Forward and Inverse Kinematics Model for Robotic Welding Process Using KR-16KS KUKA Robot

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Abstract— **This paper aims to model the forward and inverse kinematics of a KUKA KR-16KS robotic arm in the application of a simple welding process. A simple welding task to weld a block onto a metal sheet is carried out in order to investigate the forward and inverse kinematics models of KR-16KS. A movement flow planning is designed and further developed into the KR-16KS programming. Eleven points of movement are studied for the forward kinematic modeling. A summary of calculation is obtained. A general D-H representation of forward and inverse matrix is obtained. This can be used in each of the welding operation movement based on KUKA KR-16KS robotic arm. A forward kinematic and an inverse kinematic aspect of KUKA KR-16KS is successfully modeled based on a simple welding task.**

***Keywords*— Forward Kinematics, Inverse Kinematics, KUKA Robot, Robotic Arm, Robotic Welding Process**

I. Introduction

Nowadays, industrial robots are employed in a various applications and are available with a wide range of configurations, drive systems, physical sizes and payloads. However, the numbers in service in the whole world are much less than predicted over 20 years ago [1]. Whilst varying degrees of success are seen, the application of many advanced technologies has often been severely restricted in commercial systems due to the limitations of their controllers rather than their manipulator arms [2]. Nevertheless, modern commercial robotic systems are still very complex. They integrate many sensors and effectors, have many interacting degrees-of freedom (DOF) and most of them require operator interfaces, programming tools and real-time capabilities [3].

Welding robots, with the proper setup and a welding end effecter, are used to weld two pieces together. With the development of industry robot technology, welding robot and robot automatic welding technology are used in mechanical fields for metal components sewing, manufacture and many more. In order to satisfied different requirements of welding tasks, functions of welding robot control system are researched and improved as time goes by [4, 5], such as intelligent control system [6], welding seam tracking technology [7], advanced adaptive capability [8]. Besides the functions mentioned above, robot control systems are designed more open for users to add different external equipments, develop and expand functions of robots.

It is important to analyse the kinematic solution and to plan the trajectory of the robot during the course from its design to experiment. The kinematic problems, however, are very complex with difficult computing due to the multi-degree-of–freedom and multilink space mechanisms of the robot. Many new research theories and technologies are developed on it with the improvement of the robot [9]. The capabilities of early robotic manipulators were limited by the servo-control of separate joint axes. The modern industrial robotic systems, on the other hand, should implement the task-level control that simplifies the manufacturing task definition for the end users [10]. This results in making a kinematic control module as a built-in part of the hierarchical control system. Nowadays, there are various types of robots suitable for the welding task available in market, including Motoman, Fanuc, and many other robotic arms. One of them is the KUKA KR-16KS.

In this paper the focus will be on the kinematics and inverse kinematics modeling of a KUKA KR-16KS robotic arm. Generally, KUKA Robotics is a leading German producer of industrial robots for a variety of industrial processes. The robotic arm comes with a control panel that has a display and an integrated mouse, with which the manipulator is moved, positions are saved (TouchUp), or where modules, functions, and data lists are created and modified. The connection to the controller is a VGA interface and a CAN-bus. Controls for the latest control panel use the Windows XP operating system. The KR-16KS is a 6-axes robotic arm weighting 235kg with the payload up to 16kg [11]. This research will focus specifically on the use of this type of robotic arm in a simple welding process.

II. Methodology

A. The Work Pieces

This is a research is focused on the kinematics and inverse kinematics of a simple welding process using KUKA robotic arm. Thus the work pieces used are a simple small rectangular block and a flat metal sheet. The dimension of the rectangular block is 0.34m x 0.17m x 0.1m as shown in Fig 1 below. The idea is to weld the block onto the centre top of the metal sheet. The work piece is not the main concern in this

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study, as it is just a dummy object used to carry out the research. The focus is more on the operation flow and the kinematics of the KUKA robotic arm.

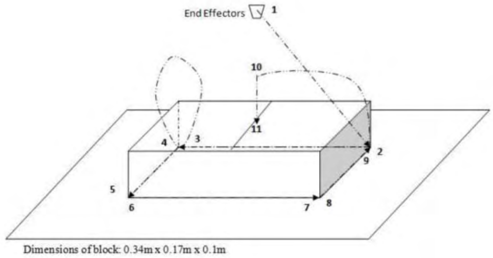

Fig. 1 Dimension of the work piece and the motion path of the end effecter.

B. *Operation Flow and the Programming*

The outline of the motion or operation was developed before the programming took place. The overall operation flow of the KUKA robotic arm is as shown in Fig 2. The programming of the motion was then successfully developed using KUKA Control Panel.

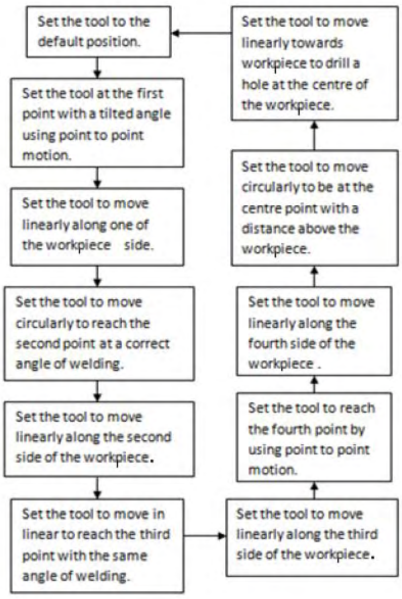

Fig. 2 The operation flow of the developed welding process.

III. KINEMATICS ANALYSIS AND DISCUSSION

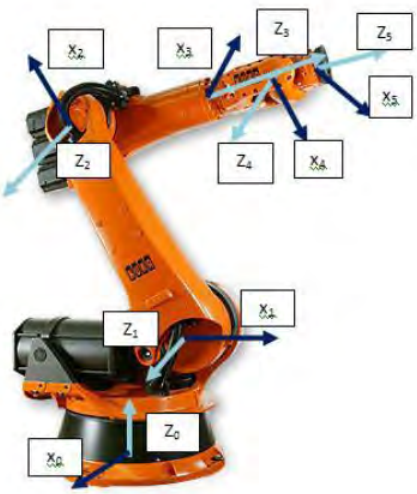

Fig. 3 Joints of KUKA KR 16KS robotic arm.

Referring to Fig. 3 above, beginning at joint 1, Z_0 represents the first joint, which is a revolute joint. X_0 is chosen to be parallel to the reference frame x-axis. Next, Z_1 is assigned at joint 2. X_1 will be normal to Z_0 and Z_1, since these two axes are intersecting. X_2 will be in the direction of the common normal between Z_1 and Z_2. X_3 is in the direction of the common normal between Z_3 and Z_5. Similarly X_4 is in the direction of the common normal between Z_2 and Z_4. Finally Z_5 and Z_6 are in same direction because they are parallel and collinear. Z_5 represent the motions of joint 6, while Z_6 represents the motions of the end effectors. Next, the assigned coordinate frames are followed to fill out the parameters as shown in Table I below.

TABLE I
PARAMETERS OF ALL LINKS AND JOINTS

#	θ	d	a	α
1	θ_1	0	a_1	90
2	θ_2	0	a_2	0
3	θ_3	d_3	0	90
4	θ_4	0	0	-90
5	θ_5	0	0	90
6	θ_6	0	0	0

A. *The D-H Parameters Table*

Denavit and Hartenberg put forwards to a matrix method to build the attached coordinate system on each link in the joint chains of the robot to describe the relationship of translation or rotation between the contiguous links way back in 1955 [12]. This robot kinematic model is based on the D-H Coordination system. The transformations between each two successive joints can be written by simply substituting the parameters from the parameters table into the A matrix.

$$A_{n+1} = \begin{bmatrix} C\theta_{n+1} & -S\theta_{n+1}C\alpha_{n+1} & S\theta_{n+1}S\alpha_{n+1} & a_{n+1}C\theta_{n+1} \\ S\theta_{n+1} & C\theta_{n+1}C\alpha_{n+1} & -C\theta_{n+1}S\alpha_{n+1} & a_{n+1}S\theta_{n+1} \\ 0 & S\alpha_{n+1} & C\alpha_{n+1} & d_{n+1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

$$A_1 = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & a_1C\theta_1 \\ S\theta_1 & 0 & -C\theta_1 & a_1S\theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & a_2C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & a_2S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$A_3 = \begin{bmatrix} C\theta_3 & 0 & S\theta_3 & 0 \\ S\theta_3 & 0 & -C\theta_3 & 0 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$A_4 = \begin{bmatrix} C\theta_4 & 0 & -S\theta_4 & 0 \\ S\theta_4 & 0 & C\theta_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$A_5 = \begin{bmatrix} C\theta_5 & 0 & S\theta_5 & 0 \\ S\theta_5 & 0 & -C\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$A_6 = \begin{bmatrix} C\theta_6 & -S\theta_6 & 0 & 0 \\ S\theta_6 & C\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

At the base of the robot, it can be started with the first joint and then transform to the second joint, then to the third until to the arm-end of the robot, and eventually to the end effectors. The total transformation between the base of the robot and the hand is

$$^{R}T_{H} = A_1A_2A_3A_4A_5A_6 \quad (3)$$

$$A_1A_2 = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & a_1C\theta_1 \\ S\theta_1 & 0 & -C\theta_1 & a_1S\theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & a_2C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & a_2S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_1C_2 & -C_1S_2 & S_1 & a_2C_1C_2 + a_1C_1 \\ S_1C_2 & -S_1S_2 & -C_1 & a_2S_1C_2 + a_1S_1 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3A_4 = \begin{bmatrix} C\theta_3 & 0 & S\theta_3 & 0 \\ S\theta_3 & 0 & -C\theta_3 & 0 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_4 & 0 & -S\theta_4 & 0 \\ S\theta_4 & 0 & C\theta_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_3C_4 & -S_3 & -S_4C_3 & 0 \\ S_3C_4 & C_3 & -S_4S_3 & 0 \\ S_4 & 0 & C_4 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5A_6 = \begin{bmatrix} C\theta_5 & 0 & S\theta_5 & 0 \\ S\theta_5 & 0 & -C\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_6 & -S\theta_6 & 0 & 0 \\ S\theta_6 & C\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_5C_6 & -S_6C_5 & S_5 & 0 \\ S_5C_6 & -S_5S_6 & -C_5 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (4)$$

$$^{R}T_H = A_1A_2A_3A_4A_5A_6 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (5)$$

$$\begin{aligned}
n_x &= C_1[C_{23}(C_4C_5C_6 - S_4S_6) - S_{23}S_5C_6] + S_1(S_4C_5C_6 + C_4S_6) \\
n_y &= S_1[C_{23}(C_4C_5C_6 - S_4S_6) - S_{23}S_5C_6] - C_1(S_4C_5C_6 + C_4S_6) \\
n_z &= S_{23}(C_4C_5C_6 - S_4S_6) + C_{23}S_5C_6 \\
o_x &= C_1[- C_{23}(C_4C_5C_6+S_4C_6) + S_{23}S_5S_6] + S_1(-S_4S_5S_6 + C_4C_6) \\
o_y &= S_1[- C_{23}(C_4C_5C_6+S_4C_6) + S_{23}S_5S_6] - C_1(-S_4C_5S_6 + C_4C_6) \\
o_z &= -S_{23}(C_4C_5C_6+S_4C_6) - C_{23}S_5S_6 \\
a_x &= C_1[C_{23}C_4S_5 + S_{23}C_5] - C_1S_4S_5 \\
a_y &= S_1[C_{23}C_4S_5 + S_{23}C_5] - C_1S_4S_5 \\
a_z &= S_{23}C_4S_5 - C_{23}C_5 \\
p_x &= C_1(C_2a_2 + a_1) + S_1d_3 \\
p_y &= S_1(C_2a_2 + a_1) - C_1d_3 \\
p_z &= S_2a_2
\end{aligned} \qquad (6)$$

B. *Forward Kinematics Analysis*

The kinematic analysis mainly includes two aspects, namely the forward kinematic analysis and the inverse kinematic. The forward kinematic analysis means that the location and pose of the end of manipulator in a given references coordinates system can be worked out with the given geometry parameters of the links and the variables of the joints for a robot [9]. Table II below shows the descriptions of the point-to-point movement of the developed programming. The point numbering is based on the figure shown in Fig. 1.

TABLE III
POINT-TO-POINT MOVEMENT DESCRIPTION BASED ON THE DEVELOPED PROGRAMMING

Points	Descriptions (based on program)
1	Home Position
2	Cylindrical Coordinate movement
3	Cartesian Coordinate movement
4	Spherical Coordinate movement
5	Cartesian Coordinate movement
6	Spherical Coordinate orientation
7	Cartesian Coordinate movement
8	Spherical Coordinate orientation
9	Cartesian Coordinate movement
10	Spherical Coordinate movement
11	Cartesian Coordinate movement for drilling

(a)Point 1

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1.3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (7)$$

(b)Point 2

By making axis 4, 5 and 6 static and overall transformation performs as cylindrical; the axes are move 0.17m along x-axis, -0.5m along z-axis and rotate 50° about z-axis.

$$\begin{aligned}
^{1}T_2 &= T_{cyl}(r,\alpha,l) \\
&= Trans(0,0,l)Rot(z,\alpha)Trans(r,0,0)
\end{aligned}$$

$$= \begin{bmatrix} C\alpha & -S\alpha & 0 & rC\alpha \\ S\alpha & C\alpha & 0 & rS\alpha \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} cos50 & -sin50 & 0 & 0.17cos50 \\ sin50 & cos50 & 0 & 0.17sin50 \\ 0 & 0 & 1 & -0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.6428 & -0.7660 & 0 & 0.1093 \\ 0.7660 & 0.6428 & 0 & 0.1302 \\ 0 & 0 & 1 & -0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (8)$$

$$^{0}T_2 = P \times {}^{1}T_2$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1.3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.6428 & -0.7660 & 0 & 0.1093 \\ 0.7660 & 0.6428 & 0 & 0.1302 \\ 0 & 0 & 1 & -0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.6428 & -0.7660 & 0 & 0.1093 \\ 0.7660 & 0.6428 & 0 & 1.1302 \\ 0 & 0 & 1 & 0.8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (9)$$

(c)Point 3

By making axis 2, 4 and 6 static and overall transformation performs as Cartesian, a movement is made -0.34m along x-axis.

$$^{2}T_3 = T_{cart} = \begin{bmatrix} 1 & 0 & 0 & P_x \\ 0 & 1 & 0 & P_y \\ 0 & 0 & 1 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & -0.34 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (10)$$

$$^{0}T_3 = {}^{0}T_2 \times {}^{2}T_3$$

$$= \begin{bmatrix} 0.6428 & -0.7660 & 0 & 0.1093 \\ 0.7660 & 0.6428 & 0 & 1.1302 \\ 0 & 0 & 1 & 0.8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -0.34 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.6428 & -0.7660 & 0 & -0.1093 \\ 0.7660 & 0.6428 & 0 & 0.8698 \\ 0 & 0 & 1 & 0.8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (11)$$

(d)Point 4

By making axis 2, 3 and 6 static and overall transformation performs as spherical; movements are made 0.2m along z-axis, rotate 23° about y-axis and 27° about z-axis.

$$^{3}T_4 = T_{sph}(r,\beta,\gamma) = Rot(z,\gamma)Rot(y,\beta)Trans(0,0,r)$$

$$= \begin{bmatrix} C\beta.C\gamma & -S\gamma & S\beta.C\gamma & rS\beta.C\gamma \\ C\beta.S\gamma & C\gamma & S\beta.S\gamma & rS\beta.S\gamma \\ -S\beta & 0 & C\beta & rC\beta \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} cos23.cos27 & -sin27 & sin23.cos27 & 0.2sin23.cos27 \\ cos23.sin27 & cos27 & sin23.sin27 & 0.2sin23.sin27 \\ -sin23 & 0 & cos23 & 0.2cos23 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8202 & -0.4540 & 0.3481 & 0.0696 \\ 0.4179 & 0.8910 & 0.1774 & 0.0355 \\ -0.3907 & 0 & 0.9205 & 0.1841 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (12)$$

(e)Point 5

By making axis 1, 4 and 6 as static and overall transformation performs as Cartesian; movement is made 0.17m along y-axis.

$$ {}^{4}T_5 = T_{cart} = \begin{bmatrix} 1 & 0 & 0 & P_x \\ 0 & 1 & 0 & P_y \\ 0 & 0 & 1 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0.17 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{13} $$

$$ {}^{0}T_5 = {}^{0}T_4 \text{ x } {}^{4}T_5 $$

$$ = \begin{bmatrix} 0.2071 & -0.9743 & 0.0879 & -0.0917 \\ 0.8969 & 0.2250 & 0.3807 & 0.9459 \\ -0.3907 & 0 & 0.9205 & 0.9841 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0.17 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.2071 & -0.9743 & 0.0879 & -0.2573 \\ 0.8969 & 0.2250 & 0.3807 & 0.9841 \\ -0.3907 & 0 & 0.9205 & 0.9841 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14} $$

(f)Point 6

By making axis 1, 2 and 6 static and overall transformation performs as spherical; the end effectors orientation angle is moved with the rotation 40° about y-axis and 32° about z-axis.

$$ {}^{5}T_6 = T_{sph}(r,\beta,\gamma) = Rot(z,\gamma)Rot(y,\beta)Trans(0,0,r) $$

$$ = \begin{bmatrix} C\beta.C\gamma & -S\gamma & S\beta.C\gamma & rS\beta.C\gamma \\ C\beta.S\gamma & C\gamma & S\beta.S\gamma & rS\beta.S\gamma \\ -S\beta & 0 & C\beta & rC\beta \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} cos40.cos32 & -sin32 & sin40.cos32 & 0.sin40.cos32 \\ cos40.sin32 & cos32 & sin40.sin32 & 0.sin40.sin32 \\ -sin40 & 0 & cos40 & 0.cos40 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.6496 & -0.5299 & 0.5451 & 0 \\ 0.4059 & 0.8480 & 0.3406 & 0 \\ -0.6428 & 0 & 0.7660 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{15} $$

$$ {}^{0}T_6 = {}^{0}T_5 \text{ x } {}^{5}T_6 $$

$$ = \begin{bmatrix} 0.2071 & -0.9743 & 0.0879 & -0.2573 \\ 0.8969 & 0.2250 & 0.3807 & 0.9841 \\ -0.3907 & 0 & 0.9205 & 0.9841 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.6496 & -0.5299 & 0.5451 & 0 \\ 0.4059 & 0.8480 & 0.3406 & 0 \\ -0.6428 & 0 & 0.7660 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -0.3180 & -0.9360 & -0.1517 & -0.2573 \\ 0.4268 & -0.2845 & 0.8571 & 0.9841 \\ -0.8444 & 0.2070 & 0.4921 & 0.9841 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{16} $$

(g) Point 7

By making axis 2, 4 and 6 static and overall transformation performs as Cartesian; movement is made 0.34m along x-axis.

$$ {}^{6}T_7 = T_{cart} = \begin{bmatrix} 1 & 0 & 0 & P_x \\ 0 & 1 & 0 & P_y \\ 0 & 0 & 1 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0.34 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{17} $$

$$ {}^{0}T_7 = {}^{0}T_6 \text{ x } {}^{6}T_7 $$

$$ = \begin{bmatrix} -0.3180 & -0.9360 & -0.1517 & -0.2573 \\ 0.4268 & -0.2845 & 0.8571 & 0.9841 \\ -0.8444 & 0.2070 & 0.4921 & 0.9841 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0.34 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -0.3180 & -0.9360 & -0.1517 & -0.3655 \\ 0.4268 & -0.2845 & 0.8571 & 1.1293 \\ -0.8444 & 0.2070 & 0.4921 & 0.6970 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{18} $$

(h) Point 8

By making axis 2, 3 and 6 static and overall transformation performs as spherical; the end effectors orientation angle is moved with the rotation -34° about y-axis and -35° about z-axis.

$$ {}^{7}T_8 = T_{sph}(r,\beta,\gamma) = Rot(z,\gamma)Rot(y,\beta)Trans(0,0,r) $$

$$ = \begin{bmatrix} C\beta.C\gamma & -S\gamma & S\beta.C\gamma & rS\beta.C\gamma \\ C\beta.S\gamma & C\gamma & S\beta.S\gamma & rS\beta.S\gamma \\ -S\beta & 0 & C\beta & rC\beta \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} cos(-34).cos(-35) & -sin(-35) & sin(-34).cos(-35) & 0 \\ cos(-34).sin(-35) & cos(-35) & sin(-34).sin(-35) & 0 \\ -sin(-34) & 0 & cos(-34) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.6791 & -0.5736 & -0.4581 & 0 \\ -0.4755 & 0.8192 & 0.3207 & 0 \\ 0.5592 & 0 & 0.8290 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{19} $$

$$ {}^{0}T_8 = {}^{0}T_7 \text{ x } {}^{7}T_8 $$

$$ = \begin{bmatrix} -0.3180 & -0.9360 & -0.1517 & -0.3655 \\ 0.4268 & -0.2845 & 0.8571 & 1.1293 \\ -0.8444 & 0.2070 & 0.4921 & 0.6970 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.6791 & -0.5736 & -0.4581 & 0 \\ -0.4755 & 0.8192 & 0.3207 & 0 \\ 0.5592 & 0 & 0.8290 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.1443 & -0.5844 & -0.2802 & -0.3655 \\ 0.9044 & -0.4779 & 0.4238 & 1.1293 \\ -0.3967 & 0.6540 & 0.8612 & 0.6970 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{20} $$

(i)Point 9

By making axis 1, 4 and 6 static and overall transformation performs as Cartesian; movement is made -0.17m along y-axis.

$$ {}^{8}T_9 = T_{cart} = \begin{bmatrix} 1 & 0 & 0 & P_x \\ 0 & 1 & 0 & P_y \\ 0 & 0 & 1 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -0.17 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{21} $$

$$ {}^{0}T_9 = {}^{0}T_8 \text{ x } {}^{8}T_9 $$

$$ = \begin{bmatrix} 0.1443 & -0.5844 & -0.2802 & -0.3655 \\ 0.9044 & -0.4779 & 0.4238 & 1.1293 \\ -0.3967 & 0.6540 & 0.8612 & 0.6970 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -0.17 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.1443 & -0.5844 & -0.2802 & -0.2661 \\ 0.9044 & -0.4779 & 0.4238 & 1.2105 \\ -0.3967 & 0.6540 & 0.8612 & 0.5858 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{22} $$

(j)Point 10

By making axis 3, 4 and 6 static and overall transformation performs as spherical; movement is made 0.22m along z-axis, rotate -43° about y-axis and 17° about z-axis.

$$ {}^{9}T_{10} = T_{sph}(r,\beta,\gamma) = Rot(z,\gamma)Rot(y,\beta)Trans(0,0,r) $$

$$ = \begin{bmatrix} C\beta.C\gamma & -S\gamma & S\beta.C\gamma & rS\beta.C\gamma \\ C\beta.S\gamma & C\gamma & S\beta.S\gamma & rS\beta.S\gamma \\ -S\beta & 0 & C\beta & rC\beta \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} cos(-43).cos17 & -sin17 & sin(-43).cos17 & 0.22sin(-43).cos17 \\ cos(-43).sin17 & cos17 & sin(-43).sin17 & 0.22sin(-43).sin17 \\ -sin(-43) & 0 & cos23 & 0.22cos(-43) \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.6994 & -0.2924 & 0.6522 & -0.1435 \\ 0.2138 & 0.9563 & -0.1994 & -0.0439 \\ 0.6820 & 0 & 0.9205 & 0.1609 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{23} $$

$$ {}^{0}T_{10} = {}^{0}T_9 \text{ x } {}^{9}T_{10} $$

$$ = \begin{bmatrix} 0.1443 & -0.5844 & -0.2802 & -0.2661 \\ 0.9044 & -0.4779 & 0.4238 & 1.2105 \\ -0.3967 & 0.6540 & 0.8612 & 0.5858 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.6994 & -0.2924 & 0.6522 & -0.1435 \\ 0.2138 & 0.9563 & -0.1994 & -0.0439 \\ 0.6820 & 0 & 0.9205 & 0.1609 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -0.2151 & -0.6011 & -0.0473 & -0.3062 \\ 0.8194 & -0.7215 & 1.0753 & 1.1699 \\ 0.4497 & 0.7414 & 0.4036 & 0.7526 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{24} $$

(k)Point 11

By making axis 1, 4 and 6 static and overall transformation performs as Cartesian; movement is made -0.12m along z-axis.

$$ {}^{2}T_3 = T_{cart} = \begin{bmatrix} 1 & 0 & 0 & P_x \\ 0 & 1 & 0 & P_y \\ 0 & 0 & 1 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -0.12 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (25)$$

$$ {}^{0}T_{11} = {}^{0}T_{10} \times {}^{9}T_{11} $$

$$ = \begin{bmatrix} -0.2151 & -0.6011 & -0.0473 & -0.3062 \\ 0.8194 & -0.7215 & 1.0753 & 1.1699 \\ 0.4497 & 0.7414 & 0.4036 & 0.7526 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -0.12 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ = \begin{bmatrix} -0.2151 & -0.6011 & -0.0473 & -0.3006 \\ 0.8194 & -0.7215 & 1.0753 & 1.0409 \\ 0.4497 & 0.7414 & 0.4036 & 0.7042 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (26)$$

C. *Inverse Kinematics Analysis*

Inverse kinematic analysis is the opposite of the forward kinematic analysis. The corresponding variables of each joint could found with the given location requirement of the end of the manipulator in the given references coordinates system. Inverse kinematic analysis is done by multiplying each inverse matrix of T matrices on the left side of above equation and then equalizing the corresponding elements of the equal matrices of both ends [9]. With inverse kinematic solutions, the value of each joint can be determined in order to place the arm at a desired position and orientation. To decouple the angles, the ${}^{R}T_H$ matrix is routinely premultiplied with the individual A_n^{-1} matrices.

To solve for the angles, we will successively premultiply the two matrices with the A_n^{-1} matrices, starting with A_1^{-1}:

$$ A_1^{-1} \times \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_2 A_3 A_4 A_5 A_6 $$

$$ \begin{bmatrix} n_xC_1 + n_yS_1 & o_xC_1 + o_yS_1 & a_xC_1 + a_yS_1 & P_xC_1 + P_yS_1 - a_1(C_1C_1 + S_1S_1) \\ n_z & o_z & a_z & P_z \\ n_xS_1 - n_yC_1 & o_xS_1 - o_yC_1 & a_xS_1 - a_yC_1 & P_xS_1 - P_yC_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ = \begin{bmatrix} C_{23}(C_4C_5C_6 - S_4C_6) - S_{23}S_5C_6 & C_{23}(-C_4C_5C_6 - S_4C_6) - S_{23}S_5S_6 & C_{23}C_4S_5 + S_{23}C_5 & a_2C_2 \\ S_{23}(C_4C_5C_6 - S_4S_6) - C_{23}S_5C_6 & S_{23}(-C_4C_5C_6 - S_4S_6) - C_{23}S_5S_6 & S_{23}C_4S_5 - C_{23}C_5 & a_2S_2 \\ S_4C_5C_6 + C_4S_6 & -S_4C_5S_6 + C_4C_6 & S_5S_4 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (27)$$

From (2,4) elements,

$$ P_z = a_2S_2 \quad (28)$$

$$ S_2 = P_z/a_2 \quad (29)$$

$$ \boldsymbol{\theta_2 = sin^{-1}(P_z/a_2)} \quad (30)$$

From (3,4) elements,

$$ P_xS_1 - P_yC_1 = d_3 \quad (31)$$

$$ S_1 = \frac{d_3 + P_yC_1}{P_x} \quad (32)$$

From (1,4) elements,

$$ P_xC_1 + P_yS_1 - a_1 = a_2C_2 \quad (33)$$

From equation 32 and 33,

$$ C_1 = \frac{P_x(a_2C_2 + a_1) - P_yd_3}{P_x^2 + P_y^2} \quad (34)$$

$$ \boldsymbol{\theta_1 = cos^{-1}\left(\frac{P_x(a_2C_2 + a_1) - P_yd_3}{P_x^2 + P_y^2}\right)} \quad (35)$$

Next step is to premultiply by the inverse of A_1 through A_3.

$$ A_3^{-1}A_2^{-1}A_1^{-1} \times \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_4A_5A_6 \quad (36)$$

$$ \begin{bmatrix} n_xC_1C_{23} + n_yS_1C_{23} + n_zS_{23} & o_xC_1C_{23} + o_yS_1C_{23} + o_zS_{23} & a_xC_1C_{23} + a_yS_1C_{23} + a_zS_{23} & (P_xC_1 + P_yS_1 - a_1)C_{23} + P_zS_{23} - a_2C_3 \\ n_xS_1 - n_yC_1 & o_xS_1 - o_yC_1 & a_xS_1 - a_yC_1 & P_xS_1 - P_yC_1 - d_3 \\ n_xC_1S_{23} + n_yS_1S_{23} - n_zC_{23} & o_xC_1S_{23} + o_yS_1S_{23} - o_zC_{23} & a_xC_1S_{23} + a_yS_1S_{23} - a_zC_{23} & (P_xC_1 + P_yS_1 - a_1)S_{23} - P_zC_{23} - a_2S_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ = \begin{bmatrix} C_4C_5S_6 - S_4C_6 & -C_4C_5S_6 - S_4C_6 & C_4C_5 & 0 \\ S_4C_5C_6 + C_4S_6 & -S_4C_5S_6 + C_4C_6 & S_4S_5 & 0 \\ -S_5C_6 & S_5S_6 & C_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (37)$$

From element (1,4) and (3,4),

$$ (P_xC_1 + P_yS_1 - a_1)C_{23} + P_zS_{23} - a_2C_3 = 0 \quad (38)$$

$$ (P_xC_1 + P_yS_1 - a_1)s_{23} - P_zC_{23} - a_2S_3 = 0 \quad (39)$$

From equation 38 and 39,

$$ C_{23} = \sqrt{\frac{P_z^2 - a_2^2 + [P_xC_1 + P_yS_1 - a_1]^2}{2P_z^2}} \quad (40)$$

$$ \theta_{23} = cos^{-1}\sqrt{\frac{P_z^2 - a_2^2 + [P_xC_1 + P_yS_1 - a_1]^2}{2P_z^2}} $$

$$ \boldsymbol{\theta_3 = \theta_{23} - \theta_2} \quad (41)$$

From element (1,3) and (2,3),

$$ a_xS_1 - a_yC_1 = S_4S_5 \quad (42)$$

$$ a_xC_1C_{23} + a_yS_1C_{23} + a_yS_1C_{23} + a_zS_{23} = C_4S_5 \quad (43)$$

From equation 42 and 43,

$$ \boldsymbol{\theta_4 = tan^{-1}\left[\frac{a_xS_1 - a_yC_1}{a_xC_1C_{23} + a_yS_1S_{23} + a_zS_{23}}\right]} \quad (44)$$

From element (3,1) and (3,2),

$$ o_xC_1S_{23} + o_yS_1S_{23} - o_zC_{23} = S_5S_6 \quad (45)$$

$$ n_xC_1S_{23} + n_yS_1S_{23} - n_zC_{23} = -S_5S_6 \quad (46)$$

From equation 7 and 8,

$$ \boldsymbol{\theta_6 = tan^{-1}\left[\frac{o_zC_{23} - o_xC_1S_{23} - o_yS_1S_{23}}{n_xC_1S_{23} + n_yS_1S_{23} - n_zC_{23}}\right]} \quad (47)$$

Since,

$$ A_4^{-1}A_3^{-1}A_2^{-1}A_1^{-1} \times \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_5A_6 \quad (48)$$

From element (1, 3) and (2, 3),

$$ a_xC_1C_{23}C_4 + a_yS_1S_{23}C_4 + a_2S_{23}C_4 + a_xS_1S_4 - a_yC_1S_4 = S_5 \quad (49)$$

$$ -a_xS_{23} - a_yS_1S_{23} + a_zC_{23} = -C_5 \quad (50)$$

From equation 9 and 10,

$$ \boldsymbol{\theta_5 = -tan^{-1}\left[\frac{a_xC_1C_{23}C_4 + a_yS_1S_{23}C_4 + a_2S_{23}C_4 + a_xS_1S_4 - a_yC_1S_4 = S_5}{-a_xS_{23} - a_yS_1S_{23} + a_zC_{23}}\right]} \quad (51)$$

As a summary,

$$ \boldsymbol{\theta_2 = sin^{-1}(P_z/a_2)} \quad (52)$$

$$ \boldsymbol{\theta_1 = cos^{-1}\left(\frac{P_x(a_2C_2 + a_1) - P_yd_3}{P_x^2 + P_y^2}\right)} \quad (53)$$

$$\theta_{23} = cos^{-1}\sqrt{\frac{P_z^2 - a_2^2 + [P_x C_1 + P_y S_1 - a_1]^2}{2P_z^2}} \tag{54}$$

$$\theta_3 = \theta_{23} - \theta_2 \tag{55}$$

$$\theta_4 = tan^{-1}\left[\frac{a_x S_1 - a_y C_1}{a_x C_1 C_{23} + a_y S_1 S_{23} + a_z S_{23}}\right] \tag{56}$$

$$\theta_6 = tan^{-1}\left[\frac{o_z C_{23} - o_x C_1 S_{23} - o_y S_1 S_{23}}{n_x C_1 S_{23} + n_y S_1 S_{23} - n_z C_{23}}\right] \tag{57}$$

$$\theta_5 = -tan^{-1}\left[\frac{a_z C_1 C_{23} C_4 + a_y S_1 S_{23} C_4 + a_2 S_{23} C_4 + a_x S_1 S_4 - a_y C_1 S_4 = S_5}{-a_x S_{23} - a_y S_1 S_{23} + a_z C_{23}}\right] \tag{58}$$

D. Total Transformations

$n_x = C_1[C_{23}(C_4C_5C_6 - S_4S_6)-S_{23}S_5C_6] + S_1(S_4C_5C_6 + C_4S_6)$

$n_y = S_1[C_{23}(C_4C_5C_6 - S_4S_6)- S_{23}S_5C_6] - C_1(S_4C_5C_6 + C_4S_6)$

$n_z = S_{23}(C_4C_5C_6 - S_4S_6) + C_{23}S_5C_6$

$o_x = C_1[-C_{23}(C_4C_5C_6+S_4C_6) + S_{23}S_5S_6] + S_1(-S_4S_5S_6 + C_4C_6)$

$o_y = S_1[-C_{23}(C_4C_5C_6+S_4C_6) + S_{23}S_5S_6] - C_1(-S_4C_5S_6 + C_4S_6)$

$o_z = -S_{23} (C_4C_5C_6+S_4C_6) - C_{23}S_5S_6$

$a_x = C_1[C_{23}C_4S_5 + S_{23}C_5] -C_1S_4S_5$

$a_y = S_1[C_{23}C_4S_5 + S_{23}C_5] -C_1S_4S_5$

$a_z = S_{23}C_4S_5 – C_{23}C_5$

$p_x = C_1(C_2a_2 + a_1) + S_1d_3$

$p_y = S_1(C_2a_2 + a_1) - C_1d_3$

$p_z = S_2a_2$

$${}^0T_{11} = {}^0T_1 \text{ x } {}^1T_2 \text{ x } {}^2T_3 \text{ x } {}^3T_4 \text{ x } {}^4T_5 \text{ x } {}^5T_6 \text{ x } {}^6T_7 \text{ x } {}^7T_8 \text{ x } {}^8T_9 \text{ x } {}^9T_{10} \text{ x } {}^{10}T_{11}$$

$$= \begin{bmatrix} -0.2151 & -0.6011 & -0.0473 & -0.3006 \\ 0.8194 & -0.7215 & 1.0753 & 1.0409 \\ 0.4497 & 0.7414 & 0.4036 & 0.7042 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

IV. CONCLUSIONS

Based on relevant theories on robotic arms movement, a forward and an inverse kinematic model are successfully developed with the application of KUKA KR 16-KS robotic arm in a simple welding process in this paper. A simple welding task to weld a block onto a metal sheet is carried out in order to investigate the forward and inverse kinematics models of KR-16KS. A movement flow planning is designed and further developed into the KR-16KS programming. Eleven points of movement are studied for the forward kinematic modeling. A summary of calculation is obtained. A general D-H representation of forward and inverse matrix is obtained. This can be used in each of the welding operation movement based on KUKA KR-16KS robotic arm. A forward kinematic and an inverse kinematic aspect of KUKA KR-16KS is successfully modeled based on a simple welding task.

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