

## Seminari 3

### MATEMATIKA ZA EKONOMISTE 2

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#### Zadatak 2

Riješite neodređeni integral  $\int \frac{(x-3)^2}{x^5} dx$ .

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

#### Rješenje

$$\begin{aligned}\int \frac{(x-3)^2}{x^5} dx &= \int \frac{x^2 - 6x + 9}{x^5} dx = \int \left( \frac{1}{x^3} - \frac{6}{x^4} + \frac{9}{x^5} \right) dx = \\ &= \int x^{-3} dx - 6 \int x^{-4} dx + 9 \int x^{-5} dx = \\ &= \frac{x^{-2}}{-2} - 6 \cdot \frac{x^{-3}}{-3} + 9 \cdot \frac{x^{-4}}{-4} + C = \\ &= -\frac{1}{2x^2} + \frac{2}{x^3} - \frac{9}{4x^4} + C, \quad C \in \mathbb{R}\end{aligned}$$

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#### Zadatak 1

Riješite neodređeni integral  $\int \frac{dx}{\sqrt[4]{x^3}}$ .

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

#### Rješenje

$$\sqrt[n]{x^m} = x^{\frac{m}{n}}$$

$$\begin{aligned}\int \frac{dx}{\sqrt[4]{x^3}} &= \int x^{-\frac{3}{4}} dx = \frac{x^{-\frac{3}{4}+1}}{-\frac{3}{4}+1} + C = \\ &= \frac{x^{\frac{1}{4}}}{\frac{1}{4}} + C = 4\sqrt[4]{x} + C, \quad C \in \mathbb{R}\end{aligned}$$

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#### Zadatak 3

Riješite neodređeni integral  $\int (5e^x - 3 \sin x) dx$ .

#### Rješenje

$$\begin{aligned}\int (5e^x - 3 \sin x) dx &= 5 \int e^x dx - 3 \int \sin x dx = \\ &= 5e^x - 3 \cdot (-\cos x) + C = \\ &= 5e^x + 3 \cos x + C, \quad C \in \mathbb{R}\end{aligned}$$

$$\int e^x dx = e^x + C$$

$$\int \sin x dx = -\cos x + C$$

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**Zadatak 4**

Riješite neodređeni integral  $\int 3^x e^x dx$ .

**Rješenje**

$$\int 3^x e^x dx = \int (3e)^x dx = \frac{(3e)^x}{\ln(3e)} + C, \quad C \in \mathbb{R}$$

$$(ab)^n = a^n b^n$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

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**Zadatak 5**

Riješite neodređeni integral  $\int (3 - 2x)^8 dx$ .

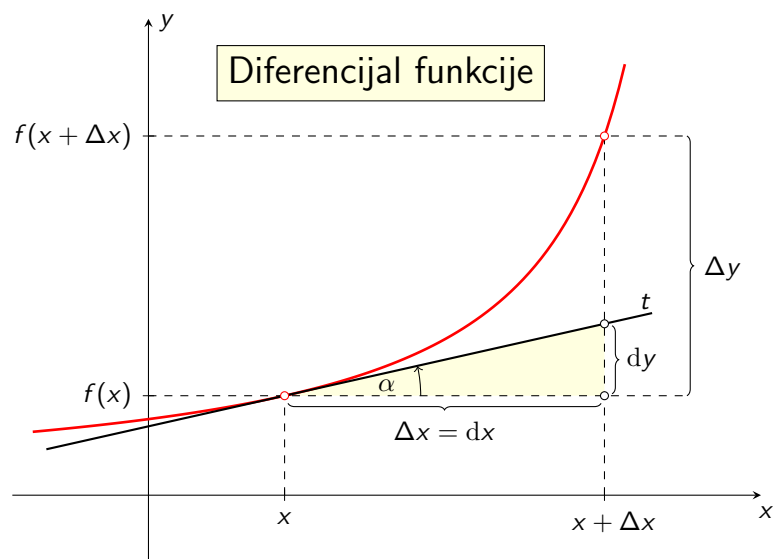
**Rješenje**

$$\begin{aligned} \int (3 - 2x)^8 dx &= \left[ \begin{array}{l} 3 - 2x = t / ' \\ -2 dx = dt \end{array} \right] = \int t^8 \cdot \frac{dt}{-2} = \\ &= -\frac{1}{2} \int t^8 dt = -\frac{1}{2} \cdot \frac{t^9}{9} + C = -\frac{1}{18} t^9 + C = \\ &= -\frac{1}{18} (3 - 2x)^9 + C, \quad C \in \mathbb{R} \end{aligned}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\begin{array}{l} t = f(x) \\ dt = f'(x) dx \end{array}$$

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**Diferencijal funkcije**

$$y = f(x), \quad f'(x) = \operatorname{tg} \alpha = \frac{dy}{dx}, \quad dy = f'(x) dx$$

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**Zadatak 6**

Riješite neodređeni integral  $\int \sqrt[4]{(x-2)^3} dx$ .

**Rješenje**

$$\begin{aligned} \int \sqrt[4]{(x-2)^3} dx &= \int (x-2)^{\frac{3}{4}} dx = \left[ \begin{array}{l} x-2 = t / ' \\ dx = dt \end{array} \right] = \\ &= \int t^{\frac{3}{4}} dt = \frac{t^{\frac{7}{4}}}{\frac{7}{4}} + C = \frac{4}{7} t^{\frac{7}{4}} + C = \\ &= \frac{4}{7} (x-2)^{\frac{7}{4}} + C, \quad C \in \mathbb{R} \end{aligned}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

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**Zadatak 7**

Riješite neodređeni integral  $\int x \cdot 7^{x^2} dx$ .

**Rješenje**

$$\int x \cdot 7^{x^2} dx = \left[ \begin{array}{l} x^2 = t / ' \\ 2x dx = dt \end{array} \right] = \int 7^t \cdot \frac{dt}{2} = \frac{1}{2} \int 7^t dt =$$

$$= \frac{1}{2} \cdot \frac{7^t}{\ln 7} + C = \frac{7^t}{2 \ln 7} + C = \frac{7^{x^2}}{2 \ln 7} + C, \quad C \in \mathbb{R}$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

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**Zadatak 9**

Riješite neodređeni integral  $\int \frac{\sqrt[3]{1 + \ln x}}{x} dx$ .

**Rješenje**

$$\int \frac{\sqrt[3]{1 + \ln x}}{x} dx = \left[ \begin{array}{l} 1 + \ln x = t / ' \\ \frac{1}{x} dx = dt \end{array} \right] = \int \sqrt[3]{t} dt = \int t^{\frac{1}{3}} dt =$$

$$= \frac{t^{\frac{4}{3}}}{\frac{4}{3}} + C = \frac{3}{4} t^{\frac{4}{3}} + C = \frac{3}{4} (1 + \ln x)^{\frac{4}{3}} + C, \quad C \in \mathbb{R}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

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**Zadatak 8**

Riješite neodređeni integral  $\int \frac{x dx}{\sqrt{1 - x^2}}$ .

**Rješenje**

$$\int \frac{x dx}{\sqrt{1 - x^2}} = \left[ \begin{array}{l} 1 - x^2 = t / ' \\ -2x dx = dt \end{array} \right] = \int \frac{-\frac{dt}{2}}{\sqrt{t}} = -\frac{1}{2} \int t^{-\frac{1}{2}} dt =$$

$$= -\frac{1}{2} \cdot \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C = -\sqrt{t} + C = -\sqrt{1 - x^2} + C, \quad C \in \mathbb{R}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

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**Napomena**

$$(\ln |x|)' = \frac{1}{x}, \quad x \neq 0$$

- Ako je  $x > 0$ , tada je  $|x| = x$  pa znamo da vrijedi

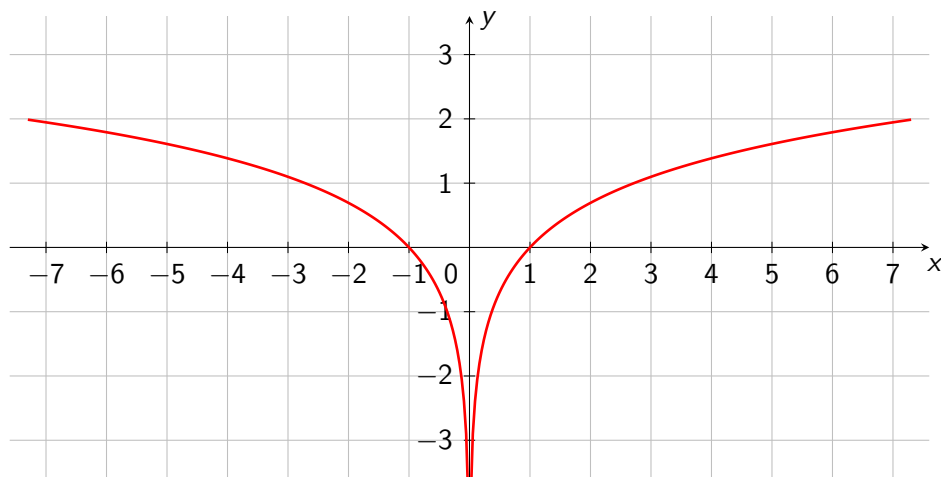
$$(\ln x)' = \frac{1}{x}$$

- Ako je  $x < 0$ , tada je  $|x| = -x$  pa korištenjem pravila za derivaciju složene funkcije ponovo dobivamo

$$(\ln(-x))' = \frac{1}{-x} \cdot (-x)' = \frac{1}{-x} \cdot (-1) = \frac{1}{x}$$

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## Graf funkcije $f(x) = \ln|x|$



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### Zadatak 10

Riješite neodređeni integral  $\int \frac{dx}{3-2x}$ .

#### Rješenje

$$\int \frac{dx}{3-2x} = \left[ \begin{array}{l} 3-2x = t \\ -2dx = dt \end{array} \right] \int \frac{\frac{dt}{-2}}{t} = -\frac{1}{2} \int \frac{dt}{t} = -\frac{1}{2} \ln|t| + C = -\frac{1}{2} \ln|3-2x| + C, \quad C \in \mathbb{R}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{dx}{x} = \ln|x| + C$$

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### Zadatak 11

Riješite neodređeni integral  $\int \frac{1-3x}{3+2x} dx$ .

#### Rješenje

$$P_1(x) = P_2(x)Q(x) + R(x)$$

$$\begin{aligned} \int \frac{1-3x}{3+2x} dx &= \int \frac{-3x+1}{2x+3} dx = \int \left( -\frac{3}{2} + \frac{\frac{11}{2}}{2x+3} \right) dx = \\ &= -\frac{3}{2} \int dx + \frac{11}{2} \int \frac{dx}{2x+3} = -\frac{3}{2}x + \frac{11}{2} \cdot \frac{1}{2} \ln|2x+3| + C = \\ &= -\frac{3}{2}x + \frac{11}{4} \ln|2x+3| + C, \quad C \in \mathbb{R} \end{aligned}$$

$$\int \frac{dx}{ax+b} = \frac{1}{a} \ln|ax+b| + C$$

$$\frac{P_1(x)}{P_2(x)} = Q(x) + \frac{R(x)}{P_2(x)}$$

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$$\begin{array}{l} (-3x+1) : (2x+3) = -\frac{3}{2} \\ \frac{3x+\frac{9}{2}}{2x+3} \\ \frac{11}{2} \end{array}$$

$Q(x)$   
 $R(x)$

### Zadatak 12

Riješite neodređeni integral  $\int \frac{x^2+5x-4}{5x+3} dx$ .

#### Rješenje

$$(x^2+5x-4) : (5x+3) = \frac{1}{5}x + \frac{22}{25}$$

$Q(x)$

$$\begin{array}{r} -x^2 - \frac{3}{5}x \\ \hline \frac{22}{5}x - 4 \\ -\frac{22}{5}x - \frac{66}{25} \\ \hline \end{array}$$

$$R(x) \rightarrow -\frac{166}{25}$$

$$P_1(x) = P_2(x)Q(x) + R(x)$$

$$\frac{P_1(x)}{P_2(x)} = Q(x) + \frac{R(x)}{P_2(x)}$$

$$\frac{x^2+5x-4}{5x+3} = \frac{1}{5}x + \frac{22}{25} + \frac{-\frac{166}{25}}{5x+3}$$

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$$\int \frac{x^2 + 5x - 4}{5x + 3} dx = \int \left( \frac{1}{5}x + \frac{22}{25} + \frac{-\frac{166}{25}}{5x + 3} \right) dx =$$

$$= \frac{1}{5} \int x dx + \frac{22}{25} \int dx - \frac{166}{25} \int \frac{dx}{5x + 3} =$$

$$= \frac{1}{5} \cdot \frac{x^2}{2} + \frac{22}{25}x - \frac{166}{25} \cdot \frac{1}{5} \ln |5x + 3| + C =$$

$$= \frac{1}{10}x^2 + \frac{22}{25}x - \frac{166}{125} \ln |5x + 3| + C, \quad C \in \mathbb{R}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{dx}{ax + b} = \frac{1}{a} \ln |ax + b| + C$$

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### Zadatak 14

Riješite neodređeni integral  $\int \frac{dx}{e^x + 2}$ .

### Rješenje

$$\int \frac{dx}{e^x + 2} = \frac{1}{2} \int \frac{(e^x + 2) - e^x}{e^x + 2} dx = \frac{1}{2} \int \left( 1 - \frac{e^x}{e^x + 2} \right) dx =$$

$$= \frac{1}{2} \cdot \left( \int dx - \int \frac{e^x}{e^x + 2} dx \right) = \frac{1}{2}x - \frac{1}{2} \int \frac{(e^x + 2)'}{e^x + 2} dx =$$

$$= \frac{1}{2}x - \frac{1}{2} \ln (e^x + 2) + C, \quad C \in \mathbb{R}$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + C$$

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### Zadatak 13

Riješite neodređeni integral  $\int \frac{1 - \sin x}{x + \cos x} dx$ .

### Rješenje

$$\int \frac{1 - \sin x}{x + \cos x} dx = \int \frac{(x + \cos x)'}{x + \cos x} dx = \ln |x + \cos x| + C, \quad C \in \mathbb{R}$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + C$$

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