CS201 Data Structures and Algorithms

Summations

Prove $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$.

By induction on n.

Basis: Let $n = 1$.

$$\sum_{k=1}^{n} k = \tfrac{n(n+1)}{2}$$

$$\sum_{k=1}^{1} k = \tfrac{1(1+1)}{2}$$

$$1 = \tfrac{1\times 2}{2} = \tfrac{2}{2}$$

$$1 = 1$$

$\blacksquare$

Inductive Hypothesis: Assume true for $1 < k < n$.

Induction:

$$\sum_{k=1}^{n} k = \sum_{k=1}^{n-1} k + n$$

$$\sum_{k=1}^{n} k = \tfrac{(n-1)((n-1)+1)}{2} + n \text{ (By The Inductive Hypothesis)}$$

$$\sum_{k=1}^{n} k = \tfrac{(n-1)n}{2} + n$$

$$\sum_{k=1}^{n} k = \tfrac{n^2-n}{2} + \tfrac{2n}{2}$$

$$\sum_{k=1}^{n} k = \tfrac{n^2-n+2n}{2}$$

$$\sum_{k=1}^{n} k = \tfrac{n^2+n}{2}$$

$$\sum_{k=1}^{n} k = \tfrac{n(n+1)}{2}$$

$\blacksquare$

Prove $\sum_{k=0}^{n} 2^k = 2^{n+1} - 1$.

By induction on n.

Basis: Let $n = 0$.

$$\sum_{k=0}^{n} 2^k = 2^{n+1} - 1$$

$$\sum_{k=0}^{0} 2^k = 2^{0+1} - 1$$

$2^0 = 2^1 - 1 = 2 - 1$

$1 = 1$

∎

Inductive Hypothesis: Assume true for $1 < d < n$.

Induction:

$$\sum_{k=0}^{n} 2^k = \sum_{k=0}^{n-1} 2^k + 2^n$$

$$\sum_{k=0}^{n} k = 2^{(n-1)+1} - 1 + 2^n \text{ (By The Inductive Hypothesis)}$$

$$\sum_{k=0}^{n} k = 2^n - 1 + 2^n$$

$$\sum_{k=0}^{n} k = 2 \times 2^n - 1$$

$$\sum_{k=0}^{n} k = 2^{n+1} - 1$$

∎

Problems:

- 2.2-3
- 2.2-7
- 3.1-1
- 3.1-5
- 3.1-7