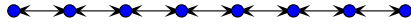


Fun with Grammars



Fun with grammars

Here are a set of abstract languages and grammars which can generate them.

Set of all strings over A,B ending in A

```
s : zA
z : Az | Bz | empty
```

Note that the non-terminal z generates all strings composed of A's and B's. The non-terminal s simply appends all strings generated by z with an A to give the desired result.

Set of all strings over A,B beginning and ending in A

```
s : AzA | A
z : Az | Bz | *empty*
```

Here, z plays the same role as before. In this case, s simply prepends an A as well to give the desired result.

Set of all strings over A,B with three consecutive A's

```
s : zAAAz
z : Az | Bz | *empty*
```

Now, s places a z at both ends to ensure that there are three consecutive A's somewhere in the sentence.

Set of all strings over A,B such that there is a pair of A's separated by $4i$, $i \geq 0$, characters

```
s : zAlAz
z : Az | Bz | *empty*
l : fl | *empty*
f : Ag | Bg
g : Ah | Bh
h : Ai | Bi
i : A | B
```

Similar to before, but note that l generates zero or more f strings. Note that an i string is composed of a single character. Therefore, an h string is composed of two characters, a g string is composed of three characters and a f string is composed of four characters. Since there are zero or more f strings between the two A's in an s string, we get the desired language.

Set of all strings over A,B such that at no two A's and no two B's are adjacent

```
s : a | b
b : Ba | *empty*
a : Ab | *empty*
```

The non-terminals a and b flip flop back and forth to ensure no two like characters are adjacent.

Palindromes over A,B

```
s : A | B | AsA | BsB | *empty*
```

If we place an A in the front, we must place one in the back. Likewise for B's.

Balanced parentheses

```
s : (s) | ss | *empty*
```

or

```
s : (s)s | *empty*
```

It is a temptation to define s as (s) . This disallows such balanced strings as $((())())$.

Set of strings over A, B so that the number of A s equals the number of B 's

$s : Ab \mid Ba \mid \text{empty}$

$a : As \mid Baa$

$b : Bs \mid Abb$

Note that b stands for strings with 1 more B than A and a stands for strings with 1 more A than B. These non-terminals arise naturally out of the observation that the strings must start with an A or B and the consequences of that starting character on the remainder of the string.