overrightarrow{AD} = \lambda \overrightarrow{BD} \\
\overrightarrow{AD} = 0 \cdot \overrightarrow{BD} \\
\overrightarrow{AD} = \overrightarrow{0} \\
D = A
\end{array}$$

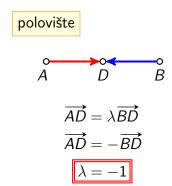


$$\begin{array}{ccc}
\lambda = 0 \\
D \\
A \\
\overrightarrow{AD} = \lambda \overrightarrow{BD} \\
\overrightarrow{AD} = 0 \cdot \overrightarrow{BD} \\
\overrightarrow{AD} = \overrightarrow{0} \\
D = A
\end{array}$$

polovište

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$
 $\overrightarrow{AD} = -\overrightarrow{BD}$
 $\lambda = -1$

$$\begin{array}{ccc}
\lambda = 0 \\
D \\
A & B \\
\overrightarrow{AD} = \lambda \overrightarrow{BD} \\
\overrightarrow{AD} = 0 \cdot \overrightarrow{BD} \\
\overrightarrow{AD} = \overrightarrow{0} \\
D = A
\end{array}$$



$$\begin{array}{ccc}
\lambda = 0 \\
D \\
A \\
\overrightarrow{AD} = \lambda \overrightarrow{BD} \\
\overrightarrow{AD} = 0 \cdot \overrightarrow{BD} \\
\overrightarrow{AD} = \overrightarrow{0} \\
D = A
\end{array}$$

polovište

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$
 $\overrightarrow{AD} = -\overrightarrow{BD}$
 $\lambda = -1$

$$\lambda = 1$$



polovište
$$\overrightarrow{A} D B$$

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$

$$\overrightarrow{AD} = -\overrightarrow{BD}$$

$$\lambda = -1$$

$$\lambda = 1$$

A B

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$

$$\lambda = 0$$

$$D$$

$$A D$$

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$

 $\overrightarrow{AD} = \overrightarrow{BD}$

$$\lambda = 0$$

$$D$$

$$A \longrightarrow B$$

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0 \cdot \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0$$

$$\overrightarrow{AD} = 0$$

$$\overrightarrow{AD} = -\overrightarrow{BD}$$

$$\overrightarrow{AD} = -\overrightarrow{BD}$$

$$\lambda = -1$$

$$\lambda = 1$$

 \overrightarrow{A} \overrightarrow{B}

$$\lambda = 0$$

$$D$$

$$A D$$

 $D \quad A \quad B$

$$\begin{array}{c|c}
\lambda = 0 \\
D \\
\overrightarrow{A} \\
\overrightarrow{AD} = \lambda \overrightarrow{BD} \\
\overrightarrow{AD} = 0 \cdot \overrightarrow{BD} \\
\overrightarrow{AD} = \overrightarrow{0} \\
D = A
\end{array}$$

$$\begin{array}{c|c}
\overrightarrow{AD} = \lambda \overrightarrow{BD} \\
\overrightarrow{AD} = \lambda \overrightarrow{BD} \\
\overrightarrow{AD} = -\overrightarrow{BD} \\
\overrightarrow{\lambda} = -1
\end{array}$$

$$\begin{array}{c|c}
\lambda = 1
\end{array}$$

 $D \stackrel{\circ}{A} \stackrel{\circ}{B} \stackrel{\circ}{D}$

$$\lambda = 0$$

$$D$$

$$A$$

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0 \cdot \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0$$

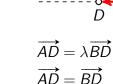
$$\overrightarrow{AD} = 0$$

$$\overrightarrow{AD} = -\overrightarrow{BD}$$

$$\overrightarrow{AD} = -\overrightarrow{BD}$$

$$\lambda = -1$$

$$\lambda = 1$$





$$\lambda = 0$$

$$D$$

$$A$$

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0 \cdot \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0$$

$$\overrightarrow{AD} = 0$$

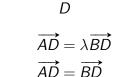
$$\overrightarrow{AD} = 0$$

$$\overrightarrow{AD} = -\overrightarrow{BD}$$

$$\overrightarrow{AD} = -\overrightarrow{BD}$$

$$\lambda = -1$$

$$\lambda = 1$$



 \overrightarrow{A} \overrightarrow{BD}

Hm?

 $\overrightarrow{AD} = \overrightarrow{BD}$

$$\lambda = 0$$

$$D$$

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0 \cdot \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0$$

$$\overrightarrow{AD} = -\overrightarrow{BD}$$

$$\overrightarrow{AD} = -1$$

$$\lambda = -1$$

$$\lambda = 1$$

A B B

$$\lambda = 0$$

$$D$$

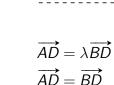
$$A$$

N D D

$$\lambda = 0$$

$$D$$

$$A D$$



A B D

$$\lambda = 0$$

$$D$$

$$A$$

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0 \cdot \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0$$

$$\overrightarrow{AD} = 0$$

$$\overrightarrow{AD} = 0$$

$$\overrightarrow{AD} = -\overrightarrow{BD}$$

$$\overrightarrow{AD} = -\overrightarrow{BD}$$

$$\lambda = -1$$

$$\lambda = 1$$

 $=\lambda \overrightarrow{BD}$

$$\lambda = 0$$

$$D$$

$$A$$

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0 \cdot \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0$$

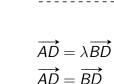
$$\overrightarrow{AD} = 0$$

$$\overrightarrow{AD} = -\overrightarrow{BD}$$

$$\overrightarrow{AD} = -\overrightarrow{BD}$$

$$\lambda = -1$$

$$\lambda = 1$$





$$\lambda = 0$$

$$D$$

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0 \cdot \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0$$

$$\lambda = 0$$

$$D$$

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0 \cdot \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0$$

$$\overrightarrow{AD} = 0$$

$$\overrightarrow{AD} = 0$$

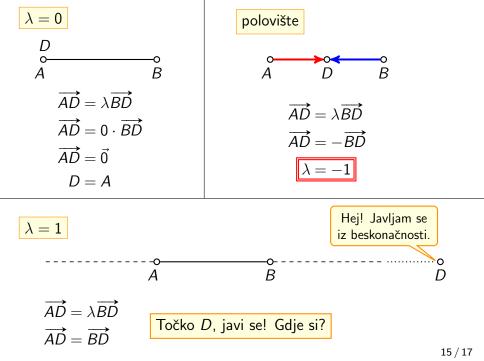
$$\overrightarrow{AD} = -\overrightarrow{BD}$$

$$\lambda = -1$$

$$\lambda = 1$$

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$
 $\overrightarrow{AD} = \overrightarrow{BD}$

 \overrightarrow{BD} Točko D, javi se! Gdje si?



$$\lambda = 0$$

$$D$$

$$A$$

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0 \cdot \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0$$

$$\overrightarrow{AD} = 0$$

$$\overrightarrow{AD} = 0$$

$$\overrightarrow{AD} = -\overrightarrow{BD}$$

$$\lambda = -1$$

$$\lambda = 1$$

 $\overrightarrow{AD} = \lambda \overrightarrow{BD}$ Točka D je beskonačno daleka točka na pravcu AB.

$$\lambda = 0$$

$$\overrightarrow{A} \qquad \overrightarrow{B}$$

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0 \cdot \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0$$

$$\overrightarrow{AD} = 0$$

$$\overrightarrow{AD} = -\overrightarrow{BD}$$

$$\overrightarrow{AD} = -\overrightarrow{BD}$$

$$\lambda = -1$$

$$\lambda = 1$$

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$
 Točka D je beskonačno daleka točka na pravcu AB .

$$\lambda = 0$$

$$D$$

$$A$$

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0 \cdot \overrightarrow{BD}$$

$$\overrightarrow{AD} = 0$$

$$D = A$$

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$

$$\overrightarrow{AD} = -\overrightarrow{BD}$$

$$\lambda = -1$$

$$\lambda = 1$$

$$\overrightarrow{AD} = \lambda \overrightarrow{BD}$$
 Točka D je beskonačno

 $\overrightarrow{AD} = \overrightarrow{BD}$ daleka točka na pravcu \overrightarrow{AB} .

$$\overrightarrow{AD} = -\frac{2}{5}\overrightarrow{BD}$$

$$\overrightarrow{AD} = -\frac{2}{5}\overrightarrow{BD}$$

$$\frac{|AD|}{|BD|} = \frac{2}{5}$$

$$\overrightarrow{AD} = -\frac{2}{5}\overrightarrow{BD}$$

$$\overrightarrow{A} \qquad \overrightarrow{B}$$

$$\frac{|AD|}{|BD|} = \frac{2}{5} \quad 5 + 2 = 7$$

$$\overrightarrow{AD} = -\frac{2}{5}\overrightarrow{BD}$$

$$\overrightarrow{A} \qquad \overrightarrow{B}$$

$$\frac{|AD|}{|BD|} = \frac{2}{5} \quad 5 + 2 = 7$$

$$\overrightarrow{AD} = -\frac{2}{5}\overrightarrow{BD}$$

$$\overrightarrow{A} \xrightarrow{D} \xrightarrow{B}$$

$$\frac{|AD|}{|BD|} = \frac{2}{5} \quad 5 + 2 = 7$$

$$\overrightarrow{AD} = -\frac{2}{5}\overrightarrow{BD}$$

$$\overrightarrow{D}$$

$$B$$

$$\frac{|AD|}{|BD|} = \frac{2}{5} \quad 5+2=7$$

$$\overrightarrow{AD} = \frac{2}{5}\overrightarrow{BD}$$



$$\overrightarrow{AD} = -\frac{2}{5}\overrightarrow{BD}$$

$$\overrightarrow{D}$$

$$B$$

$$\frac{|AD|}{|BD|} = \frac{2}{5}$$
 5+2=7

$$\overrightarrow{AD} = \frac{2}{5}\overrightarrow{BD}$$



$$\overrightarrow{AD} = -\frac{2}{5}\overrightarrow{BD}$$

$$\overrightarrow{D}$$

$$B$$

$$\frac{|AD|}{|BD|} = \frac{2}{5}$$
 5+2=7

$$\overrightarrow{AD} = \frac{2}{5}\overrightarrow{BD}$$

$$\frac{|AD|}{|BD|} = \frac{2}{5}$$

$$\overrightarrow{AD} = -\frac{2}{5}\overrightarrow{BD}$$

$$\overrightarrow{D}$$

$$B$$

$$\frac{|AD|}{|BD|} = \frac{2}{5}$$
 5 + 2 = 7

$$\overrightarrow{AD} = \frac{2}{5}\overrightarrow{BD}$$

$$\frac{|AD|}{|BD|} = \frac{2}{5}$$
 5 - 2 = 3

$$\overrightarrow{AD} = -\frac{2}{5}\overrightarrow{BD}$$

$$\overrightarrow{D}$$

$$B$$

$$\frac{|AD|}{|BD|} = \frac{2}{5}$$
 5 + 2 = 7

$$\overrightarrow{AD} = \frac{2}{5}\overrightarrow{BD}$$

$$\frac{|AD|}{|BD|} = \frac{2}{5}$$
 5 - 2 = 3

$$\overrightarrow{AD} = -\frac{2}{5}\overrightarrow{BD}$$

$$\overrightarrow{AD} = \frac{2}{5}\overrightarrow{BD}$$

$$\overrightarrow{AD} = -\frac{2}{5}\overrightarrow{BD}$$

$$\overrightarrow{A} \xrightarrow{D} \xrightarrow{B}$$

$$\frac{|AD|}{|BD|} = \frac{2}{5} \quad 5 + 2 = 7$$

$$\overrightarrow{AD} = \frac{2}{5}\overrightarrow{BD}$$

$$\overrightarrow{D} \qquad \overrightarrow{A} \qquad \overrightarrow{B}$$

$$\frac{|AD|}{|BD|} = \frac{2}{5} \quad 5 - 2 = 3$$

$$\overrightarrow{AD} = -\frac{2}{5}\overrightarrow{BD}$$

$$\overrightarrow{AD} = \frac{2}{5}\overrightarrow{BD}$$

$$\overrightarrow{AD} = -\frac{5}{2}\overrightarrow{BD}$$

$$\overrightarrow{AD} = -\frac{2}{5}\overrightarrow{BD}$$

$$\overrightarrow{AD} = \frac{2}{5}\overrightarrow{BD}$$

$$\overrightarrow{AD} = \frac{2}{5}\overrightarrow{BD}$$

$$\overrightarrow{AD} = \frac{2}{5}\overrightarrow{BD}$$

$$\frac{|AD|}{|BD|} = \frac{2}{5} \quad 5 + 2 = 7$$

$$\frac{|AD|}{|BD|} = \frac{2}{5} \quad 5 - 2 = 3$$

$$\overrightarrow{AD} = -\frac{5}{2}\overrightarrow{BD}$$

$$\begin{array}{c}
O \\
A
\end{array}$$

$$\frac{|AD|}{|BD|} = \frac{5}{2}$$

$$\overrightarrow{AD} = -\frac{2}{5}\overrightarrow{BD}$$

$$\overrightarrow{AD} = \frac{2}{5}\overrightarrow{BD}$$

$$\overrightarrow{AD} = -\frac{5}{2}\overrightarrow{BD}$$

$$\overrightarrow{A} \qquad \overrightarrow{B}$$

$$\frac{|AD|}{|BD|} = \frac{5}{2} \quad 5 + 2 = 7$$

$$\overrightarrow{AD} = -\frac{2}{5}\overrightarrow{BD}$$

$$\overrightarrow{AD} = -\frac{2}{5}\overrightarrow{BD}$$

$$\frac{|AD|}{|BD|} = \frac{2}{5} \quad 5 + 2 = 7$$

$$\overrightarrow{AD} = -\frac{5}{2}\overrightarrow{BD}$$

$$\overrightarrow{AD} = -\frac{5}{2}\overrightarrow{BD}$$

 $\frac{|AD|}{|BD|} = \frac{2}{5} \quad 5 - 2 = 3$

 $\overrightarrow{AD} = \frac{2}{5}\overrightarrow{BD}$

$$\overrightarrow{AD} = -\frac{2}{5}\overrightarrow{BD}$$

$$\overrightarrow{AD} = \frac{2}{5}\overrightarrow{BD}$$

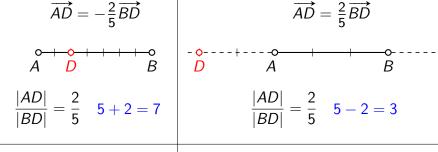
$$\overrightarrow{AD} = -\frac{5}{2}\overrightarrow{BD}$$

$$\overrightarrow{A} \qquad \overrightarrow{D} \qquad \overrightarrow{B}$$

$$\frac{|AD|}{|BD|} = \frac{5}{2} \quad 5 + 2 = 7$$

$\overrightarrow{AD} = -\frac{2}{5}\overrightarrow{BD}$

Primjer



$$\overrightarrow{AD} = -\frac{5}{2}\overrightarrow{BD}$$

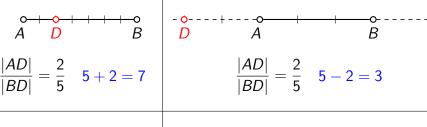
$$\overrightarrow{AD} = \frac{5}{2}\overrightarrow{BD}$$

$$B$$

$$\begin{array}{ccc}
 & & & & & & & & \\
A & & & & & & & \\
\hline
|AD| & & & & & \\
|BD| & & & & \\
\hline
\end{bmatrix} = \frac{5}{2} \quad 5 + 2 = 7$$

$\overrightarrow{AD} = -\frac{2}{5}\overrightarrow{BD}$

Primjer

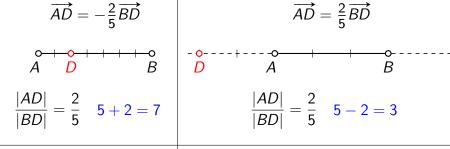


$$\overrightarrow{AD} = -\frac{5}{2}\overrightarrow{BD}$$

$$\overrightarrow{A} \qquad \overrightarrow{D} \qquad \overrightarrow{B}$$

 $\overrightarrow{AD} = \frac{5}{2}\overrightarrow{BD}$ $\overrightarrow{A} \qquad \overrightarrow{B}$

 $\overrightarrow{AD} = \frac{2}{5}\overrightarrow{BD}$



$$\overrightarrow{AD} = -\frac{5}{2}\overrightarrow{BD}$$

$$\overrightarrow{A}$$

$$\overrightarrow{D}$$

$$\overrightarrow{B}$$

$$\overrightarrow{AD} = \frac{5}{2}\overrightarrow{BD}$$

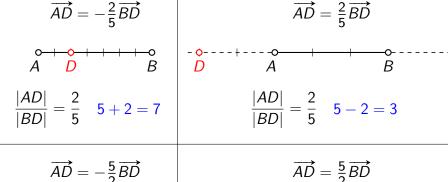
$$\overrightarrow{A} \qquad \overrightarrow{B}$$

$$A \qquad D \qquad B$$

$$\frac{|AD|}{|BD|} = \frac{5}{2} \quad 5 + 2 = 7$$

$\overrightarrow{AD} = -\frac{2}{5}\overrightarrow{BD}$

Primjer

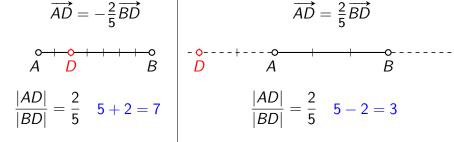


 $\overrightarrow{AD} = -\frac{5}{2}\overrightarrow{BD}$

$$\frac{|AD|}{|BD|} = \frac{5}{2} \quad 5 - 2 = 3$$

$$A \qquad D \qquad B$$

$$\frac{|AD|}{|BD|} = \frac{5}{2} \quad 5 + 2 = 7$$



$$\overrightarrow{AD} = -\frac{5}{2}\overrightarrow{BD}$$

$$\overrightarrow{A} \qquad \overrightarrow{D} \qquad \overrightarrow{B}$$

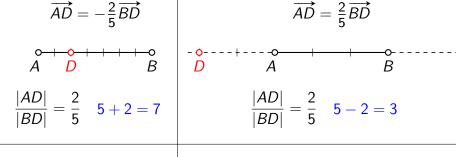
$$\overrightarrow{AD} = \frac{5}{2}\overrightarrow{BD}$$

$$\overrightarrow{A}$$

$$A \qquad D \qquad B$$

$$\frac{|AD|}{|BD|} = \frac{5}{2} \quad 5 + 2 = 7$$

$$\frac{|AD|}{|BD|} = \frac{5}{2} \quad 5 - 2 = 3$$



 $\overrightarrow{AD} = -\frac{5}{2}\overrightarrow{BD}$

 $\frac{|AD|}{|BD|} = \frac{5}{2}$ 5+2=7

$$\overrightarrow{AD} = \frac{5}{2}\overrightarrow{BD}$$

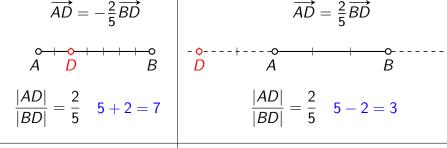
$$A \qquad B$$

$$|AD| \qquad 5$$

 $\frac{|AD|}{|BD|} = \frac{5}{2} \quad 5 - 2 = 3$

_

Primjer



$$\overrightarrow{AD} = -\frac{5}{2}\overrightarrow{BD}$$

$$\overrightarrow{AD} = \frac{5}{2}\overrightarrow{BD}$$

$$\overrightarrow{A}$$

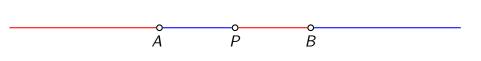
$$\overrightarrow{A}$$

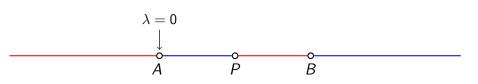
$$\overrightarrow{B}$$

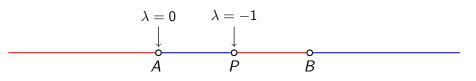
$$\frac{|AD|}{|BD|} = \frac{5}{2} \quad 5 - 2 = 3$$

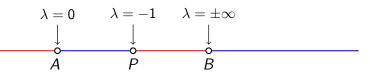
$$A \qquad D \qquad B$$

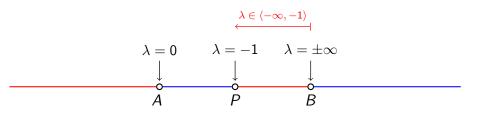
$$\frac{|AD|}{|BD|} = \frac{5}{2} \quad 5 + 2 = 7$$

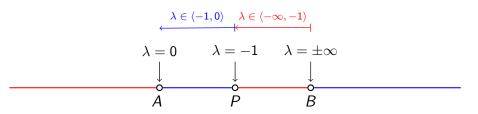


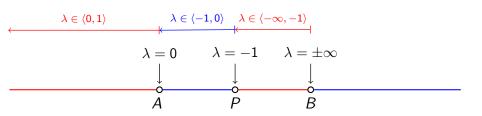


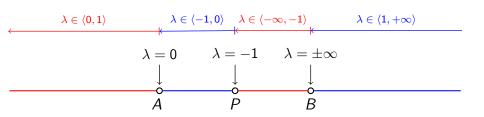


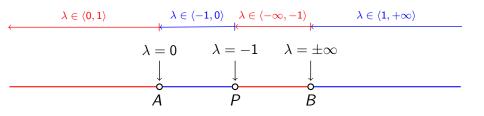




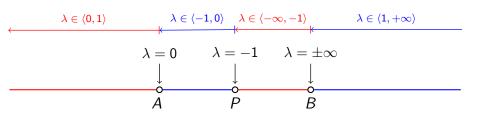


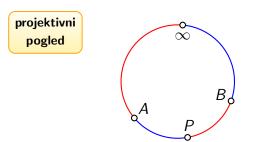


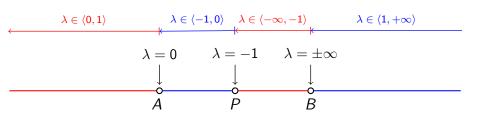


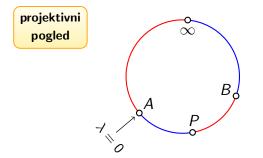


projektivni pogled







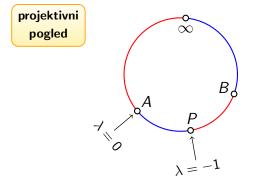


$$\lambda \in \langle 0, 1 \rangle \qquad \lambda \in \langle -1, 0 \rangle \qquad \lambda \in \langle -\infty, -1 \rangle \qquad \lambda \in \langle 1, +\infty \rangle$$

$$\lambda = 0 \qquad \lambda = -1 \qquad \lambda = \pm \infty$$

$$\downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow$$

$$A \qquad P \qquad B$$

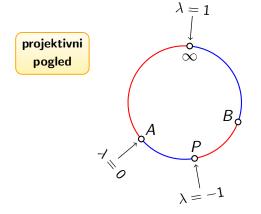


$$\lambda \in \langle 0, 1 \rangle \qquad \lambda \in \langle -1, 0 \rangle \qquad \lambda \in \langle -\infty, -1 \rangle \qquad \lambda \in \langle 1, +\infty \rangle$$

$$\lambda = 0 \qquad \lambda = -1 \qquad \lambda = \pm \infty$$

$$\downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow$$

$$A \qquad P \qquad B$$



$$\lambda \in \langle 0, 1 \rangle \qquad \lambda \in \langle -1, 0 \rangle \qquad \lambda \in \langle -\infty, -1 \rangle \qquad \lambda \in \langle 1, +\infty \rangle$$

$$\lambda = 0 \qquad \lambda = -1 \qquad \lambda = \pm \infty$$

$$\downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow$$

$$A \qquad P \qquad B$$

