Realne funkcije realne varijable – 2. dio

Matematika 2

Damir Horvat

FOI, Varaždin

Sadržaj

definicija funkcije

prvi zadatak

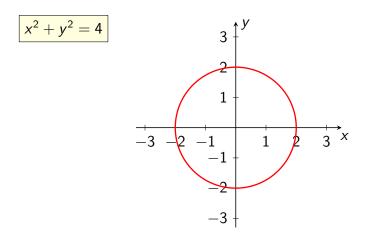
drugi zadatak

treći zadatak

četvrti zadatak

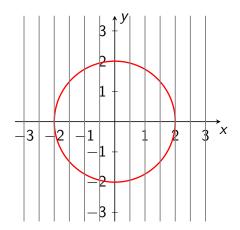
peti zadatak

šesti zadatak

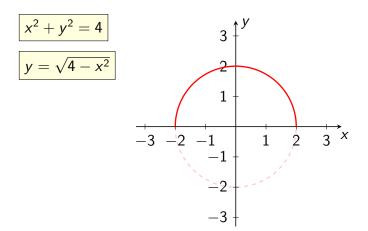


Kružnica $x^2 + y^2 = 4$ nije graf niti jedne funkcije y = f(x) jer

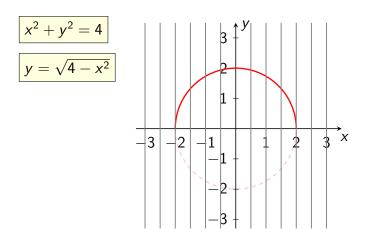




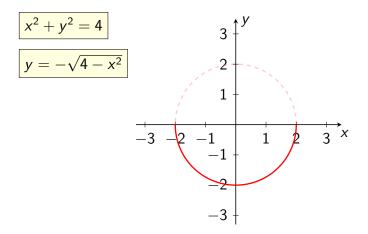
Kružnica $x^2 + y^2 = 4$ nije graf niti jedne funkcije y = f(x) jer postoje paralele s y-osi koje sijeku tu krivulju u više od jedne točke.



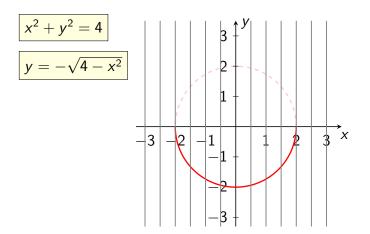
Gornja polukružnica jest graf funkcije y = f(x) jer



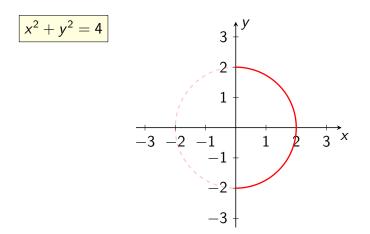
Gornja polukružnica jest graf funkcije y = f(x) jer svaka paralela s y-osi siječe tu krivulju u najviše jednoj točki.



Donja polukružnica jest graf funkcije y = f(x) jer

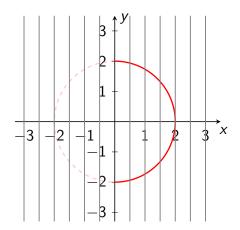


Donja polukružnica jest graf funkcije y = f(x) jer svaka paralela s y-osi siječe tu krivulju u najviše jednoj točki.

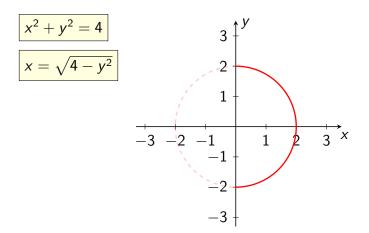


Desna polukružnica nije graf niti jedne funkcije y = f(x) jer

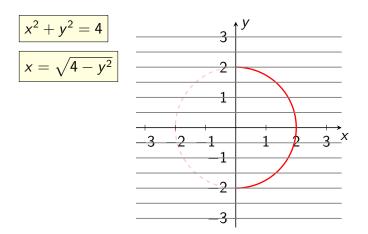




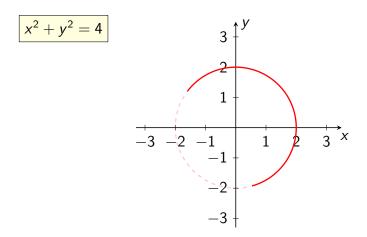
Desna polukružnica nije graf niti jedne funkcije y=f(x) jer postoje paralele s y-osi koje sijeku tu krivulju u više od jedne točke.



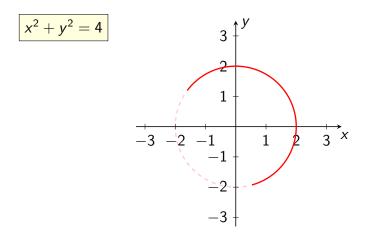
Desna polukružnica jest graf funkcije x = f(y) jer



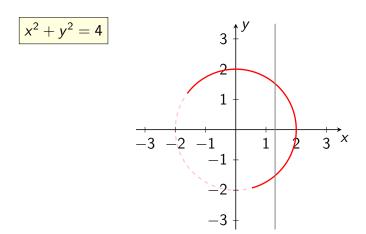
Desna polukružnica jest graf funkcije x = f(y) jer svaka paralela s x-osi siječe tu krivulju u najviše jednoj točki.



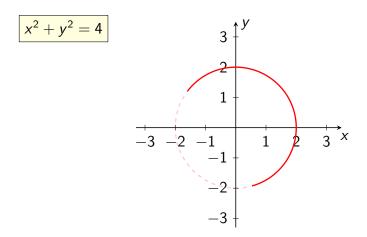
Dio kružnice prikazan na slici



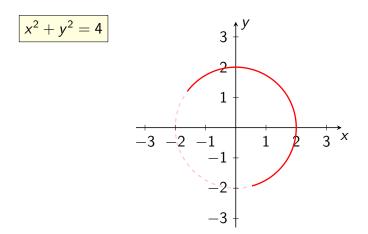
Dio kružnice prikazan na slici nije graf niti jedne funkcije y = f(x)



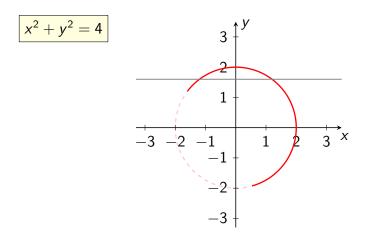
Dio kružnice prikazan na slici nije graf niti jedne funkcije y = f(x) jer postoji paralela s y-osi koja siječe tu krivulju u više od jedne točke.



Dio kružnice prikazan na slici



Dio kružnice prikazan na slici nije graf niti jedne funkcije x = f(y)

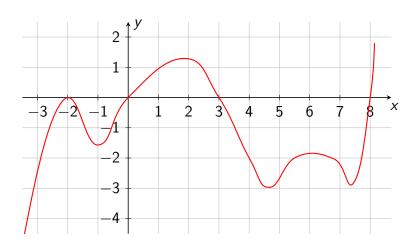


Dio kružnice prikazan na slici nije graf niti jedne funkcije x = f(y) jer postoji paralela s x-osi koja siječe tu krivulju u više od jedne točke.

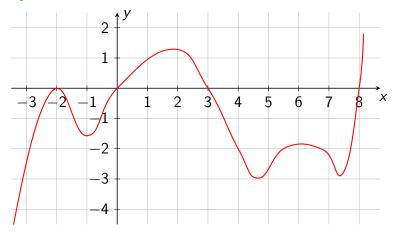
prvi zadatak

Zadatak 1

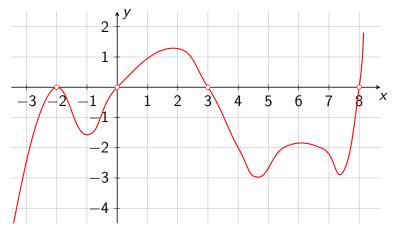
Zadan je graf funkcije f.



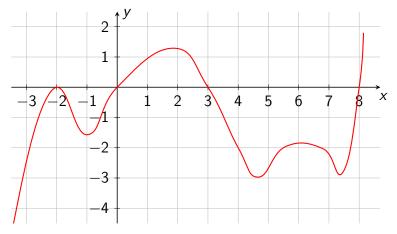
- a) Odredite nultočke funkcije f.
- b) Navedite neki interval na kojemu je funkcija f pozitivna.
- c) Navedite neki interval na kojemu je funkcija f negativna.
- d) Napišite neki interval na kojemu funkcija f pada.
- e) Napišite neki interval na kojemu funkcija f raste.
- f) Napišite neki interval na kojemu funkcija f nije monotona.
- g) Napišite neki interval na kojemu je $f(x) \leq -1$.
- h) Koliko lokalnih ekstrema ima funkcija f?
- i) Koliko rješenja ima jednadžba f(x) = 1 na segmentu [-3, 9]?
- j) Koliko rješenja ima jednadžba f(x) = 1 na segmentu [-3, 8]?



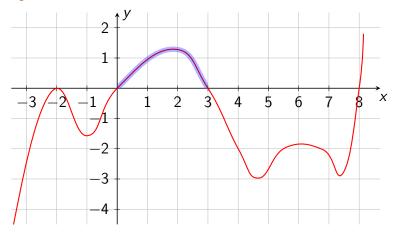
a) Nultočke funkcije f su:



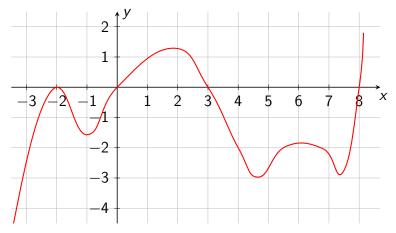
a) Nultočke funkcije f su: -2,0,3,8



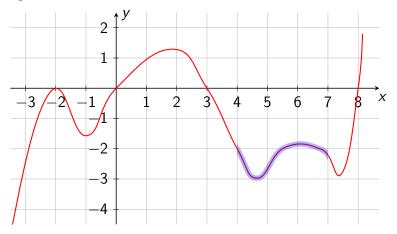
b) Funkcija f je pozitivna na primjer na intervalu



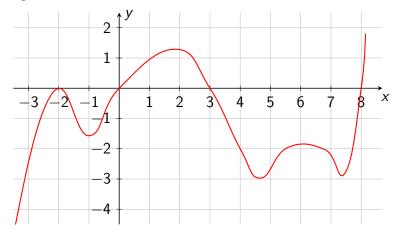
b) Funkcija f je pozitivna na primjer na intervalu $\langle 0,3 \rangle$.



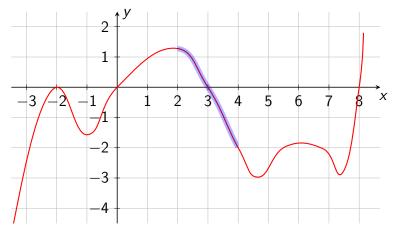
c) Funkcija f je negativna na primjer na intervalu



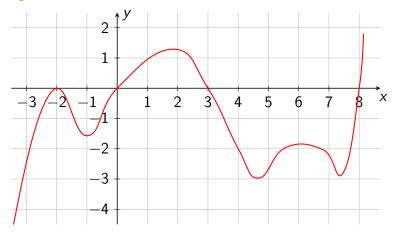
c) Funkcija f je negativna na primjer na intervalu $\langle 4,7 \rangle$.



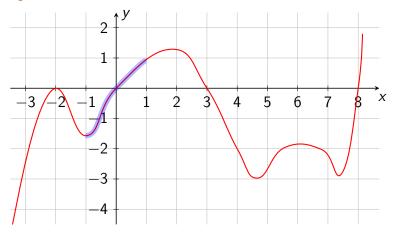
d) Funkcija f pada na primjer na intervalu



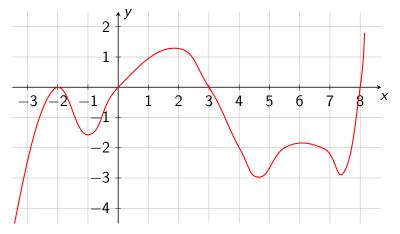
d) Funkcija f pada na primjer na intervalu $\langle 2,4 \rangle$.



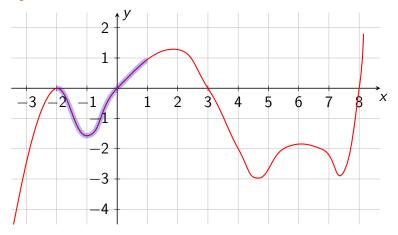
e) Funkcija f raste na primjer na intervalu



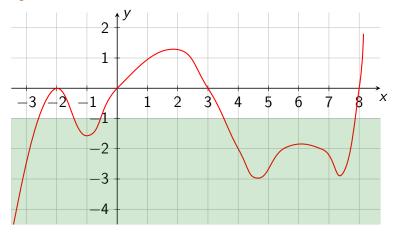
e) Funkcija f raste na primjer na intervalu $\langle -1, 1 \rangle$.



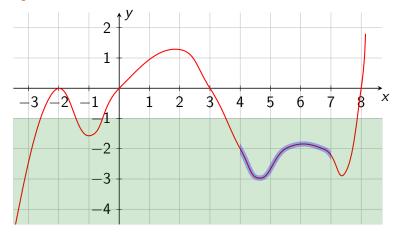
f) Funkcija f nije monotona na primjer na intervalu



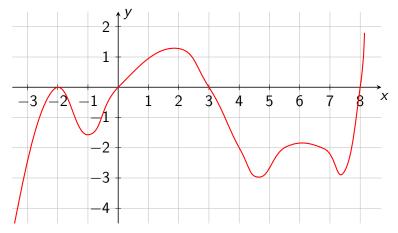
f) Funkcija f nije monotona na primjer na intervalu $\langle -2, 1 \rangle$.



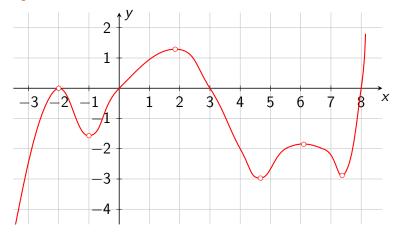
g) $f(x) \leqslant -1$ na primjer na intervalu



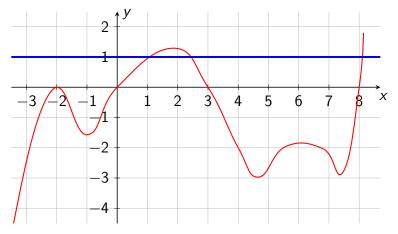
g) $f(x) \leqslant -1$ na primjer na intervalu $\langle 4, 7 \rangle$.



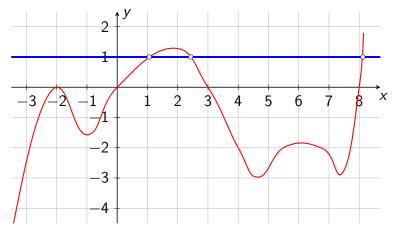
h) f ima ukupno ___ lokalnih ekstrema.



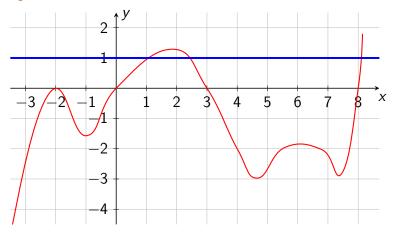
h) f ima ukupno <u>6</u> lokalnih ekstrema.



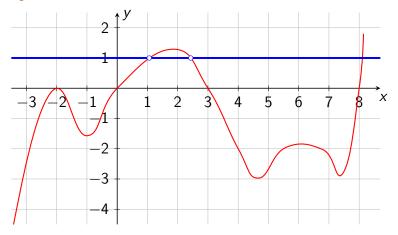
i) Jednadžba f(x) = 1 ima ukupno ___ rješenja na segmentu [-3, 9].



i) Jednadžba f(x) = 1 ima ukupno 3 rješenja na segmentu [-3, 9].



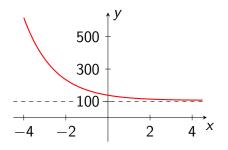
j) Jednadžba f(x) = 1 ima ukupno ___ rješenja na segmentu [-3,8].



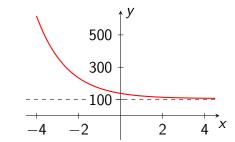
j) Jednadžba f(x) = 1 ima ukupno 2 rješenja na segmentu [-3, 8].

drugi zadatak

Zadana je funkcija h svojim grafom na donjoj slici.

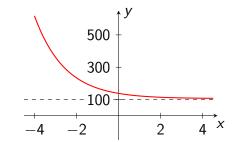


monotonost



monotonost

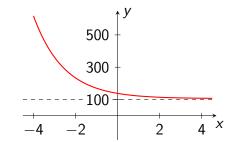
Funkcija *h* je monotona funkcija jer strogo pada.



monotonost

Funkcija *h* je monotona funkcija jer strogo pada.

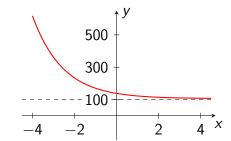
omeđenost



monotonost

Funkcija *h* je monotona funkcija jer strogo pada.

omeđenost
$$m \leqslant h(x) \leqslant M$$

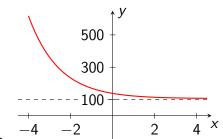


monotonost

Funkcija *h* je monotona funkcija jer strogo pada.

omeđenost
$$m \leqslant h(x) \leqslant M$$

Funkcija *h* nije omeđena odozgo jer je

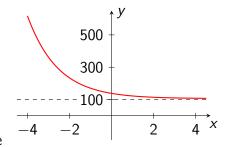


monotonost

Funkcija *h* je monotona funkcija jer strogo pada.

omeđenost
$$m \leqslant h(x) \leqslant M$$

Funkcija h nije omeđena odozgo jer je



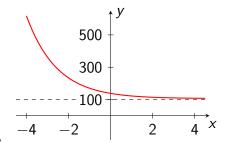
monotonost

Funkcija *h* je monotona funkcija jer strogo pada.

omeđenost
$$m \leqslant h(x) \leqslant M$$

Funkcija h nije omeđena odozgo jer je

$$\lim_{x \to -\infty} h(x) = +\infty$$
. Kraći zapis



monotonost

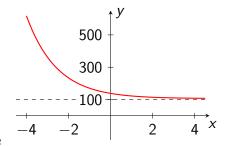
Funkcija *h* je monotona funkcija jer strogo pada.

omeđenost
$$m \leqslant h(x) \leqslant M$$

Funkcija h nije omeđena odozgo jer je

$$\lim_{x \to -\infty} h(x) = +\infty$$
. Kraći zapis

Funkcija h je omeđena odozdo jer je



monotonost

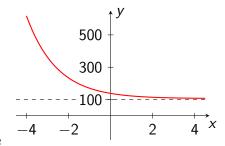
Funkcija *h* je monotona funkcija jer strogo pada.

omeđenost
$$m \leqslant h(x) \leqslant M$$

Funkcija h nije omeđena odozgo jer je

$$\lim_{x \to -\infty} h(x) = +\infty$$
. Kraći zapis

Funkcija h je omeđena odozdo jer je $h(x) \ge 100$,



monotonost

Funkcija *h* je monotona funkcija jer strogo pada.

omeđenost
$$m \leqslant h(x) \leqslant M$$

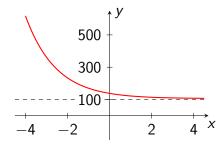
$$m \leqslant n(x) \leqslant N$$

Funkcija h nije omeđena odozgo jer je

$$\lim_{x \to -\infty} h(x) = +\infty. \leftarrow \text{kraći zapis}$$

Funkcija h je omeđena odozdo jer je

$$h(x) \geqslant 100$$
, tj. $m = 100$ je jedna donja međa funkcije h .



monotonost

Funkcija *h* je monotona funkcija jer strogo pada.

omeđenost
$$m \leqslant h(x) \leqslant M$$

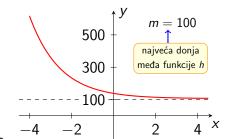
Funkcija h nije omeđena odozgo jer je

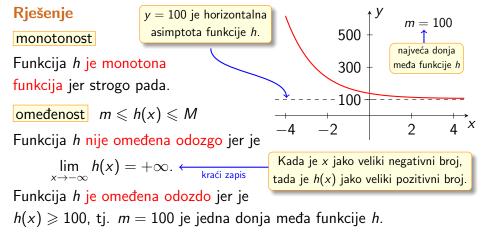
$$\lim_{x \to -\infty} h(x) = +\infty$$
. Kraći zapis

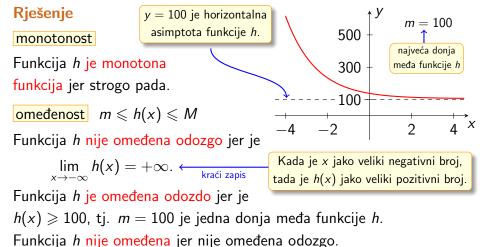
Kada je x jako veliki negativni broj, tada je h(x) jako veliki pozitivni broj.

Funkcija h je omeđena odozdo jer je

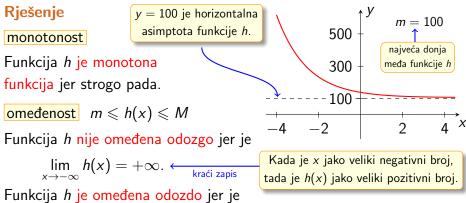
$$h(x) \geqslant 100$$
, tj. $m = 100$ je jedna donja međa funkcije h .







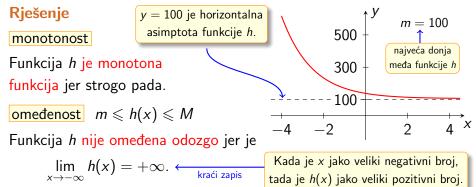
6 / 23



 $h(x) \ge 100$, tj. m = 100 je jedna donja međa funkcije h.

Funkcija h nije omeđena jer nije omeđena odozgo.

parnost/neparnost

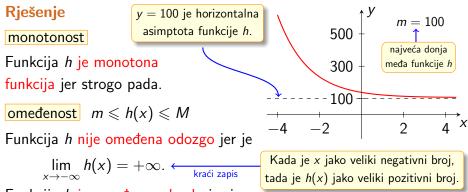


 $h(x) \geqslant 100$, tj. m = 100 je jedna donja međa funkcije h.

Funkcija *h* nije omeđena jer nije omeđena odozgo.

parnost/neparnost

Funkcija h nije parna jer

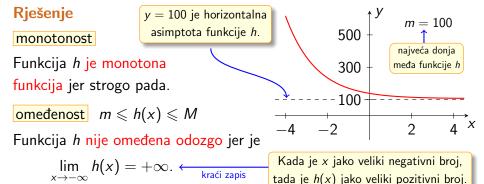


$$h(x) \geqslant 100$$
, tj. $m = 100$ je jedna donja međa funkcije h .

Funkcija h nije omeđena jer nije omeđena odozgo.

parnost/neparnost

Funkcija h nije parna jer njezin graf nije simetričan s obzirom na os y.



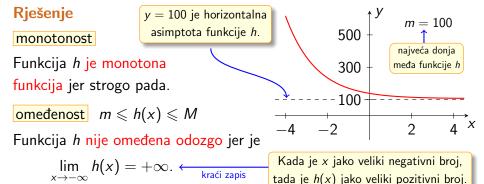
$$h(x) \geqslant 100$$
, tj. $m = 100$ je jedna donja međa funkcije h .

Funkcija *h* nije omeđena jer nije omeđena odozgo.

parnost/neparnost

Funkcija h nije parna jer njezin graf nije simetričan s obzirom na os y.

Funkcija h nije neparna jer



 $h(x) \geqslant 100$, tj. m = 100 je jedna donja međa funkcije h.

Funkcija h nije omeđena jer nije omeđena odozgo.

parnost/neparnost

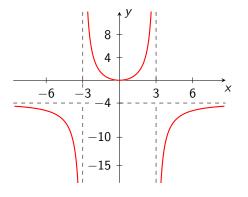
Funkcija h nije parna jer njezin graf nije simetričan s obzirom na os y.

Funkcija *h* nije neparna jer njezin graf nije simetričan s obzirom na ishodište koordinatnog sustava.

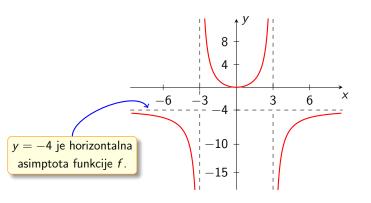
6/23

treći zadatak

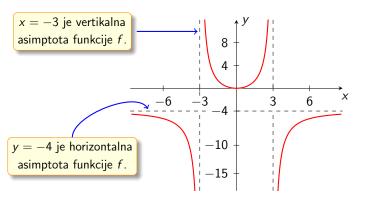
Zadana je funkcija f svojim grafom na donjoj slici.



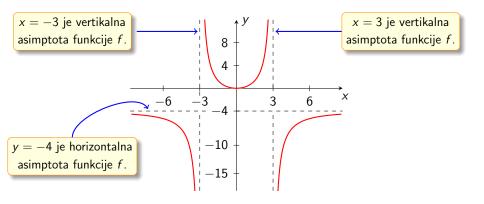
Zadana je funkcija f svojim grafom na donjoj slici.



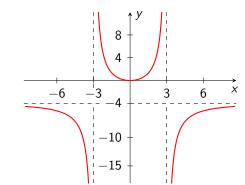
Zadana je funkcija f svojim grafom na donjoj slici.



Zadana je funkcija f svojim grafom na donjoj slici.

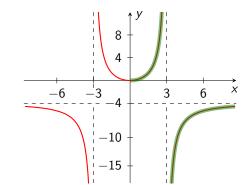


monotonost



monotonost

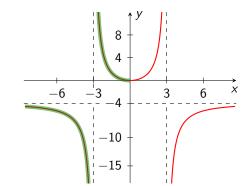
Funkcija f raste na intervalima $\langle 0,3 \rangle$ i $\langle 3,+\infty \rangle$.



monotonost

Funkcija f raste na intervalima $\langle 0, 3 \rangle$ i $\langle 3, +\infty \rangle$.

Funkcija f pada na intervalima $\langle -\infty, -3 \rangle$ i $\langle -3, 0 \rangle$.

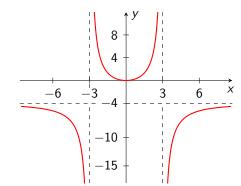


monotonost

Funkcija f raste na intervalima $\langle 0, 3 \rangle$ i $\langle 3, +\infty \rangle$.

Funkcija f pada na intervalima $\langle -\infty, -3 \rangle$ i $\langle -3, 0 \rangle$.

Funkcija *f* nije monotona funkcija na svojoj domeni.

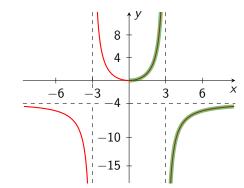


monotonost

Funkcija f raste na intervalima $\langle 0, 3 \rangle$ i $\langle 3, +\infty \rangle$.

Funkcija f pada na intervalima $\langle -\infty, -3 \rangle$ i $\langle -3, 0 \rangle$.

Funkcija *f* nije monotona funkcija na svojoj domeni.



Budite iznimno oprezni

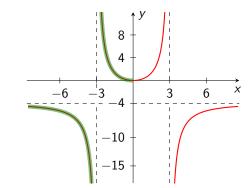
Funkcija f ne raste na skupu $\langle 0,3\rangle \cup \langle 3,+\infty\rangle$.

monotonost

Funkcija f raste na intervalima $\langle 0, 3 \rangle$ i $\langle 3, +\infty \rangle$.

Funkcija f pada na intervalima $\langle -\infty, -3 \rangle$ i $\langle -3, 0 \rangle$.

Funkcija *f* nije monotona funkcija na svojoj domeni.



Budite iznimno oprezni

Funkcija f ne raste na skupu $\langle 0,3\rangle \cup \langle 3,+\infty\rangle$.

Budite iznimno oprezni

Funkcija f ne pada na skupu $\langle -\infty, -3 \rangle \cup \langle -3, 0 \rangle$.

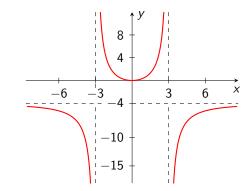
monotonost

Funkcija f raste na intervalima (0,3) i $(3,+\infty)$.

Funkcija f pada na intervalima $\langle -\infty, -3 \rangle$ i $\langle -3, 0 \rangle$.

Funkcija *f* nije monotona funkcija na svojoj domeni.

parnost/neparnost



Budite iznimno oprezni

Funkcija f ne raste na skupu $\langle 0,3 \rangle \cup \langle 3,+\infty \rangle.$

Budite iznimno oprezni

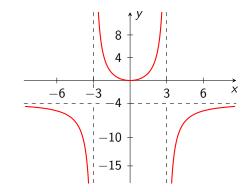
Funkcija f ne pada na skupu $\langle -\infty, -3 \rangle \cup \langle -3, 0 \rangle$.

monotonost

Funkcija f raste na intervalima $\langle 0, 3 \rangle$ i $\langle 3, +\infty \rangle$.

Funkcija f pada na intervalima $\langle -\infty, -3 \rangle$ i $\langle -3, 0 \rangle$.

Funkcija *f* nije monotona funkcija na svojoj domeni.



parnost/neparnost

Funkcija f je parna jer je njezin graf simetričan s obzirom na os y.

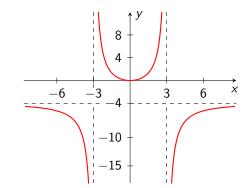
Budite iznimno oprezni

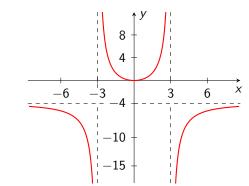
Funkcija f ne raste na skupu $\langle 0,3\rangle \cup \langle 3,+\infty \rangle$.

Budite iznimno oprezni

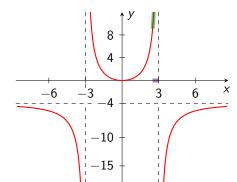
Funkcija f ne pada na skupu $\langle -\infty, -3 \rangle \cup \langle -3, 0 \rangle$.

omeđenost



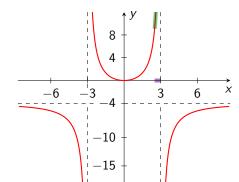


Funkcija *f* nije omeđena odozgo jer u okolini broja 3 s lijeve (minus) strane poprima beskonačno velike pozitivne vrijednosti,



Funkcija f nije omeđena odozgo jer u okolini broja 3 s lijeve (minus) strane poprima beskonačno velike pozitivne vrijednosti, tj.

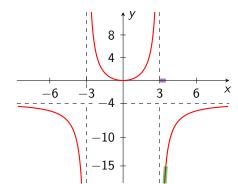
$$\lim_{x\to 3-} f(x) = +\infty.$$



Funkcija f nije omeđena odozgo jer u okolini broja 3 s lijeve (minus) strane poprima beskonačno velike pozitivne vrijednosti, tj.

$$\lim_{x \to 3-} f(x) = +\infty.$$

Funkcija f nije omeđena odozdo jer u okolini broja 3 s desne (plus) strane poprima beskonačno velike negativne vrijednosti,

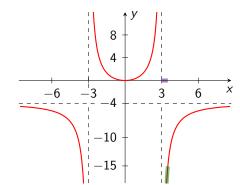


Funkcija f nije omeđena odozgo jer u okolini broja 3 s lijeve (minus) strane poprima beskonačno velike pozitivne vrijednosti, tj.

$$\lim_{x\to 3-} f(x) = +\infty.$$

Funkcija f nije omeđena odozdo jer u okolini broja 3 s desne (plus) strane poprima beskonačno velike negativne vrijednosti, tj.

$$\lim_{x\to 3+} f(x) = -\infty.$$



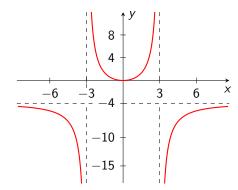
Funkcija f nije omeđena odozgo jer u okolini broja 3 s lijeve (minus) strane poprima beskonačno velike pozitivne vrijednosti, tj.

$$\lim_{x\to 3-} f(x) = +\infty.$$

Funkcija f nije omeđena odozdo jer u okolini broja 3 s desne (plus) strane poprima beskonačno velike negativne vrijednosti, tj.

$$\lim_{x\to 3+} f(x) = -\infty.$$

Funkcija f nije omeđena jer nije omeđena niti odozgo niti odozdo.



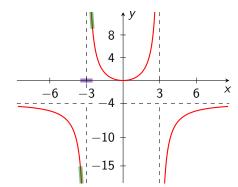
Funkcija *f* nije omeđena odozgo jer u okolini broja 3 s lijeve (minus) strane poprima beskonačno velike pozitivne vrijednosti, tj.

$$\lim_{x\to 3-} f(x) = +\infty.$$

Funkcija f nije omeđena odozdo jer u okolini broja 3 s desne (plus) strane poprima beskonačno velike negativne vrijednosti, tj.

$$\lim_{x \to 3+} f(x) = -\infty.$$

Funkcija *f* nije omeđena jer nije omeđena niti odozgo niti odozdo.



Slično je u okolini broja
$$-3$$

$$\lim_{x \to -3-} f(x) = -\infty$$

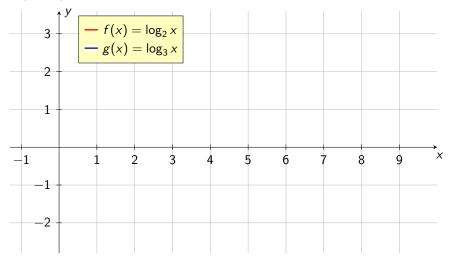
$$\lim_{x \to -3+} f(x) = +\infty$$

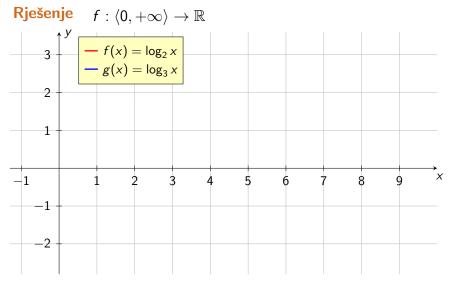
četvrti zadatak

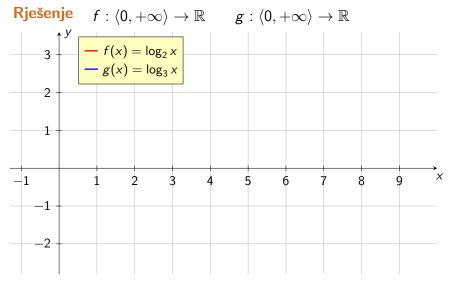
Zadatak 4

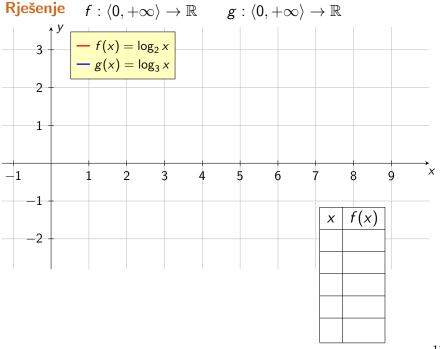
Zadane su funkcije $f(x) = \log_2 x$ i $g(x) = \log_3 x$.

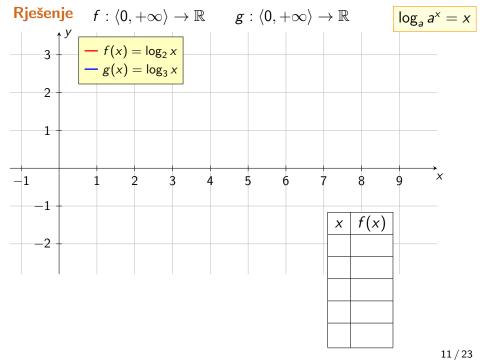
- a) Na kojim dijelovima domena vrijedi nejednakost $f(x) \ge g(x)$?
- b) Na kojim dijelovima domena vrijedi nejednakost $f(x) \leq g(x)$?
- c) Na kojem dijelu domene vrijedi $1 \le f(x) \le 2$?
- d) Na kojem dijelu domene vrijedi $1 \le g(x) \le 2$?
- e) Na kojim dijelovima domena vrijedi nejednakost $f^{-1}(x) \geqslant g^{-1}(x)$?
- f) Na kojim dijelovima domena vrijedi nejednakost $f^{-1}(x) \leq g^{-1}(x)$?
- g) Usporedite funkcije f, g, f^{-1} i g^{-1} na intervalu $(0, +\infty)$ s linearnom funkcijom h(x) = x.

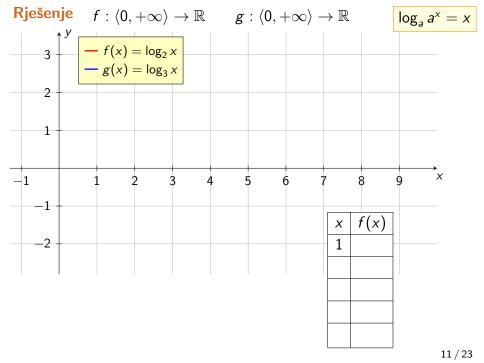


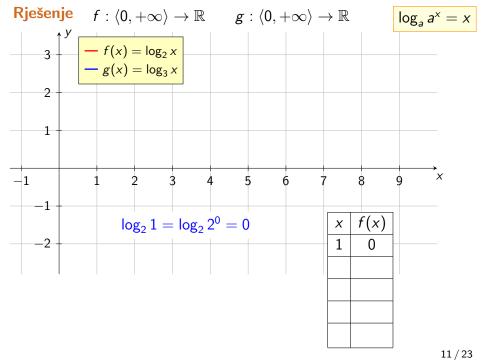


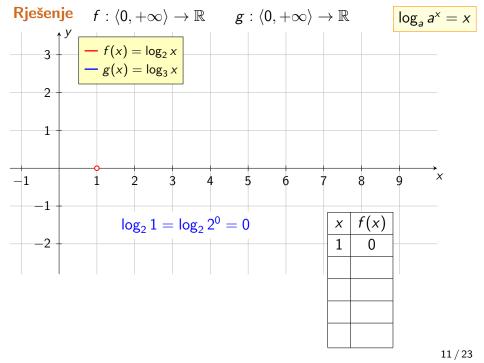


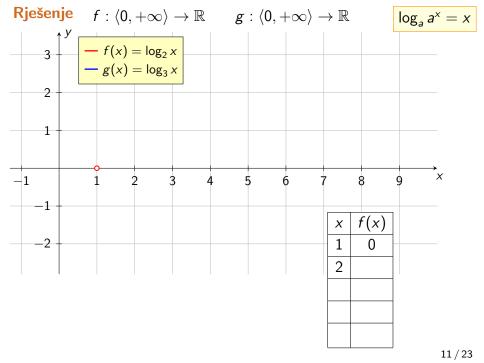


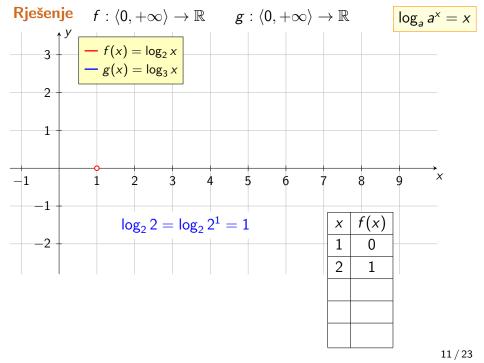


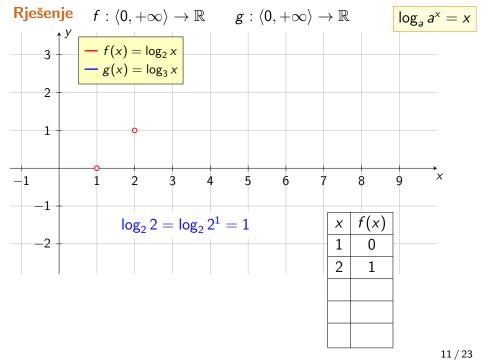


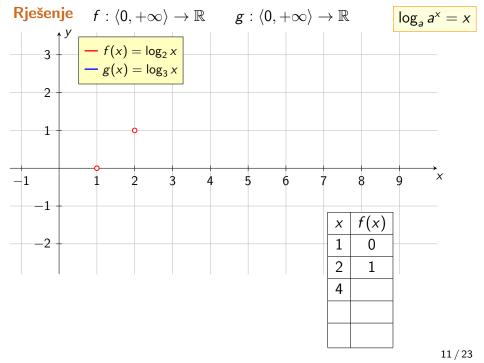


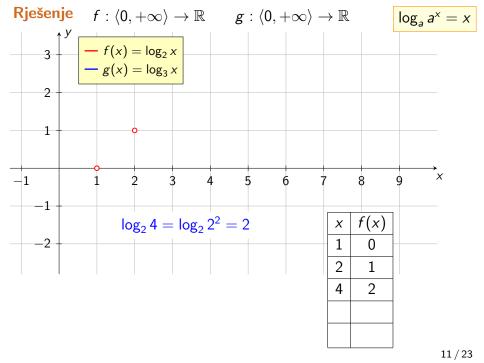


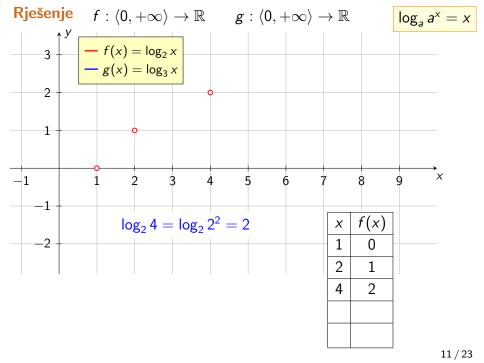


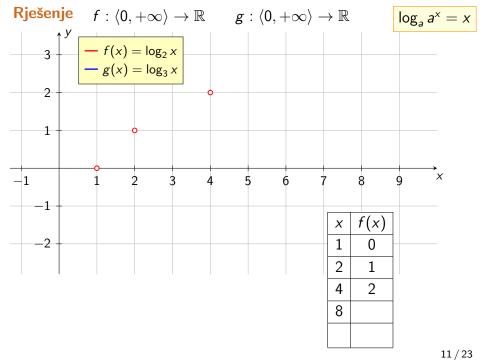


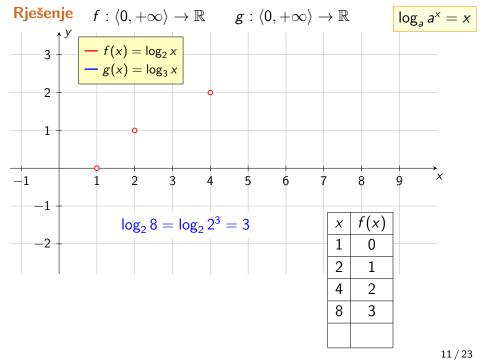


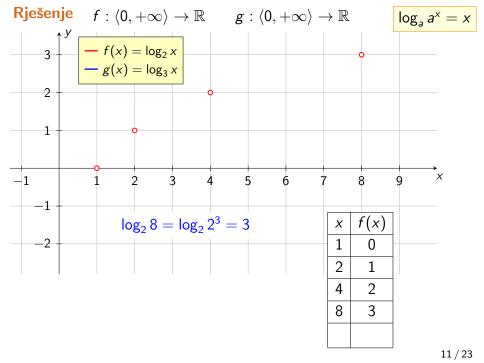


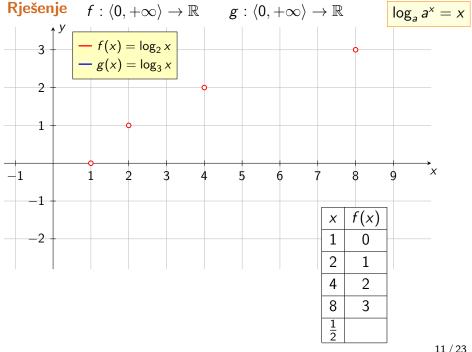


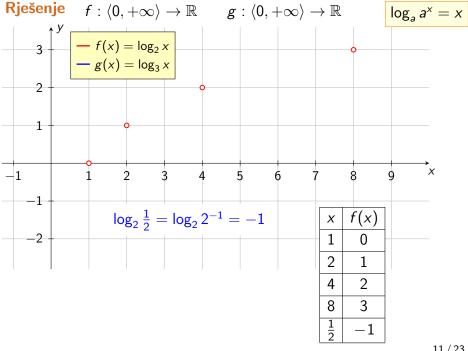


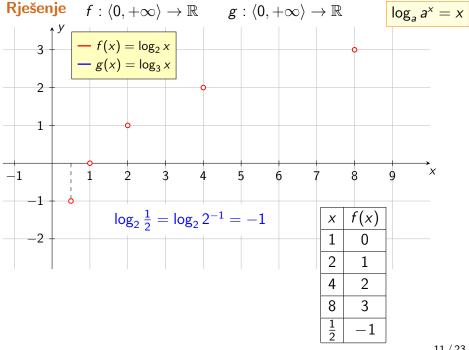


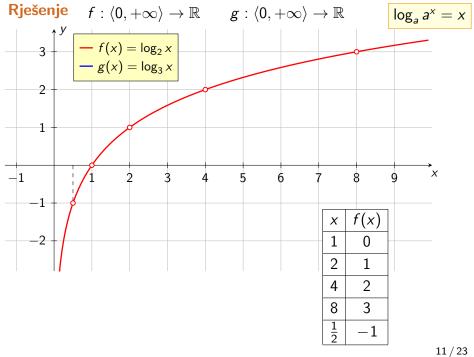


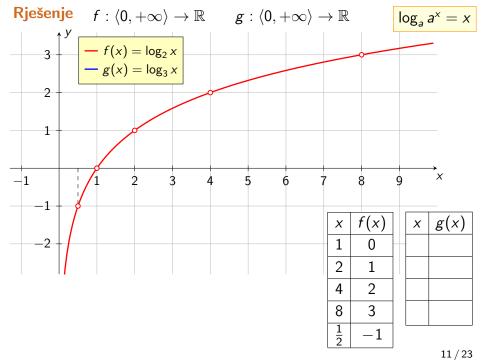


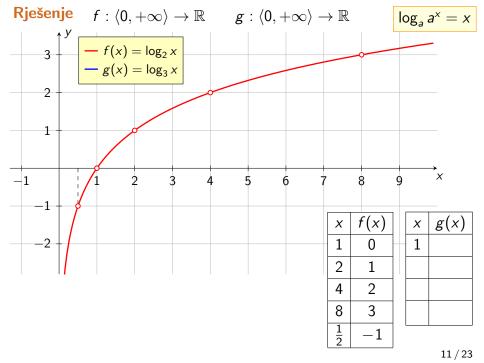


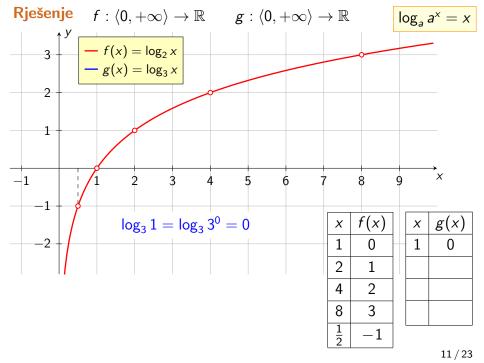


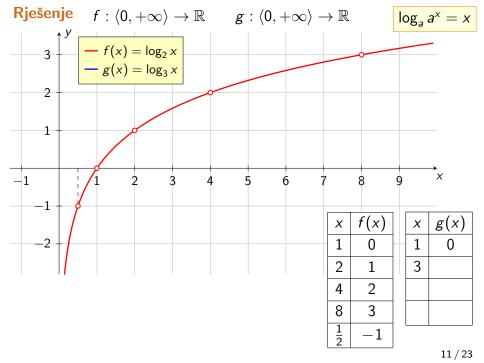


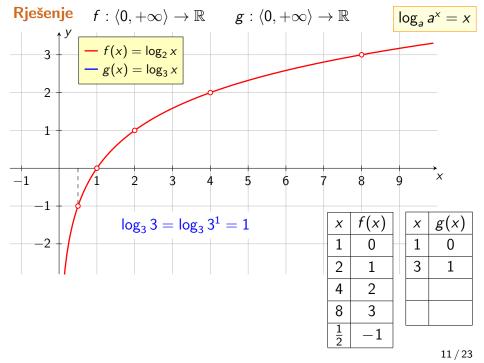


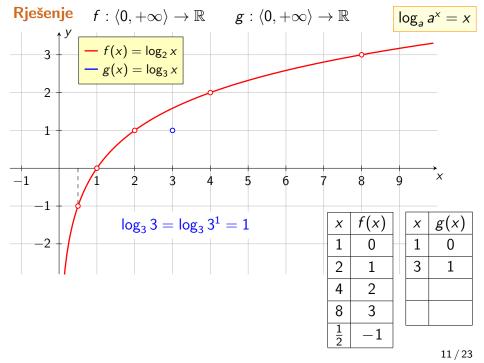


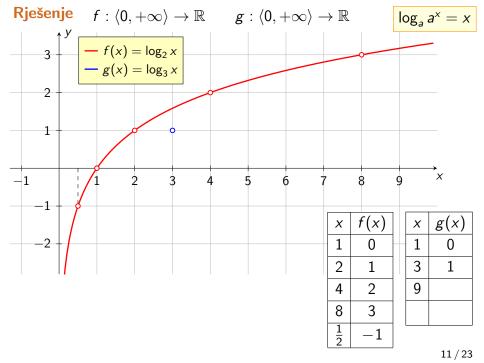


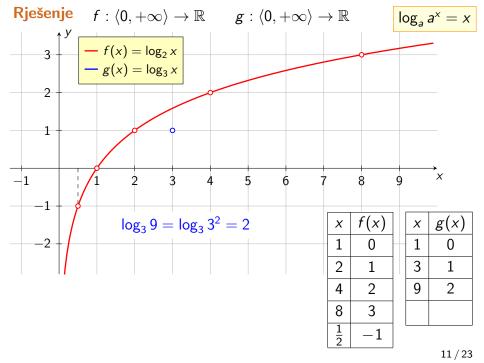


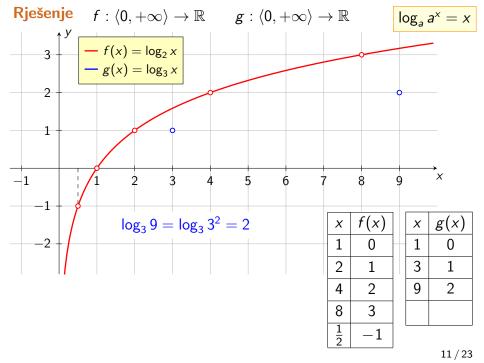


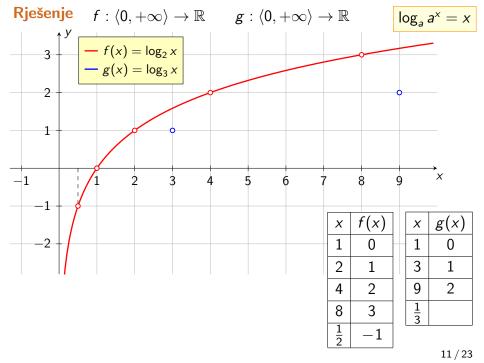


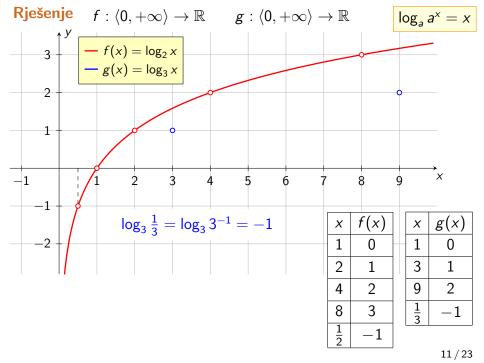


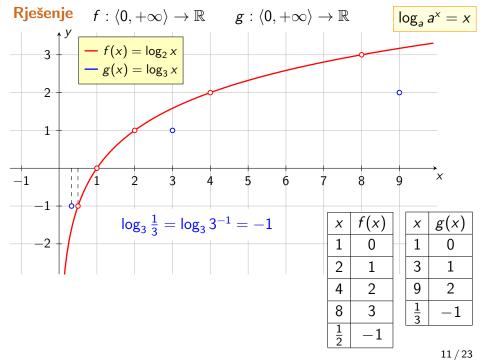


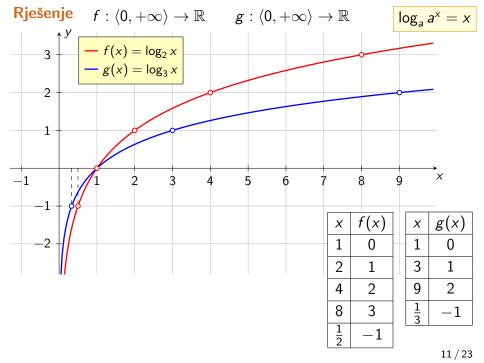


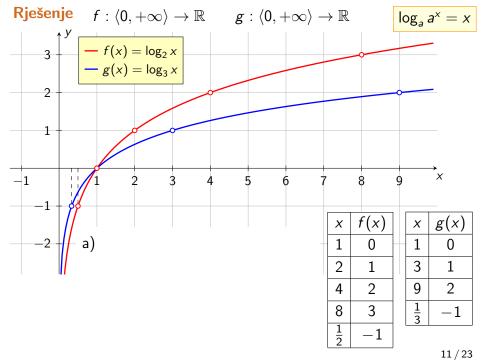


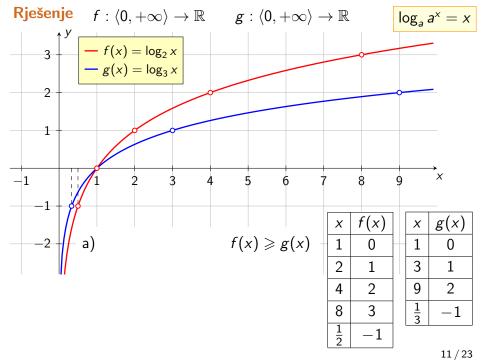


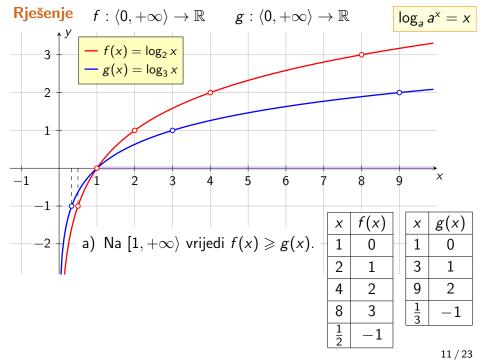


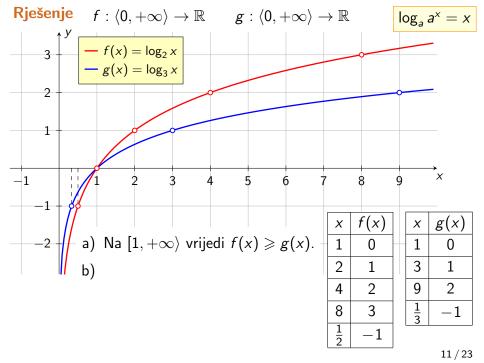


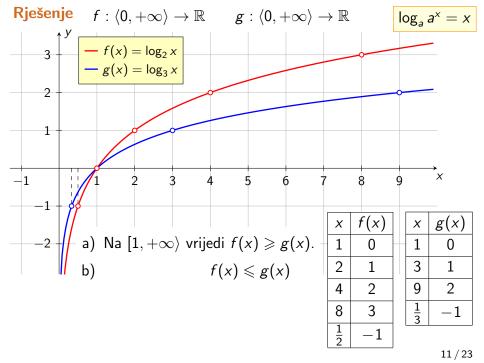


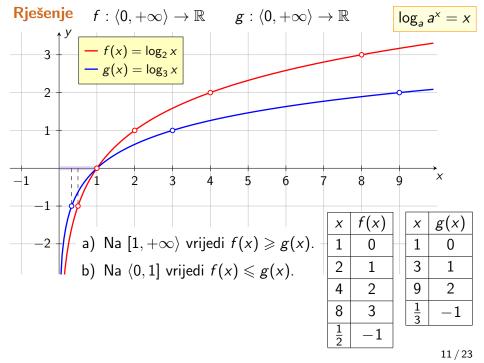


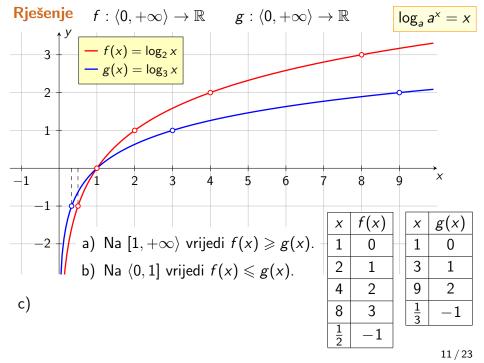


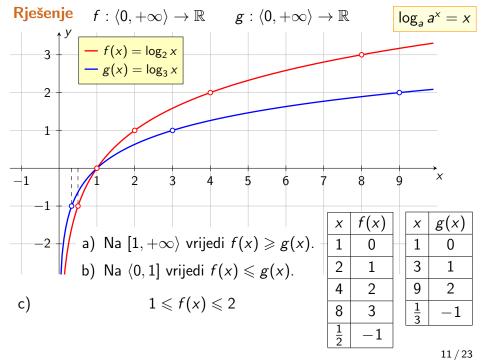


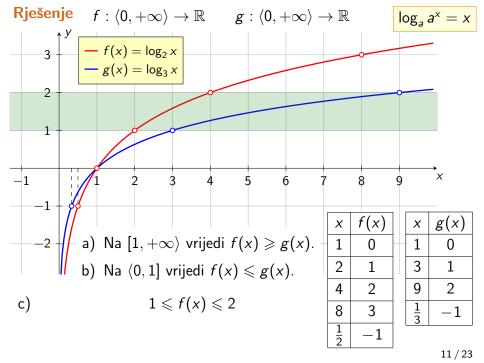


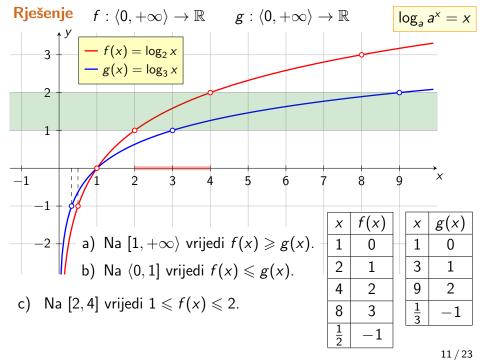


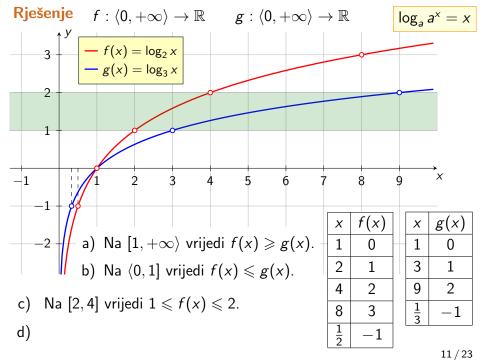


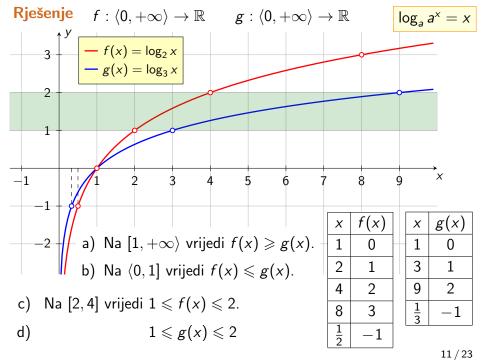


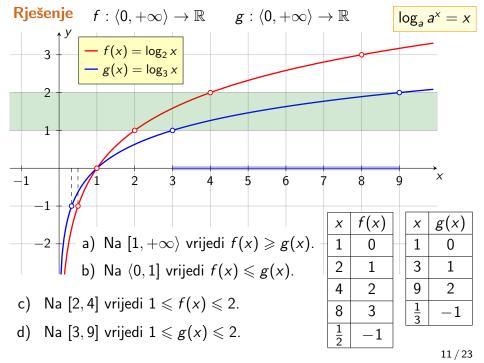


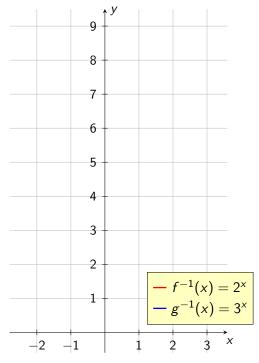


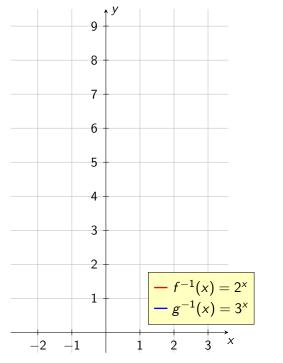




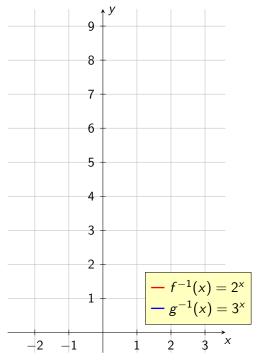






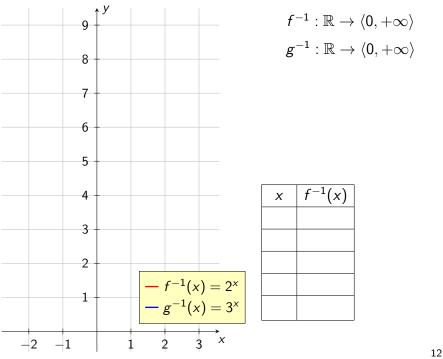


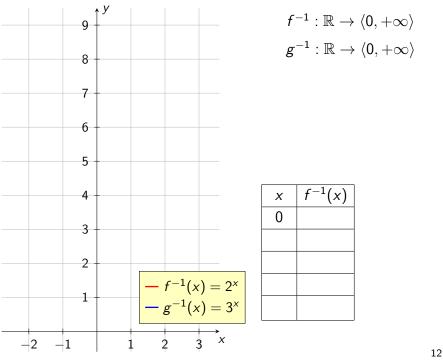
 $f^{-1}:\mathbb{R}\to\langle 0,+\infty
angle$

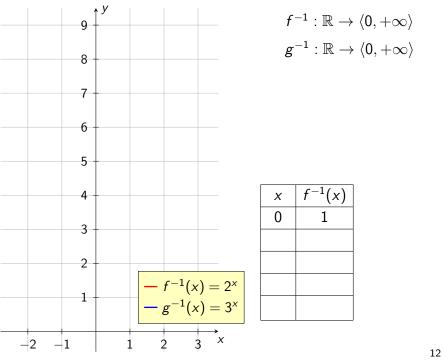


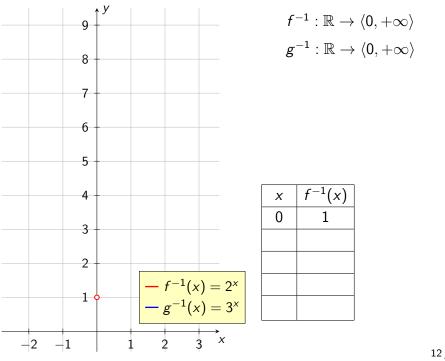
 $g^{-1}: \mathbb{R} o \langle 0, +\infty
angle$

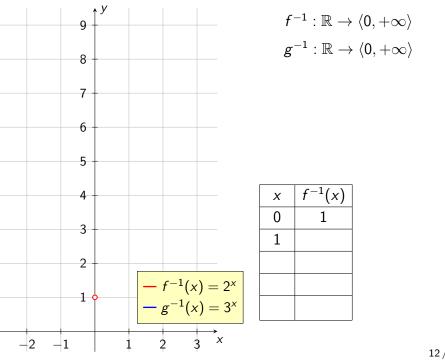
 $f^{-1}: \mathbb{R} \to \langle 0, +\infty \rangle$

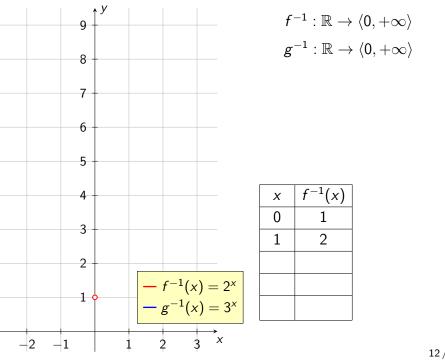


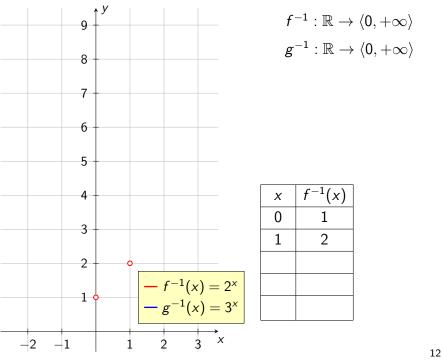


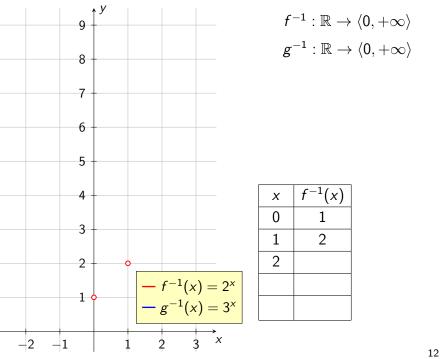


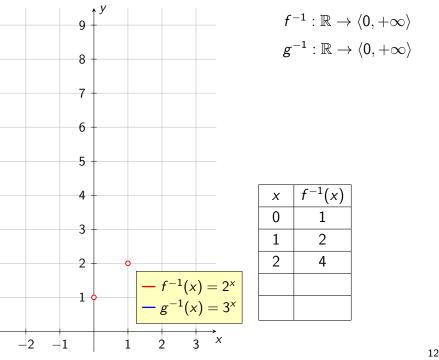


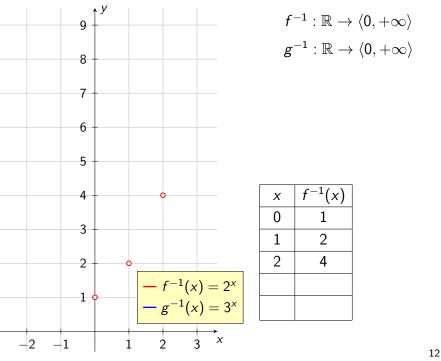


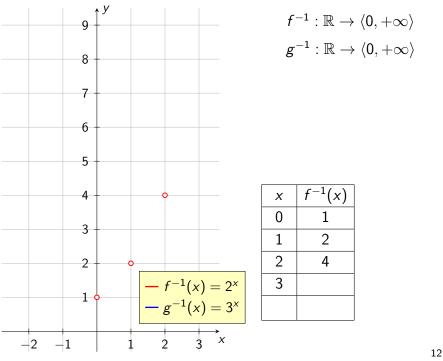


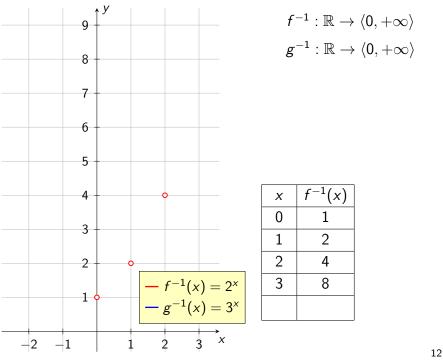


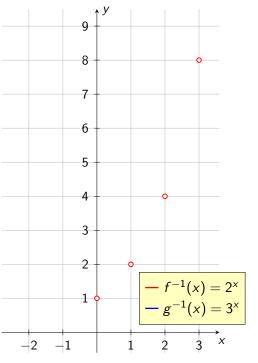








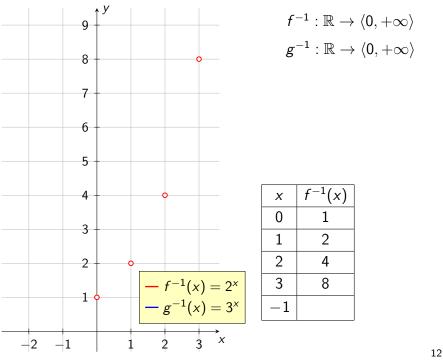


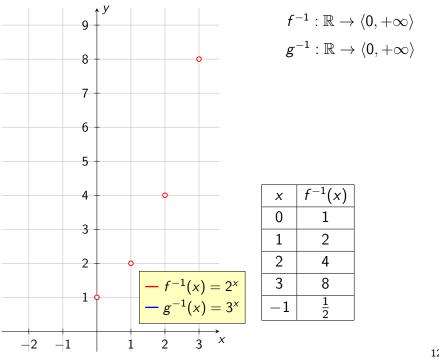


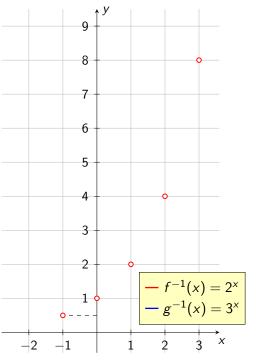
$$g^{-1}:\mathbb{R} o \langle 0,+\infty
angle$$

 $f^{-1}: \mathbb{R} \to \langle 0, +\infty \rangle$

X	$f^{-1}(x)$
0	1
1	2
2	4
3	8



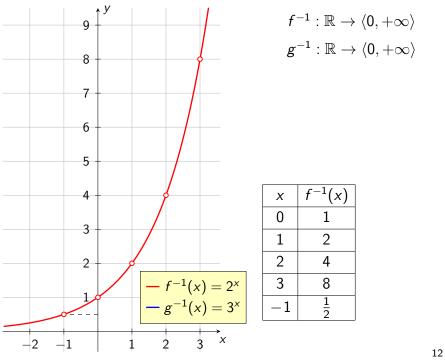


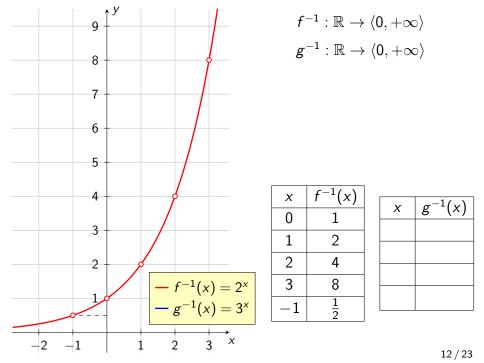


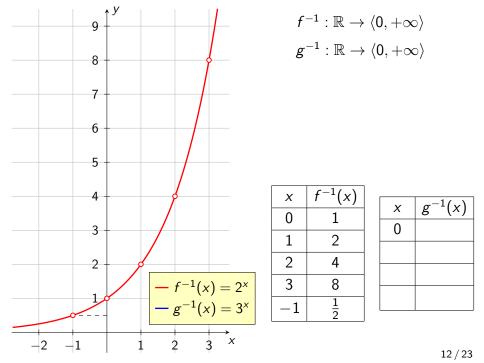
$$g^{-1}: \mathbb{R} o \langle 0, +\infty
angle$$

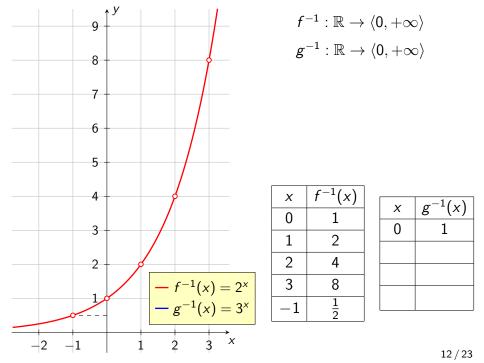
 $f^{-1}:\mathbb{R}\to\langle 0,+\infty
angle$

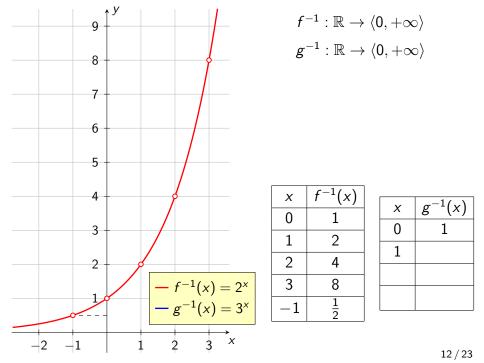
X	$ t^{-1}(x) $
0	1
1	2
2	4
3	8
-1	$\frac{1}{2}$

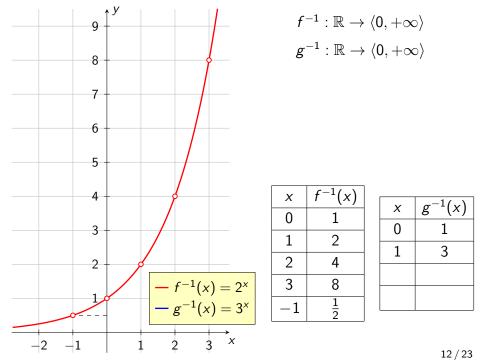


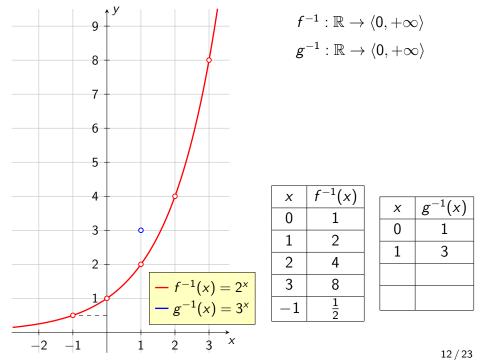


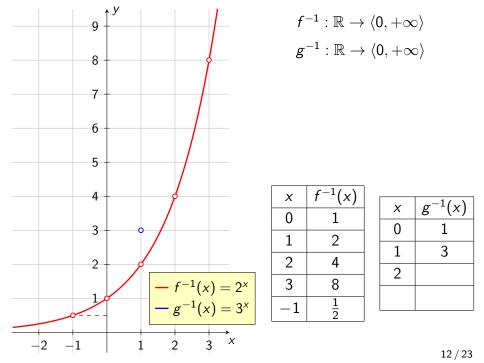


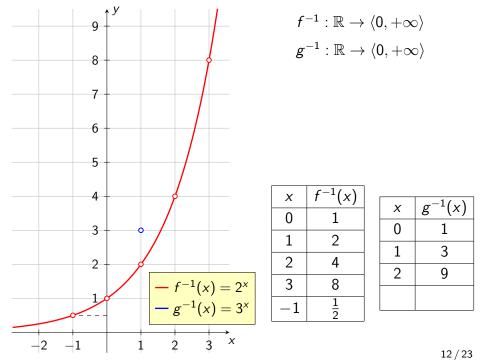


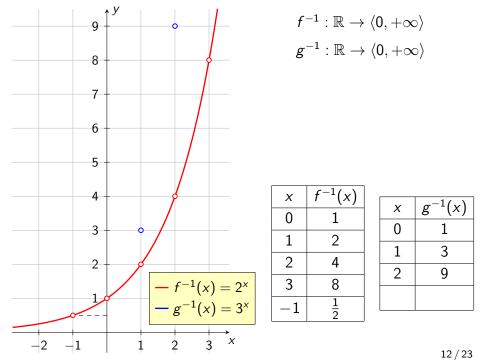


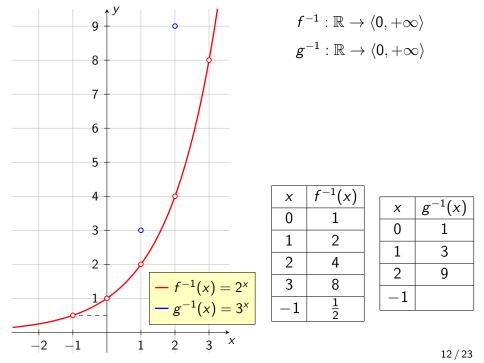


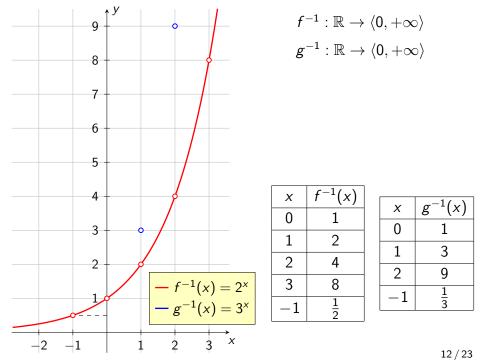


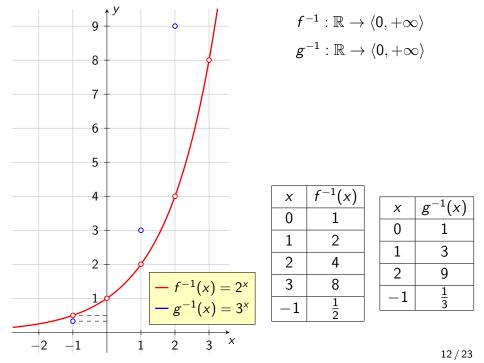


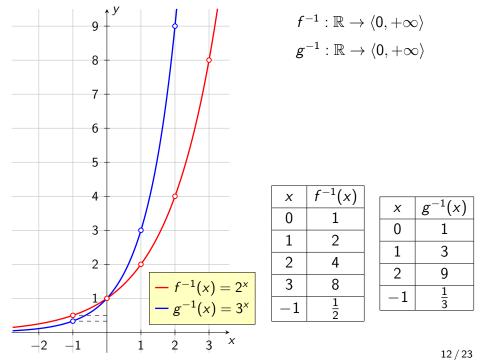


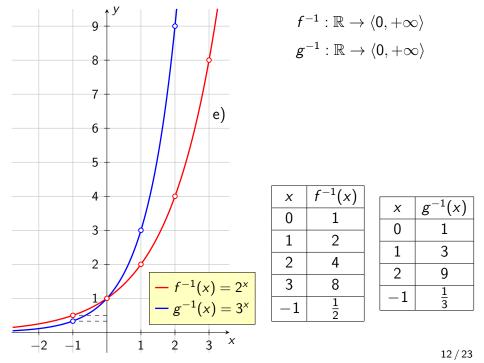


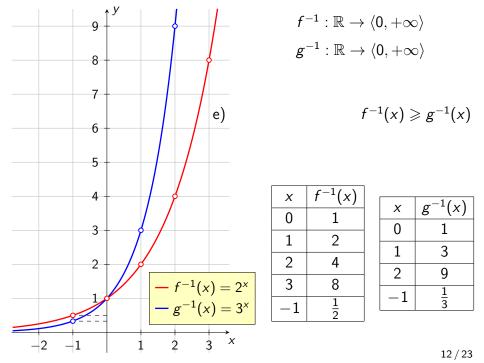


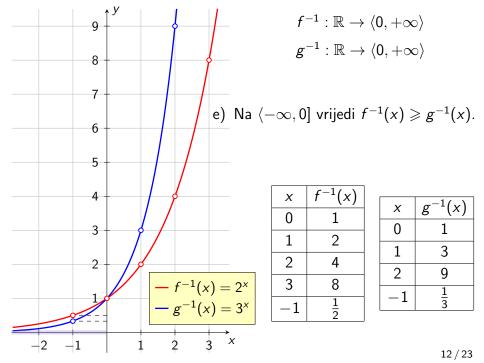


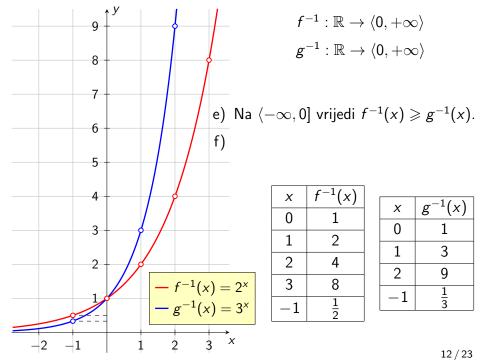


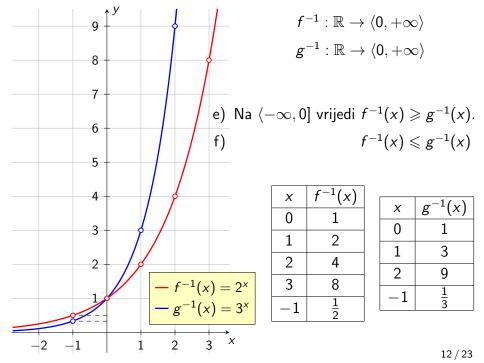


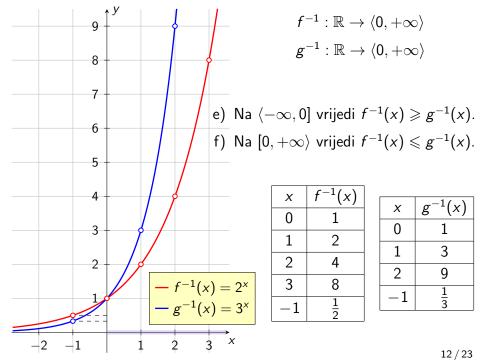


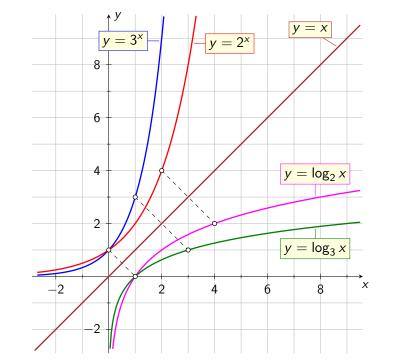










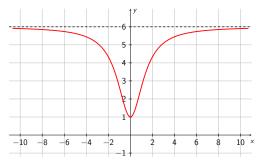


g)

peti zadatak

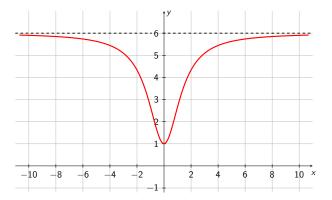
Zadatak 5

Zadan je graf funkcije $g: \mathbb{R} \to \operatorname{Im} g$, a funkcije g_1, g_2 i g_3 imaju isto pravilo pridruživanja kao i funkcija g.



- a) Ispitajte omeđenost funkcije g.
- b) Je li funkcija $g: \mathbb{R} \to \operatorname{Im} g$ bijekcija?
- c) Je li funkcija $g_1: \langle -\infty, 0] \to \mathbb{R}$ bijekcija?
- d) Je li funkcija $g_2:\langle -\infty,0] \to [1,6\rangle$ bijekcija?
- e) Je li funkcija $g_3:[0,+\infty)\to [1,6]$ bijekcija?

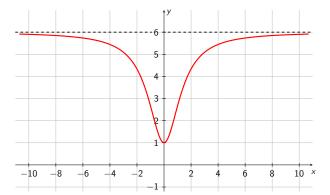
Rješenje



omeđenost
$$m \leqslant g(x) \leqslant M$$

a)

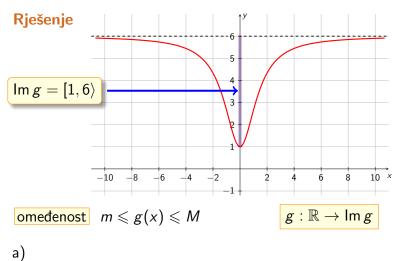
Rješenje



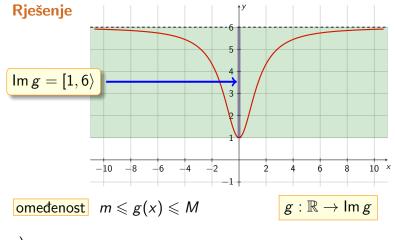
omeđenost
$$m \leqslant g(x) \leqslant M$$

$$g:\mathbb{R} o \operatorname{Im} g$$

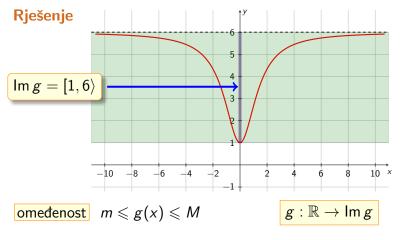
a)



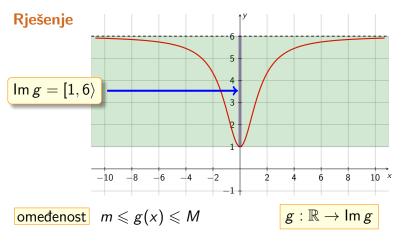
15 / 23



a)

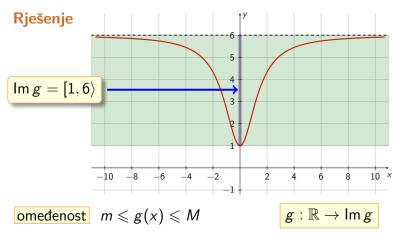


a) Funkcija g je omeđena jer je $1 \leqslant g(x) \leqslant 6$.



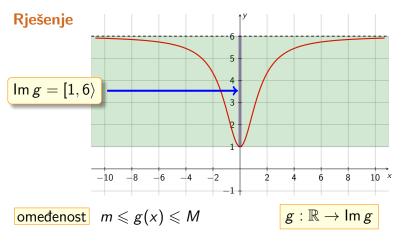
a) Funkcija g je omeđena jer je $1 \leqslant g(x) \leqslant 6$.

$$m = 1$$



a) Funkcija g je omeđena jer je $1 \le g(x) \le 6$.

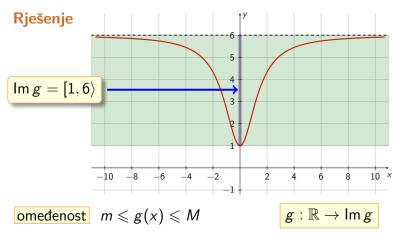
m=1 — najveća donja međa funkcije g



a) Funkcija g je omeđena jer je $1 \leqslant g(x) \leqslant 6$.

$$m=1$$
 — najveća donja međa funkcije g

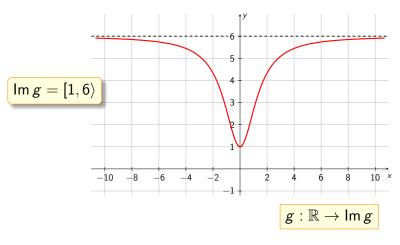
$$M = 6$$

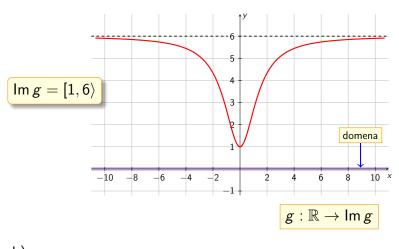


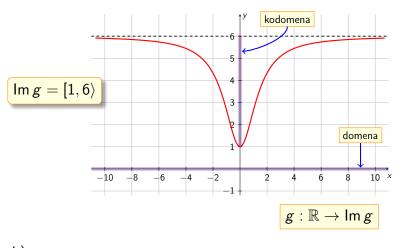
a) Funkcija g je omeđena jer je $1 \le g(x) \le 6$.

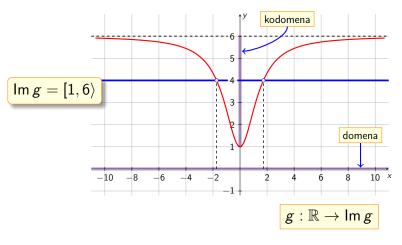
m=1 — najveća donja međa funkcije g

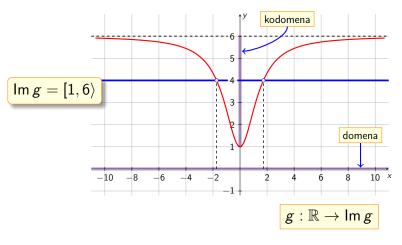
M=6 — najmanja gornja međa funkcije g



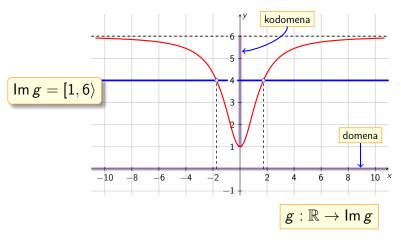






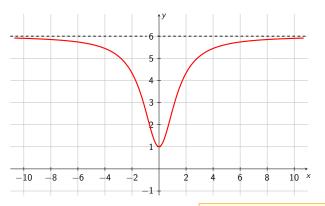


- \implies g nije injekcija jer, na primjer, pravac y=4 siječe graf funkcije g u više od jedne točke.

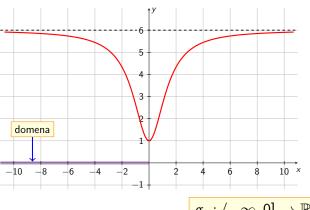


- b) Funkcija g nije bijekcija (jer nije injekcija).

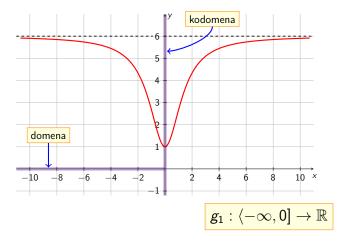
 - $\implies g$ je surjekcija jer je njezina kodomena jednaka $\operatorname{Im} g$.



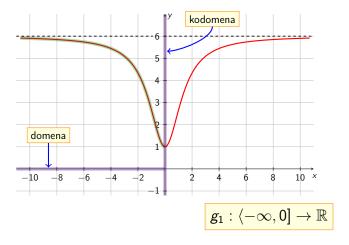
$$g_1: \langle -\infty, 0] o \mathbb{R}$$

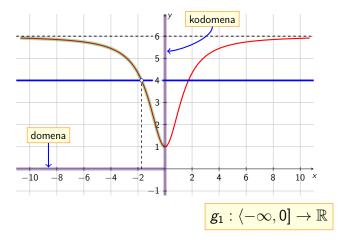


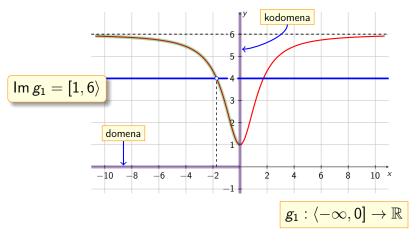
$$g_1:\langle -\infty,0] \to \mathbb{R}$$

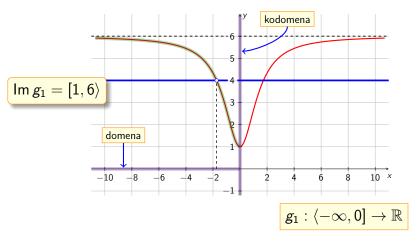


C)

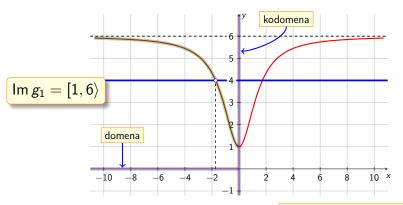








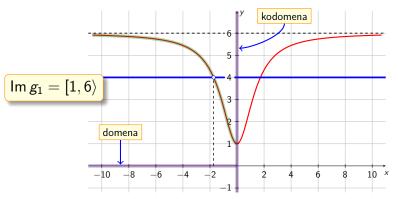
- $riangleq g_1$ nije surjekcija jer je $\operatorname{Im} g_1
 eq \mathbb{R}$.



 $g_1:\langle -\infty,0] \to \mathbb{R}$

c)

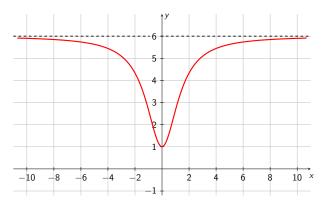
- $riangleq g_1$ nije surjekcija jer je $\operatorname{Im} g_1
 eq \mathbb{R}$.



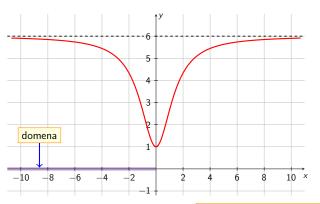
$$g_1:\langle -\infty,0] \to \mathbb{R}$$

- c) Funkcija g_1 nije bijekcija (jer nije surjekcija).

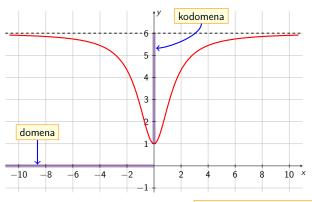
 - $\implies g_1$ nije surjekcija jer je $\operatorname{Im} g_1 \neq \mathbb{R}$.



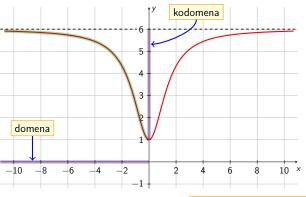
$$g_2:\langle -\infty,0] \to [1,6\rangle$$



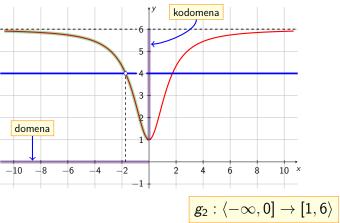
$$g_2:\langle -\infty,0] \to [1,6\rangle$$



$$g_2:\langle -\infty,0] \to [1,6\rangle$$

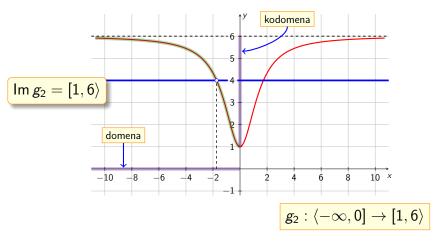


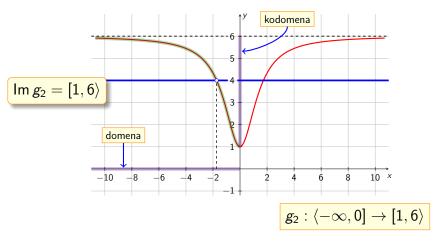
$$g_2:\langle -\infty,0] \to [1,6\rangle$$



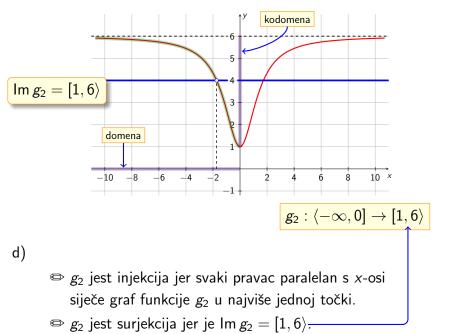
$$g_2:\langle -\infty,0]\to [1,6\rangle$$

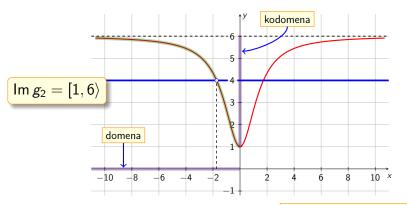
⇔ g₂ jest injekcija jer svaki pravac paralelan s x-osi siječe graf funkcije g_2 u najviše jednoj točki.





- \implies g_2 jest injekcija jer svaki pravac paralelan s x-osi siječe graf funkcije g_2 u najviše jednoj točki.
- $\implies g_2$ jest surjekcija jer je $\text{Im } g_2 = [1, 6\rangle$.

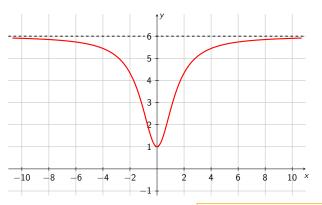




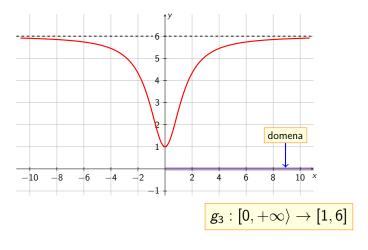
$$g_2:\langle -\infty,0]\to [1,6\rangle$$

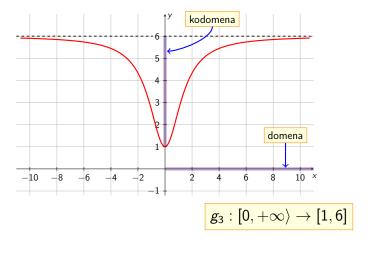
- d) Funkcija g_2 je bijekcija.

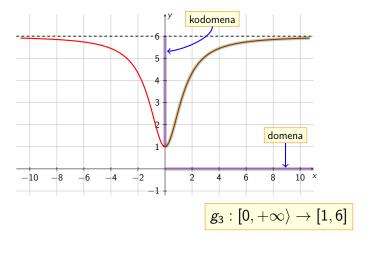
 - $\implies g_2$ jest surjekcija jer je $\text{Im } g_2 = [1, 6)$.

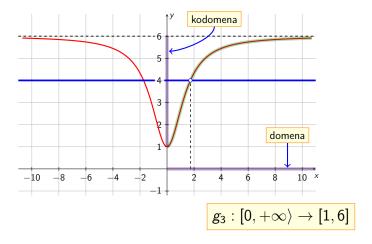


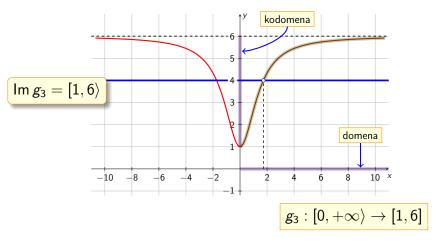
$$g_3:[0,+\infty\rangle \to [1,6]$$

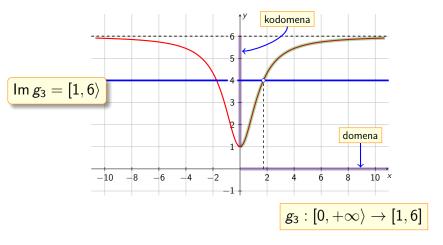




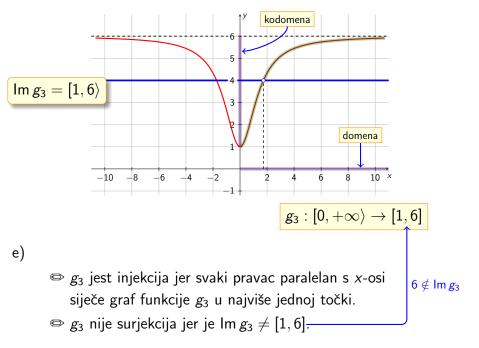


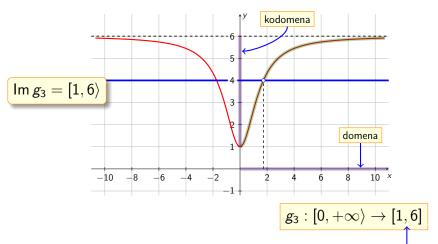






- $\implies g_3$ nije surjekcija jer je $\text{Im }g_3 \neq [1,6].$





- e) Funkcija g_3 nije bijekcija (jer nije surjekcija).

 - $\implies g_3$ nije surjekcija jer je $\text{Im } g_3 \neq [1,6]$ -

 $6 \notin \operatorname{Im} g_3$



šesti zadatak

Zadatak 6

Ispitajte parnost sljedećih funkcija:

a)
$$f(x) = \frac{2x^2}{3-x^2}$$

b)
$$h(x) = 2^{5-x} + 50$$

c)
$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

Zadatak 6

Ispitajte parnost sljedećih funkcija:

a)
$$f(x) = \frac{2x^2}{3-x^2}$$

b)
$$h(x) = 2^{5-x} + 50$$

c)
$$g(x) = \log_4 \frac{3 + 2x}{3 - 2x}$$

Parna funkcija

- $x \in D_f \Rightarrow -x \in D_f$
- $f(-x) = f(x), \forall x \in D_f$

Zadatak 6

Ispitajte parnost sljedećih funkcija:

a)
$$f(x) = \frac{2x^2}{3-x^2}$$

b)
$$h(x) = 2^{5-x} + 50$$

c)
$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

Parna funkcija

- $x \in D_f \Rightarrow -x \in D_f$
- $f(-x) = f(x), \forall x \in D_f$

Neparna funkcija

- $x \in D_f \Rightarrow -x \in D_f$
- $f(-x) = -f(x), \forall x \in D_f$

$$f(x) = \frac{2x^2}{3 - x^2}$$

$$3-x^2\neq 0$$

$$f(x) = \frac{2x^2}{3 - x^2}$$

$$3 - x^2 \neq 0 \xrightarrow{} x^2 \neq 3$$

$$f(x) = \frac{2x^2}{3 - x^2}$$

$$f(x) = \frac{2x^2}{3 - x^2}$$

$$3 - x^2 \neq 0 \xrightarrow{} x^2 \neq 3 \xrightarrow{} x \neq \pm \sqrt{3}$$

a) domena
$$D_f =$$

a) domena
$$D_f = \mathbb{R} \setminus \{-\sqrt{3}, \sqrt{3}\}$$

$$f(x) = \frac{2x^2}{3 - x^2}$$

$$3 - x^2 \neq 0 \xrightarrow{} x^2 \neq 3 \xrightarrow{} x \neq \pm \sqrt{3}$$

a) domena
$$D_f = \mathbb{R} \setminus \left\{ -\sqrt{3}, \sqrt{3} \right\}$$

$$f(x) = \frac{2x^2}{3 - x^2}$$

$$3 - x^2 \neq 0 \xrightarrow{} x^2 \neq 3 \xrightarrow{} x \neq \pm \sqrt{3}$$

$$f(-x) =$$

a) domena
$$D_f = \mathbb{R} \setminus \left\{ -\sqrt{3}, \sqrt{3} \right\}$$

$$f(x) = \frac{2x^2}{3 - x^2}$$

$$3 - x^2 \neq 0 \xrightarrow{} x^2 \neq 3 \xrightarrow{} x \neq \pm \sqrt{3}$$

a) domena
$$D_f = \mathbb{R} \setminus \left\{ -\sqrt{3}, \sqrt{3} \right\}$$

$$f(x) = \frac{2x^2}{3 - x^2}$$

$$3 - x^2 \neq 0 \xrightarrow{} x^2 \neq 3 \xrightarrow{} x \neq \pm \sqrt{3}$$

$$f(-x) = \frac{2 \cdot (-x)^2}{}$$

a) domena
$$D_f = \mathbb{R} \setminus \left\{ -\sqrt{3}, \sqrt{3} \right\}$$

$$f(x) = \frac{2x^2}{3 - x^2}$$

domena)
$$D_f = \mathbb{R} \setminus \{-\sqrt{3}, \sqrt{3}\}$$

$$3 - x^2 \neq 0 \xrightarrow{} x^2 \neq 3 \xrightarrow{} x \neq \pm \sqrt{3}$$

$$f(-x) = \frac{2 \cdot (-x)^2}{3 - (-x)^2}$$

a) domena
$$D_f = \mathbb{R} \setminus \left\{ -\sqrt{3}, \sqrt{3} \right\}$$

$$f(x) = \frac{2x^2}{3 - x^2}$$

$$f(-x) = \frac{2 \cdot (-x)^2}{3 - (-x)^2} = ----$$

a) domena
$$D_f = \mathbb{R} \setminus \left\{ -\sqrt{3}, \sqrt{3} \right\}$$

$$f(x) = \frac{2x^2}{3-x^2}$$

$$3 - x^2 \neq 0 \xrightarrow{} x^2 \neq 3 \xrightarrow{} x \neq \pm \sqrt{3}$$

$$f(-x) = \frac{2 \cdot (-x)^2}{3 - (-x)^2} = \frac{2x^2}{}$$

a) domena
$$D_f = \mathbb{R} \setminus \left\{ -\sqrt{3}, \sqrt{3} \right\}$$

$$f(x) = \frac{2x^2}{3 - x^2}$$

$$3 - x^2 \neq 0 \xrightarrow{} x^2 \neq 3 \xrightarrow{} x \neq \pm \sqrt{3}$$

$$f(-x) = \frac{2 \cdot (-x)^2}{3 - (-x)^2} = \frac{2x^2}{3 - x^2}$$

a) domena
$$D_f = \mathbb{R} \setminus \left\{ -\sqrt{3}, \sqrt{3} \right\}$$

$$f(x) = \frac{2x^2}{3 - x^2}$$

$$3 - x^2 \neq 0 \xrightarrow{} x^2 \neq 3 \xrightarrow{} x \neq \pm \sqrt{3}$$

$$f(-x) = \frac{2 \cdot (-x)^2}{3 - (-x)^2} = \frac{2x^2}{3 - x^2} = f(x)$$

a) domena
$$D_f = \mathbb{R} \setminus \left\{ -\sqrt{3}, \sqrt{3} \right\}$$

$$f(x)=\frac{2x^2}{3-x^2}$$

$$3 - x^2 \neq 0 \longrightarrow x^2 \neq 3 \longrightarrow x \neq \pm \sqrt{3}$$

$$f(-x) = \frac{2 \cdot (-x)^2}{3 - (-x)^2} = \frac{2x^2}{3 - x^2} = f(x)$$

a) domena
$$D_f = \mathbb{R} \setminus \left\{ -\sqrt{3}, \sqrt{3} \right\}$$

$$f(x) = \frac{2x^2}{3 - x^2}$$

$$3 - x^2 \neq 0 \xrightarrow{} x^2 \neq 3 \xrightarrow{} x \neq \pm \sqrt{3}$$

$$f(-x) = \frac{2 \cdot (-x)^2}{3 - (-x)^2} = \frac{2x^2}{3 - x^2} = f(x)$$

$$h(x) = 2^{5-x} + 50$$

a) domena
$$D_f = \mathbb{R} \setminus \left\{ -\sqrt{3}, \sqrt{3} \right\}$$

$$f(x) = \frac{2x^2}{3 - x^2}$$

$$3 - x^2 \neq 0 \longrightarrow x^2 \neq 3 \longrightarrow x \neq \pm \sqrt{3}$$

$$f(-x) = \frac{2 \cdot (-x)^2}{3 - (-x)^2} = \frac{2x^2}{3 - x^2} = f(x)$$

b) domena
$$D_h = \mathbb{R}$$

$$h(x)=2^{5-x}+50$$

a) domena
$$D_f = \mathbb{R} \setminus \left\{ -\sqrt{3}, \sqrt{3} \right\}$$

$$f(x) = \frac{2x^2}{3 - x^2}$$

$$3 - x^2 \neq 0 \xrightarrow{} x^2 \neq 3 \xrightarrow{} x \neq \pm \sqrt{3}$$

$$f(-x) = \frac{2 \cdot (-x)^2}{3 - (-x)^2} = \frac{2x^2}{3 - x^2} = f(x)$$

Funkcija f je parna funkcija.

b) domena $D_h = \mathbb{R}$

$$h(-x) =$$

$$h(x)=2^{5-x}+50$$

a) domena
$$D_f = \mathbb{R} \setminus \left\{ -\sqrt{3}, \sqrt{3} \right\}$$

$$f(x) = \frac{2x^2}{3 - x^2}$$

$$3 - x^2 \neq 0 \xrightarrow{} x^2 \neq 3 \xrightarrow{} x \neq \pm \sqrt{3}$$

$$f(-x) = \frac{2 \cdot (-x)^2}{3 - (-x)^2} = \frac{2x^2}{3 - x^2} = f(x)$$

Funkcija f je parna funkcija.

b) domena $D_h = \mathbb{R}$

$$h(-x) = 2^{5-(-x)}$$

$$h(x) = 2^{5-x} + 50$$

a) domena
$$D_f = \mathbb{R} \setminus \left\{ -\sqrt{3}, \sqrt{3} \right\}$$

$$f(x) = \frac{2x^2}{3 - x^2}$$

$$3 - x^2 \neq 0 \xrightarrow{} x^2 \neq 3 \xrightarrow{} x \neq \pm \sqrt{3}$$

$$f(-x) = \frac{2 \cdot (-x)^2}{3 - (-x)^2} = \frac{2x^2}{3 - x^2} = f(x)$$

Funkcija f je parna funkcija.

b) domena $D_h = \mathbb{R}$

$$h(-x) = 2^{5-(-x)} + 50$$

$$h(x) = 2^{5-x} + 50$$

a) domena
$$D_f = \mathbb{R} \setminus \left\{ -\sqrt{3}, \sqrt{3} \right\}$$

$$f(x) = \frac{2x^2}{3 - x^2}$$

$$3 - x^2 \neq 0 \longrightarrow x^2 \neq 3 \longrightarrow x \neq \pm \sqrt{3}$$

$$f(-x) = \frac{2 \cdot (-x)^2}{3 - (-x)^2} = \frac{2x^2}{3 - x^2} = f(x)$$

b) domena
$$D_h = \mathbb{R}$$

$$h(-x) = 2^{5-(-x)} + 50 = 2^{5+x} + 50$$

$$h(x)=2^{5-x}+50$$

a) domena
$$D_f = \mathbb{R} \setminus \left\{ -\sqrt{3}, \sqrt{3}
ight\}$$

$$f(x) = \frac{2x^2}{3 - x^2}$$

$$3 - x^2 \neq 0 \xrightarrow{} x^2 \neq 3 \xrightarrow{} x \neq \pm \sqrt{3}$$

$$f(-x) = \frac{2 \cdot (-x)^2}{3 - (-x)^2} = \frac{2x^2}{3 - x^2} = f(x)$$

b) domena
$$D_h = \mathbb{R}$$

$$h(x)=2^{5-x}+50$$

$$h(-x) = 2^{5-(-x)} + 50 = 2^{5+x} + 50 \neq \pm h(x)$$

a) domena
$$D_f = \mathbb{R} \setminus \left\{ -\sqrt{3}, \sqrt{3} \right\}$$

$$f(x) = \frac{2x^2}{3 - x^2}$$

$$3 - x^2 \neq 0 \xrightarrow{} x^2 \neq 3 \xrightarrow{} x \neq \pm \sqrt{3}$$

$$f(-x) = \frac{2 \cdot (-x)^2}{3 - (-x)^2} = \frac{2x^2}{3 - x^2} = f(x)$$

Funkcija f je parna funkcija.

b) domena
$$D_h = \mathbb{R}$$

$$h(x) = 2^{5-x} + 50$$

$$h(-x) = 2^{5-(-x)} + 50 = 2^{5+x} + 50 \neq \pm h(x)$$

Funkcija *h* nije niti parna niti neparna.

a) domena
$$D_f = \mathbb{R} \setminus \left\{ -\sqrt{3}, \sqrt{3} \right\}$$

$$f(x) = \frac{2x^2}{3 - x^2}$$

$$3 - x^2 \neq 0 \xrightarrow{} x^2 \neq 3 \xrightarrow{} x \neq \pm \sqrt{3}$$

$$f(-x) = \frac{2 \cdot (-x)^2}{3 - (-x)^2} = \frac{2x^2}{3 - x^2} = f(x)$$

Funkcija f je parna funkcija.

b) domena $D_h = \mathbb{R}$

$$h(x) = 2^{5-x} + 50$$

$$h(-x) = 2^{5-(-x)} + 50 = 2^{5+x} + 50 \neq \pm h(x)$$

Funkcija h nije niti parna niti neparna.

Riešenje

a) domena
$$D_f = \mathbb{R} \setminus \{-\sqrt{3}, \sqrt{3}\}$$

$$f(x) = \frac{2x^2}{3 - x^2}$$

 $h(x) = 2^{5-x} + 50$

$$3 - x^2 \neq 0 \xrightarrow{} x^2 \neq 3 \xrightarrow{} x \neq \pm \sqrt{3}$$

$$f(-x) = \frac{2 \cdot (-x)^2}{3 - (-x)^2} = \frac{2x^2}{3 - x^2} = f(x)$$

Funkcija f je parna funkcija.

b) domena $D_h = \mathbb{R}$

$$h(-x) = 2^{5-(-x)} + 50 = 2^{5+x} + 50 \neq \pm h(x)$$

Funkcija h nije niti parna niti neparna.

$$h(1) =$$

Riešenje

a) domena
$$D_f = \mathbb{R} \setminus \{-\sqrt{3}, \sqrt{3}\}$$

$$f(x) = \frac{2x^2}{3 - x^2}$$

 $h(x) = 2^{5-x} + 50$

$$3 - x^2 \neq 0 \xrightarrow{} x^2 \neq 3 \xrightarrow{} x \neq \pm \sqrt{3}$$

$$f(-x) = \frac{2 \cdot (-x)^2}{3 - (-x)^2} = \frac{2x^2}{3 - x^2} = f(x)$$

Funkcija f je parna funkcija.

b) domena
$$D_h = \mathbb{R}$$

$$h(-x) = 2^{5-(-x)} + 50 = 2^{5+x} + 50 \neq \pm h(x)$$

Funkcija h nije niti parna niti neparna.

$$h(1) = 2^4 + 50$$

Riešenje

a) domena
$$D_f = \mathbb{R} \setminus \{-\sqrt{3}, \sqrt{3}\}$$

$$f(x) = \frac{2x^2}{3 - x^2}$$

 $h(x) = 2^{5-x} + 50$

$$3 - x^2 \neq 0 \xrightarrow{} x^2 \neq 3 \xrightarrow{} x \neq \pm \sqrt{3}$$

$$f(-x) = \frac{2 \cdot (-x)^2}{3 - (-x)^2} = \frac{2x^2}{3 - x^2} = f(x)$$

Funkcija f je parna funkcija.

b) domena $D_h = \mathbb{R}$

$$h(-x) = 2^{5-(-x)} + 50 = 2^{5+x} + 50 \neq \pm h(x)$$

Funkcija h nije niti parna niti neparna.

$$h(1) = 2^4 + 50 = 66$$

a) domena
$$D_f = \mathbb{R} \setminus \left\{ -\sqrt{3}, \sqrt{3} \right\}$$

$$f(x) = \frac{2x^2}{3 - x^2}$$

 $h(x) = 2^{5-x} + 50$

$$3 - x^2 \neq 0 \xrightarrow{} x^2 \neq 3 \xrightarrow{} x \neq \pm \sqrt{3}$$

$$f(-x) = \frac{2 \cdot (-x)^2}{3 - (-x)^2} = \frac{2x^2}{3 - x^2} = f(x)$$

Funkcija f je parna funkcija.

b) domena $D_h = \mathbb{R}$

$$h(-x) = 2^{5-(-x)} + 50 = 2^{5+x} + 50 \neq \pm h(x)$$

Funkcija h nije niti parna niti neparna.

$$h(1) = 2^4 + 50 = 66$$
, $h(-1) =$

a) domena
$$D_f = \mathbb{R} \setminus \{-\sqrt{3}, \sqrt{3}\}$$
 $3 - x^2 \neq 0 \xrightarrow{} x^2 \neq 3 \xrightarrow{} x \neq \pm \sqrt{3}$

$$f(x) = \frac{2x^2}{3 - x^2}$$

$$2 (...)^2 2...^2$$

$$f(-x) = \frac{2 \cdot (-x)^2}{3 - (-x)^2} = \frac{2x^2}{3 - x^2} = f(x)$$

Funkcija f je parna funkcija.

$$h(x)=2^{5-x}+50$$

b) domena
$$D_h = \mathbb{R}$$

$$h(-x) = 2^{5-(-x)} + 50 = 2^{5+x} + 50 \neq \pm h(x)$$

Funkcija h nije niti parna niti neparna.

$$h(1) = 2^4 + 50 = 66, \quad h(-1) = 2^6 + 50$$

a) domena
$$D_f = \mathbb{R} \setminus \{-\sqrt{3}, \sqrt{3}\}$$
 $3 - x^2 \neq 0 \xrightarrow{} x^2 \neq 3 \xrightarrow{} x \neq \pm \sqrt{3}$

$$f(x) = \frac{2x^2}{3 - x^2}$$

$$f(-x) = \frac{2 \cdot (-x)^2}{3 - (-x)^2} = \frac{2x^2}{3 - x^2} = f(x)$$

Funkcija f je parna funkcija.

 $h(x)=2^{5-x}+50$

b) domena
$$D_h = \mathbb{R}$$

$$h(-x) = 2^{5-(-x)} + 50 = 2^{5+x} + 50 \neq \pm h(x)$$

Funkcija *h* nije niti parna niti neparna.

$$h(1) = 2^4 + 50 = 66, \quad h(-1) = 2^6 + 50 = 114$$

a) domena
$$D_f = \mathbb{R} \setminus \{-\sqrt{3}, \sqrt{3}\}$$
 $3 - x^2 \neq 0 \xrightarrow{} x^2 \neq 3 \xrightarrow{} x \neq \pm \sqrt{3}$

$$f(x) = \frac{2x^2}{3 - x^2}$$

 $h(x) = 2^{5-x} + 50$

$$f(-x) = \frac{2 \cdot (-x)^2}{3 - (-x)^2} = \frac{2x^2}{3 - x^2} = f(x)$$

Funkcija f je parna funkcija.

b) domena $D_h = \mathbb{R}$

$$h(-x) = 2^{5-(-x)} + 50 = 2^{5+x} + 50 \neq \pm h(x)$$

Funkcija h nije niti parna niti neparna.

Protuprimjer
$$h(-1) \neq \pm h(1)$$

$$h(1) = 2^4 + 50 = 66, \quad h(-1) = 2^6 + 50 = 114$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

c) domena

$$g(x) = \log_4 \frac{3 + 2x}{3 - 2x}$$

c) domena

$$\frac{3+2x}{3-2x}>0$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x} > 0$$

$$3 + 2x = 0$$

$$x = -\frac{3}{2}$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{2}$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$



$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

$\overline{3+2x}$	
$\overline{3-2x}$	

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

$$\begin{array}{c|c}
3+2x \\
\hline
3-2x \\
\hline
\frac{3+2x}{3-2x}
\end{array}$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

$$\begin{array}{c|c}
-\infty \\
3 + 2x \\
3 - 2x \\
\hline
\frac{3+2x}{3-2x}
\end{array}$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

$$\begin{array}{c|c}
-\infty & +\infty \\
3+2x & \\
3-2x & \\
\hline
\frac{3+2x}{3-2x} & \\
\end{array}$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

$$\begin{array}{c|cccc}
-\frac{3}{2} & +\infty \\
\hline
3+2x & \\
\hline
3-2x & \\
\hline
\frac{3+2x}{3-2x} & \\
\end{array}$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

$$=\frac{1}{2}$$
 $\frac{3}{3}$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

-с	∞ - - - - - - - - - -	$\frac{3}{2}$ $\frac{3}{2}$	$\frac{3}{2}$ $+\infty$)
3 + 2x	_			_
3 - 2x				
$\frac{3+2x}{3-2x}$				

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{}$$
 $x = \frac{3}{}$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

$$\frac{3+2x}{3-2x}$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

$$\frac{3+2x}{3-2x}$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

$$D_g = \left\langle -\frac{3}{2}, \, \frac{3}{2} \right\rangle$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

$$D_{g}=\left\langle -rac{3}{2},\,rac{3}{2}
ight
angle$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x} > 0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

$$D_{g}=\left\langle -rac{3}{2},\,rac{3}{2}
ight
angle$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

$$g(-x) =$$

$$D_{g}=\left\langle -rac{3}{2},\,rac{3}{2}
ight
angle$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

$$g(-x) = \log_{4}$$

$$D_{g}=\left\langle -rac{3}{2},\,rac{3}{2}
ight
angle$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

$$g(-x) = \log_4 \frac{3 + 2 \cdot (-x)}{2}$$

$$D_{g}=\left\langle -rac{3}{2},\,rac{3}{2}
ight
angle$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$

$$x = -\frac{3}{2} \qquad \qquad x = \frac{3}{2}$$

$$g(-x) = \log_4 \frac{3 + 2 \cdot (-x)}{3 - 2 \cdot (-x)}$$

$$D_{g}=\left\langle -rac{3}{2},\,rac{3}{2}
ight
angle$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$g(-x) = \log_4 \frac{3 + 2 \cdot (-x)}{3 - 2 \cdot (-x)} = \log_4 - \cdots$$

$$D_{g}=\left\langle -rac{3}{2},\,rac{3}{2}
ight
angle$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x} > 0$$

$$3+2x = 0 \quad 3-2x = 0$$

$$x = -\frac{3}{2} \quad x = \frac{3}{2}$$

$$3 + 2x = 0 \quad 3 - 2x = 0$$

$$3 + 2x = 0 \quad 3 - 2x = 0$$

$$3 + 2x = 0 \quad 3 - 2x = 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0$$

$$4 + 2x = 0$$

$$5 + 2x = 0$$

$$5 + 2x = 0$$

$$5 + 2x = 0$$

$$7 + 2x = 0$$

$$7 + 2x = 0$$

$$8 + 2x =$$

$$g(-x) = \log_4 \frac{3 + 2 \cdot (-x)}{3 - 2 \cdot (-x)} = \log_4 \frac{3 - 2x}{3 - 2x}$$

$$D_{g}=\left\langle -rac{3}{2},\,rac{3}{2}
ight
angle$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$g(-x) = \log_4 \frac{3 + 2 \cdot (-x)}{3 - 2 \cdot (-x)} = \log_4 \frac{3 - 2x}{3 + 2x}$$

$$D_{g}=\left\langle -rac{3}{2},\,rac{3}{2}
ight
angle$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x} > 0$$

$$3+2x = 0 \quad 3-2x = 0$$

$$x = -\frac{3}{2} \quad x = \frac{3}{2}$$

$$3 + 2x = 0 \quad 3 - 2x = 0$$

$$3 + 2x = 0 \quad 3 - 2x = 0$$

$$3 + 2x = 0 \quad 3 - 2x = 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0$$

$$4 + 2x = 0$$

$$4 + 2x = 0$$

$$4 + 2x = 0$$

$$5 + 2x = 0$$

$$5 + 2x = 0$$

$$5 + 2x = 0$$

$$7 + 2x = 0$$

$$7 + 2x = 0$$

$$8 + 2x$$

$$g(-x) = \log_4 \frac{3 + 2 \cdot (-x)}{3 - 2 \cdot (-x)} = \log_4 \frac{3 - 2x}{3 + 2x} = \log_4$$

$$D_{g}=\left\langle -rac{3}{2},\,rac{3}{2}
ight
angle$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$g(-x) = \log_4 \frac{3 + 2 \cdot (-x)}{3 - 2 \cdot (-x)} = \log_4 \frac{3 - 2x}{3 + 2x} = \log_4 \left(\frac{3 + 2x}{3 - 2x}\right)^{-1}$$

$$D_{g}=\left\langle -rac{3}{2},\,rac{3}{2}
ight
angle$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x} > 0$$

$$3+2x = 0 \quad 3-2x = 0$$

$$x = -\frac{3}{2} \quad x = \frac{3}{2}$$

$$3 + 2x = 0 \quad 3 - 2x = 0$$

$$3 + 2x = 0 \quad 3 - 2x = 0$$

$$3 + 2x = 0 \quad 3 - 2x = 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0 \quad 4 + 0$$

$$3 + 2x = 0$$

$$4 + 2x = 0$$

$$4 + 2x = 0$$

$$4 + 2x = 0$$

$$5 + 2x = 0$$

$$7 + 2x = 0$$

$$7 + 2x = 0$$

$$7 + 2x = 0$$

$$8 + 2x = 0$$

$$8$$

$$g(-x) = \log_4 \frac{3 + 2 \cdot (-x)}{3 - 2 \cdot (-x)} = \log_4 \frac{3 - 2x}{3 + 2x} = \log_4 \left(\frac{3 + 2x}{3 - 2x}\right)^{-1} =$$

$$\log_a x^k = k \cdot \log_a x$$

$$D_g = \left\langle -rac{3}{2}, \, rac{3}{2}
ight
angle$$

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$g(-x) = \log_4 \frac{3 + 2 \cdot (-x)}{3 - 2 \cdot (-x)} = \log_4 \frac{3 - 2x}{3 + 2x} = \log_4 \left(\frac{3 + 2x}{3 - 2x}\right)^{-1} =$$

$$\log_a x^k = k \cdot \log_a x$$

 $D_{g}=\left\langle -rac{3}{2},\,rac{3}{2}
ight
angle$

 $g(x) = \log_4 \frac{3+2x}{3-2x}$

$$\frac{3+2x}{3-2x}>0$$

$$3 + 2x = 0$$
 $3 - 2x = 0$
 $x = -\frac{3}{2}$ $x = \frac{3}{2}$

$$x = \frac{3}{2}$$

$$g(-x) = \log_4 \frac{3 + 2 \cdot (-x)}{3 - 2 \cdot (-x)} = \log_4 \frac{3 - 2x}{3 + 2x} = \log_4 \left(\frac{3 + 2x}{3 - 2x}\right)^{-1} =$$
$$= -\log_4 \frac{3 + 2x}{3 - 2x}$$

$$\frac{3+2}{3-2}$$

$$\log_a x^k = k \cdot \log_a x$$

 $D_{g}=\left\langle -rac{3}{2},\,rac{3}{2}
ight
angle$

 $g(x) = \log_4 \frac{3+2x}{3-2x}$

$$\frac{3+2x}{3-2x}>0$$

$$\frac{2x}{2x} > 0 \qquad \qquad -\infty$$

$$3 - 2x = 0$$

$$3 + 2x$$

$$\frac{3}{2}$$
 $\frac{3}{2}$ $+\infty$ $+$ $+$ $+$ $-$

$$3 + 2x = 0$$
 $3 - 2x = 0$
 $x = -\frac{3}{2}$ $x = \frac{3}{2}$

$$x=\frac{3}{2}$$

$$\frac{3+2x}{3-2x}$$

3 - 2x

$$g(-x) = \log_4 \frac{3 + 2 \cdot (-x)}{3 - 2 \cdot (-x)} = \log_4 \frac{3 - 2x}{3 + 2x} = \log_4 \left(\frac{3 + 2x}{3 - 2x}\right)^{-1} =$$

$$= -\log_4 \frac{3 + 2x}{3 + 2x} = -g(x)$$

$$\log_a x^k = k \cdot \log_a x$$

 $D_{g}=\left\langle -\frac{3}{2},\,\frac{3}{2}\right
angle$

3 + 2x

3 - 2x

$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$\frac{3+2x}{3-2x} > 0$$

$$3 - 2x = 0$$

$$\frac{3}{2}$$
 $\frac{3}{2}$ $+\infty$ $+$ $+$ $+$ $-$

$$3 + 2x = 0$$
 $3 - 2x = 0$
 $x = -\frac{3}{2}$ $x = \frac{3}{2}$

$$x = \frac{3}{2}$$

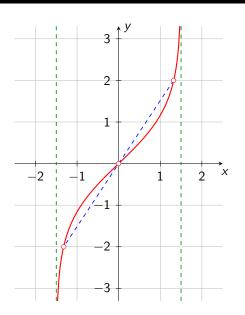
$$g(-x) = \log_4 \frac{3 + 2 \cdot (-x)}{3 - 2 \cdot (-x)} = \log_4 \frac{3 - 2x}{3 + 2x} = \log_4 \left(\frac{3 + 2x}{3 - 2x}\right)^{-1} =$$

$$=-\log_4\frac{3+2x}{3-2x}=-g(x)$$

g je neparna funkcija

22 / 23

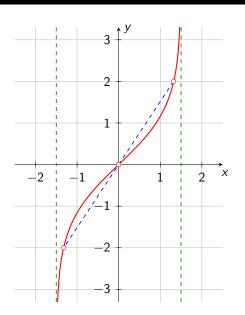
Graf funkcije *g*



$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$D_g = \left\langle -\frac{3}{2}, \frac{3}{2} \right\rangle$$

Graf funkcije *g*

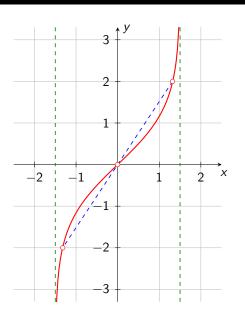


$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$D_g = \left\langle -\frac{3}{2}, \, \frac{3}{2} \right\rangle$$

$$\lim_{x\to\frac{3}{2}-}g(x)=+\infty$$

Graf funkcije *g*



$$g(x) = \log_4 \frac{3+2x}{3-2x}$$

$$D_g = \left\langle -\frac{3}{2}, \, \frac{3}{2} \right\rangle$$

$$\lim_{x\to\frac{3}{2}-}g(x)=+\infty$$

$$\lim_{x\to -\frac{3}{2}+}g(x)=-\infty$$