Analysis of Algorithms

Binomial and Fibonacci Heaps

If more than one question appears correct, choose the more specific answer, unless otherwise instructed.

Assume min-heaps unless otherwise directed.

Binomial heaps

1. After 1 insertion into an empty binomial heap, how many subheaps are in the root list?
   (A) 3
   (B) 1
   (C) 2
   (D) 0
2. After 2 consecutive insertions into an empty binomial heap, how many subheaps are in the root list?
   (A) 1
   (B) 0
   (C) 3
   (D) 2
3. After 7 consecutive insertions into an empty binomial heap, how many subheaps are in the root list?
   (A) 3
   (B) 2
   (C) 1
   (D) 0
4. After 8 consecutive insertions into an empty binomial heap, how many subheaps are in the root list?
   (A) 3
   (B) 2
   (C) 0
   (D) 1
5. Continuing with the previous question, how many subheaps are in the root list if we perform an *extractMin*?
   (A) 2
   (B) 0
   (C) 3
   (D) 1
6. How many elements are in a binomial heap with only a degree 0 subheap?
   (A) 1
   (B) 2
   (C) 0
   (D) 3
7. How many elements are in a binomial heap with only a degree 2 and a degree 3 subheap?
   (A) 0
   (B) 8
   (C) 4
   (D) 12
8. How many elements are in a binomial heap with only a degree 2 and a degree 5 subheap?
   (A) 0
   (B) 18
   (C) 54
   (D) 36
9. What is the binary representation of 29?
   (A) 0001 1101
   (B) 0010 1001
   (C) 0010 0010
   (D) 0000 0000
10. How many subheaps are in a binomial heap with 29 elements?
    (A) 2
    (B) 3
    (C) 0
    (D) 4
    (E) 1

11. What is the binary representation of 8?

(A) 0010
(B) 0100
(C) 0000
(D) 1000
(E) 0001

12. What is the degree of the subheap in a binomial heap with 8 elements?

(A) 2
(B) 3
(C) 1
(D) 0

13. **T** or **F**: You can delete an element from a binomial heap without consolidating.

14. **T** or **F**: You can decrease a key in a binomial heap without consolidating.

15. **T** or **F**: Sometimes, a decrease key operation in a binomial heap can cause consolidation.

16. After inserting a 0 and a 1 into an empty binomial min-heap, which element(s) will be in the root list?

(A) 1
(B) None
(C) 0
(D) Both of them

17. Continuing with the previous question, inserting a 2 yields which values in the root list?

(A) 2
(B) 0
(C) None
(D) 1 and 2
(E) 0 and 1
(F) 0, 1, and 2
(G) 1
(H) 0 and 2

18. Continuing with the previous question, inserting a 3 yields which values in the root list?

(A) 0
(B) None
(C) 3
(D) 0, 1, 2, and 3
(E) 2
(F) 1 and 2
(G) 0 and 3
(H) 1

19. Continuing with the previous question, performing an *extractMin* yields which values in the root list?

(A) 3
(B) 1
(C) 2 and 3
(D) 1, 2, and 3
(E) 2
(F) None
(G) 1 and 2
(H) 1 and 3

20. After 7 consecutive inserts into an empty binomial heap, how many subheaps are in the root list?

(A) 7
(B) 4
(C) 1
(D) 8
(E) 5
(F) 6
(G) 2
(H) 3

21. After 9 insertions into an empty binomial heap, how many subheaps are there?

(A) 6
(B) 5
(C) 3
(D) 9
(E) 4
(F) 8
(G) 2
(H) 7

22. **T** or **F**: Consider a sequence of n insertions into an empty binomial heap, followed by a single *extractMin* operation. At this point, the number of subheaps in the root list can be calculated from n.

23. Consider inserting the following values, in the order given:

    ```
    3 2 9 5 6 4 1 0
    ```

    into an empty binomial heap. The value 0 is found in a subheap whose root has value:

    (A) 7
    (B) 2
    (C) 5
    (D) 0
    (E) 6
    (F) 4
    (G) 1
    (H) 3

24. Consider inserting the consecutive integers from 0 to 12, inclusive and in increasing order, into an empty binomial heap. After 3 *extractMin* operations, the value 12 can be found in the subheap whose root has value:

    (A) 3
    (B) 7
    (C) 4
    (D) 1
    (E) 0
    (F) 6
    (G) 2
    (H) 5

25. Consider inserting the consecutive integers from 0 to 12, inclusive and in increasing order, into an empty binomial heap. After deleting the value 5, the value 12 can be found in the subheap whose root has value:

    (A) 6
    (B) 4
    (C) 3
    (D) 7
    (E) 1
    (F) 2
    (G) 5
    (H) 0

26. One expects to insert a value into a binomial heap in time that is:

    (A) log-linear
    (B) constant
    (C) log log
    (D) linear
    (E) log

27. Consider inserting the consecutive integers from 0 to 12, inclusive and not necessarily in order, into an empty binomial heap. What is the largest root value possible after all values have been inserted?

Fibonacci heaps

28. After one insertion into an empty fibonacci heap, how many subheaps are in the root list?

    (A) 2
    (B) 0
    (C) 3
    (D) 1

29. After two consecutive insertions into an empty fibonacci heap, how many subheaps are in the root list?

    (A) 1
    (B) 3
    (C) 2
    (D) 4

30. After four consecutive insertions into an empty fibonacci heap, how many subheaps are in the root list?

    (A) 3
    (B) 1
    (C) 2
    (D) 4

31. After 8 consecutive insertions into an empty fibonacci heap, how many subheaps are in the root list?

    (A) 8
    (B) 5
    (C) 7
    (D) 6

32. Continuing with the previous question, how many subheaps are in the root list if we perform an *extractMin*?

(A) 6
(B) 8
(C) 1
(D) 3

33. After inserting 0 and 1 into an empty fibonacci min-heap, which element(s) will be in the root list?

(A) 1
(B) None
(C) 0
(D) 0 and 1

34. Continuing with the previous question, inserting a 2 yields which values in the root list?

(A) 0, 1, and 2
(B) 0 and 1
(C) 2
(D) 0 and 2
(E) 0
(F) 1 and 2
(G) 1
(H) None

35. Continuing with the previous question, inserting a 3 yields which values in the root list?

(A) 2
(B) 1
(C) 0 and 3
(D) 0, 1, 2, and 3
(E) 1 and 2
(F) 0
(G) None
(H) 3

36. Continuing with the previous question, performing an *extractMin* yields which values in the root list?

(A) 1
(B) 2
(C) 2 and 3
(D) 3
(E) 1 and 2
(F) 1 and 3
(G) None
(H) 1, 2, and 3

Assume for fibonacci heaps that newly inserted values are inserted at the beginning (leftmost) of the root list and that consolidation runs from left to right. Assume that in the union of heaps A and B, the root list of B is appended to the root list of A. Assume that *extractMin* appends the child list of the extracted value to the root list. Assume that after consolidation, the root list is ordered with lower degree subheaps to the left of higher degree subheaps.

37. After 7 consecutive inserts into an empty fibonacci heap, how many subheaps are in the root list?

(A) 4
(B) 2
(C) 1
(D) 8
(E) 5
(F) 3
(G) 6
(H) 7

38. **T** or **F**: Consider a sequence of n insertions into an empty fibonacci heap, followed by a single *extractMin* operation. At this point, the number of subheaps in the root list can be calculated from n.

39. **T** or **F**: Consider a sequence of n insertions into an empty fibonacci heap, followed by two consecutive *extractMin* operations. At this point, the number of subheaps in the root list can be calculated from n.

40. Consider inserting the following values, in the order given:

```
3 2 9 5 6 4 1 0
```

into an empty fibonacci heap. The value 0 is found in a subheap whose root has value:

(A) 5
(B) 3
(C) 6
(D) 4
(E) 1
(F) 2
(G) 0
(H) 7

41. Consider inserting the consecutive integers from 0 to 12, inclusive and in increasing order, into an empty fibonacci heap. After 3 *extractMin* operations, the value 12 can be found in the subheap whose root has value:

(A) 4
(B) 7
(C) 5
(D) 0
(E) 1
(F) 2
(G) 6
(H) 3

42. Consider inserting the consecutive integers from 0 to 12, inclusive and in increasing order, into an empty fibonacci heap. After deleting the value 5, the value 12 can be found in the subheap whose root has value:

(A) 2
(B) 3
(C) 4
(D) 6
(E) 5
(F) 1
(G) 7
(H) 0

43. Consider inserting the consecutive integers from 0 to 12 into an empty fibonacci heap. After deleting the value 5, which node is marked?

(A) 3
(B) 0
(C) 4
(D) the answer is not given
(E) 2
(F) 1
(G) 5
(H) no node is marked

44. One expects to find a value in a Fibonacci heap in amortized:

(A) $\Theta(\log n \times \log n)$ time
(B) $\Theta(n)$ time
(C) $\Theta(\log(\log n))$ time
(D) $\Theta(\log n)$ time
(E) $\Theta(n \log n)$ time
(F) $\Theta(1)$ time

45. One expects to find a value in a Fibonacci heap in the worst case in:

(A) $\Theta(\log n \times \log n)$ time
(B) $\Theta(\log(\log n))$ time
(C) $\Theta(\log n)$ time
(D) $\Theta(n)$ time
(E) $\Theta(1)$ time
(F) $\Theta(n \log n)$ time

46. Consider this set of operations: 15 inserts and one extraction of the minimum (in any order). What is the fewest / most number of subheaps found after the set is performed on an initially empty fibonacci heap?

(A) 3 / 14
(B) 3 / 15
(C) 3 / 13
(D) 4 / 12
(E) 3 / 12
(F) 4 / 13
(G) 4 / 15
(H) 4 / 14

47. **T** or **F**: A *decreaseKey* operation can be performed upon the root node of a subheap.

48. **T** or **F**: A *decreaseKey* operation can cause a root node to lose multiple children.

49. **T** or **F**: A single cut (not cascading) can remove more than one node from a subheap.

50. What degree is the lowest degree subheap in which a cascading cut can be performed?

(A) 4
(B) 1
(C) 2
(D) 5
(E) 0
(F) 3

51. How many nodes is it possible to cut from a subheap of degree 2?

(A) 0
(B) 4
(C) 1
(D) 2
(E) 5
(F) 3

52. How many nodes is it possible to cut from a subheap of degree 3?

(A) 4
(B) 0
(C) 1
(D) 2
(E) 5
(F) 3

53. How many nodes is it possible to cut from a subheap of degree 4?

(A) 6
(B) 11
(C) 8
(D) 9
(E) 7
(F) 10

54. How many nodes is it possible to cut from a subheap of degree d?

(A) $2^d - fib(d+2)$
(B) $d^2 - fib(d+1)$
(C) $2^d - fib(d)$
(D) $d^2 - fib(d+2)$
(E) $2^d - fib(d+1)$
(F) $d^2 - fib(d)$

55. Consider cuts that only remove a single node. How many of these cuts can be made in a subheap of degree 3 without causing a cascading cut?

(A) 3
(B) 2
(C) 4
(D) 6
(E) 5
(F) 8
(G) 1
(H) 7

56. Consider cuts that only remove a single node. How many of these cuts can be made in a subheap of degree n without causing a cascading cut?

(A) $2^{n-1} + 1$
(B) 2^{n+1}
(C) n^2
(D) $n^2 - 1$

When cutting a node out of a Fibonacci subheap, the degree of the parent is decremented. In addition, the parent of the node (unless it is a root) is either marked, if it hasn't been previously marked, or cut, if it has.

57. What is the most number of nodes that can be cut in a single cascade on a subheap of degree 4?

(A) 5
(B) 2
(C) 1
(D) 3
(E) 4
(F) 6
(G) 8
(H) 7

58. What is the most number of marked nodes that can exist in a subheap of degree 4?

(A) 15
(B) 8
(C) 16
(D) 3
(E) 4
(F) 7
(G) 2
(H) 5

59. What are the most number of nodes that can be cut from a Fibonacci subheap of degree 4 such that the degree of the root is not reduced?

(A) 8
(B) 10
(C) 3
(D) 4
(E) 6
(F) 7
(G) 5
(H) 9

60. What are the most number of nodes that can be cut from a Fibonacci subheap of degree 4 such that the degree of the root is reduced by exactly one?

(A) 12
(B) 10
(C) 14
(D) 13
(E) 9
(F) 11
(G) 7
(H) 8

Priority Queues

61. Consider a priority queue (max) based upon a singly lined list. What is the time complexity of the insert operation?

(A) logarithmic
(B) log linear
(C) constant
(D) linear

62. Consider a priority queue (max) based upon a normal heap. What is the time complexity of the insert operation?

(A) logarithmic
(B) linear
(C) constant
(D) log linear

63. Consider a priority queue (max) based upon a singly lined list. What is the time complexity of the increase-key operation?

(A) linear
(B) logarithmic
(C) constant
(D) log linear

64. Consider a priority queue (max) based upon a normal heap. What is the time complexity of the increase-key operation?

(A) logarithmic
(B) log linear
(C) linear
(D) constant

65. Consider a priority queue (max) based upon a singly lined list. What is the time complexity of the decrease-key operation? Assume the link list is sort

(A) logarithmic
(B) linear
(C) constant
(D) log linear

66. Consider a priority queue based upon a normal heap. What is the time complexity of the decrease-key operation?

(A) log linear
(B) logarithmic
(C) constant
(D) linear