

## STATISTICAL RETHINKING 2026

### A05 SOLUTIONS

We need two models. The first just regresses weight on sex. That will estimate the total effect of sex. The second model must include sex and height, so we can estimate the direct effect of sex.

Let's standardize the data.

```
library(rethinking)
data(Howell1)
d <- Howell1
d <- d[ d$age < 13 , ]

dat <- list(
  H = standardize(d$height),
  W = standardize(d$weight),
  S = d$male + 1
)
```

Here's the first model.


```
m1 <- quap(
  alist(
    W ~ dnorm( mu , sigma ),
    mu <- a[S],
    a[S] ~ dnorm(0,1),
    sigma ~ dexp(1)
  ), data=dat )
precis(m1,depth=2)
```

	mean	sd	5.5%	94.5%
a[1]	-0.13	0.11	-0.31	0.05
a[2]	0.14	0.12	-0.04	0.33
sigma	0.98	0.06	0.89	1.07

Now we need to contrast between the two intercepts:

```
post1 <- extract.samples(m1)
post1$WdoS <- with( post1 , a[,2] - a[,1] )
precis(post1)
```

quap posterior: 10000 samples from m1

	mean	sd	5.5%	94.5%	histogram
sigma	0.98	0.06	0.89	1.07	

WdoS 0.28 0.16 0.02 0.53



The total causal effect of sex (switch from sex 1 to sex 2) has a posterior mean of 0.28 standard deviations (these are standardized data), but note the width of the uncertainty: 0.02 to 0.53. This is probably because of the age range—the effect of sex is not constant across age from birth to 13 years of age.

Now the direct effect:

```
m2 <- quap(
  alist(
    W ~ dnorm( mu , sigma ),
    mu <- a[S] + b[S]*H,
    a[S] ~ dnorm(0,1),
    b[S] ~ dnorm(0,0.5),
    sigma ~ dexp(1)
  ), data=dat )
precis(m2,depth=2)
```

	mean	sd	5.5%	94.5%
a[1]	-0.04	0.03	-0.09	0.00
a[2]	0.03	0.03	-0.02	0.08
b[1]	0.90	0.03	0.86	0.95
b[2]	1.02	0.03	0.97	1.07
sigma	0.25	0.01	0.23	0.27

Now to get the direct effect, we need to simulate individuals of the same height but different sex.

```
post2 <- extract.samples(m2)
# expected weight for individual with average height and sex=1
EW_H0_S1 <- with( post2 , a[,1] + b[,1]*0 )
# expected weight for individual with average height and sex=2
EW_H0_S2 <- with( post2 , a[,2] + b[,2]*0 )
# contrast
post2$WdoS_H0 <- EW_H0_S2 - EW_H0_S1
precis(post2)
```

quap posterior: 10000 samples from m2

	mean	sd	5.5%	94.5%	histogram
sigma	0.25	0.01	0.23	0.27	
WdoS_H0	0.08	0.04	0.01	0.14	

A small positive effect, ranging from 0.01 to 0.14 standard deviations. Much smaller than the total effect, in other words. But larger than the direct effect in adults.