Realne funkcije realne varijable – 3. dio

Matematika 2

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Sadržaj

prvi zadatak

drugi zadatak

treći zadatak

četvrti zadatak

peti zadatak

prvi zadatak

Zadatak 1

Zadane su funkcije $f(x) = \ln(x-3)$ i $g(x) = x^2 + x + 1$.

- a) Odredite pravila pridruživanja funkcija $f \circ g$ i $g \circ f$.
- b) Na kojim su domenama od funkcija f i g kompozicije $f \circ g$ i $g \circ f$ dobro definirane?

 $g(x) = x^2 + x + 1$ $f(x) = \ln(x - 3)$

Rješenje

$$g(x) = x^2 + x + 1$$
 $f(x) = \ln(x - 3)$

$$g(x) = x^2 + x + 1$$
 $f(x) = \ln(x - 3)$

 $ln = log_e$

Rješenje

$$(f \circ g)(x) =$$

$$g(x) = x^2 + x + 1$$
 $f(x) = \ln(x - 3)$

 $ln = log_e$ a)

 $(f \circ g)(x) = f($

$$g(x) = x^2 + x + 1$$
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$$(f\circ g)(x)=f(g(x))$$

$$g(x) = x^2 + x + 1$$
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$$(f \circ g)(x) = f(g(x)) = f($$

$$g(x) = x^2 + x + 1$$
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 $ln = log_e$

$$(f \circ g)(x) = f(g(x)) = f(x^2 + x + 1)$$

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 $ln = log_e$

$$(f \circ g)(x) = f(g(x)) = f(x^2 + x + 1) =$$

= $\ln ($

$$g(x) = x^2 + x + 1$$
 $f(x) = \ln(x - 3)$

 $ln = log_e$

$$(f \circ g)(x) = f(g(x)) = f(x^2 + x + 1) =$$

= $\ln ((x^2 + x + 1))$

$$g(x) = x^2 + x + 1$$
 $f(x) = \ln(x - 3)$

 $ln = log_e$

$$(f \circ g)(x) = f(g(x)) = f(x^2 + x + 1) =$$

= $\ln((x^2 + x + 1) - 3)$

$$g(x) = x^2 + x + 1$$
 $f(x) = \ln(x - 3)$

 $ln = log_e$

$$(f \circ g)(x) = f(g(x)) = f(x^2 + x + 1) =$$

= $\ln((x^2 + x + 1) - 3) = \ln(x^2 + x - 2)$

$$g(x) = x^2 + x + 1$$
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$$(g \circ f)(x) = g(f(x)) = g(\ln(x-3))$$

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$$= ($$

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= $\ln((x^2 + x + 1) - 3) = \ln(x^2 + x - 2)$

$$(g \circ f)(x) = g(f(x)) = g(\ln(x-3)) =$$

= $(\ln(x-3))^2$

$$g(x) = x^2 + x + 1$$
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$$(f \circ g)(x) = f(g(x)) = f(x^2 + x + 1) =$$

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= $(\ln(x-3))^2 + \ln(x-3)$

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= $(\ln(x-3))^2 + \ln(x-3) + 1$

$$g(x) = x^2 + x + 1$$

$$f(x) = \ln(x - 3)$$

$$\ln = \log_e$$

a)

Riešenie

$$(f \circ g)(x) = f(g(x)) = f(x^2 + x + 1) =$$

= $\ln((x^2 + x + 1) - 3) = \ln(x^2 + x - 2)$

$$(g \circ f)(x) = g(f(x)) = g(\ln(x-3)) =$$

= $(\ln(x-3))^2 + \ln(x-3) + 1 =$
= $\ln^2(x-3)$

$$\log_a^k x = \left(\log_a x\right)^k$$

$$g(x) = x^2 + x + 1$$

$$f(x) = \ln(x - 3)$$

$$\ln = \log_e$$

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Riešenie

$$(f \circ g)(x) = f(g(x)) = f(x^2 + x + 1) =$$

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$$= \ln^2(x-3) + \ln(x-3)$$

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$$(g \circ f)(x) = g(f(x)) = g(\ln(x-3)) =$$

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$$= \ln^{2}(x-3) + \ln(x-3) + 1$$

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$$= (\ln(x-3))^{2} + \ln(x-3) + 1 =$$

$$= \ln^{2}(x-3) + \ln(x-3) + 1$$

Budite jako oprezni $(\log_2 x)^k \neq \log_2 x^k$

 $\log_a^k x = \left(\log_a x\right)^k$

$$f(x) = \ln(x-3)$$
 $(g \circ f)(x) = \ln^2(x-3) + \ln(x-3) + 1$

$$g(x) = x^2 + x + 1$$
 $(f \circ g)(x) = \ln(x^2 + x - 2)$

b)

$$f(x) = \ln(x-3)$$
 $(g \circ f)(x) = \ln^2(x-3) + \ln(x-3) + 1$

$$g(x) = x^2 + x + 1$$
 $(f \circ g)(x) = \ln(x^2 + x - 2)$

$$f(x) = \ln(x-3) \left| (g \circ f)(x) = \ln^2(x-3) + \ln(x-3) + 1 \right|$$

$$g(x) = x^2 + x + 1$$
 $(f \circ g)(x) = \ln(x^2 + x - 2)$

b) domena funkcije
$$g \circ f$$

 $x - 3 > 0$

$$f(x) = \ln(x-3)$$

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$$g(x) = x^2 + x + 1$$

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 $(f \circ g)(x) = \ln(x^2 + x - 2)$

$$x - 3 > 0 - x > 3$$

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$$x - 3 > 0 \longrightarrow x > 3$$

$$D_{g\circ f}=\langle 3,+\infty \rangle$$

$$f(x) = \ln(x - 3)$$

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domena funkcije $f \circ g$

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$$D_{g\circ f}=\langle 3,+\infty \rangle$$

domena funkcije $f \circ g$

$$x^2 + x - 2 > 0$$

$$f(x) = \ln\left(x - 3\right)$$

$$f(x) = \ln(x-3) \mid (g \circ f)(x) = \ln^2(x-3) + \ln(x-3) + 1$$

$$g(x) = x^2 + x + 1$$

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$$x - 3 > 0 - x > 3$$

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$$x^2 + x - 2 > 0$$

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$$x - 3 > 0 \longrightarrow x > 3$$

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$$x^2 + x - 2 > 0$$

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$$x_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2}$$

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$$g \circ f$$

$$x - 3 > 0 \xrightarrow{} x > 3$$

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$$x - 3 > 0 - x > 3$$

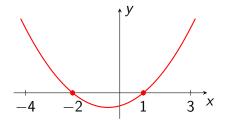
$$D_{g\circ f}=\langle 3,+\infty\rangle$$

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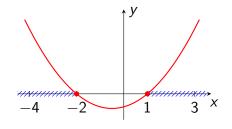
$$D_{g\circ f}=\langle 3,+\infty\rangle$$

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$$x-3 > 0 \longrightarrow x > 3$$

 $D_{\sigma \circ f} = \langle 3, +\infty \rangle$

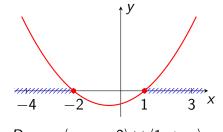
domena funkcije
$$f \circ g$$

$$x^2 + x - 2 > 0$$

$$x^2 + x - 2 = 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2}$$

$$x_1 = 1, \ x_2 = -2$$



$$D_{f \circ g} = \langle -\infty, -2 \rangle \cup \langle 1, +\infty \rangle$$

$$f(x) = \ln(x-3)$$

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$$x - 3 > 0 \longrightarrow x > 3$$

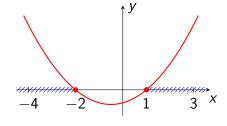
 $D_{rof} = \langle 3, +\infty \rangle$

domena funkcije
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$$D_{f \circ g} = \langle -\infty, -2 \rangle \cup \langle 1, +\infty \rangle$$

Funkcije f i g moramo gledati na sljedeći način:

$$f(x) = \ln(x - 3)$$

$$f(x) = \ln(x-3) \quad (g \circ f)(x) = \ln^2(x-3) + \ln(x-3) + 1$$

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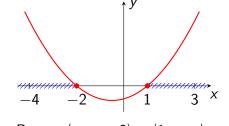
 $D_{\sigma \circ f} = \langle 3, +\infty \rangle$

domena funkcije
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$$D_{f \circ g} = \langle -\infty, -2 \rangle \cup \langle 1, +\infty \rangle$$

Funkcije f i g moramo gledati na sljedeći način:

 $\operatorname{Im} f \subseteq D_g$

$$f(x) = \ln(x - 3)$$

$$f(x) = \ln(x-3) \qquad (g \circ f)(x) = \ln^2(x-3) + \ln(x-3) + 1$$

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$$+x-2)$$

$$x - 3 > 0 \longrightarrow x > 3$$

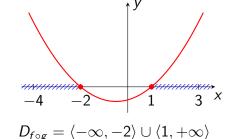
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domena funkcije
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$$D_{f \circ g} = \langle -\infty, -2 \rangle \cup \langle 1, +\infty \rangle$$

Funkcije f i g moramo gledati na sljedeći način:

$$f:\langle 3,+\infty\rangle\to\mathbb{R}$$

 $\operatorname{Im} f \subseteq D_g$

$$\begin{array}{c}
f(x) = \ln(x-3) \\
g(x) = x^2 + x + 1
\end{array}$$

$$\begin{array}{c}
(g \circ f)(x) = \ln^2(x-3) + \ln(x-3) + 1 \\
(f \circ g)(x) = \ln(x^2 + x - 2)
\end{array}$$

$$\begin{array}{c}
\ln f \subseteq D_g
\end{array}$$

b) domena funkcije
$$g \circ f$$

 $x - 3 > 0 \longrightarrow x > 3$

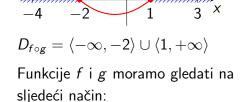
$$D_{g \circ f} = \langle 3, +\infty \rangle$$

domena funkcije
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 $f:\langle 3,+\infty\rangle\to\mathbb{R}$

 $\operatorname{Im} g \subseteq D_f \mid {}_{\uparrow} y$

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b) domena funkcije
$$g \circ f$$

$$x - 3 > 0 \xrightarrow{} x > 3$$

$$D_{g \circ f} = \langle 3, +\infty \rangle$$
domena funkcije $f \circ g$

$$x^2 + x - 2 > 0$$

$$x^2 + x - 2 = 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2}$$

$$x_1 = 1, x_2 = -2$$

$$Funkcije f i g moramo gledati na sljedeći način:
$$f : \langle 3, +\infty \rangle \to \mathbb{R}$$

$$g : \langle -\infty, -2 \rangle \cup \langle 1, +\infty \rangle \to \mathbb{R}$$$$

 $f(x) = \ln(x-3) \mid (g \circ f)(x) = \ln^2(x-3) + \ln(x-3) + 1$

 $g(x) = x^2 + x + 1$ $| (f \circ g)(x) = \ln(x^2 + x - 2)$

drugi zadatak

Zadatak 2

Dana su pravila pridruživanja funkcija f i g s

$$f(x) = \log_3 x - 2$$
 i $g(x) = \sqrt{1 - x}$.

- a) Pronađite inverzne funkcije od f i g te komentirajte na kojim su domenama i kodomenama funkcije f i g bijekcije.
- b) Nacrtajte na istoj slici graf funkcije f i graf funkcije f^{-1} .
- c) Nacrtajte na istoj slici graf funkcije g i graf funkcije g^{-1} .

$$y = f(x) \Leftrightarrow x = f^{-1}(y)$$

a)

$$f(x) = \log_3 x - 2$$

$$y = f(x) \Leftrightarrow x = f^{-1}(y)$$

a)

$$f(x) = \log_3 x - 2$$
$$y = \log_3 x - 2$$

$$y = f(x) \Leftrightarrow x = f^{-1}(y)$$

$$f(x) = \log_3 x - 2$$
$$y = \log_3 x - 2$$
$$\log_3 x - 2$$

$$-\log_3 x =$$

$$y = f(x) \Leftrightarrow x = f^{-1}(y)$$

$$f(x) = \log_3 x - 2$$
$$y = \log_3 x - 2$$
$$-\log_3 x = -x - 2$$

$$-\log_3 x = -y - 2$$

$$y = f(x) \Leftrightarrow x = f^{-1}(y)$$

$$f(x) = \log_3 x - 2$$
$$y = \log_3 x - 2$$
$$-\log_3 x = -y - 2 / \cdot (-1)$$

$$y = f(x) \Leftrightarrow x = f^{-1}(y)$$

$$f(x) = \log_3 x - 2$$
$$y = \log_3 x - 2$$
$$-\log_3 x = -y - 2 / \cdot (-1)$$
$$\log_3 x = y + 2$$

$$\log_a x = b \longrightarrow x = a^b$$

$$f(x) = \log_3 x - 2$$

$$y = \log_3 x - 2$$

$$-\log_3 x = -y - 2 / \cdot (-1)$$

$$\log_3 x = y + 2$$

$$= y + 2$$

$$\log_3 x = y + 2$$

$$\log_a x = b \longrightarrow x = a^b$$

$$f(x) = \log_3 x - 2$$

$$y = \log_3 x - 2$$

$$-\log_3 x = -y - 2 / \cdot (-1)$$

$$\log_3 x = y + 2$$

$$x =$$

 $\log_a x = b \longrightarrow x = a^b$

$$f(x) = \log_3 x - 2$$

$$y = \log_3 x - 2$$

$$-\log_3 x = -y - 2 / \cdot (-1)$$

$$\log_3 x = y + 2$$

$$x = 3^{y+2}$$

$$\log_a x = b \longrightarrow x = a^b$$

$$f(x) = \log_3 x - 2$$

$$y = \log_3 x - 2$$

$$-\log_3 x = -y - 2 / \cdot (-1)$$

$$\log_3 x = y + 2$$

$$x = 3^{y+2}$$

$$f^{-1}(y) = 3^{y+2}$$

$$(y) = 3^{y+2}$$

 $\log_a x = b \longrightarrow x = a^b$

$$y = f(x) \Leftrightarrow x = f^{-1}(y)$$

$$f(x) = \log_3 x - 2$$

$$y = \log_3 x - 2$$

$$-\log_3 x = -y - 2 / \cdot (-1)$$

$$\log_3 x = y + 2$$

$$x = 3^{y+2}$$

$$f^{-1}(y) = 3^{y+2}$$

$$f^{-1}(x) = 3^{x+2}$$

 $\log_a x = b \longrightarrow x = a^b$

$$f(x) = \log_3 x - 2$$

$$y = \log_3 x - 2$$

$$-\log_3 x = -y - 2 / \cdot (-1)$$

$$\log_3 x = y + 2$$

$$x = 3^{y+2}$$

$$x=3^{y+2}$$

$$f^{-1}(y) = 3^{y+2}$$

$$f^{-1}(x) = 3^{x+2}$$

 $\log_a x = b \longrightarrow x = a^b$

$$f(x) = \log_3 x - 2$$

$$y = \log_3 x - 2$$

$$-\log_3 x = -y - 2 / \cdot (-1)$$

$$\log_3 x = y + 2$$

$$x = 3^{y+2}$$

$$f^{-1}(y) = 3^{y+2}$$

$$f^{-1}(x) = 3^{x+2}$$

f

$$\log_a x = b \longrightarrow x = a^b$$

$$f(x) = \log_3 x - 2$$

$$I(x) = \log_3 x -$$

$$y = \log_3 x - 2$$
$$-\log_3 x = -y - 2 / \cdot (-1)$$
$$\log_3 x = y + 2$$

$$x = 3^{y+2}$$

$$f^{-1}(y) = 3^{y+2}$$

$$f^{-1}(x) = 3^{x+2}$$

$$f:\langle 0,+\infty \rangle$$

$$\log_a x = b \longrightarrow x = a^b$$

$$f(x) = \log_3 x - 2$$

$$y = \log_3 x - 2$$
$$y = \log_3 x - 2$$

$$y = \log_3 x - 2$$

$$-\log_3 x = -y - 2 / \cdot (-1)$$

$$\log_3 x = y + 2$$

$$x=3^{y+2}$$

$$f^{-1}(y) = 3^{y+2}$$

 $f^{-1}(x) = 3^{x+2}$

$$f:\langle 0,+\infty\rangle \to$$

 $\log_a x = b \longrightarrow x = a^b$

 $y = f(x) \Leftrightarrow x = f^{-1}(y)$

$$f(x) = \log_3 x - 2$$

$$y = \log_3 x - 2$$

$$-\log_3 x = -y - 2 / \cdot (-1)$$

$$\log_3 x = y + 2$$

$$x = 3^{y+2}$$

$$f^{-1}(y) = 3^{y+2}$$

$$f:\langle 0,+\infty\rangle \to \mathbb{R}$$

 $f^{-1}(x) = 3^{x+2}$

$$\log_a x = b \longrightarrow x = a^b$$

 $y = f(x) \Leftrightarrow x = f^{-1}(y)$

$$f(x) = \log_3 x - 2$$

$$y = \log_3 x - 2$$
$$-\log_3 x = -y - 2 / \cdot (-1)$$

$$\log_3 x = y + 2$$

$$-y + 1 = 3^{y+2}$$

$$x=3^{y+2}$$

$$-3y+2$$

$$= 3^{y+2}$$

$$f:\langle 0,+\infty\rangle \to \mathbb{R}$$

 f^{-1} :

$$f^{-1}(y) = 3^{y+2}$$
$$f^{-1}(x) = 3^{x+2}$$

 $\log_a x = b \longrightarrow x = a^b$ $f(x) = \log_3 x - 2$

 $y = f(x) \Leftrightarrow x = f^{-1}(y)$

$$y = \log_3 x - 2$$

$$-\log_3 x = -y - 2 / \cdot (-1)$$

$$\log_3 x = y + 2$$

$$x = 3^{y+2}$$

$$x = y + 2$$

 $x = y + 2$
 $x = 3^{y+2}$
 $y = 3^{y+2}$

$$f^{-1}(y) = 3^{y+2}$$
$$f^{-1}(x) = 3^{x+2}$$

$$f:\langle 0,+\infty
angle
ightarrow\mathbb{R}$$

 $f^{-1}:\mathbb{R}$

$$f^{-1}(x) = 3^{x+2}$$
$$f: \langle 0, +\infty \rangle \to \mathbb{R}$$

 $\log_a x = b \longrightarrow x = a^b$

$$f(x) = \log_3 x - 2$$

$$y = \log_3 x - 2$$

$$-\log_3 x = -y - 2 / \cdot (-1)$$

$$\log_3 x = y + 2$$

$$x = 3^{y+2}$$

$$x = 3^{y+2}$$

$$f^{-1}(y) = 3^{y+2}$$

$$y)=3^{y+2}$$

$$f:\langle 0,+\infty
angle
ightarrow\mathbb{R}$$

 $f^{-1}:\mathbb{R} \to$

$$f^{-1}(y) = 3^{x+2}$$
$$f^{-1}(x) = 3^{x+2}$$

Rješenje
$$\log_a x = b \longrightarrow x = a^b$$

 $y = f(x) \Leftrightarrow x = f^{-1}(y)$

$$f(x) = \log_3 x - 2$$

$$y = \log_3 x - 2$$

$$-\log_3 x = -y - 2 / \cdot (-1)$$

$$\log_3 x = y + 2$$

$$x = 3^{y+2}$$

$$x = 3^{y+2}$$

$$x^{-1}(y) = 3^{y+2}$$

$$x - 3^{-1}(y) = 3^{y+2}$$
 $x - 3^{-1}(x) = 3^{x+2}$

 $f:\langle 0,+\infty\rangle\to\mathbb{R}$

 $f^{-1}: \mathbb{R} \to \langle 0, +\infty \rangle$

$$f^{-1}(y) = 3^{y+2}$$
$$f^{-1}(x) = 3^{x+2}$$

$$0 = 3^{y+2}$$

 $0 = 3^{x+2}$

 $\log_a x = b \longrightarrow x = a^b$ $f(x) = \log_3 x - 2$

 $y = f(x) \Leftrightarrow x = f^{-1}(y)$

$$x = 3^{y+2}$$
$$f^{-1}(y) = 3^{y+2}$$

$$x = 3^{y+2}$$

 $f:\langle 0,+\infty\rangle\to\mathbb{R}$

 $f^{-1}: \mathbb{R} \to \langle 0, +\infty \rangle$

$$x = 3^{y+2}$$

 $f^{-1}(x) = 3^{x+2}$

$$\log_3 x = y + 2$$
$$x = 3^{y+2}$$

$$= -y - 2 /$$
$$= y + 2$$

$$y = \log_3 x - 2$$
$$-\log_3 x = -y - 2 / \cdot (-1)$$

$$= \log_3 x - 2$$
$$= -y - 2/x$$

$$=\log_3 x - 2$$

$$= \log_3 x - 2$$

$$y = \log_3 x - 2$$

$$x-2$$

$$g(x) = \sqrt{1-x}$$

Rješenje

$\log_a x = b \longrightarrow x = a^b$ $f(x) = \log_3 x - 2$

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$$-\log_3 x = -y - 2 / \cdot (-1)$$

$$\log_3 x = y + 2$$

$$x = 3^{y+2}$$

 $\log_3 x = y + 2$

$$x = 3^{y+2}$$

$$y) = 3^{y+2}$$

 $y = \log_3 x - 2$

$$f^{-1}(y) = 3^{y+2}$$

$$(y) = 3^{y+2}$$

 $(x) = 3^{x+2}$

 $f:\langle 0,+\infty\rangle\to\mathbb{R}$

 $f^{-1}: \mathbb{R} \to \langle 0, +\infty \rangle$

$$f^{-1}(x)$$

$$f^{-1}(x) = 3^{x+2}$$

$$f^{-1}(x) = 3^{x+2}$$

$$f^{-1}(x) = 3^{x+2}$$

$$= 3^{x+2}$$

 $g(x) = \sqrt{1-x}$

 $y = \sqrt{1 - x}$

Rješenje

 $\log_a x = b \longrightarrow x = a^b$ $f(x) = \log_3 x - 2$

 $y=\sqrt{1-x}/^2$

 $g(x) = \sqrt{1-x}$

 $-\log_3 x = -y - 2 / \cdot (-1)$ $\log_3 x = y + 2$ $x = 3^{y+2}$

 $y = \log_3 x - 2$

 $f^{-1}(y) = 3^{y+2}$

 $f^{-1}(x) = 3^{x+2}$

$$f^{-1}(x) = 3^{x+2}$$

 $f:\langle 0,+\infty\rangle\to\mathbb{R}$

 $f^{-1}: \mathbb{R} \to \langle 0, +\infty \rangle$

$$=3^{x+2}$$

$$f(x) = \log_3 x - 2$$
 $g(x) = \sqrt{1 - x}$ $y = \log_3 x - 2$ $y = \sqrt{1 - x} / 2$ uz uvjet $y = \log_3 x = -y - 2 / (-1)$ $y \ge 0$ $y = \sqrt{1 - x} / 2$ $y \ge 0$

$$f^{-1}(f^{-1}(f^{-1}))$$

 $x = 3^{y+2}$

 $f:\langle 0,+\infty\rangle\to\mathbb{R}$

 $f^{-1}: \mathbb{R} \to \langle 0, +\infty \rangle$

Rješenje

a)

$$f^{-1}(y) = 3^{y+2}$$
$$f^{-1}(x) = 3^{x+2}$$

 $\log_a x = b \longrightarrow x = a^b$

$$f^{-1}(x) = 3^{x+2}$$

 $y = f(x) \Leftrightarrow x = f^{-1}(y)$

$$f(x) = \log_3 x - 2$$
 $g(x) = \sqrt{1 - x}$ $y = \log_3 x - 2$ $y = \sqrt{1 - x} / 2$ uz uvjet $-\log_3 x = -y - 2 / \cdot (-1)$ $y \ge 0$ $\log_3 x = y + 2$ $x = 3^{y+2}$ $f^{-1}(y) = 3^{y+2}$ $f^{-1}(x) = 3^{x+2}$

Rješenje

 $f:\langle 0,+\infty\rangle\to\mathbb{R}$

 $f^{-1}: \mathbb{R} \to \langle 0, +\infty \rangle$

a)

$$f(x) = \log_3 x - 2$$
 $g(x) = \sqrt{1 - x}$ $y = \log_3 x - 2$ $y = \sqrt{1 - x} / 2$ uz uvjet $-\log_3 x = -y - 2 / \cdot (-1)$ $y \ge 0$ $\log_3 x = y + 2$ $x = 3^{y+2}$ $f^{-1}(y) = 3^{y+2}$ $f^{-1}(x) = 3^{x+2}$

Rješenje

 $f:\langle 0,+\infty\rangle\to\mathbb{R}$

 $f^{-1}: \mathbb{R} \to \langle 0, +\infty \rangle$

a)

$$f(x) = \log_3 x - 2$$
 $g(x) = \sqrt{1 - x}$ $y = \log_3 x - 2$ $y = \sqrt{1 - x}/2$ uz uvjet $-\log_3 x = -y - 2/\cdot (-1)$ $y \ge 0$ $\log_3 x = y + 2$ $\log_3 x = y +$

Rješenje

 $f:\langle 0,+\infty\rangle\to\mathbb{R}$

 $f^{-1}: \mathbb{R} \to \langle 0, +\infty \rangle$

a)

$$f(x) = \log_3 x - 2$$
 $g(x) = \sqrt{1-x}$
 $y = \log_3 x - 2$ $y = \sqrt{1-x}/^2$ uz uvjet
 $-\log_3 x = -y - 2/\cdot (-1)$ $y \ge 0$
 $\log_3 x = y + 2$ $\log_3 x = y +$

Rješenje

 $f:\langle 0,+\infty\rangle\to\mathbb{R}$

 $f^{-1}: \mathbb{R} \to \langle 0, +\infty \rangle$

a)

a)
$$f(x) = \log_3 x - 2 \qquad g(x) = \sqrt{1-x}$$

$$y = \log_3 x - 2 \qquad y = \sqrt{1-x} /^2 \quad \text{uz uvjet}$$

$$-\log_3 x = -y - 2 / \cdot (-1) \qquad y \geqslant 0$$

$$\log_3 x = y + 2 \qquad \text{laž}$$

$$x = 3^{y+2}$$

$$f^{-1}(y) = 3^{y+2}$$

$$f^{-1}(x) = 3^{x+2}$$

$$istina$$

$$f: \langle 0, +\infty \rangle \to \mathbb{R}$$

Rješenje

 $f^{-1}: \mathbb{R} \to \langle 0, +\infty \rangle$

$$f(x) = \log_3 x - 2$$

$$y = \log_3 x - 2$$

$$-\log_3 x = -y - 2 / \cdot (-1)$$

$$\log_3 x = y + 2$$

$$x = 3^{y+2}$$

$$f^{-1}(y) = 3^{y+2}$$

$$f^{-1}(x) = 3^{x+2}$$

$$g(x) = \sqrt{1-x}$$

$$y = \sqrt{1-x} /^2 \quad \text{uz uvjet}$$

$$y^2 = 1 - x \quad y \geqslant 0$$

$$\log_3 x = y + 2$$

$$25 = 5 /^2$$

$$25 = 25$$
istina

Rješenje

 $f^{-1}: \mathbb{R} \to \langle 0, +\infty \rangle$

a)

a)
$$f(x) = \log_3 x - 2 \qquad g(x) = \sqrt{1 - x}$$

$$y = \log_3 x - 2 \qquad y = \sqrt{1 - x} /^2 \quad \text{uz uvjet}$$

$$-\log_3 x = -y - 2 / \cdot (-1) \qquad y^2 = 1 - x \qquad y \geqslant 0$$

$$\log_3 x = y + 2 \qquad x = \frac{1}{2}$$

$$x = 3^{y+2}$$

$$f^{-1}(y) = 3^{y+2}$$

$$f^{-1}(x) = 3^{x+2}$$

$$f: \langle 0, +\infty \rangle \to \mathbb{R}$$

Rješenje

 $f^{-1}: \mathbb{R} \to \langle 0, +\infty \rangle$

a)
$$f(x) = \log_3 x - 2 \qquad g(x) = \sqrt{1 - x}$$

$$y = \log_3 x - 2 \qquad y = \sqrt{1 - x} / 2^x \quad \text{uz uvjet}$$

$$-\log_3 x = -y - 2 / \cdot (-1) \qquad y^2 = 1 - x \qquad y \geqslant 0$$

$$\log_3 x = y + 2 \qquad x = 1 - y^2 \qquad \text{laž}$$

$$f^{-1}(y) = 3^{y+2} \qquad \text{istina}$$

$$f: \langle 0, +\infty \rangle \to \mathbb{R}$$

 $y = f(x) \Leftrightarrow x = f^{-1}(y)$

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Rješenje

 $f^{-1}: \mathbb{R} \to \langle 0, +\infty \rangle$

a)
$$f(x) = \log_3 x - 2 \qquad g(x) = \sqrt{1 - x}$$

$$y = \log_3 x - 2 \qquad y = \sqrt{1 - x} /^2 \quad \text{uz uvjet}$$

$$-\log_3 x = -y - 2 / \cdot (-1) \qquad y^2 = 1 - x \qquad y \geqslant 0$$

$$\log_3 x = y + 2 \qquad x = 1 - y^2 \qquad \text{laž}$$

$$x = 3^{y+2} \qquad g^{-1}(y) = 1 - y^2 \qquad -5 = 5 /^2$$

$$f^{-1}(y) = 3^{y+2} \qquad \text{istina}$$

$$f: \langle 0, +\infty \rangle \to \mathbb{R}$$

Rješenje

 $f^{-1}: \mathbb{R} \to \langle 0, +\infty \rangle$

a)
$$f(x) = \log_3 x - 2 \qquad g(x) = \sqrt{1 - x}$$

$$y = \log_3 x - 2 \qquad y = \sqrt{1 - x} /^2 \quad \text{uz uvjet}$$

$$-\log_3 x = -y - 2 / \cdot (-1) \qquad y^2 = 1 - x \qquad y \geqslant 0$$

$$\log_3 x = y + 2 \qquad x = 1 - y^2 \qquad \text{laž}$$

$$x = 3^{y+2} \qquad g^{-1}(y) = 1 - y^2 \qquad -5 = 5 /^2$$

$$f^{-1}(y) = 3^{y+2} \qquad g^{-1}(x) = 1 - x^2 \qquad \text{istina}$$

$$f: \langle 0, +\infty \rangle \to \mathbb{R}$$

Rješenje

 $f^{-1}: \mathbb{R} \to \langle 0, +\infty \rangle$

a)
$$f(x) = \log_3 x - 2 \qquad g(x) = \sqrt{1 - x}$$

$$y = \log_3 x - 2 \qquad y = \sqrt{1 - x} /^2 \quad \text{uz uvjet}$$

$$-\log_3 x = -y - 2 / \cdot (-1) \qquad y^2 = 1 - x \qquad y \geqslant 0$$

$$\log_3 x = y + 2 \qquad x = 1 - y^2 \qquad \text{laž}$$

$$x = 3^{y+2} \qquad g^{-1}(y) = 1 - y^2 \qquad \text{f}^{-1}(x) = 3^{y+2}$$

$$f^{-1}(x) = 3^{x+2} \qquad g^{-1}(x) = 1 - x^2$$

$$f : \langle 0, +\infty \rangle \to \mathbb{R}$$

Rješenje

 $f^{-1}: \mathbb{R} \to \langle 0, +\infty \rangle$

$$f(x) = \log_3 x - 2$$

$$y = \log_3 x - 2$$

$$-\log_3 x = -y - 2 / \cdot (-1)$$

$$\log_3 x = y + 2$$

$$x = 3^{y+2}$$

$$f^{-1}(1 - x \ge 0)$$

$$f^{-1}(1 - x \ge -1)$$

$$f^{-1}(1 - x \ge 0)$$

$$f^{-1}(1 -$$

 $y = f(x) \Leftrightarrow x = f^{-1}(y)$

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$$f(x) = \log_3 x - 2$$

$$y = \log_3 x - 2$$

$$-\log_3 x = -y - 2 / \cdot (-1)$$

$$\log_3 x = y + 2$$

$$x = 3^{y+2}$$

$$f^{-1}(x) = \log_3 x - 2$$

$$f^{-1}(x) = 1 - x$$

$$g(x) = \sqrt{1 - x}$$

$$y = \sqrt{1 - x} / 2$$

$$y = 1 - x / 2$$

$$y = \sqrt{1 - x} /$$

 $y = f(x) \Leftrightarrow x = f^{-1}(y)$

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$$f(x) = \log_3 x - 2$$

$$y = \log_3 x - 2$$

$$-\log_3 x = -y - 2 / \cdot (-1)$$

$$\log_3 x = y + 2$$

$$x = 3^{y+2}$$

$$f^{-1}(1 - x \ge 0)$$

$$f^{-1}(1 - x \ge 1)$$

$$f : (0, +\infty) \to \mathbb{R}$$

$$f^{-1} : \mathbb{R} \to \langle 0, +\infty \rangle$$

$$g(x) = \sqrt{1 - x}$$

$$y = \sqrt{1 - x} / 2 \quad \text{uz uvjet}$$

$$y^2 = 1 - x \quad y \ge 0$$

$$x = 1 - y^2 \quad \text{laž}$$

$$x = 1 - y^2 \quad \text{laž}$$

$$g^{-1}(y) = 1 - y^2$$

$$25 = 25$$

$$g^{-1}(x) = 1 - x^2$$

$$g : \text{stina}$$

 $y = f(x) \Leftrightarrow x = f^{-1}(y)$

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Kyesenje
a)
$$f(x) = \log_3 x - 2$$

$$y = \log_3 x - 2$$

$$-\log_3 x = -y - 2/\cdot (-1)$$

$$\log_3 x = y + 2$$

$$x = 3^{y+2}$$

$$f^{-1}(1 - x \geqslant 0)$$

$$f^{-1}(1 - x \geqslant 0)$$

$$f^{-1}(1 - x \geqslant 1)$$

$$f : (0, +\infty) \to \mathbb{R}$$

$$f^{-1} : \mathbb{R} \to \langle 0, +\infty \rangle$$

$$y = f(x) \Leftrightarrow x = f^{-1}(y)$$

$$y = \sqrt{1 - x}$$

$$y = 0$$

$$x = 1 - y^2$$

$$y = f(x) \Leftrightarrow x = f^{-1}(y)$$

Reserve
$$| \log_{a} x = b \longrightarrow x = a^{b}$$

$$| y = f(x) \Leftrightarrow x = f^{-1}(y)$$

$$| f(x) = \log_{3} x - 2$$

$$| y = \log_{3} x - 2$$

$$| y = \sqrt{1 - x} \rangle^{2}$$

$$|$$

Rješenje
a)
$$f(x) = \log_3 x - 2$$

$$y = \log_3 x - 2$$

$$-\log_3 x = -y - 2/\cdot (-1)$$

$$\log_3 x = y + 2$$

$$x = 3^{y+2}$$

$$f^{-1}(y) = 3^{y+2}$$

$$f^{-1}(x) = 3^{x+2}$$

$$f^{-1}: \mathbb{R} \to \langle 0, +\infty \rangle$$

$$g(x) = \sqrt{1-x}$$

$$y = \sqrt{1-x} / 2$$

$$y$$

Rješenje
a)
$$f(x) = \log_3 x - 2$$

$$y = \log_3 x - 2$$

$$-\log_3 x = -y - 2/\cdot (-1)$$

$$\log_3 x = y + 2$$

$$x = 3^{y+2}$$

$$f^{-1}(y) = 3^{y+2}$$

$$f^{-1}(x) = 3^{x+2}$$

$$f^{-1}: \mathbb{R} \to \langle 0, +\infty \rangle$$

$$y = f(x) \Leftrightarrow x = f^{-1}(y)$$

$$g(x) = \sqrt{1-x}$$

$$y = \sqrt{1-x}/^2 \quad \text{uz uvjet}$$

$$y^2 = 1-x \quad y \geqslant 0$$

$$x = 1-y^2 \quad \text{laž}$$

$$g^{-1}(y) = 1-y^2 \quad -5 \stackrel{?}{=} 5/^2$$

$$25 = 25$$

$$g^{-1}(x) = 1-x^2$$

$$g: \langle -\infty, 1] \to [0, +\infty \rangle$$

$$g^{-1}: [0, +\infty \rangle$$

Rješenje
a)
$$f(x) = \log_3 x - 2$$

$$y = \log_3 x - 2$$

$$-\log_3 x = -y - 2/\cdot (-1)$$

$$\log_3 x = y + 2$$

$$x = 3^{y+2}$$

$$f^{-1}(y) = 3^{y+2}$$

$$f^{-1}(x) = 3^{x+2}$$

$$f^{-1}: \mathbb{R} \to \langle 0, +\infty \rangle$$

$$g(x) = \sqrt{1-x}$$

$$y = \sqrt{1-x} / 2$$

$$y$$

Rješenje
a)
$$f(x) = \log_3 x - 2$$

$$y = \log_3 x - 2$$

$$-\log_3 x = -y - 2/\cdot (-1)$$

$$\log_3 x = y + 2$$

$$x = 3^{y+2}$$

$$f^{-1}(y) = 3^{y+2}$$

$$f^{-1}(x) = 3^{x+2}$$

$$f^{-1}: \mathbb{R} \to \langle 0, +\infty \rangle$$

$$g(x) = \sqrt{1-x}$$

$$y = \sqrt{1-x}/^2 \quad \text{uz uvjet}$$

$$y^2 = 1 - x \quad y \geqslant 0$$

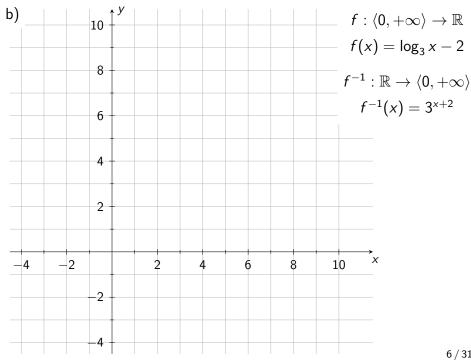
$$x = 1 - y^2 \quad \text{laž}$$

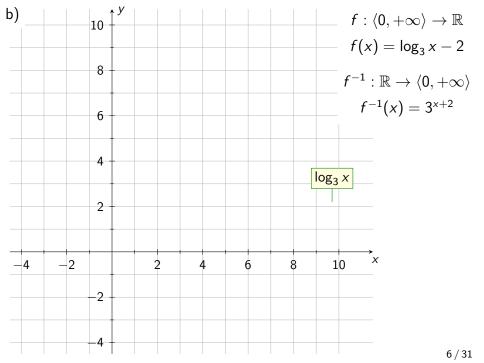
$$x = 1 - y^2 \quad \text{laž}$$

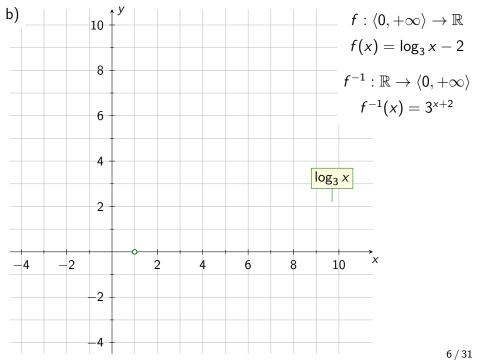
$$g^{-1}(y) = 1 - y^2 \quad -5 = 5/^2$$

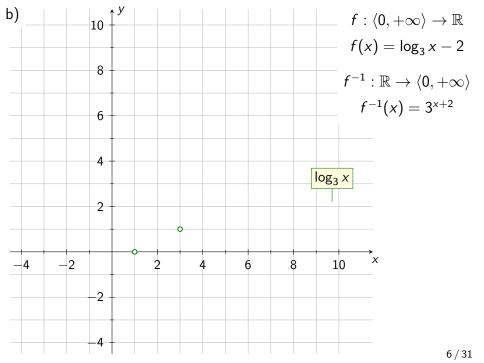
$$25 = 25$$

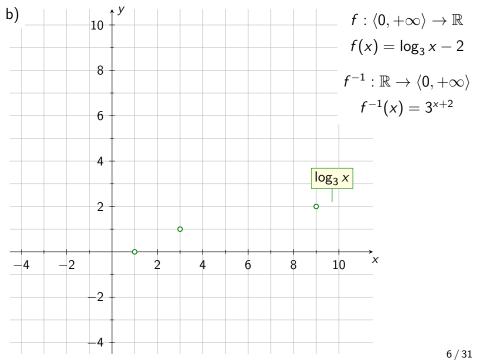
$$g^{-1}(x) = 1 - x^2$$

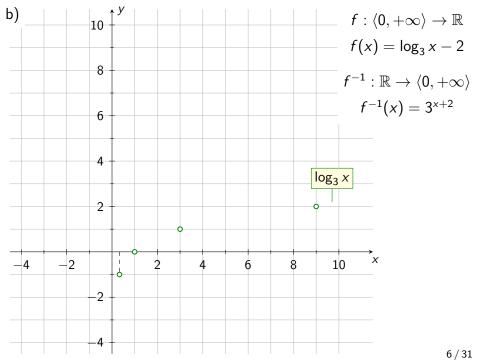


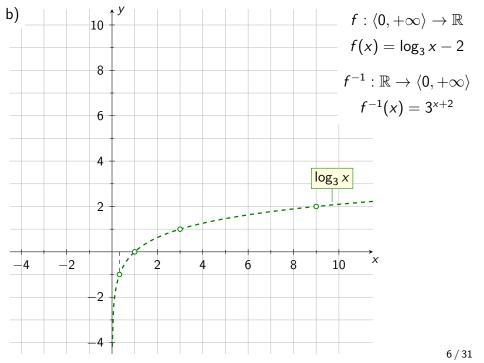


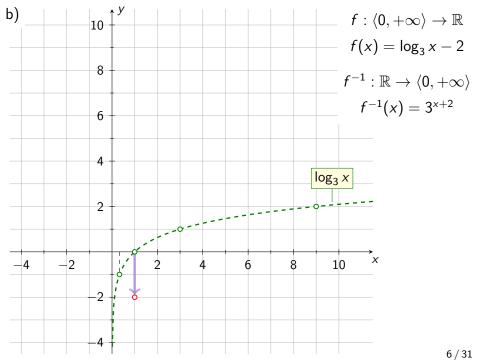


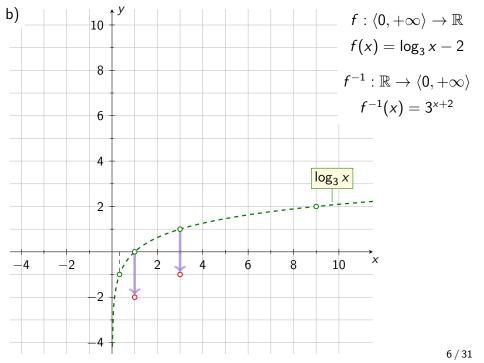


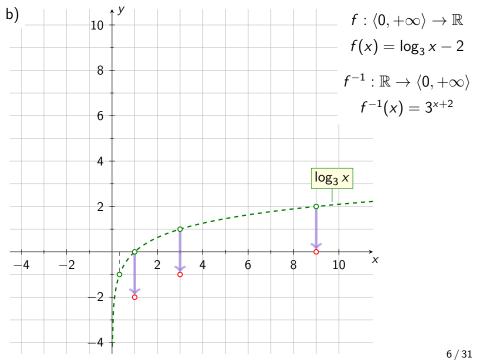


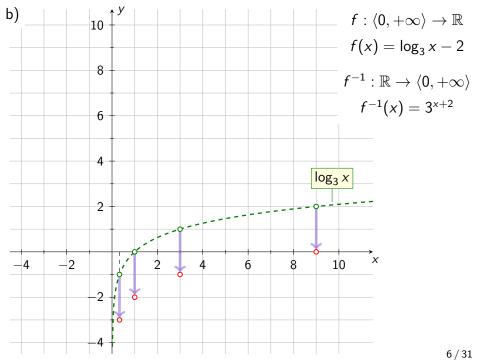


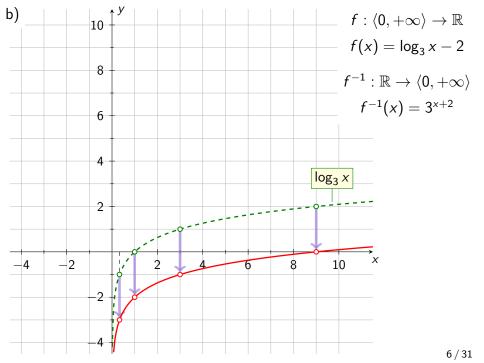


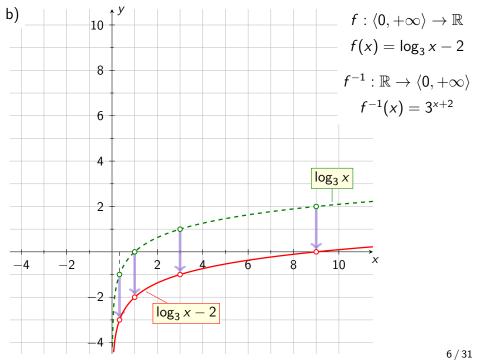


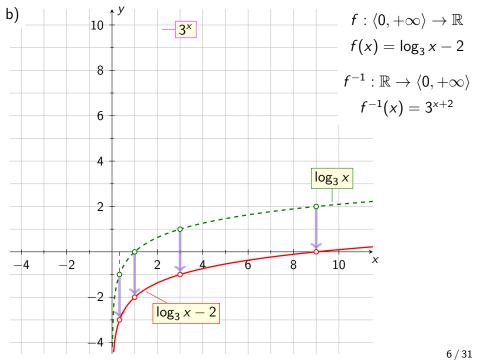


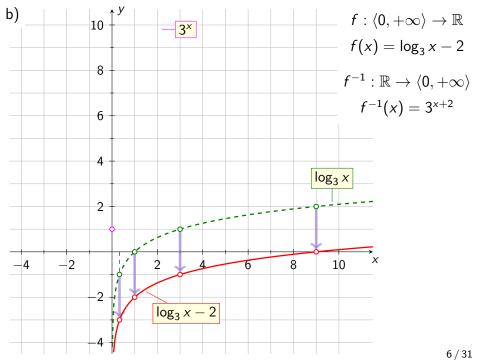


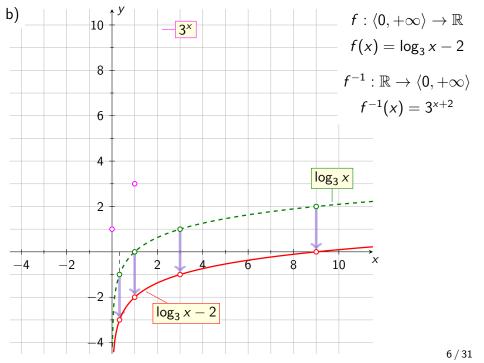


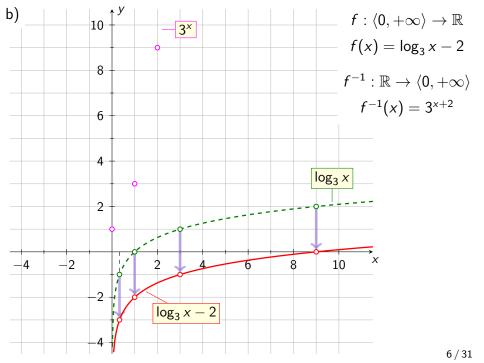


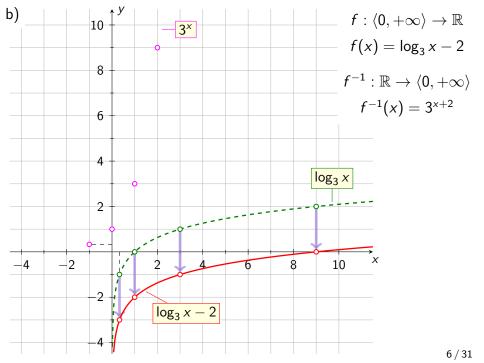


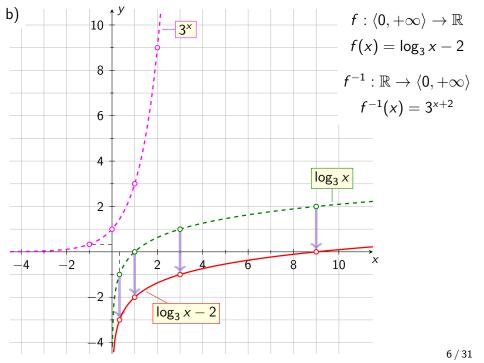


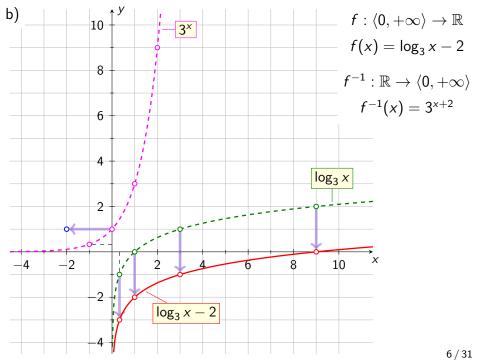


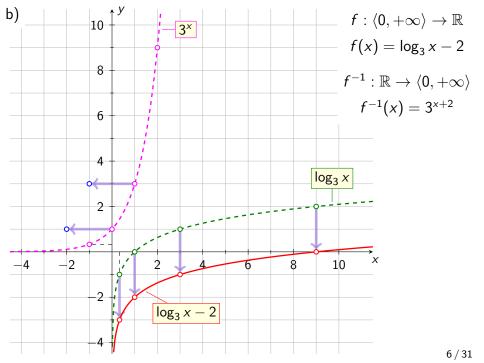


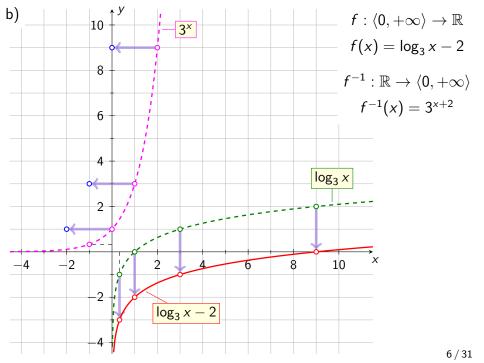


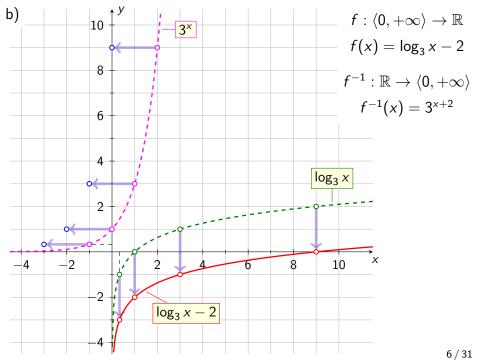


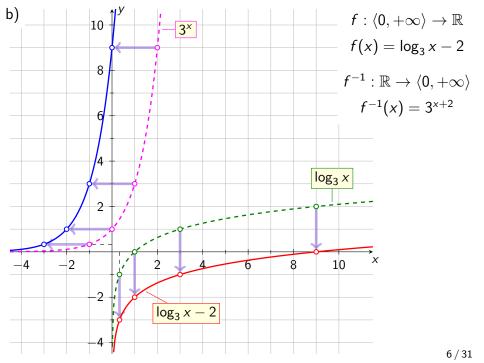


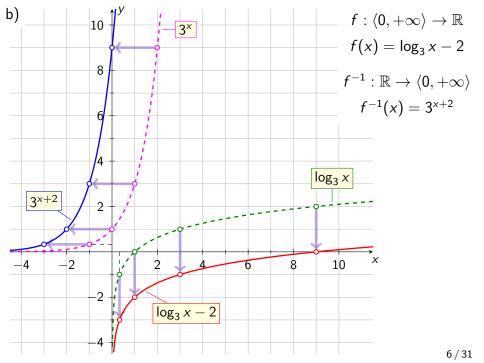


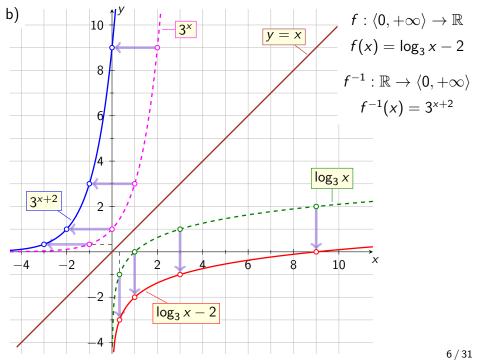


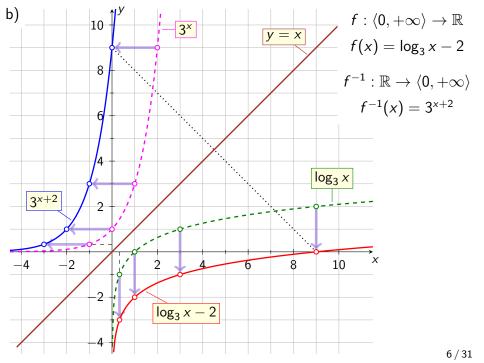


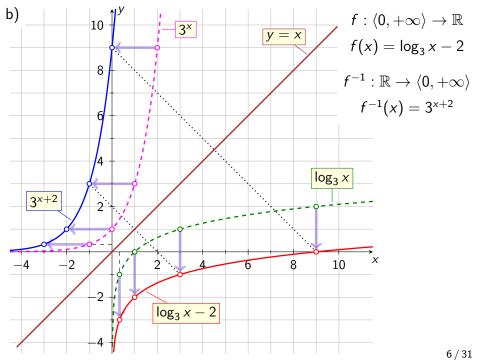


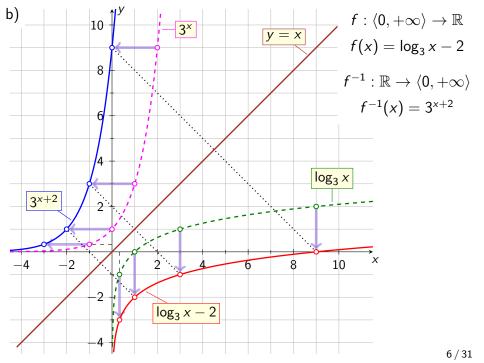


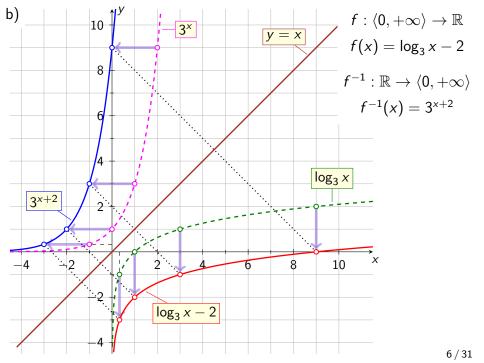


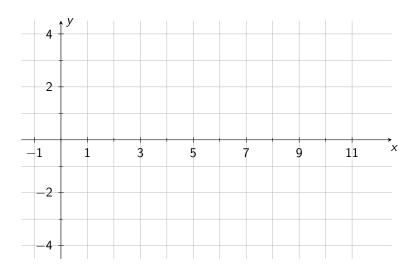


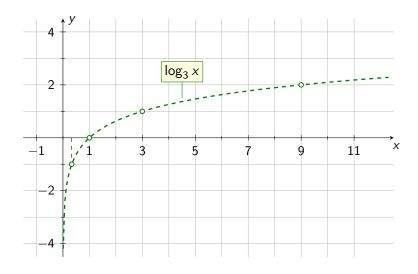


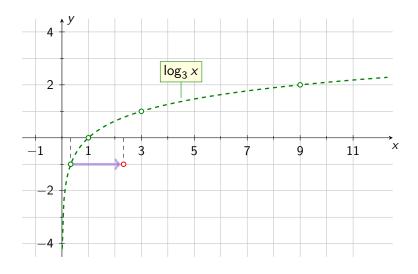


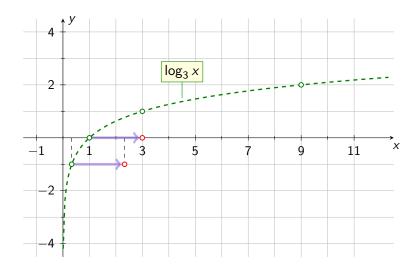


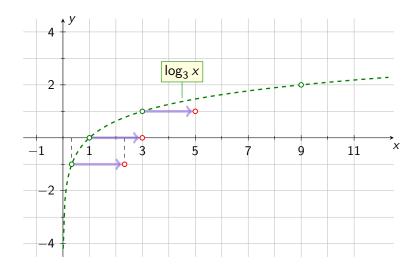


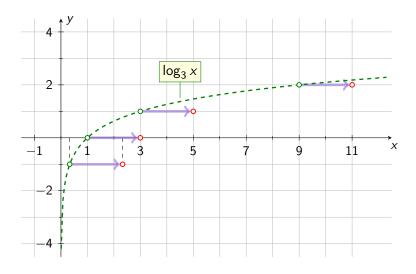


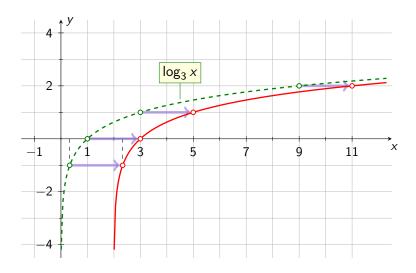


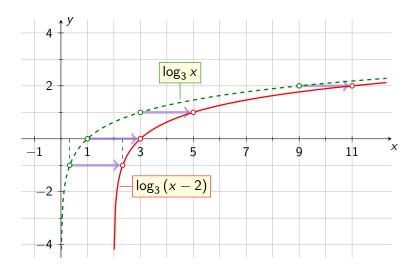


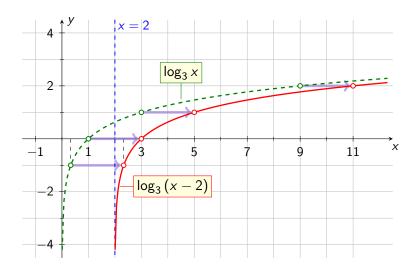


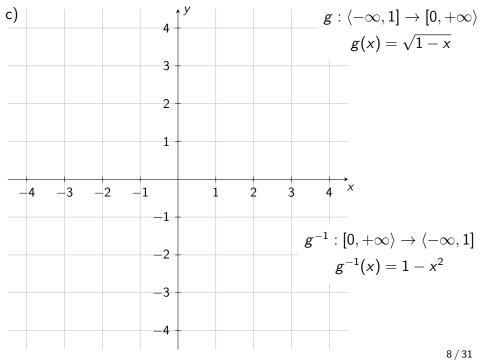


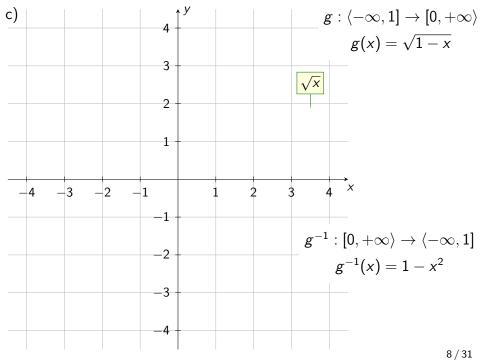


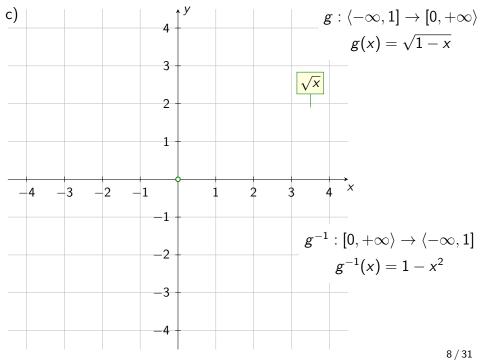


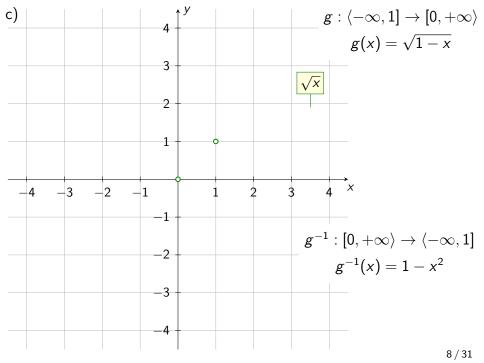


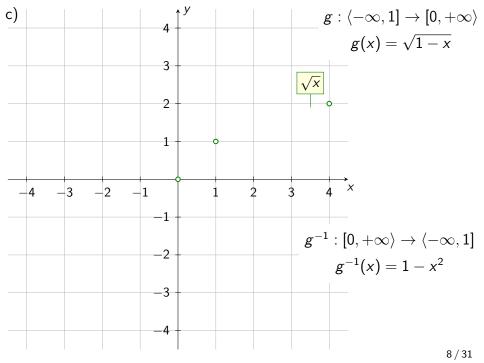


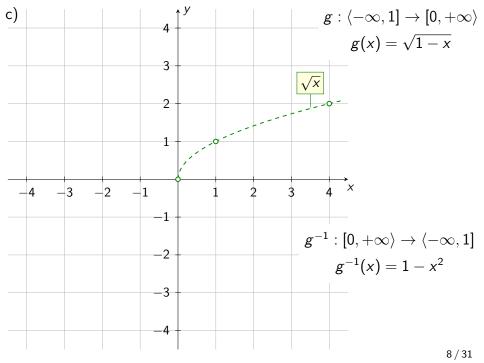


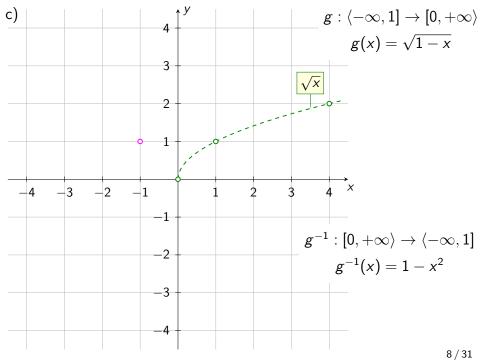


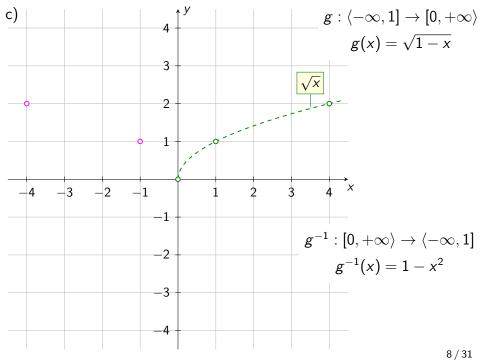


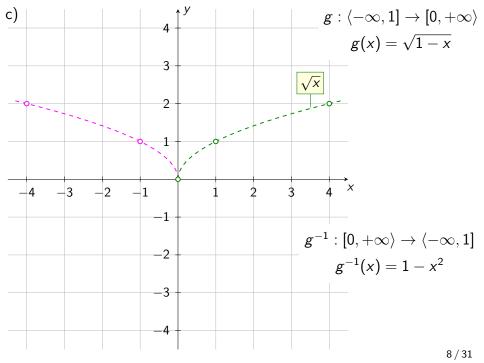


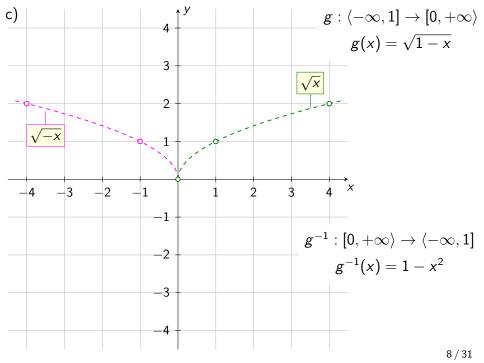


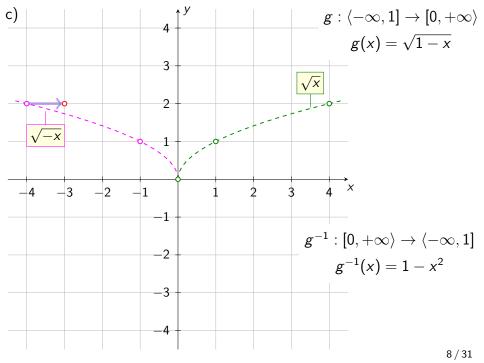


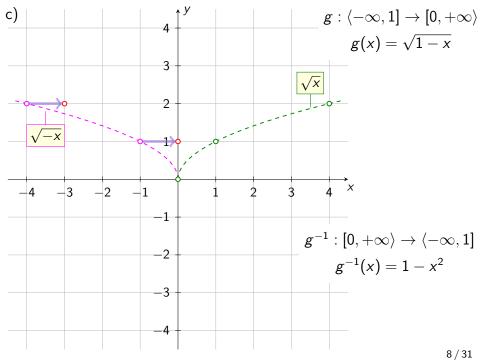


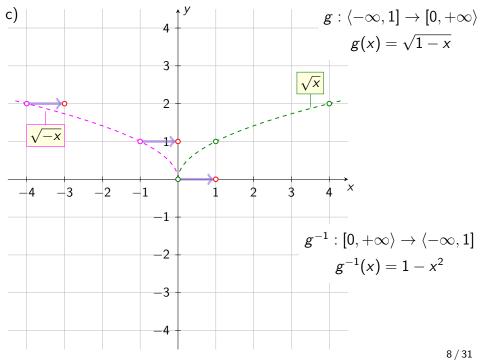


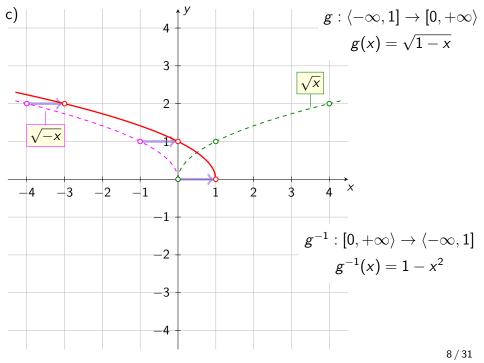


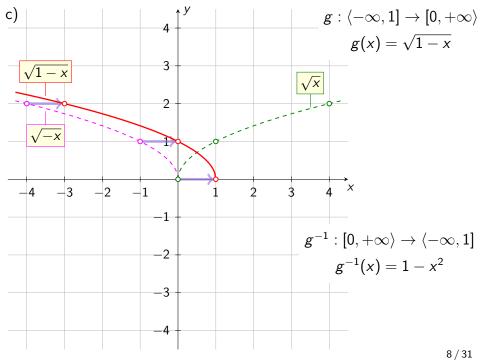


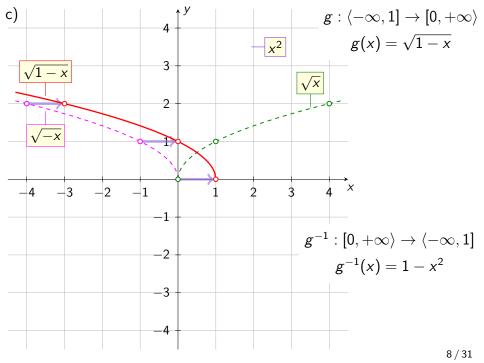


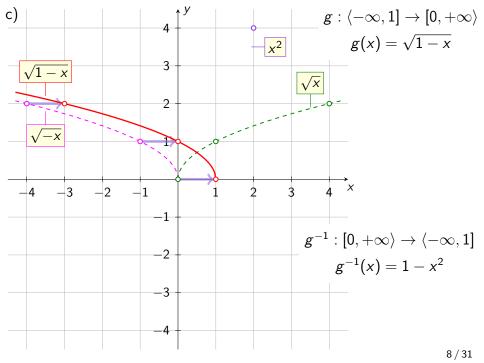


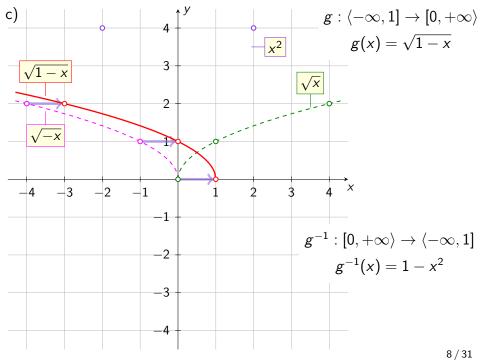


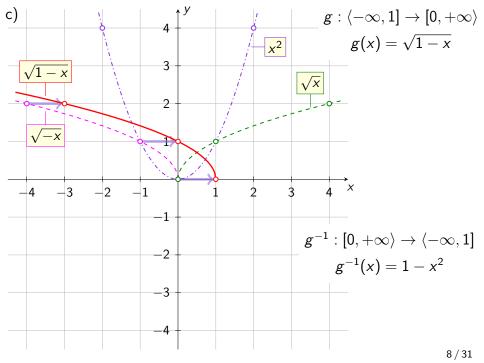


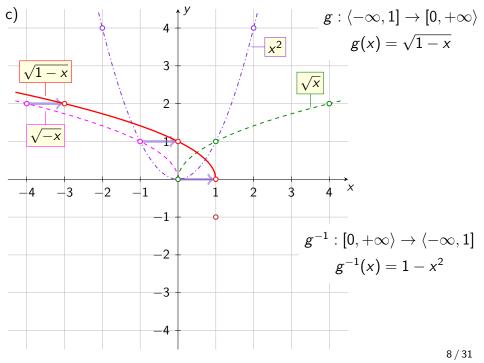


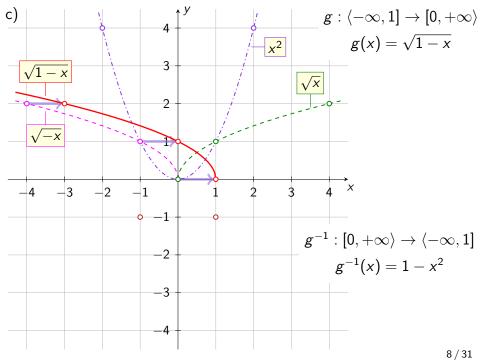


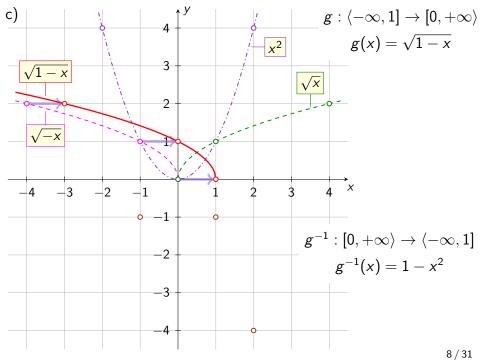


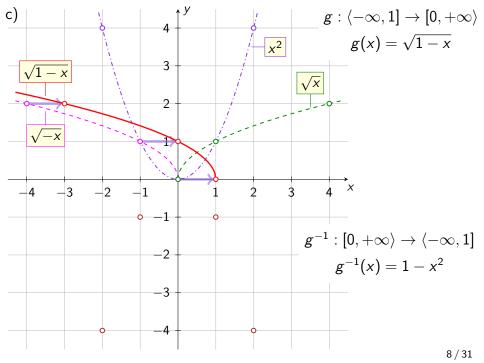


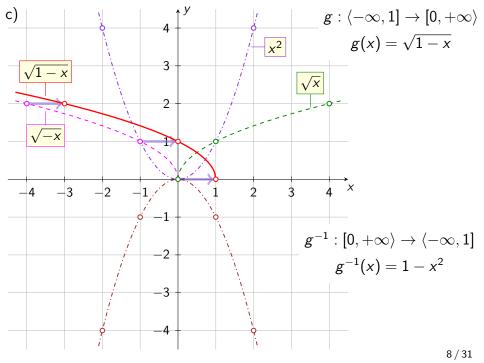


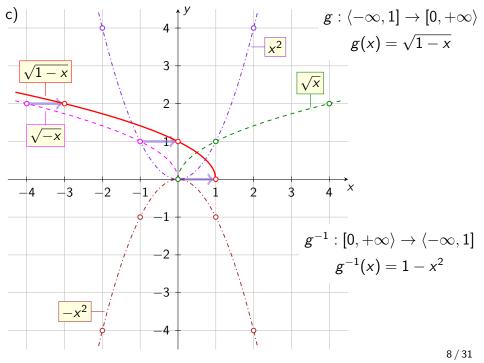


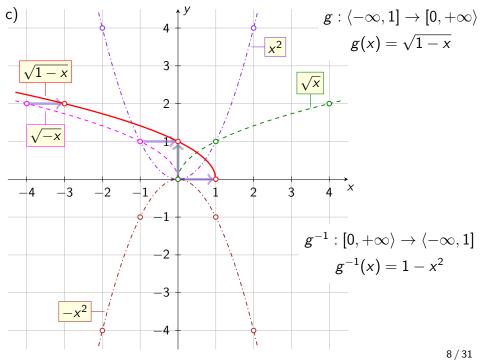


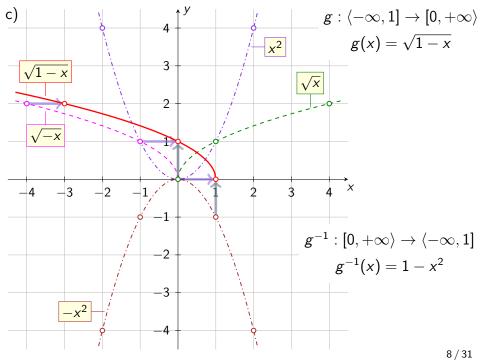


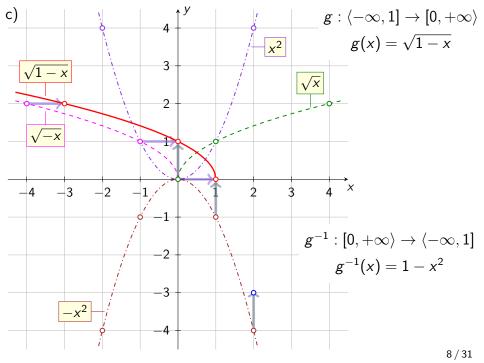


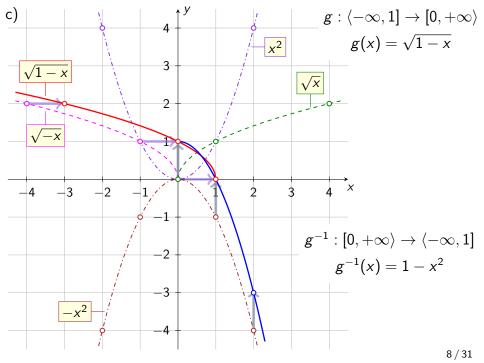


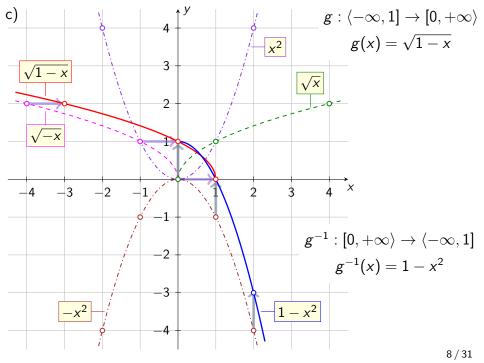


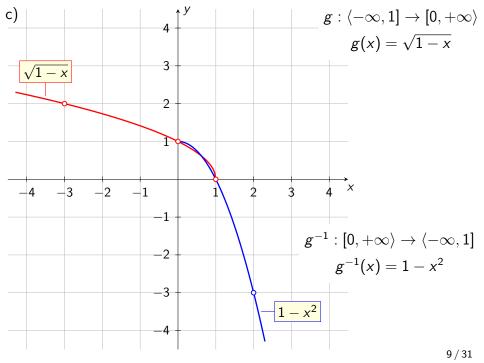


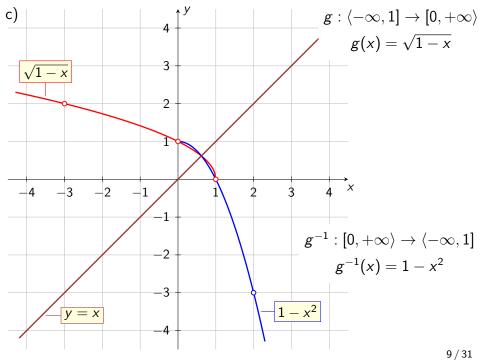


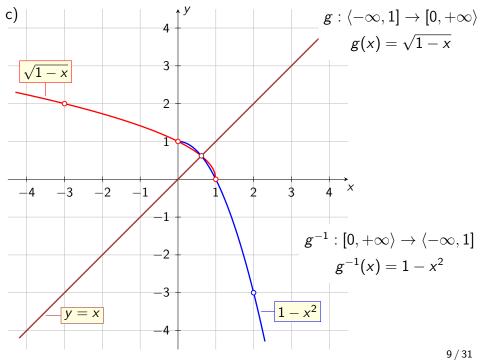


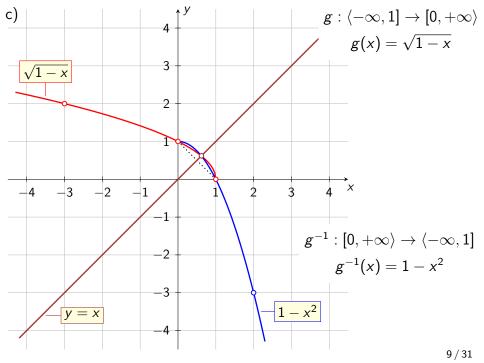


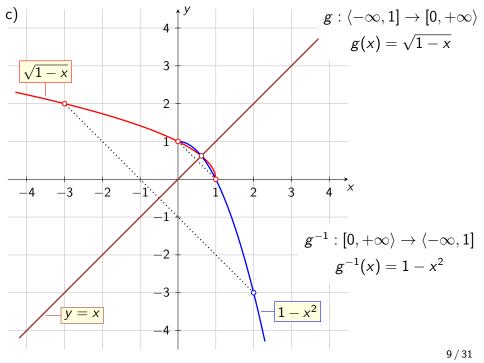












treći zadatak

Zadana je funkcija
$$f: \mathbb{R} \setminus \{1\} \to \mathbb{R}$$
, $f(x) = \frac{x+2}{x-1}$.

- a) Dokažite da je funkcija f monotona na intervalu $(1, +\infty)$.
- b) Dokažite da je funkcija f monotona na intervalu $\langle -\infty, 1 \rangle$.
- c) Dokažite da funkcija f nije monotona na svojoj prirodnoj domeni.

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$$f(x) = \frac{x+2}{x-1} = -$$

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Rješenje

$$f(x) = \frac{x+2}{x-1} = \frac{(x-1)}{x-1}$$

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Rješenje

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Rješenje

$$f(x) = \frac{x+2}{x-1} = \frac{(x-1)+3}{x-1} = \frac{x-1}{x-1} + \frac{3}{x-1}$$

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Rješenje

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Rješenje

$$f(x) = \frac{x+2}{x-1} = \frac{(x-1)+3}{x-1} = \frac{x-1}{x-1} + \frac{3}{x-1} = 1$$

Zadana je funkcija $f: \mathbb{R} \setminus \{1\} \to \mathbb{R}$, $f(x) = \frac{x+2}{x-1}$.

- a) Dokažite da je funkcija f monotona na intervalu $(1, +\infty)$.
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Rješenje

$$f(x) = \frac{x+2}{x-1} = \frac{(x-1)+3}{x-1} = \frac{x-1}{x-1} + \frac{3}{x-1} = 1 + \frac{3}{x-1}$$

a) Neka su $x_1, x_2 \in \langle 1, +\infty \rangle$ takvi da je $x_1 < x_2$. $\left| f(x) = 1 + \frac{3}{x-1} \right|$ $x_1 < x_2$

$$x_1 < 1$$

$$x_1 < x_2 \implies$$

 $x_1 - 1$

 $x_1 < x_2 \implies$

 $x_1 - 1 <$

 $x_1 < x_2 \implies$

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$$x_1 < x_2 \implies$$

$$x_1 < x_2 =$$

- $x_1 1 < x_2 1$

$$x_1 < x_2 \implies$$

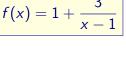
$$x_2 =$$

$$x_1 - 1 < x_2 - 1 / : (x_1 - 1)(x_2 - 1)$$

$$y=1+\frac{1}{x-1}$$

$$x_1 < x_2 \implies > 0$$

 $x_1 - 1 < x_2 - 1 / : (x_1 - 1)(x_2 - 1)$



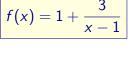
$$x_1 < x_2 \implies 0$$
 $x_1 - 1 < x_2 - 1 / : (x_1 - 1)(x_2 - 1)$

$$f(x) = 1 + \frac{3}{x-1}$$

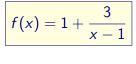
$$x_1 < x_2 \implies 0 > 0$$
 $x_1 - 1 < x_2 - 1 / : (x_1 - 1)(x_2 - 1)$

$$f(x) = 1 + \frac{3}{x - 1}$$

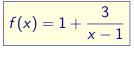
Neka su
$$x_1, x_2 \in \langle 1, +\infty \rangle$$
 takvi da jo $x_1 < x_2 \Longrightarrow 0 > 0$ $x_1 - 1 < x_2 - 1 / : (x_1 - 1)(x_2 - 1)$



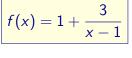
Neka su
$$x_1, x_2 \in \langle 1, +\infty \rangle$$
 takvi da je $x_1 < x_2 \Longrightarrow 0 \longrightarrow 0$ $x_1 - 1 < x_2 - 1 / : \underbrace{(x_1 - 1)(x_2 - 1)}_{>0}$



$$x_1 < x_2 \implies >0 >0$$
 $x_1 - 1 < x_2 - 1 /: (x_1 - 1)(x_2 - 1)$

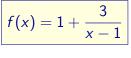


$$x_1 < x_2 \Longrightarrow 0 \longrightarrow 0$$
 $x_1 - 1 < x_2 - 1 / : \underbrace{(x_1 - 1)(x_2 - 1)}_{>0} \Longrightarrow$



$$x_1 < x_2 \Longrightarrow \underbrace{>_0}_{>0} \underbrace{>_0}_{>0}$$

$$x_1 - 1 < x_2 - 1 / : \underbrace{(x_1 - 1)(x_2 - 1)}_{>0} \Longrightarrow$$

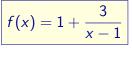


$$x_1 < x_2 \implies \underbrace{>_0}_{>0} >_0$$

$$x_1 - 1 < x_2 - 1 / : \underbrace{(x_1 - 1)(x_2 - 1)}_{>0} \implies$$

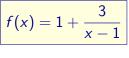
$$f(x)=1+\frac{3}{x-1}$$

$$x_{1} < x_{2} \implies 0 > 0 x_{1} - 1 < x_{2} - 1 / : \underbrace{(x_{1} - 1)(x_{2} - 1)}_{>0} \implies \frac{1}{x_{2} - 1} < \frac{1}{x_{1} - 1}$$



$$x_1 < x_2 \implies 0 > 0$$
 $x_1 - 1 < x_2 - 1 / : (x_1 - 1)(x_2 - 1) \implies 0$
 $\frac{1}{x_2 - 1} < \frac{1}{x_1 - 1} / \cdot 3$

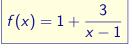
Ovdje je iznimno bitna pretpostavka da su oba broja
$$x_1$$
 i x_2 strogo veći od 1. Zbog toga su faktori x_1-1 i x_2-1 pozitivni pa je njihov produkt pozitivan. Stoga nejednadžbu dijelimo s pozitivnim brojem pa znak nejednakosti ostaje sačuvan.



$$x_{1} < x_{2} \Longrightarrow \underbrace{\begin{array}{c} > 0 \\ > 0 \\ x_{1} - 1 < x_{2} - 1 / : \underbrace{\left(x_{1} - 1\right)\left(x_{2} - 1\right)}_{> 0} \Longrightarrow \\ \frac{1}{x_{2} - 1} < \frac{1}{x_{1} - 1} / \cdot 3 \Longrightarrow \underbrace{\begin{array}{c} > 0 \\ > 0 \\ > 0 \end{array}}$$

Ovdje je iznimno bitna pretpostavka da su oba broja
$$x_1$$
 i x_2 strogo veći od 1. Zbog toga su faktori x_1-1 i x_2-1 pozitivni pa je njihov produkt pozitivan. Stoga nejednadžbu dijelimo s pozitivnim brojem pa znak nejednakosti ostaje sačuvan.

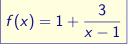
a) Neka su
$$x_1, x_2 \in \langle 1, +\infty \rangle$$
 takvi da je $x_1 < x_2$.



$$\frac{1}{x_2-1} < \frac{1}{x_1-1} / \cdot 3 \implies$$

$$\frac{3}{x_2-1}$$

a) Neka su
$$x_1, x_2 \in \langle 1, +\infty \rangle$$
 takvi da je $x_1 < x_2$.



$$x_1 < x_2 \Longrightarrow 0 \longrightarrow 0$$

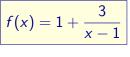
$$x_1 - 1 < x_2 - 1 / : \underbrace{(x_1 - 1)(x_2 - 1)}_{>0} \Longrightarrow$$

$$\frac{1}{x_2-1} < \frac{1}{x_1-1} / \cdot 3 \implies$$

$$\frac{3}{3}$$

Ovdje je iznimno bitna pretpostavka da su oba broja
$$x_1$$
 i x_2 strogo veći od 1. Zbog toga su faktori x_1-1 i x_2-1 pozitivni pa je njihov produkt pozitivan. Stoga nejednadžbu dijelimo s pozitivnim brojem pa znak nejednakosti ostaje sačuvan.

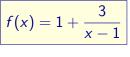
a) Neka su
$$x_1, x_2 \in \langle 1, +\infty \rangle$$
 takvi da je $x_1 < x_2$.



$$\frac{1}{x_2-1} < \frac{1}{x_1-1} / \cdot 3 \implies$$

$$\frac{3}{x_2 - 1} < \frac{3}{x_1 - 1}$$

a) Neka su
$$x_1, x_2 \in \langle 1, +\infty \rangle$$
 takvi da je $x_1 < x_2$.



$$x_1 < x_2 \Longrightarrow \underbrace{>_0}_{>0} \underbrace{>_0}_{>0} \Longrightarrow$$
 $x_1 - 1 < x_2 - 1 / : \underbrace{(x_1 - 1)(x_2 - 1)}_{>0} \Longrightarrow$

$$\frac{1}{\mathbf{r}_0 - 1} < \frac{1}{\mathbf{r}_1 - 1} / \cdot 3 \implies$$

$$\frac{3}{x_2-1} < \frac{3}{x_1-1} \implies$$

a) Neka su
$$x_1, x_2 \in \langle 1, +\infty \rangle$$
 takvi da je $x_1 < x_2$.

 $f(x) = 1 + \frac{3}{x-1}$

$$x_1 < x_2 \Longrightarrow \underbrace{>_0}_{>0} >_0$$

 $x_1 - 1 < x_2 - 1 / : \underbrace{(x_1 - 1)(x_2 - 1)}_{>0} \Longrightarrow$

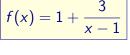
$$x_1 - 1 < x_2 - 1 / : (x_1 - 1)(x_2 - 1) = \frac{1}{x_2 - 1} < \frac{1}{x_1 - 1} / : 3 \Longrightarrow$$

$$\frac{3}{x_2-1} < \frac{3}{x_1-1} \Longrightarrow$$

Ovdje je iznimno bitna pretpostavka da su oba broja
$$x_1$$
 i x_2 strogo veći od 1. Zbog toga su faktori x_1-1 i x_2-1 pozitivni pa je njihov produkt pozitivan. Stoga nejednadžbu dijelimo s pozitivnim brojem pa znak nejednakosti ostaje sačuvan.

$$1+\frac{3}{x_2-}$$

a) Neka su
$$x_1, x_2 \in \langle 1, +\infty \rangle$$
 takvi da je $x_1 < x_2$.



$$x_1 < x_2 \Longrightarrow \underbrace{> 0} > 0$$
 $x_1 - 1 < x_2 - 1 / : \underbrace{(x_1 - 1)(x_2 - 1)} \Longrightarrow$

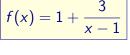
$$\frac{1}{\mathbf{r}_0 - 1} < \frac{1}{\mathbf{r}_1 - 1} / \cdot 3 \implies$$

Ovdje je iznimno bitna pretpostavka da su oba broja
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pa znak nejednakosti ostaje sačuvan.

$$\frac{3}{x_2 - 1} < \frac{3}{x_1 - 1} \implies 1 + \frac{3}{x_1 - 1} <$$

a) Neka su
$$x_1, x_2 \in \langle 1, +\infty \rangle$$
 takvi da je $x_1 < x_2$.



$$x_1 < x_2 \Longrightarrow \underbrace{>_0}_{>0} \underbrace{>_0}_{>0}$$

$$x_1 - 1 < x_2 - 1 / : \underbrace{(x_1 - 1)(x_2 - 1)}_{>0} \Longrightarrow$$

$$\frac{1}{x_2-1} < \frac{1}{x_1-1} / \cdot 3 \implies$$

$$\frac{3}{x_2 - 1} < \frac{3}{x_1 - 1} \implies \frac{3}{x_1 - 1} < \frac{3}{x_1 - 1}$$

$$1 + \frac{3}{x_2 - 1} < 1 + \frac{3}{x_1 - 1}$$

a) Neka su
$$x_1, x_2 \in \langle 1, +\infty \rangle$$
 takvi da je $x_1 < x_2$.

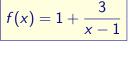
$$x_1 < x_2 \implies 0 > 0$$
 $x_1 - 1 < x_2 - 1 / : (x_1 - 1)(x_2 - 1) \implies$

$$\frac{1}{x_2-1} < \frac{1}{x_1-1} / \cdot 3 \implies$$

$$\frac{3}{x_2-1} < \frac{3}{x_1-1} \Longrightarrow$$

$$1 + \frac{3}{x_2 - 1} < 1 + \frac{3}{x_1 - 1} \implies$$

a) Neka su
$$x_1, x_2 \in \langle 1, +\infty \rangle$$
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$$\frac{1}{x_2 - 1} < \frac{1}{x_1 - 1} / \cdot 3 \implies$$

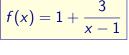
 $f(x_2)$

$$\frac{3}{x_2-1}<\frac{3}{x_1-1}\implies$$

$$1 + \frac{3}{x_2 - 1} < 1 + \frac{3}{x_1 - 1} \implies$$

Ovdje je iznimno bitna pretpostavka da su oba broja
$$x_1$$
 i x_2 strogo veći od 1. Zbog toga su faktori $x_1 - 1$ i $x_2 - 1$ pozitivni pa je njihov produkt pozitivan. Stoga nejednadžbu dijelimo s pozitivnim brojem pa znak nejednakosti ostaje sačuvan.

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 takvi da je $x_1 < x_2$.



$$x_1 < x_2 \implies \underbrace{>_0}_{>0} >_0$$

 $x_1 - 1 < x_2 - 1 / : (x_1 - 1)(x_2 - 1) \implies$

$$\frac{1}{x_2-1} < \frac{1}{x_1-1} / \cdot 3 \implies$$

$$\frac{3}{x_2-1}<\frac{3}{x_1-1}\implies$$

$$1 + \frac{3}{x_2 - 1} < 1 + \frac{3}{x_1 - 1} \implies$$

$$f(x_2) <$$

a) Neka su
$$x_1, x_2 \in \langle 1, +\infty \rangle$$
 takvi da je $x_1 < x_2$.

$$f(x) = 1 + \frac{3}{x-1}$$

$$x_1 < x_2 \implies 0 > 0$$

 $x_1 - 1 < x_2 - 1 / : (x_1 - 1)(x_2 - 1) \implies$

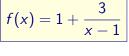
$$\frac{1}{x_2-1} < \frac{1}{x_1-1} / \cdot 3 \implies$$

$$\frac{3}{x_2-1}<\frac{3}{x_1-1}\implies$$

$$1 + \frac{3}{x_2 - 1} < 1 + \frac{3}{x_1 - 1} \implies$$

$$f(x_2) < f(x_1)$$

a) Neka su
$$x_1, x_2 \in \langle 1, +\infty \rangle$$
 takvi da je $x_1 < x_2$.



$$\frac{1}{x_2-1} < \frac{1}{x_1-1} / \cdot 3 \implies$$

$$\frac{3}{x_2-1}<\frac{3}{x_1-1}\implies$$

$$1 + \frac{3}{x_2 - 1} < 1 + \frac{3}{x_2 - 1} \implies$$

$$f(x_2) < f(x_1) \implies$$

$$f(x) = 1 + \frac{3}{x-1}$$

$$x_1 < x_2 \Longrightarrow 0 \longrightarrow 0$$

$$x_1 - 1 < x_2 - 1 / : \underbrace{(x_1 - 1)(x_2 - 1)}_{0} \Longrightarrow$$

$$\frac{1}{x_2 - 1} < \frac{1}{x_1 - 1} / \cdot 3 \implies$$

$$\frac{3}{x_2 - 1} < \frac{3}{x_1 - 1} \implies$$

$$\frac{3}{1-1} =$$

$$1 + \frac{3}{x_2 - 1} < 1 + \frac{3}{x_1 - 1} \implies$$

$$f(x_2) < f(x_1) \implies$$

$$f(x_1)$$

$$f(x) = 1 + \frac{3}{x-1}$$

$$x_{1} < x_{2} \Longrightarrow 0 \longrightarrow 0$$

$$x_{1} - 1 < x_{2} - 1 / : \underbrace{(x_{1} - 1)(x_{2} - 1)}_{>0} \Longrightarrow$$

$$\frac{1}{x_{2} - 1} < \frac{1}{x_{1} - 1} / \cdot 3 \Longrightarrow$$

$$\frac{3}{x_2-1} < \frac{3}{x_1-1} \Longrightarrow$$

$$\frac{3}{x_2 - 1} < \frac{3}{x_1 - 1} \Longrightarrow$$

$$1 + \frac{3}{x_2 - 1} < 1 + \frac{3}{x_2 - 1} \Longrightarrow$$

$$f(x_2) < f(x_1) \implies$$

$$f(x_2) < f(x_1) =$$

$$f(x_1) >$$

oba broja
$$x_1$$
 i x_2 strogo veći od 1. Zbog toga su faktori x_1-1 i x_2-1 pozitivni pa je njihov produkt pozitivan. Stoga nejednadžbu dijelimo s pozitivnim brojem pa znak nejednakosti ostaje sačuvan.

Ovdje je iznimno bitna pretpostavka da su

$$f(x) = 1 + \frac{3}{x - 1}$$

$$x_{1} < x_{2} \Longrightarrow 0 > 0$$

$$x_{1} - 1 < x_{2} - 1 / : \underbrace{(x_{1} - 1)(x_{2} - 1)}_{> 0} \Longrightarrow$$

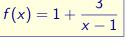
$$\frac{1}{x_{2} - 1} < \frac{1}{x_{1} - 1} / \cdot 3 \Longrightarrow$$

$$\frac{3}{x_2-1} < \frac{3}{x_1-1} \Longrightarrow$$

$$1 + \frac{3}{x_2 - 1} < 1 + \frac{3}{x_1 - 1} \implies$$

$$f(x_2) < f(x_1) \implies$$

$$f(x_1) > f(x_2)$$



$$x_{1} < x_{2} \Longrightarrow 0 > 0$$

$$x_{1} - 1 < x_{2} - 1 / : (x_{1} - 1)(x_{2} - 1) \Longrightarrow$$

$$\frac{1}{x_{2} - 1} < \frac{1}{x_{1} - 1} / \cdot 3 \Longrightarrow$$

$$\frac{3}{x_2-1} < \frac{3}{x_1-1} \Longrightarrow$$

$$1 + \frac{3}{x_2 - 1} < 1 + \frac{3}{x_1 - 1} \implies$$

$$f(x_2) < f(x_1) \implies$$

$$f(x_1) > f(x_2)$$

$$x_1 < x_2 \Longrightarrow 0 > 0$$

$$x_1 - 1 < x_2 - 1 / : (x_1 - 1)(x_2 - 1) \Longrightarrow$$

$$\frac{1}{x_2 - 1} < \frac{1}{x_1 - 1} / \cdot 3 \implies$$

$$\frac{3}{x_2 - 1} < \frac{3}{x_1 - 1} \implies$$

$$\frac{3}{1-1} =$$

$$1 + \frac{3}{x_2 - 1} < 1 + \frac{3}{x_1 - 1} \implies$$
$$f(x_2) < f(x_1) \implies$$

$$f(x_1) > f(x_2)$$

$$(x) = 1 + \frac{1}{x - 1}$$

Ovdje je iznimno bitna pretpostavka da su

 $(\forall x_1, x_2 \in \mathcal{D}_f)(x_1 < x_2 \Rightarrow f(x_1) > f(x_2))$

$$\begin{vmatrix} x_1 < x_2 \end{vmatrix} \Longrightarrow \begin{vmatrix} 0 & 0 \\ x_1 - 1 < x_2 - 1 / \vdots \underbrace{(x_1 - 1)(x_2 - 1)}_{0 > 0} \Longrightarrow$$

$$\frac{1}{x_2 - 1} < \frac{1}{x_1 - 1} / \cdot 3 \implies$$

$$\frac{3}{x_1 - 1} < \frac{3}{x_1 - 1} \implies$$

$$\frac{3}{x_2-1}<\frac{3}{x_1-1}\Longrightarrow \qquad \qquad \text{oba broja x_1 i x_2 strogo veći od 1. Zbog} \\ 1+\frac{3}{x_2-1}<1+\frac{3}{x_1-1}\Longrightarrow \qquad \qquad \text{oba broja x_1 i x_2 strogo veći od 1. Zbog} \\ 1+\frac{3}{x_2-1}<1+\frac{3}{x_1-1}\Longrightarrow \qquad \qquad \text{nejednadžbu dijelimo s pozitivnim brojem} \\ pa znak nejednakosti ostaje sačuvan.}$$

$$f(x_2) < f(x_1) \implies$$

$$f(x_2) < f(x_1) = f(x_1) > f(x_2)$$

$$x_1 - 1 < x_2 - 1 / : \underbrace{(x_1 - 1)(x_2 - 1)}_{>0} \Longrightarrow$$

$$\frac{1}{x_2 - 1} < \frac{1}{x_1 - 1} / \cdot 3 \Longrightarrow$$

$$\frac{3}{x_2-1} < \frac{3}{x_1-1} \Longrightarrow \qquad \text{oba broja } x_1 \text{ i } x_2 \text{ strogo veći od } 1. \text{ Zbog} \\ \text{toga su faktori } x_1-1 \text{ i } x_2-1 \text{ pozitivni pa} \\ \text{je njihov produkt pozitivan. Stoga} \\ 1+\frac{3}{x_2-1} < 1+\frac{3}{x_1-1} \Longrightarrow \qquad \text{nejednadžbu dijelimo s pozitivnim brojem} \\ \text{pa znak nejednakosti ostaje sačuvan.}$$

$$f(x_2) < f(x_1) \Longrightarrow$$

$$f(x_1) > f(x_2)$$
Stroggo
$$(\forall x_1, x_2)$$

Strogo padajuća funkcija
$$(\forall x_1, x_2 \in \mathcal{D}_f)(x_1 < x_2 \Rightarrow f(x_1) > f(x_2))$$

Ovdje je iznimno bitna pretpostavka da su

f strogo pada na $\langle 1, +\infty \rangle$

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 $x_1 < x_2$

 $x_1 < x_2 \implies$

 $x_1 - 1$

 $x_1 < x_2 \implies$

 $x_1 - 1 <$

 $x_1 < x_2 \implies$

$$x_1 < x_2 \implies$$

 $x_1 - 1 < x_2 - 1$

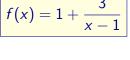
$$x-1$$

$$x_1 < x_2 \implies$$

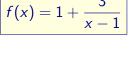
$$x_1 - 1 < x_2 - 1 / : (x_1 - 1)(x_2 - 1)$$

$$f(x) = 1 + \frac{3}{x-1}$$

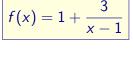
$$x_1 < x_2 \implies \underbrace{<0}_{x_1 - 1 < x_2 - 1 / : (x_1 - 1)(x_2 - 1)}$$



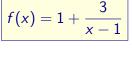
$$x_1 < x_2 \implies 0$$
 $x_1 - 1 < x_2 - 1 / : (x_1 - 1)(x_2 - 1)$



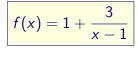
$$x_1 < x_2 \implies 0 < 0$$
 $x_1 - 1 < x_2 - 1 / : (x_1 - 1)(x_2 - 1)$



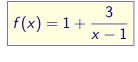
Neka su
$$x_1, x_2 \in \langle -\infty, 1 \rangle$$
 takvi da jo $x_1 < x_2 \Longrightarrow 0$ $x_1 - 1 < x_2 - 1 / : (x_1 - 1)(x_2 - 1)$



Neka su
$$x_1, x_2 \in \langle -\infty, 1 \rangle$$
 takvi da je $x_1 < x_2 \Longrightarrow \begin{cases} 0 & < 0 \\ x_1 - 1 < x_2 - 1 / : \underbrace{(x_1 - 1)(x_2 - 1)}_{>0} \end{cases}$

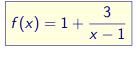


$$x_1 < x_2 \implies 0 < 0$$
 $x_1 - 1 < x_2 - 1 / : (x_1 - 1)(x_2 - 1)$
 $x_1 < x_2 = 0$

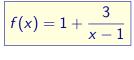


$$x_1 < x_2 \implies \underbrace{\begin{array}{c} 0 \\ 0 \\ 0 \\ \end{array}}_{>0} \implies$$

$$x_1 - 1 < x_2 - 1 / : \underbrace{\left(x_1 - 1\right)\left(x_2 - 1\right)}_{>0} \implies$$



$$x_1 < x_2 \implies \underbrace{\begin{array}{c} <0 \\ <0 \\ x_1 - 1 < x_2 - 1 \) : \underbrace{(x_1 - 1)(x_2 - 1)}_{>0} \implies \\ \underbrace{1}$$

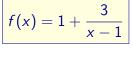


$$x_{1} < x_{2} \Longrightarrow \underbrace{\begin{array}{c} <0 \\ <0 \\ <1 \end{array}} \Longrightarrow \underbrace{\begin{array}{c} <1 \\ =1 \\ >0 \end{array}} \Longrightarrow \underbrace{\begin{array}{c} 1 \\ <0 \\ <1 \end{array}} \Longrightarrow \underbrace{\begin{array}{c} <0 \\ <1 \\ >0 \end{array}} \Longrightarrow \underbrace{\begin{array}{c} <0 \\ <1 \\ >0 \end{array}} \Longrightarrow \underbrace{\begin{array}{c} <0 \\ <1 \\ <1 \end{array}} \Longrightarrow \underbrace{\begin{array}{c} <0 \\ <1 \end{array}} \Longrightarrow \underbrace{\begin{array}{c} <0 \\ <1 \\ <1 \end{array}} \Longrightarrow \underbrace{\begin{array}{c} <0 \\ <1 \\ <1 \end{array}} \Longrightarrow \underbrace{\begin{array}{c} <0 \\ <1 \end{array}} \Longrightarrow \underbrace{\begin{array}{c} <0 \\ <1 \\ <1 \end{array}} \Longrightarrow \underbrace{\begin{array}{c} <0 \\ <1 \end{array}} \Longrightarrow \underbrace{\begin{array}{c} <0$$

$$f(x) = 1 + \frac{3}{x-1}$$

$$x_{1} < x_{2} \Longrightarrow \underbrace{\begin{array}{c} 0 \\ 0 \\ x_{1} - 1 < x_{2} - 1 \end{array}}_{>0} \Longrightarrow \underbrace{\begin{array}{c} 1 \\ x_{2} - 1 \end{array}}_{>0} < \underbrace{\begin{array}{c} 1 \\ x_{1} - 1 \end{array}}_{>0} \Longrightarrow \underbrace{\begin{array}{c} 1 \\ x_{2} - 1 \end{array}}_{>0}$$

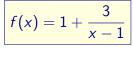
b) Neka su
$$x_1, x_2 \in \langle -\infty, 1 \rangle$$
 takvi da je $x_1 < x_2$.



$$x_{1} < x_{2} \Longrightarrow \underbrace{\begin{array}{c} 0 \\ < 0 \\ < 1 \end{array}}_{>0} \Longrightarrow \underbrace{\frac{1}{x_{2} - 1} < \frac{1}{x_{1} - 1} / \cdot 3}$$

Ovdje je iznimno bitna pretpostavka da su oba broja
$$x_1$$
 i x_2 strogo manji od 1. Zbog toga su faktori $x_1 - 1$ i $x_2 - 1$ negativni pa je njihov produkt pozitivan. Stoga nejednadžbu dijelimo s pozitivnim brojem pa znak nejednakosti ostaje sačuvan.

b) Neka su
$$x_1, x_2 \in \langle -\infty, 1 \rangle$$
 takvi da je $x_1 < x_2$.



$$x_{1} < x_{2} \Longrightarrow \underbrace{\begin{array}{c} 0 \\ < 0 \\ < 1 \end{array}}_{x_{1} - 1} < x_{2} - 1 / : \underbrace{\left(x_{1} - 1\right)\left(x_{2} - 1\right)}_{> 0} \Longrightarrow \underbrace{\begin{array}{c} 1 \\ x_{2} - 1 \\ < 1 \end{array}}_{> 0} < \underbrace{\begin{array}{c} 1 \\ x_{1} - 1 \\ < 1 \end{array}}_{> 0} = \underbrace{\begin{array}{c} 1 \\ > 0 \\ > 0 \end{array}}_{> 0}$$

b) Neka su
$$x_1, x_2 \in \langle -\infty, 1 \rangle$$
 takvi da je $x_1 < x_2$.

$$f(x)=1+\frac{3}{x-1}$$

$$x_1 < x_2 \implies \underbrace{0 < 0}_{<0}$$

$$x_1 - 1 < x_2 - 1 / : \underbrace{(x_1 - 1)(x_2 - 1)}_{>0} \implies$$

$$\frac{1}{x_2-1} < \frac{1}{x_1-1} / \cdot 3 \implies$$

$$\frac{3}{x_2-1}$$

b) Neka su
$$x_1, x_2 \in \langle -\infty, 1 \rangle$$
 takvi da je $x_1 < x_2$.

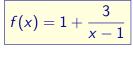
$$f(x)=1+\frac{3}{x-1}$$

$$x_{1} < x_{2} \Longrightarrow \underbrace{\begin{array}{c} <0 \\ <0 \\ x_{1}-1 < x_{2}-1 \end{array}}_{>0} \Longrightarrow \underbrace{\begin{array}{c} <0 \\ (x_{1}-1)(x_{2}-1) \end{array}}_{>0} \Longrightarrow$$

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pa znak nejednakosti ostaje sačuvan.

b) Neka su
$$x_1, x_2 \in \langle -\infty, 1 \rangle$$
 takvi da je $x_1 < x_2$.



$$x_1 < x_2 \Longrightarrow \underbrace{<0}_{<0} \underbrace{<0}_{<1}$$

$$x_1 - 1 < x_2 - 1 / : \underbrace{(x_1 - 1)(x_2 - 1)}_{>0} \Longrightarrow$$

$$\frac{1}{x_0-1} < \frac{1}{x_1-1} / \cdot 3 \implies$$

$$\frac{3}{x_2 - 1} < \frac{3}{x_1 - 1}$$

b) Neka su
$$x_1, x_2 \in \langle -\infty, 1 \rangle$$
 takvi da je $x_1 < x_2$.

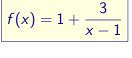
$$f(x)=1+\frac{3}{x-1}$$

$$x_1 < x_2 \Longrightarrow \underbrace{}_{<0} \underbrace{}_{<0} \times \underbrace{}_{$$

$$\frac{1}{\mathbf{r}_0 - 1} < \frac{1}{\mathbf{r}_1 - 1} / \cdot 3 \implies$$

$$\frac{3}{x_2-1}<\frac{3}{x_1-1}\implies$$

b) Neka su
$$x_1, x_2 \in \langle -\infty, 1 \rangle$$
 takvi da je $x_1 < x_2$.



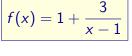
$$x_1 < x_2 \implies \begin{cases} 0 & < 0 \\ < 1 & < 1 \end{cases} = \begin{cases} x_1 - 1 < x_2 - 1 / : (x_1 - 1)(x_2 - 1) \end{cases} \implies$$

$$\frac{1}{x_2-1} < \frac{1}{x_1-1} / \cdot 3 \implies$$

$$\frac{3}{x_2 - 1} < \frac{3}{x_1 - 1} \implies 3$$

$$1+\frac{3}{x_2-1}$$

b) Neka su
$$x_1, x_2 \in \langle -\infty, 1 \rangle$$
 takvi da je $x_1 < x_2$.



$$x_1 < x_2 \implies 0 < 0$$

 $x_1 - 1 < x_2 - 1 / : (x_1 - 1)(x_2 - 1) \implies$

$$\frac{1}{x_2-1}<\frac{1}{x_1-1}\Big/\cdot 3\implies$$

Ovdje je iznimno bitna pretpostavka da su oba broja
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nejednadžbu dijelimo s pozitivnim brojem pa znak nejednakosti ostaje sačuvan.

$$\frac{3}{x_2-1} < \frac{3}{x_1-1} \implies$$

$$1 + \frac{3}{x_1 - 1} <$$

b) Neka su
$$x_1, x_2 \in \langle -\infty, 1 \rangle$$
 takvi da je $x_1 < x_2$.

$$f(x) = 1 + \frac{3}{x-1}$$

$$x_1 < x_2 \implies \underbrace{<0}_{<0} \underbrace{<0}_{<1}$$

$$x_1 - 1 < x_2 - 1 / : \underbrace{(x_1 - 1)(x_2 - 1)}_{<0} \implies$$

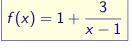
$$\frac{1}{x_2-1} < \frac{1}{x_1-1} / \cdot 3 \implies$$

$$\frac{3}{x_2-1}<\frac{3}{x_1-1}\implies$$

$$1 + \frac{3}{x_2 - 1} < 1 + \frac{3}{x_1 - 1}$$

Ovdje je iznimno bitna pretpostavka da su oba broja
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$$x_1, x_2 \in \langle -\infty, 1 \rangle$$
 takvi da je $x_1 < x_2$.



$$\frac{1}{x_2-1} < \frac{1}{x_1-1} / \cdot 3 \implies$$

$$\frac{3}{x_2-1} < \frac{3}{x_1-1} \implies$$

$$1 + \frac{3}{x_2 - 1} < 1 + \frac{3}{x_1 - 1} \implies$$

b) Neka su
$$x_1, x_2 \in \langle -\infty, 1 \rangle$$
 takvi da je $x_1 < x_2$.

$$x_1 < x_2 \implies \underbrace{\begin{array}{c} \\ <0 \\ <1 \end{array}}_{<0} = \underbrace{\begin{array}{c} \\ <0 \\ <1 \end{array}}_{<0} \implies \underbrace{\begin{array}{c} \\ <0 \\ <1 \end{array}}_{>0} \implies \underbrace{\begin{array}{c} \\ <0 \\ <1 \end{array}}_{>0} = \underbrace{\begin{array}{c} \\ <0 \\ <0 \end{array}}_{>0} = \underbrace{\begin{array}{c} \\ <0 \\ <1 \end{array}}_{>0} = \underbrace{\begin{array}{c} \\ <0 \\ <0 \end{array}}_{>0} = \underbrace{\begin{array}{c} \\ <0 \\ <1 \end{array}}_{>0} = \underbrace{\begin{array}{c}$$

$$\frac{1}{x_2 - 1} < \frac{1}{x_1 - 1} / \cdot 3 \implies$$

$$\frac{3}{x_2 - 1} < \frac{3}{x_1 - 1} \implies$$

$$1 + \frac{3}{x_2 - 1} < 1 + \frac{3}{x_1 - 1} \implies$$

 $f(x_2)$

$$<1+\frac{3}{x_1-1}$$

b) Neka su
$$x_1, x_2 \in \langle -\infty, 1 \rangle$$
 takvi da je $x_1 < x_2$.

$$x_1 < x_2 \implies \underbrace{\phantom{\left(\begin{array}{c} 0 \\ < 0 \\ < 1 \end{array}\right)}}_{< 0} < \underbrace{\phantom{\left(\begin{array}{c} 0 \\ < 1 \end{array}\right)}}_{< 0} = \underbrace{\phantom{\left(\begin{array}{c} 0$$

$$\frac{1}{x_2 - 1} < \frac{1}{x_1 - 1} / \cdot 3 \implies$$

$$\frac{3}{x_2 - 1} < \frac{3}{x_1 - 1} \implies$$

$$\frac{3}{-1} =$$

$$1 + \frac{3}{x_2 - 1} < 1 + \frac{3}{x_1 - 1} \implies$$

$$f(x_2) <$$

b) Neka su
$$x_1, x_2 \in \langle -\infty, 1 \rangle$$
 takvi da je $x_1 < x_2$.

$$x_1 < x_2 \implies \underbrace{\phantom{\left(\begin{array}{c} 0 \\ 0 \\ 0 \end{array}\right)}}_{<0} = x_1 - 1 < x_2 - 1 / : \underbrace{\left(x_1 - 1\right)\left(x_2 - 1\right)}_{>0} \implies$$

$$\frac{1}{x_2 - 1} < \frac{1}{x_1 - 1} / \cdot 3 \implies$$

$$\frac{3}{x_2 - 1} < \frac{3}{x_1 - 1} \implies$$

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$$1 + \frac{3}{x_2 - 1} < 1 + \frac{3}{x_1 - 1} \implies$$

$$f(x_2) < f(x_1)$$

b) Neka su
$$x_1, x_2 \in \langle -\infty, 1 \rangle$$
 takvi da je $x_1 < x_2$.

$$x_1 < x_2 \implies \underbrace{\begin{array}{c} \\ <0 \\ <1 \end{array}} = \underbrace{\begin{array}{c} \\ <0 \\ (x_1-1)(x_2-1) \end{array}} \implies \underbrace{\begin{array}{c} \\ <0 \\ <1 \end{array}} = \underbrace{\begin{array}{c} \\ <0 \\ <1 \end{array}$$

$$\frac{1}{x_2 - 1} < \frac{1}{x_1 - 1} / \cdot 3 \implies$$

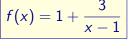
$$\frac{3}{x_2 - 1} < \frac{3}{x_1 - 1} \implies$$

$$\frac{3}{-1} =$$

$$1 + \frac{3}{x_2 - 1} < 1 + \frac{3}{x_1 - 1} \implies$$

$$f(x_2) < f(x_1) \implies$$

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$$x_1, x_2 \in \langle -\infty, 1 \rangle$$
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$$x_1 < x_2 \implies \underbrace{<0}_{<0} < \underbrace{<0}_{<1} + \underbrace{<0}_{<1} < \underbrace{<0}_{<1} + \underbrace{<0}_{<1} = \underbrace{<0}_{<1} + \underbrace{<0}_{<1} = \underbrace{<0}_{<1} + \underbrace{<0}_{<1} = \underbrace{<0}_{<0}_{<1} = \underbrace{<0}_{<1} =$$

$$\frac{1}{x_2 - 1} < \frac{1}{x_1 - 1} / \cdot 3 \implies$$

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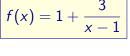
$$1 + \frac{3}{x_2 - 1} < 1 + \frac{3}{x_1 - 1} \implies$$
$$f(x_2) < f(x_1) \implies$$

pa znak nejednakosti ostaje sačuvan.

$$f(x_2) < f(x_1) \implies$$

$$f(x_1)$$

b) Neka su
$$x_1, x_2 \in \langle -\infty, 1 \rangle$$
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$$x_1 < x_2 \Longrightarrow \underbrace{<0}_{<0} \underbrace{<0}_{<1}$$
 $x_1 - 1 < x_2 - 1 / : \underbrace{(x_1 - 1)(x_2 - 1)}_{0} \Longrightarrow$

$$\frac{1}{x_2 - 1} < \frac{1}{x_1 - 1} / \cdot 3 \implies$$

$$\frac{3}{x_2 - 1} < \frac{3}{x_1 - 1} \implies$$

$$\frac{3}{-1} =$$

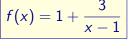
$$1 + \frac{3}{x_2 - 1} < 1 + \frac{3}{x_1 - 1} \implies$$

$$f(x_2) < f(x_1) \implies$$

$$f(x_2) < f(x_1) \implies$$

$$f(x_1) >$$

b) Neka su
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$$x_1 < x_2 \Longrightarrow \underbrace{\begin{array}{c} <0 \\ <0 \end{array}}_{<0} = \underbrace{\begin{array}{c} <0 \\ (x_1-1)(x_2-1) \end{array}}_{>0} \Longrightarrow$$

$$\frac{1}{x_2 - 1} < \frac{1}{x_1 - 1} / \cdot 3 \implies$$

$$\frac{3}{x_2 - 1} < \frac{3}{x_1 - 1} \implies$$

$$\frac{1}{-1} =$$

$$f(x_2) < f(x_1) \Longrightarrow$$

$$f(x_1) > f(x_2)$$

 $1 + \frac{3}{x_2 - 1} < 1 + \frac{3}{x_2 - 1} \implies$

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$$x_1, x_2 \in \langle -\infty, 1 \rangle$$
 takvi da je $x_1 < x_2$.

 $f(x) = 1 + \frac{3}{x-1}$

$$|x_1 < x_2| \Longrightarrow \langle 0 \rangle \langle 0 \rangle \langle 0 \rangle \langle 1 \rangle \langle$$

$$\frac{1}{x_2-1}<\frac{1}{x_1-1}\Big/\cdot 3\implies$$

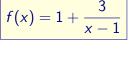
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b) Neka su
$$x_1, x_2 \in \langle -\infty, 1 \rangle$$
 takvi da je $x_1 < x_2$.



$$|x_1 < x_2| \Longrightarrow \langle 0 \rangle$$

$$|x_1 - 1| < |x_2 - 1| : (x_1 - 1)(x_2 - 1) \rangle \Longrightarrow$$

$$\frac{1}{x_2 - 1} < \frac{1}{x_1 - 1} / \cdot 3 \implies$$

$$\frac{3}{x_2-1}<\frac{3}{x_1-1}\implies$$

$$1 + \frac{3}{x_2 - 1} < 1 + \frac{3}{x_1 - 1} \implies$$

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b) Neka su
$$x_1, x_2 \in \langle -\infty, 1 \rangle$$
 takvi da je $x_1 < x_2$.

× 1

$$x_{1} - 1 < x_{2} - 1 / : \underbrace{\begin{pmatrix} 0 & < 0 \\ (x_{1} - 1)(x_{2} - 1) \end{pmatrix}}_{> 0} \Longrightarrow$$

$$\frac{1}{x_{2} - 1} < \frac{1}{x_{1} - 1} / \cdot 3 \Longrightarrow$$

$$\frac{3}{x_2-1} < \frac{3}{x_1-1} \Longrightarrow$$

Ovdje je iznimno bitna pretpostavka da su oba broja x_1 i x_2 strogo manji od 1. Zbog toga su faktori x_1-1 i x_2-1 negativni pa je njihov produkt pozitivan. Stoga nejednadžbu dijelimo s pozitivnim brojem pa znak nejednakosti ostaje sačuvan.

 $(\forall x_1, x_2 \in \mathcal{D}_f)(x_1 < x_2 \Rightarrow f(x_1) > f(x_2))$

$$1 + \frac{3}{x_2 - 1} < 1 + \frac{3}{x_1 - 1} \implies$$

$$f(x_2) < f(x_1) \implies$$

 $f(x_2) < f(x_1)$ $f(x_1) > f(x_2)$

b) Neka su
$$x_1, x_2 \in \langle -\infty, 1 \rangle$$
 takvi da je $x_1 < x_2$.

x - 1

$$x_1 - 1 < x_2 - 1 / : \underbrace{(x_1 - 1)(x_2 - 1)}_{>0} \Longrightarrow$$

$$\frac{1}{x_2 - 1} < \frac{1}{x_1 - 1} / \cdot 3 \Longrightarrow$$

$$\frac{3}{x_2-1}<\frac{3}{x_1-1}\Longrightarrow \qquad \qquad \text{oba broja }x_1\text{ i }x_2\text{ strogo manji od }1\text{. Zbog} \\ \log a \text{ su faktori }x_1-1\text{ i }x_2-1\text{ negativni pa} \\ je njihov produkt pozitivan. Stoga} \\ 1+\frac{3}{x_2-1}<1+\frac{3}{x_1-1}\Longrightarrow \qquad \qquad \text{nejednadžbu dijelimo s pozitivnim brojem} \\ pa \text{ znak nejednakosti ostaje sačuvan.}$$

$$f(x_2) < f(x_1) \implies$$
 $f(x_1) > f(x_2)$

Strogo padajuća funkcija
$$(\forall x_1, x_2 \in \mathcal{D}_f)(x_1 < x_2 \Rightarrow f(x_1) > f(x_2))$$

Ovdje je iznimno bitna pretpostavka da su

f strogo pada na $\langle -\infty, 1 \rangle$

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$$f(x) = 1 + \frac{3}{x - 1}$$

$$x_1 < x_2$$

$$x_1 < x_2 \implies$$

$$f(x) = 1 + \frac{1}{x-1}$$

$$x_1 < x_2 \implies$$

$$x_1 < x_2 =$$

 $x_1 - 1$

$$(x) = 1 + \frac{1}{x - 1}$$

$$x_1 < x_2 \implies$$

 $x_1 - 1 <$

$$x_1 < x_2 =$$

$$f(x) = 1 + \frac{3}{x - 1}$$

$$x_1 < x_2 \implies$$

$$x_1 - 1 < x_2 - 1$$

$$f(x) = 1 + \frac{3}{x - 1}$$

$$x_1 < x_2 \implies$$

$$x_1 - 1 < x_2 - 1 / : (x_1 - 1)(x_2 - 1)$$

$$f(x) = 1 + \frac{3}{x-1}$$

$$x_1 < x_2 \implies$$
 $x_1 - 1 < x_2 - 1 / : (x_1 - 1)(x_2 - 1)$

Kako su x_1 i x_2 bilo koji brojevi različiti od 1, faktori $x_1 - 1$ i $x_2 - 1$ mogu biti istog ili različitog predznaka. Stoga i njihov produkt može biti pozitivan ili negativan. Dakle, nakon dijeljenja nejednadžbe s $(x_1-1)(x_2-1)$ znak nejednakosti može, ali i ne mora ostati sačuvan.

$$f(x)=1+\frac{3}{x-1}$$

$$x_1 < x_2 \implies$$
 $x_1 - 1 < x_2 - 1 / : (x_1 - 1)(x_2 - 1)$

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Ako faktori x_1-1 i x_2-1 imaju iste predznake, tada znak nejednakosti ostaje sačuvan, a u protivnom se znak nejednakosti preokreće. To zapravo znači da funkcija f nije monotona na $\mathbb{R} \setminus \{1\}$.

$$f(x) = 1 + \frac{3}{x-1}$$

$$x_1 < x_2 \implies$$
 $x_1 - 1 < x_2 - 1 / : (x_1 - 1)(x_2 - 1)$

Kako su x_1 i x_2 bilo koji brojevi različiti od 1, faktori $x_1 - 1$ i $x_2 - 1$ mogu biti istog ili različitog predznaka. Stoga i njihov produkt može biti pozitivan ili negativan. Dakle, nakon dijeljenja nejednadžbe s $(x_1-1)(x_2-1)$ znak nejednakosti može, ali i ne mora ostati sačuvan.

Ako faktori $x_1 - 1$ i $x_2 - 1$ imaju iste predznake, tada znak nejednakosti ostaje sačuvan, a u protivnom se znak nejednakosti preokreće. To zapravo znači da funkcija f nije monotona na $\mathbb{R} \setminus \{1\}$.

Monotone funkcije moraju stalno čuvati znak nejednakosti (rastuće funkcije) ili ga moraju stalno preokretati (padajuće funkcije) za bilo koji izbor dva elementa x_1 i x_2 iz domene.

c) Možemo direktno protuprimjerom pokazati da funkcija f nije monotona na $\mathbb{R} \setminus \{1\}$.

$$f(x) = \frac{x+2}{x-1}$$

$$\mathbb{R}\setminus\{1\}=\langle -\infty,1\rangle\cup\langle 1,+\infty\rangle$$

c) Možemo direktno protuprimjerom pokazati da funkcija f nije monotona na $\mathbb{R}\setminus\{1\}$.

$$f(x) = \frac{x+2}{x-1}$$

$$f(-2) =$$

$$\mathbb{R}\setminus\{1\}=\langle-\infty,1\rangle\cup\langle1,+\infty\rangle$$

c) Možemo direktno protuprimjerom pokazati da funkcija f nije monotona na $\mathbb{R}\setminus\{1\}.$

$$f(-2) = \frac{-2+2}{-2-1}$$

$$(x) = \frac{x+2}{x-1}$$

$$\mathbb{R}\setminus\{1\}=\langle -\infty,1\rangle\cup\langle 1,+\infty\rangle$$

c) Možemo direktno protuprimjerom pokazati da funkcija f nije monotona na $\mathbb{R} \setminus \{1\}$.

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 nije monotona na $\mathbb{R}\setminus\{1\}.$

 $f(-2) = \frac{-2+2}{-2-1} = 0$

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$$f(x) = \frac{x+2}{x-1}$$

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, $f(0) = \frac{0+2}{0-1} = -2$, $f(2) = -2$

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$$-2 < 0$$

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$$-2 < 0 \implies f(-2) > f(0)$$

$$\mathbb{R}\setminus\{1\}=\langle-\infty,1\rangle\cup\langle1,+\infty\rangle$$

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$$-2 < 0 \implies f(-2) > f(0) \xrightarrow{\text{www}} f \text{ nije rastuća na } \mathbb{R} \setminus \{1\}$$

$$\downarrow | \qquad \qquad | \qquad \qquad | \text{jer nije sačuvan znak}$$

$$0 \qquad -2 \qquad \qquad \qquad \text{nejednakosti}$$

$$\mathbb{R}\setminus\{1\}=\langle-\infty,1\rangle\cup\langle1,+\infty\rangle$$

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$$0 \qquad -2 \qquad \qquad \text{nejednakosti}$$

$$\mathbb{R}\setminus\{1\}=\langle -\infty,1\rangle\cup\langle 1,+\infty\rangle$$

$$f(x) = \frac{x+2}{x-1}$$

$$0 < 2 \implies f(0) < f(2)$$

$$\mathbb{R}\setminus\{1\}=\langle-\infty,1
angle\cup\langle1,+\infty
angle$$

$$f(x) = \frac{x+2}{x-1}$$

$$f(-2) = \frac{-2+2}{-2-1} = 0, \quad f(0) = \frac{0+2}{0-1} = -2, \quad f(2) = \frac{2+2}{2-1} = 4$$

$$-2 < 0 \implies f(-2) > f(0) \xrightarrow{\text{local properties of mije rastuća na } \mathbb{R} \setminus \{1\}$$

$$\downarrow \text{jer nije sačuvan znak}$$

$$0 \qquad -2 \qquad \text{nejednakosti}$$

$$\mathbb{R}\setminus\{1\}=\langle-\infty,1
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$$-2 < 0 \implies f(-2) > f(0) \xrightarrow{\text{www}} f \text{ nije rastuća na } \mathbb{R} \setminus \{1\}$$

$$\parallel \qquad \qquad \parallel \qquad \qquad \text{jer nije sačuvan znak}$$

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$$\mathbb{R}\setminus\{1\}=\langle-\infty,1
angle\cup\langle1,+\infty
angle$$

$$f(x) = \frac{x+2}{x-1}$$

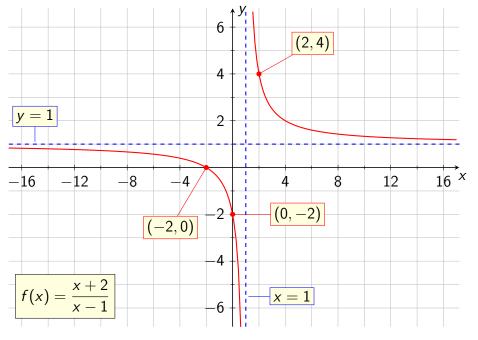
$$f(-2) = \frac{-2+2}{-2-1} = 0, \quad f(0) = \frac{0+2}{0-1} = -2, \quad f(2) = \frac{2+2}{2-1} = 4$$

$$-2 < 0 \implies f(-2) > f(0) \xrightarrow{\text{output}} f \text{ nije rastuća na } \mathbb{R} \setminus \{1\}$$

$$\downarrow | | | | \text{ jer nije sačuvan znak}$$

$$0 \qquad -2 \qquad \text{nejednakosti}$$

$$\mathbb{R}\setminus\{1\}=\langle-\infty,1\rangle\cup\langle1,+\infty
angle$$



četvrti zadatak

Zadatak 4

Zadana je funkcija $f: \mathbb{R} \to [0, +\infty)$, $f(x) = \frac{3x^2}{1+x^2}$.

- a) Ispitajte je li funkcija f surjekcija.
- b) Ispitajte je li funkcija f injekcija.
- c) Neka je $g: \langle -\infty, 0] \to K$ funkcija koja ima isto pravilo pridruživanja kao i funkcija f. Odredite $K \subseteq \mathbb{R}$ tako da g bude bijekcija i odredite pravilo pridruživanja njezine inverzne funkcije.

finite function
$$f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1+x^2}$$

a) Neka je $y \in [0, +\infty)$ proizvoljan element iz kodomene.

$$f: \mathbb{R} \to [0, +\infty\rangle, \ f(x) = \frac{3x^2}{1+x^2}$$

Definicija surjekcije

Funkcija
$$f: D \to K$$
 je surjekcija ako je $\operatorname{Im} f = K$, tj. $(\forall y \in K) (\exists x \in D) (f(x) = y)$.

a) Neka je $y \in [0, +\infty)$ proizvoljan element iz kodomene.

 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1+x^2}$

Funkcija
$$f: D \to K$$
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vrijedi f(x) = y.

 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1+x^2}$ a) Neka je $y \in [0, +\infty)$ proizvoljan element iz kodomene. Tražimo barem jedan $x \in \mathbb{R}$ iz domene za koji

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Funkcija
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element iz kodomene. Tražimo barem jedan $x \in \mathbb{R}$ iz domene za koji vrijedi f(x) = y.

Funkcija $f: D \to K$ je surjekcija ako je Im f = K, tj.

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 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1+x^2}$ em jedan $x \in \mathbb{R}$ iz domene za koji

$$\frac{3x^2}{1+x^2} = y$$

vrijedi f(x) = y.

Funkcija f:D o K je surjekcija ako je $\mathsf{Im}\, f=K$, tj.

a) Neka je $y \in [0, +\infty)$ proizvoljan element iz kodomene. Tražimo barem jedan $x \in \mathbb{R}$ iz domene za koji vrijedi f(x) = y.

 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1+x^2}$

$$\frac{3x^2}{1+x^2} = y$$
$$3x^2 = (1+x^2)y$$

Funkcija
$$f: D \to K$$
 je surjekcija ako je $\text{Im } f = K$, tj. $(\forall y \in K) (\exists x \in D) (f(x) = y)$.

 $3x^2 - x^2y = y$

 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1+x^2}$ a) Neka je $y \in [0, +\infty)$ proizvoljan element iz kodomene. Tražimo barem jedan $x \in \mathbb{R}$ iz domene za koji vrijedi f(x) = y.

vrijedi
$$f(x) = y$$
.
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$$f(x) = y$$
.

$$\frac{3x^2}{1+x^2} = y$$

$$3x^2 = (1+x^2)y$$

$$3x^2 - x^2y = y$$

$$x^2(3-y) = y$$

Definicija surjekcije
Funkcija
$$f: D \to K$$
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$$(\forall y \in K) (\exists x \in D) (f(x) = y).$$

 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1+x^2}$

Rješenje a) Neka je

a) Neka je $y \in [0, +\infty)$ proizvoljan element iz kodomene. Tražimo barem jedan $x \in \mathbb{R}$ iz domene za koji

vrijedi
$$f(x) = y$$
.
$$\frac{3x^2}{1 + x^2} = y$$

$$3x^{2} = (1 + x^{2})y$$
$$3x^{2} - x^{2}y = y$$

$$x^2(3-y) = y$$
$$x^2 = \frac{y}{3-y}$$

Funkcija
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a) Neka je $y \in [0, +\infty)$ proizvoljan

Rješenje

element iz kodomene. Tražimo barem jedan $x \in \mathbb{R}$ iz domene za koji vrijedi f(x) = y.

 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1+x^2}$

$$\frac{3x^2}{1+x^2} = y$$

$$3x^2 = (1+x^2)y$$

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$$x^2(3-y) = y$$

 $x^2 = \frac{y}{3 - v}$

Definicija surjekcije

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$$\frac{3x^2}{1+x^2} = y$$
$$3x^2 = (1+x^2)y$$
$$3x^2 - x^2y = y$$

 $x^{2}(3-y)=y$

$$x^2 = \frac{y}{3 - y}$$

a) Neka je $y \in [0, +\infty)$ proizvoljan

element iz kodomene. Tražimo barem jedan $x \in \mathbb{R}$ iz domene za koji

Dobili smo da postoje čak dva takva elementa iz do-

mene koji se preslikaju u odabrani element y iz kodo-

Međutim, takva dva elementa postoje jedino uz uvjet $\frac{y}{3-y} \geqslant 0$ jer u protivnom ne možemo izvaditi drugi

 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1+x^2}$

vrijedi
$$f(x) = y$$
.
$$\frac{3x^2}{1+x^2} = y$$

$$3x^2 = (1 + x^2)y$$
$$3x^2 - x^2y = y$$

 $x^2(3-y) = y$

 $x^2 = \frac{y}{3 - v}$

 $(\forall y \in K) (\exists x \in D) (f(x) = y).$

korijen.

$$y$$
. $x = \pm \sqrt{\frac{1}{2}}$

Definicija surjekcije
Funkcija
$$f:D o K$$
 je surjekcija ako je Im $f=K$, tj.

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a) Neka je $y \in [0, +\infty)$ proizvoljan

element iz kodomene. Tražimo barem jedan
$$x \in \mathbb{R}$$
 iz domene za koji

vrijedi f(x) = y. $\frac{3x^2}{1+x^2}=y$

$$y$$
.

$$\frac{y}{3-y} \geqslant 0$$

Dobili smo da postoje čak dva takva elementa iz do-

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 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1+x^2}$

$$3x^2 - x^2y = y$$
$$x^2(3 - y) = y$$

 $x^2 = \frac{y}{3 - v}$

 $3x^2 = (1+x^2)y$

korijen.

Funkcija $f: D \to K$ je surjekcija ako je Im f = K, tj.

$$(\forall y \in K)(\exists x \in D)(f(x) = y).$$

a) Neka je $y \in [0, +\infty)$ proizvoljan

element iz kodomene. Tražimo barem jedan $x \in \mathbb{R}$ iz domene za koji

 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1 + x^2}$

vrijedi
$$f(x) = y$$
.

$$\frac{3x^2}{1+x^2} = y$$
$$3x^2 = (1+x^2)y$$

$$3x^2 - x^2y = y$$
$$x^2(3 - y) = y$$

 $x^2 = \frac{y}{3 - v}$

$$x = \pm \sqrt{\frac{y}{3 - y}} \qquad \frac{y}{3 - y} \geqslant 0$$

$$y = 0$$

Dobili smo da postoje čak dva takva elementa iz domene koji se preslikaju u odabrani element y iz kodo-Međutim, takva dva elementa postoje jedino uz uvjet $\frac{y}{3-y} \geqslant 0$ jer u protivnom ne možemo izvaditi drugi korijen.

Definicija surjekcije

a) Neka je $y \in [0, +\infty)$ proizvoljan

element iz kodomene. Tražimo barem jedan
$$x \in \mathbb{R}$$
 iz domene za koji vrijedi $f(x) = y$.

 $\frac{3x^2}{1+x^2} = y$

$$3x^2 = (1 + x^2)y$$
$$3x^2 - x^2y = y$$

 $x^2(3-y)=y$

$$x^2 = \frac{y}{3 - y}$$

 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1+x^2}$ arem jedan $x \in \mathbb{R}$ iz domene za koji

$$\frac{y}{3-y} \geqslant 0$$

$$y = 0 \quad 3-y = 0$$

Dobili smo da postoje čak dva takva elementa iz domene koji se preslikaju u odabrani element y iz kodomene. Međutim, takva dva elementa postoje jedino uz uvjet $\frac{y}{3-y}\geqslant 0$ jer u protivnom ne možemo izvaditi drugi korijen.

Definicija surjekcije

a) Neka je $y \in [0, +\infty)$ proizvoljan

element iz kodomene. Tražimo barem jedan
$$x\in \mathbb{R}$$
 iz domene za koji

vrijedi f(x) = y. $\frac{3x^2}{1+x^2} = y$

$$3x^{2} = (1 + x^{2})y$$
$$3x^{2} - x^{2}y = y$$

$$x^2(3-y)=y$$

$$x^2 = \frac{y}{3 - y}$$
 korijen.

 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1+x^2}$ em jedan $x \in \mathbb{R}$ iz domene za koji

em jedan
$$x \in \mathbb{R}$$
 iz domene za l
$$\frac{y}{3-y} \geqslant 0 \qquad y = 3$$
$$y = 0 \qquad 3 - y = 0$$

Dobili smo da postoje čak dva takva elementa iz domene koji se preslikaju u odabrani element y iz kodomene. Međutim, takva dva elementa postoje jedino uz uvjet $\frac{y}{3-y}\geqslant 0$ jer u protivnom ne možemo izvaditi drugi korijen.

Definicija surjekcije

Funkcija $f: D \to K$ je surjekcija ako je Im f = K, tj. $(\forall y \in K) (\exists x \in D) (f(x) = y)$.

17 / 31

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 proizvoljan element iz kodomene. Tražimo h

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 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1+x^2}$

vrijedi
$$f(x) = y$$
.

 $\frac{3x^2}{1+x^2} = y$
 $3x^2 = (1+x^2)y$
 $3x^2 - x^2y = y$
 $x^2(3-y) = y$
 $x^2 = \frac{y}{3-y}$
 $x = \pm \sqrt{\frac{y}{3-y}}$
 $y = 0$
 $y = 3$
 $y = 0$
 $y = 3$
 $y = 0$

Definicija surjekcije Funkcija $f: D \to K$ je surjekcija ako je Im f = K, tj.

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 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1+x^2}$

Vrijedi
$$f(x) = y$$
.

$$\frac{3x^2}{1+x^2} = y$$

$$3x^2 = (1+x^2)y$$

$$3x^2 - x^2y = y$$

$$x^{2}(3-y) = y$$

$$x^{2} = \frac{y}{3-y}$$

Rješenje

Definicija surjekcije
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 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1 + x^2}$

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 $3x^2 - x^2y = y$
 $x^2(3-y) = y$
 $x^2 = \frac{y}{3-y}$
 $y = 0$
 $y = 3$
 $y = 0$
 $y = 3$
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Rješenje

Definicija surjekcije

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 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1+x^2}$

vrijedi
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 $y = 0$
 $y = 3$
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Rješenje

Definicija surjekcije Funkcija $f: D \to K$ je surjekcija ako je Im f = K, tj.

 $(\forall y \in K) (\exists x \in D) (f(x) = y).$

17/31

a) Neka je
$$y \in [0, +\infty)$$
 proizvoljan element iz kodomene. Tražimo barem jedan $x \in \mathbb{R}$ iz domene za koji

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$$x = \pm \sqrt{\frac{y}{3-y}}$$

$$y = 0$$

$$y = 3$$

$$y = 0$$

$$-\infty$$

$$y$$

$$\frac{y}{3-y} = 0$$

Rješenje

Definicija surjekcije

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 proizvoljan element iz kodomene. Tražimo h

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Rješenje

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 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1+x^2}$

vrijedi
$$f(x) = y$$
.
$$\frac{3x^2}{1+x^2} = y$$

$$3x^2 = (1+x^2)y$$

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Rješenje

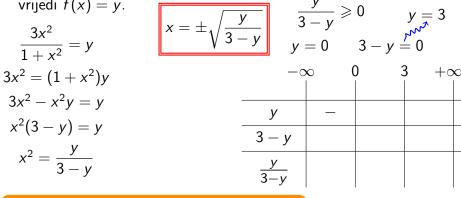
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 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1+x^2}$

vrijedi
$$f(x) = y$$
.
$$\frac{3x^2}{1+x^2} = y$$

$$x = \pm \sqrt{\frac{3}{3}}$$



Rješenje

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.
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$$3x^2 - x^2y = y$$

$$x = \pm \sqrt{\frac{y}{3-y}} \qquad \begin{cases} \frac{y}{3-y} \geqslant 0 & y = 3 \\ y = 0 & 3-y = 0 \end{cases}$$

$$-\infty \qquad 0 \qquad 3 \qquad + \infty$$

$$y \qquad - \qquad + \qquad \frac{3-y}{3-y}$$

_

 $x^{2}(3-y)=y$

 $x^2 = \frac{y}{3 - v}$

Rješenje

Definicija surjekcije

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 y

Rješenje

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$$-\infty$$

$$0$$

$$3 + \infty$$

$$\frac{y}{3-y} = 0$$

$$\frac{y}{3-y} = 0$$

Rješenje

a) Neka je
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 proizvoljan

$$=\pm\sqrt{\frac{y}{3-y}}$$

3 - v

$$\frac{y}{3-y} \geqslant 0 \qquad y = 3$$

$$y = 0 \qquad 3 - y = 0$$

$$-\infty \qquad 0 \qquad 3 \qquad + y = 0$$

$$y = 1 \qquad y = 3$$

 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1+x^2}$

$$x^2 = \frac{y}{3 - y}$$

Rješenje

 $\frac{3x^2}{1+x^2}=y$

 $3x^2 = (1+x^2)y$ $3x^2 - x^2y = y$ $x^{2}(3-y)=y$

Definicija surjekcije

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$$3 + \infty$$

$$\frac{y}{3-y} \Rightarrow 0 \qquad y = 3$$

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Rješenje

element iz kodomene. Tražimo barem jedan $x \in \mathbb{R}$ iz domene za koji vrijedi f(x) = y.

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$$f(x) = y$$
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Rješenje

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$$\sqrt{\frac{y}{3-y}}$$

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 $\frac{y}{3-y} \geqslant 0 \qquad y = 3$ $y = 0 \qquad 3 - y = 0$

$$\operatorname{Im} f = [0, 3\rangle$$

$$x^{2}(3-y) = y$$
 $\lim f \neq [0, +\infty\rangle$ $x^{2} = \frac{y}{3-y}$ f nije surjekcija

3-y $x^2 = \frac{y}{3 - v}$

Rješenje

 $\frac{3x}{1+x^2}=y$

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 $(\forall y \in K) (\exists x \in D) (f(x) = y).$

Funkcija $f: D \to K$ je injekcija ako $(\forall x_1, x_2 \in D) (f(x_1) = f(x_2) \Rightarrow x_1 = x_2).$

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 $f: \mathbb{R} \to [0, +\infty\rangle, \ f(x) = \frac{3x^2}{1+x^2}$

Funkcija f:D o K je injekcija ako

 $f(x_1)=f(x_2)$

$$(\forall x_1, x_2 \in D)(f(x_1) = f(x_2) \Rightarrow x_1 = x_2).$$

Funkcija $f: D \rightarrow K$ je injekcija ako

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 $3x_1^2(1+x_2^2)$

 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1+x^2}$

 $\frac{3x_1^2}{1+x_1^2} = \frac{3x_2^2}{1+x_2^2}$

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 $3x_1^2(1+x_2^2) =$

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 $3x_1^2 + 3x_1^2x_2^2$

 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1+x^2}$

 $\frac{3x_1^2}{1+x_1^2} = \frac{3x_2^2}{1+x_2^2}$

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Funkcija $f: D \to K$ je injekcija ako

 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1+x^2}$

 $(\forall x_1, x_2 \in D)(f(x_1) = f(x_2) \Rightarrow x_1 = x_2).$ 18/31

Definicija injekcije

Funkcija
$$f:D \to K$$
 je injekcija ako

 $\frac{3x_1^2}{1+x_1^2} = \frac{3x_2^2}{1+x_2^2}$

 $3x_1^2(1+x_2^2) = 3x_2^2(1+x_1^2)$

 $3x_1^2 + 3x_1^2x_2^2 = 3x_2^2 + 3x_1^2x_2^2$

 $3x_1^2 = 3x_2^2$

 $(\forall x_1, x_2 \in D)(f(x_1) = f(x_2) \Rightarrow x_1 = x_2).$

 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1 + \sqrt{2}}$

$$3x_1^2 \left(1 + x_2^2\right) = 3x_2^2 \left(1 + x_1^2\right)$$
 $3x_1^2 + 3x_1^2x_2^2 = 3x_2^2 + 3x_1^2x_2^2$
 $3x_1^2 = 3x_2^2$
 $x_1^2 = x_2^2$

Definicija injekcije
Funkcija $f: D \to K$ je injekcija ako

 $(\forall x_1, x_2 \in D)(f(x_1) = f(x_2) \Rightarrow x_1 = x_2).$

 $f(x_1) = f(x_2)$

 $\frac{3x_1^2}{1+x_1^2} = \frac{3x_2^2}{1+x_2^2}$

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 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1 + \sqrt{2}}$

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$$3x_1^2 = 3x_2^2$$

$$x_1^2 = x_2^2$$

$$x_1 = \pm x_2$$
Definicija injekcije
Funkcija $f: D \to K$ je injekcija ako
$$(\forall x_1, x_2 \in D) (f(x_1) = f(x_2) \Rightarrow x_1 = x_2).$$

 $\frac{3x_1^2}{1+x_1^2} = \frac{3x_2^2}{1+x_2^2}$

 $3x_1^2(1+x_2^2) = 3x_2^2(1+x_1^2)$

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 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1 + \sqrt{2}}$

$$x_1^2 = x_2^2$$

$$x_1 = \pm x_2 \leftarrow$$

 $\frac{3x_1^2}{1+x_1^2} = \frac{3x_2^2}{1+x_2^2}$

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 $3x_1^2 + 3x_1^2x_2^2 = 3x_2^2 + 3x_1^2x_2^2$

 $3x_1^2 = 3x_2^2$

$$x_1 = -x_2.$$

 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1 + x^2}$

Na ovaj zaključak utječe domena funkcije f koja

je jednaka \mathbb{R} . U domeni se nalazi barem jedan par suprotnih brojeva pa onda iz $x_1^2 = x_2^2$ ne

mora nužno slijediti da je $x_1 = x_2$, može biti i

Funkcija $f: D \to K$ je injekcija ako

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$$3x_1^2 = 3x_2^2$$

$$x_1^2 = x_2^2$$

$$x_1 = \pm x_2$$

$$f \text{ nije injekcija}$$
Na ovaj zaključak utječe domena funkcije f koja je jednaka \mathbb{R} . U domeni se nalazi barem jedan par suprotnih brojeva pa onda iz $x_1^2 = x_2^2$ ne mora nužno slijediti da je $x_1 = x_2$, može biti i $x_1 = -x_2$.

je jednaka \mathbb{R} . U domeni se nalazi barem jedan par suprotnih brojeva pa onda iz $x_1^2 = x_2^2$ ne mora nužno slijediti da je $x_1 = x_2$, može biti i $x_1 = -x_2$.

 $f: \mathbb{R} \to [0, +\infty\rangle, \ f(x) = \frac{3x^2}{1 + x^2}$

Funkcija $f: D \to K$ je injekcija ako

 $f(x_1) = f(x_2)$

$$(\forall x_1, x_2 \in D)(f(x_1) = f(x_2) \Rightarrow x_1 = x_2).$$

$$3x_1^2 = 3x_2^2$$

$$x_1^2 = x_2^2$$

$$x_1 = x_2^2$$

 $\frac{3x_1^2}{1+x_1^2} = \frac{3x_2^2}{1+x_2^2}$

 $3x_1^2(1+x_2^2) = 3x_2^2(1+x_1^2)$

 $3x_1^2 + 3x_1^2x_2^2 = 3x_2^2 + 3x_1^2x_2^2$

Definicija injekcije

f nije injekcija

 $x_1 = \pm x_2$

icite.

par suprotnih brojeva pa onda iz $x_1^2=x_2^2$ ne mora nužno slijediti da je $x_1=x_2$, može biti i $x_1=-x_2$.

Na ovaj zaključak utječe domena funkcije f koja

je jednaka \mathbb{R} . U domeni se nalazi barem jedan

 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1 + x^2}$

Kako je f parna funkcija, možemo u ovom slučaju protuprimjerom brže dokazati da nije injekcija. Na primjer: $-1 \neq 1$, ali f(-1) = f(1). Različiti elementi domene ne preslikavaju se uvijek u različite elemente kodomene.

Definicija injekci

Funkcija $f:D\to K$ je injekcija ako

$$(\forall x_1, x_2 \in D)(f(x_1) = f(x_2) \Rightarrow x_1 = x_2).$$



- Funkcija $f: D \to K$ je injekcija ako

- $3x_1^2 = 3x_2^2$

 $\frac{3x_1^2}{1+x_1^2} = \frac{3x_2^2}{1+x_2^2}$

 $x_1^2 = x_2^2$

f nije injekcija

 $x_1 = \pm x_2$

- $3x_1^2(1+x_2^2) = 3x_2^2(1+x_1^2)$

 $x_1 = -x_2$.

elemente kodomene.

- $3x_1^2 + 3x_1^2x_2^2 = 3x_2^2 + 3x_1^2x_2^2$

- ovaj način prolazi jedino ako želimo dokazati

 $f: \mathbb{R} \to [0, +\infty), \ f(x) = \frac{3x^2}{1 + x^2}$

Na ovaj zaključak utječe domena funkcije f koja

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da funkcija nije injekcija

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 $(\forall x_1, x_2 \in D)(f(x_1) = f(x_2) \Rightarrow x_1 = x_2).$

C

$$x = \pm \sqrt{\frac{y}{3 - y}}$$

$$\operatorname{Im} f = [0, 3)$$

 $f: \mathbb{R} \to [0, +\infty\rangle, \ f(x) = \frac{3x^2}{1+x^2}$

 $g: \langle -\infty, 0] \to K, \ g(x) = \frac{3x^2}{1+x^2}$

19/31

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Im f = [0, 3)

Funkcije f i g imaju isto pravilo pridruživanja.

g(x) = y

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Kako su u domeni funkcije g samo brojevi manji ili jednaki od nule, za zadani $y \in K$ pos-

toji najviše jedan $x \in \langle -\infty, 0 \rangle$ takav da je

$$g:\langle -\infty,0]\to K,\ g(x)=\frac{3x^2}{1+x^2}$$

$$+x^2$$

$$g(x) = y$$
 i pritom je $x = -\sqrt{\frac{y}{3-y}}$.

 $f: \mathbb{R} \to [0, +\infty\rangle, \ f(x) = \frac{3x^2}{1+x^2}$

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19/31

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$$g:\langle -\infty,0]\to K,\ g(x)=\frac{3x^2}{1+x^2}$$

$$g(x)=y$$
 i pritom je $x=-\sqrt{\frac{y}{3-y}}$. Stoga je također Im $g=[0,3)$ pa mora biti $K=[0,3)$

 $f: \mathbb{R} \to [0, +\infty\rangle, \ f(x) = \frac{3x^2}{1+x^2}$

 $x = \pm \sqrt{\frac{y}{3-y}}$

Im f = [0, 3)

tako da g bude surjekcija.

$$=\frac{3x^2}{1+x^2}$$

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Funkcije f i g imaju isto pravilo pridruživanja.

cija jer u njezinoj domeni nema niti jednog para suprotnih brojeva koji su "kvarili" injektivnost

 $x = \pm \sqrt{\frac{y}{3-y}}$ Kako su u domeni funkcije g samo brojevi manji ili jednaki od nule, za zadani $y \in K$ pos-Im f = [0, 3)toji najviše jedan $x \in \langle -\infty, 0 \rangle$ takav da je g(x) = y i pritom je $x = -\sqrt{\frac{y}{3-y}}$. Stoga je

 $f: \mathbb{R} \to [0, +\infty\rangle, \ f(x) = \frac{3x^2}{1+x^2}$

 $g:\langle -\infty,0]\to K,\ g(x)=\frac{3x^2}{1+x^2}$

kod funkcije f.

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također Im g = [0,3) pa mora biti K = [0,3)tako da g bude surjekcija. Funkcija g je injek-

Funkcije f i g imaju isto pravilo pridruživanja.

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Kako su u domeni funkcije
$$g$$
 samo brojevi manji ili jednaki od nule, za zadani $y \in K$ postoji najviše jedan $x \in \langle -\infty, 0]$ takav da je $g(x) = y$ i pritom je $x = -\sqrt{\frac{y}{3-y}}$. Stoga je također Im $g = [0, 3\rangle$ na mora biti $K = [0, 3\rangle$

kod funkcije
$$f$$
.
$$g: \langle -\infty, 0] \to K, \ g(x) = \frac{3x^2}{1+x^2}$$

nji ili jednaki od nule, za zadani
$$y \in K$$
 postoji najviše jedan $x \in \langle -\infty, 0 \rangle$ takav da je

g(x) = y i pritom je $x = -\sqrt{\frac{y}{3-v}}$. Stoga je

također Im g = [0,3) pa mora biti $K = \overline{(0,3)}$ tako da g bude surjekcija. Funkcija g je injek-

Funkcije f i g imaju isto pravilo pridruživanja.

Kako su u domeni funkcije g samo brojevi ma-

cija jer u njezinoj domeni nema niti jednog para suprotnih brojeva koji su "kvarili" injektivnost kod funkcije f.

 $g:\langle -\infty,0]\to K,\ g(x)=\frac{3x^2}{1+x^2}$

Im f = [0, 3)

 $x = \pm \sqrt{\frac{y}{3-y}}$

$$g^{-1}:[0,3)$$

c)

Kako su u domeni funkcije
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 samo brojevi manji ili jednaki od nule, za zadani $y \in K$ pos-

Funkcije f i g imaju isto pravilo pridruživanja.

nji ili jednaki od nule, za zadani $y \in K$ postoji najviše jedan $x \in \langle -\infty, 0 \rangle$ takav da je g(x) = y i pritom je $x = -\sqrt{\frac{y}{3-y}}$. Stoga je također Im g = [0,3) pa mora biti $K = \overline{(0,3)}$ tako da g bude surjekcija. Funkcija g je injekcija jer u njezinoj domeni nema niti jednog para

suprotnih brojeva koji su "kvarili" injektivnost

Im f = [0, 3)

 $x = \pm \sqrt{\frac{y}{3-y}}$

$$g^{-1}:[0,3\rangle\to\langle-\infty,0]$$

kod funkcije
$$f$$
.
$$g: \langle -\infty, 0] \to K, \ g(x) = \frac{3x^2}{1+x^2}$$

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Kako su u domeni funkcije
$$g$$
 samo brojevi manji ili jednaki od nule, za zadani $y \in K$ postoji pojiviše jedan $x \in A$ os 01 takov da je

toji najviše jedan $x \in \langle -\infty, 0 \rangle$ takav da je g(x) = y i pritom je $x = -\sqrt{\frac{y}{3-y}}$. Stoga je također Im g = [0,3) pa mora biti $K = \overline{(0,3)}$

cija jer u njezinoj domeni nema niti jednog para

Funkcije f i g imaju isto pravilo pridruživanja.

 $\operatorname{Im} f = [0, 3]$ tako da g bude surjekcija. Funkcija g je injek-

 $f: \mathbb{R} \to [0, +\infty\rangle, \ f(x) = \frac{3x^2}{1+x^2}$

suprotnih brojeva koji su "kvarili" injektivnost kod funkcije
$$f$$
.
$$g: \langle -\infty, 0] \to K, \ g(x) = \frac{3x^2}{1+x^2}$$

 $g^{-1}(x) = -\sqrt{\frac{x}{3-x}}$

 $g^{-1}:[0,3\rangle\to\langle-\infty,0]$

 $x = \pm \sqrt{\frac{y}{3-y}}$

c)

Kako su u domeni funkcije g samo brojevi manji ili jednaki od nule, za zadani $y \in K$ postoji najviše jedan $x \in \langle -\infty, 0]$ takav da je g(x) = y i pritom je $x = -\sqrt{\frac{y}{3-y}}$. Stoga je također Im g = [0,3) pa mora biti K = [0,3) tako da g bude surjekcija. Funkcija g je injek-

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Funkcije f i g imaju isto pravilo pridruživanja.

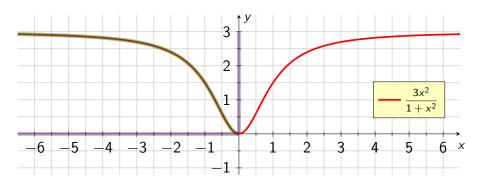
 $\operatorname{Im} f = [0, 3\rangle$

 $x = \pm \sqrt{\frac{y}{3 - v}}$

suprotnih brojeva koji su "kvarili" injektivnost kod funkcije
$$f$$
. $g^{-1}(x) = -\sqrt{\frac{x}{3-x}}$

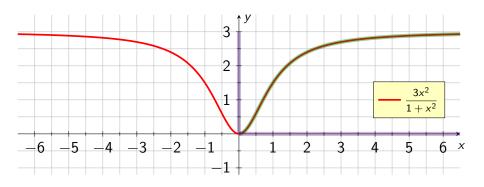
kod funkcije
$$f$$
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$$g^{-1}:[0,3\rangle\to\langle-\infty,0]$$



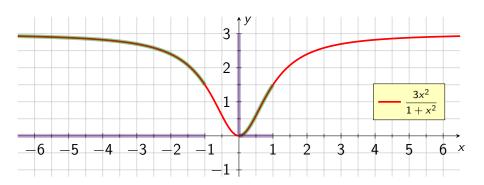
$$g:\langle -\infty,0] \to [0,3\rangle, \quad g(x) = \frac{3x^2}{1+x^2}$$

$$g^{-1}:[0,3)\to \langle -\infty,0], \quad g^{-1}(x)=-\sqrt{\frac{x}{3-x}}$$



$$h: [0, +\infty) \to [0, 3), \quad h(x) = \frac{3x^2}{1+x^2}$$

$$h^{-1}:[0,3)\to [0,+\infty), \quad h^{-1}(x)=\sqrt{\frac{x}{3-x}}$$



$$k:\langle -\infty, -1] \cup [0, 1\rangle \rightarrow [0, 3\rangle, \quad k(x) = \frac{3x^2}{1+x^2}$$

$$k^{-1}: [0,3) \to \langle -\infty, -1] \cup [0,1\rangle, \quad k^{-1}(x) = \begin{cases} \sqrt{\frac{x}{3-x}}, & x \in \left[0, \frac{3}{2}\right) \\ -\sqrt{\frac{x}{3-x}}, & x \in \left[\frac{3}{2}, 3\right) \end{cases}$$

peti zadatak

$$f(x) = A\sin(\omega x + \varphi)$$

- $A \neq 0$, $\omega \neq 0$
- Funkcija f je periodična s temeljnim periodom $T=rac{2\pi}{|\omega|}$.
- Graf funkcije f se dobiva na sljedeći način:
 - Najprije se graf sinusoide $y = \sin x$ "zgusne" ili "rastegne" tako da dobijemo graf funkcije $x \mapsto \sin(\omega x)$ koja ima temeljni period $T = \frac{2\pi}{|\omega|}$.
 - Nakon toga se dobiveni graf skalira duž y-osi tako da "titra" između pravaca y=-|A| i y=|A|. Na taj način dobijemo graf funkcije $x\mapsto A\sin(\omega x)$.
 - Konačno, dobiveni graf se translatira za vektor $\left(-\frac{\varphi}{\omega},0\right)$.

$$f(x) = A\cos(\omega x + \varphi)$$

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- Funkcija f je periodična s temeljnim periodom $T=rac{2\pi}{|\omega|}$.
- Graf funkcije f se dobiva na sljedeći način:
 - Najprije se graf sinusoide $y=\cos x$ "zgusne" ili "rastegne" tako da dobijemo graf funkcije $x\mapsto\cos\left(\omega x\right)$ koja ima temeljni period $T=\frac{2\pi}{|\omega|}.$
 - Nakon toga se dobiveni graf skalira duž y-osi tako da "titra" između pravaca y = -|A| i y = |A|. Na taj način dobijemo graf funkcije $x \mapsto A \cos(\omega x)$.
 - Konačno, dobiveni graf se translatira za vektor $\left(-\frac{\varphi}{\omega},0\right)$.

Zadatak 5

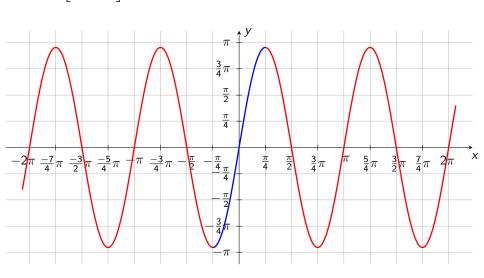
Odredite inverzne funkcije sljedećih funkcija:

a)
$$f_1: \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \to [-3, 3], \ f_1(x) = 3\sin 2x,$$

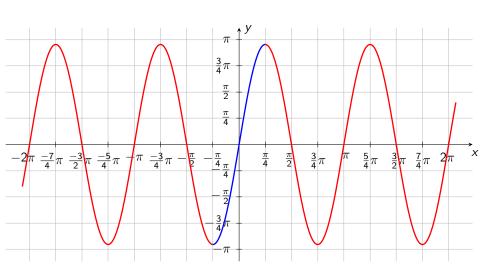
b)
$$f_2: \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \to [-3, 3], f_2(x) = -3\sin 2x,$$

c)
$$f_3: \left| -\frac{5}{4}\pi, -\frac{3}{4}\pi \right| \to [-3, 3], f_3(x) = 3\sin 2x,$$

d)
$$f_4: \left[\frac{5}{4}\pi, \frac{7}{4}\pi\right] \to [-3, 3], f_4(x) = 3\sin 2x.$$

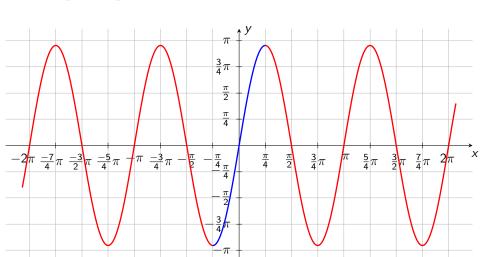


$$T = \frac{2\pi}{|\omega|}$$

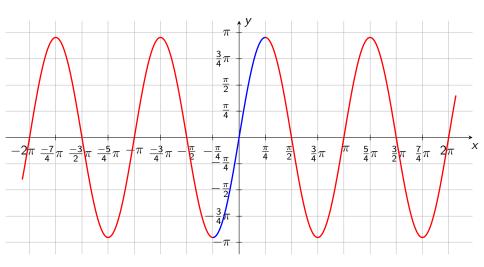


 ω

 $T = \frac{2\pi}{|\omega|} = \frac{2\pi}{|2|}$



 $T = \frac{2\pi}{|\omega|} = \frac{2\pi}{|2|} = \pi$



a)
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$$y = 3 \sin 2x$$

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$$\sin 2x = \frac{y}{3}$$

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$$3\sin 2x = y$$

$$\sin 2x = \frac{y}{3}$$

$$2x =$$

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$$2x = \arcsin \frac{y}{3}$$

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$$y = 3\sin 2x$$

$$3\sin 2x = y$$

$$\sin 2x = \frac{y}{3}$$

$$2x = \arcsin \frac{y}{3}, \quad 2x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

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$$x = \frac{1}{2}\arcsin \frac{y}{3}$$

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$$f_1^{-1}: [-3,3] \to \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$$

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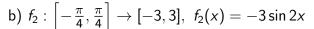
$$3\sin 2x = y$$

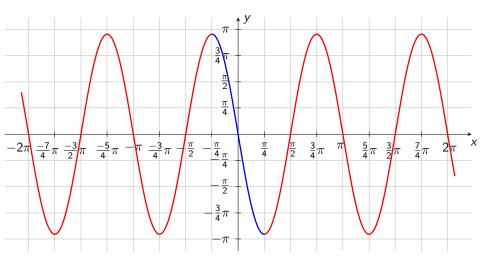
$$\sin 2x = \frac{y}{3}$$

$$2x = \arcsin \frac{y}{3}, \quad 2x \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$$

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$$f_1^{-1}: [-3,3] \to \left[-\frac{\pi}{4}, \frac{\pi}{4}\right], \ f_1^{-1}(x) = \frac{1}{2} \arcsin \frac{x}{3}$$





b)
$$f_2: \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \to [-3, 3], \ f_2(x) = -3\sin 2x$$

$$y = -3\sin 2x$$

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 $x = \frac{1}{2}\arcsin\left(-\frac{y}{2}\right), \quad x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

b)
$$f_2: \left[-\frac{\pi}{4}, \frac{\pi}{4} \right] \to [-3, 3], \ f_2(x) = -3\sin 2x$$

$$y = -3\sin 2x$$

$$-3\sin 2x = y$$

$$\sin 2x = -\frac{y}{3}$$

$$2x = \arcsin\left(-\frac{y}{3}\right), \quad 2x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

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$$f_2^{-1}: [-3, 3] \to \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$$

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$$\left[-\frac{\pi}{4},\frac{\pi}{4}\right]$$

$$\left[-\frac{\pi}{4},\frac{\pi}{4}\right]$$

$$\frac{\pi}{2}$$

$$-1$$

$$\frac{\pi}{2}$$

$$-\arcsin x$$

$$f_2^{-1}: [-3,3] o \left[-\frac{\pi}{4}, \frac{\pi}{4}\right], \ f_2^{-1}(x) = \frac{1}{2}\arcsin\left(-\frac{x}{3}\right)$$

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$$x = \frac{1}{2}\arcsin\left(-\frac{y}{3}\right), \quad x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$$

$$\frac{\pi}{2}$$

$$\frac{\pi}{2}$$

$$\frac{\pi}{2}$$

$$\frac{\pi}{2}$$
arcsin je neparna funkcija

$$-\sin\left(-\frac{X}{x}\right)$$

$$f_2^{-1}: [-3,3] \to \left[-\frac{\pi}{4}, \frac{\pi}{4}\right], \ f_2^{-1}(x) = \frac{1}{2}\arcsin\left(-\frac{x}{3}\right)$$

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$$f_2: \left[-\frac{\pi}{4}, \frac{\pi}{4} \right] \to [-3, 3], \ f_2(x) = -3\sin 2x$$

$$y = -3\sin 2x$$
$$-3\sin 2x = y$$
$$\sin 2x = -\frac{y}{2}$$

$$=-\frac{y}{3}$$

$$=-\frac{1}{3}$$

$$2x = \arcsin\left(-\frac{y}{2}\right), \quad 2x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$x = \frac{1}{2}\arcsin\left(-\frac{y}{3}\right), \quad x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$$

$$x = \frac{1}{2} \arcsin\left(-\frac{\pi}{3}\right), \quad x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$$
 $\arcsin\left(-\frac{x}{3}\right)$

$$f_2^{-1}: \left[-3, 3\right] \to \left[-\frac{\pi}{4}, \frac{\pi}{4}\right], \quad f_2^{-1}(x) = \frac{1}{2} \arcsin\left(-\frac{x}{3}\right)$$

$$\frac{\pi}{2}$$

$$\frac{\pi}{2}$$

$$\frac{\pi}{2}$$

$$\frac{\pi}{2}$$
arcsin je neparna funkcija

 $\arcsin(-x) = -\arcsin x$

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$$y = -3\sin 2x$$

$$-3\sin 2x = y$$

$$-1$$
1

b) $f_2: \left[-\frac{\pi}{4}, \frac{\pi}{4} \right] \to [-3, 3], \ f_2(x) = -3\sin 2x$

$$\sin 2x = -\frac{y}{3}$$

$$2x = \arcsin\left(-\frac{y}{3}\right), \quad 2x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$x = \frac{1}{2}\arcsin\left(-\frac{y}{3}\right), \quad x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$$

$$\arcsin(-x) = -\arcsin x$$

$$2x = \arcsin\left(-\frac{x}{3}\right), \quad 2x \in \left[-\frac{x}{2}, \frac{x}{2}\right] \qquad \text{arcsin je neparna funkcija}$$

$$x = \frac{1}{2}\arcsin\left(-\frac{y}{3}\right), \quad x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \qquad \text{arcsin}\left(-x\right) = -\arcsin x$$

$$f_2^{-1}: \left[-3, 3\right] \to \left[-\frac{\pi}{4}, \frac{\pi}{4}\right], \quad f_2^{-1}(x) = \frac{1}{2}\arcsin\left(-\frac{x}{3}\right)$$

$$: [-3,3] \to \left[-\frac{\pi}{4}, \frac{\pi}{4} \right], \ f_2^{-1}(x) = \frac{1}{2} \arcsin\left(-\frac{x}{3} \right)$$
$$f_2^{-1}(x) = -\frac{1}{2} \arcsin\frac{x}{3}$$

