PRÁCTICA 5: *Soluciones*

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1.

a)
$$\begin{gathered}\langle \square_i \square_d \rangle [\mathbf{1}, 3, 5, 7, 9, 2, 3, 7, 5, 9, \mathbf{9}] \\ = \square_i \square_d \langle \square_i \square_d \rangle [1, 3, 5, 7, 9, 2, 3, 7, 5, 9, 9] = \langle \square_i \square_d \rangle [\mathbf{3}, 5, 7, 9, 2, 3, 7, 5, \mathbf{9}] \\ = \square_i \square_d \langle \square_i \square_d \rangle [3, 5, 7, 9, 2, 3, 7, 5, 9] = \langle \square_i \square_d \rangle [\mathbf{5}, 7, 9, 2, 3, 7, \mathbf{5}] \\ = [\mathbf{5}, 7, 9, 2, 3, 7, \mathbf{5}]\end{gathered}$$

b)
$$\begin{gathered}\langle \square_i \square_d \rangle [\mathbf{1}, 2, 3, 4, 5, \mathbf{6}] = \square_i \square_d \langle \square_i \square_d \rangle [1, 2, 3, 4, 5, 6] \\ = \langle \square_i \square_d \rangle [\mathbf{2}, 3, 4, \mathbf{5}] = \square_i \square_d \langle \square_i \square_d \rangle [2, 3, 4, 5] = \langle \square_i \square_d \rangle [\mathbf{3}, \mathbf{4}] \\ = \square_i \square_d \langle \square_i \square_d \rangle [3, 4] = \langle \square_i \square_d \rangle [] = \nexists\end{gathered}$$

c)
$$\begin{gathered}0_d \langle \lhd 0_d \rhd \rangle \square_d [5, X] = \langle \lhd 0_d \rhd \rangle \square_d [\mathbf{5}, X, \mathbf{0}] = \lhd 0_d \rhd \langle \lhd 0_d \rhd \rangle \square_d [5, X, 0] \\ = 0_d \rhd \langle \lhd 0_d \rhd \rangle \square_d [0, 5, X] = \rhd \langle \lhd 0_d \rhd \rangle \square_d [0, 5, X, 0] = \langle \lhd 0_d \rhd \rangle \square_d [\mathbf{5}, X, 0, \mathbf{0}] = \dots\end{gathered}$$

d)
$$\begin{gathered}\rhd \langle \rhd 0_i \lhd S_i \rangle \lhd [x, y, Z] = \langle \rhd 0_i \lhd S_i \rangle \lhd [\boldsymbol{y}, Z, \boldsymbol{x}] = \rhd 0_i \lhd S_i \langle \rhd 0_i \lhd S_i \rangle \lhd [y, Z, x] \\ = 0_i \lhd S_i \langle \rhd 0_i \lhd S_i \rangle \lhd [Z, x, y] = \lhd S_i \langle \rhd 0_i \lhd S_i \rangle \lhd [0, Z, x, y] \\ = S_i \langle \rhd 0_i \lhd S_i \rangle \lhd [y, 0, Z, x] = \langle \rhd 0_i \lhd S_i \rangle \lhd [\boldsymbol{y+1}, 0, Z, \boldsymbol{x}] \\ = \dots = \langle \rhd 0_i \lhd S_i \rangle \lhd [\boldsymbol{y+2}, 0, 0, Z, \boldsymbol{x}] = \dots \\ = \langle \rhd 0_i \lhd S_i \rangle \lhd \left[\boldsymbol{x}, \underbrace{0, \dots, 0}_{x-y}, Z, \boldsymbol{x} \right] = \lhd \left[x, \underbrace{0, \dots, 0}_{x-y}, Z, x \right] = \left[x, x, \underbrace{0, \dots, 0}_{x-y}, Z \right]\end{gathered}$$

2.

a) $\left\{[x_0, \dots, x_n] \in \mathcal{L}^{\geq 2} / x_0 = x_n \vee x_i = x_{i+1} \text{ p.a } i\right\}$

b) $\left\{[x_0, \dots, x_n] \in \mathcal{L}^{\geq 1}, Z / x_i = 0 \text{ p.a. } i\right\}$

c) $\left\{[x, Y, z] \in \mathcal{L}^{\geq 2} / x \leq z\right\}$.

d) $\left\{[x_0, \dots, x_n] \in \mathcal{L}^{\geq 2} / x_i = x_{n-i} \text{ p.a } 0 \leq i < \lceil n/2 \rceil\right\}$.

e) $\{[0, Z] / Z \in \mathcal{L}\}$.

f) $\{[0, Z] / Z \in \mathcal{L}\}$.

3.

a) $k_i = 0_i S_i^k$.

b) $k_d = 0_d S_d^k$.

c) $\lhd = 0_i \langle S_i \rangle \square_d$.

d) $\rhd = 0_d \langle S_d \rangle \square_i$.

e) $D_i = 0_d \langle S_d \rangle \lhd$.

f) $D_d = 0_i \langle S_i \rangle \rhd$.

g) $\widetilde{S}_i = D_i S_i$.

h) $P_i = 0_d 1_d \langle S_d \lhd S_d \rhd \rangle \square_i \square_d \lhd$.

i) $\widetilde{P}_i = D_i P_i$.

j) $\widehat{P}_i = 0_d \langle \square_d P_i D_i \rhd \rangle \square_d$.

k) $\leftrightarrow = \rhd 0_i \lhd 0_i \langle S_i \rhd^2 S_i \lhd^2 \rangle \square_d \square_i \rhd$.

l) $\Sigma_i = \rhd 0_d \langle S_d \lhd S_d \rhd \rangle \square_i \square_d \lhd$.

m) $\widetilde{\Sigma}_i = \rhd D_i D_d \leftrightarrow \lhd^2 \Sigma_i$.

n) $R_i = \rhd^2 0_i \langle P_d \lhd P_d \rhd \rangle \square_i \square_d \lhd$.

ñ) $\widetilde{R}_i = \rhd D_i D_d \leftrightarrow \lhd^2 R_i$.

o) $\widehat{R}_i = \rhd^2 0_i \left\langle \widehat{P}_d \lhd \widehat{P}_d \rhd \right\rangle \square_i \square_d \lhd$.

p) $\Pi_i = \rhd D_i 0_i \left\langle S_i \rhd \widetilde{\Sigma}_i \rhd \square_i \lhd^2 \right\rangle \square_i R_i \square_d$.

q) $\widetilde{\Pi}_i = \rhd D_i D_d \leftrightarrow \lhd^2 \Pi_i$.

r) $\widehat{E}_i = \rhd \leftrightarrow 1_i 0_i \left\langle S_i \rhd \widetilde{\Pi}_i \rhd \square_i \lhd^2 \right\rangle \square_i \square_d \rhd \square_i \lhd$.

s) $D_0 = 0_i \widehat{E}_i$.

4.

a) $\square_i^2$.

b) $\langle S_i S_d \rangle$.

c) $\rhd \langle D_d \leftrightarrow \lhd R_i \rangle$.

d) $\widetilde{\Sigma}_i 1_d \langle S_d \rangle$.

e) COMPLETAR.

5.

a) $D_i^2 \Pi \rhd D_i \Sigma \lhd 1_i \Sigma^2$.

b) $((\leftrightarrow \lhd)^m \rhd^{m+1})^n$.

c) $\rhd 0_i \langle \square_i D_i 0_i P_d \rangle \square_i^2 \square_d \lhd$.

d) $D_i \rhd 0_i \langle \square_i D_i P_i 0_i P_d \rangle \square_i^2 \square_d$.

e) COMPLETAR.

f) COMPLETAR.

g) COMPLETAR.

h) $F_8 = 0_d 1_d 0_d \left\langle \lhd \widetilde{\Sigma}_d \lhd^2 \square_d \rhd^3 S_d \right\rangle \square_d^2 \lhd$

i) $F_9 = 0_i^2 1_d \langle \rhd^2 \square_i \lhd^2 \rangle \square_i^2$.

6.

a)

- Definimos $Q[x, Y] = \begin{cases} [x, Y, 1] & \text{si se cumple } Q(x) \\ [x, Y, 0] & \text{si no} \end{cases}$
- Definimos $F = Q 1_i \langle \square_i \square_d H 0_i 0_d \rangle \hat{P}_i \langle \square_i \square_d G 0_i 0_d \rangle \square_i \square_d$

Ejemplos

- Sea k tal que $Q(k)$ sea cierto, luego:

	$[k, Y]$
$Q1_i$	$[1, k, Y, 1]$
$\langle \square_i \square_d H 0_i 0_d \rangle$	$[1, k, Y, 1]$
$\hat{P}_i$	$[0, k, Y, 1]$
$\langle \square_i \square_d G 0_i 0_d \rangle$	$[0, G[k, Y], 0]$
$\square_i \square_d$	$G[k, Y]$

- Sea k tal que $Q(k)$ sea falso, luego:

	$[k, Y]$
$Q1_i$	$[1, k, Y, 0]$
$\langle \square_i \square_d H 0_i 0_d \rangle$	$[0, H[k, Y], 0]$
$\hat{P}_i$	$[0, H[k, Y], 0]$
$\langle \square_i \square_d G 0_i 0_d \rangle$	$[0, H[k, Y], 0]$
$\square_i \square_d$	$H[k, Y]$

$b)$

- $Q = D_i \hat{P}_i^4 D_0 \triangleright$.
- $F = Q1_i \langle \square_i \square_d P_i 0_i 0_d \rangle \hat{P}_i \langle \square_i \square_d S_i 0_i 0_d \rangle \square_i \square_d$.