## Seminari 2

Matematika za ekonomiste 2

Damir Horvat

FOI, Varaždin

# Sadržaj

prvi zadatak

drugi zadatak

treći zadatak

četvrti zadatak

peti zadatak

prvi zadatak

#### Zadatak 1

Zadana je funkcija troškova  $T = 0.001x^3 + 10x + 2000$ .

- a) Odredite funkcije prosječnih i graničnih troškova i nacrtajte njihove grafove na istoj slici na segmentu [0, 250].
- b) Odredite za koju količinu proizvodnje su prosječni troškovi minimalni. U kakvom su odnosu granični i prosječni troškovi za tu količinu proizvodnje? Kako se taj odnos može vidjeti na grafu?

$$T_p(x) =$$

$$T_{\rho}(x) = \frac{T(x)}{x}$$

$$T_p(x) = \frac{T(x)}{x} = \frac{0.001x^3 + 10x + 2000}{x}$$

$$T_p(x) = \frac{T(x)}{x} = \frac{0.001x^3 + 10x + 2000}{x}$$
$$T_p(x) = 0.001x^2 + 10 + \frac{2000}{x}$$

• Funkcija prosječnih troškova

$$T_p(x) = \frac{T(x)}{x} = \frac{0.001x^3 + 10x + 2000}{x}$$

$$T_p(x) = 0.001x^2 + 10 + \frac{2000}{x}$$

Funkcija graničnih troškova

• Funkcija prosječnih troškova

$$T_p(x) = \frac{T(x)}{x} = \frac{0.001x^3 + 10x + 2000}{x}$$

$$T_p(x) = 0.001x^2 + 10 + \frac{2000}{x}$$

Funkcija graničnih troškova

$$T_g(x) = T'(x)$$

• Funkcija prosječnih troškova

$$T_p(x) = \frac{T(x)}{x} = \frac{0.001x^3 + 10x + 2000}{x}$$
 $T_p(x) = 0.001x^2 + 10 + \frac{2000}{x}$ 

Funkcija graničnih troškova

$$T_g(x) = T'(x)$$
  
 $T_g(x) = 0.003x^2 + 10$ 

$$T_p(x) = 0.001x^2 + 10 + \frac{2000}{x}$$

$$T_p(x) = 0.001x^2 + 10 + \frac{2000}{x}$$

$$T_p(x) = 0.001x^2 + 10 + 2000x^{-1}$$

$$T_p(x) = 0.001x^2 + 10 + \frac{2000}{x}$$

 $T_p'(x) =$ 

$$T_p(x) = 0.001x^2 + 10 + 2000x^{-1}$$

$$T_p(x) = 0.001x^2 + 10 + \frac{2000}{x}$$

$$T_p(x) = 0.001x^2 + 10 + 2000x^{-1}$$
  
 $T'_p(x) = 0.002x - 2000x^{-2}$ 

$$0.002x - 2000x^{-2} = 0$$

$$T_p(x) = 0.001x^2 + 10 + \frac{2000}{x}$$
 $T_p(x) = 0.001x^2 + 10 + 2000x^{-1}$ 

$$T_p'(x) = 0.002x - 2000x^{-2}$$

$$T_p(x) = 0.001x^2 + 10 + \frac{2000}{x}$$
  
 $T_p(x) = 0.001x^2 + 10 + 2000x^{-1}$ 

$$T_p'(x) = 0.002x - 2000x^{-2}$$

$$0.002x - 2000x^{-2} = 0 / \cdot x^{2}$$
$$0.002x^{3} - 2000 = 0$$

$$T_p(x) = 0.001x^2 + 10 + \frac{2000}{x}$$
 0.  
 $T_p(x) = 0.001x^2 + 10 + 2000x^{-1}$   $T'_p(x) = 0.002x - 2000x^{-2}$ 

$$0.002x - 2000x^{-2} = 0 / \cdot x^{2}$$
$$0.002x^{3} - 2000 = 0$$
$$0.002x^{3} = 2000$$

 $T_p'(x) = 0.002x - 2000x^{-2}$ 

 $T_p(x) = 0.001x^2 + 10 + 2000x^{-1}$ 

 $0.002x - 2000x^{-2} = 0 / x^2$ 

 $0.002x^3 = 2000 / : 0.002$ 

 $x^3 = 1000000$ 

$$T_p'(x) = 0.002x - 2000x^{-2}$$

 $T_p(x) = 0.001x^2 + 10 + 2000x^{-1}$ 

 $0.002x - 2000x^{-2} = 0 / x^2$ 

 $0.002x^3 = 2000 / : 0.002$ 

 $x^3 = 1000000$ 

x = 100

 $0.002x^3 - 2000 = 0$ 

3/26

 $T_p'(x) = 0.002x - 2000x^{-2}$ 

 $T_{D}(x) = 0.001x^{2} + 10 + 2000x^{-1}$ 

 $0.002x - 2000x^{-2} = 0 / x^2$ 

 $0.002x^3 = 2000 / : 0.002$ 

 $x^3 = 1000000$ 

x = 100

$$egin{array}{c|c|c|c} 0 & 100 & +\infty \ \hline T_{
ho}' & - & & \ \hline T_{
ho} & & & \ \hline \end{array}$$

 $T_p'(x) = 0.002x - 2000x^{-2}$ 

 $T_p(x) = 0.001x^2 + 10 + 2000x^{-1}$ 

 $0.002x - 2000x^{-2} = 0 / x^2$ 

 $0.002x^3 = 2000 / : 0.002$ 

 $x^3 = 1000000$ 

x = 100

$$egin{array}{c|c|c|c} 0 & 100 & +\infty \ \hline T_p' & - & + \ \hline T_p & & & \end{array}$$

 $T_p'(x) = 0.002x - 2000x^{-2}$ 

 $T_{D}(x) = 0.001x^{2} + 10 + 2000x^{-1}$ 

 $0.002x - 2000x^{-2} = 0 / x^2$ 

 $0.002x^3 = 2000 / : 0.002$ 

 $x^3 = 1000000$ 

x = 100

$$\begin{array}{c|cccc}
0 & 100 & +\infty \\
\hline
T'_{p} & - & + \\
\hline
T_{p} & \searrow & \\
\end{array}$$

 $T_p'(x) = 0.002x - 2000x^{-2}$ 

 $T_{D}(x) = 0.001x^{2} + 10 + 2000x^{-1}$ 

 $0.002x - 2000x^{-2} = 0 / x^2$ 

 $0.002x^3 = 2000 / : 0.002$ 

 $x^3 = 1000000$ 

x = 100

$$\begin{array}{c|cccc}
0 & 100 & +\infty \\
\hline
T'_{p} & - & + \\
\hline
T_{p} & \nearrow & \end{array}$$

 $T_p'(x) = 0.002x - 2000x^{-2}$ 

 $T_{D}(x) = 0.001x^{2} + 10 + 2000x^{-1}$ 

 $0.002x - 2000x^{-2} = 0 / x^2$ 

 $0.002x^3 = 2000 / : 0.002$ 

 $x^3 = 1000000$ 

x = 100

$$\begin{array}{c|cccc}
0 & 100 & +\infty \\
\hline
T'_p & - & + \\
\hline
T_p & \nearrow
\end{array}$$

 $T_p'(x) = 0.002x - 2000x^{-2}$ 

 $T_p(x) = 0.001x^2 + 10 + 2000x^{-1}$ 

 $0.002x - 2000x^{-2} = 0 / x^2$ 

 $0.002x^3 = 2000 / : 0.002$ 

 $x^3 = 1000000$ 

x = 100

$$T_p(x) = 0.001x^2 + 10 + 2000x^{-1}$$

$$T'_p(x) = 0.002x - 2000x^{-2}$$

$$0.002x^3 = 2000 /: 0.002$$

$$x^3 = 1000 000$$

$$x = 100$$

 $0.002x^3 - 2000 = 0$ 

Prosječni troškovi su minimalni za 100 proizvoda.

 $T_p(x) = 0.001x^2 + 10 + \frac{2000}{}$ 

$$T_p(x) = 0.001x^2 + 10 + 2000x^{-1}$$

$$T'_p(x) = 0.002x - 2000x^{-2}$$

$$0.002x^3 = 2000 /: 0.002$$

$$x^3 = 1000 000$$

$$x = 100$$

$$T_p(100) =$$

 $0.002x^3 - 2000 = 0$ 

Prosječni troškovi su minimalni za 100 proizvoda.

 $T_p(x) = 0.001x^2 + 10 + \frac{2000}{}$ 

$$T_{p}(100) = 0.001 \cdot 100^{2} + 10 + \frac{2000}{100}$$
 $T_{p}' \mid - \mid + \mid$ 
 $T_{p} \mid \searrow \mid \nearrow \mid$ 
 $T_{p}(100) = 0.001 \cdot 100^{2} + 10 + \frac{2000}{100}$ 

 $0.002x^3 = 2000 / : 0.002$ 

 $x^3 = 1000000$ 

x = 100

 $0.002x^3 - 2000 = 0$ 

Prosječni troškovi su minimalni za 100 proizvoda.

 $T_p(x) = 0.001x^2 + 10 + \frac{2000}{.}$ 

 $T_p'(x) = 0.002x - 2000x^{-2}$ 

 $T_p(x) = 0.001x^2 + 10 + 2000x^{-1}$ 

$$T_{
ho}(100) = 0.001 \cdot 100^{2} + 10 + rac{2000}{100}$$
 $T_{
ho}' - + T_{
ho}(100) = 40$ 
minimum

 $0.002x^3 = 2000 / : 0.002$ 

 $x^3 = 1000000$ 

x = 100

 $0.002x^3 - 2000 = 0$ 

Prosječni troškovi su minimalni za 100 proizvoda.

 $T_p(x) = 0.001x^2 + 10 + \frac{2000}{}$ 

 $T_p'(x) = 0.002x - 2000x^{-2}$ 

 $T_p(x) = 0.001x^2 + 10 + 2000x^{-1}$ 

$$T_{\rho}(x) = 0.001x^{2} + 10 + 2000x^{-1}$$

$$T'_{\rho}(x) = 0.002x - 2000x^{-2}$$

$$T_{g}(x) = 0.003x^{2} + 10$$

$$0 \quad 100 \quad +\infty$$

$$T'_{\rho}(x) = 0.003x^{2} + 10$$

$$T_{\rho}(100) = 0.001 \cdot 100^{2} + 10 + \frac{2000}{100}$$

$$T_{\rho}(100) = 40$$

$$T_{g}(100) = 0.001 \cdot 100^{2} + 10 + \frac{2000}{100}$$

$$T_{g}(100) = 0.001 \cdot 100^{2} + 10 + \frac{2000}{100}$$

$$T_{g}(100) = 0.001 \cdot 100^{2} + 10 + \frac{2000}{100}$$

$$T_{g}(100) = 0.001 \cdot 100^{2} + 10 + \frac{2000}{100}$$

$$T_{g}(100) = 0.001 \cdot 100^{2} + 10 + \frac{2000}{100}$$

$$T_{g}(100) = 0.001 \cdot 100^{2} + 10 + \frac{2000}{100}$$

 $0.002x - 2000x^{-2} = 0 / x^2$ 

3/26

$$T_{p}(x) = 0.001x^{2} + 10 + 2000x^{-1}$$

$$T'_{p}(x) = 0.002x - 2000x^{-2}$$

$$T_{g}(x) = 0.003x^{2} + 10$$

$$0 \quad 100 \quad +\infty$$

$$T'_{p}(x) = 0.003x^{2} + 10$$

$$T_{p}(100) = 0.001 \cdot 100^{2} + 10 + \frac{2000}{100}$$

$$T_{p}(100) = 40$$

$$T_{p}(100) = 0.003 \cdot 100^{2} + 10$$

$$T_{g}(100) = 0.003 \cdot 100^{2} + 10$$
Prosječni troškovi su minimalni za 100 proizvoda.

 $0.002x - 2000x^{-2} = 0 / x^2$ 

 $0.002x^3 - 2000 = 0$ 

3/26

$$T_{p}(x) = 0.001x^{2} + 10 + \frac{2000}{x}$$

$$T_{p}(x) = 0.001x^{2} + 10 + 2000x^{-1}$$

$$T'_{p}(x) = 0.002x - 2000x^{-2}$$

$$T'_{p}(x) = 0.003x^{2} + 10$$

$$0 \quad 100 \quad +\infty$$

$$T'_{p}(x) = 0.003x^{2} + 10$$

$$T_{p}(100) = 0.001 \cdot 100^{2} + 10 + \frac{2000}{100}$$

$$T_{p}(100) = 40$$

$$T_{p}(100) = 0.003 \cdot 100^{2} + 10$$

$$T_{p}(100) = 40$$

$$T_{p}(100) = 40$$

Prosječni troškovi su minimalni za 100 proizvoda.

$$T_{p}(x) = 0.001x^{2} + 10 + \frac{2000}{x}$$

$$T_{p}(x) = 0.001x^{2} + 10 + 2000x^{-1}$$

$$T'_{p}(x) = 0.002x - 2000x^{-2}$$

$$T'_{p}(x) = 0.003x^{2} + 10$$

$$0 \quad 100 \quad +\infty$$

$$T'_{p}(x) = 0.003x^{2} + 10$$

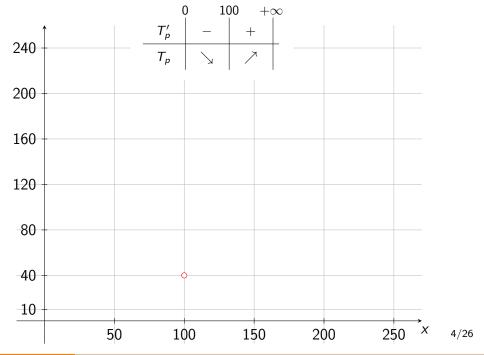
$$T'_{p}(x) = 0.003x^{2} + 10$$

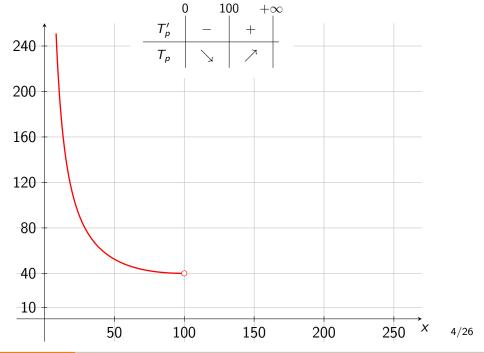
$$T_{p}(100) = 0.001 \cdot 100^{2} + 10 + \frac{2000}{100}$$

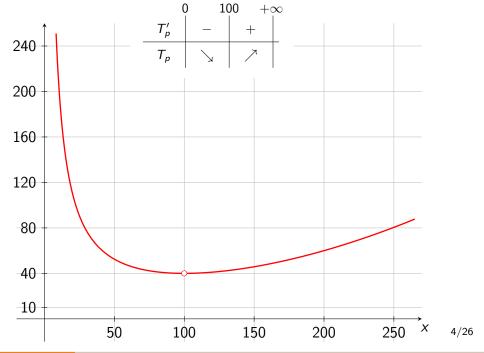
$$T_{p}(100) = 40$$

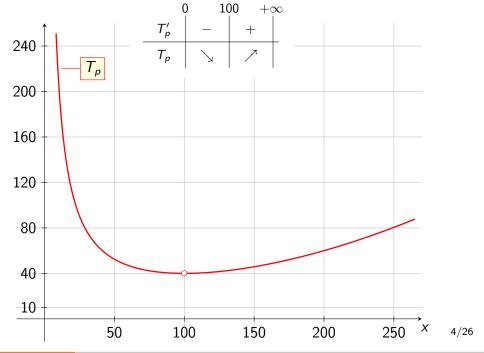
$$T_{p}(100) = 40$$

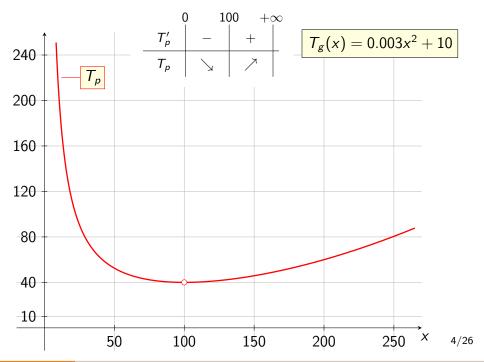
$$T_{p}(100) = 40$$
Prosječni troškovi su minimalni za 100 proizvoda. Za ovu količinu proizvodnje granični troškovi su jednaki prosječnim troškovima i iznose 40 novčanih jedinica.

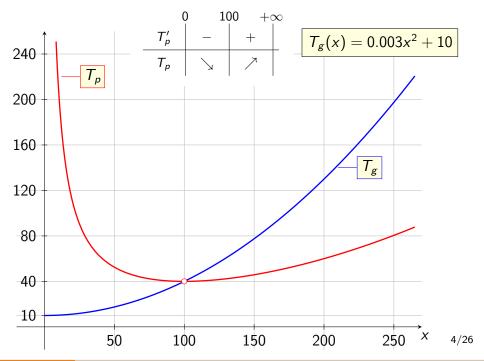












$$T_{p}(x) = \frac{T(x)}{x}$$

$$T_p(x) = \frac{T(x)}{x}$$

$$T'_p(x) =$$

$$T_p(x) = \frac{T(x)}{x}$$
 $T'_p(x) = ----$ 

$$T_p(x) = \frac{T(x)}{x}$$
$$T'_p(x) = \frac{1}{x^2}$$

$$T_{\rho}(x) = \frac{T(x)}{x}$$

$$T'_{\rho}(x) = \frac{T'(x)}{x^2}$$

$$T_{\rho}(x) = \frac{T(x)}{x}$$

$$T'_{\rho}(x) = \frac{T'(x) \cdot x^2}{x^2}$$

$$T_{\rho}(x) = \frac{T(x)}{x}$$

$$T'_{\rho}(x) = \frac{T'(x) \cdot x}{x^2}$$

$$T_{p}(x) = \frac{T(x)}{x}$$

$$T'_{p}(x) = \frac{T'(x) \cdot x - x^{2}}{x^{2}}$$

$$T_{p}(x) = \frac{T(x)}{x}$$

$$T'_{p}(x) = \frac{T'(x) \cdot x - T(x)}{x^{2}}$$

$$T_{p}(x) = \frac{T(x)}{x}$$

$$T'_{p}(x) = \frac{T'(x) \cdot x - T(x) \cdot x}{x^{2}}$$

$$T_{p}(x) = \frac{T(x)}{x}$$

$$T'_{p}(x) = \frac{T'(x) \cdot x - T(x) \cdot 1}{x^{2}}$$

$$T_{p}(x) = \frac{T(x)}{x}$$

$$T'_{p}(x) = \frac{T'(x) \cdot x - T(x) \cdot 1}{x^{2}}$$

$$T'_{p}(x) = \frac{T'(x) \cdot x - T(x) \cdot 1}{x^{2}}$$

$$T_{p}(x) = \frac{T(x)}{x}$$

$$T'_{p}(x) = \frac{T'(x) \cdot x - T(x) \cdot 1}{x^{2}}$$

$$T'_{p}(x) = \frac{T'(x) \cdot x - T(x) \cdot 1}{x^{2}}$$

$$T_{p}(x) = \frac{T(x)}{x}$$

$$T'_{p}(x) = \frac{T'(x) \cdot x - T(x) \cdot 1}{x^{2}}$$

$$T'_{p}(x) = \frac{x \cdot \left(\begin{array}{c} \\ \\ x^{2} \end{array}\right)}{x^{2}}$$

$$T_p(x) = \frac{T(x)}{x}$$

$$T'_p(x) = \frac{T'(x) \cdot x - T(x) \cdot 1}{x^2}$$

$$T'_p(x) = \frac{x \cdot \left(T'(x)\right)}{x^2}$$

$$T_p(x) = \frac{T(x)}{x}$$

$$T'_p(x) = \frac{T'(x) \cdot x - T(x) \cdot 1}{x^2}$$

$$T'_p(x) = \frac{x \cdot \left(T'(x) - \frac{1}{x^2}\right)}{x^2}$$

$$T_p(x) = \frac{T(x)}{x}$$

$$T'_p(x) = \frac{T'(x) \cdot x - T(x) \cdot 1}{x^2}$$

$$T'_p(x) = \frac{x \cdot \left(T'(x) - \frac{T(x)}{x}\right)}{x^2}$$

$$T_{\rho}(x) = \frac{T(x)}{x}$$

$$T'_{\rho}(x) = \frac{T'(x) \cdot x - T(x) \cdot 1}{x^2}$$

$$T'_{\rho}(x) = \frac{x \cdot \left(T'(x) - \frac{T(x)}{x}\right)}{x^2}$$

$$T'_{\rho}(x) = ------$$

$$T_{p}(x) = \frac{T(x)}{x}$$

$$T'_{p}(x) = \frac{T'(x) \cdot x - T(x) \cdot 1}{x^{2}}$$

$$T'_{p}(x) = \frac{x \cdot \left(T'(x) - \frac{T(x)}{x}\right)}{x^{2}}$$

$$T'_{p}(x) = \frac{x \cdot \left(T'(x) - \frac{T(x)}{x}\right)}{x}$$

$$T_{p}(x) = \frac{T(x)}{x}$$

$$T'_{p}(x) = \frac{T'(x) \cdot x - T(x) \cdot 1}{x^{2}}$$

$$T'_{p}(x) = \frac{x \cdot \left(T'(x) - \frac{T(x)}{x}\right)}{x^{2}}$$

$$T'_{p}(x) = \frac{T_{g}(x)}{x}$$

$$T_{p}(x) = \frac{T(x)}{x}$$

$$T'_{p}(x) = \frac{T'(x) \cdot x - T(x) \cdot 1}{x^{2}}$$

$$T'_{p}(x) = \frac{x \cdot \left(T'(x) - \frac{T(x)}{x}\right)}{x^{2}}$$

$$T'_{p}(x) = \frac{T_{g}(x) - \frac{T(x)}{x}}{x}$$

$$T_{p}(x) = \frac{T(x)}{x}$$

$$T'_{p}(x) = \frac{T'(x) \cdot x - T(x) \cdot 1}{x^{2}}$$

$$T'_{p}(x) = \frac{x \cdot \left(T'(x) - \frac{T(x)}{x}\right)}{x^{2}}$$

$$T'_{p}(x) = \frac{T_{g}(x) - T_{p}(x)}{x}$$

$$T_{p}(x) = \frac{T(x)}{x}$$

$$T'_{p}(x) = \frac{T'(x) \cdot x - T(x) \cdot 1}{x^{2}}$$

$$T'_{p}(x) = \frac{x \cdot \left(T'(x) - \frac{T(x)}{x}\right)}{x^{2}}$$

$$T'_{p}(x) = \frac{T_{g}(x) - T_{p}(x)}{x}$$

 Ako su granični troškovi veći od prosječnih troškova, tada prosječni troškovi rastu.

$$T'_{p}(x) = \frac{T'(x) \cdot x - T(x) \cdot 1}{x^{2}}$$

$$T'_{p}(x) = \frac{x \cdot \left(T'(x) - \frac{T(x)}{x}\right)}{x^{2}}$$

$$T'_{p}(x) = \frac{T_{g}(x) - T_{p}(x)}{x}$$

 $T_p(x) = \frac{T(x)}{x}$ 

 Ako su granični troškovi veći od prosječnih troškova, tada prosječni troškovi rastu.

5/26

 Ako su granični troškovi manji od prosječnih troškova, tada prosječni troškovi padaju.

$$T'_{p}(x) = \frac{T'(x) \cdot x - T(x)}{x^{2}} = \frac{T_{g}(x) - T_{p}(x)}{x}$$

$$T'_{p}(x) = \frac{T'(x) \cdot x - T(x)}{x^{2}} = \frac{T_{g}(x) - T_{p}(x)}{x}$$

• Iz  $T'_p(x_0) = 0$  slijedi

$$T'(x_0) = \frac{T(x_0)}{x_0}.$$

$$T'_{p}(x) = \frac{T'(x) \cdot x - T(x)}{x^{2}} = \frac{T_{g}(x) - T_{p}(x)}{x}$$

• Iz  $T'_p(x_0) = 0$  slijedi

$$T'(x_0)=\frac{T(x_0)}{x_0}.$$

ullet Jednadžba tangente na graf funkcije troškova u točki  $x_0$ 

$$t \dots y - T(x_0) = T'(x_0) \cdot (x - x_0)$$

$$T'_{p}(x) = \frac{T'(x) \cdot x - T(x)}{x^{2}} = \frac{T_{g}(x) - T_{p}(x)}{x}$$

• Iz  $T'_p(x_0) = 0$  slijedi

$$T'(x_0)=\frac{T(x_0)}{x_0}.$$

• Jednadžba tangente na graf funkcije troškova u točki  $x_0$ 

$$t \dots y - T(x_0) = T'(x_0) \cdot (x - x_0)$$
$$t \dots y = \frac{T(x_0)}{x_0} \cdot x$$

$$T_p'(x) = \frac{T'(x) \cdot x - T(x)}{x^2} = \frac{T_g(x) - T_p(x)}{x}$$

• Iz  $T'_{p}(x_0) = 0$  slijedi

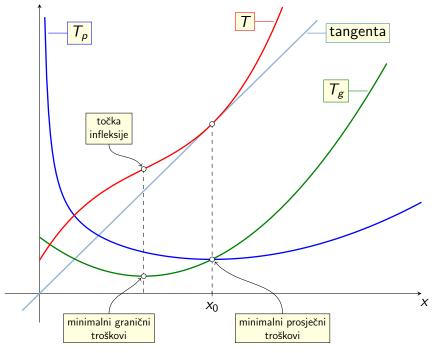
$$T'(x_0) = \frac{T(x_0)}{x_0}.$$

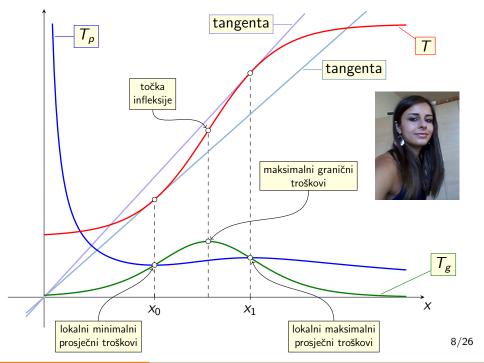
ullet Jednadžba tangente na graf funkcije troškova u točki  $x_0$ 

$$t \dots y - T(x_0) = T'(x_0) \cdot (x - x_0)$$

$$t \dots y = \frac{T(x_0)}{x_0} \cdot x$$

tangenta prolazi kroz ishodište koordinatnog sustava





## Neka svojstva funkcije troškova

- Funkcija troškova je u pravilu rastuća funkcija na  $[0, +\infty)$ .
- Ako je funkcija troškova konveksna, tada se granični troškovi povećavaju s porastom proizvodnje.
- Ako je funkcija troškova konkavna, tada se granični troškovi smanjuju s porastom proizvodnje.
- Tangenta na graf funkcije troškova u točki u kojoj su prosječni troškovi minimalni prolazi kroz ishodište koordinatnog sustava, a funkcija troškova je konveksna u toj točki.
- Razina proizvodnje za koju su prosječni troškovi minimalni, prosječni troškovi su jednaki graničnim troškovima.
- Prosječni troškovi padaju ako su veći od graničnih troškova.
   Prosječni troškovi rastu ako su manji od graničnih troškova.

### Zadatak (Domaća zadaća)

Zadana je linearna funkcija troškova T = 2x + 5.

- a) Odredite funkcije prosječnih i graničnih troškova.
- b) Nacrtajte grafove funkcije troškova, prosječnih troškova i graničnih troškova na istoj slici.
- c) Za koju razinu proizvodnje su prosječni troškovi minimalni? Objasnite što se događa u ovom slučaju.
- d) Kako se odmah mogao donijeti zaključak o minimalnim prosječnim troškovima na temelju zadane funkcije troškova? Koristite svojstvo tangente koje smo ranije spomenuli.

# 

#### Zadatak 2

Zadana je funkcija troškova  $T = 45\sqrt{3Q + 2209}$ .

- a) Odredite fiksne, varijabilne i prosječne troškove.
- b) Koliko je proizvoda proizvedeno ako su troškovi jednaki 5000?
- c) Odredite granične troškove za 10 proizvoda i interpretirajte rezultat.
- d) Odredite funkciju elastičnosti troškova.
- e) Odredite elastičnost troškova za 10 proizvoda i interpretirajte rezultat.

## $T=45\sqrt{3Q+2209}$

### Rješenje

a) Fiksni troškovi

 $T_F =$ 

## $T = 45\sqrt{3Q + 2209}$

#### Rješenje

$$T_F = T(0)$$

## $T = 45\sqrt{3Q + 2209}$

#### Rješenje

$$T_F = T(0) = 45\sqrt{3 \cdot 0 + 2209}$$

$$T_F = T(0) = 45\sqrt{3 \cdot 0 + 2209} = 45 \cdot 47$$

$$T_F = T(0) = 45\sqrt{3 \cdot 0 + 2209} = 45 \cdot 47 = 2115$$

a) Fiksni troškovi

$$T_F = T(0) = 45\sqrt{3 \cdot 0 + 2209} = 45 \cdot 47 = 2115$$

Varijabilni troškovi

a) Fiksni troškovi

$$T_F = T(0) = 45\sqrt{3 \cdot 0 + 2209} = 45 \cdot 47 = 2115$$

Varijabilni troškovi

$$T_V = T - T_F$$

a) Fiksni troškovi

$$T_F = T(0) = 45\sqrt{3 \cdot 0 + 2209} = 45 \cdot 47 = 2115$$

Varijabilni troškovi

$$T_V = T - T_F = 45\sqrt{3Q + 2209} - 2115$$

a) Fiksni troškovi

$$T_F = T(0) = 45\sqrt{3 \cdot 0 + 2209} = 45 \cdot 47 = 2115$$

Varijabilni troškovi

$$T_V = T - T_F = 45\sqrt{3Q + 2209} - 2115$$

Prosječni troškovi

a) Fiksni troškovi

$$T_F = T(0) = 45\sqrt{3 \cdot 0 + 2209} = 45 \cdot 47 = 2115$$

Varijabilni troškovi

$$T_V = T - T_F = 45\sqrt{3Q + 2209} - 2115$$

Prosječni troškovi

$$T_p = \frac{T}{Q}$$

a) Fiksni troškovi

$$T_F = T(0) = 45\sqrt{3 \cdot 0 + 2209} = 45 \cdot 47 = 2115$$

Varijabilni troškovi

$$T_V = T - T_F = 45\sqrt{3Q + 2209} - 2115$$

Prosječni troškovi

$$T_p = \frac{T}{Q} = \frac{45\sqrt{3Q + 2209}}{Q}$$

T = 5000

$$T = 5000$$

$$45\sqrt{3Q + 2209} = 5000$$

$$T = 5000$$

$$45\sqrt{3Q + 2209} = 5000 / : 5$$

$$9\sqrt{3Q + 2209} = 1000$$

$$T = 5000$$
 $45\sqrt{3Q + 2209} = 5000 / : 5$ 
 $9\sqrt{3Q + 2209} = 1000 /^{2}$ 
 $81 \cdot (3Q + 2209) = 1000000$ 

$$T = 5000$$
 $45\sqrt{3Q + 2209} = 5000 / : 5$ 
 $9\sqrt{3Q + 2209} = 1000 /^2$ 
 $81 \cdot (3Q + 2209) = 1000000$ 
 $243Q + 178929 = 1000000$ 

$$T = 5000$$
 $45\sqrt{3Q + 2209} = 5000 / : 5$ 
 $9\sqrt{3Q + 2209} = 1000 /^2$ 
 $81 \cdot (3Q + 2209) = 1000000$ 
 $243Q + 178929 = 1000000$ 
 $243Q = 821071$ 

$$T = 5000$$

$$45\sqrt{3Q + 2209} = 5000 / : 5$$

$$9\sqrt{3Q + 2209} = 1000 /^{2}$$

$$81 \cdot (3Q + 2209) = 1000000$$

$$243Q + 178929 = 1000000$$

$$243Q = 821071$$

$$Q = \frac{821071}{243}$$

$$T = 5000$$
 $45\sqrt{3Q + 2209} = 5000 / : 5$ 
 $9\sqrt{3Q + 2209} = 1000 /^2$ 
 $81 \cdot (3Q + 2209) = 1000000$ 
 $243Q + 178929 = 1000000$ 
 $243Q = 821071$ 
 $Q = \frac{821071}{243}$ 
 $Q \approx 3378.89$ 

$$T = 5000$$

$$45\sqrt{3Q + 2209} = 5000 /: 5$$

$$9\sqrt{3Q + 2209} = 1000 /^{2}$$

$$81 \cdot (3Q + 2209) = 1000000$$

$$243Q + 178929 = 1000000$$

$$243Q = 821071$$

$$Q = \frac{821071}{243}$$

$$Q \approx 3378.89$$

Proizvedeno je približno 3379 proizvoda.

$$T' =$$

$$\left(\sqrt{\mathsf{ne ilde{s}to}}\,
ight)' = rac{1}{2\sqrt{\mathsf{ne ilde{s}to}}} \cdot (\mathsf{ne ilde{s}to})'$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$'=rac{45}{2\sqrt{3Q+2209}}$$

$$\left(\sqrt{\mathsf{ne ilde{s}to}}\,\right)' = rac{1}{2\sqrt{\mathsf{ne ilde{s}to}}}\cdot (\mathsf{ne ilde{s}to})'$$
  $\left(\sqrt{x}\,\right)' = rac{1}{2\sqrt{x}}$ 

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$a' = \frac{45}{2\sqrt{3Q + 2209}}$$

$$\left(\sqrt{\mathsf{ne iny sto}}
ight)' = rac{1}{2\sqrt{\mathsf{ne iny sto}}}\cdot (\mathsf{ne iny sto})'$$
  $\left(\sqrt{x}\,\right)' = rac{1}{2\sqrt{x}}$ 

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$T' = \frac{45}{2\sqrt{3Q + 2209}} \cdot (3Q + 2209)'$$

$$\left(\sqrt{\mathsf{ne imesto}}\,
ight)' = rac{1}{2\sqrt{\mathsf{ne imesto}}} \cdot (\mathsf{ne imesto})' \qquad (\sqrt{x}\,)' = rac{1}{2\sqrt{x}}$$

$$(\bar{\kappa})' = \frac{1}{2\sqrt{x}}$$

$$T' = rac{45}{2\sqrt{3Q + 2209}} \cdot (3Q + 2209)' = rac{45}{2\sqrt{3Q + 2209}}$$

$$\left(\sqrt{\mathsf{ne iny sto}}
ight)' = rac{1}{2\sqrt{\mathsf{ne iny sto}}}\cdot (\mathsf{ne iny sto})' \qquad (\sqrt{x}\,)' = rac{1}{2\sqrt{x}}$$

$$(\bar{x})' = \frac{1}{2\sqrt{x}}$$

$$T' = \frac{45}{2\sqrt{3Q + 2209}} \cdot (3Q + 2209)' = \frac{45}{2\sqrt{3Q + 2209}} \cdot 3$$

$$\left(\sqrt{\mathsf{ne iny sto}}
ight)' = rac{1}{2\sqrt{\mathsf{ne iny sto}}}\cdot (\mathsf{ne iny sto})'$$
  $\left(\sqrt{x}\,
ight)' = rac{1}{2\sqrt{x}}$ 

$$)' = \frac{1}{2\sqrt{x}}$$

$$T' = \frac{45}{2\sqrt{3Q + 2209}} \cdot (3Q + 2209)' = \frac{45}{2\sqrt{3Q + 2209}} \cdot 3 = 135$$

$$\left(\sqrt{\mathsf{ne ilde{s}to}}\,\right)' = rac{1}{2\sqrt{\mathsf{ne ilde{s}to}}}\cdot (\mathsf{ne ilde{s}to})'$$
  $\left(\sqrt{x}\,\right)' = rac{1}{2\sqrt{x}}$ 

 $=\frac{1}{2\sqrt{3Q+2209}}$ 

$$T' = \frac{45}{2\sqrt{3Q + 2209}} \cdot (3Q + 2209)' = \frac{45}{2\sqrt{3Q + 2209}} \cdot 3 = 135$$

$$T'(10) =$$

$$\left(\sqrt{\mathsf{ne iny sto}}\,\right)' = rac{1}{2\sqrt{\mathsf{ne iny sto}}}\cdot (\mathsf{ne iny sto})'$$
  $\left(\sqrt{x}\,\right)' = rac{1}{2\sqrt{x}}$ 

 $=\frac{1}{2\sqrt{3Q+2209}}$ 

$$y'=\frac{1}{2\sqrt{x}}$$

$$T' = \frac{45}{2\sqrt{3Q + 2209}} \cdot (3Q + 2209)' = \frac{45}{2\sqrt{3Q + 2209}} \cdot 3 = 135$$

$$T'(10) = \frac{135}{2\sqrt{3\cdot 10 + 2209}}$$

 $=\frac{2\sqrt{3Q+2209}}{2\sqrt{3Q+2209}}$ 

$$\left(\sqrt{\mathsf{ne ilde{s}to}}\,
ight)' = rac{1}{2\sqrt{\mathsf{ne ilde{s}to}}} \cdot \left(\mathsf{ne ilde{s}to}
ight)' \qquad \left(\sqrt{x}\,
ight)' = rac{1}{2\sqrt{x}}$$

$$)' = \frac{1}{2\sqrt{x}}$$

$$T' = \frac{45}{2\sqrt{3Q + 2209}} \cdot (3Q + 2209)' = \frac{45}{2\sqrt{3Q + 2209}} \cdot 3 =$$
$$= \frac{135}{2\sqrt{3Q + 2209}}$$

$$T'(10) = \frac{135}{2\sqrt{3 \cdot 10 + 2209}} = \frac{135}{2\sqrt{2239}}$$

$$\left(\sqrt{\mathsf{ne imesto}}\,\right)' = rac{1}{2\sqrt{\mathsf{ne imesto}}} \cdot (\mathsf{ne imesto})'$$
  $\left(\sqrt{x}\,\right)' = rac{1}{2\sqrt{x}}$ 

$$)' = \frac{1}{2\sqrt{x}}$$

$$T' = rac{45}{2\sqrt{3Q + 2209}} \cdot (3Q + 2209)' = rac{45}{2\sqrt{3Q + 2209}} \cdot 3 = = rac{135}{2\sqrt{3Q + 2209}}$$

$$T'(10) = \frac{135}{2\sqrt{3 \cdot 10 + 2209}} = \frac{135}{2\sqrt{2239}} \approx 1.43$$

$$\left(\sqrt{\mathsf{ne imesto}}\,\right)' = rac{1}{2\sqrt{\mathsf{ne imesto}}} \cdot (\mathsf{ne imesto})'$$
  $\left(\sqrt{x}\,\right)' = rac{1}{2\sqrt{x}}$ 

$$)' = \frac{1}{2\sqrt{x}}$$

$$T' = rac{45}{2\sqrt{3Q + 2209}} \cdot (3Q + 2209)' = rac{45}{2\sqrt{3Q + 2209}} \cdot 3 =$$

$$= rac{135}{2\sqrt{3Q + 2209}}$$

$$T'(10) = \frac{135}{2\sqrt{3 \cdot 10 + 2209}} = \frac{135}{2\sqrt{2239}} \approx 1.43$$

Ako na razini proizvodnje od 10 proizvoda proizvodnju povećamo za jedan proizvod, troškovi će se povećati za 1.43 novčane jedinice.

$$(\sqrt{\text{nešto}})' = \frac{1}{2\sqrt{\text{nešto}}} \cdot (\text{nešto})'$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$=\frac{1}{2\sqrt{x}}$$

$$T' = \frac{135}{2\sqrt{3Q + 2209}}$$

$$T=45\sqrt{3Q+2209}$$

$$E_{T,Q} = \frac{Q}{T} \cdot T'$$

$$T' = \frac{135}{2\sqrt{3Q + 2209}}$$

$$T=45\sqrt{3Q+2209}$$

$$E_{T,Q} = \frac{Q}{T} \cdot T' = -$$

$$T' = \frac{135}{2\sqrt{3Q + 2209}}$$

$$T=45\sqrt{3Q+2209}$$

$$E_{T,Q} = \frac{Q}{T} \cdot T' = \frac{Q}{Q}$$

$$T' = \frac{135}{2\sqrt{3Q + 2209}}$$

$$T=45\sqrt{3Q+2209}$$

$$E_{T,Q} = \frac{Q}{T} \cdot T' = \frac{Q}{45\sqrt{3Q + 2209}}$$

$$T' = \frac{135}{2\sqrt{3Q + 2209}}$$

$$T = 45\sqrt{3Q + 2209}$$

$$E_{T,Q} = \frac{Q}{T} \cdot T' = \frac{Q}{45\sqrt{3Q + 2209}} \cdot$$

$$T' = \frac{135}{2\sqrt{3Q + 2209}}$$
 
$$T = 45\sqrt{3Q + 2209}$$

$$T=45\sqrt{3Q+2209}$$

$$E_{T,Q} = \frac{Q}{T} \cdot T' = \frac{Q}{45\sqrt{3Q + 2209}} \cdot \frac{135}{2\sqrt{3Q + 2209}}$$

$$T' = \frac{135}{2\sqrt{3Q + 2209}}$$
 
$$T = 45\sqrt{3Q + 2209}$$

$$E_{T,Q} = \frac{Q}{T} \cdot T' = \frac{Q}{45\sqrt{3Q + 2209}} \cdot \frac{135}{2\sqrt{3Q + 2209}} =$$

$$= \frac{Q}{\sqrt{3Q + 2209}} \cdot \frac{135}{\sqrt{3Q + 2209}} = \frac{Q}{\sqrt{3Q +$$

$$T' = \frac{135}{2\sqrt{3Q + 2209}}$$

 $T = 45\sqrt{3Q + 2209}$ 

$$E_{T,Q} = \frac{Q}{T} \cdot T' = \frac{Q}{45\sqrt{3Q + 2209}} \cdot \frac{135}{2\sqrt{3Q + 2209}} = \frac{3Q}{2\sqrt{3Q + 2209}}$$

$$T' = \frac{135}{2\sqrt{3Q + 2209}}$$
 
$$T = 45\sqrt{3Q + 2209}$$

$$T=45\sqrt{3Q+2209}$$

$$E_{T,Q} = \frac{Q}{T} \cdot T' = \frac{Q}{45\sqrt{3Q + 2209}} \cdot \frac{135}{2\sqrt{3Q + 2209}} = \frac{3Q}{2(3Q + 2209)}$$

$$T' = \frac{135}{2\sqrt{3Q + 2209}}$$
 
$$T = 45\sqrt{3Q + 2209}$$

$$E_{T,Q} = \frac{Q}{T} \cdot T' = \frac{Q}{45\sqrt{3Q + 2209}} \cdot \frac{135}{2\sqrt{3Q + 2209}} = \frac{3Q}{2(3Q + 2209)} = \frac{3Q}{6Q + 4418}$$

$$T' = \frac{135}{2\sqrt{3Q + 2209}}$$
 
$$T = 45\sqrt{3Q + 2209}$$

$$=45\sqrt{3Q+2209}$$

d)

$$E_{T,Q} = \frac{Q}{T} \cdot T' = \frac{Q}{45\sqrt{3Q + 2209}} \cdot \frac{135}{2\sqrt{3Q + 2209}} = \frac{3Q}{2(3Q + 2209)} = \frac{3Q}{6Q + 4418}$$

e)

$$E_{T,Q}(10) = \frac{3 \cdot 10}{6 \cdot 10 + 4418}$$

$$T' = \frac{135}{2\sqrt{3Q + 2209}}$$

$$T = 45\sqrt{3Q + 2209}$$

$$= 45\sqrt{3Q + 2209}$$

d)

$$E_{T,Q} = \frac{Q}{T} \cdot T' = \frac{Q}{45\sqrt{3Q + 2209}} \cdot \frac{135}{2\sqrt{3Q + 2209}} = \frac{3Q}{2(3Q + 2209)} = \frac{3Q}{6Q + 4418}$$

e)

$$E_{T,Q}(10) = \frac{3 \cdot 10}{6 \cdot 10 + 4418} = \frac{15}{2239} \approx 0.0067$$

$$T' = \frac{135}{2\sqrt{3Q + 2209}}$$

$$T = 45\sqrt{3Q + 2209}$$

$$C = 45\sqrt{3Q + 2209}$$

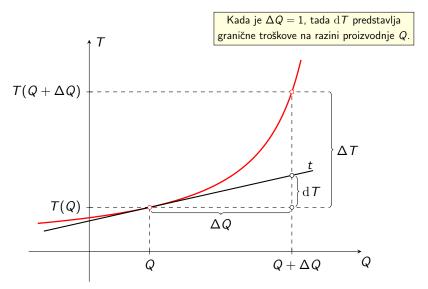
$$E_{T,Q} = \frac{Q}{T} \cdot T' = \frac{Q}{45\sqrt{3Q + 2209}} \cdot \frac{135}{2\sqrt{3Q + 2209}} = \frac{3Q}{2(3Q + 2209)} = \frac{3Q}{6Q + 4418}$$

e)

$$E_{T,Q}(10) = \frac{3 \cdot 10}{6 \cdot 10 + 4418} = \frac{15}{2239} \approx 0.0067$$

Ako na razini proizvodnje od 10 proizvoda proizvodnju povećamo za 1%, troškovi će se povećati za 0.0067%.

# Vizualizacija graničnih troškova



treći zadatak

Zadana je funkcija ponude  $q = 2.43 \sqrt[4]{18 + 0.2p^2}$ .

- a) Koliko se proizvoda nudi ako je cijena jednog proizvoda 100 kn.
- b) Odredite funkciju elastičnosti ponude.
- c) Odredite elastičnost ponude na razini cijene od 30 kn i interpretirajte rezultat.
- d) Odredite cijenu za koju je elastičnost ponude jednaka  $\frac{1}{22}$ .

Zadana je funkcija ponude  $q = 2.43 \sqrt[4]{18 + 0.2p^2}$ .

- a) Koliko se proizvoda nudi ako je cijena jednog proizvoda 100 kn.
- b) Odredite funkciju elastičnosti ponude.
- c) Odredite elastičnost ponude na razini cijene od 30 kn i interpretirajte rezultat.
- d) Odredite cijenu za koju je elastičnost ponude jednaka  $\frac{1}{22}$ .

#### Rješenje

a) 
$$q(100) = 2.43\sqrt[4]{18 + 0.2 \cdot 100^2}$$

Zadana je funkcija ponude  $q = 2.43 \sqrt[4]{18 + 0.2p^2}$ .

- a) Koliko se proizvoda nudi ako je cijena jednog proizvoda 100 kn.
- b) Odredite funkciju elastičnosti ponude.
- c) Odredite elastičnost ponude na razini cijene od 30 kn i interpretirajte rezultat.
- d) Odredite cijenu za koju je elastičnost ponude jednaka  $\frac{1}{22}$ .

## Rješenje

a) 
$$q(100) = 2.43\sqrt[4]{18 + 0.2 \cdot 100^2} \approx 16.29$$

Zadana je funkcija ponude  $q = 2.43 \sqrt[4]{18 + 0.2p^2}$ .

- a) Koliko se proizvoda nudi ako je cijena jednog proizvoda 100 kn.
- b) Odredite funkciju elastičnosti ponude.
- c) Odredite elastičnost ponude na razini cijene od 30 kn i interpretirajte rezultat.
- d) Odredite cijenu za koju je elastičnost ponude jednaka  $\frac{1}{22}$ .

## Rješenje

a)  $q(100) = 2.43\sqrt[4]{18 + 0.2 \cdot 100^2} \approx 16.29$ Ako je cijena jednog proizvoda 100 kn, nudi se približno 16 proizvoda.

b) 
$$q = 2.43 \cdot (18 + 0.2p^2)^{\frac{1}{4}}$$
  
 $q' =$ 

b) 
$$q = 2.43 \cdot (18 + 0.2p^2)^{\frac{1}{4}}$$
  
 $q' = 2.43 \cdot$ 

$$(x^n)' = nx^{n-1}$$

$$((\text{nešto})^n)' = n(\text{nešto})^{n-1} \cdot (\text{nešto})'$$
 18/26

$$q' = 2.43 \cdot \frac{1}{4} \left( 18 + 0.2p^2 \right)^{-\frac{3}{4}}$$

b)  $q = 2.43 \cdot (18 + 0.2p^2)^{\frac{1}{4}}$ 

 $\left((\mathsf{ne ext{sto}})^n\right)' = n(\mathsf{ne ext{sto}})^{n-1} \cdot (\mathsf{ne ext{sto}})'$ 

 $(x^n)' = nx^{n-1}$ 

b) 
$$q = 2.43 \cdot (18 + 0.2p^2)^{\frac{1}{4}}$$
 
$$q = 2.43 \sqrt[4]{18 + 0.2p^2}$$
$$q' = 2.43 \cdot \frac{1}{4}(18 + 0.2p^2)^{-\frac{3}{4}}$$

$$q' = 2.43 \cdot \frac{1}{4} \left( 18 + 0.2p^2 \right)^{-\frac{3}{4}} \cdot$$

$$(x^n)' = nx^{n-1}$$

$$((\text{nešto})^n)' = n(\text{nešto})^{n-1} \cdot (\text{nešto})'$$
 18/26

$$q' = 2.43 \cdot \frac{1}{4} (18 + 0.2p^2)^{-\frac{3}{4}} \cdot (18 + 0.2p^2)'$$

b)  $q = 2.43 \cdot (18 + 0.2p^2)^{\frac{1}{4}}$ 

 $ig((\mathsf{ne ext{ iny sto}})^nig)' = \mathit{n}(\mathsf{ne ext{ iny sto}})^{n-1}\cdot(\mathsf{ne ext{ iny sto}})'$ 

 $(x^n)' = nx^{n-1}$ 

b) 
$$q = 2.43 \cdot \left(18 + 0.2p^2\right)^{\frac{1}{4}}$$
 
$$q' = 2.43 \cdot \frac{1}{4} \left(18 + 0.2p^2\right)^{-\frac{3}{4}} \cdot \left(18 + 0.2p^2\right)' =$$

$$=2.43\cdot\frac{1}{4}(18+0.2p^2)^{-\frac{3}{4}}$$

$$= 2.43 \cdot \frac{10}{4} (10 + 0.2p)^{-4}$$

$$(x^n)' = nx^{n-1}$$

$$((\text{nešto})^n)' = n(\text{nešto})^{n-1} \cdot (\text{nešto})'$$
 18/2

b) 
$$q = 2.43 \cdot \left(18 + 0.2p^2\right)^{\frac{1}{4}}$$
 
$$q' = 2.43 \cdot \frac{1}{4} \left(18 + 0.2p^2\right)^{-\frac{3}{4}} \cdot \left(18 + 0.2p^2\right)' =$$

$$= 2.43 \cdot \frac{1}{4} (18 + 0.2p^{2})^{-\frac{3}{4}} \cdot 0.4p$$

 $((\text{nešto})^n)' = n(\text{nešto})^{n-1} \cdot (\text{nešto})'$ 

 $q = 2.43\sqrt[4]{18 + 0.2p^2}$ 

$$= 2.43 \cdot \frac{1}{4} (18 + 0.2p^{2})^{-\frac{3}{4}} \cdot 0.4p =$$

$$= 0.243p(18 + 0.2p^{2})^{-\frac{3}{4}}$$

$$(x^{n})' = nx^{n-1}$$

 $q' = 2.43 \cdot \frac{1}{4} (18 + 0.2p^2)^{-\frac{3}{4}} \cdot (18 + 0.2p^2)' =$ 

b)  $q = 2.43 \cdot (18 + 0.2p^2)^{\frac{1}{4}}$ 

 $((\text{nešto})^n)' = n(\text{nešto})^{n-1} \cdot (\text{nešto})'$  18/2

b) 
$$q = 2.43 \cdot \left(18 + 0.2p^2\right)^{\frac{1}{4}}$$
 
$$q' = 2.43 \cdot \frac{1}{4} \left(18 + 0.2p^2\right)^{-\frac{3}{4}} \cdot \left(18 + 0.2p^2\right)' =$$

$$=2.43\cdot\frac{1}{4}\big(18+0.2p^2\big)^{-\frac{3}{4}}\cdot0.4p=$$

$$= 0.243p(18 + 0.2p^2)^{-\frac{3}{4}}$$

$$E_{q,p} = rac{
ho}{q} \cdot q'$$

$$\mathsf{E}_{q,p} = -\cdot q$$

$$((\text{nešto})^n)' = n(\text{nešto})^{n-1} \cdot (\text{nešto})'$$

b) 
$$q = 2.43 \cdot \left(18 + 0.2p^2\right)^{\frac{1}{4}}$$
 
$$q' = 2.43 \cdot \frac{1}{4} \left(18 + 0.2p^2\right)^{-\frac{3}{4}} \cdot \left(18 + 0.2p^2\right)' =$$

$$=2.43\cdot\frac{1}{4}\big(18+0.2p^2\big)^{-\frac{3}{4}}\cdot0.4p=$$

$$=0.243p(18+0.2p^2)^{-\frac{3}{4}}$$

$$=0.243p(18+0.2p^2)^{-\frac{3}{4}}$$

$$E_{q,p} = \frac{p}{q} \cdot q' = -$$

$$((\text{nešto})^n)' = n(\text{nešto})^{n-1} \cdot (\text{nešto})'$$
 18/2

b) 
$$q = 2.43 \cdot \left(18 + 0.2p^2\right)^{\frac{1}{4}}$$
 
$$q' = 2.43 \cdot \frac{1}{4} \left(18 + 0.2p^2\right)^{-\frac{3}{4}} \cdot \left(18 + 0.2p^2\right)' =$$

$$=2.43\cdot\frac{1}{4}\big(18+0.2p^2\big)^{-\frac{3}{4}}\cdot0.4p=$$

$$= 0.243p(18 + 0.2p^2)^{-\frac{3}{4}}$$

$$E_{q,p} = \frac{p}{q} \cdot q' = \frac{p}{q}$$

$$((\text{nešto})^n)' = n(\text{nešto})^{n-1} \cdot (\text{nešto})'$$
 18/

b) 
$$q = 2.43 \cdot \left(18 + 0.2p^2\right)^{\frac{1}{4}}$$
 
$$q' = 2.43 \cdot \frac{1}{4} \left(18 + 0.2p^2\right)^{-\frac{3}{4}} \cdot \left(18 + 0.2p^2\right)' =$$

$$=2.43\cdot\frac{1}{4}\big(18+0.2p^2\big)^{-\frac{3}{4}}\cdot0.4p=$$

$$= 0.243 p \left(18 + 0.2 p^2\right)^{-\frac{3}{4}}$$

$$E_{q,p} = \frac{p}{q} \cdot q' = \frac{p}{2.43 \cdot \left(18 + 0.2p^2\right)^{\frac{1}{4}}}$$

$$((\text{nešto})^n)' = n(\text{nešto})^{n-1} \cdot (\text{nešto})'$$
18

b) 
$$q = 2.43 \cdot \left(18 + 0.2p^2\right)^{\frac{1}{4}}$$
 
$$q' = 2.43 \cdot \frac{1}{4} \left(18 + 0.2p^2\right)^{-\frac{3}{4}} \cdot \left(18 + 0.2p^2\right)' =$$

$$=2.43\cdot\frac{1}{4}\big(18+0.2p^2\big)^{-\frac{3}{4}}\cdot0.4p=$$

$$= 0.243 p (18 + 0.2p^2)^{-\frac{3}{4}}$$

$$p = \frac{p}{q} \cdot q' =$$

$$E_{q,p} = rac{p}{q} \cdot q' = rac{p}{2.43 \cdot \left(18 + 0.2p^2\right)^{rac{1}{4}}} \cdot$$

b) 
$$q = 2.43 \cdot \left(18 + 0.2p^2\right)^{\frac{1}{4}}$$
 
$$q' = 2.43 \cdot \frac{1}{4} \left(18 + 0.2p^2\right)^{-\frac{3}{4}} \cdot \left(18 + 0.2p^2\right)' =$$

$$= 2.43 \cdot \frac{1}{4} (18 + 0.2p^{2})^{-\frac{3}{4}} \cdot 0.4p =$$

$$(x^{n})' = nx^{n-1}$$

$$= 0.243p(18 + 0.2p^{2})^{-\frac{3}{4}}$$

$$= 0.243p(18 + 0.2p^{2})^{-\frac{3}{4}}$$

$$= 0.243p(18 + 0.2p^{2})^{-\frac{3}{4}}$$

$$E_{q,p} = \frac{p}{q} \cdot q' = \frac{p}{2.43 \cdot \left(18 + 0.2p^2\right)^{\frac{1}{4}}} \cdot 0.243p \left(18 + 0.2p^2\right)^{-\frac{3}{4}}$$

$$((\text{nešto})^n)' = n(\text{nešto})^{n-1} \cdot (\text{nešto})'$$
18

b) 
$$q = 2.43 \cdot \left(18 + 0.2p^2\right)^{\frac{1}{4}}$$
 
$$q' = 2.43 \cdot \frac{1}{4} \left(18 + 0.2p^2\right)^{-\frac{3}{4}} \cdot \left(18 + 0.2p^2\right)' =$$

$$=2.43\cdot\frac{1}{4}\big(18+0.2p^2\big)^{-\frac{3}{4}}\cdot0.4p=$$

$$= 0.243p(18 + 0.2p^2)^{-\frac{3}{4}} \qquad (x^n)' = nx^{n-1}$$

$$E_{q,p} = \frac{p}{q} \cdot q' = \frac{p}{2.43 \cdot \left(18 + 0.2p^2\right)^{\frac{1}{4}}} \cdot 0.243p \left(18 + 0.2p^2\right)^{-\frac{3}{4}} =$$

b) 
$$q = 2.43 \cdot (18 + 0.2p^2)^{\frac{1}{4}}$$
 
$$q' = 2.43 \cdot \frac{1}{4} (18 + 0.2p^2)^{-\frac{3}{4}} \cdot (18 + 0.2p^2)' =$$

$$=2.43\cdot\frac{1}{4}(18+0.2p^2)^{-\frac{3}{4}}\cdot0.4p=$$

$$= 0.243p(18 + 0.2p^{2})^{-\frac{3}{4}}$$

$$E_{n,p} = \frac{p}{m} \cdot q' = \frac{p}{m} \cdot 0.243p(18 + 0.2p^{2})^{-\frac{3}{4}} = 0.243p(18 + 0.2p^{2})^{-\frac{3}{4}}$$

$$E_{q,p} = \frac{p}{q} \cdot q' = \frac{p}{2.43 \cdot \left(18 + 0.2p^2\right)^{\frac{1}{4}}} \cdot 0.243p \left(18 + 0.2p^2\right)^{-\frac{3}{4}} = p^2$$

$$=$$
  $p^2$ 

$$q' = 2.43 \cdot \frac{1}{4} (18 + 0.2p^2)^{-\frac{3}{4}} \cdot (18 + 0.2p^2)' =$$

$$= 2.43 \cdot \frac{1}{4} (18 + 0.2p^2)^{-\frac{3}{4}} \cdot 0.4p =$$

$$(x^n)' = nx^{n-1}$$

$$E_{q,p} = \frac{p}{q} \cdot q' = \frac{p}{2.43 \cdot \left(18 + 0.2p^2\right)^{\frac{1}{4}}} \cdot 0.243p \left(18 + 0.2p^2\right)^{-\frac{3}{4}} =$$

 $=0.243p(18+0.2p^2)^{-\frac{3}{4}}$ 

$$=\frac{p^2}{10}$$

b)  $q = 2.43 \cdot (18 + 0.2p^2)^{\frac{1}{4}}$ 

$$((\text{nešto})^n)' = n(\text{nešto})^{n-1} \cdot (\text{nešto})'$$
 18,

$$q' = 2.43 \cdot \frac{1}{4} (18 + 0.2p^2)^{-\frac{3}{4}} \cdot (18 + 0.2p^2)' =$$

$$= 2.43 \cdot \frac{1}{4} (18 + 0.2p^2)^{-\frac{3}{4}} \cdot 0.4p =$$

$$= 0.243p(18 + 0.2p^2)^{-\frac{3}{4}}$$

$$(x^n)' = nx^{n-1}$$

$$E_{q,p} = \frac{p}{q} \cdot q' = \frac{p}{2.43 \cdot (18 + 0.2p^2)^{\frac{1}{4}}} \cdot 0.243p(18 + 0.2p^2)^{-\frac{3}{4}} =$$

$$= \frac{p^2}{10 \cdot (18 + 0.2p^2)^{\frac{1}{4}}}$$

b)  $q = 2.43 \cdot (18 + 0.2p^2)^{\frac{1}{4}}$ 

$$\left( \left( \mathsf{ne\breve{s}to} \right)^n \right)' = n \left( \mathsf{ne\breve{s}to} \right)^{n-1} \cdot \left( \mathsf{ne\breve{s}to} \right)'$$
 18/26

 $q = 2.43\sqrt[4]{18 + 0.2p^2}$ 

$$q' = 2.43 \cdot \frac{1}{4} (18 + 0.2p^2)^{-\frac{3}{4}} \cdot (18 + 0.2p^2)' =$$

$$= 2.43 \cdot \frac{1}{4} (18 + 0.2p^2)^{-\frac{3}{4}} \cdot 0.4p =$$

$$= 0.243p(18 + 0.2p^2)^{-\frac{3}{4}}$$

$$(x^n)' = nx^{n-1}$$

 $q = 2.43\sqrt[4]{18 + 0.2p^2}$ 

b)  $q = 2.43 \cdot (18 + 0.2p^2)^{\frac{1}{4}}$ 

$$= \frac{}{10 \cdot \left(18 + 0.2 \rho^2\right)^{\frac{1}{4}} \cdot \left(18 + 0.2 \rho^2\right)^{\frac{3}{4}}}$$

 $E_{q,p} = \frac{p}{q} \cdot q' = \frac{p}{2.43 \cdot \left(18 + 0.2p^2\right)^{\frac{1}{4}}} \cdot 0.243p \left(18 + 0.2p^2\right)^{-\frac{3}{4}} =$ 

$$((\text{nešto})^n)' = n(\text{nešto})^{n-1} \cdot (\text{nešto})'$$
 18/26

$$q' = 2.43 \cdot \frac{1}{4} (18 + 0.2p^{2})^{-\frac{3}{4}} \cdot (18 + 0.2p^{2})' =$$

$$= 2.43 \cdot \frac{1}{4} (18 + 0.2p^{2})^{-\frac{3}{4}} \cdot 0.4p =$$

$$= 0.243p (18 + 0.2p^{2})^{-\frac{3}{4}}$$

$$E_{q,p} = \frac{p}{q} \cdot q' = \frac{p}{2.43 \cdot (18 + 0.2p^{2})^{\frac{1}{4}}} \cdot 0.243p (18 + 0.2p^{2})^{-\frac{3}{4}} =$$

b)  $q = 2.43 \cdot (18 + 0.2p^2)^{\frac{1}{4}}$ 

$$=\frac{p^2}{10\cdot \left(18+0.2p^2\right)^{\frac{1}{4}}\cdot \left(18+0.2p^2\right)^{\frac{3}{4}}}=\frac{p^2}{10\cdot \left(18+0.2p^2\right)}$$

$$\frac{\left((\text{nešto})^n\right)' = n(\text{nešto})^{n-1} \cdot (\text{nešto})'}{(\text{nešto})^n}$$
 18/26

 $q = 2.43\sqrt[4]{18 + 0.2p^2}$ 

b) 
$$q = 2.43 \cdot (18 + 0.2p^2)^{\frac{1}{4}}$$
  $q = 2.43 \sqrt[4]{18 + 0.2p^2}$   $q' = 2.43 \cdot \frac{1}{4} (18 + 0.2p^2)^{-\frac{3}{4}} \cdot (18 + 0.2p^2)' =$   $= 2.43 \cdot \frac{1}{4} (18 + 0.2p^2)^{-\frac{3}{4}} \cdot 0.4p =$   $= 0.243p(18 + 0.2p^2)^{-\frac{3}{4}}$   $(x^n)' = nx^{n-1}$   $E_{q,p} = \frac{p}{q} \cdot q' = \frac{p}{2.43 \cdot (18 + 0.2p^2)^{\frac{1}{4}}} \cdot 0.243p(18 + 0.2p^2)^{-\frac{3}{4}} =$   $= \frac{p^2}{10 \cdot (18 + 0.2p^2)^{\frac{1}{4}} \cdot (18 + 0.2p^2)^{\frac{3}{4}}} = \frac{p^2}{10 \cdot (18 + 0.2p^2)} =$   $= \frac{p^2}{2p^2 + 180}$   $((\text{nešto})^n)' = n(\text{nešto})^{n-1} \cdot (\text{nešto})'$   $= (\text{nešto})^n$ 

$$E_{q,p} = \frac{p^2}{2p^2 + 180}$$

$$E_{q,p} = \frac{p^2}{2p^2 + 180}$$

$$E_{q,p}(30) = \frac{30^2}{2 \cdot 30^2 + 180}$$

$$E_{q,p} = \frac{p^2}{2p^2 + 180}$$

$$E_{q,p}(30) = \frac{30^2}{2 \cdot 30^2 + 180}$$

$$E_{q,p}(30) = \frac{5}{11}$$

$$E_{q,p} = \frac{p^2}{2p^2 + 180}$$

$$E_{q,p}(30) = \frac{30^2}{2 \cdot 30^2 + 180}$$

$$E_{q,p}(30) = \frac{5}{11} \approx 0.45$$

$$E_{q,p} = rac{p^2}{2p^2 + 180}$$
  $E_{q,p}(30) = rac{30^2}{2 \cdot 30^2 + 180}$ 

$$E_{q,p}(30) = \frac{5}{11} \approx 0.45$$

Ako na razini cijene od 30 kn cijenu povećamo za 1%, ponuda će porasti za 0.45%.

$$E_{q,p}=rac{1}{22}$$

$$E_{q,p} = \frac{1}{22}$$
$$\frac{p^2}{2p^2 + 180} = \frac{1}{22}$$

$$E_{q,p} = \frac{1}{22}$$

$$\frac{p^2}{2p^2 + 180} = \frac{1}{22}$$

$$22p^2 = 2p^2 + 180$$

$$E_{q,p} = rac{1}{22}$$
  $rac{p^2}{2p^2 + 180} = rac{1}{22}$   $22p^2 = 2p^2 + 180$   $20p^2 = 180$ 

$$E_{q,p} = rac{1}{22}$$
  $rac{p^2}{2p^2 + 180} = rac{1}{22}$   $22p^2 = 2p^2 + 180$   $20p^2 = 180 \ / : 20$   $p^2 = 9$ 

$$E_{q,p} = rac{1}{22}$$
  $rac{p^2}{2p^2 + 180} = rac{1}{22}$   $22p^2 = 2p^2 + 180$   $20p^2 = 180 / : 20$   $p^2 = 9$   $p = 3$ 

$$E_{q,p} = \frac{1}{22}$$
 $\frac{p^2}{2p^2 + 180} = \frac{1}{22}$ 
 $22p^2 = 2p^2 + 180$ 
 $20p^2 = 180 / : 20$ 
 $p^2 = 9$ 
 $p = 3$ 

Na razini cijene p=3 elastičnost ponude jednaka je  $\frac{1}{2^2}$ .

## \_\_\_\_\_

četvrti zadatak

## Zadatak 4

Zadana je funkcija ponude  $q = (2p + 1) \log_4 (5p)$ .

- a) Izračunajte elastičnost ponude za cijenu p=10 i interpretirajte rezultat.
- b) Odredite koliko se proizvoda nudi po cijeni p = 18. Da li je za tu cijenu ponuda elastična ili neelastična?

 $q = (2p+1)\log_4(5p)$ 

Rješenje

a)

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

$$q'=(2p+1)'$$

$$q'=(2p+1)'\cdot$$

 $q = (2p+1)\log_4{(5p)}$ 

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

$$q' = (2p+1)' \cdot \log_4(5p)$$

$$q' = (2p+1)' \cdot \log_4(5p) +$$

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1)$$

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot$$

 $q = (2p+1)\log_4{(5p)}$ 

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

 $q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot (\log_4(5p))'$ 

 $q = (2p+1)\log_4(5p)$ 

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

$$=2\log_4{(5p)}$$

 $q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot (\log_4(5p))' =$ 

$$=2\log_4{(5p)}+(2p+1)\cdot$$

 $q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot (\log_4(5p))' =$ 

 $q = (2p+1)\log_4{(5p)}$ 

 $(\log_a(\text{nešto}))' = \frac{1}{\text{nešto} \cdot \ln a} \cdot (\text{nešto})'$ 

 $\left(\log_a x\right)' = \frac{1}{x \ln a}$ 

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot (\log_4(5p))' =$$

$$= 2\log_4(5p) + (2p+1) \cdot \frac{1}{5p \ln 4}$$

$$\left(\log_a x\right)' = \frac{1}{x \ln a}$$

$$\left(\log_a (\text{nešto})\right)' = \frac{1}{\text{nešto} \cdot \ln a} \cdot (\text{nešto})'$$

$$q' = (2p+1)' \cdot \log_4{(5p)} + (2p+1) \cdot \left(\log_4{(5p)}\right)' =$$

 $= 2\log_4{(5p)} + (2p+1) \cdot \frac{1}{5p \ln 4} \cdot 5$ 

 $q = (2p+1)\log_4{(5p)}$ 

$$\left(\log_a(\text{nešto})\right)' = \frac{1}{\text{nešto} \cdot \ln a} \cdot (\text{nešto})'$$

 $\left(\log_a x\right)' = \frac{1}{x \ln a}$ 

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot (\log_4(5p))' =$$

 $q = (2p+1)\log_4{(5p)}$ 

 $=2\log_4{(5p)}+(2p+1)\cdot\frac{1}{5p\ln 4}\cdot 5=$ 

$$= 2\log_4(5p) + \frac{2p+1}{p\ln 4} \qquad \left(\log_a x\right)' = \frac{1}{x\ln a}$$

$$\left(\log_a(\text{nešto})\right)' = \frac{1}{\text{nešto} \cdot \ln a} \cdot (\text{nešto})'$$

$$q = (2p+1)\log_4(5p)$$

a)

$$q' = (2p+1)' \cdot \log_4 (5p) + (2p+1) \cdot (\log_4 (5p))' =$$

$$= 2\log_4 (5p) + (2p+1) \cdot \frac{1}{5p \ln 4} \cdot 5 =$$

$$= 2\log_4 (5p) + \frac{2p+1}{p \ln 4}$$

$$E_{q,p}(10) = \frac{10}{q(10)} \cdot q'(10) =$$

$$E_{q,p} = \frac{p}{q} \cdot q'$$

$$q = (2p+1)\log_4(5p)$$

a)

$$\begin{aligned} q' &= (2p+1)' \cdot \log_4 (5p) + (2p+1) \cdot (\log_4 (5p))' = \\ &= 2\log_4 (5p) + (2p+1) \cdot \frac{1}{5p \ln 4} \cdot 5 = \\ &= 2\log_4 (5p) + \frac{2p+1}{p \ln 4} \end{aligned}$$

$$E_{q,p}(10) = \frac{10}{g(10)} \cdot g'(10) =$$

 $E_{q,p} = \frac{p}{q} \cdot q'$ 

$$q(10) = (2 \cdot 10 + 1) \cdot \log_4 (5 \cdot 10)$$

22/26

$$q = (2p+1)\log_4(5p)$$

$$=2\log_4{(5p)}+(2p+1)\cdotrac{1}{5p\ln 4}\cdot 5= \ =2\log_4{(5p)}+rac{2p+1}{p\ln 4}$$

$$E_{q,p}(10) = rac{10}{g(10)} \cdot q'(10) =$$

 $q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot (\log_4(5p))' =$ 

$$E_{q,p} = \frac{p}{q} \cdot q'$$

 $\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}$ 

$$q(10) = (2 \cdot 10 + 1) \cdot \log_4 (5 \cdot 10) = 21 \log_4 50$$

$$q = (2p+1)\log_4(5p)$$

$$= 2 \log_4 (5p) + (2p+1) \cdot \frac{1}{5p \ln 4} \cdot 5 =$$

$$= 2 \log_4 (5p) + \frac{2p+1}{p \ln 4}$$

 $q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot (\log_4(5p))' =$ 

$$E_{q,p}(10) = rac{10}{g(10)} \cdot g'(10) =$$

 $E_{q,p} = \frac{p}{q} \cdot q'$ 

 $\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}$ 

$$q(10) = (2 \cdot 10 + 1) \cdot \log_4 (5 \cdot 10) = 21 \log_4 50 \approx 59.26$$

$$(0) = (2 \cdot 10 + 1) \cdot \log_4 (5 \cdot 10) = 21 \log_4 50 \approx 59.$$

$$q = (2p+1)\log_4(5p)$$

$$\log_a x = \frac{\log x}{\log a} =$$

 $= 2 \log_4 (5p) + (2p+1) \cdot \frac{1}{5p \ln 4} \cdot 5 =$ 

 $q(10) = (2 \cdot 10 + 1) \cdot \log_4 (5 \cdot 10) = 21 \log_4 50 \approx 59.26$ 

 $\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}$ 

$$E_{q,p}(10) = rac{10}{q(10)} \cdot q'(10) =$$

 $q'(10) = 2\log_4(5 \cdot 10) + \frac{2 \cdot 10 + 1}{10 \ln 4}$ 

 $=2\log_4{(5p)}+\frac{2p+1}{p\ln 4}$ 

 $E_{q,p} = \frac{p}{q} \cdot q'$ 

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot (\log_4(5p))' =$$

 $q = (2p+1)\log_4{(5p)}$ 

$$= 2 \log_4 (5p) + (2p+1) \cdot \frac{1}{5p \ln 4} \cdot 5 =$$

$$= 2 \log_4 (5p) + \frac{2p+1}{p \ln 4}$$

$$E_{q,p}(10) = \frac{10}{q(10)} \cdot q'(10) =$$

 $E_{q,p} = \frac{p}{q} \cdot q'$ 

 $\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}$ 

$$q(10) = (2 \cdot 10 + 1) \cdot \log_4 (5 \cdot 10) = 21 \log_4 50 \approx 59.26$$

$$q'(10) = 2 \log_4 (5 \cdot 10) + \frac{2 \cdot 10 + 1}{10 \ln 4} = 2 \log_4 50 + \frac{21}{10 \ln 4}$$

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot (\log_4(5p))' =$$

 $q = (2p+1)\log_4{(5p)}$ 

 $\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}$ 

 $E_{q,p} = \frac{p}{q} \cdot q'$ 

$$= 2 \log_4 (5p) + (2p+1) \cdot \frac{1}{5p \ln 4} \cdot 5 =$$

$$= 2 \log_4 (5p) + \frac{2p+1}{p \ln 4}$$

 $E_{q,p}(10) = \frac{10}{q(10)} \cdot q'(10) =$ 

$$q(10) = (2 \cdot 10 + 1) \cdot \log_4 (5 \cdot 10) = 21 \log_4 50 \approx 59.26$$

$$q'(10) = 2 \log_4 (5 \cdot 10) + \frac{2 \cdot 10 + 1}{10 \ln 4} = 2 \log_4 50 + \frac{21}{10 \ln 4} \approx 7.16$$

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot (\log_4(5p))' =$$

 $q = (2p+1)\log_4{(5p)}$ 

 $\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}$ 

 $E_{q,p} = \frac{p}{q} \cdot q'$ 

$$= 2 \log_4 (5p) + (2p+1) \cdot \frac{1}{5p \ln 4} \cdot 5 =$$

$$= 2 \log_4 (5p) + \frac{2p+1}{p \ln 4}$$

$$E_{q,p}(10) = \frac{10}{g(10)} \cdot q'(10) = \frac{10}{59.26} \cdot 7.16$$

$$q(10) = (2 \cdot 10 + 1) \cdot \log_4 (5 \cdot 10) = 21 \log_4 50 \approx 59.26$$

 $q'(10) = 2\log_4(5 \cdot 10) + \frac{2 \cdot 10 + 1}{10 \ln 4} = 2\log_4 50 + \frac{21}{10 \ln 4} \approx 7.16$ 

$$\boxed{q = (2p+1)\log_4(5p)} \qquad \boxed{\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}}$$
$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot (\log_4(5p))' =$$

 $q = (2p+1)\log_4{(5p)}$ 

$$= 2 \log_4 (5p) + (2p+1) \cdot \frac{1}{5p \ln 4} \cdot 5 =$$

$$= 2 \log_4 (5p) + \frac{2p+1}{p \ln 4}$$

$$E_{q,p}(10)$$

$$E_{q,p}(10) = \frac{10}{q(10)} \cdot q'(10) = \frac{10}{59.26} \cdot 7.16 \approx 1.21$$
  $E_{q,p} = \frac{p}{q} \cdot q'$ 

$$q(10) = (2 \cdot 10 + 1) \cdot \log_4{(5 \cdot 10)} = 21 \log_4{50} \approx 59.26$$

 $q'(10) = 2\log_4(5 \cdot 10) + \frac{2 \cdot 10 + 1}{10 \ln 4} = 2\log_4 50 + \frac{21}{10 \ln 4} \approx 7.16$ 

$$q' = (2p+1)' \cdot \log_4(5p) + (2p+1) \cdot (\log_4(5p))' =$$

$$= 2\log_4(5p) + (2p+1) \cdot \frac{1}{5p\ln 4} \cdot 5 =$$

$$= 2\log_4(5p) + \frac{2p+1}{p\ln 4}$$

 $q = (2p+1)\log_4{(5p)}$ 

Rješenje

a)

 $\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}$ 

$$q(10) = (2 \cdot 10 + 1) \cdot \log_4 (5 \cdot 10) = 21 \log_4 50 \approx 59.26$$

$$q'(10) = 21 \log_4 (5 \cdot 10) + 2 \log_4 (5 \cdot 10) = 21 \log_4 50 \approx 59.26$$

 $E_{q,p}(10) = \frac{10}{g(10)} \cdot g'(10) = \frac{10}{59.26} \cdot 7.16 \approx 1.21$   $E_{q,p} = \frac{p}{q} \cdot q'$ 

 $q'(10) = 2\log_4(5\cdot 10) + \frac{2\cdot 10 + 1}{10\ln 4} = 2\log_4 50 + \frac{21}{10\ln 4} \approx 7.16$ Ako na razini cijene p=10 cijenu povećamo za 1%, ponuda će se

Ako na razini cijene p=10 cijenu povećamo za 1%, ponuda će se povećati za 1.21%.

$$q = (2p+1)\log_4{(5p)}$$

$$q' = 2\log_4(5p) + \frac{2p+1}{p\ln 4}$$

b) 
$$q(18) = (2 \cdot 18 + 1) \log_4 (5 \cdot 18) =$$

$$q = (2p+1)\log_4(5p)$$

$$q' = 2\log_4(5p) + \frac{2p+1}{p\ln 4}$$

b) 
$$q(18) = (2 \cdot 18 + 1) \log_4 (5 \cdot 18) = 37 \log_4 90$$

$$q = (2p+1)\log_4(5p)$$

$$q' = 2\log_4(5p) + \frac{2p+1}{p\ln 4}$$

b) 
$$q(18) = (2 \cdot 18 + 1) \log_4 (5 \cdot 18) = 37 \log_4 90 \approx 120.099$$

$$q = (2p+1)\log_4(5p)$$

$$q' = 2\log_4(5p) + \frac{2p+1}{p\ln 4}$$

b) 
$$q(18) = (2 \cdot 18 + 1) \log_4 (5 \cdot 18) = 37 \log_4 90 \approx 120.099$$
  
Po cijeni  $p = 18$  nudi se oko 120 proizvoda.

$$q=(2p+1)\log_4(5p)$$

$$q' = 2\log_4(5p) + \frac{2p+1}{p\ln 4}$$

b) 
$$q(18) = (2 \cdot 18 + 1) \log_4 (5 \cdot 18) = 37 \log_4 90 \approx 120.099$$

$$E_{q,p} = \frac{p}{q} \cdot q'$$

$$E_{q,p}(18) = rac{18}{q(18)} \cdot q'(18) =$$

$$q = (2p+1)\log_4(5p)$$

$$q'=2\log_4(5p)+\frac{2p+1}{p\ln 4}$$

b) 
$$q(18) = (2 \cdot 18 + 1) \log_4 (5 \cdot 18) = 37 \log_4 90 \approx 120.099$$

$$E_{q,p} = \frac{p}{q} \cdot q'$$

$$E_{q,p}(18) = \frac{18}{q(18)} \cdot q'(18) =$$

$$q'(18) = 2\log_4(5 \cdot 18) + \frac{2 \cdot 18 + 1}{18 \ln 4}$$

$$q = (2p+1)\log_4(5p)$$

$$q' = 2\log_4(5p) + \frac{2p+1}{p\ln 4}$$

b) 
$$q(18) = (2 \cdot 18 + 1) \log_4 (5 \cdot 18) = 37 \log_4 90 \approx 120.099$$

$$E_{q,p} = \frac{p}{q} \cdot q'$$

$$E_{q,p}(18) = \frac{18}{q(18)} \cdot q'(18) =$$

$$q'(18) = 2\log_4(5 \cdot 18) + \frac{2 \cdot 18 + 1}{18\ln 4} = 2\log_4 90 + \frac{37}{18\ln 4}$$

$$q = (2p+1)\log_4(5p)$$

$$q'=2\log_4(5p)+\frac{2p+1}{p\ln 4}$$

b) 
$$q(18) = (2 \cdot 18 + 1) \log_4 (5 \cdot 18) = 37 \log_4 90 \approx 120.099$$

$$\boxed{E_{q,p} = \frac{p}{q} \cdot q'}$$

$$E_{q,p}(18) = \frac{18}{g(18)} \cdot g'(18) =$$

$$q'(18) = 2\log_4(5 \cdot 18) + \frac{2 \cdot 18 + 1}{18\ln 4} = 2\log_4 90 + \frac{37}{18\ln 4} \approx 7.97$$

$$q = (2p+1)\log_4(5p)$$

$$q'=2\log_4(5p)+\frac{2p+1}{p\ln 4}$$

b) 
$$q(18) = (2 \cdot 18 + 1) \log_4 (5 \cdot 18) = 37 \log_4 90 \approx 120.099$$

$$E_{q,p} = \frac{p}{q} \cdot q'$$

$$E_{q,p}(18) = \frac{18}{q(18)} \cdot q'(18) = \frac{18}{120.099} \cdot 7.97$$

$$q'(18) = 2\log_4(5 \cdot 18) + \frac{2 \cdot 18 + 1}{18\ln 4} = 2\log_4 90 + \frac{37}{18\ln 4} \approx 7.97$$

$$q = (2p+1)\log_4(5p)$$

$$q'=2\log_4(5p)+\frac{2p+1}{p\ln 4}$$

b) 
$$q(18) = (2 \cdot 18 + 1) \log_4 (5 \cdot 18) = 37 \log_4 90 \approx 120.099$$

$$E_{q,p} = \frac{p}{q} \cdot q'$$

$$E_{q,p}(18) = \frac{18}{q(18)} \cdot q'(18) = \frac{18}{120.099} \cdot 7.97 \approx \overline{1.195}$$

$$q'(18) = 2\log_4(5 \cdot 18) + \frac{2 \cdot 18 + 1}{18\ln 4} = 2\log_4 90 + \frac{37}{18\ln 4} \approx 7.97$$

$$q = (2p+1)\log_4(5p)$$

$$q' = 2\log_4(5p) + \frac{2p+1}{p\ln 4}$$

b) 
$$q(18) = (2 \cdot 18 + 1) \log_4 (5 \cdot 18) = 37 \log_4 90 \approx 120.099$$

$$\boxed{E_{q,p} = \frac{p}{q} \cdot q'}$$

$$E_{q,p}(18) = \frac{18}{q(18)} \cdot q'(18) = \frac{18}{120.099} \cdot 7.97 \approx 1.195$$

$$q'(18) = 2\log_4(5\cdot 18) + \frac{2\cdot 18 + 1}{18\ln 4} = 2\log_4 90 + \frac{37}{18\ln 4} \approx 7.97$$

Kako je  $|E_{q,p}(18)|=|1.195|=1.195>1$ , zaključujemo da je ponuda elastična za cijenu p=18.

# \_\_\_\_\_

peti zadatak

#### Zadatak 5

Funkcija potražnje zadana je s  $Q = 2005 + 25 \cdot 0.7^{p}$ .

- a) Odredite elastičnost potražnje za p = 4 i interpretirajte rezultat.
- b) Ako se cijena na razini p=4 poveća za 1%, da li će prihod porasti ili će se smanjiti? Objasnite!
- c) Odredite cijenu za koju je potražnja jednaka 2010.

- a)
- Q' =

 $E_{Q,p} = rac{p}{Q} \cdot Q'$ 

a)

$$Q'=0+$$

 $E_{Q,p} = rac{p}{Q} \cdot Q'$ 

a)

$$Q'=0+25\cdot$$

 $E_{Q,p} = rac{p}{Q} \cdot Q'$ 

Rješenje

 $(a^x)' = a^x \ln a$ 

$$Q' = 0 + 25 \cdot 0.7^p \cdot \ln 0.7$$

 $Q = 2005 + 25 \cdot 0.7^{p}$ 

#### Ĭ.

Rješenje

 $(a^x)' = a^x \ln a$ 

 $E_{Q,p}=rac{p}{Q}\cdot Q'$ 

a) 
$$Q' = 0 + 25 \cdot 0.7^p \cdot \ln 0.7$$
 
$$Q' = 25 \cdot 0.7^p \cdot \ln 0.7$$

a)  $Q' = 0 + 25 \cdot 0.7^p \cdot \ln 0.7$ 

 $Q' = 25 \cdot 0.7^p \cdot \ln 0.7$ 

 $(a^x)' = a^x \ln a$ 

$$E_{Q,p}(4)=\frac{4}{Q(4)}\cdot Q'(4)$$

 $Q = 2005 + 25 \cdot 0.7^{p}$ 

a)

 $Q' = 0 + 25 \cdot 0.7^p \cdot \ln 0.7$ 

$$Q' = 25 \cdot 0.7^p \cdot \ln 0.7$$

$$Q' = 25 \cdot 0.7^p \cdot \ln 0.7$$
$$Q(4) = 2005 + 25 \cdot 0.7^4$$

.7 
$$E_{Q,p}(4) = \frac{4}{Q(4)} \cdot Q'(4)$$

 $(a^x)' = a^x \ln a$ 

 $E_{Q,p} = rac{p}{Q} \cdot Q'$ 

 $Q' = 0 + 25 \cdot 0.7^p \cdot \ln 0.7$ 

$$Q' = 0 + 25 \cdot 0.7^{p} \cdot \ln 0.7$$
  
 $Q' = 25 \cdot 0.7^{p} \cdot \ln 0.7$ 

$$Q' = 25 \cdot 0.7^p \cdot \ln 0.7$$

$$Q = 25 \cdot 0.7^{2} \cdot 100.7$$

$$Q(4) = 2005 + 25 \cdot 0.7^{4}$$

Q(4) = 2011.0025

$$E_{Q,p}(4)=\frac{4}{Q(4)}\cdot Q'(4)$$

 $(a^x)' = a^x \ln a$ 

 $Q = 2005 + 25 \cdot 0.7^{p}$ 

 $\overline{E_{Q,p}} = \frac{p}{Q} \cdot Q'$ 

 $(a^x)' = a^x \ln a$ 

 $Q' = 0 + 25 \cdot 0.7^p \cdot \ln 0.7$ 

 $Q' = 25 \cdot 0.7^p \cdot \ln 0.7$ 

 $Q(4) = 2005 + 25 \cdot 0.7^4$ 

Q(4) = 2011.0025

 $Q'(4) = 25 \cdot 0.7^4 \cdot \ln 0.7$ 

 $E_{Q,p}(4) = \frac{4}{Q(4)} \cdot Q'(4)$ 

 $Q = 2005 + 25 \cdot 0.7^{p}$ 

 $(a^{\times})' = a^{\times} \ln a$ 

 $\overline{E_{Q,p}} = \frac{p}{Q} \cdot Q'$ 

$$Q' = 0 + 25 \cdot 0.7^{p} \cdot \ln 0.7$$

$$Q'=25\cdot 0.7^p\cdot \ln 0.7$$

$$E_{Q,p}(4)=\frac{4}{Q(4)}\cdot Q'(4)$$

$$Q(4) = 2005 + 25 \cdot 0.7^4$$

$$Q(4) = 2011.0025$$
$$Q'(4) = 25 \cdot 0.7^4 \cdot \ln 0.7$$

$$Q'(4) = 23 \cdot 0.7 \cdot 1110.7$$
  
 $Q'(4) = -2.14094$ 

 $Q' = 0 + 25 \cdot 0.7^p \cdot \ln 0.7$ 

$$Q' = 0 + 25 \cdot 0.7^{p} \cdot \ln 0.7$$
  
 $Q' = 25 \cdot 0.7^{p} \cdot \ln 0.7$ 

$$Q' = 25 \cdot 0.7^{p} \cdot \ln 0.7$$

$$Q(4) = 2005 + 25 \cdot 0.7^{4}$$

$$Q(4) = 2011.0025$$

$$4) = 2011.0025$$

$$Q'(4) = 25 \cdot 0.7^4 \cdot \ln 0.7$$

$$Q'(4) = 25 \cdot 0.7^4 \cdot \ln 0.7$$
  
 $Q'(4) = -2.14094$ 

$$=\frac{1}{Q}$$

 $E_{Q,p}(4) = ---$ 

 $(a^x)' = a^x \ln a$ 

$$E_{Q,p}(4) = \frac{4}{Q(4)} \cdot Q'(4)$$

 $Q = 2005 + 25 \cdot 0.7^{p}$ 

 $Q' = 0 + 25 \cdot 0.7^p \cdot \ln 0.7$  $Q' = 25 \cdot 0.7^p \cdot \ln 0.7$ 

$$= 2005 + 25 \cdot 0.7^4$$

$$Q(4) = 2005 + 25 \cdot 0.7^4$$

$$Q(4) = 2011.0025$$

$$Q'(4) = 25 \cdot 0.7^4 \cdot \ln 0.7$$

$$Q'(4) = 25 \cdot 0.7^4 \cdot \ln 0.7$$
  
 $Q'(4) = -2.14094$ 

$$E_{Q,p}(4) = \frac{4}{2011,0025}$$

 $(a^x)' = a^x \ln a$ 

$$E_{Q,p}(4)=\frac{4}{Q(4)}\cdot Q'(4)$$

 $Q = 2005 + 25 \cdot 0.7^{p}$ 

a) 
$$O' =$$

$$Q' = 0 + 25 \cdot 0.7^{\rho} \cdot \ln 0.7$$
  
 $Q' = 25 \cdot 0.7^{\rho} \cdot \ln 0.7$ 

$$Q' = 25 \cdot 0.7^{p} \cdot \ln 0.7$$

$$Q(4) = 2005 + 25 \cdot 0.7^{4}$$

$$Q(4) = 2005 + 25 \cdot 0.7^4$$

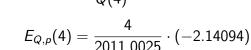
$$Q(4) = 2011 \cdot 0025$$

$$Q(4) = 2005 + 25 \cdot 0.7^4$$
$$Q(4) = 2011.0025$$

$$25 \cdot 0.7^4 \cdot \ln 0.7$$

$$25 \cdot 0.7^4 \cdot \ln 0.7$$

$$Q'(4) = 25 \cdot 0.7^4 \cdot \ln 0.7$$
  
 $Q'(4) = -2.14094$ 



 $(a^x)' = a^x \ln a$ 

$$Q(4) = \frac{1}{Q(4)}$$

$$E_{Q,p}(4) = \frac{4}{Q(4)} \cdot Q'(4)$$

 $Q = 2005 + 25 \cdot 0.7^{p}$ 

$$\frac{1}{25} \cdot \left(-2\right)$$

$$\frac{1}{5} \cdot (-2)$$

Q'(4) = -2.14094

$$Q' = 0 + 25 \cdot 0.7^{p} \cdot \ln 0.7$$
  
 $Q' = 25 \cdot 0.7^{p} \cdot \ln 0.7$ 

$$Q(4) = 2005 + 25 \cdot 0.7^4$$

$$Q(4) = 2011.0025$$

$$Q'(4) = 25 \cdot 0.7^4 \cdot \ln 0.7$$

$$E_{Q,p}(4) = \frac{4}{2011,0025} \cdot (-2.14094)$$

 $(a^x)' = a^x \ln a$ 

$$\overline{Q}(Q) = \overline{Q}(Q)$$

$$E_{Q,p}(4) = \frac{4}{Q(4)} \cdot Q'(4)$$

$$\frac{1}{5} \cdot (-2)$$

 $Q = 2005 + 25 \cdot 0.7^{p}$ 

$$\frac{1}{5} \cdot (-2)$$

$$\frac{1}{5} \cdot (-1)$$

$$E_{Q,p}(4) \approx -0.0043$$

$$Q' = 0 + 25 \cdot 0.7^p \cdot \ln 0.7$$

 $(a^{x})' = a^{x} \ln a$ 

25/26

 $Q' = 25 \cdot 0.7^p \cdot \ln 0.7$  $Q(4) = 2005 + 25 \cdot 0.7^4$   $E_{Q,p}(4) = \frac{4}{Q(4)} \cdot Q'(4)$ 

 $E_{Q,p}(4) = \frac{4}{2011\ 0025} \cdot (-2.14094)$ 

 $Q = 2005 + 25 \cdot 0.7^{p}$ 

 $E_{Q,p} = \frac{p}{Q} \cdot Q'$ 

Ako na razini cijene p=4 cijenu povećamo za 1%, potražnja će se

 $E_{Q,p}(4) \approx -0.0043$ 

Q(4) = 2011.0025

smanjiti za 0.0043%.

 $Q'(4) = 25 \cdot 0.7^4 \cdot \ln 0.7$ 

Q'(4) = -2.14094





a)  $Q' = 0 + 25 \cdot 0.7^p \cdot \ln 0.7$ 

 $(a^{x})' = a^{x} \ln a$   $Q = 2005 + 25 \cdot 0.7^{p}$ 

 $E_{Q,p}(4) = \frac{4}{2011\ 0025} \cdot (-2.14094)$ 

 $E_{Q,p}(4) = \frac{4}{Q(4)} \cdot Q'(4)$ 

Riešenje

 $Q' = 25 \cdot 0.7^p \cdot \ln 0.7$ 

 $Q(4) = 2005 + 25 \cdot 0.7^4$ 

smanjiti za 0.0043%.

Q(4) = 2011.0025

$$Q'(4) = 25 \cdot 0.7^4 \cdot \ln 0.7$$
  $E_{Q,p}(4) \approx -0.0043$   $Q'(4) = -2.14094$ 

Ako na razini cijene p=4 cijenu povećamo za 1%, potražnja će se

jer je potražnja na toj razini cijene neelastična, tj.  $|E_{q,p}(4)| < 1$ .

b) Ako se cijena na razini p=4 poveća za 1%, prihod će se povećati

```
Q = 2005 + 25 \cdot 0.7^{p}
```

prvi način

$$2005 + 25 \cdot 0.7^p = 2010$$

```
Q = 2005 + 25 \cdot 0.7^{p}
```

prvi način

$$2005 + 25 \cdot 0.7^p = 2010$$
$$25 \cdot 0.7^p = 5$$

```
Q = 2005 + 25 \cdot 0.7^{p}
```

```
c)
```

prvi način

$$2005 + 25 \cdot 0.7^{p} = 2010$$
$$25 \cdot 0.7^{p} = 5 /: 25$$
$$0.7^{p} = 0.2$$

```
Q = 2005 + 25 \cdot 0.7^{p}
```

$$2005 + 25 \cdot 0.7^{p} = 2010$$
$$25 \cdot 0.7^{p} = 5 /: 25$$
$$0.7^{p} = 0.2$$
$$p = \log_{0.7} 0.2$$

$$2005 + 25 \cdot 0.7^{p} = 2010$$

$$25 \cdot 0.7^{p} = 5 / : 25$$

$$0.7^{p} = 0.2$$

$$p = \log_{0.7} 0.2$$

$$p = \frac{\log 0.2}{\log 0.7}$$

$$2005 + 25 \cdot 0.7^{p} = 2010$$
$$25 \cdot 0.7^{p} = 5 / : 25$$
$$0.7^{p} = 0.2$$
$$p = \log_{0.7} 0.2$$

$$p = 4.51$$

$$2005 + 25 \cdot 0.7^{p} = 2010$$
$$25 \cdot 0.7^{p} = 5 /: 25$$

$$0.7^p = 0.2$$

$$p = \log_{0.7} 0.2$$

$$p = \frac{\log 0.2}{\log 0.7}$$

$$p = 4.51$$

$$2005 + 25 \cdot 0.7^p = 2010$$

$$2005 + 25 \cdot 0.7^p = 2010$$

$$25 \cdot 0.7^p = 5 / : 25$$
  
 $0.7^p = 0.2$ 

$$p = \log_{0.7} 0.2$$

$$p = \frac{\log 0.2}{\log 0.7}$$

$$p = 4.51$$

$$2005 + 25 \cdot 0.7^p = 2010$$
$$25 \cdot 0.7^p = 5$$

$$2005 + 25 \cdot 0.7^{p} = 2010$$
$$25 \cdot 0.7^{p} = 5 /: 25$$
$$0.7^{p} = 0.2$$
$$p = \log_{0.7} 0.2$$

$$\rho = \log 0.7$$

$$p = 4.51$$

$$2005 + 25 \cdot 0.7^{p} = 2010$$
$$25 \cdot 0.7^{p} = 5 /: 25$$
$$0.7^{p} = 0.2$$

$$2005 + 25 \cdot 0.7^{p} = 2010$$
$$25 \cdot 0.7^{p} = 5 /: 25$$

$$0.7^p = 0.2$$

$$p = \log_{0.7} 0.2$$

$$p = \frac{\log 0.2}{\log 0.2}$$

$$p = 4.51$$

$$2005 + 25 \cdot 0.7^{p} = 2010$$
$$25 \cdot 0.7^{p} = 5 /: 25$$
$$0.7^{p} = 0.2 / \log \log 0.7^{p} = \log 0.2$$

$$2005 + 25 \cdot 0.7^{p} = 2010$$
$$25 \cdot 0.7^{p} = 5 /: 25$$
$$0.7^{p} = 0.2$$
$$p = \log_{0.7} 0.2$$

$$p = \frac{\log 0.2}{\log 0.7}$$

$$p = 4.51$$

$$2005 + 25 \cdot 0.7^{p} = 2010$$
$$25 \cdot 0.7^{p} = 5 /: 25$$
$$0.7^{p} = 0.2 / \log \log 0.7^{p} = \log 0.2$$

$$p \cdot \log 0.7 = \log 0.2$$

$$Q = 2005 + 25 \cdot 0.7^{p}$$

$$2005 + 25 \cdot 0.7^{p} = 2010$$
$$25 \cdot 0.7^{p} = 5 /: 25$$

$$p = \log_{0.7} 0.2$$

$$p = \frac{\log 0.2}{\log 0.7}$$

 $0.7^p = 0.2$ 

$$p = 4.51$$

$$2005 + 25 \cdot 0.7^p = 2010$$

$$25 \cdot 0.7^p = 5 / : 25$$
  
 $0.7^p = 0.2 / \log$ 

$$\log 0.7^p = \log 0.2$$

$$p \cdot \log 0.7 = \log 0.2 / : \log 0.7$$

$$p = \frac{\log 0.2}{\log 0.7}$$

$$Q = 2005 + 25 \cdot 0.7^{p}$$

$$2005 + 25 \cdot 0.7^{p} = 2010$$
$$25 \cdot 0.7^{p} = 5 /: 25$$

$$0.7^p = 0.2$$

$$p = \log_{0.7} 0.2$$

$$p = \frac{6}{\log 0.7}$$

p = 4.51

$$2005 + 25 \cdot 0.7^p = 2010$$

$$25 \cdot 0.7^p = 5 / : 25$$
  
 $0.7^p = 0.2 / \log$ 

$$\log 0.7^p = \log 0.2$$

$$p \cdot \log 0.7 = \log 0.2 / : \log 0.7$$

$$p = \frac{\log 0.2}{\log 0.7}$$

$$\frac{\log 0.2}{\log 0.7}$$

$$p = 4.51$$