Seminari 8

Matematika za ekonomiste 2

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Sadržaj

prvi zadatak

drugi zadatak

treći zadatak

četvrti zadatak

prvi zadatak

Zadatak 1

Zadana je funkcija proizvodnje

$$Q(L, K) = 0.24L^{0.45}K^{0.37}$$

- u ovisnosti o radu L i kapitalu K.
 - a) Provjerite da je Q homogena funkcija i odredite njezin stupanj homogenosti.
 - b) Koristeći Eulerov teorem odredite sumu parcijalnih elastičnosti proizvodnje u odnosu na rad i kapital.
 - c) Odredite sumu parcijalnih elastičnosti direktno bez korištenja Eulerovog teorema.
 - d) Kakav tip prinosa određuje zadana funkcija proizvodnje?
 - e) Za koliko se promijeni količina proizvodnje ako rad i kapital povećamo za 10%?

 $Q(\lambda L, \lambda K) =$

 $Q(\lambda L, \lambda K) = \lambda^{\alpha} Q(L, K)$

 $Q(L,K) = 0.24L^{0.45}K^{0.37}$

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 $Q(\lambda L, \lambda K) = 0.24$ ·

 $Q(\lambda L, \lambda K) = \lambda^{\alpha} Q(L, K)$

$$Q(\lambda L, \lambda K) = 0.24 \cdot (\lambda L)^{0.45}$$

 $Q(\lambda L, \lambda K) = 0.24 \cdot (\lambda L)^{0.45} \cdot (\lambda K)^{0.37}$

 $Q(\lambda L, \lambda K) = \lambda^{\alpha} Q(L, K)$

a)

 $Q(\lambda L, \lambda K) = \lambda^{\alpha} Q(L, K)$

 $Q(\lambda L, \lambda K) = 0.24 \cdot (\lambda L)^{0.45} \cdot (\lambda K)^{0.37} =$

 $= 0.24 \cdot$

a)

$$Q(\lambda L, \lambda K) = \lambda^{\alpha} Q(L, K)$$

$$Q(\lambda L, \lambda K) = \lambda^{\alpha} Q(L, K)$$

 $= 0.24 \cdot \lambda^{0.45} L^{0.45} \cdot \lambda^{0.37} K^{0.37}$

$$Q(\lambda L, \lambda K) = 0.24 \cdot (\lambda L)^{0.45} \cdot (\lambda K)^{0.37} =$$

$$Q(\lambda L, \lambda K) = \lambda^{\alpha} Q(L, K)$$

- a)
 - $Q(\lambda L, \lambda K) = 0.24 \cdot (\lambda L)^{0.45} \cdot (\lambda K)^{0.37} =$
 - $= 0.24 \cdot \lambda^{0.45} L^{0.45} \cdot \lambda^{0.37} K^{0.37} =$

 $Q(L,K) = 0.24L^{0.45}K^{0.37}$

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$$Q(\lambda L, \lambda K) = \lambda^{\alpha} Q(L, K)$$

 $=\lambda^{0.82}\cdot 0.24$

 $= 0.24 \cdot \lambda^{0.45} L^{0.45} \cdot \lambda^{0.37} K^{0.37} =$

 $Q(L,K) = 0.24L^{0.45}K^{0.37}$

a)

$$Q(\lambda L, \lambda K) = 0.24 \cdot (\lambda L)^{0.45} \cdot (\lambda K)^{0.37} =$$

 $=\lambda^{0.82}\cdot 0.24L^{0.45}$

 $= 0.24 \cdot \lambda^{0.45} L^{0.45} \cdot \lambda^{0.37} K^{0.37} =$

a) $Q(\lambda L, \lambda K) = 0.24 \cdot (\lambda L)^{0.45} \cdot (\lambda K)^{0.37} =$

 $=\lambda^{0.82}\cdot 0.24L^{0.45}K^{0.37}$

$$Q(\lambda L, \lambda K) = 0.24 \cdot (\lambda L)^{0.45} \cdot (\lambda K)^{0.37} =$$

$$= 0.24 \cdot \lambda^{0.45} L^{0.45} \cdot \lambda^{0.37} K^{0.37} =$$

 $Q(L,K) = 0.24L^{0.45}K^{0.37}$

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$$= 0.24 \cdot \lambda^{0.45} L^{0.45} \cdot \lambda^{0.37} K^{0.37} =$$

 $=\lambda^{0.82}$.

 $=\lambda^{0.82}\cdot 0.24L^{0.45}K^{0.37}=$

$$Q(\lambda L, \lambda K) = 0.24 \cdot (\lambda L)^{0.45} \cdot (\lambda K)^{0.37} =$$

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$$= \lambda^{0.82} \cdot 0.24 L^{0.45} K^{0.37} =$$

$$=\lambda^{0.82}\cdot Q(L,K)$$

$$Q(L, K) = 0.24L^{0.45}K^{0.37}$$

$$Q(\lambda L, \lambda K) = 0.24 \cdot (\lambda L)^{0.45} \cdot (\lambda K)^{0.37} =$$

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$$= \lambda^{0.82} \cdot 0.24 L^{0.45} K^{0.37} =$$

 $=\lambda^{0.82}\cdot Q(L,K)$

Q je homogena funkcija stupnja homogenosti lpha=0.82.

Rješenje

$Q(\lambda L, \lambda K) = \lambda^{\alpha} Q(L, K)$

a)
$$Q(\lambda L, \lambda K) = 0.24 \cdot (\lambda L)^{0.45} \cdot (\lambda K)^{0.37} =$$

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$$= \lambda^{0.82} \cdot 0.24 L^{0.45} K^{0.37} =$$

$$=\lambda^{0.82}\cdot Q(L,K)$$
 Q je homogena funkcija stupnja homogenosti $lpha=0.82.$



$$E_{Q,L} + E_{Q,K} =$$

Rješenje

 $Q(\lambda L, \lambda K) = \lambda^{\alpha} Q(L, K)$

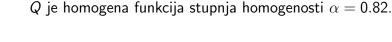
 $Q(L, K) = 0.24L^{0.45}K^{0.37}$

 $=\lambda^{0.82}\cdot 0.24L^{0.45}K^{0.37}=$

 $= 0.24 \cdot \lambda^{0.45} L^{0.45} \cdot \lambda^{0.37} K^{0.37} =$

$$Q(\lambda L, \lambda K) = 0.24 \cdot (\lambda L)^{0.45} \cdot (\lambda K)^{0.37} =$$

$$=\lambda^{0.82}\cdot Q(L,K)$$



$$E_{Q,L} + E_{Q,K} = \alpha$$

Rješenje

a)

 $Q(\lambda L, \lambda K) = \lambda^{\alpha} Q(L, K)$

 $Q(\lambda L, \lambda K) = 0.24 \cdot (\lambda L)^{0.45} \cdot (\lambda K)^{0.37} =$

 $=\lambda^{0.82}\cdot 0.24I^{0.45}K^{0.37}=$ $=\lambda^{0.82}\cdot Q(L,K)$

$$= 0.24 \cdot \lambda^{0.45} L^{0.45} \cdot \lambda^{0.37} K^{0.37} =$$

$$= \lambda^{0.82} \cdot 0.24 L^{0.45} K^{0.37} =$$

$$Q$$
 je homogena funkcija stupnja homogenosti $lpha=0.82.$

$$= \alpha$$

)
$$E_{Q,L} + E_{Q,K} = \alpha$$

$$E_{Q,L} + E_{Q,K} = 0.82$$

c)
$$E_{Q,L} + E_{Q,K} =$$

$$E_{Q,L} =$$

c)
$$E_{Q,L} + E_{Q,K} =$$

$${\it E_{Q,L}} = rac{{\it L}}{{\it Q}} \cdot {\it Q_L}$$

c)
$$E_{Q,L} + E_{Q,K} =$$

$$E_{Q,L} = \frac{L}{Q} \cdot Q_L = ----$$

$$E_{Q,L} = rac{L}{Q} \cdot Q_L = rac{L}{Q}$$

c) $E_{Q,L} + E_{Q,K} =$

c)
$$E_{Q,L} + E_{Q,K} =$$

c)
$$L_{Q,L} + L_{Q,K}$$

$$E_{Q,L} = rac{L}{Q} \cdot Q_L = rac{L}{0.24L^{0.45}K^{0.37}}$$

 $Q(L,K) = 0.24L^{0.45}K^{0.37}$ c) $E_{Q,L} + E_{Q,K} =$

$$E_{Q,L} + E_{Q,K}$$

$$E_{Q,L} = rac{L}{Q} \cdot Q_L = rac{L}{0.24L^{0.45}K^{0.37}} \, .$$

 $Q(L,K) = 0.24L^{0.45}K^{0.37}$ c) $E_{Q,L} + E_{Q,K} =$

 $E_{Q,L} = \frac{L}{Q} \cdot Q_L = \frac{L}{0.24L^{0.45}K^{0.37}} \cdot 0.24K^{0.37}$

 $Q(L,K) = 0.24L^{0.45}K^{0.37}$ c) $E_{Q,L} + E_{Q,K} =$

)
$$E_{Q,L} + E_{Q,K}$$

$$E_{Q,L} = \frac{L}{Q} \cdot Q_L = \frac{L}{0.24L^{0.45}K^{0.37}} \cdot 0.24K^{0.37} \cdot 0.45L^{-0.55}$$

c)
$$E_{Q,L} + E_{Q,K} =$$

$$E_{Q,L} = \frac{L}{Q} \cdot Q_L = \frac{L}{0.24L^{0.45}K^{0.37}} \cdot 0.24K^{0.37} \cdot 0.45L^{-0.55}$$
 $E_{Q,L} =$

c)
$$E_{Q,L} + E_{Q,K} =$$

$$E_{Q,L} = \frac{L}{Q} \cdot Q_L = \frac{L}{0.24L^{0.45}K^{0.37}} \cdot 0.24K^{0.37} \cdot 0.45L^{-0.55}$$

$$E_{Q,L} = 0.45$$

c)
$$E_{Q,L} + E_{Q,K} =$$

$$E_{Q,L} = \frac{L}{Q} \cdot Q_L = \frac{L}{0.24L^{0.45}K^{0.37}} \cdot 0.24K^{0.37} \cdot 0.45L^{-0.55}$$
 $E_{Q,L} = 0.45$

$$E_{Q,K} =$$

c)
$$E_{Q,L} + E_{Q,K} =$$

$$E_{Q,L} = \frac{L}{Q} \cdot Q_L = \frac{L}{0.24L^{0.45}K^{0.37}} \cdot 0.24K^{0.37} \cdot 0.45L^{-0.55}$$

$$E_{Q,L} = 0.45$$

$$E_{Q,K} = \frac{K}{Q} \cdot Q_K$$

c)
$$E_{Q,L} + E_{Q,K} =$$

$$\overline{Q(L,K)} = 0.24L^{0.45}K^{0.37}$$

$$E_{Q,L} = \frac{L}{Q} \cdot Q_L = \frac{L}{0.24L^{0.45}K^{0.37}} \cdot 0.24K^{0.37} \cdot 0.45L^{-0.55}$$

$$E_{Q,L} = 0.45$$

$$E_{Q,K} = \frac{K}{Q} \cdot Q_K = -$$

c)
$$E_{Q,L} + E_{Q,K} =$$

$$Q, \kappa =$$

$$E_{Q,L} = \frac{L}{Q} \cdot Q_L = \frac{L}{0.24L^{0.45}K^{0.37}} \cdot 0.24K^{0.37} \cdot 0.45L^{-0.55}$$

$$E_{Q,L} = 0.45$$

c)
$$E_{Q,L} + E_{Q,K} =$$

$$E_{Q,L} = \frac{L}{Q} \cdot Q_L = \frac{L}{0.24L^{0.45}K^{0.37}} \cdot 0.24K^{0.37} \cdot 0.45L^{-0.55}$$
 $E_{Q,L} = 0.45$

$$= 0.4$$

$$E_{Q,K} = \frac{K}{Q} \cdot Q_K = \frac{K}{0.24L^{0.45}K^{0.37}}$$

c)
$$E_{Q,L} + E_{Q,K} =$$

$$E_{Q,L} = \frac{L}{Q} \cdot Q_L = \frac{L}{0.24L^{0.45}K^{0.37}} \cdot 0.24K^{0.37} \cdot 0.45L^{-0.55}$$

$$E_{Q,L}=0.45$$

$$E_{Q,K} = \frac{K}{Q} \cdot Q_K = \frac{K}{0.24L^{0.45}K^{0.37}} \cdot$$

c)
$$E_{Q,L} + E_{Q,K} =$$

$$E_{Q,L} = \frac{L}{Q} \cdot Q_L = \frac{L}{0.24L^{0.45}K^{0.37}} \cdot 0.24K^{0.37} \cdot 0.45L^{-0.55}$$
 $E_{Q,L} = 0.45$

$$= 0.4$$

$$E_{Q,K} = \frac{K}{Q} \cdot Q_K = \frac{K}{0.24L^{0.45}K^{0.37}} \cdot 0.24L^{0.45} \cdot$$

c)
$$E_{Q,L} + E_{Q,K} =$$

$$E_{Q,L} = \frac{L}{Q} \cdot Q_L = \frac{L}{0.24L^{0.45}K^{0.37}} \cdot 0.24K^{0.37} \cdot 0.45L^{-0.55}$$

$$E_{Q,L}=0.45$$

$$E_{Q,K} = \frac{K}{Q} \cdot Q_K = \frac{K}{0.24L^{0.45}K^{0.37}} \cdot 0.24L^{0.45} \cdot 0.37K^{-0.63}$$

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c)
$$E_{Q,L} + E_{Q,K} =$$

$$E_{Q,L} = \frac{L}{Q} \cdot Q_L = \frac{L}{0.24L^{0.45}K^{0.37}} \cdot 0.24K^{0.37} \cdot 0.45L^{-0.55}$$

 $E_{Q,K} = \frac{K}{Q} \cdot Q_K = \frac{K}{0.24I^{0.45}K^{0.37}} \cdot 0.24L^{0.45} \cdot 0.37K^{-0.63}$

$$E_{Q,L} = 0.45$$

 $E_{Q,K} =$

c)
$$E_{Q,L} + E_{Q,K} =$$

$$E_{Q,L} = \frac{L}{Q} \cdot Q_L = \frac{L}{0.24L^{0.45}K^{0.37}} \cdot 0.24K^{0.37} \cdot 0.45L^{-0.55}$$

$$E_{Q,L} = 0.45$$

$$Q_K = \frac{1}{0}$$

$$E_{Q,K} = \frac{K}{Q} \cdot Q_K = \frac{K}{0.24L^{0.45}K^{0.37}} \cdot 0.24L^{0.45} \cdot 0.37K^{-0.63}$$

$E_{Q,K} = 0.37$

$$0.24L^{0.45}K^{0.3}$$

c)
$$E_{Q,L} + E_{Q,K} = 0.45 + 0.37$$

$$E_{Q,L} = \frac{L}{Q} \cdot Q_L = \frac{L}{0.24L^{0.45}K^{0.37}} \cdot 0.24K^{0.37} \cdot 0.45L^{-0.55}$$
 $E_{Q,L} = 0.45$

$$= 0.45$$

$$E_{Q,K} = \frac{K}{Q} \cdot Q_K = \frac{K}{0.24L^{0.45}K^{0.37}} \cdot 0.24L^{0.45} \cdot 0.37K^{-0.63}$$

$$E_{Q,K}=0.37$$

c) $E_{O.L} + E_{O.K} = 0.45 + 0.37 = 0.82$

$$E_{Q,L} = \frac{L}{Q} \cdot Q_L = \frac{L}{0.24L^{0.45}K^{0.37}} \cdot 0.24K^{0.37} \cdot 0.45L^{-0.55}$$
 $E_{Q,L} = 0.45$

$$L_{Q,L} = 0.43$$

$$E_{Q,K} = \frac{K}{Q} \cdot Q_K = \frac{K}{0.24L^{0.45}K^{0.37}} \cdot 0.24L^{0.45} \cdot 0.37K^{-0.63}$$
 $E_{Q,K} = 0.37$

c)
$$E_{Q,L} + E_{Q,K} = 0.45 + 0.37 = 0.82$$

$$E_{Q,L} = \frac{L}{Q} \cdot Q_L = \frac{L}{0.24L^{0.45}K^{0.37}} \cdot 0.24K^{0.37} \cdot 0.45L^{-0.55}$$
 $E_{Q,L} = 0.45$

$$E_{Q,K} = \frac{K}{Q} \cdot Q_K = \frac{K}{0.24L^{0.45}K^{0.37}} \cdot 0.24L^{0.45} \cdot 0.37K^{-0.63}$$
 $E_{Q,K} = 0.37$

d) Stupanj homogenosti:
$$\alpha = 0.82$$
, $0 < \alpha < 1$

 α) Stupanj nomogenosti. $\alpha=0.02, \quad 0<\alpha$

c)
$$E_{Q,L} + E_{Q,K} = 0.45 + 0.37 = 0.82$$

 $E_{O,I} = 0.45$

proizvodnje ima padajuće prinose.

$$E_{Q,K}=rac{K}{Q}\cdot Q_K=rac{K}{0.24L^{0.45}K^{0.37}}\cdot 0.24L^{0.45}\cdot 0.37K^{-0.63}$$
 $E_{Q,K}=0.37$ d) Stupanj homogenosti: $lpha=0.82,\quad 0$

Kako je stupanj homogenosti između 0 i 1, zadana funkcija

 $E_{Q,L} = \frac{L}{Q} \cdot Q_L = \frac{L}{0.2410.45 \, \text{k}' 0.37} \cdot 0.24 \, \text{K}^{0.37} \cdot 0.45 \, L^{-0.55}$

L 10% povećanja rada

L — 10% povećanja rada L + 0.1L = 1.1L

L $\xrightarrow{10\%$ povećanja rada \longrightarrow L+0.1L=1.1L \longrightarrow K+0.1K=1.1K

10% povećanja rada

Κ

10% povećanja rada

10% povećanja kapitala

Q(1.1L, 1.1K) =

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 $Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$

L + 0.1L = 1.1L

K + 0.1K = 1.1K

K

10% povećanja rada

10% povećanja kapitala

 $Q(1.1L, 1.1K) = 1.1^{0.82} \cdot Q(L, K)$

 $Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$

L + 0.1L = 1.1L

K + 0.1K = 1.1K

10% povećanja rada

Promjena proizvodnje:

e)

 $Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$

L + 0.1L = 1.1L

• Promjena proizvodnje: Q(1.1L, 1.1K) - Q(L, K)

10% povećanja rada

e)

 $Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$

L + 0.1L = 1.1L

 $Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$

L + 0.1L = 1.1L

Promjena proizvodnje u postocima:

10% povećanja kapitala

10% povećanja rada

$$K$$
 — 10% povećanja kapitala $ext{ } K+0.1K=1.1K$

$$Q(1.1L,1.1K) = 1.1^{0.82} \cdot Q(L,K)$$
• Promjena proizvodnje: $Q(1.1L,1.1K) - Q(L,K)$

 $Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$

L + 0.1L = 1.1L

Promjena proizvodnje u postocima:

$$-$$
 10% povećanja kapitala \longrightarrow \mathcal{K} $+$

10% povećanja rada

$$K$$
 $\xrightarrow{10/6}$ povecarija Kapitara \longrightarrow $K+0.1K=1.1K$ $Q(1.1L,1.1K)=1.1^{0.82}\cdot Q(L,K)$

• Promjena proizvodnje:
$$Q(1.1L, 1.1K) - Q(L, K)$$

K

 $Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$

L + 0.1L = 1.1L

K + 0.1K = 1.1K

 Promjena proizvodnje u postocima: Q(L,K)

 $Q(1.1L, 1.1K) = 1.1^{0.82} \cdot Q(L, K)$

10% povećanja rada

10% povećanja kapitala

• Promjena proizvodnje: Q(1.1L, 1.1K) - Q(L, K)

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 $Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$

 $L \longrightarrow 10\%$ povećanja rada L + 0.1L = 1.1L

$$K$$
 $\xrightarrow{10\%$ povecanja kapitala \longrightarrow $K+0.1K=1.1K$ $Q(1.1L,1.1K)=1.1^{0.82}\cdot Q(L,K)$

Promiona projevodnje:
$$O(1.11.1.1K)$$
 $O(1.K)$

- Promjena proizvodnje: Q(1.1L, 1.1K) Q(L, K)
- Promjena proizvodnje u postocima:

$$\frac{Q(1.1L, 1.1K) - Q(L, K)}{Q(L, K)}$$

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 $Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$

$$10\%$$
 povećanja rada $L+0.1L=1.1L$ 10% povećanja kapitala $L+0.1L=1.1L$

$$K \longrightarrow K + 0.1K = 1.1K$$

$$Q(1.1L, 1.1K) = 1.1^{0.82} \cdot Q(L, K)$$

• Promjena proizvodnje:
$$Q(1.1L, 1.1K) - Q(L, K)$$

- Promjena proizvodnje: Q(1.1L, 1.1K) Q(L, K)
- Promjena proizvodnje u postocima:

$$\frac{Q(1.1L,1.1K)-Q(L,K)}{Q(L,K)}=\frac{}{Q(L,K)}$$

 $L \longrightarrow 10\%$ povećanja rada L+0.1L=1.1L

$$\mathcal{K} \quad \xrightarrow{10\% \text{ povećanja kapitala}} \quad \mathcal{K} + 0.1 \mathcal{K} = 1.1 \mathcal{K}$$

 $Q(1.1L, 1.1K) = 1.1^{0.82} \cdot Q(L, K)$

• Promjena proizvodnje:
$$Q(1.1L, 1.1K) - Q(L, K)$$

 $=\frac{100y}{x}$

 $Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$

• Promjena proizvodnje u postocima:

$$\frac{Q(1.1L, 1.1K) - Q(L, K)}{Q(L, K)} = \frac{1.1^{0.82} \cdot Q(L, K)}{Q(L, K)}$$

 $L \longrightarrow 10\%$ povećanja rada L+0.1L=1.1L

 $Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$

K

$$rac{10\%$$
 povećanja kapitala $\qquad \qquad K+0.1K=1.1K$ $Q(1.1L,1.1K)=1.1^{0.82}\cdot Q(L,K)$

- Promjena proizvodnje: Q(1.1L, 1.1K) Q(L, K)
- Promjena proizvodnje u postocima:

$$\frac{Q(1.1L, 1.1K) - Q(L, K)}{Q(L, K)} = \frac{1.1^{0.82} \cdot Q(L, K) - Q(L, K)}{Q(L, K)}$$

K

 $Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$

L + 0.1L = 1.1L

K + 0.1K = 1.1K

• Promjena proizvodnje: Q(1.1L, 1.1K) - Q(L, K)

 $Q(1.1L, 1.1K) = 1.1^{0.82} \cdot Q(L, K)$

Promjena proizvodnje u postocima:

$$\frac{Q(1.1L, 1.1K) - Q(L, K)}{Q(L, K)} = \frac{1.1^{0.82} \cdot Q(L, K) - Q(L, K)}{Q(L, K)} = \frac{Q(L, K) - Q(L, K)}{Q(L, K)}$$

10% povećanja rada

10% povećanja kapitala

K — 10% povećanja kapitala

 $Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$

L + 0.1L = 1.1L

K + 0.1K = 1.1K

 $Q(1.1L, 1.1K) = 1.1^{0.82} \cdot Q(L, K)$

10% povećanja rada

- Promjena proizvodnje: Q(1.1L, 1.1K) Q(L, K)
- Promjena proizvodnje u postocima:

$$egin{split} rac{Q(1.1L,1.1K)-Q(L,K)}{Q(L,K)} &= rac{1.1^{0.82} \cdot Q(L,K)-Q(L,K)}{Q(L,K)} = \ &= rac{(1.1^{0.82}-1) \cdot Q(L,K)}{Q(L,K)} \end{split}$$

K — $\frac{10\%$ povećanja kapitala \longrightarrow K+0.1K=1.1K

 $Q(1.1L, 1.1K) = 1.1^{0.82} \cdot Q(L, K)$

• Promjena proizvodnje:
$$Q(1.1L, 1.1K) - Q(L, K)$$

10% povećanja rada

 $=\frac{100y}{x}$

 $Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$

L + 0.1L = 1.1L

Promjena proizvodnje u postocima:

$$egin{split} rac{Q(1.1L,1.1K)-Q(L,K)}{Q(L,K)} &= rac{1.1^{0.82} \cdot Q(L,K)-Q(L,K)}{Q(L,K)} = \ &= rac{(1.1^{0.82}-1) \cdot Q(L,K)}{Q(L,K)} = 1.1^{0.82}-1 \end{split}$$

 $Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$

L + 0.1L = 1.1L

K + 0.1K = 1.1K

 $Q(1.1L, 1.1K) = 1.1^{0.82} \cdot Q(L, K)$

K

• Promjena proizvodnje: Q(1.1L, 1.1K) - Q(L, K)

Promjena proizvodnje u postocima:

$$egin{split} rac{Q(1.1L,1.1K)-Q(L,K)}{Q(L,K)} &= rac{1.1^{0.82}\cdot Q(L,K)-Q(L,K)}{Q(L,K)} = \ &= rac{(1.1^{0.82}-1)\cdot Q(L,K)}{Q(L,K)} = 1.1^{0.82}-1pprox 0.08129 \end{split}$$

10% povećanja rada

10% povećanja kapitala

 $Q(1.1L,1.1K) = 1.1^{0.82} \cdot Q(L,K)$ • Promjena proizvodnje: Q(1.1L,1.1K) - Q(L,K) $p = \frac{100y}{R}$

 $\frac{Q(1.1L, 1.1K) - Q(L, K)}{Q(L, K)} = \frac{1.1^{0.82} \cdot Q(L, K) - Q(L, K)}{Q(L, K)} =$

10% povećanja rada

10% povećanja kapitala

Promjena proizvodnje u postocima:

e)

K

 $Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$

L + 0.1L = 1.1L

K + 0.1K = 1.1K

$$=\frac{(1.1^{0.82}-1)\cdot Q(L,K)}{Q(L,K)}=1.1^{0.82}-1\approx 0.08129$$
 Ako rad i kapital povećamo za 10%, proizvodnja će se povećati za

8.129%.

10% povećanja kapitala K K + 0.1K = 1.1K $Q(1.1L, 1.1K) = 1.1^{0.82} \cdot Q(L, K)$ • Promjena proizvodnje: Q(1.1L, 1.1K) - Q(L, K) Promjena proizvodnje u postocima: $\frac{Q(1.1L, 1.1K) - Q(L, K)}{Q(L, K)} = \frac{1.1^{0.82} \cdot Q(L, K) - Q(L, K)}{Q(L, K)} =$ $=\frac{(1.1^{0.82}-1)\cdot Q(L,K)}{Q(L,K)}=1.1^{0.82}-1\approx 0.08129$

Ako rad i kapital povećamo za 10%, proizvodnja će se povećati za

padajući prinosi

10% povećanja rada

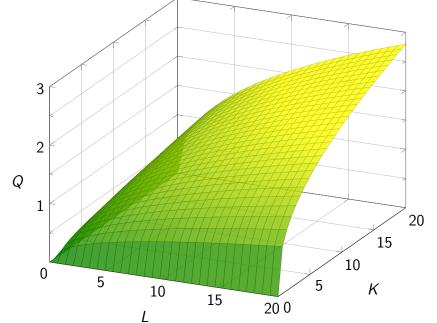
e)

8.129%,

 $Q(\lambda L, \lambda K) = \lambda^{0.82} Q(L, K)$

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L + 0.1L = 1.1L



5/21

drugi zadatak

Zadatak 2

Zadana je funkcija proizvodnje

$$Q(L, K) = 2L^{0.25}K^{0.5}$$

u ovisnosti o radu L i kapitalu K.

- a) Odredite funkciju granične produktivnosti rada i interpretirajte rezultat na nivou $L=10,\ K=5.$
- b) Odredite funkciju granične produktivnosti kapitala i interpretirajte rezultat na nivou $L=10,\ K=5.$
- c) Izvedite jednadžbu izokvante L = L(K) na nivou proizvodnje Q = 30.

Rješenje

a)

$$Q_L =$$

 $Q(L,K) = 2L^{0.25}K^{0.5}$

Rješenje

 $Q_L = 2K^{0.5}$.

a)

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 $Q(L,K) = 2L^{0.25}K^{0.5}$

Rješenje

a)

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75}$$

 $Q(L,K) = 2L^{0.25}K^{0.5}$

a)

 $Q(L,K) = 2L^{0.25}K^{0.5}$

 $Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$

a)

Rješenje

 $Q(L,K) = 2L^{0.25}K^{0.5}$

 $Q_L = 2 \mathcal{K}^{0.5} \cdot 0.25 L^{-0.75} = 0.5 L^{-0.75} \mathcal{K}^{0.5}$ $Q_L(10,5) =$

,

a)

Rješenje

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$

 $Q_L(10, 5) = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5}$

.

Rješenje

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$
 $Q_L(10,5) = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5} = 0.1988 \cdot \cdot \cdot$

a)

Rješenje

 $Q(L, K) = 2L^{0.25}K^{0.5}$

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$

$$Q_L(10,5) = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5} = 0.1988 \dots \approx 0.2$$

 $Q(L,K) = 2L^{0.25}K^{0.5}$

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$

 $Q_L(10, 5) = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5} = 0.1988 \cdot \cdot \cdot \approx 0.2$

Ako na nivou L=10, K=5 rad povećamo za jednu jedinicu, proizvodnja će se povećati za 0.2 jedinice.

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$
 $Q_L(10,5) = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5} = 0.1988 \cdots \approx 0.2$

Ako na nivou $L=10,\ K=5$ rad povećamo za jednu jedinicu, proizvodnja će se povećati za 0.2 jedinice.

$$Q_{\mathcal{K}} =$$

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

a)

$$Q_L = 2 \mathcal{K}^{0.5} \cdot 0.25 L^{-0.75} = 0.5 L^{-0.75} \mathcal{K}^{0.5}$$

 $Q_L(10,5) = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5} = 0.1988 \cdots \approx 0.2$

Ako na nivou $L=10,\,K=5$ rad povećamo za jednu jedinicu, proizvodnja će se povećati za 0.2 jedinice.

$$Q_{K}=2L^{0.25}.$$

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

a)

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$

 $L \quad K$
 $Q_L(10,5) = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5} = 0.1988 \cdots \approx 0.2$

Ako na nivou L=10, K=5 rad povećamo za jednu jedinicu, proizvodnja će se povećati za 0.2 jedinice.

$$Q_{K} = 2L^{0.25} \cdot 0.5K^{-0.5}$$

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$

 $L K$
 $Q_L(10,5) = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5} = 0.1988 \cdots \approx 0.2$

Ako na nivou $L=10,\,K=5$ rad povećamo za jednu jedinicu, proizvodnja će se povećati za 0.2 jedinice.

$$Q_{K} = 2L^{0.25} \cdot 0.5K^{-0.5} = L^{0.25}K^{-0.5}$$

 $Q(L,K) = 2L^{0.25}K^{0.5}$

a)

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$

 $L \quad K$
 $Q_L(10,5) = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5} = 0.1988 \cdots \approx 0.2$

Ako na nivou $L=10,\ K=5$ rad povećamo za jednu jedinicu, proizvodnja će se povećati za 0.2 jedinice.

$$Q_{\mathcal{K}} = 2L^{0.25} \cdot 0.5 \mathcal{K}^{-0.5} = L^{0.25} \mathcal{K}^{-0.5}$$
 $Q_{\mathcal{K}}(10,5) =$

 $Q(L,K) = 2L^{0.25}K^{0.5}$

a)

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$

 $L K$
 $Q_L(10,5) = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5} = 0.1988 \cdots \approx 0.2$

Ako na nivou L=10, K=5 rad povećamo za jednu jedinicu, proizvodnja će se povećati za 0.2 jedinice.

$$Q_{\mathcal{K}} = 2L^{0.25} \cdot 0.5 \mathcal{K}^{-0.5} = L^{0.25} \mathcal{K}^{-0.5}$$
 $Q_{\mathcal{K}}(10,5) = 10^{0.25} \cdot 5^{-0.5}$

 $Q(L,K) = 2L^{0.25}K^{0.5}$

a)

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$

 $Q_L(10, 5) = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5} = 0.1988 \cdots \approx 0.2$

Ako na nivou L=10, K=5 rad povećamo za jednu jedinicu, proizvodnja će se povećati za 0.2 jedinice.

$$Q_{\mathcal{K}} = 2L^{0.25} \cdot 0.5K^{-0.5} = L^{0.25}K^{-0.5}$$
 $Q_{\mathcal{K}}(10,5) = 10^{0.25} \cdot 5^{-0.5} = 0.79527 \cdots$

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

$$Q_L = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$$

 $L K$
 $Q_L(10,5) = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5} = 0.1988 \dots \approx 0.2$

Ako na nivou $L=10,\,K=5$ rad povećamo za jednu jedinicu, proizvodnja će se povećati za 0.2 jedinice.

$$Q_{\mathcal{K}} = 2L^{0.25} \cdot 0.5 \mathcal{K}^{-0.5} = L^{0.25} \mathcal{K}^{-0.5}$$
 $Q_{\mathcal{K}}(10,5) = 10^{0.25} \cdot 5^{-0.5} = 0.79527 \cdots \approx 0.8$

a)

Riešenie

$$\cdot \cdot \approx 0.2$$

 $Q(L,K) = 2L^{0.25}K^{0.5}$

$$Q_L(10,5) = 0.5 \cdot 10^{-0.75} \cdot 5^{0.5} = 0.1988 \cdots \approx 0.2$$

 $Q_t = 2K^{0.5} \cdot 0.25L^{-0.75} = 0.5L^{-0.75}K^{0.5}$

Ako na nivou L=10, K=5 rad povećamo za jednu jedinicu, proizvodnja će se povećati za 0.2 jedinice.

b)

$$Q_{\mathcal{K}} = 2L^{0.25} \cdot 0.5 \mathcal{K}^{-0.5} = L^{0.25} \mathcal{K}^{-0.5}$$
 $Q_{\mathcal{K}}(10,5) = 10^{0.25} \cdot 5^{-0.5} = 0.79527 \cdots \approx 0.8$

Ako na nivou L=10, K=5 kapital povećamo za jednu jedinicu, proizvodnja će se povećati za 0.8 jedinica.

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

$$Q = 30$$

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

$$Q = 30$$
 $2L^{0.25}K^{0.5} = 30$

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

$$Q = 30$$

 $2L^{0.25}K^{0.5} = 30 /: 2$
 $L^{0.25}K^{0.5} = 15$

$$Q = 30$$

 $2L^{0.25}K^{0.5} = 30 /: 2$
 $L^{0.25}K^{0.5} = 15 /^4$

$$Q = 30$$

 $2L^{0.25}K^{0.5} = 30 /: 2$
 $L^{0.25}K^{0.5} = 15 /^4$
 $LK^2 = 50625$

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

L = L(K)

$$Q = 30$$

 $2L^{0.25}K^{0.5} = 30 /: 2$
 $L^{0.25}K^{0.5} = 15 /^4$
 $LK^2 = 50625$

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

L = L(K)

$$Q = 30$$

$$2L^{0.25}K^{0.5} = 30 /: 2$$

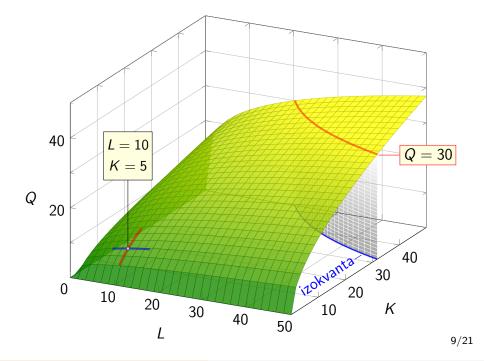
$$L^{0.25}K^{0.5} = 15 /^{4}$$

$$LK^{2} = 50625$$

$$L = \frac{50625}{K^{2}}$$

$$Q(L,K) = 2L^{0.25}K^{0.5}$$

$$Q = 30$$
 $2L^{0.25}K^{0.5} = 30 \ / : 2$
 $L^{0.25}K^{0.5} = 15 \ /^4$
 $LK^2 = 50625$
 $L = \frac{50625}{K^2}$
jednadžba izokvante



treći zadatak

Zadatak 3

Zadana je funkcija proizvodnje

$$Q(L,K)=3L^{\frac{1}{2}}K$$

u ovisnosti o radu L i kapitalu K.

- a) Jedna jedinica rada košta 10 €, a jedna jedinica kapitala košta 15 €. Ako poduzeće ima na raspolaganju 20 000 €, pronađite kombinaciju rada i kapitala za koje se uz maksimalno iskorištenje kapaciteta ostvaruje maksimalna proizvodnja. Koliko iznosi maksimalna proizvodnja?
- b) Na istoj slici prikažite budžetsko ograničenje i izokvantu na nivou maksimalne proizvodnje. Što možete reći o njihovom odnosu?

a) budžetsko ograničenje:

$$Q(L,K)=3L^{\frac{1}{2}}K$$

a) budžetsko ograničenje: $10L + 15K = 20\,000$

a) budžetsko ograničenje: $10L + 15K = 20\,000$

$$10L + 15K = 20\,000$$

a) budžetsko ograničenje: $10L+15K=20\,000$

$$10L + 15K = 20000 / : 5$$

 $2L + 3K = 4000$

a) budžetsko ograničenje: $10L + 15K = 20\,000$

$$10L + 15K = 20000 /: 5$$

 $2L + 3K = 4000$
 $2L = 4000 - 3K$

$$Q(L,K)=3L^{\frac{1}{2}}K$$

a) budžetsko ograničenje: $10L + 15K = 20\,000$

$$10L + 15K = 20000 /: 5$$

$$2L + 3K = 4000$$

$$2L = 4000 - 3K /: 2$$

$$L = 2000 - \frac{3}{2}K$$

 $Q(L,K)=3L^{\frac{1}{2}}K$

a) budžetsko ograničenje: $10L + 15K = 20\,000$

$$10L + 15K = 20000 /: 5$$

$$2L + 3K = 4000$$

$$2L = 4000 - 3K /: 2$$

$$L = 2000 - \frac{3}{2}K$$

a) budžetsko ograničenje: $10L + 15K = 20\,000$

$$10L + 15K = 20000 /: 5$$

$$2L + 3K = 4000$$

$$2L = 4000 - 3K /: 2$$

$$L = 2000 - \frac{3}{2}K$$

$$Q\left(2000 - \frac{3}{2}K, K\right) =$$

a) budžetsko ograničenje: $10L + 15K = 20\,000$

cenje:
$$10L + 15K = 20\,000$$

$$10L + 15K = 20\,000 /: 5$$

$$2L + 3K = 4000$$

$$2L = 4000 - 3K /: 2$$

$$L = 2000 - \frac{3}{2}K$$

$$Q\left(2000 - \frac{3}{2}K, K\right) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}K$$

a) budžetsko ograničenje: 10L + 15K = 20000

Rješenje

10L + 15K = 20000 / : 5

$$2L + 3K = 4000$$

$$2L = 4000 - 3K / : 2$$

$$L = 2000 - \frac{3}{2}K$$

$$O\left(2000 - \frac{3}{5}\kappa \kappa\right) -$$

$$Q\left(2000 - \frac{3}{2}K, K\right) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}K$$

$$\left(\frac{3}{2}K\right)^{\frac{1}{2}}K$$

$$f(K) = 3K \left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

Rješenje

a) budžetsko ograničenje: 10L + 15K = 20000

$$2000 - \frac{3}{2}K \geqslant 0$$
$$-\frac{3}{2}K \geqslant -2000$$

$$\frac{1}{2}K \geqslant -2000$$

$$K \leqslant \frac{4000}{3}$$

$$10L + 15K = 20\,000 / : 5$$

$$2L = 4000 - 3K / : 2$$

2L + 3K = 4000

$$L = 2000 - \frac{3}{2}K$$

$$Q\left(2000 - \frac{3}{2}K, K\right) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}K$$

$$\left(2000 - \frac{3}{2}K, K\right) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f(K) = 3K\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

 $Q(L,K) = 3L^{\frac{1}{2}}K$

$$f(K) = 3K \left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) =$$

$$f(K) = 3K \left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3$$

$$f(K) = 3K \left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f(K) = 3K \left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} +$$

$$f(K) = 3K \left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K$$

$$f(K) = 3K \left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2}\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}}$$

$$f(K) = 3K \left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

 $f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2}\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}}$

$$f(K) = 3K \left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3K \left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2} \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \frac{-3}{2}$$

$$f(K) = 3K \left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2}\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \frac{-3}{2}$$

$$f'(K) =$$

$$f(K) = 3K \left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2}\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \frac{-3}{2}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f(K) = 3K \left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2}\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \frac{-3}{2}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} - \frac{9}{4}K$$

$$f(K) = 3K \left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2}\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \frac{-3}{2}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} - \frac{9}{4}K\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}}$$

$$f(K) = 3K \left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2}\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \frac{-3}{2}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} - \frac{9}{4}K\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}}$$

$$f'(K) =$$

$$f(K) = 3K \left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2}\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \frac{-3}{2}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} - \frac{9}{4}K\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}}$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left[$$

$$f(K) = 3K \left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2}\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \frac{-3}{2}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} - \frac{9}{4}K\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}}$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \left[3\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}}\right]$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left[3\left(2000 - \frac{3}{2}K\right)\right]$$

$$f(K) = 3K \left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2}\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \frac{-3}{2}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} - \frac{9}{4}K\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}}$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left[3\left(2000 - \frac{3}{2}K\right) - \right]$$

$$f(K) = 3K \left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2}\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \frac{-3}{2}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} - \frac{9}{4}K\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}}$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left[3\left(2000 - \frac{3}{2}K\right) - \frac{9}{4}K\right]$$

$$f(K) = 3K \left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2}\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \frac{-3}{2}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} - \frac{9}{4}K\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}}$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left[3\left(2000 - \frac{3}{2}K\right) - \frac{9}{4}K\right]$$

$$f(K) = 3K \left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2}\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \frac{-3}{2}$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} - \frac{9}{4}K\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}}$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left[3\left(2000 - \frac{3}{2}K\right) - \frac{9}{4}K\right]$$

$$f'(K) =$$

$$f(K) = 3K \left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

 $f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}}$

 $f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2}\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \frac{-3}{2}$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} - \frac{9}{4}K\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}}$$
$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left[3\left(2000 - \frac{3}{2}K\right) - \frac{9}{4}K\right]$$

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$$f(K) = 3K \left(2000 - \frac{1}{2}\right)$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2}\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \frac{-3}{2}$$

$$f(K) = 3K \left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

 $f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} - \frac{9}{4}K\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}}$

 $f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left[3\left(2000 - \frac{3}{2}K\right) - \frac{9}{4}K\right]$

- $f'(K) = \left(2000 \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000\right)$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left[3\left(2000 - \frac{3}{2}K\right) - \frac{9}{4}K\right]$$
$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$

$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} - \frac{9}{4}K\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}}$$

$$f(K) = 3K \left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$
$$f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2}\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \frac{-3}{2}$$

$$f(K) = 3K \left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}}$$

 $f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} + 3K \cdot \frac{1}{2}\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \frac{-3}{2}$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left[3\left(2000 - \frac{3}{2}K\right) - \frac{9}{4}K\right]$$

 $f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$

 $f'(K) = 3\left(2000 - \frac{3}{2}K\right)^{\frac{1}{2}} - \frac{9}{4}K\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}}$

 $f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$

 $f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$

 $\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$
$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

$$6000 - \frac{27}{4}K = 0$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$
$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

$$6000 - \frac{27}{4}K = 0 / \cdot 4$$
$$24000 - 27K = 0$$

$$24\,000 - 27K = 0$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$
$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

$$6000 - \frac{27}{4}K = 0 / \cdot 4$$

$$24\,000 - 27K = 0$$

$$K = \frac{8000}{0}$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$
$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

$$6000 - \frac{27}{4}K = 0 / \cdot 4$$

$$24\,000 - 27K = 0$$

$$K = \frac{8000}{9}$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$
$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

$$6000 - \frac{27}{4}K = 0 / \cdot 4$$

$$24\,000 - 27K = 0$$

$$K = \frac{8000}{9} \quad K \approx 888.89$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$

$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

$$6000 - \frac{27}{4}K = 0 / \cdot 4$$

$$24\,000 - 27K = 0$$

$$K = \frac{8000}{9} \quad K \approx 888.89$$

$$0 \quad \frac{8000}{9} \quad \frac{4000}{3}$$

$$f' \quad | \quad | \quad |$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$
$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

$$6000 - \frac{27}{4}K = 0 / \cdot 4$$

$$24 000 - 27K = 0$$

$$K = \frac{8000}{9} \quad K \approx 888.89$$

$$0 \quad \frac{8000}{9} \quad \frac{4000}{3}$$

$$f' \quad + \quad | \quad |$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$
$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

$$6000 - \frac{27}{4}K = 0 / \cdot 4$$

$$24\,000 - 27K = 0$$

$$K = \frac{8000}{9}$$
 $K \approx 888.89$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$

$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

$$6000 - \frac{27}{4}K = 0 / \cdot 4$$

$$24 000 - 27K = 0$$

$$K = \frac{8000}{9} \quad K \approx 888.89$$

$$0 \quad \frac{8000}{9} \quad \frac{4000}{3}$$

$$f' \quad + \quad -$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$
$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

$$6000 - \frac{27}{4}K = 0 / \cdot 4$$

$$24 \ 000 - 27K = 0$$

$$K = \frac{8000}{9} \quad K \approx 888.89$$

$$0 \quad \frac{8000}{9} \quad \frac{4000}{3}$$

$$f' \quad + \quad - \quad +$$

$$f \quad \nearrow$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$

$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

$$6000 - \frac{27}{4}K = 0 / \cdot 4$$

$$24\,000 - 27K = 0$$

$$K = \frac{8000}{9}$$
 $K \approx 888.89$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$

$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

$$6000 - \frac{27}{4}K = 0 / \cdot 4 \qquad L = 2000 - \frac{3}{2}K$$

 $24\,000 - 27K = 0$

$$K = \frac{8000}{9}$$
 $K \approx 888.89$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$

$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

$$6000 - \frac{27}{4}K = 0 / \cdot 4 \qquad L = 2000 - \frac{3}{2}K$$

$$24\,000 - 27K = 0 \qquad L = 2000 - \frac{3}{2} \cdot \frac{8000}{9}$$

$$K = \frac{8000}{9} \quad K \approx 888.89$$

$$0 \quad \frac{8000}{9} \quad \frac{4000}{3}$$

$$f' \quad + \quad - \quad |$$

$$f \quad | \qquad |$$
globalni maksimum

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$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$

$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

$$6000 - \frac{27}{4}K = 0 / \cdot 4 \qquad L = 2000 - \frac{3}{2}K$$

$$24\ 000 - 27K = 0$$

$$K = \frac{8000}{9}$$

$$0 \quad \frac{8000}{9}$$

$$L = 2000 - \frac{3}{2} \cdot \frac{8000}{9}$$

$$L = \frac{2000}{3}$$

globalni maksimum

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$

$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

$$6000 - \frac{27}{4}K = 0 / \cdot 4 \qquad L = 2000 - \frac{3}{2}K$$

$$24\,000 - 27K = 0 \qquad L = 2000 - \frac{3}{2} \cdot \frac{8000}{9}$$

$$K = \frac{8000}{9} \quad K \approx 888.89$$

$$0 \quad \frac{8000}{9} \quad \frac{4000}{3}$$

$$f' \quad + \quad - \quad \text{globalni maksimum}$$

$$f'(K) = \left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right)$$

$$\left(2000 - \frac{3}{2}K\right)^{-\frac{1}{2}} \cdot \left(6000 - \frac{27}{4}K\right) = 0$$

$$6000 - \frac{27}{4}K = 0 / \cdot 4$$

$$L = 2000 - \frac{3}{2}K$$

$$24000 - 27K = 0$$

$$K = \frac{8000}{9}$$

$$K \approx 888.89$$

$$L = 2000 - \frac{3}{2} \cdot \frac{8000}{9}$$

$$L = \frac{2000}{3}$$

$$L \approx 666.67$$

globalni maksimum

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$$Q(L,K)=3L^{\frac{1}{2}}K$$

$$Q\left(\frac{2000}{3},\frac{8000}{9}\right) =$$

$$Q(L,K)=3L^{\frac{1}{2}}K$$

$$Q\left(\frac{2000}{3}, \frac{8000}{9}\right) = 3 \cdot \left(\frac{2000}{3}\right)^{\frac{1}{2}} \cdot \frac{8000}{9}$$

$$Q(L,K)=3L^{\frac{1}{2}}K$$

$$Q\left(\frac{2000}{3}, \frac{8000}{9}\right) = 3 \cdot \left(\frac{2000}{3}\right)^{\frac{1}{2}} \cdot \frac{8000}{9} \approx 68853.04$$

$$Q(L,K)=3L^{\frac{1}{2}}K$$

$$Q\left(\frac{2000}{3}, \frac{K}{9000}\right) = 3 \cdot \left(\frac{2000}{3}\right)^{\frac{1}{2}} \cdot \frac{8000}{9} \approx 68853.04$$

$$Q(L,K)=3L^{\frac{1}{2}}K$$

$$Q\left(\frac{\frac{L}{2000}}{3}, \frac{\frac{K}{8000}}{9}\right) = 3 \cdot \left(\frac{2000}{3}\right)^{\frac{1}{2}} \cdot \frac{8000}{9} \approx 68\,853.04$$

$$Q = 68853.04$$

$$Q(L,K)=3L^{\frac{1}{2}}K$$

$$Q\left(\frac{2000}{3}, \frac{K}{9000}\right) = 3 \cdot \left(\frac{2000}{3}\right)^{\frac{1}{2}} \cdot \frac{8000}{9} \approx 68853.04$$

$$Q = 68853.04$$

$$3L^{\frac{1}{2}}K = 68\,853.04$$

$$Q(L,K)=3L^{\frac{1}{2}}K$$

$$Q\left(\frac{2000}{3}, \frac{K}{9000}\right) = 3 \cdot \left(\frac{2000}{3}\right)^{\frac{1}{2}} \cdot \frac{8000}{9} \approx 68853.04$$

$$Q = 68853.04$$

$$3L^{\frac{1}{2}}K = 68\,853.04$$

$$\frac{3853.04}{31^{\frac{1}{2}}}$$

$$Q(L,K)=3L^{\frac{1}{2}}K$$

• Maksimalna proizvodnja
$$Q(L, K)$$
:
$$Q\left(\frac{2000}{3}, \frac{8000}{9}\right) = 3 \cdot \left(\frac{2000}{3}\right)^{\frac{1}{2}} \cdot \frac{8000}{9} \approx 68\,853.04$$

$$Q = 68853.04$$

$$3L^{\frac{1}{2}}K = 68\,853.04$$

$$\frac{853.04}{31^{\frac{1}{2}}}$$

$$K = K(L)$$

$$Q(L,K) = 3L^{\frac{1}{2}}K$$

• Maksimalna proizvodnja
$$Q(L, K)$$
 : $Q\left(\frac{2000}{3}, \frac{8000}{9}\right) = 3 \cdot \left(\frac{2000}{3}\right)^{\frac{1}{2}} \cdot \frac{8000}{9} \approx 68\,853.04$

Budžetsko ograničenje

$$Q = 68\,853.04$$
$$3L^{\frac{1}{2}}K = 68\,853.04$$

$$K = \frac{68853.04}{3L^{\frac{1}{2}}}$$

$$Q(L,K) = 3L^{\frac{1}{2}}K$$

• Maksimaina proizvodnja
$$Q(L, K)$$
 : $Q\left(\frac{2000}{3}, \frac{8000}{9}\right) = 3 \cdot \left(\frac{2000}{3}\right)^{\frac{1}{2}} \cdot \frac{8000}{9} \approx 68\,853.04$

K = K(L)

$$Q = 68853.04$$

$$3L^{\frac{1}{2}}K = 68\,853.04$$

$$\frac{353.04}{31^{\frac{1}{2}}}$$

Budžetsko ograničenje

$$10L + 15K = 20\,000 / : 5$$

$$Q(L,K) = 3L^{\frac{1}{2}}K$$

$$Q\left(\frac{2000}{3}, \frac{8000}{9}\right) = 3 \cdot \left(\frac{2000}{3}\right)^{\frac{1}{2}} \cdot \frac{8000}{9} \approx 68853.04$$

$$Q = 68\,853.04$$
$$3L^{\frac{1}{2}}K = 68\,853.04$$

$$K = \frac{68853.04}{8L^{\frac{1}{2}}}$$

$$K = \frac{68853.04}{3L^{\frac{1}{2}}}$$

Budžetsko ograničenje

10L + 15K = 20000 / : 5

2L + 3K = 4000 / : 4000

$$Q(L,K) = 3L^{\frac{1}{2}}K$$

$$Q\left(\frac{L}{3}, \frac{K}{9}\right) = 3 \cdot \left(\frac{2000}{3}\right)^{\frac{1}{2}} \cdot \frac{8000}{9} \approx 68853.04$$

$$Q = 68853.04$$

$$3L^{\frac{1}{2}}K = 68\,853.04$$

$$K = \frac{68853.04}{8L^{\frac{1}{2}}}$$

$$K = \frac{68853.04}{3L^{\frac{1}{2}}}$$

Budžetsko ograničenje

$$10L + 15K = 20000 / : 5$$

$$\frac{6}{10} = 1$$

2L + 3K = 4000 / : 4000

$$Q(L,K) = 3L^{\frac{1}{2}}K$$

$$Q\left(\frac{2000}{3}, \frac{8000}{9}\right) = 3 \cdot \left(\frac{2000}{3}\right)^{\frac{1}{2}} \cdot \frac{8000}{9} \approx 68853.04$$

$$Q = 68\,853.04$$

$$3L^{\frac{1}{2}}K = 68\,853.04$$

$$3L^{2}K = 68853.04$$

$$K = \frac{68853.04}{3L^{\frac{1}{2}}}$$

Budžetsko ograničenje

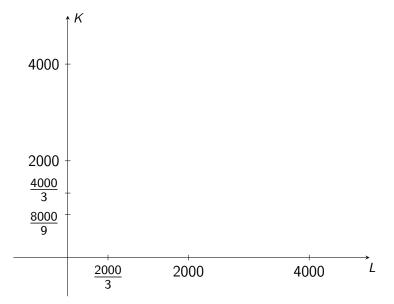
10L + 15K = 20000 / : 5

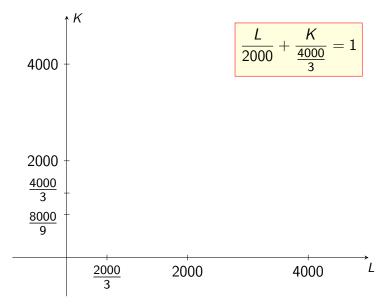
$$3L + 15K = 20000 / : 5$$

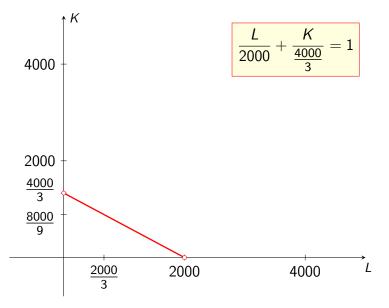
 $2L + 3K = 4000 / : 4000$

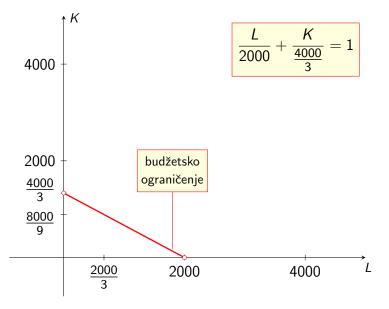
$$\frac{L}{2000} + \frac{K}{\frac{4000}{2}} = 1$$

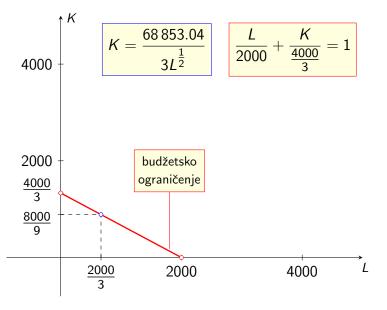
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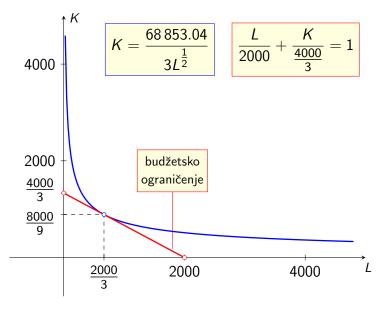


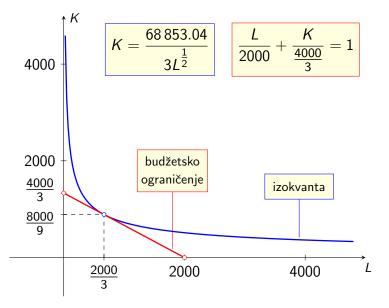


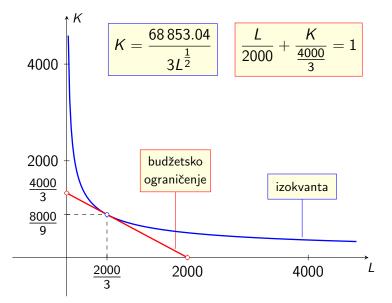




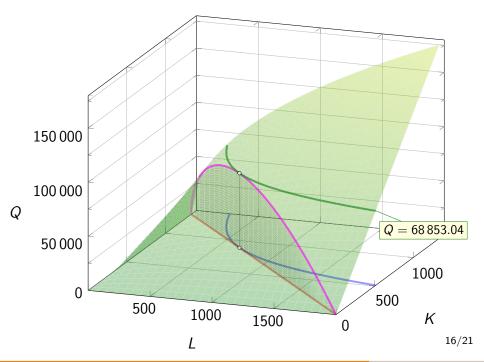


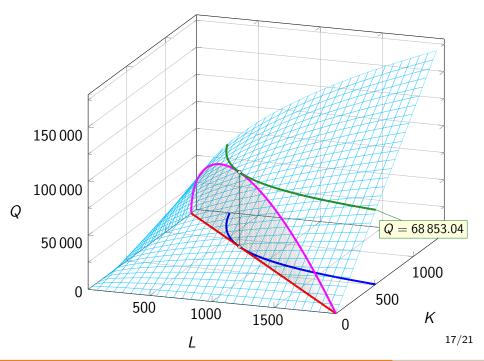






Budžetsko ograničenje je tangenta na izokvantu na nivou maksimalne proizvodnje u točki u kojoj se postiže maksimalna proizvodnja. 15/21





četvrti zadatak

Zadatak 4

Cijena jedinice rada iznosi $1 \in$, cijena jedinice kapitala iznosi $2 \in$, a fiksni troškovi su $10 \in$. Funkcija proizvodnje u ovisnosti o radu L i kapitalu K dana je s

$$Q(L,K) = \sqrt{0.5}LK^{\frac{1}{2}}.$$

Na nivou proizvodnje Q=8 pronađite optimalnu kombinaciju rada i kapitala tako da troškovi budu minimalni. Koliko iznose minimalni troškovi?

• Funkcija troškova

$$T(L,K) =$$

• Funkcija troškova

$$T(L,K)=1\cdot L$$

Funkcija troškova

$$T(L,K)=1\cdot L+2\cdot K$$

 $Q(L,K) = \sqrt{0.5}LK^{\frac{1}{2}}$

Funkcija troškova

$$T(L,K)=1\cdot L+2\cdot K+10$$

ullet Funkcija troškova $T(L,K) = 1 \cdot L + 2 \cdot K + 10$

$$T(L,K)=L+2K+10$$

. .

Rješenje

Funkcija troškova

 $Q(L,K) = \sqrt{0.5}LK^{\frac{1}{2}}$

 $T(L,K) = 1 \cdot L + 2 \cdot K + 10$

T(L,K) = L + 2K + 10

Uvjet

.

Rješenje

- Funkcija troškova
- $T(L,K) = 1 \cdot L + 2 \cdot K + 10$ T(L,K) = L + 2K + 10

Uvjet

• Funkcija troškova

 $T(L,K) = 1 \cdot L + 2 \cdot K + 10$ T(L,K) = L + 2K + 10

• Uvjet

ullet Funkcija troškova $\mathcal{T}(L,\mathcal{K})=1\cdot L+2\cdot \mathcal{K}+10$

$$T(L,K)=L+2K+10$$

Uvjet

$$Q = 8$$

$$\sqrt{0.5}LK^{\frac{1}{2}} = 8$$

ullet Funkcija troškova $\mathcal{T}(L,K) = 1 \cdot L + 2 \cdot K + 10$

 $Q(L,K) = \sqrt{0.5}LK^{\frac{1}{2}}$

$$T(L,K)=L+2K+10$$

Uvjet

$$\sqrt{0.5}LK^{\frac{1}{2}} = 8$$

$$L = \frac{8}{\sqrt{0.5}}$$

ullet Funkcija troškova $T(L,K)=1\cdot L+2\cdot K+10$

$$T(L,K)=L+2K+10$$

Uvjet

$$\sqrt{0.5}LK^{\frac{1}{2}} = 8$$

$$L = \frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}}$$

Q = 8

Funkcija troškova

$$Q(L,K) = \sqrt{0.5}LK^{\frac{1}{2}}$$

$$T(L,K) = 1 \cdot L + 2 \cdot K + 10$$
$$T(L,K) = L + 2K + 10$$

Uvjet

$$\sqrt{0.5}LK^{\frac{1}{2}} = 8$$

$$L = \frac{8}{\sqrt{0.5}}K^{-1}$$

Q = 8

 $Q(L,K) = \sqrt{0.5}LK^{\frac{1}{2}}$

Funkcija troškova

$$T(L,K) = 1 \cdot L + 2 \cdot K + 10$$
$$T(L,K) = L + 2K + 10$$

Uvjet

$$Q = 8$$

$$\sqrt{0.5}LK^{\frac{1}{2}} = 8$$

$$L = \frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}}$$

$$T\left(\frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}},K\right)=$$

 $Q(L,K) = \sqrt{0.5}LK^{\frac{1}{2}}$

Funkcija troškova

$$T(L,K) = 1 \cdot L + 2 \cdot K + 10$$
$$T(L,K) = L + 2K + 10$$

Uvjet

$$Q = 8$$

$$\sqrt{0.5}LK^{\frac{1}{2}} = 8$$

$$L = \frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}}$$

$$T\left(\frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}},K\right) = \frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}} + 2K + 10$$

Uvjet

Funkcija troškova

$$T(L,K) = 1 \cdot L + 2 \cdot K + 10$$

$$Q = \sqrt{0.5}LK^{\frac{1}{2}} =$$

$$L = \frac{8}{\sqrt{0.5}} K^{-\frac{1}{2}}$$

$$T\left(\frac{8}{\sqrt{0.5}} K^{-\frac{1}{2}}, K\right) = \frac{8}{\sqrt{0.5}} K^{-\frac{1}{2}} + 2K + 10$$

 $f(K) = \frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}} + 2K + 10$

T(L,K) = L + 2K + 10

$$T\left(\frac{8}{\sqrt{8}}K^{-\frac{1}{2}},\right)$$

19/21

T(L,K) = L + 2K + 10

f'(K) =

T(L,K) = L + 2K + 10

 $f'(K) = \frac{8}{\sqrt{0.5}}$

$$f(K) = \frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}} + 2K + 10$$
$$f'(K) = \frac{8}{\sqrt{0.5}} \cdot \frac{-1}{2}K^{-\frac{3}{2}}$$

T(L,K) = L + 2K + 10

 $f(K) = \frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}} + 2K + 10$

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20/21

 $-\frac{4}{\sqrt{0.5}}K^{-\frac{3}{2}}+2=0$

T(L,K) = L + 2K + 10

 $f(K) = \frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}} + 2K + 10$

$$f'(K) = \frac{1}{\sqrt{0.5}} \cdot \frac{1}{2} \cdot K^{-\frac{3}{2}} + 2$$

$$-\frac{4}{\sqrt{0.5}} K^{-\frac{3}{2}} = -2$$

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$$f'(K) = \frac{1}{\sqrt{0.5}} \left(\frac{1}{2} - \frac{1}{\sqrt{0.5}} K^{-\frac{3}{2}} + 2 \right) - \frac{4}{\sqrt{0.5}} K^{-\frac{3}{2}} = -2 / \cdot \frac{-\sqrt{0.5}}{4}$$

20/21

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$$K^{-\frac{3}{2}} = \frac{\sqrt{0.5}}{2}$$

20/21

 $-\frac{4}{\sqrt{0.5}}K^{-\frac{3}{2}}+2=0$

T(L,K) = L + 2K + 10

 $f(K) = \frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}} + 2K + 10$

$$f'(K) = \frac{\sqrt{0.5}}{\sqrt{0.5}} 2^{-1} + 2$$

$$-\frac{4}{\sqrt{0.5}} K^{-\frac{3}{2}} = -2 / \cdot \frac{-\sqrt{0.5}}{4}$$

$$K^{-\frac{3}{2}} = \frac{\sqrt{0.5}}{2} / \frac{-\frac{2}{3}}{3}$$

20/21

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$$K^{-\frac{3}{2}} = \frac{\sqrt{0.5}}{2} / \frac{-\frac{2}{3}}{4}$$

$$K = \left(\frac{\sqrt{0.5}}{2}\right)^{-\frac{2}{3}}$$

 $-\frac{4}{\sqrt{0.5}}K^{-\frac{3}{2}}+2=0$

 $L = \frac{8}{\sqrt{0.5}} K^{-\frac{1}{2}}$

20/21

T(L,K) = L + 2K + 10

$$f(K) = \frac{8}{\sqrt{0.5}} K^{-\frac{1}{2}} + 2K + 10$$

$$f'(K) = \frac{8}{\sqrt{0.5}} \int_{-\frac{2}{3}}^{-\frac{2}{3}} = K = \frac{1}{2}$$

$$K = \left(\frac{\sqrt{0.5}}{2}\right)^{-\frac{2}{3}}$$

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$$f(K) = \frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}} + 2K + 10$$

$$-\frac{4}{-1}K^{-\frac{3}{2}} + 2 = 0$$

$$f'\left(\frac{\sqrt{0.5}}{2}\right)^{-\frac{2}{3}} = \left(\frac{\sqrt{2^{-1}}}{2}\right)^{-\frac{2}{3}}$$

$$K = \left(\frac{\sqrt{0.5}}{2}\right)^{-\frac{2}{3}}$$

$$20/21$$

$$f(K) = \frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}} + 2K + 10$$

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$$f'\left(\frac{\sqrt{0.5}}{2}\right)^{-\frac{2}{3}} = \left(\frac{\sqrt{2^{-1}}}{2}\right)^{-\frac{2}{3}} = \left(\frac{2^{-\frac{1}{2}}}{2}\right)^{-\frac{2}{3}}$$

$$K = \left(\frac{\sqrt{0.5}}{2}\right)^{-\frac{2}{3}}$$

$$20/21$$

$$f' = \left(\frac{\sqrt{0.5}}{2}\right)^{-\frac{2}{3}} = \left(\frac{\sqrt{2^{-1}}}{2}\right)^{-\frac{2}{3}} = \left(\frac{2^{-\frac{1}{2}}}{2}\right)^{-\frac{2}{3}} = \left(2^{-\frac{3}{2}}\right)^{-\frac{2}{3}}$$

$$K = \left(\frac{\sqrt{0.5}}{2}\right)^{-\frac{2}{3}}$$

$$20/21$$

 $\frac{4}{6} \kappa^{-\frac{3}{2}} + 2 = 0$

T(L,K) = L + 2K + 10

$$f' \left(\frac{\sqrt{0.5}}{2}\right)^{-\frac{2}{3}} = \left(\frac{\sqrt{2^{-1}}}{2}\right)^{-\frac{2}{3}} = \left(\frac{2^{-\frac{1}{2}}}{2}\right)^{-\frac{2}{3}} = \left(2^{-\frac{3}{2}}\right)^{-\frac{2}{3}} = 2$$

$$K = \left(\frac{\sqrt{0.5}}{2}\right)^{-\frac{2}{3}}$$

$$20/21$$

 $\frac{4}{6} \kappa^{-\frac{3}{2}} + 2 = 0$

T(L,K) = L + 2K + 10

$$f'(K) = \frac{8}{\sqrt{0.5}} \cdot \frac{-1}{2} K^{-\frac{3}{2}} + 2$$

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K=2

T(L,K) = L + 2K + 10

 $f(K) = \frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}} + 2K + 10$

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20/21

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$$K = 2$$

 $L = \frac{8}{\sqrt{0.5}} \cdot 2^{-\frac{1}{2}}$ 20/21

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minimum
$$K = 2$$

$$L = \frac{8}{\sqrt{0.5}} \cdot 2^{-\frac{1}{2}} = 8$$
20/21

$$f(K) = \frac{8}{\sqrt{0.5}}K^{-\frac{1}{2}} + 2K + 10$$

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$$20/21$$

