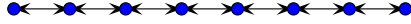


Summations



Prove $\sum_{k=1}^n k = \frac{n(n+1)}{2}$.

By induction on n .

Basis: Let $n = 1$.

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^1 k = \frac{1(1+1)}{2}$$

$$1 = \frac{1 \times 2}{2} = \frac{2}{2}$$

$$1 = 1$$

■

Inductive Hypothesis: Assume true for $1 < k < n$.

Induction:

$$\sum_{k=1}^n k = \sum_{k=1}^{n-1} k + n$$

$$\sum_{k=1}^n k = \frac{(n-1)((n-1)+1)}{2} + n \text{ (BY THE INDUCTIVE HYPOTHESIS)}$$

$$\sum_{k=1}^n k = \frac{(n-1)n}{2} + n$$

$$\sum_{k=1}^n k = \frac{n^2 - n}{2} + \frac{2n}{2}$$

$$\sum_{k=1}^n k = \frac{n^2 - n + 2n}{2}$$

$$\sum_{k=1}^n k = \frac{n^2 + n}{2}$$

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

■

Prove $\sum_{k=0}^n 2^k = 2^{n+1} - 1$.

By induction on n .

Basis: Let $n = 0$.

$$\sum_{k=0}^n 2^k = 2^{n+1} - 1$$

$$\sum_{k=0}^0 2^k = 2^{0+1} - 1$$

$$2^0 = 2^1 - 1 = 2 - 1$$

$$1 = 1$$

■

Inductive Hypothesis: Assume true for $1 < d < n$.

Induction:

$$\sum_{k=0}^n 2^k = \sum_{k=0}^{n-1} 2^k + 2^n$$

$$\sum_{k=0}^n k = 2^{(n-1)+1} - 1 + 2^n \text{ (BY THE INDUCTIVE HYPOTHESIS)}$$

$$\sum_{k=0}^n k = 2^n - 1 + 2^n$$

$$\sum_{k=0}^n k = 2 \times 2^n - 1$$

$$\sum_{k=0}^n k = 2^{n+1} - 1$$

■

Problems:

- 2.2-3
- 2.2-7
- 3.1-1
- 3.1-5
- 3.1-7