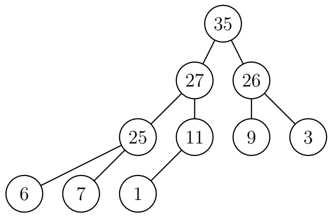
CS201 Data Structures and Algorithms

Heaps

Introduction

Some heap facts:

- A heap is a *complete* binary tree.



- For a *max*-heap, a parent's value is always *greater* than or equal to the values of its children.
- For a *min*-heap, a parent's value is always *less* than or equal to the values of its children.
- There is no relationship between the values of sibling nodes.

We will assume a max-heap unless otherwise stated.

Problems:

- 7.1-1
- 7.1-2
- 7.1-4 (all elements distinct)
- 7.1-5
- 7.1-6

Correcting point errors in a heap

Often in the course of manipulating a heap, heap ordering is disrupted. Typically, these are *point errors*. That is to say, only one value in the heap is incorrect; absent that error, the heap has the correct ordering.

There are two types of point errors that arise during some heap operations. One is that a parent value is modified and turns out to be too small. Since it is a point error, we know that the modified value needs to move lower down in the heap. The other is that a node gains a child that has a value that is larger than it should be. In this case, the child value needs to move up.

To remedy the first case, the *heapify* or *pushDown* operation for max-heaps pushes a small value down the heap until it resides at its proper level (*mutatis mutandis* for min-heaps). The sub-heap rooted at the original node will be a heap after this operation completes:

```
function heapify(i) //i is the ith node
    {
    var next = i;
    if (hasLeft(i) && value(left(i)) > value(next))
        next = left(i);
    if (hasRight(i) && value(right(i)) > value(next))
        next = right(i);
    if (next != i)
        {
        swapValues(i,next);
        heapify(next);
        }
    }
```

To correct the second case, the *fixHeap* or *bubbleUp* operation for max-heaps moves a large value up the heap until it resides at its proper level (*mutatis mutandis* for min-heaps).

```
function fixHeap(i)
    {
    if (hasParent(i) && value(i) > value(parent(i)))
        {
        swapValues(i,parent(i));
        fixHeap(parent(i));
        }
    }
```

Problems:

- 7.2-1
- 7.2-3
- 7.2-4

Constructing heaps

The *buildHeap* operation is:

- a routine which turns an unordered complete tree into a max-heap
- works from the bottom up

```
function buildHeap(h)
    {
    for each node n in h from rightmost leaf to root by level
        heapify(n)
    }
```

Some questions:

- How would you traverse an array-based heap?
- How would you traverse a tree-based heap?
- Can you ignore the leaves?
- The obvious running time for *buildHeap* is $O(n \log n)$. Is this bound tight?

Problems:

- 7.3-1
- 7.3-3