## Seminari 12

Matematičke metode za informatičare

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# Sadržaj

prvi zadatak

drugi zadatak

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# Eliminacija cjelobrojnih i racionalnih kandidata

#### **Teorem**

Ako je  $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$  polinom s cjelobrojnim koeficijentima i  $\alpha$  njegova cjelobrojna nultočka, tada je za svaki  $k \in \mathbb{Z}$  broj f(k) djeljiv s  $\alpha - k$ .

## Teorem

Ako je M(p,q)=1 i  $\alpha=\frac{p}{q}$  racionalna nultočka polinoma f(x) s cjelobrojnim koeficijentima, tada je za svaki cijeli broj k broj f(k) djeljiv s p-kq.

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

Rješenje 
$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\left(x^2\right)$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\left(x^2 - \frac{1}{2}x\right)^2$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2}$$

Ferrarijevom metodom riješite jednadžbu  $x^4 - x^3 + 9x^2 - 13x = -24$ .

## Rješenje

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2}_{X^4}$$

Ferrarijevom metodom riješite jednadžbu  $x^4 - x^3 + 9x^2 - 13x = -24$ .

## Rješenje

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2}_{x^4 + \frac{1}{4}x^2}$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2}_{x^4 + \frac{1}{4}x^2 + y^2}$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3}$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y}$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[ x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy \right]}_{}$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^{2} - \frac{1}{2}x + y\right)^{2} - \left[x^{4} + \frac{1}{4}x^{2} + y^{2} - x^{3} + 2x^{2}y - xy\right]}_{2}$$

Rješenje 
$$x^{4} - x^{3} + 9x^{2} - 13x + 24 = 0$$

$$(x^{2} - \frac{1}{2}x + y)^{2} - [() x^{2}$$

$$x^{4} + \frac{1}{4}x^{2} + y^{2} - x^{3} + 2x^{2}y - xy$$

$$x^{4} - x^{3} + 9x^{2} - 13x + 24 = 0$$

$$\underbrace{\left(x^{2} - \frac{1}{2}x + y\right)^{2} - \left[\left(\frac{1}{4} + 2y\right) x^{2}\right]}_{x}$$

$$x^4 + \frac{1}{4}x^2 + y^2 - x^3 + \underline{2x^2y} - xy$$

$$x^{4} - x^{3} + 9x^{2} - 13x + 24 = 0$$

$$\left(x^{2} - \frac{1}{2}x + y\right)^{2} - \left[\left(\frac{1}{4} + 2y - 9\right)x^{2}\right]$$

$$x^4 + \frac{1}{4}x^2 + y^2 - x^3 + \underline{2x^2y} - xy$$

$$x^{4} - x^{3} + 9x^{2} - 13x + 24 = 0$$

$$\left(x^{2} - \frac{1}{2}x + y\right)^{2} - \left[\left(\frac{1}{4} + 2y - 9\right)x^{2}\right]$$

$$x^4 + \frac{1}{4}x^2 + y^2 - x^3 + \underline{2x^2y} - \underline{xy}$$

Rješenje 
$$x^{4} - x^{3} + 9x^{2} - 13x + 24 = 0$$

$$\underbrace{\left(x^{2} - \frac{1}{2}x + y\right)^{2} - \left[\left(\frac{1}{4} + 2y - 9\right)x^{2} + ( )x\right]}_{x^{4} + \frac{1}{4}x^{2} + y^{2} - x^{3} + 2x^{2}y - xy}$$

Rješenje 
$$x^{4} - x^{3} + 9x^{2} - 13x + 24 = 0$$

$$\underbrace{\left(x^{2} - \frac{1}{2}x + y\right)^{2} - \left[\left(\frac{1}{4} + 2y - 9\right)x^{2} + (-y)x\right]}_{x^{4} + \frac{1}{4}x^{2} + y^{2} - x^{3} + 2x^{2}y - xy}$$

Rješenje 
$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$
$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x\right]$$

$$x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy$$

Rješenje  

$$x^{4} - x^{3} + 9x^{2} - 13x + 24 = 0$$

$$\left(x^{2} - \frac{1}{2}x + y\right)^{2} - \left[\left(\frac{1}{4} + 2y - 9\right)x^{2} + (-y + 13)x\right]$$

$$x^{4} + \frac{1}{4}x^{2} + y^{2} - x^{3} + 2x^{2}y - xy$$

Rješenje 
$$x^{4} - x^{3} + 9x^{2} - 13x + 24 = 0$$

$$\underbrace{\left(x^{2} - \frac{1}{2}x + y\right)^{2} - \left[\left(\frac{1}{4} + 2y - 9\right)x^{2} + (-y + 13)x + (\right)\right]}_{x^{4} + \frac{1}{4}x^{2} + y^{2} - x^{3} + 2x^{2}y - xy}$$

Rješenje 
$$x^{4} - x^{3} + 9x^{2} - 13x + 24 = 0$$

$$\underbrace{\left(x^{2} - \frac{1}{2}x + y\right)^{2} - \left[\left(\frac{1}{4} + 2y - 9\right)x^{2} + (-y + 13)x + (y^{2})\right]}_{x^{4} + \frac{1}{4}x^{2} + y^{2} - x^{3} + 2x^{2}y - xy}$$

Rješenje  

$$x^{4} - x^{3} + 9x^{2} - 13x + 24 = 0$$

$$(x^{2} - \frac{1}{2}x + y)^{2} - [(\frac{1}{4} + 2y - 9)x^{2} + (-y + 13)x + (y^{2} - 24)]$$

$$x^{4} + \frac{1}{4}x^{2} + y^{2} - x^{3} + 2x^{2}y - xy$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + \frac{y^2}{2} - x^3 + \frac{2x^2y}{2} - \frac{xy}{2}}$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \underbrace{\frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}_{\mathbf{x}^2}}$$
$$\left(x^2 - \frac{1}{2}x + y\right)^2$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right]}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy} = 0$$

$$\left(x^2-\frac{1}{2}x+y\right)^2-\left[$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2\right]$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x\right]$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + \left(y^2 - 24\right)\right]$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + \left(y^2 - 24\right)\right] = 0$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$b^2 - 4ac = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + \left(y^2 - 24\right)\right] = 0$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$b^2 - 4ac = 0$$

$$x^{4} + \frac{1}{4}x^{2} + y^{2} - x^{3} + \frac{2x^{2}y - xy}{2}$$

$$\left(x^{2} - \frac{1}{2}x + y\right)^{2} - \left[\left(2y - \frac{35}{4}\right)x^{2} + \left(-y + 13\right)x + \left(y^{2} - 24\right)\right] = 0$$

$$a^2 - 4ac = 0$$

$$(-y+13)^2$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$b^2 - 4ac = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + \left(y^2 - 24\right)\right] = 0$$

$$(-y+13)^2-4$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$b^2 - 4ac = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + \left(y^2 - 24\right)\right] = 0$$
$$(-y + 13)^2 - 4\left(2y - \frac{35}{4}\right)$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$b^2 - 4ac = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + \left(y^2 - 24\right)\right] = 0$$
$$(-y + 13)^2 - 4\left(2y - \frac{35}{4}\right)\left(y^2 - 24\right)$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$b^2 - 4ac = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + \left(y^2 - 24\right)\right] = 0$$
$$(-y + 13)^2 - 4\left(2y - \frac{35}{4}\right)\left(y^2 - 24\right) = 0$$

$$b^2 - 4ac = 0$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$b^2 - 4ac = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0$$
$$(-y + 13)^2 - 4\left(2y - \frac{35}{4}\right)(y^2 - 24) = 0$$
$$(-y + 13)^2$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$b^2 - 4ac = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0$$
$$(-y + 13)^2 - 4\left(2y - \frac{35}{4}\right)(y^2 - 24) = 0$$
$$(-y + 13)^2 - (8y - 35)$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$b^2 - 4ac = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0$$
$$(-y + 13)^2 - 4\left(2y - \frac{35}{4}\right)(y^2 - 24) = 0$$
$$(-y + 13)^2 - (8y - 35)(y^2 - 24)$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$b^2 - 4ac = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0$$
$$(-y + 13)^2 - 4\left(2y - \frac{35}{4}\right)(y^2 - 24) = 0$$
$$(-y + 13)^2 - (8y - 35)(y^2 - 24) = 0$$

Ferrarijevom metodom riješite jednadžbu  $x^4 - x^3 + 9x^2 - 13x = -24$ .

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$b^2 - 4ac = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + \left(y^2 - 24\right)\right] = 0$$
$$(-y + 13)^2 - 4\left(2y - \frac{35}{4}\right)\left(y^2 - 24\right) = 0$$

$$(-y+13)^2-(8y-35)(y^2-24)=0$$

.

Ferrarijevom metodom riješite jednadžbu  $x^4 - x^3 + 9x^2 - 13x = -24$ .

 $v^2 - 26v$ 

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$b^2 - 4ac = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + \left(y^2 - 24\right)\right] = 0$$
$$(-y + 13)^2 - 4\left(2y - \frac{35}{4}\right)\left(y^2 - 24\right) = 0$$
$$(-y + 13)^2 - \left(8y - 35\right)\left(y^2 - 24\right) = 0$$

 $y^2 - 26y + 169$ 

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$b^2 - 4ac = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0$$
$$(-y + 13)^2 - 4\left(2y - \frac{35}{4}\right)(y^2 - 24) = 0$$
$$(-y + 13)^2 - (8y - 35)(y^2 - 24) = 0$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$b^2 - 4ac = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + \left(y^2 - 24\right)\right] = 0$$

$$(-y + 13)^2 - 4\left(2y - \frac{35}{4}\right)\left(y^2 - 24\right) = 0$$

$$(-y + 13)^2 - (8y - 35)\left(y^2 - 24\right) = 0$$

$$v^2 - 26v + 169 - 8v^3$$

Ferrarijevom metodom riješite jednadžbu  $x^4 - x^3 + 9x^2 - 13x = -24$ .

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$b^2 - 4ac = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + \left(y^2 - 24\right)\right] = 0$$
$$(-y + 13)^2 - 4\left(2y - \frac{35}{4}\right)\left(y^2 - 24\right) = 0$$
$$(-y + 13)^2 - \left(8y - 35\right)\left(y^2 - 24\right) = 0$$

 $y^2 - 26y + 169 - 8y^3 + 192y$ 

Ferrarijevom metodom riješite jednadžbu  $x^4 - x^3 + 9x^2 - 13x = -24$ .

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$b^2 - 4ac = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + \left(y^2 - 24\right)\right] = 0$$
$$(-y + 13)^2 - 4\left(2y - \frac{35}{4}\right)\left(y^2 - 24\right) = 0$$
$$(-y + 13)^2 - \left(8y - 35\right)\left(y^2 - 24\right) = 0$$

 $y^2 - 26y + 169 - 8y^3 + 192y + 35y^2$ 

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$b^2 - 4ac = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + \left(y^2 - 24\right)\right] = 0$$
$$(-y + 13)^2 - 4\left(2y - \frac{35}{4}\right)\left(y^2 - 24\right) = 0$$

$$(-y+13)^2 - (8y-35)(y^2-24) = 0$$
$$y^2 - 26y + 169 - 8y^3 + 192y + 35y^2 - 840$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$b^2 - 4ac = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0$$

$$(-y + 13)^2 - 4\left(2y - \frac{35}{4}\right)(y^2 - 24) = 0$$

$$(-y+13)^2 - (8y-35)(y^2-24) = 0$$
$$y^2 - 26y + 169 - 8y^3 + 192y + 35y^2 - 840 = 0$$

Ferrarijevom metodom riješite jednadžbu  $x^4 - x^3 + 9x^2 - 13x = -24$ .

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2} - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0$$

 $x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy$ 

$$b^2 - 4ac = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + \left(y^2 - 24\right)\right] = 0$$

$$(-y+13)^2-4\left(2y-\frac{35}{4}\right)\left(y^2-24\right)=0$$

$$= 0$$

$$(-y+13)^2 - (8y-35)(y^2-24) = 0$$
$$y^2 - 26y + 169 - 8y^3 + 192y + 35y^2 - 840 = 0$$

 $-8v^{3}$ 

Ferrarijevom metodom riješite jednadžbu  $x^4 - x^3 + 9x^2 - 13x = -24$ .

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right]}_{2} = 0$$

$$b^2 - 4ac = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0$$

 $x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy$ 

$$(-y+13)^2 - 4\left(2y - \frac{35}{4}\right)(y^2 - 24) = 0$$
$$(-y+13)^2 - (8y-35)(y^2 - 24) = 0$$

$$y^2 - 26y + 169 - 8y^3 + 192y + 35y^2 - 840 = 0$$

Ferrarijevom metodom riješite jednadžbu  $x^4 - x^3 + 9x^2 - 13x = -24$ .

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}$$

$$b^2 - 4ac = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + \left(y^2 - 24\right)\right] = 0$$
$$(-y + 13)^2 - 4\left(2y - \frac{35}{4}\right)\left(y^2 - 24\right) = 0$$

 $x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy$ 

$$(-y+13)^2 - (8y-35)(y^2-24) = 0$$

$$y^2 - 26y + 169 - 8y^3 + 192y + 35y^2 - 840 = 0$$
$$-8y^3 + 36y^2 + 166y$$

Ferrarijevom metodom riješite jednadžbu  $x^4 - x^3 + 9x^2 - 13x = -24$ .

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$b^2 - 4ac = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + \left(y^2 - 24\right)\right] = 0$$

$$(-y+13)^2 - 4\left(2y - \frac{35}{4}\right)(y^2 - 24) = 0$$

$$(-y+13)^2 - (8y-35)(y^2-24) = 0$$
$$y^2 - 26y + 169 - 8y^3 + 192y + 35y^2 - 840 = 0$$

 $-8v^3 + 36v^2 + 166v - 671$ 

 $b^2 - 4ac = 0$ 

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$b^2 - 4ac = 0$$

$$b^2 - 4ac = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + \left(y^2 - 24\right)\right] = 0$$
$$(-y + 13)^2 - 4\left(2y - \frac{35}{4}\right)\left(y^2 - 24\right) = 0$$

$$(-y+13)^2 - 4(2y-\frac{\pi}{4})(y-24) = 0$$
  
 $(-y+13)^2 - (8y-35)(y^2-24) = 0$ 

$$(-y+13)^2 - (8y-35)(y^2-24) = 0$$
$$y^2 - 26y + 169 - 8y^3 + 192y + 35y^2 - 840 = 0$$

$$y^2 - 20y + 109 - 8y^3 + 192y + 35y^2 - 8y^3 + 36y^2 + 166y - 671 = 0$$

Ferrarijevom metodom riješite jednadžbu  $x^4 - x^3 + 9x^2 - 13x = -24$ .

**Rješenje** 
$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right]}_{} = 0$$

$$x^{4} + \frac{1}{4}x^{2} + y^{2} - x^{3} + 2x^{2}y - xy$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + \left(-y + 13\right)x + \left(y^2 - 24\right)\right] = 0$$

$$(-y+13)^2-4\left(2y-\frac{35}{4}\right)\left(y^2-24\right)=0$$

$$(-y+13)^2-(8y-35)(y^2-24)=0$$

$$y^{2} - 26y + 169 - 8y^{3} + 192y + 35y^{2} - 840 = 0$$
$$-8y^{3} + 36y^{2} + 166y - 671 = 0$$

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 $b^2 - 4ac = 0$ 

Ferrarijevom metodom riješite jednadžbu  $x^4 - x^3 + 9x^2 - 13x = -24$ .

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\underbrace{\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(\frac{1}{4} + 2y - 9\right)x^2 + (-y + 13)x + (y^2 - 24)\right] = 0}_{x^4 + \frac{1}{4}x^2 + y^2 - x^3 + 2x^2y - xy}$$

$$b^2 - 4ac = 0$$

$$b^2 - 4ac = 0$$

Ferrarijeva

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + \left(y^2 - 24\right)\right] = 0$$

$$(-y+13)^2-4\left(2y-\frac{35}{4}\right)(y^2-24)=0$$

$$(-y+13)^2-(8y-35)(y^2-24)=0$$

$$y^{2} - 26y + 169 - 8y^{3} + 192y + 35y^{2} - 840 = 0$$

$$-8y^{3} + 36y^{2} + 166y - 671 = 0$$

$$-8y^3 + 36y^2 + 166y - 671 = 0$$

$$-8y^3 + 36y^2 + 166y - 671 = 0$$

 $\frac{p}{q}$ :

$$-8y^3 + 36y^2 + 166y - 671 = 0$$

 $671=11\cdot 61$ 

<u>p</u> :

 $-8y^3 + 36y^2 + 166y - 671 = 0$ 

 $671 = 11 \cdot 61$ 

$$p: 1, -1,$$

 $671 = 11 \cdot 61$ 

 $-8y^3 + 36y^2 + 166y - 671 = 0$ 

<u>p</u> 9

$$p:1,-1,11,-11,$$

 $-8y^3 + 36y^2 + 166y - 671 = 0$ 

 $671 = 11 \cdot 61$ 

$$p:1,-1,11,-11,61,-61,$$

 $-8y^3 + 36y^2 + 166y - 671 = 0$ 

 $671 = 11 \cdot 61$ 

<u>!</u>

$$p: 1, -1, 11, -11, 61, -61, 671, -671$$

 $671 = 11 \cdot 61$ 

 $-8y^3 + 36y^2 + 166y - 671 = 0$ 

$$rac{P}{q}$$
:

p: 1, -1, 11, -11, 61, -61, 671, -671

 $-8y^3 + 36y^2 + 166y - 671 = 0$ 

 $671 = 11 \cdot 61$ 

 $-8y^3 + 36y^2 + 166y - 671 = 0$ 

 $671 = 11 \cdot 61$ 

q:1,

p: 1, -1, 11, -11, 61, -61, 671, -671

q: 1, 2,

p: 1, -1, 11, -11, 61, -61, 671, -671

 $-8y^3 + 36y^2 + 166y - 671 = 0$ 

 $671 = 11 \cdot 61$ 

 $671 = 11 \cdot 61$ 

q:1,2,4,

 $671 = 11 \cdot 61$ 

q:1,2,4,8

$$\frac{p}{q}$$
:  $\frac{1}{1}$ ,

 $671 = 11 \cdot 61$ 

q:1,2,4,8

$$\frac{p}{q}$$
:  $\frac{1}{1}$ ,  $\frac{1}{2}$ ,

 $671 = 11 \cdot 61$ 

q:1,2,4,8

$$\frac{p}{q}: \frac{1}{1}, \frac{1}{2}, \frac{1}{4},$$

 $671 = 11 \cdot 61$ 

q:1,2,4,8

$$\frac{p}{q}$$
:  $\frac{1}{1}$ ,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,

 $671 = 11 \cdot 61$ 

q:1,2,4,8

$$\frac{p}{q}: \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1},$$

 $671 = 11 \cdot 61$ 

q:1,2,4,8

$$\frac{p}{q}$$
:  $\frac{1}{1}$ ,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{-1}{1}$ ,  $\frac{-1}{2}$ ,

 $671 = 11 \cdot 61$ 

q:1,2,4,8

$$\frac{p}{q}$$
:  $\frac{1}{1}$ ,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{-1}{1}$ ,  $\frac{-1}{2}$ ,  $\frac{-1}{4}$ ,

 $671 = 11 \cdot 61$ 

q:1,2,4,8

$$\frac{p}{q}$$
:  $\frac{1}{1}$ ,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{-1}{1}$ ,  $\frac{-1}{2}$ ,  $\frac{-1}{4}$ ,  $\frac{-1}{8}$ ,

 $671 = 11 \cdot 61$ 

q:1,2,4,8

$$\frac{p}{q}$$
:  $\frac{1}{1}$ ,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{-1}{1}$ ,  $\frac{-1}{2}$ ,  $\frac{-1}{4}$ ,  $\frac{-1}{8}$ ,  $\frac{11}{1}$ ,

 $671 = 11 \cdot 61$ 

q:1,2,4,8

$$\frac{p}{q}$$
:  $\frac{1}{1}$ ,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{-1}{1}$ ,  $\frac{-1}{2}$ ,  $\frac{-1}{4}$ ,  $\frac{-1}{8}$ ,  $\frac{11}{1}$ ,  $\frac{11}{2}$ ,

 $671 = 11 \cdot 61$ 

q:1,2,4,8

$$\frac{p}{q}: \ \frac{1}{1}, \ \frac{1}{2}, \ \frac{1}{4}, \ \frac{1}{8}, \ \frac{-1}{1}, \ \frac{-1}{2}, \ \frac{-1}{4}, \ \frac{-1}{8}, \ \frac{11}{1}, \ \frac{11}{2}, \ \frac{11}{4},$$

 $671 = 11 \cdot 61$ 

q:1,2,4,8

$$\frac{p}{q}$$
:  $\frac{1}{1}$ ,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{-1}{1}$ ,  $\frac{-1}{2}$ ,  $\frac{-1}{4}$ ,  $\frac{-1}{8}$ ,  $\frac{11}{1}$ ,  $\frac{11}{2}$ ,  $\frac{11}{4}$ ,

 $671 = 11 \cdot 61$ 

q:1,2,4,8

*p* :

$$\begin{array}{c} q: 1, 2, 4, 8 \\ p: 1, -1, 11, -11, 61, -61, 671, -671 \end{array} -8y^3 + 36y^2 + 166y - 671 = 0 \\ 671 = 11 \cdot 61 \\ \end{array}$$

 $671 = 11 \cdot 61$ 

$$\frac{p}{q}: \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4},$$

$$\frac{p}{a}: \frac{11}{8},$$

$$\frac{p}{q}: \ \frac{1}{1}, \ \frac{1}{2}, \ \frac{1}{4}, \ \frac{1}{8}, \ \frac{-1}{1}, \ \frac{-1}{2}, \ \frac{-1}{4}, \ \frac{-1}{8}, \ \frac{11}{1}, \ \frac{11}{2}, \ \frac{11}{4},$$

 $671 = 11 \cdot 61$ 

q:1,2,4,8

p: 1, -1, 11, -11, 61, -61, 671, -671

 $\frac{p}{a}: \frac{11}{8}, \frac{-11}{1},$ 

$$671 = 11 \cdot 61$$

$$\frac{p}{a}: \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4},$$

p: 1, -1, 11, -11, 61, -61, 671, -671

$$\frac{p}{q}$$
:  $\frac{11}{8}$ ,  $\frac{-11}{1}$ ,  $\frac{-11}{2}$ ,

q:1,2,4,8

$$671 = 11 \cdot 61$$

$$\frac{p}{a} : \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4},$$

$$\frac{p}{q}$$
:  $\frac{11}{8}$ ,  $\frac{-11}{1}$ ,  $\frac{-11}{2}$ ,  $\frac{-11}{4}$ ,

p: 1, -1, 11, -11, 61, -61, 671, -671

q:1,2,4,8

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$$\frac{p:1,-1,11,-11,61,-61,671,-671}{671=11\cdot 61}$$

 $\frac{p}{a}$ :  $\frac{1}{1}$ ,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{-1}{1}$ ,  $\frac{-1}{2}$ ,  $\frac{-1}{4}$ ,  $\frac{-1}{8}$ ,  $\frac{11}{1}$ ,  $\frac{11}{2}$ ,  $\frac{11}{4}$ ,

q:1,2,4,8

 $\frac{p}{q}$ :  $\frac{11}{8}$ ,  $\frac{-11}{1}$ ,  $\frac{-11}{2}$ ,  $\frac{-11}{4}$ ,  $\frac{-11}{8}$ ,

 $-8y^3 + 36y^2 + 166y - 671 = 0$ 

$$\begin{array}{c} q: 1, 2, 4, 8 \\ p: 1, -1, 11, -11, 61, -61, 671, -671 \end{array} -8y^3 + 36y^2 + 166y - 671 = 0 \\ 671 = 11 \cdot 61 \end{array}$$

 $\frac{p}{a}$ :  $\frac{1}{1}$ ,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{-1}{1}$ ,  $\frac{-1}{2}$ ,  $\frac{-1}{4}$ ,  $\frac{-1}{8}$ ,  $\frac{11}{1}$ ,  $\frac{11}{2}$ ,  $\frac{11}{4}$ ,

$$\frac{p}{q}$$
:  $\frac{11}{8}$ ,  $\frac{-11}{1}$ ,  $\frac{-11}{2}$ ,  $\frac{-11}{4}$ ,  $\frac{-11}{8}$ ,  $\frac{61}{1}$ ,

q:1,2,4,8

 $671 = 11 \cdot 61$ 

$$\begin{array}{c} q: 1, 2, 4, 8 \\ p: 1, -1, 11, -11, 61, -61, 671, -671 \end{array} -8y^3 + 36y^2 + 166y - 671 = 0$$

 $671 = 11 \cdot 61$ 

$$\frac{p}{q}: \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2},$$

$$\begin{array}{c} q: 1, 2, 4, 8 \\ p: 1, -1, 11, -11, 61, -61, 671, -671 \end{array} -8y^3 + 36y^2 + 166y - 671 = 0$$

 $671 = 11 \cdot 61$ 

$$\frac{p}{q}$$
:  $\frac{11}{8}$ ,  $\frac{-11}{1}$ ,  $\frac{-11}{2}$ ,  $\frac{-11}{4}$ ,  $\frac{-11}{8}$ ,  $\frac{61}{1}$ ,  $\frac{61}{2}$ ,  $\frac{61}{4}$ ,

$$p:1,-1,11,-11,61,-61,671,-671$$

 $671 = 11 \cdot 61$ 

 $-8y^3 + 36y^2 + 166y - 671 = 0$ 

$$\frac{p}{q}: \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8},$$

$$p:1,-1,11,-11,61,-61,671,-671$$

 $-8y^3 + 36y^2 + 166y - 671 = 0$ 

 $671 = 11 \cdot 61$ 

$$\frac{p}{a}: \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1},$$

 $\frac{p}{a}$ :  $\frac{1}{1}$ ,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{-1}{1}$ ,  $\frac{-1}{2}$ ,  $\frac{-1}{4}$ ,  $\frac{-1}{8}$ ,  $\frac{11}{1}$ ,  $\frac{11}{2}$ ,  $\frac{11}{4}$ ,

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$$p: 1, -1, 11, -11, 61, -61, 671, -671$$

 $\frac{p}{a}$ :  $\frac{1}{1}$ ,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{-1}{1}$ ,  $\frac{-1}{2}$ ,  $\frac{-1}{4}$ ,  $\frac{-1}{8}$ ,  $\frac{11}{1}$ ,  $\frac{11}{2}$ ,  $\frac{11}{4}$ ,

 $671 = 11 \cdot 61$ 

 $-8y^3 + 36y^2 + 166y - 671 = 0$ 

$$rac{
ho}{q}$$
 :

 $\frac{p}{a}: \ \frac{11}{8}, \ \frac{-11}{1}, \ \frac{-11}{2}, \ \frac{-11}{4}, \ \frac{-11}{8}, \ \frac{61}{1}, \ \frac{61}{2}, \ \frac{61}{4}, \ \frac{61}{8}, \ \frac{-61}{1},$ 

$$p:1,-1,11,-11,61,-61,671,-671$$

 $671 = 11 \cdot 61$ 

 $-8y^3 + 36y^2 + 166y - 671 = 0$ 

$$\frac{p}{q}$$
:  $\frac{1}{1}$ ,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{-1}{1}$ ,  $\frac{-1}{2}$ ,  $\frac{-1}{4}$ ,  $\frac{-1}{8}$ ,  $\frac{11}{1}$ ,  $\frac{11}{2}$ ,  $\frac{11}{4}$ ,

 $\frac{p}{q}$ :  $\frac{11}{8}$ ,  $\frac{-11}{1}$ ,  $\frac{-11}{2}$ ,  $\frac{-11}{4}$ ,  $\frac{-11}{8}$ ,  $\frac{61}{1}$ ,  $\frac{61}{2}$ ,  $\frac{61}{4}$ ,  $\frac{61}{8}$ ,  $\frac{-61}{1}$ ,

$$\frac{p}{a}: \frac{-61}{2}$$

q:1,2,4,8

$$p: 1, -1, 11, -11, 61, -61, 671, -671$$

$$\frac{p}{q}$$
:  $\frac{1}{1}$ ,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{-1}{1}$ ,  $\frac{-1}{2}$ ,  $\frac{-1}{4}$ ,  $\frac{-1}{8}$ ,  $\frac{11}{1}$ ,  $\frac{11}{2}$ ,  $\frac{11}{4}$ ,

$$\frac{p}{a}$$
:  $\frac{11}{8}$ ,  $\frac{-11}{1}$ ,  $\frac{-11}{2}$ ,  $\frac{-11}{4}$ ,  $\frac{61}{8}$ ,  $\frac{61}{1}$ ,  $\frac{61}{2}$ ,  $\frac{61}{4}$ ,  $\frac{61}{8}$ ,  $\frac{-61}{1}$ ,

 $\frac{p}{a}: \frac{-61}{2}, \frac{-61}{4},$ 

q:1,2,4,8

$$\frac{-11}{4}$$
,  $\frac{-1}{8}$ 

$$\frac{61}{2}$$
,

$$\frac{61}{4}$$
,

 $-8y^3 + 36y^2 + 166y - 671 = 0$ 

 $671 = 11 \cdot 61$ 

$$p:1,-1,11,-11,61,-61,671,-671$$

 $\frac{p}{a}: \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8},$ 

q:1,2,4,8

 $671 = 11 \cdot 61$ 

 $-8y^3 + 36y^2 + 166y - 671 = 0$ 

$$\frac{p}{q}$$
:  $\frac{11}{8}$ ,  $\frac{-11}{1}$ ,  $\frac{-11}{2}$ ,  $\frac{-11}{4}$ ,  $\frac{-11}{8}$ ,  $\frac{61}{1}$ ,  $\frac{61}{2}$ ,  $\frac{61}{4}$ ,  $\frac{61}{8}$ ,  $\frac{-61}{1}$ ,

 $\frac{p}{a}$ :  $\frac{1}{1}$ ,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{-1}{1}$ ,  $\frac{-1}{2}$ ,  $\frac{-1}{4}$ ,  $\frac{-1}{8}$ ,  $\frac{11}{1}$ ,  $\frac{11}{2}$ ,  $\frac{11}{4}$ ,

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$$p:1,-1,11,-11,61,-61,671,-671$$

 $671 = 11 \cdot 61$ 

$$\frac{p}{q}: \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4},$$

$$\frac{p}{a}: \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1},$$

$$\frac{p}{q}: \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1},$$

$$\frac{61}{2}$$
,

$$\frac{61}{4}$$
,

 $-8y^3 + 36y^2 + 166y - 671 = 0$ 

$$p: 1, -1, 11, -11, 61, -61, 671, -671$$

 $\frac{p}{a}: \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4},$ 

$$\frac{p}{q}$$
:  $\frac{11}{8}$ ,  $\frac{-11}{1}$ ,  $\frac{-11}{2}$ ,  $\frac{-11}{4}$ ,  $\frac{-11}{8}$ ,  $\frac{61}{1}$ ,  $\frac{61}{2}$ ,  $\frac{61}{4}$ ,  $\frac{61}{8}$ ,  $\frac{-61}{1}$ ,

 $\frac{p}{q}$ :  $\frac{-61}{2}$ ,  $\frac{-61}{4}$ ,  $\frac{-61}{8}$ ,  $\frac{671}{1}$ ,  $\frac{671}{2}$ ,

$$\frac{61}{2}$$
,

$$\frac{61}{4}$$
,

 $-8y^3 + 36y^2 + 166y - 671 = 0$ 

$$\frac{61}{8}$$
,

 $671 = 11 \cdot 61$ 

$$p:1,-1,11,-11,61,-61,671,-671$$

 $671 = 11 \cdot 61$ 

 $-8y^3 + 36y^2 + 166y - 671 = 0$ 

$$\frac{p}{q}$$
:  $\frac{11}{8}$ ,  $\frac{-11}{1}$ ,  $\frac{-11}{2}$ ,  $\frac{-11}{4}$ ,  $\frac{-11}{8}$ ,  $\frac{61}{1}$ ,  $\frac{61}{2}$ ,  $\frac{61}{4}$ ,  $\frac{61}{8}$ ,  $\frac{-61}{1}$ ,

 $\frac{p}{a}$ :  $\frac{-61}{2}$ ,  $\frac{-61}{4}$ ,  $\frac{-61}{8}$ ,  $\frac{671}{1}$ ,  $\frac{671}{2}$ ,  $\frac{671}{4}$ ,

$$p:1,-1,11,-11,61,-61,671,-671$$

 $671 = 11 \cdot 61$ 

 $-8y^3 + 36y^2 + 166y - 671 = 0$ 

$$\frac{p}{q}: \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1},$$

$$\frac{p}{q}: \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8},$$

$$p: 1, -1, 11, -11, 61, -61, 671, -671$$

 $-8y^3 + 36y^2 + 166y - 671 = 0$ 

 $671 = 11 \cdot 61$ 

$$\frac{p}{q}$$
:  $\frac{1}{1}$ ,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{-1}{1}$ ,  $\frac{-1}{2}$ ,  $\frac{-1}{4}$ ,  $\frac{-1}{8}$ ,  $\frac{11}{1}$ ,  $\frac{11}{2}$ ,  $\frac{11}{4}$ ,

 $\frac{p}{a}$ :  $\frac{11}{8}$ ,  $\frac{-11}{1}$ ,  $\frac{-11}{2}$ ,  $\frac{-11}{4}$ ,  $\frac{-11}{8}$ ,  $\frac{61}{1}$ ,  $\frac{61}{2}$ ,  $\frac{61}{4}$ ,  $\frac{61}{8}$ ,  $\frac{-61}{1}$ ,

 $\frac{p}{q}$ :  $\frac{-61}{2}$ ,  $\frac{-61}{4}$ ,  $\frac{-61}{8}$ ,  $\frac{671}{1}$ ,  $\frac{671}{2}$ ,  $\frac{671}{4}$ ,  $\frac{671}{8}$ ,  $\frac{-671}{1}$ ,

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$$p:1,-1,11,-11,61,-61,671,-671$$

 $-8y^3 + 36y^2 + 166y - 671 = 0$ 

 $671 = 11 \cdot 61$ 

$$\frac{p}{q}$$
:  $\frac{1}{1}$ ,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{-1}{1}$ ,  $\frac{-1}{2}$ ,  $\frac{-1}{4}$ ,  $\frac{-1}{8}$ ,  $\frac{11}{1}$ ,  $\frac{11}{2}$ ,  $\frac{11}{4}$ ,

$$\frac{p}{q}: \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1},$$

 $\frac{p}{a}$ :  $\frac{-61}{2}$ ,  $\frac{-61}{4}$ ,  $\frac{-61}{8}$ ,  $\frac{671}{1}$ ,  $\frac{671}{2}$ ,  $\frac{671}{4}$ ,  $\frac{671}{8}$ ,  $\frac{-671}{1}$ ,

$$\frac{p}{q}: \frac{-01}{2}, \frac{-01}{4}, \frac{-0}{8}$$

 $\frac{p}{a}$ : 3/21

$$p:1,-1,11,-11,61,-61,671,-671$$

 $\frac{p}{a}$ :  $\frac{11}{8}$ ,  $\frac{-11}{1}$ ,  $\frac{-11}{2}$ ,  $\frac{-11}{4}$ ,  $\frac{-11}{8}$ ,  $\frac{61}{1}$ ,  $\frac{61}{2}$ ,  $\frac{61}{4}$ ,  $\frac{61}{8}$ ,  $\frac{-61}{1}$ ,

 $\frac{p}{a}: \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4},$ 

 $-8y^3 + 36y^2 + 166y - 671 = 0$ 

 $671 = 11 \cdot 61$ 

 $\frac{p}{a}$ :  $\frac{-61}{2}$ ,  $\frac{-61}{4}$ ,  $\frac{-61}{8}$ ,  $\frac{671}{1}$ ,  $\frac{671}{2}$ ,  $\frac{671}{4}$ ,  $\frac{671}{8}$ ,  $\frac{-671}{1}$ ,

 $\frac{p}{a}: \frac{-671}{2},$ 

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$$p:1,-1,11,-11,61,-61,671,-671$$

 $-8y^3 + 36y^2 + 166y - 671 = 0$ 

 $671 = 11 \cdot 61$ 

$$\frac{p}{q}$$
:  $\frac{1}{1}$ ,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{-1}{1}$ ,  $\frac{-1}{2}$ ,  $\frac{-1}{4}$ ,  $\frac{-1}{8}$ ,  $\frac{11}{1}$ ,  $\frac{11}{2}$ ,  $\frac{11}{4}$ ,

$$\frac{1}{1}$$
,  $\frac{-11}{1}$ ,  $\frac{-11}{2}$ ,  $\frac{-11}{4}$ ,

$$\frac{p}{q}$$
:  $\frac{11}{8}$ ,  $\frac{-11}{1}$ ,  $\frac{-11}{2}$ ,  $\frac{-11}{4}$ ,  $\frac{-11}{8}$ ,  $\frac{61}{1}$ ,  $\frac{61}{2}$ ,  $\frac{61}{4}$ ,  $\frac{61}{8}$ ,  $\frac{-61}{1}$ ,

$$\frac{p}{q}$$
:  $\frac{-61}{2}$ ,  $\frac{-61}{4}$ ,  $\frac{-61}{8}$ ,  $\frac{671}{1}$ ,  $\frac{671}{2}$ ,  $\frac{671}{4}$ ,  $\frac{671}{8}$ ,  $\frac{-671}{1}$ ,

$$\frac{p}{q}$$
:  $\frac{-671}{2}$ ,  $\frac{-671}{4}$ ,

$$\frac{p:1,-1,11,-11,61,-61,671,-671}{671}$$

$$\frac{p}{q}:\frac{1}{1},\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{-1}{1},\frac{-1}{2},\frac{-1}{4},\frac{-1}{8},\frac{11}{1},\frac{11}{2},\frac{11}{4},$$

 $\frac{p}{q}: \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1},$   $\frac{p}{q}: \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1},$ 

 $\frac{p}{q}$ :  $\frac{-671}{2}$ ,  $\frac{-671}{4}$ ,  $\frac{-671}{8}$ 

 $-8y^3 + 36y^2 + 166y - 671 = 0$ 

 $671 = 11 \cdot 61$ 

$$p - kq \xrightarrow{k = -1} p + q$$

$$\frac{p}{q} : \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4},$$

$$\frac{p}{q} : \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1},$$

$$\frac{p}{q} : \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1},$$

$$\frac{p}{q} : \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}$$

 $-8y^3 + 36y^2 + 166y - 671 = 0$ 

 $671 = 11 \cdot 61$ 

q:1,2,4,8

p: 1, -1, 11, -11, 61, -61, 671, -671

$$p - kq \xrightarrow{k = -1} p + q \qquad f(-1) =$$

$$\frac{p}{q} : \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4},$$

$$\frac{p}{q} : \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1},$$

$$\frac{p}{q} : \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1},$$

$$\frac{p}{q} : \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}$$

$$p - kq \xrightarrow{k = -1} p + q \qquad f(-1) = -793$$

$$\frac{p}{q} : \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4},$$

$$\frac{p}{q} : \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1},$$

$$\frac{p}{q} : \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1},$$

$$\frac{p}{q} : \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}$$

$$p - kq \xrightarrow{k = -1} p + q \qquad f(-1) = -793$$

$$\frac{p}{q} : \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4},$$

$$p + q :$$

$$\frac{p}{q} : \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1},$$

$$\frac{p}{q} : \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1},$$

$$\frac{p}{q} : \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}$$

$$3/21$$

$$\frac{p}{q}: \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4}, 
p+q: 2,$$

$$\frac{p}{q}: \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1}, 
\frac{p}{q}: \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1},$$

$$\frac{p}{q}: \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}$$

$$3/21$$

 $p-kq \xrightarrow{k=-1} p+q$  f(-1)=-793

$$\frac{p}{q}: \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4}, 
p+q: 2, 3, 
\frac{p}{q}: \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1}, 
\frac{p}{q}: \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1}, 
\frac{p}{q}: \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}$$

$$3/21$$

 $p-kq \xrightarrow{k=-1} p+q$  f(-1)=-793

$$p - kq \xrightarrow{k = -1} p + q \qquad f(-1) = -793$$

$$\frac{p}{q} : \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4},$$

$$p + q : 2, 3, 5,$$

$$\frac{p}{q} : \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1},$$

$$\frac{p}{q} : \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1},$$

$$\frac{p}{q} : \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}$$

$$3/21$$

$$\frac{p}{q}: \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4}, \\
p+q: 2, 3, 5, 9, \\
\frac{p}{q}: \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1}, \\
\frac{p}{q}: \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1}, \\
\frac{p}{q}: \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}$$

$$3/21$$

 $p - kq \xrightarrow{k = -1} p + q$  f(-1) = -793

$$\frac{p}{q}: \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4}, 
p+q: 2, 3, 5, 9, 0, 
\frac{p}{q}: \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1}, 
\frac{p}{q}: \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1}, 
\frac{p}{q}: \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}$$

$$3/21$$

 $p - kq \xrightarrow{k = -1} p + q$  f(-1) = -793

$$p - kq \xrightarrow{k = -1} p + q \qquad f(-1) = -793$$

$$\frac{p}{q} : \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4},$$

$$p + q : 2, 3, 5, 9, 0, 1, 3,$$

$$\frac{p}{q} : \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1},$$

$$\frac{p}{q} : \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1},$$

$$\frac{p}{q} : \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}$$

$$3/21$$

$$p - kq \xrightarrow{k = -1} p + q \qquad f(-1) = -793$$

$$\frac{p}{q} : \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4},$$

$$p + q : 2, 3, 5, 9, 0, 1, 3, 7,$$

$$\frac{p}{q} : \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1},$$

$$\frac{p}{q} : \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1},$$

$$\frac{p}{q} : \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}$$

$$3/21$$

$$p - kq \xrightarrow{k = -1} p + q \qquad f(-1) = -793$$

$$\frac{p}{q} : \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4},$$

$$p + q : 2, 3, 5, 9, 0, 1, 3, 7, 12,$$

$$\frac{p}{q} : \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1},$$

$$\frac{p}{q} : \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1},$$

$$\frac{p}{q} : \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}$$

$$p - kq \xrightarrow{k = -1} p + q \qquad f(-1) = -793$$

$$\frac{p}{q} : \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4},$$

$$p + q : 2, 3, 5, 9, 0, 1, 3, 7, 12, 13,$$

$$\frac{p}{q} : \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1},$$

$$\frac{p}{q} : \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1},$$

$$\frac{p}{q} : \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}$$

$$p - kq \xrightarrow{k = -1} p + q \qquad f(-1) = -793$$

$$\frac{p}{q} : \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4},$$

$$p + q : 2, 3, 5, 9, 0, 1, 3, 7, 12, 13, 15,$$

$$\frac{p}{q} : \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1},$$

$$\frac{p}{q} : \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1},$$

$$\frac{p}{q} : \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}$$

$$p - kq \xrightarrow{k = -1} p + q \qquad f(-1) = -793$$

$$\frac{p}{q} : \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4},$$

$$p + q : 2, 3, 5, 9, 0, 1, 3, 7, 12, 13, 15,$$

$$\frac{p}{q} : \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1},$$

$$p + q :$$

$$\frac{p}{q} : \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1},$$

$$\frac{p}{q} : \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}$$

$$p - kq \xrightarrow{k = -1} p + q \qquad f(-1) = -793$$

$$\frac{p}{q} : \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4},$$

$$p + q : 2, 3, 5, 9, 0, 1, 3, 7, 12, 13, 15,$$

$$\frac{p}{q} : \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1},$$

$$p + q : 19,$$

$$\frac{p}{q} : \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1},$$

$$\frac{p}{q} : \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}$$

$$p - kq \xrightarrow{k = -1} p + q \qquad f(-1) = -793$$

$$\frac{p}{q} : \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4},$$

$$p + q : 2, 3, 5, 9, 0, 1, 3, 7, 12, 13, 15,$$

$$\frac{p}{q} : \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1},$$

$$p + q : 19, -10,$$

$$\frac{p}{q} : \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1},$$

$$\frac{p}{q} : \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}$$

$$p - kq \xrightarrow{k = -1} p + q \qquad f(-1) = -793$$

$$\frac{p}{q} : \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4},$$

$$p + q : 2, 3, 5, 9, 0, 1, 3, 7, 12, 13, 15,$$

$$\frac{p}{q} : \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1},$$

$$p + q : 19, -10, -9,$$

$$\frac{p}{q} : \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1},$$

$$\frac{p}{q} : \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}$$

$$p - kq \xrightarrow{k = -1} p + q \qquad f(-1) = -793$$

$$\frac{p}{q} : \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4},$$

$$p + q : 2, 3, 5, 9, 0, 1, 3, 7, 12, 13, 15,$$

$$\frac{p}{q} : \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1},$$

$$p + q : 19, -10, -9, -7,$$

$$\frac{p}{q} : \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1},$$

$$\frac{p}{q} : \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}$$

$$\begin{array}{c}
q: 1, 2, 4, 8 \\
p: 1, -1, 11, -11, 61, -61, 671, -671
\end{array}$$

$$\begin{array}{c}
-8y^3 + 36y^2 + 166y - 671 = 0 \\
f(y) & 671 = 11 \cdot 61
\end{array}$$

$$\begin{array}{c}
p - kq \xrightarrow{k = -1} & p + q & f(-1) = -793
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4}, \\
p + q: 2, 3, 5, 9, 0, 1, 3, 7, 12, 13, 15,
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1}, \\
p + q: 19, -10, -9, -7, -3,
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1},
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}
\end{array}$$

$$p - kq \xrightarrow{k = -1} p + q \qquad f(-1) = -793$$

$$\frac{p}{q} : \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4},$$

$$p + q : 2, 3, 5, 9, 0, 1, 3, 7, 12, 13, 15,$$

$$\frac{p}{q} : \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1},$$

$$p + q : 19, -10, -9, -7, -3, 62,$$

$$\frac{p}{q} : \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1},$$

$$\frac{p}{q} : \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}$$

$$p - kq \xrightarrow{k = -1} p + q \qquad f(-1) = -793$$

$$\frac{p}{q} : \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4},$$

$$p + q : 2, 3, 5, 9, 0, 1, 3, 7, 12, 13, 15,$$

$$\frac{p}{q} : \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1},$$

$$p + q : 19, -10, -9, -7, -3, 62, 63,$$

$$\frac{p}{q} : \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1},$$

$$\frac{p}{q} : \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}$$

$$p - kq \xrightarrow{k = -1} p + q \qquad f(-1) = -793$$

$$\frac{p}{q} : \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4},$$

$$p + q : 2, 3, 5, 9, 0, 1, 3, 7, 12, 13, 15,$$

$$\frac{p}{q} : \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1},$$

$$p + q : 19, -10, -9, -7, -3, 62, 63, 65,$$

$$\frac{p}{q} : \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1},$$

$$\frac{p}{q} : \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}$$

$$p - kq \xrightarrow{k = -1} p + q \qquad f(-1) = -793$$

$$\frac{p}{q} : \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4},$$

$$p + q : 2, 3, 5, 9, 0, 1, 3, 7, 12, 13, 15,$$

$$\frac{p}{q} : \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1},$$

$$p + q : 19, -10, -9, -7, -3, 62, 63, 65, 69,$$

$$\frac{p}{q} : \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1},$$

$$\frac{p}{q} : \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}$$

$$\frac{q:1,2,4,8}{p:1,-1,11,-11,61,-61,671,-671} = 0$$

$$\frac{k=-1}{p-kq} \xrightarrow{k=-1} p+q \qquad f(-1) = -793$$

$$\frac{p}{q}:\frac{1}{1},\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{-1}{1},\frac{-1}{2},\frac{-1}{4},\frac{-1}{8},\frac{11}{1},\frac{11}{2},\frac{11}{4},$$

$$p+q:2,3,5,9,0,1,3,7,12,13,15,$$

$$\frac{p}{q}:\frac{11}{8},\frac{-11}{1},\frac{-11}{2},\frac{-11}{4},\frac{-11}{8},\frac{61}{1},\frac{61}{2},\frac{61}{4},\frac{61}{8},\frac{-61}{1},$$

$$p+q:19,-10,-9,-7,-3,62,63,65,69,-60,$$

$$\frac{p}{q}:\frac{-61}{2},\frac{-61}{4},\frac{-61}{8},\frac{671}{1},\frac{671}{2},\frac{671}{4},\frac{671}{8},\frac{-671}{1},$$

$$\frac{p}{q}:\frac{-671}{2},\frac{-671}{4},\frac{-671}{8},\frac{-671}{4},\frac{-671}{8}$$

$$\begin{array}{c}
q: 1, 2, 4, 8 \\
p: 1, -1, 11, -11, 61, -61, 671, -671
\end{array}$$

$$\begin{array}{c}
-8y^3 + 36y^2 + 166y - 671 = 0 \\
f(y) & 671 = 11 \cdot 61
\end{array}$$

$$\begin{array}{c}
p - kq \xrightarrow{k = -1} & p + q & f(-1) = -793
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4}, \\
p + q: 2, 3, 5, 9, 0, 1, 3, 7, 12, 13, 15,
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1}, \\
p + q: 19, -10, -9, -7, -3, 62, 63, 65, 69, -60,
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1}, \\
p + q: -59,
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}
\end{array}$$

$$\begin{array}{c}
q: 1, 2, 4, 8 \\
p: 1, -1, 11, -11, 61, -61, 671, -671
\end{array}$$

$$\begin{array}{c}
-8y^3 + 36y^2 + 166y - 671 = 0 \\
f(y) & 671 = 11 \cdot 61
\end{array}$$

$$\begin{array}{c}
p - kq \xrightarrow{k = -1} & p + q & f(-1) = -793
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4}, \\
p + q: 2, 3, 5, 9, 0, 1, 3, 7, 12, 13, 15,
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1}, \\
p + q: 19, -10, -9, -7, -3, 62, 63, 65, 69, -60,
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1}, \\
p + q: -59, -57,
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}
\end{array}$$

$$\begin{array}{c}
q: 1, 2, 4, 8 \\
p: 1, -1, 11, -11, 61, -61, 671, -671
\end{array}$$

$$\begin{array}{c}
-8y^3 + 36y^2 + 166y - 671 = 0 \\
f(y) & 671 = 11 \cdot 61
\end{array}$$

$$\begin{array}{c}
p - kq \xrightarrow{k = -1} & p + q & f(-1) = -793
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4}, \\
p + q: 2, 3, 5, 9, 0, 1, 3, 7, 12, 13, 15,
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1}, \\
p + q: 19, -10, -9, -7, -3, 62, 63, 65, 69, -60,
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1}, \\
p + q: -59, -57, -53,
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}
\end{array}$$

$$\begin{array}{c}
q: 1, 2, 4, 8 \\
p: 1, -1, 11, -11, 61, -61, 671, -671
\end{array}$$

$$\begin{array}{c}
-8y^3 + 36y^2 + 166y - 671 = 0 \\
f(y) & 671 = 11 \cdot 61
\end{array}$$

$$\begin{array}{c}
p - kq \xrightarrow{k = -1} & p + q & f(-1) = -793
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4}, \\
p + q: 2, 3, 5, 9, 0, 1, 3, 7, 12, 13, 15,
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1}, \\
p + q: 19, -10, -9, -7, -3, 62, 63, 65, 69, -60,
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1}, \\
p + q: -59, -57, -53, 672,
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}
\end{array}$$

$$\begin{array}{c}
q: 1, 2, 4, 8 \\
p: 1, -1, 11, -11, 61, -61, 671, -671
\end{array}$$

$$\begin{array}{c}
-8y^3 + 36y^2 + 166y - 671 = 0 \\
f(y) & 671 = 11 \cdot 61
\end{array}$$

$$\begin{array}{c}
p - kq \xrightarrow{k = -1} & p + q & f(-1) = -793
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4}, \\
p + q: 2, 3, 5, 9, 0, 1, 3, 7, 12, 13, 15,
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1}, \\
p + q: 19, -10, -9, -7, -3, 62, 63, 65, 69, -60,
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1}, \\
p + q: -59, -57, -53, 672, 673,
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}
\end{array}$$

$$\begin{array}{c}
q: 1, 2, 4, 8 \\
p: 1, -1, 11, -11, 61, -61, 671, -671
\end{array}$$

$$\begin{array}{c}
-8y^3 + 36y^2 + 166y - 671 = 0 \\
f(y) & 671 = 11 \cdot 61
\end{array}$$

$$\begin{array}{c}
p - kq \xrightarrow{k = -1} & p + q & f(-1) = -793
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4}, \\
p + q: 2, 3, 5, 9, 0, 1, 3, 7, 12, 13, 15,
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1}, \\
p + q: 19, -10, -9, -7, -3, 62, 63, 65, 69, -60,
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1}, \\
p + q: -59, -57, -53, 672, 673, 675,
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}
\end{array}$$

$$\begin{array}{c}
q: 1, 2, 4, 8 \\
p: 1, -1, 11, -11, 61, -61, 671, -671
\end{array}$$

$$\begin{array}{c}
-8y^3 + 36y^2 + 166y - 671 = 0 \\
f(y) & 671 = 11 \cdot 61
\end{array}$$

$$\begin{array}{c}
p - kq \xrightarrow{k = -1} & p + q & f(-1) = -793
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4}, \\
p + q: 2, 3, 5, 9, 0, 1, 3, 7, 12, 13, 15,
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1}, \\
p + q: 19, -10, -9, -7, -3, 62, 63, 65, 69, -60,
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1}, \\
p + q: -59, -57, -53, 672, 673, 675, 679,
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}
\end{array}$$

$$\begin{array}{c}
q: 1, 2, 4, 8 \\
p: 1, -1, 11, -11, 61, -61, 671, -671
\end{array}$$

$$\begin{array}{c}
-8y^3 + 36y^2 + 166y - 671 = 0 \\
f(y) & 671 = 11 \cdot 61
\end{array}$$

$$\begin{array}{c}
p - kq \xrightarrow{k = -1} & p + q & f(-1) = -793
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4}, \\
p + q: 2, 3, 5, 9, 0, 1, 3, 7, 12, 13, 15,
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{11}{8}, \frac{-11}{1}, \frac{-11}{2}, \frac{-11}{4}, \frac{-11}{8}, \frac{61}{1}, \frac{61}{2}, \frac{61}{4}, \frac{61}{8}, \frac{-61}{1}, \\
p + q: 19, -10, -9, -7, -3, 62, 63, 65, 69, -60,
\end{array}$$

$$\begin{array}{c}
\frac{p}{q}: \frac{-61}{2}, \frac{-61}{4}, \frac{-61}{8}, \frac{671}{1}, \frac{671}{2}, \frac{671}{4}, \frac{671}{8}, \frac{-671}{1}, \\
p + q: -59, -57, -53, 672, 673, 675, 679, -670,
\end{array}$$

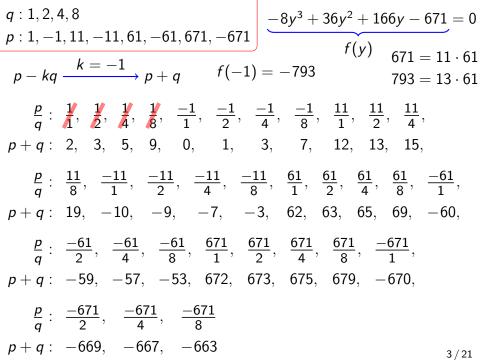
$$\begin{array}{c}
\frac{p}{q}: \frac{-671}{2}, \frac{-671}{4}, \frac{-671}{8}
\end{array}$$

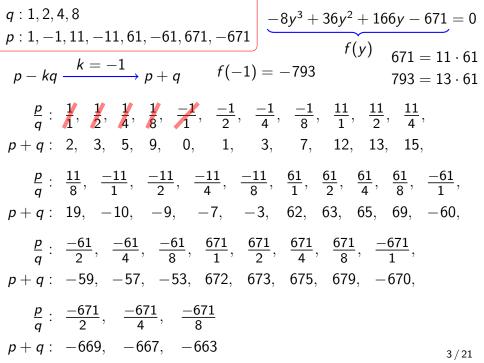
$$\begin{array}{c} q:1,2,4,8 \\ p:1,-1,11,-11,61,-61,671,-671 \end{array} \begin{array}{c} -8y^3+36y^2+166y-671=0 \\ \hline p-kq & k=-1 \\ \hline p+q & f(-1)=-793 \end{array} \end{array} \begin{array}{c} f(y) \\ 671=11\cdot 61 \\ \hline p+q: \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4}, \\ p+q: 2, 3, 5, 9, 0, 1, 3, 7, 12, 13, 15, \\ \hline p+q: 19, -10, -9, -7, -3, 62, 63, 65, 69, -60, \\ \hline p+q: -61 & 671 &$$

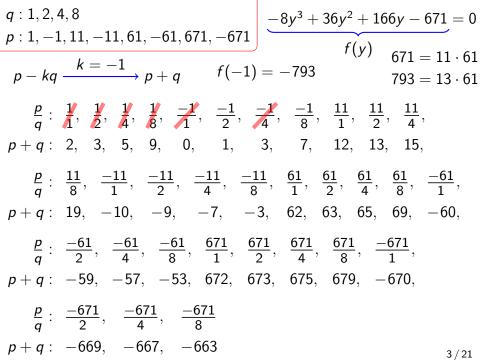
$$\begin{array}{c} q:1,2,4,8 \\ p:1,-1,11,-11,61,-61,671,-671 \end{array} \begin{array}{c} -8y^3+36y^2+166y-671=0 \\ \hline p-kq & k=-1 \\ \hline p+q & f(-1)=-793 \end{array} \end{array} \begin{array}{c} f(y) \\ 671=11\cdot 61 \\ \hline p+q: \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{-1}{1}, \frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \frac{11}{1}, \frac{11}{2}, \frac{11}{4}, \\ p+q: 2, 3, 5, 9, 0, 1, 3, 7, 12, 13, 15, \\ \hline p+q: 19, -10, -9, -7, -3, 62, 63, 65, 69, -60, \\ \hline p+q: -61 & 671 &$$

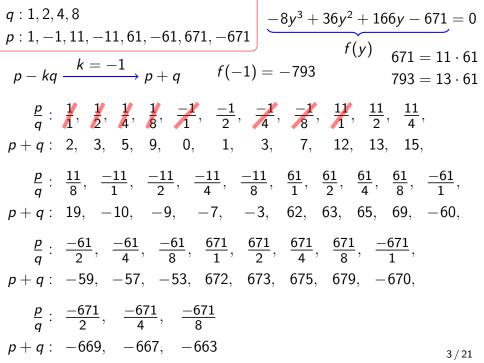
$$\begin{array}{c} q:1,2,4,8\\ p:1,-1,11,-11,61,-61,671,-671 \end{array} \begin{array}{c} -8y^3+36y^2+166y-671=0\\ \hline p-kq & k=-1\\ \hline p+q & f(-1)=-793 \end{array} \end{array} \begin{array}{c} f(y) & 671=11\cdot 61\\ \hline p+q:1,\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{-1}{1},\frac{-1}{2},\frac{-1}{4},\frac{-1}{8},\frac{11}{1},\frac{11}{2},\frac{11}{4},\\ \hline p+q:2,3,5,9,0,1,3,7,12,13,15,\\ \hline p+q:19,-10,-9,-7,-3,62,63,65,69,-60,\\ \hline p+q:19,-10,-9,-7,-3,62,63,65,69,-60,\\ \hline p+q:-59,-57,-53,672,673,675,679,-670,\\ \hline p+q:-669,-667,-663 \end{array} \begin{array}{c} -8y^3+36y^2+166y-671=0\\ \hline f(y) & 671=11\cdot 61 \end{array}$$

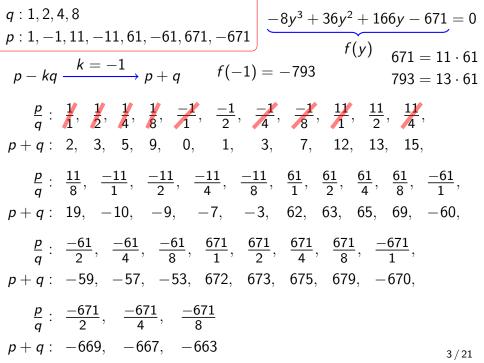
$$\begin{array}{c} q:1,2,4,8\\ p:1,-1,11,-11,61,-61,671,-671 \end{array} \begin{array}{c} -8y^3+36y^2+166y-671=0\\ \hline p-kq \xrightarrow{k=-1} p+q & f(-1)=-793 & f(y)\\ \hline p+q:\frac{1}{1},\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{-1}{1},\frac{-1}{2},\frac{-1}{4},\frac{-1}{8},\frac{11}{1},\frac{11}{2},\frac{11}{4},\\ p+q:2,3,5,9,0,1,3,7,12,13,15,\\ \hline p+q:19,-10,-9,-7,-3,62,63,65,69,-60,\\ \hline p+q:-61/2,\frac{-61}{4},\frac{-61}{8},\frac{671}{1},\frac{671}{2},\frac{671}{4},\frac{671}{8},\frac{-671}{1},\\ p+q:-59,-57,-53,672,673,675,679,-670,\\ \hline p+q:-669,-667,-663 & 3/21 \end{array}$$

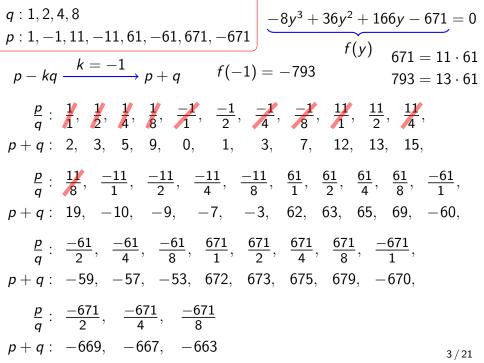


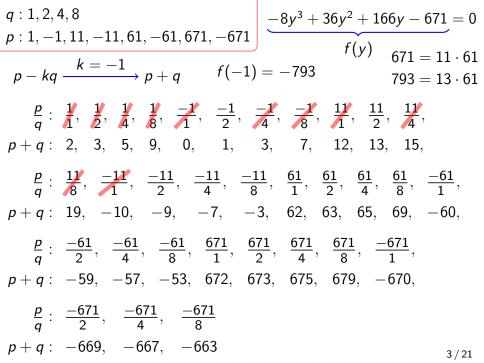


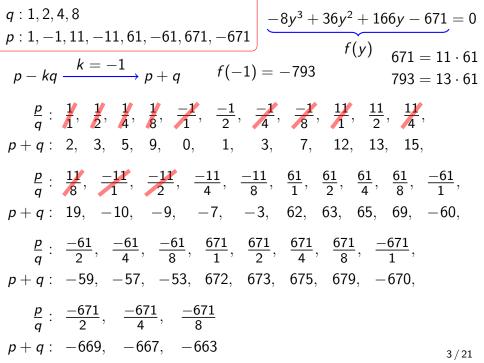


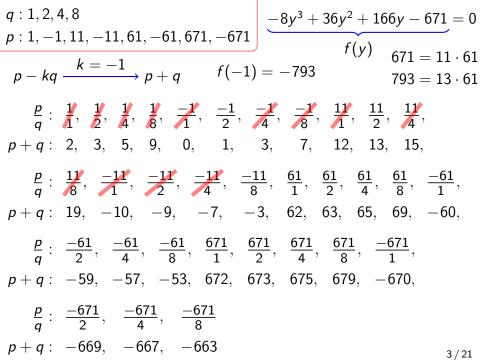


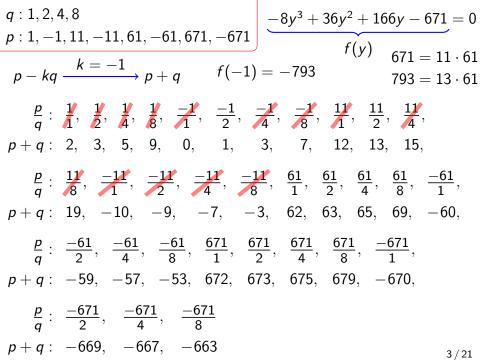


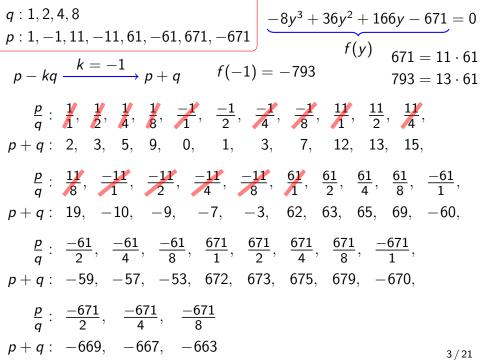


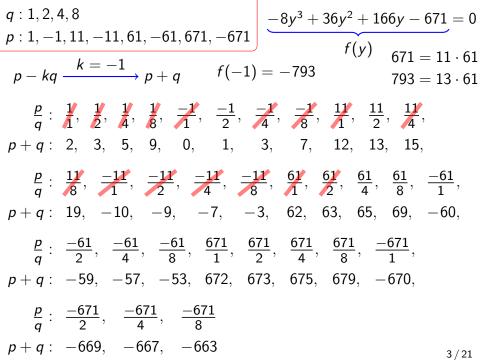


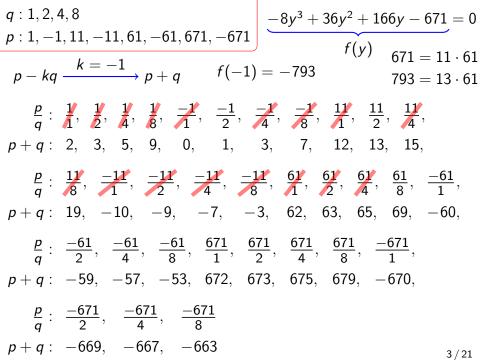


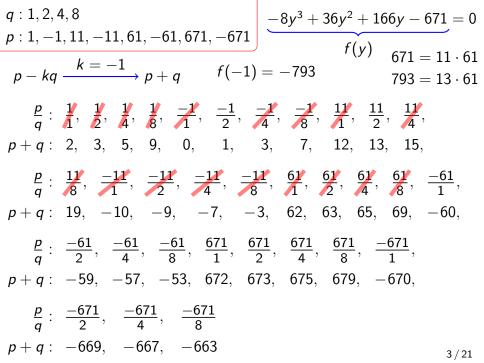


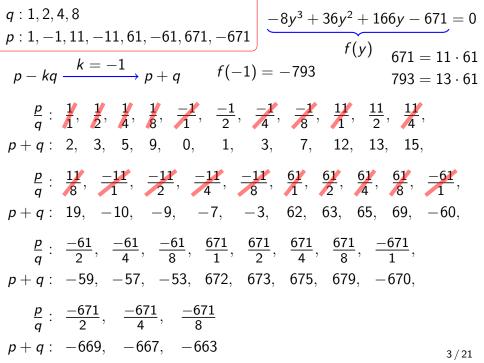


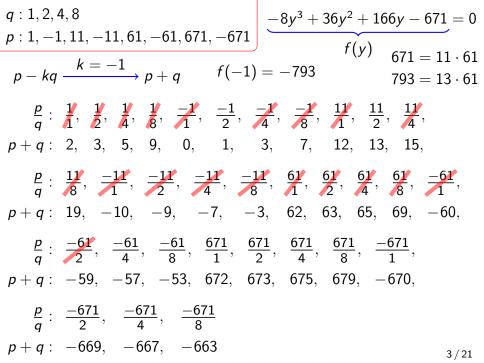


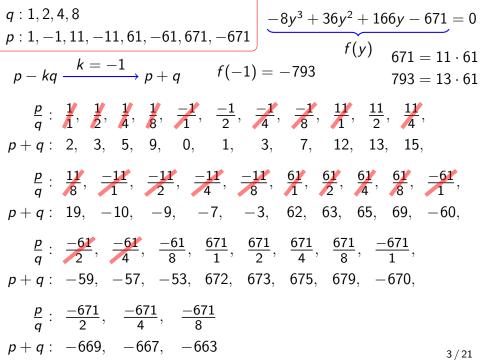


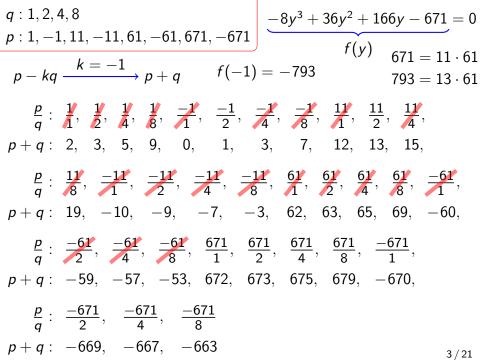


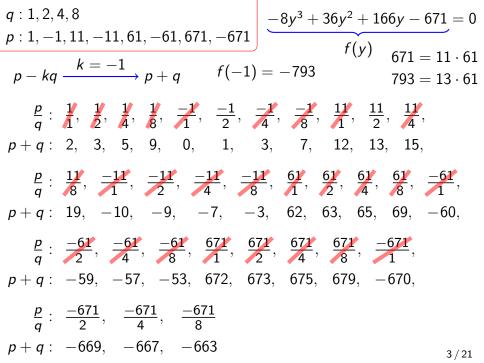


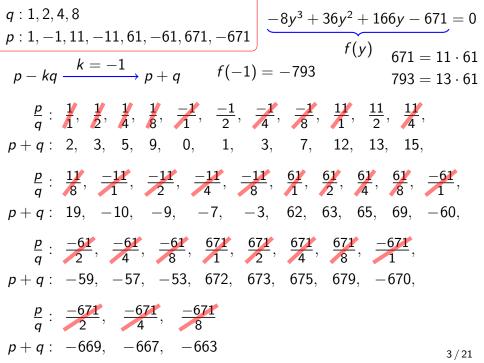


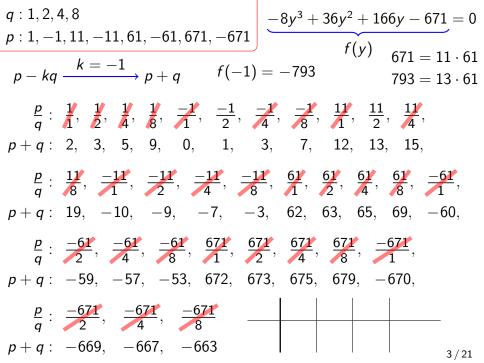


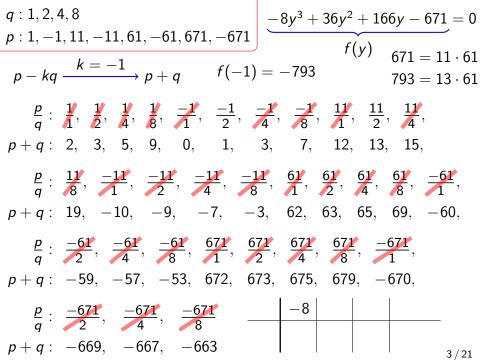


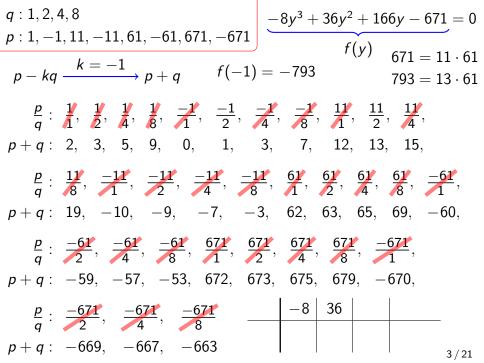


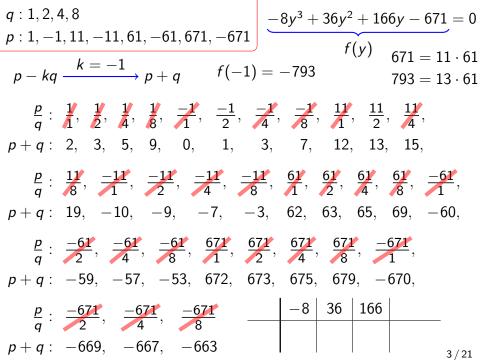


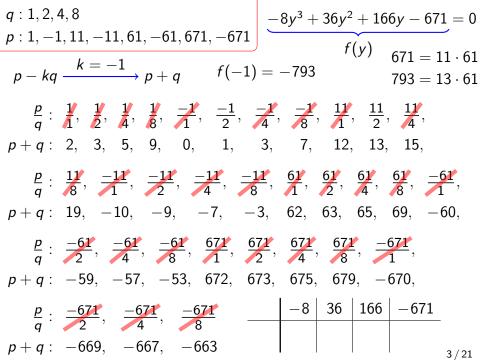


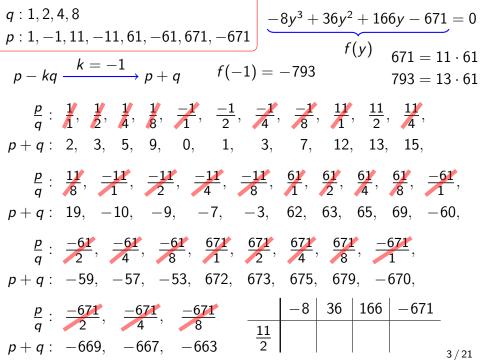


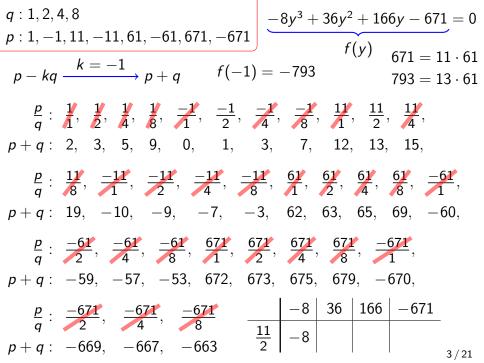


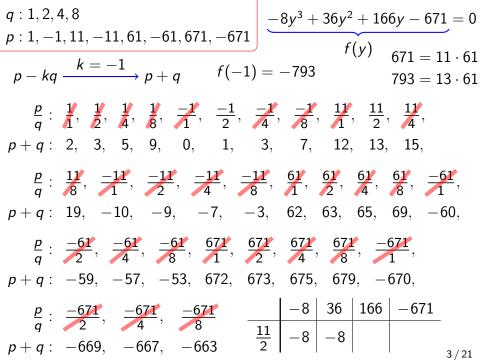


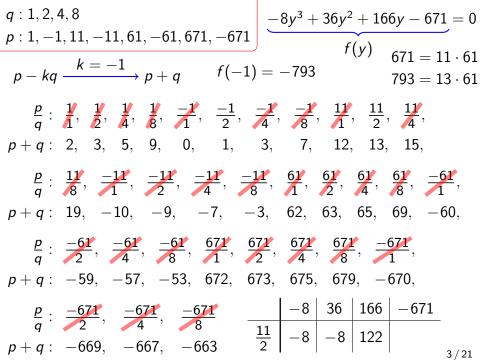


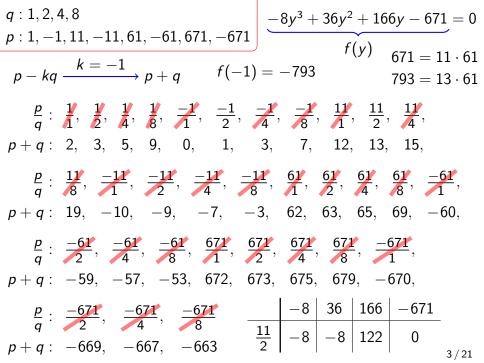


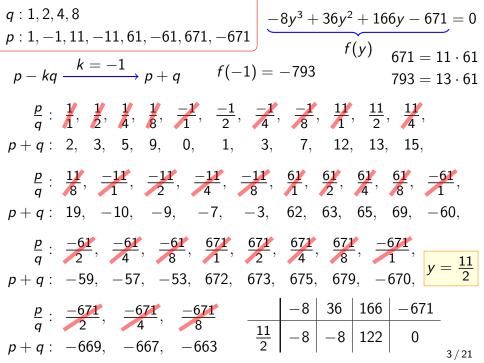












$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + \left(y^2 - 24\right)\right] = 0$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$(2.1)^2$$
 [(2.35) 2.4 (2.21) ...

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + \left(-y + 13\right)x + \left(y^2 - 24\right)\right] = 0$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$(v^2 - 1v + v)^2 - [(2v - 35)v^2 + (-v + 13)v + (v^2 - 24)] = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + \left(-y + 13\right)x + \left(y^2 - 24\right)\right] = 0$$

 $\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right)^2$ 

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{2}\right)x^2 + \left(-y + 13\right)x + \left(y^2 - 24\right)\right] = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + \left(-y + 13\right)x + \left(y^2 - 24\right)\right] = 0$$

 $\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right)^2 - \left[$ 

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + \left(-y + 13\right)x + \left(y^2 - 24\right)\right] = 0$$

 $\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right)^2 - \left[\frac{9}{4}x^2\right]$ 

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$(v^2 - 1v + v)^2 - [(2v - 35)v^2 + (v + 12)v + (v^2 - 24)] = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + \left(y^2 - 24\right)\right] = 0$$

 $\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right)^2 - \left[\frac{9}{4}x^2 + \frac{15}{2}x\right]$ 

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 $\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right)^2 - \left[\frac{9}{4}x^2 + \frac{15}{2}x + \frac{25}{4}\right] \qquad y = \frac{11}{2}$ 

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$(2 1 1)^2 [(2 35) 2 ( 12) ( 24)] 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + \left(-y + 13\right)x + \left(y^2 - 24\right)\right] = 0$$

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$$\left(x^{2} - \frac{1}{2}x + \frac{11}{2}\right)^{2} - \left[\frac{9}{4}x^{2} + \frac{15}{2}x + \frac{25}{4}\right] = 0$$

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 $\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right)^2 -$ 

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$$\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right)^2 - \left[\frac{9}{4}x^2 + \frac{15}{2}x + \frac{25}{4}\right] = 0$$

$$\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right)^2 - \left(\frac{3}{4}x + \frac{5}{4}\right)^2$$

$$\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right)^2 - \left(\frac{3}{2}x + \frac{5}{2}\right)^2$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + \left(-y + 13\right)x + \left(y^2 - 24\right)\right] = 0$$

$$\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right)^2 - \left[\frac{9}{4}x^2 + \frac{15}{2}x + \frac{25}{4}\right] = 0$$
$$\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right)^2 - \left(\frac{3}{4}x + \frac{5}{2}\right)^2 = 0$$

$$\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right)^2 - \left(\frac{3}{2}x + \frac{5}{2}\right)^2 = 0$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\left(x^{2} - \frac{1}{2}x + y\right)^{2} - \left[\left(2y - \frac{35}{4}\right)x^{2} + (-y + 13)x + \left(y^{2} - 24\right)\right] = 0$$

$$\left(x^{2} - \frac{1}{2}x + \frac{11}{2}\right)^{2} - \left[\frac{9}{4}x^{2} + \frac{15}{2}x + \frac{25}{4}\right] = 0$$

$$\left(x^{2} - \frac{1}{2}x + \frac{11}{2}\right)^{2} - \left(\frac{3}{2}x + \frac{5}{2}\right)^{2} = 0$$

$$a^2 - b^2 = (a + b)(a - b)$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\left(x^{2} - \frac{1}{2}x + y\right)^{2} - \left[\left(2y - \frac{35}{4}\right)x^{2} + (-y + 13)x + \left(y^{2} - 24\right)\right] = 0$$

$$\left(x^{2} - \frac{1}{2}x + \frac{11}{2}\right)^{2} - \left[\frac{9}{4}x^{2} + \frac{15}{2}x + \frac{25}{4}\right] = 0$$

$$y = \frac{11}{2}$$

$$\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right)^2 - \left(\frac{3}{2}x + \frac{5}{2}\right)^2 = 0$$

$$\left[\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right)\right]$$

$$a^2 - b^2 = (a + b)(a - b)$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

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$$\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right)^2 - \left(\frac{3}{2}x + \frac{5}{2}\right)^2 = 0$$

$$\left[\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right) + \right]$$

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$$\left[ \left( x^2 - \frac{1}{2}x + \frac{11}{2} \right) + \left( \frac{3}{2}x + \frac{5}{2} \right) \right]$$

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$$y = \frac{11}{2}$$

$$\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right)^2 - \left(\frac{3}{2}x + \frac{5}{2}\right)^2 = 0$$
$$\left[\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right) + \left(\frac{3}{2}x + \frac{5}{2}\right)\right] \left[\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right)\right]$$

$$a^2 - b^2 = (a + b)(a - b)$$

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$$\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right)^2 - \left[\frac{9}{4}x^2 + \frac{15}{2}x + \frac{25}{4}\right] = 0$$

$$y = \frac{11}{2}$$

$$\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right)^2 - \left(\frac{3}{2}x + \frac{5}{2}\right)^2 = 0$$
$$\left[\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right) + \left(\frac{3}{2}x + \frac{5}{2}\right)\right] \left[\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right) - \frac{1}{2}x + \frac{11}{2}\right]$$

$$a^2 - b^2 = (a + b)(a - b)$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + \left(y^2 - 24\right)\right] = 0$$

$$\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right)^2 - \left[\frac{9}{4}x^2 + \frac{15}{2}x + \frac{25}{4}\right] = 0$$

$$y = \frac{11}{2}$$

$$\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right)^2 - \left(\frac{3}{2}x + \frac{5}{2}\right)^2 = 0$$

$$\left[\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right) + \left(\frac{3}{2}x + \frac{5}{2}\right)\right] \left[\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right) - \left(\frac{3}{2}x + \frac{5}{2}\right)\right]$$

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$$y = \frac{11}{2}$$

$$\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right)^2 - \left(\frac{3}{2}x + \frac{5}{2}\right)^2 = 0$$

$$\left( x - \frac{1}{2}x + \frac{1}{2} \right) - \left( \frac{1}{2}x + \frac{1}{2} \right) = 0$$

$$\left[ \left( x^2 - \frac{1}{2}x + \frac{11}{2} \right) + \left( \frac{3}{2}x + \frac{5}{2} \right) \right] \left[ \left( x^2 - \frac{1}{2}x + \frac{11}{2} \right) - \left( \frac{3}{2}x + \frac{5}{2} \right) \right] = 0$$

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$$(x^{2} - \frac{1}{2}x + y) - \left[ (2y - \frac{35}{4})x^{2} + (-y + 13)x + (y^{2} - 24) \right] = 0$$

$$(x^{2} - \frac{1}{2}x + \frac{11}{2})^{2} - \left[ \frac{9}{4}x^{2} + \frac{15}{2}x + \frac{25}{4} \right] = 0$$

$$y = \frac{11}{2}$$

$$\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right)^2 - \left(\frac{3}{2}x + \frac{5}{2}\right)^2 = 0$$

$$\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right) - \left(\frac{3}{2}x + \frac{5}{2}\right) = 0$$

$$\left[\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right) + \left(\frac{3}{2}x + \frac{5}{2}\right)\right] \left[\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right) - \left(\frac{3}{2}x + \frac{5}{2}\right)\right] = 0$$

$$(x^2 + x + 8)$$

$$a^2 - b^2 = (a + b)(a - b)$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + \left(-y + 13\right)x + \left(y^2 - 24\right)\right] = 0$$

$$\left(x^{2} - \frac{1}{2}x + \frac{11}{2}\right)^{2} - \left[\frac{9}{4}x^{2} + \frac{15}{2}x + \frac{25}{4}\right] = 0$$

$$\left(x^{2} - \frac{1}{2}x + \frac{11}{2}\right)^{2} - \left(\frac{3}{4}x + \frac{5}{4}\right)^{2} = 0$$

$$\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right)^2 - \left(\frac{3}{2}x + \frac{5}{2}\right)^2 = 0$$

$$\left[\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right) + \left(\frac{3}{2}x + \frac{5}{2}\right)\right] \left[\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right) - \left(\frac{3}{2}x + \frac{5}{2}\right)\right] = 0$$

$$(x^2 + x + 8)(x^2 - 2x + 3)$$

$$a^2 - b^2 = (a + b)(a - b)$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\left(x^2 - \frac{1}{2}x + y\right)^2 - \left[\left(2y - \frac{35}{4}\right)x^2 + (-y + 13)x + \left(y^2 - 24\right)\right] = 0$$

$$\left(x^{2} - \frac{1}{2}x + \frac{11}{2}\right)^{2} - \left[\frac{9}{4}x^{2} + \frac{15}{2}x + \frac{25}{4}\right] = 0$$

$$y = \frac{11}{2}$$

$$\left(x^2 - \frac{1}{2}x + \frac{11}{2}\right)^2 - \left(\frac{3}{2}x + \frac{5}{2}\right)^2 = 0$$

$$\left[ \left( x^2 - \frac{1}{2}x + \frac{11}{2} \right) + \left( \frac{3}{2}x + \frac{5}{2} \right) \right] \left[ \left( x^2 - \frac{1}{2}x + \frac{11}{2} \right) - \left( \frac{3}{2}x + \frac{5}{2} \right) \right] = 0$$

$$\left( x^2 + x + 8 \right) \left( x^2 - 2x + 3 \right) = 0$$

$$a^2 - b^2 = (a + b)(a - b)$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$(x^2 + x + 8)(x^2 - 2x + 3) = 0$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$(x^{2} + x + 8)(x^{2} - 2x + 3) = 0$$

$$x^{2} + x + 8 = 0$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$(x^{2} + x + 8)(x^{2} - 2x + 3) = 0$$

$$x^{2} + x + 8 = 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1 - 32}}{2}$$

$$\sqrt{-31} = \sqrt{31}i$$

$$\sqrt{31}i \qquad \qquad x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$(x^{2} + x + 8)(x^{2} - 2x + 3) = 0$$

$$x^{2} + x + 8 = 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1 - 32}}{2}$$

$$x_{1} = -\frac{1}{2} + \frac{\sqrt{31}}{2}i$$

$$\sqrt{-31} = \sqrt{31}i$$

$$x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$(x^{2} + x + 8)(x^{2} - 2x + 3) = 0$$

$$x^{2} + x + 8 = 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1 - 32}}{2}$$

$$x_1 = -\frac{1}{2} + \frac{\sqrt{31}}{2}i$$

$$x_2 = -\frac{1}{2} - \frac{\sqrt{31}}{2}i$$

$$\sqrt{-31} = \sqrt{31}i$$

$$\sqrt{31}i \qquad \qquad x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$(x^{2} + x + 8)(x^{2} - 2x + 3) = 0$$

$$x^{2} + x + 8 = 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1 - 32}}{2}$$

$$x_{1} = -\frac{1}{2} + \frac{\sqrt{31}}{2}i$$

$$x_{2} = -\frac{1}{2} - \frac{\sqrt{31}}{2}i$$

$$\sqrt{-31} = \sqrt{31}i$$

$$\sqrt{31}i \qquad \qquad x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$(x^{2} + x + 8)(x^{2} - 2x + 3) = 0$$

$$x^{2} + x + 8 = 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1 - 32}}{2}$$

$$x_{1,2} = \frac{-1 \pm \sqrt{31}}{2}i$$

$$x_{2} = -\frac{1}{2} - \frac{\sqrt{31}}{2}i$$

$$x_{2} = -\frac{1}{2} - \frac{\sqrt{31}}{2}i$$

$$\sqrt{-31} = \sqrt{31}i \qquad x^4 - x^3 + 9x^2 - 13x + 24 = 0$$

$$\sqrt{-8} = \sqrt{8}i = 2\sqrt{2}i$$

$$(x^2 + x + 8)(x^2 - 2x + 3) = 0$$

$$x^2 + x + 8 = 0 \qquad x^2 - 2x + 3 = 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1 - 32}}{2} \qquad x_{3,4} = \frac{2 \pm \sqrt{4 - 12}}{2}$$

$$x_1 = -\frac{1}{2} + \frac{\sqrt{31}}{2}i \qquad x_{3,4} = \frac{2 \pm 2\sqrt{2}i}{2}$$

$$x_2 = -\frac{1}{2} - \frac{\sqrt{31}}{2}i$$

$$\sqrt{-31} = \sqrt{31}i$$

$$x^{4} - x^{3} + 9x^{2} - 13x + 24 = 0$$

$$\sqrt{-8} = \sqrt{8}i = 2\sqrt{2}i$$

$$(x^{2} + x + 8)(x^{2} - 2x + 3) = 0$$

$$x^{2} + x + 8 = 0$$

$$x^{2} - 2x + 3 = 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1 - 32}}{2}$$

$$x_{3,4} = \frac{2 \pm \sqrt{4 - 12}}{2}$$

$$x_{1} = -\frac{1}{2} + \frac{\sqrt{31}}{2}i$$

$$x_{3,4} = \frac{2 \pm 2\sqrt{2}i}{2}$$

 $x_2 = -\frac{1}{2} - \frac{\sqrt{31}}{2}i$ 

 $x_3 = 1 + \sqrt{2}i$ 

$$\sqrt{-31} = \sqrt{31}i$$

$$\sqrt{-8} = \sqrt{8}i = 2\sqrt{2}i$$

$$(x^2 + x + 8)(x^2 - 2x + 3) = 0$$

$$x^{2} + x + 8 = 0$$

$$x^{2} - 2x + 3 = 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1 - 32}}{2}$$

$$x_{3,4} = \frac{2 \pm \sqrt{4 - 12}}{2}$$

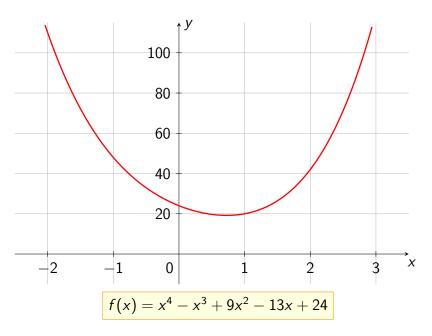
$$x_{1} = -\frac{1}{2} + \frac{\sqrt{31}}{2}i$$

$$x_{2} = -\frac{1}{2} - \frac{\sqrt{31}}{2}i$$

$$x_{3,4} = \frac{2 \pm 2\sqrt{2}i}{2}$$

$$x_{3} = 1 + \sqrt{2}i$$

$$x_{4} = 1 - \sqrt{2}i$$



## \_\_\_\_

drugi zadatak

## Zadatak 2

Zadana je jednadžba  $x^3 + 6x - 2 = 0$ .

- a) Bez direktnog rješavanja jednadžbe komentirajte koliko ima realnih, a koliko pravih kompleksnih rješenja.
- b) Pomoću Cardanove formule riješite zadanu jednadžbu.

a)

a)

 $x^3 + 6x - 2 = 0$ 

 $x^3 + px + q = 0$ 

a)

 $x^3 + px + q = 0$ 

 $x^3 + 6x - 2 = 0$ 

$$x^3 + px + q = 0$$

$$\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$$

$$x^3 + px + q = 0$$

 $\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$ 

$$\Delta =$$

 $x^3 + 6x - 2 = 0$ 

$$x^3 + px + q = 0$$

$$\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$$

$$\Delta = \left(\frac{-2}{2}\right)^2$$

$$x^3 + px + q = 0$$

$$\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$$

$$\Delta = \left(\frac{-2}{2}\right)^2 +$$

,

$$x^3 + px + q = 0$$

$$\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$$

$$\Delta = \left(\frac{-2}{2}\right)^2 + \left(\frac{6}{3}\right)^3$$

 $x^3 + 6x - 2 = 0$ 

 $x^3 + px + q = 0$ 

$$\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$$
$$\Delta = \left(\frac{-2}{2}\right)^2 + \left(\frac{6}{3}\right)^3$$

$$\Delta =$$

 $x^3 + 6x - 2 = 0$ 

$$x^3 + px + q = 0$$

$$\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$$

$$\Delta = \left(\frac{-2}{2}\right)^2 + \left(\frac{6}{3}\right)^3$$

$$\Delta = 1$$

$$\Delta = 1$$

$$x^3 + px + q = 0$$

$$\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$$

$$\Delta = \left(\frac{-2}{2}\right)^2 + \left(\frac{6}{3}\right)^3$$

$$\Delta = 1 + 8$$

,

 $x^3 + px + q = 0$ 

$$\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$$

$$\Delta = \left(\frac{-2}{2}\right)^2 + \left(\frac{6}{3}\right)^3$$

$$\Delta = 1 + 8 = 9$$

 $x^3 + 6x - 2 = 0$ 

$$x^3 + px + q = 0$$

$$\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$$
$$\Delta = \left(\frac{-2}{2}\right)^2 + \left(\frac{6}{3}\right)^3$$

$$\Delta = \left(\frac{2}{2}\right) + \left(\frac{3}{3}\right)$$

$$\Delta = 1 + 8 = 9$$

$$\Delta > 0$$

a)

$$x^3 + px + q = 0$$

$$\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$$

$$\Delta = \left(\frac{-2}{2}\right)^2 + \left(\frac{6}{3}\right)^3$$

$$\Delta = 1 + 8 = 9$$

 $\Delta>0$  ---------- jednadžba ima jedno realno i dva konjugirano kompleksna rješenja

 $x^3 + 6x - 2 = 0$  $p = 6, \quad q = -2$ 

9/21

b)

 $x^3 + 6x - 2 = 0$  $p = 6, \quad q = -2$ 

 $x^3 + 6x - 2 = 0$ p = 6, q = -2

9/21

b)  $u_0 = \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}$ 

 $u_0 =$ 

 $u_0 = \sqrt[3]{}$ 

 $x^3 + 6x - 2 = 0$ p = 6, q = -2

 $u_0 = \sqrt[3]{-\frac{-2}{2}}$ 

 $x^3 + 6x - 2 = 0$ p = 6, q = -2

 $u_0 = \sqrt[3]{-\frac{-2}{2}} +$ 

 $x^3 + 6x - 2 = 0$ p = 6, q = -2

 $x^3 + 6x - 2 = 0$ p = 6, q = -2

 $\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$ 

b)  $u_0 = \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}$ 

 $u_0 = \sqrt[3]{-\frac{-2}{2}} +$ 

 $u_0 = \sqrt[3]{-\frac{-2}{2}} +$ 

 $x^3 + 6x - 2 = 0$ p = 6, q = -2

 $\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$ 

 $\Delta = 9$ 

 $u_0 = \sqrt[3]{-\frac{-2}{2} + \sqrt{9}}$ 

 $x^3 + 6x - 2 = 0$ p = 6, q = -2

 $\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$ 

 $\Delta = 9$ 

$$u_0 =$$

p = 6, q = -2

 $\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$ 

 $\Delta = 9$ 

9/21

b)  $u_0 = \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}$ 

 $u_0 = \sqrt[3]{-\frac{-2}{2} + \sqrt{9}}$ 

$$u_0 = \sqrt[3]{1+3}$$

$$\Delta = 9$$

 $u_0 = \sqrt[3]{-\frac{-2}{2} + \sqrt{9}}$ 

 $x^3 + 6x - 2 = 0$ 

p = 6, q = -2

 $\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$ 

$$u_0 = \sqrt[3]{1+3}$$
  $u_0 = \sqrt[3]{4}$ 

 $u_0 = \sqrt[3]{-\frac{-2}{2} + \sqrt{9}}$ 

 $x^3 + 6x - 2 = 0$ p = 6, q = -2

 $\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$ 

 $\Delta = 9$ 

$$u_0 = \sqrt[3]{1+3} \qquad u_0 = \sqrt[3]{4}$$

p = 6, q = -2

 $\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$ 

 $\Delta = 9$ 

b)  $u_0 = \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}$ 

 $u_0 = \sqrt[3]{-\frac{-2}{2} + \sqrt{9}}$ 

$$u_0 = \sqrt[3]{-\frac{2}{2} + \sqrt{9}}$$

$$u_0 = \sqrt[3]{1+3}$$

$$u_0 = \sqrt[3]{4}$$

$$u_0 = \sqrt[3]{4}$$

$$\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$$

$$\Delta = 9$$

 $u_0 = \sqrt[3]{-\frac{-2}{2}} + \sqrt{9}$ 

 $u_0 = \sqrt[3]{1+3}$   $u_0 = \sqrt[3]{4}$ 

 $x^3 + 6x - 2 = 0$  $p = 6, \quad q = -2$ 

 $v_0 =$ 

 $x^3 + 6x - 2 = 0$ 

 $\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$ 

b)  $u_0 = \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}$ 

 $u_0 = \sqrt[3]{-\frac{-2}{2}+\sqrt{9}}$ 

 $u_0 = \sqrt[3]{1+3}$   $u_0 = \sqrt[3]{4}$ 

 $x^3 + 6x - 2 = 0$  $p = 6, \quad q = -2$ 

 $\Delta = 9$ 

 $v_0 = -\frac{6}{3\sqrt[3]{4}} \quad \Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$ 

b)  $u_0 = \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}$ 

 $u_0 = \sqrt[3]{-\frac{-2}{2}+\sqrt{9}}$ 

 $u_0 = \sqrt[3]{1+3}$   $u_0 = \sqrt[3]{4}$ 

 $x^3 + 6x - 2 = 0$ p = 6, q = -2

 $\Delta = \left(rac{q}{2}
ight)^2 + \left(rac{p}{3}
ight)^3$ 

 $\Delta = 9$ 

b)  $u_0 = \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}$ 

 $u_0 = \sqrt[3]{-\frac{-2}{2} + \sqrt{9}}$ 

 $u_0 = \sqrt[3]{1+3}$ 

 $u_0 = \sqrt[3]{4}$ 

 $u_0 = \sqrt[3]{-\frac{-2}{2} + \sqrt{9}}$ 

 $u_0 = \sqrt[3]{1+3}$ 

 $x^3 + 6x - 2 = 0$ 

 $\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$ 

 $\Delta = 9$ 

$$x_1 = u_0 + v_0$$

 $u_0 = \sqrt[3]{4}$ 

 $u_0 = \sqrt[3]{-\frac{-2}{2} + \sqrt{9}}$ 

 $u_0 = \sqrt[3]{1+3}$ 

 $x^3 + 6x - 2 = 0$ 

 $\Delta = \left(rac{q}{2}
ight)^2 + \left(rac{p}{3}
ight)^3$ 

 $\Delta = 9$ 

$$x_1 = u_0 + v_0 = \sqrt[3]{4} - \frac{2}{\sqrt[3]{4}}$$

 $u_0 = \sqrt[3]{4}$ 

 $u_0 = \sqrt[3]{-\frac{-2}{2} + \sqrt{9}}$ 

 $u_0 = \sqrt[3]{1+3}$ 

 $x^3 + 6x - 2 = 0$ 

 $\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$ 

 $\Delta = 9$ 

$$u_0 = \sqrt[3]{1+3}$$
  $u_0 = \sqrt[3]{4}$   $v_0 = -\frac{3}{\sqrt[3]{4}}$   $v_1 = u_0 + v_0 = \sqrt[3]{4} - \frac{2}{\sqrt[3]{4}}$   $v_2 = 0.32748$ 

 $u_0 = \sqrt[3]{-\frac{-2}{2}} + \sqrt{9}$ 

 $x^3 + 6x - 2 = 0$ 

 $\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$ 

 $\Delta = 9$ 

$$u_0 = \sqrt[3]{1+3}$$
  $u_0 = \sqrt[3]{4}$   $v_0 = -\frac{3}{\sqrt[3]{4}}$   $v_1 = u_0 + v_0 = \sqrt[3]{4} - \frac{2}{\sqrt[3]{4}}$   $v_2 = u_0 \varepsilon + v_0 \bar{\varepsilon}$ 

 $u_0 = \sqrt[3]{-\frac{-2}{2} + \sqrt{9}}$ 

 $x^3 + 6x - 2 = 0$ 

 $\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$ 

 $\Delta = 9$ 

$$u_{0} = \sqrt[3]{1+3} \qquad u_{0} = \sqrt[3]{4}$$

$$v_{0} = -\frac{1}{\sqrt[3]{4}}$$

$$\varepsilon = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$x_{1} = u_{0} + v_{0} = \sqrt[3]{4} - \frac{2}{\sqrt[3]{4}}$$

$$x_{2} = u_{0}\varepsilon + v_{0}\bar{\varepsilon}$$

 $u_0 = \sqrt[3]{-\frac{-2}{2} + \sqrt{9}}$ 

 $x^3 + 6x - 2 = 0$ 

 $\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$ 

$$x_1 = u_0 + v_0 = \sqrt[3]{4} - \frac{2}{\sqrt[3]{4}}$$
  $x_1 \approx 0.32748$   
 $x_2 = u_0 \varepsilon + v_0 \bar{\varepsilon} = \sqrt[3]{4} \left( -\frac{1}{2} - \frac{\sqrt{3}}{2} i \right)$ 

 $u_0 = \sqrt[3]{4}$ 

 $u_0 = \sqrt[3]{-\frac{-2}{2} + \sqrt{9}}$ 

 $u_0 = \sqrt[3]{1+3}$ 

 $x^3 + 6x - 2 = 0$ 

 $\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$ 

 $\varepsilon = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$ 

$$x_{1} = u_{0} + v_{0} = \sqrt[3]{4} - \frac{2}{\sqrt[3]{4}}$$

$$x_{1} \approx 0.32748$$

$$x_{2} = u_{0}\varepsilon + v_{0}\bar{\varepsilon} = \sqrt[3]{4} \left( -\frac{1}{2} - \frac{\sqrt{3}}{2}i \right) - \frac{2}{\sqrt[3]{4}} \left( -\frac{1}{2} + \frac{\sqrt{3}}{2}i \right)$$

 $u_0 = \sqrt[3]{4}$ 

 $u_0 = \sqrt[3]{-\frac{-2}{2} + \sqrt{9}}$ 

 $u_0 = \sqrt[3]{1+3}$ 

 $x^3 + 6x - 2 = 0$ 

 $\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$ 

 $\varepsilon = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$ 

$$u_{0} = \sqrt[3]{-\frac{2}{2} + \sqrt{9}}$$

$$u_{0} = \sqrt[3]{1+3}$$

$$u_{0} = \sqrt[3]{4}$$

$$v_{0} = -\frac{3}{3\sqrt[3]{4}}$$

$$v_{0} = -\frac{2}{3\sqrt[3]{4}}$$

 $x_2 =$ 

 $x^3 + 6x - 2 = 0$ 

$$u_{0} = \sqrt[3]{-\frac{2}{2} + \sqrt{9}}$$

$$u_{0} = \sqrt[3]{1+3}$$

$$u_{0} = \sqrt[3]{4}$$

$$v_{0} = -\frac{3}{3}\sqrt[3]{4}$$

$$v_{0} = -\frac{2}{3}\sqrt[3]{4}$$

$$v_{0} = -\frac{2}\sqrt[3]{4}$$

$$v_{0} = -\frac{2}{3}\sqrt[3]{4}$$

$$v_{0} = -\frac{2}{3}\sqrt[3]{4}$$

$$v$$

 $x^3 + 6x - 2 = 0$ 

$$\begin{aligned} u_0 &= \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} \\ u_0 &= \sqrt[3]{-\frac{2}{2} + \sqrt{9}} \\ u_0 &= \sqrt[3]{1+3} \end{aligned} \qquad \begin{aligned} v_0 &= -\frac{i}{3u_0} \\ v_0 &= -\frac{6}{3\sqrt[3]{4}} \\ v_0 &= -\frac{2}{\sqrt[3]{4}} \end{aligned} \qquad \begin{aligned} \lambda &= \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3 \\ \lambda &= 9 \\ \varepsilon &= -\frac{1}{2} - \frac{\sqrt{3}}{2}i \end{aligned}$$

$$x_1 &= u_0 + v_0 = \sqrt[3]{4} - \frac{2}{\sqrt[3]{4}} \qquad x_1 \approx 0.32748$$

$$x_2 &= u_0 \varepsilon + v_0 \bar{\varepsilon} = \sqrt[3]{4} \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) - \frac{2}{\sqrt[3]{4}} \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \end{aligned}$$

$$x_2 &= \left(\frac{1}{\sqrt[3]{4}} - \frac{1}{2}\sqrt[3]{4}\right) - \left(\frac{\sqrt{3} \cdot \sqrt[3]{4}}{2} + \frac{\sqrt{3}}{\sqrt[3]{4}}\right)i \end{aligned}$$

 $v_0 = -\frac{\rho}{3u_0}$ 

 $x^3 + 6x - 2 = 0$ 

p = 6, q = -2

b)
$$u_{0} = \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^{2} + \left(\frac{p}{3}\right)^{3}}}$$

$$v_{0} = -\frac{p}{3u_{0}}$$

$$v_{0} = \frac{p}{3u_{0}}$$

$$v_{0} = -\frac{6}{3\sqrt[3]{4}}$$

$$v_{0} = -\frac{6}{3\sqrt[3]{4}}$$

$$v_{0} = -\frac{6}{3\sqrt[3]{4}}$$

$$v_{0} = -\frac{2}{\sqrt[3]{4}}$$

$$v$$

$$x_3 = u_0 \bar{\varepsilon} + v_0 \varepsilon$$

 $x^3 + 6x - 2 = 0$ 

 $\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$ 

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b)  $u_0 = \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}$ 

 $u_0 = \sqrt[3]{-\frac{-2}{2} + \sqrt{9}}$ 

 $u_0 = \sqrt[3]{1+3}$ 

$$x_3 = u_0\bar{\varepsilon} + v_0\varepsilon = \sqrt[3]{4}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

 $x^3 + 6x - 2 = 0$ 

 $\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$ 

10/21

b)  $u_0 = \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}$ 

 $u_0 = \sqrt[3]{4}$ 

 $u_0 = \sqrt[3]{-\frac{-2}{2} + \sqrt{9}}$ 

 $u_0 = \sqrt[3]{1+3}$ 

$$x_3 = u_0\bar{\varepsilon} + v_0\varepsilon = \sqrt[3]{4} \left( -\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) - \frac{2}{\sqrt[3]{4}} \left( -\frac{1}{2} - \frac{\sqrt{3}}{2}i \right)$$

 $u_0 = \sqrt[3]{4}$ 

 $u_0 = \sqrt[3]{-rac{-2}{2} + \sqrt{9}}$ 

 $u_0 = \sqrt[3]{1+3}$ 

 $x^3 + 6x - 2 = 0$ 

 $\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$ 

$$x_3 = u_0\bar{\varepsilon} + v_0\varepsilon = \sqrt[3]{4}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) - \frac{2}{\sqrt[3]{4}}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$$

$$x_3 =$$

 $x^3 + 6x - 2 = 0$ 

 $\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$ 

10/21

b)  $u_0 = \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}$ 

 $u_0 = \sqrt[3]{4}$ 

 $u_0 = \sqrt[3]{-rac{-2}{2} + \sqrt{9}}$ 

 $u_0 = \sqrt[3]{1+3}$ 

$$x_{3} = u_{0}\bar{\varepsilon} + v_{0}\varepsilon = \sqrt[3]{4} \left( -\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) - \frac{2}{\sqrt[3]{4}} \left( -\frac{1}{2} - \frac{\sqrt{3}}{2}i \right)$$

$$x_{3} = \left( \frac{1}{\sqrt[3]{4}} - \frac{1}{2}\sqrt[3]{4} \right)$$

 $u_0 = \sqrt[3]{4}$ 

 $u_0 = \sqrt[3]{-\frac{-2}{2} + \sqrt{9}}$ 

 $u_0 = \sqrt[3]{1+3}$ 

 $x^3 + 6x - 2 = 0$ 

 $\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$ 

$$x_{3} = u_{0}\bar{\varepsilon} + v_{0}\varepsilon = \sqrt[3]{4} \left( -\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) - \frac{2}{\sqrt[3]{4}} \left( -\frac{1}{2} - \frac{\sqrt{3}}{2}i \right)$$

$$x_{3} = \left( \frac{1}{\sqrt[3]{4}} - \frac{1}{2}\sqrt[3]{4} \right) + \left( \frac{\sqrt{3} \cdot \sqrt[3]{4}}{2} + \frac{\sqrt{3}}{\sqrt[3]{4}} \right)i$$

 $v_0 = -\frac{2}{\sqrt[3]{4}}$ 

 $u_0 = \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}$ 

 $u_0 = \sqrt[3]{1+3}$   $u_0 = \sqrt[3]{4}$ 

 $u_0 = \sqrt[3]{-rac{-2}{2} + \sqrt{9}}$ 

 $x^3 + 6x - 2 = 0$ 

 $\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$ 

$$u_{0} = \sqrt[3]{-\frac{2}{2} + \sqrt{9}}$$

$$u_{0} = \sqrt[3]{1+3}$$

$$u_{0} = \sqrt[3]{4}$$

$$v_{0} = -\frac{2}{3\sqrt[3]{4}}$$

$$v_{0} = -\frac{2}{3\sqrt[3]{4}}$$

$$v_{0} = -\frac{2}{3\sqrt[3]{4}}$$

$$\Delta = 9$$

$$\varepsilon = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

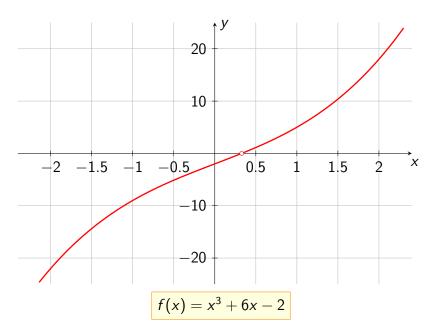
$$x_{3} = u_{0}\bar{\varepsilon} + v_{0}\varepsilon = \sqrt[3]{4} \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) - \frac{2}{\sqrt[3]{4}} \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$$

$$x_{3} = \left(\frac{1}{\sqrt[3]{4}} - \frac{1}{2}\sqrt[3]{4}\right) + \left(\frac{\sqrt{3} \cdot \sqrt[3]{4}}{2} + \frac{\sqrt{3}}{\sqrt[3]{4}}\right)i$$

$$x_{3} \approx -0.16374 + 2.46585i$$

$$10/21$$

 $x^3 + 6x - 2 = 0$ 



# treći zadatak

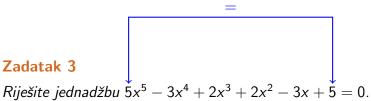
#### Zadatak 3

Riješite jednadžbu  $5x^5 - 3x^4 + 2x^3 + 2x^2 - 3x + 5 = 0$ .

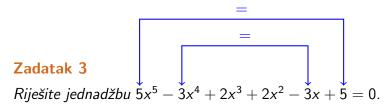
#### Zadatak 3

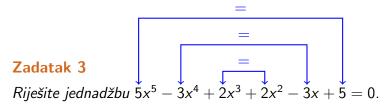
Riješite jednadžbu  $5x^5 - 3x^4 + 2x^3 + 2x^2 - 3x + 5 = 0$ .

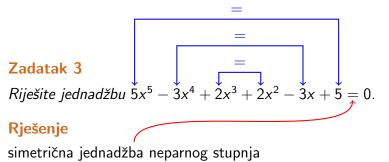
#### Rješenje



Zadatak 3



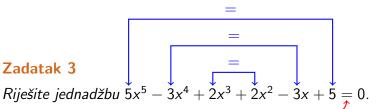




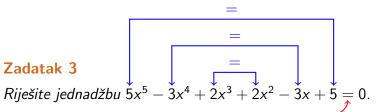
# Riješite jednadžbu $5x^5 - 3x^4 +$

#### Rješenje

Zadatak 3

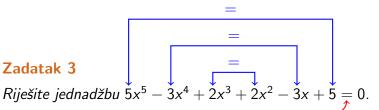


Zadatak 3



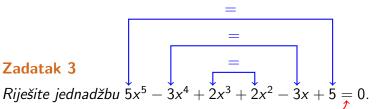
simetrična jednadžba neparnog stupnja extstyle extst





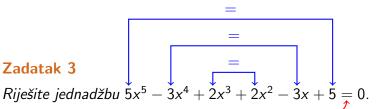
Zadatak 3

	5			
-				



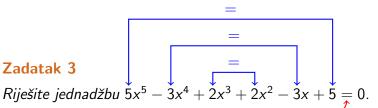
Zadatak 3

	5	-3		
-				



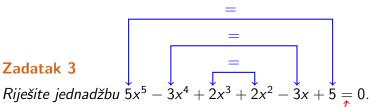
Zadatak 3

	5	-3	2		



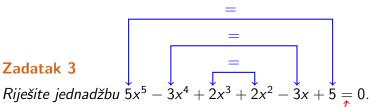
Zadatak 3

	5	-3	2	2	



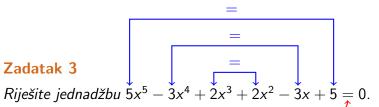
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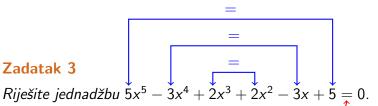
	5	-3	2	2	-3	



Zadatak 3

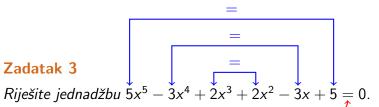
5	-3	2	2	-3	5

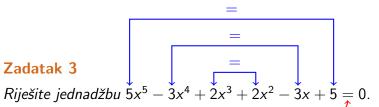


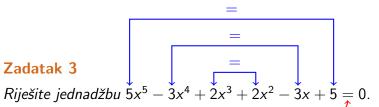


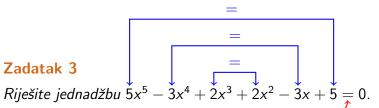
Zadatak 3

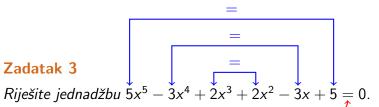
	5	-3	2	2	-3	5
-1	5					

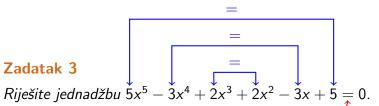




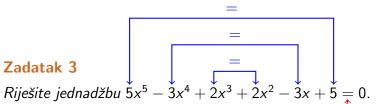




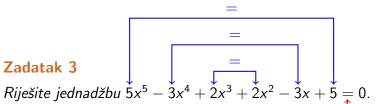




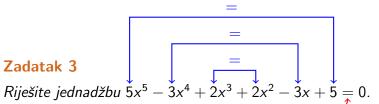
$$(x + 1)$$



$$(x+1)(5x^4-8x^3+10x^2-8x+5)$$



$$(x+1)(5x^4 - 8x^3 + 10x^2 - 8x + 5) = 0$$



$$(x+1)(5x^4 - 8x^3 + 10x^2 - 8x + 5) = 0$$
$$5x^4 - 8x^3 + 10x^2 - 8x + 5 = 0$$



#### Riešenie

$$(x+1)(5x^4 - 8x^3 + 10x^2 - 8x + 5) = 0$$

$$5x^4 - 8x^3 + 10x^2 - 8x + 5 = 0$$

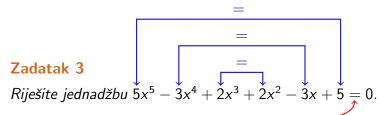


#### Riešenie

$$(x+1)(5x^4 - 8x^3 + 10x^2 - 8x + 5) = 0$$

$$5x^4 - 8x^3 + 10x^2 - 8x + 5 = 0$$

$$=$$



simetrična jednadžba neparnog stupnja extstyle extst

$$(x+1)(5x^4 - 8x^3 + 10x^2 - 8x + 5) = 0$$

$$5x^4 - 8x^3 + 10x^2 - 8x + 5 = 0$$

$$\Rightarrow parnog stupnja$$

$$5x^4 - 8x^3 + 10x^2 - 8x + 5 = 0$$

$$5x^4 - 8x^3 + 10x^2 - 8x + 5 = 0 / : x^2$$

$$5x^4 - 8x^3 + 10x^2 - 8x + 5 = 0 / : x^2$$
  
 $5x^2$ 

$$5x^4 - 8x^3 + 10x^2 - 8x + 5 = 0 / : x^2$$
  
 $5x^2 - 8x$ 

$$5x^4 - 8x^3 + 10x^2 - 8x + 5 = 0 / : x^2$$
  
 $5x^2 - 8x + 10$ 

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 / : x^{2}$$
$$5x^{2} - 8x + 10 - \frac{8}{x}$$

$$5x^4 - 8x^3 + 10x^2 - 8x + 5 = 0 / : x^2$$
$$5x^2 - 8x + 10 - \frac{8}{x} + \frac{5}{x^2}$$

$$5x^4 - 8x^3 + 10x^2 - 8x + 5 = 0 / : x^2$$
$$5x^2 - 8x + 10 - \frac{8}{x} + \frac{5}{x^2} = 0$$

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 / : x^{2}$$
$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$

$$5\left(x^2+\frac{1}{x^2}\right)$$

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$
$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$
$$5\left(x^{2} + \frac{1}{x^{2}}\right) - 8\left(x + \frac{1}{x}\right)$$

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$
$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$
$$5\left(x^{2} + \frac{1}{x^{2}}\right) - 8\left(x + \frac{1}{x}\right) + 10$$

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$
$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$
$$5\left(x^{2} + \frac{1}{x^{2}}\right) - 8\left(x + \frac{1}{x}\right) + 10 = 0$$

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$

$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$

$$5\left(x^{2} + \frac{1}{x^{2}}\right) - 8\left(x + \frac{1}{x}\right) + 10 = 0$$

$$x + \frac{1}{x} = t$$

$$= t$$

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$
$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$
$$5\left(x^{2} + \frac{1}{x^{2}}\right) - 8\left(x + \frac{1}{x}\right) + 10 = 0$$

$$x + \frac{1}{x} = t$$

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$

$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$

$$5\left(x^{2} + \frac{1}{x^{2}}\right) - 8\left(x + \frac{1}{x}\right) + 10 = 0$$

$$x + \frac{1}{x} = t$$

$$=t$$

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$

$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$

$$5\left(x^{2} + \frac{1}{x^{2}}\right) - 8\left(x + \frac{1}{x}\right) + 10 = 0$$

$$x + \frac{1}{x} = t$$

x

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$

$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$

$$5\left(x^{2} + \frac{1}{x^{2}}\right) - 8\left(x + \frac{1}{x}\right) + 10 = 0$$

$$x + \frac{1}{x} = t$$

$$=t$$

$$x^{2} + 2$$

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$

$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$

$$5\left(x^{2} + \frac{1}{x^{2}}\right) - 8\left(x + \frac{1}{x}\right) + 10 = 0$$

$$x + \frac{1}{x} = t$$

$$x + \frac{1}{x} = t$$

$$x^2 + 2 + \frac{1}{x^2}$$

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$

$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$

$$5\left(x^{2} + \frac{1}{x^{2}}\right) - 8\left(x + \frac{1}{x}\right) + 10 = 0$$

$$x + \frac{1}{x} = t$$

$$x^2 + 2 + \frac{1}{x^2} = t^2$$

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$

$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$

$$5\left(x^{2} + \frac{1}{x^{2}}\right) - 8\left(x + \frac{1}{x}\right) + 10 = 0$$

$$x + \frac{1}{x} = t$$

$$x^2 + 2 + \frac{1}{x^2} = t^2 \longrightarrow x^2 + \frac{1}{x^2} =$$

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$

$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$

$$5\left(x^{2} + \frac{1}{x^{2}}\right) - 8\left(x + \frac{1}{x}\right) + 10 = 0$$

$$x + \frac{1}{x} = t$$

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$

$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$

$$5\left(x^{2} + \frac{1}{x^{2}}\right) - 8\left(x + \frac{1}{x}\right) + 10 = 0$$

$$x + \frac{1}{x} = t$$

$$x + \frac{1}{x} = t$$

$$x^2 + 2 + \frac{1}{x^2} = t^2 \longrightarrow x^2 + \frac{1}{x^2} = t^2 - 2$$

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 / : x^{2}$$

$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$

$$5\left(x^{2} + \frac{1}{x^{2}}\right) - 8\left(x + \frac{1}{x}\right) + 10 = 0$$

$$\boxed{x + \frac{1}{x} = t} /^{2}$$

$$x^{2} + 2 + \frac{1}{x^{2}} = t^{2} \xrightarrow{} x^{2} + \frac{1}{x^{2}} = t^{2} - 2$$

 $5(t^2-2)$ 

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$

$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$

$$5\left(x^{2} + \frac{1}{x^{2}}\right) - 8\left(x + \frac{1}{x}\right) + 10 = 0$$

$$\boxed{x + \frac{1}{x} = t} /^{2}$$

$$x^{2} + 2 + \frac{1}{x^{2}} = t^{2} \xrightarrow{} x^{2} + \frac{1}{x^{2}} = t^{2} - 2$$

$$5(t^{2} - 2) - 8t$$

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$

$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$

$$5\left(x^{2} + \frac{1}{x^{2}}\right) - 8\left(x + \frac{1}{x}\right) + 10 = 0$$

$$\boxed{x + \frac{1}{x} = t} /^{2}$$

$$x^{2} + 2 + \frac{1}{x^{2}} = t^{2} \xrightarrow{} x^{2} + \frac{1}{x^{2}} = t^{2} - 2$$

$$5(t^{2} - 2) - 8t + 10$$

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$

$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$

$$5\left(x^{2} + \frac{1}{x^{2}}\right) - 8\left(x + \frac{1}{x}\right) + 10 = 0$$

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$$x^{2} + 2 + \frac{1}{x^{2}} = t^{2} \xrightarrow{} x^{2} + \frac{1}{x^{2}} = t^{2} - 2$$

$$5(t^{2} - 2) - 8t + 10 = 0$$

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$

$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$

$$5\left(x^{2} + \frac{1}{x^{2}}\right) - 8\left(x + \frac{1}{x}\right) + 10 = 0$$

$$x + \frac{1}{x} = t /^{2}$$

$$x^{2} + 2 + \frac{1}{x^{2}} = t^{2} \xrightarrow{} x^{2} + \frac{1}{x^{2}} = t^{2} - 2$$

$$5(t^{2} - 2) - 8t + 10 = 0$$

$$5t^{2} - 8t = 0$$

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$

$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$

$$5\left(x^{2} + \frac{1}{x^{2}}\right) - 8\left(x + \frac{1}{x}\right) + 10 = 0$$

$$x + \frac{1}{x} = t /^{2}$$

$$x^{2} + 2 + \frac{1}{x^{2}} = t^{2} \xrightarrow{} x^{2} + \frac{1}{x^{2}} = t^{2} - 2$$

$$5(t^{2} - 2) - 8t + 10 = 0$$

$$5t^{2} - 8t = 0$$

$$t(5t - 8) = 0$$

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$

$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$

$$5\left(x^{2} + \frac{1}{x^{2}}\right) - 8\left(x + \frac{1}{x}\right) + 10 = 0$$

$$x + \frac{1}{x} = t /^{2}$$

$$x^{2} + 2 + \frac{1}{x^{2}} = t^{2} \xrightarrow{\text{cov}} x^{2} + \frac{1}{x^{2}} = t^{2} - 2$$

$$5(t^{2} - 2) - 8t + 10 = 0$$

$$5t^{2} - 8t = 0$$

$$t(5t - 8) = 0$$

$$t_{1} = 0,$$

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$

$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$

$$5\left(x^{2} + \frac{1}{x^{2}}\right) - 8\left(x + \frac{1}{x}\right) + 10 = 0$$

$$x + \frac{1}{x} = t /^{2}$$

$$x^{2} + 2 + \frac{1}{x^{2}} = t^{2} \xrightarrow{} x^{2} + \frac{1}{x^{2}} = t^{2} - 2$$

$$5(t^{2} - 2) - 8t + 10 = 0$$

$$5t^{2} - 8t = 0$$

$$t(5t - 8) = 0$$

$$t_{1} = 0, \quad t_{2} = \frac{8}{5}$$

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$

$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$

$$5\left(x^{2} + \frac{1}{x^{2}}\right) - 8\left(x + \frac{1}{x}\right) + 10 = 0$$

$$x + \frac{1}{x} = t /^{2}$$

$$x^{2} + 2 + \frac{1}{x^{2}} = t^{2} \xrightarrow{} x^{2} + \frac{1}{x^{2}} = t^{2} - 2$$

$$5(t^{2} - 2) - 8t + 10 = 0$$

$$5t^{2} - 8t = 0$$

$$t(5t - 8) = 0$$

$$t_{1} = 0, \quad t_{2} = \frac{8}{5}$$

$$x + \frac{1}{x} = t$$

$$x^{2} + 2 + \frac{1}{x^{2}} = t^{2} \xrightarrow{x^{2} + \frac{1}{x^{2}}} = t^{2} - 2$$

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$$t_{1} = 0, \quad t_{2} = \frac{8}{5}$$

 $5x^2 - 8x + 10 - \frac{8}{x} + \frac{5}{x^2} = 0$ 

 $5\left(x^2 + \frac{1}{x^2}\right) - 8\left(x + \frac{1}{x}\right) + 10 = 0$ 

 $x + \frac{1}{x} = 0$ 

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$$5\left(x^{2} + \frac{1}{x^{2}}\right) - 8\left(x + \frac{1}{x}\right) + 10 = 0$$

$$x + \frac{1}{x} = t$$

$$x^{2} + 2 + \frac{1}{x^{2}} = t^{2} \xrightarrow{x^{2}} x^{2} + \frac{1}{x^{2}} = t^{2} - 2$$

$$5(t^{2} - 2) - 8t + 10 = 0$$

$$5t^{2} - 8t = 0$$

$$t(5t - 8) = 0$$

 $5x^2 - 8x + 10 - \frac{8}{x} + \frac{5}{x^2} = 0$ 

 $t_1 = 0, \quad t_2 = \frac{8}{5}$ 

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 $x + \frac{1}{x} = 0 / \cdot x$ 

$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$

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 $x + \frac{1}{x} = 0 / \cdot x$ 

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 $x^2 + 1 = 0$ 

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$

$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$

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 $x^2 = -1$ 

$$5\left(x^{2} + \frac{1}{x^{2}}\right) - 8\left(x + \frac{1}{x}\right) + 10 = 0$$

$$x + \frac{1}{x} = t$$

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$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$

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$$t(5t - 8) = 0$$

$$t_{1} = 0, \quad t_{2} = \frac{8}{5}$$

 $x + \frac{1}{x} = 0 / \cdot x$ 

 $x^2 + 1 = 0$  $x^2 = -1$ 

 $x_2 = i, \quad x_3 = -i$ 

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$

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$$5t^{2} - 8t = 0$$

$$t(5t - 8) = 0$$

$$t_{1} = 0, \quad t_{2} = \frac{8}{5}$$

 $x + \frac{1}{x} = \frac{8}{5}$ 

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 $x + \frac{1}{x} = 0 / \cdot x$ 

 $x^2 + 1 = 0$  $x^2 = -1$ 

 $x_2=i, \quad x_3=-i$ 

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$

$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$

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$$t_{1} = 0, \quad t_{2} = \frac{8}{5}$$

 $x_2 = i, \quad x_3 = -i$  $x + \frac{1}{x} = \frac{8}{5} / \cdot 5x$ 

 $x + \frac{1}{x} = 0 / \cdot x$ 

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 $x + \frac{1}{x} = 0 / \cdot x$ 

 $x^2 + 1 = 0$  $x^2 = -1$ 

 $x_2 = i, \quad x_3 = -i$ 

 $5x^2$ 

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 / : x^{2}$$

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$$t(5t - 8) = 0$$

$$t_{1} = 0, \quad t_{2} = \frac{8}{5}$$

$$5x^2 - 8x$$

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 $x + \frac{1}{x} = 0 / \cdot x$ 

 $x^2 + 1 = 0$  $x^2 = -1$ 

 $x_2 = i, \quad x_3 = -i$ 

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 / : x^{2}$$

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 $5x^2 - 8x + 5$ 

 $x + \frac{1}{x} = 0 / \cdot x$ 

 $x^2 + 1 = 0$ 

 $x^2 = -1$ 

 $x_2 = i, \quad x_3 = -i$ 

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 / : x^{2}$$

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 $x + \frac{1}{x} = 0 / \cdot x$ 

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 $5x^2 - 8x + 5 = 0$  $x_{4,5} = \frac{8 \pm \sqrt{64 - 100}}{10}$ 

 $x + \frac{1}{x} = 0 / \cdot x$ 

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$$x_{4,5} = \frac{8 \pm 6i}{10}$$

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 $5x^2 - 8x + 10 - \frac{8}{x} + \frac{5}{x^2} = 0$ 

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 $5x^2 - 8x + 10 - \frac{8}{x} + \frac{5}{x^2} = 0$ 

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$$t_{1} = 0, \quad t_{2} = \frac{8}{5}$$

 $x_{2} = i, \quad x_{3} = -i$   $x + \frac{1}{x} = \frac{8}{5} / \cdot 5x$   $5x^{2} - 8x + 5 = 0$   $x_{4,5} = \frac{8 \pm \sqrt{64 - 100}}{10}$ 

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 $x + \frac{1}{x} = 0 / \cdot x$ 

 $x^2 + 1 = 0$  $x^2 = -1$ 

 $x_{4,5} = \frac{8 \pm 6i}{10} = \frac{4 \pm 3i}{5}$ 

 $x_4 = \frac{4}{5} + \frac{3}{5}i$ 

$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$

$$5x^{2} - 8x + 10 - \frac{8}{x} + \frac{5}{x^{2}} = 0$$

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$$x + \frac{1}{x} = 0 / \cdot x$$

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$$5x^{2} - 8x + 5 = 0$$

$$x_{4,5} = \frac{8 \pm \sqrt{64 - 100}}{10}$$

$$x_{4,5} = \frac{8 \pm 6i}{10} = \frac{4 \pm 3i}{5}$$

$$x_{4} = \frac{4}{5} + \frac{3}{5}i$$

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$$5x^{4} - 8x^{3} + 10x^{2} - 8x + 5 = 0 /: x^{2}$$

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$$x + \frac{1}{x} = \frac{8}{5} / \cdot 5x$$

$$5x^{2} - 8x + 5 = 0$$

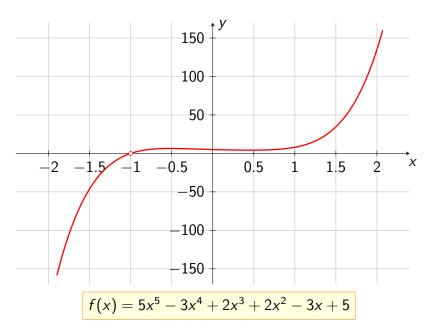
$$x_{4,5} = \frac{8 \pm \sqrt{64 - 100}}{10}$$

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$$x_{4} = \frac{4}{5} + \frac{3}{5}i$$

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$$x_{5} = \frac{4}{5} - \frac{3}{5}i$$



četvrti zadatak

### **Oznake**

• Funkcija dvije varijable: 
$$z = z(x, y)$$

• Parcijalna derivacija po varijabli x

$$z_x z_x' \frac{\partial z}{\partial x}$$

• Parcijalna derivacija po varijabli y

$$z_y z_y' \frac{\partial z}{\partial y}$$

# Parcijalne derivacije drugog reda – oznake

• Funkcija dvije varijable: z = z(x, y)

$$z_{xx}$$
  $z'_{xx}$   $\frac{\partial^2 z}{\partial x^2}$ 
 $z_{xy}$   $z'_{xy}$   $\frac{\partial^2 z}{\partial x \partial y}$ 
 $z_{yx}$   $z'_{yx}$   $\frac{\partial^2 z}{\partial y \partial x}$ 
 $z_{yy}$   $z'_{yy}$   $\frac{\partial^2 z}{\partial y^2}$ 

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x,y) = x^2 + y^2$$
 c)  $z = \frac{y}{x}$   
b)  $g(x,y) = 3x^2 + xy + \sqrt{y}$ 

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x,y) = x^2 + y^2$$

c) 
$$z = \frac{y}{x}$$

b)  $g(x,y) = 3x^2 + xy + \sqrt{y}$ 

### Rješenje

a)  $f_x =$ 

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Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x,y) = x^2 + y^2$$

c) 
$$z = \frac{y}{x}$$

b)  $g(x,y) = 3x^2 + xy + \sqrt{y}$ 

## Rješenje

a)  $f_x = 2x$ 

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Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x,y) = x^2 + y^2$$

c) 
$$z = \frac{y}{x}$$

b)  $g(x, y) = 3x^2 + xy + \sqrt{y}$ 

### Rješenje

a) 
$$f_x = 2x +$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x,y) = x^2 + y^2$$

c) 
$$z = \frac{y}{x}$$

b)  $g(x,y) = 3x^2 + xy + \sqrt{y}$ 

### Rješenje

a)  $f_x = 2x + 0$ 

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Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x,y) = x^2 + y^2$$

c) 
$$z = \frac{y}{x}$$

b)  $g(x,y) = 3x^2 + xy + \sqrt{y}$ 

### Rješenje

a) 
$$f_x = 2x + 0 = 2x$$

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Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x, y) = x^2 + y^2$$
  
b)  $g(x, y) = 3x^2 + xy + \sqrt{y}$ 

c) 
$$z = \frac{y}{x}$$

## Rješenje

a) 
$$f_x = 2x + 0 = 2x$$

$$f_y =$$

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Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x,y) = x^2 + y^2$$

b) 
$$g(x, y) = 3x^2 + xy + \sqrt{y}$$

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$$z = \frac{y}{x}$$

### Rješenje

a) 
$$f_x = 2x + 0 = 2x$$

$$f_y = 0$$

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Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x,y) = x^2 + y^2$$

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$$z = \frac{y}{x}$$

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### Rješenje

a) 
$$f_x = 2x + 0 = 2x$$

$$f_y = 0 +$$

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Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x,y) = x^2 + y^2$$

b) 
$$g(x, y) = 3x^2 + xy + \sqrt{y}$$

$$(x^n)'=nx^{n-1}$$

### Rješenje

a) 
$$f_x = 2x + 0 = 2x$$

$$f_y=0+2y$$

c)  $z = \frac{y}{x}$ 

Odredite parcijalne derivacije sljedećih funkcija:

$$(x^n)' = nx^{n-1}$$

a) 
$$f(x,y) = x^2 + y^2$$
  
b)  $g(x,y) = 3x^2 + xy + \sqrt{y}$ 

c) 
$$z = \frac{y}{x}$$

a) 
$$f_x = 2x + 0 = 2x$$

$$f_y=0+2y=2y$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x, y) = x^2 + y^2$$
 c)  $z = \frac{y}{x^2}$ 

c) 
$$z = \frac{y}{x}$$
  $(cu)'(x) = c \cdot u'(x)$ 

 $(x^n)' = nx^{n-1}$ 

b) 
$$g(x,y) = 3x^2 + xy + \sqrt{y}$$

## Rješenje

a) 
$$f_x = 2x + 0 = 2x$$

 $f_{v} = 0 + 2v = 2v$ 

b) 
$$g_x =$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x, y) = x^2 + y^2$$
 c)  $z = \frac{y}{x^2}$ 

c) 
$$z = \frac{y}{x}$$
  $(cu)'(x) = c \cdot u'(x)$ 

 $f_{v} = 0 + 2v = 2v$ 

 $(x^n)' = nx^{n-1}$ 

b) 
$$g(x,y) = 3x^2 + xy + \sqrt{y}$$

a) 
$$f_x = 2x + 0 = 2x$$

b) 
$$g_x = 6x$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x, y) = x^2 + y^2$$
 c)  $z = 3$ 

 $f_{v} = 0 + 2v = 2v$ 

c) 
$$z = \frac{y}{x}$$
  $(cu)'(x) = c \cdot u'(x)$ 

 $(x^n)' = nx^{n-1}$ 

b) 
$$g(x,y) = 3x^2 + xy + \sqrt{y}$$

a) 
$$f_x = 2x + 0 = 2x$$

$$+ 0 = 2x$$

b) 
$$g_x = 6x +$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x,y) = x^2 + y^2$$

b) 
$$g(x, y) = 3x^2 + xy + \sqrt{y}$$

 $(x^n)' = nx^{n-1}$ 

c) 
$$z = \frac{y}{x}$$
  $(cu)'(x) = c \cdot u'(x)$ 

a) 
$$f_x = 2x + 0 = 2x$$

b) 
$$g_x = 6x + y$$

$$f_y=0+2y=2y$$

Odredite parcijalne derivacije sljedećih funkcija:

$$(cu)'(x) = c \cdot u'(x)$$

 $(x^n)' = nx^{n-1}$ 

a) 
$$f(x,y) = x^2 + y^2$$
  
b)  $g(x,y) = 3x^2 + xy + \sqrt{y}$ 

c)  $z = \frac{y}{y}$ 

 $f_{v} = 0 + 2v = 2v$ 

a) 
$$f_x = 2x + 0 = 2x$$

b) 
$$g_x = 6x + y +$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x,y) = x^2 + y^2$$
 c)  $z = \frac{y}{x}$ 

$$(cu)'(x) = c \cdot u'(x)$$

 $f_{v} = 0 + 2v = 2v$ 

 $(x^n)' = nx^{n-1}$ 

b) 
$$g(x, y) = 3x^2 + xy + \sqrt{y}$$

a) 
$$f_x = 2x + 0 = 2x$$

b) 
$$g_x = 6x + y + 0$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x, y) = x^2 + y^2$$
  
b)  $g(x, y) = 3x^2 + xy + \sqrt{y}$ 

 $(x^n)' = nx^{n-1}$ 

c) 
$$z = \frac{y}{x}$$
  $(cu)'(x) = c \cdot u'(x)$ 

a) 
$$f_x = 2x + 0 = 2x$$

$$f_v = 0 + 2y = 2y$$

b) 
$$g_x = 6x + y + 0 = 6x + y$$

Odredite parcijalne derivacije sljedećih funkcija:

c) 
$$z = \frac{y}{x}$$
  $(cu)'(x) = c \cdot u'(x)$ 

 $(x^n)' = nx^{n-1}$ 

a) 
$$f(x,y) = x^2 + y^2$$

b) 
$$g(x,y) = 3x^2 + xy + \sqrt{y}$$

a) 
$$f_x = 2x + 0 = 2x$$

$$f_y=0+2y=2y$$

b) 
$$g_x = 6x + y + 0 = 6x + y$$

$$g_y =$$

Odredite parcijalne derivacije sljedećih funkcija:

c) 
$$z = \frac{y}{x}$$
  $(cu)'(x) = c \cdot u'(x)$ 

 $(x^n)' = nx^{n-1}$ 

a) 
$$f(x,y) = x^2 + y^2$$

$$c)$$
  $z = \frac{y}{z}$ 

b)  $g(x, y) = 3x^2 + xy + \sqrt{y}$ 

a) 
$$f_x = 2x + 0 = 2x$$

$$f_y = 0 + 2y = 2y$$

b) 
$$g_x = 6x + y + 0 = 6x + y$$

$$g_y = 0$$

Odredite parcijalne derivacije sljedećih funkcija:

$$c(x) = c \cdot u'(x)$$

 $(x^n)' = nx^{n-1}$ 

a) 
$$f(x,y) = x^2 + y^2$$

b) 
$$g(x, y) = 3x^2 + xy + \sqrt{y}$$

c) 
$$z = \frac{y}{x}$$
  $(cu)'(x) = c \cdot u'(x)$ 

a) 
$$f_x = 2x + 0 = 2x$$

$$f_y = 0 + 2y = 2y$$

b) 
$$g_x = 6x + y + 0 = 6x + y$$

$$g_{v} = 0 +$$

Odredite parcijalne derivacije sljedećih funkcija:

c) 
$$z = \frac{y}{x}$$
  $(cu)'(x) = c \cdot u'(x)$ 

 $(x^n)' = nx^{n-1}$ 

a) 
$$f(x,y) = x^2 + y^2$$

b) 
$$g(x,y) = 3x^2 + xy + \sqrt{y}$$

a) 
$$f_x = 2x + 0 = 2x$$

$$f_y=0+2y=2y$$

b) 
$$g_x = 6x + y + 0 = 6x + y$$

$$g_y = 0 + x$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x,y) = x^2 + y^2$$

b) 
$$g(x,y) = 3x^2 + xy + \sqrt{y}$$

c)  $z = \frac{y}{y}$ 

$$(cu)'(x) = c \cdot u'(x)$$

$$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

 $(x^n)' = nx^{n-1}$ 

a) 
$$f_x = 2x + 0 = 2x$$

$$f_y=0+2y=2y$$

b) 
$$g_x = 6x + y + 0 = 6x + y$$

$$g_v = 0 + x +$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x,y) = x^2 + y^2$$

b) 
$$g(x,y) = 3x^2 + xy + \sqrt{y}$$

c)  $z = \frac{y}{x}$ 

$$(cu)'(x) = c \cdot u'(x)$$

$$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

 $(x^n)' = nx^{n-1}$ 

a) 
$$f = 2x + 0$$

a) 
$$f_x = 2x + 0 = 2x$$

$$f_y=0+2y=2y$$

b) 
$$g_x = 6x + y + 0 = 6x + y$$
  $g_y = 0 + x + \frac{1}{2\sqrt{y}}$ 

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x, y) = x^2 + y^2$$
  
b)  $g(x, y) = 3x^2 + xy + \sqrt{y}$ 

c)  $z = \frac{y}{y}$ 

 $(cu)'(x) = c \cdot u'(x)$ 

 $(x^n)' = nx^{n-1}$ 

$$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

## Rješenje

a) 
$$f_x = 2x + 0 = 2x$$
  $f_y = 0 + 2y$ 

b)  $g_x = 6x + y + 0 = 6x + y$ 

$$f_y = 0 + 2y = 2y$$

 $g_y = 0 + x + \frac{1}{2\sqrt{y}} = x + \frac{1}{2\sqrt{y}}$ 

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x, y) = x^2 + y^2$$
  
b)  $g(x, y) = 3x^2 + xy + \sqrt{y}$ 

c) 
$$z = \frac{y}{x}$$

$$(cu)'(x) = c \cdot u'(x)$$
$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

 $(x^n)' = nx^{n-1}$ 

## Rješenje

 $f_{v} = 0 + 2y = 2y$ 

a) 
$$f_x = 2x + 0 = 2x$$
  $f_y = 0$ 

b) 
$$g_x = 6x + y + 0 = 6x + y$$
  $g_y = 0 + x + \frac{1}{2\sqrt{y}} = x + \frac{1}{2\sqrt{y}}$ 

c)  $z_x =$ 

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x,y) = x^2 + y^2$$
 c)  $z =$ 

$$v^2$$
 c)  $z = \frac{y}{2}$ 

$$(cu)'(x) = c \cdot u'(x)$$
$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

 $(x^n)' = nx^{n-1}$ 

b) 
$$g(x,y) = 3x^2 + xy + \sqrt{y}$$

## Rješenje

a) 
$$f_x = 2x + 0 =$$

a) 
$$f_x = 2x + 0 = 2x$$

$$f_y=0+2y=2y$$

c)  $z_x = y$ 

b) 
$$\sigma = 6x \pm y \pm 1$$

b) 
$$g_x = 6x + y + 0 = 6x + y$$

$$y + y = 0$$

$$g_y = 0 + x + \frac{1}{2\sqrt{y}} = x + \frac{1}{2\sqrt{y}}$$

$$+2y=2y$$

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Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x,y) = x^2 + y^2$$

b)  $g(x, y) = 3x^2 + xy + \sqrt{y}$ 

c) 
$$z = \frac{y}{x}$$

$$(cu)'(x) = c \cdot u'(x)$$
$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

 $(x^n)' = nx^{n-1}$ 

## Rješenje

a)  $f_x = 2x + 0 = 2x$  $f_{v} = 0 + 2y = 2y$ 

a) 
$$t_x = 2x + 0$$

b) 
$$g_x = 6x + y + 0 = 6x + y$$

c) 
$$z_x = y$$

$$0 + 2y = 2y$$
  
 $g_y = 0 + x + \frac{1}{2\sqrt{y}} = x + \frac{1}{2\sqrt{y}}$ 

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Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x, y) = x^2 + y^2$$
 c)  $z = \frac{y}{y^2}$ 

b) 
$$g(x, y) = 3x^2 + xy + \sqrt{y}$$

$$z - \frac{y}{z}$$

$$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

 $(x^n)' = nx^{n-1}$ 

 $(cu)'(x) = c \cdot u'(x)$ 

a) 
$$t_x = 2x + 0 = 2x$$

$$0+2y=2$$

a) 
$$f_x = 2x + 0 = 2x$$
  $f_y = 0 + 1$ 

$$b) \sigma_{x} = 6x + y +$$

$$f_y = 0 + 2y = 2y$$

$$x = x + \frac{1}{2}$$

b) 
$$g_x = 6x + y + 0 = 6x + y$$
  $g_y = 0 + x$ 

c) 
$$z_x = y \cdot (-x^{-2})$$

$$g_y = 0 + x + \frac{1}{2\sqrt{y}} = x + \frac{1}{2\sqrt{y}}$$

$$y = 0 + x$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x,y) = x^2 + y^2$$

b) 
$$g(x,y) = 3x^2 + xy + \sqrt{y}$$

$$(cu)'(x) = c \cdot u'(x)$$
$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

 $(x^n)' = nx^{n-1}$ 

## Rješenje

a) 
$$f_x = 2x + 0 = 2x$$

$$+ 0 = 2x$$

b)  $g_x = 6x + y + 0 = 6x + y$ 

c)  $z_x = y \cdot (-x^{-2}) = -\frac{y}{x^2}$ 

$$0=2x$$

$$y = 0$$

$$f_y = 0 + 2y = 2y$$

$$g_y = 0 + x + \frac{1}{2\sqrt{y}} = x + \frac{1}{2\sqrt{y}}$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x,y) = x^2 + y^2$$
  
b)  $g(x,y) = 3x^2 + xy + \sqrt{y}$ 

$$y^2$$
 c)  $z = \frac{y}{x}$ 

$$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

 $(x^n)' = nx^{n-1}$ 

 $(cu)'(x) = c \cdot u'(x)$ 

Rješenje

a) 
$$f_x = 2x + 0 = 2x$$

$$f_y=0+2y=2y$$

$$a_j = i\chi = 2\chi + 0$$

c)  $z_x = y \cdot (-x^{-2}) = -\frac{y}{x^2}$ 

b) 
$$g_x = 6x + y + 0 = 6x + y$$

$$i_y = 0 + 2y = 2$$

$$t_y = 0 +$$

$$g_y = 0 +$$

 $z_v =$ 

$$+\frac{1}{2\sqrt{2}}$$

$$x = x + \frac{1}{2}$$

$$\frac{1}{2\sqrt{y}} = x + \frac{1}{2}$$

$$g_y = 0 + x + \frac{1}{2\sqrt{y}} = x + \frac{1}{2\sqrt{y}}$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x,y) = x^2 + y^2$$
 c)  $z = \frac{y}{x^2}$ 

$$z = \frac{y}{x}$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

 $(cu)'(x) = c \cdot u'(x)$ 

 $(x^n)' = nx^{n-1}$ 

Rješenje

a) 
$$f_x = 2x + 0 = 2x$$

$$f_y=0+2y=2y$$

a) 
$$T_X = 2$$

b) 
$$g_x = 6x + y + 0$$

b) 
$$g_x = 6x + y + 0 = 6x + y$$

 $z_y = x^{-1}$ 

b) 
$$g_x = 6x + y + 0 = 6x + y$$

c)  $z_x = y \cdot (-x^{-2}) = -\frac{y}{x^2}$ 

b)  $g(x, y) = 3x^2 + xy + \sqrt{y}$ 

$$g_y = 0 + x + \frac{1}{2\sqrt{y}} = x + \frac{1}{2\sqrt{y}}$$

$$f_y = 0 + 2y =$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x,y) = x^2 + y^2$$

 $(cu)'(x) = c \cdot u'(x)$  $\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$ 

b)  $g(x, y) = 3x^2 + xy + \sqrt{y}$ 

# Rješenje

a)  $f_x = 2x + 0 = 2x$ 

c)  $z_x = y \cdot (-x^{-2}) = -\frac{y}{x^2}$ 

$$f_y=0+2y=2y$$

$$\sigma = 0 + v$$

$$2y = 2y$$

 $z_{v}=x^{-1}$ .

 $(x^n)' = nx^{n-1}$ 

$$g_y = 0 + x + \frac{1}{2\sqrt{y}} = x + \frac{1}{2\sqrt{y}}$$

$$\frac{1}{2\sqrt{y}}$$

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$$+x+\frac{1}{2}$$

b) 
$$g_x = 6x + y + 0 = 6x + y$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x,y) = x^2 + y^2$$
 c)  $z = \frac{y}{x^2}$ 

$$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

 $(cu)'(x) = c \cdot u'(x)$ 

 $(x^n)' = nx^{n-1}$ 

Rješenje

$$0=2x$$

b)  $g(x, y) = 3x^2 + xy + \sqrt{y}$ 

c)  $z_x = y \cdot (-x^{-2}) = -\frac{y}{x^2}$ 

 $f_{v} = 0 + 2y = 2y$ 

a) 
$$f_x = 2x + 0 = 2x$$

 $g_y = 0 + x + \frac{1}{2\sqrt{y}} = x + \frac{1}{2\sqrt{y}}$ 

b) 
$$g_x = 6x + y + 0 = 6x + y$$

$$z_v = x^{-1} \cdot 1$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$f(x,y) = x^2 + y^2$$
 c)  $z = \frac{y}{x^2}$ 

$$=\frac{y}{x}$$

$$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

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 $(x^n)' = nx^{n-1}$ 

 $(cu)'(x) = c \cdot u'(x)$ 

Rješenje

a) 
$$f_x = 2x + 0 = 2x$$

b)  $g(x, y) = 3x^2 + xy + \sqrt{y}$ 

$$f_y =$$

$$f_y=0+2y=2y$$

 $z_y = x^{-1} \cdot 1 = \frac{1}{z}$ 

$$0 + 2y = 2y$$
  $g_y = 0 + x + \frac{1}{2\sqrt{y}} = x + \frac{1}{2\sqrt{y}}$ 

b) 
$$g_x = 6x + y + 0 = 6x + y$$



c)  $z_x = y \cdot (-x^{-2}) = -\frac{y}{x^2}$ 

peti zadatak

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^y$$

$$z = ye^y +$$

b) 
$$z = ye^{y} + \sqrt{x}$$
 c)  $u(x, y) = \frac{2x - y}{x + y}$ 

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^y$$

$$(1) \quad z = ve^y \pm \sqrt{v}$$

b) 
$$z = ye^{y} + \sqrt{x}$$
 c)  $u(x, y) = \frac{2x - y}{x + y}$ 

a) 
$$z_x =$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^y$$

$$) z = ve^{y} + \sqrt{x}$$

b) 
$$z = ye^{y} + \sqrt{x}$$
 c)  $u(x, y) = \frac{2x - y}{x + y}$ 

### Rješenje

a) 
$$z_x = e^y$$

 $(cu)'(x) = c \cdot u'(x)$ 

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^y$$

$$(x) \quad z = ye^y \perp \sqrt{y}$$

b) 
$$z = ye^{y} + \sqrt{x}$$
 c)  $u(x, y) = \frac{2x - y}{x + y}$ 

a) 
$$z_x = e^y$$

$$(cu)'(x) = c \cdot u'(x)$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^y$$

$$) z = ye^y +$$

b) 
$$z = ye^{y} + \sqrt{x}$$
 c)  $u(x, y) = \frac{2x - y}{x + y}$ 

a) 
$$z_x = e^y \cdot 1$$

$$(cu)'(x) = c \cdot u'(x)$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^y$$

$$) z = ye^y + \sqrt{x}$$

b) 
$$z = ye^{y} + \sqrt{x}$$
 c)  $u(x, y) = \frac{2x - y}{x + y}$ 

a) 
$$z_x = e^y \cdot 1 = e^y$$

$$(cu)'(x) = c \cdot u'(x)$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^y$$

b) 
$$z = ye^y + \sqrt{x}$$

c) 
$$u(x,y) = \frac{2x-y}{x+y}$$

a) 
$$z_x = e^y \cdot 1 = e^y$$
  $z_y =$ 

$$(cu)'(x) = c \cdot u'(x)$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^y$$

b) 
$$z = ye^y + \sqrt{x}$$

c) 
$$u(x,y) = \frac{2x-y}{x+y}$$

a) 
$$z_x = e^y \cdot 1 = e^y$$

$$= e^y z_y = x$$

$$(cu)'(x) = c \cdot u'(x)$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^y$$
 b)  $z = ye^y + \sqrt{x}$ 

b) 
$$z = ye^{y} + \sqrt{x}$$
 c)  $u(x, y) = \frac{2x - y}{x + y}$ 

### Rješenje

a) 
$$z_x = e^y \cdot 1 = e^y$$

 $(e^{x})'=e^{x}$ 

 $(cu)'(x) = c \cdot u'(x)$ 

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^y$$
 b)  $z = ye^y + \sqrt{x}$ 

b) 
$$z = ye^{y} + \sqrt{x}$$
 c)  $u(x, y) = \frac{2x - y}{x + y}$ 

### Rješenje

a) 
$$z_x = e^y \cdot 1 = e^y$$

 $z_v = x \cdot e^y$ 

$$(cu)'(x) = c \cdot u'(x)$$

$$(e^{x})' = e^{x}$$

a)  $z = xe^y$ 

Odredite parcijalne derivacije sljedećih funkcija:

b) 
$$z = ye^{y} + \sqrt{x}$$
 c)  $u(x, y) = \frac{2x - y}{x + y}$ 

a) 
$$z_x = e^y \cdot 1 = e^y$$

$$z_y = x \cdot e^y = xe^y$$
  $(cu)'(x) = c \cdot u'(x)$ 

$$(e^{x})' = e^{x}$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^y$$
 b)  $z = ye^y + \sqrt{x}$ 

c) 
$$u(x,y) = \frac{2x-y}{x+y}$$

 $z_y = x \cdot e^y = xe^y$   $(cu)'(x) = c \cdot u'(x)$ 

#### Diexeni

a) 
$$z_x = e^y \cdot 1 = e^y$$

b) 
$$z_x =$$

$$(e^{x})' = e^{x}$$

Odredite parcijalne derivacije sljedećih funkcija:

b) 
$$z = ye^y + \sqrt{x}$$

 $z_y = x \cdot e^y = xe^y$   $(cu)'(x) = c \cdot u'(x)$ 

c) 
$$u(x,y) = \frac{2x-y}{x+y}$$

a)  $z = xe^y$ 

#### Rješenje

b)  $z_{x} = 0$ 

a)  $z_{x} = e^{y} \cdot 1 = e^{y}$ 

 $(e^{x})'=e^{x}$ 

# Odredite parcijalne derivacije sljedećih funkcija:

Rješenje

b)  $z_x = 0 +$ 

Zadatak 5

a)  $z = xe^y$ 

a)  $z_x = e^y \cdot 1 = e^y$ 

b)  $z = ye^y + \sqrt{x}$ 

$$z_y = x \cdot e^y = xe^y$$
  $(cu)'(x) = c \cdot u'(x)$ 

c) 
$$u(x,y) = \frac{2x-y}{x+y}$$

 $\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$ 

#### Zadatak 5 Odredite parcijalne derivacije sljedećih funkcija:

a)  $z = xe^y$ 

Rješenje

a)  $z_x = e^y \cdot 1 = e^y$ 

 $b) z_x = 0 + \frac{1}{2\sqrt{x}}$ 

b)  $z = ye^y + \sqrt{x}$ 

$$z_y = x \cdot e^y = xe^y$$
  $(cu)'(x) = c \cdot u'(x)$ 

c) 
$$u(x,y) = \frac{2x-y}{x+y}$$

 $(e^{x})'=e^{x}$ 

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Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^{y}$$
 b)  $z = ye^{y} + \sqrt{x}$ 

c) 
$$u(x,y) = \frac{2x-y}{x+y}$$

a) 
$$z_x = e^y \cdot 1 = e^y$$

$$z_y = x \cdot e^y = x$$

$$z_y = x \cdot e^y = xe^y$$
  $(cu)'(x) = c \cdot u'(x)$ 

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$(e^{x})' = e^{x}$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^y$$

b) 
$$z = ye^y + \sqrt{x}$$

c) 
$$u(x,y) = \frac{2x-y}{x+y}$$

#### Rješenje

a) 
$$z_x = e^y \cdot 1 = e^y$$

$$z_y = x \cdot e^y = xe^y$$

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$z_y =$$

b) 
$$z_x$$

$$z_y =$$

$$(cu)'(x) = c \cdot u'(x)$$

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

 $(e^x)'=e^x$ 

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Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^y$$
 b)  $z = ye^y + \sqrt{x}$ 

c) 
$$u(x,y) = \frac{2x-y}{x+y}$$

### Rješenje

a) 
$$z_x = e^y \cdot 1 = e^y$$

$$z_y = x \cdot e^y = xe^y$$

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$\frac{1}{2\sqrt{x}}$$

$$z_y = 1$$

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

$$(cu)'(x) = c \cdot u'(x)$$

 $(e^x)'=e^x$ 

18/21

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### Rješenje

a) 
$$z_x = e^y \cdot 1 = e^y$$

$$z_y = x \cdot e^y = xe^y$$

a) 
$$z_x = e^y \cdot 1 = e^y$$

$$=\frac{1}{2\sqrt{x}}$$

a) 
$$z_x = e^y \cdot 1 = e^y$$
  $z_y = x \cdot e^y = xe^y$ 

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$
  $z_y = 1$ .

$$\sqrt{x}$$

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

$$(cu)'(x) = c \cdot u'(x)$$

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Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^{y}$$
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### Rješenje

a)  $z_x = e^y \cdot 1 = e^y$ 

$$z_y = x \cdot e^y = xe^y$$

$$y = xe^y$$

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

$$(cu)'(x) = c \cdot u'(x)$$

$$(e^{x})'=e^{x}$$

$$(e^x)' = e^x$$
$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$
  $z_y = 1 \cdot e^y$ 

$$z_y = 1 \cdot e^y$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^y$$
 b)  $z = ye^y + \sqrt{x}$ 

c) 
$$u(x,y) = \frac{2x - y}{x + y}$$

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

#### Rješenje

a) 
$$z_x = e^y \cdot 1 = e^y$$

$$z_y = x \cdot e^y = xe^y$$

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$= 1 \cdot e^y +$$

b) 
$$z_x = 0$$

$$z_y = 1 \cdot e^y +$$

$$(e^{x})'=\epsilon$$

 $(cu)'(x) = c \cdot u'(x)$ 

$$(e^x)' = e^x$$
$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^{y}$$
 b)  $z = ye^{y} + \sqrt{x}$ 

c) 
$$u(x,y) = \frac{2x-y}{x+y}$$

### Rješenje

a) 
$$z_v =$$

a) 
$$z_x = e^y \cdot 1 = e^y$$

$$z_y = x \cdot e^y = xe^y$$

$$z_y = 1 \cdot e^y + y$$

b) 
$$z_x = 0$$

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$z_y = 1 \cdot e^y +$$

$$z_{x} = x \cdot e^{y} = xe^{y}$$

$$z_y = x \cdot e^y = xe^y$$
  $(cu)'(x) = c \cdot u'(x)$ 

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

 $(e^x)'=e^x$ 

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Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^y$$
 b)  $z = ye^y + \sqrt{x}$ 

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$$u(x,y) = \frac{2x-y}{x+y}$$

### Rješenje

a) 
$$z_x = e^y \cdot 1 = e^y$$

$$z_y = x \cdot e^y = xe^y$$
  $(cu)'(x) = c \cdot u'(x)$ 

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$z_y = 1 \cdot e^y + y \cdot$$

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

 $(e^x)'=e^x$ 

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^y$$

b) 
$$z = ye^y + \sqrt{x}$$

c) 
$$u(x,y) = \frac{2x - y}{x + y}$$

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

a) 
$$z_x = e^y \cdot 1 = e^y$$

$$= x \cdot e^y = xe^y$$

$$z_y = x \cdot e^y = xe^y$$
  $(cu)'(x) = c \cdot u'(x)$ 

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$z_y = 1 \cdot e^y +$$

$$(e^{x})' = e^{x}$$

$$z_x = 0 + \frac{1}{2\sqrt{x}} = 0$$

$$z_y = 1 \cdot e^y + y \cdot e^y$$

$$(e^{x})' = e^{x}$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$z_y = 1 \cdot e^y + 1$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^y$$

b) 
$$z = ye^y + \sqrt{x}$$

c) 
$$u(x,y) = \frac{2x-y}{x+y}$$

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

### Rješenje

# a) $z_x = e^y \cdot 1 = e^y$

$$z_y = x \cdot e^y = xe^y$$
  $(cu)'(x) = c \cdot u'(x)$ 

$$z_y = 1 \cdot e^y + y \cdot e^y +$$

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$
  $z_y = 1 \cdot e^{\frac{1}{2}}$ 

$$e^{y} + (e^{x})' =$$

$$2\sqrt{x}$$

$$(e^{x})' = e^{x}$$
$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$\frac{1}{x} = \frac{1}{2\sqrt{x}}$$
  $2y = 1 \cdot e^{-x}$ 

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^y$$
 b)  $z = ye^y + \sqrt{x}$ 

c) 
$$u(x,y) = \frac{2x-y}{x+y}$$

#### Rješenje

a) 
$$z_x = e^y \cdot 1 = e^y$$

a) 
$$z_x = e^y \cdot 1 = e^y$$
  
b)  $z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$ 

$$z_y = x \cdot e^y = xe^y$$
  $(cu)'(x) = c \cdot u'(x)$ 

$$z_y = 1 \cdot e^y + y \cdot e^y + 0$$

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

 $(e^x)'=e^x$ 

 $\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$ 

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^{y}$$
 b)  $z = ye^{y} + \sqrt{x}$ 

c) 
$$u(x,y) = \frac{2x-y}{x+y}$$

### Rješenje

a) 
$$z_x = e^y \cdot 1 = e^y$$

a) 
$$z_x = e^y \cdot 1 = e^y$$
  
b)  $z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$ 

$$y = xe^y$$

$$(x)'(x) = c \cdot a$$

$$z_y = x \cdot e^y = xe^y$$
  $(cu)'(x) = c \cdot u'(x)$ 

$$0 = (1 + y)$$

$$z_y = 1 \cdot e^y + y \cdot e^y + 0 = (1+y)e^y$$

$$(e^{x})'=e^{x}$$

- $\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$
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c)  $u_x =$ 

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^y$$
 b)  $z = ye^y + \sqrt{x}$ 

c) 
$$u(x,y) = \frac{2x - y}{x + y}$$

$$\mathsf{nje} \ \left(\frac{u}{v}\right)^{r}(x) = \frac{1}{2}$$

$$v(x)^{2}$$

$$z_{y} = x \cdot e^{y} = xe^{y} \qquad (cu)'(x) = c \cdot u'(x)$$

$$z_y = 1 \cdot e^y + y \cdot e^y + 0 = (1+y)e^y$$

$$e^y + y$$

 $(e^x)'=e^x$ 

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 $\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$ 

b) 
$$z = ye^y + \sqrt{x}$$

Rješenje 
$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$

$$(x) = -$$

$$v(x)^2$$

a) 
$$z_x = e^y \cdot 1 = e^y$$

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$Z_y$$

$$z_y = x \cdot e$$

$$\cdot v'(x)$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^y$$
 b)  $z = ye^y + \sqrt{x}$ 

c) 
$$u(x,y) = \frac{2x-y}{x+y}$$

ješenje 
$$\left(\frac{a}{v}\right)(x) =$$

Rješenje 
$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$

a) 
$$z_x = e^y \cdot 1 = e^y$$

$$z_y = x \cdot e^y = xe^y$$
  $(cu)'(x) = c \cdot u'(x)$ 

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$
  $z_y = 0$ 

$$e^{y} + 0 = (1 + y)$$

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2}$$

c)  $u_{x} = -$ 

$$z_y = 1 \cdot e^y + y \cdot e^y + 0 = (1+y)e^y$$

 $(e^x)'=e^x$ 

 $\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$ 

a) 
$$z_x = e^y \cdot 1 = e^y$$
  $z_y = x \cdot e^y =$ 

$$z_v = 1 \cdot e^y + 1$$

$$\frac{u'(x)\cdot v(x)-u(x)\cdot v'(x)}{v(x)^2}$$

$$\frac{(x)\cdot v(x)-u(x)\cdot v'}{v(x)^2}$$

$$u'(x) \cdot v(x) - u(x) \cdot v'(x)$$

b) 
$$z = ye^y + \sqrt{x}$$

b) 
$$z = ye^y + \sqrt{x}$$

b) 
$$z = ye^y \perp \sqrt{y}$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^y$$
 b)  $z = ye^y + \sqrt{x}$ 

$$z_{v} = x \cdot e^{y} = z$$

$$z_y = 1 \cdot e^y$$

b) 
$$z_x = 0 +$$

c)  $u_x = ---$ 

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

 $(x+y)^2$ 

$$z_v = 1 \cdot e^y +$$

$$z_y = 1 \cdot e^y + y \cdot e^y + 0 = (1+y)e^y$$

 $(e^x)'=e^x$ 

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 $\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$ 

$$\frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$e^y + 0 = (1 +$$

a) 
$$z_x = e^y \cdot 1 = e^y$$
  $z_y = x \cdot e^y = xe^y$   $(cu)'(x) = c \cdot u'(x)$ 

Rješenje 
$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$

$$v(x)^2$$

$$(x) \cdot y(x) = y(x) \cdot y'(x)$$

$$\cdot v'(x)$$

$$D) \ Z = ye^{x} + \sqrt{x}$$

c) 
$$u(x,y) = \frac{2x-y}{x+y}$$

$$u(x,y) = \frac{2x-y}{x+y}$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^y$$
 b)  $z = ye^y + \sqrt{x}$ 

c) 
$$u(x,y) = \frac{2x-y}{x+y}$$

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

$$\left(\frac{u}{v}\right)'(x) =$$

Rješenje 
$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$

a) 
$$z_x = e^y \cdot 1 = e^y$$

$$z_y = x \cdot e^y = xe^y$$
  $(cu)'(x) = c \cdot u'(x)$ 

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$e^y + y$$
.

b) 
$$z_x = 0 + \frac{1}{2\sqrt{z}} = \frac{1}{2\sqrt{z}}$$
  $z_y = 1 \cdot e^y$ 

$$z_y = 1 \cdot e^y + y \cdot e^y + 0 = (1+y)e^y$$

c) 
$$u_x = \frac{2}{(x+y)^2}$$

$$e^y + 0 = (1 +$$

$$0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$e^{y} + 0 = (1 + y)e^{y}$$

$$(e^{x})' = e^{x}$$

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

b) 
$$z_x = 0 +$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^y$$
 b)  $z = ye^y$ 

b) 
$$z = ye^y + \sqrt{x}$$

Rješenje 
$$\frac{\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2} }{v(x)^2}$$

Rješenje 
$$z_x = e^y \cdot 1 = e^y$$

c)  $u_x = \frac{2 \cdot (x+y)^2}{(x+y)^2}$ 

$$=e^{y}$$

$$z_y = x \cdot e^y = xe^y \qquad (cu)'(x) = c \cdot u'(x)$$

$$z_y = 1 \cdot \epsilon$$

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$z_y = 1 \cdot e^y + y \cdot e^y + 0 = (1+y)e^y$$

$$\cdot e^{y} + v \cdot$$

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

c)  $u(x, y) = \frac{2x - y}{x + y}$ 

 $(e^x)'=e^x$ 

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$$u'(x) = c \cdot u'(x)$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^{y}$$
 b)  $z = ye^{y} +$ 

b) 
$$z = ye^y + \sqrt{x}$$

Rješenje 
$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$

c)  $u_x = \frac{2 \cdot (x + y)}{(x + v)^2}$ 

a) 
$$z_x = e^y \cdot 1 = e^y$$

$$z_y = x \cdot e^y = xe^y$$
  $(cu)'(x) = c \cdot u'(x)$ 

$$z_y = 1 \cdot e^y + y \cdot e^y + 0 = (1+y)e^y$$

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$z_y = x \cdot e^{x}$$

$$\cdot e^y = x$$

$$= xe^y$$

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

c)  $u(x, y) = \frac{2x - y}{x + y}$ 

$$(e^{x})' = e^{x}$$
$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = xe^y$$
 b)  $z = ye^y + \sqrt{x}$ 

Rješenje 
$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$

 $z_v = x \cdot e^y = xe^y$   $(cu)'(x) = c \cdot u'(x)$ a)  $z_x = e^y \cdot 1 = e^y$ 

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

 $z_v = 1 \cdot e^y + y \cdot e^y + 0 = (1+y)e^y$ 

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$
  
c)  $u_x = \frac{2 \cdot (x+y) - (x+y)^2}{(x+y)^2}$ 

$$e^{y} + 0 = (1 + y)e^{y}$$

$$(e^{x})' = e^{x}$$

 $\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$ 

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a) 
$$z_x = e^{x} \cdot 1 = e^{x}$$

$$z_y = x \cdot e^{x} = xe^{x}$$

$$z_{v} = x \cdot e^{y} = xe^{y}$$

$$u'(x) \cdot v(x) - u(x) \cdot v'(x)$$

b) 
$$z = ye^{y} + \sqrt{x}$$

b) 
$$z = ye^y + \sqrt{x}$$

$$\sqrt{x}$$
 c)  $u(x,y) = \frac{2x-y}{x+y}$ 

h) 
$$z = ve^y + \sqrt{x}$$

Odredite parcijalne derivacije sljedećih funkcija:

c)  $u(x, y) = \frac{2x - y}{x + y}$ 

a)  $z = xe^y$ Rješenje  $\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$ 

a)  $z_x = e^y \cdot 1 = e^y$ 

b)  $z = ye^y + \sqrt{x}$ 

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

b)  $z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$ 

 $z_v = 1 \cdot e^y + y \cdot e^y + 0 = (1 + y)e^y$ 

 $z_v = x \cdot e^y = xe^y$   $(cu)'(x) = c \cdot u'(x)$ 

 $(e^x)'=e^x$ 

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 $\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$ 

c)  $u_x = \frac{2 \cdot (x+y) - (2x-y)}{(x+y)^2}$ 

Odredite parcijalne derivacije sljedećih funkcija:

b)  $z = ye^y + \sqrt{x}$ 

c) 
$$u(x,y) = \frac{2x-y}{x+y}$$

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

a)  $z = xe^y$ Rješenje  $\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$ 

$$z_v = x \cdot e$$

$$= xe^y$$

b)  $z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$ 

$$\frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$e^y+0=(1-$$

$$z_y = x \cdot e^y = xe^y$$
  $(cu)'(x) = c \cdot u'(x)$ 

$$+0=(1+v)$$

$$y' + 0 = (1 + y)$$

$$+0=(1+y)$$

$$+0=(1+y)$$

$$x^{2} + 0 = (1 + y)^{2}$$

$$z_y = 1 \cdot e^y + y \cdot e^y + 0 = (1+y)e^y$$

$$(e^x)'=e^x$$

$$(e^x)' = e^x$$
$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

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$$\frac{1}{\sqrt{z}} = \frac{1}{2\sqrt{z}} \qquad z_y = 1 \cdot e^y + y$$

# a) $z_x = e^y \cdot 1 = e^y$

c)  $u_x = \frac{2 \cdot (x+y) - (2x-y) \cdot (x+y)^2}{(x+y)^2}$ 

Odredite parcijalne derivacije sljedećih funkcija:

b) 
$$z = ye^y + \sqrt{x}$$

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

c) 
$$u(x,y) = \frac{2x - y}{x + y}$$

$$\left(\frac{u}{v}\right)$$

c)  $u_x = \frac{2 \cdot (x+y) - (2x-y) \cdot 1}{(x+y)^2}$ 

Rješenje 
$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$
a)  $z_x = e^y \cdot 1 = e^y$   $z_y = x \cdot e^y = xe^y$   $(cu)'(x) = c \cdot u'(x)$ 

a) 
$$z_x = e^y \cdot 1 = e^y$$

a)  $z = xe^y$ 

$$Z_y = x$$
.

b) 
$$z_x = 0 +$$

$$z_v = 1 \cdot e^y + y \cdot e^y + 0 = (1+y)e^y$$

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$= xe^y$$

$$= xe^y$$

$$=(1+y)e$$

$$(1+y)e^{y}$$

$$(e^{x})'=e^{x}$$

 $\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$ 

Odredite parcijalne derivacije sljedećih funkcija:

b)  $z = ye^y + \sqrt{x}$ 

a)  $z = xe^y$ 

Rješenje  $\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$ 

a)  $z_x = e^y \cdot 1 = e^y$ 

b)  $z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$ 

 $z_v = x \cdot e^y = xe^y$   $(cu)'(x) = c \cdot u'(x)$ 

c)  $u_x = \frac{2 \cdot (x+y) - (2x-y) \cdot 1}{(x+y)^2} = \frac{1}{(x+y)^2}$ 

 $z_v = 1 \cdot e^y + y \cdot e^y + 0 = (1 + y)e^y$ 

 $(e^x)'=e^x$ 

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 $\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$ 

c)  $u(x, y) = \frac{2x - y}{x + y}$ 

Odredite parcijalne derivacije sljedećih funkcija:

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

c)  $u(x, y) = \frac{2x - y}{x + y}$ 

 $z_v = 1 \cdot e^y + y \cdot e^y + 0 = (1 + y)e^y$ 

 $(e^x)'=e^x$ 

b)  $z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$ 

 $z_v = x \cdot e^y = xe^y$   $(cu)'(x) = c \cdot u'(x)$ a)  $z_x = e^y \cdot 1 = e^y$ 

a)  $z = xe^y$ 

b)  $z = ye^y + \sqrt{x}$ Rješenje  $\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$ 

c)  $u_x = \frac{2 \cdot (x+y) - (2x-y) \cdot 1}{(x+y)^2} = \frac{3y}{(x+y)^2}$  $\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$ 18/21

#### Zadatak 5 Odredite parcijalne derivacije sljedećih funkcija:

c) 
$$u(x, y) = \frac{2x - y}{x + y}$$

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

b) 
$$z = ye^y + \sqrt{x}$$

Rješenje 
$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$
a)  $z = e^y \cdot 1 - e^y$   $z = x \cdot e^y - xe^y$  (4)

a) 
$$z_x = e^y \cdot 1 = e^y$$
  $z_y = x \cdot e^y = xe^y$   $(cu)'(x) = c \cdot u'(x)$ 

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$
  $z_y = 1 \cdot e^y + y \cdot e^y$ 

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$
  $z_y = 1 \cdot e^y + y \cdot e^y$ 

b) 
$$z_{y} = 0 + \frac{1}{z_{y}} = \frac{1}{z_{y}}$$
  $z_{y} = 1 \cdot e^{y} + \frac{1}{z_{y}}$ 

c)  $u_x = \frac{2 \cdot (x+y) - (2x-y) \cdot 1}{(x+y)^2} = \frac{3y}{(x+y)^2}$ 

 $u_{v} =$ 

$$z_y = 1 \cdot e^y + y \cdot e^y + 0 = (1+y)e^y$$

$$\frac{1}{\sqrt{x}} \qquad z_y = 1 \cdot e^y + y$$

$$(e^{x})' = e^{x}$$
$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$\frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}} \qquad z_y = 1 \cdot e^y +$$

$$z_x = 0 + \frac{1}{2\sqrt{z}} = \frac{1}{2\sqrt{z}}$$
  $z_y = 1 \cdot e^y + 1$ 

$$(u)' \qquad u'(x) \cdot v(x) - u(x) \cdot v'$$

a) 
$$z = xe^y$$
 b)  $z = ye^y + \sqrt{x}$ 

b) 
$$z = ye^y + \sqrt{x}$$

b) 
$$z = ve^y + \sqrt{x}$$

#### $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ Zadatak 5 Odredite parcijalne derivacije sljedećih funkcija:

b) 
$$z = ye^y + \sqrt{x}$$

a) 
$$z = xe^{y}$$
 b)  $z = ye^{y} + \sqrt{x}$ 

Rješenje 
$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$

a) 
$$z_x = e^y \cdot 1 = e^y$$
  $z_y = x \cdot e^y = xe^y$   $(cu)'(x) = c \cdot u'(x)$ 

c)  $u_x = \frac{2 \cdot (x+y) - (2x-y) \cdot 1}{(x+y)^2} = \frac{3y}{(x+y)^2}$ 

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$z_y = 1 \cdot e^y$$

$$z_y = 1 \cdot e^y + y \cdot e^y + 0 = (1+y)e^y$$

$$\cdot e^{y} + y \cdot$$

$$\cdot e^y + y \cdot$$

c) 
$$u(x,y) = \frac{2x-y}{x+y}$$

 $(e^x)'=e^x$ 

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a) 
$$z = xe^y$$
 b)  $z = ye^y + \sqrt{x}$ 

 $(x+y)^2$ 

c) 
$$u(x,y) = \frac{2x}{x+1}$$

Rješenje 
$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$

a) 
$$z_x = e^y \cdot 1 = e^y$$

$$y = x \cdot e^y = x$$

$$z_y = 1 \cdot e^y + y \cdot e^y + 0 = (1+y)e^y$$

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$
  $z_y = 1 \cdot e^y + \frac{1}{2\sqrt{x}}$   
c)  $u_x = \frac{2 \cdot (x+y) - (2x-y) \cdot 1}{(x+y)^2} = \frac{3y}{(x+y)^2}$ 

$$z_y = 1 \cdot e^y$$

$$(e^{x})' = e^{y} + 0 = (1+y)e^{y}$$
$$(e^{x})' = e^{x}$$

 $\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$ 

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$$+\frac{1}{2\sqrt{x}}=\frac{1}{2\sqrt{x}}$$
  $z_y=1\cdot e^y+1$ 

$$c \cdot 1 = e^y$$
  $z_y = x \cdot e^y = xe^y$ 

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

Zadatak 5

$$z_y = 1 \cdot e^y + y$$

$$z_y = x \cdot e^y = 1$$

$$v(x)^2$$

$$z_y = x \cdot e^y = x\epsilon$$

$$z_y = x \cdot e^y = xe^y \qquad (cu)'(x) = c \cdot u'(x)$$

$$v'(x) \cdot v(x) - u(x) \cdot v'(x)$$

$$(x) \cdot v'(x)$$

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

$$y = ye^{x} + \sqrt{x}$$

$$(x,y) = x + (x)$$

b) 
$$z = ye^y + \sqrt{x}$$

$$x + y$$

c) 
$$u(x, y) = \frac{2x - y}{x + y}$$

a) 
$$z = xe^{y}$$
 b)  $z = ye^{y} + \sqrt{x}$ 

c) 
$$u(x,y) = \frac{2x - y}{x + y}$$

Rješenje 
$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$
a)  $z_x = e^y \cdot 1 = e^y$   $z_y = x \cdot e^y = xe^y$ 

$$z_y = x \cdot e^y = xe^y$$
  $(cu)'(x) = c \cdot u'(x)$ 

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

Zadatak 5

$$\cdot e^y + y \cdot$$

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

 $u_y = \frac{-1}{(x+y)^2}$ 

$$\frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$e^y + y$$
.

$$z_y = 1 \cdot e^y + y \cdot e^y + 0 = (1+y)e^y$$

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$
  $z_y = 1 \cdot e^y + y \cdot e^y$ 

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

 $(e^x)'=e^x$ 

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 $\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$ 

a) 
$$z = xe^{y}$$
 b)  $z = ye^{y} + \sqrt{x}$ 

c) 
$$u(x,y) = \frac{2x-y}{x+y}$$

Rješenje 
$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$

c)  $u_x = \frac{2 \cdot (x+y) - (2x-y) \cdot 1}{(x+y)^2} = \frac{3y}{(x+y)^2}$ 

a) 
$$z_x = e^y \cdot 1 = e^y$$

 $u_y = \frac{-1 \cdot (x+y)^2}{}$ 

$$z_y = 1 \cdot e^y + y \cdot e^y + 0 = (1+y)e^y$$

 $(e^x)'=e^x$ 

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 $\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$ 

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

Zadatak 5

$$z_y = 1 \cdot e^{y}$$

$$e^y + y$$
.

$$z_y = x \cdot e^y = xe^y$$
  $(cu)'(x) = c \cdot u'(x)$ 

$$\frac{1}{2\sqrt{z}}$$
 z

$$\cdot e^y + 1$$

$$+y\cdot$$

$$\cdot e^y$$

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

a) 
$$z = xe^{y}$$
 b)  $z = ye^{y} + \sqrt{x}$ 

Rješenje 
$$\frac{\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2} }{v(x)^2}$$

c)  $u_x = \frac{2 \cdot (x+y) - (2x-y) \cdot 1}{(x+y)^2} = \frac{3y}{(x+y)^2}$ 

 $u_y = \frac{-1 \cdot (x+y)}{(x+y)^2}$ 

Rješenje 
$$v'$$
a)  $z_x = e^y \cdot 1 = e^y$ 

Zadatak 5

$$Z_{j}$$

$$Z_y$$

$$z_y = x \cdot e^y = xe^y \qquad (cu)'(x) = c \cdot u'(x)$$
$$z_y = 1 \cdot e^y + y \cdot e^y + 0 = (1+y)e^y$$

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

 $(e^x)'=e^x$ 

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 $\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$ 

c) 
$$u(x,y) = \frac{2x-y}{x+y}$$

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

a) 
$$z = xe^y$$
 b)  $z = ye^y + \sqrt{x}$ 

Rješenje 
$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$

a) 
$$z_x = e^y \cdot 1 = e^y$$

Zadatak 5

$$z_y = x \cdot e^y = xe^y$$
  $(cu)'(x) = c \cdot u'(x)$ 

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

 $u_y = \frac{-1 \cdot (x+y) - (x+y)^2}{(x+y)^2}$ 

$$z_{
m v}=1\cdot{
m e}^{
m v}$$

$$z_y = 1 \cdot e^y + y \cdot e^y + 0 = (1+y)e^y$$

c) 
$$u_x = \frac{2 \cdot (x+y) - (2x-y) \cdot 1}{(x+y)^2} = \frac{3y}{(x+y)^2}$$

$$z_y = 1 \cdot e^{-\frac{1}{2}}$$

$$=1\cdot e^{y}$$

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

 $(e^x)'=e^x$ 

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 $\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$ 

a) 
$$z = xe^{y}$$
 b)  $z = ye^{y} + \sqrt{x}$ 

a) 
$$z = xe^y$$
 b)  $z = ye^y + \sqrt{x}$  c)  $u(x, y) = \frac{2x - y}{x + y}$ 

Rješenje 
$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$

a) 
$$z_x = e^y \cdot 1 = e^y$$
  $z_y = x \cdot e^y = xe^y$   $(cu)'(x) = c \cdot u'(x)$ 

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

Zadatak 5

$$z_y = 1 \cdot e^y$$

$$1 \cdot e^y +$$

$$z_y = 1 \cdot e^y + y \cdot e^y + 0 = (1+y)e^y$$

$$\frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

c)  $u_x = \frac{2 \cdot (x+y) - (2x-y) \cdot 1}{(x+y)^2} = \frac{3y}{(x+y)^2}$ 

 $u_y = \frac{-1 \cdot (x+y) - (2x-y)}{(x+y)^2}$ 

$$e^y + y$$
.

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

 $(e^x)'=e^x$ 

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 $\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$ 

a) 
$$z = xe^{y}$$
 b)  $z = ye^{y} + \sqrt{x}$ 

c) 
$$u(x,y) = \frac{2x - y}{x + y}$$

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

Zadatak 5

$$e^y + y$$
.

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$
  $z_y = 1 \cdot e^y + y \cdot e^y + 0 = (1+y)e^y$ 

c) 
$$u_x = \frac{2 \cdot (x+y) - (2x-y) \cdot 1}{(x+y)^2} = \frac{3y}{(x+y)^2}$$

 $u_y = \frac{-1 \cdot (x+y) - (2x-y) \cdot }{(x+y)^2}$ 

$$z_y = 1 \cdot e^y + y$$

$$(e^{x})'=e^{x}$$

 $\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$ 

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$$z_y = 1 \cdot e^y + 1$$

a) 
$$z = xe^y$$
 b)  $z = ye^y + \sqrt{x}$ 

Rješenje 
$$\frac{\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2} }{v(x)^2}$$
a)  $z_x = e^y \cdot 1 = e^y$   $z_y = x \cdot e^y = xe^y$ 

$$\frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$z_y = x \cdot e^y = xe^y$$
  $(cu)'(x) = c \cdot u'(x)$ 

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$
  $z_y = 1 \cdot e^y + y$ 

$$\frac{1}{2y} = x \cdot e^y + y$$

$$\frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

 $u_y = \frac{-1 \cdot (x+y) - (2x-y) \cdot 1}{(x+y)^2}$ 

Zadatak 5

$$z_y = 1 \cdot e^y + y \cdot e^y + 0 = (1+y)e^y$$

$$(e^x)' = e^x$$

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$$z_y = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$
  $z_y = 1 \cdot e^y + y$ 

$$\frac{(e^x)' = e^x}{(x+y)^2}$$

$$\frac{(x+y)-(2x-y)\cdot 1}{(x+y)^2}=\frac{3y}{(x+y)^2}$$

c) 
$$u_x = \frac{2 \cdot (x+y) - (2x-y) \cdot 1}{(x+y)^2} = \frac{3y}{(x+y)^2}$$

$$(e^{x})' = e^{x}$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$\frac{(x+y)-(2x-y)\cdot 1}{(x+y)^2} = \frac{3y}{(x+y)^2}$$

$$\frac{-(2x-y)\cdot 1}{+y)^2} = \frac{3y}{(x+y)^2}$$

$$d_x = \frac{2 \cdot (x+y) - (2x-y) \cdot 1}{(x+y)^2} = \frac{3y}{(x+y)^2}$$

c) 
$$u_x = \frac{2(x+y)(2x-y)^2}{(x+y)^2} = \frac{3y}{(x+y)^2}$$

a) 
$$z = xe^y$$
 b)  $z = ye^y + \sqrt{x}$ 

Zadatak 5

Rješenje 
$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$

a) 
$$z_x = e^y \cdot 1 = e^y$$
  $z_y = x \cdot e^y = xe^y$   $(cu)'(x) = c \cdot u'(x)$ 

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$
  $z_y = 1 \cdot e^y + y \cdot e^y$ 

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$
  $z_y = 1 \cdot e^y + y$ 

c)  $u_x = \frac{2 \cdot (x+y) - (2x-y) \cdot 1}{(x+y)^2} = \frac{3y}{(x+y)^2}$ 

 $u_y = \frac{-1 \cdot (x+y) - (2x-y) \cdot 1}{(x+y)^2} = \frac{1}{(x+y)^2}$ 

$$\frac{1}{\sqrt{x}} \qquad z_y = 1 \cdot e^y + y$$

$$z_y = 1 \cdot e^y + y \cdot e^y + 0 = (1+y)e^y$$

$$(cu)'(x) = c \cdot u'(x)$$

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

 $(e^x)'=e^x$ 

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 $\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$ 

$$cu)(x) = c \cdot u(x)$$

a) 
$$z = xe^y$$
 b)  $z = ye^y + \sqrt{x}$ 

Rješenje 
$$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$$

a) 
$$z_x = e^y \cdot 1 = e^y$$
  $z_y = x \cdot e^y = xe^y$   $(cu)'(x) = c \cdot u'(x)$ 

 $u_y = \frac{-1 \cdot (x+y) - (2x-y) \cdot 1}{(x+y)^2} = \frac{-3x}{(x+y)^2}$ 

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$
  $z_y = 1 \cdot e$ 

b) 
$$z_x = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$2 \cdot (x + y) - (2x + y) = \frac{1}{2\sqrt{x}}$$

Zadatak 5

$$-\frac{1}{2\sqrt{z}} = \frac{1}{2\sqrt{z}} \qquad z_{y} = 1 \cdot e^{y} + y$$

$$z_y = 1 \cdot e^y + y \cdot e^y$$

$$z_y = 1 \cdot e^y + y$$

$$z_y = 1 \cdot e^y + y \cdot e^y + 0 = (1+y)e^y$$
  
( $e^x$ )' =  $e^x$ 

$$(e^{x})'=e^{x}$$

$$= \frac{3y}{(e^x)^2}$$

 $(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ 

$$\frac{(e^x)' = e^x}{(\sqrt{x})' = \frac{1}{2\sqrt{x}}}$$

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$$\frac{(2x-y)\cdot 1}{(x+y)^2} = \frac{3y}{(x+y)^2}$$

c) 
$$u_x = \frac{2 \cdot (x+y) - (2x-y) \cdot 1}{(x+y)^2} = \frac{3y}{(x+y)^2}$$

$$(x-y)\cdot 1 = \frac{3y}{(x-y)^2}$$

$$7 - 1 \circ V + V$$

$$x = x \cdot e^y = xe^y$$
 (



šesti zadatak

#### Zadatak 6

Odredite parcijalne derivacije sljedećih funkcija:

a) 
$$z = 2^{\sin \frac{y}{x}}$$

b) 
$$z = x^y$$

c) 
$$f(x, y, z) = e^{2xz} - \ln(yz) + 1$$

$$(a^x)'=a^x$$

$$(a^x)' = a^x \ln a$$
  $(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$ 

$$z_x =$$

$$z=2^{\sin\frac{y}{x}}$$

$$(a^x)' = a^x \ln$$

$$(a^x)' = a^x \ln a$$
  $(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$ 

$$a) z_x = 2^{\sin\frac{y}{x}} \ln 2$$

$$z=2^{\sin\frac{y}{x}}$$

$$(a^x)' = a^x \ln$$

$$(a^x)' = a^x \ln a$$
  $(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$ 

$$a) z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot$$

$$z=2^{\sin\frac{y}{x}}$$

$$(a^x)'=a^x \ln a$$

$$(a^{x})' = a^{x} \ln a$$
  $(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$ 

a) 
$$z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)'_x$$

$$z=2^{\sin\frac{y}{x}}$$

$$(a^x)'=a^x \ln a$$

$$(a^{x})' = a^{x} \ln a$$
  $(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$ 

a) 
$$z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)_x' = 2^{\sin \frac{y}{x}} \ln 2$$

$$z=2^{\sin\frac{y}{x}}$$

$$(a^{x})' = a^{x} \ln a$$
  $(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$ 

$$(\sin x) = \cos x$$

 $z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)' = 2^{\sin \frac{y}{x}} \ln 2 \cdot$ a)

$$z=2^{\sin\frac{y}{x}}$$

$$(a^x)' = a^x \ln a$$
  $(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$ 

$$(\sin x)' = \cos x$$

$$(\sin x)' = \cos x$$
 
$$(\sin (\text{nešto}))' = \cos (\text{nešto}) \cdot (\text{nešto})'$$

$$a = a \sin \frac{y}{2} + a \cos \frac{y}{2}$$

a) 
$$z_x = 2^{\sin\frac{y}{x}} \ln 2 \cdot \left(\sin\frac{y}{x}\right)_x' = 2^{\sin\frac{y}{x}} \ln 2 \cdot \cos\frac{y}{x}$$

$$z=2^{\sin\frac{y}{x}}$$

$$(a^{x})' = a^{x} \ln a$$
  $(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$ 

$$(\sin x)' = \cos x$$
  $(\sin (\text{nešto}))' = \cos (\text{nešto}) \cdot (\text{nešto})'$ 

$$z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x}$$

$$z=2^{\sin\frac{y}{x}}$$

a)

$$(a^{x})' = a^{x} \ln a \qquad (a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$$

a) 
$$z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \left(\frac{y}{x}\right)'$$

$$z=2^{\sin\frac{y}{x}}$$

$$(a^{x})' = a^{x} \ln a$$
 
$$(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$$

$$(3112) = 032$$

a) 
$$z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \left(\frac{y}{x}\right)' =$$

$$z = 2^{\sin \frac{y}{x}} = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x}$$

$$\left(\frac{9}{x}\right)_x = 2^{\sin x} \ln 2 \cdot \cos \frac{9}{x} \cdot \left(\frac{9}{x}\right)_x =$$

$$(a^{x})' = a^{x} \ln a$$
  $(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$ 

$$(\sin x)' = \cos x$$
 
$$(\sin (\text{nešto}))' = \cos (\text{nešto}) \cdot (\text{nešto})'$$

$$y$$
)'  $a\sin \frac{y}{2}$   $a\sin \frac{y}{2}$   $a\sin \frac{y}{2}$ 

$$2\sin \frac{y}{y}$$
 In  $2\cos \frac{y}{y}$ 

$$z_{x} = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)_{x}' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \left(\frac{y}{x}\right)_{x}' =$$

$$z = 2^{\sin \frac{y}{x}} = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x}.$$

$$(a^{x})' = a^{x} \ln a$$
  $(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$ 

$$(\sin x)' = \cos x$$
 
$$(\sin (\text{nešto}))' = \cos (\text{nešto}) \cdot (\text{nešto})'$$

a) 
$$z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{y}\right)' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{y} \cdot \left(\frac{y}{y}\right)' =$$

$$z = 2^{\sin \frac{y}{x}} = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \frac{-y}{x^2}$$

a)

$$(a^x)' = a^x \ln a$$
  $(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$ 

 $z = 2^{\sin \frac{y}{x}} = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \frac{-y}{x^2} = -\frac{y}{x^2}$ 

 $(\sin x)' = \cos x$   $(\sin (\text{nešto}))' = \cos (\text{nešto}) \cdot (\text{nešto})'$ 

 $z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{y}\right)' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{y} \cdot \left(\frac{y}{y}\right)' =$ 

$$(a^{x})' = a^{x} \ln a$$
 
$$(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$$

$$(\sin x) = \cos x$$

a) 
$$z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)_x' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \left(\frac{y}{x}\right)_x' =$$

$$z = 2^{\sin \frac{y}{x}} = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \frac{-y}{x^2} = -\frac{y}{x^2} \cdot 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x}$$

a)

 $z_v =$ 

$$(a^x)' = a^x \ln a$$
  $(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$ 

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- $(\sin x)' = \cos x$   $(\sin (\text{nešto}))' = \cos (\text{nešto}) \cdot (\text{nešto})'$
- $z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \left(\frac{y}{x}\right)' =$

$$(a^{x})' = a^{x} \ln a \qquad (a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$$

$$(\sin x)' = \cos x$$

a) 
$$z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)_x' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \left(\frac{y}{x}\right)_x' =$$

$$z = 2^{\sin \frac{y}{x}} = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \frac{-y}{x^2} = -\frac{y}{x^2} \cdot 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x}$$

$$z_{\rm v}=2^{\sin{rac{y}{x}}}\ln{2}$$

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$$(a^{x})' = a^{x} \ln a$$
  $(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$ 

$$(\sin x)' = \cos x$$
  $(\sin (\text{nešto}))' = \cos (\text{nešto}) \cdot (\text{nešto})'$ 

$$z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{y}\right)' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{y} \cdot \left(\frac{y}{y}\right)' =$$

a) 
$$z_{x} = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)$$

$$z = 2^{\sin \frac{y}{x}} = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \frac{-y}{x^2} = -\frac{y}{x^2} \cdot 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x}$$

$$z_{\rm v}=2^{\sin{\frac{y}{x}}}\ln{2}$$
 .

$$(a^{x})' = a^{x} \ln a$$
  $(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$ 

$$(\sin x) = \cos x$$

a) 
$$z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{y}\right)' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{y} \cdot \left(\frac{y}{y}\right)' =$$

$$z = 2^{\sin \frac{y}{x}} = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \frac{-y}{x^2} = -\frac{y}{x^2} \cdot 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x}$$

$$z_y = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)'$$

$$(a^{x})' = a^{x} \ln a$$
  $(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$ 

$$(3112) = 032$$

a) 
$$z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{y}\right)' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{y} \cdot \left(\frac{y}{y}\right)' =$$

$$z = 2^{\sin \frac{y}{x}} = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \frac{-y}{x^2} = -\frac{y}{x^2} \cdot 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x}$$

$$z_y = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)_y' = 2^{\sin \frac{y}{x}} \ln 2$$

$$(a^{x})' = a^{x} \ln a$$
 
$$(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$$

$$(311 \times) = 03 \times$$

a) 
$$z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \left(\frac{y}{x}\right)' =$$

$$z = 2^{\sin \frac{y}{x}} = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \frac{-y}{x^2} = -\frac{y}{x^2} \cdot 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x}$$

$$z_y = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)_x' = 2^{\sin \frac{y}{x}} \ln 2 \cdot$$

$$(a^{x})' = a^{x} \ln a$$
  $(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$ 

$$(\sin x)' = \cos x$$
  $(\sin (\text{nešto}))' = \cos (\text{nešto}) \cdot (\text{nešto})'$ 

$$\sin^y$$
)' —  $2\sin^y$  In 2  $\cos^y$  (y)' —

a) 
$$z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)_x' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \left(\frac{y}{x}\right)_x' =$$

$$z = 2^{\sin \frac{y}{x}} = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \frac{-y}{x^2} = -\frac{y}{x^2} \cdot 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x}$$

$$z_y = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)_x' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x}$$

$$(a^{x})' = a^{x} \ln a$$
  $(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$ 

$$(\sin x) = \cos x$$

a) 
$$z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \left(\frac{y}{x}\right)' =$$

$$z = 2^{\sin \frac{y}{x}} = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \frac{-y}{x^2} = -\frac{y}{x^2} \cdot 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x}$$

$$z_y = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)_y' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x}$$

$$(a^x)' = a^x \ln a$$
  $(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$ 

$$(\sin x)^{r} = \cos x$$

a) 
$$z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \left(\frac{y}{x}\right)' =$$

$$z = 2^{\sin \frac{y}{x}} = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \frac{-y}{x^2} = -\frac{y}{x^2} \cdot 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x}$$

$$z_y = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)_y' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \left(\frac{y}{x}\right)_y'$$

$$(a^{x})' = a^{x} \ln a$$
  $(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$ 

$$(\sin x)' = \cos x$$
 
$$(\sin (\text{nešto}))' = \cos (\text{nešto}) \cdot (\text{nešto})'$$

a) 
$$z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \left(\frac{y}{x}\right)' =$$

$$z = 2^{\sin \frac{y}{x}} = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \frac{-y}{x^2} = -\frac{y}{x^2} \cdot 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x}$$

$$z_y = 2^{\sin\frac{y}{x}} \ln 2 \cdot \left(\sin\frac{y}{x}\right)_y' = 2^{\sin\frac{y}{x}} \ln 2 \cdot \cos\frac{y}{x} \cdot \left(\frac{y}{x}\right)_y' =$$
$$= 2^{\sin\frac{y}{x}} \ln 2 \cdot \cos\frac{y}{x}$$

a)

$$(a^{x})' = a^{x} \ln a \qquad (a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$$

 $(\sin x)' = \cos x$   $(\sin (\text{nešto}))' = \cos (\text{nešto}) \cdot (\text{nešto})'$ 

$$(\sin x) = \cos x$$

$$z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \left(\frac{y}{x}\right)' =$$

$$z = 2^{\sin \frac{y}{x}} = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \frac{-y}{x^2} = -\frac{y}{x^2} \cdot 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x}$$

$$z_y = 2^{\sin\frac{y}{x}} \ln 2 \cdot \left(\sin\frac{y}{x}\right)_y' = 2^{\sin\frac{y}{x}} \ln 2 \cdot \cos\frac{y}{x} \cdot \left(\frac{y}{x}\right)_y' =$$
$$= 2^{\sin\frac{y}{x}} \ln 2 \cdot \cos\frac{y}{x} \cdot \left(\frac{y}{x}\right)_y' =$$

a)

$$(a^{x})' = a^{x} \ln a \qquad (a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$$

$$(\sin x)' = \cos x$$
  $(\sin (\text{nešto}))' = \cos (\text{nešto}) \cdot (\text{nešto})'$ 

$$z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \left(\frac{y}{x}\right)' =$$

$$z_y = 2^{\sin\frac{y}{x}} \ln 2 \cdot \left(\sin\frac{y}{x}\right)_y' = 2^{\sin\frac{y}{x}} \ln 2 \cdot \cos\frac{y}{x} \cdot \left(\frac{y}{x}\right)_y' =$$

$$= 2^{\sin\frac{y}{x}} \ln 2 \cdot \cos\frac{y}{x} \cdot \frac{1}{x}$$

$$(a^{x})' = a^{x} \ln a \qquad (a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$$

$$(\sin x)' = \cos x$$
  $(\sin (\text{nešto}))' = \cos (\text{nešto}) \cdot (\text{nešto})'$ 

a) 
$$z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \left(\frac{y}{x}\right)' =$$

$$z_y = 2^{\sin\frac{y}{x}} \ln 2 \cdot \left(\sin\frac{y}{x}\right)_y' = 2^{\sin\frac{y}{x}} \ln 2 \cdot \cos\frac{y}{x} \cdot \left(\frac{y}{x}\right)_y' =$$

$$= 2^{\sin\frac{y}{x}} \ln 2 \cdot \cos\frac{y}{x} \cdot \frac{1}{x} = \frac{1}{x}.$$

$$(a^{x})' = a^{x} \ln a$$
  $(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$ 

$$(\sin x)' = \cos x$$
 
$$(\sin (\text{nešto}))' = \cos (\text{nešto}) \cdot (\text{nešto})'$$

a) 
$$z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{y}\right)' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{y} \cdot \left(\frac{y}{y}\right)' =$$

$$z = 2^{\sin \frac{y}{x}} = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \frac{-y}{x^2} = -\frac{y}{x^2} \cdot 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x}$$

$$z_y = 2^{\sin\frac{y}{x}} \ln 2 \cdot \left(\sin\frac{y}{x}\right)_y' = 2^{\sin\frac{y}{x}} \ln 2 \cdot \cos\frac{y}{x} \cdot \left(\frac{y}{x}\right)_y' =$$

$$= 2^{\sin\frac{y}{x}} \ln 2 \cdot \cos\frac{y}{x} \cdot \frac{1}{x} = \frac{1}{x} \cdot 2^{\sin\frac{y}{x}} \ln 2 \cdot \cos\frac{y}{x}$$

$$(x^n)' = nx^{n-1} \qquad (\sin x)' = \cos x \qquad (\sin (\text{nešto}))' = \cos (\text{nešto}) \cdot (\text{nešto})'$$

 $(a^x)' = a^x \ln a$   $(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$ 

a) 
$$z_x$$

 $z = x^y$ 

b)  $z_x$ 

 $z_y = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \left(\frac{y}{x}\right)' =$ 

 $=2^{\sin\frac{y}{x}}\ln 2\cdot\cos\frac{y}{x}\cdot\frac{1}{x}=\frac{1}{x}\cdot2^{\sin\frac{y}{x}}\ln 2\cdot\cos\frac{y}{x}$ 

 $z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \left(\frac{y}{x}\right)' =$ 

 $z = 2^{\sin \frac{y}{x}} = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \frac{-y}{x^2} = -\frac{y}{x^2} \cdot 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x}$ 

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 $(x^n)' = nx^{n-1}$   $(\sin x)' = \cos x$   $(\sin (\text{nešto}))' = \cos (\text{nešto}) \cdot (\text{nešto})'$ 

$$(3112) = 6032$$

a)

a) 
$$z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)$$

a) 
$$z_x = 2^{\sin \frac{x}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)$$

$$z_x = 2^{\sin x} \ln 2 \cdot \left(\sin \frac{x}{x}\right)$$

$$2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)$$

 $z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \left(\frac{y}{x}\right)' =$ 

 $(a^x)' = a^x \ln a$   $(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$ 

$$\frac{y}{2} \cdot 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x}$$

$$z = 2^{\sin \frac{y}{x}} = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \frac{-y}{x^2} = -\frac{y}{x^2} \cdot 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x}$$

$$z_y = 2^{\sin\frac{y}{x}} \ln 2 \cdot \left(\sin\frac{y}{y}\right)' = 2^{\sin\frac{y}{x}} \ln 2 \cdot \cos\frac{y}{y} \cdot \left(\frac{y}{y}\right)' =$$

$$= 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \frac{1}{1} = \frac{1}{1} \cdot 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x}$$

$$= 2^{\sin\frac{y}{x}} \ln 2 \cdot \cos\frac{y}{x} \cdot \frac{1}{x} = \frac{1}{x} \cdot 2^{\sin\frac{y}{x}} \ln 2 \cdot \cos\frac{y}{x}$$

b)  $z_x = vx^{y-1}$ 

 $z = x^y$ 

b)  $z_{x} = yx^{y-1}$ 

- $(x^n)' = nx^{n-1}$   $(\sin x)' = \cos x$   $(\sin (\text{nešto}))' = \cos (\text{nešto}) \cdot (\text{nešto})'$
- $z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \left(\frac{y}{x}\right)' =$ a)
- $z = 2^{\sin \frac{y}{x}} = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \frac{-y}{x^2} = -\frac{y}{x^2} \cdot 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x}$

 $z_v =$ 

 $(a^x)' = a^x \ln a$   $(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$ 

- $z_y = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \left(\frac{y}{x}\right)' =$ 

  - $= 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \frac{1}{x} = \frac{1}{x} \cdot 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x}$

 $z = x^y$ 

b)  $z_{x} = yx^{y-1}$ 

- $(x^n)' = nx^{n-1}$   $(\sin x)' = \cos x$   $(\sin (\text{nešto}))' = \cos (\text{nešto}) \cdot (\text{nešto})'$

$$osin \frac{y}{2} + o (y)$$

- $z_x = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \left(\frac{y}{x}\right)' =$ a)

$$= 2^{\sin \frac{\pi}{x}} \ln 2 \cdot \left(\sin \frac{\pi}{x}\right)$$

$$\left(\frac{y}{x}\right)$$

 $(a^x)' = a^x \ln a$   $(a^{\text{nešto}})' = a^{\text{nešto}} \ln a \cdot (\text{nešto})'$ 

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- $z_y = 2^{\sin \frac{y}{x}} \ln 2 \cdot \left(\sin \frac{y}{x}\right)' = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \left(\frac{y}{x}\right)' =$

 $=2^{\sin\frac{y}{x}}\ln 2\cdot\cos\frac{y}{x}\cdot\frac{1}{x}=\frac{1}{x}\cdot2^{\sin\frac{y}{x}}\ln 2\cdot\cos\frac{y}{x}$ 

 $z_v = x^y \ln x$ 

- $z = 2^{\sin \frac{y}{x}} = 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x} \cdot \frac{-y}{x^2} = -\frac{y}{x^2} \cdot 2^{\sin \frac{y}{x}} \ln 2 \cdot \cos \frac{y}{x}$

$$f(x,y,z) = e^{2xz} - \ln(yz) + 1$$

 $f_x = e^{2xz}$ 

$$f(x,y,z) = e^{2xz} - \ln(yz) + 1$$

 $f_x = e^{2xz} \cdot (2xz)_x'$ 

$$f(x, y, z) = e^{2xz} - \ln(yz) + 1$$

 $f_x = e^{2xz} \cdot (2xz)_x' - 0$ 

$$f(x,y,z) = e^{2xz} - \ln(yz) + 1$$

 $f_x = e^{2xz} \cdot (2xz)_x' - 0 + 0$ 

$$f(x,y,z)=e^{2xz}-\ln{(yz)}+1$$

 $(e^{\text{nešto}})' = e^{\text{nešto}} \cdot (\text{nešto})'$   $(e^x)' = e^x$ 

 $f_x = e^{2xz} \cdot (2xz)_x' - 0 + 0 = e^{2xz}$ 

$$f(x,y,z) = e^{2xz} - \ln(yz) + 1$$

 $(e^{\text{nešto}})' = e^{\text{nešto}} \cdot (\text{nešto})'$   $(e^{x})' = e^{x}$ 

 $f_x = e^{2xz} \cdot (2xz)_x' - 0 + 0 = e^{2xz}$ 

$$f(x,y,z)=e^{2xz}-\ln{(yz)}+1$$

 $(e^{\text{nešto}})' = e^{\text{nešto}} \cdot (\text{nešto})'$   $(e^{x})' = e^{x}$ 

 $f_x = e^{2xz} \cdot (2xz)_x' - 0 + 0 = e^{2xz} \cdot 2z$ 

$$f(x,y,z)=e^{2xz}-\ln{(yz)}+1$$

 $(e^{\mathsf{nešto}})' = e^{\mathsf{nešto}} \cdot (\mathsf{nešto})'$   $(e^x)' = e^x$ 

$$f(x, y, z) = e^{2xz} - \ln(yz) + 1$$

 $f_x = e^{2xz} \cdot (2xz)_x' - 0 + 0 = e^{2xz} \cdot 2z = 2ze^{2xz}$   $f_y =$ 

$$f(x,y,z) = e^{2xz} - \ln(yz) + 1$$

 $(e^{\mathsf{nešto}})' = e^{\mathsf{nešto}} \cdot (\mathsf{nešto})'$   $(e^x)' = e^x$ 

 $f_x = e^{2xz} \cdot (2xz)_x' - 0 + 0 = e^{2xz} \cdot 2z = 2ze^{2xz}$  $f_y = 0$ 

$$f(x,y,z)=e^{2xz}-\ln{(yz)}+1$$

 $(e^{\mathsf{nešto}})' = e^{\mathsf{nešto}} \cdot (\mathsf{nešto})'$   $(e^x)' = e^x$ 

$$f_x = e^{2xz} \cdot (2xz)'_x - 0 + 0 = e^{2xz} \cdot 2z = 2ze^{2xz}$$

 $f_{v} = 0 -$ 

$$f(x,y,z)=e^{2xz}-\ln{(yz)}+1$$

 $(e^{\text{nešto}})' = e^{\text{nešto}} \cdot (\text{nešto})'$   $(e^x)' = e^x$ 

 $\left(\ln\left(\text{nešto}\right)\right)' = \frac{1}{\text{nešto}} \cdot \left(\text{nešto}\right)'$   $\left(\ln x\right)' = \frac{1}{x}$ 

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$$f_x = e^{2xz} \cdot (2xz)'_x - 0 + 0 = e^{2xz} \cdot 2z = 2ze^{2xz}$$

 $f(x, y, z) = e^{2xz} - \ln(yz) + 1$ 

 $(e^{\text{nešto}})' = e^{\text{nešto}} \cdot (\text{nešto})'$   $(e^{\text{x}})' = e^{\text{x}}$ 

 $\left(\ln\left(\text{nešto}\right)\right)' = \frac{1}{\text{nešto}} \cdot \left(\text{nešto}\right)'$   $\left(\ln x\right)' = \frac{1}{x}$ 

 $f_y=0-\frac{1}{vz}$ 

$$\left(\ln\left(\text{nešto}\right)\right)' = \frac{1}{\text{nešto}} \cdot \left(\text{nešto}\right)'$$

$$f_x = e^{2xz} \cdot (2xz)'_x - 0 + 0 = e^{2xz} \cdot 2z = 2ze^{2xz}$$

 $(e^{\text{nešto}})' = e^{\text{nešto}} \cdot (\text{nešto})'$   $(e^{\text{x}})' = e^{\text{x}}$ 

$$f(x, y, z) = e^{2xz} - \ln(yz) + 1$$

 $f_y = 0 - \frac{1}{vz} \cdot (yz)'_y$ 

$$\left(\ln\left(\text{nešto}\right)\right)' = \frac{1}{\text{nešto}} \cdot \left(\text{nešto}\right)'$$

$$f_x = e^{2xz} \cdot (2xz)'_x - 0 + 0 = e^{2xz} \cdot 2z = 2ze^{2xz}$$

$$f_y = 0 - \frac{1}{vz} \cdot (yz)_y' + 0$$

$$f(x,y,z)=e^{2xz}-\ln{(yz)}+1$$

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 $f_x = e^{2xz} \cdot (2xz)'_x - 0 + 0 = e^{2xz} \cdot 2z = 2ze^{2xz}$ 

 $f_y = 0 - \frac{1}{vz} \cdot (yz)'_y + 0 = -\frac{1}{vz}$ 

 $f(x, y, z) = e^{2xz} - \ln(yz) + 1$ 

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$$f_x = e^{2xz} \cdot (2xz)'_x - 0 + 0 = e^{2xz} \cdot 2z = 2ze^{2xz}$$

 $f_y = 0 - \frac{1}{vz} \cdot (yz)'_y + 0 = -\frac{1}{vz} \cdot$ 

$$f(x,y,z)=e^{2xz}-\ln{(yz)}+1$$

$$f_y = 0 - \frac{1}{vz} \cdot (yz)'_y + 0 = -\frac{1}{vz} \cdot z$$

 $\left(\ln\left(\text{nešto}\right)\right)' = \frac{1}{\text{nešto}} \cdot \left(\text{nešto}\right)'$   $\left(\ln x\right)' = \frac{1}{x}$ 

 $f(x, y, z) = e^{2xz} - \ln(yz) + 1$ 

$$f_y = 0 - \frac{1}{vz} \cdot (yz)'_y + 0 = -\frac{1}{vz} \cdot z = -\frac{1}{v}$$

yz yz yz yz yz

 $f_x = e^{2xz} \cdot (2xz)'_x - 0 + 0 = e^{2xz} \cdot 2z = 2ze^{2xz}$ 

$$f(x,y,z)=e^{2xz}-\ln{(yz)}+1$$

 $(e^{\text{nešto}})' = e^{\text{nešto}} \cdot (\text{nešto})'$   $(e^x)' = e^x$ 

 $\left(\ln\left(\text{nešto}\right)\right)' = \frac{1}{\text{nešto}} \cdot (\text{nešto})'$   $\left(\ln x\right)' = \frac{1}{x}$ 

$$\left(\ln\left(\text{nešto}\right)\right)' = \frac{1}{\text{nešto}} \cdot \left(\text{nešto}\right)'$$
  $\left(\ln x\right)' = \frac{1}{x}$ 

$$f_x = e^{2xz} \cdot (2xz)'_x - 0 + 0 = e^{2xz} \cdot 2z = 2ze^{2xz}$$

$$f_y = 0 - \frac{1}{yz} \cdot (yz)'_y + 0 = -\frac{1}{yz} \cdot z = -\frac{1}{y}$$
 $f_z = 0$ 

$$f(x, y, z) = e^{2xz} - \ln(yz) + 1$$

$$f_y = 0 - \frac{1}{vz} \cdot (yz)'_y + 0 = -\frac{1}{vz} \cdot z = -\frac{1}{v}$$

 $\left(\ln\left(\text{nešto}\right)\right)' = \frac{1}{\text{nešto}} \cdot \left(\text{nešto}\right)'$   $\left(\ln x\right)' = \frac{1}{x}$ 

 $f_z = e^{2xz}$ 

$$f(x,y,z)=e^{2xz}-\ln{(yz)}+1$$

 $f_{x} = e^{2xz} \cdot (2xz)'_{x} - 0 + 0 = e^{2xz} \cdot 2z = 2ze^{2xz}$  $f_y = 0 - \frac{1}{vz} \cdot (yz)'_y + 0 = -\frac{1}{vz} \cdot z = -\frac{1}{v}$ 

$$f_z = e^{2xz} \cdot (2xz)'_z$$

$$f(x,y,z)=e^{2xz}-\ln{(yz)}+1$$

 $(e^{\text{nešto}})' = e^{\text{nešto}} \cdot (\text{nešto})'$   $(e^x)' = e^x$ 

 $\left(\ln\left(\text{nešto}\right)\right)' = \frac{1}{\text{nešto}} \cdot (\text{nešto})'$   $\left(\ln x\right)' = \frac{1}{x}$ 

$$f_y = 0 - \frac{1}{vz} \cdot (yz)'_y + 0 = -\frac{1}{vz} \cdot z = -\frac{1}{v}$$

 $\left(\ln\left(\text{nešto}\right)\right)' = \frac{1}{\text{nešto}} \cdot \left(\text{nešto}\right)'$   $\left(\ln x\right)' = \frac{1}{x}$ 

 $f_z = e^{2xz} \cdot (2xz)_z' -$ 

$$f(x,y,z) = e^{2xz} - \ln(yz) + 1$$

$$f_y = 0 - \frac{1}{yz} \cdot (yz)'_y + 0 = -\frac{1}{yz} \cdot z = -\frac{1}{y}$$

 $\left(\ln\left(\text{nešto}\right)\right)' = \frac{1}{\text{nešto}} \cdot \left(\text{nešto}\right)'$   $\left(\ln x\right)' = \frac{1}{x}$ 

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 $f(x, y, z) = e^{2xz} - \ln(yz) + 1$ 

 $f_{x} = e^{2xz} \cdot (2xz)'_{x} - 0 + 0 = e^{2xz} \cdot 2z = 2ze^{2xz}$ 

 $f_z = e^{2xz} \cdot (2xz)_z' - \frac{1}{vz}$ 

$$z'_{x} + 0 = -\frac{1}{x} \cdot z = -\frac{1}{x}$$

 $\left(\ln\left(\text{nešto}\right)\right)' = \frac{1}{\text{nešto}} \cdot \left(\text{nešto}\right)'$   $\left(\ln x\right)' = \frac{1}{x}$ 

$$f_y = 0 - \frac{1}{vz} \cdot (yz)'_y + 0 = -\frac{1}{vz} \cdot z = -\frac{1}{v}$$

$$f_z = e^{2xz} \cdot (2xz)_z' - \frac{1}{\sqrt{z}} \cdot (yz)_z'$$

$$f(x, y, z) = e^{2xz} - \ln(yz) + 1$$

$$-\frac{1}{2} \cdot z = -\frac{1}{2}$$

 $\left(\ln\left(\text{nešto}\right)\right)' = \frac{1}{\text{nešto}} \cdot \left(\text{nešto}\right)'$   $\left(\ln x\right)' = \frac{1}{x}$ 

 $f_y = 0 - \frac{1}{yz} \cdot (yz)'_y + 0 = -\frac{1}{yz} \cdot z = -\frac{1}{y}$ 

$$f_z = e^{2xz} \cdot (2xz)'_z - \frac{1}{vz} \cdot (yz)'_z + 0$$

$$f(x, y, z) = e^{2xz} - \ln(yz) + 1$$

$$0' + 0 - \frac{1}{2} \cdot z - \frac{1}{2}$$

 $f_y = 0 - \frac{1}{yz} \cdot (yz)'_y + 0 = -\frac{1}{yz} \cdot z = -\frac{1}{y}$ 

 $f_{x} = e^{2xz} \cdot (2xz)'_{x} - 0 + 0 = e^{2xz} \cdot 2z = 2ze^{2xz}$ 

$$f_z = e^{2xz} \cdot (2xz)'_z - \frac{1}{vz} \cdot (yz)'_z + 0 = e^{2xz}$$

$$f(x, y, z) = e^{2xz} - \ln(yz) + 1$$

 $(e^{\text{nešto}})' = e^{\text{nešto}} \cdot (\text{nešto})'$   $(e^x)' = e^x$ 

 $\left(\ln\left(\text{nešto}\right)\right)' = \frac{1}{\text{nešto}} \cdot \left(\text{nešto}\right)'$   $\left(\ln x\right)' = \frac{1}{x}$ 

$$f_y = 0 - \frac{1}{vz} \cdot (yz)'_y + 0 = -\frac{1}{vz} \cdot z = -\frac{1}{v}$$

 $\left(\ln\left(\text{nešto}\right)\right)' = \frac{1}{\text{nešto}} \cdot \left(\text{nešto}\right)'$   $\left(\ln x\right)' = \frac{1}{x}$ 

 $f_z = e^{2xz} \cdot (2xz)'_z - \frac{1}{yz} \cdot (yz)'_z + 0 = e^{2xz}$ 

$$f(x, y, z) = e^{2xz} - \ln(yz) + 1$$

$$f_y = 0 - \frac{1}{vz} \cdot (yz)'_y + 0 = -\frac{1}{vz} \cdot z = -\frac{1}{v}$$

 $\left(\ln\left(\text{nešto}\right)\right)' = \frac{1}{\text{nešto}} \cdot \left(\text{nešto}\right)'$   $\left(\ln x\right)' = \frac{1}{x}$ 

 $f(x, y, z) = e^{2xz} - \ln(yz) + 1$ 

 $f_{x} = e^{2xz} \cdot (2xz)'_{x} - 0 + 0 = e^{2xz} \cdot 2z = 2ze^{2xz}$ 

 $f_z = e^{2xz} \cdot (2xz)'_z - \frac{1}{vz} \cdot (yz)'_z + 0 = e^{2xz} \cdot 2x$ 

$$f_y = 0 - \frac{1}{vz} \cdot (yz)'_y + 0 = -\frac{1}{vz} \cdot z = -\frac{1}{v}$$

 $\left(\ln\left(\text{nešto}\right)\right)' = \frac{1}{\text{nešto}} \cdot \left(\text{nešto}\right)'$   $\left(\ln x\right)' = \frac{1}{x}$ 

$$f_z = e^{2xz} \cdot (2xz)'_{-} - \frac{1}{-} \cdot (vz)'_{-} + 0 = e^{2xz} \cdot 2x - \frac{1}{-}$$

$$f_z = e^{2xz} \cdot (2xz)'_z - \frac{1}{yz} \cdot (yz)'_z + 0 = e^{2xz} \cdot 2x - \frac{1}{yz}$$

$$f(x, y, z) = e^{2xz} - \ln(yz) + 1$$

$$f_y = 0 - \frac{1}{vz} \cdot (yz)'_y + 0 = -\frac{1}{vz} \cdot z = -\frac{1}{v}$$

 $\left(\ln\left(\text{nešto}\right)\right)' = \frac{1}{\text{nešto}} \cdot \left(\text{nešto}\right)'$   $\left(\ln x\right)' = \frac{1}{x}$ 

 $f_x = e^{2xz} \cdot (2xz)_x' - 0 + 0 = e^{2xz} \cdot 2z = 2ze^{2xz}$ 1 1 1

$$yz \qquad yz \qquad y$$

$$f_z = e^{2xz} \cdot (2xz)_z' - \frac{1}{vz} \cdot (yz)_z' + 0 = e^{2xz} \cdot 2x - \frac{1}{vz}$$

 $f(x, y, z) = e^{2xz} - \ln(yz) + 1$ 

$$z'_{y} + 0 = -\frac{1}{z} \cdot z = -\frac{1}{z}$$

 $\left(\ln\left(\text{nešto}\right)\right)' = \frac{1}{\text{nešto}} \cdot \left(\text{nešto}\right)'$   $\left(\ln x\right)' = \frac{1}{x}$ 

 $f_y = 0 - \frac{1}{vz} \cdot (yz)'_y + 0 = -\frac{1}{vz} \cdot z = -\frac{1}{v}$ 

$$f_z = e^{2xz} \cdot (2xz)'_z - \frac{1}{vz} \cdot (yz)'_z + 0 = e^{2xz} \cdot 2x - \frac{1}{vz} \cdot y$$

$$f(x,y,z) = e^{2xz} - \ln(yz) + 1$$

$$y'_{y} + 0 = -\frac{1}{yz} \cdot z = -\frac{1}{yz}$$

 $\left(\ln\left(\text{nešto}\right)\right)' = \frac{1}{\text{nešto}} \cdot \left(\text{nešto}\right)'$   $\left(\ln x\right)' = \frac{1}{x}$ 

 $f_{x} = e^{2xz} \cdot (2xz)'_{x} - 0 + 0 = e^{2xz} \cdot 2z = 2ze^{2xz}$ 

$$f_x = e^{-1} \cdot (2xz)_x - 0 + 0 = e^{-1} \cdot 2z = 2ze^{-1}$$

 $f_y = 0 - \frac{1}{vz} \cdot (yz)'_y + 0 = -\frac{1}{vz} \cdot z = -\frac{1}{v}$ 

$$f_y = 0 - \frac{1}{yz} \cdot (yz)'_y + 0 = -\frac{1}{yz} \cdot z = -\frac{1}{yz}$$

 $f_z = e^{2xz} \cdot (2xz)_z' - \frac{1}{vz} \cdot (yz)_z' + 0 = e^{2xz} \cdot 2x - \frac{1}{vz} \cdot y = 2xe^{2xz}$ 

$$f(x, y, z) = e^{2xz} - \ln(yz) + 1$$

$$f_y = 0 - \frac{1}{vz} \cdot (yz)'_y + 0 = -\frac{1}{vz} \cdot z = -\frac{1}{v}$$

 $\left(\ln\left(\text{nešto}\right)\right)' = \frac{1}{\text{nešto}} \cdot \left(\text{nešto}\right)'$   $\left(\ln x\right)' = \frac{1}{x}$ 

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 $f(x, y, z) = e^{2xz} - \ln(yz) + 1$ 

 $f_z = e^{2xz} \cdot (2xz)'_z - \frac{1}{vz} \cdot (yz)'_z + 0 = e^{2xz} \cdot 2x - \frac{1}{vz} \cdot y = 2xe^{2xz} - \frac{1}{z}$