Edu-Risk International

Financial Risk Management & Training
Justin Clarke
+353 87 901 4483
justin.clarke@edurisk.ie
www.edurisk.ie

Constructing the OIS Curve

Introduction

During June 2010, LCH.Clearnet announced their intention to value swaps off the OIS curve instead of using the Swap Curve as was previously done. The reason for the move to the OIS curve was related to the collateral and margining arrangements of the majority of ISDA CSA agreements of trades cleared through LCH.Clearnet. A discussion on the rationale and effects of the change to OIS discounting is covered in another technical note produced by Edu-Risk International which can be found at the following link: Swap Discounting and the OIS Curve

This document describes the process that can be used to bootstrap or construct an OIS curve. Once an OIS curve has been created, the following two things can be achieved:

1. Libor-based interest generating curves can be bootstrapped relative to the OIS curve to correctly generate forward Libor interest flows on interest rate derivatives which are required for swap pricing (also described in an Edu-Risk technical note: Bootstrapping Libor Curve Off The OIS Curve)
2. Swap cash flow discounting can be performed using the OIS curve.

The OIS Market

An Overnight Index Swap (OIS) is a fixed-floating rate interest rate swap where the floating rate is indexed to an overnight interest rate (normally a cash-collateralised central bank accommodation rate or, in some countries, an interbank rate for the most creditworthy banks). The floating rate index rates in the major markets are the Fed Funds Rate (USD), SONIA (GBP) and EONIA (EUR). A more complete list of currencies and their index rates are given in a table below:

Currency	OIS Index
AUD	AONIA
CAD	CORRA
DKK	DKKOIS
EUR	EONIA
GBP	SONIA
HKD	HONIX
JPY	TONA
NZD	NZIONA
SEK	SIOR
USD	FED FUNDS
ZAR	SAFEX O/N Dep Rate

The fixed rate for OIS trades is normally a simple rate with interest at maturity for OIS's with maturities of less than 1 year and for OIS's longer than 1 year, the fixed rate is an annual fixed rate.

In the past, OIS have not been as liquid as Libor based swaps, but in recent times the liquidity and maturities of OIS markets has increased.

Calculation of OIS Floating Rate Interest

Most OIS markets use the following (ISDA) standard for computation of the floating interest rate during an interest period, this standard is equivalent to interest compounding on a business day basis:

$$Int = Nom\left[\prod_{i=1}^{d_n}\left(1+\frac{n_i REF_i}{d_y}\right)-1\right] \qquad 1$$

Where:

- d_n is the number of business days in the interest period.
- d_y is the number of days in the year normal for that currency (360/365).
- n_i is the number days between business day di and the next business day (e.g. for Fridays $n_i = 3$)
- REF_i is the reference date for business day i (valid until the next business day), normally published on business day i+1.
- The final settlement of an OIS occurs a day after the maturity date of the OIS because of the delay in publishing the reference date for the maturity date (the next morning).

This approach for calculating interest is normal for most currencies. One notable exception is for ZAR where floating rate interest is calculated on the basis of daily interest accruals, but only compounded monthly (as found in a call account). ZAR OIS's are called RODS (rand overnight Deposit swaps). A way of calculating the ZAR floating rate interest is to compute a daily average Reference Rate for each month (or fraction of months for partial months at the start and end of the OIS) and then to calculate monthly compounded interest using the month-by-month average rates.

The property of the OIS Reference Rates is that they are relatively constant between Meeting Dates[1], particularly when the rate is closely tied to a central bank policy rate. This is a direct consequence of the policy rate only being changed at Meeting Dates. However, in the case of extremely volatile markets there is always a risk of policy rates being changed at an unscheduled meeting, but this is unusual.

The following graph shows the Fed Funds rate between 2002 and 2006. It can be seen that when policy rates are steady, the Fed Funds rate stays relatively constant. Furthermore when the policy rates are in the process of being increased or decreased, the Fed Fund rates follow a step function. Superimposed upon the constant Fed Fund rates between Meeting Dates, there are some variations in the rate which would coincide with periods in the market where liquidity issues may predominate (such as year or month ends or tax payment dates or times of market stress)

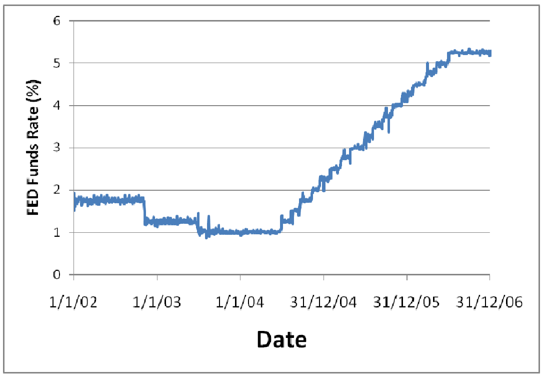


Figure 1: Graph showing the Fed Funds rate between 2002 and 2006.

General Issues Relating to OIS Curve Construction

The quasi-static behaviour of Reference Rates between Meeting Dates has a large influence on the construction of an OIS curve.

In essence, boot strapping the OIS curve faces a number of challenges, which would affect the construction of the curve as well as interpolating OIS rates from the curve.

1. **Short Term OIS Curve Rates:** In this part of the curve careful attention is paid to the quasi-static behaviour of Reference Rates between Meeting Dates. The jump-behaviour in Reference Rates around Meeting Dates is best handled in the curve modelling process with daily discount factors. Daily discount factors allow for correct interpolation of OIS rates across Meeting Dates.
2. **Seasonality in Reference Rates:** It is possible that Reference Rates may exhibit seasonality, for example, at month, quarter and year ends or on dates where there are large structural flows (e.g. tax payment dates). If these effects are present in the currency concerned, then short-term Reference Rates need to be seasonally adjusted in the construction of the short-end of the OIS curve.
3. **Longer Term OIS Rates:** These rates can be treated similarly to swap rates where direct interpolation between quoted OIS rates introduces no significant errors.
4. **Coupons:** OIS rates greater than 1 year generally pay annual interest. A traditional bootstrapping approach should be used to back out the OIS curve for OIS rates greater than 1 year.
5. **Smooth Forward Rates & Splining:** The same challenges exist in constructing the OIS curve in respect of constructing a curve with smooth forward rates. However, smoothing should be used with caution at the short end of the curve due to the quasi-static behaviour of Reference Rates.

Approach to OIS Curve Construction

The problem of OIS curve bootstrapping can be broken down into three segments where different approaches should be used to bootstrap the curve:

1. **Short Dated Region:** In this region, the effect of expected rate changes on Meeting Dates predominates. Seasonal behaviour in the Reference Rate can also be significant for short-dated or irregular dated OIS's. Daily Reference Rates and discount factors are used to interpolate between any two dates in this region. Depending on the currency, this region may last from to 3 to 6 months. The approach to calculating daily Reference Rates is described in a section below.
2. **Medium Dated Region:** In this portion of the curve, from the end of the Short dated region to 1 year (but no longer), OIS rates can be used directly to bootstrap the curve, with interpolation for other dates. Normally, there is little benefit from continuing the daily discount process as used in the Short Dated Region (although it can still be used).
3. **Long Dated Region:** This region is for OIS rates greater than 1 year, where OIS's pay annual interest. A traditional bootstrapping process is used which is described later in this paper.

[1] When we use the term 'Meeting Dates' we refer to the Monetary Policy Meeting dates (US: FOMC dates, UK MPC dates) relevant to each currency. These meeting dates fix short-term accommodation policy rates and overnight interest rates follow the direction of the policy rates.

Construction of the Short Dated Portion of the OIS Curve

In this section we will look at the process of bootstrapping the short-end of the OIS curve. Assume that we have quoted OIS rates for the first number of Meeting Dates (Meeting Dates are M1, M2 etc.), let these rates be r_{M1} ,r_{M2} etc. Also assume that we have regular tenor OIS rates (r_{T1} and r_{T2}) at times T1 and T2 – standard tenor dates may be 1-month, 2-month etc. This can be seen in Figure 2.

Also assume that (as indicated in Figure 2, that M2 <T1 <M3 and M3<T2<M4. The basic premise is that in any of periods between any of the date segments, M1..M4, T1..T2 the Reference Rate is constant.

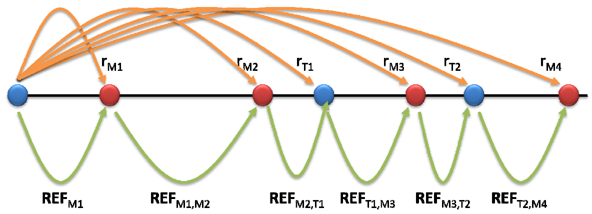


Figure 2: Diagram illustrating quoted short-term OIS rates on Meeting dates and standard tenor dates.

Assume one can make seasonality adjustments s_t on a daily basis to the quasi static Reference Rate, where the Reference Rate at time t, $REF_t = REF_{Static} + s_t$. The seasonality adjustment could be determined in a number of different ways, but in the formulas below it is expressed as an additive adjustment to the reference data on particular days. It is also assumed that the s_i's are known.

Using a seasonally adjusted version of equation 1, we have for the period t_0 to t_{M1} (consisting of d_{M1} workdays):

$$\frac{r_{M1}(t_{M1}-t_0)}{d_y}=\prod_{i=1}^{d_{M1}}\left(1+\frac{n_i(REF_i+s_i)}{d_y}\right)-1 \qquad 2$$

Equation 2 relates the interest on the fixed leg (left hand term) to the interest on the floating leg (right hand term). The interest on the fixed term is assumed to be simple interest. If we assume that in the period t_0 to t_M, REF_i is considered constant (REF_{M1}) for all i, then equation 2 can be expressed as:

$$\frac{r_{M1}(t_{M1}-t_0)}{d_y}=\left[\prod_{i=1}^{d_{M1}}\left(1+\frac{n_i(REF_{M1}+s_i)}{d_y}\right)-1\right] \qquad 3$$

Equation 3 can be solved numerically for REF_{M1}. Once REF_{M1} is known, then the constant Reference Rate between M1 and M2 ($REF_{M1,M2}$) can be determined numerically using the following equation:

$$\frac{r_{M2}(t_{M2}-t_0)}{d_y}=\prod_{i=1}^{d_{M1}}\left(1+\frac{n_i(REF_{M1}+s_i)}{d_y}\right)\prod_{i=d_{M1}}^{d_{M1,M2}}\left(1+\frac{n_i(REF_{M1,M2}+s_i)}{d_y}\right)-1$$

Similarly once $REF_{M1,M2}$ has been determined, the Reference Rate $REF_{M2,T1}$ can be determined thus:

$$\frac{r_{T1}(t_{T1}-t_0)}{d_y}=\prod_{i=1}^{d_{M1}}\left(1+\frac{n_i(REF_{M1}+s_i)}{d_y}\right)\prod_{i=d_{M1}}^{d_{M1,M2}}\left(1+\frac{n_i(REF_{M1,M2}+s_i)}{d_y}\right)\prod_{i=d_{M2}}^{d_{M2,T1}}\left(1+\frac{n_i(REF_{M2,T1}+s_i)}{d_y}\right)-1$$

By successive application of the above process, one can calculate all quasi-static Reference Rates in the short-term portion of the curve.

Forward Starting OIS Rates Between Meeting Dates

In certain markets, forward starting OISs rates are quoted between meeting dates rather than from the spot date to the meeting date. If this is the case, then the methodology presented above can be modified to account for these forward starting OIS rates between meeting dates.

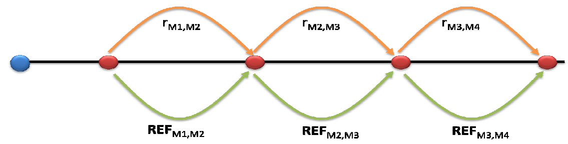


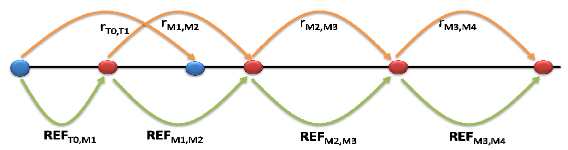
Figure 3: Diagram Indicating forward starting OIS rates quoted between meeting dates.

The following equation can be used to determine numerically the average reference rate $Ref_{M1,M2}$ between meeting dates (M1 and M2) for a forward starting OIS with fixed rate $r_{M1,M2}$:

$$\frac{r_{M1,M2}(t_{M2}-t_{M1})}{d_y}=\prod_{i=d_{M1}}^{d_{M2}}\left(1+\frac{n_i(REF_{M1,M2}+s_i)}{d_y}\right)-1$$

A similar approach would be used to determine $REF_{M2,M3}$ and $Ref_{M3,M4}$.

The presence of forward starting OIS rates between meeting dates, does simplify the calculation of the short end of the OIS curve, but it would be a good idea to ascertain whether OIS rates quoted on normal tenor dates price consistently with the meeting date OIS.



In addition, one could use one of the normal tenor OIS rates ($r_{T0,T1}$)and the first meeting date OIS rate to compute the average reference rate from T0 to M1 ($Ref_{T0,M1}$), assuming that the first tenor date T1 lies between M1 and M2. The equation to do this would be:

$$\frac{r_{T0,T1}(t_{T1}-t_{T0})}{d_y}=\prod_{i=d_{T0}}^{d_{M1}}\left(1+\frac{n_i(REF_{T0,M1}+s_i)}{d_y}\right)\prod_{i=d_{M1}}^{d_{T1}}\left(1+\frac{n_i(REF_{M1,M2}+s_i)}{d_y}\right)-1$$

Where $REF_{T0,M1}$ is determined numerically, all other variables being known.

Interpolating OIS Rates in the Short End of the Curve

In order to compute OIS rates for dates other than the M1-M4 and T1-T2 dates as indicated in the figure 2, we would compound up the daily implied Reference Rates (including seasonality adjustments) computed for each time segment over the required period. Using an equation similar to those above, one would then determine the applicable OIS rates for the relevant period.

It is important that a process of compounding daily implied Reference Rates are used to interpolate rates because errors will occur if OIS rates are implied over Meeting Dates from OIS quotes instead of using the daily implied because of the step-function nature of the Reference Rates. Additionally, seasonality will not be able to be applied unless the daily Reference Rates are used. The shorter the term of the OIS being priced, the greater the possible error could be.

Medium Term OIS Curve Bootstrapping

Normally the approach described for the short term region would be applied up until the last maturing meeting-date OIS. Between the last maturing meeting-date OIS and the 1 year OIS, normal interpolation of OIS rates can be used.

Longer Term OIS Curve Boot-strapping

For OIS's with maturities of greater than 1 year, a bootstrapping approach should be used to generate the OIS curve. Assume that one has annually quoted OIS rates up to 10 years (Each currency would have a series of OIS's quoted out to varying maturities). One would use the shorter dated OIS rates to first determine the curve discount factors for shorter maturities and then use these discount factors to aid in the determination of the next longer discount factor. We will demonstrate this process algebraically and then extend the formulation to a matrix equation which allows for easier computation.

Assume a set of annual interest paying OIS_i rates quoted for each annual maturity i. Assume that the relevant day count factor (e.g. days/365) for year i is τ_i, then the interest payable on OIS in year j for OIS_i is $OIS_i\tau_j$, provided that j≤i, the interest is = 0 otherwise.

Assuming that the OIS trades at par, then the following equation holds for all i:

$$100 = \sum_{k=1}^{i} OIS_i \tau_k f_{0,k} + 100 f_{0,i}$$

Where $f_{0,k}$ is the discount factor from time t_0 to time t_k. It can be seen that from starting at year 1 and using OIS_1, one can easily calculate $f_{0,1}$. Once $f_{0,1}$ has been calculated, then $f_{0,2}$ can be calculated from OIS_2 and $f_{0,1}$.

The calculation of the $f_{0,k}$'s is best done in a matrix (the example below is for the first 4 years only and $f_{0,k}$ is denoted as f_k for convenience):

$$\begin{bmatrix} OIS_1\tau_1+100 & & & \\ OIS_2\tau_1 & OIS_2\tau_2+100 & & \\ OIS_3\tau_1 & OIS_3\tau_2 & OIS_3\tau_3+100 & \\ OIS_4\tau_1 & OIS_4\tau_2 & OIS_4\tau_3 & OIS_4\tau_4+100 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \end{bmatrix}$$

To calculate the curve discount factors (f_1 to f_4), one solves the matrix equation as follows:

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \begin{bmatrix} OIS_1\tau_1+100 & & & \\ OIS_2\tau_1 & OIS_2\tau_2+100 & & \\ OIS_3\tau_1 & OIS_3\tau_2 & OIS_3\tau_3+100 & \\ OIS_4\tau_1 & OIS_4\tau_2 & OIS_4\tau_3 & OIS_4\tau_4+100 \end{bmatrix}^{-1} \begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \end{bmatrix}$$

Conclusion

The methodology presented in this document provides a framework for computing OIS curves. Constructing of OIS curves is an essential requirement for the pricing of all collateralised interest rate derivative products in today's markets. The OIS curve is fast becoming the primary interest rate benchmark curve used in the interest rate derivative market.

Edu-Risk International

Edu-Risk International is an Irish based financial risk management consultancy and training company owned and managed by Justin Clarke. Justin has over 20 years experience in the banking industry, mainly in the area of Risk Management and related fields.

More information may be obtained at:

http://www.edurisk.ie