Seminari 13

Matematičke metode za informatičare

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FOI, Varaždin

Sadržaj

prvi zadatak

drugi zadatak

treći zadatak

četvrti zadatak

peti zadatak

prvi zadatak

Zadatak 1

Zadana je funkcija $f(x, y) = \ln(x + y^2)$.

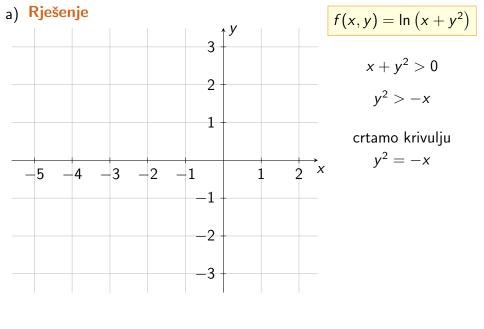
- a) Prikažite grafički domenu funkcije f.
- b) Odredite nivo-linije funkcije f i specijalno nacrtajte nivo-liniju za vrijednost $z = \ln 5$.
- c) Odredite nultočke funkcije f.
- d) Odredite parcijalne derivacije funkcije f.
- e) Odredite $\frac{\partial^4 f}{\partial x^3 \partial y}$.

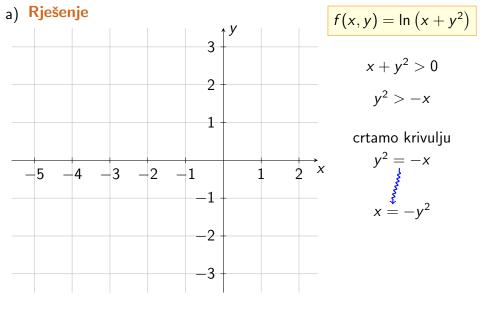
$$x+y^2>0$$

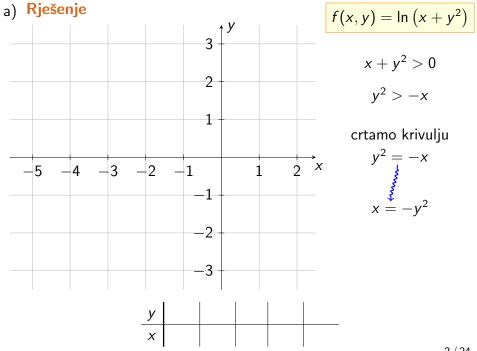
a) Rješenje

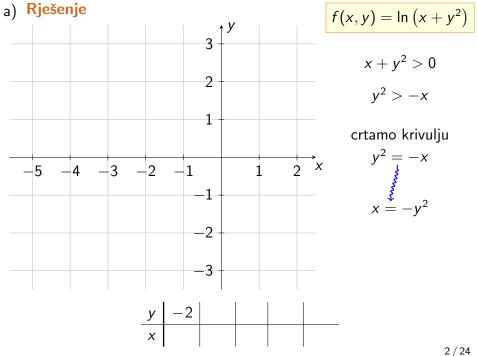
$$x + y^2 > 0$$
$$y^2 > -x$$

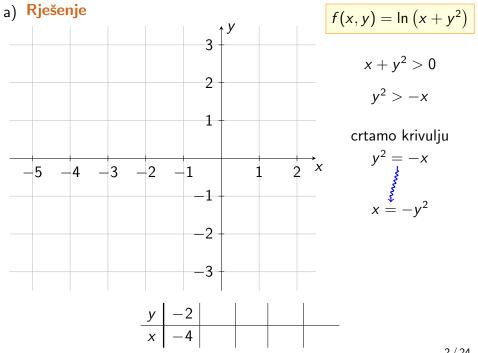
a) Rješenje

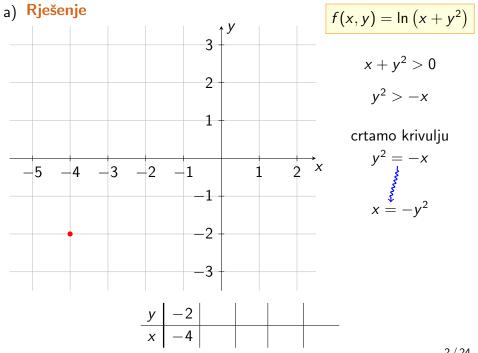


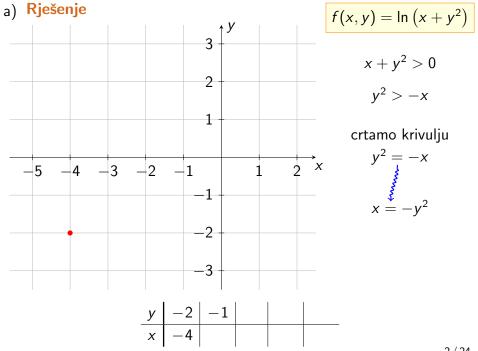


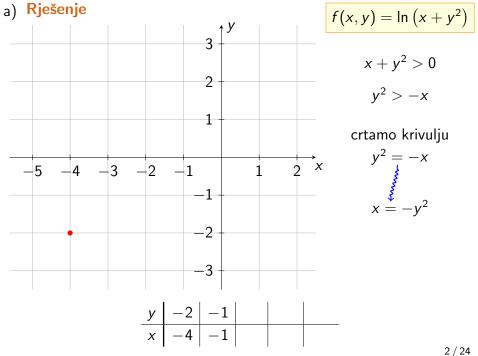


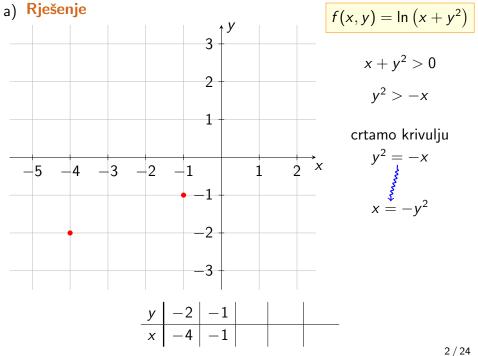


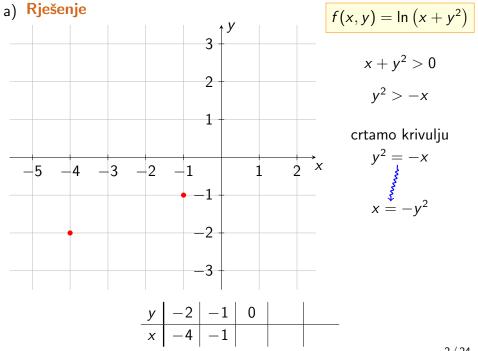


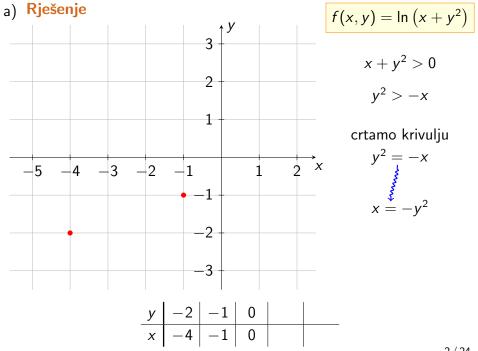


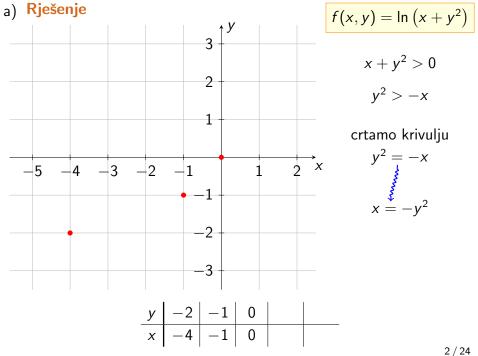


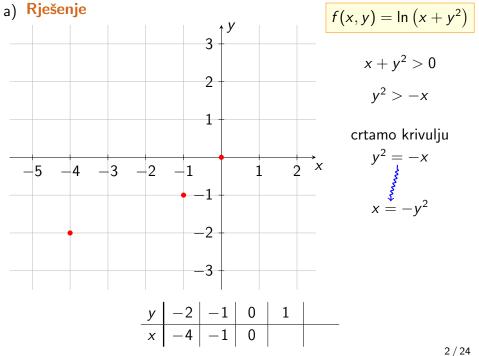


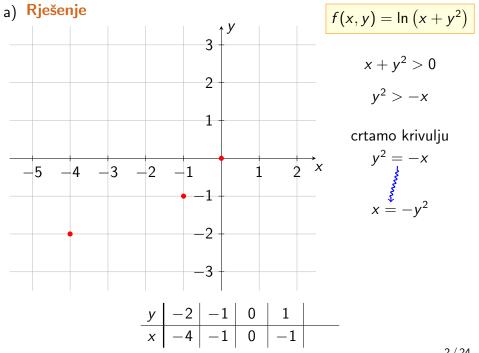


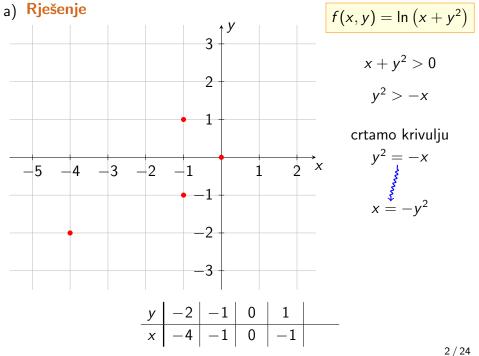


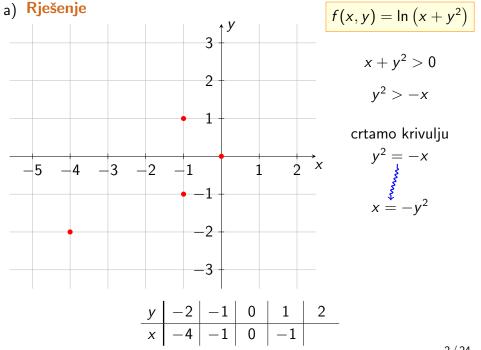


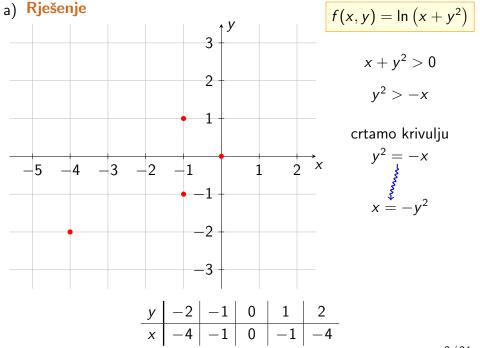


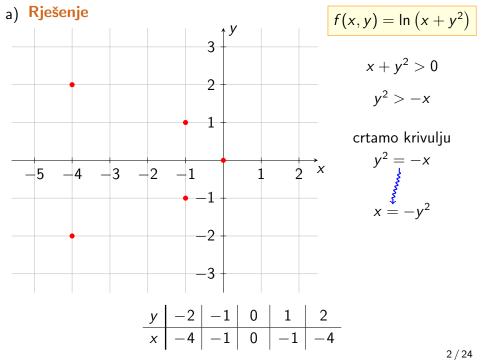


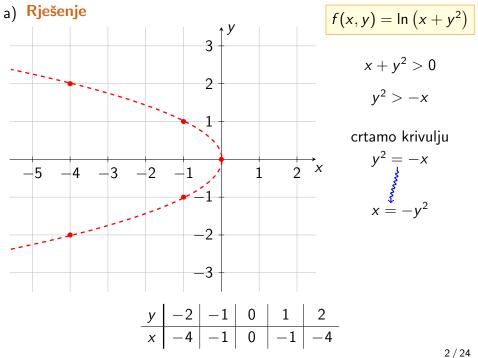


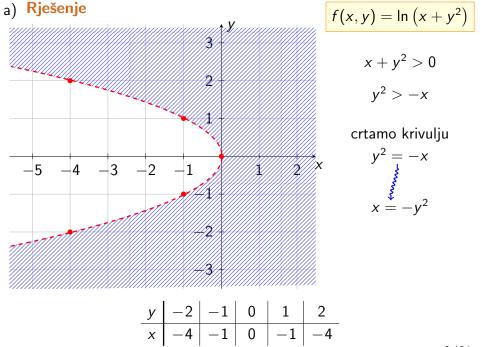












$$\ln\left(x+y^2\right)=C$$

 $\log_a x = b \longrightarrow x = a^b$ $\ln (x + y^2) = C$

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$$x + y^2 =$$

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$$\ln (x + y^2) = C$$

$$x + y^2 = e^C$$

$$\ln(x + y^2) = C$$

$$x + y^2 = e^C$$

 $y^2 = -x + e^C$

$$\log_a x = b \longrightarrow x = a^b$$

$$\log_a x = b \xrightarrow{} x = a^b$$

$$\ln (x + y^2) = C$$

$$x + y^2 = e^C$$

 $v^2 = -x + e^C$. $C \in \mathbb{R}$

$$\ln(x + y^2) = C$$

$$x + y^2 = e^C$$

$$y^2 = -x + e^C, C \in \mathbb{R}$$

$$\text{nivo-linije}$$

$$\text{su parabole}$$

su parabole
$$C = \ln 5$$

 $\ln\left(x+y^2\right)=C$

$$x + y^{2} = e^{C}$$

$$y^{2} = -x + e^{C}, C \in \mathbb{R}$$

$$\text{nivo-linije}$$

$$\text{su parabole}$$

$$C = \ln 5$$

$$\ln(x + y^2) = C$$

$$x + y^2 = e^C$$

$$y^2 = -x + e^C, C \in \mathbb{R}$$

$$\text{nivo-linije}$$

$$\text{su parabole}$$

$$C = \ln 5$$

b)

$$f(x,y) = \ln(x + y^2)$$

$$\ln(x + y^2) = C$$

$$x + y^2 = e^C$$

$$y^2 = -x + e^C, C \in \mathbb{R}$$

$$\text{nivo-linije}$$

$$\text{su parabole}$$

$$C = \ln 5$$

$$y^2 = -x + e^{\ln 5}$$

b)

$$\ln(x + y^2) = C$$

$$x + y^2 = e^C$$

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$$\text{nivo-linije}$$

$$\text{su parabole}$$

$$C = \ln 5$$

$$y^2 = -x + e^{\ln 5}$$

 $f(x,y) = \ln\left(x + y^2\right)$

 $v^2 = -x + 5$

 $a^{\log_a x} = x$

$$\ln(x + y^2) = C$$

$$x + y^2 = e^C$$

$$y^2 = -x + e^C, C \in \mathbb{R}$$

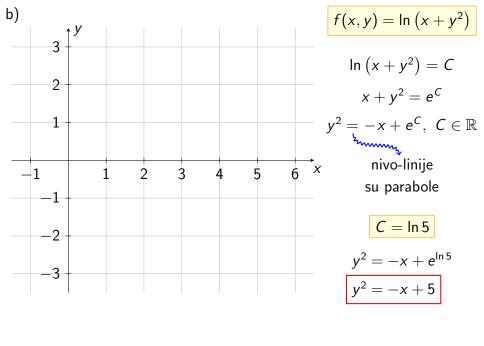
$$\text{nivo-linije}$$

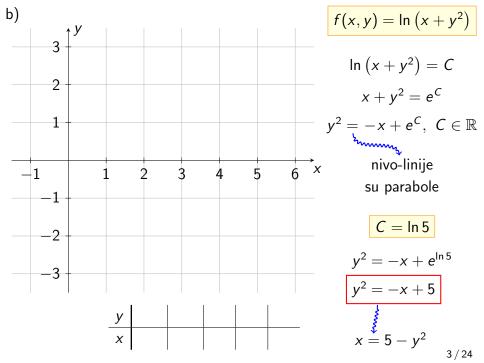
$$\text{su parabole}$$

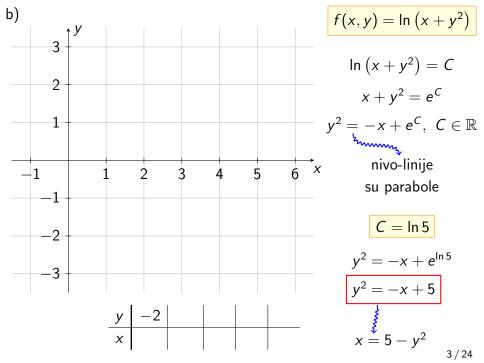
$$C = \ln 5$$

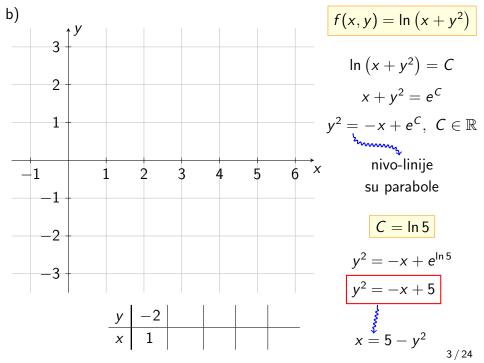
$$y^2 = -x + e^{\ln 5}$$

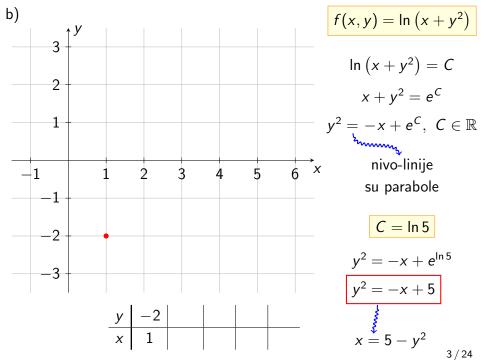
 $y^2 = -x + 5$

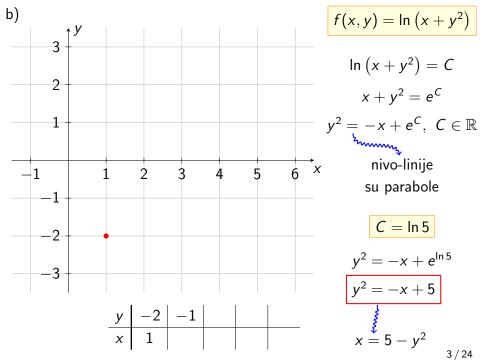


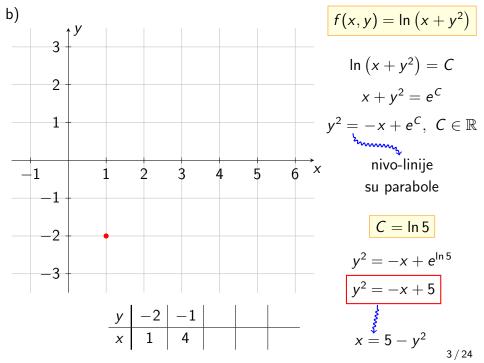


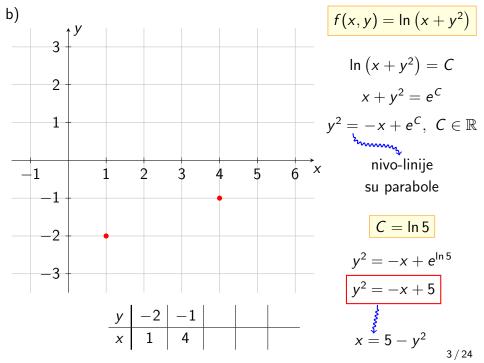


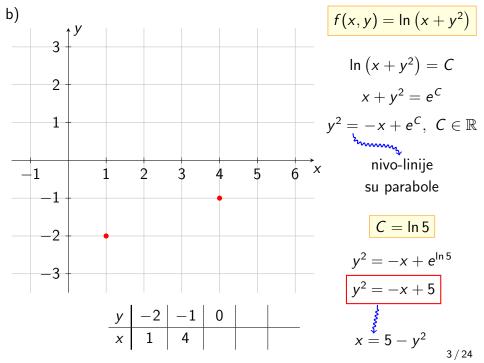


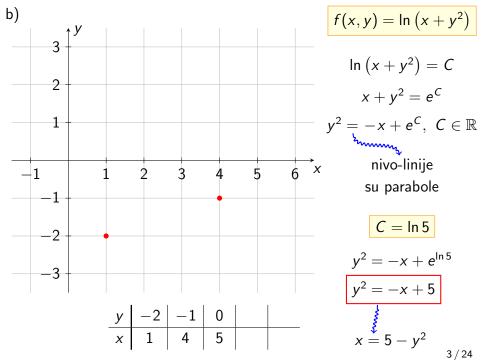


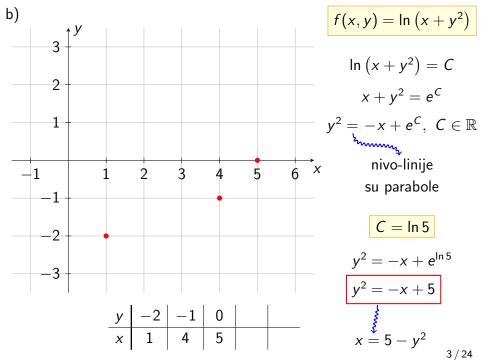


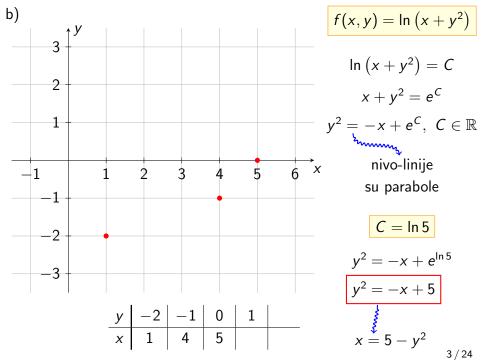


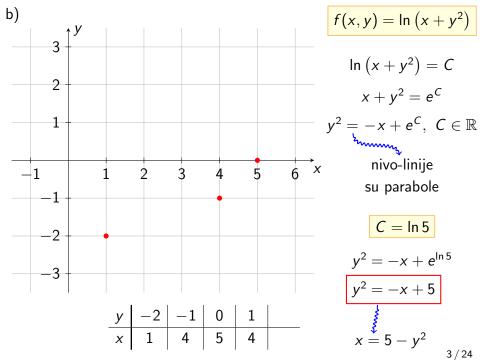


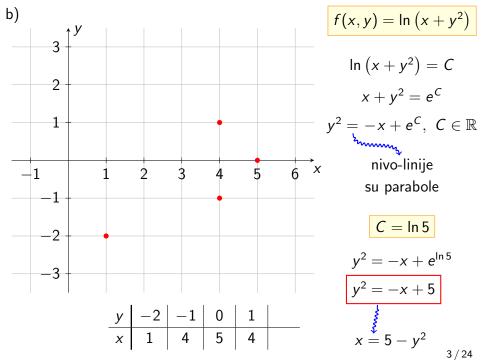


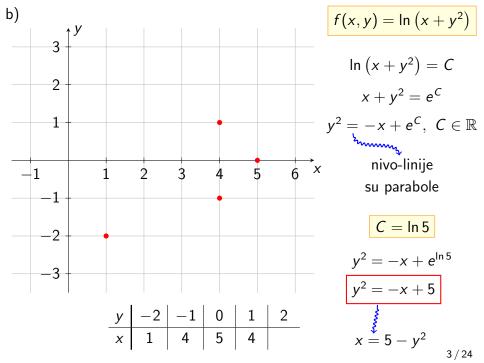


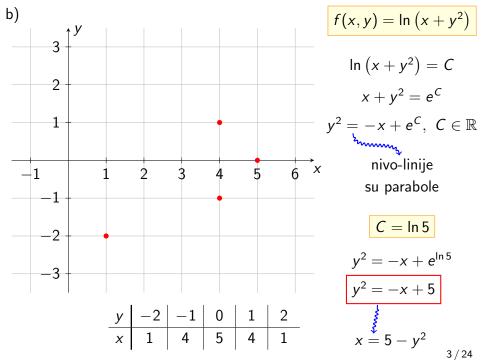


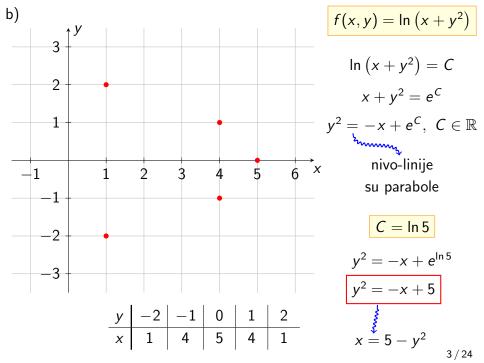


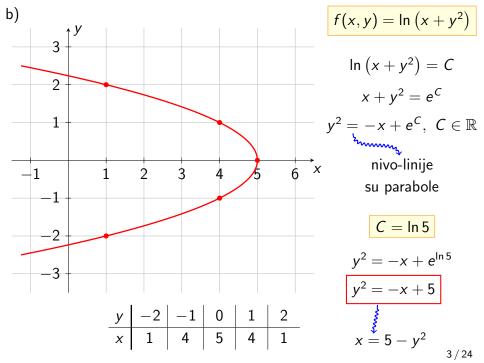














 $y^2 = -x + e^C$, $C \in \mathbb{R}$

nultočke





 $y^2 = -x + e^C$, $C \in \mathbb{R}$

nultočke C = 0



 $v^2 = -x + e^C$, $C \in \mathbb{R}$

nultočke C = 0

 $y^2 = -x + e^0$

 $v^2 = -x + e^C$, $C \in \mathbb{R}$

nultočke C = 0

 $y^2 = -x + e^0$

 $v^2 = -x + 1$



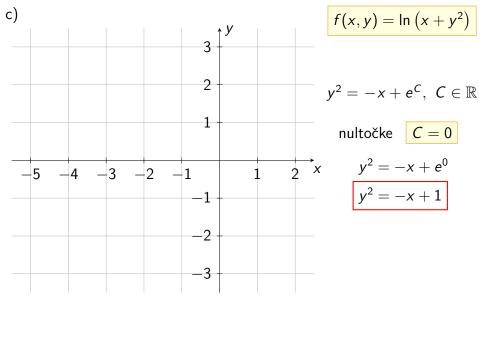
 $v^2 = -x + e^C$, $C \in \mathbb{R}$

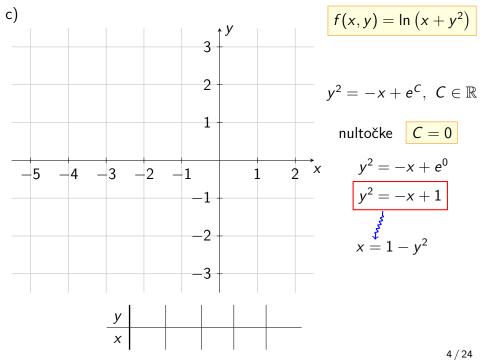
nultočke C = 0

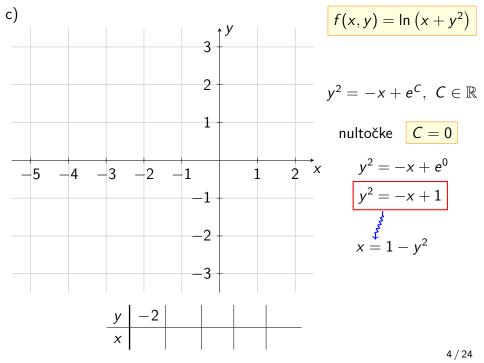
 $y^2 = -x + e^0$

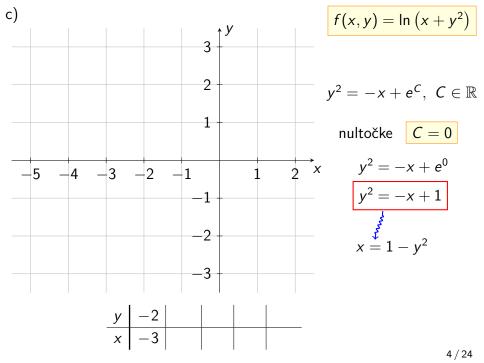
 $y^2 = -x + 1$

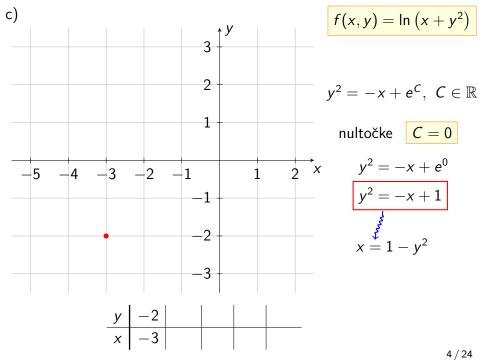


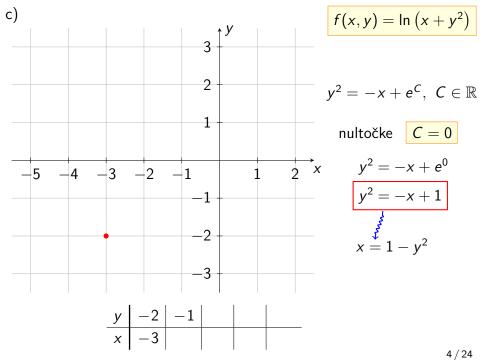


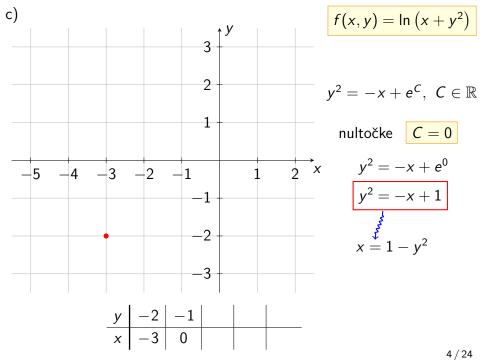


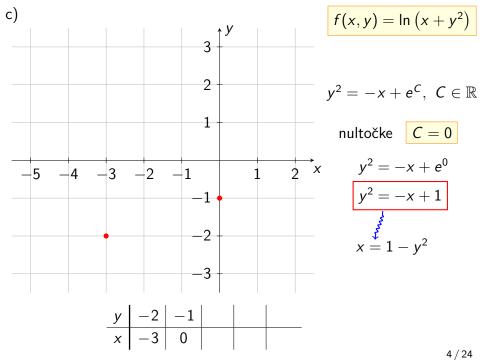


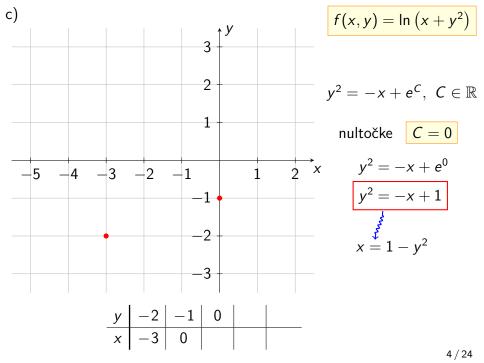


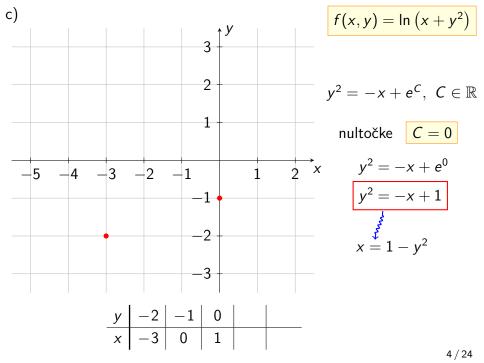


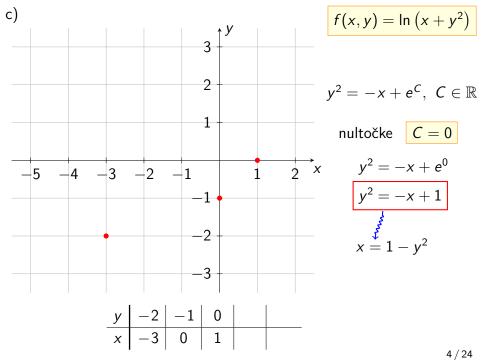


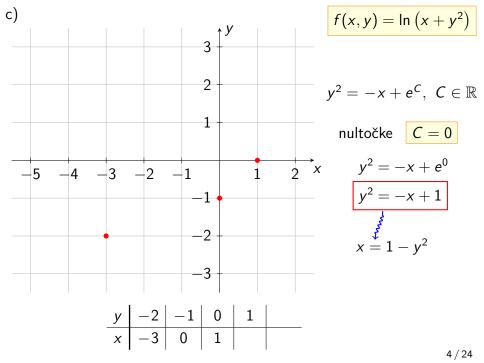


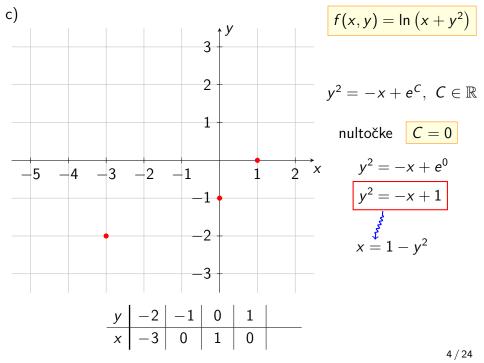


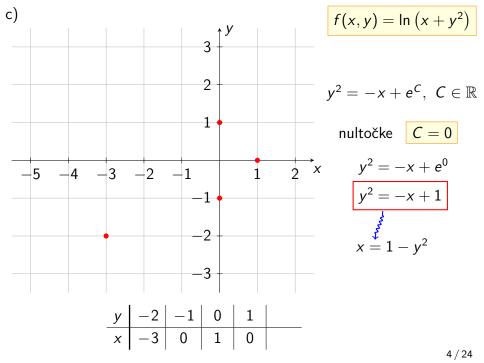


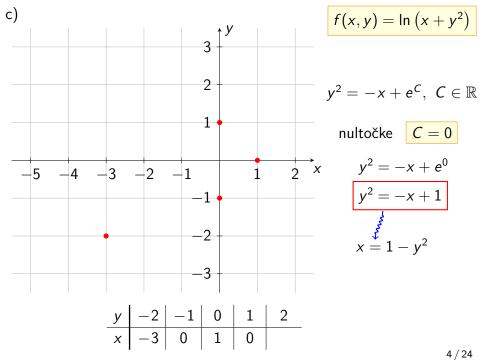


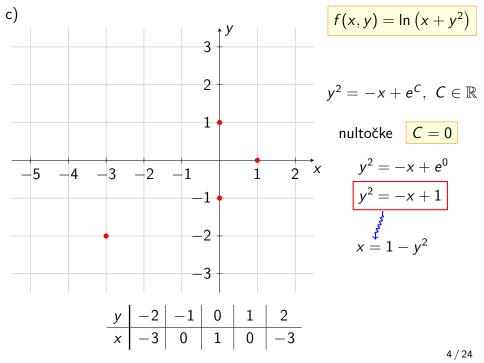


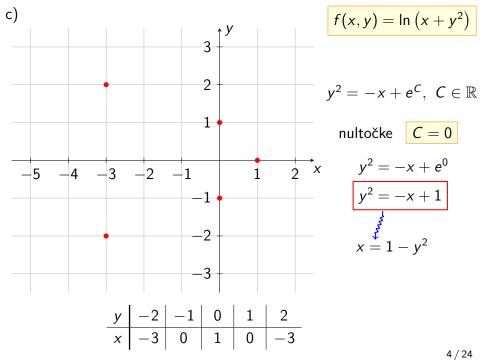


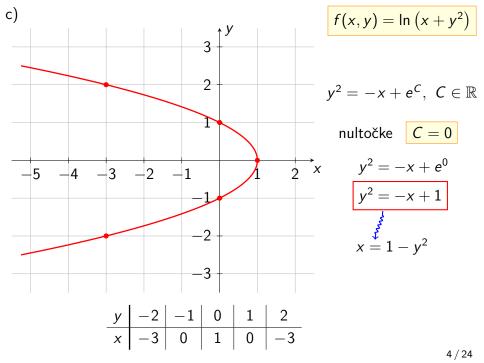


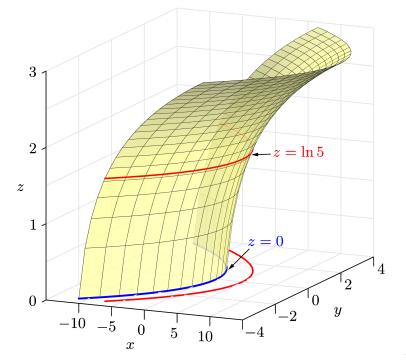


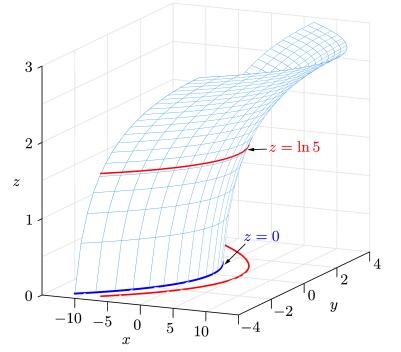












d)
$$\frac{\partial f}{\partial x}$$

$$(\ln x)' = \frac{1}{x}$$

 $(\ln x)' = \frac{1}{x}$

 $\frac{\partial f}{\partial x} = \frac{1}{x + y^2} \cdot \left(x + y^2 \right)_x'$

 $(\ln x)' = \frac{1}{x}$

 $\frac{\partial f}{\partial x} = \frac{1}{x + y^2} \cdot \left(x + y^2 \right)_x' = \frac{1}{x + y^2} \cdot$

 $(\ln x)' = \frac{1}{x}$

 $(x^n)' = nx^{n-1}$

 $\frac{\partial f}{\partial x} = \frac{1}{x + y^2} \cdot \left(x + y^2 \right)_x' = \frac{1}{x + y^2} \cdot 1$

 $(\ln x)' = \frac{1}{x}$

 $(x^n)' = nx^{n-1}$

 $\frac{\partial f}{\partial x} = \frac{1}{x + y^2} \cdot (x + y^2)'_x = \frac{1}{x + y^2} \cdot 1 = \frac{1}{x + y^2}$

 $(x^n)' = nx^{n-1}$

$$\frac{\partial f}{\partial y} = \frac{1}{x}$$

 $\frac{\partial f}{\partial x} = \frac{1}{x + y^2} \cdot (x + y^2)'_x = \frac{1}{x + y^2} \cdot 1 = \frac{1}{x + y^2}$

d)

d)

 $f(x,y) = \ln\left(x + y^2\right)$

 $(\ln x)' = \frac{1}{x}$

$$(x^n)' = nx^{n-1}$$

 $\frac{\partial f}{\partial y} = \frac{1}{x + y^2} \cdot \left(x + y^2 \right)_y'$

 $f(x,y) = \ln\left(x + y^2\right)$

$$(x^n)' = nx^{n-1}$$

 $\frac{\partial f}{\partial y} = \frac{1}{x + y^2} \cdot \left(x + y^2 \right)_y' = \frac{1}{x + y^2} \cdot$

 $f(x,y) = \ln\left(x + y^2\right)$

$$(x^n)' = nx^{n-1}$$

 $\frac{\partial f}{\partial y} = \frac{1}{x + y^2} \cdot \left(x + y^2\right)_y' = \frac{1}{x + y^2} \cdot 2y$

 $f(x,y) = \ln\left(x + y^2\right)$

$$(x^n)' = nx^{n-1}$$

 $\frac{\partial f}{\partial y} = \frac{1}{x + y^2} \cdot (x + y^2)'_y = \frac{1}{x + y^2} \cdot 2y = \frac{2y}{x + y^2}$

 $f(x,y) = \ln\left(x + y^2\right)$

e)
$$\frac{\partial^4 f}{\partial x^3 \partial y} \to f_{xxxy}$$

 $\frac{\partial f}{\partial v} = \frac{1}{x + v^2} \cdot (x + y^2)'_y = \frac{1}{x + y^2} \cdot 2y = \frac{2y}{x + y^2}$

 $f(x,y) = \ln\left(x + y^2\right)$

e)
$$\frac{\partial^4 f}{\partial x^3 \partial y} \to f_{xxxy}$$

$$\frac{\partial^2 f}{\partial x^3 \partial y} = \frac{\partial^2 f}{\partial x} = \frac{\partial^2 f}{\partial x$$

 $\frac{\partial f}{\partial v} = \frac{1}{x + v^2} \cdot (x + y^2)'_y = \frac{1}{x + y^2} \cdot 2y = \frac{2y}{x + y^2}$

d)

 $f(x,y) = \ln(x+y^2)$

e)
$$\frac{\partial^4 f}{\partial x^3 \partial y} \to f_{xxxy}$$

$$\frac{\partial^2 f}{\partial y^2} = f_{xx}$$

$$(x^n)' = nx^{n-1}$$

 $\frac{\partial f}{\partial v} = \frac{1}{x + v^2} \cdot (x + y^2)'_y = \frac{1}{x + y^2} \cdot 2y = \frac{2y}{x + y^2}$

d)

 $f(x,y) = \ln(x+y^2)$

e)
$$\frac{\partial^4 f}{\partial x^3 \partial y} \to f_{xxxy}$$

$$\frac{\partial^2 f}{\partial y^2} = f_{xx} = (f_x)_x$$

 $\frac{\partial f}{\partial v} = \frac{1}{x + v^2} \cdot (x + y^2)'_y = \frac{1}{x + y^2} \cdot 2y = \frac{2y}{x + y^2}$

d)

 $f(x,y) = \ln(x+y^2)$

 $(\ln x)' = \frac{1}{x}$

e)
$$\frac{\partial^4 f}{\partial x^3 \partial y} \to f_{xxxy}$$

$$\frac{\partial^2 f}{\partial x^2} = f_{xx} = (f_x)_x$$

 $\frac{\partial f}{\partial v} = \frac{1}{x + v^2} \cdot (x + y^2)'_y = \frac{1}{x + y^2} \cdot 2y = \frac{2y}{x + y^2}$

d)

 $f(x,y) = \ln\left(x + y^2\right)$

 $(\ln x)' = \frac{1}{x}$

e)
$$\frac{\partial^4 f}{\partial x^3 \partial y} \to f_{xxxy}$$

$$\frac{\partial^2 f}{\partial x^2} = f_{xx} = (f_x)_x = -(x + y^2)^{-2}$$

 $\frac{\partial f}{\partial v} = \frac{1}{x + v^2} \cdot (x + y^2)'_y = \frac{1}{x + y^2} \cdot 2y = \frac{2y}{x + y^2}$

d)

 $f(x,y) = \ln\left(x + y^2\right)$

 $(\ln x)' = \frac{1}{x}$

$$\frac{\partial^2 f}{\partial x^2} = f_{xx} = (f_x)_x = -(x+y^2)^{-2} \cdot (x+y^2)'_x$$

 $\frac{\partial f}{\partial v} = \frac{1}{x + v^2} \cdot (x + y^2)'_y = \frac{1}{x + v^2} \cdot 2y = \frac{2y}{x + v^2}$

 $\frac{\partial^4 f}{\partial x^3 \partial y} \rightarrow f_{xxxy}$

d)

 $f(x,y) = \ln\left(x + y^2\right)$

 $(\ln x)' = \frac{1}{x}$

e)
$$\frac{\partial^4 f}{\partial x^3 \partial y} \to f_{xxxy}$$

$$= 1$$

$$\frac{\partial^2 f}{\partial x^2} = f_{xx} = (f_x)_x = -(x+y^2)^{-2} \cdot (x+y^2)_x'$$

 $\frac{\partial f}{\partial v} = \frac{1}{x + v^2} \cdot (x + y^2)'_y = \frac{1}{x + y^2} \cdot 2y = \frac{2y}{x + y^2}$

d)

 $f(x,y) = \ln\left(x + y^2\right)$

 $(\ln x)' = \frac{1}{x}$

$$\frac{\partial^2 f}{\partial x^2} = f_{xx} = (f_x)_x = -(x + y^2)^{-2} \cdot \left[(x + y^2)_x' \right] = -(x + y^2)^{-2}$$

 $\frac{\partial f}{\partial v} = \frac{1}{x + v^2} \cdot (x + y^2)'_y = \frac{1}{x + y^2} \cdot 2y = \frac{2y}{x + y^2}$

 $\frac{\partial^4 f}{\partial x^3 \partial y} \rightarrow f_{xxxy}$

d)

 $f(x,y) = \ln\left(x + y^2\right)$

$$\frac{\partial^2 f}{\partial x^2} = f_{xx} = (f_x)_x = -(x + y^2)^{-2} \cdot \left[(x + y^2)_x' \right] = -(x + y^2)^{-2}$$

$$\partial^3 f$$

 $\frac{\partial f}{\partial v} = \frac{1}{x + v^2} \cdot (x + y^2)'_y = \frac{1}{x + y^2} \cdot 2y = \frac{2y}{x + y^2}$

 $\frac{\partial^4 f}{\partial x^3 \partial y} \rightarrow f_{xxxy}$

d)

 $f(x,y) = \ln\left(x + y^2\right)$

$$\frac{\partial^2 f}{\partial x^2} = f_{xx} = (f_x)_x = -(x+y^2)^{-2} \cdot \boxed{(x+y^2)_x'} = -(x+y^2)^{-2}$$
$$\frac{\partial^3 f}{\partial x^3} = f_{xxx}$$

 $\frac{\partial f}{\partial x} = \frac{1}{x + v^2} \cdot (x + y^2)'_x = \frac{1}{x + v^2} \cdot 1 = \frac{1}{x + y^2} = (x + y^2)^{-1}$

 $\frac{\partial f}{\partial v} = \frac{1}{x + v^2} \cdot (x + y^2)'_y = \frac{1}{x + y^2} \cdot 2y = \frac{2y}{x + y^2}$

 $\frac{\partial^4 f}{\partial x^3 \partial y} \rightarrow f_{xxxy}$

d)

 $f(x,y) = \ln\left(x + y^2\right)$

$$\frac{\partial^2 f}{\partial x^2} = f_{xx} = (f_x)_x = -(x+y^2)^{-2} \cdot \left[(x+y^2)_x' \right] = -(x+y^2)^{-2}$$

$$\frac{\partial^3 f}{\partial x^3} = f_{xxx} = (f_{xx})_x$$

 $\frac{\partial f}{\partial x} = \frac{1}{x + v^2} \cdot (x + y^2)'_x = \frac{1}{x + v^2} \cdot 1 = \frac{1}{x + y^2} = (x + y^2)^{-1}$

 $\frac{\partial f}{\partial v} = \frac{1}{x + v^2} \cdot (x + y^2)'_{y} = \frac{1}{x + y^2} \cdot 2y = \frac{2y}{x + y^2}$

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 $\frac{\partial f}{\partial x} = \frac{1}{x + y^2} \cdot (x + y^2)'_x = \frac{1}{x + y^2} \cdot 1 = \frac{1}{x + y^2} = (x + y^2)^{-1}$

 $\frac{\partial f}{\partial v} = \frac{1}{x + v^2} \cdot (x + y^2)'_y = \frac{1}{x + y^2} \cdot 2y = \frac{2y}{x + y^2}$

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 $\frac{\partial f}{\partial x} = \frac{1}{x + y^2} \cdot (x + y^2)'_x = \frac{1}{x + y^2} \cdot 1 = \frac{1}{x + y^2} = (x + y^2)^{-1}$

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 $\frac{\partial^4 f}{\partial x^3 \partial y} \rightarrow f_{xxxy}$

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$$\frac{\partial^2 f}{\partial x^2} = f_{xx} = (f_x)_x = -(x+y^2)^{-2} \cdot (x+y^2)_x' = -(x+y^2)^{-2}$$

$$= 1$$

$$\frac{\partial^3 f}{\partial x^3} = f_{xxx} = (f_{xx})_x = -(-2)(x+y^2)^{-3} \cdot (x+y^2)_x'$$

 $\frac{\partial f}{\partial x} = \frac{1}{x + v^2} \cdot (x + y^2)'_x = \frac{1}{x + y^2} \cdot 1 = \frac{1}{x + y^2} = (x + y^2)^{-1}$

 $\frac{\partial f}{\partial v} = \frac{1}{x + v^2} \cdot (x + y^2)'_y = \frac{1}{x + y^2} \cdot 2y = \frac{2y}{x + y^2}$

 $\frac{\partial^4 f}{\partial x^3 \partial y} \to f_{xxxy}$

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 $f(x,y) = \ln(x+y^2)$

$$\frac{\partial^2 f}{\partial x^2} = f_{xx} = (f_x)_x = -(x+y^2)^{-2} \cdot (x+y^2)_x' = -(x+y^2)^{-2}$$

$$= 1$$

$$\frac{\partial^3 f}{\partial x^3} = f_{xxx} = (f_{xx})_x = -(-2)(x+y^2)^{-3} \cdot (x+y^2)_x' = 2(x+y^2)^{-3}$$

 $\frac{\partial f}{\partial x} = \frac{1}{x + y^2} \cdot (x + y^2)'_x = \frac{1}{x + y^2} \cdot 1 = \frac{1}{x + y^2} = (x + y^2)^{-1}$

 $\frac{\partial f}{\partial y} = \frac{1}{x + y^2} \cdot (x + y^2)'_y = \frac{1}{x + y^2} \cdot 2y = \frac{2y}{x + y^2}$

 $\frac{\partial^4 f}{\partial x^3 \partial y} \rightarrow f_{xxxy}$

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$$\frac{\partial f}{\partial x} = \frac{1}{x + y^2} \cdot (x + y^2)'_x = \frac{1}{x + y^2} \cdot 1 = \frac{1}{x + y^2} = (x + y^2)^{-1}$$

$$\frac{\partial f}{\partial y} = \frac{1}{x + y^2} \cdot (x + y^2)'_y = \frac{1}{x + y^2} \cdot 2y = \frac{2y}{x + y^2}$$

$$(\ln x)' = \frac{1}{x}$$

 $\frac{\partial^4 f}{\partial x^3 \partial y} \to f_{xxxy}$

$$\frac{\partial^2 f}{\partial x^2} = f_{xx} = (f_x)_x = -(x+y^2)^{-2} \cdot \underbrace{(x+y^2)'_x}_{} = -(x+y^2)^{-2}$$

$$= 1$$

$$\frac{\partial^3 f}{\partial x^3} = f_{xxx} = (f_{xx})_x = -(-2)(x+y^2)^{-3} \cdot \underbrace{(x+y^2)'_x}_{} = 2(x+y^2)^{-3}$$

$$\frac{\partial f}{\partial x} = \frac{1}{x + y^2} \cdot (x + y^2)'_x = \frac{1}{x + y^2} \cdot 1 = \frac{1}{x + y^2} = (x + y^2)^{-1}$$

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 $\frac{\partial^4 f}{\partial x^3 \partial y} \to f_{xxxy}$

 $f(x,y) = \ln\left(x + y^2\right)$

$$\frac{\partial^2 f}{\partial x^2} = f_{xx} = (f_x)_x = -(x+y^2)^{-2} \cdot \boxed{(x+y^2)_x'} = -(x+y^2)^{-2}$$

$$= 1$$

$$\frac{\partial^3 f}{\partial x^3} = f_{xxx} = (f_{xx})_x = -(-2)(x+y^2)^{-3} \cdot \boxed{(x+y^2)_x'} = 2(x+y^2)^{-3}$$

 $\frac{\partial^4 f}{\partial x^3 \partial y} = f_{xxxy}$

$$\frac{\partial f}{\partial x} = \frac{1}{x + y^2} \cdot (x + y^2)'_x = \frac{1}{x + y^2} \cdot 1 = \frac{1}{x + y^2} = (x + y^2)^{-1}$$

$$\frac{\partial f}{\partial y} = \frac{1}{x + y^2} \cdot (x + y^2)'_y = \frac{1}{x + y^2} \cdot 2y = \frac{2y}{x + y^2}$$

$$(\ln x)' = \frac{1}{x}$$

 $\frac{\partial^2 f}{\partial x^2} = f_{xx} = (f_x)_x = -(x+y^2)^{-2} \cdot \underbrace{\left(x+y^2\right)_x'}_{==-(x+y^2)^{-2}} = -(x+y^2)^{-2}$ = 1 $\frac{\partial^3 f}{\partial x^3} = f_{xxx} = (f_{xx})_x = -(-2)(x+y^2)^{-3} \cdot \underbrace{\left(x+y^2\right)_x'}_{==2(x+y^2)^{-3}}$

 $\frac{\partial^4 f}{\partial x^3 \partial y} \to f_{xxxy}$

 $\frac{\partial^4 f}{\partial x^3 \partial v} = f_{xxxy} = (f_{xxx})_y$

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$$\frac{\partial f}{\partial x} = \frac{1}{x + y^2} \cdot (x + y^2)'_x = \frac{1}{x + y^2} \cdot 1 = \frac{1}{x + y^2} = (x + y^2)^{-1}$$

$$\frac{\partial f}{\partial y} = \frac{1}{x + y^2} \cdot (x + y^2)'_y = \frac{1}{x + y^2} \cdot 2y = \frac{2y}{x + y^2}$$

$$(\ln x)' = \frac{1}{x}$$

 $\frac{\partial^4 f}{\partial x^3 \partial y} \rightarrow f_{xxxy}$

$$\frac{\partial^2 f}{\partial x^2} = f_{xx} = (f_x)_x = -(x+y^2)^{-2} \cdot \frac{1}{(x+y^2)_x'} = -(x+y^2)^{-2}$$

$$= 1$$

$$\frac{\partial^3 f}{\partial x^2} = f_{xx} = (f_x)_x = -(f_x)_x = -(x+y^2)^{-3} \cdot \frac{1}{(x+y^2)_x'} = -(x+y^2)^{-3}$$

$$\frac{\partial^3 f}{\partial x^3} = f_{xxx} = (f_{xx})_x = -(-2)(x + y^2)^{-3} \cdot (x + y^2)_x' = 2(x + y^2)^{-3}$$

$$\frac{\partial f}{\partial x} = \frac{1}{x + y^2} \cdot (x + y^2)'_x = \frac{1}{x + y^2} \cdot 1 = \frac{1}{x + y^2} = (x + y^2)^{-1}$$

$$\frac{\partial f}{\partial y} = \frac{1}{x + y^2} \cdot (x + y^2)'_y = \frac{1}{x + y^2} \cdot 2y = \frac{2y}{x + y^2} \qquad (\ln x)' = \frac{1}{x}$$

$$\frac{\partial^4 f}{\partial x^3 \partial y} \to f_{xxxy}$$

$$= 1$$

 $\frac{\partial^2 f}{\partial x^2} = f_{xx} = (f_x)_x = -(x+y^2)^{-2} \cdot \left[(x+y^2)_x' \right] = -(x+y^2)^{-2}$ = 1 $\frac{\partial^3 f}{\partial x^2} = f_{xx} = (f_x)_x = -(x+y^2)^{-2} \cdot \left[(x+y^2)_x' \right] = -(x+y^2)^{-2}$

$$\frac{\partial^{3} f}{\partial x^{3}} = f_{xxx} = (f_{xx})_{x} = -(-2)(x + y^{2})^{-3} \cdot \underbrace{(x + y^{2})'_{x}}^{=2} = 2(x + y^{2})^{-3}$$
$$\frac{\partial^{4} f}{\partial x^{3} \partial y} = f_{xxxy} = (f_{xxx})_{y} = -6(x + y^{2})^{-4} \cdot (x + y^{2})'_{y}$$

$$\frac{\partial f}{\partial x} = \frac{1}{x + y^2} \cdot (x + y^2)_x' = \frac{1}{x + y^2} \cdot 1 = \frac{1}{x + y^2} = (x + y^2)^{-1}$$

$$\frac{\partial f}{\partial y} = \frac{1}{x + y^2} \cdot (x + y^2)_y' = \frac{1}{x + y^2} \cdot 2y = \frac{2y}{x + y^2}$$

$$(\ln x)' = \frac{1}{x}$$

$$\frac{\partial^4 f}{\partial x^3 \partial y} \to f_{xxxy}$$

$$= 1$$

$$\frac{\partial^2 f}{\partial x^2} = f_{xx} = (f_x)_x = -(x+y^2)^{-2} \cdot (x+y^2)_x' = -(x+y^2)^{-2}$$

$$= 1$$

$$\frac{\partial^3 f}{\partial x^2} = f_{xx} = (f_{xx})_x = -(-2)(x+y^2)^{-3} \cdot (x+y^2)_x' = 2(x+y^2)^{-2}$$

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$$\frac{\partial^4 f}{\partial x^3} = f_{xxx} = (f_{xx})_x = -(-2)(x + y^2) \cdot (x + y^2)_x = 2(x + y^2)$$

$$\frac{\partial^4 f}{\partial x^3} = f_{xxx} = (f_{xx})_x = -6(x + y^2)^{-4} \cdot (x + y^2)'$$

 $\frac{\partial^4 f}{\partial x^3 \partial y} = f_{xxxy} = (f_{xxx})_y = -6(x + y^2)^{-4} \cdot \sqrt{(x + y^2)_y'}$

$$\frac{\partial f}{\partial x} = \frac{1}{x + y^2} \cdot (x + y^2)_x' = \frac{1}{x + y^2} \cdot 1 = \frac{1}{x + y^2} = (x + y^2)^{-1}$$

$$\frac{\partial f}{\partial y} = \frac{1}{x + y^2} \cdot (x + y^2)_y' = \frac{1}{x + y^2} \cdot 2y = \frac{2y}{x + y^2}$$

$$(\ln x)' = \frac{1}{x}$$

$$\frac{\partial^4 f}{\partial x^3 \partial y} \to f_{xxxy}$$

$$(x^n)' = nx^{n-1}$$

$$= 1$$

$$\frac{\partial^2 f}{\partial x^2} = f_{xx} = (f_x)_x = -(x+y^2)^{-2} \cdot \frac{1}{(x+y^2)_x'} = -(x+y^2)^{-2}$$

$$= 1$$

$$\frac{\partial^3 f}{\partial x^2} = f_{xx} = (f_x)_x = -(x+y^2)^{-2} \cdot \frac{1}{(x+y^2)_x'} = -(x+y^2)^{-2}$$

$$\frac{\partial^3 f}{\partial x^3} = f_{xxx} = (f_{xx})_x = -(-2)(x + y^2)^{-3} \cdot \frac{1}{(x + y^2)_x'} = 2(x + y^2)^{-3}$$

$$\frac{\partial^4 f}{\partial x^3 \partial y} = f_{xxxy} = (f_{xxx})_x = -(-2)(x+y') \qquad (x+y')_x = 2(x+y')$$

$$\frac{\partial^4 f}{\partial x^3 \partial y} = f_{xxxy} = (f_{xxx})_y = -6(x+y^2)^{-4} \cdot (x+y^2)_y' = \frac{-12y}{(x+y^2)^4}$$

$$= 2y$$

drugi zadatak

Zadatak 2

Zadana je ploha $z = x^3 + y^3$.

- a) Odredite na zadanoj plohi sve točke kojima je x-koordinata jednaka 1 i u kojima su tangencijalne ravnine plohe okomite na ravninu x+y+51z=0.
- b) U svim tako pronađenim točkama napišite jednadžbe tangencijalnih ravnina i jednadžbe normala zadane plohe.

Rješenje a) $z = x^3 + y^3$

a)
$$z = x^3 + y^3$$

$$\Sigma \ldots x + y + 51z = 0$$

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a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\Sigma \dots x + y + 51z = 0$$

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$$\Pi_t \perp \Sigma$$

a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\Sigma \dots x + y + 51z = 0$$

$$\Pi_t \perp \Sigma \iff \vec{n}_t \cdot \vec{n}_{\Sigma} = 0$$

a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\Sigma \ldots x + y + 51z = 0$$

$$\Pi_t \perp \Sigma \Leftrightarrow \boxed{\vec{n}_t \cdot \vec{n}_{\Sigma} = 0}$$

a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\Sigma \dots x + y + 51z = 0$$

$$\Pi_t \perp \Sigma \Leftrightarrow \boxed{\vec{n}_t \cdot \vec{n}_{\Sigma} = 0}$$

$$\vec{n}_t = \left(\frac{\partial z}{\partial x}, \, \frac{\partial z}{\partial y}, \, -1\right)$$

a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\Sigma \dots x + y + 51z = 0$$

$$\Pi_t \perp \Sigma \Leftrightarrow \boxed{\vec{n}_t \cdot \vec{n}_{\Sigma} = 0}$$

$$\vec{n_t} = \left(\frac{\partial z}{\partial x}, \, \frac{\partial z}{\partial y}, \, -1\right)$$

$$ec{n}_t = ($$

a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\Sigma \dots x + y + 51z = 0$$

$$\Pi_t \perp \Sigma \Leftrightarrow \boxed{\vec{n}_t \cdot \vec{n}_{\Sigma} = 0}$$

$$\vec{n}_t = \left(\frac{\partial z}{\partial x}, \, \frac{\partial z}{\partial y}, \, -1\right)$$

$$\vec{n}_t = (3x^2,$$

a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\Sigma \dots x + y + 51z = 0$$

$$\Pi_t \perp \Sigma \Leftrightarrow \boxed{\vec{n}_t \cdot \vec{n}_{\Sigma} = 0}$$

$$\vec{n}_t = \left(\frac{\partial z}{\partial x}, \, \frac{\partial z}{\partial y}, \, -1\right)$$

$$\vec{n}_t = \left(3x^2,\,3y^2,\,$$

a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\Sigma \dots x + y + 51z = 0$$

$$\Pi_t \perp \Sigma \Leftrightarrow \boxed{\vec{n}_t \cdot \vec{n}_{\Sigma} = 0}$$

$$\vec{n}_t = \left(\frac{\partial z}{\partial x}, \, \frac{\partial z}{\partial y}, \, -1\right)$$

$$\vec{n}_t = \left(3x^2, \, 3y^2, \, -1\right)$$

a) $z = x^3 + y^3$, T(1, y, z)

$$\Sigma \dots x + y + 51z = 0$$

$$\Pi_t \perp \Sigma \Leftrightarrow \boxed{\vec{n}_t \cdot \vec{n}_{\Sigma} = 0}$$

$$\vec{n}_t = \left(\frac{\partial z}{\partial x}, \, \frac{\partial z}{\partial y}, \, -1\right)$$

$$\vec{n}_t = \left(3x^2, \, 3y^2, \, -1\right)$$

$$\vec{n}_{\Sigma} = (1, 1, 51)$$

a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\Sigma \dots x + y + 51z = 0$$

$$\Pi_t \perp \Sigma \Leftrightarrow \vec{n}_t \cdot \vec{n}_{\Sigma} = 0$$

$$\vec{n}_t = \left(\frac{\partial z}{\partial x}, \, \frac{\partial z}{\partial y}, \, -1\right)$$

$$ec{n}_t = ig(3x^2, \, 3y^2, \, -1ig)$$
 $ec{n}_\Sigma = ig(1, 1, 51ig)$

$$n_{\Sigma}=(1,1,51)$$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) = 0$$

a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\Sigma \dots x + y + 51z = 0$$

$$\Pi_t \perp \Sigma \Leftrightarrow \vec{n}_t \cdot \vec{n}_{\Sigma} = 0$$

$$\vec{n_t} = \left(\frac{\partial z}{\partial x}, \, \frac{\partial z}{\partial y}, \, -1\right)$$

$$ec{n}_t = \left(3x^2, \, 3y^2, \, -1
ight)$$
 $ec{n}_\Sigma = \left(1, 1, 51
ight)$

$$\vec{n}_{\Sigma}=(1,1,51)$$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) = 0$$

$$3x^2$$

a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

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$$\Pi_t \perp \Sigma \Leftrightarrow \vec{n}_t \cdot \vec{n}_{\Sigma} = 0$$

$$\vec{n_t} = \left(\frac{\partial z}{\partial x}, \, \frac{\partial z}{\partial y}, \, -1\right)$$

$$ec{n}_t = \left(3x^2, \, 3y^2, \, -1
ight)$$
 $ec{n}_{\Sigma} = \left(1, 1, 51
ight)$

$$\vec{n}_{\Sigma}=(1,1,51)$$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) = 0$$

$$3x^2 + 3y^2$$

a) $z = x^3 + y^3$, T(1, y, z)

$$\Sigma \ldots x + y + 51z = 0$$

$$\Pi_t \perp \Sigma \Leftrightarrow \vec{n}_t \cdot \vec{n}_{\Sigma} = 0$$

$$\vec{n_t} = \left(\frac{\partial z}{\partial x}, \, \frac{\partial z}{\partial y}, \, -1\right)$$

$$ec{n}_t = \left(3x^2, \, 3y^2, \, -1
ight)$$
 $ec{n}_{\Sigma} = \left(1, 1, 51
ight)$

$$\vec{n}_{\Sigma}=(1,1,51)$$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) \stackrel{\checkmark}{=} 0$$

$$3x^2 + 3y^2 - 51$$

a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\Sigma \dots x + y + 51z = 0$$

$$\Pi_t \perp \Sigma \Leftrightarrow \vec{n}_t \cdot \vec{n}_{\Sigma} = 0$$

$$\vec{n_t} = \left(\frac{\partial z}{\partial x}, \, \frac{\partial z}{\partial y}, \, -1\right)$$

$$ec{n}_t = \left(3x^2, \, 3y^2, \, -1
ight)$$
 $ec{n}_{\Sigma} = \left(1, 1, 51
ight)$

$$\vec{n}_{\Sigma}=(1,1,51)$$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) = 0$$

$$3x^2 + 3v^2 - 51 = 0$$

a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\Sigma \dots x + y + 51z = 0$$

$$\Pi_t \perp \Sigma \Leftrightarrow \vec{n}_t \cdot \vec{n}_{\Sigma} = 0$$

$$\vec{n_t} = \left(\frac{\partial z}{\partial x}, \, \frac{\partial z}{\partial y}, \, -1\right)$$

$$ec{n}_t = \left(3x^2, \, 3y^2, \, -1
ight)$$
 $ec{n}_\Sigma = \left(1, 1, 51
ight)$

$$\vec{n}_{\Sigma}=(1,1,51)$$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) = 0$$

$$3x^{2} + 3y^{2} - 51 = 0$$

$$x = 1$$

a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\Sigma \ldots x + y + 51z = 0$$

$$\Pi_t \perp \Sigma \Leftrightarrow \vec{n}_t \cdot \vec{n}_{\Sigma} = 0$$

$$\vec{n_t} = \left(\frac{\partial z}{\partial x}, \, \frac{\partial z}{\partial y}, \, -1\right)$$

$$ec{n}_t = \left(3x^2, \, 3y^2, \, -1
ight)$$
 $ec{n}_\Sigma = \left(1, 1, 51
ight)$

$$\vec{n}_{\Sigma}=(1,1,51)$$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) \stackrel{\checkmark}{=} 0$$

$$-51 = 0$$

$$3x^2 + 3y^2 - 51 = 0$$

$$x = 1$$

$$3y^2 = 48$$

a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\Sigma \ldots x + y + 51z = 0$$

$$\Pi_t \perp \Sigma \Leftrightarrow \vec{n}_t \cdot \vec{n}_{\Sigma} = 0$$

$$\vec{n_t} = \left(\frac{\partial z}{\partial x}, \, \frac{\partial z}{\partial y}, \, -1\right)$$

$$\vec{n}_t = (3x^2, 3y^2, -1)$$
 $\vec{n}_{\Sigma} = (1, 1, 51)$

$$\vec{n}_{\Sigma}=(1,1,51)$$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) = 0$$

$$-51 = 0$$

$$3x^{2} + 3y^{2} - 51 = 0$$

$$x = 1$$

$$3y^{2} = 48 \longrightarrow y^{2} = 16$$

$$\longrightarrow y^2 = 16$$

a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\sum \ldots x + y + 51z = 0$$

$$\Pi_t \perp \Sigma \Leftrightarrow \vec{n}_t \cdot \vec{n}_{\Sigma} = 0$$

$$\vec{n}_t = \left(\frac{\partial z}{\partial x}, \, \frac{\partial z}{\partial y}, \, -1\right)$$

$$\vec{n}_t = (3x^2, 3y^2, -1)$$
 $\vec{n}_{\Sigma} = (1, 1, 51)$

$$n_{\Sigma}=(1,1,51)$$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) \stackrel{\checkmark}{=} 0$$

$$-51 = 0$$

$$3x^{2} + 3y^{2} - 51 = 0$$

$$x = 1$$

$$3y^{2} = 48 \implies y^{2} = 16$$

$$y_1 = 4$$

a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\Sigma \dots x + y + 51z = 0$$

$$\Pi_t \perp \Sigma \Leftrightarrow \vec{n_t} \cdot \vec{n_{\Sigma}} = 0$$

$$\vec{n_t} = \left(\frac{\partial z}{\partial x}, \, \frac{\partial z}{\partial y}, \, -1\right)$$

$$ec{n}_t = \left(3x^2, \, 3y^2, \, -1
ight)$$
 $ec{n}_{\Sigma} = \left(1, 1, 51
ight)$

$$n_{\Sigma} = (1, 1, 51)$$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) = 0$$

$$3x^2 + 3y^2 - 51 = 0$$

$$x = 1$$

$$3y^2 = 48 \longrightarrow y^2 = 16$$

$$v_1 = 4$$
, $v_2 = -4$

a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\Sigma \dots x + y + 51z = 0$$

$$\Pi_t \perp \Sigma \Leftrightarrow \vec{n_t} \cdot \vec{n_{\Sigma}} = 0$$

$$\vec{n_t} = \left(\frac{\partial z}{\partial x}, \, \frac{\partial z}{\partial y}, \, -1\right)$$

$$ec{n}_t = \left(3x^2, \, 3y^2, \, -1
ight)$$
 $ec{n}_{\Sigma} = \left(1, 1, 51
ight)$

$$n_{\Sigma}=(1,1,51)$$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) \stackrel{\checkmark}{=} 0$$

$$3x^2 + 3y^2 - 51 = 0$$

$$x = 1$$
 $3y^2 = 48 - y^2 = 16$

$$y_1 = 4$$
, $y_2 = -4$, $z_1 =$

a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\sum \ldots x + y + 51z = 0$$

$$\Pi_t \perp \Sigma \Leftrightarrow \vec{n}_t \cdot \vec{n}_{\Sigma} = 0$$

$$\vec{n}_t = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1\right)$$

$$ec{n}_t = \left(3x^2, \, 3y^2, \, -1
ight)$$
 $ec{n}_{\Sigma} = \left(1, 1, 51
ight)$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) = 0$$

$$1,31) = 0$$

$$3x^2 + 3y^2 - 51 = 0$$

$$x = 1$$
 $3y^2 = 48 - y^2 = 16$

$$y_1 = 4$$
, $y_2 = -4$, $z_1 = 1^3 + 4^3$

a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\sum \dots x + y + 51z = 0$$

$$\Pi_t \perp \Sigma \Leftrightarrow \boxed{\vec{n}_t \cdot \vec{n}_{\Sigma} = 0} - \eta_t$$

$$\vec{n_t} = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1\right)$$

$$ec{n}_t = \left(3x^2, \, 3y^2, \, -1
ight)$$
 $ec{n}_{\Sigma} = \left(1, 1, 51
ight)$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) = 0$$

$$3x^2 + 3y^2 - 51 = 0$$

$$x = 1$$
 $3y^2 = 48 - y^2 = 16$

$$y_1 = 4$$
, $y_2 = -4$, $z_1 = 1^3 + 4^3 = 65$

a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\Sigma \dots x + y + 51z = 0$$

$$\Pi_t \perp \Sigma \Leftrightarrow \vec{n_t} \cdot \vec{n_\Sigma} = 0$$

$$\vec{n_t} = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1\right)$$

$$\vec{z} = \left(2x^2, 2x^2, -1\right)$$

$$ec{n}_t = \left(3x^2, \, 3y^2, \, -1
ight)$$
 $ec{n}_{\Sigma} = \left(1, 1, 51
ight)$

$$n_{\Sigma} = (1, 1, 51)$$

 $(3x^2, 3y^2, -1) \cdot (1, 1, 51) = 0$

$$3x^2 + 3y^2 - 51 = 0$$

$$x = 1$$
 $3y^2 = 48 - y^2 = 16$

$$y_1 = 4$$
, $y_2 = -4$, $z_1 = 1^3 + 4^3 = 65$, $z_2 = 1^3 + 4^3 = 65$

a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\Sigma \dots x + y + 51z = 0$$

$$\Pi_t \perp \Sigma \Leftrightarrow \vec{n_t} \cdot \vec{n_\Sigma} = 0$$

$$ec{n_t} = \left(rac{\partial z}{\partial x}, rac{\partial z}{\partial y}, -1
ight)$$
 $ec{n_t} = \left(3x^2, 3y^2, -1
ight)$
 $ec{n_{\Sigma}} = \left(1, 1, 51
ight)$

$$\vec{n}_{\Sigma} = (1, 1, 51)$$

$$ec{\eta}_{\Sigma} = (1,1,51)$$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) = 0$$

$$3x^2 + 3y^2 - 51 = 0$$

$$\Rightarrow y^2 = 1$$

$$x = 1$$
 $3y^2 = 48 \longrightarrow y^2 = 16$
 $y_1 = 4$, $y_2 = -4$, $z_1 = 1^3 + 4^3 = 65$, $z_2 = 1^3 + (-4)^3$

a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\sum \dots x + y + 51z = 0$$

$$\Pi_t \perp \Sigma \Leftrightarrow \vec{n_t} \cdot \vec{n_\Sigma} = 0$$

$$ec{n_t} = \left(rac{\partial z}{\partial x}, rac{\partial z}{\partial y}, -1
ight)$$
 $ec{n_t} = \left(3x^2, 3y^2, -1
ight)$
 $ec{n_{\Sigma}} = \left(1, 1, 51
ight)$

$$3x^2 + 3v^2 - 51 = 0$$

$$x = 1$$
 $3y^2 = 48 - y^2 = 16$

 $(3x^2, 3y^2, -1) \cdot (1, 1, 51) = 0$

$$y_1 = 4$$
, $y_2 = -4$, $z_1 = 1^3 + 4^3 = 65$, $z_2 = 1^3 + (-4)^3 = -63$

a) $z = x^3 + y^3$, T(1, y, z)

$$\frac{\sum \dots x + y + 51z = 0}{\prod_{t} \perp \sum \Leftrightarrow \boxed{\vec{n}_{t} \cdot \vec{n}_{\Sigma} = 0}}$$

$$\vec{n}_{t} = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1\right)$$

$$(3x^2, 3y^2, -$$

$$(1, 1, 31)$$
 $(1, 1, 51)$

$$3y^2$$
, -1) · $(1, 1, 51) = 0$
 $3x^2 + 3y^2 - 51 = 0$

$$ec{n}_t = \left(3x^2, \, 3y^2, \, -1
ight)$$
 $ec{n}_{\Sigma} = \left(1, 1, 51
ight)$

$$\begin{pmatrix}
-1 \\
1 \\
1 \\
1
\end{pmatrix} = 0$$

$$(1,51) = 0$$

 $51 = 0$

 $T_1(1, 4, 65)$

$$3x^{2} + 3y^{2} - 51 = 0$$

$$x = 1$$

$$3y^{2} = 48 - y^{2} = 16$$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) \stackrel{?}{=} 0$$

 $3x^2 + 3y^2 - 51 = 0$

$$(51) = 0$$
 $(51) = 0$

$$0 = 0$$

$$y_1 = 4$$
, $y_2 = -4$, $z_1 = 1^3 + 4^3 = 65$, $z_2 = 1^3 + (-4)^3 = -63$

a) $z = x^3 + y^3$, T(1, y, z)

$$\frac{\sum \dots x + y + 51z = 0}{\prod_{t} \perp \sum \Leftrightarrow \boxed{\vec{n}_{t} \cdot \vec{n}_{\Sigma} = 0}}$$

$$\vec{n}_{t} = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1\right)$$

$$ec{n}_t = \left(3x^2, \, 3y^2, \, -1
ight)$$
 $ec{n}_{\Sigma} = \left(1, 1, 51
ight)$

$$=(1,1,51)$$

$$-1$$
) · (1, 1,

$$-, -1$$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) \stackrel{?}{=} 0$$

$$-1$$
) · (1, 1, 5

$$3x^{2}, 3y^{2}, -1) \cdot (1, 1, 51) = 0$$

$$3x^{2} + 3y^{2} - 51 = 0$$

$$x = 1$$

$$3y^{2} = 48 \longrightarrow y^{2} = 16$$

$$(-1) \cdot (1, 1, 51)$$

$$, -1)$$

$$, -1)$$

$$-1$$
)

$$-1$$
)

$$-1$$
) $\frac{1}{2}$

 $T_1(1,4,65)$

$$-1$$
)

 $y_1 = 4$, $y_2 = -4$, $z_1 = 1^3 + 4^3 = 65$, $z_2 = 1^3 + (-4)^3 = -63$

a) $z = x^3 + y^3$, T(1, y, z)

$$\begin{array}{c} \Sigma \ldots x + y + 51z = 0 \\ \hline \Pi_t \perp \Sigma \iff \begin{array}{c} \vec{n_t} \cdot \vec{n_{\Sigma}} = 0 \end{array} \end{array}$$

$$\vec{n_t} = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1 \right)$$

$$ec{n}_t = \left(3x^2, 3y^2, -1\right)$$
 $ec{n}_{\Sigma} = \left(1, 1, 51\right)$

$$\frac{1}{1} = (1)$$

$$(-1)$$

$$-1)$$

$$3y^2$$
, -1) · $(1, 1, 51)$
 $3x^2 + 3y^2 - 51 = 0$

x = 1 $3v^2 = 48 \longrightarrow v^2 = 16$

$$\vec{n}_{\Sigma} = (1, 1, 51)$$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) = 0$$

$$3y^2, -1$$

$$3y^2, -1$$

$$(2, -1)$$

$$\begin{pmatrix} -1 \end{pmatrix}$$

 $y_1 = 4$, $y_2 = -4$, $z_1 = 1^3 + 4^3 = 65$, $z_2 = 1^3 + (-4)^3 = -63$

 $T_1(1,4,65)$ $T_2(1,-4,-63)$

$$-1$$

$$-1$$

$$-1$$

$$-1$$

$$-1$$

$$-1$$

Riešenje

a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$egin{aligned} \Sigma \dots x + y + 51z &= 0 \ \hline \Pi_t \perp \Sigma &\Leftrightarrow egin{aligned} ec{n}_t \cdot ec{n}_\Sigma &= 0 \end{aligned}$$

$$\vec{n}_t = (3x^2, 3y^2, -1)$$
 $\vec{n}_{\Sigma} = (1, 1, 51)$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) \stackrel{\checkmark}{=} 0$$

$$(1)^{2}$$

$$+3y$$

$$3x^{2} + 3y^{2} - 51 = 0$$

$$x = 1$$

$$3y^{2} = 48 \implies y^{2} = 16$$

$$3x^2 + 3y^2 - 51 = 0$$

 $y_1 = 4$, $y_2 = -4$, $z_1 = 1^3 + 4^3 = 65$, $z_2 = 1^3 + (-4)^3 = -63$

 $T_2(1, -4, -63)$

 $T_1(1,4,65)$

$$3y^2$$
,

$$\overline{\partial y}$$
,

$$\frac{\partial z}{\partial v}$$
, -

Rješenje
a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\frac{\sum \dots x + y + 51z = 0}{\prod_t \perp \sum \Leftrightarrow \overrightarrow{n_t} \cdot \overrightarrow{n_\Sigma} = 0}$$

$$\overrightarrow{n_t} = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1\right)$$

$$\overrightarrow{n_t} = (3x^2, 3y^2, -1)$$

$$\overrightarrow{n_{\Sigma}} = (1, 1, 51)$$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) = 0$$

$$3x^2 + 3y^2 - 51 = 0$$

$$x = 1$$

$$3y^2 = 48 \implies y^2 = 16$$

$$y_1 = 4, \quad y_2 = -4, \quad z_1 = 1^3 + 4^3 = 65, \quad z_2 = 1^3 + (-4)^3 = -63$$

$$T_1(1, 4, 65)$$

$$T_2(1, -4, -63)$$

Rješenje
a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\frac{\sum \dots x + y + 51z = 0}{\prod_t \perp \sum \Leftrightarrow \overrightarrow{n_t} \cdot \overrightarrow{n_\Sigma} = 0}$$
 $\overrightarrow{n_t} = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1\right)$
 $\overrightarrow{n_t} = (3x^2, 3y^2, -1)$
 $\overrightarrow{n_\tau} = (1, 1, 51)$
 $(3x^2, 3y^2, -1) \cdot (1, 1, 51) = 0$
 $3x^2 + 3y^2 - 51 = 0$
 $x = 1$
 $3y^2 = 48 \implies y^2 = 16$
 $y_1 = 4$, $y_2 = -4$, $z_1 = 1^3 + 4^3 = 65$, $z_2 = 1^3 + (-4)^3 = -63$

$$\boxed{T_1(1, 4, 65)}$$

Rješenje
a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\underline{\Sigma \dots x + y + 51z = 0}$$
 $\vec{n}_t \perp \Sigma \Leftrightarrow \vec{n}_t \cdot \vec{n}_\Sigma = 0$
 $\vec{n}_t = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1\right)$
 $\vec{n}_t = (3x^2, 3y^2, -1)$
 $\vec{n}_\Sigma = (1, 1, 51)$
 $(3x^2, 3y^2, -1) \cdot (1, 1, 51) = 0$
 $3x^2 + 3y^2 - 51 = 0$
 $x = 1$
 $3y^2 = 48 \implies y^2 = 16$
 $y_1 = 4, \quad y_2 = -4, \quad z_1 = 1^3 + 4^3 = 65, \quad z_2 = 1^3 + (-4)^3 = -63$
 $T_1(1, 4, 65)$

Rješenje
a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\sum \dots x + y + 51z = 0$$

$$\vec{\Pi}_t \perp \Sigma \Leftrightarrow \vec{n}_t \cdot \vec{n}_\Sigma = 0$$

$$\vec{n}_t = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1\right)$$

$$\vec{n}_t = (3x^2, 3y^2, -1)$$

$$\vec{n}_\Sigma = (1, 1, 51)$$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) = 0$$

$$3x^2 + 3y^2 - 51 = 0$$

$$x = 1$$

$$3y^2 = 48 \implies y^2 = 16$$

$$y_1 = 4, \quad y_2 = -4, \quad z_1 = 1^3 + 4^3 = 65, \quad z_2 = 1^3 + (-4)^3 = -63$$

$$T_1(1, 4, 65)$$

$$T_2(1, -4, -63)$$

$$T_2(1, -4, -63)$$

Rješenje
a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\sum \dots x + y + 51z = 0$$

$$\vec{\Pi}_t \perp \Sigma \Leftrightarrow \vec{n}_t \cdot \vec{n}_\Sigma = 0$$

$$\vec{n}_t = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1\right)$$

$$\vec{n}_t = (3x^2, 3y^2, -1)$$

$$\vec{n}_\Sigma = (1, 1, 51)$$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) = 0$$

$$3x^2 + 3y^2 - 51 = 0$$

$$x = 1$$

$$3y^2 = 48 \implies y^2 = 16$$

$$y_1 = 4, \quad y_2 = -4, \quad z_1 = 1^3 + 4^3 = 65, \quad z_2 = 1^3 + (-4)^3 = -63$$

$$T_1(1, 4, 65)$$

$$T_2(1, -4, -63)$$

$$T_2(1, -4, -63)$$

Rješenje
a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\sum ... x + y + 51z = 0$$

$$\vec{n}_t \perp \Sigma \Leftrightarrow \vec{n}_t \cdot \vec{n}_\Sigma = 0$$

$$\vec{n}_t = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1\right)$$

$$\vec{n}_t = (3x^2, 3y^2, -1)$$

$$\vec{n}_\Sigma = (1, 1, 51)$$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) = 0$$

$$3x^2 + 3y^2 - 51 = 0$$

$$x = 1$$

$$3y^2 = 48 \implies y^2 = 16$$

$$y_1 = 4, \quad y_2 = -4, \quad z_1 = 1^3 + 4^3 = 65, \quad z_2 = 1^3 + (-4)^3 = -63$$

$$T_1(1, 4, 65)$$

$$T_2(1, -4, -63)$$

Rješenje
a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\sum \dots x + y + 51z = 0$$

$$\overrightarrow{\Pi}_t \perp \Sigma \Leftrightarrow \overrightarrow{\overrightarrow{n}_t} \cdot \overrightarrow{\overrightarrow{n}_\Sigma} = 0$$

$$\overrightarrow{\vec{n}_t} = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1\right)$$

$$\overrightarrow{\vec{n}_t} = (3x^2, 3y^2, -1)$$

$$\overrightarrow{\vec{n}_t} = (3x^2, 3y^2, -1)$$

$$\overrightarrow{\vec{n}_t} = (1, 1, 51)$$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) = 0$$

$$3x^2 + 3y^2 - 51 = 0$$

$$x = 1$$

$$3y^2 = 48 \longrightarrow y^2 = 16$$

$$y_1 = 4, \quad y_2 = -4, \quad z_1 = 1^3 + 4^3 = 65, \quad z_2 = 1^3 + (-4)^3 = -63$$

$$T_1(1, 4, 65)$$

$$\overrightarrow{T_2(1, -4, -63)}$$

$$T_2(1, -4, -63)$$

Rješenje
a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\sum \dots x + y + 51z = 0$$

$$\overrightarrow{\Pi}_t \perp \Sigma \Leftrightarrow \overrightarrow{\overrightarrow{n}_t} \cdot \overrightarrow{\overrightarrow{n}_\Sigma} = 0$$

$$\overrightarrow{\vec{n}_t} = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1\right)$$

$$\overrightarrow{\vec{n}_t} = (3x^2, 3y^2, -1)$$

$$\overrightarrow{\vec{n}_t} = (1, 1, 51)$$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) = 0$$

$$3x^2 + 3y^2 - 51 = 0$$

$$x = 1$$

$$3y^2 = 48 \implies y^2 = 16$$

$$y_1 = 4, \quad y_2 = -4, \quad z_1 = 1^3 + 4^3 = 65, \quad z_2 = 1^3 + (-4)^3 = -63$$

$$T_2(1, -4, -63)$$

$$T_2(1, -4, -63)$$

$$T_2(1, -4, -63)$$

Rješenje
a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\sum \dots x + y + 51z = 0$$

$$\overrightarrow{\Pi}_t \perp \Sigma \Leftrightarrow \overrightarrow{\overrightarrow{n}_t} \cdot \overrightarrow{\overrightarrow{n}_\Sigma} = 0$$

$$\overrightarrow{\vec{n}_t} = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1\right)$$

$$\overrightarrow{\vec{n}_t} = (3x^2, 3y^2, -1)$$

$$\overrightarrow{\vec{n}_t} = (3x^2, 3y^2, -1)$$

$$\overrightarrow{\vec{n}_t} = (1, 1, 51)$$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) = 0$$

$$3x^2 + 3y^2 - 51 = 0$$

$$x = 1$$

$$3y^2 = 48 \implies y^2 = 16$$

$$y_1 = 4, \quad y_2 = -4, \quad z_1 = 1^3 + 4^3 = 65, \quad z_2 = 1^3 + (-4)^3 = -63$$

$$T_1(1, 4, 65)$$
b) $x_0 y_0 z_0 \\
T_1(1, 4, 65)$

$$\overrightarrow{\vec{n}_{t_1}} = (3, 48, -1)$$

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

$$3(x - 1) + 48(y - 4) - 1 \cdot (z - 65) = 0$$

$$\overline{\vec{n}_1} \dots 3x + 48y - z - 130 = 0$$

$$\overline{\vec{n}_1} \dots 3x + 48y - z - 130 = 0$$

$$\overline{\vec{n}_1} \dots 3x + 48y - z - 130 = 0$$

$$\overline{\vec{n}_1} \dots 3x + 48y - z - 130 = 0$$

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$$\overline{\vec{n}_1} \dots 3x + 48y - z - 130 = 0$$

$$\overline{\vec{n}_1} \dots 3x + 48y - z - 130 = 0$$

$$\overline{$$

Rješenje
a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\sum \dots x + y + 51z = 0$$

$$\overrightarrow{\Pi}_t \perp \Sigma \Leftrightarrow \overrightarrow{\overrightarrow{n}_t} \cdot \overrightarrow{\overrightarrow{n}_\Sigma} = 0$$

$$\overrightarrow{n}_t = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1\right)$$

$$\overrightarrow{n}_t = (3x^2, 3y^2, -1)$$

$$\overrightarrow{n}_\Sigma = (1, 1, 51)$$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) = 0$$

$$3x^2 + 3y^2 - 51 = 0$$

$$x = 1$$

$$3y^2 = 48 \implies y^2 = 16$$

$$y_1 = 4, \quad y_2 = -4, \quad z_1 = 1^3 + 4^3 = 65, \quad z_2 = 1^3 + (-4)^3 = -63$$

$$T_2(1, -4, -63)$$
b) $x_0 y_0 z_0 \\ \overrightarrow{T}_{1}(1, 4, 65)$

$$\overrightarrow{n}_{t_1} = (3, 48, -1)$$

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

$$3(x - 1) + 48(y - 4) - 1 \cdot (z - 65) = 0$$

$$n_1 \dots 3x + 48y - z - 130 = 0$$

$$n_1 \dots 3x = \frac{1}{48} = \frac{1}{-1}$$

Rješenje
a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\sum \dots x + y + 51z = 0$$

$$\overrightarrow{\Pi}_t \perp \Sigma \Leftrightarrow \overrightarrow{\overrightarrow{n}_t} \cdot \overrightarrow{\overrightarrow{n}_\Sigma} = 0$$

$$\overrightarrow{n}_t = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1\right)$$

$$\overrightarrow{n}_t = (3x^2, 3y^2, -1)$$

$$\overrightarrow{n}_\Sigma = (1, 1, 51)$$

$$(3x^2, 3y^2, -1) \cdot (1, 1, 51) = 0$$

$$3x^2 + 3y^2 - 51 = 0$$

$$x = 1$$

$$3y^2 = 48 \implies y^2 = 16$$

$$y_1 = 4, \quad y_2 = -4, \quad z_1 = 1^3 + 4^3 = 65, \quad z_2 = 1^3 + (-4)^3 = -63$$

$$T_2(1, -4, -63)$$
b) $x_0 y_0 z_0 \\ \overrightarrow{T}_{1}(1, 4, 65)$

$$\overrightarrow{n}_{t_1} = (3, 48, -1)$$

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$$n_1 \dots \frac{x-1}{3} = \frac{1}{48} = \frac{1}{-1}$$

Rješenje
a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\sum \dots x + y + 51z = 0$$

$$\overrightarrow{\Pi}_t \perp \Sigma \Leftrightarrow \overrightarrow{\overrightarrow{n}_t} \cdot \overrightarrow{\overrightarrow{n}_\Sigma} = 0$$

$$\overrightarrow{n}_t = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1\right)$$

$$\overrightarrow{n}_t = (3x^2, 3y^2, -1)$$

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Rješenje
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$$\overrightarrow{T_2(1, -4, -63)} \qquad \overrightarrow{T_2(1, -4, -63)}$$

Rješenje
a)
$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\sum_{\Sigma \dots x + y + 51z = 0} T_1(1, 4, 65) \qquad \vec{n}_{t_1} = (3, 48, -1)$$

$$\vec{n}_t \perp \Sigma \Leftrightarrow \vec{n}_t \cdot \vec{n}_{\Sigma} = 0$$

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$$x = 1$$

$$3y^2 = 48 \implies y^2 = 16$$
b) $x_0 y_0 z_0 \qquad A B C$

$$T_2(1, -4, -63) \qquad \vec{n}_{t_2} = (3, 48, -1)$$

$$T_2(1, -4, -63) \qquad \vec{n}_{t_2} = (3, 48, -1)$$

$$3(x - 1) + 48(y - 4) - 1 \cdot (z - 65) = 0$$

$$T_2(1, -4, -63) \qquad \vec{n}_{t_2} = (3, 48, -1)$$

$$3(x - 1) + 48(y + 4) - 1 \cdot (z + 63) = 0$$

$$T_2 \dots 3x + 48y - z + 126 = 0$$

$$T_2 \dots 3x + 48y - z + 126 = 0$$

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$$T_2 \dots 3x + 48y - z + 126 = 0$$

$$T_2 \dots 3x + 48y - z$$

Rješenje
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$$z = x^3 + y^3$$
, $T(1, y, z)$

$$\sum \ldots x + y + 51z = 0$$

$$\overline{\Pi_t \perp \Sigma} \Leftrightarrow \overline{n_t \cdot n_\Sigma} = 0$$

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b) $X_0 y_0 z_0$

$$T_{1}(1, 4, 65)$$

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$$T_{2}(1, -4, -63)$$

$$\overline{n_{t_2}} = (3, 48, -1)$$

$$x - 1 - 3x + 48y - z - 130 = 0$$

$$T_{2}(1, -4, -63)$$

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$$T_{3}(1, -4, -63)$$

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$$x - 1 - 3x + 48y - z - 130 = 0$$

$$T_{2}(1, -4, -63)$$

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$$T_{3} = \frac{48}{48} = \frac{-1}{-1}$$

$$T_{2}(1, -4, -63)$$

Rješenje
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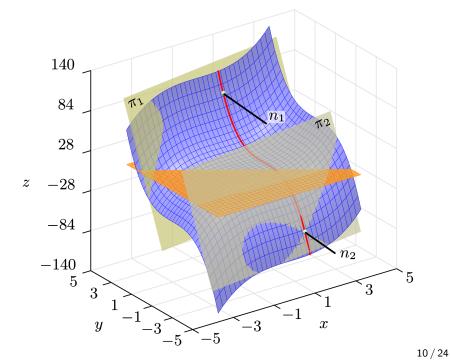
$$T_1(1, 4, 65)$$

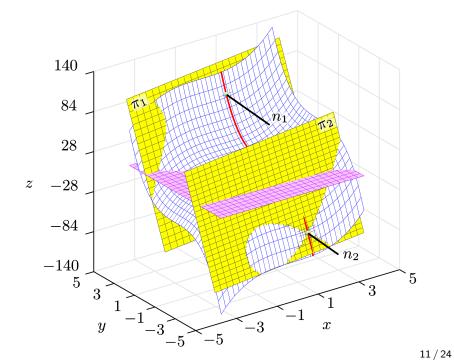
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$$T_1(1, 4, 65)$$

$$T_2(1, 4, 65)$$





treći zadatak

Zadatak 3

Zadana je ploha $x^2z + y^2z = 9$ i pravac

$$p \dots \frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1}.$$

Odredite jednadžbe tangencijalnih ravnina i normala na zadanu plohu u točkama u kojima zadani pravac siječe tu plohu.

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1}$$

$$x^2z + y^2z = 9$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^2z + y^2z = 9$$

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$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^2z + y^2z = 9$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x = t + 1$$

$$x^2z + y^2z = 9$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \end{cases}$$

$$x^2z + y^2z = 9$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^2z + y^2z = 9 \longleftarrow$$

$$-\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^2z + y^2z = 9 \leftrightarrow (t+1)^2$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^2z + y^2z = 9 \longleftarrow (t+1)^2(t-1)$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^2z + y^2z = 9 \leftarrow (t+1)^2(t-1) +$$

$$\begin{cases} x = t+1 \\ y = t-2 \\ z = t-1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9 \leftarrow (t+1)^{2}(t-1) + (t-2)^{2}$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9 \leftarrow (t+1)^{2}(t-1) + (t-2)^{2}(t-1)$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9$$
 $(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9$$
 $(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$
 $(t-1)($

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^2z + y^2z = 9$$
 $(t+1)^2(t-1) + (t-2)^2(t-1) = 9$
 $(t-1)((t+1)^2)$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^2z + y^2z = 9$$
 $(t+1)^2(t-1) + (t-2)^2(t-1) = 9$
 $(t-1)((t+1)^2 +)$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9 \leftarrow (t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$
$$(t-1)((t+1)^{2} + (t-2)^{2})$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9 \leftarrow (t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$
$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9 \leftarrow (t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$
$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$
$$(t-1)($$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9 \leftarrow (t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$
$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$
$$(t-1)(t^{2}$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9 \leftarrow (t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$
$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$
$$(t-1)(t^{2} + 2t)$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9$$

$$(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$

$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

$$(t-1)(t^{2} + 2t + 1)$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9$$

$$(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$

$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

$$(t-1)(t^{2} + 2t + 1 + t^{2})$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9$$

$$(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$

$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

 $(t-1)(t^2+2t+1+t^2-4t)$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9$$

$$(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$

$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

$$(t-1)(t^{2} + 2t + 1 + t^{2} - 4t + 4)$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9$$

$$(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$

$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

$$(t-1)(t^{2} + 2t + 1 + t^{2} - 4t + 4) = 9$$

$$-\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9$$

$$(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$

$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

$$(t-1)(t^{2} + 2t + 1 + t^{2} - 4t + 4) = 9$$

$$(t-1)($$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9$$

$$(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$

$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

$$(t-1)(t^{2} + 2t + 1 + t^{2} - 4t + 4) = 9$$

$$(t-1)(2t^{2}$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9$$

$$(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$

$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

$$(t-1)(t^{2} + 2t + 1 + t^{2} - 4t + 4) = 9$$

$$(t-1)(2t^{2} - 2t)$$

$$-\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9$$

$$(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$

$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

$$(t-1)(t^{2} + 2t + 1 + t^{2} - 4t + 4) = 9$$

$$(t-1)(2t^{2} - 2t + 5)$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9$$

$$(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$

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$$(t-1)(2t^{2} - 2t + 5) = 9$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

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$$(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$

$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

$$(t-1)(t^{2} + 2t + 1 + t^{2} - 4t + 4) = 9$$

$$(t-1)(2t^{2} - 2t + 5) = 9$$

$$2t^{3}$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9$$

$$(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$

$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

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$$(t-1)(2t^{2} - 2t + 5) = 9$$

$$2t^{3} - 2t^{2}$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9$$

$$(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$

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$$(t-1)(t^{2} + 2t + 1 + t^{2} - 4t + 4) = 9$$

$$(t-1)(2t^{2} - 2t + 5) = 9$$

$$2t^{3} - 2t^{2} + 5t$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9$$

$$(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$

$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

$$(t-1)(t^{2} + 2t + 1 + t^{2} - 4t + 4) = 9$$

$$(t-1)(2t^{2} - 2t + 5) = 9$$

$$2t^{3} - 2t^{2} + 5t - 2t^{2}$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9$$

$$(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$

$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

$$(t-1)(t^{2} + 2t + 1 + t^{2} - 4t + 4) = 9$$

$$(t-1)(2t^{2} - 2t + 5) = 9$$

$$2t^{3} - 2t^{2} + 5t - 2t^{2} + 2t$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9$$

$$(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$

$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

$$(t-1)(t^{2} + 2t + 1 + t^{2} - 4t + 4) = 9$$

$$(t-1)(2t^{2} - 2t + 5) = 9$$

$$2t^{3} - 2t^{2} + 5t - 2t^{2} + 2t - 5$$

$$\begin{cases}
x = t + 1 \\
y = t - 2 \\
z = t - 1
\end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9$$

$$(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$

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$$(t-1)(t^{2} + 2t + 1 + t^{2} - 4t + 4) = 9$$

$$(t-1)(2t^{2} - 2t + 5) = 9$$

$$2t^{3} - 2t^{2} + 5t - 2t^{2} + 2t - 5 - 9$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9$$

$$(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$

$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

$$(t-1)(t^{2} + 2t + 1 + t^{2} - 4t + 4) = 9$$

$$(t-1)(2t^{2} - 2t + 5) = 9$$

$$2t^{3} - 2t^{2} + 5t - 2t^{2} + 2t - 5 - 9 = 0$$

$$-\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9$$

$$(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$

$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

$$(t-1)(t^{2} + 2t + 1 + t^{2} - 4t + 4) = 9$$

$$(t-1)(2t^{2} - 2t + 5) = 9$$

$$2t^{3} - 2t^{2} + 5t - 2t^{2} + 2t - 5 - 9 = 0$$

$$2t^{3}$$

$$\begin{cases}
x = t + 1 \\
y = t - 2 \\
z = t - 1
\end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9$$

$$(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$

$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

$$(t-1)(t^{2} + 2t + 1 + t^{2} - 4t + 4) = 9$$

$$(t-1)(2t^{2} - 2t + 5) = 9$$

$$2t^{3} - 2t^{2} + 5t - 2t^{2} + 2t - 5 - 9 = 0$$

$$2t^{3} - 4t^{2}$$

$$-\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9$$

$$(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$

$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

$$(t-1)(t^{2} + 2t + 1 + t^{2} - 4t + 4) = 9$$

$$(t-1)(2t^{2} - 2t + 5) = 9$$

$$2t^{3} - 2t^{2} + 5t - 2t^{2} + 2t - 5 - 9 = 0$$

$$2t^{3} - 4t^{2} + 7t$$

$$-\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9$$

$$(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$

$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

$$(t-1)(t^{2} + 2t + 1 + t^{2} - 4t + 4) = 9$$

$$(t-1)(2t^{2} - 2t + 5) = 9$$

$$2t^{3} - 2t^{2} + 5t - 2t^{2} + 2t - 5 - 9 = 0$$

$$2t^{3} - 4t^{2} + 7t - 14$$

$$-\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9$$

$$(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$

$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

$$(t-1)(t^{2} + 2t + 1 + t^{2} - 4t + 4) = 9$$

$$(t-1)(2t^{2} - 2t + 5) = 9$$

$$2t^{3} - 2t^{2} + 5t - 2t^{2} + 2t - 5 - 9 = 0$$

$$2t^{3} - 4t^{2} + 7t - 14 = 0$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9 \leftarrow (t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$
$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$
$$(t-1)(t^{2} + 2t + 1 + t^{2} - 4t + 4) = 9$$
$$(t-1)(2t^{2} - 2t + 5) = 9$$
$$2t^{3} - 2t^{2} + 5t - 2t^{2} + 2t - 5 - 9 = 0$$
$$2t^{3} - 4t^{2} + 7t - 14 = 0$$
$$1, -1,$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9 \leftarrow (t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$
$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$
$$(t-1)(t^{2} + 2t + 1 + t^{2} - 4t + 4) = 9$$
$$(t-1)(2t^{2} - 2t + 5) = 9$$
$$2t^{3} - 2t^{2} + 5t - 2t^{2} + 2t - 5 - 9 = 0$$
$$2t^{3} - 4t^{2} + 7t - 14 = 0$$
$$1, -1, 2, -2,$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9 \leftarrow (t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$
$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$
$$(t-1)(t^{2} + 2t + 1 + t^{2} - 4t + 4) = 9$$
$$(t-1)(2t^{2} - 2t + 5) = 9$$
$$2t^{3} - 2t^{2} + 5t - 2t^{2} + 2t - 5 - 9 = 0$$
$$2t^{3} - 4t^{2} + 7t - 14 = 0$$
$$1, -1, 2, -2, 7, -7,$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9$$

$$(t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$

$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

$$(t-1)(t^{2} + 2t + 1 + t^{2} - 4t + 4) = 9$$

$$(t-1)(2t^{2} - 2t + 5) = 9$$

$$2t^{3} - 2t^{2} + 5t - 2t^{2} + 2t - 5 - 9 = 0$$

$$2t^{3} - 4t^{2} + 7t - 14 = 0$$

$$1, -1, 2, -2, 7, -7, 14, -14$$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9 \leftarrow (t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$
$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

$$(t-1)^2(t-1) + (t-2)^2(t-1) = 0$$

 $(t-1)((t+1)^2 + (t-2)^2) = 9$

$$(t-1)(t^2+2t+1+t^2-4t+4)=9$$

$$(t-1)(2t^2-2t+5)=9$$

$$2t^3 - 2t^2 + 5t - 2t^2 + 2t - 5 - 9 = 0$$

$$2t^3 - 4t^2 + 7t - 14 = 0$$

$$1, -1, 2, -2, 7, -7, 14, -14$$



$$\begin{cases}
x = t + 1 \\
y = t - 2 \\
z = t - 1
\end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

 $-\begin{cases} x = t+1 \\ y = t-2 \\ z = t-1 \end{cases}$

$$x^{2}z + y^{2}z = 9 \leftarrow (t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$
$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

$$(-2)^2(t-1) = 9$$

$$(t-1)(t^2+2t+1+t^2-4t+4)=9$$

$$(t-1)(2t^2-2t+5)=9$$

$$2t^3 - 2t^2 + 5t - 2t^2 + 2t - 5 - 9 = 0$$

$$2t^3 - 4t^2 + 7t - 14 = 0$$

$$1, -1, 2, -2, 7, -7, 14, -14$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9 \leftarrow (t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9$$
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$$-\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

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$$(t-1)(t-1)+(t-2)(t-1)=$$
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$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

$$(t-1)\big((t+1)^2+(t-2)^2\big)=9$$

$$(t-1)(t^2+2t+1+t^2-4t+4)=9$$
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$$1, -1, 2, -2, 7, -7, 14, -14$$

$$\begin{vmatrix} 2 & -4 & 7 & -14 \\ 2 & 2 & 0 & 7 & 0 \end{vmatrix}$$

$$-\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

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$$1,\ -1,\ 2,\ -2,\ 7,\ -7,\ 14,\ -14$$

$$-\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$(t-2)$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

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$$(t-1)((t+1)^{2} + (t-2)^{2}) = 9$$

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$$1, -1, 2, -2, 7, -7, 14, -14$$

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$$(t-2)\big(2t^2+0\cdot t+7\big)$$

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1, -1, 2, -2, 7, -7, 14, -14

Rješenje presjek pravca i plohe
$$x^2z + y^2z = 9$$

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$(t+1)^2(t-1) + (t-2)^2(t-1) = 9$$

 $(t-1)((t+1)^2 + (t-2)^2) = 9$
 $(t-1)(t^2 + 2t + 1 + t^2 - 4t + 4) = 9$

$$\begin{cases} x = t + 1 \\ y = t - 2 \\ z = t - 1 \end{cases}$$

$$(t-1)(2t^2 - 2t + 5) = 9$$
$$2t^3 - 2t^2 + 5t - 2t^2 + 2t - 5 - 9 = 0$$

$$(t-2)(2t^2-$$

$$2t^3 - 4t^2 + 7t - 14 = 0$$
1, -1, 2, -2, 7, -7, 14, -14

$$(t-2)(2t^2+0\cdot t+7)=0$$
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Rješenje presjek pravca i plohe
$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

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$$(t-1)(t^{2} + 2t + 1 + t^{2} - 4t + 4) = 9$$

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$$2t^{3} - 2t^{2} + 5t - 2t^{2} + 2t - 5 - 9 = 0$$

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$$1, -1, 2, -2, 7, -7, 14, -14$$

$$\begin{vmatrix} 2 & -4 & 7 & -14 \\ 2 & 2 & 0 & 7 & 0 \end{cases}$$

$$t = 2$$

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Rješenje presjek pravca i plohe
$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

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$$1, -1, 2, -2, 7, -7, 14, -14$$

$$|2| -4 |7| -14$$

$$|2| -4 |7| -14$$

$$|2| -4 |7| -14$$

$$|2| -4 |7| -14$$

$$|2| -4 |7| -14$$

$$|2| -4 |7| -14$$

$$|2| -4 |7| -14$$

$$|2| -4 |7| -14$$

$$|2| -4 |7| -14$$

Rješenje presjek pravca i plohe
$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^{2}z + y^{2}z = 9 \leftarrow \begin{cases} (t+1)^{2}(t-1) + (t-2)^{2}(t-1) = 9 \\ (t-1)((t+1)^{2} + (t-2)^{2}) = 9 \end{cases}$$

$$(t-1)(t^{2} + 2t + 1 + t^{2} - 4t + 4) = 9$$

$$(t-1)(2t^{2} - 2t + 5) = 9$$

$$2t^{3} - 2t^{2} + 5t - 2t^{2} + 2t - 5 - 9 = 0$$

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$$1, -1, 2, -2, 7, -7, 14, -14$$

$$\begin{vmatrix} 2 & -4 & 7 & -14 \\ 2 & 2 & 0 & 7 & 0 \end{cases}$$

$$(t-2)(2t^{2} + 0 \cdot t + 7) = 0$$

$$(t-2)(2t^{2} + 7) = 0$$

$$2t^{2} + 7 = 0$$

Rješenje presjek pravca i plohe
$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{1} = t$$

$$x^2z + y^2z = 9 \leftarrow (t+1)^2(t-1) + (t-2)^2(t-1) = 9$$

$$(t-1)((t+1)^2 + (t-2)^2) = 9$$

$$(t-1)(t^2 + 2t + 1 + t^2 - 4t + 4) = 9$$

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$$1, -1, 2, -2, 7, -7, 14, -14$$

$$|2| -4 |7| -14$$

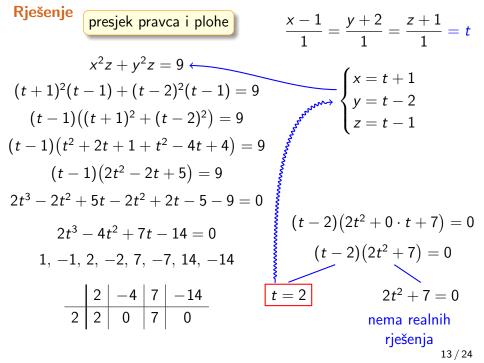
$$|2| -4 |7| -14$$

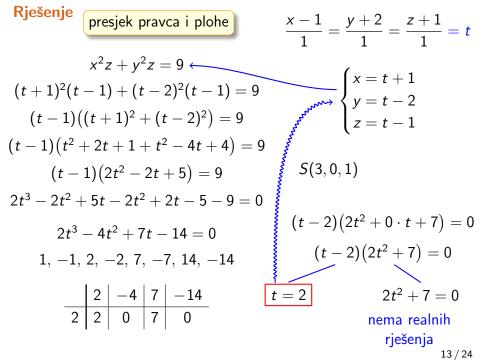
$$|2| -4 |7| -14$$

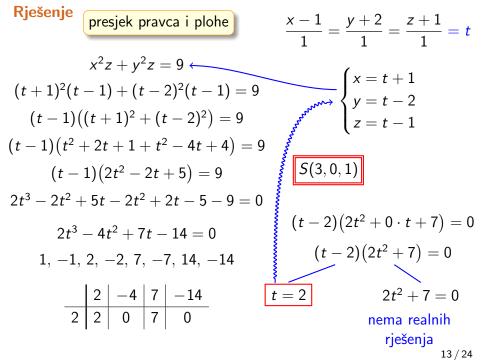
$$|2| 2 |0| 7 |0$$

$$|2t^2 + 7 = 0$$

$$|2t^2$$







 $x^2z + y^2z = 9$

$$x^2z + v^2z = 9$$

$$x^2z + y^2z = 9$$
 $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$

$$x^2z+y$$

$$x^2z + y^2z = 9$$
 $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$

$$x^2z + y^2z = 9$$
 $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$

$$x^2z + y^2z = 9$$
 $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$

 x_0 y_0 z_0

$$x^2z + y^2z - 9 = 0$$
 $x_0 \ y_0 \ z_0$ $S(3, 0, 1)$

$$x^2z + y^2z = 9$$
 $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$
 $x^2z + y^2z - 9 = 0$ $x_0 y_0 z_0$

$$F(x, y, z) = x^2z + y^2z - 9$$

$$x^2z + y^2z = 9$$
 $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$

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 $F(x, y, z) = x^2z + y^2z - 9$
 $x_0 \ y_0 \ z_0$
 $S(3, 0, 1)$

$$F_x =$$

$$x^2z + y^2z = 9$$
 $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$

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 $F(x, y, z) = x^2z + y^2z - 9$
 $x_0 \ y_0 \ z_0$
 $S(3, 0, 1)$

 $F_x = 2xz$

$$x^2z + y^2z = 9$$
 $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$

$$x^2z + y^2z - 9 = 0$$

 $F(x, y, z) = x^2z + y^2z - 9$
 $x_0 \ y_0 \ z_0$
 $S(3, 0, 1)$

$$F_x = 2xz$$
, $F_y =$

$$x^2z + y^2z = 9$$
 $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$

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 $S(3, 0, 1)$

$$F_x = 2xz$$
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 $S(3, 0, 1)$

$$F_x = 2xz$$
, $F_y = 2yz$, $F_z =$

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 $x_0 \ y_0 \ z_0$
 $S(3, 0, 1)$

$$F_x = 2xz$$
, $F_y = 2yz$, $F_z = x^2 + y^2$

$$x^2z + y^2z = 9$$
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$$F_x = 2xz$$
, $F_y = 2yz$, $F_z = x^2 + y^2$ $\vec{n}_t = (F_x, F_y, F_z)$

$$x^2z + y^2z = 9$$
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$$x^2z + y^2z - 9 = 0$$
 $x_0 y_0 z_0$
 $F(x, y, z) = x^2z + y^2z - 9$ $S(3, 0, 1)$

$$F_x = 2xz, \quad F_y = 2yz, \quad F_z = x^2 + y^2$$

 $\vec{n}_t = (F_x, F_y, F_z), \quad \vec{n}_t = ($

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 $\vec{n}_t = (F_x, F_y, F_z), \quad \vec{n}_t = (2xz,$

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 $x_0 \ y_0 \ z_0$
 $S(3, 0, 1)$

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 $F(x, y, z) = x^2z + y^2z - 9$
 $x_0 \ y_0 \ z_0$
 $S(3, 0, 1)$

$$F_x = 2xz, \quad F_y = 2yz, \quad F_z = x^2 + y^2$$
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 $x_0 y_0 z_0$
 $F(x, y, z) = x^2z + y^2z - 9$ $S(3, 0, 1)$

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 $\vec{n_t} = (F_x, F_y, F_z), \quad \vec{n_t} = (2xz, 2yz, x^2 + y^2)$ $\vec{n_t} = (6,$

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 $x_0 \ y_0 \ z_0$
 $S(3, 0, 1)$

$$F_x = 2xz, \quad F_y = 2yz, \quad F_z = x^2 + y^2$$
 $\vec{n}_t = (F_x, F_y, F_z), \quad \vec{n}_t = (2xz, 2yz, x^2 + y^2)$ $\vec{n}_t = (6, 0, 0)$

$$x^2z + y^2z = 9$$
 $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$

$$x^2z + y^2z - 9 = 0$$

 $F(x, y, z) = x^2z + y^2z - 9$
 $x_0 \ y_0 \ z_0$
 $S(3, 0, 1)$

$$F_x = 2xz, \quad F_y = 2yz, \quad F_z = x^2 + y^2$$
 $\vec{n}_t = (F_x, F_y, F_z), \quad \vec{n}_t = (2xz, 2yz, x^2 + y^2)$ $\vec{n}_t = (6, 0, 9)$

$$x^2z + y^2z = 9$$
 $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$

$$x^2z + y^2z - 9 = 0$$

 $F(x, y, z) = x^2z + y^2z - 9$
 $x_0 \ y_0 \ z_0$
 $S(3, 0, 1)$

$$F_x = 2xz$$
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 $\vec{n}_t = (F_x, F_y, F_z)$, $\vec{n}_t = (2xz, 2yz, x^2 + y^2)$
 $\vec{n}_t = (6, 0, 9) = 3 \cdot (2, 0, 3)$

$$x^{2}z + y^{2}z = 9$$

$$x^{2}z + y^{2}z - 9 = 0$$

$$x_{0} y_{0} z_{0}$$

$$S(3, 0, 1)$$

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 $S(3, 0, 1)$
 $F_{x} = 2xz, \quad F_{y} = 2yz, \quad F_{z} = x^{2} + y^{2}$

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 $2 \cdot (x - 3) + 0 \cdot (y - 0) + 3 \cdot (z - 1) = 0$

$$x^2z + y^2z = 9$$
 $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$
 $x^2z + y^2z - 9 = 0$ $x_0 y_0 z_0$

S(3,0,1)

$$F_x = 2xz, \quad F_y = 2yz, \quad F_z = x^2 + y^2$$
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 $F(x, y, z) = x^2z + y^2z - 9$

$$x^{2}z + y^{2}z = 9$$

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$$S(3, 0, 1)$$

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$$\boxed{\Pi_t \dots 2x + 3z - 9 = 0}$$

$$\vec{n}_t \dots \frac{1}{2} = \frac{1}{0} = \frac{1}{3}$$

$$x^2z + y^2z = 9$$
 $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$

$$x^{2}z + y^{2}z - 9 = 0$$

 $F(x, y, z) = x^{2}z + y^{2}z - 9$
 $x_{0} y_{0} z_{0}$
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$$\vec{n}_{t} = (6, 0, 9) = 3 \cdot (2, 0, 3)$$

$$2 \cdot (x - 3) + 0 \cdot (y - 0) + 3 \cdot (z - 1) = 0$$

$$\boxed{\Pi_{t} \dots 2x + 3z - 9 = 0}$$

$$n \dots \frac{x - 3}{2} = \frac{1}{0} = \frac{1}{3}$$

$$x^2z + y^2z = 9$$
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$$n\ldots\frac{x-3}{2}=\frac{y}{0}=\frac{z-1}{3}$$

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 $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$

$$x^{2}z + y^{2}z - 9 = 0$$

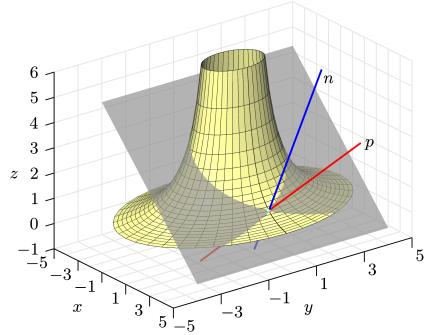
$$x_{0} y_{0} z_{0}$$

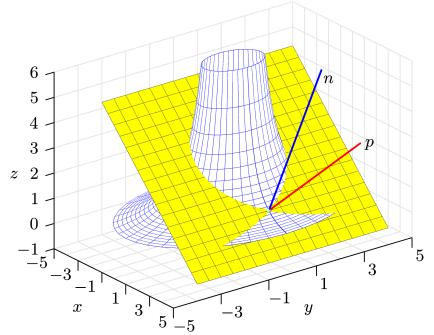
$$5(3, 0, 1)$$

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$$n \dots \frac{x-3}{2} = \frac{y}{0} = \frac{z-1}{3}$$





$$z = f(x, y)$$

$$z=f(x,y)$$
 \leftarrow eksplicitni oblik jednadžbe plohe $f(x,y)-z=0$ \leftarrow implicitni oblik jednadžbe plohe

$$z=f(x,y)$$
 ceksplicitni oblik jednadžbe plohe $f(x,y)-z=0$ complicitni oblik jednadžbe plohe $F(x,y,z)=f(x,y)-z$

 $F_{\star} =$

$$z=f(x,y)$$
 ceksplicitni oblik jednadžbe plohe $f(x,y)-z=0$ complicitni oblik jednadžbe plohe $F(x,y,z)=f(x,y)-z$

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 eksplicitni oblik jednadžbe plohe $f(x,y)-z=0$ emerimental implicitni oblik jednadžbe plohe $F(x,y,z)=f(x,y)-z$ $F_x=f_x,\quad F_y=f_y,\quad F_z=$

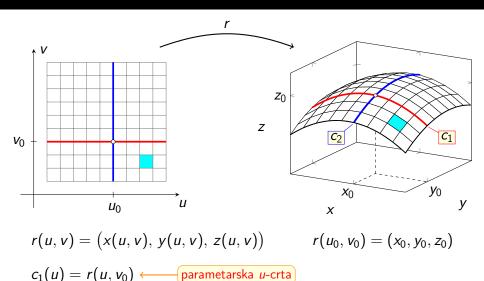
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$$z=f(x,y)$$
 constant eksplicitni oblik jednadžbe plohe $f(x,y)-z=0$ constant implicitni oblik jednadžbe plohe $F(x,y,z)=f(x,y)-z$ $F_x=f_x,\quad F_y=f_y,\quad F_z=-1$ $\vec{n}_t=(f_x,f_y,-1)$

$$z=f(x,y)$$
 eksplicitni oblik jednadžbe plohe $f(x,y)-z=0$ implicitni oblik jednadžbe plohe $F(x,y,z)=f(x,y)-z$ $F_x=f_x, \quad F_y=f_y, \quad F_z=-1$ $\vec{n}_t=(f_x,f_y,-1)$ vektor normale tangencijalne ravnine

četvrti zadatak

Parametrizacija plohe



 $c_2(v) = r(u_0, v) \leftarrow parametarska v-crta$

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Zadatak 4

Zadana je ploha

$$r(u,v) = (\sin u, \sin v, \sin (u+v))$$

i točka A na toj plohi s parametrima $u = \frac{\pi}{3}$, $v = \frac{\pi}{6}$.

- a) Odredite Kartezijeve koordinate točke A.
- b) Odredite dva vektora koji razapinju tangencijalnu ravninu zadane plohe u točki A.
- c) Nađite jednadžbu tangencijalne ravnine plohe u točki A.

Rješenje $A \longrightarrow u = \frac{\pi}{3}, \ v = \frac{\pi}{6}$ $r(u, v) = (\sin u, \sin v, \sin (u + v))$

a)

Rješenje
$$A \longrightarrow u = \frac{\pi}{3}, \ v = \frac{\pi}{6}$$
 $r(u, v) = (\sin u, \sin v, \sin (u + v))$

 $r\left(\frac{\pi}{3},\frac{\pi}{6}\right) =$

Rješenje
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$$u = \frac{\pi}{3}, v = \frac{\pi}{6}$$

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$$r(u, v) = \left(\sin u, \sin v, \sin \left(u + v\right)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \frac{\pi}{6}\right)$$

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$$u = \frac{\pi}{3}, \quad v = \frac{\pi}{6}$$

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Rješenje $A \longrightarrow u = \frac{\pi}{3}, \ v = \frac{\pi}{6}$ $r(u, v) = (\sin u, \sin v, \sin (u + v))$

a) $r\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right)$

Rješenje
$$A \longrightarrow u = \frac{\pi}{3}, \ v = \frac{\pi}{6} \qquad r(u, v) = \left(\sin u, \sin v, \sin \left(u + v\right)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right)$$

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$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{3}\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\left(\frac{\pi}{3} - r\left(\frac{\pi}{3}, \frac{\pi}{6}\right)\right)\right)$$
$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right)$$
$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) =$$

Rješenje
$$A \longrightarrow u = \frac{\pi}{3}, \ v = \frac{\pi}{6} \qquad r(u, v) = (\sin u, \sin v, \sin (u + v))$$

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$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right)$$
$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{\pi}{6}\right)$$

Rješenje
$$A \longrightarrow u = \frac{\pi}{3}, \ v = \frac{\pi}{6} \qquad r(u, v) = \left(\sin u, \sin v, \sin \left(u + v\right)\right)$$

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$$r\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right)$$

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$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{6}\right)$$

 $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, \frac{1}{2}\right)$

Rješenje
$$A \longrightarrow u = \frac{\pi}{3}, \quad v = \frac{\pi}{6} \qquad r(u, v) = \left(\sin u, \sin v, \sin \left(u + v\right)\right)$$

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$$r\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

Rješenje $A \longrightarrow u = \frac{\pi}{3}, \ v = \frac{\pi}{6}$ $r(u, v) = (\sin u, \sin v, \sin (u + v))$

a) $r\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right)$

Rješenje
$$A \longrightarrow u = \frac{\pi}{3}, \ v = \frac{\pi}{6}$$
 $r(u,v) = \left(\sin u, \sin v, \sin \left(u+v\right)\right)$ a) $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right)$ b)

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

Rješenje
$$A \longrightarrow u = \frac{\pi}{3}, \ v = \frac{\pi}{6} \qquad r(u, v) = \left(\sin u, \sin v, \sin \left(u + v\right)\right)$$

$$\left(\frac{u}{x}, \frac{v}{x}\right) - \left(\sin \frac{\pi}{x}, \sin \frac{\pi}{x}, \sin \left(\frac{\pi}{x} + \frac{\pi}{x}\right)\right) = 0$$

a)
$$r\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right) \begin{vmatrix} b \\ r_u = c \end{vmatrix}$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{3}\right) \begin{vmatrix} b \\ r_u = c \end{vmatrix}$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right)$$

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$$A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

a)
$$r\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right) \mid b) r_u = (\cos u, \cos u)$$

$$n\left(\frac{\pi}{3}+\frac{\pi}{6}\right)$$
 $r_u=(\cos u,$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$\frac{1}{2}$$
, 1)

Rješenje
$$A \longrightarrow u = \frac{\pi}{3}, \ v = \frac{\pi}{6} \qquad r(u, v) = (\sin u, \sin v, \sin (u + v))$$

$$\begin{pmatrix} u & v \\ (\pi & \pi) \end{pmatrix} \qquad (\sin \pi, \sin \pi, \sin (\pi + \pi)) \qquad b)$$

a)
$$r\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right) \begin{vmatrix} b \\ r_u = (\cos u, 0, \\ r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right) \end{vmatrix}$$

 $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$ $A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$

Rješenje
$$A \longrightarrow u = \frac{\pi}{3}, \ v = \frac{\pi}{6} \qquad r(u, v) = (\sin u, \sin v, \sin (u + v))$$

$$v = (\sin u, \sin v, \sin (u + v))$$

$$v = (\sin u, \sin v, \sin (u + v))$$

$$v = (\sin u, \sin v, \sin (u + v))$$

a)
$$r\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right) \begin{vmatrix} b \\ r_u = (\cos u, 0, \cos(u + v)) \end{vmatrix}$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

a)
$$r\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right) \begin{vmatrix} b \\ r_u = (\cos u, 0, \cos(u + v)) \end{vmatrix}$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right) \qquad r_v =$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right) \qquad A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

a)
$$r\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right) \begin{vmatrix} b \\ r_u = (\cos u, 0, \cos(u + v)) \end{vmatrix}$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right) \begin{vmatrix} r_v = (0, 0, \cos(u + v)) \end{vmatrix}$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right) \begin{vmatrix} A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right) \end{vmatrix}$$

a)
$$r\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right) \begin{vmatrix} b \\ r_u = (\cos u, 0, \cos(u + v)) \end{vmatrix}$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right) \qquad r_v = (0, \cos v, 0)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right) \qquad A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

a)
$$r\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right)$$
 $r_u = \left(\cos u, 0, \cos\left(u + v\right)\right)$ $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right)$ $r_v = \left(0, \cos v, \cos\left(u + v\right)\right)$

 $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$ $A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$

a)
$$r\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right) \begin{vmatrix} b \\ r_u = (\cos u, 0, \cos(u+v)) \end{vmatrix}$$

 $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right) \begin{vmatrix} r_v = (0, \cos v, \cos(u+v)) \end{vmatrix}$

 $r_u\left(\frac{\pi}{3},\frac{\pi}{6}\right) =$

 $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$ $A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$

a)
$$r\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right) \begin{vmatrix} b \\ r_u = (\cos u, 0, \cos(u+v)) \end{vmatrix}$$
$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right) \begin{vmatrix} r_v = (0, \cos v, \cos(u+v)) \end{vmatrix}$$

 $r\left(\frac{\pi}{3},\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2},\,\frac{1}{2},\,1\right) \quad A\left(\frac{\sqrt{3}}{2},\,\frac{1}{2},\,1\right)$

 $r_u\left(\frac{\pi}{3},\frac{\pi}{6}\right) =$

a)
$$r\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right) \begin{vmatrix} b \\ r_u = (\cos u, 0, \cos(u+v)) \end{vmatrix}$$
$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right) \begin{vmatrix} r_v = (0, \cos v, \cos(u+v)) \end{vmatrix}$$

 $r\left(\frac{\pi}{3},\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2},\,\frac{1}{2},\,1\right) \quad A\left(\frac{\sqrt{3}}{2},\,\frac{1}{2},\,1\right)$

 $r_u\left(\frac{\pi}{3},\frac{\pi}{6}\right)=\left(\frac{1}{2},\right)$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$r_{v} = \left(0, \cos v, \cos\left(u + v\right)\right)$$

$$r_{u}\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\frac{1}{2}, 0, \frac{v}{6}\right)$$

a) $r\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right) \mid b) r_u = \left(\cos u, 0, \cos\left(u + v\right)\right)$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$r_{v} = \left(0, \cos v, \cos\left(u + v\right)\right)$$

$$r_{u}\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\frac{1}{2}, 0, 0\right)$$

a) $r\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right)$

b) $r_u = (\cos u, 0, \cos(u + v))$

Rješenje
a)
$$u = \frac{\pi}{3}, \quad v = \frac{\pi}{6}$$

$$r(u, v) = \left(\sin u, \sin v, \sin (u + v)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right) \begin{vmatrix} b \\ r_u = (\cos u, 0, \cos (u + v)) \end{vmatrix}$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right)$$

$$r\left(\pi, \pi\right) = \left(\sqrt{3}, 1, 1\right)$$

$$\frac{\pi}{3}, \frac{\pi}{6} = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right)$$

$$\frac{\pi}{3}, \frac{\pi}{6} = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$r_u\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\frac{1}{2}, 0, 0\right)$$
 $r_v\left(\frac{\pi}{3}, \frac{\pi}{6}\right) =$

 $r_v = (0, \cos v, \cos (u + v))$

a)
$$r\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right)$$
 $r_u = \left(\cos u, 0, \cos\left(u + v\right)\right)$ $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right)$ $r_v = \left(0, \cos v, \cos\left(u + v\right)\right)$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right) A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right) \begin{cases} r_{u}\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\frac{1}{2}, 0, 0\right) \\ r_{v}\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{1}{2}, \frac{v}{6}\right) = \left(\frac{v}{3}, \frac{v}{6}\right) = \left$$

a)
$$r\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right) \begin{vmatrix} b \\ r_u = (\cos u, 0, \cos(u+v)) \end{vmatrix}$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right) \begin{vmatrix} r_v = (0, \cos v, \cos(u+v)) \end{vmatrix}$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right) \begin{vmatrix} A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right) \end{vmatrix}$$

$$A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right) \begin{vmatrix} r_v = (0, \cos v, \cos(u+v)) \end{vmatrix}$$

$$r_v = \left(\frac{u}{3}, \frac{v}{6}\right) = \left(\frac{1}{2}, 0, 0\right)$$

 $r_{\nu}\left(\frac{u}{3},\frac{v}{6}\right)=\left(0,\right)$

Rješenje A
$$\longrightarrow u = \frac{\pi}{3}, \ v = \frac{\pi}{6}$$
 $r(u,v) = (\sin u, \sin v, \sin (u+v))$

$$r(\frac{\pi}{3}, \frac{\pi}{6}) = (\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin (\frac{\pi}{3} + \frac{\pi}{6})) \begin{vmatrix} b \\ r_u = (\cos u, 0, \cos (u+v)) \end{vmatrix}$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right)$$
$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{3}, \frac{1}{3}, \frac{1}{3}\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right) A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

 $r_v = (0, \cos v, \cos (u + v))$

Rješenje
a)
$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right) \begin{vmatrix} b \\ r_u = (\cos u, 0, \cos(u+v)) \end{vmatrix}$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right)$$

$$r_{v} = \left(0, \cos v, \cos\left(u + v\right)\right)$$

$$r_{v} = \left(0, \cos v, \cos\left(u + v\right)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$r_u\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{1}{2}, 0, 0\right)$$

$$r_v\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(0, \frac{\sqrt{3}}{2}, 0\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$r_{v} = \left(0, \cos v, \cos\left(u + v\right)\right)$$

$$r_{u} \left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{1}{2}, 0, 0\right)$$

$$r_{v} \left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(0, \frac{\sqrt{3}}{2}, 0\right)$$

a) $r\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right)$

b) $r_u = (\cos u, 0, \cos (u + v))$

 $r_v = (0, \cos v, \cos (u + v))$

$$r\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right) \begin{vmatrix} b \\ r_u = (\cos u, 0, \cos(u+v)) \end{vmatrix}$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right) \begin{vmatrix} r_v = (0, \cos v, \cos(u+v)) \end{vmatrix}$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right) \begin{vmatrix} A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right) \end{vmatrix}$$

$$r_u = \left(\cos u, 0, \cos(u+v)\right) \begin{vmatrix} r_v = (0, \cos v, \cos(u+v)) \end{vmatrix}$$

$$r_v = \left(\frac{u}{3}, \frac{v}{6}\right) = \left(\frac{1}{2}, 0, 0\right)$$

c)
$$r_u\left(\frac{\pi}{3}, \frac{\pi}{6}\right) \times r_v\left(\frac{\pi}{3}, \frac{\pi}{6}\right) =$$

 $r\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right)$

$$r_{\nu}\left(\frac{u}{3},\frac{v}{6}\right) = \left(0,\frac{\sqrt{3}}{2},0\right)$$

Rješenje A
$$\longrightarrow u = \frac{\pi}{3}, \ v = \frac{\pi}{6}$$

$$r(u,v) = \left(\sin u, \sin v, \sin \left(u+v\right)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \frac{\pi}{2}\right)$$

$$r_v = \left(0, \cos v, \cos \left(u+v\right)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sin\frac{\pi}{3}}{3}, \frac{\sin\frac{\pi}{6}}{6}, \frac{\sin\frac{\pi}{2}}{2}\right)$$

$$A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$r_u\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\frac{1}{2}, 0, 0\right)$$

$$r_u\left(\frac{u}{3}, \frac{v}{6}\right) = \left(0, \frac{\sqrt{3}}{2}, 0, 0\right)$$

$$\frac{c}{r_{\nu}\left(\frac{\pi}{3},\frac{\pi}{6}\right) \times r_{\nu}\left(\frac{\pi}{3},\frac{\pi}{6}\right) = \left(0,\frac{\sqrt{3}}{2},0\right)}$$

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Rješenje
A
$$\longrightarrow u = \frac{\pi}{3}, \ v = \frac{\pi}{6}$$

$$r(u,v) = \left(\sin u, \sin v, \sin (u+v)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \frac{\pi}{2}\right)$$

$$r_v = \left(0, \cos v, \cos (u+v)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right)$$
$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{3}, \frac{1}{3}, 1\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

c)
$$r_{u}\left(\frac{\pi}{3}, \frac{\pi}{6}\right) \times r_{v}\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ & & \end{vmatrix}$$

 $r_u\left(\frac{u}{3},\frac{v}{6}\right)=\left(\frac{1}{2},0,0\right)$

$$r_{\nu}\left(\frac{u}{3},\frac{v}{6}\right)=\left(0,\frac{\sqrt{3}}{2},0\right)$$

Rješenje A
$$\longrightarrow u = \frac{\pi}{3}, \ v = \frac{\pi}{6}$$

$$r(u,v) = \left(\sin u, \sin v, \sin \left(u+v\right)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \frac{\pi}{2}\right)$$

$$r_v = \left(0, \cos v, \cos \left(u+v\right)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$r_u\left(\frac{\pi}{3}, \frac{\pi}{6}\right) \times r_v\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{1}{2} & 0 & 0 \end{vmatrix}$$

 $r_u\left(\frac{u}{3},\frac{v}{6}\right)=\left(\frac{1}{2},0,0\right)$ $r_{\nu}\left(\frac{\pi}{3},\frac{\pi}{6}\right)=\left(0,\frac{\sqrt{3}}{2},0\right)$

Rješenje
a)
$$A \leadsto u = \frac{\pi}{3}, \quad v = \frac{\pi}{6}$$

$$r(u,v) = \left(\sin u, \sin v, \sin \left(u+v\right)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \frac{\pi}{2}\right)$$

$$r_v = \left(0, \cos v, \cos \left(u+v\right)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$r_u\left(\frac{\pi}{3}, \frac{\pi}{6}\right) \times r_v\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{vmatrix}$$

Rješenje
A
$$\longrightarrow u = \frac{\pi}{3}, \ v = \frac{\pi}{6}$$

$$r(u,v) = \left(\sin u, \sin v, \sin (u+v)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right) \begin{vmatrix} b \\ r_u = (\cos u, 0, \cos (u+v)) \end{vmatrix}$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin\frac{\pi}{3}, \sin\frac{\pi}{6}, \sin\frac{\pi}{2}\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$r_{v} = \left(0, \cos v, \cos\left(u + v\right)\right)$$

$$r_{u}\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\frac{1}{2}, 0, 0\right)$$

$$\begin{array}{c|c}
A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right) \\
\hline
r_{\nu}\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(0, \frac{\sqrt{3}}{2}, 0\right)
\end{array}$$

$$\frac{\mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{6}\right) \times \mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{6}\right) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{vmatrix} = \begin{pmatrix} \mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{6}\right) = \left(0,\frac{\sqrt{3}}{2},0\right) \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{vmatrix} = \begin{pmatrix} \mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{6}\right) = \left(0,\frac{\sqrt{3}}{2},0\right) \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{vmatrix} = \begin{pmatrix} \mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{6}\right) = \left(0,\frac{\sqrt{3}}{2},0\right) \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{vmatrix} = \begin{pmatrix} \mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{6}\right) = \left(0,\frac{\sqrt{3}}{2},0\right) \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{vmatrix} = \begin{pmatrix} \mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{6}\right) = \left(0,\frac{\sqrt{3}}{2},0\right) \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{vmatrix} = \begin{pmatrix} \mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{6}\right) = \left(0,\frac{\sqrt{3}}{2},0\right) \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{pmatrix} = \begin{pmatrix} \mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{6}\right) = \left(0,\frac{\sqrt{3}}{2},0\right) \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{pmatrix} = \begin{pmatrix} \mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{6}\right) = \left(0,\frac{\sqrt{3}}{2},0\right) \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{pmatrix} = \begin{pmatrix} \mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{6}\right) = \left(0,\frac{\sqrt{3}}{2},0\right) \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{pmatrix} = \begin{pmatrix} \mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{6}\right) = \left(0,\frac{\sqrt{3}}{2},0\right) \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{pmatrix} = \begin{pmatrix} \mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{6}\right) = \left(0,\frac{\sqrt{3}}{2},0\right) \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{pmatrix} = \begin{pmatrix} \mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{6}\right) = \left(0,\frac{\sqrt{3}}{2},0\right) \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{pmatrix} = \begin{pmatrix} \mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{6}\right) = \left(0,\frac{\sqrt{3}}{2},0\right) \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{pmatrix} = \begin{pmatrix} \mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{6}\right) = \left(0,\frac{\sqrt{3}}{2},0\right) \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{pmatrix} = \begin{pmatrix} \mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{6}\right) = \left(0,\frac{\sqrt{3}}{2},0\right) \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{pmatrix} = \begin{pmatrix} \mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{6}\right) = \left(0,\frac{\sqrt{3}}{2},0\right) \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{pmatrix} = \begin{pmatrix} \mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{6}\right) = \left(0,\frac{\sqrt{3}}{2},0\right) \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{pmatrix} = \begin{pmatrix} \mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{6}\right) = \left(0,\frac{\sqrt{3}}{2},0\right) \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{pmatrix} = \begin{pmatrix} \mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{6}\right) = \left(0,\frac{\sqrt{3}}{2},0\right) \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{pmatrix} = \begin{pmatrix} \mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{6}\right) = \left(0,\frac{\sqrt{3}}{2},0\right) \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{pmatrix} = \begin{pmatrix} \mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{6}) + \frac{\pi}{3} & \frac{\pi}{3} & \frac{\pi}{3} \\ 0 & \frac{\pi}{3} & \frac{\pi}{3} & \frac{\pi}{3} & \frac{\pi}{3} & \frac{\pi}{3} \end{pmatrix} = \begin{pmatrix} \mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{3}\right) + \frac{\pi}{3} & \frac{\pi}{3}$$

Rješenje
a)
$$u = \frac{\pi}{3}$$
, $v = \frac{\pi}{6}$ $r(u,v) = \left(\sin u, \sin v, \sin \left(u + v\right)\right)$
a) $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right)$ $r_u = \left(\cos u, 0, \cos \left(u + v\right)\right)$
 $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \frac{\pi}{2}\right)$ $r_v = \left(0, \cos v, \cos \left(u + v\right)\right)$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$r_{u}\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\frac{1}{2}, 0, 0\right)$$

$$r_{v}\left(\frac{u}{3}, \frac{v}{6}\right) = \left(0, \frac{\sqrt{3}}{2}, 0\right)$$

$$\begin{array}{c}
\overrightarrow{c} \\
r_{u}\left(\frac{\pi}{3}, \frac{\pi}{6}\right) \times r_{v}\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \begin{vmatrix} \overrightarrow{i} & \overrightarrow{j} & \overrightarrow{k} \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{vmatrix} = \begin{pmatrix} 0, \frac{1}{2}, 0 \end{pmatrix}$$

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Rješenje
a)
$$u = \frac{\pi}{3}$$
, $v = \frac{\pi}{6}$ $r(u, v) = (\sin u, \sin v, \sin (u + v))$
a) $v = (\sin u, \sin v, \sin (u + v))$
 $v = (\sin u, \sin v, \sin (u + v))$
 $v = (\cos u, \cos (u + v))$
 $v = (\cos u, \cos (u + v))$
 $v = (\cos u, \cos (u + v))$

 $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$ $A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$ $r_{u}\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{1}{2}, 0, 0\right)$ $r_{v}\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(0, \frac{\sqrt{3}}{2}, 0\right)$

$$\begin{array}{c}
\overrightarrow{c} \\
r_{u}\left(\frac{\pi}{3}, \frac{\pi}{6}\right) \times r_{v}\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \begin{vmatrix} \overrightarrow{i} & \overrightarrow{j} & \overrightarrow{k} \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{vmatrix} = \begin{pmatrix} 0, \frac{\sqrt{3}}{2}, 0 \end{pmatrix}$$

20 / 24

Rješenje
a)
$$u = \frac{\pi}{3}, \quad v = \frac{\pi}{6}$$

$$r(u, v) = \left(\sin u, \sin v, \sin (u + v)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \frac{\pi}{2}\right)$$

$$r_v = \left(0, \cos v, \cos (u + v)\right)$$

 $r_{v}\left(\frac{u}{3},\frac{v}{6}\right) = \left(0,\frac{\sqrt{3}}{2},0\right)$

$$\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sin \frac{\pi}{3}}{3}, \frac{\sin \frac{\pi}{6}}{5}, \frac{\sin \frac{\pi}{2}}{2}\right)$$

$$\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$r_u\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{1}{2}, 0, 0\right)$$

$$r_v\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(0, \frac{\sqrt{3}}{2}, 0\right)$$

$$\frac{\mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{6}\right)}{\mathbf{r}_{u}\left(\frac{\pi}{3},\frac{\pi}{6}\right)\times\mathbf{r}_{v}\left(\frac{\pi}{3},\frac{\pi}{6}\right)} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{vmatrix} = \begin{pmatrix} 0, 0, \frac{\sqrt{3}}{4} \end{pmatrix}$$

Rješenje
A
$$\longrightarrow u = \frac{\pi}{3}, \ v = \frac{\pi}{6}$$

$$r(u,v) = \left(\sin u, \sin v, \sin(u+v)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \frac{\pi}{2}\right)$$

$$r_v = \left(0, \cos v, \cos(u+v)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$r_{u}\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{1}{2}, 0, 0\right)$$

$$r_{v}\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(0, \frac{\sqrt{3}}{2}, 0\right)$$

 $\Pi_t \dots A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$

 $r_{\nu}\left(\frac{u}{3},\frac{v}{6}\right) = \left(0,\,\frac{\sqrt{3}}{2},\,0\right)$

$$\frac{\vec{r}_{v}\left(\frac{\pi}{3}, \frac{\pi}{6}\right) \times \vec{r}_{v}\left(\frac{\pi}{3}, \frac{\pi}{6}\right)}{\vec{r}_{u}\left(\frac{\pi}{3}, \frac{\pi}{6}\right)} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{vmatrix} = \begin{pmatrix} 0, \frac{\sqrt{3}}{4} \end{pmatrix} = \begin{pmatrix} 0, \frac{\sqrt{3}}{4} \end{pmatrix}$$

Rješenje

A
$$\longrightarrow u = \frac{\pi}{3}, \ v = \frac{\pi}{6}$$

$$r(u,v) = \left(\sin u, \sin v, \sin (u+v)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \frac{\pi}{2}\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{1}{2}, 0, 0\right)$$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right) \quad \begin{array}{c} x_0 & y_0 & z_0 \\ A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right) \\ \hline c) & |\vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 2 \\ \end{array}$$

 $r_{\nu}\left(\frac{u}{3},\frac{v}{6}\right)=\left(0,\frac{\sqrt{3}}{2},0\right)$

c)
$$r_{u}\left(\frac{\pi}{3}, \frac{\pi}{6}\right) \times r_{v}\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{vmatrix} = \left(0, 0, \frac{\sqrt{3}}{4}\right)$$

$$\Pi_t \dots A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$$

Rješenje a)
$$u = \frac{\pi}{3}$$
, $v = \frac{\pi}{6}$ $r(u,v) = \left(\sin u, \sin v, \sin \left(u+v\right)\right)$ a) $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right)$ b) $r_u = \left(\cos u, 0, \cos \left(u+v\right)\right)$ $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \frac{\pi}{2}\right)$ $r_v = \left(0, \cos v, \cos \left(u+v\right)\right)$ $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$ $r_v = \left(\frac{u}{3}, \frac{v}{6}\right) = \left(\frac{1}{2}, 0, 0\right)$

$$r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right) \qquad x_0 \quad y_0 \quad z_0$$

$$A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right) \qquad r_u\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{1}{2}, 0, 0\right)$$

$$r_v\left(\frac{u}{3}, \frac{v}{6}\right) = \left(0, \frac{\sqrt{3}}{2}, 0\right)$$

c)
$$r_{u}\left(\frac{\pi}{3}, \frac{\pi}{6}\right) \times r_{v}\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{vmatrix} = \left(0, 0, \frac{\sqrt{3}}{4}\right) = \frac{\sqrt{3}}{4} \cdot (0, 0, 1)$$

$$r_u\left(\frac{\pi}{3}, \frac{\pi}{6}\right) \times r_v\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \begin{vmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{vmatrix} = \left(0, 0, \frac{\sqrt{3}}{4}\right) = \frac{\sqrt{3}}{4} \cdot (0, 0, 1)$$

 $\Pi_t \dots A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$

Rješenje a)
$$u = \frac{\pi}{3}$$
, $v = \frac{\pi}{6}$ $r(u,v) = \left(\sin u, \sin v, \sin \left(u+v\right)\right)$ a) $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right)$ b) $r_u = \left(\cos u, 0, \cos \left(u+v\right)\right)$ $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \frac{\pi}{2}\right)$ $r_v = \left(0, \cos v, \cos \left(u+v\right)\right)$ $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$ $r_v = \left(\frac{u}{3}, \frac{v}{6}\right) = \left(\frac{1}{2}, 0, 0\right)$

 $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right) \left| \begin{array}{c} x_0 & y_0 & z_0 \\ A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right) \end{array} \right|$ $r_{\nu}\left(\frac{u}{3},\frac{v}{6}\right)=\left(0,\frac{\sqrt{3}}{2},0\right)$

c)
$$r_{u}\left(\frac{\pi}{3}, \frac{\pi}{6}\right) \times r_{v}\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{vmatrix} = \left(0, 0, \frac{\sqrt{3}}{4}\right) = \frac{\sqrt{3}}{4} \cdot \begin{pmatrix} A B C \\ 0, 0, 1 \end{pmatrix}$$

 $\Pi_t \dots A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$

Rješenje

A
$$\longrightarrow u = \frac{\pi}{3}, \ v = \frac{\pi}{6}$$
 $r(u,v) = (\sin u, \sin v, \sin (u+v))$
 $r(\frac{\pi}{3}, \frac{\pi}{6}) = (\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin (\frac{\pi}{3} + \frac{\pi}{6}))$
 $r(\frac{\pi}{3}, \frac{\pi}{6}) = (\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \frac{\pi}{2})$
 $r(\frac{\pi}{3}, \frac{\pi}{6}) = (\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \frac{\pi}{2})$
 $r(\frac{\pi}{3}, \frac{\pi}{6}) = (\frac{\sqrt{3}}{2}, \frac{1}{2}, 1)$
 $r(\frac{\pi}{3}, \frac{\pi}{6}) = (\frac{\sqrt{3}}{2}, \frac{1}{2}, 1)$
 $r(\frac{\pi}{3}, \frac{\pi}{6}) = (\frac{\sqrt{3}}{2}, \frac{1}{2}, 1)$
 $r(\frac{\pi}{3}, \frac{\pi}{6}) = (\frac{1}{2}, 0, 0)$
 $r(\frac{\pi}{3}, \frac{\pi}{6}) = (0, \frac{\sqrt{3}}{2}, 0)$

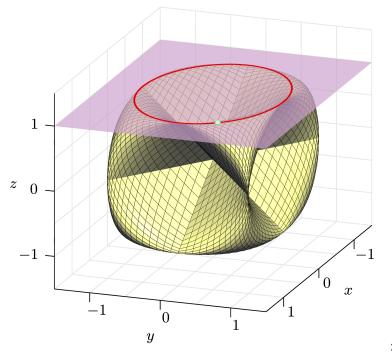
 $0 \cdot \left(x - \frac{\sqrt{3}}{2}\right) + 0 \cdot \left(y - \frac{1}{2}\right) + 1 \cdot (z - 1) = 0$

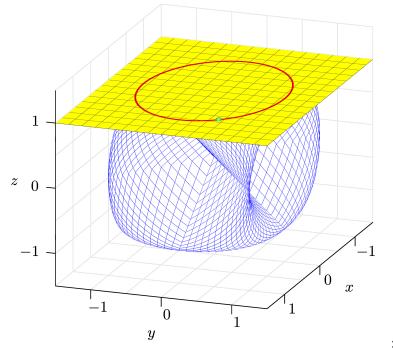
Rješenje

A
$$\longrightarrow u = \frac{\pi}{3}, \ v = \frac{\pi}{6}$$
 $r(u,v) = (\sin u, \sin v, \sin (u+v))$
 $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right)$
 $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \frac{\pi}{2}\right)$
 $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \frac{\pi}{2}\right)$
 $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$
 $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$
 $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$
 $r\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\frac{1}{2}, 0, 0\right)$
 $r\left(\frac{u}{3}, \frac{v}{6}\right) = \left(\frac{1}{2}, 0, 0\right)$
 $r\left(\frac{u}{3}, \frac{v}{6}\right) = \left(0, \frac{\sqrt{3}}{2}, \frac{v}{6}\right)$
 $r\left(\frac{v}{3}, \frac{v}{6}\right) = \left(0, \frac{\sqrt{3}}{3}, \frac{v}{6}\right)$
 $r\left(\frac{v}{3}, \frac{v}{6}\right) = \left(0, \frac{v}{3}, \frac{v}{6}\right)$
 $r\left(\frac{v}{3}, \frac{v}{6}\right) = \left$

Rješenje

A
$$\longrightarrow u = \frac{\pi}{3}, \ v = \frac{\pi}{6}$$
 $r(u,v) = (\sin u, \sin v, \sin (u+v))$
 $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right)$
 $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \frac{\pi}{2}\right)$
 $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\sin \frac{\pi}{3}, \sin \frac{\pi}{6}, \sin \frac{\pi}{2}\right)$
 $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$
 $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$
 $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{1}{2}, 0, 0\right)$
 $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\pi}{3}, \frac{\pi}{6}\right)$
 $r\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \left(\frac{\pi}{3},$





peti zadatak

Zadatak 5

Susjedne stranice pravokutnika imaju duljine 10 cm i 24 cm. Kako će se promijeniti duljina dijagonale tog pravokutnika ako prvu stranicu produljimo za 4 mm, a drugu stranicu skratimo za 1 mm? Usporedite približnu promjenu dobivenu pomoću diferencijala sa stvarnom promjenom.

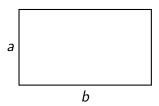
Zadatak 5

Susjedne stranice pravokutnika imaju duljine 10 cm i 24 cm. Kako će se promijeniti duljina dijagonale tog pravokutnika ako prvu stranicu produljimo za 4 mm, a drugu stranicu skratimo za 1 mm? Usporedite približnu promjenu dobivenu pomoću diferencijala sa stvarnom promjenom.

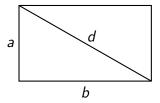
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Riešenie

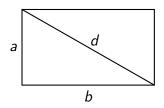
Susjedne stranice pravokutnika imaju duljine 10 cm i 24 cm. Kako će se promijeniti duljina dijagonale tog pravokutnika ako prvu stranicu produljimo za 4 mm, a drugu stranicu skratimo za 1 mm? Usporedite približnu promjenu dobivenu pomoću diferencijala sa stvarnom promjenom.



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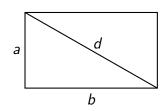


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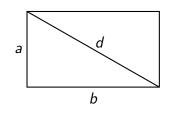
$$d=\sqrt{a^2+b^2}$$

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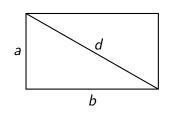


$$d = \sqrt{a^2 + b^2}$$

$$f(x, y) = \sqrt{x^2 + y^2}$$

$$x = 10 \text{ cm},$$

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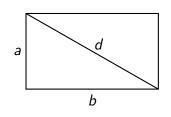


$$d = \sqrt{a^2 + b^2}$$

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$$x = 10 \text{ cm}, \quad y = 24 \text{ cm}$$

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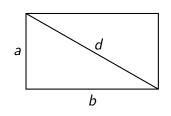
$$d = \sqrt{a^2 + b^2}$$

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$$x = 10 \text{ cm}, \quad y = 24 \text{ cm}$$

$$(10, 24)$$

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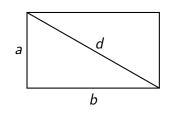
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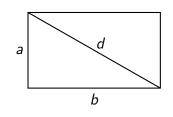
$$f(x, y) = \sqrt{x^2 + y^2}$$

$$x = 10 \text{ cm}, \quad y = 24 \text{ cm}$$

$$\begin{pmatrix} x & y \\ (10, 24) \end{pmatrix}$$

$$\Delta x = 0.4 \, \mathrm{cm}$$

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$$d=\sqrt{a^2+b^2}$$
 $f(x,y)=\sqrt{x^2+y^2}$
 $x=10\,\mathrm{cm},\quad y=24\,\mathrm{cm}$
 $x=0.4\,\mathrm{cm},\quad \Delta y=-0.1\,\mathrm{cm}$

$$\Delta x = 0.4 \,\mathrm{cm}, \quad \Delta y = -0.1 \,\mathrm{cm}$$

$$f(x,y) = \sqrt{x^2 + y^2}, \quad x = 10, \ y = 24, \quad \Delta x = 0.4, \ \Delta y = -0.1$$

$$\Delta f =$$

$$\Delta f = f(x + \Delta x, y + \Delta y)$$

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$$\Delta f = f(x + \Delta x, y + \Delta y) - f(x, y)$$
 $\Delta f = \sqrt{679.37}$
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$$\Delta f = f(x + \Delta x, y + \Delta y) - f(x, y) \qquad \Delta f = \sqrt{679.37} - \frac{1}{2}$$

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približna promjena dijagonale

 $\Delta f = \sqrt{10.4^2 + 23.9^2} - \sqrt{10^2 + 24^2}$

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$$\Delta f = \sqrt{10.4^2 + 23.9^2} - \sqrt{10^2 + 24^2}$$

$$\frac{\partial \tau}{\partial x} = \frac{\partial \tau}{\partial x}$$

$$\Delta f = 0.064727\cdots$$

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 $f(x, y) = \sqrt{x^2 + y^2}$, x = 10, y = 24, $\Delta x = 0.4$, $\Delta y = -0.1$

točna promjena dijagonale

 $\Delta f = f(x + \Delta x, y + \Delta y) - f(x, y)$

 $\Delta f = f(10.4, 23.9) - f(10, 24)$

približna promjena dijagonale

1

$$2\sqrt{x}$$

 $\Delta f = \sqrt{679.37} - \sqrt{676}$

 $\Delta f = 0.064727 \cdots$

 $\Delta f = \sqrt{10.4^2 + 23.9^2} - \sqrt{10^2 + 24^2}$ $\partial f = 1$

 $f(x, y) = \sqrt{x^2 + y^2}$, x = 10, y = 24, $\Delta x = 0.4$, $\Delta y = -0.1$

približna promjena dijagonale $\frac{\partial t}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}}$

točna promjena dijagonale

 $\Delta f = f(x + \Delta x, y + \Delta y) - f(x, y)$

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približna promjena dijagonale $\frac{\partial t}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x$

točna promjena dijagonale

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približna promjena dijagonale
$$\frac{\partial f}{\partial f} = \frac{\partial f}{\partial f}$$

$$\partial f$$

točna promjena dijagonale

$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial f}{\partial y}$$

$$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

točna promjena dijagonale

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 ∂f 1

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točna promjena dijagonale $\Delta f = f(x + \Delta x, y + \Delta y) - f(x, y) \qquad \Delta f = \sqrt{679.37} - \sqrt{676}$

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$$\partial f \qquad \qquad 1$$

$$\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y$$

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približna promjena dijagonale

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$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$$
$$\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\frac{y}{2+y^2}$$

$$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

24 / 24

$$f(x,y)=\sqrt{x^2+y^2}, \quad x=10, \ y=24, \quad \Delta x=0.4, \ \Delta y=-0.1$$

$$\Delta f = f(x + \Delta x, y + \Delta y) - f(x, y)$$
 $\Delta f = \sqrt{679.37} - \sqrt{676}$
 $\Delta f = f(10.4, 23.9) - f(10, 24)$ $\Delta f = 0.064727 \cdots$

$$\Delta f = \sqrt{10.4^2 + 23.9^2} - \sqrt{10^2 + 24^2}$$

približna promjena dijagonale

oribližna promjena dijagonale
$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial f}{\partial x} = \frac{1}{\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\Delta f \approx \mathrm{d}f$$

$$\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2}} \qquad \Delta f \approx \mathrm{d}f$$

$$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

 $\Delta f = f(10.4, 23.9) - f(10, 24)$ $\Delta f = \sqrt{10.4^2 + 23.9^2} - \sqrt{10^2 + 24^2}$ $\Delta f = 0.064727 \cdots$ $df = f_x dx + f_y dy$

 $f(x, y) = \sqrt{x^2 + y^2}, \quad x = 10, \ y = 24, \quad \Delta x = 0.4, \ \Delta y = -0.1$

točna promjena dijagonale

 $\Delta f = f(x + \Delta x, y + \Delta y) - f(x, y)$

 $\Delta f = \sqrt{10.4^2 + 23.9^2} - \sqrt{10^2 + 24^2} \qquad \qquad \text{d}f = f_x \, \text{d}x + f_y \, \text{d}y$ $\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$

$$\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2}} \qquad \Delta f \approx \mathrm{d}f$$

$$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

$$\Delta f = \sqrt{10.4^2 + 23.9^2} - \sqrt{10^2 + 24^2}$$

$$\frac{\mathrm{d}f = f_x \, \mathrm{d}x + f_y \, \mathrm{d}y}{\partial x}$$

$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$$

točna promjena dijagonale

 $\Delta f = f(x + \Delta x, y + \Delta y) - f(x, y)$

 $\Delta f = f(10.4, 23.9) - f(10, 24)$

$$\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2}} \qquad \Delta f \approx \mathrm{d}f$$

$$\Delta f \approx$$

$$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

 $\Delta f = \sqrt{679.37} - \sqrt{676}$

točna promjena dijagonale

 $\Delta f = f(x + \Delta x, y + \Delta y) - f(x, y)$

 $\Delta f = f(10.4, 23.9) - f(10, 24)$

približna promjena dijagonale $\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$ $\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$

$$\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\Delta f \approx df$$

$$\Delta f pprox rac{x}{\sqrt{x^2 + y^2}} \, \mathrm{d}x$$

$$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

 $\Delta f = \sqrt{679.37} - \sqrt{676}$

$$\Delta f = \sqrt{10.4^2 + 23.9^2} - \sqrt{10^2 + 24^2}$$

$$\frac{\partial f}{\partial x} = \frac{1}{2x - \frac{x}{2x}}$$
približna promiena dijagonale

točna promjena dijagonale

 $\Delta f = f(x + \Delta x, y + \Delta y) - f(x, y)$

 $\Delta f = f(10.4, 23.9) - f(10, 24)$

 $\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$ približna promjena dijagonale

približna promjena dijagonale
$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{\lambda}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{\lambda}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{\lambda}{\sqrt{x^2 + y^2}}$$

 $\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2}}$ $\Delta f \approx \mathrm{d}f$

$$\Delta f pprox rac{x}{\sqrt{x^2 + y^2}} \, \mathrm{d}x + \left(\sqrt{x}\right)' = rac{1}{2\sqrt{x}}$$

 $\Delta f = \sqrt{679.37} - \sqrt{676}$

$$\Delta f = \sqrt{10.4^2 + 23.9^2} - \sqrt{10^2 + 24^2}$$

$$\frac{\mathrm{d}f = f_x \, \mathrm{d}x + f_y \, \mathrm{d}y}{\partial x}$$

$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$$

približna promjena dijagonale

 $\Delta f = f(10.4, 23.9) - f(10, 24)$

 $\Delta f = f(x + \Delta x, y + \Delta y) - f(x, y)$

točna promjena dijagonale

$$\frac{\partial x}{\partial x} = \frac{1}{2\sqrt{2}}$$

 $\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2}}$ $\Delta f \approx \mathrm{d}f$

$$\Delta f \approx \frac{x}{\sqrt{x^2 + y^2}} \, \mathrm{d}x + \frac{y}{\sqrt{x^2 + y^2}} \, \mathrm{d}y$$

$$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

 $\Delta f = \sqrt{679.37} - \sqrt{676}$

$$\Delta f = \sqrt{10.4^2 + 23.9^2} - \sqrt{10^2 + 24^2}$$

$$df = f_x dx + f_y dy$$

 $\Delta f = \sqrt{679.37} - \sqrt{676}$

 $\Delta f = 0.064727 \cdots$

 $f(x, y) = \sqrt{x^2 + y^2}, \quad x = 10, \ y = 24, \quad \Delta x = 0.4, \ \Delta y = -0.1$

točna promjena dijagonale

 $\Delta f = f(x + \Delta x, y + \Delta y) - f(x, y)$

 $\Delta f = f(10.4, 23.9) - f(10, 24)$

približna promjena dijagonale
$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$$

približna promjena dijagonale
$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2}}$$

$$x \qquad \qquad V$$

$$\Delta f \approx df$$

$$\Delta f \approx \frac{x}{\sqrt{x^2 + y^2}} \, \mathrm{d}x + \frac{y}{\sqrt{x^2 + y^2}} \, \mathrm{d}y$$

$$\Delta f \approx \frac{1}{2\sqrt{x}} \, \mathrm{d}x + \frac{y}{\sqrt{x^2 + y^2}} \, \mathrm{d}y$$

 $\Delta f = \sqrt{10.4^2 + 23.9^2} - \sqrt{10^2 + 24^2}$ $df = f_x dx + f_y dy$

 $f(x, y) = \sqrt{x^2 + y^2}, \quad x = 10, \ y = 24, \quad \Delta x = 0.4, \ \Delta y = -0.1$

približna promjena dijagonale
$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\Delta f \approx \mathrm{d}f$$

 $\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2}}$ $\Delta f \approx \frac{x}{\sqrt{x^2 + y^2}} dx + \frac{y}{\sqrt{x^2 + y^2}} dy$

točna promjena dijagonale

 $\Delta f = f(x + \Delta x, y + \Delta y) - f(x, y)$

 $\Delta f = f(10.4, 23.9) - f(10, 24)$

$$\Delta t \approx \frac{1}{\sqrt{x^2 + y^2}} dx + \frac{1}{\sqrt{x^2 + y^2}} dy$$

$$\Delta f \approx \frac{1}{2\sqrt{x}}$$

 $\Delta f = \sqrt{679.37} - \sqrt{676}$

 $\Delta f = \sqrt{10.4^2 + 23.9^2} - \sqrt{10^2 + 24^2}$ $\mathrm{d}f = f_x \, \mathrm{d}x + f_y \, \mathrm{d}y$

 $f(x, y) = \sqrt{x^2 + y^2}, \quad x = 10, \ y = 24, \quad \Delta x = 0.4, \ \Delta y = -0.1$

točna promjena dijagonale

 $\Delta f = f(x + \Delta x, y + \Delta y) - f(x, y)$

 $\Delta f = f(10.4, 23.9) - f(10, 24)$

$$\Delta t = \sqrt{10.4^2 + 23.9^2} - \sqrt{10^2 + 24^2}$$

$$\frac{dt}{dt} = \frac{t_x}{t_x} \frac{dx}{dx} + \frac{t_y}{t_y} \frac{dy}{dy}$$
približna promjena dijagonale
$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{2x+2}} \cdot 2x = \frac{x}{\sqrt{2x+2}}$$

približna promjena dijagonale
$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\Delta f \approx df$$

$$\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\Delta f \approx \frac{x}{\sqrt{x^2 + y^2}} \, \mathrm{d}y$$

$$\Delta f \approx \frac{x}{\sqrt{x^2 + y^2}} \, \mathrm{d}x + \frac{y}{\sqrt{x^2 + y^2}} \, \mathrm{d}y$$

$$\Delta f \approx \frac{10}{\sqrt{x}} \left(\sqrt{x} \right)' = \frac{1}{2\sqrt{x}}$$

24 / 24

 $\Delta f = \sqrt{679.37} - \sqrt{676}$

$$\Delta f = f(10.4, 23.9) - f(10, 24)$$

$$\Delta f = \sqrt{10.4^2 + 23.9^2} - \sqrt{10^2 + 24^2}$$

$$\Delta f = f(10.4, 23.9) - f(10, 24)$$

$$\Delta f = 0.064727 \cdots$$

$$\Delta f = f(10.4, 23.9) - f(10, 24)$$

približna promjena dijagonale
$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2}}$$

 $\Delta f = f(x + \Delta x, y + \Delta y) - f(x, y)$

točna promjena dijagonale

 $\Delta f \approx \mathrm{d}f$

 $\Delta f = \sqrt{679.37} - \sqrt{676}$

$$\frac{y}{\sqrt{x^2 + y^2}} \, \mathrm{d}y$$

$$\Delta f \approx \frac{x}{\sqrt{x^2 + y^2}} \, \mathrm{d}x + \frac{y}{\sqrt{x^2 + y^2}} \, \mathrm{d}y$$
$$\Delta f \approx \frac{10}{\sqrt{10^2 + 24^2}}$$

24 / 24

 $\Delta f = f(x + \Delta x, y + \Delta y) - f(x, y)$ $\Delta f = \sqrt{679.37} - \sqrt{676}$

$$\Delta f = f(10.4, 23.9) - f(10, 24)$$

$$\Delta f = \sqrt{10.4^2 + 23.9^2} - \sqrt{10^2 + 24^2}$$

$$\Delta f = f(10.4, 23.9) - f(10, 24)$$

$$\Delta f = 0.064727 \cdots$$

$$\Delta f = f(x) + f(x$$

 $f(x, y) = \sqrt{x^2 + y^2}, \quad x = 10, \ y = 24, \quad \Delta x = 0.4, \ \Delta y = -0.1$

približna promjena dijagonale $\frac{\partial t}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$

točna promjena dijagonale

 $\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2}} \qquad \Delta f \approx \mathrm{d}f$

 $\Delta f \approx \frac{x}{\sqrt{x^2 + y^2}} \, \mathrm{d}x + \frac{y}{\sqrt{x^2 + y^2}} \, \mathrm{d}y$

$$\Delta f \approx \frac{10}{\sqrt{x^2 + y^2}} \, \mathrm{d}x + \frac{1}{\sqrt{x^2 + y^2}} \, \mathrm{d}y$$

$$\Delta f \approx \frac{10}{\sqrt{10^2 + 24^2}} \cdot 0.4$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

24 / 24

$$\Delta f = f(10.4, 23.9) - f(10, 24)$$

$$\Delta f = \sqrt{10.4^2 + 23.9^2} - \sqrt{10^2 + 24^2}$$

$$\Delta f = f(10.4, 23.9) - f(10, 24)$$

$$\Delta f = 0.064727 \cdots$$

$$\Delta f = f(10.4, 23.9) - f(10, 24)$$

točna promjena dijagonale

 $\Delta f = f(x + \Delta x, y + \Delta y) - f(x, y)$

 $\Delta t = \sqrt{10.4^2 + 23.9^2} - \sqrt{10^2 + 24^2}$ $\frac{dt}{dt} = t_x \, dx + t_y \, dy$ $\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$

$$\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2}} \qquad \Delta f \approx \mathrm{d}f$$

$$\partial y = 2\sqrt{x^2 + y^2} \quad 2y = \sqrt{x^2 + y^2}$$

$$\Delta f \approx \frac{x}{\sqrt{x^2 + y^2}} \, \mathrm{d}x + \frac{y}{\sqrt{x^2 + y^2}} \, \mathrm{d}y$$

$$\Delta f \approx \frac{10}{\sqrt{x^2 + y^2}} \operatorname{d}x + \frac{1}{\sqrt{x^2 + y^2}} \operatorname{d}y$$

$$\Delta f \approx \frac{10}{\sqrt{10^2 + 24^2}} \cdot 0.4 + \frac{1}{2\sqrt{x}}$$

 $\Delta f = f(10.4, 23.9) - f(10, 24)$ $\Delta f = \sqrt{10.4^2 + 23.9^2} - \sqrt{10^2 + 24^2}$ $\Delta f = 0.064727 \cdots$ $df = f_x dx + f_y dy$

 $f(x, y) = \sqrt{x^2 + y^2}, \quad x = 10, \ y = 24, \quad \Delta x = 0.4, \ \Delta y = -0.1$

približna promjena dijagonale
$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\Delta f \approx df$$

 $\frac{\partial t}{\partial y} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2}}$ $\Delta f \approx \frac{x}{\sqrt{x^2 + y^2}} dx + \frac{y}{\sqrt{x^2 + y^2}} dy$

točna promjena dijagonale

 $\Delta f = f(x + \Delta x, y + \Delta y) - f(x, y)$

$$\Delta f \approx \frac{x}{\sqrt{x^2 + y^2}} \, \mathrm{d}x + \frac{y}{\sqrt{x^2 + y^2}} \, \mathrm{d}y$$

$$\Delta f \approx \frac{10}{\sqrt{10^2 + 24^2}} \cdot 0.4 + \frac{1}{\sqrt{x^2 + y^2}} \, \mathrm{d}y$$

24 / 24

 $\Delta f = f(10.4, 23.9) - f(10, 24)$ $\Delta f = \sqrt{10.4^2 + 23.9^2} - \sqrt{10^2 + 24^2}$ $\Delta f = f(10.4, 23.9) - f(10, 24)$ $\Delta f = 0.064727 \cdots$ $\Delta f = f(10.4, 23.9) - f(10, 24)$

 $f(x, y) = \sqrt{x^2 + y^2}, \quad x = 10, \ y = 24, \quad \Delta x = 0.4, \ \Delta y = -0.1$

točna promjena dijagonale

 $\Delta f = f(x + \Delta x, y + \Delta y) - f(x, y)$

približna promjena dijagonale
$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial x}{\partial y} = \frac{2\sqrt{x^2 + y^2}}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2}} \qquad \Delta f \approx \mathrm{d}f$$

$$\frac{\partial y}{\partial y} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{\Delta t}{\sqrt{x^2 + y^2}}$$

$$\Delta f \approx \frac{x}{\sqrt{x^2 + y^2}} dx + \frac{y}{\sqrt{x^2 + y^2}} dy$$

$$\Delta f \approx \frac{x}{\sqrt{x^2 + y^2}} \, \mathrm{d}x + \frac{y}{\sqrt{x^2 + y^2}} \, \mathrm{d}y$$

$$\Delta f \approx \frac{10}{\sqrt{10^2 + 24^2}} \cdot 0.4 + \frac{24}{\sqrt{x^2 + y^2}} \, \mathrm{d}y$$

24 / 24

$$\Delta f = f(10.4, 23.9) - f(10, 24)$$

$$\Delta f = \sqrt{10.4^2 + 23.9^2} - \sqrt{10^2 + 24^2}$$

$$\Delta f = 0.064727 \cdots$$

$$df = f_x dx + f_y dy$$

točna promjena dijagonale

 $\Delta f = f(x + \Delta x, y + \Delta y) - f(x, y)$

 $\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$

$$\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2}} \qquad \Delta f \approx \mathrm{d}f$$

$$\Delta f \approx \frac{x}{\sqrt{x^2 + y^2}} \, \mathrm{d}x + \frac{y}{\sqrt{x^2 + y^2}} \, \mathrm{d}y$$

$$\Delta f \approx \frac{10}{\sqrt{10^2 + 24^2}} \cdot 0.4 + \frac{24}{\sqrt{10^2 + 24^2}}$$
 $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$

 $\Delta f = f(10.4, 23.9) - f(10, 24)$ $\Delta f = 0.064727 \cdots$ $\Delta f = \sqrt{10.4^2 + 23.9^2} - \sqrt{10^2 + 24^2}$ $\mathrm{d}f = f_x \, \mathrm{d}x + f_y \, \mathrm{d}y$

 $f(x, y) = \sqrt{x^2 + y^2}, \quad x = 10, \ y = 24, \quad \Delta x = 0.4, \ \Delta y = -0.1$

točna promjena dijagonale

 $\Delta f = f(x + \Delta x, y + \Delta y) - f(x, y)$

 $\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$ približna promjena dijagonale

$$\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2}} \qquad \Delta f \approx \mathrm{d}f$$

$$\frac{\partial y}{\partial y} = \frac{2\sqrt{x^2 + y^2}}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{\Delta r}{\sqrt{x^2 + y^2}}$$

$$\Delta f \approx \frac{x}{\sqrt{x^2 + y^2}} \, dx + \frac{y}{\sqrt{x^2 + y^2}} \, dy$$

$$\Delta f \approx \frac{x}{\sqrt{x^2 + y^2}} \, \mathrm{d}x + \frac{y}{\sqrt{x^2 + y^2}} \, \mathrm{d}y$$

$$(\sqrt{x})' = \frac{1}{\sqrt{x^2 + y^2}}$$

$$\Delta f \approx \frac{\lambda}{\sqrt{x^2 + y^2}} \, dx + \frac{y}{\sqrt{x^2 + y^2}} \, dy$$

$$\Delta f \approx \frac{10}{\sqrt{10^2 + 24^2}} \cdot 0.4 + \frac{24}{\sqrt{10^2 + 24^2}} \cdot (-0.1)$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

 $\Delta f = 0.064727 \cdots$ $\Delta f = f(10.4, 23.9) - f(10, 24)$ $\Delta f = \sqrt{10.4^2 + 23.9^2} - \sqrt{10^2 + 24^2}$ $\mathrm{d}f = f_x \, \mathrm{d}x + f_y \, \mathrm{d}y$

 $f(x, y) = \sqrt{x^2 + y^2}, \quad x = 10, \ y = 24, \quad \Delta x = 0.4, \ \Delta y = -0.1$

točna promjena dijagonale

 $\Delta f = f(x + \Delta x, y + \Delta y) - f(x, y)$

 $\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$ približna promjena dijagonale

$$\frac{\partial x}{\partial y} = \frac{1}{2\sqrt{2+3}} \cdot 2y = \frac{y}{\sqrt{2+3}}$$

$$\Delta f \approx df$$

$$\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2}} \qquad \Delta f \approx \mathrm{d}f$$

$$\Delta f \approx x \qquad x \qquad dy + y \qquad dy$$

$$\Delta f \approx \frac{x}{\sqrt{x^2 + y^2}} \, \mathrm{d}x + \frac{y}{\sqrt{x^2 + y^2}} \, \mathrm{d}y$$

$$\Delta f \approx \frac{1}{\sqrt{x^2 + y^2}} \, \mathrm{d}x + \frac{y}{\sqrt{x^2 + y^2}} \, \mathrm{d}y$$

$$\Delta f \approx \frac{\lambda}{\sqrt{x^2 + y^2}} \, dx + \frac{y}{\sqrt{x^2 + y^2}} \, dy$$

$$\Delta f \approx \frac{10}{\sqrt{10^2 + 24^2}} \cdot 0.4 + \frac{24}{\sqrt{10^2 + 24^2}} \cdot (-0.1)$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$\Delta f = f(10.4, 23.9) - f(10, 24)$$

$$\Delta f = \sqrt{10.4^2 + 23.9^2} - \sqrt{10^2 + 24^2}$$

$$\Delta f = f(10.4, 23.9) - f(10, 24)$$

$$\Delta f = 0.064727 \cdots$$

$$\Delta f = f_x \, dx + f_y \, dy$$

 $\Delta f = \sqrt{679.37} - \sqrt{676}$

 $f(x, y) = \sqrt{x^2 + y^2}, \quad x = 10, \ y = 24, \quad \Delta x = 0.4, \ \Delta y = -0.1$

točna promjena dijagonale

 $\Delta f = f(x + \Delta x, y + \Delta y) - f(x, y)$

približna promjena dijagonale $\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$

$$\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\Delta f \approx df$$

$$\frac{\partial y}{\partial y} = 2\sqrt{x^2 + y^2} \qquad \sqrt{x^2 + y^2}$$

$$\Delta f \approx \frac{x}{\sqrt{x^2 + y^2}} dx + \frac{y}{\sqrt{x^2 + y^2}} dy$$

$$\Delta f \approx 0.061538 \cdots$$

$$\Delta f \approx \frac{x}{\sqrt{x^2 + y^2}} \, dx + \frac{y}{\sqrt{x^2 + y^2}} \, dy$$

$$\Delta f \approx \frac{10}{\sqrt{10^2 + 24^2}} \cdot 0.4 + \frac{24}{\sqrt{10^2 + 24^2}} \cdot (-0.1)$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

 $\sqrt{10^2 + 24^2}$ $\sqrt{10^2 + 24^2}$ $\sqrt{24/24}$

 $\Delta f = 0.064727 \cdots$ $\Delta f = f(10.4, 23.9) - f(10, 24)$ $\Delta f = \sqrt{10.4^2 + 23.9^2} - \sqrt{10^2 + 24^2}$ $\mathrm{d}f = f_x \, \mathrm{d}x + f_y \, \mathrm{d}y$

 $f(x, y) = \sqrt{x^2 + y^2}, \quad x = 10, \ y = 24, \quad \Delta x = 0.4, \ \Delta y = -0.1$

točna promjena dijagonale

 $\Delta f = f(x + \Delta x, y + \Delta y) - f(x, y)$

$$\Delta f = \sqrt{10.4^2 + 23.9^2} - \sqrt{10^2 + 24^2} \qquad \qquad \text{d}f = f_x \, \text{d}x + f_y \, \text{d}y$$
približna promjena dijagonale
$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{2x-3}} \cdot 2x = \frac{x}{\sqrt{2x-3}}$$

približna promjena dijagonale
$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2}} \qquad \Delta f \approx \mathrm{d}f$$

$$\Delta f \approx x \quad x \quad y \quad \Delta f \approx 0.061538 \cdots$$

$$\Delta f \approx \frac{x}{\sqrt{x^2 + y^2}} \, \mathrm{d}x + \frac{y}{\sqrt{x^2 + y^2}} \, \mathrm{d}y$$

$$\Delta f \approx \frac{10}{\sqrt{x^2 + y^2}} \, dx + \frac{3}{\sqrt{x^2 + y^2}} \, dy$$

$$\Delta f \approx \frac{10}{\sqrt{10^2 + 24^2}} \cdot 0.4 + \frac{24}{\sqrt{10^2 + 24^2}} \cdot (-0.1)$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$