Alpha-Beta (/Alpha-Beta)

Alpha Beta [2]

The **Alpha-Beta** algorithm (Alpha-Beta Pruning, Alpha-Beta Heuristic [1]) is a significant enhancement to the minimax search algorithm that eliminates the need to search large portions of the game tree applying a branch-and-bound technique. Remarkably, it does this without any potential of overlooking a better move. If one already has found a quite good move and search for alternatives, **one** refutation is enough to avoid it. No need to look for even stronger refutations. The algorithm maintains two values, alpha and beta. They represent the minimum score that the maximizing player is assured of and the maximum score that the minimizing player is assured of respectively. Consider the following example...

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How it works

Say it is White's turn to move, and we are searching to a depth of 2 (that is, we are consider all of White's moves, and all of Black's responses to each of those moves.) First we pick one of White's possible moves - let's call this Possible Move #1. We consider this move and every possible response to this move by black. After this analysis, we determine that the result of making Possible Move #1 is an even position. Then, we move on and consider another of White's possible moves (Possible Move #2.) When we consider the first possible counter-move by black, we discover that playing this results in black winning a Rook! In this situation, we can safely ignore all of Black's other possible responses to Possible Move #2 because we already know that Possible Move #1 is better. We really don't care *exactly* how much worse Possible Move #2 is. Maybe another possible response wins a Queen, but it doesn't matter because we know that we can achieve *at least* an even game by playing Possible Move #1. The full analysis of Possible Move #1 gave us a lower bound. We know that we can achieve at least that, so anything that is clearly worse can be ignored.

The situation becomes even more complicated, however, when we go to a search depth of 3 or greater, because now both players can make choices affecting the game tree. Now we have to maintain both a lower bound and an upper bound (called Alpha and Beta.) We maintain a lower bound because if a move is too bad we don't consider it. But we also have to maintain an upper bound because if a move at depth 3 or higher leads to a continuation that is too good, the other player won't allow it, because there was a better move higher up on the game tree that he could have played to avoid this situation. One player's lower bound is the other player's upper bound.

Savings

The savings of alpha beta can be considerable. If a standard minimax search tree has **x** nodes, an alpha beta tree in a well-written program can have a node count close to the square-root of **x**. How many nodes you can actually cut, however, depends on how well ordered your game tree is. If you always search the best possible move first, you eliminate the most of the nodes. Of course, we don't always know what the best move is, or we wouldn't have to search in the first place. Conversely, if we always searched worse moves before the better moves, we wouldn't be able to cut any part of the tree at all! For this reason, good move ordering is very important, and is the focus of a lot of the effort of writing a good chess program. As pointed out by Levin in 1961, assuming constantly **b** moves for each node visited and search depth **n**, the maximal number of leaves in alpha-beta is equivalent to minimax, **b** ^ **n**. Considering always the best move first, it is **b** ^ ceil(n/2) plus **b** ^ floor(n/2) minus one. The minimal number of leaves is shown in following table which also demonstrates the odd-even effect:

	number of leaves with depth n and b = 40	
depth	**worst case**	**best case**
n	b^n	$b^{\lceil n/2 \rceil} + b^{\lfloor n/2 \rfloor} - 1$
0	1	1
1	40	40
2	1,600	79
3	64,000	1,639
4	2,560,000	3,199
5	102,400,000	65,569
6	4,096,000,000	127,999
7	163,840,000,000	2,623,999
8	6,553,600,000,000	5,119,999

History

Alpha-Beta was invented independently by several researchers and pioneers from the 50s [3], and further research until the 80s, most notable by

- John McCarthy proposed the idea of Alpha-Beta in 1955 at the Dartmouth Conference [4]
- Allen Newell and Herbert Simon Approximation in 1958
- Arthur Samuel Approximation in 1959
- Daniel Edwards and Timothy Hart, Description in 1961 [5]
- Alexander Brudno, Description in 1963
- Samuel Fuller, John Gaschnig, James Gillogly, Analysis 1973 [6]
- Donald Knuth, Analysis in 1975
  Knuth and Moore's famous Function F2, aka AlphaBeta
  Knuth already introduced an iterative solution , see Iterative Search
  Knuth's node types
- Gérard M. Baudet, Analysis in 1978

Quotes

McCarthy

Quote by John McCarthy from *Human-Level AI is harder than it seemed in 1955* [7]:

> Chess programs catch some of the human chess playing abilities but rely on the limited effective branching of the chess move tree. The ideas that work for chess are inadequate for go. Alpha-beta pruning characterizes human play, but it wasn't noticed by early chess programmers - Turing, Shannon, Pasta and Ulam, and Bernstein. We humans are not very good at identifying the heuristics we ourselves use. Approximations to alpha-beta used by Samuel, Newell and Simon, McCarthy. Proved equivalent to minimax by Hart and Levin, independently by Brudno. Knuth gives details.

Ershov and Shura-Bura

Quote from *The Early Development of Programming in the USSR* by Andrey Ershov, Mikhail R. Shura-Bura [8]

> At the end of the 1950's a group of Moscow mathematicians began a study of computerized chess. Sixteen years later, the studies would lead to victory in the first world chess tournament for computer programs held in Stockholm during the 1974 IFIP Congress. An important component of this success was a deep study of the problems of information organization in computer memory and of various search heuristics. G. M. Adelson-Velsky and E. M. Landis invented the binary search tree ("dichotomic inquiry") and A. L. Brudno, independent of J. McCarthy, discovered the (α,β)-heuristic for reducing search times on a game tree.

Knuth

Quote by Knuth [9] :

> It is interesting to convert this recursive procedure to an iterative (non-recursive) form by a sequence of mechanical transformations, and to apply simple optimizations which preserve program correctness. The resulting procedure is surprisingly simple, but not as easy to prove correct as the recursive form.

Implementation

Max versus Min

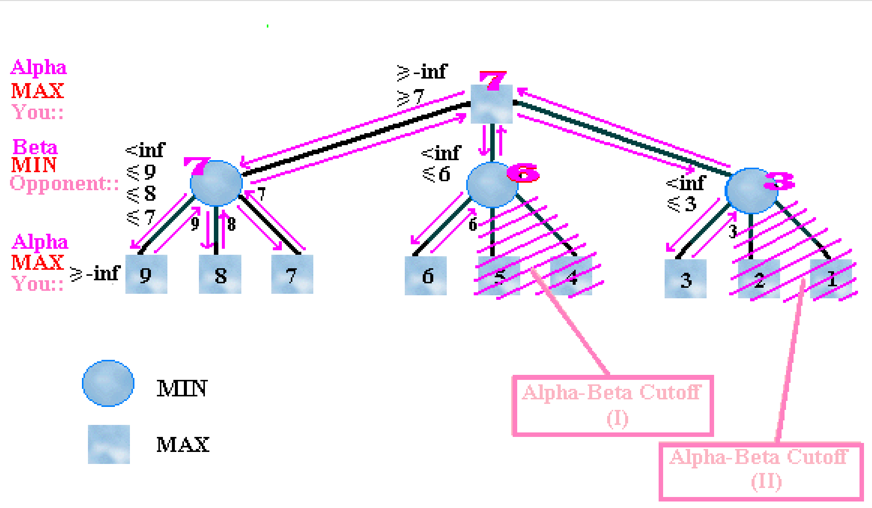
A C-like pseudo code implementation of the alpha-beta algorithm with distinct indirect recursive routines for the max- and min-player, similar to the minimax routines. Making and unmaking moves is omitted, and should be done before and after the recursive calls. So called beta-cutoffs occur for the max-play, alpha-cutoffs for the min-player.

```
int alphaBetaMax( int alpha, int beta, int depthleft ) {
   if ( depthleft == 0 ) return evaluate();
   for ( all moves) {
      score = alphaBetaMin( alpha, beta, depthleft - 1 );
      if( score >= beta )
         return beta;   // fail hard beta-cutoff
      if( score > alpha )
         alpha = score; // alpha acts like max in MiniMax
   }
   return alpha;
}

int alphaBetaMin( int alpha, int beta, int depthleft ) {
   if ( depthleft == 0 ) return -evaluate();
   for ( all moves) {
      score = alphaBetaMax( alpha, beta, depthleft - 1 );
      if( score <= alpha )
         return alpha; // fail hard alpha-cutoff
      if( score < beta )
         beta = score; // beta acts like min in MiniMax
   }
   return beta;
}
```

With this call from the Root:

```
   score = alphaBetaMax(-oo, +oo, depth);
```



Alpha-beta search tree with two alpha-cuts at min nodes [10]

Negamax Framework

Inside a negamax framework the routine looks simpler, but is not necessarily simpler to understand. Despite negating the returned score of the direct recursion, alpha of the min-player becomes minus beta of the max-player and vice versa, and the term alpha-cutoff or alpha-pruning is somehow diminished.

```
int alphaBeta( int alpha, int beta, int depthleft ) {
   if( depthleft == 0 ) return quiesce( alpha, beta );
   for ( all moves)  {
      score = -alphaBeta( -beta, -alpha, depthleft - 1 );
      if( score >= beta )
         return beta;   //  fail hard beta-cutoff
      if( score > alpha )
```

```
         alpha = score; // alpha acts like max in MiniMax
   }
   return alpha;
}
```


Note #1: Notice the call to quiesce(). This performs a quiescence search, which makes the alpha-beta search much more stable.

Note #2: This function only returns the score for the position, not the best move. Normally, a different (but very similar) function is used for searching the root node. The SearchRoot function calls the alphaBeta function and returns both a score and a best move. Also, most search functions collect the principal variation not only for display purposes, but for a good guess as the leftmost path of the next iteration inside an iterative deepening framework.

Fail hard

Since alpha and beta act as hard bounds of the return value if depth left is greater zero in the above code samples, this is referred to a fail-hard-framework.

Outside the Bounds

Fail-Soft Alpha-Beta [11] may return scores outside the bounds, that is either greater than beta or less than alpha. It has to keep track of the best score, which might be below alpha.

```
int alphaBeta( int alpha, int beta, int depthleft ) {
   int bestscore = -oo;
   if( depthleft == 0 ) return quiesce( alpha, beta );
   for ( all moves)  {
      score = -alphaBeta( -beta, -alpha, depthleft - 1 );
      if( score >= beta )
         return score;  // fail-soft beta-cutoff
      if( score > bestscore ) {
         bestscore = score;
         if( score > alpha )
            alpha = score;
      }
   }
   return bestscore;
}
```

Enhancements

The alpha-beta algorithm can also be improved. These enhancements come from the fact that if you restrict the window of scores that are interesting, you can achieve more cutoffs. Since move ordering is so much important, a technique applied outside of the search for this is iterative deepening boosted by a transposition table, and possibly aspiration windows. Other enhancements, applied within the search function, are further discussed.

Obligatory

- Transposition Table
- Iterative Deepening
- Aspiration Windows

Selectivity

- Quiescence Search
- Selectivity

Scout and Friends

- Scout
- NegaScout
- Principal Variation Search

Alpha-Beta goes Best-First

- Alpha-Beta Conspiracy Search
- MTD(f)
- NegaC*
- SSS* and Dual* as MT

See also

- Alpha
- Beta

- Beta-Cutoff
- Bound
- CPW-Engine, search
- Fail-Low
- Fail-High
- Gamma-Algorithm
- Iterative Search
- KC Chess
- MCαβ
- Minimax
- Negamax
- Node Types
- Odd-Even Effect
- Parallel Alpha-Beta
  Parallel Alpha-Beta in Cilk
- Search Explosion
- Theorem-Proving and M & N procedure
- Window

Videos

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  Re: Computer Chess (LONG) by Andy Walker, rgc, September 16, 1993
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- Dumb question about alpha-beta by Daylen Yang, CCC, March 04, 2014

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- Search algorithm in it's simplest forum by Mahmoud Uthman, CCC, February 25, 2015
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