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%%%%%%%%%%%%%% Main Entrance
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%%%%%%%%%%%%%% By Max Yi Ren and Emrah Bayrak
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% Instruction: Please read through the code and
fill in blanks
% (marked by ***). Note that you need to do so for
every involved
% function, i.e., m files.

%% Optional overhead
clear; % Clear the workspace
close all; % Close all windows

%% Optimization settings
% Here we specify the objective function by giving
the function handle to a
% variable, for example:
f = @(x)x(1)^2 + (x(2)-3)^2; % replace with your
objective function
% In the same way, we also provide the gradient of
the
% objective:
df = @(x)[2*x(1), 2*(x(2)-3)]; % replace
accordingly

g = @(x)[x(2)^2-2*x(1);(x(2)-1)^2+5*x(1)-15];
dg = @(x)[-2, 2*x(2);5, 2*(x(2)-1)];

% Note that explicit gradient and Hessian
information is only optional.
% However, providing these information to the
search algorithm will save
% computational cost from finite difference
calculations for them.

% % Specify algorithm
opt.alg = 'matlabqp'; % 'myqp' or 'matlabqp'
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% Turn on or off line search. You could turn on
line search once other
% parts of the program are debugged.
opt.linesearch = true; % false or true

% Set the tolerance to be used as a termination
criterion:
opt.eps = 1e-3;

% Set the initial guess:
x0 = [0;0];

% Feasibility check for the initial point.
if max(g(x0)>0)
    errordlg('Infeasible intial point! You need to
start from a feasible one!');
    return
end
%% Run optimization
% Run your implementation of SQP algorithm. See
mysqp.m
solution = mysqp(f, df, g, dg, x0, opt);
in = 1:length(solution.x(1,:));
for i=1:length(in)
f_n(i)= f(solution.x(:,i));
end
sub1 = g(solution.x(:,i));
x1 = num2str(solution.x(1,i));
x2 = num2str(solution.x(2,i));
figure
subplot(2,1,1)
plot(in,f_n)
xlabel('iteration')
ylabel('f(x)')
title(['@x=(',x1,',',x2,') g1= ',
num2str(sub1(1)),'g2=', num2str(sub1(2))])

subplot(2,1,2)
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plot(solution.x(1,:),solution.x(2,:),'*-')
xlabel('x1')
ylabel('x2')
title(['global minimum @x=(',x1,',',x2,')
f(x)=',num2str(f_n(i))])

%% Report
%report(solution,f,g);

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%%%%%%%%%%%%%% Sequential Quadratic Programming
Implementation with BFGS %%%%%%%%%%%%%%%%
%%%%%%%%%%%%%% By Max Yi Ren and Emrah Bayrak
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function solution = mysqp(f, df, g, dg, x0, opt)
    % Set initial conditions

    x = x0; % Set current solution to the initial
guess

    % Initialize a structure to record search
process
    solution = struct('x',[]);
    solution.x = [solution.x, x]; % save current
solution to solution.x

    % Initialization of the Hessian matrix
    W = eye(numel(x));              % Start with an
identity Hessian matrix
    % Initialization of the Lagrange multipliers
    mu_old = zeros(size(g(x)));     % Start with
zero Lagrange multiplier estimates
    % Initialization of the weights in merit
function
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    w = zeros(size(g(x)));          % Start with
zero weights

    % Set the termination criterion
    gnorm = norm(df(x) + mu_old'*dg(x)); % norm of
Largangian gradient

    while gnorm>opt.eps % if not terminated

        % Implement QP problem and solve
        if strcmp(opt.alg, 'myqp')
            % Solve the QP subproblem to find s and
mu (using your own method)
            [s, mu_new] = solveqp(x, W, df, g, dg);
        else
            % Solve the QP subproblem to find s and
mu (using MATLAB's solver)
            qpalg = optimset('Algorithm', 'active-
set', 'Display', 'off');
            [s,~,~,~,lambda] =
quadprog(W,[df(x)]',dg(x),-g(x),[], [], [],
[],[0;0], [],  qpalg);
            mu_new = lambda.ineqlin;
        end

        % opt.linesearch switches line search on or
off.
        % You can first set the variable "a" to
different constant values and see how it
        % affects the convergence.
        if opt.linesearch
            [a, w] = lineSearch(f, df, g, dg, x, s,
mu_old, w);
        else
            a = 0.1;
        end

        % Update the current solution using the
step
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        dx = a*s;                    % Step for x
        x = x + dx;                  % Update x using
the step

        % Update Hessian using BFGS. Use equations
(7.36), (7.73) and (7.74)
        % Compute y_k
        y_k = [df(x) + mu_new'*dg(x) - df(x-dx) -
mu_new'*dg(x-dx)]';
        % Compute theta
        if dx'*y_k >= 0.2*dx'*W*dx
            theta = 1;
        else
            theta = (0.8*dx'*W*dx)/(dx'*W*dx-
dx'*y_k);
        end
        % Compute  dg_k
        dg_k = theta*y_k + (1-theta)*W*dx;
        % Compute new Hessian
        W = W + (dg_k*dg_k')/(dg_k'*dx) -
((W*dx)*(W*dx)')/(dx'*W*dx);

        % Update termination criterion:
        gnorm = norm(df(x) + mu_new'*dg(x)); % norm
of Largangian gradient
        mu_old = mu_new;

        % save current solution to solution.x
        solution.x = [solution.x, x];
    end

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%The following code performs line search on the
merit function

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% Armijo line search
function [a, w] = lineSearch(f, df, g, dg, x, s,
mu_old, w_old)
    t = 0.1; % scale factor on current gradient:
[0.01, 0.3]
    b = 0.8; % scale factor on backtracking: [0.1,
0.8]
    a = 1; % maximum step length

    D = s;                  % direction for x

    % Calculate weights in the merit function using
eaution (7.77)
    w = max(abs(mu_old), 0.5*(w_old+abs(mu_old)));
    % terminate if line search takes too long
    count = 0;
    while count<100
        % Calculate phi(alpha) using merit function
in (7.76)
        phi_a = f(x + a*D) + w'*abs(min(0, -
g(x+a*D)));

        % Caluclate psi(alpha) in the line search
using phi(alpha)
        phi0 = f(x) + w'*abs(min(0, -g(x)));
% phi(0)
        dphi0 = df(x)*D + w'*((dg(x)*D).*(g(x)>0));
% phi'(0)
        psi_a = phi0 +  t*a*dphi0;
% psi(alpha)
        % stop if condition satisfied
        if phi_a<psi_a;
            break;
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        else
            % backtracking
            a = a*b;
            count = count + 1;
        end
    end
end

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%The following code solves the QP subproblem using
active set strategy

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function [s, mu0] = solveqp(x, W, df, g, dg)
    % Implement an Active-Set strategy to solve the
QP problem given by
    % min     (1/2)*s'*W*s + c'*s
    % s.t.    A*s-b <= 0
    %
    % where As-b is the linearized active contraint
set

    % Strategy should be as follows:
    % 1-) Start with empty working-set
    % 2-) Solve the problem using the working-set
    % 3-) Check the constraints and Lagrange
multipliers
    % 4-) If all constraints are staisfied and
Lagrange multipliers are positive, terminate!
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    % 5-) If some Lagrange multipliers are negative
or zero, find the most negative one
    %     and remove it from the active set
    % 6-) If some constraints are violated, add the
most violated one to the working set
    % 7-) Go to step 2


    % Compute c in the QP problem formulation
    c = [df(x)]';

    % Compute A in the QP problem formulation
    A0 = dg(x);

    % Compute b in the QP problem formulation
    b0 = -g(x);

    % Initialize variables for active-set strategy
    stop = 0;           % Start with stop = 0
    % Start with empty working-set
    A = [];         % A for empty working-set
    b = [];         % b for empty working-set
    % Indices of the constraints in the working-set
    active = [];    % Indices for empty-working set

    while ~stop  % Continue until stop = 1
        % Initialize all mu as zero and update the
mu in the working set
        mu0 = zeros(size(g(x)));

        % Extact A corresponding to the working-set
        A = A0(active,:);
        % Extract b corresponding to the working-
set
        b = b0(active);

        % Solve the QP problem given A and b
        [s, mu] = solve_activeset(x, W, c, A, b);
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        % Round mu to prevent numerical errors
(Keep this)
        mu = round(mu*1e12)/1e12;

        % Update mu values for the working-set
using the solved mu values
        mu0(active) = mu;

        % Calculate the constraint values using the
solved s values
        gcheck = A0*s-b0;

        % Round constraint values to prevent
numerical errors (Keep this)
        gcheck = round(gcheck*1e12)/1e12;

        % Variable to check if all mu values make
sense.
        mucheck = 0;          % Initially set to 0

        % Indices of the constraints to be added to
the working set
        Iadd = [];                % Initialize as
empty vector
        % Indices of the constraints to be added to
the working set
        Iremove = [];             % Initialize as
empty vector

        % Check mu values and set mucheck to 1 when
they make sense
        if (numel(mu) == 0)
            % When there no mu values in the set
            mucheck = 1;          % OK
        elseif min(mu) > 0
            % When all mu values in the set
positive
            mucheck = 1;          % OK
        else
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            % When some of the mu are negative
            % Find the most negaitve mu and remove
it from acitve set
            [~,Iremove] = min(mu);  % Use Iremove
to remove the constraint
        end

        % Check if constraints are satisfied
        if max(gcheck) <= 0
            % If all constraints are satisfied
            if mucheck == 1
                % If all mu values are OK,
terminate by setting stop = 1
                stop = 1;
            end
        else
            % If some constraints are violated
            % Find the most violated one and add it
to the working set
            [~,Iadd] = max(gcheck); % Use Iadd to
add the constraint
        end
        % Remove the index Iremove from the
working-set
        active = setdiff(active, active(Iremove));
        % Add the index Iadd to the working-set
        active = [active, Iadd];

        % Make sure there are no duplications in
the working-set (Keep this)
        active = unique(active);
    end
end

function [s, mu] = solve_activeset(x, W, c, A, b)
    % Given an active set, solve QP
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    % Create the linear set of equations given in
equation (7.79)
    M = [W, A'; A, zeros(size(A,1))];
    U = [-c; b];
    sol = M\U;          % Solve for s and mu

    s = sol(1:numel(x));                % Extract s
from the solution
    mu = sol(numel(x)+1:numel(sol));    % Extract
mu from the solution

end

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end
function createfigure(X1, Y1, X2, Y2)
%CREATEFIGURE(X1, Y1, X2, Y2)
%  X1:  vector of x data
%  Y1:  vector of y data
%  X2:  vector of x data
%  Y2:  vector of y data

%  Auto-generated by MATLAB on 09-Dec-2021 00:02:49

% Create figure
figure1 = figure;

% Create subplot
subplot1 = subplot(2,1,1,'Parent',figure1);
hold(subplot1,'on');

% Create plot
plot(X1,Y1,'Parent',subplot1);

% Create ylabel
ylabel('f(x)');

% Create xlabel
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xlabel('iteration');

% Create title
title('@x=(1.0602,1.4561) g1= 7.5767e-06g2=-
9.4911');

% Uncomment the following line to preserve the X-
limits of the axes
% xlim(subplot1,[1 9]);
% Uncomment the following line to preserve the Y-
limits of the axes
% ylim(subplot1,[0 10]);
box(subplot1,'on');
hold(subplot1,'off');
% Create subplot
subplot2 = subplot(2,1,2,'Parent',figure1);
hold(subplot2,'on');

% Create plot
plot(X2,Y2,'Parent',subplot2,'Marker','*');

% Create ylabel
ylabel('x2');

% Create xlabel
xlabel('x1');

% Create title
title('global minimum @x=(1.0602,1.4561)
f(x)=3.5075');

box(subplot2,'on');
hold(subplot2,'off');
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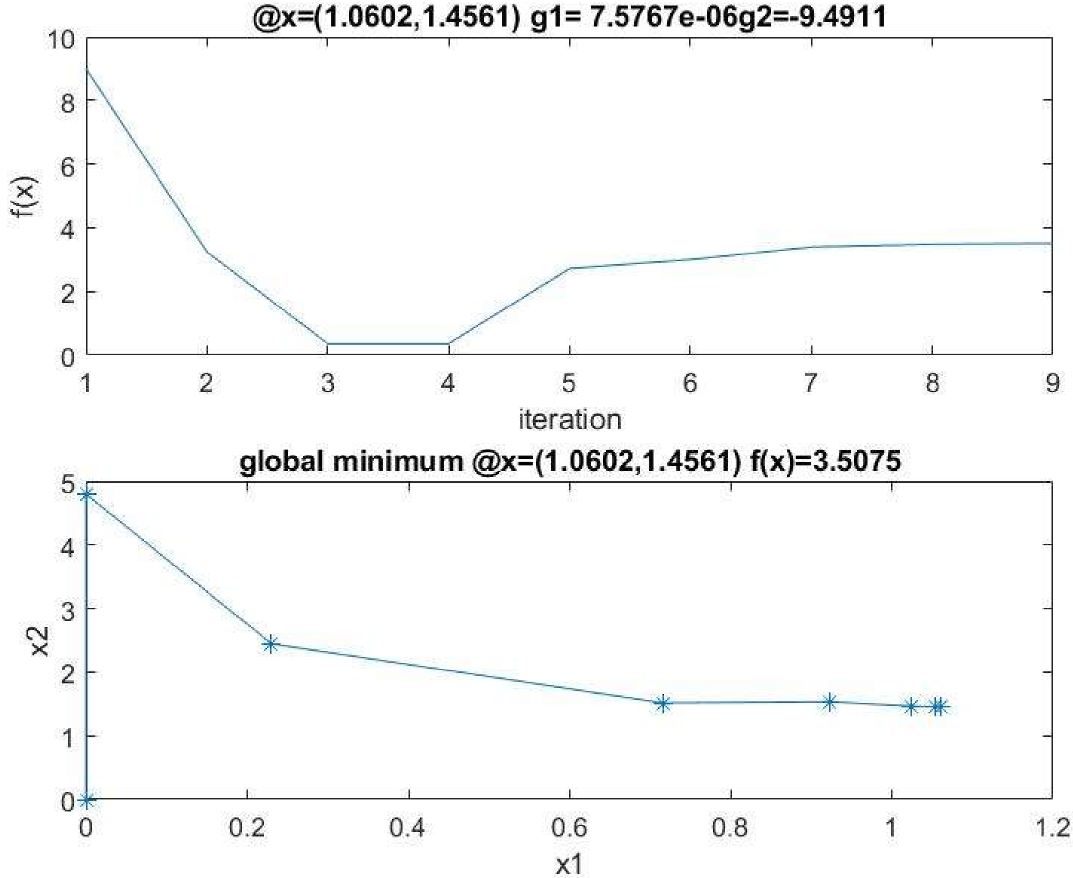
@x=(1.0602,1.4561) g1= 7.5767e-06g2=-9.4911
f(x)
iteration
global minimum @x=(1.0602,1.4561) f(x)=3.5075
x2
x1