Analysis of Algorithms

Recurrence Equations

If more than one answer appears correct, choose the more specific answer, unless otherwise instructed.

Concept: *recurrence equations*

1. Stooge sort has the following algorithm. Recursively sort the lower two-thirds of an array, then recursively sort the upper two-thirds, then recursively sort the lower two-thirds again. The recursion stops when the array consists of two or fewer elements. If the array size is two, the elements are swapped if necessary. Which of the following recurrence equations describe stooge sort?

   (A) $T(n) = 3T(\frac{n}{2}) + \Theta(n)$
   (B) $T(n) = 3T(2\frac{n}{3}) + \Theta(1)$
   (C) $T(n) = 3T(\frac{n}{2}) + \Theta(\log n)$
   (D) $T(n) = 2T(\frac{n}{3}) + \Theta(1)$
   (E) $T(n) = 2T(3\frac{n}{2}) + \Theta(1)$

2. Merge sort has the following algorithm. Recursively sort the lower half of an array, then recursively sort the upper half, then merge the two halves. When merging, a single pass is made through each half. The recursion stops when the array consists of one element. Which of the following recurrence equations describe merge sort?

   (A) $T(n) = 4T(\frac{n}{2}) + \Theta(1)$
   (B) $T(n) = T(n-1) + \Theta(n)$
   (C) $T(n) = 2T(\frac{n}{2}) + \Theta(1)$
   (D) $T(n) = 2T(\frac{n}{2}) + \Theta(n)$

3. Binary search has the following algorithm. Look at the middle element in an array. If the element is the one begin looked for (the target), return FOUND. Otherwise, if the element is greater than the target, recursively search the lower half of the array. Otherwise, recursively search the upper half of the array. If the array consists of three or fewer elements, perform a linear search of the array, returning FOUND if found and MISSING otherwise. Which of the following recurrence equations describe binary search?

   (A) $T(n) = T(\frac{n}{2}) + \Theta(n)$
   (B) $T(n) = 2T(\frac{n}{2}) + \Theta(n)$
   (C) $T(n) = 2T(\frac{n}{2}) + \Theta(\log n)$
   (D) $T(n) = 2T(n-1) + \Theta(1)$
   (E) $T(n) = 2T(\frac{n}{2}) + \Theta(1)$
   (F) $T(n) = T(\frac{n}{2}) + \Theta(1)$
   (G) $T(n) = T(\frac{n}{2}) + \Theta(\log n)$

4. Quicksort can have the following algorithm: find the median value of the array and partition the array into two regions, one region holds all the values less than the median and the other holds all values greater than the median. Median finding and partitioning can both be done in linear time. Finally, the two regions are recursively sorted. Which of the following recurrence equations describe this version of quicksort?

   (A) $T(n) = 2T(\frac{n}{2}) + \Theta(n)$
   (B) $T(n) = 2T(\frac{n}{2}) + \Theta(1)$
   (C) $T(n) = T(\frac{n}{2}) + \Theta(\log n)$
   (D) $T(n) = T(\frac{n}{2}) + \Theta(n^2)$
   (E) $T(n) = 2T(n-1) + \Theta(1)$
   (F) $T(n) = T(\frac{n}{2}) + \Theta(n)$
   (G) $T(n) = 2T(\frac{n}{2}) + \Theta(n^2)$

5. The recurrence equation $T(n) = T(n-1) + \Theta(n)$ describes which sort, in the worst case?

   (A) merge sort
   (B) radix sort
   (C) quick sort
   (D) heap sort

6. Consider using selection sort to sort a previously sorted array. Which of the following recurrence equations describes this situation?

   (A) $T(n) = T(n-1) + \Theta(n)$
   (B) $T(n) = 2T(n-1) + \Theta(n)$
   (C) $T(n) = 2T(\frac{n}{2}) + \Theta(n)$
   (D) $T(n) = 2T(\frac{n}{2}) + \Theta(1)$
   (E) $T(n) = 2T(n-1) + \Theta(1)$
   (F) $T(n) = T(n-1) + \Theta(1)$

Concept: *recursion trees*

Level 0 refers to the apex of recursion tree. Level 1 refers to the level immediately below level 0, and so on. Assume at the bottom level, the size of the problem is 1. Assume that it takes $\Theta(1)$ time to solve a problem of size 1. Assume the problem size is always evenly divisible when divided.

7. Given the recurrence $T(n) = 4T(\frac{n}{2}) + \log n$, what is the work done at the level 0?

   (A) $n^{\log_2 4}$
   (B) $4n$
   (C) $4 \log \frac{n}{2}$
   (D) $4^{log_2 n}$
   (E) $\log n$
   (F) n^4

8. Given the recurrence $T(n) = 4T(\frac{n}{2}) + \log n$, what is the work done at level 1?

   (A) $4^{log_2 \frac{n}{2}}$
   (B) $4 \log \frac{n}{2}$
   (C) n^4
   (D) $2n$
   (E) $\log n$
   (F) $\frac{n}{2}^{\log_2 4}$

9. Given the recurrence $T(n) = 4T(\frac{n}{3}) + n$, how many nodes are level 2?

   (A) 2
   (B) 16
   (C) 6
   (D) 8
   (E) 4
   (F) 12

10. Given the recurrence $T(n) = 4T(\frac{n}{3}) + n$, how many nodes are at the bottom level?

    (A) $3^{\log_3 n}$
    (B) $4^{\log_4 n}$
    (C) $3^{\log_4 n}$
    (D) $4^{\log_3 n}$

11. Given the recurrence $T(n) = 4T(\frac{n}{3}) + \log n$, how much work is done at the bottom level?

    (A) $\Theta(4^{\log_3 n} \log n)$
    (B) $\Theta(3^{\log_3 n} \log n)$
    (C) $\Theta(3^{\log_4 n} \log n)$
    (D) $\Theta(3^{\log_4 n})$
    (E) $\Theta(4^{\log_4 n})$
    (F) $\Theta(4^{\log_4 n} \log n)$
    (G) $\Theta(3^{\log_3 n})$
    (H) $\Theta(4^{\log_3 n})$

The following twelve questions concern a recurrence tree for the following equation: $T(n) = 2T(\frac{n}{3}) + n$.

12. What is the root of the recurrence tree?

    (A) $\frac{n}{3}$
    (B) n
    (C) $\frac{2n^2}{3}$
    (D) $\frac{2n}{3}$

13. How many nodes are on level 1?

    (A) 3
    (B) 2
    (C) n
    (D) $\frac{n}{3}$

14. What is the work of each individual node on level 1?

    (A) n
    (B) $\frac{2n}{3}$
    (C) $\frac{n}{3}$
    (D) $\frac{2n^2}{3}$

15. What is the total work of the nodes on level 1?

    (A) $\frac{n}{3}$
    (B) $\frac{2n}{3}$
    (C) n
    (D) $\frac{2n^2}{3}$

16. How many nodes are on level 2?

    (A) 9
    (B) 4
    (C) n
    (D) $2n$

17. What is the work of each individual node on level 2?

   (A) $\frac{2n}{9}$
   (B) $\frac{4n}{9}$
   (C) $\frac{2n}{3}$
   (D) $\frac{n}{9}$

18. What is the total work of the nodes on level 2?

   (A) $\frac{n}{3}$
   (B) n
   (C) $\frac{2n^2}{3}$
   (D) $\frac{4n}{9}$
   (E) $\frac{4n}{3}$
   (F) $\frac{2n}{3}$

19. $a^{log_b c}$ is equivalent to:

   (A) $c^{log_b a}$
   (B) $b^{log_a c}$
   (C) $c^{log_a b}$
   (D) $b^{log_c a}$
   (E) $a^{log_c b}$
   (F) none of the other answers are correct

20. What is the total work on the bottom level?

   (A) n^2
   (B) n^3
   (C) $n^{log_2 3}$
   (D) $n^{log_3 2}$

21. What is the total work on the bottom level?

   (A) $2^{log_3 n}$
   (B) 2^n
   (C) $3^{log_2 n}$
   (D) 3^n

22. **T** or **F**: The work done by each node on the bottom level of the tree is T(1).

23. **T** or **F**: The total work on the bottom level is equal to $\theta(k)$ where k is the number of nodes on the bottom level.

Concept: ***identifying cases of the master recurrence theorem***

For the following questions, consider recurrence equations of the form $T(n) = aT(\frac{n}{b}) + f(n)$.

24. **T** or **F**: The values $a = \sqrt{2}$ and $b = \sqrt{5}$ fit the constraints of the Master method.

25. **T** or **F**: The values $a = 9$ and $b = \sqrt{5}$ fit the constraints of the Master method.

26. **T** or **F**: The values $a = 2$ and $b = \frac{3}{4}$ fit the constraints of the Master method.

27. **T** or **F**: The values $a = 4$ and $b = \sqrt{5}$ do not fit the constraints of the Master method.

28. **T** or **F**: The values $a = -4$ and $b = 3$ fit the constraints of the Master method.

29. **T** or **F**: The values $a = 5$ and $b = 1$ fit the constraints of the Master method.

30. **T** or **F**: The total work of a recurrence equation $T(n) = aT(\frac{n}{b}) + f(n)$ with $a \geq 1$ and $b > 1$ is always $\Theta(n^{log_b a})$.

31. **T** or **F**: The total work of a recurrence equation $T(n) = aT(\frac{n}{b}) + f(n)$ with $a = 1$ and $b > 1$ is always $\Theta(n^{log_b a})$.

32. **T** or **F**: The total work of a recurrence equation $T(n) = aT(\frac{n}{b}) + f(n)$ with $a > 1$ and $b > 1$ and $f = \theta(1)$ is always $\Theta(n^{log_b a})$.

33. **T** or **F**: The total work of a recurrence equation $T(n) = aT(\frac{n}{b}) + f(n)$ with $a \geq 1$ and $b > 1$ is always $\Theta(f(n))$.

34. **T** or **F**: The total work of a recurrence equation $T(n) = aT(\frac{n}{b}) + f(n)$ with $a = 1$ and $b > 1$ and $f = \omega \log n$ is always $\Theta(f(n))$.

35. **T** or **F**: The total work of a recurrence equation $T(n) = aT(\frac{n}{b}) + f(n)$ with $a = 1$ and $b > 1$ and $f = \omega 1$ is always $\Theta(f(n))$.

36. In terms of the master recurrence theorem, where does the equation $T(n) = 4T(\frac{n}{4}) + n^2$ fall?

(A) case 3
(B) between case 2 and case 3
(C) case 1
(D) case 2
(E) the equation is not in the correct form
(F) between case 1 and case 2

37. In terms of the master recurrence theorem, where does the equation $T(n) = 3T(\frac{n}{2}) + n^2$ fall?

(A) case 1
(B) the equation is not in the correct form
(C) case 3
(D) case 2
(E) between case 2 and case 3
(F) between case 1 and case 2

38. In terms of the master recurrence theorem, where does the equation $T(n) = 4T(\frac{n}{2}) + n^2$ fall?

(A) the equation is not in the correct form
(B) case 3
(C) case 2
(D) between case 1 and case 2
(E) between case 2 and case 3
(F) case 1

39. In terms of the master recurrence theorem, where does the equation $T(n) = T(\frac{n}{2}) + 2^n$ fall?

(A) between case 2 and case 3
(B) case 2
(C) case 3
(D) between case 1 and case 2
(E) case 1
(F) the equation is not in the correct form

40. In terms of the master recurrence theorem, where does the equation $T(n) = 2^n T(\frac{n}{2}) + n^n$ fall?

(A) case 3
(B) between case 1 and case 2
(C) between case 2 and case 3
(D) the equation is not in the correct form
(E) case 1
(F) case 2

41. In terms of the master recurrence theorem, where does the equation $T(n) = T(n-1) + 1$ fall?

(A) case 2
(B) case 1
(C) the equation is not in the correct form
(D) between case 2 and case 3
(E) between case 1 and case 2
(F) case 3

42. In terms of the master recurrence theorem, where does the equation $T(n) = 2T(n-2) + n$ fall?

(A) case 3
(B) between case 2 and case 3
(C) case 3
(D) case 2
(E) case 1
(F) between case 1 and case 2
(G) the equation is not in the correct form

43. In terms of the master recurrence theorem, where does the equation $T(n) = 16T(\frac{n}{4}) + n$ fall?

(A) the equation is not in the correct form
(B) case 1
(C) case 2
(D) case 3
(E) between case 1 and case 2
(F) between case 2 and case 3

44. In terms of the master recurrence theorem, where does the equation $T(n) = 2T(\frac{n}{2}) + n \log n$ fall?

(A) the equation is not in the correct form
(B) between case 1 and case 2
(C) case 2
(D) case 3
(E) case 1
(F) between case 2 and case 3

45. In terms of the master recurrence theorem, where does the equation $T(n) = 3T(\frac{n}{2}) + n \log n$ fall?

(A) between case 2 and case 3
(B) between case 1 and case 2
(C) case 2
(D) the equation is not in the correct form
(E) case 1
(F) case 3

46. In terms of the master recurrence theorem, where does the equation $T(n) = 2T(\frac{n}{2}) + \frac{n}{\log n}$ fall?

(A) between case 2 and case 3
(B) case 1
(C) between case 1 and case 2
(D) the equation is not in the correct form
(E) case 3
(F) case 2

47. In terms of the master recurrence theorem, where does the equation $T(n) = 2T(\frac{n}{4}) + n^{0.51}$ fall?

(A) between case 1 and case 2
(B) case 1
(C) between case 2 and case 3
(D) case 3
(E) the equation is not in the correct form
(F) case 2

48. In terms of the master recurrence theorem, where does the equation $T(n) = \frac{1}{2}T(\frac{n}{2}) + 1$ fall?

(A) case 2
(B) between case 2 and case 3
(C) the equation is not in the correct form
(D) case 3
(E) between case 1 and case 2
(F) case 1

49. In terms of the master recurrence theorem, where does the equation $T(n) = 16T(\frac{n}{4}) + n!$ fall?

(A) case 2
(B) case 3
(C) case 1
(D) the equation is not in the correct form
(E) between case 1 and case 2
(F) between case 2 and case 3

50. In terms of the master recurrence theorem, where does the equation $T(n) = \sqrt{2}T(\frac{n}{2}) + \log n$ fall?

(A) between case 2 and case 3
(B) the equation is not in the correct form
(C) case 3
(D) case 2
(E) case 1
(F) between case 1 and case 2

51. In terms of the master recurrence theorem, where does the equation $T(n) = 3T(\frac{n}{2}) + n$ fall?

(A) between case 1 and case 2
(B) case 2
(C) the equation is not in the correct form
(D) case 1
(E) between case 2 and case 3
(F) case 3

52. In terms of the master recurrence theorem, where does the equation $T(n) = 3T(\frac{n}{3}) + \sqrt{n}$ fall?

(A) between case 2 and case 3
(B) case 3
(C) between case 1 and case 2
(D) the equation is not in the correct form
(E) case 2
(F) case 1

53. In terms of the master recurrence theorem, where does the equation $T(n) = 4T(\frac{n}{2}) + n$ fall?

(A) case 3
(B) the equation is not in the correct form
(C) between case 1 and case 2
(D) case 2
(E) case 1
(F) between case 2 and case 3

54. In terms of the master recurrence theorem, where does the equation $T(n) = 3T(\frac{n}{4}) + n \log n$ fall?

(A) case 3
(B) between case 2 and case 3
(C) case 1
(D) between case 1 and case 2
(E) the equation is not in the correct form
(F) case 2

55. In terms of the master recurrence theorem, where does the equation $T(n) = 3T(\frac{n}{3}) + \frac{n}{2}$ fall?

(A) between case 1 and case 2
(B) case 1
(C) case 3
(D) case 2
(E) between case 2 and case 3
(F) the equation is not in the correct form

56. In terms of the master recurrence theorem, where does the equation $T(n) = 6T(\frac{n}{3}) + n^2 \log n$ fall?

(A) case 3
(B) the equation is not in the correct form
(C) between case 1 and case 2
(D) case 2
(E) case 1
(F) between case 2 and case 3

57. In terms of the master recurrence theorem, where does the equation $T(n) = 4T(\frac{n}{2}) + \frac{n}{\log n}$ fall?

(A) between case 2 and case 3
(B) between case 1 and case 2
(C) case 2
(D) the equation is not in the correct form
(E) case 1
(F) case 3

58. In terms of the master recurrence theorem, where does the equation $T(n) = 64T(\frac{n}{8}) - \frac{n}{\log n}$ fall?

(A) between case 2 and case 3
(B) case 3
(C) the equation is not in the correct form
(D) case 2
(E) between case 1 and case 2
(F) case 1

59. In terms of the master recurrence theorem, where does the equation $T(n) = 7T(\frac{n}{\sqrt{7}}) + n^2$ fall?

(A) the equation is not in the correct form
(B) case 1
(C) case 3
(D) between case 2 and case 3
(E) case 2
(F) between case 1 and case 2

60. In terms of the master recurrence theorem, where does the equation $T(n) = 7T(\frac{n}{2}) + n^2$ fall?

(A) case 2
(B) between case 1 and case 2
(C) the equation is not in the correct form
(D) case 3
(E) between case 2 and case 3
(F) case 1

61. In terms of the master recurrence theorem, where does the equation $T(n) = 7T(\frac{n}{3}) + n^2$ fall?

(A) between case 1 and case 2
(B) the equation is not in the correct form
(C) case 3
(D) case 1
(E) between case 2 and case 3
(F) case 2

62. In terms of the master recurrence theorem, where does the equation $T(n) = 4T(\frac{n}{2}) + \log n$ fall?

(A) between case 1 and case 2
(B) case 3
(C) case 1
(D) the equation is not in the correct form
(E) between case 2 and case 3
(F) case 2

63. In terms of the master recurrence theorem, where does the equation $T(n) = 5T(\frac{n}{2}) + \log n$ fall?

(A) case 2
(B) case 1
(C) between case 2 and case 3
(D) the equation is not in the correct form
(E) case 3
(F) between case 1 and case 2

64. In terms of the master recurrence theorem, where does the equation $T(n) = 5T(\frac{n}{2}) + n$ fall?

(A) between case 2 and case 3
(B) case 3
(C) the equation is not in the correct form
(D) between case 1 and case 2
(E) case 1
(F) case 2

65. In terms of the master recurrence theorem, where does the equation $T(n) = 5T(\frac{n}{\sqrt[3]{5}}) + n^3$ fall?

(A) case 3
(B) between case 1 and case 2
(C) between case 2 and case 3
(D) case 2
(E) case 1
(F) the equation is not in the correct form

66. In terms of the master recurrence theorem, where does the equation $T(n) = 5T(\frac{n}{\sqrt[3]{5}}) + \frac{n^3}{\log n}$ fall?

(A) the equation is not in the correct form
(B) case 1
(C) case 3
(D) case 2
(E) between case 2 and case 3
(F) between case 1 and case 2

67. In terms of the master recurrence theorem, where does the equation $T(n) = 5T(\frac{n}{\sqrt{5}}) + n^2 \log n$ fall?

(A) between case 2 and case 3
(B) the equation is not in the correct form
(C) case 1
(D) case 3
(E) between case 1 and case 2
(F) case 2

68. In terms of the master recurrence theorem, where does the equation $T(n) = 5T(\frac{n}{\sqrt{5}}) + n \log n$ fall?

(A) case 3
(B) the equation is not in the correct form
(C) case 1
(D) between case 2 and case 3
(E) case 2
(F) between case 1 and case 2

Concept: *using the master recurrence theorem*

69. Using the master recurrence theorem, what is the solution of $T(n) = 4T(\frac{n}{4}) + n^2$?

(A) $\Theta(n^3 \log n)$
(B) the master theorem cannot be used
(C) $\Theta(n^2 \log n)$
(D) $\Theta(n^3)$
(E) $\Theta(n^2)$
(F) $\Theta(n^4)$

70. Using the master recurrence theorem, what is the solution of $T(n) = 3T(\frac{n}{2}) + n^2$?

(A) $\Theta(n^2)$
(B) $\Theta(n^2 \log n)$
(C) the master theorem cannot be used
(D) $\Theta(n^3 \log n)$
(E) $\Theta(n^{\frac{3}{2}})$
(F) $\Theta(n^3)$

71. Using the master recurrence theorem, what is the solution of $T(n) = 4T(\frac{n}{2}) + n^2$?

(A) $\Theta(n^2 \log n)$
(B) the master theorem cannot be used
(C) $\Theta(n^2)$
(D) $\Theta(n^3)$
(E) $\Theta(n^4)$
(F) $\Theta(n^3 \log n)$

72. Using the master recurrence theorem, what is the solution of $T(n) = T(\frac{n}{2}) + 2^n$?

(A) the master theorem cannot be used
(B) $\Theta(n^2 \log n)$
(C) $\Theta(2^n \log n)$
(D) $\Theta(2^n)$
(E) $\Theta(2^{n^{\log n}})$
(F) $\Theta(n^2)$

73. Using the master recurrence theorem, what is the solution of $T(n) = 2^n T(\frac{n}{2}) + n^n$?

(A) $\Theta(2^n)$
(B) $\Theta((n^{n^{\log n}})$
(C) the master theorem cannot be used
(D) $\Theta(n^n)$
(E) $\Theta(2^n \log n)$
(F) $\Theta(n^n \log n)$

74. Using the master recurrence theorem, what is the solution of $T(n) = T(n-1) + 1$?

(A) $\Theta(1)$
(B) $\Theta(n)$
(C) $\Theta(\log n)$
(D) $\Theta(\log n^{\log n})$
(E) $\Theta(n \log n)$
(F) the master theorem cannot be used

75. Using the master recurrence theorem, what is the solution of $T(n) = 2T(n-2) + n$?

(A) the master theorem cannot be used
(B) $\Theta(\log n)$
(C) $\Theta((\log n)^2)$
(D) $\Theta(1)$
(E) $\Theta(n \log n)$
(F) $\Theta(n)$

76. Using the master recurrence theorem, what is the solution of $T(n) = 16T(\frac{n}{4}) + n$?

(A) $\Theta(n^2 \log n)$
(B) $\Theta(\log n)$
(C) the master theorem cannot be used
(D) $\Theta(n)$
(E) $\Theta(n \log n)$
(F) $\Theta(n^2)$

77. Using the master recurrence theorem, what is the solution of $T(n) = 2T(\frac{n}{2}) + n \log n$?

(A) $\Theta(n^2 \log n)$
(B) $\Theta(n^2)$
(C) $\Theta(\log n)$
(D) $\Theta(n \log n)$
(E) $\Theta(n)$
(F) the master theorem cannot be used

78. Using the master recurrence theorem, what is the solution of $T(n) = 3T(\frac{n}{2}) + n \log n$?

(A) $\Theta(n \log_{\frac{3}{2}}(n))$
(B) the master theorem cannot be used
(C) $\Theta(n^{\log_2(3)})$
(D) $\Theta(n^{\frac{3}{2}})$
(E) $\Theta(n^{\log_3(2)})$
(F) $\Theta(n \log n)$

79. Using the master recurrence theorem, what is the solution of $T(n) = 2T(\frac{n}{2}) + \frac{n}{\log n}$?

(A) $\Theta(\log n)$
(B) $\Theta(n \log n)$
(C) $\Theta(n^2)$
(D) $\Theta(n)$
(E) the master theorem cannot be used
(F) $\Theta(n^2 \log n)$

80. Using the master recurrence theorem, what is the solution of $T(n) = 2T(\frac{n}{4}) + n^{0.51}$?

(A) $\Theta(n^{0.51})$
(B) $\Theta(\sqrt{n} \log n)$
(C) $\Theta(\sqrt{n})$
(D) $\Theta(\log n)$
(E) $\Theta(n)$
(F) the master theorem cannot be used

81. Using the master recurrence theorem, what is the solution of $T(n) = \frac{1}{2}T(\frac{n}{2}) + 1$?

(A) $\Theta(\log n)$
(B) the master theorem cannot be used
(C) $\Theta(n^{\frac{1}{2}})$
(D) $\Theta(\log_2 \frac{1}{2})$
(E) $\Theta(n)$
(F) $\Theta(n^{\log_2 \frac{1}{2}})$

82. Using the master recurrence theorem, what is the solution of $T(n) = 16T(\frac{n}{4}) + n!$?

(A) $\Theta(n! \log n)$
(B) the master theorem cannot be used
(C) $\Theta(n!)$
(D) $\Theta(n^2)$
(E) $\Theta((n^2)!)$
(F) $\Theta(n^2 n!)$

83. Using the master recurrence theorem, what is the solution of $T(n) = \sqrt{2}T(\frac{n}{2}) + \log n$?

(A) $\Theta(n \log n)$
(B) $\Theta(\frac{n}{\log(2)})$
(C) $\Theta(\log n)$
(D) $\Theta(\sqrt{n})$
(E) $\Theta(\sqrt{n} \log n)$
(F) the master theorem cannot be used

84. Using the master recurrence theorem, what is the solution of $T(n) = 3T(\frac{n}{2}) + n$?

(A) $\Theta(n^{\log_2(3)})$
(B) $\Theta(n^{\log_3(2)})$
(C) $\Theta(n)$
(D) $\Theta(n \log n)$
(E) the master theorem cannot be used
(F) $\Theta(n \log_{\frac{3}{2}} n)$

85. Using the master recurrence theorem, what is the solution of $T(n) = 3T(\frac{n}{3}) + \sqrt{n}$?

(A) the master theorem cannot be used
(B) $\Theta(\log n)$
(C) $\Theta(n \log \sqrt{n})$
(D) $\Theta(\sqrt{n} \log n)$
(E) $\Theta(n \log n)$
(F) $\Theta(n)$

86. Using the master recurrence theorem, what is the solution of $T(n) = 4T(\frac{n}{2}) + n$?

(A) the master theorem cannot be used
(B) $\Theta(\log n)$
(C) $\Theta(n)$
(D) $\Theta(n^2 \log n)$
(E) $\Theta(n \log n)$
(F) $\Theta(n^2)$

87. Using the master recurrence theorem, what is the solution of $T(n) = 3T(\frac{n}{4}) + n \log n$?

(A) $\Theta(n)$
(B) $\Theta(\log n)$
(C) $\Theta(n^{\log_4(3)})$
(D) the master theorem cannot be used
(E) $\Theta(n \log n)$
(F) $\Theta(n^{\log_3(4)})$

88. Using the master recurrence theorem, what is the solution of $T(n) = 3T(\frac{n}{3}) + \frac{n}{2}$?

(A) $\Theta(n^2 \log n)$
(B) the master theorem cannot be used
(C) $\Theta(n)$
(D) $\Theta(n^2)$
(E) $\Theta(n \log n)$
(F) $\Theta(\log n)$

89. Using the master recurrence theorem, what is the solution of $T(n) = 6T(\frac{n}{3}) + n^2 \log n$?

(A) $\Theta(n^{\log_3(6)})$
(B) $\Theta(n^{\log_6(3)})$
(C) $\Theta(n^{\log_6(3)} \log n)$
(D) $\Theta(n^2 \log n)$
(E) $\Theta(n^{\log_3(6)} \log n)$
(F) the master theorem cannot be used

90. Using the master recurrence theorem, what is the solution of $T(n) = 4T(\frac{n}{2}) + \frac{n}{\log n}$?

(A) $\Theta(n \log n)$
(B) the master theorem cannot be used
(C) $\Theta(\log n)$
(D) $\Theta(\frac{n^2}{\log n})$
(E) $\Theta(n^2 \log n)$
(F) $\Theta(n^2)$

91. Using the master recurrence theorem, what is the solution of $T(n) = 64T(\frac{n}{8}) - \frac{n}{\log n}$?

(A) the master theorem cannot be used
(B) $\Theta(n \log n)$
(C) $\Theta(n)$
(D) $\Theta(n^2)$
(E) $\Theta(\log n)$
(F) $\Theta(n^2 \log n)$

92. Using the master recurrence theorem, what is the solution of $T(n) = 7T(\frac{n}{\sqrt{7}}) + n^2$?

(A) $\Theta(n^2 \log n)$
(B) $\Theta(n^2)$
(C) $\Theta(\frac{n}{\log_{\sqrt{7}}(7)})$
(D) the master theorem cannot be used
(E) $\Theta(n^{\log_{\sqrt{7}}(7)})$
(F) $\Theta(n \log n)$

93. Using the master recurrence theorem, what is the solution of $T(n) = 7T(\frac{n}{2}) + n^2$?

(A) $\Theta(n^{2+\log_2(7)})$
(B) $\Theta(n^2)$
(C) the master theorem cannot be used
(D) $\Theta(n^{\log_2(7)})$
(E) $\Theta(n^{\log_7(2)})$
(F) $\Theta(n^{2+\log_7(2)})$

94. Using the master recurrence theorem, what is the solution of $T(n) = 7T(\frac{n}{3}) + n^2$?

(A) the master theorem cannot be used
(B) $\Theta(n^{2+\log_7(3)})$
(C) $\Theta(n^{\log_7(3)})$
(D) $\Theta(n^{\log_3(7)})$
(E) $\Theta(n^{2+\log_3(7)})$
(F) $\Theta(n^2)$

95. Using the master recurrence theorem, what is the solution of $T(n) = 4T(\frac{n}{2}) + \log n$?

(A) $\Theta(\log n)$
(B) $\Theta(n \log n)$
(C) $\Theta(n^2)$
(D) $\Theta(n^2 \log n)$
(E) the master theorem cannot be used
(F) $\Theta(n)$

96. Using the master recurrence theorem, what is the solution of $T(n) = 5T(\frac{n}{2}) + \log n$?

(A) $\Theta(\log n)$
(B) the master theorem cannot be used
(C) $\Theta(n^{\log_5(2)} \log n)$
(D) $\Theta(n^{\log_5(2)})$
(E) $\Theta(n^{\log_2(5)} \log n)$
(F) $\Theta(n^{\log_2(5)})$

97. Using the master recurrence theorem, what is the solution of $T(n) = 5T(\frac{n}{2}) + n$?

(A) $\Theta(n^{\log_5(2)})$
(B) $\Theta(n^{\log_5(2)} \log n)$
(C) $\Theta(n \log n)$
(D) the master theorem cannot be used
(E) $\Theta(n^{\log_2(5)})$
(F) $\Theta(n^{\log_2(5)} \log n)$

98. Using the master recurrence theorem, what is the solution of $T(n) = 5T(\frac{n}{\sqrt[3]{5}}) + n^3$?

(A) $\Theta(n^3 \log n)$
(B) $\Theta(n^{\log_5 \sqrt[3]{5}} \times \log n)$
(C) $\Theta(n^{\log_{\sqrt[3]{5}}(5)})$
(D) $\Theta(n^{\log_5 \sqrt[3]{5}})$
(E) $\Theta(n^{\log_{\sqrt[3]{5}}(5)} \times \log n)$
(F) the master theorem cannot be used

99. Using the master recurrence theorem, what is the solution of $T(n) = 5T(\frac{n}{\sqrt[3]{5}}) + \frac{n^3}{\log n}$?

(A) $\Theta(n^{\log_5 \sqrt[3]{5}})$
(B) $\Theta(n^{\log_5 \sqrt[3]{5}} \times \log n)$
(C) $\Theta(n^{\log_{\sqrt[3]{5}}(5)})$
(D) $\Theta(n^{\log_{\sqrt[3]{5}}(5)} \times \log n)$
(E) the master theorem cannot be used
(F) $\Theta(n^3 \log n)$

100. Using the master recurrence theorem, what is the solution of $T(n) = 5T(\frac{n}{\sqrt{5}}) + n^2 \log n$?

(A) $\Theta(n^{\log_{\sqrt{5}}(5)} \times \log n)$
(B) $\Theta(n^{\log_5 \sqrt{5}} \times \log n)$
(C) $\Theta(n^{\log_{\sqrt{5}}(5)})$
(D) $\Theta(n^{\log_5 \sqrt{5}})$
(E) $\Theta(n^2 \log n)$
(F) the master theorem cannot be used

101. Using the master recurrence theorem, what is the solution of $T(n) = 5T(\frac{n}{\sqrt{5}}) + n \log n$?

(A) $\Theta(n^{2+\log_5 \sqrt{5}})$
(B) $\Theta(n \log n)$
(C) $\Theta(n^2)$
(D) $\Theta(n^{\log_{\sqrt{5}}(5)} \times \log n)$
(E) $\Theta(n^{\log_5 \sqrt{5}} \times \log n)$
(F) the master theorem cannot be used
(G) $\Theta(n^{2+\log_{\sqrt{5}}(5)})$

Determining ϵ in the M.R.T.

102. Consider using the master recurrence theorem to solve the equation $T(n) = 4T(\frac{n}{4}) + \log n$. What is the largest listed value of ϵ that can be used in the solution?

(A) 0.1
(B) ϵ is not used in the solution
(C) 0.25
(D) no legal value is listed
(E) the master theorem cannot be used
(F) 0.5

103. Consider using the master recurrence theorem to solve the equation $T(n) = 3T(\frac{n}{2}) + n^2$? What is the largest listed value of ϵ that can be used in the solution? The log (base 2) of 3 is 1.585 and the log (base 3) of 2 is 0.631.

(A) the master theorem cannot be used
(B) no legal value is listed
(C) ϵ is not used in the solution
(D) 0.25
(E) 0.1
(F) 0.5

104. Consider using the master recurrence theorem to solve the equation $T(n) = 4T(\frac{n}{2}) + n^2$? What is the largest listed value of ϵ that can be used in the solution? The log (base 4) of 2 is 0.5 and the log (base 2) of 4 is 2.

(A) 0.5
(B) 0.25
(C) 1.0
(D) the master theorem cannot be used
(E) no legal value is listed
(F) ϵ is not used in the solution

105. Consider using the master recurrence theorem to solve the equation $T(n) = T(\frac{n}{2}) + 2^n$? What is the largest listed value of ϵ that can be used in the solution?

(A) 1.0
(B) 0.1
(C) ϵ is not used in the solution
(D) the master theorem cannot be used
(E) 0.5
(F) no legal value is listed

106. Consider using the master recurrence theorem to solve the equation $T(n) = 2^n T(\frac{n}{2}) + n^n$? What is the largest listed value of ϵ that can be used in the solution?

(A) no legal value is listed
(B) 1.0
(C) 0.1
(D) ϵ is not used in the solution
(E) 0.5
(F) the master theorem cannot be used

107. Consider using the master recurrence theorem to solve the equation $T(n) = 16T(\frac{n}{4}) + n$? What is the largest listed value of ϵ that can be used in the solution? The log (base 16) of 4 is 0.5 and log (base 4) of 16 is 2.

(A) 0.25
(B) no legal value is listed
(C) 0.5
(D) 0.75
(E) ϵ is not used in the solution
(F) the master theorem cannot be used

108. Consider using the master recurrence theorem to solve the equation $T(n) = 2T(\frac{n}{2}) + n \log n$? What is the largest listed value of ϵ that can be used in the solution?

(A) 1.0
(B) 0.5
(C) no legal value is listed
(D) 0.75
(E) ϵ is not used in the solution
(F) the master theorem cannot be used

109. Consider using the master recurrence theorem to solve the equation $T(n) = 3T(\frac{n}{2}) + n \log n$? What is the largest listed value of ϵ that can be used in the solution? The log (base 2) of 3 is 1.585 and the log (base 3) of 2 is 0.631.

(A) 0.5
(B) 1.0
(C) 0.25
(D) no legal value is listed
(E) ϵ is not used in the solution
(F) the master theorem cannot be used

110. Consider using the master recurrence theorem to solve the equation $T(n) = 8T(\frac{n}{2}) + n$? What is the largest listed value of ϵ that can be used in the solution? The log (base 8) of 2 is 0.333 and the log (base 2) of 8 is 3.

(A) the master theorem cannot be used
(B) 0.100
(C) no legal value is listed
(D) ϵ is not used in the solution
(E) 0.025
(F) 0.05

111. Consider using the master recurrence theorem to solve the equation $T(n) = 2T(\frac{n}{4}) + n^{0.51}$? What is the largest listed value of ϵ that can be used in the solution? The log (base 4) of 2 is 0.5 and log (base 2) of 4 is 2.

(A) the master theorem cannot be used
(B) ϵ is not used in the solution
(C) no legal value is listed
(D) 0.050
(E) 0.075
(F) 0.100

112. Consider using the master recurrence theorem to solve the equation $T(n) = \frac{1}{2}T(\frac{n}{2}) + 1$? What is the largest listed value of ϵ that can be used in the solution?

(A) 0.05
(B) the master theorem cannot be used
(C) 0.01
(D) ϵ is not used in the solution
(E) 0.10
(F) no legal value is listed

113. Consider using the master recurrence theorem to solve the equation $T(n) = 16T(\frac{n}{4}) + n!$? What is the largest listed value of ϵ that can be used in the solution? The log (base 16) of 4 is 0.5 and the log (base 4) of 16 is 2.

(A) the master theorem cannot be used
(B) 1
(C) 10
(D) ϵ is not used in the solution
(E) no legal value is listed
(F) 100

114. Consider using the master recurrence theorem to solve the equation $T(n) = \sqrt{2}T(\frac{n}{2}) + \log n$? What is the largest listed value of ϵ that can be used in the solution? The log (base 2) of $\sqrt{2}$ is 0.5 and the log (base $\sqrt{2}$) of 2 is 2.

(A) ϵ is not used in the solution
(B) 0.25
(C) the master theorem cannot be used
(D) no legal value is listed
(E) 0.50
(F) 1.0

115. Consider using the master recurrence theorem to solve the equation $T(n) = 3T(\frac{n}{2}) + n$? What is the largest listed value of ϵ that can be used in the solution? The log (base 3) of 2 is 0.631 and the log (base 2) of 3 is 1.585.

(A) ϵ is not used in the solution
(B) the master theorem cannot be used
(C) no legal value is listed
(D) 0.25
(E) 1.0
(F) 0.10

116. Consider using the master recurrence theorem to solve the equation $T(n) = 3T(\frac{n}{3}) + \sqrt{n}$? What is the largest listed value of ϵ that can be used in the solution?

(A) no legal value is listed
(B) the master theorem cannot be used
(C) 0.9
(D) 0.3
(E) ϵ is not used in the solution
(F) 0.6

117. Consider using the master recurrence theorem to solve the equation $T(n) = 4T(\frac{n}{2}) + n$? What is the largest listed value of ϵ that can be used in the solution? The log (base 2) of 4 is 2 and the log (base 4) of 2 is 0.5.

(A) 0.9
(B) the master theorem cannot be used
(C) 0.6
(D) no legal value is listed
(E) 0.3
(F) ϵ is not used in the solution

118. Consider using the master recurrence theorem to solve the equation $T(n) = 3T(\frac{n}{4}) + n \log n$? What is the largest listed value of ϵ that can be used in the solution? The log (base 3) of 4 is 1.262 and the log (base 4) of 3 is 0.792.

(A) the master theorem cannot be used
(B) ϵ is not used in the solution
(C) 0.6
(D) 0.9
(E) 0.3
(F) no legal value is listed

119. Consider using the master recurrence theorem to solve the equation $T(n) = 3T(\frac{n}{3}) + \frac{n}{2}$? What is the largest listed value of ϵ that can be used in the solution?

(A) the master theorem cannot be used
(B) ϵ is not used in the solution
(C) 0.3
(D) no legal value is listed
(E) 0.6
(F) 0.9

120. Consider using the master recurrence theorem to solve the equation $T(n) = 6T(\frac{n}{3}) + n^2 \log n$? What is the largest listed value of ϵ that can be used in the solution? The log (base 3) of 6 is 1.631 and the log (base 6) of 3 is 0.613.

(A) no legal value is listed
(B) ϵ is not used in the solution
(C) 0.3
(D) the master theorem cannot be used
(E) 0.4
(F) 0.5

121. Consider using the master recurrence theorem to solve the equation $T(n) = 4T(\frac{n}{2}) + \frac{n}{\log n}$? What is the largest listed value of ϵ that can be used in the solution? The log (base 4) of 2 is 0.5 and the log (base 2) of 4 is 2.

(A) 1.0
(B) the master theorem cannot be used
(C) 0.5
(D) ϵ is not used in the solution
(E) no legal value is listed
(F) 0.25

122. Consider using the master recurrence theorem to solve the equation $T(n) = 64T(\frac{n}{8}) - \frac{n}{\log n}$? What is the largest listed value of ϵ that can be used in the solution? The log (base 8) of 64 is 2 and the log (base 64) of 8 is 0.5.

(A) ϵ is not used in the solution
(B) 0.5
(C) the master theorem cannot be used
(D) no legal value is listed
(E) 1.0
(F) 0.25

123. Consider using the master recurrence theorem to solve the equation $T(n) = 7T(\frac{n}{\sqrt{7}}) + n^2$? What is the largest listed value of ϵ that can be used in the solution? The log (base 7) of $\sqrt{7}$ is 0.5 and the log (base $\sqrt{7}$) of 7 is 2.

(A) no legal value is listed
(B) 1.0
(C) 0.5
(D) the master theorem cannot be used
(E) 0.25
(F) ϵ is not used in the solution

124. Consider using the master recurrence theorem to solve the equation $T(n) = 7T(\frac{n}{2}) + n^2$? What is the largest listed value of ϵ that can be used in the solution? The log (base 2) of 7 is 2.807 and the log (base 7) of 2 is 0.356.

(A) no legal value is listed
(B) 0.8
(C) the master theorem cannot be used
(D) ϵ is not used in the solution
(E) 1.0
(F) 0.6

125. Consider using the master recurrence theorem to solve the equation $T(n) = 7T(\frac{n}{3}) + n^2$? What is the largest listed value of ϵ that can be used in the solution? The log (base 3) of 7 is 0.565 and the log (base 7) of 3 is 1.771.

(A) ϵ is not used in the solution
(B) 0.6
(C) 1.0
(D) the master theorem cannot be used
(E) 0.8
(F) no legal value is listed

126. Consider using the master recurrence theorem to solve the equation $T(n) = 4T(\frac{n}{2}) + \log n$? What is the largest listed value of ϵ that can be used in the solution? The log (base 2) of 4 is 2 and the log (base 4) of 2 is 0.5.

(A) 0.5
(B) the master theorem cannot be used
(C) no legal value is listed
(D) 1.0
(E) ϵ is not used in the solution
(F) 2.0

127. Consider using the master recurrence theorem to solve the equation $T(n) = 5T(\frac{n}{2}) + \log n$? What is the largest listed value of ϵ that can be used in the solution? The log (base 5) of 2 is 0.431 and the log (base 2) of 5 is 2.322.

(A) ϵ is not used in the solution
(B) 2.0
(C) 1.0
(D) the master theorem cannot be used
(E) 0.5
(F) no legal value is listed

128. Consider using the master recurrence theorem to solve the equation $T(n) = 5T(\frac{n}{2}) + n$? What is the largest listed value of ϵ that can be used in the solution? The log (base 2) of 5 is 2.322 and the log (base 5) of 2 is 0.431.

(A) no legal value is listed
(B) 2.0
(C) 0.5
(D) 1.0
(E) the master theorem cannot be used
(F) ϵ is not used in the solution

129. Consider using the master recurrence theorem to solve the equation $T(n) = 5T(\frac{n}{\sqrt[3]{5}}) + n^3$? What is the largest listed value of ϵ that can be used in the solution? The log (base 5) of $\sqrt[3]{5}$ is 0.333 and the log (base $\sqrt[3]{5}$) of 5 is 3.

(A) 2.0
(B) 1.0
(C) ϵ is not used in the solution
(D) 0.5
(E) the master theorem cannot be used
(F) no legal value is listed

130. Consider using the master recurrence theorem to solve the equation $T(n) = 5T(\frac{n}{\sqrt[3]{5}}) + \frac{n^3}{\log n}$? What is the largest listed value of ϵ that can be used in the solution? The log (base $\sqrt[3]{5}$) of 5 is 3 and the log (base 5) of $\sqrt[3]{5}$ is 0.333.

(A) ϵ is not used in the solution
(B) the master theorem cannot be used
(C) no legal value is listed
(D) 0.5
(E) 1.0
(F) 2.0

131. Consider using the master recurrence theorem to solve the equation $T(n) = 5T(\frac{n}{\sqrt{5}}) + n^2 \log n$? What is the largest listed value of ϵ that can be used in the solution? The log (base 5) of $\sqrt{5}$ is 0.5 and the log (base $\sqrt{5}$) of 5 is 2.

(A) 2.0
(B) ϵ is not used in the solution
(C) 1.0
(D) no legal value is listed
(E) the master theorem cannot be used
(F) 0.5

132. Consider using the master recurrence theorem to solve the equation $T(n) = 5T(\frac{n}{\sqrt{5}}) + n \log n$? What is the largest listed value of ϵ that can be used in the solution? The log (base $\sqrt{5}$) of 5 is 2 and the log (base 5) of $\sqrt{5}$ is 0.5.

(A) no legal value is listed
(B) ϵ is not used in the solution
(C) 0.5
(D) 2.0
(E) the master theorem cannot be used
(F) 1.0

Determining c for M.R.T., case 3

133. Consider using the master recurrence theorem and the regularity condition for case 3 to solve the equation $T(n) = 8T(\frac{n}{8}) + n^2$. What is the smallest legal value of the constant c listed for the solution?

(A) no legal value is listed
(B) $\frac{1}{9}$
(C) $\frac{1}{5}$
(D) $\frac{1}{7}$
(E) case 3 does not apply

134. Consider using the master recurrence theorem and the regularity condition for case 3 to solve the equation $T(n) = 7T(\frac{n}{8}) + n^2$. What is the smallest legal value of the constant c listed for the solution?

(A) $\frac{7}{64}$
(B) $\frac{11}{64}$
(C) case 3 does not apply
(D) no legal value is listed
(E) $\frac{9}{64}$

135. Consider using the master recurrence theorem and the regularity condition for case 3 to solve the equation $T(n) = 6T(\frac{n}{8}) + n^2$. What is the smallest legal value of the constant c listed for the solution?

(A) $\frac{3}{64}$
(B) case 3 does not apply
(C) $\frac{5}{64}$
(D) $\frac{1}{64}$
(E) no legal value is listed

136. Consider using the master recurrence theorem and the regularity condition for case 3 to solve the equation $T(n) = 64(\frac{n}{8}) + n^2$. What is the smallest legal value of the constant c listed for the solution?

(A) $\frac{65}{64}$
(B) $\frac{61}{64}$
(C) no legal value is listed
(D) $\frac{63}{64}$
(E) case 3 does not apply

137. Consider using the master recurrence theorem and the regularity condition for case 3 to solve the equation $T(n) = 32(\frac{n}{8}) + n^2$. What is the smallest legal value of the constant c listed for the solution?

(A) no legal value is listed
(B) $\frac{63}{64}$
(C) case 3 does not apply
(D) $\frac{8}{64}$
(E) $\frac{16}{64}$

Inductive Proofs

Here is a recursive version of mergesort, which works for all arrays with size one or greater:

```
mergeSort(array, start, end)
    {
    if (end-start < 2) return;

    m = (start + end) / 2;
```

```
mergesort(array,start,m);
mergesort(array,m,end);
merge(array,start,m,end);
}
```

Assume that the recurrence for merge sort is described as:

$T(2^0) = 1$

$T(2^n) = 2T(2^{n-1}) + 2^n$

Consider a proof, by induction, that $T(2^n) = (n+1)2^n$ where 2^n is the size of the array and b is the base case value of n.

138. To completely prove the base case value b, you would need to show:

(A) $2T(2^{b-1}) = 2$
(B) $T(2^b) = 4$
(C) $2T(2^{b-1}) + 2^b = (b+1)2^b$
(D) $b2^b = 2$
(E) $b2^b = 2^{b+1}$
(F) $2T(2^{b-1}) = 2^b$

139. Assuming b is the largest base case value and you wish to prove the inductive case n, an inductive hypothesis for a *strong* inductive proof would be:

(A) Assume true for $i \leq n$.
(B) Assume true for $i < n$.
(C) Assume true for n.
(D) Assume true for $n + 1$.
(E) Assume true for $b < i \leq n$.
(F) Assume true for $b < i < n$.

140. Assuming b is the largest base case value and you wish to prove the inductive case *n+1*, an inductive hypothesis for a *weak* inductive proof would be:

(A) Assume true for n.
(B) Assume true for $n + 2$.
(C) Assume true for $n - 1$.
(D) Assume true for $n + 1$.

141. Assuming you wish to prove the inductive case n, the first line of the inductive case for a *strong* inductive proof would be:

(A) $T(2^{n-1}) = 2T(2^{(n-1)-1}) + 2^{n-1}$
(B) $T(2^{n-1}) = ((n-1)+1)2^{n-1}$
(C) $T(2^{n+1}) = 2T(2^{(n+1)-1}) + 2^{n+1}$
(D) $T(2^{n+1}) = ((n+1)+1)2^{n+1}$
(E) $T(2^n) = 2T(2^{n-1}) + 2^n$
(F) $T(2^n) = (n+1)2^n$

142. Assuming you wish to prove the inductive case *n+1*, the first line of the inductive case for a *weak* inductive proof would be:

(A) $T(2^{n-1}) = ((n-1)+1)2^{n-1}$
(B) $T(2^{n+1}) = 2T(2^{(n+1)-1}) + 2^{n+1}$
(C) $T(2^{n+1}) = ((n+1)+1)2^{n+1}$
(D) $T(2^n) = (n+1)2^n$
(E) $T(2^{n-1}) = 2T(2^{(n-1)-1}) + 2^{n-1}$
(F) $T(2^n) = 2T(2^{n-1}) + 2^n$

143. Assuming you wish to prove the inductive case n, the inductive step for a *strong* inductive proof would be:

(A) substitute $n2^{n-1}$ for $T(2^{n-1})$
(B) substitute $T(2^{(n-1)-1})$ for $(n-1)2^{n-2}$
(C) substitute $(n+1)2^n$ for $T(2^{(n+1)-1})$
(D) substitute $T(2^{n-1})$ for $n2^{n-1}$
(E) substitute $T(2^{(n+1)-1})$ for $(n+1)2^n$
(F) substitute $(n-1)2^{n-2}$ for $T(2^{(n-1)-1})$

144. Assuming you wish to prove the inductive case *n+1*, the inductive step for a *weak* inductive proof would be:

(A) substitute $T(2^{(n+1)-1})$ for $(n+1)2^n$
(B) substitute $T(2^{(n-1)-1})$ for $(n-1)2^{n-2}$
(C) substitute $(n+1)2^n$ for $T(2^{(n+1)-1})$
(D) substitute $T(2^{n-1})$ for $n2^{n-1}$
(E) substitute $n2^{n-1}$ for $T(2^{n-1})$
(F) substitute $(n-1)2^{n-2}$ for $T(2^{(n-1)-1})$

145. **T** or **F**: For *strong* inductive proofs, proving the inductive case makes use of the previously proved base cases.

146. **T** or **F**: *Weak* inductive proofs are more closely related to recursive functions than *strong* inductive proofs.

147. The more formal names for *weak* and *strong* inductive proofs are:

(A) simple and mathematical
(B) mathematical and complete
(C) complex and simple
(D) simple and complex
(E) complete and mathematical
(F) mathematical and simple

Concept: *inductive proofs*

148. (6 pts) Prove by strong induction, with n as the inductive case, that $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$.

Base case:

Inductive Hypothesis:

Inductive Case:

149. (6 pts) Prove by strong induction, with n as the inductive case, that $T(n) = \frac{n(n+1)}{2}$, where:

$T(0) = 0$
$T(n) = T(n-1) + n$

Base case:

Inductive Hypothesis:

Inductive Case:

150. (6 pts) Prove by strong induction, with n as the inductive case, that $\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.

Base case:

Inductive Hypothesis:

Inductive Case:

151. (6 pts) Prove by strong induction, with n as the inductive case, that $T(n) = \frac{n(n+1)(2n+1)}{6}$, where:

$T(0) = 0$
$T(n) = T(n-1) + n^2$

Base case:

Inductive Hypothesis:

Inductive Case:

152. (6 pts) Prove by strong induction that $\sum_{k=0}^{n} k^3 = \frac{n^2(n+1)^2}{4}$.

Base case:

Inductive Hypothesis:

Inductive Case:

153. (6 pts) Prove by strong induction that $T(n) = \frac{n^2(n+1)^2}{4}$, where:

$T(0) = 0$
$T(n) = T(n-1) + n^3$

Base case:

Inductive Hypothesis:

Inductive Case:

154. (6 pts) Prove by strong induction that $\displaystyle\sum_{k=0}^{n} x^k = \frac{x^{n+1}-1}{x-1}$.

Base case:

Inductive Hypothesis:

Inductive Case:

155. (6 pts) Prove by strong induction that $T(x,n) = \frac{x^{n+1}-1}{x-1}$, where:

$T(x,0) = 1$
$T(x,n) = T(x,n-1) + x^n$

Base case:

Inductive Hypothesis:

Inductive Case:

156. (6 pts) Prove by strong induction that the number of nodes in a perfect binary tree with n levels is $2^n - 1$.

Base case:

Inductive Hypothesis:

Inductive Case:

157. (6 pts) Prove by strong induction that the number of nodes in a perfect quaternary tree with n levels is $\frac{4^n-1}{3}$.

Base case:

Inductive Hypothesis:

Inductive Case:

158. (6 pts) Prove by strong induction that the number of nodes in a perfect binary tree with n levels is $2^n - 1$.

Base case:

Inductive Hypothesis:

Inductive Case:

159. (6 pts) Prove by strong induction that the number of interior nodes in a perfect binary tree with n levels is $2^{n-1} - 1$.

Base case:

Inductive Hypothesis:

Inductive Case:

160. (6 pts) Prove by strong induction that the number of child pointers in an arbitrary binary tree with n nodes is $n - 1$.

Base case:

Inductive Hypothesis:

Inductive Case: