Analysis of Algorithms

Graphs and Graph Algorithms

If more than one question appears correct, choose the more specific answer, unless otherwise instructed.

Concept: *Graphs*

1. What is the primary characteristic of a directed graph?

   (A) the edges are bi-directional
   (B) at least one vertex is unreachable
   (C) the vertices are all reachable
   (D) the edges are uni-directional

2. What is the primary characteristic of a weighted graph?

   (A) each edge has an associated weight
   (B) the vertex count exceeds some threshold
   (C) each vertex has an associated weight
   (D) the edge count exceeds vertex count

3. What is the primary characteristic of an undirected, simple graph?

   (A) there is at most one edge between any two vertices
   (B) the edge count is less than the vertex count
   (C) the edge count exceeds or equals the vertex count
   (D) the termini of an edge may be the same vertex

4. What is the degree of a vertex in an undirected, simple graph?

   (A) the count of vertices not reachable from that vertex
   (B) the total number of paths emanating from the vertex
   (C) the total number of edges emanating from the vertex
   (D) the count of vertices reachable from that vertex

5. What is the primary characteristic of a undirected, simple, connected graph?

   (A) there is at least one edge
   (B) an edge exists between every pair of vertices
   (C) each vertex has a edge that connects to itself
   (D) a path exists between every pair of vertices

6. What is the primary characteristic of an undirected, simple, regular graph?

   (A) all vertices have the same weight
   (B) all edges terminate at the same vertex
   (C) all edges have the same weight
   (D) all vertices have the same degree

7. What is the primary characteristic of an undirected, simple, complete graph? Choose the most general answer.

   (A) each vertex has two edges or no edges
   (B) all vertices have a degree greater than some threshold
   (C) there exists a path between every pair of vertices
   (D) there exists an edge between every pair of vertices

8. What is the primary characteristic of an undirected, simple, connected, acyclic graph? Choose the most general answer.

   (A) the vertex count is equal to the edge count
   (B) no edge has a single vertex as its termini
   (C) the vertex count is one greater than the edge count
   (D) there are at least two unique paths between any two vertices

9. What is the primary characteristic of a planar graph?

   (A) the maximum degree of a vertex is 4
   (B) it can be drawn in a plane with no crossed edges
   (C) when drawn in a plane, it must have crossed edges
   (D) the maximum degree of a vertex is 3

Graph traversals

10. What is a walk? Choose the most general answer.

    (A) a sequence of vertices with an edge between v_i and v_{i+1}
    (B) a sequence of vertices with no edge between v_i and v_{i+1}
    (C) a set of edges that have one vertex in common
    (D) a sequence of edges such that e_i and e_{i+1} have no common vertex

11. What is a trail? Choose the most general answer.

    (A) a walk with at least one vertex appearing twice (or more)
    (B) a walk with at least one edge appearing twice (or more)
    (C) a walk with no vertex appearing more than once
    (D) a walk with no edge appearing more than once

12. What is the primary characteristic of a path?

    (A) a trail with all vertices appearing at least once
    (B) a trail with all edges appearing at least once
    (C) a trail with no vertex appearing more than twice
    (D) a trail with no vertex appearing more than once

13. What is an Euler trail? Choose the most general answer.

    (A) a trail which invovles every edge exactly once
    (B) the longest trail in a graph
    (C) a trail which invovles every vertex exactly once
    (D) a trail which invovles every vertex, except one, exactly once

14. What is a Hamiltonian path? Choose the most general answer.

    (A) a path which involves every edge
    (B) the shortest path in a graph
    (C) the longest path in a graph
    (D) a path which involves every vertex

15. **T** or **F**: An Euler trail is always the longest trail in an undirected graph.

16. **T** or **F**: All undireted graphs have an Euler trail.

17. **T** or **F**: A Hamiltonian path is always the longest path in an undirected graph.

18. **T** or **F**: All undireted graphs have a Hamiltonian path.

19. The longest path in an undirected graph is bounded by:

    (A) the number of vertices
    (B) the maximum degree of a graph
    (C) the minimum degree of a graph
    (D) the number of edges

20. **T** or **F**: An undirected graph with a min-degree of 2 must have a cycle.

Spanning trees and shortest paths

21. **T** or **F**: All connected, undirected graphs have a spanning tree.

22. Refering to graphs, Kruskal's algorithm is used to find:

    (A) a minimum spanning tree
    (B) all-pairs shortest paths
    (C) the longest path with the least weight
    (D) the shortest path between two vertices

23. Refering to graphs, Dijkstra's algorithm is used to find:

    (A) the longest path with the least weight
    (B) the shortest path between a source vertex and the other vertices
    (C) all-pairs shortest paths
    (D) a minimum spanning tree

24. Refering to graphs, Prim's algorithm is used to find:

(A) the longest path with the least weight
(B) the shortest path between two vertices
(C) a minimum spanning tree
(D) all-pairs shortest paths

25. Refering to graphs, the Floyd-Warshall algorithm is used to find:

(A) the longest path with the least weight
(B) the shortest path
(C) a minimum spanning tree
(D) all-pairs shortest paths

26. Suppose $E = \omega(V)$. What is the asymptotic running time of Kruskal's Algorithm? Choose the simplest answer.

(A) $\Theta(E + V \log V)$
(B) $\Theta(E \log E)$
(C) $\Theta(E + V)$
(D) $\Theta(E \log E + V \log V)$
(E) $\Theta(E \log E + V \log E)$
(F) $\Theta(E)$

27. Suppose $E = \Theta(V)$. What is the asymptotic running time of Prim's Algorithm? Choose the simplest answer.

(A) $\Theta(E \log E)$
(B) $\Theta(E + V)$
(C) $\Theta(E \log E + V \log E)$
(D) $\Theta(E + V \log V)$
(E) $\Theta(E \log E + V \log V)$
(F) $\Theta(E)$

28. **T** or **F**: Suppose you kept track of the level number for each vertex w in a breadth-first search of a simple, undirected, unweighted graph, starting from a vertex v. The level numbers of each w would correspond to the shortest path distance.

29. **T** or **F**: For a simple, undirected graph, $\Theta(E \log E) = \Theta(E \log V)$

30. Consider running Kruskal's algorithm on the complete graph K_n, processing the first i edges. What is the smallest value of i that could yield the final result?

(A) $\frac{n(n+1)}{2}$
(B) n-1
(C) $\frac{n(n-1)}{2} - 1$
(D) n
(E) $\frac{n(n-1)}{2}$
(F) $\frac{n(n+1)}{2} - 1$

31. Consider running Dijkstra's algorithm using a linked list (with a tail pointer) as the basis for a priority queue. What is the asymptotic run time for the algorithm?

(A) the correct answer is not listed
(B) $\Theta(V^3 \log V)$
(C) $\Theta(V^2 \log V)$
(D) $\Theta(V^2 E)$
(E) $\Theta(V E^2)$
(F) $\Theta(V^3 E)$