Analysis of Algorithms

Amortized Analysis

If more than one question appears correct, choose the more specific answer, unless otherwise instructed.

Concept: *amortized analysis*

1. Suppose a dynamic array was implemented so that growing the array increased the capacity by 1000 elements. What is the worst-case cost of the append operation?

   (A) constant
   (B) log linear
   (C) quadratic
   (D) linear

2. Suppose a dynamic array was implemented so that growing the array increased the capacity by 2%. What is the amortized and worst-case costs of the append operation?

   (A) constant and linear
   (B) quadratic and quadratic
   (C) log and linear
   (D) linear and linear

3. Suppose a dynamic array was implemented so that growing the array increased the capacity by 100 elements. What is the amortized and worst-case costs of the append operation?

   (A) constant and linear
   (B) linear and linear
   (C) log and linear
   (D) quadratic and quadratic

4. Consider a dynamic fillable array which grows by doubling in size. Which is a valid equation for calculating the total cost incurred when insertion $2^i + 1$ is made? Assume the individual cost of an insert when there is room is 1 and the individual cost of an insert when there is no room is $2^i + 1$.

   (A) $2n - 1$
   (B) $2n - 2$
   (C) $3n - 2$
   (D) $2n - 3$
   (E) $3n - 3$
   (F) $3n - 1$

5. Consider a dynamic fillable array which grows from size n to size $2n + 1$ each time the array is filled. Which is a valid equation for calculating the total cost incurred when insertion 2^i is made? Assume the individual cost of an insert when there is room is 1 and the individual cost of an insert when there is no room is 2^i.

   (A) $3n - \log n - 2$
   (B) $3n - 6$
   (C) $2n - 1$
   (D) $2n$
   (E) $3n - 1$
   (F) $3n - 3$

6. Consider a dynamic fillable array which grows by doubling in size. Let S represent the number of filled slots and E, the number of empty slots, and C, the capacity of the array. Which is a valid potential function for proving the amoritized cost of an insert is constant?

   (A) $S + E$
   (B) $S - E$
   (C) $S + 2E$
   (D) $2S - C$
   (E) $E + 2C$
   (F) $2S - E$

7. Consider implementing a queue with two stacks. *Enqueues* are translated to pushes onto the first stack. For a *dequeue*, if the second stack is empty, each element on the first stack is popped and pushed, in turn, onto the second stack. In either case, the item popped from the second stack is returned.

   The worst-case times for *enqueue* and *dequeue* are:

   (A) $\Theta(1)$ and $\Theta(1)$
   (B) $\Theta(1)$ and $\Theta(n)$
   (C) $\Theta(n)$ and $\Theta(1)$
   (D) $\Theta(n)$ and $\Theta(n)$

8. Consider implementing a queue with two stacks. *Enqueues* are translated to pushes onto the first stack, V. For a *dequeue*, if the second stack, W, is empty, each element on the first stack is popped and pushed, in turn, onto the second stack. The total work of this transfer is the number of elements popped plus the number of elements pushed. In either case, the item popped from the second stack is returned. The number of elements on stack V is denoted V_S and the number of elements on stack W is denoted W_S.

   Which of the following potential functions can be used to show an amortized bound of $\Theta(1)$ for operations on this kind of queue?

   (A) $\Phi = V_s + W_s$
   (B) $\Phi = V_s$
   (C) $\Phi = 2V_s$
   (D) $\Phi = V_s + 2W_s$

9. Suppose a data structure has operation A with a real cost of 1 and operation B with a real cost of $2n+1$. After an A operation, n increases by 1 while after a B operation, n decreases to zero.

   Which of the following potential functions can be used to show an amortized bound of $\Theta(1)$ for operations A and B on this data structure?

   (A) $Phi = 2n$.
   (B) $\Phi = 3n$
   (C) $Phi = n$.

10. Suppose a data structure has operation A with a real cost of $3n+1$ and operation B with a real cost of $\frac{3}{2}n+1$. After an A operation, n decreases to $\frac{1}{4}n$ while after a B operation, n decreases to $\frac{5}{8}n$.

    Which of the following potential functions can be used to show an amortized bound of $\Theta(1)$ for operations A and B on this data structure?

    (A) $\Phi = \frac{3}{2}n$
    (B) $\Phi = 4n$
    (C) $\Phi = 3n$

11. Suppose a data structure has operation A with a real cost of 1 and operation B with a real cost of $k+n$. After an A operation, n increases by 1. After a B operation, n decreases to $k+1$.

    Which of the following potential functions can be used to show an amortized bound of $\Theta(1)$ for A operations and $\Theta(k)$ for B operations?

    (A) $\Phi = 3n$
    (B) $\Phi = n$
    (C) $\Phi = 2n$