Minimax (/Minimax)

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Max Ernst,
Little Machine
Constructed
by

Constructed by Minimax Dadamax in Person , 1919-1920 [1] **Minimax** is an algorithm used to determine the <u>score</u> in a <u>zero-sum</u> game after a certain number of moves, with best play according to an <u>evaluation</u> function.

The algorithm can be explained like this: In a one-ply-search, where only move sequences with length one are examined, the side to move (max player) can simply look at the evaluation after playing all possible moves. The move with the best evaluation is chosen. But for a two-ply search, when the opponent also moves, things become more complicated. The opponent (min player) also chooses the move that gets the best score. Therefore, the score of each move is now the score of the worst that the opponent can do.

```
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History

Jaap van den Herik's thesis (1983) [2] contains a detailed account of the known publications on that topic. It concludes that although John von Neumann is usually associated with that concept (1928) [3], primacy probably belongs to Émile Borel. Further there is a conceivable claim that the first to credit should go to Charles Babbage [4]. The original minimax as defined by Von Neumann is based on exact values from game-terminal positions, whereas the minimax search suggested by Norbert Wiener [5] is based on heuristic evaluations from positions a few moves distant, and far from the end of the game.

Implementation

Below the pseudo code for an indirect recursive depth-first search. For clarity move making and unmaking before and after the recursive call is omitted.

```
int maxi( int depth ) {
    if ( depth == 0 ) return evaluate();
    int max = -oo;
    for ( all moves) {
        score = mini( depth - 1 );
        if( score > max )
            max = score;
    }
    return max;
}

int mini( int depth ) {
    if ( depth == 0 ) return -evaluate();
```

```
int min = +oo;
for (all moves) {

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    min = score;
}
return min;
}
```

Negamax

Usually the <u>Negamax</u> algorithm is used for simplicity. This means that the evaluation of a position is equivalent to the negation of the evaluation from the opponent's viewpoint. This is because of the zero-sum property of chess: one side's win is the other side's loss.

See also

Search

- Alpha-Beta
- Minimax (program)
- Negamax
- Search Pathology
- Theorem-Proving and M & N procedure
- Theorem-Proving from Five-Year Plan

People

- John von Neumann
- Claude Shannon
- Norbert Wiener

Selected Publications

1920 ...

- Émile Borel (1921). La théorie du jeu et les équations intégrales à noyau symétrique. Comptes Rendus de Académie des Sciences, Vol. 173, pp. 1304-1308, English translation by <u>Leonard J. Savage</u> (1953). <u>The Theory of Play and Integral Equations with Skew Symmetric Kernels</u>.
- <u>John von Neumann</u> (1928). <u>Zur Theorie der Gesellschaftsspiele</u> . Berlin [6]

1940 ...

- John von Neumann, Oskar Morgenstern (1944). Theory of Games and Economic Behavior . Princeton University Press, Princeton, NJ.
- Norbert Wiener (1948). Cybernetics or Control and Communication in the Animal and the Machine MIT Press, Cambridge, MA.

1950 ...

- Claude Shannon (1950). Programming a Computer for Playing Chess , Philosophical Magazine, Ser.7, Vol. 41, No. 314 March 1950
- <u>Hermann Weyl</u> (1950). Elementary Proof of a Minimax Theorem due to von Neumann. in <u>Harold W. Kuhn</u> and <u>Albert W. Tucker</u> (eds) (1950). <u>Contributions to the Theory of Games I</u>. Princeton University Press
- <u>Émile Borel</u>, <u>Maurice R. Fréchet</u>, <u>John von Neumann</u> (1953). Discussion of the Early History of the Theory of Games, with Special Reference to the Minimax Theorem. <u>Econometrica</u>, Vol. 21
- <u>Leonard J. Savage</u> (1953). The Theory of Play and Integral Equations with Skew Symmetric Kernels. <u>Econometrica</u>, Vol. 21, pp. 101-115, English translation of <u>Émile Borel</u> (1921). <u>La théorie du jeu et les équations intégrales à noyau symétrique</u>.
- <u>David Blackwell</u> (1956). An analog of the minimax theorem for vector payoffs . Pacific Journal of Mathematics , Vol. 6, No, 1

1960 ...

- James R. Slagle (1963). Game Trees, M & N Minimaxing, and the M & N alpha-beta procedure. Artificial Intelligence Group Report 3, UCRL-4671, University of California
- <u>Donald Michie</u> (1966). *Game Playing and Game Learning Automata*. Advances in Programming and Non-Numerical Computation, <u>Leslie Fox</u> (ed.), pp. 183-200. Oxford, Pergamon. » Includes Appendix: *Rules of SOMAC* by <u>John Maynard Smith</u>, introduces <u>Expectiminimax tree</u>
- James R. Slagle, Philip Bursky (1968). Experiments With a Multipurpose, Theorem-Proving Heuristic Program
 Journal of the ACM, Vol. 15,

 No. 1
- James R. Slagle, John K. Dixon (1969). Experiments With Some Programs That Search Game Trees. Journal of the ACM, Vol 16, No. 2, pdf

1970 ...

- James R. Slagle, John K. Dixon (1970). Experiments with the M & N Tree-Searching Program. Communications of the ACM, Vol. 13, No. 3, pp. 147-154.
- Minimax in Alex Bell (1972). Games Playing with Computers .

1980 ...

- Ivan Bratko, Matjaž Gams (1982). Error Analysis of the Minimax Principle. Advances in Computer Chess 3
- Ronald L. Rivest (1987). Game Tree Searching by Min/Max Approximation. Artificial Intelligence Vol. 34, 1, pdf 1995

1990 ...

• Liwu Li, Tony Marsland (1990). On Minimax Game Tree Search Pathology and Node-value Dependence. TR90-24, University of Alberta, pdf

- Claude G. Diderich (1993). A Bibliography on Minimax Trees . ACM SIGACT News, Vol. 24, No. 4
- <u>David E. Moriarty</u>, <u>Risto Milikkulainen</u> (1994). Evolving Neural Networks to focus Minimax Search. <u>AAAI-94</u>, <u>pdf</u> » <u>Othelio</u>
- Sewels time identise to say farewell (1995) greatury; we've initiate the sage desision to desisio
- Richard Korf, Max Chickering (1996). Best-first minimax search . Artificial Intelligence , Vol. 84, Nos 1-2 » Best-First
- · Yoav Freund, Robert Schapire (1996). Game Theory, On-line Prediction and Boosting. COLT 1996, pdf
- Don Beal (1999). The Nature of MINIMAX Search. Ph.D. thesis

2000 ...

- Claude G. Diderich, Marc Gengler (2001). Minimax Game Tree Searching . Encyclopedia of Optimization , Springer
- <u>Tinne Hoff Kjeldsen</u> (2001). John von Neumann's Conception of the Minimax Theorem: A Journey Through Different Mathematical Contexts.

 <u>Archive for History of Exact Sciences</u>, Vol. 56, <u>Springer</u>
- Thomas Hauk, Michael Buro, Jonathan Schaeffer (2004). Rediscovering *-Minimax Search . CG 2004, pdf
- Mitja Luštrek, Matjaž Gams, Ivan Bratko (2005). Why Minimax Works: An Alternative Explanation . IJCAI 2005 » Search Pathology
- <u>Claude G. Diderich, Marc Gengler</u> (2009). <u>Minimax Game Tree Searching</u> . <u>Encyclopedia of Optimization</u> , <u>Springer</u>

2010 ...

• Jeff Rollason (2014). Interest Search - Another way to do Minimax . Al Factory, Summer 2014

Forum Posts

- <u>beyond minimax</u> by <u>Harm Geert Muller</u>, <u>CCC</u>, April 27, 2007
- The evaluation value and value returned by minimax search by Ma Chao, CCC, March 09, 2012 » Evaluation
- Why minimax is fundamentally flawed by Harm Geert Muller, CCC, November 09, 2014 » KRK

External Links

- Min-Max Search from Bruce Moreland's Programming Topics
- Minimax from Wikipedia
- Minimax estimator from Wikipedia
- Expectiminimax tree from Wikipedia
- Maxima and minima from Wikipedia
- Nash equilibrium from Wikipedia
- Parthasarathy's theorem from Wikipedia
- Sion's minimax theorem from Wikipedia
- Analog voltage maximizer and minimizer circuits from FreePatentsOnline

References

- 1. ^ Little Machine Constructed by Minimax Dadamax in Person from Wikipedia
- <u>Auap van den Herik</u> (1983). Computerschaak, Schaakwereld en Kunstmatige Intelligentie. Ph.D. thesis, <u>Delft University of Technology</u>.
 Academic Service, The Hague. ISBN 90 62 33 093 2 (Dutch)
- 3. <u>^ John von Neumann</u> (1928). <u>Zur Theorie der Gesellschaftsspiele</u> . Berlin
- 4. <u>^ Don Beal</u> (1999). The Nature of MINIMAX Search. Ph.D. thesis, ISBN 90-62-16-6348, pp. 2, refers Philip Morrison and Emily Morrison (1961). Charles Babbage and his Calculating Engines. Dover Publ. New York
- 5. A Norbert Wiener (1948). Cybernetics or Control and Communication in the Animal and the Machine MIT Press, Cambridge, MA.
- 6. <u>^ Alexander Reinefeld</u> (2005). Die Entwicklung der Spielprogrammierung: Von John von Neumann bis zu den hochparallelen Schachmaschinen. <u>slides as pdf</u>, Themen der Informatik im historischen Kontext Ringvorlesung an der <u>HU Berlin</u>, 02.06.2005 (English paper, German title)
- 7. ^ see Swap-off by Helmut Richter

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