Modeling and Analysis of a 6 DOF Robotic Arm Manipulator

Jamshed Iqbal, Raza ul Islam, and Hamza Khan

Abstract — *The behavior of physical systems in many situations may better be expressed with an analytical model. Robot modeling and analysis essentially involve its kinematics. For robotic manipulators having high Degrees Of Freedom (DOF) with multiple degrees in one or more joints, an analytical solution to the inverse kinematics is probably the most important topic in robot modeling. This paper develops the kinematic models a 6 DOF robotic arm and analyzes its workspace. The proposed model makes it possible to control the manipulator to achieve any reachable position and orientation in an unstructured environment. The forward kinematic model is predicated on Denavit Hartenberg (DH) parametric scheme of robot arm position placement. Given the desired position and orientation of the robot end-effector, the realized inverse kinematics model provides the required corresponding joint angles. The forward kinematic model has been validated using Robotics Toolbox for MATLAB while the inverse kinematic model has been implemented on a real robotic arm. Experimental results demonstrate that using the developed model, the end-effector of robotic arm can point to the desired coordinates within precision of ±0.5cm. The approach presented in this work can also be applicable to solve the kinematics problem of other similar kinds of robot manipulators.*

Key Words — Modeling of robot; Robot kinematics; Robotic system simulation; Analysis of Serial mechanism.

I. Introduction

Analytical prediction of the behavior of physical systems in many key situations is either extremely complicated or even impossible. Driven with the constraints to prototype a physical system, modeling finds enormous motivations to study and investigate the performance of a system.

Modeling a robot involves study of its kinematic behavior. A kinematic model is concerned with the robot's motion without considering forces producing the motions. The kinematics of a robotic arm deals with the study of the geometric and time based properties of the motion and in particular how various links of a robot move with respect to one another and with time. It provides an analytical description of the spatial movements of a robot i.e. a relationship between position and orientation of robot end-effector and its joint variables. The problem of kinematic modeling is usually categorized into two sub-problems. First is the forward or direct kinematics, which is the problem of solving the Cartesian position and orientation of a mechanism, given the knowledge of the kinematic structure and the joint coordinates. The second sub-problem is Inverse Kinematics (IK), which computes the joint variables using the given information of a robot's end-effector position and orientation. In case of serial robotic arms, IK problem is more complex than direct kinematic problem [1].

Kinematic modeling of robots also benefits the industrial automation processes by making them semi-autonomous or even fully autonomous. Because of the task nature and operational environment, the industrial robots are usually composed up of series of rigid links mounted on a base. They operate in a manner similar to that of the human arm. A 6 Degree Of Freedom (DOF) robotic arm manipulator is widely used in the industry. The most common applications of industrial robots include Spot welding, Spraying, Assembling and Manufacturing. Many of these applications actually require accomplishment of pick and place task. Implementation of this task essentially necessitates having the kinematic model of the robotic arm being employed.

In the area of robot modeling and simulation, kinematics is a rigorously researched topic. Scientific community reports various robot modeling and analysis techniques. These are usually based either on line transformation or point transformation, the later being used more commonly for robot modeling. Clothier et al. [2] proposed a geometric model to solve the unknown joint angles required for autonomous positioning of a robotic system. A new method, quaternion algebra, for solution of forward kinematic problem was derived by Sahu et al. [3]. Popovic et al. [4] developed a strategy to analyze the upper extremity movement of the arm while complete body kinematics of a radial symmetrical six-legged robot was presented by Wang et al. [5]. A mathematical approach to analyze kinematics of a humanoid robot was reported in [6]. Cubero [7] proposed an IK model to solve all the joint variables of a serial arm manipulator of any type. This model was based on forward kinematic solution. A

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virtual model robot having forward and inverse kinematic solutions was reported by Kuma in [8].

This paper first presents kinematic model of the robot in Section II. The robot has been modeled for its forward kinematics as well as IK. Section III discusses validation of forward kinematic model using MATLAB toolbox for robotics while Section IV presents the results of workspace analysis of the robotic arm. Implementation of IK model on the real robot is discussed in Section V. Finally Section VI comments on conclusion.

II. KINEMATIC MODEL

The robotic platform used in the present work is a 6 DOF robotic arm manipulator ED7220C developed by ED Corporation, Korea. The robotic arm has been extensively used in research, development and teaching. It is basically a serial manipulator having all joints as revolute. The arm geometrical configuration is made up of waist, shoulder, elbow and wrist in correspondence with the human arm joints (Figure 1). Each of these joints except the wrist has a single DOF. Wrist can move in two planes (roll and pitch), thus making the end-effector more flexible in terms of object manipulation. Constructed in a vertical articulated fashion, the robot offers visual observation of the mechanical behavior of each joint at a glance. The arm is fully-actuated with each DOF achieved by a precise servo motor (DME 38B50G-115) equipped with an optical encoder. The end-effector is a two-state gripper having rubber pads. The built in mechanical safety limits restrict the joint motion in case something in the control algorithm goes wrong. TABLE I lists salient features of the robotic arm ED7220C.

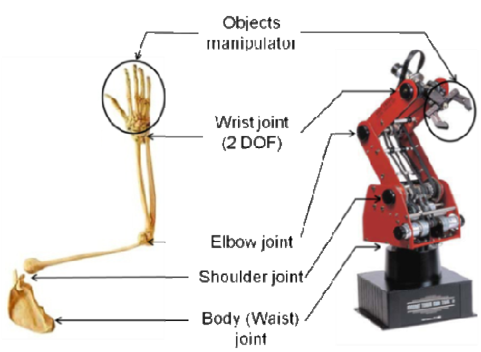


Fig. 1. Joints configurations in ED7220C.

TABLE I.
SALIENT FEATURES OF ED7220C

Feature	Description
Position precision	±0.5mm (approx.)
Movement speed	100mm/s (max.)
Load capacity	1 Kg
Weight	33 Kg
Range Of Motion (ROM)	Wrist - Pitch: 260°, Roll:360° Elbow: 172° Shoulder: 90° Waist: 310°

The overall system (Figure 2) consists of the robot, its controller interfaced with a standard PC and a teaching pendant. The function of the controller (ED-Mark IV) is to provide ports for encoded motors to optically drive the robot. The controller has more than 100 higher level kernel commands that make the platform versatile. The controller can be operated in either of the two modes: host controlled or through teaching pendant. Pendant is the instructional pad for manual commanding the robotic arm. This is used to let the robot learn about any reachable coordinates. The taught points can be saved in the controller for later retrieval and command execution. However, the robot has to be taught every time provided the location of the object to be manipulated is changed. In an attempt to make this robotic arm autonomous, an image-guided robotic system has been conceived and presented in [9].

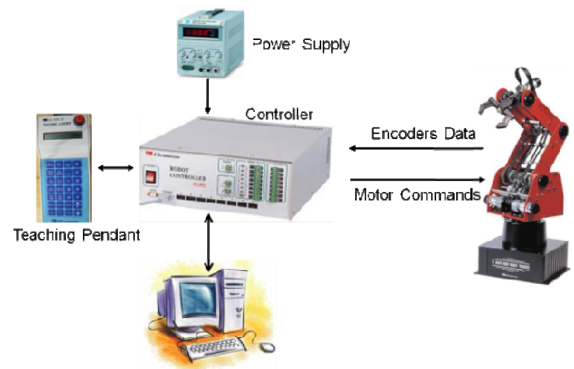


Fig. 2. Arm shown with other system components.

A. Forward Kinematic Model

The study of kinematic problem of a robot can be carried out by different methods. Two commonly used methods are based on Denavit-Hartenberg (DH) parameters and successive screw displacements. Both methods are systematic in nature and more suitable for modeling serial manipulators. Also geometric methods are frequently used by some researchers for the serial manipulators of relatively simple geometry [10]. DH method has been used to develop the kinematic model of the robot in this work because of its versatility and acceptability for modeling of any number of joints and links of a serial manipulator regardless of complexity.

Figure 3 illustrates the simplified kinematic model of the robotic arm in an inverted 'L' pose. The first three joints are used to move the tool point to its desired position, while the last two joints adjust the orientation of the end-effector. Link lengths mentioned in Figure 3 are tabulated in TABLE II.

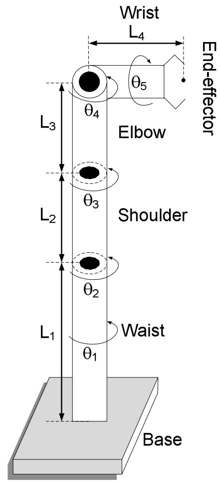


Fig. 3. ED7220C — Kinematic model.

TABLE II.
ED 7220C — LINK LENGTHS

Joint	Waist	Shoulder	Elbow	Wrist
Symbol	L_1	L_2	L_3	L_4
Link Length [mm]	385	220	220	155

DH works with quadruple $\{\alpha_{i-1}, a_{i-1}, d_i, \theta_i\}$ which represents twist angle, link length, link offset and joint angle respectively. Following DH convention, an orthonormal coordinate system has been attached to each link of the manipulator (Figure 4). TABLE III lists DH parameters for the robotic arm ED7220C.

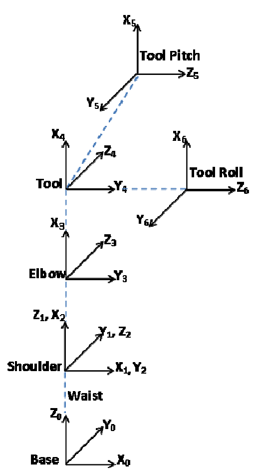


Fig. 4. ED7220C — Coordinate assignment.

TABLE III.
ED 7220C — DH PARAMETERS

Symbol	Joints (i)					
	1	2	3	4	5	6
α_{i-1}	0	-90°	0	0	-90°	0
a_{i-1}	0	0	L_2	L_3	0	0
d_i	L_1	0	0	0	0	L_4
θ_i	θ_1	θ_2 - 90°	θ_3	θ_4	θ_5	0

Using the general form of the transformation matrix for each link (expressing joint i in its previous neighboring joint i-1) derived in [11], the corresponding transformation matrices for each link of the robotic arm have been written. Based on the compound transformation property, these individual transformation matrices when multiplied yield the overall matrix representing the end-effector of the robot in terms of its base (1).

$$ {}^{0}_{6}T = \begin{bmatrix} C_1C_5S_{234} + S_1S_5 & -C_1S_{234}S_5 + S_1C_5 & C_1C_{234} & C_1A \\ -S_1C_5C_{234} - C_1S_5 & S_1C_{234}C_5 + C_1C_5 & S_1C_{234} & S_1A \\ C_{234}C_5 & -C_{234}S_5 & -S_{234} & B \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1) $$

Where

$$ \text{A} = L_2S_2 + L_3S_{23} + L_4C_{234} $$

$$ \text{B} = L_1 + L_2C_2 + L_3C_{23} - L_4S_{234} $$

In (1), the 3X3 matrix comprising of first three rows and first three columns is the rotation while the last column represents the position (x, y, z) of the end-effector w.r.t. base.

B. Inverse Kinematic Model

IK model finds more potential applications in practical robotic systems. IK model computes the joint angles required to achieve the given position and orientation. Not only in robotics, IK finds its importance in other fields like for example 3D games. In contrast to the forward kinematics, IK does not have a unique solution. The solutions which ensure collision-free operation and minimum joint motion are considered more optimum.

Analytical approach has been followed to develop IK model of ED7220C. This approach ensures that for any object within robotic arm workspace (Section IV), the model determines correct joint angles. The first four joint angles i.e. waist (θ_1), shoulder (θ_2), elbow (θ_3) and tool pitch (θ_4) are calculated using this approach while tool roll (θ_5) is directly given by the desired orientation for object manipulation.

Since transformation involves rotation as well as translation, so the general form of the transformation matrix from tool to base is given by (2).

$$ {}^{Base}_{Tool}T = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2) $$

Where the first 3x3 matrix and (p_x, p_y and p_z) representing the rotation and the translation of end-effector w.r.t base of the robot in an IK problem are known.

The developed analytical IK model after intensive mathematical computations yields equations (3), (9), (12) and (13) for the joint angles θ_1, θ_3, θ_2 and θ_4 respectively. These equations express the required joint angles in terms of given coefficients of (2).

$$ \theta_1 = Atan2(p_x, p_y) \quad (3) $$

$$s_{234} = c_1 a_x + s_1 a_y \tag{4}$$

$$c_{234} = a_z \tag{5}$$

$$\theta_{234} = Atan2(s_{234}, c_{234}) \tag{6}$$

$$c_3 = \frac{(c_1 p_x + s_1 p_y + l_4 s_{234})^2 + (p_z - l_1 + l_4 c_{234})^2 - {l_2}^2 - {l_3}^2}{2 l_2 l_3} \tag{7}$$

$$s_3 = \pm\sqrt{1 - {c_3}^2} \tag{8}$$

$$\theta_3 = Atan2(s_3, c_3) \tag{9}$$

$$c_2 = \frac{(c_1 p_x + s_1 p_y + l_4 s_{234})(c_3 l_3 + l_2) - (p_z - l_1 + l_4 c_{234}) s_3 l_3}{(c_3 l_3 + l_2)^2 + {s_3}^2 {l_3}^2} \tag{10}$$

$$s_2 = -\frac{(c_1 p_x + s_1 p_y + l_4 s_{234}) s_3 l_3 + (p_z - l_1 + l_4 c_{234})(c_3 l_3 + l_2)}{(c_3 l_3 + l_2)^2 + {s_3}^2 {l_3}^2} \tag{11}$$

$$\theta_2 = Atan2(s_2, c_2) \tag{12}$$

$$\theta_4 = \theta_{234} - \theta_2 - \theta_3 \tag{13}$$

III. VALIDATION OF FORWARD KINEMATIC MODEL

The forward kinematic model has been validated using Robotics Toolbox for MATLAB. Numerical results together with visual plot of position and orientation of a robot in MATLAB environment gives clear insight of the kinematic behaviour of a robot. Given various angle set as input to the developed forward kinematics model (1) and MATLAB toolbox, corresponding results have been compared and plotted.

Considering joint angle configuration [t_1 t_2 t_3 t_4] as [0 0 0 0]; the position and orientation of the end-effector expressed in the base coordinates, as computed from (1) is

$${}^0_6T = \begin{bmatrix} 0 & 0 & 1 & 155 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 825 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Using the command *'fkine'* in MATLAB Toolbox for Robotics, the same result has been obtained. The corresponding MATLAB plot for this joint configuration is illustrated in Figure 5.

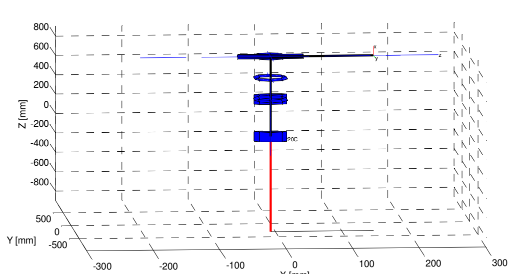


Fig. 5. Plot for joint angle configuration [0 0 0 0].

Considering another joint configuration [t_1 t_2 t_3 t_4] as [90° 90° -90° -90°], the position and orientation of the end-effector expressed in the base coordinates, as computed from model as well as MATLAB Toolbox has been found to be

$${}^0_6T = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 220 \\ 0 & 0 & 1 & 760 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Figure 6 shows MATLAB plot for this joint configuration.

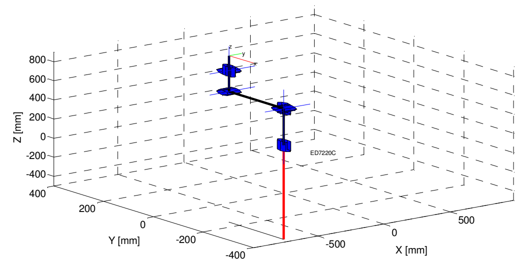


Fig. 6. Plot for joint angle configuration [90° 90° -90° -90°].

Figure 7 shows the plot for joint angle configuration [t_1 t_2 t_3 t_4] as [0 90° 90° 0] with the forward transformation as

$${}^0_6T = \begin{bmatrix} 0 & 0 & -1 & 65 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 165 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

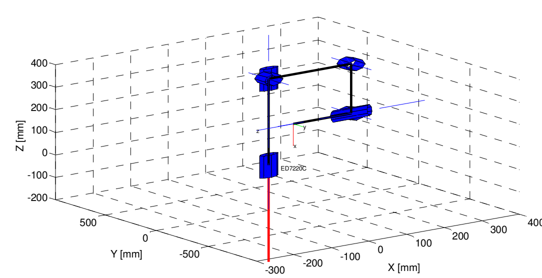


Fig.7. Plot for joint angle configuration [0 90° 90° 0].

IV. WORKSPACE ANALYSIS

With technological advancements in robotics and rapid increase in the trend to employ robots in various applications, the workspace of a robot has also become a primary performance parameter in addition to its speed, accuracy and weight. The workspace of a robot, also termed as its work envelope, actually expresses a robot's ability to reach specific area. Given the information about Range Of Motion (ROM) of joints of a robot and length of its links, workspace can be determined. The ROM of ED7220C joints are mentioned in TABLE I while the link lengths are given in TABLE II.

With these link lengths and ROM information of each joint of the robotic arm, the robot workspace has been found mathematically using equation (1). Figures 8 and 9 illustrate the robot workspace in XY and XZ coordinates respectively. As shown in Figure 8, the arm has a manipulation ability inside a circular radius of 580 mm. ROM constraints in the body joint restricts the robot functionality in the region shown as 'V' in the Figure.

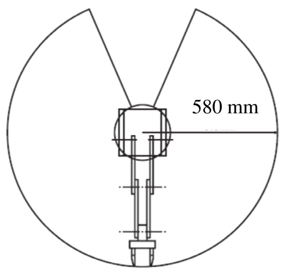


Fig. 8. Workspace in XY.

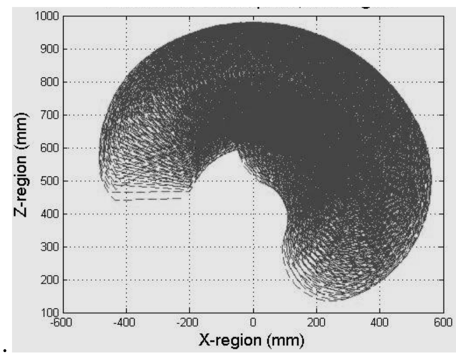


Fig. 9. Workspace in XZ.

The corresponding 3D MATLAB plot is illustrated in Figure 10.

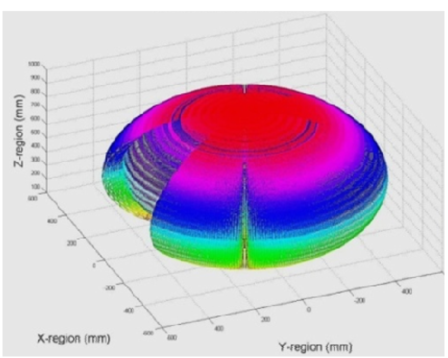


Fig. 10. 3D Workspace.

V. Implementation Of IK Model on Real Arm

Write IK model implementation here. The IK model has been implemented on the real robotic arm manipulator. An object has been placed at a known position and orientation. With this known information from a user, the developed algorithm first checks if the object lies inside the robot workspace as demonstrated in Section IV. If the object is outside the work envelope, the algorithm terminates after prompting user. Otherwise IK model computes the required joint angles pointing the end-effector as per given position and orientation. These joint angles are then mapped to low-level encoder ticks. Finally the Kernel based instructions in the program execute the command by moving the motors as per mapped encoder ticks. The flow chart of model implementation is presented in Figure 11.

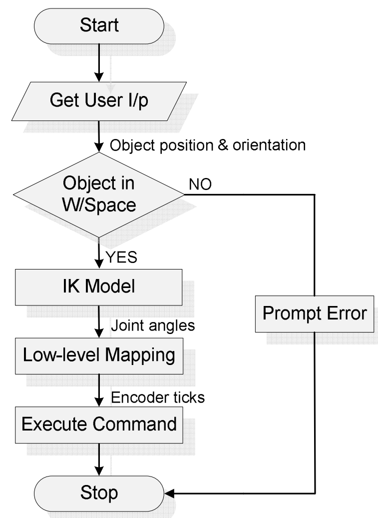
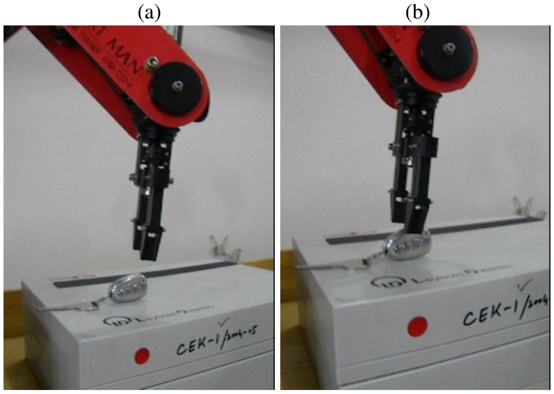


Fig. 11. IK model implementation.

To implement the developed IK model on the robotic arm, it has been ensured that the object (a car key with a key-chain) lies within the robotic arm workspace. For this purpose, the platform on which the object has been placed is raised (in height z) by placing two blocks as shown in Figure 12. The task chosen is basically picking an object from one location and placing it onto another. Both the source and destination position and orientation have been given as an input.

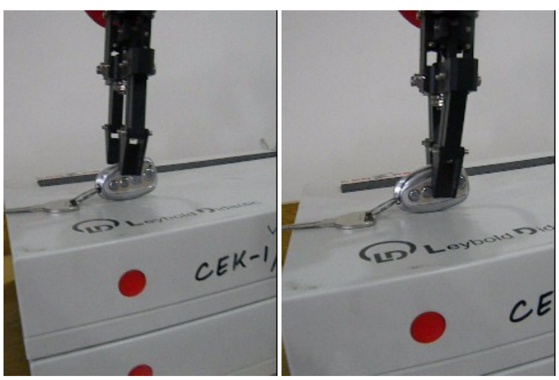
Figure 12a illustrates the arm at its 'home' position (with all encoder values as zero). Based on the object coordinates given by the user, the robot moves as per computed joint angles (by IK model). Figure 12(b-d) shows the motion of the robot toward the object. After reaching the target location, the gripper of the robot closes ultimately grasping the object. Sequence of pick-up of the object is shown in Figure 13(a-b). The robot then moves (Figure 14a-b) toward destination point, whose coordinates have also been taught by the user. The destination point should also lie inside the operational workspace of the robot. After reaching that location, the robot drops the object (Figure 15a-d) and then finds its way back to the home position (Figure 16a-b).

(a) (b)



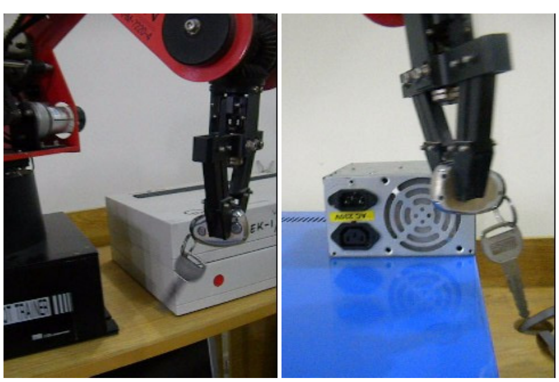
(c) (d)

Fig. 12. Moving from ‘Home’ position to the object.



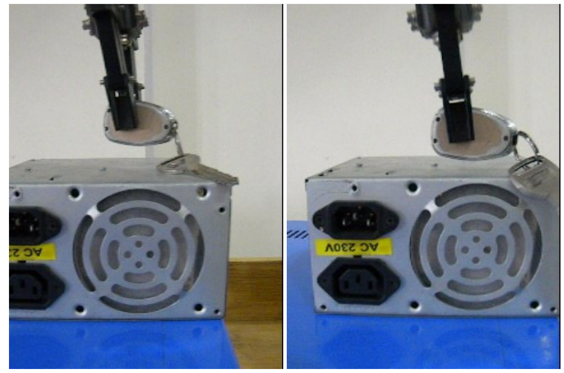
(a) (b)

Fig. 13. Picking the object.

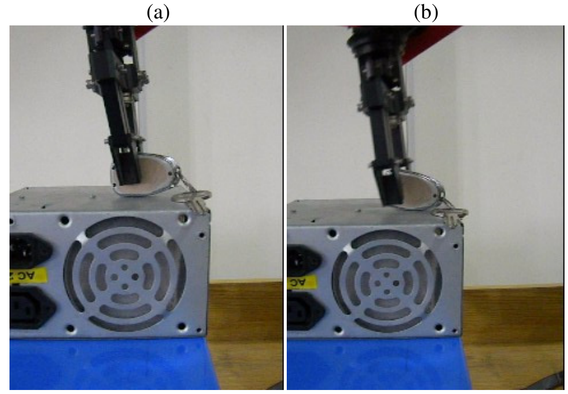


(a) (b)

Fig. 14. Moving to destination position.

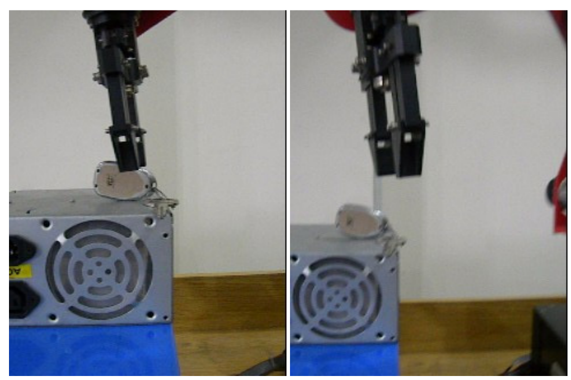


(a) (b)



(c) (d)

Fig. 15. Placing the object.



(a) (b)

Fig. 16. Moving back to ‘Home’ position.

VI. Conclusion

A widely used 6 DOF robotic arm manipulator, ED7220C has been kinematically modeled followed by the analysis of its workspace. Forward kinematic model has been validated using MATLAB. The results from the derived forward kinematic model match exactly with that of MATLAB. The IK model of the robot has also provided correct joint angles to move the arm gripper to any position and orientation within its workspace. The IK model has been implemented on the real robotic platform. Results obtained from the model have been compared with the actual performance of the robot in accomplishing a task e.g. pick and place. It has been found that with the joint angles computed by IK model, the robot achieves position precision within ±0.5cm. This little deviation is because of many reasons namely, reported platform precision (±0.5mm as mentioned in TABLE I), mechanical coupling of the joints, non-linearity in mapping angles to low-level encoder ticks. The strategy presented in this paper may also be used to model and analyze other 6 DOF robotic arms.

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REFERENCES

[1] Mark S., Seth H. and Vidyasagar M., "Robot modeling and control", John Wiley & Sons, 2006.

[2] Clothier K.E. and Shang Y, “A geometric approach for robotic arm kinematics with hardware design, electrical design and implementation”, *Journal of Robotics*, Article ID 984823, Vol. 2010.

[3] Sahu S., Biswal B. and Subudhi B., “A novel method for representing robot kinematics using quaternion theory”, *IEEE Conference on Computational Intelligence, Control and Computer Vision in Robotics & Automation*, pp. 76-82, 2008.

[4] Popovic N., Williams S, Schmitz-Rode T., Rau G., Disselhorst-Klug C., “Robot-based methodology for a kinematic and kinetic analysis of unconstrained, but reproducible upper extremity movement”, *Journal of Biomechanics*, Vol. 42 No. 10, 2009.

[5] Wang Z., Xilun D., Alberto R. and Alessandro G., “Mobility analysis of the typical gait of a radial symmetrical six-legged robot”, *Mechatronics*, Vol. 21, No. 7, pp. 1133-1146, 2011.

[6] Man C., Fan X., Li C. and Zhao Z., “Kinematics analysis based on screw theory of a humanoid robot”, *Journal of China University of Mining & Technology*, Vol. 17 No. 1, pp. 49–52, 2007.

[7] Cubero S., “An inverse kinematics method for controlling all types of serial-link robot arms, Mechatronics and Machine Vision in Practice, Springer-Verlag, Berlin, pp. 217-232.

[8] Kuma R., Kalra P and Prakash N., “A virtual RV-M1 robot system”, *Robotics and Computer-Integrated Manufacturing*, Vol. 26 No. 6, pp. 994-1000, 2011.

[9] Raza ul Islam, J. Iqbal, S. Manzoor, A. Khalid and S. Khan, “An autonomous image-guided robotic system simulating industrial applications”, *7th IEEE International Conference on System of Systems Engineering* (SoSE), Genova, Italy, pp. 314-319, 2012.

[10] Tsai L.W., “Robot Analysis: The mechanics of serial and parallel manipulators”, John Wiley & Sons, Feb. 1999.

[11] J. J. Craig, “Introduction to robotics: Mechanics and control”, Prentice Hall, 2nd edition, pp.84.

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