DATA INTELLIGENCE APPLICATIONS
PRICING & ADVERTISING

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ABSTRACT

The goal of the project is to model a scenario in which a seller exploits advertising tools in order to attract more and more users to its website, thus increasing the number of possible buyers.

The seller has to learn simultaneously the conversion rate and the number of users the advertising tools can attract.

In this report, we walk through the description of the specific scenario we have studied and the definition of the algorithm design choices we adopted in order to reach our goal. Then the achieved experimental results will be presented via some useful plots and a final conclusion summarizing the whole work.

Contents

Chapter 1

Introduction: Scenario Setup

We have studied a possible real world scenario in which a seller wants to increase the number of possible buyers by exploiting advertising tools.

The product we have considered is a particular pair of shoes which has a production cost (without loss of generality we can assume that the production cost is null) and a sell price.

The analysed campaign consists of three sub-campaigns, each with a different ad to advertise the product and each targeting a different class of users. Each class is defined by the values of pre-defined features.

The feature space we have considered is characterized by two binary features:

- *Age<30* or *Age>30*
- *Profession* that can be either *student* or *worker*

So according to the values of the above described features, we can distinguish among the following classes of users:

- *Elagant*: a worker with age>30
- *Casual*: a student with age<30
- *Sport*: a worker with age<30

Chapter 2

Advertising: Clicks Maximization

In this section we focus on the advertising and more precisely on the clicks maximization problem.
The goal of this part is to optimize the budget allocation over the three sub-campaigns in order to maximize the total number of clicks we can get.
In particular given the assignment 8 we have designed a combinatorial bandit algorithm for addreassing this task.
In the following chapter we are going to:

- describe the environment setup;
- explain the algorithm design choice;
- comment the obtained results.

2.1 Environment

The **Environment** returns the reward associated to each sub-campaign, by iterating over the days for the entire time horizon.
In our setting, it computes the optimal number of clicks knowing the real functions, one for each sub-campaign, generating the number of clicks given a bid value ***n(x)***:

$$n(x) = c_M * (v_M - e^{-\alpha x})$$

where:

- c_M : is the maximum number of clicks the considered sub-campaign can reach in a day;
- v_M and α : are values associated to each sub-campaign defining the actual bidding curve.

The Environment, given the index of the bid chosen by the Learner, returns the collection of rewards associated to each sub-campaign in a completely transparant way to the learning algorithm.
How the learner works and its implementation will be described in the following section.

2.2 Algorithm Design Choices

This section deals with the description of the algorithm design choices we adopted and it also contains the most relevant reference to the code.
In the class *BiddingEnvironment.py*, which extends the class *Environment.py* we have defined the actual environment we are working in, described in section before, with the definition of the bids space and the three curves *n(x)*.
In order to find the best budget allocation over the three sub-campaigns able to maximize the total number of clicks, we have designed a combinatorial bandit algorithm.
In the first phase of the algorithm, we learn the model of each sub-campaign from the observation we get. To do that in *GP_Learner.py* we have used a *Gaussian process regressor*.
Afterward, the model of each sub-campaign is updated using those observations and the GP will reduce the uncertainty

of its estimation. In this way, for each new collected sample, the function estimated by the GP approaches to the real function.

In order to properly used a GP regressor, we had to normalize the data. The bid space was already defined in range [0,1] and so the input variable has not to be normalized. So, we only needed to normalize the target and we have done it in the construction of the gaussian process.
A Gaussian proccess is completely defined by its mean and its covariance.Since we don't have any prior information we assumed to have zero mean and the covariance given by the squared exponential kernel function **k(x,x')**:

$$k(x, x') = \theta^2 e^{-\frac{(x-x')^2}{2l^2}}$$

where:

- l :is the *lengthscale*;
- θ: is the *scale factor*

The optimal value of the two hyperparameters have been found by maximiziation of the marginal likelihood during the fit proccess.

In the second phase of the algorithm we have used the values from the learned model to solve the problem of finding the best budget allocation to be set for the current day.
In the *Optimizer.py* class we have implemented a modified version of the dynamic programming algorithm used for solving the knapsack problem.
More precisely,we have used a matrix in which each row represents the fact that at each step a new sub-campaign enters the problem, while for the columns we have discretized the whole budget in 20 possible uniformly distributed combinations of the budget allocation.
Each cell of the matrix contains the value of the best allocation for the considered row and column. The result is given by the maximization of the sum of the values provided by the best solution of the problem solved in the previous row (i.e. without considering the new entered sub-campaign) and the value of the new considered sub-campaign (considered singularly) s.t. the daily budget over the three sub-campaigns sums to the total daily budget.
Once we have filled the entire table, we have the best solution in the last row, i.e. when all the 3 sub-campaigns are considered.
In figure 2.1 we can see, for each sub-campaign, the real function generating the number of clicks n(x), the function learned by GP regressor and its associated uncertainty.

2.3 Performance evaluation

In order to evaluate the performance of the implemented algorithm, we have computed the *cumulative regret* as the difference between the expected reward of the *Clairvoyant algorithm* and the expected reward of our combinatorial bandit algorithm.
In 2.2 the plot of the regret we obtain:

Figure 2.1: Functions generating the number of clicks

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Figure 2.2: Cumulative regret of the combinatorial bandit algorithm

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Chapter 3

Advertising: Handling Abrupt Phases

The object of this section is the design a sliding-window combinatorial bandit algorithm for optimizing the budget allocation over the three sub-campaigns in order to maximize the total number of clicks, in the case in which there are the three abrupt phases.
For addressing this assignment 8 we started from the simplified case of one single phase solved in the previous section and we extended it in order to take in consideration the more general scenario of multiple phases.

3.1 Sliding window mechanism

For this purpose we have implemented the *AbruptBiddingEnvironment.py* class which extends the *BiddingEnvironment.py* class. This class works in a scenario of multiple abrupt phases by returning, for each sub-campaign, the reward of a given pulled arm, depending on the phase we are in.

In this case, the functions generating the number of clicks given a bid value can change dynamically, according the phase we are in. The curves differ from the previous definitions only for the dependence to the phase we are in. As follows the mathematical defintion of the three curve functions:

$$n(x) = c_M[phase] * (v_M[phase] - e^{-\alpha x})$$

In order to learn the three curves in the case of multiple abrupt phases, we have implemented the *DynamicLerner.py* class as an extension of the standard GP_Learner.
It implements a *sliding window* mechanism in which we pull a new arm and add the collected rewards until the length of the window is reached. When the window is full, for each new pulled arm we get, we delete the last recent value and add the new one to the collected rewards.

3.2 Changes Detection mechanism

For solving this problem we have implemented an innovative method which is based on the concept of the *Statistical test*.
We recall that a statistical test is a procedure for deciding whether a hypothesis about a quantitative feature of a population is true or false. Then for testing a hypothesis, we draw a random sample and calculate an appropriate statistic on its items. If, in doing so, we obtain a value of the statistic that would occur rarely when the hypothesis is true, we would have reason to reject the hypothesis.
In our scenario, to detect changes, we need to compare the value of the new drawn sample with the distribution of the points belonging to the same arm in order to decide whether the hypothesis H_0,that the new drawn point belongs to that distribution, is true.
Let us suppose to draw a sample whose value is x_0 and let μ σ^2 be respectively the average and the standard deviation of the the distribution of the points belonging to the pulled arm. If the point doesn't belong to that distribution the hypothesis test should reject H_0, with a *confidence interval of 99%* So we have:

$$\mathbb{P}\left(\frac{|x_0-\mu|}{\sqrt{\sigma^2}} > h\right) = 0.01$$

We have that the critical z-score when using a 99% confidence level are ± 2.58 standard deviations.

In the *DLChangeDetect.py* class, we have implemented such change detector. When an arm is pulled more than '**min_len**' [1] times, for each pull we run the statistical test. When the test reject H_0, it means that the point doesn't belong to that distribution and so it means that it is changed. In this case we reset the arm.

3.3 Performance evaluation

In Figure 3.1 a comparison of the performance of three algorithms [2] is shown. As before, for the performance evaluations we have computed the *cumulative regret* as the difference between the expected reward of the *Clairvoyant algorithm* and the expected reward of the implemented algorithms.

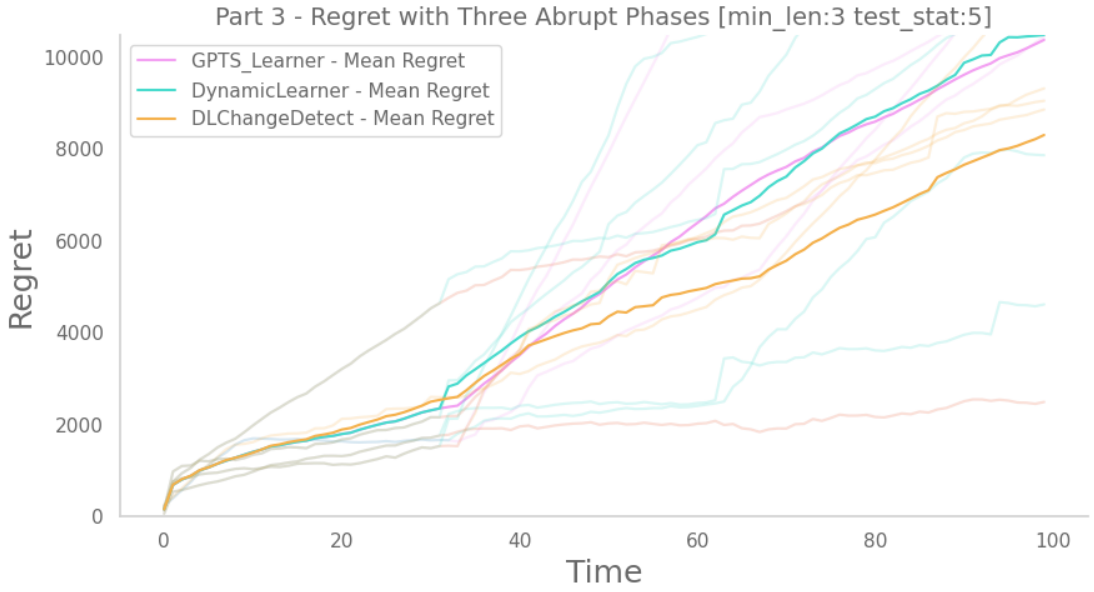

Figure 3.1: Cumulative regret in scenario with multiple abrupt phases

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[1] min_len indicates the minimum number of data we considering in an arm

[2] Combinatorial bandit algorithm without sliding window, Combinatorial bandit algorithm with sliding window and Change detection implementation

Chapter 4

Pricing: Learning the Best Price

In this chapter we are going to focus our attention on the pricing and on the optimal price, that we can propose to our customers, to maximize the total revenue.

In common scenarios, in which there is a seller that has to sell products, the retail price is fixed without exploiting information from the buyers.
Sometimes, in physical markets, the customers have the possibility to negotiate and a skillful seller can maximize his revenue keeping the price as high as possible. This is a common practice, but experience is needed, to understand the limit of the buyers, and moreover it requires time, loosing the possibility to serve other clients.
In e-commerce scenarios, we can do better exploiting information from the users, from the big data and from the software automation. In particular, given the assignments 8, we are going to design an algorithm being able to learn the optimal price for our product, to maximize the total revenue.

In the following steps, we are going to:

- define the demand curves of the users;
- describe the environment setup;
- explain the algorithm that has been chosen; and
- comment the results obtained in the end of the campaign.

4.1 Demand Curves

In the economic literature is defined the concept of **demand curve**. It is a function that maps the price to a certain probability. In other words, given a price, the function returns the probability that the product will be sold. Therefore, if we plot the demand curve, we'll obtain on the x-axis the admissible prices and on the y-axis the probability (between 0 and 1).

In our simulated scenario, we defined three classes of users with the corresponding demand curves. The shapes of the curves are motivated by the intrinsic behaviors of the classes:

- *Elegant*: they are rich, thus they will buy the product with high probability whatever is the proposed price;
- *Casual*: they are students, thus they are able to buy the product only if the price is relatively low; and
- *Sports*: they are workers, but without a particular interest on the shoes, therefore the price is relatively irrelevant and the probability is low.

In figure 4.1 there are the curves designed following the above descriptions. [1]

In the same figure, the *aggregate* curve has been reported (colored in *orange*), it represents the mean between the other curves. (It will be useful in the next subsections.)
Moreover, points representing the theoretical optimum are reported. The area below them is maximum w.r.t. the curves.

[1] A tool as been implemented to draw the curves manually. It requires few points and it performs polynomial regression to augment the sampling definition. It can be found in the attached code.

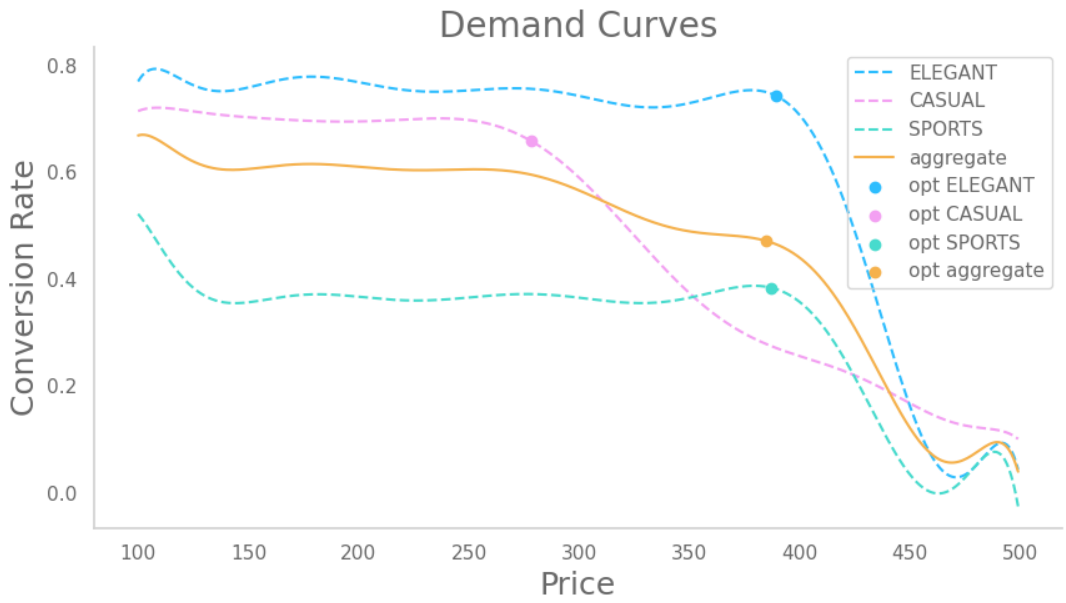


Figure 4.1: **Demand Curves**.
They are used to model the probability (*conversion rate*) that a certain class, of customers, will buy or not the product w.r.t. a proposed price.
The dashed lines represent the demand curves of the corresponding classes: *blue* for the *Elegant* class; *pink* for the *Casual* class; and *light green* for the *Sports* class.
The *orange* curve is the mean between the others and it is used to study the *aggregate* model.
The points are displayed to show the optimal price w.r.t. the conversion rate: the area below them is maximum.

4.2 Environment

In our implementation, the environment is a black box from the point of view of the learning algorithm. It is used to iterate over the days, for the entire duration of the campaign. It schedules the time and the number of users reaching our theoretic website.

Every user is randomly taken from the three classes and the class is not communicated to the learner. The environment, knowing the original class of the users, computes the optimal revenue and returns the effective reward obtained by the learner.

How the learner will handle these information will be explained in the next subsections.

4.3 Learner Algorithm Selection

This context of application can be solved in different ways, but the most common choice is to use a *multi-armed bandit (MAB)* algorithm.

We mainly tested two algorithms:

- **Thompson Sampling (TS)** algorithm; and
- **Thompson Sampling with Sliding Window (SWTS)** algorithm.

We tested both the algorithms during the implementation phase, but since the SWTS is more sensible to the window size, and finding a good one requires a lot of testing, we kept the TS as final choice.
This decision has been motivated by the fact that there aren't abrupt phases, thus there is no reason to "forgot the past" as done by SWTS.

4.4 Experiments

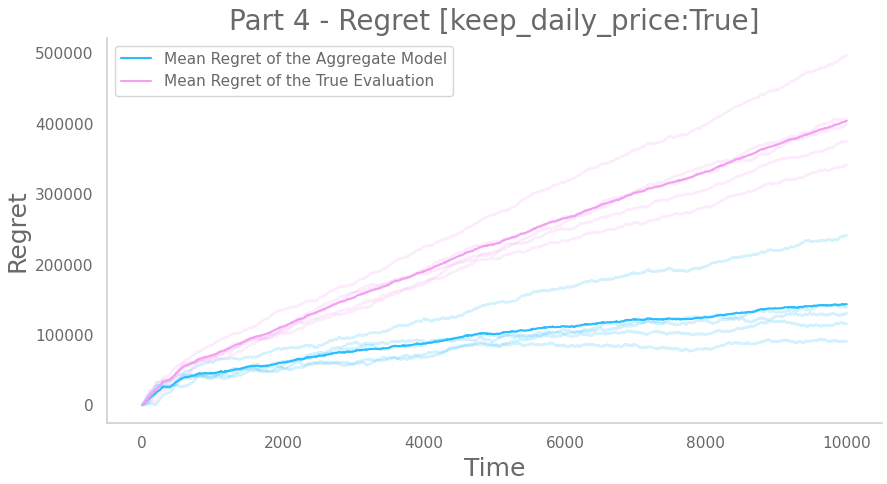
Summarizing, we have to learn the best price to propose, to the users, to maximize the total revenue.
We have discussed about the environment, that generates incoming users and allows us to iterate over the days of the campaign, and finally, we presented the learner adopted.
At this point we can explain how the tests have been initialized.

The experiments has been configured in two ways:

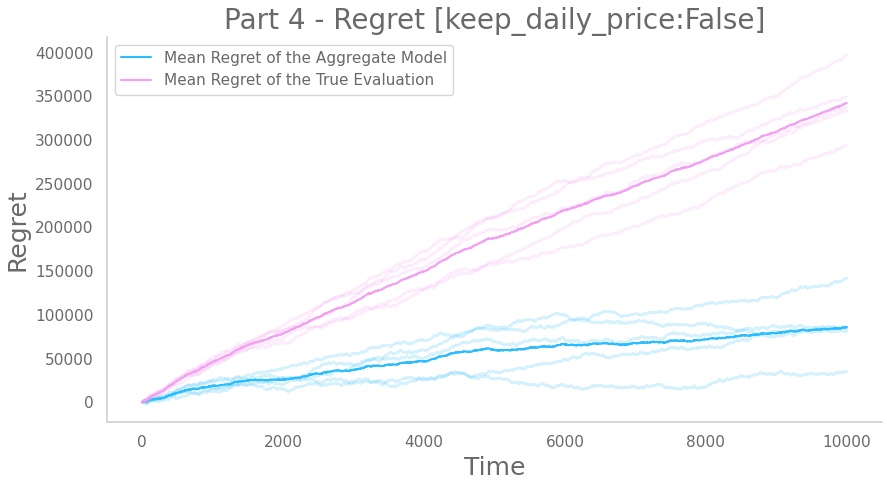
- keeping a fixed daily price: we pulled an arm only for the first user of the day; for the other users we proposed always the same price. This means that for the entire day we pull and update always the same arm of the TS;
- one price for each user: maybe, this is an unpractical option in a real world scenario, but it allows to obtain a total revenue outperforming the previous configuration.

In the end of the tests, the figure 4.2 reports the obtained regrets. The sub figure *(a)* for the first configuration and the sub figure *(b)* for the second.

We plotted two regrets computed with two different optimal revenues. The regret called **Mean Regret of the Aggregate Model** has been computed using, as optimal, the optimal point taken from the aggregate model. This is an optimistic regret because it doesn't take into consideration the information about the specific class of the users. In other words, the learner and the clairvoyant algorithm don't know the class of the customers. The learner is learning the aggregate model, thus this comparison seams fair, but
since the clairvoyant knows the original class of the users, we introduced the so called **Mean Regret of the True Evaluation**. Obviously, from the point of view of the learner, this is a pessimistic computation since it is unable to infer more information from the input data.
Anyway, these two ways, to compute the regrets, will be useful in the next chapter, since the learner will be able to infer more information from the data, creating contexts to exploit a finer profiling.
You will notice how the two regrets will be pushed down and the regret on the True Evaluation will be more similar to the regret of the Aggregate Model obtained here, proving that a finer profiling improves drastically the performances.



(a) Regret obtained proposing to the customers the same price for the entire day.



(b) Regret obtained proposing to the customers different prices for each visit. This method achieves better performances since the learner collects more data during the campaign.

Figure 4.2: Comparison between the regrets obtained during the campaign.
The **Mean Regret of the Aggregate Model** (colored in *blue*) is the regret computed keeping, as optimal, the area under the optimal point of the aggregate curve 4.1. In other words, both the learner and the clairvoyant don't know the class of the customers. This is an optimistic measure of the regret.
The **Mean Regret of the True Evaluation** (colored in *pink*) is the regret obtained computing the optimal value exploiting the original class of the customers.

Chapter 5

Pricing: Context Generation

In this part the assignment was to create a context generation algorithm for the pricing when each sub campaign had a fixed budget allocated to it. Contrary to the previous part, in which we had a single price for all three sub campaigns, here it is possible to have different prices for each context.

5.1 What is a Context?

The idea in this type of problems is that each class of users is identified by a single subset of features. In real problems we have a lot of features and so we can identify many classes of users. Context are useful because we can group these classes together based on the values of their features and we can use a price for each single context, instead of one for each single class, which might be unfeasible if the feature space is very large.

5.2 Our Approach

When we first approached the problem, we used an algorithm similar to that presented in class, with a tree structure. The idea is that we collect data for some time using the aggregate model of the previous part, then we use this data to make predictions. At the end of the week the algorithm chooses the most promising feature (the one with the highest expected reward) and splits accordingly, creating a new binary subtree, with one context for each leaf node of the tree. This process is repeated using the newly generated contexts instead of the ones used in the previous week, until it is not useful to split anymore (the expected reward is lower than without splitting or there are no more features).

However our problem has very little features (only 2) with very limited values (both are binary), so we chose a different approach. We always start with the aggregate model to collect data, but when the time to split arrives, we evaluate all possible partitions of our feature space, either obtaining again the aggregate model, splitting according to only one feature or splitting according to both features, thus producing one, two or three different contexts (it would be four but we only have three classes).

The graphs above measure the regret with respect to two different models: the Aggregate Model (we have only one price for each class) and the True Evalutation (where we use different prices for each class). As we can see, the algorithm performs very well with respect to the Aggregate Model, obtaining better results. On the other hand, the True Evaluation model performs better than the algorithm in the first weeks, but as time goes on the difference becomes smaller and smaller, as the graph grows more and more slowly.

Chapter 6

Adverting and Pricing: Combining the Algorithms

Design an optimization algorithm combining the allocation of budget and the pricing when the seller a priori knows that every subcampaign is associated with a different context and charges a different price for every context. Suggestion: the value per click to use in the knapsack-like problem depends on the pricing, that depends on the number of users of a specific class interested in buying the product. Notice that the two problems, namely, pricing and advertising, can be decomposed since each subcampaign targets a single class of users, thus allowing the computation of the value per click of a campaign only on the basis of the number of clicks generated by that subcampaign. Plot the cumulative regret when the algorithm learns both the conversion rate curves and the performance of the advertising subcampaigns.

Chapter 7

Advertising and Pricing: Fixed Prices

Do the same of Step 6 under the constraint that the seller charges a unique price to all the classes of users. Suggestion: for every possible price, fix this price and repeat the algorithm used in Step 6. Plot the cumulative regret when the algorithm learns both the conversion rate curves and the performance of the advertising subcampaigns.

Chapter 8

Assignments

Part 2: *Design a combinatorial bandit algorithm to optimize the budget allocation over the three subcampaigns to maximize the total number of clicks when, for simplicity, there is only one phase. Plot the cumulative regret.*

Part 2: *Design a sliding-window combinatorial bandit algorithm for the case, instead, in which there are the three phases aforementioned. Plot the cumulative regret and compare it with the cumulative regret that a non-sliding-window algorithm would obtain.*

Part 4: *Design a learning algorithm for pricing when the users that will buy the product are those that have clicked on the ads. Assume that the allocation of the budget over the three sub campaigns is fixed and there is only one phase (make this assumption also in the next steps). Plot the cumulative regret.*