## Seminari 9

Matematičke metode za informatičare

Damir Horvat

FOI, Varaždin

## Sadržaj

prvi zadatak

Linearni operator

drugi zadatak

treći zadatak

Domaća zadaća

prvi zadatak

#### Zadatak 1

U vektorskom prostoru  $\mathbb{R}^3$  zadane su dvije baze

$$\begin{split} \mathcal{B}_1 &= \big\{ (1,2,-1), \, (0,2,0), \, (1,1,1) \big\}, \\ \mathcal{B}_2 &= \big\{ (0,2,1), \, (2,2,0), \, (1,-1,1) \big\}. \end{split}$$

Vektor  $\vec{v} \in \mathbb{R}^3$  u bazi  $\mathcal{B}_1$  ima koordinate (3, -1, 2).

- a) Odredite koordinate vektora  $\vec{v}$  u kanonskoj bazi vektorskog prostora  $\mathbb{R}^3$ .
- b) Odredite matricu prijelaza iz baze  $\mathcal{B}_2$  u bazu  $\mathcal{B}_1$  i pomoću nje odredite koordinate vektora  $\vec{v}$  u bazi  $\mathcal{B}_2$ .

Rješenje

 $\mathcal{B}_1 = \{(1, 2, -1), (0, 2, 0), (1, 1, 1)\}$ 

a) 
$$\vec{v} = (3, -1, 2)$$

 $\mathcal{B}_1 = \{(1, 2, -1), (0, 2, 0), (1, 1, 1)\}$ 

a) 
$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1}$$

 $\mathcal{B}_1 = \{(1, 2, -1), (0, 2, 0), (1, 1, 1)\}$ 

a) 
$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1}$$

$$\mathcal{B}_{2}$$
  $\mathcal{B}_{2}$  =  $\big\{(0,2,1),\,(2,2,0),\,(1,-1,1)\big\}$   $\mathcal{B}_{\mathsf{kan}} = \big\{(1,0,0),\,(0,1,0),\,(0,0,1)\big\}$ 

a) 
$$\vec{v} = (3, -1, 2)_{B_1}$$

$$\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$$

 $\mathcal{B}_{\mathsf{kan}} = \{(1,0,0), (0,1,0), (0,0,1)\}$ 

 $\mathcal{B}_1 = \{(1, 2, -1), (0, 2, 0), (1, 1, 1)\}$ 

a) 
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$$\vec{v} = (3, -1, 2)_{B_1}$$

a) 
$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1}$$

$$\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$$

$$\mathcal{B}_{\mathsf{kan}} = \big\{ (1,0,0), \, (0,1,0), \, (0,0,1) \big\}$$

$$\vec{v} = (3, -1, 2)_{B_1} = 3 \cdot (1, 2, -1)$$

a) 
$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1}$$

$$\mathcal{B}_1 = ig\{ (1,2,-1), \, (0,2,0), \, (1,1,1) ig\}$$
  $\mathcal{B}_2 = ig\{ (0,2,1), \, (2,2,0), \, (1,-1,1) ig\}$ 

$$\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0,0), \, (0,1,0), \, (0,0,1) ig\}$$

$$\vec{v} = (3, -1, 2)_{B_1} = 3 \cdot (1, 2, -1) +$$

Rješenje

a) 
$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1}$$

 $\mathcal{B}_1 = \{(1,2,-1), (0,2,0), (1,1,1)\}$  $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

$$\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0,0), \, (0,1,0), \, (0,0,1) ig\}$$

$$\vec{v} = (3, -1, 2)_{B_1} = 3 \cdot (1, 2, -1) + (-1) \cdot (0, 2, 0)$$

a) 
$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1}$$

 $\mathcal{B}_1 = \{(1,2,-1), (0,2,0), (1,1,1)\}$  $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

$$\mathcal{B}_{kan} = \big\{ (1,0,0), \, (0,1,0), \, (0,0,1) \big\}$$

$$\vec{v} = (3, -1, 2)_{B_1} = 3 \cdot (1, 2, -1) + (-1) \cdot (0, 2, 0) +$$

a) 
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 $\mathcal{B}_1 = \{(1,2,-1), (0,2,0), (1,1,1)\}$ 

$$\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0,0), \, (0,1,0), \, (0,0,1) ig\}$$

$$\vec{v} = (3, -1, 2)_{B_1} = 3 \cdot (1, 2, -1) + (-1) \cdot (0, 2, 0) + 2 \cdot (1, 1, 1)$$

a) 
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 $\mathcal{B}_1 = \{(1, 2, -1), (0, 2, 0), (1, 1, 1)\}$ 

$$\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0,0), \, (0,1,0), \, (0,0,1) ig\}$$

$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1} = 3 \cdot (1, 2, -1) + (-1) \cdot (0, 2, 0) + 2 \cdot (1, 1, 1) =$$
  
= (

a) 
$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1}$$

 $\mathcal{B}_1 = \{(1, 2, -1), (0, 2, 0), (1, 1, 1)\}$ 

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$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1} = 3 \cdot (1, 2, -1) + (-1) \cdot (0, 2, 0) + 2 \cdot (1, 1, 1) =$$
  
= (5,

a) 
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 $\mathcal{B}_1 = \{(1, 2, -1), (0, 2, 0), (1, 1, 1)\}$ 

$$\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0,0), \, (0,1,0), \, (0,0,1) ig\}$$

$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1} = 3 \cdot (1, 2, -1) + (-1) \cdot (0, 2, 0) + 2 \cdot (1, 1, 1) =$$
  
= (5, 6,

a) 
$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1}$$

 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$  $\mathcal{B}_{\mathsf{kan}} = \{(1,0,0), (0,1,0), (0,0,1)\}$ 

 $\mathcal{B}_1 = \{(1,2,-1), (0,2,0), (1,1,1)\}$ 

$$ec{v} = (3, -1, 2)_{\mathcal{B}_1} = 3 \cdot (1, 2, -1) + (-1) \cdot (0, 2, 0) + 2 \cdot (1, 1, 1) =$$

$$= (5, 6, -1)$$

a) 
$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1}$$

 $\mathcal{B}_1 = \{(1, 2, -1), (0, 2, 0), (1, 1, 1)\}$  $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

$$\mathcal{B}_{kan} = \big\{ (1,0,0), \, (0,1,0), \, (0,0,1) \big\}$$

$$ec{v} = (3, -1, 2)_{\mathcal{B}_1} = 3 \cdot (1, 2, -1) + (-1) \cdot (0, 2, 0) + 2 \cdot (1, 1, 1) =$$

$$= (5, 6, -1)_{\mathcal{B}_{\text{tens}}}$$

a) 
$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1}$$

 $\mathcal{B}_1 = \{(1,2,-1), (0,2,0), (1,1,1)\}$  $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

$$\mathcal{B}_{\mathsf{kan}} = \big\{ (1,0,0), \, (0,1,0), \, (0,0,1) \big\}$$

$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1} = 3 \cdot (1, 2, -1) + (-1) \cdot (0, 2, 0) + 2 \cdot (1, 1, 1) =$$

$$= (5, 6, -1)_{\mathcal{B}_{kon}} \xrightarrow{\bullet}$$

a) 
$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1}$$

 $\mathcal{B}_1 = \{(1, 2, -1), (0, 2, 0), (1, 1, 1)\}$  $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

$$\mathcal{B}_{kan} = \big\{ (1,0,0), \, (0,1,0), \, (0,0,1) \big\}$$

$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1} = 3 \cdot (1, 2, -1) + (-1) \cdot (0, 2, 0) + 2 \cdot (1, 1, 1) =$$

$$= (5, 6, -1)_{\mathcal{B}_{kan}} \xrightarrow{\bullet} 5 \cdot (1, 0, 0)$$

a) 
$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1}$$

 $\mathcal{B}_1 = \{(1, 2, -1), (0, 2, 0), (1, 1, 1)\}$  $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

$$\mathcal{B}_{kan} = \big\{ (1,0,0), \, (0,1,0), \, (0,0,1) \big\}$$

$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1} = 3 \cdot (1, 2, -1) + (-1) \cdot (0, 2, 0) + 2 \cdot (1, 1, 1) =$$

$$= (5, 6, -1)_{\mathcal{B}_{km}} \xrightarrow{\bullet} 5 \cdot (1, 0, 0) +$$

Rješenje

a) 
$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1}$$

 $\mathcal{B}_1 = \{(1,2,-1), (0,2,0), (1,1,1)\}$  $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

$$\mathcal{B}_{\mathsf{kan}} = \big\{ (1,0,0), \, (0,1,0), \, (0,0,1) \big\}$$

$$ec{v} = (3, -1, 2)_{\mathcal{B}_1} = 3 \cdot (1, 2, -1) + (-1) \cdot (0, 2, 0) + 2 \cdot (1, 1, 1) =$$

$$= (5, 6, -1)_{\mathcal{B}_{\mathsf{kan}}} \xrightarrow{} 5 \cdot (1, 0, 0) + 6 \cdot (0, 1, 0)$$

a) 
$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1}$$

 $\mathcal{B}_1 = \{(1,2,-1), (0,2,0), (1,1,1)\}$  $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

$$\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0,0), \, (0,1,0), \, (0,0,1) ig\}$$

$$egin{aligned} ec{v} &= (3,-1,2)_{\mathcal{B}_1} = 3 \cdot (1,2,-1) + (-1) \cdot (0,2,0) + 2 \cdot (1,1,1) = \ &= (5,6,-1)_{\mathcal{B}_{\mathsf{kan}}} wobseteq 5 \cdot (1,0,0) + 6 \cdot (0,1,0) + \end{aligned}$$

Rješenje

a) 
$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1}$$

 $\mathcal{B}_1 = \{(1,2,-1), (0,2,0), (1,1,1)\}$  $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

$$\mathcal{B}_{\mathsf{kan}} = \big\{ (1,0,0), \, (0,1,0), \, (0,0,1) \big\}$$

$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1} = 3 \cdot (1, 2, -1) + (-1) \cdot (0, 2, 0) + 2 \cdot (1, 1, 1) =$$
  
=  $(5, 6, -1)_{\mathcal{B}_{1-1}} \longrightarrow 5 \cdot (1, 0, 0) + 6 \cdot (0, 1, 0) + (-1) \cdot (0, 0, 1)$ 

a) 
$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1}$$

 $\mathcal{B}_1 = \{(1,2,-1), (0,2,0), (1,1,1)\}$  $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

$$\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0,0), \, (0,1,0), \, (0,0,1) ig\}$$

1. način

$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1} = 3 \cdot (1, 2, -1) + (-1) \cdot (0, 2, 0) + 2 \cdot (1, 1, 1) =$$
  
=  $(5, 6, -1)_{\mathcal{B}_{1-1}} \longrightarrow 5 \cdot (1, 0, 0) + 6 \cdot (0, 1, 0) + (-1) \cdot (0, 0, 1)$ 

enje 
$$\mathcal{B}_1 = \big\{ (1,2,-1), \, (0,2,0), \, (1,1,1) \big\}$$

a) 
$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1}$$

$$\mathcal{B}_{2}=ig\{(0,2,1),\,(2,2,0),\,(1,-1,1)ig\}$$
  $\mathcal{B}_{\mathsf{kan}}=ig\{(1,0,0),\,(0,1,0),\,(0,0,1)ig\}$ 

#### 1. način

$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1} = 3 \cdot (1, 2, -1) + (-1) \cdot (0, 2, 0) + 2 \cdot (1, 1, 1) =$$
  
=  $(5, 6, -1)_{\mathcal{B}_{1-1}} \longrightarrow 5 \cdot (1, 0, 0) + 6 \cdot (0, 1, 0) + (-1) \cdot (0, 0, 1)$ 

$$X_{\mathcal{B}_{\mathsf{kan}}} = MX_{\mathcal{B}_1}$$

Rjesenje
$$\vec{v}=(3,-1,2)_{\mathcal{B}_1}$$

$$\mathcal{B}_1 = \big\{ (1,2,-1), \, (0,2,0), \, (1,1,1) \big\}$$
  $\mathcal{B}_2 = \big\{ (0,2,1), \, (2,2,0), \, (1,-1,1) \big\}$ 

$$\mathcal{B}_{\mathsf{kan}} = \big\{ (1,0,0), \, (0,1,0), \, (0,0,1) \big\}$$

1. način

$$\vec{v} = (3, -1, 2)_{\mathcal{B}_{1}} = 3 \cdot (1, 2, -1) + (-1) \cdot (0, 2, 0) + 2 \cdot (1, 1, 1) =$$

$$= (5, 6, -1)_{\mathcal{B}_{km}} \xrightarrow{\bullet} 5 \cdot (1, 0, 0) + 6 \cdot (0, 1, 0) + (-1) \cdot (0, 0, 1)$$

$$X_{\mathcal{B}_{\mathsf{kan}}} = MX_{\mathcal{B}_1}, \ \mathcal{B}_{\mathsf{kan}} \stackrel{M}{\longrightarrow} \mathcal{B}_1$$

a)  $\vec{v} = (3, -1, 2)_{R}$ 

 $\mathcal{B}_{\mathsf{kan}} = \{(1,0,0), (0,1,0), (0,0,1)\}$ 

1. način  $\vec{v} = (3, -1, 2)_{B_1} = 3 \cdot (1, 2, -1) + (-1) \cdot (0, 2, 0) + 2 \cdot (1, 1, 1) =$ 

 $= (5, 6, -1)_{\beta_{\text{total}}} \longrightarrow 5 \cdot (1, 0, 0) + 6 \cdot (0, 1, 0) + (-1) \cdot (0, 0, 1)$ 

2. način

 $X_{\mathcal{B}_{kan}} = MX_{\mathcal{B}_1}, \ \mathcal{B}_{kan} \xrightarrow{M} \mathcal{B}_1$ 

M =

 $\mathcal{B}_1 = \{(1,2,-1), (0,2,0), (1,1,1)\}$ 

a)  $\vec{v} = (3, -1, 2)_{B_1}$ 

 $\mathcal{B}_{\mathsf{kan}} = \big\{ (1,0,0), \, (0,1,0), \, (0,0,1) \big\}$ 

 $\vec{v} = (3, -1, 2)_{R_1} = 3 \cdot (1, 2, -1) + (-1) \cdot (0, 2, 0) + 2 \cdot (1, 1, 1) =$ 

$$= (5,6,-1)_{\mathcal{B}_{\mathsf{kan}}} \xrightarrow{\bullet} 5 \cdot (1,0,0) + 6 \cdot (0,1,0) + (-1) \cdot (0,0,1)$$

2. način

$$X_{\mathcal{B}_{\mathsf{kan}}} = MX_{\mathcal{B}_1}, \ \ \mathcal{B}_{\mathsf{kan}} \stackrel{M}{\longrightarrow} \mathcal{B}_1$$

$$M = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

 $\mathcal{B}_1 = \{(1,2,-1), (0,2,0), (1,1,1)\}$ 

1. način

a)  $\vec{v} = (3, -1, 2)_{B_1}$ 

 $\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0,0), \, (0,1,0), \, (0,0,1) ig\}$ 

 $\vec{v} = (3, -1, 2)_{R_1} = 3 \cdot (1, 2, -1) + (-1) \cdot (0, 2, 0) + 2 \cdot (1, 1, 1) =$ 

$$= (5,6,-1)_{\mathcal{B}_{\mathsf{kan}}} \dashrightarrow 5 \cdot (1,0,0) + 6 \cdot (0,1,0) + (-1) \cdot (0,0,1)$$

2. način

$$X_{\mathcal{B}_{\mathsf{kan}}} = MX_{\mathcal{B}_1}, \ \mathcal{B}_{\mathsf{kan}} \xrightarrow{M} \mathcal{B}_1$$

$$M = \begin{bmatrix} 1 & 0 \\ 2 & 2 \\ -1 & 0 \end{bmatrix}$$

 $\mathcal{B}_1 = \{(1,2,-1), (0,2,0), (1,1,1)\}$ 

a)  $\vec{v} = (3, -1, 2)_{B_1}$ 

$$\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0,0),\, (0,1,0),\, (0,0,1) ig\}$$

 $\mathcal{B}_1 = \{(1,2,-1), (0,2,0), (1,1,1)\}$ 

 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1} = 3 \cdot (1, 2, -1) + (-1) \cdot (0, 2, 0) + 2 \cdot (1, 1, 1) =$$

$$= (5, 6, -1)_{\mathcal{B}_{kan}} \longrightarrow 5 \cdot (1, 0, 0) + 6 \cdot (0, 1, 0) + (-1) \cdot (0, 0, 1)$$

$$X_{\mathcal{B}_{\mathsf{kan}}} = MX_{\mathcal{B}_1}, \;\; \mathcal{B}_{\mathsf{kan}} \xrightarrow{M} \mathcal{B}_1$$

$$M = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

a)  $\vec{v} = (3, -1, 2)_{B_1}$ 

 $\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0,0), \, (0,1,0), \, (0,0,1) ig\}$  1. način

 $ec{v} = (3,-1,2)_{\mathcal{B}_1} = 3 \cdot (1,2,-1) + (-1) \cdot (0,2,0) + 2 \cdot (1,1,1) =$ 

 $= (5, 6, -1)_{\beta_{\text{total}}} \longrightarrow 5 \cdot (1, 0, 0) + 6 \cdot (0, 1, 0) + (-1) \cdot (0, 0, 1)$ 

 $\mathcal{B}_1 = \{(1,2,-1), (0,2,0), (1,1,1)\}$ 

 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

 $egin{aligned} egin{aligned} 2. & \mathsf{na\check{c}in} \end{aligned} & egin{aligned} X_{\mathcal{B}_\mathsf{kan}} &= MX_{\mathcal{B}_1}, & \mathcal{B}_\mathsf{kan} & \stackrel{M}{\longrightarrow} \mathcal{B}_1 \end{aligned}$ 

$$M = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad X_{\mathcal{B}_1} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

a) 
$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1}$$

 $\vec{v} = (3, -1, 2)_{B_1} = 3 \cdot (1, 2, -1) + (-1) \cdot (0, 2, 0) + 2 \cdot (1, 1, 1) =$  $= (5, 6, -1)_{\beta_{\text{total}}} \longrightarrow 5 \cdot (1, 0, 0) + 6 \cdot (0, 1, 0) + (-1) \cdot (0, 0, 1)$ 

 $\mathcal{B}_{\mathsf{kan}} = \{(1,0,0), (0,1,0), (0,0,1)\}$ 

 $\mathcal{B}_1 = \{(1, 2, -1), (0, 2, 0), (1, 1, 1)\}$ 

$$X_{\mathcal{B}_{\mathsf{kan}}} = MX_{\mathcal{B}_1}, \ \mathcal{B}_{\mathsf{kan}} \xrightarrow{M} \mathcal{B}_1$$

$$M = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad X_{\mathcal{B}_1} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} \quad X_{\mathcal{B}_{\mathsf{kan}}} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

a)  $\vec{v} = (3, -1, 2)_{B_1}$ 

 $\mathcal{B}_1 = \{(1,2,-1), (0,2,0), (1,1,1)\}$ 

 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

 $\mathcal{B}_{\mathsf{kan}} = \{(1,0,0), (0,1,0), (0,0,1)\}$ 

 $\vec{v} = (3, -1, 2)_{B_1} = 3 \cdot (1, 2, -1) + (-1) \cdot (0, 2, 0) + 2 \cdot (1, 1, 1) =$ 

$$= (5,6,-1)_{\mathcal{B}_{\mathsf{kan}}} \dashrightarrow 5 \cdot (1,0,0) + 6 \cdot (0,1,0) + (-1) \cdot (0,0,1)$$

$$X_{\mathcal{B}_{\mathsf{kan}}} = MX_{\mathcal{B}_1}, \ \mathcal{B}_{\mathsf{kan}} \xrightarrow{M} \mathcal{B}_1$$

$$X_{\mathcal{B}_{\mathsf{kan}}} = MX_{\mathcal{B}_{1}}, \quad \mathcal{B}_{\mathsf{kan}} \xrightarrow{M} \mathcal{B}_{1}$$
 $M = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad X_{\mathcal{B}_{1}} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} \quad X_{\mathcal{B}_{\mathsf{kan}}} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$ 

$$egin{bmatrix} 1 \ 1 \ 1 \end{bmatrix} \quad X_{\mathcal{B}_1} = egin{bmatrix} \end{array}$$

1. način

a)  $\vec{v} = (3, -1, 2)_{R}$ 

$$= (5,6,-1)_{\mathcal{B}_{\mathsf{kan}}} \xrightarrow{} 5 \cdot (1,0,0) + 6 \cdot (0,1,0) + (-1) \cdot (0,0,1)$$

 $\mathcal{B}_{\mathsf{kan}} = \{(1,0,0), (0,1,0), (0,0,1)\}$ 

 $\vec{v} = (3, -1, 2)_{B_1} = 3 \cdot (1, 2, -1) + (-1) \cdot (0, 2, 0) + 2 \cdot (1, 1, 1) =$ 

$$X_{\mathcal{B}_{\mathsf{kan}}} = MX_{\mathcal{B}_1}, \;\; \mathcal{B}_{\mathsf{kan}} \stackrel{M}{\longrightarrow} \mathcal{B}_1$$

$$M = \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

 $M = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad X_{\mathcal{B}_1} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} \quad X_{\mathcal{B}_{kan}} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$ 

 $\mathcal{B}_1 = \{(1, 2, -1), (0, 2, 0), (1, 1, 1)\}$ 

 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

a)  $\vec{v} = (3, -1, 2)_{B_1}$ 

$$\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0,0), \, (0,1,0), \, (0,0,1) ig\}$$

$$= (5,6,-1)_{\mathcal{B}_{\mathsf{kan}}} \xrightarrow{\hspace{1cm}} 5 \cdot (1,0,0) + 6 \cdot (0,1,0) + (-1) \cdot (0,0,1)$$

1. način

$$X_{\mathcal{B}_{kan}} = MX_{\mathcal{B}_{1}}, \quad \mathcal{B}_{kan} \xrightarrow{M} \mathcal{B}_{1}$$

$$M = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad X_{\mathcal{B}_{1}} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} \quad X_{\mathcal{B}_{kan}} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ -1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 \\ 2 & 2 \\ -1 & 0 \end{bmatrix}$$

 $\vec{v} = (3, -1, 2)_{B_1} = 3 \cdot (1, 2, -1) + (-1) \cdot (0, 2, 0) + 2 \cdot (1, 1, 1) =$ 

 $\mathcal{B}_1 = \{(1, 2, -1), (0, 2, 0), (1, 1, 1)\}$ 

 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

b)

$$egin{aligned} \mathcal{B}_1 &= ig\{ (1,2,-1), \, (0,2,0), \, (1,1,1) ig\} \ \mathcal{B}_2 &= ig\{ (0,2,1), \, (2,2,0), \, (1,-1,1) ig\} \end{aligned}$$

b)

$$\mathcal{B}_1 = \{(1, 2, -1), (0, 2, 0), (1, 1, 1)\}$$
  
 $\mathcal{B}_2 = \{(0, 2, 1), (2, 2, 0), (1, -1, 1)\}$ 

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$

b

$$\mathcal{B}_1 = \left\{ (1, 2, -1), (0, 2, 0), (1, 1, 1) \right\}$$
  
$$\mathcal{B}_2 = \left\{ (0, 2, 1), (2, 2, 0), (1, -1, 1) \right\}$$

b) 
$$(1,2,-1) =$$

$$egin{pmatrix} \mathcal{B}_2 & \xrightarrow{\mathcal{T}} \mathcal{B}_1 \end{pmatrix} \qquad \mathcal{T} = \begin{bmatrix} & & & \\ & & & \end{bmatrix}$$

 $\mathcal{B}_2 = \big\{ (0,2,1), \, (2,2,0), \, (1,-1,1) \big\}$ 

b) 
$$(1,2,-1) = \alpha_1 \cdot (0,2,1)$$
 
$$\mathcal{B}_1 = \{(1,2,-1), (0,2,0), (1,1,1)\}$$

$$\{1,1\}$$

$$\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$$

$$egin{pmatrix} \mathcal{B}_2 & \xrightarrow{\mathcal{T}} \mathcal{B}_1 \end{pmatrix} \qquad \mathcal{T} = \left[ \begin{array}{c} & & \\ & & \end{array} \right]$$

$$\mathcal{B}_1 = \{(1, 2, -1), (0, 2, 0), (1, 1, 1)\}$$
 $\mathcal{B}_2 = \{(0, 2, 1), (2, 2, 0), (1, -1, 1)\}$ 

b)  $(1,2,-1) = \alpha_1 \cdot (0,2,1) +$ 

$$egin{pmatrix} \mathcal{B}_2 & \stackrel{\mathcal{T}}{\longrightarrow} \mathcal{B}_1 \end{pmatrix} \quad \mathcal{T} = egin{bmatrix} \end{array}$$

$$\mathcal{B}_1 = \{(1, 2, -1), (0, 2, 0), (1, 1, 1)\}$$
 $\mathcal{B}_2 = \{(0, 2, 1), (2, 2, 0), (1, -1, 1)\}$ 

b)  $(1,2,-1) = \alpha_1 \cdot (0,2,1) + \alpha_2 \cdot (2,2,0)$ 

$$\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$$

b)  $(1,2,-1) = \alpha_1 \cdot (0,2,1) + \alpha_2 \cdot (2,2,0) +$ 

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 $\mathcal{B}_1 = \{(1,2,-1), (0,2,0), (1,1,1)\}$ 

$$\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$$

b)  $(1,2,-1) = \alpha_1 \cdot (0,2,1) + \alpha_2 \cdot (2,2,0) + \alpha_3 \cdot (1,-1,1)$ 

$$egin{pmatrix} \mathcal{B}_2 & \stackrel{\mathcal{T}}{-\!\!\!\!-\!\!\!\!-\!\!\!\!-} \mathcal{B}_1 \end{pmatrix} \qquad \mathcal{T} = egin{bmatrix} \end{array}$$

 $\mathcal{B}_1 = \{(1,2,-1), (0,2,0), (1,1,1)\}$ 

 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

b)  $(1,2,-1) = \alpha_1 \cdot (0,2,1) + \alpha_2 \cdot (2,2,0) + \alpha_3 \cdot (1,-1,1)$ 

 $2\alpha_2 + \alpha_3 = 1$ 

$$egin{pmatrix} \mathcal{B}_2 & \stackrel{\mathcal{T}}{\longrightarrow} \mathcal{B}_1 \end{pmatrix} \quad \mathcal{T} = egin{bmatrix} \end{array}$$

 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

b)  $(1,2,-1) = \alpha_1 \cdot (0,2,1) + \alpha_2 \cdot (2,2,0) + \alpha_3 \cdot (1,-1,1)$ 

 $2\alpha_2 + \alpha_3 = 1$ 

 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

b)  $(1,2,-1) = \alpha_1 \cdot (0,2,1) + \alpha_2 \cdot (2,2,0) + \alpha_3 \cdot (1,-1,1)$ 

 $2\alpha_2 + \alpha_3 = 1$ 

 $\alpha_1 + \alpha_3 = -1$ 

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 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

b)  $(1,2,-1) = \alpha_1 \cdot (0,2,1) + \alpha_2 \cdot (2,2,0) + \alpha_3 \cdot (1,-1,1)$ 

 $2\alpha_2 + \alpha_3 = 1$ 

 $\alpha_1 + \alpha_3 = -1$ 

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 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

b)  $(1,2,-1) = \alpha_1 \cdot (0,2,1) + \alpha_2 \cdot (2,2,0) + \alpha_3 \cdot (1,-1,1)$ 

 $2\alpha_2 + \alpha_3 = 1$ 

 $\alpha_1 + \alpha_3 = -1$ 

 $2\alpha_1 + 2\alpha_2 - \alpha_3 = 2$ 

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 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

b)  $(1,2,-1) = \alpha_1 \cdot (0,2,1) + \alpha_2 \cdot (2,2,0) + \alpha_3 \cdot (1,-1,1)$ 

 $2\alpha_2 + \alpha_3 = 1$ 

 $\alpha_1 + \alpha_3 = -1$ 

 $2\alpha_1 + 2\alpha_2 - \alpha_3 = 2$ 

 $(0,2,0) = \beta_1 \cdot (0,2,1)$ 

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 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

b)  $(1,2,-1) = \alpha_1 \cdot (0,2,1) + \alpha_2 \cdot (2,2,0) + \alpha_3 \cdot (1,-1,1)$ 

 $2\alpha_2 + \alpha_3 = 1$ 

 $\alpha_1 + \alpha_3 = -1$ 

 $2\alpha_1 + 2\alpha_2 - \alpha_3 = 2$ 

 $(0,2,0) = \beta_1 \cdot (0,2,1) +$ 

$$\begin{array}{ccc}
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 & \alpha_1 & \\
 & \alpha_2 & \\
 & \alpha_3 & 
\end{array}$$

 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

b)  $(1,2,-1) = \alpha_1 \cdot (0,2,1) + \alpha_2 \cdot (2,2,0) + \alpha_3 \cdot (1,-1,1)$ 

 $2\alpha_2 + \alpha_3 = 1$ 

 $\alpha_1 + \alpha_3 = -1$ 

 $(0,2,0) = \beta_1 \cdot (0,2,1) + \beta_2 \cdot (2,2,0)$ 

$$\begin{array}{ccc}
 & T & \\
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\end{array}$$

 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

b)  $(1,2,-1) = \alpha_1 \cdot (0,2,1) + \alpha_2 \cdot (2,2,0) + \alpha_3 \cdot (1,-1,1)$ 

 $2\alpha_2 + \alpha_3 = 1$ 

 $\alpha_1 + \alpha_3 = -1$ 

 $(0,2,0) = \beta_1 \cdot (0,2,1) + \beta_2 \cdot (2,2,0) +$ 

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 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

b)  $(1,2,-1) = \alpha_1 \cdot (0,2,1) + \alpha_2 \cdot (2,2,0) + \alpha_3 \cdot (1,-1,1)$ 

 $(0,2,0) = \beta_1 \cdot (0,2,1) + \beta_2 \cdot (2,2,0) + \beta_3 \cdot (1,-1,1)$ 

 $2\alpha_2 + \alpha_3 = 1$ 

 $\alpha_1 + \alpha_3 = -1$ 

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1 \qquad T = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}$$

 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

b)  $(1, 2, -1) = \alpha_1 \cdot (0, 2, 1) + \alpha_2 \cdot (2, 2, 0) + \alpha_3 \cdot (1, -1, 1)$ 

 $(0,2,0) = \beta_1 \cdot (0,2,1) + \beta_2 \cdot (2,2,0) + \beta_3 \cdot (1,-1,1)$ 

 $2\alpha_2 + \alpha_3 = 1$ 

 $\alpha_1 + \alpha_3 = -1$ 

 $2\beta_2 + \beta_3 = 0$ 

$$2\beta_1 + 2\beta_2 - \beta_3 = 2 \qquad \boxed{\beta_2 \xrightarrow{T} \beta_1} \qquad T = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}$$

 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

b)  $(1, 2, -1) = \alpha_1 \cdot (0, 2, 1) + \alpha_2 \cdot (2, 2, 0) + \alpha_3 \cdot (1, -1, 1)$ 

 $(0,2,0) = \beta_1 \cdot (0,2,1) + \beta_2 \cdot (2,2,0) + \beta_3 \cdot (1,-1,1)$ 

 $2\alpha_2 + \alpha_3 = 1$ 

 $\alpha_1 + \alpha_3 = -1$ 

 $2\beta_2 + \beta_3 = 0$ 

$$2\beta_{2} + \beta_{3} = 0$$

$$2\beta_{1} + 2\beta_{2} - \beta_{3} = 2$$

$$\beta_{1} + \beta_{3} = 0$$

$$\mathcal{B}_{2} \xrightarrow{T} \mathcal{B}_{1}$$

$$T = \begin{bmatrix} \alpha_{1} \\ \alpha_{2} \\ \alpha_{3} \end{bmatrix}$$

 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

b)  $(1, 2, -1) = \alpha_1 \cdot (0, 2, 1) + \alpha_2 \cdot (2, 2, 0) + \alpha_3 \cdot (1, -1, 1)$ 

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 $2\alpha_2 + \alpha_3 = 1$ 

 $\alpha_1 + \alpha_3 = -1$ 

 $2\beta_2 + \beta_3 = 0$ 

$$2\beta_{2} + \beta_{3} = 0$$

$$2\beta_{1} + 2\beta_{2} - \beta_{3} = 2$$

$$\beta_{1} + \beta_{3} = 0$$

$$\mathcal{B}_{2} \xrightarrow{T} \mathcal{B}_{1}$$

$$T = \begin{bmatrix} \alpha_{1} & \beta_{1} \\ \alpha_{2} & \beta_{2} \\ \alpha_{3} & \beta_{3} \end{bmatrix}$$

 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

b)  $(1,2,-1) = \alpha_1 \cdot (0,2,1) + \alpha_2 \cdot (2,2,0) + \alpha_3 \cdot (1,-1,1)$ 

 $(0,2,0) = \beta_1 \cdot (0,2,1) + \beta_2 \cdot (2,2,0) + \beta_3 \cdot (1,-1,1)$ 

 $2\alpha_2 + \alpha_3 = 1$ 

 $\alpha_1 + \alpha_3 = -1$ 

 $2\beta_2 + \beta_3 = 0$ 

$$2\beta_{2} + \beta_{3} = 0$$

$$2\beta_{1} + 2\beta_{2} - \beta_{3} = 2$$

$$\beta_{1} + \beta_{3} = 0$$

$$\mathcal{B}_{2} \xrightarrow{T} \mathcal{B}_{1}$$

$$T = \begin{bmatrix} \alpha_{1} & \beta_{1} \\ \alpha_{2} & \beta_{2} \\ \alpha_{3} & \beta_{3} \end{bmatrix}$$

$$(1, 1, 1) =$$

 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

b)  $(1, 2, -1) = \alpha_1 \cdot (0, 2, 1) + \alpha_2 \cdot (2, 2, 0) + \alpha_3 \cdot (1, -1, 1)$ 

 $(0,2,0) = \beta_1 \cdot (0,2,1) + \beta_2 \cdot (2,2,0) + \beta_3 \cdot (1,-1,1)$ 

 $2\alpha_2 + \alpha_3 = 1$ 

 $\alpha_1 + \alpha_3 = -1$ 

 $2\beta_2 + \beta_3 = 0$ 

$$2\beta_{2} + \beta_{3} = 0$$

$$2\beta_{1} + 2\beta_{2} - \beta_{3} = 2$$

$$\beta_{1} + \beta_{3} = 0$$

$$\mathcal{B}_{2} \xrightarrow{T} \mathcal{B}_{1}$$

$$T = \begin{bmatrix} \alpha_{1} & \beta_{1} \\ \alpha_{2} & \beta_{2} \\ \alpha_{3} & \beta_{3} \end{bmatrix}$$

$$(1, 1, 1) = \gamma_{1} \cdot (0, 2, 1)$$

 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

b)  $(1, 2, -1) = \alpha_1 \cdot (0, 2, 1) + \alpha_2 \cdot (2, 2, 0) + \alpha_3 \cdot (1, -1, 1)$ 

 $(0,2,0) = \beta_1 \cdot (0,2,1) + \beta_2 \cdot (2,2,0) + \beta_3 \cdot (1,-1,1)$ 

 $2\alpha_2 + \alpha_3 = 1$ 

 $\alpha_1 + \alpha_3 = -1$ 

 $2\beta_2 + \beta_3 = 0$ 

$$2\beta_{2} + \beta_{3} = 0$$

$$2\beta_{1} + 2\beta_{2} - \beta_{3} = 2$$

$$\beta_{1} + \beta_{3} = 0$$

$$\mathcal{B}_{2} \xrightarrow{T} \mathcal{B}_{1}$$

$$T = \begin{bmatrix} \alpha_{1} & \beta_{1} \\ \alpha_{2} & \beta_{2} \\ \alpha_{3} & \beta_{3} \end{bmatrix}$$

$$(1, 1, 1) = \gamma_{1} \cdot (0, 2, 1) + \beta_{1}$$

 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

b)  $(1, 2, -1) = \alpha_1 \cdot (0, 2, 1) + \alpha_2 \cdot (2, 2, 0) + \alpha_3 \cdot (1, -1, 1)$ 

 $(0,2,0) = \beta_1 \cdot (0,2,1) + \beta_2 \cdot (2,2,0) + \beta_3 \cdot (1,-1,1)$ 

 $2\alpha_2 + \alpha_3 = 1$ 

 $\alpha_1 + \alpha_3 = -1$ 

$$2\beta_{2} + \beta_{3} = 0$$

$$2\beta_{1} + 2\beta_{2} - \beta_{3} = 2$$

$$\beta_{1} + \beta_{3} = 0$$

$$\mathcal{B}_{2} \xrightarrow{T} \mathcal{B}_{1}$$

$$T = \begin{bmatrix} \alpha_{1} & \beta_{1} \\ \alpha_{2} & \beta_{2} \\ \alpha_{3} & \beta_{3} \end{bmatrix}$$

$$(1, 1, 1) = \gamma_{1} \cdot (0, 2, 1) + \gamma_{2} \cdot (2, 2, 0)$$

 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

b)  $(1, 2, -1) = \alpha_1 \cdot (0, 2, 1) + \alpha_2 \cdot (2, 2, 0) + \alpha_3 \cdot (1, -1, 1)$ 

 $(0,2,0) = \beta_1 \cdot (0,2,1) + \beta_2 \cdot (2,2,0) + \beta_3 \cdot (1,-1,1)$ 

 $2\alpha_2 + \alpha_3 = 1$ 

 $\alpha_1 + \alpha_3 = -1$ 

$$2\beta_{2} + \beta_{3} = 0$$

$$2\beta_{1} + 2\beta_{2} - \beta_{3} = 2$$

$$\beta_{1} + \beta_{3} = 0$$

$$\mathcal{B}_{2} \xrightarrow{T} \mathcal{B}_{1}$$

$$T = \begin{bmatrix} \alpha_{1} & \beta_{1} \\ \alpha_{2} & \beta_{2} \\ \alpha_{3} & \beta_{3} \end{bmatrix}$$

$$(1, 1, 1) = \gamma_{1} \cdot (0, 2, 1) + \gamma_{2} \cdot (2, 2, 0) +$$

 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

b)  $(1, 2, -1) = \alpha_1 \cdot (0, 2, 1) + \alpha_2 \cdot (2, 2, 0) + \alpha_3 \cdot (1, -1, 1)$ 

 $(0,2,0) = \beta_1 \cdot (0,2,1) + \beta_2 \cdot (2,2,0) + \beta_3 \cdot (1,-1,1)$ 

 $2\alpha_2 + \alpha_3 = 1$ 

 $\alpha_1 + \alpha_3 = -1$ 

$$2\beta_{2} + \beta_{3} = 0$$

$$2\beta_{1} + 2\beta_{2} - \beta_{3} = 2$$

$$\beta_{1} + \beta_{3} = 0$$

$$\beta_{2} \xrightarrow{T} \beta_{1}$$

$$\beta_{1} = \begin{bmatrix} \alpha_{1} & \beta_{1} \\ \alpha_{2} & \beta_{2} \\ \alpha_{3} & \beta_{3} \end{bmatrix}$$

$$(1, 1, 1) = \gamma_{1} \cdot (0, 2, 1) + \gamma_{2} \cdot (2, 2, 0) + \gamma_{3} \cdot (1, -1, 1)$$

 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

b)  $(1, 2, -1) = \alpha_1 \cdot (0, 2, 1) + \alpha_2 \cdot (2, 2, 0) + \alpha_3 \cdot (1, -1, 1)$ 

 $(0,2,0) = \beta_1 \cdot (0,2,1) + \beta_2 \cdot (2,2,0) + \beta_3 \cdot (1,-1,1)$ 

 $2\alpha_2 + \alpha_3 = 1$ 

 $\alpha_1 + \alpha_3 = -1$ 

$$2\beta_{2} + \beta_{3} = 0$$

$$2\beta_{1} + 2\beta_{2} - \beta_{3} = 2$$

$$\beta_{1} + \beta_{3} = 0$$

$$\beta_{1} + \beta_{3} = 0$$

$$T = \begin{bmatrix} \alpha_{1} & \beta_{1} \\ \alpha_{2} & \beta_{2} \\ \alpha_{3} & \beta_{3} \end{bmatrix}$$

$$(1, 1, 1) = \gamma_{1} \cdot (0, 2, 1) + \gamma_{2} \cdot (2, 2, 0) + \gamma_{3} \cdot (1, -1, 1)$$

$$2\gamma_{2} + \gamma_{3} = 1$$

 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

b)  $(1, 2, -1) = \alpha_1 \cdot (0, 2, 1) + \alpha_2 \cdot (2, 2, 0) + \alpha_3 \cdot (1, -1, 1)$ 

 $(0,2,0) = \beta_1 \cdot (0,2,1) + \beta_2 \cdot (2,2,0) + \beta_3 \cdot (1,-1,1)$ 

 $2\alpha_2 + \alpha_3 = 1$ 

 $\alpha_1 + \alpha_3 = -1$ 

 $2\beta_2 + \beta_3 = 0$ 

$$2\beta_{2} + \beta_{3} = 0$$

$$2\beta_{1} + 2\beta_{2} - \beta_{3} = 2$$

$$\beta_{1} + \beta_{3} = 0$$

$$\beta_{1} + \beta_{3} = 0$$

$$T = \begin{bmatrix} \alpha_{1} & \beta_{1} \\ \alpha_{2} & \beta_{2} \\ \alpha_{3} & \beta_{3} \end{bmatrix}$$

$$(1, 1, 1) = \gamma_{1} \cdot (0, 2, 1) + \gamma_{2} \cdot (2, 2, 0) + \gamma_{3} \cdot (1, -1, 1)$$

$$2\gamma_{2} + \gamma_{3} = 1$$

 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

b)  $(1, 2, -1) = \alpha_1 \cdot (0, 2, 1) + \alpha_2 \cdot (2, 2, 0) + \alpha_3 \cdot (1, -1, 1)$ 

 $(0,2,0) = \beta_1 \cdot (0,2,1) + \beta_2 \cdot (2,2,0) + \beta_3 \cdot (1,-1,1)$ 

 $2\alpha_2 + \alpha_3 = 1$ 

 $\alpha_1 + \alpha_3 = -1$ 

 $2\beta_2 + \beta_3 = 0$ 

 $2\gamma_1 + 2\gamma_2 - \gamma_3 = 1$ 

$$2\beta_{2} + \beta_{3} = 0$$

$$2\beta_{1} + 2\beta_{2} - \beta_{3} = 2$$

$$\beta_{1} + \beta_{3} = 0$$

$$\beta_{1} + \beta_{3} = 0$$

$$\beta_{2} \xrightarrow{T} \beta_{1}$$

$$T = \begin{bmatrix} \alpha_{1} & \beta_{1} \\ \alpha_{2} & \beta_{2} \\ \alpha_{3} & \beta_{3} \end{bmatrix}$$

$$(1, 1, 1) = \gamma_{1} \cdot (0, 2, 1) + \gamma_{2} \cdot (2, 2, 0) + \gamma_{3} \cdot (1, -1, 1)$$

$$2\gamma_{2} + \gamma_{3} = 1$$

$$2\gamma_{1} + 2\gamma_{2} - \gamma_{3} = 1$$

$$\gamma_{1} + \gamma_{3} = 1$$

$$3/26$$

 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

b)  $(1, 2, -1) = \alpha_1 \cdot (0, 2, 1) + \alpha_2 \cdot (2, 2, 0) + \alpha_3 \cdot (1, -1, 1)$ 

 $(0,2,0) = \beta_1 \cdot (0,2,1) + \beta_2 \cdot (2,2,0) + \beta_3 \cdot (1,-1,1)$ 

 $2\alpha_2 + \alpha_3 = 1$ 

 $\alpha_1 + \alpha_3 = -1$ 

$$(0,2,0) = \beta_{1} \cdot (0,2,1) + \beta_{2} \cdot (2,2,0) + \beta_{3} \cdot (1,-1,1)$$

$$2\beta_{2} + \beta_{3} = 0$$

$$2\beta_{1} + 2\beta_{2} - \beta_{3} = 2$$

$$\beta_{1} + \beta_{3} = 0$$

$$(1,1,1) = \gamma_{1} \cdot (0,2,1) + \gamma_{2} \cdot (2,2,0) + \gamma_{3} \cdot (1,-1,1)$$

$$2\gamma_{2} + \gamma_{3} = 1$$

$$2\gamma_{1} + 2\gamma_{2} - \gamma_{3} = 1$$

$$\gamma_{1} + \gamma_{3} = 1$$

$$3/26$$

 $\mathcal{B}_2 = \{(0,2,1), (2,2,0), (1,-1,1)\}$ 

b)  $(1, 2, -1) = \alpha_1 \cdot (0, 2, 1) + \alpha_2 \cdot (2, 2, 0) + \alpha_3 \cdot (1, -1, 1)$ 

 $2\alpha_2 + \alpha_3 = 1$ 

 $\alpha_1 + \alpha_3 = -1$ 

$$2\alpha_{2} + \alpha_{3} = 1$$

$$2\alpha_{1} + 2\alpha_{2} - \alpha_{3} = 2$$

$$\alpha_{1} + \alpha_{3} = -1$$

$$2\beta_{2} + \beta_{3} = 0$$

$$2\beta_{1} + 2\beta_{2} - \beta_{3} = 2$$

$$\beta_{1} + \beta_{3} = 0$$

$$2\gamma_{2} + \gamma_{3} = 1$$

$$2\gamma_{1} + 2\gamma_{2} - \gamma_{3} = 1$$

$$\gamma_{1} + \gamma_{3} = 1$$

$$2\alpha_{2} + \alpha_{3} = 1$$

$$2\alpha_{1} + 2\alpha_{2} - \alpha_{3} = 2$$

$$\alpha_{1} + \alpha_{3} = -1$$

$$2\beta_{2} + \beta_{3} = 0$$

$$2\beta_{1} + 2\beta_{2} - \beta_{3} = 2$$

$$\beta_{1} + \beta_{3} = 0$$

$$2\gamma_{2} + \gamma_{3} = 1$$

$$2\gamma_{1} + 2\gamma_{2} - \gamma_{3} = 1$$

$$\gamma_{1} + \gamma_{3} = 1$$

$$2\alpha_{2} + \alpha_{3} = 1$$

$$2\alpha_{1} + 2\alpha_{2} - \alpha_{3} = 2$$

$$\alpha_{1} + \alpha_{3} = -1$$

$$2\beta_{2} + \beta_{3} = 0$$

$$2\beta_{1} + 2\beta_{2} - \beta_{3} = 2$$

$$\beta_{1} + \beta_{3} = 0$$

$$2\gamma_{2} + \gamma_{3} = 1$$

$$2\gamma_{1} + 2\gamma_{2} - \gamma_{3} = 1$$

$$\gamma_{2} + \gamma_{3} = 1$$

$$2\gamma_2 + \gamma_3 = 1$$

$$2\gamma_1 + 2\gamma_2 - \gamma_3 = 1$$

$$\gamma_1 + \gamma_3 = 1$$

$$2\alpha_{2} + \alpha_{3} = 1$$

$$2\alpha_{1} + 2\alpha_{2} - \alpha_{3} = 2$$

$$\alpha_{1} + \alpha_{3} = -1$$

$$2\beta_{2} + \beta_{3} = 0$$

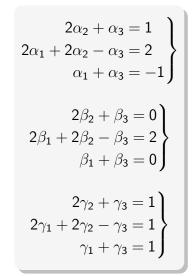
$$2\beta_{1} + 2\beta_{2} - \beta_{3} = 2$$

$$\beta_{1} + \beta_{3} = 0$$

$$2\gamma_{2} + \gamma_{3} = 1$$

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$$2\alpha_{2} + \alpha_{3} = 1$$

$$2\alpha_{1} + 2\alpha_{2} - \alpha_{3} = 2$$

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$$2\beta_{2} + \beta_{3} = 0$$

$$2\beta_{1} + 2\beta_{2} - \beta_{3} = 2$$

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$$2\gamma_{2} + \gamma_{3} = 1$$

$$2\gamma_{1} + 2\gamma_{2} - \gamma_{3} = 1$$

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$$2\alpha_{2} + \alpha_{3} = 1$$

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$$\alpha_{1} + \alpha_{3} = -1$$

$$2\beta_{2} + \beta_{3} = 0$$

$$2\beta_{1} + 2\beta_{2} - \beta_{3} = 2$$

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$$2\beta_{2} + \beta_{3} = 0$$

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$$2\alpha_{2} + \alpha_{3} = 1$$

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$$2\beta_{2} + \beta_{3} = 0$$

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$$2\gamma_{2} + \gamma_{3} = 1$$

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$$2\alpha_{2} + \alpha_{3} = 1$$

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$$2\beta_{2} + \beta_{3} = 0$$

$$2\beta_{1} + 2\beta_{2} - \beta_{3} = 2$$

$$\beta_{1} + \beta_{3} = 0$$

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$$\alpha_{1} + \alpha_{3} = -1$$

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$$2\beta_{1} + 2\beta_{2} - \beta_{3} = 2$$

$$\beta_{1} + \beta_{3} = 0$$

$$2\gamma_{2} + \gamma_{3} = 1$$

$$2\gamma_{1} + 2\gamma_{2} - \gamma_{3} = 1$$

$$\gamma_{1} + \gamma_{3} = 1$$

			$\alpha_i$	$\beta_i$	$\gamma_i$
0	2	1	1	0	1
2	2	-1	2	2	1 ← +
1	0	1	-1	0	1 /·(-2)
0	2	1	1	0	+
0	2	-3	4	2	-1/(-1)
1	0	1	-1	0	1
0	0	4	-3	-2	$\frac{}{2}/\cdot\frac{3}{4}/\cdot\frac{-1}{4}$
0	2	-3	4	2	-1
1	0	1	-1	0	1 +
0	0	4	-3	-2	
0	2	0	$\frac{7}{4}$	$\frac{1}{2}$	$\frac{1}{2}$ /: 2
1	0	0	$-\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$

La	Q	•	
$\alpha_i$	$\beta_i$	$\frac{\gamma_i}{}$	
l			

			$\alpha_i$	$\beta_i$	$\gamma_i$
0	2	1	1	0	1
2	2	-1	2	2	1 ←+
1	0	1	-1	2	1 / (-2)
0	2	1	1	0	1 ←+
0	2	-3	4	2	1 ← + -1 /·(-1)
1	0	1	-1	0	1
0	0	4	-3	-2	$\frac{1}{2} / \frac{3}{4} / \frac{-1}{4}$
0	2	<b>-3</b>	4	2	-1
1	0	1	-1	0	1 +
0	0	4	-3	-2	2 /:4
0	2	0	$\frac{7}{4}$	$\frac{1}{2}$	$\frac{1}{2}$ /:2
1	0	0	$-\frac{1}{4}$	$\frac{1}{2}$	$ \begin{array}{c c} \hline 2 /:4 \\ \hline \frac{1}{2} /:2 \\ \hline \frac{1}{2} \end{array} $

			$\alpha_i$	$\beta_i$	$\gamma_i$
1	0	0	$-\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$
			7	2	۷

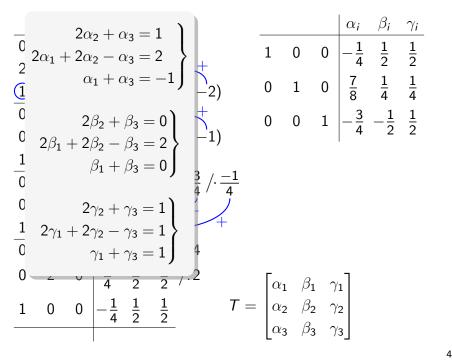
			$\alpha_i$	$\beta_i$	$\gamma_i$
0	2	1	1	0	1
2	2	-1	2	2	1 ←+
1	0	1	-1	0	1 / (-2)
0	2	1	1	0	+
0	2	-3	4	2	$1 \leftarrow +$ $-1 / \cdot (-1)$
1	0	1	-1	0	1
0	0	4	-3	-2	$\frac{}{2}/\cdot\frac{3}{4}/\cdot\frac{-1}{4}$
0	2	-3	4	2	$-1$ $\stackrel{}{\leftarrow}$ $\stackrel{}{\downarrow}$
1	0	1	-1	0	1 +
0	0	4	-3	-2	2 /:4
0	2	0	$\frac{7}{4}$	$\frac{1}{2}$	$\frac{1}{2}$ /:2
1	0	0	$-\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$ /:2

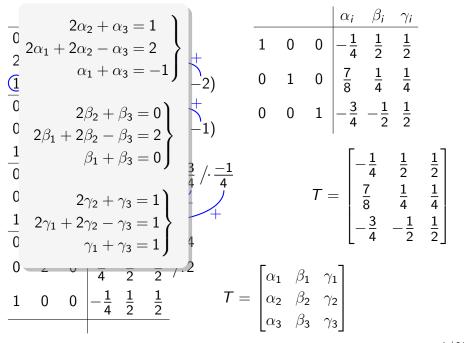
			$\alpha_i$	$\beta_i$	$\gamma_{\it i}$
1	0	0	$-\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$
0	1	0	<u>7</u> 8	$\frac{1}{4}$	$\frac{1}{4}$

			$\alpha_i$	$\beta_i$	$\gamma_i$
0	2	1	1	0	1
2	2	-1	2	2	1 ←+
1	0	1	-1	2	1 /·(-2)
0	2	1	1	0	1 ←+
0	2	-3	4	2	$-1 / \cdot (-1)$
1	0	1	-1	0	1
0	0	4	-3	-2	$\frac{1}{2} / \frac{3}{4} / \frac{-1}{4}$
0	2	-3	4	2	-1
1	0	1	-1	0	1 +
0	0	4	-3	-2	2 /:4
0	2	0	$\frac{7}{4}$	$\frac{1}{2}$	$\frac{1}{2}$ /:2
1	0	0	$-\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$ /:4 $\frac{1}{2}$ /:2 $\frac{1}{2}$

			$\alpha_i$	$\beta_i$	$\gamma_i$
1	0	0	$-\frac{1}{4}$	$\frac{1}{2}$	1/2
0	1	0	<u>7</u> 8		$\frac{1}{4}$
0	0	1	$-\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$

			$\alpha_i$	$\beta_i$	$\gamma_i$						$\alpha_i$	$\beta_i$	$\gamma_i$
0	2	1	1	0	1			1	0	0	$-\frac{1}{4}$ $\frac{7}{8}$ $-\frac{3}{4}$	1	1
2	2	-1	2	2	1 ←	<del> </del>					4	2	2
1	0	1	-1	0	1 / (-	-2)	(	)	1	0	8	$\frac{1}{4}$	$\frac{1}{4}$
0	2	1	1	0	1 /·(- 1 ← -	<del> </del>	(	)	0	1	_ 3	_1	<u>1</u>
0	2	-3	4	2	-1 / (-	-1)	`		Ū	_	4	2	2
1	0	1	-1	0	1								
0 0 1	0	4	-3	-2	$ \begin{array}{c c} \hline 2 / \frac{3}{4} \\ -1 & + \\ 1 & \longleftarrow \end{array} $	$/\cdot \frac{-1}{4}$							
0	2	-3	4	2	-1	Ĵ							
1	0	1	-1	0	1 ←	+							
0	Λ	4	-3	-2	2 /:4								
0	2	0	$\frac{7}{4}$	$\frac{1}{2}$	$\frac{1}{2}$ /:2			$\alpha_1$	$\beta_1$	$\gamma$	1		
1	0	0	$-\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$ /:2	<i>T</i> =	=	$\alpha_2$	$\beta_2$	· γ	2		
								$L^{\alpha_3}$	$ ho_3$	$\gamma$	3_		





			$\alpha_i$	$\beta_i$	$\gamma_i$						$\alpha_i$	$\beta_i$	$\gamma_i$	
0	2	1	1	Λ	1			1	0	0	$-\frac{1}{1}$	1	1	
2	2	-1	2	2	1 ←	+					4	2	2	
1	0	1	-1	0	1 1 ← 1 /· 1 ← -1 /·	(-2)		0	1	0	$-\frac{1}{4}$ $\frac{7}{8}$ $-\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	
0	2	1	1	0	1 ←	+		0	0	1	_ 3	_ 1	<u>1</u>	
0	2	-3	4	2	-1 /·	(-1)		Ū		_	4	2	2	
1	0	1	-1	0	1						Γ	1	1	17
0	0	4	-3	-2	2 /	$\frac{3}{4}$ /· ·	$\frac{-1}{4}$					4	2	2
0	2	-3	4	2	<b>-1</b> ←	ブ +  /	J			<b>T</b> =	=	<u>7</u> 8	$\frac{1}{4}$	$\frac{1}{4}$
$\begin{array}{c} 1 \\ \hline 0 \\ 0 \\ 1 \\ \end{array}$	0	1	-1	0	2 /· -1← 1 ←		+				=	<u>3</u>	_ <u>1</u>	1
^	^	4	2	_	~ /	4					L	4	2	2 ]
0	2	0	$\frac{7}{4}$	$\frac{1}{2}$	$\frac{1}{2}$ /	:2		$\alpha_1$	$\beta_1$	ιγ	1			
1	0	0	$-\frac{1}{4}$	$\frac{1}{2}$	1/2		<i>T</i> =	$\alpha_2$	$\beta_2$	$\frac{\gamma}{2}$	2			
								$L^{\alpha 3}$	$\rho_{\bar{s}}$	3 /	<b>3</b> ]			

$$\vec{v}=(3,-1,2)_{\mathcal{B}_1}$$

 $f_2 \xrightarrow{T} \mathcal{B}_1$ 

$$\vec{v} = (3, -1, 2)_{B_1}$$

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$

$$X_{\mathcal{B}_2} = T_{\mathcal{B}_2 \to \mathcal{B}_1} X_{\mathcal{B}_1}$$

$$\vec{v}=(3,-1,2)_{\mathcal{B}_1}$$

$$\left[\mathcal{B}_2 \stackrel{\mathcal{T}}{\longrightarrow} \mathcal{B}_1\right]$$

$$X_{\mathcal{B}_2} = T_{\mathcal{B}_2 \to \mathcal{B}_1} X_{\mathcal{B}_1}$$

$$X_{\mathcal{B}_2} =$$

$$\vec{v} = (3, -1, 2)_{B_1}$$

$$\left[\mathcal{B}_2 \stackrel{\mathcal{T}}{\longrightarrow} \mathcal{B}_1 \right]$$

$$X_{\mathcal{B}_2} = T_{\mathcal{B}_2 \to \mathcal{B}_1} X_{\mathcal{B}_1}$$

$$X_{\mathcal{B}_2} = egin{bmatrix} -rac{1}{4} & rac{1}{2} & rac{1}{2} \ rac{7}{8} & rac{1}{4} & rac{1}{4} \ -rac{3}{4} & -rac{1}{2} & rac{1}{2} \end{bmatrix}$$

$$\vec{v} = (3, -1, 2)_{B_1}$$

$$egin{pmatrix} \mathcal{B}_2 & \stackrel{\mathcal{T}}{-\!\!\!\!-\!\!\!\!-\!\!\!\!-} \mathcal{B}_1 \end{pmatrix}$$

$$X_{\mathcal{B}_2} = T_{\mathcal{B}_2 \to \mathcal{B}_1} X_{\mathcal{B}_1}$$

$$X_{\mathcal{B}_2} = \begin{bmatrix} -\frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\ \frac{7}{8} & \frac{1}{4} & \frac{1}{4} \\ -\frac{3}{4} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

$$\vec{v} = (3, -1, 2)_{B_1}$$

$$egin{pmatrix} \mathcal{B}_2 & \stackrel{\mathcal{T}}{-\!\!\!\!-\!\!\!\!-\!\!\!\!-} \mathcal{B}_1 \end{pmatrix}$$

$$X_{\mathcal{B}_2} = T_{\mathcal{B}_2 \to \mathcal{B}_1} X_{\mathcal{B}_1}$$

$$X_{\mathcal{B}_2} = \begin{bmatrix} -\frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\ \frac{7}{8} & \frac{1}{4} & \frac{1}{4} \\ -\frac{3}{4} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

$$X_{\mathcal{B}_2} =$$

$$\vec{v}=(3,-1,2)_{\mathcal{B}_1}$$

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$

$$X_{\mathcal{B}_2} = T_{\mathcal{B}_2 \to \mathcal{B}_1} X_{\mathcal{B}_1}$$

$$X_{\mathcal{B}_2} = \begin{bmatrix} -\frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\ \frac{7}{8} & \frac{1}{4} & \frac{1}{4} \\ -\frac{3}{4} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

$$X_{\mathcal{B}_2} = egin{bmatrix} -rac{1}{4} \ rac{23}{8} \ -rac{3}{4} \end{bmatrix}$$

## Napomena

$$\mathcal{B}_2 \xrightarrow{\ \ \, T \ \ } \mathcal{B}_1$$

## Napomena

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1 \qquad \mathcal{B}_{\mathsf{kan}} \xrightarrow{T_1} \mathcal{B}_1$$

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$
  $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_1} \mathcal{B}_1$   $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_2} \mathcal{B}_2$ 

$$\mathcal{B}_2 \stackrel{\mathcal{T}}{\longrightarrow} \mathcal{B}_1 \qquad \mathcal{B}_{\mathsf{kan}} \stackrel{\mathcal{T}_1}{\longrightarrow} \mathcal{B}_1 \qquad \mathcal{B}_{\mathsf{kan}} \stackrel{\mathcal{T}_2}{\longrightarrow} \mathcal{B}_2$$

$$\mathcal{B}_2 \stackrel{\mathcal{T}}{\longrightarrow} \mathcal{B}_1 \qquad \mathcal{B}_{\mathsf{kan}} \stackrel{\mathcal{T}_1}{\longrightarrow} \mathcal{B}_1 \qquad \mathcal{B}_{\mathsf{kan}} \stackrel{\mathcal{T}_2}{\longrightarrow} \mathcal{B}_2$$

$$\mathcal{B}_1 = ig\{ (1,2,-1),\, (0,2,0),\, (1,1,1) ig\}$$
  $\mathcal{B}_2 = ig\{ (0,2,1),\, (2,2,0),\, (1,-1,1) ig\}$   $\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0,0),\, (0,1,0),\, (0,0,1) ig\}$ 

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$
  $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_1} \mathcal{B}_1$   $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_2} \mathcal{B}_2$   $\mathcal{T}_1 = \begin{bmatrix} 1 & & \\ 2 & & \\ -1 & & \end{bmatrix}$ 

$$\mathcal{B}_1 = ig\{ (1,2,-1),\, (0,2,0),\, (1,1,1) ig\}$$
  $\mathcal{B}_2 = ig\{ (0,2,1),\, (2,2,0),\, (1,-1,1) ig\}$   $\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0,0),\, (0,1,0),\, (0,0,1) ig\}$ 

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$
  $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_1} \mathcal{B}_1$   $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_2} \mathcal{B}_2$   $\mathcal{T}_1 = \begin{bmatrix} 1 & 0 \\ 2 & 2 \\ -1 & 0 \end{bmatrix}$ 

$$\mathcal{B}_1 = ig\{ (1,2,-1),\, (0,2,0),\, (1,1,1) ig\}$$
  $\mathcal{B}_2 = ig\{ (0,2,1),\, (2,2,0),\, (1,-1,1) ig\}$   $\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0,0),\, (0,1,0),\, (0,0,1) ig\}$ 

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$
  $\mathcal{B}_{kan} \xrightarrow{T_1} \mathcal{B}_1$   $\mathcal{B}_{kan} \xrightarrow{T_2} \mathcal{B}_2$  
$$T_1 = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\mathcal{B}_1 = ig\{ (1,2,-1),\, (0,2,0),\, (1,1,1) ig\}$$
  $\mathcal{B}_2 = ig\{ (0,2,1),\, (2,2,0),\, (1,-1,1) ig\}$   $\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0,0),\, (0,1,0),\, (0,0,1) ig\}$ 

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$
  $\mathcal{B}_{kan} \xrightarrow{T_1} \mathcal{B}_1$   $\mathcal{B}_{kan} \xrightarrow{T_2} \mathcal{B}_2$   $T_1 = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$   $T_2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ 

$$\mathcal{B}_1 = ig\{ (1,2,-1),\, (0,2,0),\, (1,1,1) ig\}$$
  $\mathcal{B}_2 = ig\{ (0,2,1),\, (2,2,0),\, (1,-1,1) ig\}$   $\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0,0),\, (0,1,0),\, (0,0,1) ig\}$ 

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$
  $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_1} \mathcal{B}_1$   $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_2} \mathcal{B}_2$   $\mathcal{T}_1 = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$   $\mathcal{T}_2 = \begin{bmatrix} 0 & & & \\ 2 & & & \\ 1 & & & \end{bmatrix}$ 

$$\mathcal{B}_1 = ig\{ (1,2,-1),\, (0,2,0),\, (1,1,1) ig\}$$
  $\mathcal{B}_2 = ig\{ (0,2,1),\, (2,2,0),\, (1,-1,1) ig\}$   $\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0,0),\, (0,1,0),\, (0,0,1) ig\}$ 

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$
  $\mathcal{B}_{kan} \xrightarrow{T_1} \mathcal{B}_1$   $\mathcal{B}_{kan} \xrightarrow{T_2} \mathcal{B}_2$ 

$$T_1 = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$
  $T_2 = \begin{bmatrix} 0 & 2 \\ 2 & 2 \\ 1 & 0 \end{bmatrix}$ 

$$\mathcal{B}_1 = ig\{ (1,2,-1),\, (0,2,0),\, (1,1,1) ig\}$$
  $\mathcal{B}_2 = ig\{ (0,2,1),\, (2,2,0),\, (1,-1,1) ig\}$   $\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0,0),\, (0,1,0),\, (0,0,1) ig\}$ 

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$
  $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_1} \mathcal{B}_1$   $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_2} \mathcal{B}_2$   $T_1 = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$   $T_2 = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 2 & -1 \\ 1 & 0 & 1 \end{bmatrix}$ 

$$\mathcal{B}_1 = ig\{ (1,2,-1),\, (0,2,0),\, (1,1,1) ig\}$$
  $\mathcal{B}_2 = ig\{ (0,2,1),\, (2,2,0),\, (1,-1,1) ig\}$   $\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0,0),\, (0,1,0),\, (0,0,1) ig\}$ 

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$
  $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_1} \mathcal{B}_1$   $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_2} \mathcal{B}_2$   $T_1 = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$   $T_2 = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 2 & -1 \\ 1 & 0 & 1 \end{bmatrix}$ 

$$\mathcal{B}_2 \longrightarrow \mathcal{B}_{\mathsf{kan}} \longrightarrow \mathcal{B}_1$$

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$
  $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_1} \mathcal{B}_1$   $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_2} \mathcal{B}_2$   $T_1 = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$   $T_2 = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 2 & -1 \\ 1 & 0 & 1 \end{bmatrix}$ 

$$\mathcal{B}_2 \xrightarrow{\mathcal{T}_2^{-1}} \mathcal{B}_{\mathsf{kan}} \longrightarrow \mathcal{B}_1$$

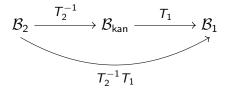
$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$
  $\mathcal{B}_{kan} \xrightarrow{T_1} \mathcal{B}_1$   $\mathcal{B}_{kan} \xrightarrow{T_2} \mathcal{B}_2$ 

$$T_1 = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$
  $T_2 = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 2 & -1 \\ 1 & 0 & 1 \end{bmatrix}$ 

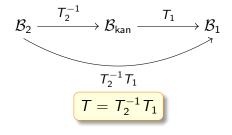
$$\mathcal{B}_2 \xrightarrow{\quad T_2^{-1} \quad} \mathcal{B}_{\mathsf{kan}} \xrightarrow{\quad T_1 \quad} \mathcal{B}_1$$

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$
  $\mathcal{B}_{kan} \xrightarrow{T_1} \mathcal{B}_1$   $\mathcal{B}_{kan} \xrightarrow{T_2} \mathcal{B}_2$ 

$$T_1 = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$
  $T_2 = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 2 & -1 \\ 1 & 0 & 1 \end{bmatrix}$ 



$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$
  $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_1} \mathcal{B}_1$   $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_2} \mathcal{B}_2$   $T_1 = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$   $T_2 = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 2 & -1 \\ 1 & 0 & 1 \end{bmatrix}$ 



$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$
  $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_1} \mathcal{B}_1$   $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_2} \mathcal{B}_2$   $T_1 = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$   $T_2 = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 2 & -1 \\ 1 & 0 & 1 \end{bmatrix}$ 

$$\mathcal{B}_{2} \xrightarrow{T_{2}^{-1}} \mathcal{B}_{kan} \xrightarrow{T_{1}} \mathcal{B}_{1}$$

$$T_{2}^{-1}T_{1}$$

$$T = T_{2}^{-1}T_{1}$$

$$T_2^{-1} = egin{bmatrix} -rac{1}{4} & rac{1}{4} & rac{1}{2} \ rac{3}{8} & rac{1}{8} & -rac{1}{4} \ rac{1}{4} & -rac{1}{4} & rac{1}{2} \end{bmatrix}$$

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$
  $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_1} \mathcal{B}_1$   $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_2} \mathcal{B}_2$   $T_1 = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$   $T_2 = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 2 & -1 \\ 1 & 0 & 1 \end{bmatrix}$ 

$$\mathcal{B}_{2} \xrightarrow{T_{2}^{-1}} \mathcal{B}_{kan} \xrightarrow{T_{1}} \mathcal{B}_{1} \qquad \boxed{DZ} \begin{bmatrix} -\frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ \frac{3}{8} & \frac{1}{8} & -\frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{2} \end{bmatrix}$$

$$T = T_{2}^{-1} T_{1}$$

$$\mathcal{B}_2 \stackrel{\mathcal{T}}{\longrightarrow} \mathcal{B}_1 \qquad \qquad \mathcal{B}_{\mathsf{kan}} \stackrel{\mathcal{T}_1}{\longrightarrow} \mathcal{B}_1 \qquad \qquad \mathcal{B}_{\mathsf{kan}} \stackrel{\mathcal{T}_2}{\longrightarrow} \mathcal{B}_2$$

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$
  $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_1} \mathcal{B}_1$   $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_2} \mathcal{B}_2$   $\mathcal{X}_{\mathcal{B}_2} = T\mathcal{X}_{\mathcal{B}_1}$ 

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$
  $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_1} \mathcal{B}_1$   $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_2} \mathcal{B}_2$   $\mathcal{X}_{\mathcal{B}_2} = T\mathcal{X}_{\mathcal{B}_1}$   $\mathcal{X}_{\mathcal{B}_{\mathsf{kan}}} = T_1\mathcal{X}_{\mathcal{B}_1}$ 

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$
  $\mathcal{B}_{kan} \xrightarrow{T_1} \mathcal{B}_1$   $\mathcal{B}_{kan} \xrightarrow{T_2} \mathcal{B}_2$ 

$$X_{\mathcal{B}_2} = TX_{\mathcal{B}_1}$$
  $X_{\mathcal{B}_{kan}} = T_1X_{\mathcal{B}_1}$   $X_{\mathcal{B}_{kan}} = T_2X_{\mathcal{B}_2}$ 

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$
  $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_1} \mathcal{B}_1$   $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_2} \mathcal{B}_2$ 
 $\mathcal{X}_{\mathcal{B}_2} = T\mathcal{X}_{\mathcal{B}_1}$   $\mathcal{X}_{\mathcal{B}_{\mathsf{kan}}} = T_1\mathcal{X}_{\mathcal{B}_1}$   $\mathcal{X}_{\mathcal{B}_{\mathsf{kan}}} = T_2\mathcal{X}_{\mathcal{B}_2}$ 

$$\vec{v}=(3,-1,2)_{\mathcal{B}_1}$$

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$
  $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_1} \mathcal{B}_1$   $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_2} \mathcal{B}_2$   $\mathcal{X}_{\mathcal{B}_2} = T\mathcal{X}_{\mathcal{B}_1}$   $\mathcal{X}_{\mathcal{B}_{\mathsf{kan}}} = T_1\mathcal{X}_{\mathcal{B}_1}$   $\mathcal{X}_{\mathcal{B}_{\mathsf{kan}}} = T_2\mathcal{X}_{\mathcal{B}_2}$ 

$$ec{v}=(3,-1,2)_{\mathcal{B}_1}$$
  $ec{v}=(5,6,-1)_{\mathcal{B}_{\mathsf{kan}}}$ 

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$
  $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_1} \mathcal{B}_1$   $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_2} \mathcal{B}_2$   $\mathcal{X}_{\mathcal{B}_2} = T\mathcal{X}_{\mathcal{B}_1}$   $\mathcal{X}_{\mathcal{B}_{\mathsf{kan}}} = T_1\mathcal{X}_{\mathcal{B}_1}$   $\mathcal{X}_{\mathcal{B}_{\mathsf{kan}}} = T_2\mathcal{X}_{\mathcal{B}_2}$   $\vec{v} = (3, -1, 2)_{\mathcal{B}_1}$   $\vec{v} = (5, 6, -1)_{\mathcal{B}_{\mathsf{kan}}}$ 

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$
  $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_1} \mathcal{B}_1$   $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T_2} \mathcal{B}_2$ 
 $X_{\mathcal{B}_2} = TX_{\mathcal{B}_1}$   $X_{\mathcal{B}_{\mathsf{kan}}} = T_1X_{\mathcal{B}_1}$   $X_{\mathcal{B}_{\mathsf{kan}}} = T_2X_{\mathcal{B}_2}$ 
 $\vec{v} = (3, -1, 2)_{\mathcal{B}_1}$ 
 $\vec{v} = (5, 6, -1)_{\mathcal{B}_{\mathsf{kan}}}$ 

$$X_{\mathcal{B}_2} =$$

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$
  $\mathcal{B}_{kan} \xrightarrow{T_1} \mathcal{B}_1$   $\mathcal{B}_{kan} \xrightarrow{T_2} \mathcal{B}_2$ 

$$X_{\mathcal{B}_2} = TX_{\mathcal{B}_1}$$
  $X_{\mathcal{B}_{kan}} = T_1X_{\mathcal{B}_1}$   $X_{\mathcal{B}_{kan}} = T_2X_{\mathcal{B}_2}$ 

$$\vec{v} = (3, -1, 2)_{\mathcal{B}_1}$$

$$\vec{v} = (5, 6, -1)_{\mathcal{B}_{kan}}$$

$$X_{\mathcal{B}_2} = \begin{bmatrix} -\frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ \frac{3}{8} & \frac{1}{8} & -\frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{2} \end{bmatrix}$$

$$\mathcal{B}_{2} \xrightarrow{T} \mathcal{B}_{1} \qquad \mathcal{B}_{kan} \xrightarrow{T_{1}} \mathcal{B}_{1} \qquad \mathcal{B}_{kan} \xrightarrow{T_{2}} \mathcal{B}_{2}$$

$$X_{\mathcal{B}_{2}} = TX_{\mathcal{B}_{1}} \qquad X_{\mathcal{B}_{kan}} = T_{1}X_{\mathcal{B}_{1}} \qquad X_{\mathcal{B}_{kan}} = T_{2}X_{\mathcal{B}_{2}}$$

$$\vec{V} = (3, -1, 2)_{\mathcal{B}_{1}}$$

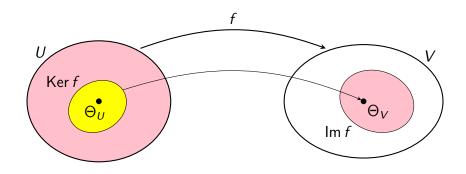
$$\vec{V} = (5, 6, -1)_{\mathcal{B}_{kan}}$$

$$\vec{V} = (5, 6, -1)_{\mathcal{B}_{kan}}$$

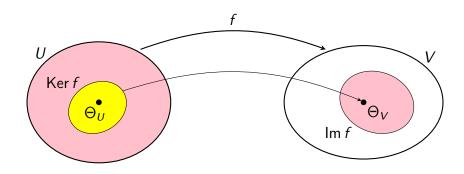
$$X_{\mathcal{B}_{2}} = \begin{bmatrix} -\frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ \frac{3}{8} & \frac{1}{8} & -\frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ -1 \end{bmatrix}$$

$$\mathcal{B}_2 \xrightarrow{T} \mathcal{B}_1$$
  $\mathcal{B}_{kan} \xrightarrow{T_1} \mathcal{B}_1$   $\mathcal{B}_{kan} \xrightarrow{T_2} \mathcal{B}_2$ 
 $X_{\mathcal{B}_2} = TX_{\mathcal{B}_1}$   $X_{\mathcal{B}_{kan}} = T_1X_{\mathcal{B}_1}$   $X_{\mathcal{B}_{kan}} = T_2X_{\mathcal{B}_2}$ 
 $\vec{v} = (3, -1, 2)_{\mathcal{B}_1}$ 
 $\vec{v} = (5, 6, -1)_{\mathcal{B}_{kan}}$ 

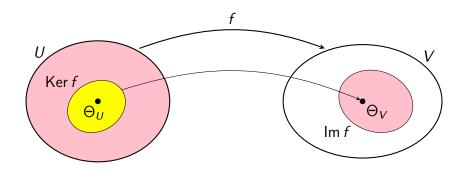
$$X_{\mathcal{B}_2} = \begin{bmatrix} -\frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ \frac{3}{8} & \frac{1}{8} & -\frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ -1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{4} \\ \frac{23}{8} \\ \frac{3}{4} \end{bmatrix}$$



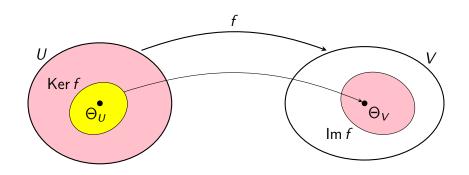
• 
$$f(\alpha a + \beta b) = \alpha f(a) + \beta f(b)$$
,  $\alpha, \beta \in F$ ,  $a, b \in U$ 



- $f(\alpha a + \beta b) = \alpha f(a) + \beta f(b)$ ,  $\alpha, \beta \in F$ ,  $a, b \in U$
- $r(f) = \dim(\operatorname{Im} f)$ ,  $d(f) = \dim(\operatorname{Ker} f)$ ,  $r(f) + d(f) = \dim U$



- $f(\alpha a + \beta b) = \alpha f(a) + \beta f(b)$ ,  $\alpha, \beta \in F$ ,  $a, b \in U$
- $r(f) = \dim(\operatorname{Im} f)$ ,  $d(f) = \dim(\operatorname{Ker} f)$ ,  $r(f) + d(f) = \dim U$
- $f: U \to V$  je injekcija  $\iff d(f) = 0$



- $f(\alpha a + \beta b) = \alpha f(a) + \beta f(b)$ ,  $\alpha, \beta \in F$ ,  $a, b \in U$
- $r(f) = \dim(\operatorname{Im} f)$ ,  $d(f) = \dim(\operatorname{Ker} f)$ ,  $r(f) + d(f) = \dim U$
- $f: U \to V$  je injekcija  $\iff d(f) = 0$
- $f: U \to V$  je surjekcija  $\iff r(f) = \dim V \pmod{V < \infty}$

# drugi zadatak

#### Zadatak 2

Zadano je preslikavanje  $h:M_2(\mathbb{R}) \to \mathbb{R}^2$  s

$$h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a+d, a-b+c).$$

- a) Dokažite da je h linearni operator.
- b) Odredite jezgru, sliku, rang i defekt operatora h.
- c) Odredite matrični zapis operatora h u paru kanonskih baza.

 $h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a+d, a-b+c)$ 

**Rješenje**  $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

a)

Rješenje 
$$h: M_2(\mathbb{R}) \to \mathbb{R}^2$$
a)  $h(\alpha A + \beta B)$   $h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a+d, a-b+c)$ 

10 / 26

 $h(\alpha A + \beta B) =$ 

 $h\left( \begin{vmatrix} a & b \\ c & d \end{vmatrix} \right) = (a+d, a-b+c)$ 

Rješenje  $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ h)  $h(\alpha A + \beta B) \stackrel{?}{=} \alpha h(A) + \beta h(B)$ 

 $h(\alpha A + \beta B) = h$ 

 $h(\alpha A + \beta B) = h \begin{pmatrix} \alpha \cdot \begin{vmatrix} a_1 & b_1 \\ c_1 & d_1 \end{vmatrix}$ 

$$h(\alpha A + \beta B) = h \left( \alpha \cdot \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \right)$$

$$h(\alpha A + \beta B) = h \left( \alpha \cdot \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \beta \cdot \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \right)$$

$$h(\alpha A + \beta B) = h \left( \alpha \cdot \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \beta \cdot \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \right)$$

$$egin{aligned} h(lpha A + eta B) &= h \left( lpha \cdot egin{bmatrix} a_1 & b_1 \ c_1 & d_1 \end{bmatrix} + eta \cdot egin{bmatrix} a_2 & b_2 \ c_2 & d_2 \end{bmatrix} 
ight) = \ &= h \left( \end{aligned}$$

$$egin{aligned} h(lpha A + eta B) &= h \left( lpha \cdot egin{bmatrix} a_1 & b_1 \ c_1 & d_1 \end{bmatrix} + eta \cdot egin{bmatrix} a_2 & b_2 \ c_2 & d_2 \end{bmatrix} 
ight) = \ &= h \left( \left[ 
ight] \end{aligned}$$

$$egin{aligned} h(lpha A + eta B) &= h \left( lpha \cdot egin{bmatrix} a_1 & b_1 \ c_1 & d_1 \end{bmatrix} + eta \cdot egin{bmatrix} a_2 & b_2 \ c_2 & d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 \end{bmatrix} 
ight] \end{aligned}$$

$$h(\alpha A + \beta B) = h \left( \alpha \cdot \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \beta \cdot \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \right) =$$

$$= h \left( \begin{bmatrix} \alpha a_1 + \beta a_2 & \alpha b_1 + \beta b_2 \end{bmatrix} \right)$$

$$egin{aligned} h(lpha A + eta B) &= h \left( lpha \cdot egin{bmatrix} a_1 & b_1 \ c_1 & d_1 \end{bmatrix} + eta \cdot egin{bmatrix} a_2 & b_2 \ c_2 & d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha b_1 + eta b_2 \ lpha c_1 + eta c_2 & \end{bmatrix} 
ight) \end{aligned}$$

$$egin{aligned} h(lpha A + eta B) &= h \left( lpha \cdot egin{bmatrix} a_1 & b_1 \ c_1 & d_1 \end{bmatrix} + eta \cdot egin{bmatrix} a_2 & b_2 \ c_2 & d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha b_1 + eta b_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \end{bmatrix} \end{aligned} 
ight)$$

$$egin{aligned} h(lpha A + eta B) &= h \left( lpha \cdot egin{bmatrix} a_1 & b_1 \ c_1 & d_1 \end{bmatrix} + eta \cdot egin{bmatrix} a_2 & b_2 \ c_2 & d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha b_1 + eta b_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \end{bmatrix} 
ight) \end{aligned}$$

$$h(\alpha A + \beta B) = h \left( \alpha \cdot \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \beta \cdot \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \right) =$$

$$= h \left( \begin{bmatrix} \alpha a_1 + \beta a_2 & \alpha b_1 + \beta b_2 \\ \alpha c_1 + \beta c_2 & \alpha d_1 + \beta d_2 \end{bmatrix} \right) =$$

$$h(\alpha A + \beta B) = h \left( \alpha \cdot \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \beta \cdot \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \right) =$$

$$= h \left( \begin{bmatrix} \alpha a_1 + \beta a_2 & \alpha b_1 + \beta b_2 \\ \alpha c_1 + \beta c_2 & \alpha d_1 + \beta d_2 \end{bmatrix} \right) =$$

$$egin{align} h(lpha A + eta B) &= h \left( lpha \cdot egin{bmatrix} a_1 & b_1 \ c_1 & d_1 \end{bmatrix} + eta \cdot egin{bmatrix} a_2 & b_2 \ c_2 & d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha b_1 + eta b_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha d_1 + eta d_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha d_1 + eta d_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \end{bmatrix} 
ight) = h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha a_1 + eta a_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \ \end{array} 
ight) = h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha a_1 + eta a_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \ \end{array} 
ight) = h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha a_1 + eta a_2 \ lpha c_1 + eta c_2 & lpha a_1 + eta c_2 \ \end{array} 
ight) = h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha a_1 + eta a_2 \ lpha a_1 + eta a_2 & lpha a_1 + eta a_2 \ \end{array} 
ight) = h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha a_2 \ a_1 + eta a_2 & lpha a_2 \ \end{array} 
ight) = h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha a_2 \ a_2 & lpha a_2 \ \end{array} 
ight) = h \left( egin{bmatrix} lpha a_2 & lpha a_2 \ a_2 & lpha a_2 \ \end{array} 
ight) = h \left( egin{bmatrix} lpha a_2 & lpha a_2 \ a_2 & lpha a_2 \ \end{array} 
ight) = h \left( egin{bmatrix} lpha a_2 & lpha a_2 \ a_2 & lpha a_2 \ \end{array} 
ight) = h \left( egin{bmatrix} lpha a_2 & lpha a_2 \ a_2 & lpha a_2 \ \end{array} 
ight) = h \left( egin{bmatrix} lpha a_2 & lpha a_2 \ a_2 & lpha a_2 \ \end{array} 
ight) = h \left( egin{bmatrix} lpha a_2 & lpha a_2 \ a_2 & lpha a_2 \ \end{array} 
ight) = h \left( egin{bmatrix} lpha a_2 & lpha a_2 \ a_2 & lpha a_2 \ \end{array} 
ight) = h \left( egin{bmatrix} lpha a_2 & lpha a_2 \ a_2 & lpha a_2 \ \end{array} 
ight) = h \left( egin{bmatrix} lpha a_2 & lpha a_2 \ a_2 & lpha a_2 \ \end{array} 
ight) = h \left( egin{bmatrix} lpha a_2 & lpha a_2 \ a_2 & lpha a_2 \ \end{array} 
ight) = h \left( egi a_2 & lpha a_2 \ a_2 & lpha a_2 \ \end{array} 
ight) = h \left( egin{bmatri$$

 $=((\alpha a_1 + \beta a_2)$ 

$$egin{align} h(lpha A + eta B) &= h \left( lpha \cdot egin{bmatrix} a_1 & b_1 \ c_1 & d_1 \end{bmatrix} + eta \cdot egin{bmatrix} a_2 & b_2 \ c_2 & d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha b_1 + eta b_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha d_1 + eta d_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha d_1 + eta d_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \end{bmatrix} 
ight) = h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha a_1 + eta a_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \ \end{array} 
ight) = h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha a_1 + eta a_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \ \end{array} 
ight) = h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha a_1 + eta a_2 \ lpha c_1 + eta c_2 & lpha a_1 + eta c_2 \ \end{array} 
ight) = h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha a_1 + eta a_2 \ lpha a_1 + eta a_2 & lpha a_1 + eta a_2 \ \end{array} 
ight) = h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha a_2 \ a_1 + eta a_2 & lpha a_2 \ \end{array} 
ight) = h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha a_2 \ a_2 & lpha a_2 \ \end{array} 
ight) = h \left( egin{bmatrix} lpha a_2 & lpha a_2 \ a_2 & lpha a_2 \ \end{array} 
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ight) = h \left( egin{bmatrix} lpha a_2 & lpha a_2 \ a_2 & lpha a_2 \ \end{array} 
ight) = h \left( egin{bmatrix} lpha a_2 & lpha a_2 \ a_2 & lpha a_2 \ \end{array} 
ight) = h \left( egin{bmatrix} lpha a_2 & lpha a_2 \ a_2 & lpha a_2 \ \end{array} 
ight) = h \left( egi a_2 & lpha a_2 \ a_2 & lpha a_2 \ \end{array} 
ight) = h \left( egin{bmatri$$

 $=((\alpha a_1 + \beta a_2) +$ 

$$egin{aligned} h(lpha A + eta B) &= h \left( lpha \cdot egin{bmatrix} a_1 & b_1 \ c_1 & d_1 \end{bmatrix} + eta \cdot egin{bmatrix} a_2 & b_2 \ c_2 & d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha b_1 + eta b_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \end{bmatrix} 
ight) = \end{aligned}$$

 $= ((\alpha \mathbf{a}_1 + \beta \mathbf{a}_2) + (\alpha \mathbf{d}_1 + \beta \mathbf{d}_2),$ 

$$h(\alpha A + \beta B) = h \left( \alpha \cdot \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \beta \cdot \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \right) =$$

$$= h \left( \begin{bmatrix} \alpha a_1 + \beta a_2 & \alpha b_1 + \beta b_2 \\ \alpha c_1 + \beta c_2 & \alpha d_1 + \beta d_2 \end{bmatrix} \right) =$$

 $= ((\alpha a_1 + \beta a_2) + (\alpha d_1 + \beta d_2), (\alpha a_1 + \beta a_2))$ 

$$egin{aligned} h(lpha A + eta B) &= h \left( lpha \cdot egin{bmatrix} a_1 & b_1 \ c_1 & d_1 \end{bmatrix} + eta \cdot egin{bmatrix} a_2 & b_2 \ c_2 & d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha b_1 + eta b_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \end{bmatrix} 
ight) = \end{aligned}$$

 $=((\alpha a_1 + \beta a_2) + (\alpha d_1 + \beta d_2), (\alpha a_1 + \beta a_2) -$ 

$$egin{aligned} h(lpha A + eta B) &= h \left( lpha \cdot egin{bmatrix} a_1 & b_1 \ c_1 & d_1 \end{bmatrix} + eta \cdot egin{bmatrix} a_2 & b_2 \ c_2 & d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha b_1 + eta b_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \end{bmatrix} 
ight) = \end{aligned}$$

 $= ((\alpha \mathbf{a}_1 + \beta \mathbf{a}_2) + (\alpha \mathbf{d}_1 + \beta \mathbf{d}_2), (\alpha \mathbf{a}_1 + \beta \mathbf{a}_2) - (\alpha \mathbf{b}_1 + \beta \mathbf{b}_2)$ 

$$egin{aligned} h(lpha A + eta B) &= h \left( lpha \cdot egin{bmatrix} a_1 & b_1 \ c_1 & d_1 \end{bmatrix} + eta \cdot egin{bmatrix} a_2 & b_2 \ c_2 & d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha b_1 + eta b_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \end{bmatrix} 
ight) = \end{aligned}$$

 $=((\alpha a_1 + \beta a_2) + (\alpha d_1 + \beta d_2), (\alpha a_1 + \beta a_2) - (\alpha b_1 + \beta b_2) +$ 

$$egin{aligned} h(lpha A + eta B) &= h \left( lpha \cdot egin{bmatrix} a_1 & b_1 \ c_1 & d_1 \end{bmatrix} + eta \cdot egin{bmatrix} a_2 & b_2 \ c_2 & d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha b_1 + eta b_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \end{bmatrix} 
ight) = \end{aligned}$$

$$egin{aligned} h(lpha A + eta B) &= h \left( lpha \cdot egin{bmatrix} a_1 & b_1 \ c_1 & d_1 \end{bmatrix} + eta \cdot egin{bmatrix} a_2 & b_2 \ c_2 & d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha b_1 + eta b_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \end{bmatrix} 
ight) = \end{aligned}$$

$$egin{aligned} h(lpha A + eta B) &= h \left( lpha \cdot egin{bmatrix} a_1 & b_1 \ c_1 & d_1 \end{bmatrix} + eta \cdot egin{bmatrix} a_2 & b_2 \ c_2 & d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha b_1 + eta b_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \end{bmatrix} 
ight) = \end{aligned}$$

 $h\left( \left| egin{matrix} a & b \\ c & d \end{array} \right| \right) = \left( a + d, \ a - b + c \right)$ 

Rješenje  $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ )  $h(\alpha A + \beta B) \stackrel{?}{=} \alpha h(A) + \beta h(B)$ 

= (

$$egin{aligned} h(lpha A + eta B) &= h \left( lpha \cdot egin{bmatrix} a_1 & b_1 \ c_1 & d_1 \end{bmatrix} + eta \cdot egin{bmatrix} a_2 & b_2 \ c_2 & d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha b_1 + eta b_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \end{bmatrix} 
ight) = \end{aligned}$$

 $h\left( \left| egin{matrix} a & b \\ c & d \end{array} \right| \right) = \left( a + d, \ a - b + c \right)$ 

Rješenje  $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ )  $h(\alpha A + \beta B) \stackrel{?}{=} \alpha h(A) + \beta h(B)$ 

 $= (\alpha(a_1 + d_1))$ 

$$h(\alpha A + \beta B) = h \left( \alpha \cdot \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \beta \cdot \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \right) =$$

$$= h \left( \begin{bmatrix} \alpha a_1 + \beta a_2 & \alpha b_1 + \beta b_2 \\ \alpha c_1 + \beta c_2 & \alpha d_1 + \beta d_2 \end{bmatrix} \right) =$$

 $h\left( \left| egin{matrix} a & b \\ c & d \end{array} \right| \right) = \left( a + d, \ a - b + c \right)$ 

Rješenje  $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ )  $h(\alpha A + \beta B) \stackrel{?}{=} \alpha h(A) + \beta h(B)$ 

 $= (\alpha(a_1 + d_1) +$ 

$$h(\alpha A + \beta B) = h \left( \alpha \cdot \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \beta \cdot \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \right) =$$

$$= h \left( \begin{bmatrix} \alpha a_1 + \beta a_2 & \alpha b_1 + \beta b_2 \\ \alpha c_1 + \beta c_2 & \alpha d_1 + \beta d_2 \end{bmatrix} \right) =$$

 $h\left( \left| egin{matrix} a & b \\ c & d \end{array} \right| \right) = \left( a + d, \ a - b + c \right)$ 

Rješenje  $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ )  $h(\alpha A + \beta B) \stackrel{?}{=} \alpha h(A) + \beta h(B)$ 

 $= (\alpha(a_1 + d_1) + \beta(a_2 + d_2),$ 

$$= h \left( \begin{bmatrix} \alpha a_1 + \beta a_2 & \alpha b_1 + \beta b_2 \\ \alpha c_1 + \beta c_2 & \alpha d_1 + \beta d_2 \end{bmatrix} \right) =$$

 $= (\alpha(a_1 + d_1) + \beta(a_2 + d_2), \alpha(a_1 - b_1 + c_1))$ 

 $h(\alpha A + \beta B) = h \left( \alpha \cdot \begin{vmatrix} a_1 & b_1 \\ c_1 & d_1 \end{vmatrix} + \beta \cdot \begin{vmatrix} a_2 & b_2 \\ c_2 & d_2 \end{vmatrix} \right) = 0$ 

$$egin{aligned} h(lpha A + eta B) &= h \left( lpha \cdot egin{bmatrix} a_1 & b_1 \ c_1 & d_1 \end{bmatrix} + eta \cdot egin{bmatrix} a_2 & b_2 \ c_2 & d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha b_1 + eta b_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \end{bmatrix} 
ight) = \end{aligned}$$

 $= (\alpha(a_1 + d_1) + \beta(a_2 + d_2), \alpha(a_1 - b_1 + c_1) +$ 

$$h(\alpha A + \beta B) = h \left( \alpha \cdot \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \beta \cdot \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \right) =$$

$$= h \left( \begin{bmatrix} \alpha a_1 + \beta a_2 & \alpha b_1 + \beta b_2 \\ \alpha c_1 + \beta c_2 & \alpha d_1 + \beta d_2 \end{bmatrix} \right) =$$

$$= ((\alpha a_1 + \beta a_2) + (\alpha d_1 + \beta d_2), (\alpha a_1 + \beta a_2) - (\alpha b_1 + \beta b_2) + (\alpha c_1 + \beta c_2)) =$$

 $= (\alpha(a_1 + d_1) + \beta(a_2 + d_2), \ \alpha(a_1 - b_1 + c_1) + \beta(a_2 - b_2 + c_2))$ 

 $h\left( \begin{vmatrix} a & b \\ c & d \end{vmatrix} \right) = (a+d, a-b+c)$ 

Rješenje  $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ )  $h(\alpha A + \beta B) \stackrel{?}{=} \alpha h(A) + \beta h(B)$ 

$$egin{aligned} h(lpha A + eta B) &= h \left( lpha \cdot egin{bmatrix} a_1 & b_1 \ c_1 & d_1 \end{bmatrix} + eta \cdot egin{bmatrix} a_2 & b_2 \ c_2 & d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha b_1 + eta b_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \end{bmatrix} 
ight) = \end{aligned}$$

 $= (\alpha(a_1 + d_1) + \beta(a_2 + d_2), \ \alpha(a_1 - b_1 + c_1) + \beta(a_2 - b_2 + c_2))$ 

 $h\left( \begin{vmatrix} a & b \\ c & d \end{vmatrix} \right) = (a+d, a-b+c)$ 

Rješenje  $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ )  $h(\alpha A + \beta B) \stackrel{?}{=} \alpha h(A) + \beta h(B)$ 

$$h(\alpha A + \beta B) = h \left( \alpha \cdot \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \beta \cdot \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \right) =$$

$$= h \left( \begin{bmatrix} \alpha a_1 + \beta a_2 & \alpha b_1 + \beta b_2 \\ \alpha c_1 + \beta c_2 & \alpha d_1 + \beta d_2 \end{bmatrix} \right) =$$

 $=(\alpha(a_1+d_1)+\beta(a_2+d_2),\ \alpha(a_1-b_1+c_1)+\beta(a_2-b_2+c_2))=$ 

 $h\left( \begin{vmatrix} a & b \\ c & d \end{vmatrix} \right) = (a+d, a-b+c)$ 

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$   $h(\alpha A + \beta B) \stackrel{?}{=} \alpha h(A) + \beta h(B)$ 

 $= \alpha \cdot ($ 

$$egin{aligned} h(lpha A + eta B) &= h \left( lpha \cdot egin{bmatrix} a_1 & b_1 \ c_1 & d_1 \end{bmatrix} + eta \cdot egin{bmatrix} a_2 & b_2 \ c_2 & d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha b_1 + eta b_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \end{bmatrix} 
ight) = \end{aligned}$$

 $=(\alpha(a_1+d_1)+\beta(a_2+d_2), \ \alpha(a_1-b_1+c_1)+\beta(a_2-b_2+c_2))=$ 

 $h\left( \begin{vmatrix} a & b \\ c & d \end{vmatrix} \right) = (a+d, a-b+c)$ 

Rješenje

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$   $h(\alpha A + \beta B) \stackrel{?}{=} \alpha h(A) + \beta h(B)$ 

 $= \alpha \cdot (a_1 + d_1,$ 

$$egin{aligned} h(lpha A + eta B) &= h \left( lpha \cdot egin{bmatrix} a_1 & b_1 \ c_1 & d_1 \end{bmatrix} + eta \cdot egin{bmatrix} a_2 & b_2 \ c_2 & d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha b_1 + eta b_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \end{bmatrix} 
ight) = \end{aligned}$$

 $=(\alpha(a_1+d_1)+\beta(a_2+d_2), \ \alpha(a_1-b_1+c_1)+\beta(a_2-b_2+c_2))=$ 

 $h\left( \begin{vmatrix} a & b \\ c & d \end{vmatrix} \right) = (a+d, a-b+c)$ 

Rješenje

Giesenje  $h: M_2(\mathbb{R}) \to \mathbb{R}^2$   $h(\alpha A + \beta B) \stackrel{?}{=} \alpha h(A) + \beta h(B)$ 

 $= \alpha \cdot (a_1 + d_1, a_1 - b_1 + c_1)$ 

$$h(\alpha A + \beta B) = h \left( \alpha \cdot \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \beta \cdot \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \right) =$$

$$= h \left( \begin{bmatrix} \alpha a_1 + \beta a_2 & \alpha b_1 + \beta b_2 \\ \alpha c_1 + \beta c_2 & \alpha d_1 + \beta d_2 \end{bmatrix} \right) =$$

 $=(\alpha(a_1+d_1)+\beta(a_2+d_2), \ \alpha(a_1-b_1+c_1)+\beta(a_2-b_2+c_2))=$ 

 $h\left( \begin{vmatrix} a & b \\ c & d \end{vmatrix} \right) = (a+d, a-b+c)$ 

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$   $h(\alpha A + \beta B) \stackrel{?}{=} \alpha h(A) + \beta h(B)$ 

 $= \alpha \cdot (a_1 + d_1, a_1 - b_1 + c_1) +$ 

$$egin{aligned} h(lpha A + eta B) &= h \left( lpha \cdot egin{bmatrix} a_1 & b_1 \ c_1 & d_1 \end{bmatrix} + eta \cdot egin{bmatrix} a_2 & b_2 \ c_2 & d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha b_1 + eta b_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \end{bmatrix} 
ight) = \end{aligned}$$

 $=(\alpha(a_1+d_1)+\beta(a_2+d_2), \ \alpha(a_1-b_1+c_1)+\beta(a_2-b_2+c_2))=$ 

 $h\left( \begin{vmatrix} a & b \\ c & d \end{vmatrix} \right) = (a+d, a-b+c)$ 

Rješenje

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$   $h(\alpha A + \beta B) \stackrel{?}{=} \alpha h(A) + \beta h(B)$ 

 $= \alpha \cdot (a_1 + d_1, a_1 - b_1 + c_1) + \beta \cdot (a_1 + d_1) + \beta \cdot (a_1$ 

$$egin{aligned} h(lpha A + eta B) &= h \left( lpha \cdot egin{bmatrix} a_1 & b_1 \ c_1 & d_1 \end{bmatrix} + eta \cdot egin{bmatrix} a_2 & b_2 \ c_2 & d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha b_1 + eta b_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \end{bmatrix} 
ight) = \end{aligned}$$

 $=(\alpha(a_1+d_1)+\beta(a_2+d_2), \ \alpha(a_1-b_1+c_1)+\beta(a_2-b_2+c_2))=$ 

 $= \alpha \cdot (a_1 + d_1, a_1 - b_1 + c_1) + \beta \cdot (a_2 + d_2, a_1)$ 

 $h\left( \begin{vmatrix} a & b \\ c & d \end{vmatrix} \right) = (a+d, a-b+c)$ 

Rješenje  $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ )  $h(\alpha A + \beta B) \stackrel{?}{=} \alpha h(A) + \beta h(B)$ 

$$egin{aligned} h(lpha A + eta B) &= h \left( lpha \cdot egin{bmatrix} a_1 & b_1 \ c_1 & d_1 \end{bmatrix} + eta \cdot egin{bmatrix} a_2 & b_2 \ c_2 & d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha b_1 + eta b_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \end{bmatrix} 
ight) = \end{aligned}$$

 $= (\alpha(a_1 + d_1) + \beta(a_2 + d_2), \ \alpha(a_1 - b_1 + c_1) + \beta(a_2 - b_2 + c_2)) =$ 

 $= \alpha \cdot (a_1 + d_1, a_1 - b_1 + c_1) + \beta \cdot (a_2 + d_2, a_2 - b_2 + c_2)$ 

 $h\left( \begin{vmatrix} a & b \\ c & d \end{vmatrix} \right) = (a+d, a-b+c)$ 

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$   $h(\alpha A + \beta B) \stackrel{?}{=} \alpha h(A) + \beta h(B)$ 

$$egin{aligned} h(lpha A + eta B) &= h \left( lpha \cdot egin{bmatrix} a_1 & b_1 \ c_1 & d_1 \end{bmatrix} + eta \cdot egin{bmatrix} a_2 & b_2 \ c_2 & d_2 \end{bmatrix} 
ight) = \ &= h \left( egin{bmatrix} lpha a_1 + eta a_2 & lpha b_1 + eta b_2 \ lpha c_1 + eta c_2 & lpha d_1 + eta d_2 \end{bmatrix} 
ight) = \end{aligned}$$

Gesenje  $h: M_2(\mathbb{R}) \to \mathbb{R}^2$   $h(\alpha A + \beta B) \stackrel{?}{=} \alpha h(A) + \beta h(B)$ 

$$= ((\alpha a_1 + \beta a_2) + (\alpha d_1 + \beta d_2), (\alpha a_1 + \beta a_2) - (\alpha b_1 + \beta b_2) + (\alpha c_1 + \beta c_2)) =$$

$$= (\alpha (a_1 + d_1) + \beta (a_2 + d_2), \alpha (a_1 - b_1 + c_1) + \beta (a_2 - b_2 + c_2)) =$$

$$= \alpha \cdot (a_1 + d_1, a_1 - b_1 + c_1) + \beta \cdot (a_2 + d_2, a_2 - b_2 + c_2) =$$

$$= \alpha \cdot$$

 $egin{aligned} \mathbf{b} & \mathbf{b} : M_2(\mathbb{R}) 
ightarrow \mathbb{R}^2 \ & \mathbf{b} (lpha A + eta B) \stackrel{?}{=} lpha \mathbf{b} (A) + eta \mathbf{b} (B) \end{aligned}$  $h\left( \begin{vmatrix} a & b \\ c & d \end{vmatrix} \right) = (a+d, a-b+c)$  $h(\alpha A + \beta B) = h \left( \alpha \cdot \begin{vmatrix} a_1 & b_1 \\ c_1 & d_1 \end{vmatrix} + \beta \cdot \begin{vmatrix} a_2 & b_2 \\ c_2 & d_2 \end{vmatrix} \right) =$ 

Rješenje

$$=h\left(\begin{bmatrix}\alpha \textbf{\textit{a}}_1+\beta \textbf{\textit{a}}_2 & \alpha \textbf{\textit{b}}_1+\beta \textbf{\textit{b}}_2\\ \alpha \textbf{\textit{c}}_1+\beta \textbf{\textit{c}}_2 & \alpha \textbf{\textit{d}}_1+\beta \textbf{\textit{d}}_2\end{bmatrix}\right)=$$

$$= ((\alpha a_1 + \beta a_2) + (\alpha d_1 + \beta d_2), (\alpha a_1 + \beta a_2) - (\alpha b_1 + \beta b_2) + (\alpha c_1 + \beta c_2)) =$$

$$= (\alpha (a_1 + d_1) + \beta (a_2 + d_2), \alpha (a_1 - b_1 + c_1) + \beta (a_2 - b_2 + c_2)) =$$

$$= (\alpha(a_1 + d_1) + \beta(a_2 + d_2), \ \alpha(a_1 - b_1 + c_1) + \beta(a_2 - b_2 + c_2)) =$$

$$= \alpha \cdot (a_1 + d_1, \ a_1 - b_1 + c_1) + \beta \cdot (a_2 + d_2, \ a_2 - b_2 + c_2) =$$

$$= \alpha \cdot (a_1 + d_1, a_1 - b_1 + c_1) + \beta \cdot (a_2 + d_2, a_2 - b_2 + c_2) =$$

 $= \alpha \cdot h \left( \begin{vmatrix} a_1 & b_1 \\ c_1 & d_1 \end{vmatrix} \right)$ 

Rješenje  $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ )  $h(\alpha A + \beta B) \stackrel{?}{=} \alpha h(A) + \beta h(B)$  $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = (a+d, a-b+c)$  $h(\alpha A + \beta B) = h \left( \alpha \cdot \begin{vmatrix} a_1 & b_1 \\ c_1 & d_1 \end{vmatrix} + \beta \cdot \begin{vmatrix} a_2 & b_2 \\ c_2 & d_2 \end{vmatrix} \right) =$ 

$$=h\left(\begin{bmatrix}\alpha a_1+\beta a_2 & \alpha b_1+\beta b_2\\ \alpha c_1+\beta c_2 & \alpha d_1+\beta d_2\end{bmatrix}\right)=$$

$$= ((\alpha a_1 + \beta a_2) + (\alpha d_1 + \beta d_2), (\alpha a_1 + \beta a_2) - (\alpha b_1 + \beta b_2) + (\alpha c_1 + \beta c_2)) =$$

$$= (\alpha (a_1 + d_1) + \beta (a_2 + d_2), \alpha (a_1 - b_1 + c_1) + \beta (a_2 - b_2 + c_2)) =$$

$$= (\alpha(a_1 + d_1) + \beta(a_2 + d_2), \ \alpha(a_1 - b_1 + c_1) + \beta(a_2 - b_2 + c_2)) =$$

$$= \alpha \cdot (a_1 + d_1, \ a_1 - b_1 + c_1) + \beta \cdot (a_2 + d_2, \ a_2 - b_2 + c_2) =$$

$$= \alpha \cdot (a_1 + d_1, a_1 - b_1 + c_1) + \beta \cdot (a_2 + d_2, a_2 - b_2 + c_2) =$$

 $= \alpha \cdot h \left( \begin{vmatrix} a_1 & b_1 \\ c_1 & d_1 \end{vmatrix} \right) + \beta \cdot$ 

Rješenje 
$$h: M_2(\mathbb{R}) \to \mathbb{R}^2$$

$$h(\alpha A + \beta B) \stackrel{?}{=} \alpha h(A) + \beta h(B) \qquad h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a+d, a-b+c)$$

$$h(\alpha A + \beta B) = h\left(\alpha \cdot \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \beta \cdot \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}\right) =$$

$$=h\left(egin{bmatrix}lpha a_1+eta a_2 & lpha b_1+eta b_2\lpha c_1+eta c_2 & lpha d_1+eta d_2\end{bmatrix}
ight)=$$

$$= ((\alpha a_1 + \beta a_2) + (\alpha d_1 + \beta d_2), (\alpha a_1 + \beta a_2) - (\alpha b_1 + \beta b_2) + (\alpha c_1 + \beta c_2)) =$$

$$= (\alpha(a_1 + d_1) + \beta(a_2 + d_2), \alpha(a_1 - b_1 + c_1) + \beta(a_2 - b_2 + c_2)) =$$

$$= (\alpha(a_1 + d_1) + \beta(a_2 + d_2), \ \alpha(a_1 - b_1 + c_1) + \beta(a_2 - b_2 + c_2)) =$$

$$= \alpha \cdot (a_1 + d_1, \ a_1 - b_1 + c_1) + \beta \cdot (a_2 + d_2, \ a_2 - b_2 + c_2) =$$

$$= \alpha \cdot (a_1 + d_1, a_1 - b_1 + c_1) + \beta \cdot (a_2 + d_2, a_2 - b_2 + c_2) =$$

$$egin{aligned} &= lpha \cdot ig( \mathsf{a_1} + \mathsf{d_1}, \ \mathsf{a_1} - \mathsf{b_1} + \mathsf{c_1} ig) + eta \cdot ig( \mathsf{a_2} + \mathsf{d_2}, \ \mathsf{a_2} - \mathsf{b_2} + \mathsf{c_2} ig) = \ &= lpha \cdot h \left( egin{bmatrix} \mathsf{a_1} & \mathsf{b_1} \ \mathsf{c_1} & \mathsf{d_1} \end{bmatrix} 
ight) + eta \cdot h \left( egin{bmatrix} \mathsf{a_2} & \mathsf{b_2} \ \mathsf{c_2} & \mathsf{d_2} \end{bmatrix} 
ight) \end{aligned}$$

Rješenje  $h: M_2(\mathbb{R}) \to \mathbb{R}^2$   $h(\alpha A + \beta B) \stackrel{?}{=} \alpha h(A) + \beta h(B)$  $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = (a+d, a-b+c)$  $h(\alpha A + \beta B) = h \left( \alpha \cdot \begin{vmatrix} a_1 & b_1 \\ c_1 & d_1 \end{vmatrix} + \beta \cdot \begin{vmatrix} a_2 & b_2 \\ c_2 & d_2 \end{vmatrix} \right) =$ 

$$= h \left( \begin{bmatrix} \alpha \mathbf{a}_1 + \beta \mathbf{a}_2 & \alpha \mathbf{b}_1 + \beta \mathbf{b}_2 \\ \alpha \mathbf{c}_1 + \beta \mathbf{c}_2 & \alpha \mathbf{d}_1 + \beta \mathbf{d}_2 \end{bmatrix} \right) =$$

 $= ((\alpha \mathbf{a}_1 + \beta \mathbf{a}_2) + (\alpha \mathbf{d}_1 + \beta \mathbf{d}_2), (\alpha \mathbf{a}_1 + \beta \mathbf{a}_2) - (\alpha \mathbf{b}_1 + \beta \mathbf{b}_2) + (\alpha \mathbf{c}_1 + \beta \mathbf{c}_2)) =$ 

$$= (\alpha(a_1 + d_1) + \beta(a_2 + d_2), \ \alpha(a_1 - b_1 + c_1) + \beta(a_2 - b_2 + c_2)) =$$

$$= \alpha \cdot (a_1 + d_1, \ a_1 - b_1 + c_1) + \beta \cdot (a_2 + d_2, \ a_2 - b_2 + c_2) =$$

$$= \alpha \cdot (a_1 + d_1, a_1 - b_1 + c_1) + \beta \cdot (a_2 + d_2, a_2 - b_2 + c_2) =$$

 $= \alpha \cdot h \left( \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} \right) + \beta \cdot h \left( \begin{vmatrix} a_2 & b_2 \\ c_2 & d_2 \end{vmatrix} \right) = \alpha h(A) + \beta h(B)$ 

$$h: M_2(\mathbb{R}) \to \mathbb{R}^2 \qquad \qquad h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \left(a+d, \ a-b+c\right)$$

b)

$$h\left(egin{bmatrix} a & b \ c & d \end{bmatrix}
ight) = \Theta_{\mathbb{R}^2}$$

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a+d, a-b+c)$ 

b)

$$h\left(egin{bmatrix} a & b \ c & d \end{bmatrix}
ight) = \Theta_{\mathbb{R}^2}$$
  $(a+d,\ a-b+c)$ 

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

Ker h

 $h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a+d, a-b+c)$ 

$$(a+d, a-b+c)=(0,0)$$

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

b)

$$(a+d, a-b+c)=(0,0)$$

a + d = 0

 $h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a+d, a-b+c)$ 

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

b)

a + d = 0a - b + c = 0

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = (a+d, a-b+c)$ 

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

(a+d, a-b+c) = (0,0)

b)

 $\begin{vmatrix}
 a+d=0 \\
 a-b+c=0
 \end{vmatrix}$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = (a+d, a-b+c)$ 

b)

Ker h

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

 $\begin{vmatrix} a+d=0 \\ a-b+c=0 \end{vmatrix} \longrightarrow d=-a$ 

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

(a+d, a-b+c) = (0,0)

b)

$$(a+d, a-b+c)=(0,0)$$

a+d=0 a-b+c=0 c=-a+b

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

$$\operatorname{\mathsf{Ker}} h =$$

a+d=0 a-b+c=0 c=-a+b

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

(a+d, a-b+c)=(0,0)

$$\operatorname{\mathsf{Ker}} h = \left\{ \left[ \begin{array}{c} \end{array} \right] \right.$$

a+d=0 a-b+c=0 c=-a+b

b)

Ker h

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

$$\operatorname{\mathsf{Ker}} h = \left\{ \left[ \begin{array}{c} a \\ \end{array} \right] \right\}$$

a+d=0 a-b+c=0 c=-a+b

b)

Ker h

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

$$\operatorname{Ker} h = \left\{ \begin{bmatrix} a & b \end{bmatrix} \right\}$$

a+d=0 a-b+c=0 c=-a+b

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

(a+d, a-b+c) = (0,0)

b)

$$(a+d, a-b+c) = (0,0)$$

$$\operatorname{Ker} h = \left\{ \begin{bmatrix} a & b \\ -a+b \end{bmatrix} \right\}$$

a+d=0 a-b+c=0 c=-a+b

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

Ker h

11/26

$$h: M_2(\mathbb{R}) \to \mathbb{R}^2 \qquad \qquad h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a+d, a-b+c)$$

$$h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \Theta_{\mathbb{R}^2} \qquad \qquad a+d=0 \} \longrightarrow d = -a$$

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

$$h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \Theta_{\mathbb{R}^2} \qquad a+d=0$$

$$a-b+c=0$$

$$(a+d, a-b+c) = (0,0)$$

$$\operatorname{Ker} h = \left\{ \begin{bmatrix} a & b \\ -a + b & -a \end{bmatrix} \right.$$

$$\begin{cases} -a + b - a \end{cases}$$

$$\operatorname{\mathsf{Ker}} h = \left\{ egin{bmatrix} a & b \ -a + b & -a \end{bmatrix} : a, b \in \mathbb{R} 
ight\}$$

a+d=0 a-b+c=0 c=-a+b

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

(a+d, a-b+c)=(0,0)

$$\operatorname{Ker} h = \left\{ \begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} : a, b \in \mathbb{R} \right\}$$

$$\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} =$$

a+d=0 a-b+c=0 c=-a+b

b)

Ker h

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

$$\operatorname{Ker} h = \left\{ \begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} : a, b \in \mathbb{R} \right\}$$

$$\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} = a \cdot \begin{bmatrix} \\ \\ \end{bmatrix}$$

a+d=0 a-b+c=0 c=-a+b

b)

Ker h

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

$$\operatorname{Ker} h = \left\{ \begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} : a, b \in \mathbb{R} \right\}$$

$$\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} = a \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

a+d=0 a-b+c=0 c=-a+b

b)

Ker h

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

$$\operatorname{Ker} h = \left\{ \begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} : a, b \in \mathbb{R} \right\}$$

$$\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} = a \cdot \begin{bmatrix} 1 & 0 \end{bmatrix}$$

a+d=0 a-b+c=0 c=-a+b

b)

Ker h

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

$$\operatorname{Ker} h = \left\{ \begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} : a, b \in \mathbb{R} \right\}$$

$$\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} = a \cdot \begin{bmatrix} 1 & 0 \\ -1 \end{bmatrix}$$

a+d=0 a-b+c=0 c=-a+b

b)

Ker h

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

$$\operatorname{Ker} h = \left\{ \begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} : a, b \in \mathbb{R} \right\}$$

$$\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} = a \cdot \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}$$

a+d=0 a-b+c=0 c=-a+b

b)

Ker h

 $h:M_2(\mathbb{R})\to\mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

$$\operatorname{Ker} h = \left\{ \begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} : a, b \in \mathbb{R} \right\}$$

$$\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} = a \cdot \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix} +$$

a+d=0 a-b+c=0 c=-a+b

b)

Ker h

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

$$\operatorname{Ker} h = \left\{ \begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} : a, b \in \mathbb{R} \right\}$$

$$\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} = a \cdot \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix} + b \cdot \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}$$

a+d=0 a-b+c=0 c=-a+b

b)

Ker h

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

$$\operatorname{Ker} h = \left\{ \begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} : a, b \in \mathbb{R} \right\}$$

$$\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} = a \cdot \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix} + b \cdot \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

a+d=0 a-b+c=0 c=-a+b

b)

Ker h

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

$$\operatorname{Ker} h = \left\{ \begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} : a, b \in \mathbb{R} \right\}$$

$$\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} = a \cdot \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix} + b \cdot \begin{bmatrix} 0 & 1 \end{bmatrix}$$

a+d=0 a-b+c=0 c=-a+b

b)

Ker h

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

$$\operatorname{Ker} h = \left\{ \begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} : a, b \in \mathbb{R} \right\}$$

$$\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} = a \cdot \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix} + b \cdot \begin{bmatrix} 0 & 1 \\ 1 & \end{bmatrix}$$

a+d=0 a-b+c=0 c=-a+b

b)

Ker h

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

(a+d, a-b+c) = (0,0)

$$\operatorname{Ker} h = \left\{ \begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} : a, b \in \mathbb{R} \right\}$$

$$\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} = a \cdot \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix} + b \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

a+d=0 a-b+c=0 c=-a+b

b)

Ker h

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

(a+d, a-b+c) = (0,0)

$$\operatorname{Ker} h = \left\{ \begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} : a, b \in \mathbb{R} \right\} \quad \mathcal{B}_{\operatorname{Ker} h} = \left\{ \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$$
$$\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} = a \cdot \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix} + b \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

a+d=0 a-b+c=0 c=-a+b

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\left|\begin{matrix} a & b \\ c & d \end{matrix}\right|\right) = \Theta_{\mathbb{R}^2}$ 

(a+d, a-b+c)=(0,0)

$$\operatorname{Ker} h = \left\{ \begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} : a, b \in \mathbb{R} \right\} \quad \mathcal{B}_{\operatorname{Ker} h} = \left\{ \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$$

$$d(h) = 2$$

$$\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} = a \cdot \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix} + b \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

a+d=0 a-b+c=0 c=-a+b

 $h:M_2(\mathbb{R})\to\mathbb{R}^2$ 

 $h\left(\left|\begin{matrix} a & b \\ c & d \end{matrix}\right|\right) = \Theta_{\mathbb{R}^2}$ 

(a+d, a-b+c)=(0,0)

$$\operatorname{Ker} h = \left\{ \begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} : a, b \in \mathbb{R} \right\} \quad \mathcal{B}_{\operatorname{Ker} h} = \left\{ \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\} \quad d(h) = 2 \neq 0$$

$$\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} = a \cdot \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix} + b \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

a+d=0 a-b+c=0 c=-a+b

 $h:M_2(\mathbb{R})\to\mathbb{R}^2$ 

 $h\left(\left|\begin{matrix} a & b \\ c & d \end{matrix}\right|\right) = \Theta_{\mathbb{R}^2}$ 

(a+d, a-b+c)=(0,0)

$$\operatorname{Ker} h = \left\{ \begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} : a, b \in \mathbb{R} \right\} \quad \mathcal{B}_{\operatorname{Ker} h} = \left\{ \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\} \quad d(h) = 2 \neq 0$$

$$\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} = a \cdot \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix} + b \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad h \text{ nije injekcija}$$

a+d=0 a-b+c=0 c=-a+b

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

 $\big(a+d,\ a-b+c\big)=(0,0)$ 

$$\operatorname{Ker} h = \left\{ \begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} : a, b \in \mathbb{R} \right\} \quad \mathcal{B}_{\operatorname{Ker} h} = \left\{ \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\} \quad d(h) = 2 \neq 0$$

$$\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} = a \cdot \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix} + b \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad h \text{ nije injekcija}$$

a+d=0 a-b+c=0 c=-a+b

 $h:M_2(\mathbb{R})\to\mathbb{R}^2$ 

 $h\left(\left|\begin{matrix} a & b \\ c & d \end{matrix}\right|\right) = \Theta_{\mathbb{R}^2}$ 

 $\big(a+d,\ a-b+c\big)=(0,0)$ 

Ker h

Im h

$$\operatorname{Ker} h = \left\{ \begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} : a, b \in \mathbb{R} \right\} \quad \mathcal{B}_{\operatorname{Ker} h} = \left\{ \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\} \quad d(h) = 2 \neq 0$$

$$\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} = a \cdot \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix} + b \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad h \text{ nije injekcija}$$

a+d=0 a-b+c=0 c=-a+b

b)

Ker h

 $h:M_2(\mathbb{R})\to\mathbb{R}^2$ 

 $r(h) + d(h) = \dim M_2(\mathbb{R})$ 

 $h\left(\left|\begin{matrix} a & b \\ c & d \end{matrix}\right|\right) = \Theta_{\mathbb{R}^2}$ 

(a+d, a-b+c)=(0,0)

$$h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \Theta_{\mathbb{R}^2} \qquad a+d=0 \\ a-b+c=0 \end{cases} \xrightarrow{\text{www}} d=-a \\ a-b+c=0 \end{cases}$$

$$(a+d, a-b+c) = (0,0)$$

$$\text{Ker } h = \left\{ \begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} : a,b \in \mathbb{R} \right\} \xrightarrow{\text{Ker } h} = \left\{ \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$$

$$d(h) = 2 \neq 0$$

$$\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} = a \cdot \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix} + b \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad h \text{ nije injekcija}$$

 $h:M_2(\mathbb{R})\to\mathbb{R}^2$ 

 $r(h) + d(h) = \dim M_2(\mathbb{R})$ 

b)

Ker h

Im h

r(h) =

11/26

$$h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \Theta_{\mathbb{R}^2} \qquad a+d=0 \\ a-b+c=0 \end{cases} \xrightarrow{\text{www}} d=-a \\ a-b+c=0 \end{cases}$$

$$(a+d, a-b+c) = (0,0)$$

$$\text{Ker } h = \left\{ \begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} : a,b \in \mathbb{R} \right\} \xrightarrow{\text{Ker } h} = \left\{ \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$$

$$d(h) = 2 \neq 0$$

$$\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} = a \cdot \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix} + b \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad h \text{ nije injekcija}$$

b)

Ker h

r(h) = 4 - 2

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

$$(a+d, a-b+c) = (0,0)$$

$$\operatorname{Ker} h = \left\{ \begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} : a, b \in \mathbb{R} \right\}$$

$$\mathcal{B}_{\operatorname{Ker} h} = \left\{ \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$$

$$d(h) = 2 \neq 0$$

$$\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} = a \cdot \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix} + b \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$h \text{ nije injekcija}$$

a+d=0 a-b+c=0 c=-a+b

b)

Ker h

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

r(h) = 4 - 2 = 2

$$h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \Theta_{\mathbb{R}^2} \qquad a+d=0 \\ a-b+c=0 \end{cases} \xrightarrow{\mathsf{www}} d = -a \\ a-b+c=0 \end{cases} \xrightarrow{\mathsf{www}} c = -a+b$$

$$(a+d, a-b+c) = (0,0)$$

$$\mathsf{Ker} \, h = \left\{ \begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} : a,b \in \mathbb{R} \right\} \, \mathcal{B}_{\mathsf{Ker} \, h} = \left\{ \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$$

b)

Ker h

 $h:M_2(\mathbb{R})\to\mathbb{R}^2$ 

 $\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} = a \cdot \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix} + b \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \begin{array}{c} d(h) = 2 \neq 0 \\ h \text{ nije injekcija} \end{array}$ 

$$r(h) = 4 - 2 = 2$$
  $r(h) = 2$ 

$$h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \Theta_{\mathbb{R}^2} \qquad a+d=0 \\ a-b+c=0 \end{cases} \xrightarrow{\text{www}} d=-a \\ a-b+c=0 \end{cases}$$

$$(a+d, a-b+c) = (0,0)$$

$$\text{Ker } h = \left\{ \begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} : a,b \in \mathbb{R} \right\} \xrightarrow{\text{Ker } h} = \left\{ \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$$

$$d(h) = 2 \neq 0$$

$$\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} = a \cdot \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix} + b \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad h \text{ nije injekcija}$$

b)

Ker h

 $h:M_2(\mathbb{R})\to\mathbb{R}^2$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = \Theta_{\mathbb{R}^2}$ 

r(h) = 4 - 2 = 2  $r(h) = 2 = \dim \mathbb{R}^2$ 

Ker 
$$h$$
:  $M_2(\mathbb{R}) \to \mathbb{R}^2$  
$$h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a+d, a-b+c)$$
$$h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \Theta_{\mathbb{R}^2} \qquad a+d=0$$
$$a-b+c=0$$
$$a+d=0$$
$$a-b+c=0$$

b)

 $h:M_2(\mathbb{R})\to\mathbb{R}^2$ 

 $\operatorname{\mathsf{Ker}} h = \left\{ \begin{bmatrix} a & b \\ -a + b & -a \end{bmatrix} : a, b \in \mathbb{R} \right\} \, \mathcal{B}_{\operatorname{\mathsf{Ker}} h} = \left\{ \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$  $\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} = a \cdot \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix} + b \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$   $d(h) = 2 \neq 0$  h nije injekcija

11/26

b) 
$$h: M_2(\mathbb{R}) \to \mathbb{R}^2$$
  $h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a+d, a-b+c)$   $h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \Theta_{\mathbb{R}^2}$   $a+d=0$   $a+d$ 

$$\begin{bmatrix} a \\ -a + b \end{bmatrix}$$

b)

 $\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} = a \cdot \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix} + b \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$   $d(h) = 2 \neq 0$  h nije injekcija

$$=\operatorname{\mathsf{dim}} \mathbb{R}^2$$

 $\operatorname{\mathsf{Im}} h = \mathbb{R}^2$ 

b) 
$$h: M_{2}(\mathbb{R}) \to \mathbb{R}^{2}$$

$$h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a+d, a-b+c)$$

$$h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \Theta_{\mathbb{R}^{2}} \qquad a+d=0 \\ a-b+c=0 \end{cases} \xrightarrow{\text{www}} d = -a$$

$$a-b+c=0 \end{cases} \xrightarrow{\text{www}} c = -a+b$$

$$(a+d, a-b+c) = (0,0)$$

$$\text{Ker } h = \left\{ \begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} : a,b \in \mathbb{R} \right\} \xrightarrow{\text{Ker } h} = \left\{ \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$$

$$d(h) = 2 \neq 0$$

$$\begin{bmatrix} a & b \\ -a+b & -a \end{bmatrix} = a \cdot \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix} + b \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \begin{array}{c} d(h) = 2 \neq 0 \\ h \text{ nije injekcija} \end{array}$$

b)

 $\mathcal{B}_{\mathsf{Im}\,h} = \{(1,0),(0,1)\}$ r(h) = 4 - 2 = 2  $r(h) = 2 = \dim \mathbb{R}^2$   $\lim h = \mathbb{R}^2$   $\lim h = \mathbb{R}^2$ 

c) 
$$h: M_2(\mathbb{R}) \to \mathbb{R}^2$$
 
$$h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a+d, a-b+c)$$

$$\mathcal{B}_1 = \left\{ egin{bmatrix} 1 & 0 \ 0 & 0 \end{bmatrix}, egin{bmatrix} 0 & 1 \ 0 & 0 \end{bmatrix}, egin{bmatrix} 0 & 0 \ 1 & 0 \end{bmatrix}, egin{bmatrix} 0 & 0 \ 0 & 1 \end{bmatrix} 
ight\}$$

$$\mathcal{B}_1 = \left\{ egin{bmatrix} 1 & 0 \ 0 & 0 \end{bmatrix}, egin{bmatrix} 0 & 1 \ 0 & 0 \end{bmatrix}, egin{bmatrix} 0 & 0 \ 1 & 0 \end{bmatrix}, egin{bmatrix} 0 & 0 \ 0 & 1 \end{bmatrix} 
ight\} \qquad \mathcal{B}_2 = \left\{ (1,0), (0,1) 
ight\}$$

$$\mathcal{B}_1 = \left\{ egin{bmatrix} 1 & 0 \ 0 & 0 \end{bmatrix}, egin{bmatrix} 0 & 1 \ 0 & 0 \end{bmatrix}, egin{bmatrix} 0 & 0 \ 1 & 0 \end{bmatrix}, egin{bmatrix} 0 & 0 \ 0 & 1 \end{bmatrix} 
ight\} \qquad \mathcal{B}_2 = \left\{ (1,0), (0,1) 
ight\} \ h\left( egin{bmatrix} 1 & 0 \ 0 & 0 \end{bmatrix} 
ight) =$$

$$egin{aligned} \mathcal{B}_1 &= \left\{ egin{bmatrix} 1 & 0 \ 0 & 0 \end{bmatrix}, egin{bmatrix} 0 & 1 \ 0 & 0 \end{bmatrix}, egin{bmatrix} 0 & 0 \ 1 & 0 \end{bmatrix}, egin{bmatrix} 0 & 0 \ 0 & 1 \end{bmatrix} 
ight\} \qquad \mathcal{B}_2 &= \left\{ (1,0), (0,1) 
ight\} \ h\left( egin{bmatrix} 1 & 0 \ 0 & 0 \end{bmatrix} 
ight) &= (1,1) \end{aligned}$$

$$h\left(\begin{bmatrix}1&0\\0&0\end{bmatrix}\right)=(1,1)=1\cdot(1,0)$$

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = (a+d, a-b+c)$ 

$$egin{aligned} \mathcal{B}_1 &= \left\{ egin{bmatrix} 1 & 0 \ 0 & 0 \end{bmatrix}, egin{bmatrix} 0 & 1 \ 0 & 0 \end{bmatrix}, egin{bmatrix} 0 & 0 \ 1 & 0 \end{bmatrix}, egin{bmatrix} 0 & 0 \ 0 & 1 \end{bmatrix} 
ight\} \qquad \mathcal{B}_2 &= \left\{ (1,0), (0,1) 
ight\} \ h\left( egin{bmatrix} 1 & 0 \ 0 & 0 \end{bmatrix} 
ight) &= (1,1) = 1 \cdot (1,0) + \end{aligned}$$

$$h\left(egin{bmatrix}1&0\0&0\end{bmatrix}
ight)=(1,1)=1\cdot(1,0)+1\cdot(0,1)$$

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = (a+d, a-b+c)$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = (a+d, a-b+c)$ 

$$egin{aligned} h\left(egin{bmatrix}1&0\0&0\end{bmatrix}
ight) &= (1,1) = 1\cdot(1,0) + 1\cdot(0,1) & H &= egin{bmatrix}1\\1&&& \end{bmatrix} \ h\left(egin{bmatrix}0&1\0&0\end{bmatrix}
ight) &= & \end{aligned}$$

 $\mathcal{B}_2 = \big\{ (1,0), (0,1) \big\}$ 

 $h:M_2(\mathbb{R})\to\mathbb{R}^2$ 

 $\mathcal{B}_1 = \left\{ egin{bmatrix} 1 & 0 \ 0 & 0 \end{bmatrix}, egin{bmatrix} 0 & 1 \ 0 & 0 \end{bmatrix}, egin{bmatrix} 0 & 0 \ 1 & 0 \end{bmatrix}, egin{bmatrix} 0 & 0 \ 0 & 1 \end{bmatrix} 
ight\}$ 

$$egin{align} h\left(egin{bmatrix}1&0\0&0\end{bmatrix}
ight) &= (1,1) = 1\cdot(1,0) + 1\cdot(0,1) \qquad H = egin{bmatrix}1\\1 \end{pmatrix} \ h\left(egin{bmatrix}0&1\0&0\end{bmatrix}
ight) &= (0,-1) \end{aligned}$$

 $\mathcal{B}_2 = \big\{ (1,0), (0,1) \big\}$ 

 $h:M_2(\mathbb{R})\to\mathbb{R}^2$ 

 $\mathcal{B}_1 = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ 

$$egin{align} h\left(egin{bmatrix}1&0\0&0\end{bmatrix}
ight) &= (1,1) = 1\cdot(1,0) + 1\cdot(0,1) \qquad H = egin{bmatrix}1\\1 \end{pmatrix} \ h\left(egin{bmatrix}0&1\0&0\end{bmatrix}
ight) &= (0,-1) = 0\cdot(1,0) \end{split}$$

 $\mathcal{B}_2 = \big\{ (1,0), (0,1) \big\}$ 

 $h:M_2(\mathbb{R}) o \mathbb{R}^2$ 

 $\mathcal{B}_1 = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ 

$$egin{align} h\left(egin{bmatrix}1&0\0&0\end{bmatrix}
ight) &= (1,1) = 1\cdot(1,0) + 1\cdot(0,1) \qquad H = egin{bmatrix}1\\1 \end{pmatrix} \ h\left(egin{bmatrix}0&1\0&0\end{bmatrix}
ight) &= (0,-1) = 0\cdot(1,0) + \end{aligned}$$

 $\mathcal{B}_2 = \big\{ (1,0), (0,1) \big\}$ 

 $h:M_2(\mathbb{R}) o \mathbb{R}^2$ 

 $\mathcal{B}_1 = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ 

$$egin{align} h\left(egin{bmatrix} 1 & 0 \ 0 & 0 \end{bmatrix}
ight) &= (1,1) = 1\cdot(1,0) + 1\cdot(0,1) \qquad H = egin{bmatrix} 1 \ 1 \end{bmatrix} \ h\left(egin{bmatrix} 0 & 1 \ 0 & 0 \end{bmatrix}
ight) &= (0,-1) = 0\cdot(1,0) + (-1)\cdot(0,1) \ \end{array}$$

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = (a+d, a-b+c)$ 

$$egin{aligned} h\left(egin{bmatrix} 1 & 0 \ 0 & 0 \end{bmatrix}
ight) &= (1,1) = 1\cdot(1,0) + 1\cdot(0,1) & H &= egin{bmatrix} 1 & 0 \ 1 & -1 \end{bmatrix} \ h\left(egin{bmatrix} 0 & 1 \ 0 & 0 \end{bmatrix}
ight) &= (0,-1) = 0\cdot(1,0) + (-1)\cdot(0,1) \end{aligned}$$

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = (a+d, a-b+c)$ 

$$egin{aligned} h\left(egin{bmatrix} 1 & 0 \ 0 & 0 \end{bmatrix}
ight) &= (1,1) = 1\cdot (1,0) + 1\cdot (0,1) & H &= egin{bmatrix} 1 & 0 \ 1 & -1 & \end{bmatrix} \ h\left(egin{bmatrix} 0 & 1 \ 0 & 0 \end{bmatrix}
ight) &= (0,-1) = 0\cdot (1,0) + (-1)\cdot (0,1) \end{aligned}$$

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = (a+d, a-b+c)$ 

 $h:M_2(\mathbb{R})\to\mathbb{R}^2$ 

c)

 $h\left(\begin{bmatrix}0&0\\1&0\end{bmatrix}\right) =$ 

$$h\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = (1,1) = 1 \cdot (1,0) + 1 \cdot (0,1) \qquad H = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$$
 $h\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right) = (0,-1) = 0 \cdot (1,0) + (-1) \cdot (0,1)$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = (a+d, a-b+c)$ 

 $h:M_2(\mathbb{R})\to\mathbb{R}^2$ 

 $h\left(\begin{bmatrix}0&0\\1&0\end{bmatrix}\right)=(0,1)$ 

c)

$$egin{aligned} h\left(egin{bmatrix}1&0\0&0\end{bmatrix}
ight) &= (1,1) = 1\cdot(1,0) + 1\cdot(0,1) & H &= egin{bmatrix}1&0\1&-1 & \end{bmatrix} \ h\left(egin{bmatrix}0&1\0&0\end{bmatrix}
ight) &= (0,-1) = 0\cdot(1,0) + (-1)\cdot(0,1) \end{aligned}$$

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = (a+d, a-b+c)$ 

 $h:M_2(\mathbb{R})\to\mathbb{R}^2$ 

 $h\left(\begin{bmatrix}0&0\\1&0\end{bmatrix}\right)=(0,1)=0\cdot(1,0)$ 

c)

$$egin{aligned} h\left(egin{bmatrix} 1 & 0 \ 0 & 0 \end{bmatrix}
ight) &= (1,1) = 1\cdot (1,0) + 1\cdot (0,1) & H &= egin{bmatrix} 1 & 0 \ 1 & -1 \end{bmatrix} \ h\left(egin{bmatrix} 0 & 1 \ 0 & 0 \end{bmatrix}
ight) &= (0,-1) = 0\cdot (1,0) + (-1)\cdot (0,1) \end{aligned}$$

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = (a+d, a-b+c)$ 

 $h:M_2(\mathbb{R})\to\mathbb{R}^2$ 

 $h\left(\begin{bmatrix}0&0\\1&0\end{bmatrix}\right)=(0,1)=0\cdot(1,0)+$ 

c)

$$egin{aligned} h\left(egin{bmatrix}1&0\0&0\end{bmatrix}
ight) &= (1,1) = 1\cdot(1,0) + 1\cdot(0,1) & H &= egin{bmatrix}1&0\1&-1 & \end{bmatrix} \ h\left(egin{bmatrix}0&1\0&0\end{bmatrix}
ight) &= (0,-1) = 0\cdot(1,0) + (-1)\cdot(0,1) \end{aligned}$$

 $\mathcal{B}_1 = \left\{ egin{bmatrix} 1 & 0 \ 0 & 0 \end{bmatrix}, egin{bmatrix} 0 & 1 \ 0 & 0 \end{bmatrix}, egin{bmatrix} 0 & 0 \ 1 & 0 \end{bmatrix}, egin{bmatrix} 0 & 0 \ 0 & 1 \end{bmatrix} 
ight\} \qquad \mathcal{B}_2 = \left\{ (1,0), (0,1) 
ight\}$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = (a+d, a-b+c)$ 

 $h:M_2(\mathbb{R}) o \mathbb{R}^2$ 

 $h\left(\begin{bmatrix}0&0\\1&0\end{bmatrix}\right)=(0,1)=0\cdot(1,0)+1\cdot(0,1)$ 

c)

$$egin{aligned} h\left(egin{bmatrix} 1 & 0 \ 0 & 0 \end{bmatrix}
ight) &= (1,1) = 1\cdot (1,0) + 1\cdot (0,1) & H &= egin{bmatrix} 1 & 0 & 0 \ 1 & -1 & 1 \end{bmatrix} \ h\left(egin{bmatrix} 0 & 1 \ 0 & 0 \end{bmatrix}
ight) &= (0,-1) = 0\cdot (1,0) + (-1)\cdot (0,1) \end{aligned}$$

 $\mathcal{B}_1 = \left\{ egin{bmatrix} 1 & 0 \ 0 & 0 \end{bmatrix}, egin{bmatrix} 0 & 1 \ 0 & 0 \end{bmatrix}, egin{bmatrix} 0 & 0 \ 1 & 0 \end{bmatrix}, egin{bmatrix} 0 & 0 \ 0 & 1 \end{bmatrix} 
ight\} \qquad \mathcal{B}_2 = \left\{ (1,0), (0,1) 
ight\}$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = (a+d, a-b+c)$ 

 $h:M_2(\mathbb{R})\to\mathbb{R}^2$ 

 $h\left(\begin{bmatrix}0&0\\1&0\end{bmatrix}\right)=(0,1)=0\cdot(1,0)+1\cdot(0,1)$ 

c)

$$h: M_2(\mathbb{R}) \to \mathbb{R}^2 \qquad \qquad h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \left(a+d, \ a-b+c\right)$$

$$\mathcal{B}_1 = \left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right\} \qquad \mathcal{B}_2 = \left\{(1,0), (0,1)\right\}$$

$$h\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = (1,1) = 1 \cdot (1,0) + 1 \cdot (0,1)$$
  $H = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 1 \end{bmatrix}$ 

$$h\left(\begin{bmatrix}0&1\\0&0\end{bmatrix}\right)=(0,-1)=0\cdot(1,0)+(-1)\cdot(0,1)$$

 $h:M_2(\mathbb{R})\to\mathbb{R}^2$ 

c)

$$\begin{pmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \end{pmatrix} = (0, -1) = 0 \cdot (1, 0)$$

$$\left( egin{bmatrix} 0 & 0 \ 1 & 0 \end{bmatrix} 
ight) = (0,1) = 0 \cdot (1,0) + 0$$

$$h\left(\begin{bmatrix}0&0\\1&0\end{bmatrix}\right)=(0,1)=0\cdot(1,0)+1\cdot(0,1)$$

$$\left( \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \right) = (0,1) = 0 \cdot (1,0) +$$

 $h\left(\begin{bmatrix}0&0\\0&1\end{bmatrix}\right) =$ 

$$\mathcal{B}_1 = \left\{ egin{bmatrix} 1 & 0 \ 0 & 0 \end{bmatrix}, egin{bmatrix} 0 & 1 \ 0 & 0 \end{bmatrix}, egin{bmatrix} 0 & 0 \ 1 & 0 \end{bmatrix}, egin{bmatrix} 0 & 0 \ 0 & 1 \end{bmatrix} 
ight\} \qquad \mathcal{B}_2 = \left\{ (1,0), (0,1) 
ight\}$$

 $h\left( \left| egin{matrix} 1 & 0 \ 0 & 0 \end{array} \right| 
ight) = (1,1) = 1 \cdot (1,0) + 1 \cdot (0,1)$  $h\left( \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right) = (0, -1) = 0 \cdot (1, 0) + (-1) \cdot (0, 1)$ 

 $h:M_2(\mathbb{R})\to\mathbb{R}^2$ 

c)

 $H = \begin{vmatrix} 1 & 0 & 0 \\ 1 & -1 & 1 \end{vmatrix}$ 

 $h\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = (a+d, a-b+c)$ 

 $h\left(\begin{bmatrix}0&0\\1&0\end{bmatrix}\right)=(0,1)=0\cdot(1,0)+1\cdot(0,1)$ 

 $h\left(\begin{bmatrix}0&0\\0&1\end{bmatrix}\right)=(1,0)$ 

12/26

$$h: M_2(\mathbb{R}) \to \mathbb{R}^2 \qquad \qquad h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a+d, a-b+c)$$

$$\mathcal{B}_1 = \left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right\} \qquad \mathcal{B}_2 = \left\{(1,0), (0,1)\right\}$$

$$h\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = (1,1) = 1 \cdot (1,0) + 1 \cdot (0,1)$$
  $H = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 1 \end{bmatrix}$ 

$$h\left(\begin{bmatrix}0&1\\0&0\end{bmatrix}\right)=(0,-1)=0\cdot(1,0)+(-1)\cdot(0,1)$$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = (0, -1) = 0 \cdot (1, 0)$$

 $h:M_2(\mathbb{R})\to\mathbb{R}^2$ 

c)

$$h\left(\begin{bmatrix}0&0\\1&0\end{bmatrix}\right)=(0,1)=0\cdot(1,0)+1\cdot(0,1)$$

$$h\left(\begin{bmatrix}1&0\end{bmatrix}\right)=(0,1)=0\cdot(1,0)+1$$
 $h\left(\begin{bmatrix}0&0\\0&1\end{bmatrix}\right)=(1,0)=1\cdot(1,0)$ 

12/26

$$h: M_2(\mathbb{R}) \to \mathbb{R}^2 \qquad \qquad h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \left(a+d, \ a-b+c\right)$$

$$\mathcal{B}_1 = \left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right\} \qquad \mathcal{B}_2 = \left\{(1,0), (0,1)\right\}$$

$$h\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right) = (0, -1) = 0 \cdot (1, 0) + (-1) \cdot (0, 1)$$
 $h\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right) = (0, 1) = 0 \cdot (1, 0) + 1 \cdot (0, 1)$ 

 $h\left(\begin{bmatrix}0&0\\0&1\end{bmatrix}\right)=(1,0)=1\cdot(1,0)+$ 

 $h\left( \left| egin{matrix} 1 & 0 \ 0 & 0 \end{array} \right| 
ight) = (1,1) = 1 \cdot (1,0) + 1 \cdot (0,1)$ 

 $h:M_2(\mathbb{R})\to\mathbb{R}^2$ 

c)

 $H = \begin{vmatrix} 1 & 0 & 0 \\ 1 & -1 & 1 \end{vmatrix}$ 

$$h: M_2(\mathbb{R}) \to \mathbb{R}^2 \qquad \qquad h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \left(a+d, \ a-b+c\right)$$

$$\mathcal{B}_1 = \left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right\} \qquad \mathcal{B}_2 = \left\{(1,0), (0,1)\right\}$$

$$h\left(\begin{bmatrix}1&0\\0&0\end{bmatrix}\right)=(1,1)=1\cdot(1,0)+1\cdot(0,1)$$

 $h:M_2(\mathbb{R})\to\mathbb{R}^2$ 

c)

 $H = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 1 \end{bmatrix}$ 

$$h\left(\begin{bmatrix}0&1\\0&0\end{bmatrix}\right)=(0,-1)=0\cdot(1,0)+(-1)\cdot(0,1)$$

$$(0) + (-1) \cdot (0,1)$$

$$h\left( egin{bmatrix} 0 & 0 \ 1 & 0 \end{bmatrix} 
ight) = (0,1) = 0 \cdot (1,0) + 1 \cdot (0,1)$$

$$\left[ egin{pmatrix} 0 & 0 \ 1 & 0 \end{bmatrix} 
ight) = (0,1) = 0 \cdot (1,0) +$$

 $h\left(\begin{bmatrix}0&0\\0&1\end{bmatrix}\right)=(1,0)=1\cdot(1,0)+0\cdot(0,1)$ 

$$(0,1) = 0 \cdot (1,0) + 1$$

$$h: M_2(\mathbb{R}) \to \mathbb{R}^2 \qquad \qquad h\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \left(a+d, \ a-b+c\right)$$

$$\mathcal{B}_1 = \left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right\} \qquad \mathcal{B}_2 = \left\{(1,0), (0,1)\right\}$$

$$h\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = (1,1) = 1 \cdot (1,0) + 1 \cdot (0,1) \qquad H = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & -1 & 1 & 0 \end{bmatrix}$$

$$h\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}\right) = (0,1) \cdot (0,1) \cdot (0,1)$$

$$h\left(\begin{bmatrix}0&1\\0&0\end{bmatrix}\right) = (0,-1) = 0 \cdot (1,0) + (-1) \cdot (0,1)$$

$$h\left(\begin{bmatrix}0&0\\0&0\end{bmatrix}\right) = (0,1) = 0 \cdot (1,0) + 1 \cdot (0,1)$$

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$
  $) = (0,1) = 0 \cdot (1,0) + 1 \cdot (1,0) = 0 \cdot (1,0) = 0 \cdot (1,0) = 0$ 

 $h: M_2(\mathbb{R}) \to \mathbb{R}^2$ 

$$h\left(\begin{bmatrix}0&0\\1&0\end{bmatrix}\right)=(0,1)=0\cdot(1,0)+1\cdot(0,1)$$

c)

 $h\left(\begin{bmatrix}0&0\\0&1\end{bmatrix}\right)=(1,0)=1\cdot(1,0)+0\cdot(0,1)$ 

treći zadatak

## Zadatak 3

Zadano je preslikavanje  $f: \mathbb{R}^4 \to \mathbb{R}^2$  s

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v).$$

- a) Dokažite da je f linearni operator.
- b) Odredite jezgru, sliku, rang i defekt operatora f.
- c) Odredite matrični prikaz operatora f u paru kanonskih baza.
- d) Odredite matrični prikaz operatora f u paru baza

$$\begin{split} \mathcal{A} &= \big\{ (1,0,0,0), \, (1,2,0,0), \, (1,2,3,0), \, (1,2,3,4) \big\}, \\ \mathcal{B} &= \big\{ (1,10), \, (1,11) \big\}. \end{split}$$

e) Odredite sliku vektora (1,0,-1,8).

Rješenje

 $f: \mathbb{R}^4 \to \mathbb{R}^2$ 

$$f(\alpha a + \beta b)$$

Rješenje

 $f: \mathbb{R}^4 \to \mathbb{R}^2$ 

a) 
$$f(\alpha a + \beta b) \stackrel{?}{=} \alpha f(a) + \beta f(b)$$
  $f: \mathbb{R}^4 \to \mathbb{R}^2$ 

$$f(\alpha a + \beta b) =$$

a)  $f(\alpha a + \beta b) \stackrel{?}{=} \alpha f(a) + \beta f(b)$ 

$$f(\alpha a + \beta b) = f($$

a)  $f(\alpha a + \beta b) \stackrel{?}{=} \alpha f(a) + \beta f(b)$ 

$$f(\alpha a + \beta b) = f(\alpha \cdot (x_1, y_1, u_1, v_1))$$

Rješenje

a)  $f(\alpha a + \beta b) \stackrel{?}{=} \alpha f(a) + \beta f(b)$ 

$$f(\alpha \mathbf{a} + \beta \mathbf{b}) = f(\alpha \cdot (\mathbf{x}_1, \mathbf{y}_1, \mathbf{u}_1, \mathbf{v}_1) +$$

Rješenje

a)  $f(\alpha a + \beta b) \stackrel{?}{=} \alpha f(a) + \beta f(b)$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

$$f(\alpha a + \beta b) = f(\alpha \cdot (x_1, y_1, u_1, v_1) + \beta \cdot (x_2, y_2, u_2, v_2)$$

Rješenje

a)  $f(\alpha a + \beta b) \stackrel{?}{=} \alpha f(a) + \beta f(b)$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

Rješenje

a)  $f(\alpha a + \beta b) \stackrel{?}{=} \alpha f(a) + \beta f(b)$ 

$$= f($$

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

Rješenje

a)  $f(\alpha a + \beta b) \stackrel{?}{=} \alpha f(a) + \beta f(b)$ 

$$=f(\alpha x_1+\beta x_2,$$

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

Rješenje

a)  $f(\alpha a + \beta b) \stackrel{?}{=} \alpha f(a) + \beta f(b)$ 

$$f(\alpha a + \beta b) = f(\alpha \cdot (x_1, y_1, u_1, v_1) + \beta \cdot (x_2, y_2, u_2, v_2)) =$$
  
=  $f(\alpha x_1 + \beta x_2, \alpha y_1 + \beta y_2,$ 

Rješenje

a)  $f(\alpha a + \beta b) \stackrel{?}{=} \alpha f(a) + \beta f(b)$ 

$$f(\alpha a + \beta b) = f(\alpha \cdot (x_1, y_1, u_1, v_1) + \beta \cdot (x_2, y_2, u_2, v_2)) =$$

$$= f(\alpha x_1 + \beta x_2, \alpha y_1 + \beta y_2, \alpha u_1 + \beta u_2,$$

Rješenje

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14 / 26

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a)  $f(\alpha a + \beta b) \stackrel{?}{=} \alpha f(a) + \beta f(b)$   $f: \mathbb{R}^4 \to \mathbb{R}^2$ 

$$f(\alpha a + \beta b) = f(\alpha \cdot (x_1, y_1, u_1, v_1) + \beta \cdot (x_2, y_2, u_2, v_2)) =$$

$$= f(\alpha x_1 + \beta x_2, \alpha y_1 + \beta y_2, \alpha u_1 + \beta u_2, \alpha v_1 + \beta v_2) =$$

$$= I(\alpha x_1 + \beta x_2, \alpha y_1 + \beta y_2, \alpha u_1 + \beta u_2, \alpha v_1 + \beta v_2) =$$

$$= ((\alpha x_1 + \beta x_2) + 2(\alpha y_1 + \beta y_2) - (\alpha u_1 + \beta u_2) - (\alpha v_1 + \beta v_2),$$
  
$$= (\alpha x_1 + \beta x_2) - 2(\alpha y_1 + \beta y_2) + (\alpha y_1 + \beta y_2) + (\alpha$$

$$-(\alpha x_1 + \beta x_2) - 2(\alpha y_1 + \beta y_2) + (\alpha u_1 + \beta u_2) + (\alpha v_1 + \beta v_2)) =$$

$$= (\alpha(x_1 + 2y_1 - u_1 - v_1) + \beta(x_2 + 2y_2 - u_2 - v_2),$$

$$\alpha(-x_1 - 2y_1 + u_2 + v_3) + \beta(-x_2 - 2y_2 + u_2 + v_3)) =$$

$$\alpha(-x_1-2y_1+u_1+v_1)+\beta(-x_2-2y_2+u_2+v_2))=$$

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 =

$$= ((\alpha x_1 + \beta x_2) + 2(\alpha y_1 + \beta y_2) - (\alpha u_1 + \beta u_2) - (\alpha v_1 + \beta v_2),$$

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14 / 26

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$$= \alpha \cdot (x_1 + 2y_1 - u_1 - v_1, -x_1 - 2y_1 + u_1 + v_1) +$$

 $+ \beta \cdot (x_2 + 2y_2 - u_2 - v_2)$ 

14/26

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$$\alpha(-x_1 - 2y_1 + u_1 + v_1) + \beta(-x_2 - 2y_2 + u_2 + v_2)) =$$

$$-\alpha \cdot (x_1 + 2y_2 - u_1 - y_2 - x_1 - 2y_2 + u_2 + y_2) +$$

$$=\alpha\cdot(x_1+2y_1-u_1-v_1,\,-x_1-2y_1+u_1+v_1)+$$

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 $+\beta \cdot (x_2 + 2y_2 - u_2 - v_2, -x_2 - 2y_2 + u_2 + v_2) =$ 

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 $= \alpha f(x_1, y_1, u_1, v_1) +$ 

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 $-(\alpha x_1 + \beta x_2) - 2(\alpha y_1 + \beta y_2) + (\alpha u_1 + \beta u_2) + (\alpha v_1 + \beta v_2)) =$   
 $= (\alpha (x_1 + 2y_1 - u_1 - v_1) + \beta (x_2 + 2y_2 - u_2 - v_2),$ 

 $\alpha(-x_1-2y_1+u_1+v_1)+\beta(-x_2-2y_2+u_2+v_2)=$ 

$$= \alpha \cdot (x_1 + 2y_1 + u_1 + v_1) + \beta (x_2 + 2y_2 + u_2 + v_2))$$

$$= \alpha \cdot (x_1 + 2y_1 - u_1 - v_1, -x_1 - 2y_1 + u_1 + v_1) + \beta (x_1 + x_2 + v_2)$$

$$=\alpha\cdot(x_1+2y_1-u_1-v_1,\,-x_1-2y_1+u_1+v_1)+$$

 $+\beta \cdot (x_2 + 2y_2 - u_2 - v_2, -x_2 - 2y_2 + u_2 + v_2) =$  $= \alpha f(x_1, y_1, u_1, v_1) + \beta f(x_2, y_2, u_2, v_2)$ 

14 / 26

Rješenje 
$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$
  
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 $= (\alpha (x_1 + 2y_1 - u_1 - v_1) + \beta (x_2 + 2y_2 - u_2 - v_2),$   
 $\alpha (-x_1 - 2y_1 + u_1 + v_1) + \beta (-x_2 - 2y_2 + u_2 + v_2)) =$   
 $= \alpha \cdot (x_1 + 2y_1 - u_1 - v_1, -x_1 - 2y_1 + u_1 + v_1) +$   
 $+ \beta \cdot (x_2 + 2y_2 - u_2 - v_2, -x_2 - 2y_2 + u_2 + v_2) =$   
 $= \alpha f(x_1, y_1, u_1, v_1) + \beta f(x_2, y_2, u_2, v_2) = \alpha f(a) + \beta f(b)$ 

14 / 26

b)

Ker f

 $f(x, y, u, v) = \Theta_{\mathbb{R}^2}$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

b)

Ker f

 $f(x, y, u, v) = \Theta_{\mathbb{R}^2}$ 

(x+2y-u-v, -x-2y+u+v)

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

b)

Ker f

 $f(x, y, u, v) = \Theta_{\mathbb{R}^2}$ (x + 2y - u - v, -x - 2y + u + v) = (0, 0)

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

b)

Ker f

$$x + 2y - u - v = 0$$

$$x + 2y - u - v = 0$$

 $f(x, y, u, v) = \Theta_{\mathbb{R}^2}$ (x+2y-u-v, -x-2y+u+v)=(0,0)

b)

Ker f

$$-x-2y+u+v=0$$

 $f(x, y, u, v) = \Theta_{\mathbb{R}^2}$ (x + 2y - u - v, -x - 2y + u + v) = (0, 0)

x + 2y - u - v = 0

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

b)

Ker f

$$\begin{cases}
 x + 2y - u - v = 0 \\
 -x - 2y + u + v = 0
 \end{cases}$$

 $f(x, y, u, v) = \Theta_{\mathbb{R}^2}$ (x + 2y - u - v, -x - 2y + u + v) = (0, 0)

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

b)

Ker f

b)

Ker f

 $f(x, y, u, v) = \Theta_{\mathbb{R}^2}$ 

(x+2y-u-v, -x-2y+u+v)=(0,0) x y

(x+2y-u-v, -x-2y+u+v)=(0,0) x y u v 1 2 -1 -1 0

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

b)

Ker f

 $f(x, y, u, v) = \Theta_{\mathbb{R}^2}$ 

x + 2y - u - v = 0-x - 2y + u + v = 0

 $f: \mathbb{R}^4 \to \mathbb{R}^2$ 

b)

Ker f

 $f(x, y, u, v) = \Theta_{\mathbb{R}^2}$ (x + 2y - u - v, -x - 2y + u + v) = (0, 0)

b)

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 $f(x, y, u, v) = \Theta_{\mathbb{R}^2}$ (x + 2y - u - v, -x - 2y + u + v) = (0, 0)

> x + 2y - u - v = 0-x - 2y + u + v = 0

b)

Ker f

 $f(x, y, u, v) = \Theta_{\mathbb{R}^2}$  (x + 2y - u - v, -x - 2y + u + v) = (0, 0)

x + 2y - u - v = 0-x - 2y + u + v = 0

b)

Ker f

 $f(x, y, u, v) = \Theta_{\mathbb{R}^2}$ 

(x+2y-u-v, -x-2y+u+v)=(0,0)

 $\begin{vmatrix}
 x + 2y - u - v = 0 \\
 -x - 2y + u + v = 0
 \end{vmatrix}$ 

 $f: \mathbb{R}^4 \to \mathbb{R}^2$ 

$$\begin{cases}
 x + 2y - u - v = 0 \\
 -x - 2y + u + v = 0
 \end{cases}$$

b)

Ker f

 $f(x, y, u, v) = \Theta_{\mathbb{R}^2}$ 

(x+2y-u-v, -x-2y+u+v)=(0,0)

 $f: \mathbb{R}^4 \to \mathbb{R}^2$ 

$$f(x, y, u, v) = \Theta_{\mathbb{R}^{2}}$$

$$(x + 2y - u - v, -x - 2y + u + v) = (0, 0)$$

$$x + 2y - u - v = 0$$

$$-x - 2y + u + v = 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0 \quad / \cdot 1$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

b)

Ker f

 $f: \mathbb{R}^4 \to \mathbb{R}^2$ 

$$f(x, y, u, v) = \Theta_{\mathbb{R}^{2}}$$

$$(x + 2y - u - v, -x - 2y + u + v) = (0, 0)$$

$$x + 2y - u - v = 0$$

$$-x - 2y + u + v = 0$$

$$0 \quad 0 \quad 0 \quad 0$$

$$x \quad y \quad u \quad v$$

$$-1 - 2 \quad 1 \quad 1 \quad 0$$

$$1 \quad 2 \quad -1 - 1 \quad 0$$

$$0 \quad 0 \quad 0 \quad 0$$

$$f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$f(x, y, u, v) = \Theta_{\mathbb{R}^2}$$

$$(x + 2y - u - v, -x - 2y + u + v) = (0, 0)$$

$$x + 2y - u - v = 0$$

$$-x - 2y + u + v = 0$$

$$0 \quad 0 \quad 0 \quad 0$$

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$$(x + 2y - u - v, -x - 2y + u + v) = (0, 0)$$

$$x + 2y - u - v = 0$$

$$-x - 2y + u + v = 0$$

$$x + 2y - u - v = 0$$

$$x + 2y - u - v = 0$$

$$x + 2y - u - v = 0$$

$$0 \quad 0 \quad 0 \quad 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

$$f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$f(x, y, u, v) = \Theta_{\mathbb{R}^2}$$

$$(x + 2y - u - v, -x - 2y + u + v) = (0, 0)$$

$$x + 2y - u - v = 0$$

$$-x - 2y + u + v = 0$$

$$x + 2y - u - v = 0 \longrightarrow x = -2y + u + v$$

$$0 \quad 0 \quad 0 \quad 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

$$f: \mathbb{R}^{4} \to \mathbb{R}^{2}$$

$$f(x, y, u, v) = \Theta_{\mathbb{R}^{2}}$$

$$(x + 2y - u - v, -x - 2y + u + v) = (0, 0)$$

$$x + 2y - u - v = 0$$

$$-x - 2y + u + v = 0$$

$$x + 2y - u - v = 0 \longrightarrow x = -2y + u + v$$

$$x + 2y - u - v = 0 \longrightarrow x = -2y + u + v$$

$$x + 2y - u - v = 0 \longrightarrow x = -2y + u + v$$

$$x + 2y - u - v = 0 \longrightarrow x = -2y + u + v$$

$$x + 2y - u - v = 0 \longrightarrow x = -2y + u + v$$

b)

 $\operatorname{Ker} f =$ 

$$f: \mathbb{R}^{4} \to \mathbb{R}^{2}$$

$$f(x, y, u, v) = \Theta_{\mathbb{R}^{2}}$$

$$(x + 2y - u - v, -x - 2y + u + v) = (0, 0)$$

$$x + 2y - u - v = 0$$

$$-x - 2y + u + v = 0$$

$$x + 2y - u - v = 0 \longrightarrow x = -2y + u + v$$

$$x + 2y - u - v = 0 \longrightarrow x = -2y + u + v$$

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F / 26

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b)

15/26

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b)

15/26

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b)

5 / 26

Ker 
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  $f(x, y, u, v) = \Theta_{\mathbb{R}^2}$   $(x + 2y - u - v, -x - 2y + u + v) = (0, 0)$   $x + 2y - u - v = 0$   $x + 2y + u + v = 0$   $x + 2y - u - v = 0$   $x + 2y -$ 

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b)

5 / 26

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$$x + 2y - u - v = 0$$

$$-x - 2y + u + v = 0$$

$$x + 2y - u - v = 0 \longrightarrow x = -2y + u + v$$

$$\frac{0 \quad 0 \quad 0 \quad 0}{1 \quad 2 \quad -1 \quad -1 \quad 0}$$

b)

$$(-2y+u+v,y,u,v)=y\cdot (-2,1,0,0)+u\cdot (1,0,1,0)+v\cdot (1,0,0,0)$$

$$f: \mathbb{R}^{4} \to \mathbb{R}^{2}$$

$$f(x, y, u, v) = \Theta_{\mathbb{R}^{2}}$$

$$(x + 2y - u - v, -x - 2y + u + v) = (0, 0)$$

$$x + 2y - u - v = 0$$

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$$x + 2y - u - v = 0 \longrightarrow x = -2y + u + v$$

$$\frac{0 \quad 0 \quad 0 \quad 0}{1 \quad 2 \quad -1 \quad -1 \quad 0}$$

 $(-2y + u + v, y, u, v) = y \cdot (-2, 1, 0, 0) + u \cdot (1, 0, 1, 0) + v \cdot (1, 0, 0, 1)$ 

 $Ker f = \{(-2y + u + v, y, u, v) : y, u, v \in \mathbb{R}\}$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

b)

/ 26

$$F: \mathbb{R}^{4} \to \mathbb{R}^{2}$$

$$f(x, y, u, v) = \Theta_{\mathbb{R}^{2}}$$

$$(x + 2y - u - v, -x - 2y + u + v) = (0, 0)$$

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$$x + 2y - u - v = 0 \longrightarrow x = -2y + u + v$$

$$0 \quad 0 \quad 0 \quad 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

$$0 \quad 0 \quad 0 \quad 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

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$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

 $\mathcal{B}_{\mathsf{Ker}\,f} = \{(-2,1,0,0), (1,0,1,0), (1,0,0,1)\}$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

b)

15 / 26

b) 
$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

$$f : \mathbb{R}^{4} \to \mathbb{R}^{2}$$

$$f(x, y, u, v) = \Theta_{\mathbb{R}^{2}}$$

$$(x + 2y - u - v, -x - 2y + u + v) = (0, 0)$$

$$x + 2y - u - v = 0$$

$$-x - 2y + u + v = 0$$

$$x + 2y - u - v = 0 \xrightarrow{\text{max}} x = -2y + u + v$$

$$\frac{0 \quad 0 \quad 0 \quad 0}{1 \quad 2 \quad -1 \quad -1 \quad 0}$$

$$\text{Ker } f = \left\{ (-2y + u + v, y, u, v) : y, u, v \in \mathbb{R} \right\}$$

$$(-2y + u + v, y, u, v) = y \cdot (-2, 1, 0, 0) + u \cdot (1, 0, 1, 0) + v \cdot (1, 0, 0, 1)$$

$$\mathcal{B}_{\text{Ker } f} = \left\{ (-2, 1, 0, 0), (1, 0, 1, 0), (1, 0, 0, 1) \right\}$$

$$d(f) = 3 \neq 0$$

b) 
$$f(x,y,u,v) = (x+2y-u-v, -x-2y+u+v)$$

$$f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$f(x,y,u,v) = \Theta_{\mathbb{R}^2}$$

$$(x+2y-u-v, -x-2y+u+v) = (0,0)$$

$$x+2y-u-v=0$$

$$-x-2y+u+v=0$$

$$x+2y-u-v=0 \longrightarrow x = -2y+u+v$$

$$0 \quad 0 \quad 0 \quad 0 \quad 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

$$x+2y-u-v=0 \longrightarrow x = -2y+u+v$$

$$0 \quad 0 \quad 0 \quad 0 \quad 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

$$1 \quad 2 \quad -1 \quad -1 \quad 0$$

$$1 \quad 3 \quad -1 \quad -1 \quad 0$$

$$1 \quad 3 \quad -1 \quad -1 \quad 0$$

$$1 \quad 3 \quad -1 \quad -1 \quad 0$$

$$1 \quad 3 \quad -1 \quad -1 \quad 0$$

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$$1 \quad 3 \quad -1 \quad -1 \quad 0$$

$$1 \quad 3 \quad -1 \quad -1 \quad 0$$

$$1 \quad 3 \quad -1 \quad -1 \quad 0$$

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

 $f:\mathbb{R}^4 o\mathbb{R}^2$ 

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

$$f: \mathbb{R}^4 o \mathbb{R}^2$$

$$r(f) + d(f) = \dim \mathbb{R}^4$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

$$f: \mathbb{R}^4 o \mathbb{R}^2$$

$$r(f) + d(f) = \dim \mathbb{R}^4$$
$$r(f) + 3 = 4$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

$$r(f) + d(f) = \dim \mathbb{R}^4$$
  
 $r(f) + 3 = 4$   
 $r(f) = 1$ 

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

$$r(f) + d(f) = \dim \mathbb{R}^4$$
 $r(f) + 3 = 4$   $r(f) \neq \dim \mathbb{R}^2$ 
 $r(f) = 1$ 

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

$$\operatorname{Im} f$$

r(f) = 1

$$r(f)+d(f)=\dim\mathbb{R}^4$$
  $r(f)+3=4$   $r(f)
eq \dim\mathbb{R}^2\longrightarrow f$  nije surjekcija

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

$$r(f)+d(f)=\dim\mathbb{R}^4$$
  $r(f)+3=4$   $r(f)
eq \dim\mathbb{R}^2\longrightarrow f$  nije surjekcija  $r(f)=1$ 

$$(x + 2y - u - v, -x - 2y + u + v) =$$

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$$(x + 2y - u - v, -x - 2y + u + v) =$$
  
=  $x \cdot ($ 

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

$$\operatorname{Im} f$$

$$r(f)+d(f)=\dim\mathbb{R}^4$$
  $r(f)+3=4$   $r(f)
eq \dim\mathbb{R}^2 \longrightarrow f$  nije surjekcija  $r(f)=1$ 

$$(x + 2y - u - v, -x - 2y + u + v) =$$
  
=  $x \cdot (1,$ 

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

 $\operatorname{Im} f$ 

$$f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$r(f)+d(f)=\dim\mathbb{R}^4$$
  $r(f)+3=4$   $r(f)
eq \dim\mathbb{R}^2\longrightarrow f$  nije surjekcija  $r(f)=1$ 

$$(x + 2y - u - v, -x - 2y + u + v) =$$
  
=  $x \cdot (1, -1)$ 

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

Im *f* 

$$f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$r(f)+d(f)=\dim\mathbb{R}^4$$
  $r(f)+3=4$   $r(f)\neq\dim\mathbb{R}^2\longrightarrow f$  nije surjekcija  $r(f)=1$ 

$$(x + 2y - u - v, -x - 2y + u + v) =$$
  
=  $x \cdot (1, -1) +$ 

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$$r(f)+d(f)=\dim\mathbb{R}^4$$
  $r(f)+3=4$   $r(f)
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$$(x + 2y - u - v, -x - 2y + u + v) =$$
  
=  $x \cdot (1, -1) + y \cdot ($ 

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Im *f* 

$$r(f)+d(f)=\dim\mathbb{R}^4$$
  $r(f)+3=4$   $r(f)\neq\dim\mathbb{R}^2\longrightarrow f$  nije surjekcija  $r(f)=1$ 

$$(x + 2y - u - v, -x - 2y + u + v) =$$

$$= x \cdot (1, -1) + y \cdot (2, -1)$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

$$\operatorname{Im} f$$

$$f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$r(f)+d(f)=\dim\mathbb{R}^4$$
  $r(f)+3=4$   $r(f)\neq\dim\mathbb{R}^2\longrightarrow f$  nije surjekcija  $r(f)=1$ 

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$$f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$r(f)+d(f)=\dim\mathbb{R}^4$$
  $r(f)+3=4$   $r(f)
eq \dim\mathbb{R}^2\longrightarrow f$  nije surjekcija  $r(f)=1$ 

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$$r(f)+d(f)=\dim\mathbb{R}^4$$
  $r(f)+3=4$   $r(f)
eq dim\,\mathbb{R}^2\longrightarrow f$  nije surjekcija  $r(f)=1$ 

$$(x + 2y - u - v, -x - 2y + u + v) =$$

$$= x \cdot (1, -1) + y \cdot (2, -2) + u \cdot ($$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

$$\operatorname{Im} f$$

$$f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$r(f)+d(f)=\dim\mathbb{R}^4$$
  $r(f)+3=4$   $r(f)
eq dim\,\mathbb{R}^2\longrightarrow f$  nije surjekcija  $r(f)=1$ 

$$(x + 2y - u - v, -x - 2y + u + v) =$$

$$= x \cdot (1, -1) + y \cdot (2, -2) + u \cdot (-1, -1)$$

$$-x \cdot (1,-1) + y \cdot (2,-2) + u \cdot (-1)$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

Im *f* 

$$r(f)+d(f)=\dim\mathbb{R}^4$$
  $r(f)+3=4$   $r(f)
eq 1$   $r(f) \neq \dim\mathbb{R}^2 \longrightarrow f$  nije surjekcija

$$(x + 2y - u - v, -x - 2y + u + v) =$$

$$= x \cdot (1, -1) + y \cdot (2, -2) + u \cdot (-1, 1)$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

$$\operatorname{Im} f$$

$$f:\mathbb{R}^4 o\mathbb{R}^2$$
  $r(f)+d(f)=\dim\mathbb{R}^4$   $r(f)+3$  . A surjection  $r(f)$  /  $dim\,\mathbb{R}^2$  . A piin surjection

$$r(f)+3=4$$
  $r(f)
eq \dim \mathbb{R}^2 \longrightarrow f$  nije surjekcija $r(f)=1$ 

$$(x + 2y - u - v, -x - 2y + u + v) =$$

$$= x \cdot (1, -1) + y \cdot (2, -2) + u \cdot (-1, 1) +$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

Im *f* 

$$r(f)+d(f)=\dim\mathbb{R}^4$$
  $r(f)+3=4$   $r(f)\neq\dim\mathbb{R}^2\longrightarrow f$  nije surjekcija  $r(f)=1$ 

$$(x+2y-u-v, -x-2y+u+v) =$$

$$= x \cdot (1,-1) + y \cdot (2,-2) + u \cdot (-1,1) + v \cdot (-1,1) + v$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

$$r(f)+d(f)=\dim\mathbb{R}^4$$
  $r(f)+3=4$   $r(f)=1$   $r(f)=f$  nije surjekcija

$$= x \cdot (1 - 1) + y \cdot (2 - 2) + u \cdot (-1 1) + y \cdot (-1 1)$$

(x + 2y - u - v, -x - 2y + u + v) =

$$= x \cdot (1, -1) + y \cdot (2, -2) + u \cdot (-1, 1) + v \cdot (-1, 1)$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

$$r(f)+d(f)=\dim\mathbb{R}^4$$
  $r(f)+3=4$   $r(f)
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$$(x+2y-u-v, -x-2y+u+v) =$$

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$$\operatorname{Im} f$$

$$r(f)+d(f)=\dim\mathbb{R}^4$$
  $r(f)+3=4$   $r(f)=1$   $r(f)=f$  nije surjekcija

$$(x + 2y - u - v, -x - 2y + u + v) =$$

$$= x \cdot (1, -1) + y \cdot (2, -2) + u \cdot (-1, 1) + v \cdot (-1, 1)$$

$$= x \cdot (1,-1) + y \cdot (2,-2) + u \cdot (-1,1) + v \cdot (-1,1)$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

$$f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$r(f)+d(f)=\dim\mathbb{R}^4$$
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eq \dim\mathbb{R}^2\longrightarrow f$  nije surjekcija  $r(f)=1$ 

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$$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

$$r(f)+d(f)=\dim\mathbb{R}^4$$
  $r(f)+3=4$   $r(f)
eq \dim\mathbb{R}^2\longrightarrow f$  nije surjekcija  $r(f)=1$ 

$$(x+2y-u-v, -x-2y+u+v) =$$

$$= x \cdot (1,-1) + y \cdot (2,-2) + u \cdot (-1,1) + v \cdot (-1,1)$$

$$\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

$$r(f)+d(f)=\dim\mathbb{R}^4$$
  $r(f)+3=4$   $r(f)
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$$(x+2y-u-v, -x-2y+u+v) =$$

$$= x \cdot (1,-1) + y \cdot (2,-2) + u \cdot (-1,1) + v \cdot (-1,1)$$

$$\begin{bmatrix} 1 & 2 & -1 \\ -1 & -2 & 1 \end{bmatrix}$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

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$$\begin{bmatrix} 1 & 2 & -1 & -1 \\ -1 & -2 & 1 & 1 \end{bmatrix}$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

$$\operatorname{Im} f$$

$$r(f)+d(f)=\dim\mathbb{R}^4$$
  $r(f)+3=4$   $r(f)
eq \dim\mathbb{R}^2\longrightarrow f$  nije surjekcija  $r(f)=1$ 

$$(x+2y-u-v, -x-2y+u+v) =$$

$$= x \cdot (1,-1) + y \cdot (2,-2) + u \cdot (-1,1) + v \cdot (-1,1)$$

$$\begin{bmatrix} 1 & 2 & -1 & -1 \\ -1 & -2 & 1 & 1 \end{bmatrix}$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

$$\operatorname{Im} f$$

$$r(f)+d(f)=\dim\mathbb{R}^4$$
  $r(f)+3=4$   $r(f)
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$$\begin{bmatrix} \textcircled{1} & 2 & -1 & -1 \\ -1 & -2 & 1 & 1 \end{bmatrix} / \cdot 1$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

$$f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$r(f)+d(f)=\dim\mathbb{R}^4$$
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$$\begin{bmatrix} \textcircled{1} & 2 & -1 & -1 \\ -1 & -2 & 1 & 1 \end{bmatrix} / \cdot \underbrace{1}_{+}$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

$$f: \mathbb{R}^4 o \mathbb{R}^2$$

$$r(f)+d(f)=\dim\mathbb{R}^4$$
  $r(f)+3=4$   $r(f)
eq \dim\mathbb{R}^2\longrightarrow f$  nije surjekcija  $r(f)=1$ 

$$(x+2y-u-v, -x-2y+u+v) =$$

$$= x \cdot (1,-1) + y \cdot (2,-2) + u \cdot (-1,1) + v \cdot (-1,1)$$

$$\begin{bmatrix}
1 & 2 & -1 & -1 \\
-1 & -2 & 1 & 1
\end{bmatrix} / \cdot \frac{1}{} \sim$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

$$f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$r(f)+d(f)=\dim\mathbb{R}^4$$
  $r(f)+3=4$   $r(f)
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$$\begin{bmatrix} \textcircled{1} & 2 & -1 & -1 \\ -1 & -2 & 1 & 1 \end{bmatrix} / \cdot \underbrace{1}_{\leftarrow} \sim \begin{bmatrix} 1 & 2 & -1 & -1 \\ \end{bmatrix}$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

$$r(f)+d(f)=\dim\mathbb{R}^4$$
  $r(f)+3=4$   $r(f)
eq \dim\mathbb{R}^2 \longrightarrow f$  nije surjekcija  $r(f)=1$ 

$$(x+2y-u-v, -x-2y+u+v) =$$

$$= x \cdot (1,-1) + y \cdot (2,-2) + u \cdot (-1,1) + v \cdot (-1,1)$$

$$\begin{bmatrix} \textcircled{1} & 2 & -1 & -1 \\ -1 & -2 & 1 & 1 \end{bmatrix} / \cdot \begin{array}{c} 1 \\ \\ \end{array} \sim \begin{bmatrix} 1 & 2 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

$$f: \mathbb{R}^4 o \mathbb{R}^2$$

$$r(f)+d(f)=\dim\mathbb{R}^4$$
  $r(f)+3=4$   $r(f)
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$$\begin{bmatrix} 1 & 2 & -1 & -1 \\ -1 & -2 & 1 & 1 \end{bmatrix} / \cdot \frac{1}{} \sim \begin{bmatrix} 1 & 2 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

$$f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$r(f)+d(f)=\dim\mathbb{R}^4$$
  $r(f)+3=4$   $r(f)
eq \dim\mathbb{R}^2\longrightarrow f$  nije surjekcija

$$r(f) = 1$$
  
 $(x + 2y - u - v, -x - 2y + u + v) =$ 

$$= x \cdot (1, -1) + y \cdot (2, -2) + u \cdot (-1, 1) + v \cdot (-1, 1)$$

$$\begin{bmatrix} \textcircled{1} & 2 & -1 & -1 \end{bmatrix} / \cdot 1 \qquad \begin{bmatrix} \textcircled{1} & 2 & -1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 & -1 \\ -1 & -2 & 1 & 1 \end{bmatrix} / \cdot \frac{1}{1} \sim \begin{bmatrix} 1 & 2 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathcal{B}_{\mathsf{Im}\,f} = \big\{ (1,-1) \big\}$$

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

$$\mathcal{A}_{\mathsf{kan}} = ig\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) ig\}$$

 $\mathcal{B}_{\mathsf{kan}} = \{(1,0), (0,1)\}$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

 $A_{kan} = \{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\}$ 

 $\mathcal{B}_{\mathsf{kan}} = \{(1,0),(0,1)\}$ 

f(1,0,0,0) =

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

 $A_{kan} = \{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\}$ 

 $\mathcal{B}_{\mathsf{kan}} = \{(1,0),(0,1)\}$ 

f(1,0,0,0) = (1,-1)

17 / 26

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

$$f(1,0,0,0) = (1,-1) = 1 \cdot (1,0)$$

 $\mathcal{B}_{\mathsf{kan}} = \{(1,0),(0,1)\}$ 

$$(1,0,0,0) = (1, 1) = 1 (1,0)$$

 $A_{\mathsf{kan}} = \{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\}$ 

$$\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0), (0,1) ig\}$$
  $f(1,0,0,0) = (1,-1) = 1 \cdot (1,0) +$ 

 $A_{\mathsf{kan}} = \{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\}$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

$$f(1,0,0,0) = (1,-1) = 1 \cdot (1,0) + (-1) \cdot (0,1)$$

 $\mathcal{B}_{\mathsf{kan}} = \{(1,0),(0,1)\}$ 

 $\mathcal{A}_{\mathsf{kan}} = \{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\}$ 

$$extstyle F_{(\mathcal{A}_{\mathsf{kan}},\,\mathcal{B}_{\mathsf{kan}})} = igg[$$

$$\mathcal{A}_{\mathsf{kan}} = ig\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) ig\}$$
  $\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0), (0,1) ig\}$ 

$$f(1,0,0,0) = (1,-1) = 1 \cdot (1,0) + (-1) \cdot (0,1)$$

$$f(1,0,0,0) = (1,-1) = 1 \cdot (1,0) + (-1) \cdot (0,1)$$

$$extstyle F_{(\mathcal{A}_{\mathsf{kan}},\,\mathcal{B}_{\mathsf{kan}})} = egin{bmatrix} 1 \ -1 \end{bmatrix}$$

$$\mathcal{A}_{\mathsf{kan}} = ig\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) ig\}$$
  $\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0), (0,1) ig\}$ 

$$\mathcal{B}_{\mathsf{kan}} = \{(1,0),(0,1)\}$$

$$f(1,0,0,0) = (1,-1) = 1 \cdot (1,0) + (-1) \cdot (0,1)$$

$$extstyle F_{(\mathcal{A}_{\mathsf{kan}},\,\mathcal{B}_{\mathsf{kan}})} = egin{bmatrix} 1 \ -1 \end{bmatrix}$$

$$egin{aligned} \mathcal{A}_{\mathsf{kan}} &= ig\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) ig\} \ \mathcal{B}_{\mathsf{kan}} &= ig\{ (1,0), (0,1) ig\} \end{aligned}$$

$$f(1,0,0,0) = (1,-1) = 1 \cdot (1,0) + (-1) \cdot (0,1)$$
  
 $f(0,1,0,0) =$ 

$$extstyle F_{(\mathcal{A}_{\mathsf{kan}},\,\mathcal{B}_{\mathsf{kan}})} = egin{bmatrix} 1 \ -1 \end{bmatrix}$$

 $\mathcal{A}_{\mathsf{kan}} = \{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\}$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

$$\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0), (0,1) ig\}$$
  $f(1,0,0,0) = (1,-1) = 1 \cdot (1,0) + (-1) \cdot (0,1)$ 

$$f(1,0,0,0) = (1,-1) = 1 \cdot (1,0) + (-1) \cdot (0,1)$$
  
 $f(0,1,0,0) = (2,-2)$ 

$$extstyle F_{(\mathcal{A}_{\mathsf{kan}},\mathcal{B}_{\mathsf{kan}})} = egin{bmatrix} 1 \ -1 \end{bmatrix}$$

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

$$\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0), (0,1) ig\}$$
  $f(1,0,0,0) = (1,-1) = 1 \cdot (1,0) + (-1) \cdot (0,1)$ 

 $f(0,1,0,0) = (2,-2) = 2 \cdot (1,0)$ 

$$extstyle \mathcal{F}_{(\mathcal{A}_{\mathsf{kan}},\,\mathcal{B}_{\mathsf{kan}})} = egin{bmatrix} 1 \ -1 \end{bmatrix}$$

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

$$\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0), (0,1) ig\}$$

$$f(1,0,0,0) = (1,-1) = 1 \cdot (1,0) + (-1) \cdot (0,1)$$
  
 $f(0,1,0,0) = (2,-2) = 2 \cdot (1,0) +$ 

$$extstyle F_{(\mathcal{A}_{\mathsf{kan}},\mathcal{B}_{\mathsf{kan}})} = egin{bmatrix} 1 \ -1 \end{bmatrix}$$

$$egin{aligned} \mathcal{A}_{\mathsf{kan}} &= ig\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) ig\} \ \mathcal{B}_{\mathsf{kan}} &= ig\{ (1,0), (0,1) ig\} \end{aligned}$$

$$f(1,0,0,0) = (1,-1) = 1 \cdot (1,0) + (-1) \cdot (0,1)$$
  
 $f(0,1,0,0) = (2,-2) = 2 \cdot (1,0) + (-2) \cdot (0,1)$ 

$$egin{align} F_{(\mathcal{A}_{\mathsf{kan}},\,\mathcal{B}_{\mathsf{kan}})} &= egin{bmatrix} 1 & 2 \ -1 & -2 \end{bmatrix} \ && \ \mathcal{A}_{\mathsf{kan}} &= \{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\} \end{aligned}$$

$$\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0), (0,1) ig\}$$
  $f(1,0,0,0) = (1,-1) = 1 \cdot (1,0) + (-1) \cdot (0,1)$ 

 $f(0,1,0,0) = (2,-2) = 2 \cdot (1,0) + (-2) \cdot (0,1)$ 

$$F_{(\mathcal{A}_{\mathsf{kan}},\,\mathcal{B}_{\mathsf{kan}})} = egin{bmatrix} 1 & 2 \ -1 & -2 \end{bmatrix}$$

c)

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

$$egin{aligned} \mathcal{B}_{\mathsf{kan}} &= ig\{ (1,0), (0,1) ig\} \ & f(1,0,0,0) = (1,-1) = 1 \cdot (1,0) + (-1) \cdot (0,1) \ & f(0,1,0,0) = (2,-2) = 2 \cdot (1,0) + (-2) \cdot (0,1) \ & f(0,0,1,0) = \end{aligned}$$

$$F_{(\mathcal{A}_{\mathsf{kan}},\,\mathcal{B}_{\mathsf{kan}})} = egin{bmatrix} 1 & 2 \ -1 & -2 \end{bmatrix}$$

c)

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

$$\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0), (0,1) ig\}$$
  $f(1,0,0,0) = (1,-1) = 1 \cdot (1,0) + (-1) \cdot (0,1)$   $f(0,1,0,0) = (2,-2) = 2 \cdot (1,0) + (-2) \cdot (0,1)$ 

f(0,0,1,0) = (-1,1)

$$F_{(\mathcal{A}_{\mathsf{kan}},\,\mathcal{B}_{\mathsf{kan}})} = egin{bmatrix} 1 & 2 \ -1 & -2 \end{bmatrix}$$

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

$$egin{aligned} \mathcal{B}_{\mathsf{kan}} &= ig\{ (1,0), (0,1) ig\} \ & f(1,0,0,0) = (1,-1) = 1 \cdot (1,0) + (-1) \cdot (0,1) \ & f(0,1,0,0) = (2,-2) = 2 \cdot (1,0) + (-2) \cdot (0,1) \end{aligned}$$

 $f(0,0,1,0) = (-1,1) = -1 \cdot (1,0)$ 

$$F_{(\mathcal{A}_{\mathsf{kan}},\,\mathcal{B}_{\mathsf{kan}})} = egin{bmatrix} 1 & 2 \ -1 & -2 \end{bmatrix}$$

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

$$\mathcal{B}_{\mathsf{kan}} = ig\{ (1,0), (0,1) ig\}$$
  $f(1,0,0,0) = (1,-1) = 1 \cdot (1,0) + (-1) \cdot (0,1)$   $f(0,1,0,0) = (2,-2) = 2 \cdot (1,0) + (-2) \cdot (0,1)$ 

 $f(0,0,1,0) = (-1,1) = -1 \cdot (1,0) +$ 

$$egin{align} F_{(\mathcal{A}_{\mathsf{kan}},\,\mathcal{B}_{\mathsf{kan}})} &= egin{bmatrix} 1 & 2 \ -1 & -2 \end{bmatrix} \ && \ \mathcal{A}_{\mathsf{kan}} &= \{(1,0,0,0),(0,1,0,0),(0,0,1,0),(0,0,0,1)\} \end{cases}$$

$$egin{aligned} \mathcal{B}_{\mathsf{kan}} &= ig\{ (1,0), (0,1) ig\} \ & f(1,0,0,0) = (1,-1) = 1 \cdot (1,0) + (-1) \cdot (0,1) \ & f(0,1,0,0) = (2,-2) = 2 \cdot (1,0) + (-2) \cdot (0,1) \end{aligned}$$

 $f(0,0,1,0) = (-1,1) = -1 \cdot (1,0) + 1 \cdot (0,1)$ 

$$\mathcal{A}_{\mathsf{kan}} = \{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\}$$

 $F_{(\mathcal{A}_{\mathsf{kan}},\,\mathcal{B}_{\mathsf{kan}})} = egin{bmatrix} 1 & 2 & -1 \ -1 & -2 & 1 \end{bmatrix}$ 

 $\mathcal{B}_{kan} = \{(1,0),(0,1)\}$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

$$f(1,0,0,0) = (1,-1) = 1 \cdot (1,0) + (-1) \cdot (0,1)$$
  
 $f(0,1,0,0) = (2,-2) = 2 \cdot (1,0) + (-2) \cdot (0,1)$   
 $f(0,0,1,0) = (-1,1) = -1 \cdot (1,0) + 1 \cdot (0,1)$ 

f(0,0,0,1) =

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

c)

$$f(1,0,0,0) = (1,-1) = 1 \cdot (1,0) + (-1) \cdot (0,1)$$

$$f(0,1,0,0) = (2,-2) = 2 \cdot (1,0) + (-2) \cdot (0,1)$$

$$f(0,0,1,0) = (-1,1) = -1 \cdot (1,0) + 1 \cdot (0,1)$$

f(0,0,0,1) = (-1,1)

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

c)

$$f(1,0,0,0) = (1,-1) = 1 \cdot (1,0) + (-1) \cdot (0,1)$$
  
 $f(0,1,0,0) = (2,-2) = 2 \cdot (1,0) + (-2) \cdot (0,1)$   
 $f(0,0,1,0) = (-1,1) = -1 \cdot (1,0) + 1 \cdot (0,1)$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

c)

$$egin{aligned} f(1,0,0,0) &= (1,-1) = 1 \cdot (1,0) + (-1) \cdot (0,1) \ f(0,1,0,0) &= (2,-2) = 2 \cdot (1,0) + (-2) \cdot (0,1) \ f(0,0,1,0) &= (-1,1) = -1 \cdot (1,0) + 1 \cdot (0,1) \end{aligned}$$

 $f(0,0,0,1) = (-1,1) = -1 \cdot (1,0)$ 

c)

 $\mathcal{B}_{kan} = \{(1,0),(0,1)\}$  $f(1,0,0,0) = (1,-1) = 1 \cdot (1,0) + (-1) \cdot (0,1)$  $f(0,1,0,0) = (2,-2) = 2 \cdot (1,0) + (-2) \cdot (0,1)$ 

 $f(0,0,1,0) = (-1,1) = -1 \cdot (1,0) + 1 \cdot (0,1)$ 

$$f(0,0,0,1) = (-1,1) = -1 \cdot (1,0) +$$

c)

$$egin{aligned} \mathcal{B}_{\mathsf{kan}} &= ig\{ (1,0), (0,1) ig\} \ & f(1,0,0,0) = (1,-1) = 1 \cdot (1,0) + (-1) \cdot (0,1) \ & f(0,1,0,0) = (2,-2) = 2 \cdot (1,0) + (-2) \cdot (0,1) \ & f(0,0,1,0) = (-1,1) = -1 \cdot (1,0) + 1 \cdot (0,1) \end{aligned}$$

 $f(0,0,0,1) = (-1,1) = -1 \cdot (1,0) + 1 \cdot (0,1)$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

c)

$$f(1,0,0,0) = (1,-1) = 1 \cdot (1,0) + (-1) \cdot (0,1)$$
  
 $f(0,1,0,0) = (2,-2) = 2 \cdot (1,0) + (-2) \cdot (0,1)$   
 $f(0,0,1,0) = (-1,1) = -1 \cdot (1,0) + 1 \cdot (0,1)$ 

 $f(0,0,0,1) = (-1,1) = -1 \cdot (1,0) + 1 \cdot (0,1)$ 

1. način

$$\mathcal{A} = \big\{ (1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4) \big\} \qquad f: \mathbb{R}^4 o \mathbb{R}$$

1. način

 $\mathcal{A} = \big\{ (1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4) \big\} \qquad f: \mathbb{R}^4 \to \mathbb{R}^2$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

1. način

 $\mathcal{A} = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\}$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

1. način

 $f:\mathbb{R}^4 o\mathbb{R}^2$ 

 $\mathcal{A} = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\}$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

1. način

f(1,0,0,0) =

 $f:\mathbb{R}^4 o\mathbb{R}^2$ 

 $\mathcal{A} = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\}$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

1. način

f(1,0,0,0) = (1,-1)

 $f:\mathbb{R}^4 o\mathbb{R}^2$ 

 $A = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\}$ 

 $f(1,0,0,0) = (1,-1) = \alpha_1 \cdot (1,10)$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

1. način

 $f:\mathbb{R}^4 o\mathbb{R}^2$ 

 $A = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\}$ 

 $f(1,0,0,0) = (1,-1) = \alpha_1 \cdot (1,10) +$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

1. način

 $f: \mathbb{R}^4 o \mathbb{R}^2$ 

 $f(1,0,0,0) = (1,-1) = \alpha_1 \cdot (1,10) + \alpha_2 \cdot (1,11)$   $\mathcal{B} = \{(1,10),(1,11)\}$ 

1. način

 $f(1,0,0,0) = (1,-1) = \alpha_1 \cdot (1,10) + \alpha_2 \cdot (1,11)$   $\mathcal{B} = \{(1,10),(1,11)\}$ 

1. način

 $\alpha_1 + \alpha_2 = 1$ 

 $f(1,0,0,0) = (1,-1) = \alpha_1 \cdot (1,10) + \alpha_2 \cdot (1,11) \mathcal{B} = \{(1,10),(1,11)\}$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

1. način

 $\alpha_1 + \alpha_2 = 1$  $10\alpha_1 + 11\alpha_2 = -1$ 

$$10lpha_1+11lpha_2=-1$$
 
$$F_{(\mathcal{A},\,\mathcal{B})}=igg[$$

 $f(1,0,0,0) = (1,-1) = \alpha_1 \cdot (1,10) + \alpha_2 \cdot (1,11) \mathcal{B} = \{(1,10),(1,11)\}$ 

1. način

 $\alpha_1 + \alpha_2 = 1$ 

$$\alpha_1 + \alpha_2 = 1$$

$$10\alpha_1 + 11\alpha_2 = -1$$

 $A = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\}$ 

1. način

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

$$extstyle F_{(\mathcal{A},\,\mathcal{B})} = egin{bmatrix} lpha_1 \ lpha_2 \end{bmatrix}$$

 $f:\mathbb{R}^4 o\mathbb{R}^2$ 

$$10lpha_1+11lpha_2=-1$$
  $f(1,2,0,0)=$  
$$F_{(\mathcal{A},\mathcal{B})}=egin{bmatrix}lpha_1\\lpha_2 \end{matrix}$$

 $A = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\}$ 

1. način

 $\alpha_1 + \alpha_2 = 1$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

 $f: \mathbb{R}^4 o \mathbb{R}^2$ 

$$lpha_1 + lpha_2 = 1$$
 $10lpha_1 + 11lpha_2 = -1$ 
 $f(1, 2, 0, 0) = (5, -5)$ 

 $\mathcal{A} = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\}$ 

1. način

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

 $F_{(A,B)} = \begin{vmatrix} \alpha_1 \\ \alpha_2 \end{vmatrix}$ 

 $f: \mathbb{R}^4 o \mathbb{R}^2$ 

$$lpha_1 + lpha_2 = 1 \ 10lpha_1 + 11lpha_2 = -1 \ f(1,2,0,0) = (5,-5) = eta_1 \cdot (1,10)$$

 $\mathcal{A} = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\}$ 

1. način

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

$$F_{(\mathcal{A},\mathcal{B})} = egin{bmatrix} lpha_1 \ lpha_2 \end{bmatrix}$$

 $f: \mathbb{R}^4 o \mathbb{R}^2$ 

$$lpha_1 + lpha_2 = 1 \ 10lpha_1 + 11lpha_2 = -1 \ f(1,2,0,0) = (5,-5) = eta_1 \cdot (1,10) +$$

 $\mathcal{A} = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\}$ 

1. način

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

 $F_{(A,B)} = \begin{vmatrix} \alpha_1 \\ \alpha_2 \end{vmatrix}$ 

 $f: \mathbb{R}^4 \to \mathbb{R}^2$ 

$$egin{align} lpha_1 + lpha_2 &= 1 \ 10lpha_1 + 11lpha_2 &= -1 \ f(1,2,0,0) &= (5,-5) = eta_1 \cdot (1,10) + eta_2 \cdot (1,11) \ \end{pmatrix}$$

 $f(1,0,0,0) = (1,-1) = \alpha_1 \cdot (\overline{1,10) + \alpha_2 \cdot (1,11)} \mathcal{B} = \{(1,10),(1,11)\}$ 

1. način

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

 $F_{(A,B)} = \begin{vmatrix} \alpha_1 \\ \alpha_2 \end{vmatrix}$ 

$$10lpha_1 + 11lpha_2 = -1$$
 $f(1,2,0,0) = (5,-5) = eta_1 \cdot (1,10) + eta_2 \cdot (1,11)$ 
 $eta_1 + eta_2 = 5$ 
 $F_{(\mathcal{A},\mathcal{B})} = egin{bmatrix} lpha_1 \\ lpha_2 \end{bmatrix}$ 

 $f(1,0,0,0) = (1,-1) = \alpha_1 \cdot (\overline{1,10) + \alpha_2 \cdot (1,11)} \mathcal{B} = \{(1,10),(1,11)\}$ 

1. način

 $\alpha_1 + \alpha_2 = 1$ 

$$10\alpha_{1} + 11\alpha_{2} = -1$$

$$f(1, 2, 0, 0) = (5, -5) = \beta_{1} \cdot (1, 10) + \beta_{2} \cdot (1, 11)$$

$$\beta_{1} + \beta_{2} = 5$$

$$10\beta_{1} + 11\beta_{2} = -5$$

$$F_{(\mathcal{A}, \mathcal{B})} = \begin{bmatrix} \alpha_{1} \\ \alpha_{2} \end{bmatrix}$$

 $A = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\}$ 

1. način

 $\alpha_1 + \alpha_2 = 1$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

 $f: \mathbb{R}^4 \to \mathbb{R}^2$ 

$$10\alpha_{1} + 11\alpha_{2} = -1$$

$$f(1, 2, 0, 0) = (5, -5) = \beta_{1} \cdot (1, 10) + \beta_{2} \cdot (1, 11)$$

$$\beta_{1} + \beta_{2} = 5$$

$$10\beta_{1} + 11\beta_{2} = -5$$

$$F_{(A, B)} = \begin{bmatrix} \alpha_{1} & \beta_{1} \\ \alpha_{2} & \beta_{2} \end{bmatrix}$$

 $f(1,0,0,0) = (1,-1) = \alpha_1 \cdot (1,10) + \alpha_2 \cdot (1,11) \mathcal{B} = \{(1,10),(1,11)\}$ 

1. način

 $\alpha_1 + \alpha_2 = 1$ 

$$10\alpha_{1} + 11\alpha_{2} = -1$$

$$f(1, 2, 0, 0) = (5, -5) = \beta_{1} \cdot (1, 10) + \beta_{2} \cdot (1, 11)$$

$$\beta_{1} + \beta_{2} = 5$$

$$10\beta_{1} + 11\beta_{2} = -5$$

$$F(A, B) = \begin{bmatrix} \alpha_{1} & \beta_{1} \\ \alpha_{2} & \beta_{2} \end{bmatrix}$$

$$f(1, 2, 3, 0) =$$

 $f(1,0,0,0) = (1,-1) = \alpha_1 \cdot (\overline{1,10) + \alpha_2 \cdot (1,11)} \mathcal{B} = \{(1,10),(1,11)\}$ 

1. način

 $\alpha_1 + \alpha_2 = 1$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

18/26

$$10lpha_1 + 11lpha_2 = -1$$
 $f(1,2,0,0) = (5,-5) = eta_1 \cdot (1,10) + eta_2 \cdot (1,11)$ 
 $eta_1 + eta_2 = 5$ 
 $10eta_1 + 11eta_2 = -5$ 
 $f(1,2,3,0) = (2,-2)$ 
 $F(A,B) = \begin{bmatrix} lpha_1 & eta_1 \\ lpha_2 & eta_2 \end{bmatrix}$ 

 $f(1,0,0,0) = (1,-1) = \alpha_1 \cdot (1,10) + \alpha_2 \cdot (1,11) \mathcal{B} = \{(1,10),(1,11)\}$ 

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 $\alpha_1 + \alpha_2 = 1$ 

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$$f(1, 2, 0, 0) = (5, -5) = \beta_{1} \cdot (1, 10) + \beta_{2} \cdot (1, 11)$$

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$$f(1, 2, 3, 0) = (2, -2) = \gamma_{1} \cdot (1, 10)$$

$$F_{(A, B)} = \begin{bmatrix} \alpha_{1} & \beta_{1} \\ \alpha_{2} & \beta_{2} \end{bmatrix}$$

 $f(1,0,0,0) = (1,-1) = \alpha_1 \cdot (\overline{1,10) + \alpha_2 \cdot (1,11)} \mathcal{B} = \{(1,10),(1,11)\}$ 

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f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

18/26

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18/26

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18/26

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		$\alpha_i$	$\beta_i$	$\gamma_i$	$\delta_i$
1	1	1	5	2	-2
10	11	-1	-5	-2	2

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10\beta_{1} + 11\beta_{2} = -5$$

$$\gamma_{1} + \gamma_{2} = 2 
10\gamma_{1} + 11\gamma_{2} = -2$$

$$\delta_{1} + \delta_{2} = -2 
10\delta_{1} + 11\delta_{2} = 2$$

$$\alpha_{1} + \alpha_{2} = 1 
10\alpha_{1} + 11\alpha_{2} = -1$$

$$\beta_{1} + \beta_{2} = 5 
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10\gamma_{1} + 11\gamma_{2} = -2$$

$$\delta_{1} + \delta_{2} = -2 
10\delta_{1} + 11\delta_{2} = 2$$

		$\alpha_i$	$\beta_i$	$\gamma_{i}$	$\delta_i$	
1	1					_/·(-10)
10	11	-1	-5	-2	2	+
1	1		5		-2	_
0	1	-11	-55	-22	22	
						_

$$\alpha_{1} + \alpha_{2} = 1 
10\alpha_{1} + 11\alpha_{2} = -1$$

$$\beta_{1} + \beta_{2} = 5 
10\beta_{1} + 11\beta_{2} = -5$$

$$\gamma_{1} + \gamma_{2} = 2 
10\gamma_{1} + 11\gamma_{2} = -2$$

$$\delta_{1} + \delta_{2} = -2 
10\delta_{1} + 11\delta_{2} = 2$$

		$  \alpha_i  $	$\beta_i$	$\gamma_i$	$\delta_i$	
1		1	5	2	-2	- /·(-10)
10	11	-1	<b>-5</b>	-2	2	<b>←</b>
1	1	1	5	2	-2	_
0	1	-11	-55	_ _22	22	
						-

$$\alpha_{1} + \alpha_{2} = 1 
10\alpha_{1} + 11\alpha_{2} = -1$$

$$\beta_{1} + \beta_{2} = 5 
10\beta_{1} + 11\beta_{2} = -5$$

$$\gamma_{1} + \gamma_{2} = 2 
10\gamma_{1} + 11\gamma_{2} = -2$$

$$\delta_{1} + \delta_{2} = -2 
10\delta_{1} + 11\delta_{2} = 2$$

$$\alpha_{1} + \alpha_{2} = 1 
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10\delta_{1} + 11\delta_{2} = 2$$

		$\alpha_i$	$\beta_i$	$\gamma_{i}$	$\delta_i$	
1	1	1	5		-2	/·(-10)
10	11	-1	-5	-2	2	+
1	1	1	5	2	-2	/·(-1)
0	1	-11	-55	-22	22	/·(-1)
						_

$$\alpha_{1} + \alpha_{2} = 1 
10\alpha_{1} + 11\alpha_{2} = -1$$

$$\beta_{1} + \beta_{2} = 5 
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$$\delta_{1} + \delta_{2} = -2 
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10\delta_{1} + 11\delta_{2} = 2$$

$$F_{(\mathcal{A},\mathcal{B})} = \begin{bmatrix} \alpha_1 & \beta_1 & \gamma_1 & \delta_1 \\ \alpha_2 & \beta_2 & \gamma_2 & \delta_2 \end{bmatrix}$$

$$\alpha_{1} + \alpha_{2} = 1 
10\alpha_{1} + 11\alpha_{2} = -1$$

$$\beta_{1} + \beta_{2} = 5 
10\beta_{1} + 11\beta_{2} = -5$$

$$\gamma_{1} + \gamma_{2} = 2 
10\gamma_{1} + 11\gamma_{2} = -2$$

$$\delta_{1} + \delta_{2} = -2 
10\delta_{1} + 11\delta_{2} = 2$$

$$F_{(\mathcal{A},\mathcal{B})} = \begin{bmatrix} \alpha_1 & \beta_1 & \gamma_1 & \delta_1 \\ \alpha_2 & \beta_2 & \gamma_2 & \delta_2 \end{bmatrix}$$

$$F_{(\mathcal{A},\mathcal{B})} = \begin{bmatrix} 12 & 60 & 24 & -24 \\ -11 & -55 & -22 & 22 \end{bmatrix}$$

$$\alpha_{1} + \alpha_{2} = 1 
10\alpha_{1} + 11\alpha_{2} = -1$$

$$\beta_{1} + \beta_{2} = 5 
10\beta_{1} + 11\beta_{2} = -5$$

$$\gamma_{1} + \gamma_{2} = 2 
10\gamma_{1} + 11\gamma_{2} = -2$$

$$f: \mathbb{R}^4 o \mathbb{R}^2$$

$$\mathcal{A}_{\mathsf{kan}} = \left\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \right\}$$

 $f: \mathbb{R}^4 \to \mathbb{R}^2$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

 $\mathcal{B} = \{(1, 10), (1, 11)\}$   $\mathcal{B}_{\mathsf{kan}} = \{(1, 0), (0, 1)\}$ 

 $A = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\}$ 

20 / 26

$$(\mathcal{A}_{\mathsf{kan}},\mathcal{B}_{\mathsf{kan}})^{\mathsf{S}}$$

 $\mathcal{A} = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\}$ 

 $\mathcal{A}_{\mathsf{kan}} = \big\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \big\}$ 

 $\mathcal{B} = \{(1, 10), (1, 11)\}$   $\mathcal{B}_{kan} = \{(1, 0), (0, 1)\}$ 

 $F_{(\mathcal{A},\mathcal{B})} = T^{-1}F_{(\mathcal{A}_{\mathsf{kan}},\mathcal{B}_{\mathsf{kan}})}S$ 

$$A_{\text{kan}}, \mathcal{B}_{\text{kan}})$$
  $S$ 

 $\mathcal{B} = \{(1, 10), (1, 11)\}$   $\mathcal{B}_{kan} = \{(1, 0), (0, 1)\}$  $F_{(A,B)} = T^{-1}F_{(A_{kan},B_{kan})}S$  $\mathcal{A}_{kan} \xrightarrow{S} \mathcal{A}$ 

 $A = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\}$ 

 $\mathcal{A}_{\mathsf{kan}} = \big\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \big\}$ 

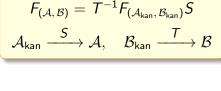
 $A = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\}$ 

$$\mathcal{A}_{kan} = \big\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \big\}$$

$$egin{aligned} & \mathcal{B} = \{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,0), \\ & \mathcal{B} = \{(1,10), (1,11)\} & \mathcal{B}_{\mathsf{kan}} = \{(1,0), (0,1)\} \end{aligned}$$

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

$$,1)\}$$



2. način 
$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$
$$\mathcal{A} = \{(1, 0, 0, 0), (1, 2, 0, 0), (1, 2, 3, 0), (1, 2, 3, 4)\} \qquad f: \mathbb{R}^4 \to \mathbb{R}^2$$

 $\mathcal{A}_{\mathsf{kan}} = \big\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \big\}$ 

 $F_{(\mathcal{A},\mathcal{B})} = T^{-1}F_{(\mathcal{A}_{\mathsf{kan}},\mathcal{B}_{\mathsf{kan}})}S$   $\mathcal{A}_{\mathsf{kan}} \xrightarrow{S} \mathcal{A}, \quad \mathcal{B}_{\mathsf{kan}} \xrightarrow{T} \mathcal{B}$ 

 $\mathcal{B} = \{(1,10),(1,11)\}$   $\mathcal{B}_{\mathsf{kan}} = \{(1,0),(0,1)\}$   $\mathcal{S} = \{(1,0),(0,1)\}$ 

20 / 26

$$\mathcal{A} = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\} \qquad f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$= \{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\} \qquad \boxed{1}$$

$$\mathcal{A}_{\mathsf{kan}} = \left\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \right\} \\ \mathcal{B} = \left\{ (1,10), (1,11) \right\} \quad \mathcal{B}_{\mathsf{kan}} = \left\{ (1,0), (0,1) \right\} \quad S = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \\ F_{(\mathcal{A},\mathcal{B})} = T^{-1} F_{(\mathcal{A}_{\mathsf{kan}},\mathcal{B}_{\mathsf{kan}})} S \\ S = T \quad T \quad D$$

$$\{(0,1)\} \hspace{0.5cm} \mathcal{S} = egin{bmatrix} 0 \ 0 \ 0 \end{bmatrix}$$

$$\mathcal{B} = \{(1, 10), (1, 11)\}$$
  $\mathcal{B}_{kan} = \mathcal{B}_{kan}$ 

$$\mathcal{B}_{(A, \mathcal{B})} = \mathcal{B}_{(A_{kan}, \mathcal{B}_{kan})} \mathcal{S}$$

$$\mathcal{A}_{kan} \xrightarrow{\mathcal{S}} \mathcal{A}, \quad \mathcal{B}_{kan} \xrightarrow{\mathcal{T}} \mathcal{B}$$

$$\mathcal{A} = \left\{ (1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4) \right\}$$

$$\mathcal{A} = \left\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \right\}$$

$$\mathcal{A}_{\mathsf{kan}} = \left\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \right\} \\ \mathcal{B} = \left\{ (1,10), (1,11) \right\} \quad \mathcal{B}_{\mathsf{kan}} = \left\{ (1,0), (0,1) \right\} \quad S = \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$F_{(\mathcal{A},\mathcal{B})} = T^{-1}F_{(\mathcal{A}_{\mathsf{kan}},\mathcal{B}_{\mathsf{kan}})}S$$

$$A_{\mathsf{loc}} \xrightarrow{S} A \quad \mathcal{B}_{\mathsf{loc}} \xrightarrow{T} \mathcal{B}$$

$$\{(0,1)\}$$
  $S = \begin{bmatrix} 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ 

2. način 
$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$
$$\mathcal{A} = \{(1, 0, 0, 0), (1, 2, 0, 0), (1, 2, 3, 0), (1, 2, 3, 4)\} \qquad f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$\mathcal{A}_{\mathsf{kan}} = \left\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \right\}$$

$$\mathcal{B} = \left\{ (1,10), (1,11) \right\} \quad \mathcal{B}_{\mathsf{kan}} = \left\{ (1,0), (0,1) \right\} \quad S = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathcal{B} = \left\{ (1,10), (1,11) \right\} \quad \mathcal{B}_{\mathsf{kan}} =$$
 $F_{(\mathcal{A},\mathcal{B})} = \mathcal{T}^{-1} F_{(\mathcal{A}_{\mathsf{kan}},\mathcal{B}_{\mathsf{kan}})} S$ 
 $\mathcal{A}_{\mathsf{kan}} \xrightarrow{S} \mathcal{A}, \quad \mathcal{B}_{\mathsf{kan}} \xrightarrow{\mathcal{T}} \mathcal{B}$ 

$$\{0,1\} \qquad S = \begin{bmatrix} 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

2. način 
$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$
$$\mathcal{A} = \{(1, 0, 0, 0), (1, 2, 0, 0), (1, 2, 3, 0), (1, 2, 3, 4)\} \qquad f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$\mathcal{A}_{\mathsf{kan}} = \left\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \right\} \\ \mathcal{B} = \left\{ (1,10), (1,11) \right\} \quad \mathcal{B}_{\mathsf{kan}} = \left\{ (1,0), (0,1) \right\} \quad S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$\mathcal{B} = \{(1, 10), (1, 11)\}$$
  $\mathcal{B}_{kan} =$ 

$$F_{(\mathcal{A}, \mathcal{B})} = T^{-1}F_{(\mathcal{A}_{kan}, \mathcal{B}_{kan})}S$$

$$S_{kan} = \{(1,0),(0,1)\} \qquad S = \begin{bmatrix} 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

2. način 
$$f(x,y,u,v) = (x+2y-u-v, -x-2y+u+v)$$
 
$$\mathcal{A} = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\} \qquad f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$\mathcal{A}_{\mathsf{kan}} = \big\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \big\} \qquad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \end{bmatrix}$$

$$\mathcal{B} = \{(1,10), (1,11)\} \quad \mathcal{B}_{\mathsf{kan}} = \{(1,0), (0,1)\} \quad S = \begin{bmatrix} 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$\mathcal{A}_{\mathsf{kan}} = \left\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \right\} \\ \mathcal{B} = \left\{ (1,10), (1,11) \right\} \quad \mathcal{B}_{\mathsf{kan}} = \left\{ (1,0), (0,1) \right\} \quad S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix} \\ \mathcal{A}_{\mathsf{kan}} \xrightarrow{S} \mathcal{A}, \quad \mathcal{B}_{\mathsf{kan}} \xrightarrow{T} \mathcal{B}$$

$$T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

2. način 
$$f(x,y,u,v) = (x+2y-u-v, -x-2y+u+v)$$
 
$$\mathcal{A} = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\} \qquad f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$\mathcal{A}_{\mathsf{kan}} = \left\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \right\} \qquad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \end{bmatrix}$$

$$\mathcal{B} = \big\{ (1,10), (1,11) \big\} \hspace{0.5cm} \mathcal{B}_{\mathsf{kan}} = \big\{ (1,0), (0,1) \big\} \hspace{0.5cm} \mathcal{S} = \big\{ (1,0), (0,1) \big\} \hspace{0.5cm} \mathcal{S$$

$$\mathcal{A}_{\mathsf{kan}} = \left\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \right\} \ \mathcal{B} = \left\{ (1,10), (1,11) \right\} \ \mathcal{B}_{\mathsf{kan}} = \left\{ (1,0), (0,1) \right\} \ \mathcal{S} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix} \ \mathcal{A}_{\mathsf{kan}} \xrightarrow{\mathcal{S}} \mathcal{A}, \ \mathcal{B}_{\mathsf{kan}} \xrightarrow{\mathcal{T}} \mathcal{B}$$
  $T = \begin{bmatrix} 1 \\ 10 \end{bmatrix}$ 

$$T = \begin{bmatrix} 1 & F_{(A_{kan}, B_{kan})} \\ F_{(A_{kan}, B_{kan})} & F_{(A_{kan}, B_{kan})} \\ F_{(A_{ka$$

2. način 
$$f(x,y,u,v) = (x+2y-u-v, -x-2y+u+v)$$
 
$$\mathcal{A} = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\} \qquad f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$\mathcal{A}_{\mathsf{kan}} = \big\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \big\} \qquad \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\mathcal{A}_{\mathsf{kan}} = \{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\}$$
 $\mathcal{B} = \{(1,10), (1,11)\}$ 
 $\mathcal{B}_{\mathsf{kan}} = \{(1,0), (0,1)\}$ 
 $S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ 
 $\mathcal{B}_{\mathsf{kan}} = \{(1,0), (0,1)\}$ 
 $\mathcal{B}_{\mathsf{kan}} = \{(1,0), (0,1)\}$ 

$$F_{(\mathcal{A},\mathcal{B})} = T^{-1}F_{(\mathcal{A}_{kan},\mathcal{B}_{kan})}S$$

$$\mathcal{A}_{kan} \xrightarrow{S} \mathcal{A}, \quad \mathcal{B}_{kan} \xrightarrow{T} \mathcal{B}$$

$$T = \begin{bmatrix} 1 & 1 \\ 10 & 11 \end{bmatrix}$$

$$0 \quad 0 \quad 0 \quad 4$$

$$T = \begin{bmatrix} 1 & 1 \\ 10 & 11 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 1 \\ 10 & 11 \end{bmatrix}$$

$$\mathcal{A} = \left\{ (1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4) \right\} \qquad f : \mathbb{R}^4 \to \mathbb{R}^2$$

$$\mathcal{A}_{\mathsf{kan}} = \left\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \right\} \qquad \qquad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$\mathcal{B} = \left\{ (1,10), (1,11) \right\} \qquad \mathcal{B}_{\mathsf{kan}} = \left\{ (1,0), (0,1) \right\} \qquad S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$\mathcal{A}_{\mathsf{kan}} \xrightarrow{S} \mathcal{A}, \quad \mathcal{B}_{\mathsf{kan}} \xrightarrow{T} \mathcal{B}$$

$$T = \begin{bmatrix} 1 & 1 \\ 10 & 11 \end{bmatrix}$$

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

 $F_{(A,B)} =$ 

2. način

20 / 26

$$f(x,y,u,v) = (x+2y-u-v, -x-2y+u+v)$$

$$\mathcal{A} = \left\{ (1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4) \right\} \qquad f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$\mathcal{A}_{\mathsf{kan}} = \left\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \right\} \qquad g = \left\{ (1,10), (1,11) \right\} \qquad \mathcal{B}_{\mathsf{kan}} = \left\{ (1,0), (0,1) \right\} \qquad S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$F_{(\mathcal{A},\mathcal{B})} = T^{-1}F_{(\mathcal{A}_{\mathsf{kan}},\mathcal{B}_{\mathsf{kan}})}S \qquad T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

 $\begin{array}{c}
F_{(\mathcal{A},\mathcal{B})} = T^{-1}F_{(\mathcal{A}_{kan},\mathcal{B}_{kan})}S \\
\mathcal{A}_{kan} \xrightarrow{S} \mathcal{A}, \quad \mathcal{B}_{kan} \xrightarrow{T} \mathcal{B}
\end{array}$   $T = \begin{bmatrix} 1 & 1 \\ 10 & 11 \end{bmatrix}$ 

$$F_{(\mathcal{A},\mathcal{B})}=rac{1}{2}$$

$$f(x,y,u,v) = (x+2y-u-v, -x-2y+u+v)$$

$$\mathcal{A} = \left\{ (1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4) \right\} \qquad f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$\mathcal{A}_{\mathsf{kan}} = \left\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \right\} \qquad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$F_{(\mathcal{A},\mathcal{B})} = T^{-1}F_{(\mathcal{A}_{\mathsf{kan}},\mathcal{B}_{\mathsf{kan}})}S \qquad T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$F_{(\mathcal{A},\mathcal{B})}=rac{1}{1}$$

 $egin{aligned} F_{(\mathcal{A},\mathcal{B})} &= T^{-1}F_{(\mathcal{A}_{\mathsf{kan}},\mathcal{B}_{\mathsf{kan}})}\mathcal{S} \ \mathcal{A}_{\mathsf{kan}} & \stackrel{\mathcal{S}}{\longrightarrow} \mathcal{A}, \quad \mathcal{B}_{\mathsf{kan}} & \stackrel{T}{\longrightarrow} \mathcal{B} \end{aligned}$ 

$$T = \begin{bmatrix} 1 & 1 \\ 10 & 11 \end{bmatrix}$$

2. način 
$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$

$$\mathcal{A} = \{(1, 0, 0, 0), (1, 2, 0, 0), (1, 2, 3, 0), (1, 2, 3, 4)\} \qquad f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$\mathcal{A}_{kan} = \{(1, 0, 0, 0), (0, 1, 0, 0), (0, 0, 1, 0), (0, 0, 0, 1)\} \qquad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \end{bmatrix}$$

$$\mathcal{B} = \{(1, 10), (1, 11)\} \qquad \mathcal{B}_{k-1} = \{(1, 0), (0, 1)\} \qquad \mathcal{S}_{k-1} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \end{bmatrix}$$

$$\mathcal{A}_{\mathsf{kan}} = \{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\}$$
 $\mathcal{B} = \{(1,10), (1,11)\}$ 
 $\mathcal{B}_{\mathsf{kan}} = \{(1,0), (0,1)\}$ 
 $S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ 
 $S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ 

$$F_{(A,B)} = \frac{1}{2} \left[ \frac{1}{2} \right]$$

$$f(x,y,u,v) = (x+2y-u-v, -x-2y+u+v)$$

$$\mathcal{A} = \left\{ (1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4) \right\} \qquad f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$\mathcal{A}_{\mathsf{kan}} = \left\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \right\} \qquad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$F_{(\mathcal{A},\mathcal{B})} = T^{-1}F_{(\mathcal{A}_{\mathsf{kan}},\mathcal{B}_{\mathsf{kan}})}S \qquad \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$F_{(\mathcal{A},\mathcal{B})} = T^{-1}F_{(\mathcal{A}_{kan},\mathcal{B}_{kan})}S$$

$$\mathcal{A}_{kan} \xrightarrow{S} \mathcal{A}, \quad \mathcal{B}_{kan} \xrightarrow{T} \mathcal{B}$$

$$T = \begin{bmatrix} 1 & 1 \\ 10 & 11 \end{bmatrix}$$

$$0 \quad 0$$

$$\mathcal{F}_{(\mathcal{A},\,\mathcal{B})}=rac{1}{1}\left[egin{array}{c}11\end{array}
ight]$$

$$f(x,y,u,v) = (x+2y-u-v, -x-2y+u+v)$$

$$\mathcal{A} = \left\{ (1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4) \right\} \qquad f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$\mathcal{A}_{\mathsf{kan}} = \left\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \right\} \qquad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \end{bmatrix}$$

$$\mathcal{A}_{\mathsf{kan}} = \{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\}$$
 $\mathcal{B} = \{(1,10), (1,11)\}$ 
 $\mathcal{B}_{\mathsf{kan}} = \{(1,0), (0,1)\}$ 
 $S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ 
 $F_{(\mathcal{A},\mathcal{B})} = T^{-1}F_{(\mathcal{A}_{\mathsf{kan}},\mathcal{B}_{\mathsf{kan}})}S$ 
 $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ 

$$F_{(\mathcal{A},\,\mathcal{B})} = rac{1}{1} \left[egin{array}{cc} 11 & & \ & 1 \end{array}
ight]$$

$$f(x,y,u,v) = (x+2y-u-v, -x-2y+u+v)$$
 
$$\mathcal{A} = \left\{ (1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4) \right\} \qquad f: \mathbb{R}^4 \to \mathbb{R}^2$$
 
$$\mathcal{A}_{\mathsf{kan}} = \left\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \right\} \qquad \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

$$\mathcal{A}_{\mathsf{kan}} = \left\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \right\} \ \mathcal{B} = \left\{ (1,10), (1,11) \right\} \ \mathcal{B}_{\mathsf{kan}} = \left\{ (1,0), (0,1) \right\} \ \mathcal{S} = \left[ egin{array}{ccccc} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{array} \right] \ \mathcal{F}_{(\mathcal{A}_{\mathsf{kBN}},\mathcal{B}_{\mathsf{kBN}})} \mathcal{S}$$

$$F_{(\mathcal{A},\mathcal{B})} = T^{-1}F_{(\mathcal{A}_{\mathsf{kan}},\mathcal{B}_{\mathsf{kan}})}S$$

$$\mathcal{A}_{\mathsf{kan}} \xrightarrow{S} \mathcal{A}, \quad \mathcal{B}_{\mathsf{kan}} \xrightarrow{T} \mathcal{B}$$

$$T = \begin{bmatrix} 1 & 1 \\ 10 & 11 \end{bmatrix}$$

$$T = \begin{bmatrix} 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$extstyle extstyle F_{(\mathcal{A},\mathcal{B})} = rac{1}{1} egin{bmatrix} 11 \ -10 & 1 \end{bmatrix}$$

$$f(x,y,u,v) = (x+2y-u-v, -x-2y+u+v)$$
 
$$\mathcal{A} = \left\{ (1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4) \right\} \qquad f: \mathbb{R}^4 \to \mathbb{R}^2$$
 
$$\mathcal{A}_{\mathsf{kan}} = \left\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \right\} \qquad \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

$$\mathcal{A}_{\mathsf{kan}} = \left\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \right\} \ \mathcal{B} = \left\{ (1,10), (1,11) \right\} \ \mathcal{B}_{\mathsf{kan}} = \left\{ (1,0), (0,1) \right\} \ \mathcal{S} = \left[ egin{array}{ccccc} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{array} \right]$$

$$F_{(\mathcal{A},\mathcal{B})} = T^{-1}F_{(\mathcal{A}_{\mathsf{kan}},\mathcal{B}_{\mathsf{kan}})}S$$

$$\mathcal{A}_{\mathsf{kan}} \xrightarrow{S} \mathcal{A}, \quad \mathcal{B}_{\mathsf{kan}} \xrightarrow{T} \mathcal{B}$$

$$T = \begin{bmatrix} 1 & 1 \\ 10 & 11 \end{bmatrix}$$

$$extstyle F_{(\mathcal{A},\,\mathcal{B})} = rac{1}{1} egin{bmatrix} 11 & -1 \ -10 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 11 & -1 \\ -10 & 1 \end{bmatrix}$$

2. način 
$$f(x,y,u,v) = (x+2y-u-v, -x-2y+u+v)$$
 
$$\mathcal{A} = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\} \qquad f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$\mathcal{A}_{\mathsf{kan}} = \left\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \right\} \\ \mathcal{B} = \left\{ (1,10), (1,11) \right\} \quad \mathcal{B}_{\mathsf{kan}} = \left\{ (1,0), (0,1) \right\} \quad S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$F_{(\mathcal{A},\mathcal{B})} = T^{-1}F_{(\mathcal{A}_{\mathsf{kan}},\mathcal{B}_{\mathsf{kan}})}S \qquad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$F_{(\mathcal{A},\mathcal{B})} = T^{-1}F_{(\mathcal{A}_{kan},\mathcal{B}_{kan})}S$$

$$\mathcal{A}_{kan} \xrightarrow{S} \mathcal{A}, \quad \mathcal{B}_{kan} \xrightarrow{T} \mathcal{B}$$

$$T = \begin{bmatrix} 1 & 1 \\ 10 & 11 \end{bmatrix}$$

$$T = \begin{bmatrix} 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$F_{(\mathcal{A},\,\mathcal{B})}=rac{1}{1}egin{bmatrix}11&-1\-10&1\end{bmatrix}egin{bmatrix}1&2&-1&-1\-1&-2&1&1\end{bmatrix}$$

2. način 
$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$
$$\mathcal{A} = \{(1, 0, 0, 0), (1, 2, 0, 0), (1, 2, 3, 0), (1, 2, 3, 4)\} \qquad f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$\mathcal{A}_{\mathsf{kan}} = \left\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \right\} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \end{bmatrix}$$

$$egin{aligned} \mathcal{A}_{\mathsf{kan}} &= \left\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \right\} \ \mathcal{B} &= \left\{ (1,10), (1,11) \right\} \quad \mathcal{B}_{\mathsf{kan}} &= \left\{ (1,0), (0,1) \right\} \quad \mathcal{S} &= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix} \ \mathcal{F}_{(\mathcal{A},\mathcal{B})} &= \mathcal{F}^{-1} \mathcal{F}_{(\mathcal{A}_{\mathsf{kan}},\mathcal{B}_{\mathsf{kan}})} \mathcal{S} \ \mathcal{A}_{\mathsf{kan}} &\xrightarrow{\mathcal{T}} \mathcal{B} \end{aligned} \qquad \mathcal{T} &= \begin{bmatrix} 1 & 1 \\ 10 & 11 \end{bmatrix}$$

$$\mathcal{A}_{\mathsf{kan}} \xrightarrow{S} \mathcal{A}, \quad \mathcal{B}_{\mathsf{kan}} \xrightarrow{T} \mathcal{B}$$

$$\begin{bmatrix} 10 & 11 \end{bmatrix}$$

$$F_{(A,B)} = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \end{bmatrix}$$

$$F_{(\mathcal{A},\mathcal{B})} = \frac{1}{1} \begin{bmatrix} 11 & -1 \\ -10 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 & -1 \\ -1 & -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$\mathcal{A} = \big\{ (1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4) \big\} \qquad f: \mathbb{R}^4 o \mathbb{R}^2 \ \mathcal{A}_{\mathsf{kan}} = \big\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \big\} \qquad \begin{bmatrix} 1 & 1 & 1 & 1 \ 0 & 2 & 2 & 2 \end{bmatrix}$$

$$egin{aligned} \mathcal{A}_{\mathsf{kan}} &= \left\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \right\} \ \mathcal{B} &= \left\{ (1,10), (1,11) \right\} \quad \mathcal{B}_{\mathsf{kan}} &= \left\{ (1,0), (0,1) \right\} \quad \mathcal{S} &= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix} \ \mathcal{F}_{(\mathcal{A},\mathcal{B})} &= \mathcal{F}^{-1} \mathcal{F}_{(\mathcal{A}_{\mathsf{kan}},\mathcal{B}_{\mathsf{kan}})} \mathcal{S} \ \mathcal{A}_{\mathsf{kan}} &\xrightarrow{\mathcal{T}} \mathcal{B} \end{aligned} \quad \mathcal{T} &= \begin{bmatrix} 1 & 1 \\ 10 & 11 \end{bmatrix}$$

$$F_{(\mathcal{A},\mathcal{B})} = rac{1}{1} egin{bmatrix} 11 & -1 \ -10 & 1 \end{bmatrix} egin{bmatrix} 1 & 2 & -1 & -1 \ -1 & -2 & 1 & 1 \end{bmatrix} egin{bmatrix} 1 & 1 & 1 & 1 \ 0 & 2 & 2 & 2 \ 0 & 0 & 3 & 3 \ 0 & 0 & 0 & 4 \end{bmatrix}$$

 $F_{(A,B)} =$ 

2. način

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

$$f(x,y,u,v) = (x+2y-u-v, -x-2y+u+v)$$

$$\mathcal{A} = \left\{ (1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4) \right\} \qquad f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$\mathcal{A}_{\mathsf{kan}} = \left\{ (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) \right\} \qquad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \end{bmatrix}$$

$$\mathcal{B} = \left\{ (1,10), (1,11) \right\} \qquad \mathcal{B}_{\mathsf{kan}} = \left\{ (1,0), (0,1) \right\} \qquad \mathcal{S} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \end{bmatrix}$$

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

$$A_{\text{kan}} \xrightarrow{S} A, \quad B_{\text{kan}} \xrightarrow{I} B$$

$$\begin{bmatrix} 10 & 11 \end{bmatrix}$$

$$F_{(A,B)} = \frac{1}{2} \begin{bmatrix} 11 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \end{bmatrix}$$

$$F_{(\mathcal{A},\mathcal{B})} = rac{1}{1} egin{bmatrix} 11 & -1 \ -10 & 1 \end{bmatrix} egin{bmatrix} 1 & 2 & -1 & -1 \ -1 & -2 & 1 & 1 \end{bmatrix} egin{bmatrix} 1 & 1 & 1 & 1 \ 0 & 2 & 2 & 2 \ 0 & 0 & 3 & 3 \ 0 & 0 & 0 & 4 \end{bmatrix}$$

 $F_{(\mathcal{A},\mathcal{B})} = \begin{bmatrix} 12 & 60 & 24 & -24 \\ -11 & -55 & -22 & 22 \end{bmatrix}$ 

$$f: \mathbb{R}^4 o \mathbb{R}^2$$

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

f(1,0,-1,8) =

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

f(1,0,-1,8) = (

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

 $f(1,0,-1,8) = (1+2\cdot 0 - (-1) - 8,$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

f(1,0,-1,8) =

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

f(1,0,-1,8) = (-6,6)

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

f(1,0,-1,8) = (-6,6)

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

f(1,0,-1,8) = (-6,6)

2. način

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

 $f: \mathbb{R}^4 \to \mathbb{R}^2$ 

$$Y_{\mathcal{B}_{\mathsf{kan}}} = F_{(\mathcal{A}_{\mathsf{kan}},\mathcal{B}_{\mathsf{kan}})} X_{\mathcal{A}_{\mathsf{kan}}}$$

f(1,0,-1,8) = (-6,6)

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

 $f: \mathbb{R}^4 \to \mathbb{R}^2$ 

2. način
$$Y_{\mathcal{B}_{kan}} = F_{(\mathcal{A}_{kan},\mathcal{B}_{kan})} X_{\mathcal{A}_{kan}}$$

f(1,0,-1,8) = (-6,6)

 $Y_{\mathcal{B}_{\mathsf{kan}}} =$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

$$Y_{\mathcal{B}_{\mathsf{kan}}} = F_{(\mathcal{A}_{\mathsf{kan}},\mathcal{B}_{\mathsf{kan}})} X_{\mathcal{A}_{\mathsf{kan}}}$$

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

e) 1. način

f(1,0,-1,8) = (-6,6)

 $Y_{\mathcal{B}_{\mathsf{kan}}} = egin{bmatrix} 1 & 2 & -1 & -1 \ -1 & -2 & 1 & 1 \end{bmatrix}$ 

 $f: \mathbb{R}^4 \to \mathbb{R}^2$ 

$$egin{aligned} Y_{\mathcal{B}_{\mathsf{kan}}} &= F_{(\mathcal{A}_{\mathsf{kan}},\mathcal{B}_{\mathsf{kan}})} X_{\mathcal{A}_{\mathsf{kan}}} \ Y_{\mathcal{B}_{\mathsf{kan}}} &= egin{bmatrix} 1 & 2 & -1 & -1 \ -1 & -2 & 1 & 1 \end{bmatrix} egin{bmatrix} 1 \ 0 \ -1 \ 8 \end{bmatrix} \end{aligned}$$

f(1,0,-1,8) = (-6,6)

2. način

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

 $f: \mathbb{R}^4 \to \mathbb{R}^2$ 

$$egin{aligned} Y_{\mathcal{B}_{\mathsf{kan}}} &= F_{(\mathcal{A}_{\mathsf{kan}},\mathcal{B}_{\mathsf{kan}})} X_{\mathcal{A}_{\mathsf{kan}}} \ Y_{\mathcal{B}_{\mathsf{kan}}} &= egin{bmatrix} 1 & 2 & -1 & -1 \ -1 & -2 & 1 & 1 \end{bmatrix} egin{bmatrix} 1 \ 0 \ -1 \ 0 \end{bmatrix} & Y_{\mathcal{B}_{\mathsf{kan}}} &= egin{bmatrix} -6 \ 6 \end{bmatrix} \end{aligned}$$

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

e) 1. način

2. način

f(1,0,-1,8)=(-6,6)

 $f: \mathbb{R}^4 \to \mathbb{R}^2$ 

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$
  
 $f: \mathbb{R}^4 \to \mathbb{R}^2$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

 $f: \mathbb{R}^4 \to \mathbb{R}^2$ 

$$(1,0,-1,8) f: \mathbb{R}^4 \to \mathbb{R}^2$$

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

3. način 
$$f(x,y,u,v) = (x+2y-u-v, -x-2y+u+v)$$
$$f: \mathbb{R}^4 \to \mathbb{R}^2$$

 $\mathcal{A}_{kan} \xrightarrow{S} \mathcal{A} \qquad \boxed{Y_{\mathcal{B}} = F_{(\mathcal{A},\mathcal{B})} X_{\mathcal{A}}}$ 

 $\mathcal{A}_{\mathsf{kan}} \xrightarrow{S} \mathcal{A} \qquad Y_{\mathcal{B}} = F_{(\mathcal{A},\mathcal{B})} X_{\mathcal{A}}$ 

3. način

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

$$S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

 $f: \mathbb{R}^4 \to \mathbb{R}^2$ 

 $A_{\text{kan}} \xrightarrow{S} A$   $A \xrightarrow{S^{-1}} A_{\text{kan}}$ 

3. način

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

$$S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

 $f: \mathbb{R}^4 \to \mathbb{R}^2$ 

 $A_{\text{kan}} \xrightarrow{S} A$   $Y_{\mathcal{B}} = F_{(\mathcal{A}, \mathcal{B})} X_{\mathcal{A}}$   $A \xrightarrow{S^{-1}} A_{\text{kan}}$ 

3. način

$$X_{\mathcal{A}} = S^{-1} X_{\mathcal{A}_{\mathsf{kan}}}$$
  $S = egin{bmatrix} 1 & 1 & 1 & 1 \ 0 & 2 & 2 & 2 \ 0 & 0 & 3 & 3 \ 0 & 0 & 0 & 4 \end{bmatrix}$ 

f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)

 $f: \mathbb{R}^4 \to \mathbb{R}^2$ 

3. način 
$$f(x,y,u,v)=(x+2y-u-v,-x-2y+u+v)$$
  $(1,0,-1,8)$  moramo pronaći koordinate u bazi  $\mathcal{A}$   $f:\mathbb{R}^4\to\mathbb{R}^2$ 

$$S^{-1} = \begin{bmatrix} 1 & -\frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & -\frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix} \qquad \begin{matrix} \mathcal{A}_{\text{kan}} & \overset{S}{\longrightarrow} \mathcal{A} \\ \mathcal{A} & \overset{S^{-1}}{\longrightarrow} \mathcal{A}_{\text{kan}} \\ \mathcal{X}_{\mathcal{A}} = S^{-1} \mathcal{X}_{\mathcal{A}_{\text{kan}}} \end{matrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & \frac{1}{4} \end{bmatrix} \qquad X_{\mathcal{A}} = S^{-1} X_{\mathcal{A}_{\mathsf{kan}}}$$
 
$$S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

3. način

$$\begin{array}{c}
\mathsf{DZ} \\
S^{-1} = \begin{bmatrix}
1 & -\frac{1}{2} & 0 & 0 \\
0 & \frac{1}{2} & -\frac{1}{3} & 0 \\
0 & 0 & \frac{1}{3} & -\frac{1}{4} \\
0 & 0 & 0 & \frac{1}{4}
\end{bmatrix} & \mathcal{A}_{\mathsf{kan}} \xrightarrow{S^{-1}} \mathcal{A}_{\mathsf{kan}} \\
\mathcal{X}_{\mathcal{A}} = S^{-1} \mathcal{X}_{\mathcal{A}_{\mathsf{kan}}}
\end{array}$$

$$\begin{bmatrix} 0 & 0 & 0 & rac{1}{4} \end{bmatrix}$$
  $X_{\mathcal{A}} = S^{-1} X_{\mathcal{A}_{\mathsf{kan}}}$   $S = egin{bmatrix} 1 & 1 & 1 & 1 \ 0 & 2 & 2 & 2 \ 0 & 0 & 3 & 3 \ 0 & 0 & 0 & 4 \end{bmatrix}$ 

$$(1,0,-1,8)$$
 ----- moramo pronaći koordinate u bazi  $\mathcal{A}$   $f:\mathbb{R}^4 o\mathbb{R}^2$ 

3. način

$$\begin{array}{c}
\mathsf{DZ} \\
S^{-1} = \begin{bmatrix}
1 & -\frac{1}{2} & 0 & 0 \\
0 & \frac{1}{2} & -\frac{1}{3} & 0 \\
0 & 0 & \frac{1}{3} & -\frac{1}{4} \\
0 & 0 & 0 & \frac{1}{4}
\end{bmatrix} & \mathcal{A}_{\mathsf{kan}} \xrightarrow{S} \mathcal{A} \\
\mathcal{A} \xrightarrow{S^{-1}} \mathcal{A}_{\mathsf{kan}} \\
\mathcal{X}_{\mathcal{A}} = S^{-1} \mathcal{X}_{\mathcal{A}_{\mathsf{kan}}}
\end{array}$$

$$S = egin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

3. način 
$$f(x,y,u,v) = (x+2y-u-v, -x-2y+u+v)$$

$$(1,0,-1,8) \xrightarrow{\text{moramo pronaći koordinate u bazi } \mathcal{A}} f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$DZ \begin{bmatrix} 1 & -\frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & -\frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix} \qquad \mathcal{A}_{\text{kan}} \xrightarrow{S^{-1}} \mathcal{A}_{\text{kan}}$$

$$X_{\mathcal{A}} = S^{-1} X_{\mathcal{A}_{\text{kan}}}$$

$$X_{\mathcal{A}} = \begin{bmatrix} 1 & -\frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & -\frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

3. način 
$$f(x,y,u,v) = (x+2y-u-v, -x-2y+u+v)$$
  $(1,0,-1,8)$  moramo pronaći koordinate u bazi  $\mathcal{A}$   $f: \mathbb{R}^4 \to \mathbb{R}^2$  
$$DZ \begin{bmatrix} 1 & -\frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & -\frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix} \qquad \mathcal{A}_{kan} \xrightarrow{S^{-1}} \mathcal{A}_{kan}$$
  $X_{\mathcal{A}} = S^{-1}X_{\mathcal{A}_{kan}}$  
$$\begin{bmatrix} 1 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$$

$$X_{\mathcal{A}} = \begin{bmatrix} 1 & -\frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & -\frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 8 \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

3. način 
$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$
  $(1, 0, -1, 8)$  moramo pronaći koordinate u bazi  $\mathcal{A}$   $f: \mathbb{R}^4 \to \mathbb{R}^2$  
$$DZ \begin{bmatrix} 1 & -\frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & -\frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix} \qquad \mathcal{A}_{kan} \xrightarrow{S^{-1}} \mathcal{A}_{kan}$$
  $\mathcal{A}_{kan}$ 

$$X_{\mathcal{A}} = \begin{bmatrix} 1 & -\frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & -\frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 8 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{3} \\ -\frac{7}{3} \\ 2 \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$f: \mathbb{R}^4 o \mathbb{R}^2$$

$$Y_{\mathcal{B}} = F_{(\mathcal{A},\,\mathcal{B})}X_{\mathcal{A}}$$

$$Y_{\mathcal{B}} =$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$
  
 $f: \mathbb{R}^4 \to \mathbb{R}^2$ 

$$Y_{\mathcal{B}}=F_{(\mathcal{A},\mathcal{B})}X_{\mathcal{A}}$$

$$Y_{\mathcal{B}} = \begin{bmatrix} 12 & 60 & 24 & -24 \\ -11 & -55 & -22 & 22 \end{bmatrix}$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$
  
 $f: \mathbb{R}^4 \to \mathbb{R}^2$ 

$$\frac{1}{(AB)}X_A$$

$$(Y_{\mathcal{B}} = F_{(\mathcal{A},\,\mathcal{B})}X_{\mathcal{A}})$$

$$Y_{\mathcal{B}} = egin{bmatrix} 12 & 60 & 24 & -24 \ -11 & -55 & -22 & 22 \end{bmatrix} egin{bmatrix} 1 \ rac{1}{3} \ -rac{7}{3} \ \end{bmatrix}$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$
$$f \cdot \mathbb{R}^4 \to \mathbb{R}^2$$

$$F_{(A,B)}X_A$$

$$Y_{\mathcal{B}} = F_{(\mathcal{A},\,\mathcal{B})}X_{\mathcal{A}}$$

$$Y_{\mathcal{B}} = \begin{bmatrix} 12 & 60 & 24 & -24 \\ -11 & -55 & -22 & 22 \end{bmatrix} \begin{bmatrix} 1 \\ \frac{1}{3} \\ -\frac{7}{3} \\ 2 \end{bmatrix} = \begin{bmatrix} -72 \\ 66 \end{bmatrix}$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$
$$f \cdot \mathbb{R}^4 \to \mathbb{R}^2$$

$$F_{(A B)}X_A$$

$$Y_{\mathcal{B}} = F_{(\mathcal{A},\,\mathcal{B})}X_{\mathcal{A}}$$

$$\begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix}$$

$$Y_{\mathcal{B}} = \begin{bmatrix} 12 & 60 & 24 & -24 \\ -11 & -55 & -22 & 22 \end{bmatrix} \begin{bmatrix} 1 \\ \frac{1}{3} \\ -\frac{7}{3} \\ 2 \end{bmatrix} = \begin{bmatrix} -72 \\ 66 \end{bmatrix}$$

$$\begin{bmatrix} -11 & -55 & -22 & 22 \end{bmatrix} \begin{bmatrix} -\frac{7}{3} \\ 2 \end{bmatrix} \begin{bmatrix} 6 \end{bmatrix}$$

$$(-72,66)_{\mathcal{B}} =$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$
$$f \cdot \mathbb{R}^4 \to \mathbb{R}^2$$

$$\mathcal{B} = \{(1, 10), (1, 11)\}$$

$$(A,B)X_A$$

$$Y_{\mathcal{B}} = F_{(\mathcal{A}, \mathcal{B})} X_{\mathcal{A}}$$

$$Y_{\mathcal{B}} = \begin{bmatrix} 12 & 60 & 24 & -24 \\ -11 & -55 & -22 & 22 \end{bmatrix} \begin{bmatrix} 1 \\ \frac{1}{3} \\ -\frac{7}{3} \\ 2 \end{bmatrix} = \begin{bmatrix} -72 \\ 66 \end{bmatrix}$$

$$(-72,66)_{\mathcal{B}} =$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$
$$f \cdot \mathbb{R}^4 \to \mathbb{R}^2$$

$$\mathcal{B} = \{(1, 10), (1, 11)\}$$

$$Y_{\mathcal{B}} = F_{(\mathcal{A},\mathcal{B})} X_{\mathcal{A}}$$

$$Y_{\mathcal{B}} = \begin{bmatrix} 12 & 60 & 24 & -24 \\ -11 & -55 & -22 & 22 \end{bmatrix} \begin{bmatrix} 1 \\ \frac{1}{3} \\ -\frac{7}{3} \\ 2 \end{bmatrix} = \begin{bmatrix} -72 \\ 66 \end{bmatrix}$$

$$(-72,66)_{\mathcal{B}} = -72 \cdot (1,10)$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$
$$f \cdot \mathbb{R}^4 \to \mathbb{R}^2$$

$$\mathcal{B} = \{(1, 10), (1, 11)\}$$

$$Y_{\mathcal{B}} = F_{(\mathcal{A},\mathcal{B})} X_{\mathcal{A}}$$

$$Y_{\mathcal{B}} = \begin{bmatrix} 12 & 60 & 24 & -24 \\ -11 & -55 & -22 & 22 \end{bmatrix} \begin{bmatrix} 1 \\ \frac{1}{3} \\ -\frac{7}{3} \\ 2 \end{bmatrix} = \begin{bmatrix} -72 \\ 66 \end{bmatrix}$$

$$(-72,66)_{\mathcal{B}} = -72 \cdot (1,10) +$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$
$$f : \mathbb{R}^4 \to \mathbb{R}^2$$

$$\mathcal{B} = \{(1, 10), (1, 11)\}$$

$$Y_{\mathcal{B}} = F_{(\mathcal{A},\mathcal{B})} X_{\mathcal{A}}$$

$$Y_{\mathcal{B}} = \begin{bmatrix} 12 & 60 & 24 & -24 \\ -11 & -55 & -22 & 22 \end{bmatrix} \begin{bmatrix} 1 \\ \frac{1}{3} \\ -\frac{7}{3} \\ 2 \end{bmatrix} = \begin{bmatrix} -72 \\ 66 \end{bmatrix}$$

$$(-72,66)_{\mathcal{B}} = -72 \cdot (1,10) + 66 \cdot (1,11)$$

$$f(x,y,u,v) = (x+2y-u-v, -x-2y+u+v)$$
$$f: \mathbb{R}^4 \to \mathbb{R}^2$$

$$\mathcal{B} = \{(1, 10), (1, 11)\}$$

$$Y_{\mathcal{B}} = F_{(\mathcal{A},\mathcal{B})} X_{\mathcal{A}}$$

$$Y_{\mathcal{B}} = \begin{bmatrix} 12 & 60 & 24 & -24 \\ -11 & -55 & -22 & 22 \end{bmatrix} \begin{bmatrix} 1 \\ \frac{1}{3} \\ -\frac{7}{3} \\ 2 \end{bmatrix} = \begin{bmatrix} -72 \\ 66 \end{bmatrix}$$

$$(-72,66)_{\mathcal{B}} = -72 \cdot (1,10) + 66 \cdot (1,11) = (-6,6)$$

$$f(x, y, u, v) = (x + 2y - u - v, -x - 2y + u + v)$$
$$f \cdot \mathbb{R}^4 \to \mathbb{R}^2$$

$$\mathcal{B} = \{(1, 10), (1, 11)\}$$

$$Y_{\mathcal{B}} = F_{(\mathcal{A},\mathcal{B})} X_{\mathcal{A}}$$

$$Y_{\mathcal{B}} = \begin{bmatrix} 12 & 60 & 24 & -24 \\ -11 & -55 & -22 & 22 \end{bmatrix} \begin{bmatrix} 1 \\ \frac{1}{3} \\ -\frac{7}{3} \\ 2 \end{bmatrix} = \begin{bmatrix} -72 \\ 66 \end{bmatrix}$$

$$(-72,66)_{\mathcal{B}} = -72 \cdot (1,10) + 66 \cdot (1,11) = (-6,6)_{\mathcal{B}_{\mathsf{kan}}}$$

Domaća zadaća

## Domaća zadaća

## Zadatak 4

Zadano je preslikavanje  $f: \mathbb{R}^3 \to \mathbb{R}^3$  definirano s

$$f(x, y, z) = (x - 2y, z, x + y).$$

- a) Dokažite da je f linearni operator.
- b) Odredite jezgru, sliku, rang i defekt operatora f.
- c) Odredite matrični prikaz operatora f u kanonskoj bazi.
- d) Odredite matrični prikaz operatora f u bazi

$$\mathcal{B} = \{(1,0,0), (1,1,0), (1,1,1)\}.$$

e) Odredite sliku vektora (2, 1, -3).

## Rješenje

b) Ker  $f = \{(0,0,0)\}, d(f) = 0, \text{Im } f = \mathbb{R}^3, r(f) = 3$ 

Baza za Ker f ne postoji jer je jezgra u ovom slučaju trivijalni vektorski prostor.

 $\mathcal{B}_{\text{Im }f} = \{(1,0,0), (0,1,0), (0,0,1)\}$ 

Linearni operator f je izomorfizam.

c)  $F_{\mathcal{B}_{\mathsf{kan}}} = egin{bmatrix} 1 & -2 & 0 \ 0 & 0 & 1 \ 1 & 1 & 0 \end{bmatrix}$ 

d) Zadatak riješite na dva načina: bez korištenja matrice prijelaza i pomoću matrice prijelaza.

$$F_{\mathcal{B}} = \begin{bmatrix} 1 & -1 & -2 \\ -1 & -2 & -1 \\ 1 & 2 & 2 \end{bmatrix} \quad T = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad T^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F_{\mathcal{B}} = T^{-1}F_{\mathcal{B}_{\mathsf{kan}}}T$$
  $\mathcal{B}_{\mathsf{kan}} \xrightarrow{T} \mathcal{B}$ 

e) 1. način (uvrštavanje u formulu kojom je zadan linearni operator) f(2,1,-3) = (0,-3,3)

2. način 
$$Y_{\mathcal{B}_{\mathsf{kan}}} = F_{\mathcal{B}_{\mathsf{kan}}} X_{\mathcal{B}_{\mathsf{kan}}}$$

$$Y_{\mathcal{B}_{\mathsf{kan}}} = egin{bmatrix} 1 & -2 & 0 \ 0 & 0 & 1 \ 1 & 1 & 0 \end{bmatrix} egin{bmatrix} 2 \ 1 \ -3 \end{bmatrix} = egin{bmatrix} 0 \ -3 \ 3 \end{bmatrix}$$

3. način 
$$Y_{\mathcal{B}} = F_{\mathcal{B}} X_{\mathcal{B}}, \quad X_{\mathcal{B}} = T^{-1} X_{\mathcal{B}_{\mathsf{kar}}}$$

$$Y_{\mathcal{B}} = \begin{bmatrix} 1 & -1 & -2 \\ -1 & -2 & -1 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix} = \begin{bmatrix} 3 \\ -6 \\ 3 \end{bmatrix}$$

$$(3,-6,3)_{\mathcal{B}} = 3 \cdot (1,0,0) + (-6) \cdot (1,1,0) + 3 \cdot (1,1,1) = (0,-3,3)_{\mathcal{B}_{\mathsf{kan}}}$$