

Realne funkcije realne varijable – 1. dio

MATEMATIKA 2

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Zadatak 1

Odredite domene i nultočke sljedećih funkcija:

- a) $f(x) = \sqrt[4]{\frac{x-3}{x+2}} - 2 - 1$ b) $g(x) = (2 + x - x^2)^{\frac{1}{5}}$
 c) $h(x) = \log(10^{x-1} - 5)$ d) $k(x) = \sqrt{\log_{\frac{1}{2}}(x+2)}$

Rješenje

a) **domena**

$$\frac{x-3}{x+2} - 2 \geq 0$$

$$\frac{x-3-2(x+2)}{x+2} \geq 0$$

$$\frac{-x-7}{x+2} \geq 0$$

$$\begin{aligned} -x-7 &= 0 & x+2 &= 0 \\ x &= -7 & x &= -2 \end{aligned}$$

$$x+2 \neq 0$$

uključeno u ovom uvjetu

$$f(x) = \sqrt[4]{\frac{x-3}{x+2}} - 2 - 1$$

	$-\infty$	-7	-2	$+\infty$
$-x-7$		+	-	-
$x+2$		-	-	+
$\frac{-x-7}{x+2}$		-	+	-

RJEŠENJE: $x \in [-7, -2)$

$$D_f = [-7, -2)$$

nultočke

$$\sqrt[4]{\frac{x-3}{x+2}} - 2 - 1 = 0$$

$$\sqrt[4]{\frac{x-3}{x+2}} - 2 = 1 \quad / \quad ^4$$

$$\frac{x-3}{x+2} - 2 = 1$$

$$\frac{x-3}{x+2} = 3 \quad / \cdot (x+2)$$

$$x-3 = 3x+6$$

$$-2x = 9$$

$$x = -\frac{9}{2}$$


$$f(x) = \sqrt[4]{\frac{x-3}{x+2}} - 2 - 1$$

$$D_f = [-7, -2)$$

jest nultočka
jer pripada domeni

b) **domena**

$$g(x) = \sqrt[5]{2+x-x^2}$$

 $D_g = \mathbb{R}$  neparni korijen je definiran za sve realne brojeve
nultočke

$$\begin{aligned} \sqrt[5]{2+x-x^2} &= 0 / ^5 \\ -x^2 + x + 2 &= 0 \\ x_{1,2} &= \frac{-1 \pm \sqrt{1^2 - 4 \cdot (-1) \cdot 2}}{2 \cdot (-1)} \\ x_{1,2} &= \frac{-1 \pm 3}{-2} \\ x_1 &= -1, \quad x_2 = 2 \end{aligned}$$

$$g(x) = (2+x-x^2)^{\frac{1}{5}}$$

$$\begin{aligned} ax^2 + bx + c &= 0 \\ x_{1,2} &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{aligned}$$

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Ako je $a > 1$

$$\log_a x > \log_a y \Leftrightarrow x > y$$

$$\log_a a^x = x$$

$$k(x) = \sqrt{\log_{\frac{1}{2}}(x+2)}$$

d) **domena**

$$\bullet x+2 > 0 \quad \text{zbog } \log_{\frac{1}{2}}$$

$$\log_{\frac{1}{2}}(x+2) \geq 0$$

$$\bullet \log_{\frac{1}{2}}(x+2) \geq 0 \quad \text{zbog } \sqrt{}$$

$$\log_{\frac{1}{2}}(x+2) \geq \log_{\frac{1}{2}}\left(\frac{1}{2}\right)^0$$

$$x+2 > 0$$

$$x+2 \leq \left(\frac{1}{2}\right)^0$$

$$x > -2$$

$$x+2 \leq 1$$

presjek rješenja

$$x \in \langle -2, -1 \rangle$$

$$x \leq -1$$

Ako je $0 < a < 1$

$$\log_a x > \log_a y \Leftrightarrow x < y$$

$$D_k = \langle -2, -1 \rangle$$

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c) **domena**

$$a^{\log_a x} = x$$

$$\log = \log_{10}$$

$$h(x) = \log(10^{x-1} - 5)$$

$$10^{x-1} - 5 > 0$$

$$10^{x-1} > 5$$

$$10^{x-1} > 10^{\log 5}$$

$$x-1 > \log 5$$

$$x > 1 + \log 5$$

nultočke

$$D_h = \langle 1 + \log 5, +\infty \rangle$$

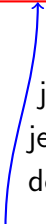
$$\log(10^{x-1} - 5) = 0$$

$$10^{x-1} - 5 = 10^0$$

$$10^{x-1} = 6$$

$$x-1 = \log 6$$

$$x = 1 + \log 6$$

 jest nultočka
jer pripada
domeni
Ako je $a > 1$

$$a^x > a^y \Leftrightarrow x > y$$

Ako je $0 < a < 1$

$$a^x > a^y \Leftrightarrow x < y$$

$$\log_a x = b \rightsquigarrow x = a^b$$

$$a^x = b \rightsquigarrow x = \log_a b$$

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nultočke

$$\sqrt{\log_{\frac{1}{2}}(x+2)} = 0 / ^2$$

$$\log_{\frac{1}{2}}(x+2) = 0$$

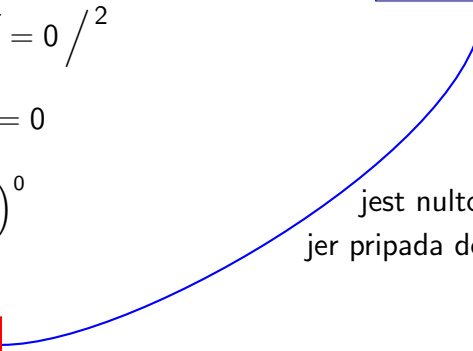
$$x+2 = \left(\frac{1}{2}\right)^0$$

$$x+2 = 1$$

$$x = -1$$

$$k(x) = \sqrt{\log_{\frac{1}{2}}(x+2)}$$

$$D_k = \langle -2, -1 \rangle$$

 jest nultočka
jer pripada domeni

$$\log_a x = b \rightsquigarrow x = a^b$$

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Zadatak 2

Odredite nultočke funkcija

$$f(x) = 2^{5-x} + 50 \text{ i } g(x) = 2^{5-x} - 50.$$

Rješenje

nultočke od f

$$2^{5-x} + 50 = 0$$

$$2^{5-x} = -50$$

$$5 - x = \log_2(-50)$$

funkcija f nema nultočki

Ups!



egzaktna
vrijednost
nultočke

aproksimacija
nultočke na
5 decimala

nultočke od g

$$2^{5-x} - 50 = 0$$

$$2^{5-x} = 50$$

$$5 - x = \log_2 50$$

$$-x = -5 + \log_2 50 / \cdot (-1)$$

$$x = 5 - \log_2 50$$

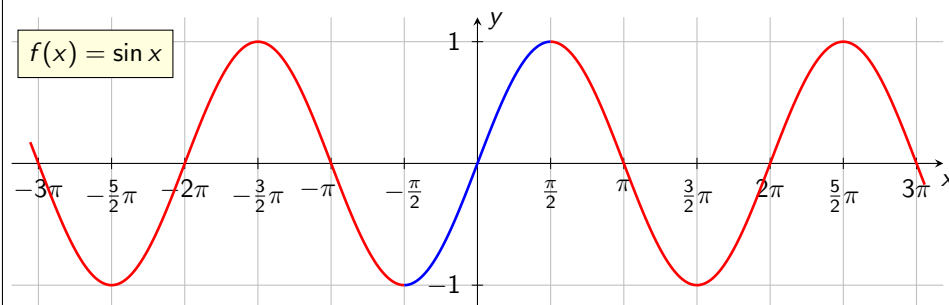
$$x = 5 - \frac{\log 50}{\log 2}$$

$$x \approx -0.64386$$

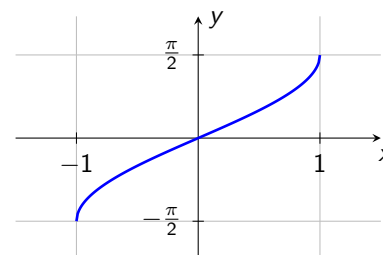
$$a^x = b \rightsquigarrow x = \log_a b$$

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$$f(x) = \sin x$$



$$f^{-1}(x) = \arcsin x$$



$$\sin x = 0 \Leftrightarrow x = k\pi, k \in \mathbb{Z}$$

$$\arcsin x = 0 \Leftrightarrow x = 0$$

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Nultočke funkcije g

1. način

$$2^{5-x} - 50 = 0$$

$$2^{5-x} = 50 / \log_2$$

$$5 - x = \log_2 50$$

$$-x = -5 + \log_2 50 / \cdot (-1)$$

$$x = 5 - \log_2 50$$

$$x = 5 - \frac{\log 50}{\log 2}$$

$$x \approx -0.64386$$

$$a^x = b \rightsquigarrow x = \log_a b$$

2. način

$$2^{5-x} - 50 = 0$$

$$2^{5-x} = 50 / \log$$

$$\log 2^{5-x} = \log 50$$

$$(5 - x) \log 2 = \log 50 / : \log 2$$

$$5 - x = \frac{\log 50}{\log 2}$$

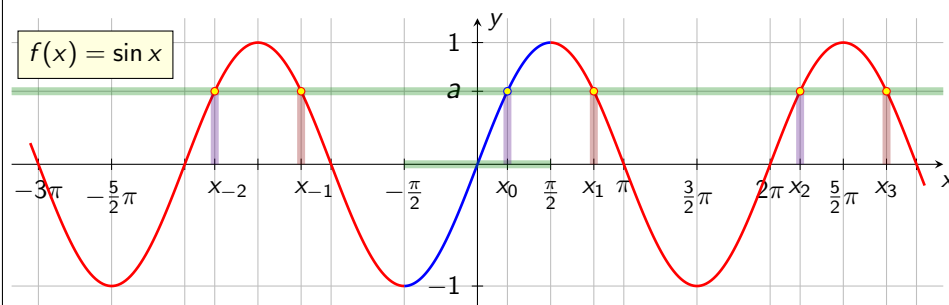
$$-x = -5 + \frac{\log 50}{\log 2} / \cdot (-1)$$

$$x = 5 - \frac{\log 50}{\log 2}$$

$$\log_a x^k = k \cdot \log_a x$$

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$$f(x) = \sin x$$



Rješenja jednadžbe $\sin x = a$ za $|a| \leq 1$

$$x_0 = \arcsin a$$

$$x_1 = \pi - \arcsin a$$

$$\bullet x_k^{(1)} = \arcsin a + 2k\pi, k \in \mathbb{Z}$$

$$x_k^{(1)} = x_{2k} = x_0 + 2k\pi$$

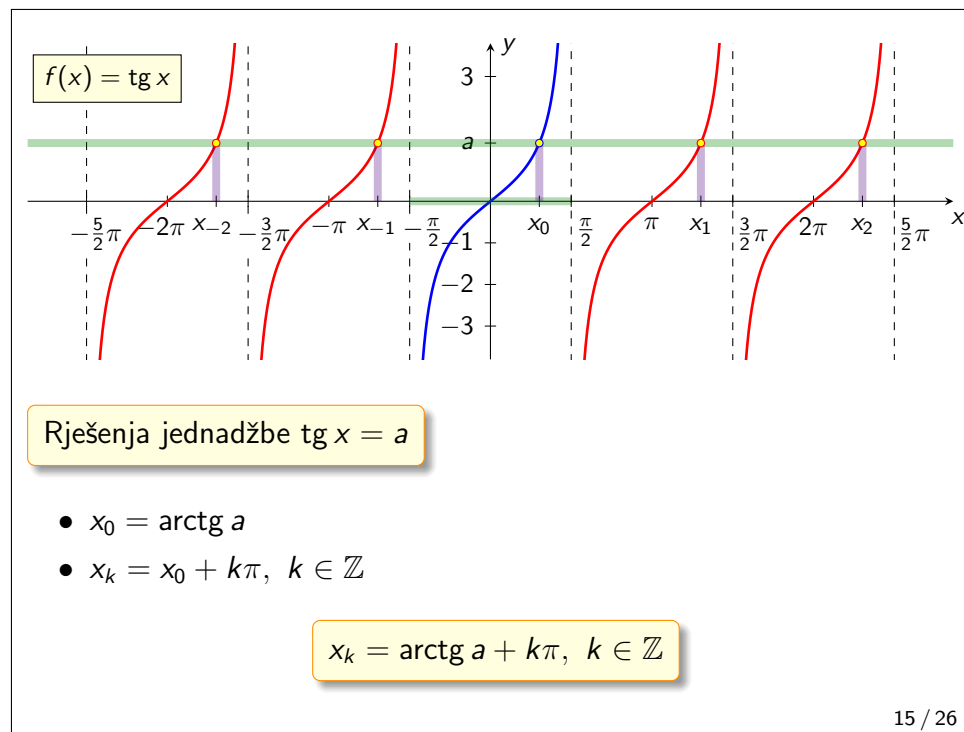
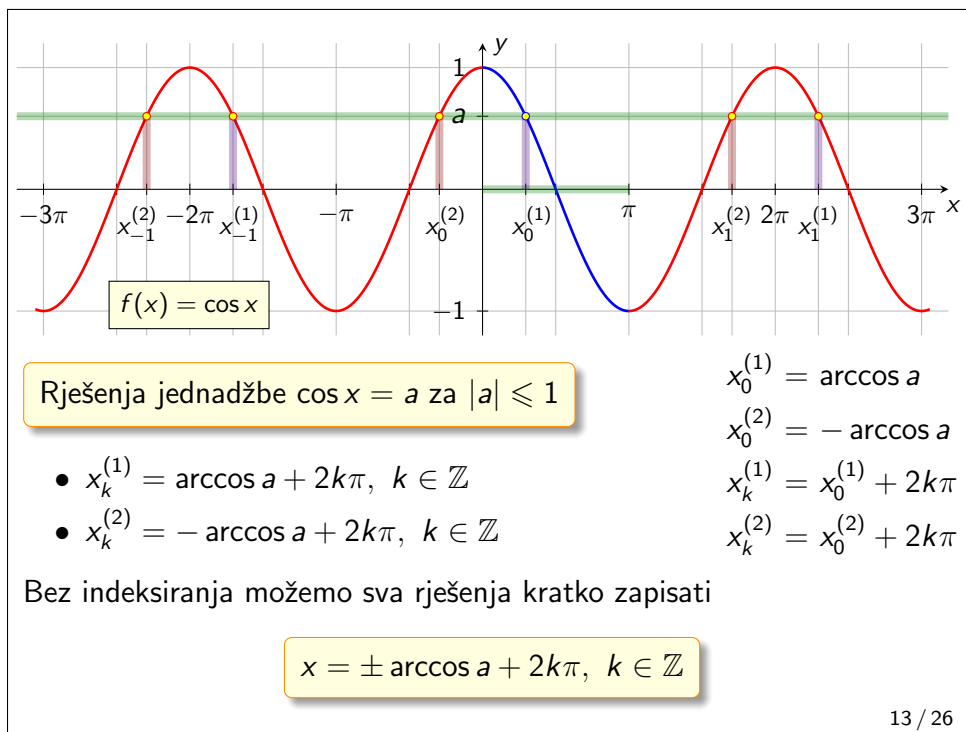
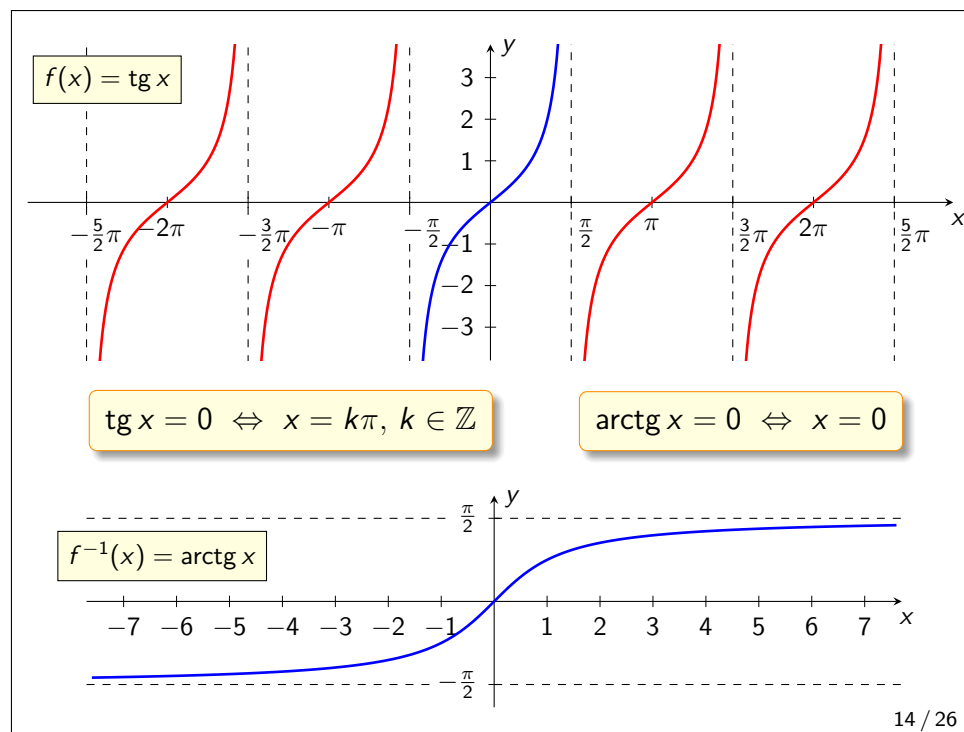
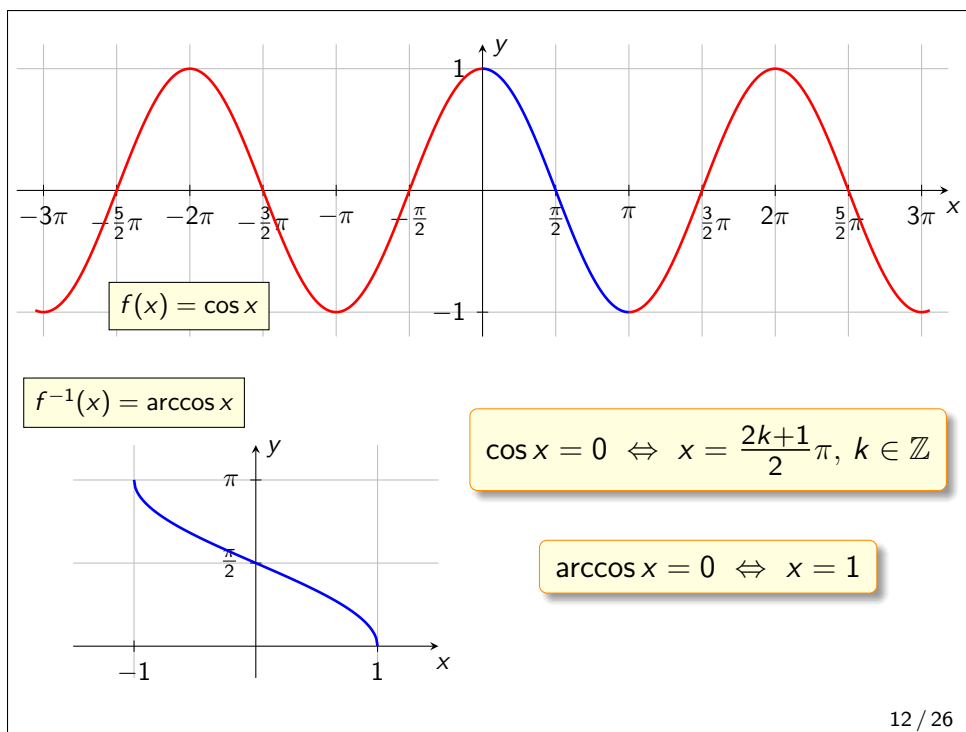
$$\bullet x_k^{(2)} = \pi - \arcsin a + 2k\pi, k \in \mathbb{Z}$$

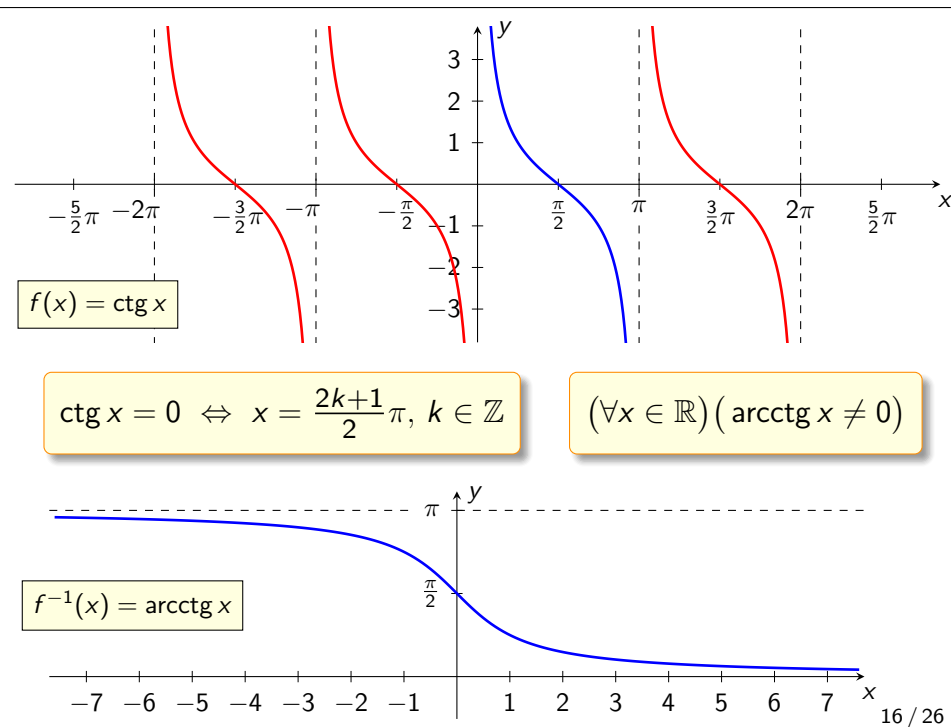
$$x_k^{(2)} = x_{2k+1} = x_1 + 2k\pi$$

Možemo sva rješenja zapisati pomoću jedne formule

$$x_k = (-1)^k \arcsin a + k\pi, k \in \mathbb{Z}$$

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Zadatak 3

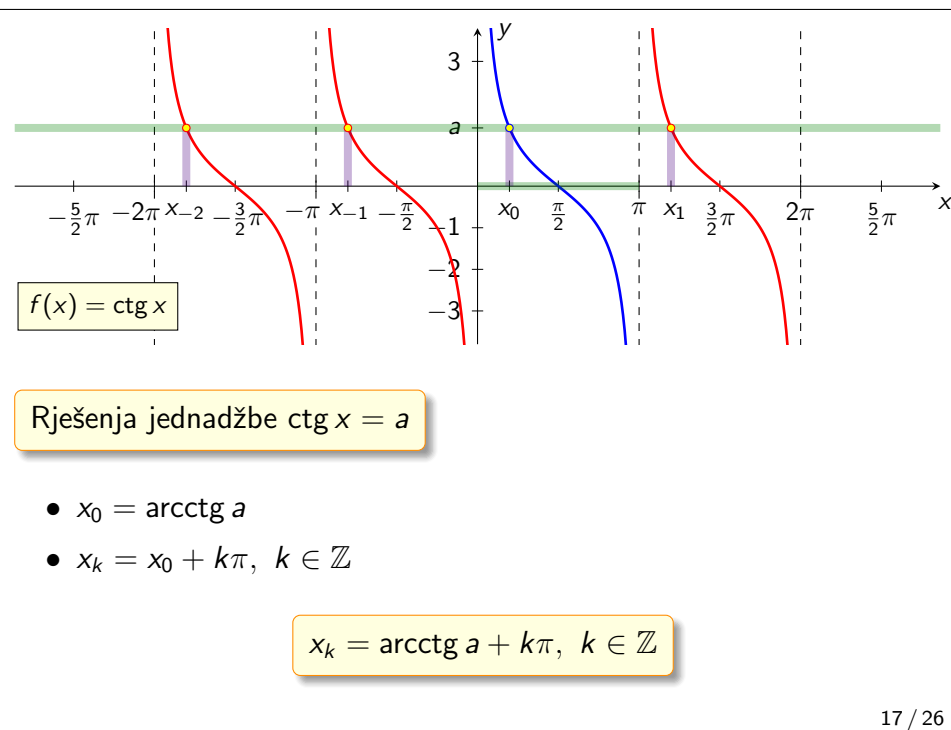
Odredite domenu i nultočke sljedećih funkcija:

a) $h(x) = \text{ctg } (\pi x + 2)$

b) $f(x) = \sqrt{\sin 3x + \frac{1}{2}}$

c) $g(x) = \frac{\arccos(x^2 - 3)}{x - 2}$

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Rješenje

a) **domena**

$$\pi x + 2 \neq k\pi, k \in \mathbb{Z}$$

$$\pi x \neq k\pi - 2 : \pi$$

$$x \neq \frac{k\pi - 2}{\pi}$$

$$x \neq k - \frac{2}{\pi}, k \in \mathbb{Z}$$

$$D_h = \mathbb{R} \setminus \left\{ k - \frac{2}{\pi} : k \in \mathbb{Z} \right\}$$

ekvivalentni zapis

$$D_h = \bigcup_{k \in \mathbb{Z}} \left\langle k - \frac{2}{\pi}, k + 1 - \frac{2}{\pi} \right\rangle$$

nultočke

$$\text{ctg } (\pi x + 2) = 0$$

$$\pi x + 2 = \frac{2k+1}{2}\pi, k \in \mathbb{Z}$$

$$\pi x = \frac{2k+1}{2}\pi - 2 : \pi$$

$$x = \frac{2k+1}{2} - \frac{2}{\pi}, k \in \mathbb{Z}$$

jesu nultočke
jer pripadaju domeni

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$$\text{b) } \arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6} \quad \pi - \left(-\frac{\pi}{6}\right) = \frac{7}{6}\pi$$

$$f(x) = \sqrt{\sin 3x + \frac{1}{2}}$$

Domena

$$\sin 3x + \frac{1}{2} \geq 0$$

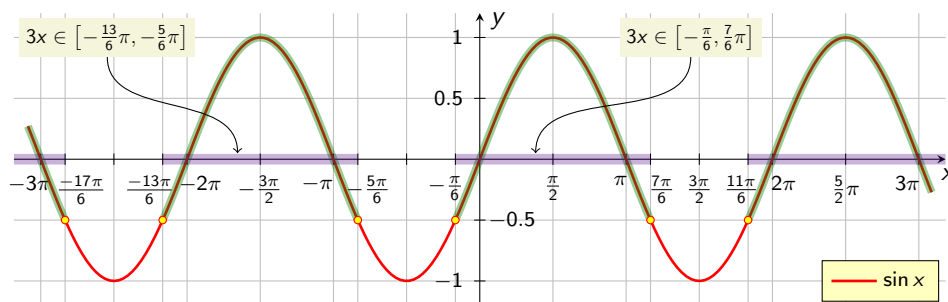
$$\sin 3x \geq -\frac{1}{2}$$

$$3x \in \bigcup_{k \in \mathbb{Z}} \left[-\frac{\pi}{6} + 2k\pi, \frac{7}{6}\pi + 2k\pi \right] / :3$$

$$x \in \bigcup_{k \in \mathbb{Z}} \left[-\frac{\pi}{18} + \frac{2}{3}k\pi, \frac{7}{18}\pi + \frac{2}{3}k\pi \right]$$

$$D_f = \bigcup_{k \in \mathbb{Z}} \left[\frac{12k-1}{18}\pi, \frac{12k+7}{18}\pi \right]$$

$$x \in \bigcup_{k \in \mathbb{Z}} \left[\frac{12k-1}{18} \pi, \frac{12k+7}{18} \pi \right]$$



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$$x = \frac{6k + (-1)^{k+1}}{18}\pi, \quad k \in \mathbb{Z}$$

$k = 2s$ za neki $s \in \mathbb{Z}$
 k je paran

$k = 2s + 1$ za neki $s \in \mathbb{Z}$
 k je neparan

$$x = \frac{6 \cdot 2s + (-1)^{2s+1}}{18} \pi$$

$$x = \frac{6 \cdot (2s+1) + (-1)^{2s+2}}{18} \pi$$

$$x = \frac{12s + (-1)^{\text{neparan}}}{18} \pi$$

$$x = \frac{12s + 6 + (-1)^{\text{paran}}}{18} \pi$$

$$x = \frac{12s - 1}{18}\pi$$

$$x = \frac{12s + 7}{18}\pi$$

$$D_f = \bigcup_{k \in \mathbb{Z}} \left[\frac{12k-1}{18}\pi, \frac{12k+7}{18}\pi \right]$$

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$$\text{b) } \arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6} \quad \pi - \left(-\frac{\pi}{6}\right) = \frac{7}{6}\pi$$

$$f(x) = \sqrt{\sin 3x + \frac{1}{2}}$$

Nultočky

$$\sqrt{\sin 3x + \frac{1}{2}} = 0 \quad / ^2$$

$$\sin 3x + \frac{1}{2} = 0$$

$$\sin 3x = -\frac{1}{2}$$

$$3x = (-1)^k \arcsin\left(-\frac{1}{2}\right) + k\pi \quad / : 3$$

$$x = \frac{(-1)^k}{3} \cdot \frac{-\pi}{6} + \frac{k\pi}{3}$$

$$x = \frac{(-1)^k \cdot (-1) \cdot \pi}{18} + \frac{k\pi}{3}$$

$$x = \frac{(-1)^{k+1}}{18}\pi + \frac{k\pi}{3}$$

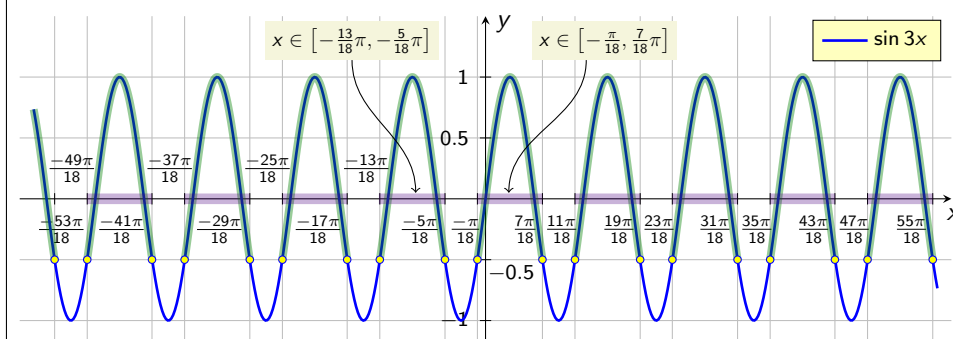
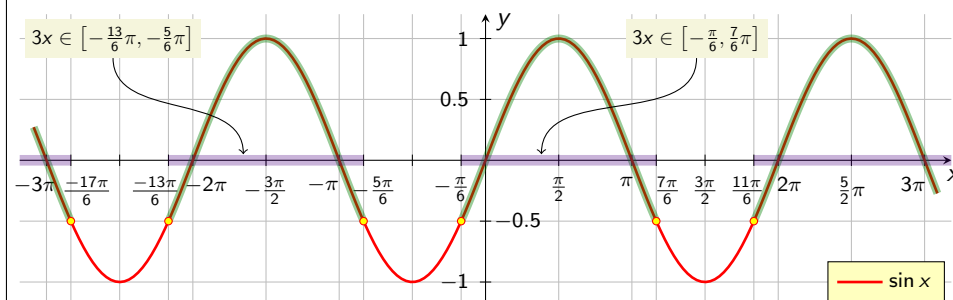
$$D_f = \bigcup_{k \in \mathbb{Z}} \left[\frac{12k-1}{18} \pi, \frac{12k+7}{18} \pi \right]$$

jesu nultočke
jer pripadaju domeni

$$x = \frac{6k + (-1)^{k+1}}{18}\pi, \quad k \in \mathbb{Z}$$

$$\sin x = a \Leftrightarrow x = (-1)^k \arcsin a + k\pi, \quad k \in \mathbb{Z}$$

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c) **Domena** $\Rightarrow x^2 - 3 \geq -1$
 $\Rightarrow x^2 - 3 \leq 1$
 $\Rightarrow x - 2 \neq 0$

$g(x) = \frac{\arccos(x^2 - 3)}{x - 2}$

domena funkcije arccos je segment $[-1, 1]$

zbog nazivnika

$x \neq 2$

presjek rješenja

$x \in \langle -\infty, -\sqrt{2} \rangle \cup [\sqrt{2}, +\infty \rangle$

$x \in [-2, 2]$

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c) **Nultočke** $\arccos x = 0 \Leftrightarrow x = 1$

$g(x) = \frac{\arccos(x^2 - 3)}{x - 2}$

$\frac{\arccos(x^2 - 3)}{x - 2} = 0$
 $\arccos(x^2 - 3) = 0$
 $x^2 - 3 = 1$
 $x^2 = 4$
 $x_1 = -2$ $x_2 = 2$

nije nultočka jer ne pripada domeni

jest nultočka jer pripada domeni

$D_g = x \in [-2, -\sqrt{2}] \cup [\sqrt{2}, 2 \rangle$

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c) **Domena** $g(x) = \frac{\arccos(x^2 - 3)}{x - 2}$

$D_g = x \in [-2, -\sqrt{2}] \cup [\sqrt{2}, 2 \rangle$

$\Rightarrow x^2 - 3 \geq -1 \rightarrow x \in \langle -\infty, -\sqrt{2} \rangle \cup [\sqrt{2}, +\infty \rangle$
 $\Rightarrow x^2 - 3 \leq 1 \rightarrow x \in [-2, 2]$
 $\Rightarrow x - 2 \neq 0 \rightarrow x \neq 2$

presjek rješenja

zamjena

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