## Seminari 1

Matematika za ekonomiste 2

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# Sadržaj

prvi zadatak

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treći zadatak

četvrti zadatak

prvi zadatak

### Zadatak 1

Zadane su funkcija potražnje  $q_1 = -3p^2 + 14p - 10$  i funkcija ponude  $q_2 = p + 2$ .

- a) Prikažite grafički zadane funkcije.
- b) Odredite cijene ekvilibrija i označite ih na grafu. Kolike su ponuda i potražnja u pojedinim cijenama ekvilibrija?

$$q_1 = -3p^2 + 14p - 10$$

### Rješenje

$$-3p^2 + 14p - 10 = 0$$

$$q_1 = -3p^2 + 14p - 10$$

 $ax^{2} + bx + c = 0$  $x_{1,2} = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$ 

### Rješenje

$$-3p^{2} + 14p - 10 = 0$$

$$p_{1,2} = \frac{-14 \pm \sqrt{14^{2} - 4 \cdot (-3) \cdot (-10)}}{2 \cdot (-3)}$$

$$q_1 = -3p^2 + 14p - 10$$

 $ax^{2} + bx + c = 0$  $x_{1,2} = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$ 

### Rješenje

$$\rho_{1,2} = \frac{-3p^2 + 14p - 10 = 0}{2 \cdot (-3)}$$

$$\rho_{1,2} = \frac{-14 \pm \sqrt{14^2 - 4 \cdot (-3) \cdot (-10)}}{2 \cdot (-3)}$$

$$\rho_{1,2} = \frac{-14 \pm \sqrt{76}}{-6}$$

$$q_1 = -3p^2 + 14p - 10$$

Rješenje

 $ax^{2} + bx + c = 0$  $x_{1,2} = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$ 

$$p_{1,2} = \frac{-3p^2 + 14p - 10 = 0}{2 \cdot (-3) \cdot (-10)}$$

$$p_{1,2} = \frac{-14 \pm \sqrt{14^2 - 4 \cdot (-3) \cdot (-10)}}{2 \cdot (-3)}$$

$$p_{1,2} = \frac{-14 \pm \sqrt{76}}{-6}$$

$$p_1 \approx 0.88, \quad p_2 \approx 3.79$$

$$q_1 = -3p^2 + 14p - 10$$

 $ax^{2} + bx + c = 0$   $x_{1,2} = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$ 

## Rješenje

$$-3p^2 + 14p - 10 = 0$$
 $p_{1,2} = \frac{-14 \pm \sqrt{14^2 - 4 \cdot (-3) \cdot (-10)}}{2 \cdot (-3)}$ 
 $p_{1,2} = \frac{-14 \pm \sqrt{76}}{-6}$  nultočke
 $p_1 \approx 0.88, \quad p_2 \approx 3.79$ 

$$q_1 = -3p^2 + 14p - 10$$
Rješenje

a) Za crtanje parabole odredimo nultočke i tjeme.

$$ho_{1,2} = rac{-3 
ho^2 + 14 
ho - 10 = 0}{2 \cdot (-3)}$$
  $ho_{1,2} = rac{-14 \pm \sqrt{14^2 - 4 \cdot (-3) \cdot (-10)}}{2 \cdot (-3)}$  nultočke  $ho_1 pprox 0.88, \quad 
ho_2 pprox 3.79$ 

$$q_1'=-6p+14$$

 $ax^{2} + bx + c = 0$   $x_{1,2} = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$ 

 $q_1 = -3p^2 + 14p - 10$ 

a) Za crtanje parabole odredimo nultočke i tjeme.

$$-3p^2 + 14p - 10 = 0$$
 $p_{1,2} = \frac{-14 \pm \sqrt{14^2 - 4 \cdot (-3) \cdot (-10)}}{2 \cdot (-3)}$ 
 $p_{1,2} = \frac{-14 \pm \sqrt{76}}{-6}$  nultočke
 $p_1 \approx 0.88, \quad p_2 \approx 3.79$ 

 $ax^{2} + bx + c = 0$   $x_{1,2} = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$ 

 $q_1' = -6p + 14$ 

$$q_1 = -3p^2 + 14p - 10$$
Rješenje

a) Za crtanje parabole odredimo nultočke i tjeme.

$$p_{1,2} = rac{-3p^2 + 14p - 10 = 0}{2 \cdot (-3)}$$
 $p_{1,2} = rac{-14 \pm \sqrt{14^2 - 4 \cdot (-3) \cdot (-10)}}{2 \cdot (-3)}$ 
 $p_{1,2} = rac{-14 \pm \sqrt{76}}{-6}$  nultočke
 $p_1 pprox 0.88, \quad p_2 pprox 3.79$ 

$$q_1' = -6p + 14 -6p + 14 = 0$$

 $ax^{2} + bx + c = 0$   $x_{1,2} = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$ 

$$-6p = -14$$

$$q_1 = -3p^2 + 14p - 10$$

Rješenje

a) Za crtanje parabole odredimo nultočke i tjeme.

$$p_{1,2} = \frac{-3p^2 + 14p - 10 = 0}{2 \cdot (-3)}$$

$$p_{1,2} = \frac{-14 \pm \sqrt{14^2 - 4 \cdot (-3) \cdot (-10)}}{2 \cdot (-3)}$$

$$p_{1,2} = \frac{-14 \pm \sqrt{76}}{-6}$$

$$p_{1} \approx 0.88, \quad p_{2} \approx 3.79$$

$$-6p + 14 = 0$$
$$-6p = -14$$
$$p = \frac{7}{3}$$

 $q_1' = -6p + 14$ 

 $ax^{2} + bx + c = 0$   $x_{1,2} = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$ 

$$-3p^2+14p-10=0$$
  $q_1'=-6p+14$   $-6p+14=0$   $p_{1,2}=rac{-14\pm\sqrt{14^2-4\cdot(-3)\cdot(-10)}}{2\cdot(-3)}$   $-6p=-14$   $p=rac{7}{3}$ 

a) Za crtanje parabole odredimo nultočke i tjeme.

 $p_{1,2} = \frac{-14 \pm \sqrt{76}}{6}$ 

 $p_1 \approx 0.88, \quad p_2 \approx 3.79 \leftarrow$ 

 $ax^{2} + bx + c = 0$   $x_{1,2} = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$ 

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$$q_1\left(\frac{7}{3}\right) =$$

 $q_1 = -3p^2 + 14p - 10$ 

Rješenje

$$-3p^2+14p-10=0 \qquad \qquad q_1'=-6p+14 \ p_{1,2}=rac{-14\pm\sqrt{14^2-4\cdot(-3)\cdot(-10)}}{2\cdot(-3)} \qquad \qquad -6p+14=0 \ -6p=-14$$

a) Za crtanje parabole odredimo nultočke i tjeme.

$$p_1 \approx 0.88, \quad p_2 \approx 3.79$$
  $q_1\left(\frac{7}{3}\right) = -3 \cdot \left(\frac{7}{3}\right)^2 + 14 \cdot \frac{7}{3} - 10$ 

 $p_{1,2} = \frac{-14 \pm \sqrt{76}}{6}$ 

 $q_1 = -3p^2 + 14p - 10$ 

Rješenje

 $ax^{2} + bx + c = 0$   $x_{1,2} = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$ 

 $p=\frac{7}{3}$ 

a) Za crtanje parabole odredimo nultočke i tjeme. 
$$-3p^2+14p-10=0 \qquad \qquad q_1'=-6p+14 \\ p_{1,2}=\frac{-14\pm\sqrt{14^2-4\cdot(-3)\cdot(-10)}}{2\cdot(-3)} \qquad \qquad -6p=-14$$

 $q_1 = -3p^2 + 14p - 10$ 

Rješenje

$$p_1 \approx 0.88, \quad p_2 \approx 3.79$$
 $q_1\left(\frac{7}{3}\right) = -3 \cdot \left(\frac{7}{3}\right)^2 + 14 \cdot \frac{7}{3} - 10 = \frac{19}{3}$ 

 $p_{1,2} = \frac{-14 \pm \sqrt{76}}{6}$ 

 $ax^{2} + bx + c = 0$   $x_{1,2} = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$ 

 $p=\frac{7}{2}$ 

a) Za crtanje parabole odredimo nultočke i tjeme. 
$$-3p^2+14p-10=0 \qquad \qquad q_1'=-6p+14 \\ p_{1,2}=\frac{-14\pm\sqrt{14^2-4\cdot(-3)\cdot(-10)}}{2\cdot(-3)} \qquad \qquad -6p=-14$$

 $q_1 = -3p^2 + 14p - 10$ 

Rješenje

$$p_1 \approx 0.88, \quad p_2 \approx 3.79$$

$$T\left(\frac{7}{3}, \frac{19}{3}\right)$$

nultočke

 $p_{1,2} = \frac{-14 \pm \sqrt{76}}{c}$ 

 $q_1\left(\frac{7}{3}\right) = -3\cdot\left(\frac{7}{3}\right)^2 + 14\cdot\frac{7}{3} - 10 = \frac{19}{3}$ 

 $ax^{2} + bx + c = 0$   $x_{1,2} = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$ 

 $p=\frac{7}{2}$ 

$$-3
ho^2+14
ho-10=0 \qquad \qquad q_1'=-6
ho+14 \ p_{1,2}=rac{-14\pm\sqrt{14^2-4\cdot(-3)\cdot(-10)}}{2\cdot(-3)} \qquad \qquad -6
ho+14=0 \ -6
ho=-14$$

a) Za crtanje parabole odredimo nultočke i tjeme.

 $p_{1,2} = \frac{-14 \pm \sqrt{76}}{\epsilon}$ 

 $p_1 \approx 0.88, \quad p_2 \approx 3.79 \leftarrow$ 

 $q_1 = -3p^2 + 14p - 10$ 

Rješenje

$$q_1\left(\frac{7}{3}\right)=-3\cdot\left(\frac{7}{3}\right)^2+14\cdot\frac{7}{3}-10=\frac{19}{3}$$
 tjeme

 $ax^{2} + bx + c = 0$   $x_{1,2} = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$ 

 $p=\frac{7}{2}$ 

 $q_2(0) =$ 

Za crtanje pravca je dovoljno odrediti neke dvije njegove točke.

$$q_2 = p+2$$

$$q_1 = -3p^2 + 14p - 10$$

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 $q_2 = p + 2$ 

$$q_2 = p + 2$$

$$q_1 = -3p^2 + 14p - 10$$

Za crtanje pravca je dovoljno odrediti neke dvije njegove točke.

 $q_2(0) = 0 + 2 = 2$ 

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 $q_2(0) = 0 + 2 = 2 \longrightarrow T_1(0,2)$ 

$$q_2(0) = 0 + 2 = 2 \longrightarrow r_1(0, 2)$$

Za crtanje pravca je dovoljno odrediti neke dvije njegove točke.

$$q_2 = p + 2$$

$$q_1 = -3p^2 + 14p - 10$$

 $q_2(0)=0+2=2\longrightarrow T_1(0,2)$ 

$$q_2(1) =$$

Za crtanje pravca je dovoljno odrediti neke dvije njegove točke.

 $q_2 = p + 2$ 

 $q_2(1) = 1 + 2 = 3$   $q_2 = p + 2$ 

 $q_2(0) = 0 + 2 = 2 \longrightarrow T_1(0,2)$ 

Za crtanje pravca je dovoljno odrediti neke dvije njegove točke.

 $q_2(0) = 0 + 2 = 2 \longrightarrow T_1(0,2)$ 

$$q_2(0) = q_2(0) = q$$

Za crtanje pravca je dovoljno odrediti neke dvije njegove točke.

$$q_2(1) = 1 + 2 = 3 \longrightarrow T_2(1,3)$$
  $q_2 = p + 2$ 

Za crtanje pravca je dovoljno odrediti neke dvije njegove točke.  $q_2(0) = 0 + 2 = 2 \longrightarrow T_1(0,2)$ 

$$q_2(0)$$
 0 | 2 |  $T_1(0,2)$ 

$$q_2(1) = 1 + 2 = 3 \longrightarrow T_2(1,3)$$

$$q_2(1)=1+2=3 \longrightarrow \mathcal{T}_2(1,3)$$
  $q_2=p+2$  b) Cijena ekvilibrija  $q_1=-3p^2+14p-10$ 

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Za crtanje pravca je dovoljno odrediti neke dvije njegove točke.  $q_2(0) = 0 + 2 = 2 \longrightarrow T_1(0,2)$ 

$$q_2(0)$$
  $0 + 2 = -1(0, 2)$ 

$$q_2(1)=1+2=3\longrightarrow T_2(1,3)$$

 $q_2 = p + 2$ 

b) Cijena ekvilibrija
$$q_1=-3p^2+14p-10$$
 $q_1=q_2$ 

 $q_2(0) = 0 + 2 = 2 \longrightarrow T_1(0, 2)$ 

$$q_2(0) = 0 + 2 = 2$$

Za crtanje pravca je dovoljno odrediti neke dvije njegove točke.

$$q_2(1)=1+2=3\longrightarrow T_2(1,3)$$

b) Cijena ekvilibrija 
$$q_1 = -3p^2 + 14p - 10$$

 $q_1 = q_2$   $-3p^2 + 14p - 10 = p + 2$ 

 $q_2 = p + 2$ 

Za crtanje pravca je dovoljno odrediti neke dvije njegove točke.

$$q_2(0) = 0 + 2 = 2 \longrightarrow T_1(0,2)$$

$$q_2(1)=1+2=3\longrightarrow T_2(1,3)$$

b) Cijena ekvilibrija 
$$q_1 = -3p^2 + 14p - 10$$

$$q_1 = q_2$$
 $-3p^2 + 14p - 10 = p + 2$ 
 $-3p^2 + 13p - 12 = 0$ 

 $q_2 = p + 2$ 

Za crtanje pravca je dovoljno odrediti neke dvije njegove točke.  $q_2(0) = 0 + 2 = 2 \longrightarrow T_1(0,2)$ 

$$q_2(1) = 1 + 2 = 3 \longrightarrow T_2(1,3)$$
  $q_2 = p + 2$ 

b) Cijena ekvilibrija 
$$q_1=q_2 \\ -3p^2+14p-10=p+2 \\ -3p^2+13p-12=0$$

$$-3p^{2} + 13p - 12 = 0$$

$$p_{1,2} = \frac{-13 \pm \sqrt{13^{2} - 4 \cdot (-3) \cdot (-12)}}{2 \cdot (-3)}$$

$$ax^{2} + bx + c = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$$

Za crtanje pravca je dovoljno odrediti neke dvije njegove točke.  $q_2(0) = 0 + 2 = 2 \longrightarrow T_1(0,2)$ 

$$q_2(1) = 1 + 2 = 3 \longrightarrow T_2(1,3)$$
  $q_2 = p + 2$ 

$$q_1 = q_2$$
 $-3p^2 + 14p - 10 = p + 2$ 
 $-3p^2 + 13p - 12 = 0$ 

$$p_{1,2} = rac{-3p^2 + 13p - 12}{2 \cdot (-3)}$$
  $p_{1,2} = rac{-13 \pm 5}{-6}$ 

$$3 \pm \sqrt{13^2 - 4 \cdot (-3) \cdot (-1)}$$
  
 $2 \cdot (-3)$ 

$$ax^{2} + bx + c = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$$

$$p_{1,2} = \frac{-13 \pm \sqrt{13^2 - 4 \cdot (-3) \cdot (-12)}}{2 \cdot (-3)}$$

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Za crtanje pravca je dovoljno odrediti neke dvije njegove točke.  $q_2(0) = 0 + 2 = 2 \longrightarrow T_1(0,2)$ 

$$q_2(1) = 1 + 2 = 3 \longrightarrow T_2(1,3)$$
  $q_2 = p + 2$ 

b) Cijena ekvilibrija 
$$q_1=-3p^2+14p-10$$
 
$$q_1=q_2$$
 
$$-3p^2+14p-10=p+2$$

$$-3p^{2} + 13p - 12 = 0$$

$$p_{1,2} = \frac{-13 \pm \sqrt{13^{2} - 4 \cdot (-3) \cdot (-12)}}{2 \cdot (-3)}$$

$$p_{1,2} = \frac{-13 \pm}{}$$

$$ax^2 + bx + c = 0$$

$$2 \cdot (-3)$$
 $-13 \pm 5$ 

$$p_{1,2} = \frac{-13 \pm 6}{6}$$

$$p_{1,2} = \frac{-13 \pm 5}{2}$$

$$p_{1,2} = \frac{-13 \pm 5}{-6}$$

$$p_{1,2} = \frac{-13 \pm 5}{-6}$$

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 $p_1=\frac{4}{3}, \quad p_2=3$ 

$$q_1 = -3p^2 + 14p - 10$$
 
$$q_2 = p + 2$$

$$q_1\left(\frac{4}{3}\right) = q_2\left(\frac{4}{3}\right)$$
$$q_1(3) = q_2(3)$$

$$q_1 = -3p^2 + 14p - 10$$
 
$$q_2 = p + 2$$

$$q_1\left(\frac{4}{3}\right) = q_2\left(\frac{4}{3}\right) = \frac{4}{3} + 2 = \frac{10}{3}$$
  
 $q_1(3) = q_2(3)$ 

$$q_1 = -3p^2 + 14p - 10$$
 
$$q_2 = p + 2$$

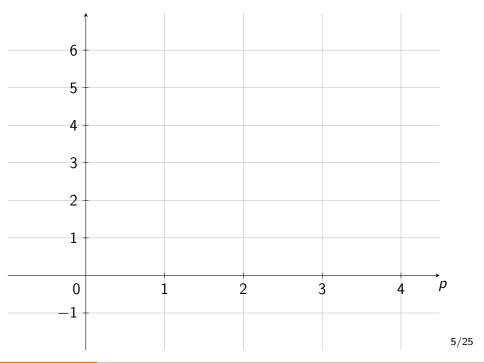
$$q_1\left(\frac{4}{3}\right) = q_2\left(\frac{4}{3}\right) = \frac{4}{3} + 2 = \frac{10}{3}$$
  
 $q_1(3) = q_2(3) = 3 + 2 = 5$ 

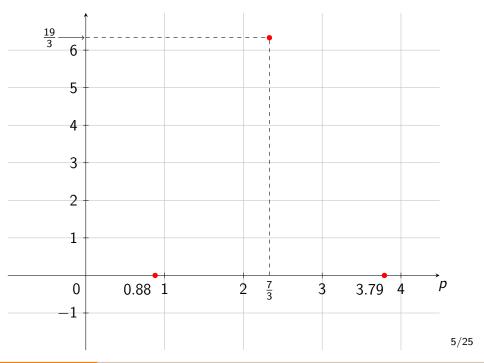
$$q_1 = -3p^2 + 14p - 10 \qquad q_2 = p + 2$$

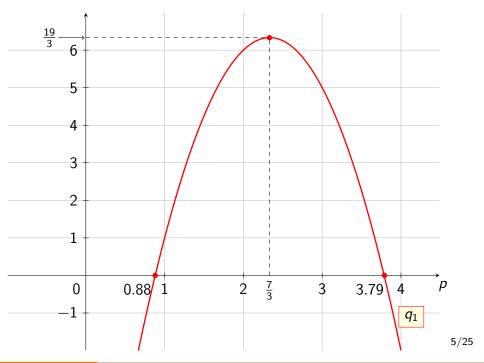
$$q_1\left(\frac{4}{3}\right) = q_2\left(\frac{4}{3}\right) = \frac{4}{3} + 2 = \frac{10}{3}$$
  
 $q_1(3) = q_2(3) = 3 + 2 = 5$ 

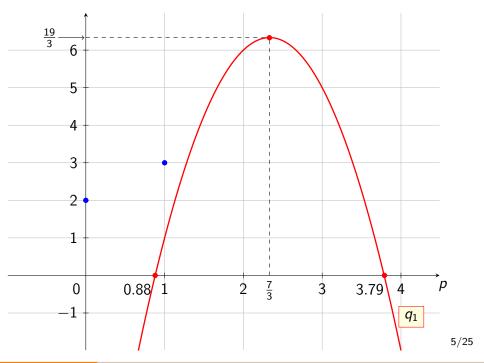
### Napomena

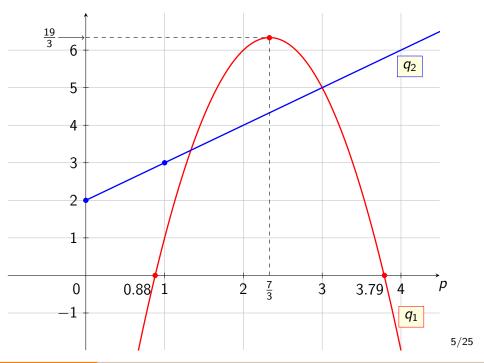
Kako je funkcija potražnje u pravilu padajuća funkcija, možemo reći da imamo samo jednu cijenu ekvilibrija p=3.

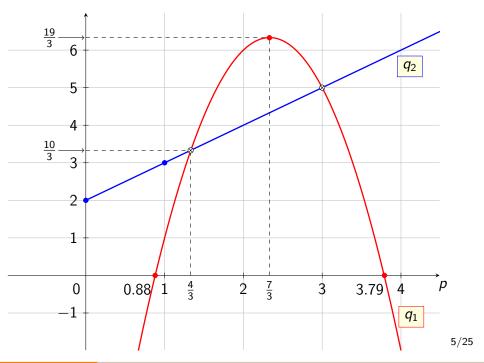


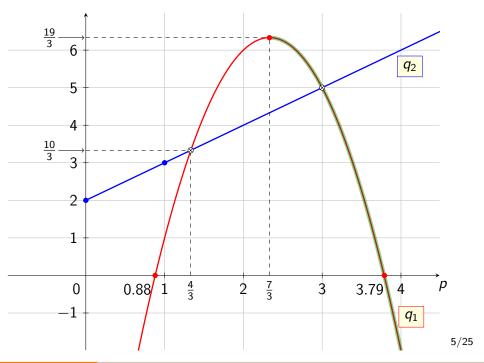


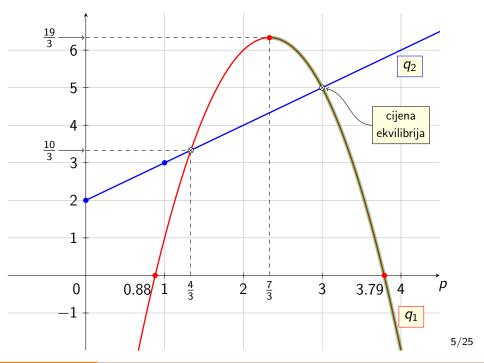












drugi zadatak

#### Zadatak 2

Da bi nešto zaradio, student kroz ljetne mjesece prodaje ogrlice na plaži. Prošlo ljeto je prodavao ogrlice po 10€ i dnevno je prodao 20 ogrlica. Međutim, kada je povećao cijenu ogrlice za 1€, prodaja se smanjila za dvije ogrlice dnevno.

- a) Pronađite funkciju potražnje za ogrlicama uz pretpostavku da se radi o linearnoj funkciji.
- b) Ako za izradu svake ogrlice student treba uložiti 6€, po kojoj cijeni treba prodavati ogrlice da bi ostvario maksimalni profit? Koliko iznosi maksimalni profit i koliko ogrlica će prodati u tom slučaju?
- c) Nacrtajte grafove funkcija potražnje, troškova, prihoda i profita u ovisnosti o cijeni proizvoda.

a) 
$$q = ap + b$$
,

$$a = \frac{\Delta q}{\Delta p}$$

a) 
$$q = ap + b$$
,  $\Delta p = 1$ ,

$$a = \frac{\Delta q}{\Delta p}$$

a) 
$$q=ap+b,$$
  $\Delta p=1,$   $\Delta q=-2$  
$$a=\frac{\Delta q}{\Delta p}$$

a) 
$$q = ap + b$$
,  $\Delta p = 1$ ,  $\Delta q = -2$ 

$$a = \frac{\Delta q}{\Delta p} = \frac{-2}{1}$$

a) 
$$q = ap + b$$
,  $\Delta p = 1$ ,  $\Delta q = -2$ 

$$a = \frac{\Delta q}{\Delta p} = \frac{-2}{1} = -2$$

a) 
$$q = ap + b$$
,  $\Delta p = 1$ ,  $\Delta q = -2$ 

$$=1, \quad \Delta q=-2$$

$$a = \frac{\Delta q}{\Delta p} = \frac{-2}{1} = -2$$

$$q=-2p+b$$

a) 
$$q=ap+b,$$
  $\Delta p=1,$   $\Delta q=-2$  
$$a=\frac{\Delta q}{2}=\frac{-2}{2}$$

$$a = \frac{\Delta q}{\Delta p} = \frac{-2}{1} = -2$$

$$q = -2p + b$$

$$q(10)=20$$

a) 
$$q=ap+b,$$
  $\Delta p=1,$   $\Delta q=-2$  
$$a=\frac{\Delta q}{\Delta p}=\frac{-2}{1}=-2$$
 
$$q=-2p+b$$

q(10) = 20

 $-2 \cdot 10 + b = 20$ 

a) 
$$q=ap+b,$$
  $\Delta p=1,$   $\Delta q=-2$  
$$a=\frac{\Delta q}{\Delta p}=\frac{-2}{1}=-2$$
 
$$q=-2p+b$$

$$q(10) = 20$$
$$-2 \cdot 10 + b = 20$$

$$b = 20$$

a) 
$$q=ap+b,$$
  $\Delta p=1,$   $\Delta q=-2$  
$$a=\frac{\Delta q}{\Delta p}=\frac{-2}{1}=-2$$
 
$$q=-2p+b$$
 
$$q(10)=20$$

$$-2 \cdot 10 + b = 20$$
$$b = 40$$

Funkcija potražnje za ogrlicama: q = -2p + 40

• Prihod kao funkcija cijene:  $P(p) = p \cdot q(p)$ 

$$P(p) =$$

• Prihod kao funkcija cijene:  $P(p) = p \cdot q(p)$ 

$$P(p) = p \cdot (-2p + 40)$$

• Prihod kao funkcija cijene:  $P(p) = p \cdot q(p)$ 

$$P(p) = p \cdot (-2p + 40) = -2p^2 + 40p$$

• Prihod kao funkcija cijene:  $P(p) = p \cdot q(p)$ 

$$P(p) = p \cdot (-2p + 40) = -2p^2 + 40p$$

Troškovi kao funkcija cijene

$$T(p) =$$

• Prihod kao funkcija cijene:  $P(p) = p \cdot q(p)$ 

$$P(p) = p \cdot (-2p + 40) = -2p^2 + 40p$$

Troškovi kao funkcija cijene

$$T(p) = 6 \cdot (-2p + 40)$$

• Prihod kao funkcija cijene:  $P(p) = p \cdot q(p)$ 

$$P(p) = p \cdot (-2p + 40) = -2p^2 + 40p$$

Troškovi kao funkcija cijene

$$T(p) = 6 \cdot (-2p + 40) = -12p + 240$$

$$P(p) = p \cdot (-2p + 40) = -2p^2 + 40p$$

Troškovi kao funkcija cijene

$$T(p) = 6 \cdot (-2p + 40) = -12p + 240$$

Profit kao funkcija cijene

$$D(p) = P(p) - T(p) =$$

$$P(p) = p \cdot (-2p + 40) = -2p^2 + 40p$$

Troškovi kao funkcija cijene

$$T(p) = 6 \cdot (-2p + 40) = -12p + 240$$

Profit kao funkcija cijene

$$D(p) = P(p) - T(p) = (-2p^2 + 40p)$$

$$P(p) = p \cdot (-2p + 40) = -2p^2 + 40p$$

Troškovi kao funkcija cijene

$$T(p) = 6 \cdot (-2p + 40) = -12p + 240$$

• Profit kao funkcija cijene

$$D(p) = P(p) - T(p) = (-2p^2 + 40p) -$$

$$P(p) = p \cdot (-2p + 40) = -2p^2 + 40p$$

Troškovi kao funkcija cijene

$$T(p) = 6 \cdot (-2p + 40) = -12p + 240$$

• Profit kao funkcija cijene

$$D(p) = P(p) - T(p) = (-2p^2 + 40p) - (-12p + 240)$$

$$P(p) = p \cdot (-2p + 40) = -2p^2 + 40p$$

Troškovi kao funkcija cijene

$$T(p) = 6 \cdot (-2p + 40) = -12p + 240$$

Profit kao funkcija cijene

$$D(p) = P(p) - T(p) = (-2p^2 + 40p) - (-12p + 240)$$
$$D(p) = -2p^2 + 52p - 240$$

$$D'(p) =$$

$$D'(p) = -4p + 52$$

$$D'(p) = -4p + 52$$
$$-4p + 52 = 0$$

$$D'(p) = -4p + 52$$
  
 $-4p + 52 = 0$   
 $-4p = -52$ 

$$D'(p) = -4p + 52$$
  
 $-4p + 52 = 0$   
 $-4p = -52 / : (-4)$ 

$$D'(p) = -4p + 52$$
  
 $-4p + 52 = 0$   
 $-4p = -52 / : (-4)$   
 $p = 13$ 

$$D'(p) = -4p + 52$$
 $-4p + 52 = 0$ 
 $-4p = -52 / : (-4)$ 
 $p = 13$ 
 $D''(p) = -4, \qquad D''(13) = -4 < 0$ 

Student mora ogrlice prodavati po cijeni od  $13 \in$  ukoliko želi ostvariti maksimalni profit.

$$D'(p) = -4p + 52$$
 $-4p + 52 = 0$ 
 $-4p = -52 / : (-4)$ 
 $p = 13$ 
 $D''(p) = -4, \qquad D''(13) = -4 < 0$ 

Student mora ogrlice prodavati po cijeni od 13€ ukoliko želi ostvariti maksimalni profit. Maksimalni profit u tom slučaju iznosi

 $D(13) = -2 \cdot 13^2 + 52 \cdot 13 - 240$ 

$$D'(p) = -4p + 52$$

$$-4p + 52 = 0$$

$$-4p = -52 /: (-4)$$

$$p = 13$$

$$D''(p) = -4, \qquad D''(13) = -4 < 0$$

$$D(13) = -2 \cdot 13^{2} + 52 \cdot 13 - 240 = 98$$

Student mora ogrlice prodavati po cijeni od 13€ ukoliko želi ostvariti maksimalni profit. Maksimalni profit u tom slučaju iznosi 98€, a proda se ukupno \_\_ ogrlica.

$$-4p + 52 = 0$$
 $-4p = -52 / : (-4)$ 
 $p = 13$ 
 $D''(p) = -4, \qquad D''(13) = -4 < 0$ 

 $D(13) = -2 \cdot 13^2 + 52 \cdot 13 - 240 = 98$ 

D'(p) = -4p + 52

 $a(13) = -2 \cdot 13 + 40$ 

$$q = -2p + 40$$

$$-4p + 52 = 0$$
 $-4p = -52 / : (-4)$ 
 $p = 13$ 
 $D''(p) = -4, \qquad D''(13) = -4 < 0$ 

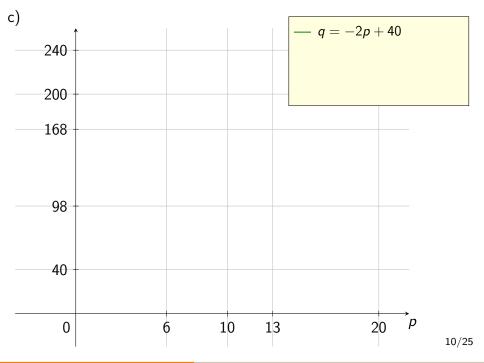
$$D(13) = -2 \cdot 13^2 + 52 \cdot 13 - 240 = 98$$
 Student mora ogrlice prodavati po cijeni od  $13 \in$  ukoliko želi ostvariti

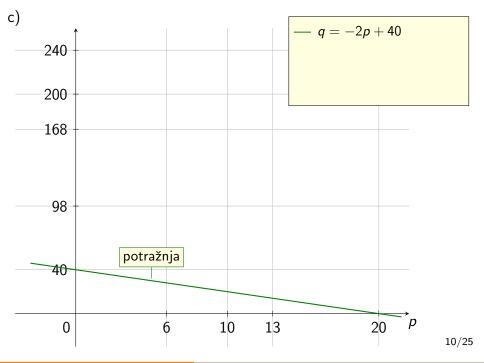
maksimalni profit. Maksimalni profit u tom slučaju iznosi 98€, a proda se ukupno <u>14</u> ogrlica.

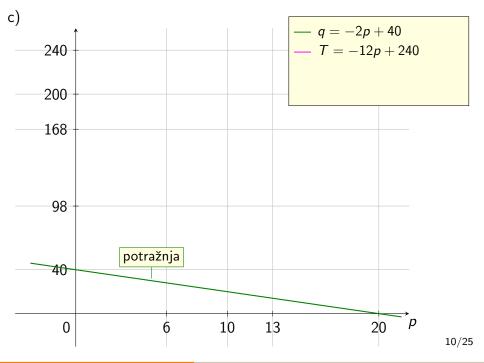
D'(p) = -4p + 52

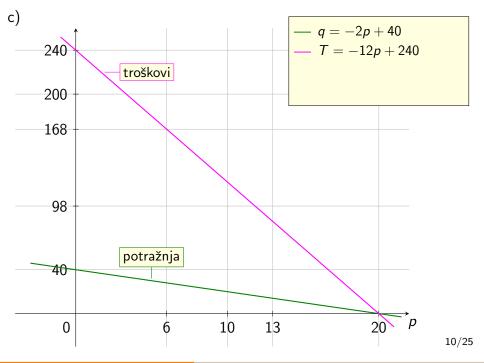
$$q(13) = -2 \cdot 13 + 40 = 14$$

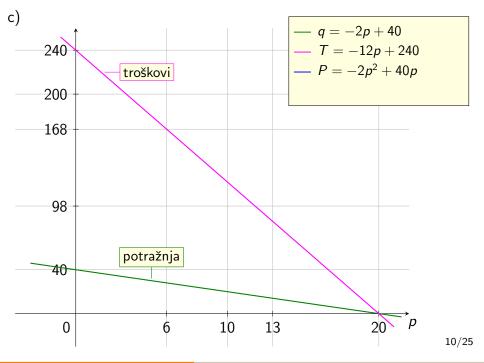
$$q=-2p+40$$

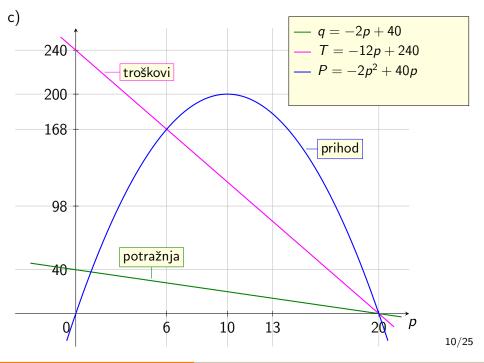


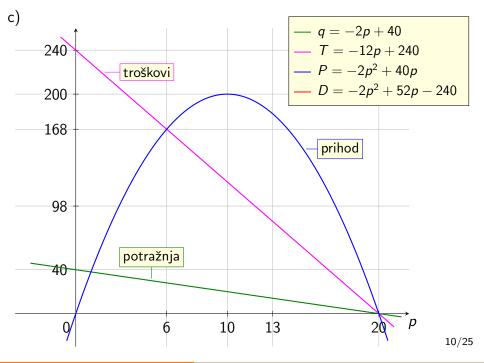


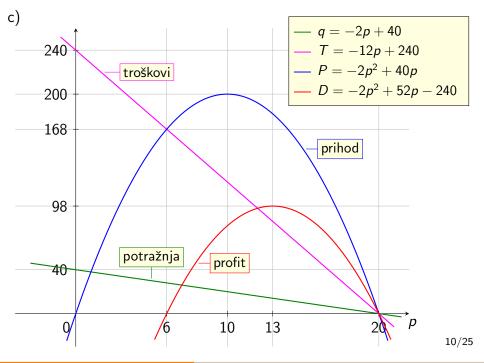












treći zadatak

#### Zadatak 3

Menadžer je utvrdio da je veza između prodajne cijene p robe i broja komada N dana s

$$N(p) = \frac{1500}{p^2 + 100}.$$

Prosječni troškovi za N komada robe jednaki su

$$T_p = 2 + \frac{14}{N}$$
.

Odredite cijenu uz koju se ostvaruje maksimalni profit i izračunajte taj profit. Odredite za koje prodajne cijene robe je profit pozitivan.

• Funkcija prosječnih troškova

$$\mathsf{prosječni} \ \mathsf{troškovi} = \frac{\mathsf{troškovi}}{\mathsf{broj} \ \mathsf{proizvoda}}$$

• Funkcija prosječnih troškova

$$\mathsf{prosječni} \ \mathsf{troškovi} = \frac{\mathsf{troškovi}}{\mathsf{broj} \ \mathsf{proizvoda}}$$

$$T_p(N) = \frac{T(N)}{N}$$

• Funkcija prosječnih troškova

$$\mathsf{prosječni} \ \mathsf{troškovi} = \frac{\mathsf{troškovi}}{\mathsf{broj} \ \mathsf{proizvoda}}$$

$$T_{\rho}(N) = \frac{T(N)}{N}$$

$$T(N) = T_p(N) \cdot N$$

• Funkcija prosječnih troškova

$$\mathsf{prosječni}\;\mathsf{troškovi} = \frac{\mathsf{troškovi}}{\mathsf{broj}\;\mathsf{proizvoda}}$$

$$T_p(N) = \frac{T(N)}{N}$$

$$T(N) = T_{\rho}(N) \cdot N = \left(2 + \frac{14}{N}\right) \cdot N$$

• Funkcija prosječnih troškova

$$\mathsf{prosječni} \ \mathsf{troškovi} = \frac{\mathsf{troškovi}}{\mathsf{broj} \ \mathsf{proizvoda}}$$

$$T_p(N) = \frac{T(N)}{N}$$

$$T(N) = T_p(N) \cdot N = \left(2 + \frac{14}{N}\right) \cdot N = 2N + 14$$

• Funkcija prosječnih troškova

prosječni troškovi = 
$$\frac{\text{troškovi}}{\text{broj proizvoda}}$$

$$T_{\rho}(N) = \frac{T(N)}{N}$$

$$T(N) = T_p(N) \cdot N = \left(2 + \frac{14}{N}\right) \cdot N = 2N + 14$$

$$T(p) = 2N(p) + 14$$

## $N(p) = \frac{1500}{p^2 + 100}$ $T_p = 2 + \frac{14}{N}$

### Riešenie

Funkcija prosječnih troškova

prosječni troškovi = 
$$\frac{\text{troškovi}}{\text{broj proizvoda}}$$

$$T_{\rho}(N) = \frac{T(N)}{N}$$

$$T(N) = T_p(N) \cdot N = \left(2 + \frac{14}{N}\right) \cdot N = 2N + 14$$

$$T(p) = 2N(p) + 14 = 2 \cdot \frac{1500}{p^2 + 100} + 14$$

# $N(p) = \frac{1500}{p^2 + 100}$ $T_p = 2 + \frac{14}{N}$

$$T_p = 2 + \frac{14}{N}$$

Riešenie

Funkcija prosječnih troškova

prosječni troškovi = 
$$\frac{\text{troškovi}}{\text{broj proizvoda}}$$

$$T_p(N) = \frac{I(N)}{N}$$

$$T(N) = T_{\rho}(N) \cdot N = \left(2 + \frac{14}{N}\right) \cdot N = 2N + 14$$

$$T(p) = 2N(p) + 14 = 2 \cdot \frac{1500}{p^2 + 100} + 14 = \frac{3000}{p^2 + 100} + 14$$

ullet Funkcija prihoda: PRIHOD = CIJENA  $\cdot$  POTRAŽNJA

$$P(p) = p \cdot N(p)$$

$$P(p) = p \cdot N(p) = p \cdot \frac{1500}{p^2 + 100}$$

$$N(p) = \frac{1500}{p^2 + 100}$$

$$P(p) = p \cdot N(p) = p \cdot \frac{1500}{p^2 + 100} = \frac{1500p}{p^2 + 100}$$

$$N(p) = \frac{1500}{p^2 + 100}$$

$$P(p) = p \cdot N(p) = p \cdot \frac{1500}{p^2 + 100} = \frac{1500p}{p^2 + 100}$$

Funkcija profita: PROFIT = PRIHOD – TROŠKOVI

$$D(p) = P(p) - T(p)$$

$$N(p) = \frac{1500}{p^2 + 100}$$

$$P(p) = p \cdot N(p) = p \cdot \frac{1500}{p^2 + 100} = \frac{1500p}{p^2 + 100}$$

Funkcija profita: PROFIT = PRIHOD – TROŠKOVI

$$D(p) = P(p) - T(p) = \frac{1500p}{p^2 + 100} - \left(\frac{3000}{p^2 + 100} + 14\right)$$

$$N(p) = \frac{1500}{p^2 + 100}$$

$$N(p) = \frac{1500}{p^2 + 100}$$
  $T(p) = \frac{3000}{p^2 + 100} + 14$ 

$$P(p) = p \cdot N(p) = p \cdot \frac{1500}{p^2 + 100} = \frac{1500p}{p^2 + 100}$$

Funkcija profita: PROFIT = PRIHOD – TROŠKOVI

$$D(p) = P(p) - T(p) = \frac{1500p}{p^2 + 100} - \left(\frac{3000}{p^2 + 100} + 14\right) =$$
$$= \frac{1500p - 3000}{p^2 + 100} - 14$$

$$N(p) = rac{1500}{p^2 + 100}$$

$$N(p) = \frac{1500}{p^2 + 100}$$
  $T(p) = \frac{3000}{p^2 + 100} + 14$ 

$$D(p) = \frac{1500p - 3000}{p^2 + 100} - 14$$

$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$

$$D'(p) =$$

$$D(p) = \frac{1500p - 3000}{p^2 + 100} - 14$$
 
$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$

$$D'(p) = -----$$

$$D(p) = \frac{1500p - 3000}{p^2 + 100} - 14$$
 
$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$

$$D'(p) = \frac{}{(p^2 + 100)^2}$$

$$D(p) = \frac{1500p - 3000}{p^2 + 100} - 14$$

$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$

$$D'(p) = \frac{1500}{(p^2 + 100)^2}$$

$$D(p) = \frac{1500p - 3000}{p^2 + 100} - 14$$

$$4 \qquad \left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$

$$D'(p) = rac{1500 \cdot }{(p^2 + 100)^2}$$

$$D(p) = \frac{1}{p^2 + 100} - 14$$

$$D(p) = \frac{1500p - 3000}{p^2 + 100} - 14 \left[ \left( \frac{u}{v} \right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2} \right]$$

$$D'(p) = rac{1500 \cdot (p^2 + 100)}{(p^2 + 100)^2}$$

$$D(p) = \frac{p^2 + 100}{p^2 + 100} - 14$$

$$D(p) = \frac{1500p - 3000}{p^2 + 100} - 14 \left[ \left( \frac{u}{v} \right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2} \right]$$

$$D'(
ho) = rac{1500 \cdot (
ho^2 + 100) -}{(
ho^2 + 100)^2}$$

$$D(p) = \frac{7}{p^2 + 100} - 14$$

$$D(p) = \frac{1500p - 3000}{p^2 + 100} - 14$$
 
$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$

$$D'(p) = \frac{1500 \cdot (p^2 + 100) - (1500p - 3000)}{(p^2 + 100)^2}$$

$$D(p) = \frac{1}{p^2 + 100} - 14$$

$$D(p) = \frac{1500p - 3000}{p^2 + 100} - 14$$
 
$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$

$$D'(p) = \frac{1500 \cdot (p^2 + 100) - (1500p - 3000) \cdot }{(p^2 + 100)^2}$$

$$D(p) = \frac{1000p}{p^2 + 100} - 14$$

$$D(p) = \frac{1500p - 3000}{p^2 + 100} - 14$$
 
$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$

$$D'(p) = rac{1500 \cdot (p^2 + 100) - (1500p - 3000) \cdot 2p}{(p^2 + 100)^2}$$

$$D(p) = \frac{1000p}{p^2 + 100} - 14$$

$$D(p) = \frac{1500p - 3000}{p^2 + 100} - 14$$
 
$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$

$$D'(p) = \frac{1500 \cdot (p^2 + 100) - (1500p - 3000) \cdot 2p}{(p^2 + 100)^2} - 0$$

$$D(p) = \frac{1000p}{p^2 + 100} - 14$$

$$D(p) = \frac{1500p - 3000}{p^2 + 100} - 14$$
 
$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$

$$D(p) = \frac{1300p}{p^2 + 100} - 14$$

$$D(p) = \frac{1500p - 3000}{p^2 + 100} - 14$$
 
$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$

$$D'(p) = \frac{1500 \cdot (p^2 + 100) - (1500p - 3000) \cdot 2p}{(p^2 + 100)^2} - 0$$

$$D'(p) = \frac{1500 \cdot (p^2 + 100)^2}{(p^2 + 100)^2}$$

$$D'(p) = \frac{}{(p^2 + 100)^2}$$

$$D(p) = \frac{1300p - 3000}{p^2 + 100} - 14$$

$$D(p) = \frac{1500p - 3000}{p^2 + 100} - 14 \left[ \left( \frac{u}{v} \right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2} \right]$$

$$D'(p) = \frac{1500 \cdot (p^2 + 100) - (1500p - 3000) \cdot 2p}{(p^2 + 100)^2} - 0$$

$$D'(p) = \frac{1500p^2 + 150000 - 3000p^2 + 6000p}{(p^2 + 100)^2}$$

$$D(p) = \frac{1300p - 3000}{p^2 + 100} - 14$$

$$D(p) = \frac{1500p - 3000}{p^2 + 100} - 14 \left[ \left( \frac{u}{v} \right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2} \right]$$

$$D'(p) = \frac{1500 \cdot (p^2 + 100) - (1500p - 3000) \cdot 2p}{(p^2 + 100)^2} - 0$$

$$D'(p) = \frac{1500p^2 + 150000 - 3000p^2 + 6000p}{(p^2 + 100)^2}$$

$$D'(p) = \frac{-1500p^2 + 6000p + 150000}{(p^2 + 100)^2}$$

$$D'(p)=0$$

$$D'(p) = 0$$

$$\frac{-1500p^2 + 6000p + 150000}{(p^2 + 100)^2} = 0$$

$$D'(p) = 0$$

$$\frac{-1500p^2 + 6000p + 150000}{(p^2 + 100)^2} = 0$$

$$-1500p^2 + 6000p + 150000 = 0$$

$$D'(p) = 0$$

$$\frac{-1500p^2 + 6000p + 150000}{(p^2 + 100)^2} = 0$$

$$-1500p^2 + 6000p + 150000 = 0 /: (-1500)$$

$$p^2 - 4p - 100 = 0$$

$$D'(p) = 0$$
  $\dfrac{-1500p^2 + 6000p + 150000}{(p^2 + 100)^2} = 0$   $-1500p^2 + 6000p + 150000 = 0 \ / : (-1500)$   $p^2 - 4p - 100 = 0$   $p_{1,2} = \dfrac{4 \pm \sqrt{16 + 400}}{2}$ 

$$D'(p) = 0$$

$$\frac{-1500p^2 + 6000p + 150000}{(p^2 + 100)^2} = 0$$

$$-1500p^2 + 6000p + 150000 = 0 /: (-1500)$$

$$p^2 - 4p - 100 = 0$$

$$p_{1,2} = \frac{4 \pm \sqrt{16 + 400}}{2}$$

$$p_1 = 12.2, \quad p_2 = -8.2$$

$$D'(p) = 0$$

$$\frac{-1500p^2 + 6000p + 150000}{(p^2 + 100)^2} = 0$$

$$-1500p^2 + 6000p + 150000 = 0 /: (-1500)$$

$$p^2 - 4p - 100 = 0$$

$$p_{1,2} = \frac{4 \pm \sqrt{16 + 400}}{2}$$

$$p_1 = 12.2, \quad p_2 = 5.2$$

$$D'(p) = 0$$

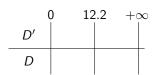
$$\frac{-1500p^2 + 6000p + 150000}{(p^2 + 100)^2} = 0$$

$$-1500p^2 + 6000p + 150000 = 0 /: (-1500)$$

$$p^2 - 4p - 100 = 0$$

$$p_{1,2} = \frac{4 \pm \sqrt{16 + 400}}{2}$$

$$p_1 = 12.2, \quad p_2 = 2$$



$$D'(p) = 0$$

$$\frac{-1500p^2 + 6000p + 150000}{(p^2 + 100)^2} = 0$$

$$-1500p^2 + 6000p + 150000 = 0 /: (-1500)$$

$$p^2 - 4p - 100 = 0$$

$$p_{1,2} = \frac{4 \pm \sqrt{16 + 400}}{2}$$

$$p_1 = 12.2, \quad p_2 = 2$$

$$D'(p) = 0$$

$$\frac{-1500p^2 + 6000p + 150000}{(p^2 + 100)^2} = 0$$

$$-1500p^2 + 6000p + 150000 = 0 /: (-1500)$$

$$p^2 - 4p - 100 = 0$$

$$p_{1,2} = \frac{4 \pm \sqrt{16 + 400}}{2}$$

$$p_1 = 12.2, \quad p_2 = 42$$

$$D'(p) = 0$$

$$\frac{-1500p^2 + 6000p + 150000}{(p^2 + 100)^2} = 0$$

$$-1500p^2 + 6000p + 150000 = 0 /: (-1500)$$

$$p^2 - 4p - 100 = 0$$

$$p_{1,2} = \frac{4 \pm \sqrt{16 + 400}}{2}$$

$$p_1 = 12.2, \quad p_2 = 3.2$$

$$\begin{array}{c|cccc}
0 & 12.2 & +\infty \\
\hline
D' & + & - \\
\hline
D & \nearrow & \\
\end{array}$$

$$D'(p) = 0$$

$$\frac{-1500p^2 + 6000p + 150000}{(p^2 + 100)^2} = 0$$

$$-1500p^2 + 6000p + 150000 = 0 /: (-1500)$$

$$p^2 - 4p - 100 = 0$$

$$p_{1,2} = \frac{4 \pm \sqrt{16 + 400}}{2}$$

$$p_1 = 12.2, \quad p_2 = 2$$

$$\begin{array}{c|cccc}
0 & 12.2 & +\infty \\
D' & + & - \\
\hline
D & \nearrow & \searrow
\end{array}$$

$$D'(p) = 0$$

$$\frac{-1500p^2 + 6000p + 150000}{(p^2 + 100)^2} = 0$$

$$-1500p^2 + 6000p + 150000 = 0 /: (-1500)$$

$$p^2 - 4p - 100 = 0$$

$$p_{1,2} = \frac{4 \pm \sqrt{16 + 400}}{2}$$

$$p_1 = 12.2, \qquad D(p) = \frac{1500p - 3000}{p^2 + 100} - 14$$

$$\begin{array}{c|ccccc}
0 & 12.2 & +0 \\
\hline
D' & + & - \\
\hline
D & \nearrow & \searrow
\end{array}$$

$$12.2) =$$

$$D'(p) = 0$$

$$\frac{-1500p^2 + 6000p + 150000}{(p^2 + 100)^2} = 0$$

$$-1500p^2 + 6000p + 150000 = 0 /: (-1500)$$

$$p^2 - 4p - 100 = 0$$

$$p_{1,2} = \frac{4 \pm \sqrt{16 + 400}}{2}$$

$$p_1 = 12.2, \qquad D(p) = \frac{1500p - 3000}{p^2 + 100} - 14$$

$$\frac{-3000}{100} - 14$$

$$D'(p) = 0$$

$$\frac{-1500p^2 + 6000p + 150000}{(p^2 + 100)^2} = 0$$

$$-1500p^2 + 6000p + 150000 = 0 /: (-1500)$$

$$p^2 - 4p - 100 = 0$$

$$p_{1,2} = \frac{4 \pm \sqrt{16 + 400}}{2}$$

$$p_1 = 12.2, \qquad D(p) = \frac{1500p - 3000}{p^2 + 100} - 14$$

$$\frac{500 \cdot 12.2 - 3000}{12.2^2 + 100} - 14 = 47.49$$

$$\frac{-1500p^2 + 6000p + 150000}{(p^2 + 100)^2} = 0$$

D'(p) = 0

$$-1500p^2 + 6000p + 150000 = 0 /: (-1500)$$

 $p^2 - 4p - 100 = 0$ 

$$p_{1,2} = rac{4 \pm \sqrt{16 + 400}}{2}$$
 $p_1 = 12.2, \quad p_2 = rac{1500p - 3000}{p^2 + 100} - 14$ 

 $D(12.2) = \frac{1500 \cdot 12.2 - 3000}{12.2^2 + 100} - 14 = 47.49$  Maksimalni profit jednak je 47.49 novčanih jedinica i ostvaruje se po cijeni od 12.2 novčanih jedinica.

$$D(p)=0$$

$$D(p) = 0$$

$$\frac{1500p - 3000}{p^2 + 100} - 14 = 0$$

$$D(p) = 0$$
 
$$\frac{1500p - 3000}{p^2 + 100} - 14 = 0 / \cdot (p^2 + 100)$$
 
$$1500p - 3000 - 14 \cdot (p^2 + 100) = 0$$

$$D(p) = 0$$

$$\frac{1500p - 3000}{p^2 + 100} - 14 = 0 / \cdot (p^2 + 100)$$

$$1500p - 3000 - 14 \cdot (p^2 + 100) = 0$$

$$-14p^2 + 1500p - 4400 = 0$$

$$D(p) = 0$$

$$\frac{1500p - 3000}{p^2 + 100} - 14 = 0 / \cdot (p^2 + 100)$$

$$1500p - 3000 - 14 \cdot (p^2 + 100) = 0$$

$$-14p^2 + 1500p - 4400 = 0 / : 2$$

$$-7p^2 + 750p - 2200 = 0$$

$$D(p) = 0$$

$$\frac{1500p - 3000}{p^2 + 100} - 14 = 0 / \cdot (p^2 + 100)$$

$$1500p - 3000 - 14 \cdot (p^2 + 100) = 0$$

$$-14p^2 + 1500p - 4400 = 0 / : 2$$

$$-7p^2 + 750p - 2200 = 0$$

$$p_{1,2} = \frac{-750 \pm \sqrt{750^2 - 4 \cdot (-7) \cdot (-2200)}}{-14}$$

$$D(p) = 0$$

$$\frac{1500p - 3000}{p^2 + 100} - 14 = 0 / \cdot (p^2 + 100)$$

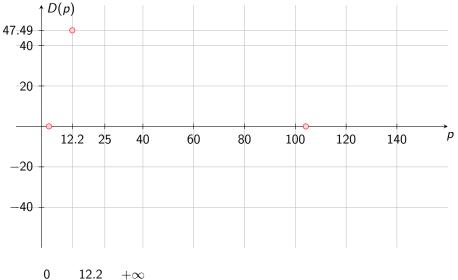
$$1500p - 3000 - 14 \cdot (p^2 + 100) = 0$$

$$-14p^2 + 1500p - 4400 = 0 / : 2$$

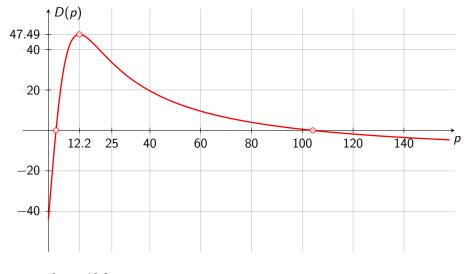
$$-7p^2 + 750p - 2200 = 0$$

$$p_{1,2} = \frac{-750 \pm \sqrt{750^2 - 4 \cdot (-7) \cdot (-2200)}}{-14}$$

$$p_1 = 3.02, \quad p_2 = 104.12$$

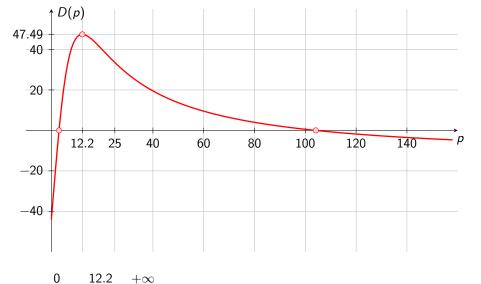


nultočke: 3.02, 104.12



$$\begin{array}{c|cccc}
0 & 12.2 & +\infty \\
D' & + & - \\
\hline
D & \nearrow & \searrow
\end{array}$$

nultočke: 3.02, 104.12



nultočke: 3.02, 104.12 Profit je pozitivan za  $p \in \langle 3.02, 104.12 \rangle$ .

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četvrti zadatak

#### Zadatak 4

Analizom tržišta zapaženo je da potrošači dnevno kupuju sljedeće količine jakni uz navedene cijene u kunama.

cijena	200	150	120	100
količina	15	22	25	30

Odredite linearnu funkciju potražnje koja najbolje aproksimira zadanu funkciju tablicom i nacrtajte na slici tablične podatke i dobivenu funkciju.

### Koristeći se dobivenim linearnim modelom procijenite:

- a) Koliko se maksimalno jakni traži dnevno? Koliko se jakni traži po cijeni od 110 kn? Po kojoj cijeni se traži 35 jakni dnevno?
- b) Za koliko se treba smanjiti cijena jakne da bi se prodale dvije jakne više dnevno ako se jakne prodaju po cijeni od 110 kn.
- c) Za koliko posto se smanji potražnja za jaknama ako se cijena na razini 110 kn poveća za 1 %? Što se u tom slučaju dogodi s ukupnim prihodom?

$p \longrightarrow$	cijena	200	150	120	100
$q \longrightarrow$	količina	15	22	25	30

• Linearna funkcija potražnje: q = ap + b

<i>'</i>	cijena	200	150	120	100
$q \longrightarrow 1$	količina	15	22	25	30

- Linearna funkcija potražnje: q = ap + b
- Na početku promatramo sustav linearnih jednadžbi AX = B.

$p \longrightarrow$	cijena	200	150	120	100
$q \longrightarrow$	količina	15	22	25	30

- Linearna funkcija potražnje: q = ap + b
- Na početku promatramo sustav linearnih jednadžbi AX = B.

$$A = \begin{bmatrix} 200 & 1 \\ 150 & 1 \\ 120 & 1 \\ 100 & 1 \end{bmatrix}$$

$p \longrightarrow$	cijena	200	150	120	100
$q \longrightarrow [$	količina	15	22	25	30

- Linearna funkcija potražnje: q = ap + b
- Na početku promatramo sustav linearnih jednadžbi AX = B.

$$A = \begin{bmatrix} 200 & 1 \\ 150 & 1 \\ 120 & 1 \\ 100 & 1 \end{bmatrix} \qquad X = \begin{bmatrix} a \\ b \end{bmatrix}$$

<b>p</b> ────────────── cijena	200	150	120	100
<b>q</b> → količina	15	22	25	30

- Linearna funkcija potražnje: q = ap + b
- Na početku promatramo sustav linearnih jednadžbi AX = B.

$$A = \begin{bmatrix} 200 & 1 \\ 150 & 1 \\ 120 & 1 \\ 100 & 1 \end{bmatrix} \qquad X = \begin{bmatrix} a \\ b \end{bmatrix} \qquad B = \begin{bmatrix} 15 \\ 22 \\ 25 \\ 30 \end{bmatrix}$$

$p \longrightarrow$	cijena	200	150	120	100
$q \longrightarrow$	količina	15	22	25	30

- Linearna funkcija potražnje: q = ap + b
- Na početku promatramo sustav linearnih jednadžbi AX = B.

$$A = \begin{bmatrix} 200 & 1 \\ 150 & 1 \\ 120 & 1 \\ 100 & 1 \end{bmatrix} \qquad X = \begin{bmatrix} a \\ b \end{bmatrix} \qquad B = \begin{bmatrix} 15 \\ 22 \\ 25 \\ 30 \end{bmatrix}$$

• Sustav AX = B je preodređen i općenito nema rješenja.

$P \longrightarrow$	cijena	200	150	120	100
$q \longrightarrow$	količina	15	22	25	30

- Linearna funkcija potražnje: q = ap + b
- Na početku promatramo sustav linearnih jednadžbi AX = B.

$$A = \begin{bmatrix} 200 & 1 \\ 150 & 1 \\ 120 & 1 \\ 100 & 1 \end{bmatrix} \qquad X = \begin{bmatrix} a \\ b \end{bmatrix} \qquad B = \begin{bmatrix} 15 \\ 22 \\ 25 \\ 30 \end{bmatrix}$$

- Sustav AX = B je preodređen i općenito nema rješenja.
- ullet Umjesto sustava AX=B rješavamo sustav normalnih jednadžbi

$$A^TAX = A^TB.$$

# $A^T A X = A^T B$

 $A^TA =$ 

$$A^TAX = A^TB$$

$$A^TA =$$

$$\begin{bmatrix} 200 & 1 \\ 150 & 1 \\ 120 & 1 \\ 100 & 1 \end{bmatrix}$$

## $A^TAX = A^TB$

$$A^{T}A = \begin{bmatrix} 200 & 150 & 120 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 200 & 1 \\ 150 & 1 \\ 120 & 1 \\ 100 & 1 \end{bmatrix}$$

$$A^TAX = A^TB$$

$$A^{T}A = \begin{bmatrix} 200 & 150 & 120 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 200 & 1 \\ 150 & 1 \\ 120 & 1 \\ 100 & 1 \end{bmatrix}$$

$$(2,4)$$

# $A^TAX = A^TB$

$$A^{T}A = \begin{bmatrix} 200 & 150 & 120 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 200 & 1 \\ 150 & 1 \\ 120 & 1 \\ 100 & 1 \end{bmatrix}$$

$$(2,4) (4,2)$$

$$A^TAX = A^TB$$

$$A^{\mathsf{T}}A = \begin{bmatrix} 200 & 150 & 120 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 200 & 1 \\ 150 & 1 \\ 120 & 1 \\ 100 & 1 \end{bmatrix} = \begin{bmatrix} 200 & 1 \\ 150 & 1 \\ 100 & 1 \end{bmatrix}$$

$$A^TAX = A^TB$$

$$A^{T}A = \begin{bmatrix} 200 & 150 & 120 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 200 & 1 \\ 150 & 1 \\ 120 & 1 \\ 100 & 1 \end{bmatrix} = \begin{bmatrix} 86\,900 \\ \end{bmatrix}$$

$$A^TAX = A^TB$$

$$A^{T}A = \begin{bmatrix} 200 & 150 & 120 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 200 & 1 \\ 150 & 1 \\ 120 & 1 \\ 100 & 1 \end{bmatrix} = \begin{bmatrix} 86\,900 & 570 \\ \end{bmatrix}$$

$$A^TAX = A^TB$$

$$A^{T}A = \begin{bmatrix} 200 & 150 & 120 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 200 & 1 \\ 150 & 1 \\ 120 & 1 \\ 100 & 1 \end{bmatrix} = \begin{bmatrix} 86\,900 & 570 \\ 570 & \end{bmatrix}$$

$$(2,4) \quad (4,2)$$

$$A^TAX = A^TB$$

$$A^{T}A = \begin{bmatrix} 200 & 150 & 120 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 200 & 1 \\ 150 & 1 \\ 120 & 1 \\ 100 & 1 \end{bmatrix} = \begin{bmatrix} 86\,900 & 570 \\ 570 & 4 \end{bmatrix}$$

$$(2,4) (4,2)$$

$$A^TAX = A^TB$$

$$A^{T}A = \begin{bmatrix} 200 & 150 & 120 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 200 & 1 \\ 150 & 1 \\ 120 & 1 \\ 100 & 1 \end{bmatrix} = \begin{bmatrix} 86\,900 & 570 \\ 570 & 4 \end{bmatrix}$$

$$(2,4) (4,2)$$

$$A^TB =$$

$$A^TAX = A^TB$$

$$A^{T}A = \begin{bmatrix} 200 & 150 & 120 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 200 & 1 \\ 150 & 1 \\ 120 & 1 \\ 100 & 1 \end{bmatrix} = \begin{bmatrix} 86\,900 & 570 \\ 570 & 4 \end{bmatrix}$$

$$(2,4) (4,2)$$

$$A^T B = \begin{bmatrix} 200 & 150 & 120 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$A^TAX = A^TB$$

$$A^{T}A = \begin{bmatrix} 200 & 150 & 120 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 200 & 1 \\ 150 & 1 \\ 120 & 1 \\ 100 & 1 \end{bmatrix} = \begin{bmatrix} 86\,900 & 570 \\ 570 & 4 \end{bmatrix}$$

$$(2, 4) (4, 2)$$

$$A^TB = \begin{bmatrix} 200 & 150 & 120 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 15 \\ 22 \\ 25 \\ 30 \end{bmatrix}$$

$$A^TAX = A^TB$$

$$A^{T}A = \begin{bmatrix} 200 & 150 & 120 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 200 & 1 \\ 150 & 1 \\ 120 & 1 \\ 100 & 1 \end{bmatrix} = \begin{bmatrix} 86\,900 & 570 \\ 570 & 4 \end{bmatrix}$$

$$(2,4) (4,2)$$

$$=$$

$$A^{T}B = \begin{bmatrix} 200 & 150 & 120 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 15 \\ 22 \\ 25 \\ 30 \end{bmatrix}$$

$$A^TAX = A^TB$$

$$A^{T}A = \begin{bmatrix} 200 & 150 & 120 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 200 & 1 \\ 150 & 1 \\ 120 & 1 \\ 100 & 1 \end{bmatrix} = \begin{bmatrix} 86\,900 & 570 \\ 570 & 4 \end{bmatrix}$$

(2,4)(4,1)

$$A^{T}B = \begin{bmatrix} 200 & 150 & 120 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 15 \\ 22 \\ 25 \\ 30 \end{bmatrix}$$

$$A^TAX = A^TB$$

$$A^{T}A = \begin{bmatrix} 200 & 150 & 120 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 200 & 1 \\ 150 & 1 \\ 120 & 1 \\ 100 & 1 \end{bmatrix} = \begin{bmatrix} 86\,900 & 570 \\ 570 & 4 \end{bmatrix}$$

$$(2,4) (4,2)$$

$$A^{T}B = \begin{bmatrix} 200 & 150 & 120 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 15 \\ 22 \\ 25 \\ 30 \end{bmatrix} = \begin{bmatrix} & & \end{bmatrix}$$

$$A^TAX = A^TB$$

$$\begin{bmatrix} 200 & 1 \\ 150 & 1 \end{bmatrix} = \begin{bmatrix} 86\,900 & 570 \end{bmatrix}$$

$$A^{T}A = \begin{bmatrix} 200 & 150 & 120 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 200 & 1 \\ 150 & 1 \\ 120 & 1 \\ 100 & 1 \end{bmatrix} = \begin{bmatrix} 86\,900 & 570 \\ 570 & 4 \end{bmatrix}$$

$$(2, 4) \quad (4, 2)$$

$$A^{T}B = \begin{bmatrix} 200 & 150 & 120 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 15 \\ 22 \\ 25 \\ 30 \end{bmatrix} = \begin{bmatrix} 12300 \end{bmatrix}$$

$$A^TAX = A^TB$$

$$A^{T}A = \begin{bmatrix} 200 & 150 & 120 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 200 & 1 \\ 150 & 1 \\ 120 & 1 \\ 100 & 1 \end{bmatrix} = \begin{bmatrix} 86\,900 & 570 \\ 570 & 4 \end{bmatrix}$$

$$(2,4) (4,2)$$

$$A^{\mathsf{T}}B = \begin{bmatrix} 200 & 150 & 120 & 100 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 15 \\ 22 \\ 25 \\ 30 \end{bmatrix} = \begin{bmatrix} 12300 \\ 92 \end{bmatrix}$$

 $A^{\mathsf{T}}A = \begin{bmatrix} 86\,900 & 570 \\ 570 & 4 \end{bmatrix} \quad A^{\mathsf{T}}B = \begin{bmatrix} 12\,300 \\ 92 \end{bmatrix}$ 

 $A^TAX = A^TB$ 

 $A^{\mathsf{T}}A = \begin{bmatrix} 86\,900 & 570 \\ 570 & 4 \end{bmatrix} \quad A^{\mathsf{T}}B = \begin{bmatrix} 12\,300 \\ 92 \end{bmatrix}$ 

 $A^TAX = A^TB$ 

 $A^TAX = A^TB$ 

 $A^T A = \begin{bmatrix} 86\,900 & 570 \\ 570 & 4 \end{bmatrix} \quad A^T B = \begin{bmatrix} 12\,300 \\ 92 \end{bmatrix}$ 

 $A^{T}AX = A^{T}B$   $(A^{T}A)^{-1} \cdot / A^{T}AX = A^{T}B$ 

$$A^{T}A = \begin{bmatrix} 86\,900 & 570 \\ 570 & 4 \end{bmatrix} \quad A^{T}B = \begin{bmatrix} 12\,300 \\ 92 \end{bmatrix}$$
$$(A^{T}A)^{-1} \cdot / A^{T}AX = A^{T}B$$
$$X = \begin{bmatrix} A^{T}AX = A^{T}B \\ X = A^{T}B \end{bmatrix}$$

 $A^{\mathsf{T}}A = \begin{bmatrix} 86\,900 & 570 \\ 570 & 4 \end{bmatrix} \quad A^{\mathsf{T}}B = \begin{bmatrix} 12\,300 \\ 92 \end{bmatrix}$ 

$$A^{T}A = \begin{bmatrix} 86\,900 & 570 \\ 570 & 4 \end{bmatrix} \quad A^{T}B = \begin{bmatrix} 12\,300 \\ 92 \end{bmatrix}$$
$$(A^{T}AX)^{-1} \cdot / A^{T}AX = A^{T}B$$
$$X = (A^{T}B)$$

 $A^{\mathsf{T}}A = \begin{bmatrix} 86\,900 & 570 \\ 570 & 4 \end{bmatrix} \quad A^{\mathsf{T}}B = \begin{bmatrix} 12\,300 \\ 92 \end{bmatrix}$ 

 $(A^{T}A)^{-1} \cdot / A^{T}AX = A^{T}B$  $X = (A^{T}A)^{-1} \cdot (A^{T}B)$ 

 $A^{T}A = \begin{vmatrix} 86\,900 & 570 \\ 570 & 4 \end{vmatrix} \quad A^{T}B = \begin{vmatrix} 12\,300 \\ 92 \end{vmatrix}$ 

 $\det (A^T A) =$ 

 $A^T A X = A^T B$ 

 $(A^{T}A)^{-1} \cdot / A^{T}AX = A^{T}B$  $X = (A^{T}A)^{-1} \cdot (A^{T}B)$ 

 $A^{T}A = \begin{vmatrix} 86\,900 & 570 \\ 570 & 4 \end{vmatrix} \quad A^{T}B = \begin{vmatrix} 12\,300 \\ 92 \end{vmatrix}$ 

 $\det (A^T A) = 86\,900 \cdot 4 - 570 \cdot 570$ 

 $(A^{T}A)^{-1} \cdot / A^{T}AX = A^{T}B$  $X = (A^{T}A)^{-1} \cdot (A^{T}B)$ 

 $\det (A^{T}A) = 86\,900 \cdot 4 - 570 \cdot 570 = 22\,700 \left| (A^{T}A)^{-1} \cdot \middle/ A^{T}AX = A^{T}B \right|$   $X = (A^{T}A)^{-1} \cdot (A^{T}B)$ 

 $A^{T}A = \begin{vmatrix} 86\,900 & 570 \\ 570 & 4 \end{vmatrix} \qquad A^{T}B = \begin{vmatrix} 12\,300 \\ 92 \end{vmatrix}$ 

 $\det (A^{T}A) = 86\,900 \cdot 4 - 570 \cdot 570 = 22\,700 \left| (A^{T}A)^{-1} \cdot \middle/ A^{T}AX = A^{T}B \right|$   $(A^{T}A)^{-1} = X = (A^{T}A)^{-1} \cdot (A^{T}B)$ 

 $A^{T}A = \begin{vmatrix} 86\,900 & 570 \\ 570 & 4 \end{vmatrix} \quad A^{T}B = \begin{vmatrix} 12\,300 \\ 92 \end{vmatrix}$ 

$$\det (A^{T}A) = 86\,900 \cdot 4 - 570 \cdot 570 = 22\,700 \left| (A^{T}A)^{-1} \cdot \middle/ A^{T}AX = A^{T}B \right|$$
$$(A^{T}A)^{-1} = \frac{1}{22\,700} \left[ \qquad \qquad \right] \qquad X = (A^{T}A)^{-1} \cdot (A^{T}B)$$

 $\det (A^{T}A) = 86\,900 \cdot 4 - 570 \cdot 570 = 22\,700 \left| (A^{T}A)^{-1} \cdot \middle/ A^{T}AX = A^{T}B \right|$ 

 $A^{T}A = \begin{vmatrix} 86\,900 & 570 \\ 570 & 4 \end{vmatrix} \quad A^{T}B = \begin{vmatrix} 12\,300 \\ 92 \end{vmatrix}$ 

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 $\det (A^{T}A) = 86\,900 \cdot 4 - 570 \cdot 570 = 22\,700 \, \Big| \big(A^{T}A\big)^{-1} \cdot \Big/ A^{T}AX = A^{T}B$ 

 $A^{T}A = \begin{vmatrix} 86\,900 & 570 \\ 570 & 4 \end{vmatrix} \quad A^{T}B = \begin{vmatrix} 12\,300 \\ 92 \end{vmatrix}$ 

 $(A^T A)^{-1} = \frac{1}{22700} \begin{vmatrix} 4 & -570 \\ -570 & 86900 \end{vmatrix}$ 

 $A^TAX = A^TB$ 

 $X = (A^T A)^{-1} \cdot (A^T B)$ 

$$(A^T A)^{-1} = \frac{1}{22700} \begin{bmatrix} 4 & -570 \\ -570 & 86900 \end{bmatrix}$$
  $X = (A^T A)^{-1} \cdot (A^T B)$ 

 $\det (A^T A) = 86\,900 \cdot 4 - 570 \cdot 570 = 22\,700 \left| \left( A^T A \right)^{-1} \cdot \middle/ A^T A X = A^T B \right|$ 

 $A^{T}A = \begin{vmatrix} 86\,900 & 570 \\ 570 & 4 \end{vmatrix} \quad A^{T}B = \begin{vmatrix} 12\,300 \\ 92 \end{vmatrix}$ 

X =

$$X = \frac{1}{22700} \begin{bmatrix} 4 & -570 \\ -570 & 86900 \end{bmatrix}$$

 $\det (A^{T}A) = 86\,900 \cdot 4 - 570 \cdot 570 = 22\,700 \left| (A^{T}A)^{-1} \cdot \middle/ A^{T}AX = A^{T}B \right|$ 

 $A^{T}A = \begin{vmatrix} 86\,900 & 570 \\ 570 & 4 \end{vmatrix} \quad A^{T}B = \begin{vmatrix} 12\,300 \\ 92 \end{vmatrix}$ 

 $(A^T A)^{-1} = \frac{1}{22700} \begin{vmatrix} 4 & -570 \\ -570 & 86900 \end{vmatrix}$ 

 $A^TAX = A^TB$ 

 $X = (A^T A)^{-1} \cdot (A^T B)$ 

$$(A^{T}A)^{-1} = \frac{1}{22700} \begin{bmatrix} 4 & -570 \\ -570 & 86900 \end{bmatrix}$$

$$X = (A^{T}A)^{-1} \cdot (A^{T}B)$$

$$X = \frac{1}{22700} \begin{bmatrix} 4 & -570 \\ -570 & 86900 \end{bmatrix} \cdot \begin{bmatrix} 12300 \\ 92 \end{bmatrix}$$

 $\det (A^{T}A) = 86\,900 \cdot 4 - 570 \cdot 570 = 22\,700 \, \Big| \big(A^{T}A\big)^{-1} \cdot \Big/ A^{T}AX = A^{T}B$ 

 $A^{T}A = \begin{vmatrix} 86\,900 & 570 \\ 570 & 4 \end{vmatrix} \quad A^{T}B = \begin{vmatrix} 12\,300 \\ 92 \end{vmatrix}$ 

$$(A^{T}A)^{-1} = \frac{1}{22700} \begin{bmatrix} 4 & -570 \\ -570 & 86900 \end{bmatrix}$$
  $X = (A^{T}A)^{-1} \cdot (A^{T}B)$ 

 $\det (A^{T}A) = 86\,900 \cdot 4 - 570 \cdot 570 = 22\,700 \left| \left( A^{T}A \right)^{-1} \cdot \middle/ A^{T}AX = A^{T}B \right|$ 

 $A^{T}A = \begin{vmatrix} 86\,900 & 570 \\ 570 & 4 \end{vmatrix} \quad A^{T}B = \begin{vmatrix} 12\,300 \\ 92 \end{vmatrix}$ 

 $X = \frac{1}{22700} \begin{vmatrix} 4 & -570 \\ -570 & 86900 \end{vmatrix} \cdot \begin{vmatrix} 12300 \\ 92 \end{vmatrix}$ 

 $A^TAX = A^TB$ 

$$X = \frac{1}{22700}$$

$$(A^{T}A)^{-1} = \frac{1}{22700} \begin{bmatrix} 4 & -570 \\ -570 & 86900 \end{bmatrix} \qquad X = (A^{T}A)^{-1} \cdot (A^{T}B)$$
(2,2)

 $\det (A^{T}A) = 86\,900 \cdot 4 - 570 \cdot 570 = 22\,700 \, \left| \left( A^{T}A \right)^{-1} \cdot \middle/ A^{T}AX = A^{T}B \right|$ 

 $A^{T}A = \begin{vmatrix} 86\,900 & 570 \\ 570 & 4 \end{vmatrix} \quad A^{T}B = \begin{vmatrix} 12\,300 \\ 92 \end{vmatrix}$ 

 $X = \frac{1}{22700} \begin{vmatrix} 4 & -570 \\ -570 & 86900 \end{vmatrix} \cdot \begin{vmatrix} 12300 \\ 92 \end{vmatrix}$ 

 $X = \frac{1}{22700}$ 22/25

$$(A^{T}A)^{-1} = \frac{1}{22700} \begin{bmatrix} 4 & -570 \\ -570 & 86900 \end{bmatrix}$$

$$X = (A^{T}A)^{-1} \cdot (A^{T}B)$$

$$(2,2) (2,1)$$

$$X = \frac{1}{22700} \begin{bmatrix} 4 & -570 \\ -570 & 86900 \end{bmatrix} \cdot \begin{bmatrix} 12300 \\ 92 \end{bmatrix}$$

 $\det (A^{T}A) = 86\,900 \cdot 4 - 570 \cdot 570 = 22\,700 \, \left| \left( A^{T}A \right)^{-1} \cdot \middle/ A^{T}AX = A^{T}B \right|$ 

 $A^{T}A = \begin{vmatrix} 86\,900 & 570 \\ 570 & 4 \end{vmatrix} \quad A^{T}B = \begin{vmatrix} 12\,300 \\ 92 \end{vmatrix}$ 

 $X = \frac{1}{22700}$ 

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$$(A^{T}A)^{-1} = \frac{1}{22700} \begin{bmatrix} 4 & -570 \\ -570 & 86900 \end{bmatrix}$$

$$= (2,2) (2,1)$$

$$X = (A^{T}A)^{-1} \cdot (A^{T}B)$$

$$= (2,2) (2,1)$$

$$X = \frac{1}{22700} \begin{bmatrix} 4 & -570 \\ -570 & 86900 \end{bmatrix} \cdot \begin{bmatrix} 12300 \\ 92 \end{bmatrix}$$

 $\det (A^{T}A) = 86\,900 \cdot 4 - 570 \cdot 570 = 22\,700 \, \big| \big(A^{T}A\big)^{-1} \cdot \Big/ A^{T}AX = A^{T}B$ 

 $A^{T}A = \begin{vmatrix} 86\,900 & 570 \\ 570 & 4 \end{vmatrix} \quad A^{T}B = \begin{vmatrix} 12\,300 \\ 92 \end{vmatrix}$ 

 $X = \frac{1}{22700} \mid$ 

 $A^TAX = A^TB$ 

$$(A^{T}A)^{-1} = \frac{1}{22700} \begin{bmatrix} 4 & -570 \\ -570 & 86900 \end{bmatrix}$$

$$= (2,2) (2,1)$$

$$X = (A^{T}A)^{-1} \cdot (A^{T}B)$$

$$= (2,2) (2,1)$$

$$X = \frac{1}{22700} \begin{bmatrix} 4 & -570 \\ -570 & 86900 \end{bmatrix} \cdot \begin{bmatrix} 12300 \\ 92 \end{bmatrix}$$

 $\det (A^{T}A) = 86\,900 \cdot 4 - 570 \cdot 570 = 22\,700 \, \big| \big(A^{T}A\big)^{-1} \cdot \Big/ A^{T}AX = A^{T}B$ 

 $A^{T}A = \begin{vmatrix} 86\,900 & 570 \\ 570 & 4 \end{vmatrix} \quad A^{T}B = \begin{vmatrix} 12\,300 \\ 92 \end{vmatrix}$ 

 $X = \frac{1}{22700} \left| -3240 \right|$ 

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$$(A^{T}A)^{-1} = \frac{1}{22700} \begin{bmatrix} 4 & -570 \\ -570 & 86900 \end{bmatrix} \qquad X = (A^{T}A)^{-1} \cdot (A^{T}B)$$

$$= (2,2) (2,1)$$

 $\det (A^{T}A) = 86\,900 \cdot 4 - 570 \cdot 570 = 22\,700 \, \big| \big(A^{T}A\big)^{-1} \cdot \Big/ A^{T}AX = A^{T}B$ 

 $A^{T}A = \begin{vmatrix} 86\,900 & 570 \\ 570 & 4 \end{vmatrix} \quad A^{T}B = \begin{vmatrix} 12\,300 \\ 92 \end{vmatrix}$ 

 $X = \frac{1}{22700} \begin{vmatrix} 4 & -570 \\ -570 & 86900 \end{vmatrix} \cdot \begin{vmatrix} 12300 \\ 92 \end{vmatrix}$ 

 $X = \frac{1}{22700} \begin{vmatrix} -3240 \\ 983800 \end{vmatrix}$ 

22/25

$$\det (A^{T}A) = 86\,900 \cdot 4 - 570 \cdot 570 = 22\,700 \begin{vmatrix} (A^{T}A)^{-1} \cdot /A^{T}AX = A^{T}B \\ (A^{T}A)^{-1} = \frac{1}{22\,700} \begin{bmatrix} 4 & -570 \\ -570 & 86\,900 \end{bmatrix} \begin{vmatrix} X = (A^{T}A)^{-1} \cdot (A^{T}B) \end{vmatrix}$$

$$X = \frac{1}{22\,700} \begin{bmatrix} 4 & -570 \\ -570 & 86\,900 \end{bmatrix} \cdot \begin{bmatrix} 12\,300 \\ 92 \end{bmatrix}$$

$$X = \frac{1}{22700} \begin{bmatrix} -3240 \\ 983800 \end{bmatrix} = \begin{bmatrix} -\frac{162}{1135} \\ \frac{9838}{227} \end{bmatrix}$$

$$(A^{T}A)^{-1} = \frac{1}{22700} \begin{bmatrix} 4 & -570 \\ -570 & 86900 \end{bmatrix}$$

$$= (2,2)(2,1)$$

$$X = \frac{1}{22700} \begin{bmatrix} 4 & -570 \\ -570 & 86900 \end{bmatrix} \cdot \begin{bmatrix} 12300 \\ 92 \end{bmatrix}$$

$$X = \frac{1}{22700} \begin{bmatrix} -3240 \\ 983800 \end{bmatrix} = \begin{bmatrix} -\frac{162}{1135} \\ \frac{9838}{227} \\ \frac{9838}{227} \\ \end{bmatrix}$$

 $\det (A^T A) = 86\,900 \cdot 4 - 570 \cdot 570 = 22\,700$ 

 $A^TAX = A^TB$ 

22/25

 $\left| \left( A^T A \right)^{-1} \cdot \middle/ A^T A X = A^T B \right|$ 

$$\det (A^{T}A) = 86\,900 \cdot 4 - 570 \cdot 570 = 22\,700 \begin{vmatrix} (A^{T}A)^{-1} \cdot /A^{T}AX = A^{T}B \\ (A^{T}A)^{-1} = \frac{1}{22\,700} \begin{bmatrix} 4 & -570 \\ -570 & 86\,900 \end{bmatrix} \begin{vmatrix} X = (A^{T}A)^{-1} \cdot (A^{T}B) \end{vmatrix}$$

$$X = \frac{1}{22\,700} \begin{bmatrix} 4 & -570 \\ -570 & 86\,900 \end{bmatrix} \cdot \begin{bmatrix} 12\,300 \\ 92 \end{bmatrix}$$

$$X = \frac{1}{22\,700} \begin{bmatrix} -3240 \\ 983\,800 \end{bmatrix} = \begin{bmatrix} -\frac{162}{1135} & -\frac{1}{22} \\ \frac{9838}{227} & -\frac{1}{22} \\ \frac{983$$

 $A^TAX = A^TB$ 

$$A^{T}A = \begin{bmatrix} 86\,900 & 570 \\ 570 & 4 \end{bmatrix} \quad A^{T}B = \begin{bmatrix} 12\,300 \\ 92 \end{bmatrix}$$

$$\det (A^{T}A) = 86\,900 \cdot 4 - 570 \cdot 570 = 22\,700$$

$$(A^{T}A)^{-1} = \frac{1}{22\,700} \begin{bmatrix} 4 & -570 \\ -570 & 86\,900 \end{bmatrix} \quad (A^{T}A)^{-1} \cdot /A^{T}AX = A^{T}B$$

$$X = \frac{1}{22\,700} \begin{bmatrix} 4 & -570 \\ -570 & 86\,900 \end{bmatrix} \cdot \begin{bmatrix} 12\,300 \\ 92 \end{bmatrix} \quad q = ap + b$$

$$X = \frac{1}{22\,700} \begin{bmatrix} 4 & -570 \\ -570 & 86\,900 \end{bmatrix} \cdot \begin{bmatrix} 12\,300 \\ 92 \end{bmatrix} \quad q = -\frac{162}{1135}p + \frac{9838}{227}$$

$$X = \frac{1}{22\,700} \begin{bmatrix} -3240 \\ 983\,800 \end{bmatrix} = \begin{bmatrix} -\frac{162}{1135} \\ \frac{9838}{227} \\ -\frac{1}{22} \end{bmatrix} \quad b$$

 $A^TAX = A^TB$ 

$$A^{T}A = \begin{bmatrix} 86\,900 & 570 \\ 570 & 4 \end{bmatrix} \quad A^{T}B = \begin{bmatrix} 12\,300 \\ 92 \end{bmatrix}$$

$$\det (A^{T}A) = 86\,900 \cdot 4 - 570 \cdot 570 = 22\,700$$

$$(A^{T}A)^{-1} = \frac{1}{22\,700} \begin{bmatrix} 4 & -570 \\ -570 & 86\,900 \end{bmatrix} \quad (A^{T}A)^{-1} \cdot /A^{T}AX = A^{T}B$$

$$X = \frac{1}{22\,700} \begin{bmatrix} 4 & -570 \\ -570 & 86\,900 \end{bmatrix} \cdot \begin{bmatrix} 12\,300 \\ 92 \end{bmatrix} \quad q = ap + b$$

$$X = \frac{1}{22\,700} \begin{bmatrix} 4 & -570 \\ -570 & 86\,900 \end{bmatrix} \cdot \begin{bmatrix} 12\,300 \\ 92 \end{bmatrix} \quad q = -\frac{162}{1135}p + \frac{9838}{227}$$

$$X = \frac{1}{22\,700} \begin{bmatrix} -3240 \\ 983\,800 \end{bmatrix} = \begin{bmatrix} -\frac{162}{1135} \\ \frac{9838}{227} \\ -\frac{1}{22} \end{bmatrix} \quad b$$

 $A^TAX = A^TB$ 

$$A^{T}A = \begin{bmatrix} 86\,900 & 570 \\ 570 & 4 \end{bmatrix} \quad A^{T}B = \begin{bmatrix} 12\,300 \\ 92 \end{bmatrix}$$

$$\det (A^{T}A) = 86\,900 \cdot 4 - 570 \cdot 570 = 22\,700 \quad (A^{T}A)^{-1} \cdot /A^{T}AX = A^{T}B$$

$$(A^{T}A)^{-1} = \frac{1}{22\,700} \begin{bmatrix} 4 & -570 \\ -570 & 86\,900 \end{bmatrix} \quad X = (A^{T}A)^{-1} \cdot (A^{T}B)$$

$$X = \frac{1}{22\,700} \begin{bmatrix} 4 & -570 \\ -570 & 86\,900 \end{bmatrix} \cdot \begin{bmatrix} 12\,300 \\ 92 \end{bmatrix} \quad q = ap + b$$

$$X = \frac{1}{22\,700} \begin{bmatrix} 4 & -570 \\ -570 & 86\,900 \end{bmatrix} \cdot \begin{bmatrix} 12\,300 \\ 92 \end{bmatrix} \quad q = -\frac{162}{1135}p + \frac{9838}{227}$$

$$X = \frac{1}{22\,700} \begin{bmatrix} -3240 \\ 983\,800 \end{bmatrix} = \begin{bmatrix} -\frac{162}{1135} & a \\ \frac{9838}{227} & b \end{bmatrix} \quad a \approx -0.1427$$

$$b \approx 43.3392$$

$$= -\frac{162}{1135}p + \frac{9838}{227}$$

a) Dnevno se maksimalno traži

$$=-\frac{162}{1135}p+\frac{9838}{227}$$

$$-\frac{162}{1135}\rho + \frac{9838}{227}$$

$$q(110) = -\frac{162}{1135} \cdot 110 + \frac{9838}{227}$$

$$-\frac{162}{1135}p + \frac{9838}{227}$$

$$q(110) = -rac{162}{1135} \cdot 110 + rac{9838}{227} = rac{6274}{227}$$

$$q = -\frac{162}{1135}p + \frac{9838}{227}$$

$$q(110) = -rac{162}{1135} \cdot 110 + rac{9838}{227} = rac{6274}{227} pprox 27.64$$

$$q = -\frac{162}{1135}p + \frac{9838}{227}$$

$$q(110) = -rac{162}{1135} \cdot 110 + rac{9838}{227} = rac{6274}{227} pprox 27.64$$

$$q = -\frac{162}{1135}p + \frac{9838}{227}$$

$$q(110) = -rac{162}{1135} \cdot 110 + rac{9838}{227} = rac{6274}{227} pprox 27.64$$

$$-\frac{162}{1135}p + \frac{9838}{227} = 35$$

$$q = -\frac{162}{1135}p + \frac{9838}{227}$$

$$q(110) = -rac{162}{1135} \cdot 110 + rac{9838}{227} = rac{6274}{227} pprox 27.64$$

$$-\frac{162}{1135}p + \frac{9838}{227} = 35 / \cdot 1135$$
$$-162p + 49190 = 39725$$

$$q = -\frac{162}{1135}p + \frac{9838}{227}$$

$$q(110) = -rac{162}{1135} \cdot 110 + rac{9838}{227} = rac{6274}{227} pprox 27.64$$

$$-\frac{162}{1135}p + \frac{9838}{227} = 35 / \cdot 1135$$
$$-162p + 49190 = 39725$$
$$-162p = -9465$$

$$q = -\frac{162}{1135}p + \frac{9838}{227}$$

$$q(110) = -\frac{162}{1135} \cdot 110 + \frac{9838}{227} = \frac{6274}{227} \approx 27.64$$

$$-\frac{162}{1135}p + \frac{9838}{227} = 35 / \cdot 1135$$
$$-162p + 49190 = 39725$$
$$-162p = -9465$$
$$p = \frac{3155}{54}$$

$$q = -\frac{162}{1135}p + \frac{9838}{227}$$

$$q(110) = -\frac{162}{1135} \cdot 110 + \frac{9838}{227} = \frac{6274}{227} \approx 27.64$$

$$-\frac{162}{1135}p + \frac{9838}{227} = 35 / \cdot 1135$$
$$-162p + 49190 = 39725$$
$$-162p = -9465$$
$$p = \frac{3155}{54} \approx 58.43$$

$$q = -\frac{162}{1135}p + \frac{9838}{227}$$

$$q(110) = -\frac{162}{1135} \cdot 110 + \frac{9838}{227} = \frac{6274}{227} \approx 27.64$$

Po cijeni od 110 kn traži se 27.64 jakni, dakle oko 28 jakni.

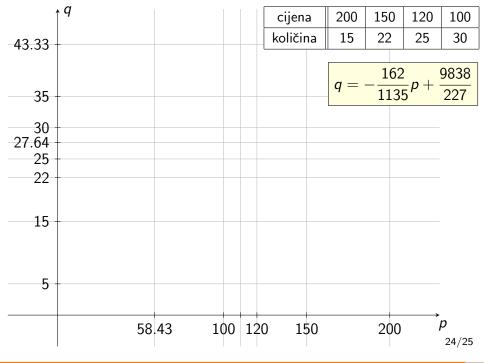
$$-\frac{162}{1135}p + \frac{9838}{227} = 35 / \cdot 1135$$
$$-162p + 49190 = 39725$$

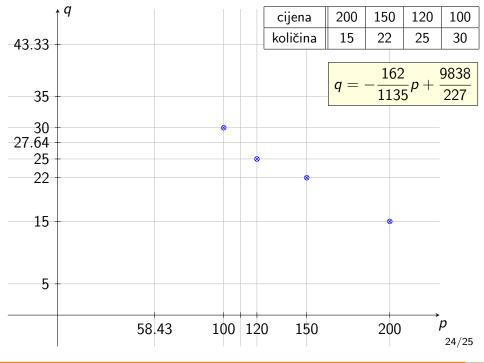
-162p = -9465

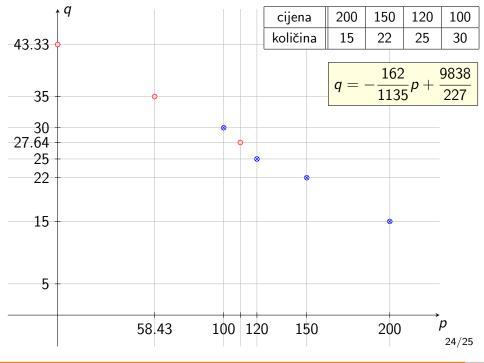
$$-162p + 49\,190 = 39\,725$$

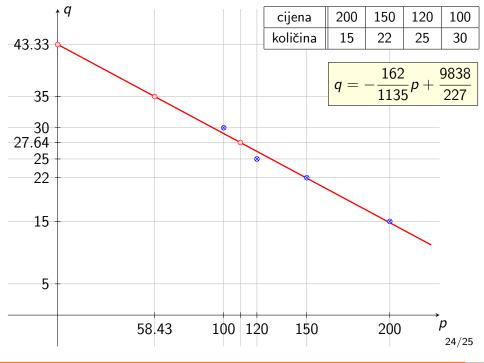
$$p = \frac{3155}{54} \approx 58.43$$

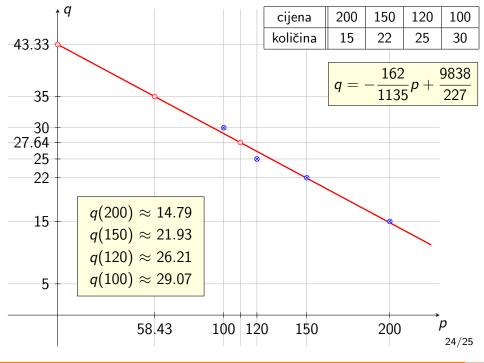
Po cijeni od 58 kuna i 43 lipe dnevno se traži 35 jakni.











b)

9838 227

25/25

 $-\frac{162}{1135}p +$ 

b)  $\Delta q = 2$ 

9838

227

 $q = -rac{162}{1135}p +$ 

$$rac{\Delta q}{\Delta p} = -rac{162}{1135}$$

b)  $\Delta q = 2$ 

162

9838

b) 
$$\Delta q = 2$$

$$\frac{\Delta q}{\Delta p} = -\frac{162}{1135}$$
$$\frac{2}{\Delta p} = -\frac{162}{1135}$$

 $\frac{1135}{1135}p +$ 

9838

b) 
$$\Delta q = 2$$

$$\frac{\Delta q}{\Delta p} = -\frac{162}{1135}$$
$$\frac{2}{\Delta p} = -\frac{162}{1135}$$

$$-162 \cdot \Delta p = 2270$$

b) 
$$\Delta q = 2$$

$$\frac{\Delta q}{\Delta p} = -\frac{162}{1135}$$
$$\frac{2}{\Delta p} = -\frac{162}{1135}$$

$$-162 \cdot \Delta p = 2270$$

$$\Delta p = -14.01$$

25/25

b) 
$$\Delta q = 2$$

$$q = -\frac{162}{1135}p + \frac{9838}{227}$$

$$\frac{\Delta q}{\Delta p} = -\frac{162}{1135}$$
$$\frac{2}{\Delta p} = -\frac{162}{1135}$$

$$-162 \cdot \Delta p = 2270$$
$$\Delta p = -14.01$$

b) 
$$\Delta q = 2$$

$$E_{q,p} = \frac{p}{q} \cdot q'$$

 $E_{q,p}(110) =$ 

$$\frac{\Delta p}{\Delta p} = -\frac{162}{1135}$$

162

$$\Delta p = -14.01$$

 $-162 \cdot \Delta p = 2270$ 

b) 
$$\Delta q = 2$$

$$\overline{E}_{q,p} = \frac{p}{q} \cdot q'$$

$$\frac{\Delta p}{\Delta p} = -\frac{1135}{1135}$$

$$\frac{2}{\Delta p} = -\frac{162}{1135}$$

$$-162 \cdot \Delta p = 2270$$

$$\Delta p = -14.01$$

b) 
$$\Delta q = 2$$

$$G_{q,p} = \frac{p}{q} \cdot q'$$

$$\frac{\Delta p}{\Delta p} = \frac{1135}{1135}$$

$$\frac{2}{\Delta p} = -\frac{162}{1135}$$

$$-162 \cdot \Delta p = 2270$$
$$\Delta p = -14.01$$

b) 
$$\Delta q = 2$$

$$E_{q,p} = \frac{p}{q} \cdot q'$$

$$E_{q,p}(110)=rac{110}{q(110)}$$

$$\frac{2}{\Delta p} = -\frac{162}{1135}$$
$$-162 \cdot \Delta p = 2270$$

 $-\frac{162}{1135}$ 

 $\Delta p = -14.01$ 

smanjiti za 14 kuna i jednu lipu. (ne ovisi o trenutnoj cijeni jakne)

b) 
$$\Delta q = 2$$

$$E_{q,p} = \frac{p}{q} \cdot q'$$

$$E_{q,p}(110) = rac{110}{q(110)} \cdot$$

$$\frac{\Delta p}{\Delta p} = \frac{1135}{1135}$$

$$\frac{2}{\Delta p} = -\frac{162}{1135}$$

$$-162 \cdot \Delta p = 2270$$
  $\Delta p = -14.01$ 

b) 
$$\Delta q = 2$$

$$\overline{E}_{q,p} = \frac{P}{q} \cdot q'$$

 $E_{q,p}(110) = \frac{110}{a(110)} \cdot q'(110)$ 

$$\frac{\Delta p}{\Delta p} = \frac{1135}{1135}$$

$$\frac{2}{\Delta p} = -\frac{162}{1135}$$

 $-162 \cdot \Delta p = 2270$ 

162

$$\Delta p = -14.01$$
• Da bi se prodale dvije jakne više dnevno, cijenu jakne treba

b) 
$$\Delta q = 2$$

$$E_{q,p} = \frac{P}{q} \cdot q'$$

$$E_{q,p}(110) = \frac{110}{}$$

 $E_{q,p}(110) = rac{110}{q(110)} \cdot q'(110)$ 

$$\frac{1}{\Delta p} = -\frac{1}{1135}$$

$$\frac{2}{\Delta p} = -\frac{162}{1135}$$

 $-162 \cdot \Delta p = 2270$ 

$$\Delta p = -14.01$$

• Da bi se prodale dvije jakne više dnevno, cijenu jakne treba

smanjiti za 14 kuna i jednu lipu. (ne ovisi o trenutnoj cijeni jakne)

b) 
$$\Delta q = 2$$

$$\overline{\xi}_{q,p} = \frac{p}{q} \cdot q'$$

$$E_{q,p}(110) = \frac{110}{27.64}$$

 $E_{q,p}(110) = rac{110}{q(110)} \cdot q'(110)$ 

$$\frac{7}{\Delta p} = -\frac{7}{1135}$$

$$\frac{2}{\Delta p} = -\frac{162}{1135}$$

$$\Delta p = -14.01$$

 $-162 \cdot \Delta p = 2270$ 

$$-162 \cdot \Delta p = 2270$$

$$\Delta p = -14.01$$

$$E_{q,p}(110) = \frac{110}{27.64} \cdot \frac{-162}{1135}$$
• Da bi se prodale dvije jakne više dnevno, cijenu jakne treba

 $\frac{162}{1135}$ 

$$E_{q,p}(110) = \frac{110}{27.64} \cdot \frac{-162}{1135}$$

$$\Delta p = -14.01$$

$$E_{q,p}(110) = \frac{110}{27.64} \cdot \frac{-162}{1135}$$

$$E_{q,p}(110) \approx -0.57$$
• Da bi se prodale dvije jakne više dnevno, cijenu jakne treba smanjiti za 14 kuna i jednu lipu. (ne ovisi o trenutnoj cijeni jakne)

 $\frac{162}{1135}$ 

b) 
$$\Delta q = 2$$

$$= -\frac{162}{1135}$$

 $\frac{162}{1135}$ 

 $-162 \cdot \Delta p = 2270$ 

 $\Delta p = -14.01$ 

$$E_{q,p} = \frac{p}{q} \cdot q'$$

$$q = -\frac{162}{1135}p + \frac{1}{2}$$

$$E_{q,p}(110) = \frac{110}{27.64} \cdot \frac{-162}{1135}$$
 $E_{q,p}(110) \approx -0.57$ 

 $E_{q,p}(110) = \frac{110}{q(110)} \cdot q'(110)$ 

- Ako se cijena jakne na razini p=110 poveća za 1%, potražnja za jaknama se smanji za 0.57%.
  - 25/25

$$egin{align} \Delta p & 1135 \ rac{2}{\Delta p} = -rac{162}{1135} \ -162 \cdot \Delta p = 2270 \ \Delta p = -14.01 \ \end{pmatrix} egin{align} E_{q,p}(110) = rac{110}{q(110)} \cdot q'(110) \ E_{q,p}(110) = rac{110}{27.64} \cdot rac{-162}{1135} \ E_{q,p}(110) pprox -0.57 \ \end{pmatrix}$$

 $=-rac{162}{1135}$ 

b)  $\Delta q = 2$ 

 $E_{q,p} = \frac{p}{q} \cdot q'$   $q = -\frac{162}{1135}p + \frac{9838}{227}$ 

smanjiti za 14 kuna i jednu lipu. (ne ovisi o trenutnoj cijeni jakne) • Ako se cijena jakne na razini p=110 poveća za 1%, potražnja za

• Da bi se prodale dvije jakne više dnevno, cijenu jakne treba

jaknama se smanji za 0.57%.

• Kako je  $|E_{a,p}(110)| < 1$ , povećanjem cijene za 1% na razini 25/25 p=110, ukupni prihod će se povećati.