# Seminari 10

Matematičke metode za informatičare

Damir Horvat

FOI, Varaždin

# Sadržaj

prvi zadatak

Problem svojstvenih vrijednosti

drugi zadatak

treći zadatak

prvi zadatak

Odredite sliku, jezgru, rang i defekt linearnog operatora  $B: \mathbb{R}^5 \to \mathbb{R}^3$  zadanog matricom

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

u paru kanonskih baza. Je li B izomorfizam?

Odredite sliku, jezgru, rang i defekt linearnog operatora  $B:\mathbb{R}^5 o \mathbb{R}^3$  zadanog matricom

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## Rješenje

Kako je dim  $\mathbb{R}^5 \neq \dim \mathbb{R}^3$ , zaključujemo da  $\mathbb{R}^5$  i  $\mathbb{R}^3$  nisu izomorfni vektorski prostori.

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Stoga ne postoji niti jedan linearni operator  $\mathbb{R}^5 o \mathbb{R}^3$  koji je bijekcija.

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Stoga ne postoji niti jedan linearni operator  $\mathbb{R}^5 \to \mathbb{R}^3$  koji je bijekcija. Dakle, linearni operator B nije izomorfizam.

Ker B

 $B:\mathbb{R}^5 o \mathbb{R}^3$ 

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$$\frac{x_{1} \quad x_{2} \quad x_{3} \quad x_{4} \quad x_{5}}{1 \quad 2 \quad 3 \quad 1 \quad 0 \quad 0} \leftarrow + \begin{pmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{pmatrix} \begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \\ x_{4} \\ x_{5} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$1 \quad 2 \quad 5 \quad 2 \quad 1 \quad 0 \quad / \cdot (-1) / \cdot (-1)$$

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$$\frac{x_{1} \quad x_{2} \quad x_{3} \quad x_{4} \quad x_{5}}{1 \quad 2 \quad 3 \quad 1 \quad 0 \quad 0} \xrightarrow{(-(-1)/(-1))} \xrightarrow{(-1)} \xrightarrow{(-1)/(-1)} \xrightarrow{$$

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2/18

2/18

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$$Y_{\mathcal{B}} = F_{(\mathcal{A}, \mathcal{B})} X_{\mathcal{A}}$$

$$2/18$$

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2/18

$$B(x) = \Theta_{\mathbb{R}^3} \longrightarrow BX = O \qquad \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5}{1 \quad 2 \quad 3 \quad 1 \quad 0 \quad 0} \longrightarrow \begin{pmatrix} (-1)/(-1) \\ 1 \quad 2 \quad 5 \quad 2 \quad 1 \quad 0 \\ \hline 0 \quad 0 \quad -2 \quad -1 \quad -1 \quad 0 \\ \hline 1 \quad 2 \quad 5 \quad 2 \quad 1 \quad 0 \\ \hline 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \\ \hline 0 \quad 0 \quad -2 \quad -1 \quad -1 \quad 0 \end{bmatrix}$$

$$\frac{Y_{\mathcal{B}} = F(\mathcal{A}, \mathcal{B})}{X_{\mathcal{A}}} X_{\mathcal{A}}$$

$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{\text{way}} BX = O \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 3 & 1 & 0 & 0 \\ \hline 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ \hline \end{pmatrix}$$

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$$2/18$$

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$$\frac{Y_{\mathcal{B}} = F_{(\mathcal{A}, \mathcal{B})} X_{\mathcal{A}}}{Y_{\mathcal{B}} = F_{(\mathcal{A}, \mathcal{B})} X_{\mathcal{A}}}$$

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$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{BX} BX = O \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{1} \xrightarrow{X_2} \xrightarrow{X_3} \xrightarrow{X_4} \xrightarrow{X_5} \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \xrightarrow{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{1} \xrightarrow{X_2} \xrightarrow{X_3} \xrightarrow{X_4} \xrightarrow{X_5} \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 0 & 0 & -2 & -1 & -1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ \hline \end{array}$$

$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{} BX = O \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5}{1 \quad 2 \quad 3 \quad 1 \quad 0 \quad 0} \xrightarrow{} (-1)/(-1)$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{} BX = O \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5}{1 \quad 2 \quad 3 \quad 1 \quad 0 \quad 0} \xrightarrow{} (-1)/(-1)$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

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$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{} BX = O \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5}{1 \quad 2 \quad 3 \quad 1 \quad 0 \quad 0} \xrightarrow{} (-1)/(-1)$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 3 \quad 1 \quad 0$$

$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{} BX = O \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5}{1 \quad 2 \quad 3 \quad 1 \quad 0 \quad 0} \xrightarrow{} (-1)/(-1)$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 3 \quad 1 \quad 0 \quad 0$$

$$\frac{1}{1} \quad 2 \quad 3 \quad 1 \quad 0 \quad 0$$

$$B: \mathbb{R}^{5} \to \mathbb{R}^{3}$$

$$B(x) = \Theta_{\mathbb{R}^{3}} \longrightarrow BX = O$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \\ x_{4} \\ x_{5} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_{1}}{1} \xrightarrow{x_{2}} \xrightarrow{x_{3}} \xrightarrow{x_{4}} \xrightarrow{x_{5}} \\ 1 & 2 & 3 & 1 & 0 & 0 \\ \hline{1} & 2 & 5 & 2 & 1 & 0 \\ \hline{0} & 0 & -2 & -1 & -1 & 0 \\ \hline{1} & 2 & 5 & 2 & 1 & 0 \\ \hline{0} & 0 & -2 & -1 & -1 & 0 \\ \hline{0} & 0 & -2 & -1 & -1 & 0 \\ \hline{1} & 2 & 5 & 2 & 1 & 0 \\ \hline{0} & 0 & -2 & -1 & -1 & 0 \\ \hline{1} & 2 & 5 & 2 & 1 & 0 \\ \hline{0} & 0 & -2 & -1 & -1 & 0 \\ \hline{1} & 2 & 3 & 1 & 0 & 0 \\ \hline$$

$$Y_{\mathcal{B}} = F_{(\mathcal{A}, \mathcal{B})} X_{\mathcal{A}}$$

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$$B: \mathbb{R}^5 \to \mathbb{R}^3$$

$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{BX} BX = O$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \\ 1 & 2 & 3 & 1 & 0 & 0 \\ \hline 0 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ \hline 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ \hline 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ \hline 1 & 2 & 3 & 1 & 0 & 0 \\ \hline \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5}{1 \quad 2 \quad 3 \quad 1 \quad 0 \quad 0} \longrightarrow \begin{pmatrix} (-1)/(-1) \\ 1 \quad 2 \quad 5 \quad 2 \quad 1 \quad 0 \\ \hline 0 \quad 0 \quad -2 \quad -1 \quad -1 \quad 0 \\ 1 \quad 2 \quad 5 \quad 2 \quad 1 \quad 0 \\ \hline 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \\ \hline 0 \quad 0 \quad -2 \quad -1 \quad -1 \quad 0 \\ \hline 1 \quad 2 \quad 5 \quad 2 \quad 1 \quad 0 \\ \hline 0 \quad 0 \quad -2 \quad -1 \quad -1 \quad 0 \\ \hline 1 \quad 2 \quad 5 \quad 2 \quad 1 \quad 0 \\ \hline 0 \quad 0 \quad -2 \quad -1 \quad -1 \quad 0 \\ \hline 1 \quad 2 \quad 3 \quad 1 \quad 0 \quad 0 \end{bmatrix}$$

$$Y_{\mathcal{B}} = F_{(\mathcal{A},\mathcal{B})} X_{\mathcal{A}}$$

$$B: \mathbb{R}^{5} \to \mathbb{R}^{3}$$

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$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \\ x_{4} \\ x_{5} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_{1} & x_{2} & x_{3} & x_{4} & x_{5} \\ 1 & 2 & 5 & 2 & 1 \\ 0 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ \hline 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ \hline 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ \hline 1 & 2 & 3 & 1 & 0 & 0 \\ \hline \end{bmatrix}$$

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$$\begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \\ x_{4} \\ x_{5} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$\begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \\ x_{4} \\ x_{5} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \\ x_{4} \\ x_{5} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \\ x_{4} \\ x_{5} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \\ x_{4} \\ x_{5} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \\ x_{4} \\ x_{5} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \\ x_{4} \\ x_{5} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$\begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \\ x_{4} \\ x_{5} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \\ x_{4} \\ x_{5} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \\ x_{4} \\ x_{5} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$\begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \\ x_{4} \\ x_{5} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$\begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \\ x_{4} \\ x_{5} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\$$

$$B: \mathbb{R}^5 \to \mathbb{R}^3$$

$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{\text{res}} BX = O$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \\ 1 & 2 & 5 & 2 & 1 \\ 0 & 0 & -2 & -1 & -1 & 0 \\ 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & 2 & 3 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 0 & 0 \\ 0 & 0 & -2 & -1 & -1 \\ 0 & 0 & -2 & -1 & -1 \\ 0 & 0 & -2 & -1 & -1 \\ 0 & 1 & 2 & 3 & 1 & 0 \\ \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$B: \mathbb{R}^5 \to \mathbb{R}^3$$

 $x_5 = -2x_3 - x_4$  $x_1 = -2x_2 - 3x_3 - x_4$ 

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$
  
$$x_1 = -2x_2 - 3x_3 - x_4$$

$$Ker B =$$

$$B:\mathbb{R}^5\to\mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$
  
$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\operatorname{\mathsf{Ker}} B = \{ ($$

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$$x_5 = -2x_3 - x_4$$
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$$\operatorname{Ker} B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3,$$

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$$x_5 = -2x_3 - x_4$$
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$$\mathsf{Ker}\, B = \big\{ \big( -2x_2 - 3x_3 - x_4, \, x_2, \, x_3, \, x_4, \,$$

$$B: \mathbb{R}^5 \to \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$
  
$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\operatorname{Ker} B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4)\}$$

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

 $x_5 = -2x_3 - x_4$  $x_1 = -2x_2 - 3x_3 - x_4$ 

$$\operatorname{\mathsf{Ker}} B = \left\{ \left( -2x_2 - 3x_3 - x_4, \, x_2, \, x_3, \, x_4, \, -2x_3 - x_4 \right) : x_2, x_3, x_4 \in \mathbb{R} \right\}$$

$$B:\mathbb{R}^5 o\mathbb{R}^3$$

 $x_5 = -2x_3 - x_4$  $x_1 = -2x_2 - 3x_3 - x_4$ 

$$\operatorname{Ker} B = \left\{ \left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) : x_2, x_3, x_4 \in \mathbb{R} \right\}$$

$$\left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) =$$

$$B:\mathbb{R}^5 o\mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$
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$$\operatorname{Ker} B = \left\{ \left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) : x_2, x_3, x_4 \in \mathbb{R} \right\}$$

$$\left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) =$$

$$= x_2 \cdot \left($$

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$
$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\operatorname{Ker} B = \left\{ \left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) : x_2, x_3, x_4 \in \mathbb{R} \right\}$$

$$\left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) =$$

$$= x_2 \cdot \left( -2, x_3 - x_4 \right) =$$

$$B:\mathbb{R}^5\to\mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$
$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\operatorname{Ker} B = \left\{ \left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) : x_2, x_3, x_4 \in \mathbb{R} \right\}$$

$$\left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) =$$

$$= x_2 \cdot \left( -2, 1, -2, 1$$

$$B: \mathbb{R}^5 \to \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$
$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\operatorname{Ker} B = \left\{ \left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) : x_2, x_3, x_4 \in \mathbb{R} \right\}$$

$$\left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) =$$

$$= x_2 \cdot \left( -2, 1, 0, -$$

$$B:\mathbb{R}^5\to\mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$
$$x_1 = -2x_2 - 3x_3 - x_4$$

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

$$\operatorname{Ker} B = \left\{ \left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) : x_2, x_3, x_4 \in \mathbb{R} \right\}$$

$$\left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) =$$

$$= x_2 \cdot \left( -2, 1, 0, 0, 0 \right)$$

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

$$Ker B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2, 1, 0, 0, 0) +$$

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

$$\operatorname{Ker} B = \left\{ \left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) : x_2, x_3, x_4 \in \mathbb{R} \right\}$$

$$\left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) =$$

$$= x_2 \cdot \left( -2, 1, 0, 0, 0 \right) + x_3 \cdot \left( -2, 1, 0, 0, 0 \right) + x_4 \cdot \left( -2, 1, 0, 0, 0 \right) + x_5 \cdot$$

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$
$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\operatorname{Ker} B = \left\{ \left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) : x_2, x_3, x_4 \in \mathbb{R} \right\}$$

$$\left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) =$$

$$= x_2 \cdot \left( -2, 1, 0, 0, 0 \right) + x_3 \cdot \left( -3, 0, 0, 0 \right)$$

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$
$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\operatorname{Ker} B = \left\{ \left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) : x_2, x_3, x_4 \in \mathbb{R} \right\}$$

$$\left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) =$$

$$= x_2 \cdot \left( -2, 1, 0, 0, 0 \right) + x_3 \cdot \left( -3, 0, 0, 0 \right)$$

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$
$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\operatorname{Ker} B = \left\{ \left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) : x_2, x_3, x_4 \in \mathbb{R} \right\}$$

$$\left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) =$$

$$= x_2 \cdot \left( -2, 1, 0, 0, 0 \right) + x_3 \cdot \left( -3, 0, 1, 0, 0 \right)$$

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

$$\operatorname{Ker} B = \left\{ \left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) : x_2, x_3, x_4 \in \mathbb{R} \right\}$$

$$\left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) =$$

$$= x_2 \cdot \left( -2, 1, 0, 0, 0 \right) + x_3 \cdot \left( -3, 0, 1, 0, 0 \right)$$

$$B: \mathbb{R}^5 \to \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$
  
$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\operatorname{Ker} B = \left\{ (-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R} \right\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2, 1, 0, 0, 0) + x_3 \cdot (-3, 0, 1, 0, -2)$$

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$
$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\operatorname{Ker} B = \left\{ (-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R} \right\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2, 1, 0, 0, 0) + x_3 \cdot (-3, 0, 1, 0, -2) +$$

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$
$$x_1 = -2x_2 - 3x_3 - x_4$$

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$
  
$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\operatorname{Ker} B = \left\{ \left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) : x_2, x_3, x_4 \in \mathbb{R} \right\}$$

$$\left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) =$$

$$= x_2 \cdot \left( -2, 1, 0, 0, 0 \right) + x_3 \cdot \left( -3, 0, 1, 0, -2 \right) + x_4 \cdot \left( -1, 0, 0, 0 \right)$$

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$
$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\operatorname{Ker} B = \left\{ \left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) : x_2, x_3, x_4 \in \mathbb{R} \right\}$$

$$\left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) =$$

$$= x_2 \cdot \left( -2, 1, 0, 0, 0 \right) + x_3 \cdot \left( -3, 0, 1, 0, -2 \right) + x_4 \cdot \left( -1, 0, 0, 0 \right)$$

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

$$\operatorname{Ker} B = \left\{ \left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) : x_2, x_3, x_4 \in \mathbb{R} \right\}$$

$$\left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) =$$

$$= x_2 \cdot \left( -2, 1, 0, 0, 0 \right) + x_3 \cdot \left( -3, 0, 1, 0, -2 \right) + x_4 \cdot \left( -1, 0, 0, 0 \right)$$

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$
$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\mathsf{Ker}\,B = \left\{ \left( -2x_2 - 3x_3 - x_4, \ x_2, \ x_3, \ x_4, \ -2x_3 - x_4 \right) : x_2, x_3, x_4 \in \mathbb{R} \right\}$$

$$\left( -2x_2 - 3x_3 - x_4, \ x_2, \ x_3, \ x_4, \ -2x_3 - x_4 \right) =$$

 $= x_2 \cdot (-2, 1, 0, 0, 0) + x_3 \cdot (-3, 0, 1, 0, -2) + x_4 \cdot (-1, 0, 0, 1, 0, -2)$ 

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$
$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\operatorname{Ker} B = \left\{ \left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) : x_2, x_3, x_4 \in \mathbb{R} \right\}$$

$$\left( -2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4 \right) =$$

$$= x_2 \cdot \left( -2, 1, 0, 0, 0 \right) + x_3 \cdot \left( -3, 0, 1, 0, -2 \right) + x_4 \cdot \left( -1, 0, 0, 1, -1 \right)$$

$$B: \mathbb{R}^5 o \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$
  
$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\operatorname{Ker} B = \left\{ \left( -2x_2 - 3x_3 - x_4, \ x_2, \ x_3, \ x_4, \ -2x_3 - x_4 \right) : x_2, x_3, x_4 \in \mathbb{R} \right\}$$

$$\left( -2x_2 - 3x_3 - x_4, \ x_2, \ x_3, \ x_4, \ -2x_3 - x_4 \right) =$$

$$= x_2 \cdot \left( -2, 1, 0, 0, 0 \right) + x_3 \cdot \left( -3, 0, 1, 0, -2 \right) + x_4 \cdot \left( -1, 0, 0, 1, -1 \right)$$

$$\mathcal{B}_{\operatorname{Ker} B} = \left\{ \left( -2, 1, 0, 0, 0, 0, (-3, 0, 1, 0, -2), (-1, 0, 0, 1, -1) \right) \right\}$$

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$
$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\operatorname{Ker} B = \left\{ (-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R} \right\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2, 1, 0, 0, 0) + x_3 \cdot (-3, 0, 1, 0, -2) + x_4 \cdot (-1, 0, 0, 1, -1)$$

$$\mathcal{B}_{\operatorname{Ker} B} = \left\{ (-2, 1, 0, 0, 0), (-3, 0, 1, 0, -2), (-1, 0, 0, 1, -1) \right\}$$

$$d(B) = 3$$

$$B: \mathbb{R}^5 \to \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$
$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\operatorname{Ker} B = \left\{ \left( -2x_2 - 3x_3 - x_4, \ x_2, \ x_3, \ x_4, \ -2x_3 - x_4 \right) : x_2, x_3, x_4 \in \mathbb{R} \right\}$$

$$\left( -2x_2 - 3x_3 - x_4, \ x_2, \ x_3, \ x_4, \ -2x_3 - x_4 \right) =$$

$$= x_2 \cdot \left( -2, 1, 0, 0, 0 \right) + x_3 \cdot \left( -3, 0, 1, 0, -2 \right) + x_4 \cdot \left( -1, 0, 0, 1, -1 \right)$$

$$\mathcal{B}_{\operatorname{Ker} B} = \left\{ \left( -2, 1, 0, 0, 0 \right), \ \left( -3, 0, 1, 0, -2 \right), \ \left( -1, 0, 0, 1, -1 \right) \right\}$$

$$d(B) = 3 \longrightarrow B \text{ nije injekcija}$$

Im B

 $B:\mathbb{R}^5\to\mathbb{R}^3$ 

$$B:\mathbb{R}^5 \to \mathbb{R}^3$$

 $B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$ 

$$B:\mathbb{R}^5\to\mathbb{R}^3$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
$$r(B) + 3 = 5$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
$$r(B) + 3 = 5$$
$$r(B) = 2$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
$$r(B) + 3 = 5$$
$$r(B) = 2$$

$$r(B) \neq \dim \mathbb{R}^3$$

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
  
 $r(B) + 3 = 5$   
 $r(B) = 2$ 

$$r(B) \neq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$B: \mathbb{R}^5 \to \mathbb{R}^3$$

$$B: \mathbb{R}^{3} \to \mathbb{R}^{3}$$

$$r(B) + d(B) = \dim \mathbb{R}^{5}$$

$$r(B) + 3 = 5$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B)=2$$
  $r(B)
eq \dim \mathbb{R}^3 \longrightarrow B$  nije surjekcija

$$B: \mathbb{R}^5 \to \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
$$r(B) + 3 = 5$$
$$r(B) = 2$$

$$r(B) \neq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$\begin{bmatrix}
1 & 2 & 3 & 1 & 0 \\
1 & 2 & 5 & 2 & 1 \\
1 & 2 & 5 & 2 & 1
\end{bmatrix}$$

$$B: \mathbb{R}^5 \to \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
$$r(B) + 3 = 5$$
$$r(B) = 2$$

$$r(B) \neq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$\begin{bmatrix}
1 & 2 & 3 & 1 & 0 \\
1 & 2 & 5 & 2 & 1 \\
1 & 2 & 5 & 2 & 1
\end{bmatrix} / \cdot (-1)$$

$$B: \mathbb{R}^5 \to \mathbb{R}^3$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
  
 $r(B) + 3 = 5$ 

r(B) = 2

$$r(B) \neq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

 $B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$ 

$$\begin{bmatrix}
1 & 2 & 3 & 1 & 0 \\
1 & 2 & 5 & 2 & 1 \\
1 & 2 & 5 & 2 & 1
\end{bmatrix}$$

$$B:\mathbb{R}^5 \to \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
$$r(B) + 3 = 5$$
$$r(B) = 2$$

$$r(B) 
eq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$\begin{bmatrix}
1 & 2 & 3 & 1 & 0 \\
1 & 2 & 5 & 2 & 1 \\
1 & 2 & 5 & 2 & 1
\end{bmatrix}$$

$$B:\mathbb{R}^5 \to \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
$$r(B) + 3 = 5$$
$$r(B) = 2$$

$$r(B) 
eq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$\begin{bmatrix}
1 & 2 & 3 & 1 & 0 \\
1 & 2 & 5 & 2 & 1 \\
1 & 2 & 5 & 2 & 1
\end{bmatrix}$$

$$B:\mathbb{R}^5 \to \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
$$r(B) + 3 = 5$$
$$r(B) = 2$$

$$r(B) 
eq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$B:\mathbb{R}^5 \to \mathbb{R}^3$$

$$B = \begin{vmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{vmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
$$r(B) + 3 = 5$$
$$r(B) = 2$$

$$r(B) 
eq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$B:\mathbb{R}^5 \to \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
  
 $r(B) + 3 = 5$   
 $r(B) = 2$ 

$$r(B) 
eq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$B: \mathbb{R}^5 \to \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
$$r(B) + 3 = 5$$
$$r(B) = 2$$

$$r(B) 
eq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$B: \mathbb{R}^5 \to \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
  
 $r(B) + 3 = 5$   
 $r(B) = 2$ 

$$r(B) 
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 nije surjekcija

$$B:\mathbb{R}^5 \to \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
  
 $r(B) + 3 = 5$   
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$$r(B) 
eq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$B:\mathbb{R}^5 \to \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
  
 $r(B) + 3 = 5$   
 $r(B) = 2$ 

$$r(B) 
eq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$\operatorname{Im} B$$

$$B:\mathbb{R}^5 \to \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
$$r(B) + 3 = 5$$
$$r(B) = 2$$

$$r(B) 
eq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
  
 $r(B) + 3 = 5$   
 $r(B) = 2$ 

$$r(B) 
eq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$B:\mathbb{R}^5 \to \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
  
 $r(B) + 3 = 5$   
 $r(B) = 2$ 

$$r(B) \neq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$\operatorname{Im} B$$

$$B:\mathbb{R}^5 \to \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
$$r(B) + 3 = 5$$
$$r(B) = 2$$

$$r(B) 
eq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$B:\mathbb{R}^5 \to \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
$$r(B) + 3 = 5$$
$$r(B) = 2$$

$$r(B) 
eq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$B:\mathbb{R}^5 \to \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
$$r(B) + 3 = 5$$
$$r(B) = 2$$

$$r(B) 
eq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$B:\mathbb{R}^5\to\mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
  
 $r(B) + 3 = 5$   
 $r(B) = 2$ 

$$r(B) 
eq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$\operatorname{Im} B$$

$$B:\mathbb{R}^5\to\mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
$$r(B) + 3 = 5$$
$$r(B) = 2$$

$$r(B) \neq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \overset{/\cdot}{\leftarrow} \overset{(-1)/\cdot}{\leftarrow} \overset{(-1)}{\leftarrow} \overset{}{\leftarrow} \overset{[1 \quad 2 \quad 3 \quad 1 \quad 0]}{\leftarrow} \overset{[1 \quad 2 \quad 3 \quad 1$$

$$B: \mathbb{R}^5 \to \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
  
 $r(B) + 3 = 5$   
 $r(B) = 2$ 

$$r(B) \neq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$\sim$$

$$B: \mathbb{R}^5 \to \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
$$r(B) + 3 = 5$$
$$r(B) = 2$$

$$r(B) \neq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$\begin{bmatrix}
1 & 2 & 3 & 1 & 0 \\
1 & 2 & 5 & 2 & 1 \\
1 & 2 & 5 & 2 & 1
\end{bmatrix}
\xrightarrow{+}
\xrightarrow{+}
\sim
\begin{bmatrix}
1 & 2 & 3 & 1 & 0 \\
0 & 0 & 2 & 1 & 1 \\
0 & 0 & 2 & 1 & 1
\end{bmatrix}
\xrightarrow{+}
\xrightarrow{+}
\sim$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ & & & & & \end{bmatrix}$$

$$B: \mathbb{R}^5 \to \mathbb{R}^3$$

$$B = \begin{vmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{vmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
$$r(B) + 3 = 5$$
$$r(B) = 2$$

$$r(B) \neq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$\begin{bmatrix}
1 & 2 & 3 & 1 & 0 \\
1 & 2 & 5 & 2 & 1 \\
1 & 2 & 5 & 2 & 1
\end{bmatrix}
\xrightarrow{/\cdot (-1)/\cdot (-1)}$$

$$\sim
\begin{bmatrix}
1 & 2 & 3 & 1 & 0 \\
0 & 0 & 2 & 1 & 1 \\
0 & 0 & 2 & 1 & 1
\end{bmatrix}
\xrightarrow{/\cdot (-1)}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 1 & 1 \end{bmatrix}$$

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
$$r(B) + 3 = 5$$
$$r(B) = 2$$

$$r(B) \neq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \xrightarrow{/\cdot (-1)/\cdot (-1)} \sim \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 2 & 1 & 1 \end{bmatrix} \xrightarrow{/\cdot (-1)} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
$$r(B) + 3 = 5$$
$$r(B) = 2$$

$$r(B) \neq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} / \cdot (-1) / \cdot (-1) \sim \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 2 & 1 & 1 \end{bmatrix} / \cdot (-1) \sim$$

$$\sim \begin{bmatrix} \boxed{1} & 2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B:\mathbb{R}^5 o \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
$$r(B) + 3 = 5$$
$$r(B) = 2$$

$$r(B) \neq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} / \cdot (-1) / \cdot (-1) \sim \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 2 & 1 & 1 \end{bmatrix} / \cdot (-1) \sim$$

$$\sim \begin{bmatrix} \boxed{1} & 2 & 3 & 1 & 0 \\ 0 & 0 & \boxed{2} & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B: \mathbb{R}^5 \to \mathbb{R}^3$$

$$B = \begin{vmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{vmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$
$$r(B) + 3 = 5$$
$$r(B) = 2$$

$$r(B) \neq \dim \mathbb{R}^3 \longrightarrow B$$
 nije surjekcija

$$\sim \begin{bmatrix} \boxed{1} & 2 & 3 & 1 & 0 \\ 0 & 0 & \boxed{2} & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathcal{B}_{\text{Im }B} = ig\{ (1,1,1), \, (3,5,5) ig\}$$

$$B : \mathbb{R}^5 \to \mathbb{R}^3$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$

$$r(B) + 3 = 5$$

$$r(B) = 2$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$1 + 2 + 3 = 5$$

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$$1 +$$

$$\sim egin{bmatrix} egin{bmatrix} 2 & 3 & 1 & 0 \ 0 & 0 & \textcircled{2} & 1 & 1 \ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \hspace{1cm} \mathcal{B}_{\mathsf{Im}\,\mathcal{B}} = ig\{ (1,1,1), \, (3,5,5) ig\} \ \end{array}$$

Ako je dim  $U = \dim V$ , je li linearni operator  $f: U \to V$  izomorfizam?

# \_\_\_\_

#### Glavne minore

Neka je  $A \in M_n(F)$  pri čemu je F polje.

- Glavna podmatrica reda r matrice A je svaka podmatrica  $A_{i_1,i_2,...,i_r}$  koja se sastoji od onih elemenata matrice A koji se nalaze na presjeku r redaka i r stupaca s istim indeksima  $i_1, i_2, \ldots, i_r$ .
- Glavnih podmatrica reda r matrice A ima ukupno  $\binom{n}{r}$ .
- Glavna minora  $\Delta_{i_1,i_2,...,i_r}$  reda r matrice A je determinanta pripadne glavne podmatrice, tj.  $\Delta_{i_1,i_2,...,i_r} = \det A_{i_1,i_2,...,i_r}$ .

• 
$$k_A^{(1)}(\lambda) = \det(A - \lambda I)$$
  
 $k_A^{(1)}(\lambda) = (-1)^n \lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^{n-2} + \ldots + a_{n-1} \lambda + a_n$ 

• 
$$k_A^{(1)}(\lambda) = \det(A - \lambda I)$$

$$k_A^{(1)}(\lambda) = (-1)^n \lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^{n-2} + \ldots + a_{n-1} \lambda + a_n$$

• 
$$k_A^{(2)}(\lambda) = \det(\lambda I - A)$$

$$k_A^{(2)}(\lambda) = \lambda^n + c_1 \lambda^{n-1} + c_2 \lambda^{n-2} + \ldots + c_{n-1} \lambda + c_n$$

• 
$$k_A^{(1)}(\lambda) = \det(A - \lambda I)$$

$$k_A^{(1)}(\lambda) = (-1)^n \lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^{n-2} + \ldots + a_{n-1} \lambda + a_n$$

• 
$$k_A^{(2)}(\lambda) = \det(\lambda I - A)$$

$$k_A^{(2)}(\lambda) = \lambda^n + c_1 \lambda^{n-1} + c_2 \lambda^{n-2} + \ldots + c_{n-1} \lambda + c_n$$

• 
$$c_r = (-1)^n a_r, \quad r = 1, 2, \dots, n$$

• 
$$k_A^{(1)}(\lambda) = \det(A - \lambda I)$$

$$k_A^{(1)}(\lambda) = (-1)^n \lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^{n-2} + \ldots + a_{n-1} \lambda + a_n$$

• 
$$k_A^{(2)}(\lambda) = \det(\lambda I - A)$$

$$k_A^{(2)}(\lambda) = \lambda^n + c_1 \lambda^{n-1} + c_2 \lambda^{n-2} + \ldots + c_{n-1} \lambda + c_n$$

• 
$$c_r = (-1)^n a_r, \quad r = 1, 2, \dots, n$$

• 
$$c_r = (-1)^r \sum_{\substack{i_1 < i_2 < \dots < i_r}} \Delta_{i_1, i_2, \dots, i_r}, \qquad \{i_1, i_2, \dots, i_r\} \subseteq \{1, 2, \dots, n\}$$

• 
$$k_A^{(1)}(\lambda) = \det(A - \lambda I)$$

$$k_A^{(1)}(\lambda) = (-1)^n \lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^{n-2} + \ldots + a_{n-1} \lambda + a_n$$

• 
$$k_{\Delta}^{(2)}(\lambda) = \det(\lambda I - A)$$

$$k_A^{(2)}(\lambda) = \lambda^n + c_1 \lambda^{n-1} + c_2 \lambda^{n-2} + \ldots + c_{n-1} \lambda + c_n$$

• 
$$c_r = (-1)^n a_r, \quad r = 1, 2, \dots, n$$

• 
$$c_r = (-1)^r \sum_{\substack{i_1 < i_2 < \dots < i_r}} \Delta_{i_1, i_2, \dots, i_r}, \qquad \{i_1, i_2, \dots, i_r\} \subseteq \{1, 2, \dots, n\}$$

• 
$$c_1 = -\operatorname{tr} A$$
,  $c_n = (-1)^n \det A$ 

- ullet  ${\cal B}$  respectively.  ${\cal B}$  respectively.

$$f:V\to V$$

- $\mathcal{B}$  ------- neka baza za vektorski prostor V

$$f: V \rightarrow V$$

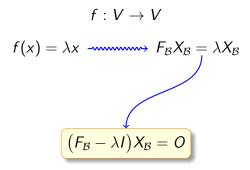
$$f(x) = \lambda x$$

- B ← neka baza za vektorski prostor V

$$f: V \to V$$

$$f(x) = \lambda x - F_{\mathcal{B}} X_{\mathcal{B}} = \lambda X_{\mathcal{B}}$$

- B ← neka baza za vektorski prostor V



$$f: V \to V$$

$$f(x) = \lambda x \xrightarrow{} F_{\mathcal{B}} X_{\mathcal{B}} = \lambda X_{\mathcal{B}}$$

$$(F_{\mathcal{B}} - \lambda I) X_{\mathcal{B}} = 0$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

drugi zadatak

#### Zadatak 2

Zadana je matrica 
$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$
.

- a) Odredite svojstvene vrijednosti matrice A.
- b) Odredite svojstvene potprostore matrice A.
- c) Odredite minimalni polinom matrice A.
- d) Izrazite  $A^{-1}$  pomoću potencija matrice A.

#### Rješenje

$$k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

#### Rješenje

a) 
$$k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 =$$

#### Rješenje

) 
$$k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right)$$

$$k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1=(-1)^1\cdot \left(\Delta_1+\Delta_2+\Delta_3
ight)=-($$

$$k_{\mathcal{A}}(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1=(-1)^1\cdot\left(\Delta_1+\Delta_2+\Delta_3\right)=-(4$$

$$k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1=(-1)^1\cdot\left(\Delta_1+\Delta_2+\Delta_3\right)=-(4+$$

$$k_{\mathcal{A}}(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4+5)^3$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4+5+$$

$$k_{\mathcal{A}}(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4+5+2)$$

$$k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4+5+2)$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4+5+2) = -11$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4+5+2) = -11$$
  $c_2 =$ 

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4+5+2) = -11$$
  $c_2 = (-1)^2 \cdot \left(\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}\right)$ 

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4+5+2) = -11$$
  $c_2 = (-1)^2 \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) =$ 

a) 
$$k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{array}{c} 4 & 1 & -1 \\ 4 & 2 & -1 \\ 4 &$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4+5+2) = -11$$

$$c_2 = (-1)^2 \cdot \left(\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}\right) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix}$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$egin{align} c_1 &= (-1)^1 \cdot \left( \Delta_1 + \Delta_2 + \Delta_3 
ight) = -(4+5+2) = -11 \ & \ c_2 &= (-1)^2 \cdot \left( \Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3} 
ight) = egin{bmatrix} 4 & 1 \ 2 & 5 \end{bmatrix} + & \ \end{array}$$

a) 
$$k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(1)$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4+5+2) = -11$$

$$c_2 = (-1)^2 \cdot \left(\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3} \right) = egin{bmatrix} 4 & 1 \ 2 & 5 \end{bmatrix} + egin{bmatrix} 4 & -1 \ 1 & 2 \end{bmatrix}$$

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$ 

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_2 = (-1)^2 \cdot ig( \Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3} ig) = egin{vmatrix} 4 & 1 \ 2 & 5 \end{bmatrix} + egin{vmatrix} 4 & -1 \ 1 & 2 \end{bmatrix} +$$

 $c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4+5+2) = -11$ 

a) 
$$k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4+5+2) = -11$$

$$c_2 = (-1)^2 \cdot \left(\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}
ight) = egin{bmatrix} 4 & 1 \ 2 & 5 \end{bmatrix} + egin{bmatrix} 4 & -1 \ 1 & 2 \end{bmatrix} + egin{bmatrix} 5 & -2 \ 1 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4+5+2) = -11$$
  $c_2 = (-1)^2 \cdot \left(\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}\right) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} =$ 

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_{1} = (-1)^{1} \cdot (\Delta_{1} + \Delta_{2} + \Delta_{3}) = -(4+5+2) = -11$$

$$c_{2} = (-1)^{2} \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} = 18$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4+5+2) = -11$$

$$c_2 = (-1)^2 \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} = 18 + 9$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4+5+2) = -11$$
 $c_2 = (-1)^2 \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} = 18 + 9 + 12$ 

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_{1} = (-1)^{1} \cdot (\Delta_{1} + \Delta_{2} + \Delta_{3}) = -(4+5+2) = -11$$

$$c_{2} = (-1)^{2} \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} =$$

$$= 18 + 9 + 12 = 39$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4+5+2) = -11$$
 $c_2 = (-1)^2 \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} = 18 + 9 + 12 = 39$ 

$$c_3 =$$

a) 
$$k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$$
  $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$ 

$$c_{1} = (-1)^{1} \cdot (\Delta_{1} + \Delta_{2} + \Delta_{3}) = -(4+5+2) = -11$$

$$c_{2} = (-1)^{2} \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} =$$

$$= 18 + 9 + 12 = 39$$

$$c_3 = (-1)^3 \cdot \Delta_{1,2,3}$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_{1} = (-1)^{1} \cdot (\Delta_{1} + \Delta_{2} + \Delta_{3}) = -(4+5+2) = -11$$

$$c_{2} = (-1)^{2} \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} =$$

$$= 18 + 9 + 12 = 39$$

$$c_3 = (-1)^3 \cdot \Delta_{1,2,3} = -1$$

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$ 

) 
$$k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$$
  $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$   $c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5 + 2) = -11$ 

 $c_2 = (-1)^2 \cdot ig( \Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3} ig) = egin{vmatrix} 4 & 1 \ 2 & 5 \end{bmatrix} + egin{vmatrix} 4 & -1 \ 1 & 2 \end{bmatrix} + egin{vmatrix} 5 & -2 \ 1 & 2 \end{bmatrix} =$ = 18 + 9 + 12 = 39 $c_3 = (-1)^3 \cdot \Delta_{1,2,3} = -1 \cdot \begin{vmatrix} 4 & 1 & -1 \ 2 & 5 & -2 \ 1 & 1 & 2 \end{vmatrix}$ 

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$ 

 $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$ 

$$c_{1} = (-1)^{1} \cdot (\Delta_{1} + \Delta_{2} + \Delta_{3}) = -(4+5+2) = -11$$

$$c_{2} = (-1)^{2} \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} =$$

$$= 18 + 9 + 12 = 39$$

 $c_3 = (-1)^3 \cdot \Delta_{1,2,3} = -1 \cdot \begin{vmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{vmatrix} = -1 \cdot 45$ 

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$ 

 $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$ 

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4+5+2) = -11$$

$$c_2 = (-1)^2 \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} = 18 + 9 + 12 = 39$$

$$c_3 = (-1)^3 \cdot \Delta_{1,2,3} = -1 \cdot \begin{vmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{vmatrix} = -1 \cdot 45 = -45$$

 $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$ 

 $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$ 

 $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$ 

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4+5+2) = -11$$

$$c_2 = (-1)^2 \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} = 18 + 9 + 12 = 39$$

 $c_3 = (-1)^3 \cdot \Delta_{1,2,3} = -1 \cdot \begin{vmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{vmatrix} = -1 \cdot 45 = -45$ 

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$ 

$$1, -1,$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3,$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5,$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$   
 $1, -1, 3, -3, 5, -5, 9, -9,$ 

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$ 

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15,\\$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$   
 $1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$ 

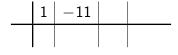
$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$   
 $1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$ 



$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$   
 $1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$ 



$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$   
 $1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$ 



$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$   
 $1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$ 

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$   
 $1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$ 

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$   
 $1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$ 

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$   
 $1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$ 

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$   
 $1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$ 

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$   
 $1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$ 

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$   
 $1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$ 

$$k_{A}(\lambda) = \lambda^{3} - 11\lambda^{2} + 39\lambda - 45$$

$$\lambda^{3} - 11\lambda^{2} + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\begin{array}{c|c} & 1 & -11 & 39 & -45 \\ \hline & 3 & 1 & -8 & 15 & 0 \end{array}$$

$$(\lambda - 3)$$

$$k_{A}(\lambda) = \lambda^{3} - 11\lambda^{2} + 39\lambda - 45$$
 $\lambda^{3} - 11\lambda^{2} + 39\lambda - 45 = 0$ 
 $1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$ 

$$\begin{array}{c|cccc} & 1 & -11 & 39 & -45 \\ \hline & 3 & 1 & -8 & 15 & 0 \end{array}$$
 $(\lambda - 3)($ 

$$k_{A}(\lambda) = \lambda^{3} - 11\lambda^{2} + 39\lambda - 45$$
 $\lambda^{3} - 11\lambda^{2} + 39\lambda - 45 = 0$ 
 $1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$ 

$$\begin{array}{c|cccc} & 1 & -11 & 39 & -45 \\ \hline & 3 & 1 & -8 & 15 & 0 \end{array}$$
 $(\lambda - 3)(\lambda^{2} - 8\lambda)$ 

$$(\lambda-3)\big(\lambda^2-8\lambda+15\big)$$

$$k_{A}(\lambda) = \lambda^{3} - 11\lambda^{2} + 39\lambda - 45$$

$$\lambda^{3} - 11\lambda^{2} + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\frac{\begin{vmatrix} 1 & -11 & 39 & -45 \\ \hline 3 & 1 & -8 & 15 & 0 \end{vmatrix}$$

$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$$k_{A}(\lambda) = \lambda^{3} - 11\lambda^{2} + 39\lambda - 45$$

$$\lambda^{3} - 11\lambda^{2} + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\frac{\begin{vmatrix} 1 & -11 & 39 & -45 \\ \hline 3 & 1 & -8 & 15 & 0 \end{vmatrix}$$

$$(\lambda - 3)(\lambda^{2} - 8\lambda + 15) = 0$$

$$\lambda_{1} = 3$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$   
 $1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$ 

$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$$\lambda_1 = 3$$

$$k_{A}(\lambda) = \lambda^{3} - 11\lambda^{2} + 39\lambda - 45$$

$$\lambda^{3} - 11\lambda^{2} + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\frac{\begin{vmatrix} 1 & -11 & 39 & -45 \\ \hline 3 & 1 & -8 & 15 & 0 \end{vmatrix}$$

$$(\lambda - 3)(\lambda^{2} - 8\lambda + 15) = 0$$

$$\lambda_{1} = 3$$

$$\lambda^{2} - 8\lambda + 15 = 0$$

$$k_{A}(\lambda) = \lambda^{3} - 11\lambda^{2} + 39\lambda - 45$$

$$\lambda^{3} - 11\lambda^{2} + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\frac{\begin{vmatrix} 1 & -11 & 39 & -45 \\ \hline 3 & 1 & -8 & 15 & 0 \end{vmatrix}$$

$$(\lambda - 3)(\lambda^{2} - 8\lambda + 15) = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^{2} - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$$\lambda_1 = 3$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2}$$

$$k_{A}(\lambda) = \lambda^{3} - 11\lambda^{2} + 39\lambda - 45$$

$$\lambda^{3} - 11\lambda^{2} + 39\lambda - 45 = 0$$
 $1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$ 

$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$$\lambda_1 = 3$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2} \qquad \lambda_2 = 5$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$   
 $1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$ 

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$$\lambda_1 = 3$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_1 = 3 \qquad \lambda^2 - 8\lambda + 15 =$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2}$$
  $\lambda_2 = 5, \ \lambda_3 = 3$ 

$$k_{A}(\lambda) = \lambda^{3} - 11\lambda^{2} + 39\lambda - 45$$

$$\lambda^{3} - 11\lambda^{2} + 39\lambda - 45 = 0$$
 $1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$ 

$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$$\lambda_1 = 3$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2}$$

$$\lambda_2 = 5, \lambda_3 = 3$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$   
 $1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$ 

$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$$\lambda_1 = 3$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_1 = 3$$

$$\lambda_2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2}$$
  $\lambda_2 = 5, \lambda_3 = 3$ 

$$k_{\mathcal{A}}(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\sigma(A) = \{3, 5\}$$

$$\frac{\begin{vmatrix} 1 & -11 & 39 & -45 \\ \hline 3 & 1 & -8 & 15 & 0 \end{vmatrix}}{(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0}$$

$$\lambda_1 = 3$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2} \qquad \lambda_{2} = 5, \ \lambda_{3} = 3$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$   
 $1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$ 

$$\sigma(A) = \{3,5\} \qquad \frac{\begin{array}{c|c|c|c} 1 & -11 & 39 & -45 \\ \hline 3 & 1 & -8 & 15 & 0 \\ \hline \end{array}}{3 & 1 & -8 & 15 & 0}$$
 algebarska kratnost jednaka je 2 
$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$
 
$$\lambda_1 = 3 \qquad \lambda^2 - 8\lambda + 15 = 0$$
 
$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2}$$
  $\lambda_2 = 5, \lambda_3 = 3$ 

algebarska kratnost jednaka je 1 
$$\lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\sigma(A) = \{3, 5\}$$

$$\frac{1 \mid -11 \mid 39 \mid -45}{3 \mid 1 \mid -8 \mid 15 \mid 0}$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

 $k_{\Delta}(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$ 

algebarska kratnost jednaka je 2 
$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$$\lambda_1 = 3$$

$$\lambda_1 = 3 \qquad \lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2}$$
  $\lambda_2 = 5, \lambda_3 = 3$ 

algebarska kratnost jednaka je 1 
$$1, -\infty$$
  $\sigma(A) = \{3, 5\}$ 

algebarska kratnost

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

 $k_{\Delta}(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$ 

jednaka je 2 
$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$$\lambda_1 = 3 \qquad \lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_1 = 3 \qquad \lambda^2 - 8\lambda + 15 = 0$$

$$\lambda^{2} - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^{2} - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2}$$
  $\lambda_2 = 5, \lambda_3 = 3$ 

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

 $k_{\Delta}(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$ 

algebarska kratnost jednaka je 2 
$$(\lambda - 1)^{-3}$$

$$(\lambda-3)(\lambda^2-8\lambda+15)=0$$

$$\lambda_1 = 3$$

$$\lambda_1 = 3 \qquad \qquad \lambda^2 - 8\lambda + 15 = 0$$

$$\lambda^{2} - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^{2} - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$3\pm 2$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2}$$
  $\lambda_2 = 5, \lambda_3 = 3$ 

$$k_A(\lambda) = (\lambda - 3)^2$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

 $k_{A}(\lambda) = \lambda^{3} - 11\lambda^{2} + 39\lambda - 45$ 

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\frac{3}{3} \frac{1}{1} - 8 \frac{15}{0}$$
algebarska kratnost

$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$$\lambda_1 = 3$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$8 \pm 2$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2}$$
  $\lambda_2 = 5, \lambda_3 = 3$ 

 $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$ 

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$
  $(F_B - \lambda I)X_B = O$ 

 $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$ (A - 3I)X = 0

 $S(\lambda) = \{x \in V : f(x) = \lambda x\}$ 

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

 $A = \begin{vmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{vmatrix}$ (A - 3I)X = 0

 $S(\lambda) = \{ x \in V : f(x) = \lambda x \}$ 

$$I(X_{\mathcal{B}} = 0)$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

 $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$ (A - 3I)X = 0

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$
  $(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = 0$ 

 $(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$ 

 $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$  (A - 3I)X = O

 $S(\lambda) = \{x \in V : f(x) = \lambda x\}$ 

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

b)  $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$ 

$$\begin{bmatrix} 1 & & \\ & 2 & \\ & & -1 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$
 
$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

 $S(\lambda) = \{x \in V : f(x) = \lambda x\}$ 

 $\begin{vmatrix} 1 & 1 & -1 \\ 2 & -2 \\ & -1 \end{vmatrix}$ 

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

 $\begin{vmatrix}
1 & 1 & -1 \\
2 & 2 & -2 \\
1 & 1 & -1
\end{vmatrix}$ 

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$
  $(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$ 

 $\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ 

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

 $S(\lambda) = \{x \in V : f(x) = \lambda x\}$ 

 $(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$ 

 $\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ 

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$
  $(F_B - \lambda I)X_B = O$ 

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = 0$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$
  $(F_B - \lambda I)X_B = O$ 

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$X_{1} \quad X_{2} \quad X_{3} \quad (A-3I)X = 0$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$
  $(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$ 

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$
  $(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$ 

$$S(\lambda) = \{ x \in V : f(x) = \lambda x \}$$
 (Figure 1)

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$$S(\lambda) = \{ x \in V : f(x) = \lambda x \}$$
 (F

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

 $S(\lambda) = \{x \in V : f(x) = \lambda x\}$ 

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

b)
$$A = \begin{bmatrix}
4 & 1 & -1 \\
2 & 5 & -2 \\
1 & 1 & -1 & 0 \\
2 & 2 & -2 & 0 /: 2 \\
\hline
1 & 1 & -1 & 0 \\
\hline
1 & 1 & -1 & 0 \\
1 & 1 & -1 & 0
\end{bmatrix}$$

$$(A - 3I)X = 0$$

$$\begin{bmatrix}
1 & 1 & -1 \\
2 & 2 & -2 \\
1 & 1 & -1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}$$

 $S(\lambda) = \{x \in V : f(x) = \lambda x\}$ 

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$
  $(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$ 

b)
$$A = \begin{bmatrix}
4 & 1 & -1 \\
2 & 5 & -2 \\
1 & 1 & -1 & 0 \\
2 & 2 & -2 & 0 / : 2 \\
\hline
1 & 1 & -1 & 0 \\
\hline
1 & 1 & -1 & 0 \\
1 & 1 & -1 & 0 \\
1 & 1 & -1 & 0
\end{bmatrix}$$

$$A = \begin{bmatrix}
4 & 1 & -1 \\
2 & 5 & -2 \\
1 & 1 & 2
\end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix}
1 & 1 & -1 \\
2 & 2 & -2 \\
1 & 1 & -1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}$$

$$1 & 1 & -1 \\
1 & 1 & -1
\end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$
  $(F_{\beta} - \lambda I)X_{\beta} = O$ 

 $(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$ 

b)
$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\frac{x_1}{1} \quad \frac{x_2}{1} \quad \frac{x_3}{1} \quad \frac{x_3}{1} \quad \frac{x_3}{1} \quad \frac{x_3}{1} \quad \frac{x_3}{1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 & 1 & -1 \\$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$
  $(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$ 

b)
$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\frac{x_1}{1} \quad \frac{x_2}{1} \quad \frac{x_3}{1} \quad \frac{x_3}{1} \quad \frac{x_3}{1} \quad \frac{x_3}{1} \quad \frac{x_3}{1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{1}{1} \quad \frac{1}{1} \quad \frac{1}{1} \quad 0 \quad \frac{1}{1} \quad 1 \quad 1 \quad 0$$

$$\frac{1}{1} \quad 1 \quad -1 \quad 0 \quad \frac{1}{1} \quad 1 \quad 1 \quad 0$$

 $x_1 + x_2 - x_3 = 0$ 

b)

 $(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$  $S(\lambda) = \{x \in V : f(x) = \lambda x\}$ 

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$X_{1} \quad X_{2} \quad X_{3} \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$X_{1} \quad X_{2} \quad X_{3} \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

1 1  $-1 \mid 0$   $x_1 + x_2 - x_3 = 0 \longrightarrow x_3 = x_1 + x_2$ 

 $S(\lambda) = \{x \in V : f(x) = \lambda x\}$ 

b)

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$$x_1 - x_3 = 0 \xrightarrow{\bullet} x_3$$

$$x_1 + x_2 - x_3 = 0 \longrightarrow x_3 = x_1 + x_2$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$
 
$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = 0$$

$$S(3)=ig\{($$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\} \qquad (F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

 $x_1 + x_2 - x_3 = 0 \longrightarrow x_3 = x_1 + x_2$ 

 $x_1 + x_2 - x_3 = 0 \longrightarrow x_3 = x_1 + x_2$   $S(3) = \{(x_1,$ 

$$S(3) = \{(x_1,$$

b)

$$S(\lambda) = \{ x \in V : f(x) = \lambda x \}$$
 (F

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$X_{1} \quad X_{2} \quad X_{3} \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = 0$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = 0$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

 $S(3) = \{(x_1, x_2,$ 

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

b)

 $(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$ 

 $x_1 + x_2 - x_3 = 0 \longrightarrow x_3 = x_1 + x_2$  $S(3) = \{(x_1, x_2, x_1 + x_2)\}$ 

$$S(3) = \{(x_1, x_2, x_1 + x_2)\}$$

 $S(\lambda) = \{x \in V : f(x) = \lambda x\}$ 

b)

 $(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$ 

$$-1 \mid 0 \qquad x_1 + x_2 - x_3 = 0 \xrightarrow{} x_3$$

 $S(3) = \{(x_1, x_2, x_1 + x_2) : x_1, x_2 \in \mathbb{R} \}$ 

$$S(\lambda) = \{ x \in V : f(x) = \lambda x \}$$
 
$$(F_{\mathcal{B}} - \lambda I) X_{\mathcal{B}} = O$$

b)
$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$x_1 \quad x_2 \quad x_3 \mid \\
1 \quad 1 \quad -1 \quad 0 \\
2 \quad 2 \quad -2 \quad 0 \quad /: 2$$

$$1 \quad 1 \quad -1 \quad 0 \\
1 \quad 1 \quad -1 \quad 0 \\
1 \quad 1 \quad -1 \quad 0 \\
1 \quad 1 \quad -1 \quad 0
\end{bmatrix} \quad \begin{bmatrix} 1 \quad 1 & -1 \\ 2 \quad 2 & -2 \\ 1 \quad 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$1 \quad 1 \quad -1 \quad 0 \quad x_1 + x_2 - x_3 = 0 \quad x_3 = x_1 + x_2$$

$$S(3) = \{(x_1, x_2, x_1 + x_2) : x_1, x_2 \in \mathbb{R}\}$$

 $(x_1, x_2, x_1 + x_2) =$ 

b)

$$(x_1, x_2, x_1 + x_2) =$$

 $S(\lambda) = \{x \in V : f(x) = \lambda x\} \qquad (F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$ 

b) 
$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$x_1 \quad x_2 \quad x_3 \quad | \quad (A-3I)X = 0$$

$$2 \quad 2 \quad -2 \quad 0 \quad /: 2$$

$$1 \quad 1 \quad -1 \quad 0$$

$$3 \quad x_1 + x_2 - x_3 = 0 \quad | \quad x_3 = x_1 + x_2$$

$$S(3) = \{(x_1, x_2, x_1 + x_2) : x_1, x_2 \in \mathbb{R}\}$$

$$(x_1, x_2, x_1 + x_2) = x_1 \cdot ($$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\} \qquad (F_{\mathcal{B}} - \lambda I) X_{\mathcal{B}} = O$$

b)
$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\frac{x_1}{1} \quad \frac{x_2}{1} \quad \frac{x_3}{1} \quad \frac{1}{1} \quad \frac{1}{1} \quad \frac{1}{1} \quad \frac{1}{2}$$

$$\frac{2}{1} \quad \frac{2}{1} \quad \frac{1}{1} \quad$$

$$(x_1, x_2, x_1 + x_2) = x_1 \cdot (1,$$

 $S(\lambda) = \{x \in V : f(x) = \lambda x\}$   $(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$ 

$$S(3) = \{(x_1, x_2, x_1 + x_2) : x_1 + x_2 \in (x_1, x_2, x_1 + x_2) = x_1 \cdot (1, 0, x_2 + x_2) =$$

$$(x_1, x_2, x_1 + x_2) = x_1 \cdot (1, 0,$$

$$(x_1, x_2, x_1 + x_2) = x_1 \cdot (1, 0,$$
 
$$S(\lambda) = \{ x \in V : f(x) = \lambda x \}$$
  $(F_B - \lambda I) X_B = O$ 

b)
$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\frac{x_1}{1} \quad \frac{x_2}{1} \quad \frac{x_3}{1} \quad \frac{1}{1} \quad \frac{1}{1} \quad \frac{1}{1} \quad \frac{1}{2}$$

$$\frac{2}{1} \quad \frac{2}{1} \quad \frac{1}{1} \quad$$

$$(x_1, x_2, x_1 + x_2) = x_1 \cdot (1, 0, 1)$$

 $S(\lambda) = \{x \in V : f(x) = \lambda x\}$   $(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$ 

b)

b)
$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\frac{x_1}{1} \quad \frac{x_2}{1} \quad \frac{x_3}{1} \quad \frac{x_3}{1} \quad \frac{x_3}{1} \quad \frac{x_3}{1} \quad \frac{x_3}{1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 & 1 & -1 \\$$

 $(x_1, x_2, x_1 + x_2) = x_1 \cdot (1, 0, 1) +$ 

$$(x_1, x_2, x_1 + x_2) = x_1 \cdot (1, 0, 1) +$$

b)

 $S(\lambda) = \{x \in V : f(x) = \lambda x\}$   $(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$ 

b) 
$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$x_{1} \quad x_{2} \quad x_{3} \quad (A-3I)X = 0$$

$$2 \quad 2 \quad -2 \quad 0 \quad /: 2$$

$$1 \quad 1 \quad -1 \quad 0$$

$$3 \quad x_{1} + x_{2} - x_{3} = 0 \quad x_{3} = x_{1} + x_{2}$$

$$S(3) = \{(x_{1}, x_{2}, x_{1} + x_{2}) : x_{1}, x_{2} \in \mathbb{R}\}$$

$$(x_{1}, x_{2}, x_{1} + x_{2}) = x_{1} \cdot (1, 0, 1) + x_{2} \cdot (1, 0, 1) + x_{2} \cdot (1, 0, 1) + x_{3} \cdot (1, 0, 1) + x_{4} \cdot (1, 0, 1) + x_{5} \cdot (1, 0, 1) +$$

 $S(\lambda) = \{x \in V : f(x) = \lambda x\} \qquad (F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$ 

b) 
$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\frac{x_1}{1} \quad \frac{x_2}{1} \quad \frac{x_3}{1} \quad \frac{x_3}{1} \quad \frac{x_3}{1} \quad \frac{x_3}{1} \quad \frac{x_3}{1} = \begin{bmatrix} 0 \\ 0 \\ 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \\ 3 & 2 & 3 \\ 3 & 3 & 0 \\ 0 & 0 \\$$

 $(x_1, x_2, x_1 + x_2) = x_1 \cdot (1, 0, 1) + x_2 \cdot (0, 1)$ 

$$(x_1, x_2, x_1 + x_2) = x_1 \cdot (1, 0, 1) + x_2 \cdot (0,$$

$$S(\lambda) = \{ x \in V : f(x) = \lambda x \}$$

$$(F_{\mathcal{B}} - \lambda I) X_{\mathcal{B}} = O$$

$$(1,0,1) + \lambda_2 \cdot (0,1)$$

b) 
$$A = \begin{bmatrix} x_1 & x_2 & x_3 \\ \hline 1 & 1 & -1 & 0 \\ 2 & 2 & -2 & 0 \\ \hline 1 & 1 & -1 & 0 \\ \hline 1 & 1 & -1 & 0 \\ \hline 1 & 1 & -1 & 0 \\ \hline 1 & 1 & -1 & 0 \\ \hline 1 & 1 & -1 & 0 \\ \hline 1 & 1 & -1 & 0 \\ \hline 1 & 1 & -1 & 0 \\ \hline 1 & 1 & -1 & 0 \\ \hline 1 & 1 & -1 & 0 \\ \hline 1 & 1 & -1 & 0 \\ \hline 1 & 1 & -1 & 0 \\ \hline 1 & 1 & -1 & 0 \\ \hline 1 & 1 & -1 & 0 \\ \hline \end{bmatrix} \begin{array}{c} (A - 3I)X = O \\ \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \\ \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \end{bmatrix} \\ \hline x_1 + x_2 - x_3 = 0 \xrightarrow{} x_3 = x_1 + x_2 \\ \\ S(3) = \{(x_1, x_2, x_1 + x_2) : x_1, x_2 \in \mathbb{R}\} \end{array}$$

$$S(3) = \{(x_1, x_2, x_1 + x_2) : x_1, x_2 \in \mathbb{R}\}$$

$$(x_1, x_2, x_1 + x_2) = x_1 \cdot (1, 0, 1) + x_2 \cdot (0, 1, 1)$$

$$(x_1, x_2, x_1 + x_2) = x_1 \cdot (1, 0, 1) + x_2 \cdot (0, 1, 1)$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\} \qquad (F_{\mathcal{B}} - \lambda I) X_{\mathcal{B}} = O$$

b) 
$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$x_1 \quad x_2 \quad x_3 \quad | \quad (A-3I)X = 0$$

$$2 \quad 2 \quad -2 \quad 0 \quad /: 2$$

$$1 \quad 1 \quad -1 \quad 0$$

$$3 \quad x_1 + x_2 - x_3 = 0 \quad x_3 = x_1 + x_2$$

$$S(3) = \{(x_1, x_2, x_1 + x_2) : x_1, x_2 \in \mathbb{R}\}$$

$$(x_1, x_2, x_1 + x_2) = x_1 \cdot (1, 0, 1) + x_2 \cdot (0, 1, 1)$$

$$(x_1, x_2, x_1 + x_2) = x_1 \cdot (1, 0, 1) + x_2 \cdot (0, 1, 1)$$

$$S(\lambda) = \{ x \in V : f(x) = \lambda x \}$$

$$(F_{\mathcal{B}} - \lambda I) X_{\mathcal{B}} = O$$

$$S(3) = \{(x_1, x_2, x_1 + x_2) : x_1, x_2 \in \mathbb{R}\}$$

$$S(3) = \{(x_1, x_2, x_1 + x_2) : x_1, x_2 \in \mathbb{R}\}$$

$$(x_1, x_2, x_1 + x_2) = x_1 \cdot (1, 0, 1) + x_2 \cdot (0, 1, 1)$$

$$(x_1, x_2, x_1 + x_2) = x_1 \cdot (1, 0, 1) + x_2 \cdot (0, 1, 1)$$

$$S(\lambda) = \left\{ x \in V : f(x) = \lambda x \right\} \qquad (F_{\mathcal{B}} - \lambda I) X_{\mathcal{B}} = O$$

b)  $\mathcal{B}_{S(3)} = \{(1,0,1), (0,1,1)\}$ 

 $\dim S(3) = 2$ 

$$S(3) = \{(x_1, x_2, x_1 + x_2) : x_1, x_2 \in \mathbb{R}\}$$

 $\mathcal{B}_{S(3)} = \{(1,0,1), (0,1,1)\}$ 

 $S(3) = \{(x_1, x_2, x_1 + x_2) : x_1, x_2 \in \mathbb{R} \}$ 

$$S(3) = \{(x_1, x_2, x_1 + x_2) : x_1, x_2 \in \mathbb{R}\}$$

$$(x_1, x_2, x_1 + x_2) = x_1 \cdot (1, 0, 1) + x_2 \cdot (0, 1, 1)$$

$$(x_1, x_2, x_1 + x_2) = x_1 \cdot (1, 0, 1) + x_2 \cdot (0, 1, 1)$$

$$S(\lambda) = \{ x \in V : f(x) = \lambda x \}$$

$$(F_{\mathcal{B}} - \lambda I) X_{\mathcal{B}} = O$$

 $S(\lambda) = \{x \in V : f(x) = \lambda x\}$   $(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$ 

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$
  $(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$ 

(A-5I)X = O  $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$ 

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\} \qquad (F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

(A-5I)X = O  $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$ 

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$
  $(F_B - \lambda I)X_B = O$ 

(A-5I)X = O  $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$ 

 $\begin{bmatrix} -1 & & \\ & 0 & \end{bmatrix}$ 

(A-5I)X = O  $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$ 

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$$3 = 0$$

(A-5I)X = O  $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$ 

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

 $\begin{bmatrix} -1 \\ 0 \\ -3 \end{bmatrix}$ 

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$
  $(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$ 

 $\begin{bmatrix} -1 & 1 & -1 \\ & 0 & -2 \\ & & -3 \end{bmatrix}$ 

(A-5I)X = O  $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$ 

(A-5I)X = O  $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$ 

 $\begin{bmatrix}
 -1 & 1 & -1 \\
 2 & 0 & -2 \\
 1 & 1 & -3
 \end{bmatrix}$ 

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$
  $(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$ 

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

(A-5I)X=O

 $\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ 

 $A = \begin{vmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{vmatrix}$ 

$$S(\lambda) = \{x \in V : f(x) = \lambda x\} \qquad (F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

 $(A-5I)X = O A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ 

$$(A-5I)X = O A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

 $x_1$ 

 $X_2$ 

 $X_3$ 

$$S(\lambda) = \{x \in V : f(x) = \lambda x\} \qquad (F_{\mathcal{B}} - \lambda I) X_{\mathcal{B}} = O$$

 $S(\lambda) = \{x \in V : f(x) = \lambda x\}$  $(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$ 

$$S(\lambda) = \{x \in V : f(x) = \lambda x\} \qquad (F_{\beta} - \lambda I)X_{\beta} = O$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$
 
$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = 0$$

 $S(\lambda) = \{x \in V : f(x) = \lambda x\}$  $(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$ 12 / 18

 $S(\lambda) = \left\{ x \in V : f(x) = \lambda x \right\} \tag{F_B - \lambda I) X_B = 0}$ 

 $S(\lambda) = \{x \in V : f(x) = \lambda x\}$ 

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

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$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

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 $X_2$ 

$$S(\lambda) = \{x \in V : f(x) = \lambda x\} \qquad (F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

*X*<sub>3</sub>

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$
 
$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

*X*<sub>3</sub>

 $k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$ 

 $k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$ 

 $(A-3I) \cdot (A-5I) =$ 

$$(A-3I)\cdot (A-5I) = \begin{bmatrix} & & \\ & & \end{bmatrix}$$

 $k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$ 

$$(A-3I)\cdot(A-5I)=\begin{bmatrix}1\\\\\end{bmatrix}$$

 $k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$ 

$$(A-3I)\cdot (A-5I) = \begin{bmatrix} 1 & & \\ & 2 & \\ & & \end{bmatrix}$$

 $k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$ 

$$(A-3I)\cdot(A-5I)=\begin{bmatrix}1\\2\\-1\end{bmatrix}$$

 $k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$ 

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 $(A-3I)\cdot (A-5I) = \begin{vmatrix} 1 & 1 & -1 \\ 2 & -2 \\ & -1 \end{vmatrix}$ 

$$(A-3I)\cdot (A-5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix}$$

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 $(A-3I)\cdot (A-5I) = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{vmatrix}$ 

 $(A-3I)\cdot (A-5I) = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{vmatrix} \begin{vmatrix} -1 \\ -1 \\ -1 \end{vmatrix}$ 

c)  $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$ 

 $k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$ 

 $(A-3I)\cdot (A-5I) = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{vmatrix} \begin{vmatrix} -1 \\ 0 \\ 0 \end{vmatrix}$ 

c)  $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$ 

 $k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$ 

 $(A-3I)\cdot (A-5I) = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{vmatrix} \begin{vmatrix} -1 \\ 0 \\ -3 \end{vmatrix}$ 

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$$(A-3I)\cdot (A-5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix}$$

 $k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$ 

$$(A-3I)\cdot (A-5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

 $k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$ 

$$m_{A}(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$$

$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

 $k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$ 

c) 
$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$   $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$   
 $m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$   $m_A(\lambda) = \lambda^2 - 8\lambda + 15$  
$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

c) 
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 $m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$   $m_A(\lambda) = \lambda^2 - 8\lambda + 15$  
$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(c) 
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 $m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$   $m_A(\lambda) = \lambda^2 - 8\lambda + 15$   $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$   
 $(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ 

d)

c) 
$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$   $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$   
 $m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$   $m_A(\lambda) = \lambda^2 - 8\lambda + 15$   $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$   
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d)

c)  $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$ 

 $k_A(A) = O$ 

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 $(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$   
d)  $k_A(A) = O$ 

 $A^3 - 11A^2 + 39A - 45I = 0$ 

c) 
$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$   $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$   
 $m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$   $m_A(\lambda) = \lambda^2 - 8\lambda + 15$   $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$   
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 $k_A(A) = O$  $A^3 - 11A^2 + 39A - 45I = 0$ 

451 =

c) 
$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$   $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$   
 $m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$   $m_A(\lambda) = \lambda^2 - 8\lambda + 15$   $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$   
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d)  $k_A(A) = O$ 

 $A^3 - 11A^2 + 39A - 45I = 0$  $45I = A^3 - 11A^2 + 39A$ 

c) 
$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$   $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$   
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d)  $k_A(A) = O$   
 $A^3 - 11A^2 + 39A - 45I = O$ 

 $45I = A^3 - 11A^2 + 39A / A^{-1}$ 

c) 
$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$   $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$   
 $m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$   $m_A(\lambda) = \lambda^2 - 8\lambda + 15$   $\begin{bmatrix} A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$   
 $(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$   
d)  $k_A(A) = O$   
 $A^3 - 11A^2 + 39A - 45I = O$   
 $45I = A^3 - 11A^2 + 39A / A^{-1}$ 

 $45A^{-1} =$ 

c) 
$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$   $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$   
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d)  $k_A(A) = O$   
 $A^3 - 11A^2 + 39A - 45I = O$   
 $45I = A^3 - 11A^2 + 39A / A^{-1}$ 

 $45A^{-1} = A^2$ 

c) 
$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$   $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$   
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d)  $k_A(A) = O$   
 $A^3 - 11A^2 + 39A - 45I = O$ 

 $45I = A^3 - 11A^2 + 39A / \cdot A^{-1}$ 

 $45A^{-1} = A^2 - 11A$ 

c) 
$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$   $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$   
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d)  $k_A(A) = O$   
 $A^3 - 11A^2 + 39A - 45I = O$ 

 $45I = A^3 - 11A^2 + 39A / \cdot A^{-1}$ 

 $45A^{-1} = A^2 - 11A + 39I$ 

c) 
$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$   $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$   
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d)  $k_A(A) = O$   
 $A^3 - 11A^2 + 39A - 45I = O$   
 $45I = A^3 - 11A^2 + 39A / A^{-1}$ 

 $45A^{-1} = A^2 - 11A + 39I / :45$ 

c) 
$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$   $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$   
 $m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$   $m_A(\lambda) = \lambda^2 - 8\lambda + 15$   $\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$   
d)  $k_A(A) = O$ 

$$A^{3} - 11A^{2} + 39A - 45I = O$$

$$45I = A^{3} - 11A^{2} + 39A / \cdot A^{-1}$$

$$45A^{-1} = A^{2} - 11A + 39I / : 45$$

 $A^{-1} =$ 

c)  $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$ 

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$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$d)$$

$$k_A(A) = O$$

$$A^3 - 11A^2 + 39A - 45I = O$$

$$45I = A^3 - 11A^2 + 39A / C$$

 $A^{-1} = \frac{1}{45}A^2$ 

$$A - 45I = O$$
  
 $A^2 + 39A / A^{-1}$ 

 $45I = A^3 - 11A^2 + 39A / \cdot A^{-1}$  $45A^{-1} = A^2 - 11A + 39I / :45$ 

$$45I = A^{3} - 11A^{2} + 39A / A$$

$$45A^{-1} = A^{2} - 11A + 39I / A$$

c)  $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$ 

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$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
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 $(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$   
d)  $k_A(A) = O$ 

$$45I = A^3 - 12$$

$$45A^{-1} - A^2 - 45A^{-1} = A^2 - 45A^$$

 $A^3 - 11A^2 + 39A - 45I = 0$  $45I = A^3 - 11A^2 + 39A / A^{-1}$ 

 $45A^{-1} = A^2 - 11A + 39I / :45$  $A^{-1} = \frac{1}{45}A^2 - \frac{11}{45}A$ 

c)  $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$ 

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d)  $k_A(A) = O$ 

$$A^3 - 11A^2 + 39A - 45I = O$$

 $45I = A^3 - 11A^2 + 39A / \cdot A^{-1}$ 

c)  $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$ 

$$45I = A^{3} - 11A^{2} + 39A / \cdot A^{-1}$$
$$45A^{-1} = A^{2} - 11A + 39I / :45$$

 $A^{-1} = \frac{1}{45}A^2 - \frac{11}{45}A + \frac{39}{45}I$ 

c) 
$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
 $k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$   $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$   
 $m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$   $m_A(\lambda) = \lambda^2 - 8\lambda + 15$   $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$   
 $(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$   
d)  $k_A(A) = O$ 

 $A^3 - 11A^2 + 39A - 45I = 0$ 

 $45I = A^3 - 11A^2 + 39A / \cdot A^{-1}$ 

$$45A^{-1} = A^{2} - 11A + 39I / :45$$

$$A^{-1} = \frac{1}{45}A^{2} - \frac{11}{45}A + \frac{39}{45}I$$

c) 
$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
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 $m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$   $m_A(\lambda) = \lambda^2 - 8\lambda + 15$   $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$   
 $(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$   
d)  $k_A(A) = O$   $m_A(A) = O$   
 $A^3 - 11A^2 + 39A - 45I = O$ 

$$45I = A^{3} - 11A^{2} + 39A / A^{-1}$$

$$45A^{-1} = A^{2} - 11A + 39I / : 45$$

$$A^{-1} = \frac{1}{45}A^{2} - \frac{11}{45}A + \frac{39}{45}I$$

c) 
$$k_{A}(\lambda) = \lambda^{3} - 11\lambda^{2} + 39\lambda - 45$$
  
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 $m_{A}(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$   $m_{A}(\lambda) = \lambda^{2} - 8\lambda + 15$   $\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix}$   $\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$   
d)  $k_{A}(A) = O$   $m_{A}(A) = O$   
 $A^{3} - 11A^{2} + 39A - 45I = O$   $A^{2} - 8A + 15I = O$   
 $45I = A^{3} - 11A^{2} + 39A / \cdot A^{-1}$ 

 $45A^{-1} = A^2 - 11A + 39I / :45$  $A^{-1} = \frac{1}{45}A^2 - \frac{11}{45}A + \frac{39}{45}I$ 

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d)  $k_{A}(A) = O$   $m_{A}(A) = O$   
 $A^{3} - 11A^{2} + 39A - 45I = O$   $A^{2} - 8A + 15I = O$   
 $45I = A^{3} - 11A^{2} + 39A / \cdot A^{-1}$   $15I = O$ 

 $45A^{-1} = A^2 - 11A + 39I / :45$ 

 $A^{-1} = \frac{1}{45}A^2 - \frac{11}{45}A + \frac{39}{45}I$ 

c) 
$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
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d)  $k_A(A) = O$   $m_A(A) = O$   
 $A^3 - 11A^2 + 39A - 45I = O$   $A^2 - 8A + 15I = O$   
 $45I = A^3 - 11A^2 + 39A / A^{-1}$   $15I = -A^2 + 8A$ 

 $45A^{-1} = A^2 - 11A + 39I / :45$ 

 $A^{-1} = \frac{1}{45}A^2 - \frac{11}{45}A + \frac{39}{45}I$ 13/18

c) 
$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
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d)  $k_A(A) = O$   $m_A(A) = O$   
 $A^3 - 11A^2 + 39A - 45I = O$   $A^2 - 8A + 15I = O$ 

 $45I = A^3 - 11A^2 + 39A / \cdot A^{-1}$  $15I = -A^2 + 8A / \cdot A^{-1}$  $45A^{-1} = A^2 - 11A + 39I / :45$  $A^{-1} = \frac{1}{45}A^2 - \frac{11}{45}A + \frac{39}{45}I$ 

c) 
$$k_{A}(\lambda) = \lambda^{3} - 11\lambda^{2} + 39\lambda - 45$$
  
 $k_{A}(\lambda) = (\lambda - 3)^{2} \cdot (\lambda - 5)$   $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$   
 $M_{A}(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$   $M_{A}(\lambda) = \lambda^{2} - 8\lambda + 15$   $M_{A}(\lambda) = (\lambda - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ 

$$M_{A}(A) = O$$

$$M_{A}$$

 $A^{-1} = \frac{1}{45}A^2 - \frac{11}{45}A + \frac{39}{45}I$ 

13/18

c) 
$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
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d)  $k_A(A) = O$   $m_A(A) = O$   
 $A^3 - 11A^2 + 39A - 45I = O$   $A^2 - 8A + 15I = O$   
 $45I = A^3 - 11A^2 + 39A / \cdot A^{-1}$   $15I = -A^2 + 8A / \cdot A^{-1}$ 

 $15A^{-1} = -A$ 

c)  $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$ 

 $45A^{-1} = A^2 - 11A + 39I / :45$ 

 $A^{-1} = \frac{1}{45}A^2 - \frac{11}{45}A + \frac{39}{45}I$ 

13/18

c) 
$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
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d)  $k_A(A) = O$   $m_A(A) = O$   
 $A^3 - 11A^2 + 39A - 45I = O$   $A^2 - 8A + 15I = O$   
 $45I = A^3 - 11A^2 + 39A / \cdot A^{-1}$   $15I = -A^2 + 8A / \cdot A^{-1}$ 

 $45A^{-1} = A^2 - 11A + 39I / :45$  $15A^{-1} = -A + 8I$  $A^{-1} = \frac{1}{45}A^2 - \frac{11}{45}A + \frac{39}{45}I$ 

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 $M_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$   $M_A(\lambda) = \lambda^2 - 8\lambda + 15$   $M_A(\lambda) = \lambda^2 - 8\lambda + 15$   $M_A(\lambda) = (\lambda - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$   
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d)  $k_A(A) = O$   $m_A(A) = O$   
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 $A^{-1} = \frac{1}{45}A^2 - \frac{11}{45}A + \frac{39}{45}I$   $A^{-1} = -\frac{1}{15}A$ 

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$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$
  
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 $45A^{-1} = A^2 - 11A + 39I / : 45$   $15A^{-1} = -A + 8I / : 15$   
 $A^{-1} = \frac{1}{45}A^2 - \frac{11}{45}A + \frac{39}{45}I$   $A^{-1} = -\frac{1}{15}A + \frac{8}{15}I$ 

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 $45I = A^3 - 11A^2 + 39A / \cdot A^{-1}$   $15I = -A^2 + 8A / \cdot A^{-1}$   
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 $A^{-1} = \frac{1}{45}A^2 - \frac{11}{45}A + \frac{39}{45}I$   $A^{-1} = -\frac{1}{15}A + \frac{8}{15}I$ 

treći zadatak

## Zadatak 3

Postoji li linearni operator  $f: \mathbb{R}^3 \to \mathbb{R}^2$  za kojeg vrijedi

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$
?

Ako postoji, odredite u tom slučaju f(0,0,1) i njegovu matricu u paru kanonskih baza.

enje 
$$f: \mathbb{R}^3 o \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

Rješenje 
$$f: \mathbb{R}^3 \to \mathbb{R}^2$$
  $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$ 

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$$

**Rješenje** 
$$f: \mathbb{R}^3 \to \mathbb{R}^2$$
  $f(1,0,0)=(1,0), \quad f(0,1,0)=(1,3), \quad f(1,1,1)=(2,4)$   $\mathcal{B}=\left\{(1,0,0),\, (0,1,0),\, (1,1,1)\right\}$ 

Rješenje 
$$f: \mathbb{R}^3 \to \mathbb{R}^2$$
  $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$ 

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$$

Rješenje 
$$f: \mathbb{R}^3 \to \mathbb{R}^2$$
  $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$ 

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Rješenje 
$$f: \mathbb{R}^3 \to \mathbb{R}^2$$
  $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$ 

Rješenje 
$$f: \mathbb{R}^3 \to \mathbb{R}^2$$
  $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$ 

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Rješenje 
$$f: \mathbb{R}^3 \to \mathbb{R}^2$$
  $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$ 

$$\mathcal{B} = \left\{ (1,0,0), \, (0,1,0), \, (1,1,1) \right\}$$
 
$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Rješenje 
$$f: \mathbb{R}^3 \to \mathbb{R}^2$$
  $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$ 

$$\mathcal{B} = \big\{ (1,0,0), \, (0,1,0), \, (1,1,1) \big\}$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$$
 je baza za  $\mathbb{R}^3$ .

$$\begin{bmatrix}
1 & 0 & 1 \\
0 & 1 & 1 \\
0 & 0 & 1
\end{bmatrix}$$

$$f: \mathbb{R}^3 o \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

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Svaki linearni operator zadan je svojim djelovanjem na nekoj bazi.

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 je baza za  $\mathbb{R}^3.$ 

$$\begin{bmatrix}
1 & 0 & 1 \\
0 & 1 & 1 \\
0 & 0 & 1
\end{bmatrix}$$

$$f(0,0,1) = ?$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$

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 je baza za  $\mathbb{R}^3.$ 

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(0,0,1) =$$

$$f(0,0,1) = ?$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \big\{ (1,0,0), \, (0,1,0), \, (1,1,1) \big\}$$
 je baza za  $\mathbb{R}^3.$ 

f(0,0,1) = ?

$$(0,0,1) = \alpha_1 \cdot (1,0,0)$$

$$f: \mathbb{R}^3 o \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \big\{ (1,0,0), \, (0,1,0), \, (1,1,1) \big\}$$
 je baza za  $\mathbb{R}^3.$ 

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(0,0,1) = \alpha_1 \cdot (1,0,0) +$$

$$f(0,0,1) = ?$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \big\{ (1,0,0), \, (0,1,0), \, (1,1,1) \big\} \text{ je baza za } \mathbb{R}^3.$$

$$(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0)$$

$$f(0,0,1) = ?$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \big\{ (1,0,0), \, (0,1,0), \, (1,1,1) \big\}$$
 je baza za  $\mathbb{R}^3.$ 

$$(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0) +$$

$$f(0,0,1) = ?$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

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 je baza za  $\mathbb{R}^3.$ 

$$(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0) + \alpha_3 \cdot (1,1,1)$$

$$f(0,0,1) = ?$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

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 je baza za  $\mathbb{R}^3.$ 

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

f(0,0,1) = ?

$$(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0) + \alpha_3 \cdot (1,1,1)$$

$$\alpha_1 + \alpha_3 = 0$$

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f(0,0,1) = ?

$$(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0) + \alpha_3 \cdot (1,1,1)$$

$$\alpha_1 + \alpha_3 = 0$$

$$\alpha_2 + \alpha_3 = 0$$

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$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

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$$f(0,0,1) = ?$$

$$(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0) + \alpha_3 \cdot (1,1,1)$$

$$\alpha_1 + \alpha_3 = 0$$

$$\alpha_2 + \alpha_3 = 0$$

$$\alpha_3 = 1$$

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$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

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$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0) + \alpha_3 \cdot (1,1,1)$$

$$\left. \begin{array}{l} \alpha_1 + \alpha_3 = 0 \\ \alpha_2 + \alpha_3 = 0 \\ \alpha_3 = 1 \end{array} \right\}$$

$$f(0,0,1) = ?$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$

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$$\alpha_1 + \alpha_3 = 0 
\alpha_2 + \alpha_3 = 0 
\alpha_3 = 1$$

$$\alpha_3 = 1$$

$$f(0,0,1) = ?$$

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$$\alpha_1 + \alpha_3 = 0 
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$$\alpha_2 = -1 
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$$f(0,0,1) = ?$$

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$$\alpha_1 + \alpha_3 = 0 
\alpha_2 + \alpha_3 = 0 
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$$\alpha_3 = 1$$

$$\alpha_3 = 1$$

$$f(0,0,1) = ?$$

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$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

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$$(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0) + \alpha_3 \cdot (1,1,1)$$

$$\alpha_1 + \alpha_3 = 0 
\alpha_2 + \alpha_3 = 0 
\alpha_3 = 1$$

$$\alpha_3 = 1$$

$$\alpha_1 = -1$$

$$f(0,0,1) = ?$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

 $\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$  je baza za  $\mathbb{R}^3$ .

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$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

f(0,0,1) = ?

$$(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0) + \alpha_3 \cdot (1,1,1)$$

$$\alpha_1 + \alpha_3 = 0 
\alpha_2 + \alpha_3 = 0 
\alpha_3 = 1$$

$$\alpha_3 = 1$$

$$\alpha_1 = -1$$

$$\alpha_1 = -1$$

$$f(0,0,1) =$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

 $\mathcal{B} = \big\{ (1,0,0), \, (0,1,0), \, (1,1,1) \big\}$  je baza za  $\mathbb{R}^3.$ 

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$$f(0,0,1) = ?$$

$$(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0) + \alpha_3 \cdot (1,1,1)$$

$$\alpha_1 + \alpha_3 = 0$$

$$\alpha_2 + \alpha_3 = 0$$

$$\alpha_3 = 1$$

$$\alpha_3 = 1$$

$$\alpha_3 = 1$$

$$f(0,0,1)=f($$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

 $\mathcal{B} = \big\{ (1,0,0), \, (0,1,0), \, (1,1,1) \big\}$  je baza za  $\mathbb{R}^3.$ 

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(0,0,1) = ?$$

$$(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0) + \alpha_3 \cdot (1,1,1)$$

$$\alpha_1 + \alpha_3 = 0 
\alpha_2 + \alpha_3 = 0 
\alpha_3 = 1$$

$$\alpha_2 = -1 
\alpha_3 = 1$$

$$f(0,0,1) = f(-1 \cdot (1,0,0))$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \big\{ (1,0,0), \, (0,1,0), \, (1,1,1) \big\}$$
 je baza za  $\mathbb{R}^3.$ 

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(0,0,1) = ?$$

$$(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0) + \alpha_3 \cdot (1,1,1)$$

$$\alpha_1 + \alpha_3 = 0 
\alpha_2 + \alpha_3 = 0 
\alpha_3 = 1$$

$$\alpha_3 = 1$$

$$\alpha_1 = -1$$

$$f(0,0,1) = f(-1 \cdot (1,0,0) +$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \big\{ (1,0,0), \, (0,1,0), \, (1,1,1) \big\}$$
 je baza za  $\mathbb{R}^3.$ 

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(0,0,1) = ?$$

$$(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0) + \alpha_3 \cdot (1,1,1)$$

$$\alpha_1 + \alpha_3 = 0$$

$$\alpha_2 + \alpha_3 = 0$$

$$\alpha_3 = 1$$

$$\alpha_3 = 1$$

$$\alpha_3 = 1$$

$$f(0,0,1) = f(-1 \cdot (1,0,0) + (-1) \cdot (0,1,0)$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

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 je baza za  $\mathbb{R}^3.$ 

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(0,0,1) = ?$$

$$(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0) + \alpha_3 \cdot (1,1,1)$$

$$\begin{array}{c} \alpha_1 + \alpha_3 = 0 \\ \alpha_2 + \alpha_3 = 0 \\ \alpha_3 = 1 \end{array}$$
 
$$\begin{array}{c} \leftarrow \alpha_2 = -1 \\ \leftarrow \alpha_3 = 1 \end{array}$$
 
$$\begin{array}{c} \alpha_1 = -1 \\ \leftarrow \alpha_3 = 1 \end{array}$$

$$f(0,0,1) = f(-1 \cdot (1,0,0) + (-1) \cdot (0,1,0) +$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(0,0,1) = ?$$

$$(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0) + \alpha_3 \cdot (1,1,1)$$

$$\alpha_1 + \alpha_3 = 0$$

$$\alpha_2 + \alpha_3 = 0$$

$$\alpha_3 = 1$$

$$\alpha_3 = 1$$

$$\alpha_3 = 1$$

$$f(0,0,1) = f\left(-1\cdot(1,0,0) + (-1)\cdot(0,1,0) + 1\cdot(1,1,1)\right)$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(0,0,1) = ?$$

$$(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0) + \alpha_3 \cdot (1,1,1)$$

$$\alpha_1 + \alpha_3 = 0 
\alpha_2 + \alpha_3 = 0 
\alpha_3 = 1$$

$$\alpha_2 = -1 
\alpha_3 = 1$$

$$f(0,0,1) = f\left(-1 \cdot (1,0,0) + (-1) \cdot (0,1,0) + 1 \cdot (1,1,1)\right)$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(0,0,1) = ?$$

$$(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0) + \alpha_3 \cdot (1,1,1)$$

$$\alpha_1 + \alpha_3 = 0 
\alpha_2 + \alpha_3 = 0 
\alpha_3 = 1$$

$$\alpha_2 = -1 
\alpha_3 = 1$$

$$f(0,0,1) = f(-1 \cdot (1,0,0) + (-1) \cdot (0,1,0) + 1 \cdot (1,1,1)) =$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

Svaki linearni operator zadan je svojim djelovanjem na nekoj bazi. Postoji jedinstveni linearni operator f koji zadovoljava zadane uvjete.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0) + \alpha_3 \cdot (1,1,1)$$

$$\begin{array}{c}
\alpha_{1} + \alpha_{3} = 0 \\
\alpha_{2} + \alpha_{3} = 0 \\
\alpha_{3} = 1
\end{array}$$

$$\begin{array}{c}
\alpha_{2} = -1 \\
\alpha_{3} = 1
\end{array}$$

$$\begin{array}{c}
\alpha_{1} = -1 \\
\alpha_{3} = 1
\end{array}$$

$$\begin{array}{c}
f \text{ je op} \\
\text{op} \\
f(0, 0, 1) = f(-1 \cdot (1, 0, 0) + (-1) \cdot (0, 1, 0) + 1 \cdot (1, 1, 1)) = 0
\end{array}$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

 $\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$  je baza za  $\mathbb{R}^3$ .

Svaki linearni operator zadan je svojim djelovanjem na nekoj bazi. Postoji jedinstveni linearni operator f koji zadovoljava zadane uvjete.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

f(0,0,1) = ?

f je linearni

$$(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0) + \alpha_3 \cdot (1,1,1)$$

$$\alpha_1 + \alpha_3 = 0 
\alpha_2 + \alpha_3 = 0 
\alpha_3 = 1$$

$$\alpha_3 = 1$$

$$\alpha_3 = 1$$

$$\alpha_1 = -1$$

$$\alpha_2 + \alpha_3 = 0$$
 $\alpha_3 = 1$ 
 $\alpha_3 = 1$ 
 $\alpha_1 = -1$ 
operator
 $\alpha_3 = 1$ 
 $\alpha_1 = -1$ 
 $\alpha_1 = -1$ 

$$f:\mathbb{R}^3 o \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

 $\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$  je baza za  $\mathbb{R}^3$ .

Svaki linearni operator zadan je svojim djelovanjem na nekoj bazi. Postoji jedinstveni linearni operator f koji zadovoljava zadane uvjete.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0) + \alpha_3 \cdot (1,1,1)$$

$$\alpha_1 + \alpha_3 = 0 
\alpha_2 + \alpha_3 = 0 
\alpha_3 = 1$$

$$\alpha_3 = 1$$

$$\alpha_3 = 1$$

$$\alpha_1 = -1$$

$$\alpha_{2} + \alpha_{3} = 0 \\ \alpha_{3} = 1$$
 
$$\alpha_{3} = 1$$
 
$$\alpha_{4} = -1$$
 
$$\alpha_{5} = -1$$
 
$$\alpha_{1} = -1$$
 
$$\alpha_{1} = -1$$
 
$$\alpha_{1} = -1$$
 
$$\alpha_{2} = -1$$
 
$$\alpha_{3} = 1$$
 
$$\alpha_{1} = -1$$
 
$$\alpha_{1} = -1$$
 
$$\alpha_{2} = -1$$
 
$$\alpha_{3} = 1$$
 
$$\alpha_{1} = -1$$
 
$$\alpha_{1} = -1$$
 
$$\alpha_{2} = -1$$
 
$$\alpha_{3} = 1$$
 
$$\alpha_{1} = -1$$
 
$$\alpha_{2} = -1$$
 
$$\alpha_{3} = 1$$
 
$$\alpha_{1} = -1$$
 
$$\alpha_{2} = -1$$
 
$$\alpha_{3} = 1$$
 
$$\alpha_{3} = 1$$
 
$$\alpha_{4} = -1$$
 
$$\alpha_{5} = -1$$
 
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$$\alpha_{5} = -1$$
 
$$\alpha_{1} = -1$$
 
$$\alpha_{1} = -1$$
 
$$\alpha_{2} = -1$$
 
$$\alpha_{3} = -1$$
 
$$\alpha_{4} = -1$$
 
$$\alpha_{5} =$$

$$= -1 \cdot f(1,0,0) + (-1) \cdot f(0,1,0)$$

$$f: \mathbb{R}^3 o \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

Svaki linearni operator zadan je svojim djelovanjem na nekoj bazi. Postoji jedinstveni linearni operator f koji zadovoljava zadane uvjete.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0) + \alpha_3 \cdot (1,1,1)$$

$$\alpha_1 + \alpha_3 = 0 
\alpha_2 + \alpha_3 = 0 
\alpha_3 = 1$$

$$\alpha_3 = 1$$

$$\alpha_1 = -1$$

$$f(0,0,1) = f(-1 \cdot (1,0,0) + (-1) \cdot (0,1,0) + 1 \cdot (1,1,1)) \stackrel{\checkmark}{=}$$
  
=  $-1 \cdot f(1,0,0) + (-1) \cdot f(0,1,0) + 1 \cdot f(1,1,1)$ 

=-(1,0)

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

 $f: \mathbb{R}^3 \to \mathbb{R}^2$ 

$$\mathcal{B} = ig\{ (1,0,0), \, (0,1,0), \, (1,1,1) ig\}$$
 je baza za  $\mathbb{R}^3.$ 

Svaki linearni operator zadan je svojim djelovanjem na nekoj bazi. Postoji jedinstveni linearni operator f koji zadovoljava zadane uvjete.

f(0,0,1) = ?

$$(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0) + \alpha_3 \cdot (1,1,1)$$

$$\begin{array}{c}
\alpha_{1} + \alpha_{3} = 0 \\
\alpha_{2} + \alpha_{3} = 0 \\
\alpha_{3} = 1
\end{array}$$

$$\begin{array}{c}
\alpha_{2} = -1 \\
\alpha_{3} = 1
\end{array}$$

$$\begin{array}{c}
\alpha_{1} = -1 \\
\alpha_{3} = 1
\end{array}$$

$$\begin{array}{c}
f \text{ je linearni operator} \\
\text{operator} \\
\text{operator} \\
\text{operator} \\
\text{operator}
\end{array}$$

$$f(0, 0, 1) = f(-1 \cdot (1, 0, 0) + (-1) \cdot (0, 1, 0) + 1 \cdot (1, 1, 1)) = (-1) \cdot f(1, 0, 0) + (-1) \cdot f(0, 1, 0) + 1 \cdot f(1, 1, 1) = (-1) \cdot f(1, 0, 0) + (-1) \cdot f(1, 0,$$

Rješenje 
$$f: \mathbb{R}^3 \to \mathbb{R}^2$$
  $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$ 

$$\mathcal{B} = \big\{ (1,0,0), \, (0,1,0), \, (1,1,1) \big\}$$
 je baza za  $\mathbb{R}^3.$ 

Svaki linearni operator zadan je svojim djelovanjem

na nekoj bazi. Postoji jedinstveni linearni operator f koji zadovoljava zadane uvjete.

 $(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0) + \alpha_3 \cdot (1,1,1)$ 

$$f(0,0,1) = f(-1 \cdot (1,0,0) + (-1) \cdot (0,1,0) + 1 \cdot (1,1,1)) \stackrel{\checkmark}{=}$$

$$= -1 \cdot f(1,0,0) + (-1) \cdot f(0,1,0) + 1 \cdot f(1,1,1) =$$

$$= -(1,0) - (1,3)$$

**Rješenje** 
$$f: \mathbb{R}^3 \to \mathbb{R}^2$$
  $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$   $\mathcal{B} = \big\{ (1,0,0), (0,1,0), (1,1,1) \big\}$  je baza za  $\mathbb{R}^3$ . Svaki linearni operator zadan je svojim djelovanjem na nekoj bazi. Postoji jedinstveni linearni operator  $f$  koji zadovoljava zadane uvjete. 
$$f(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0) + \alpha_3 \cdot (1,1,1)$$
  $\alpha_1 + \alpha_3 = 0$   $\alpha_2 + \alpha_3 = 0$   $\alpha_2 + \alpha_3 = 0$   $\alpha_3 = 1$   $\alpha_1 = -1$   $\alpha_1 = -1$   $\alpha_2 = -1$   $\alpha_3 = 1$   $\alpha_4 = -1$   $\alpha_5 = -1 \cdot f(1,0,0) + (-1) \cdot f(0,1,0) + 1 \cdot f(1,1,1) = (2,4)$ 

$$=-(1,0)-(1,3)+(2,4)$$

$$\begin{array}{c} \textbf{Rješenje} & f: \mathbb{R}^3 \to \mathbb{R}^2 \\ f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4) \\ \mathcal{B} = \big\{ (1,0,0), (0,1,0), (1,1,1) \big\} \text{ je baza za } \mathbb{R}^3. \\ \text{Svaki linearni operator zadan je svojim djelovanjem} & \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \\ \text{koji zadovoljava zadane uvjete.} \\ (0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0) + \alpha_3 \cdot (1,1,1) \\ \alpha_1 + \alpha_3 = 0 \\ \alpha_2 + \alpha_3 = 0 \\ \alpha_3 = 1 \\ \end{array}$$
 
$$\begin{array}{c} \sigma_1 + \alpha_3 = 0 \\ \alpha_2 + \alpha_3 = 0 \\ \alpha_3 = 1 \\ \end{array}$$
 
$$\begin{array}{c} \sigma_1 = -1 \\ \alpha_3 = 1 \\ \end{array}$$
 
$$\begin{array}{c} f \text{ je linearni operator} \\ \end{array}$$
 
$$\begin{array}{c} f \text{ je linearni operator} \\ \end{array}$$
 
$$\begin{array}{c} f \text{ je linearni operator} \\ \end{array}$$
 
$$\begin{array}{c} \sigma_1 + \sigma_3 = 0 \\ \sigma_2 = -1 \\ \sigma_3 = 1 \\ \end{array}$$
 
$$\begin{array}{c} \sigma_1 = -1 \\ \sigma_3 = 1 \\ \end{array}$$
 
$$\begin{array}{c} f \text{ je linearni operator} \\ \end{array}$$

= -(1,0) - (1,3) + (2,4) = (0,1)

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$
 $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$ 

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$$

$$f: \mathbb{R}^3 o \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$$
  $\mathcal{A}_{\mathsf{kan}} = \{(1,0), (0,1)\}$ 

$$f: \mathbb{R}^3 o \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \big\{ (1,0,0), \, (0,1,0), \, (1,1,1) \big\} \qquad \quad \mathcal{A}_{\mathsf{kan}} = \big\{ (1,0), \, (0,1) \big\}$$

$$F_{(\mathcal{B}, \mathcal{A}_{\mathsf{kan}})} = igg[$$

$$f:\mathbb{R}^3 o\mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = ig\{ (1,0,0), \, (0,1,0), \, (1,1,1) ig\} \qquad \mathcal{A}_{\mathsf{kan}} = ig\{ (1,0), \, (0,1) ig\}$$

$$F_{(\mathcal{B},\,\mathcal{A}_{\mathsf{kan}})} = egin{bmatrix} 1 \ 0 \end{bmatrix}$$

$$f: \mathbb{R}^3 o \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$$
  $\mathcal{A}_{\mathsf{kan}} = \{(1,0), (0,1)\}$ 

$$F_{(\mathcal{B}, \mathcal{A}_{\mathsf{kan}})} = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}$$

$$f:\mathbb{R}^3 o\mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \big\{ (1,0,0), \, (0,1,0), \, (1,1,1) \big\} \qquad \quad \mathcal{A}_{\mathsf{kan}} = \big\{ (1,0), \, (0,1) \big\}$$

$$F_{(\mathcal{B},\mathcal{A}_{kan})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$f: \mathbb{R}^3 o \mathbb{R}^2$$
  $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$   $\mathcal{B} = \big\{ (1,0,0), \, (0,1,0), \, (1,1,1) \big\} \qquad \mathcal{A}_{\mathsf{kan}} = \big\{ (1,0), \, (0,1) \big\}$ 

$$F_{(\mathcal{B}, \mathcal{A}_{\mathsf{kan}})} = egin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$f(0,0,1) = ?$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$
  $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$ 

$$\mathcal{B} = \left\{ (1,0,0), \, (0,1,0), \, (1,1,1) \right\} \qquad \mathcal{A}_{\mathsf{kan}} = \left\{ (1,0), \, (0,1) \right\}$$

$$F_{(\mathcal{B}, \mathcal{A}_{kan})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$Y_{\mathcal{A}_{kan}} = F_{(\mathcal{B}, \mathcal{A}_{kan})} X_{\mathcal{B}}$$

$$f(0, 0, 1) = ?$$

$$f: \mathbb{R}^3 o \mathbb{R}^2$$
  $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$   $\mathcal{B} = \big\{ (1,0,0), \, (0,1,0), \, (1,1,1) \big\} \qquad \mathcal{A}_{\mathsf{kan}} = \big\{ (1,0), \, (0,1) \big\}$ 

$$F_{(\mathcal{B},\mathcal{A}_{kan})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$Y_{\mathcal{A}_{\mathsf{kan}}} = F_{(\mathcal{B}, \mathcal{A}_{\mathsf{kan}})} X_{\mathcal{B}}$$
 $0, 0, 1) = ?$ 

$$f(0,0,1)=$$
?  $X_{\mathcal{B}}=egin{bmatrix} -1\ -1\ 1 \end{bmatrix}$ 

$$f: \mathbb{R}^3 o \mathbb{R}^2$$
 $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$ 
 $\mathcal{B} = \left\{ (1,0,0), (0,1,0), (1,1,1) \right\} \qquad \mathcal{A}_{\mathsf{kan}} = \left\{ (1,0), (0,1) \right\}$ 
 $F_{(\mathcal{B},\mathcal{A}_{\mathsf{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix} \qquad \begin{array}{c} Y_{\mathcal{A}_{\mathsf{kan}}} = F_{(\mathcal{B},\mathcal{A}_{\mathsf{kan}})} X_{\mathcal{B}} \\ f(0,0,1) = ? \\ X_{\mathcal{B}} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$ 

$$f: \mathbb{R}^3 o \mathbb{R}^2$$
 $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$ 
 $\mathcal{B} = \left\{ (1,0,0), (0,1,0), (1,1,1) \right\} \qquad \mathcal{A}_{\mathsf{kan}} = \left\{ (1,0), (0,1) \right\}$ 
 $F_{(\mathcal{B},\mathcal{A}_{\mathsf{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix} \qquad \qquad Y_{\mathcal{A}_{\mathsf{kan}}} = F_{(\mathcal{B},\mathcal{A}_{\mathsf{kan}})} X_{\mathcal{B}}$ 
 $f(0,0,1) = ?$ 
 $Y_{\mathcal{A}_{\mathsf{kan}}} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$ 

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$
  
 $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$ 

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$$
  $\mathcal{A}_{\mathsf{kan}} = \{(1,0), (0,1)\}$ 

$$F_{(\mathcal{B},\,\mathcal{A}_{\mathsf{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$Y_{\mathcal{A}_{\mathsf{kan}}} = egin{bmatrix} 1 & 1 & 2 \ 0 & 3 & 4 \end{bmatrix} egin{bmatrix} -1 \ -1 \ 1 \end{bmatrix}$$

$$Y_{\mathcal{A}_{\mathsf{kan}}} = F_{(\mathcal{B},\mathcal{A}_{\mathsf{kan}})} X_{\mathcal{B}}$$

$$f(0,0,1)=$$
?  $X_{\mathcal{B}}=egin{bmatrix} -1\ -1\ 1 \end{bmatrix}$ 

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$
  
 $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$ 

$$\mathcal{B} = \big\{ (1,0,0), \, (0,1,0), \, (1,1,1) \big\} \qquad \mathcal{A}_{\mathsf{kan}} = \big\{ (1,0), \, (0,1) \big\}$$

$$F_{(\mathcal{B}, \mathcal{A}_{kan})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$Y_{\mathcal{A}_{kan}} = egin{bmatrix} 1 & 1 & 2 \ 0 & 3 & 4 \end{bmatrix} egin{bmatrix} -1 \ -1 \ 1 \end{bmatrix} = egin{bmatrix} 0 \ 1 \end{bmatrix}$$

$$Y_{\mathcal{A}_{\mathsf{kan}}} = F_{(\mathcal{B}, \mathcal{A}_{\mathsf{kan}})} X_{\mathcal{B}}$$

$$f(0,0,1)=? \ X_{\mathcal{B}}=egin{bmatrix} -1 \ -1 \ 1 \end{bmatrix}$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$
 $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$ 
 $\mathcal{B} = \left\{ (1,0,0), (0,1,0), (1,1,1) \right\} \qquad \mathcal{A}_{\mathsf{kan}} = \left\{ (1,0), (0,1) \right\}$ 
 $F_{(\mathcal{B},\mathcal{A}_{\mathsf{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$ 
 $Y_{\mathcal{A}_{\mathsf{kan}}} = F_{(\mathcal{B},\mathcal{A}_{\mathsf{kan}})} X_{\mathcal{B}}$ 

f(0,0,1) = ?

$$Y_{\mathcal{A}_{kan}} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$X_{\mathcal{B}} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

$$f(0,0,1) = f(-1 \cdot (1,0,0) + (-1) \cdot (0,1,0) + 1 \cdot (1,1,1)) =$$

$$= -1 \cdot f(1,0,0) + (-1) \cdot f(0,1,0) + 1 \cdot f(1,1,1) =$$

$$= -(1,0) - (1,3) + (2,4) = (0,1)$$

$$f: \mathbb{R}^3 o \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$$
  $\mathcal{A}_{\mathsf{kan}} = \{(1,0), (0,1)\}$ 

$$F_{(\mathcal{B},\mathcal{A}_{kan})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$f:\mathbb{R}^3 o \mathbb{R}^2 \ f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \big\{ (1,0,0), \, (0,1,0), \, (1,1,1) \big\} \qquad \mathcal{A}_{\mathsf{kan}} = \big\{ (1,0), \, (0,1) \big\}$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \big\{ (1,0,0), \, (0,1,0), \, (1,1,1) \big\} \qquad \mathcal{A}_{\mathsf{kan}} = \big\{ (1,0), \, (0,1) \big\}$$

$$F_{(\mathcal{B},\mathcal{A}_{\mathsf{kan}})} = egin{bmatrix} 1 & 1 & 2 \ 0 & 3 & 4 \end{bmatrix} \hspace{1cm} \mathcal{B}_{\mathsf{kan}} = egin{bmatrix} (1,0,0), \ (0,1,0), \ (0,0,1) \end{pmatrix}$$

$$F_{(\mathcal{B}_{\mathsf{kan}},\,\mathcal{A}_{\mathsf{kan}})} = T^{-1}F_{(\mathcal{B},\,\mathcal{A}_{\mathsf{kan}})}S$$

$$f:\mathbb{R}^3 o\mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$$
  $\mathcal{A}_{\mathsf{kan}} = \{(1,0), (0,1)\}$ 

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$$
  $\mathcal{A}_{\mathsf{kan}} = \{(1,0), (0,1)\}$ 

$$\mathcal{B}_{\mathsf{kan}} = \{(1,0,0), (0,1,0), (0,0,1)\}$$

$$F_{(\mathcal{B},\mathcal{A}_{\mathsf{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$F_{(\mathcal{B}_{\mathsf{kan}},\,\mathcal{A}_{\mathsf{kan}})} = T^{-1}F_{(\mathcal{B},\,\mathcal{A}_{\mathsf{kan}})}S$$

$$\mathcal{B} \stackrel{\mathcal{S}}{\longrightarrow} \mathcal{B}_{\mathsf{kan}}$$

$$f:\mathbb{R}^3 o\mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$$
  $\mathcal{A}_{\mathsf{kan}} = \{(1,0), (0,1)\}$ 

$$\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0,0),\, (0,1,0),\, (0,0,1) ig\}$$

$$F_{(\mathcal{B},\mathcal{A}_{\mathsf{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$
  $\mathcal{B}_{\mathsf{kan}} = \{(1,0,0), (0,1,0)\}$ 

$$F_{(\mathcal{B}_{\mathsf{kan}},\,\mathcal{A}_{\mathsf{kan}})} = T^{-1}F_{(\mathcal{B},\,\mathcal{A}_{\mathsf{kan}})}S$$

$$\mathcal{B} \xrightarrow{S} \mathcal{B}_{\mathsf{kan}} \qquad \mathcal{A}_{\mathsf{kan}} \xrightarrow{T} \mathcal{A}_{\mathsf{kan}}$$

$$f:\mathbb{R}^3 o\mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$$
  $\mathcal{A}_{kan} = \{(1,0), (0,1)\}$ 

$$F_{(\mathcal{B}, \mathcal{A}_{\mathsf{kan}})} = egin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$F_{(\mathcal{B},\mathcal{A}_{\mathsf{kan}})} = egin{bmatrix} 1 & 1 & 2 \ 0 & 3 & 4 \end{bmatrix} \hspace{1cm} \mathcal{B}_{\mathsf{kan}} = egin{bmatrix} (1,0,0), \ (0,1,0), \ (0,0,1) \end{pmatrix}$$

$$\left[F_{\left(\mathcal{B}_{\mathsf{kan}},\,\mathcal{A}_{\mathsf{kan}}
ight)}=T^{-1}F_{\left(\mathcal{B},\,\mathcal{A}_{\mathsf{kan}}
ight)}S
ight]$$

$$\mathcal{B} \xrightarrow{S} \mathcal{B}_{kan} \qquad \mathcal{A}_{kan} \xrightarrow{T} \mathcal{A}_{kan}$$

$$f: \mathbb{R}^3 o \mathbb{R}^2$$
 $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$ 
 $\mathcal{B} = \left\{ (1,0,0), \, (0,1,0), \, (1,1,1) \right\} \qquad \mathcal{A}_{\mathsf{kan}} = \left\{ (1,0), \, (0,1) \right\}$ 
 $F_{(\mathcal{B},\mathcal{A}_{\mathsf{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix} \qquad \mathcal{B}_{\mathsf{kan}} = \left\{ (1,0,0), \, (0,1,0), \, (0,0,1) \right\}$ 
 $F_{(\mathcal{B}_{\mathsf{kan}},\mathcal{A}_{\mathsf{kan}})} = \mathcal{T}^{-1} F_{(\mathcal{B},\mathcal{A}_{\mathsf{kan}})} \mathcal{S} \qquad \mathcal{S}^{-1} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$ 

$$\mathcal{B}_{\overbrace{S^{-1}}} \xrightarrow{S} \mathcal{B}_{kan} \qquad \mathcal{A}_{kan} \xrightarrow{T} \mathcal{A}_{kan}$$

$$f: \mathbb{R}^3 o \mathbb{R}^2$$
 $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$ 
 $\mathcal{B} = \left\{ (1,0,0), (0,1,0), (1,1,1) \right\} \qquad \mathcal{A}_{\mathsf{kan}} = \left\{ (1,0), (0,1) \right\}$ 
 $F_{(\mathcal{B},\mathcal{A}_{\mathsf{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix} \qquad S^{-1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ 
 $S^{-1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ 

$$\mathcal{B}_{\varsigma} \xrightarrow{S} \mathcal{B}_{kan} \qquad \mathcal{A}_{kan} \xrightarrow{T} \mathcal{A}_{kan}$$

$$f: \mathbb{R}^3 o \mathbb{R}^2$$
 $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$ 
 $\mathcal{B} = \left\{ (1,0,0), (0,1,0), (1,1,1) \right\} \qquad \mathcal{A}_{\mathsf{kan}} = \left\{ (1,0), (0,1) \right\}$ 
 $F_{(\mathcal{B},\mathcal{A}_{\mathsf{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix} \qquad S^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ 

 $\mathcal{B}_{r} \xrightarrow{S} \mathcal{B}_{kan} \qquad \mathcal{A}_{kan} \xrightarrow{T} \mathcal{A}_{kan}$ 

$$f: \mathbb{R}^3 o \mathbb{R}^2$$
 $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$ 
 $\mathcal{B} = \left\{ (1,0,0), (0,1,0), (1,1,1) \right\} \qquad \mathcal{A}_{\mathsf{kan}} = \left\{ (1,0), (0,1) \right\}$ 
 $F_{(\mathcal{B},\mathcal{A}_{\mathsf{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix} \qquad S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ 
 $F_{(\mathcal{B}_{\mathsf{kan}},\mathcal{A}_{\mathsf{kan}})} = T^{-1}F_{(\mathcal{B},\mathcal{A}_{\mathsf{kan}})}S$ 

$$\mathcal{B} \xrightarrow{S} \mathcal{B}_{kan} \qquad \mathcal{A}_{kan} \xrightarrow{T} \mathcal{A}_{kan}$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$
 $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$ 
 $f(1,0,0), (0,1,0), (1,1,1), \dots, (1,0,0), (0,1), \dots$ 

$$\mathcal{B} = ig\{ (1,0,0), \, (0,1,0), \, (1,1,1) ig\} \qquad \mathcal{A}_{\mathsf{kan}} = ig\{ (1,0), \, (0,1) ig\} \ \mathcal{B}_{\mathsf{kan}} = ig\{ (1,0,0), \, (0,1,0), \, (0,0,1) ig\}$$

$$F_{(\mathcal{B}, \mathcal{A}_{\mathsf{kan}})} = egin{bmatrix} 1 & 1 & 2 \ 0 & 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} S, S_{kan} & \begin{bmatrix} 0 & 3 & 4 \end{bmatrix} \\ F_{(\mathcal{B}_{kan}, \mathcal{A}_{kan})} & S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad S = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T \longrightarrow \Delta$$

$$\mathcal{B}_{\overbrace{S^{-1}}} \xrightarrow{S} \mathcal{B}_{kan} \qquad \mathcal{A}_{kan} \xrightarrow{T} \mathcal{A}_{kan}$$

$$f: \mathbb{R}^{3} \to \mathbb{R}^{2}$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \left\{ (1,0,0), (0,1,0), (1,1,1) \right\} \qquad \mathcal{A}_{\mathsf{kan}} = \left\{ (1,0), (0,1) \right\}$$

$$F_{(\mathcal{B},\mathcal{A}_{\mathsf{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix} \qquad \mathcal{B}_{\mathsf{kan}} = \left\{ (1,0,0), (0,1,0), (0,0,1) \right\}$$

$$S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathcal{B}_{\mathsf{kan}} = \left\{ (1,0,0), (0,1,0), (0,0,1) \right\}$$

$$S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathcal{S} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathcal{B}_{r} \xrightarrow{S} \mathcal{B}_{kan} \qquad \mathcal{A}_{kan} \xrightarrow{T} \mathcal{A}_{kan}$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$$
  $\mathcal{A}_{kan} = \{(1,0), (0,1)\}$ 

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$$
  $\mathcal{A}_{\mathsf{kan}} = \{(1,0), (0,1)\}$   $\mathcal{B}_{\mathsf{kan}} = \{(1,0,0), (0,1,0), (0,0,1)\}$ 

$$F_{(\mathcal{B}, \mathcal{A}_{\mathsf{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$F_{(\mathcal{B}_{kan}, \mathcal{A}_{kan})} = T^{-1}F_{(\mathcal{B}, \mathcal{A}_{kan})}S$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{array}{c} \mathsf{DZ} \\ S = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathcal{B} \xrightarrow{S} \mathcal{B}_{kan} \qquad \mathcal{A}_{kan} \xrightarrow{T} \mathcal{A}_{kan}$$

$$T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$$
  $\mathcal{A}_{kan} = \{(1,0), (0,1)\}$ 

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$$
  $\mathcal{A}_{kan} = \{(1,0), (0,1)\}$ 

$$\mathcal{B}_{\mathsf{kan}} = \{(1,0,0),\, (0,1,0),\, (0,0,1)\}$$

$$F_{(\mathcal{B}, \mathcal{A}_{\mathsf{kan}})} = egin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$D_{kan} = \{(1,0,0), (0,1,0),$$

$$F_{(\mathcal{B},\mathcal{A}_{kan})} = \begin{bmatrix} 0 & 3 & 4 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} DZ \\ S = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F_{(\mathcal{B}_{kan}, \mathcal{A}_{kan})} = T^{-1}F_{(\mathcal{B}, \mathcal{A}_{kan})}S$$

$$T^{-1} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$
  $T = \begin{bmatrix} 1 & 0 \end{bmatrix}$ 

$$F_{(\mathcal{B}_{kan},\mathcal{A}_{kan})} = T^{-1}F_{(\mathcal{B},\mathcal{A}_{kan})}$$

$$\mathcal{B}_{\overbrace{S^{-1}}} \xrightarrow{S} \mathcal{B}_{\mathsf{kan}} \qquad \mathcal{A}_{\mathsf{kan}} \xrightarrow{\quad T \quad} \mathcal{A}_{\mathsf{kan}} \qquad T^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$
 $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$ 

$$\mathcal{B} = ig\{ (1,0,0), \, (0,1,0), \, (1,1,1) ig\} \qquad \mathcal{A}_{\mathsf{kan}} = ig\{ (1,0), \, (0,1) ig\} \ \mathcal{B}_{\mathsf{kan}} = ig\{ (1,0,0), \, (0,1,0), \, (0,0,1) ig\}$$

$$F_{(\mathcal{B},\,\mathcal{A}_{\mathsf{kan}})} = egin{bmatrix} 1 & 1 & 2 \ 0 & 3 & 4 \end{bmatrix}$$

$$F_{(\mathcal{B}_{kan}, \mathcal{A}_{kan})} = T^{-1}F_{(\mathcal{B}, \mathcal{A}_{kan})}S$$

$$S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{array}{c} \mathsf{DZ} \\ S = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F_{(\mathcal{B}_{\mathsf{kan}},\mathcal{A}_{\mathsf{kan}})}=$$

$$\mathcal{B} \xrightarrow{S} \mathcal{B}_{kan} \qquad \mathcal{A}_{kan} \xrightarrow{T} \mathcal{A}_{kan} \qquad T^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$f: \mathbb{R}^3 \to \mathbb{R}^2$$
  $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$ 

$$\mathcal{B} = \big\{ (1,0,0),\, (0,1,0),\, (1,1,1) \big\} \qquad \, \mathcal{A}_{\mathsf{kan}} = \big\{ (1,0),\, (0,1) \big\}$$

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\} \qquad \mathcal{A}_{kan} = \{(1,0), (0,1)\}$$

$$F_{(\mathcal{B},\mathcal{A}_{\mathsf{kan}})} = egin{bmatrix} 1 & 1 & 2 \ 0 & 3 & 4 \end{bmatrix} \hspace{1cm} \mathcal{B}_{\mathsf{kan}} = egin{bmatrix} (1,0,0), \ (0,1,0), \ (0,0,1) \end{pmatrix} \ & \Gamma_1 & 0 & 1 \end{bmatrix} \hspace{1cm} \mathsf{DZ} \hspace{0.2cm} \Gamma_1 & 0 & -1 \end{bmatrix}$$

$$F_{(\mathcal{B}_{kan}, \mathcal{A}_{kan})} = \begin{bmatrix} 0 & 3 & 4 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathcal{B} \xrightarrow{S} \mathcal{B}_{\mathsf{kan}} \qquad \mathcal{A}_{\mathsf{kan}} \xrightarrow{T} \mathcal{A}_{\mathsf{kan}} \qquad T^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$F_{(\mathcal{B}_{\mathsf{kan}},\mathcal{A}_{\mathsf{kan}})} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad T = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

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$$\mathcal{B} = \big\{ (1,0,0),\, (0,1,0),\, (1,1,1) \big\} \qquad \mathcal{A}_{\mathsf{kan}} = \big\{ (1,0),\, (0,1) \big\}$$
 
$$\mathcal{B}_{\mathsf{kan}} = \big\{ (1,0,0),\, (0,1,0),\, (0,0,1) \big\}$$

 $f: \mathbb{R}^3 \to \mathbb{R}^2$  $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$ 

$$F_{(\mathcal{B},\mathcal{A}_{\mathsf{kan}})} = egin{bmatrix} 1 & 1 & 2 \ 0 & 3 & 4 \end{bmatrix} \hspace{1cm} \mathcal{B}_{\mathsf{kan}} = \{(1,0,0),\,(0,1,0),\,(0,0$$

$$F_{(\mathcal{B}_{kan}, \mathcal{A}_{kan})} = \begin{bmatrix} 0 & 3 & 4 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

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$$S = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathcal{B} \xrightarrow{S} \mathcal{B}_{kan} \qquad \mathcal{A}_{kan} \xrightarrow{T} \mathcal{A}_{kan} \qquad T^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$F_{(\mathcal{B}_{kan}, \mathcal{A}_{kan})} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$F_{(\mathcal{B}, \mathcal{A}_{kan})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$F_{(\mathcal{B}_{kan}, \mathcal{A}_{kan})} = T^{-1}F_{(\mathcal{B}, \mathcal{A}_{kan})}S$$

$$S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$DZ \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

 $f: \mathbb{R}^3 \to \mathbb{R}^2$   $f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$ 

 $\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$   $\mathcal{A}_{\mathsf{kan}} = \{(1,0), (0,1)\}$ 

$$\mathcal{B} \xrightarrow{S} \mathcal{B}_{kan} \qquad \mathcal{A}_{kan} \xrightarrow{T} \mathcal{A}_{kan} \qquad T^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$F_{(\mathcal{B}_{kan}, \mathcal{A}_{kan})} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathcal{B} = \left\{ (1,0,0), (0,1,0), (1,1,1) \right\} \qquad \mathcal{A}_{\mathsf{kan}} = \left\{ (1,0), (0,1) \right\}$$

$$F_{(\mathcal{B},\mathcal{A}_{\mathsf{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix} \qquad \mathcal{B}_{\mathsf{kan}} = \left\{ (1,0,0), (0,1,0), (0,0,1) \right\}$$

$$S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathcal{D} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

 $f:\mathbb{R}^3 o \mathbb{R}^2 \ f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$ 

 $\mathcal{B} \xrightarrow{S} \mathcal{B}_{kan} \qquad \mathcal{A}_{kan} \xrightarrow{T} \mathcal{A}_{kan} \qquad T^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$   $F_{(\mathcal{B}_{kan}, \mathcal{A}_{kan})} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$ 

$$f:\mathbb{R}^3 o\mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \big\{ (1,0,0), \, (0,1,0), \, (1,1,1) \big\} \qquad \quad \mathcal{A}_{\mathsf{kan}} = \big\{ (1,0), \, (0,1) \big\}$$

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$$
  $\mathcal{A}_{\mathsf{kan}} = \{(1,0), (0,1)\}$   $\mathcal{B}_{\mathsf{kan}} = \{(1,0), (0,1)\}$   $\mathcal{B}_{\mathsf{kan}} = \{(1,0), (0,1)\}$ 

$$f:\mathbb{R}^3 o \mathbb{R}^2 \ f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

 $F_{(\mathcal{B}_{kan}, \mathcal{A}_{kan})} = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \end{vmatrix}$ 

$$B = \{(1,0,0), (0,1,0), (1,1,1)\}$$
  $A_{i} = \{(1,0), (0,1)\}$ 

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$$
  $\mathcal{A}_{\mathsf{kan}} = \{(1,0), (0,1)\}$ 

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$$
  $\mathcal{A}_{\mathsf{kan}} = \{(1,0), (0,1)\}$   $\mathcal{B}_{\mathsf{kan}} = \{(1,0,0), (0,1,0), (0,0,1)\}$   $\mathcal{B}_{\mathsf{kan}} = \{(1,0,0), (0,1,0), (0,0,1)\}$ 

$$f: \mathbb{R}^3 o \mathbb{R}^2 \ f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \big\{ (1,0,0), \, (0,1,0), \, (1,1,1) \big\} \qquad \quad \mathcal{A}_{\mathsf{kan}} = \big\{ (1,0), \, (0,1) \big\}$$

$$F_{(\mathcal{B},\mathcal{A}_{\mathsf{kan}})} = egin{bmatrix} 1 & 1 & 2 \ 0 & 3 & 4 \end{bmatrix} \hspace{1cm} \mathcal{B}_{\mathsf{kan}} = egin{bmatrix} (1,0,0), \ (0,1,0), \ (0,0,1) \end{pmatrix} \ F_{(\mathcal{B}_{\mathsf{kan}},\mathcal{A}_{\mathsf{kan}})} = egin{bmatrix} 1 & 1 & 0 \ 0 & 3 & 1 \end{bmatrix}$$

$$f(0,0,1) = ?$$

$$f(0,0,1) = ? \qquad \begin{bmatrix} 0 & 3 & 1 \end{bmatrix}$$

$$f:\mathbb{R}^3 o\mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

 $F_{(\mathcal{B}_{kan}, \mathcal{A}_{kan})} = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \end{vmatrix}$ 

$$\mathcal{B} = \big\{ (1,0,0), \, (0,1,0), \, (1,1,1) \big\} \qquad \quad \mathcal{A}_{\mathsf{kan}} = \big\{ (1,0), \, (0,1) \big\}$$

$$F_{(\mathcal{B}, \mathcal{A}_{\mathsf{kan}})} = egin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$f(0,0,1) = ?$$

$$Y_{\mathcal{A}_{\mathsf{kan}}} = F_{(\mathcal{B}_{\mathsf{kan}}, \mathcal{A}_{\mathsf{kan}})} X_{\mathcal{B}_{\mathsf{kan}}}$$

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$$f:\mathbb{R}^3 o\mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$$
  $\mathcal{A}_{kan} = \{(1,0), (0,1)\}$ 

$$\mathcal{B}_{\mathsf{kan}} = \{(1,0,0), (0,1,0), (0,0,1)\}$$

$$F_{(\mathcal{B}, \mathcal{A}_{kan})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$
 $F_{(\mathcal{B}_{kan}, \mathcal{A}_{kan})} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$ 

$$f(0,0,1) = ?$$

$$\left( \mathit{Y}_{\mathcal{A}_{\mathsf{kan}}} = \mathit{F}_{\left(\mathcal{B}_{\mathsf{kan}}, \mathcal{A}_{\mathsf{kan}}
ight)} \mathit{X}_{\mathcal{B}_{\mathsf{kan}}} 
ight)$$

$$X_{\mathcal{B}_{\mathsf{kan}}} = egin{bmatrix} 0 \ 0 \ 1 \end{bmatrix}$$

$$f: \mathbb{R}^3 o \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$$
  $\mathcal{A}_{kan} = \{(1,0), (0,1)\}$ 

$$\mathcal{B}_{\mathsf{kan}} = \{(1,0,0), (0,1,0), (0,0,1)\}$$

$$F_{(\mathcal{B}, \mathcal{A}_{kan})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$F_{(\mathcal{B}_{kan}, \mathcal{A}_{kan})} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

$$f(0,0,1) = ?$$

$$\left( \mathit{Y}_{\mathcal{A}_{\mathsf{kan}}} = \mathit{F}_{\left(\mathcal{B}_{\mathsf{kan}},\,\mathcal{A}_{\mathsf{kan}}
ight)} \mathit{X}_{\mathcal{B}_{\mathsf{kan}}} 
ight)$$

$$Y_{\mathcal{A}_{\mathsf{kan}}} = F_{(\mathcal{B}_{\mathsf{kan}}, \mathcal{A}_{\mathsf{kan}})} X_{\mathcal{B}_{\mathsf{kan}}}$$

$$Y_{A_{kan}} =$$

$$X_{\mathcal{B}_{\mathsf{kan}}} = egin{bmatrix} 0 \ 0 \ 1 \end{bmatrix}$$

$$f:\mathbb{R}^3 o\mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \big\{ (1,0,0), \, (0,1,0), \, (1,1,1) \big\} \qquad \mathcal{A}_{\mathsf{kan}} = \big\{ (1,0), \, (0,1) \big\}$$

$$F_{(\mathcal{B},\,\mathcal{A}_{\mathsf{kan}})} = egin{bmatrix} 1 & 1 & 2 \ 0 & 3 & 4 \end{bmatrix}$$

$$f(0,0,1) = ?$$

$$Y_{\mathcal{A}_{\mathsf{kan}}} = F_{(\mathcal{B}_{\mathsf{kan}}, \mathcal{A}_{\mathsf{kan}})} X_{\mathcal{B}_{\mathsf{kan}}}$$

$$Y_{\mathcal{A}_{\mathsf{kan}}} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

$$F_{(\mathcal{B}_{\mathsf{kan}},\mathcal{A}_{\mathsf{kan}})} = egin{bmatrix} 1 & 1 & 0 \ 0 & 3 & 1 \end{bmatrix}$$
  $X_{\mathcal{B}_{\mathsf{kan}}} = egin{bmatrix} 0 \ 0 \ 1 \end{bmatrix}$ 

$$f:\mathbb{R}^3 o\mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \big\{ (1,0,0),\, (0,1,0),\, (1,1,1) \big\} \qquad \mathcal{A}_{\mathsf{kan}} = \big\{ (1,0),\, (0,1) \big\}$$

$$F_{(\mathcal{B}, \mathcal{A}_{\mathsf{kan}})} = egin{bmatrix} 1 & 1 & 2 \ 0 & 3 & 4 \end{bmatrix}$$

$$f(0,0,1) = ?$$

$$Y_{\mathcal{A}_{\mathsf{kan}}} = F_{(\mathcal{B}_{\mathsf{kan}}, \mathcal{A}_{\mathsf{kan}})} X_{\mathcal{B}_{\mathsf{kan}}}$$

$$Y_{\mathcal{A}_{\mathsf{kan}}} = egin{bmatrix} 1 & 1 & 0 \ 0 & 3 & 1 \end{bmatrix} egin{bmatrix} 0 \ 0 \ 1 \end{bmatrix}$$

$$F_{(\mathcal{B}_{\mathsf{kan}}, \mathcal{A}_{\mathsf{kan}})} = egin{bmatrix} 1 & 1 & 0 \ 0 & 3 & 1 \end{bmatrix}$$
  $X_{\mathcal{B}_{\mathsf{kan}}} = egin{bmatrix} 0 \ 0 \ 1 \end{bmatrix}$ 

$$f:\mathbb{R}^3 o\mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$$
  $\mathcal{A}_{\mathsf{kan}} = \{(1,0), (0,1)\}$ 

$$F_{(\mathcal{B}, \mathcal{A}_{\mathsf{kan}})} = egin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$F_{(\mathcal{B}_{\mathsf{kan}},\mathcal{A}_{\mathsf{kan}})} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

$$f(0,0,1) = ?$$

$$Y_{\mathcal{A}_{\mathsf{kan}}} = F_{(\mathcal{B}_{\mathsf{kan}}, \mathcal{A}_{\mathsf{kan}})} X_{\mathcal{B}_{\mathsf{kan}}}$$

$$Y_{\mathcal{A}_{\mathsf{kan}}} = egin{bmatrix} 1 & 1 & 0 \ 0 & 3 & 1 \end{bmatrix} egin{bmatrix} 0 \ 0 \ 1 \end{bmatrix} = egin{bmatrix} 0 \ 1 \end{bmatrix}$$

$$X_{\mathcal{B}_{\mathsf{kan}}} = egin{bmatrix} 0 \ 0 \ 1 \end{bmatrix}$$