Accurate Cascade Graphic Equalizer

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Abstract—A graphic equalizer is a high-order filter controlling the gain of several frequency bands. For good accuracy, graphic equalizers consisting of cascaded IIR filters have been of very high order. A previously proposed parallel graphic equalizer entailing twice as many second-order filter sections as there are bands can have a maximum approximation error of less than 1 dB, but its design is complicated. This letter proposes a cascade graphic equalizer having an accuracy comparable to the best parallel graphic equalizer, although only one second-order section is assigned per command gain. A key idea is to use band filters whose interaction with the two neighboring filters at their center frequency is exactly controlled. The filter gains are obtained using the least-squares method with one iteration step, which involves linear interpolation of the target gain vector, inversion of a square matrix, and a few matrix multiplications. The proposed method is compared with previous designs and is shown to be the most accurate one. The new graphic equalizer is widely useful for audio and music processing.

Index Terms—Audio systems, equalizers, IIR filters.

I. INTRODUCTION

GRAPHIC equalization refers to filtering in which the gain at certain frequency bands can be controlled [1]. Graphic equalizers (EQ) are used in numerous audio applications, such as in music production [2] and in sound reproduction [3], [4], [1], [5]. This letter proposes a method to design a cascade graphic EQ having a magnitude frequency response which meets the command gain values closely.

A graphic EQ can be realized either in parallel [6], [7], [8], [9], [10], [1] or in cascade [6], [11], [12], [13], [1] form. Recently, a design method for a parallel graphic EQ having great precision and a maximum approximation error of less than 1 dB was proposed [10]. Unfortunately, the high-precision parallel EQ is difficult to design, because a high-resolution target response must be interpolated between the command gain settings [14], and a phase response must also be estimated for the target response.

In principle, the parallel and cascade EQs could be equivalent: using the same filter order, obtaining a very similar frequency response as well as accuracy of approximation should be possible. Additionally, for the magnitude-only design, such as the graphic equalization problem, the cascade filter design is expected to be easier than the parallel filter design, since there is no need to consider the phase of the band filters [1].

The main obstacle in graphic EQ design is the interaction between the band filters [15], [16], [17], [1]. High-order IIR filters can be used for each band to make the neighboring bands

independent of each other, but this makes the overall cascade graphic EQ a very large filter [12], [13]. Some researchers have also proposed using intermediate filters between the band filters to better cope with the interaction [15], [18]. This approach was used in the parallel filter design [10].

However, previous cascade graphic EQ designs proposed by Abel and Berners [19] and recently by Oliver and Jot [17] have reached quite good accuracy by cascading second-order filters and accounting for the interaction between the filters by solving a set of linear equations. In principle, the dB gain at each center frequency is the sum of the dB gains of all filters at that frequency. Oliver and Jot reported that the worst-case error of their design is slightly more than 2 dB [17].

Inspired by this earlier work, this letter develops an accurate design method for a cascade graphic EQ. One second-order IIR filter is set at each command frequency. The design is then based on the knowledge of the shape of each second-order filter's amplitude response on the dB scale. In addition to the center frequencies, an extra point between two adjacent center frequencies is included in the design.

This letter is organized as follows. Sec. II describes an earlier magnitude-only design method for a cascade graphic EQ, which cannot quite reach a good accuracy. Sec. III introduces new ideas which in combination lead to the accurate design method proposed in this letter. Sec. IV compares the proposed method against three previous designs in terms of approximation error, complexity of design, and computational cost. Sec. V concludes this letter.

II. INTERACTION-MATRIX-BASED DESIGN

The frequency response of a cascade graphic EQ is the product of M equalizing filter responses:

$$H_C(e^{j\omega T_s}) = G_0 \prod_{m=1}^{M} H_m(e^{j\omega T_s}), \tag{1}$$

where G_0 is a gain factor (in this work, we set $G_0 = 1$), $H_m(e^{j\omega T_s})$ are the frequency responses of equalizing filters ($m = 1, 2, 3,... M$), ω is the radial frequency, and $T_s = 1/f_s$ is the sample interval, when the sample rate is f_s. The corresponding amplitude response on the dB scale, A_C, can be written, using basic logarithmic rules, as a sum of individual filter responses:

$$A_C(e^{j\omega T_s}) = g_0 + \sum_{m=1}^{M} A_m(e^{j\omega T_s}), \tag{2}$$

where $g_0 = 20\log(G_0)$ and $A_m(e^{j\omega T_s}) = 20\log(|H_m(e^{j\omega T_s})|)$.

Abel and Berners [19] and Oliver and Jot [17] have observed that the amplitude responses $A_m(e^{j\omega T_s})$ of the individual equalizing filters are very self-similar on the dB scale. This leads to a design principle in which the samples taken from

the dB amplitude responses are used as basis functions that are weighted by their respective command gain G_m [19], [17], [1].

An M-by-M interaction matrix $\mathbf{A}$ that stores the normalized dB amplitude response of all M filters at M center frequencies can be formed:

$$\mathbf{A}_{k,m} = A_m(e^{j\omega_k T_s})/g_p, \qquad (3)$$

where $k = 1, 2, 3, ... M$ and $m = 1, 2, 3, ... M$ are the frequency and filter indices, respectively, and $g_p = 20\log(G_p)$ is the prototype dB gain common to all equalizing filters. The amplitude response of the graphic EQ in dB at the center frequencies may then be approximated as

$$\hat{\mathbf{t}} = \mathbf{A}\mathbf{g}, \qquad (4)$$

where $\mathbf{g} = [20\log(G_1)\ 20\log(G_2)\ ...\ 20\log(G_M)]^T$ is the dB gain vector (the superscript T denotes the transpose). The approximation error in (4) becomes zero when the vector $\mathbf{g}$ contains the prototype gain values used to define $\mathbf{A}$ in (3).

With the above choices, matrix $\mathbf{A}$ is square and nonsingular. Thus, the optimal dB gains in the least squares sense can be solved with the help of the inverse matrix $\mathbf{A}^{-1}$ [20]:

$$\mathbf{g}_{\text{opt}} = \mathbf{A}^{-1}\mathbf{t}, \qquad (5)$$

where $\mathbf{t} = [g_1\ g_2\ ...\ g_M]^T$ is the target amplitude response defined by command gains in dB.

The equalizing filters in earlier work used a parametric EQ filter having the midpoint gain $\sqrt{G_m}$ (and not the more traditional -3-dB gain) at the crossover points defined by the bandwidth [19], [17]. Several formulations exist for parametric EQ filters having their bandwidth defined for the midpoint gain [21], [19], [22], [23], [1]. We choose to use the following second-order IIR peak/notch filter given by Orfanidis [24] in which we have set the reference gain at dc to 1:

$$H(z) = \frac{1 + G\beta - 2\cos(\omega_c)z^{-1} + (1 - G\beta)z^{-2}}{1 + \beta - 2\cos(\omega_c)z^{-1} + (1 - \beta)z^{-2}}, \qquad (6)$$

where G is the peak gain, ω_c is the center frequency,

$$\beta = \begin{cases} \tan(B/2) & \text{when } G = 1 \\ \sqrt{\dfrac{|G_B{}^2 - 1|}{|G^2 - G_B{}^2|}} \tan\left(\dfrac{B}{2}\right) & \text{otherwise,} \end{cases} \qquad (7)$$

and G_B is the gain at bandwidth B. One filter $H_m(z)$ having the transfer function (6) is assigned to every command band, each with its own parameters G_m, $\omega_{c,m}$, B_m, and $G_{B,m}$.

To closely reproduce the octave EQ design proposed by Oliver and Jot [17], we used 10 filters of the type (6) centered at 31.25, 62.5, 125, 250, 500, 1000, 2000, 4000, 8000, and 16000 Hz. The filter bandwidths at these center frequencies were chosen to be 32.76, 65.51, 131.0, 262.1, 524.1, 1048, 2096, 4193, 7212, and 10730 Hz, respectively, to closely imitate the Q values and figures given in [17]. For the midpoint gain, we defined $G_{B,m} = G_m/2$. The interaction matrix $\mathbf{A}$ was designed using a 10-dB prototype gain for all filters, as
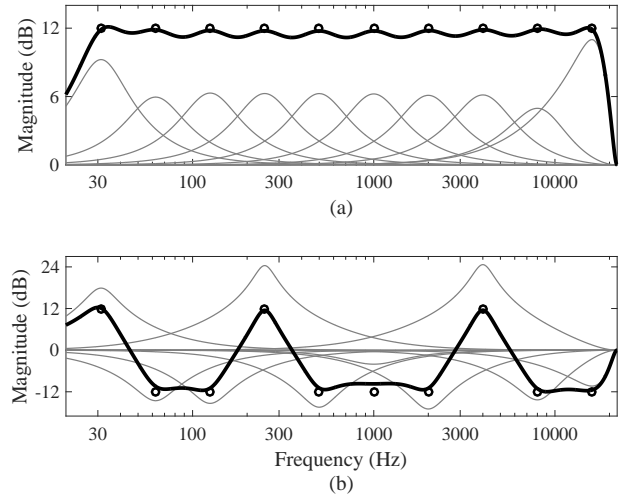

Fig. 1. Amplitude responses (thick black line) obtained with the Oliver-Jot design with $G_0 = 1$ for (a) the constant +12-dB and (b) the special zigzag settings. The circles show the command gain positions, and the gray curves show the amplitude responses of individual parametric equalizing filters.

suggested in [17]. The sample rate f_s is 44.1 kHz in this and all other design examples in this letter.

Fig. 1 shows design examples for two test cases: a constant +12-dB setting, which is common in graphic EQ testing [10], and a special zigzag setting $\mathbf{t} = [12\ -12\ -12\ 12\ -12\ -12\ -12\ 12\ 12\ -12\ -12]^T$, which Oliver and Jot [17] reported to be the most difficult case for their design. In the constant case, Fig. 1(a), the amplitude response is seen to be quite accurate at the command points and falls slightly between them. In Fig. 1(b), the worst case command gain error of 2.28 dB occurs at 1.0 kHz, as also reported in [17]. In audio, having the gain within 1 dB of the specification is desirable. Achieving this goal using the above method is impossible. In fact, before the current work this accuracy has not been achievable using a second-order filter per command band in graphic equalization.

III. PROPOSED DESIGN

This section proposes improvements to the design of a cascade graphic EQ having one filter per command band. The proposed design fulfills the stringent ±1-dB error tolerance, when the command gains vary between -12 and 12 dB.

A. Filters with Fixed Gain at Neighboring Center Frequencies

In the new design, second-order filters given by (6) are used, but this time the bandwidths for the filters are defined so that a specified dB gain $g_{B,m} = cg_m$ (where $0 < c < 1$ is a free design parameter) is reached at the neighboring center frequencies. We have experimentally found that a value $c = 0.30$ yields generally small approximation errors.

The filter bandwidth B_m may then be set as the difference of the center frequencies above and below that filter. In the case of an octave equalizer (see Sec. II), we obtain $B_m = 1.5\omega_{c,m}$. However, due to the slight asymmetry of the magnitude response of the peak/notch filter at high frequencies, the bandwidths of the three last filters, B_8, B_9, and B_{10}, must be adjusted separately. Values $B_8 = 0.93\omega_{c,8}$, $B_9 = 0.78\omega_{c,9}$,
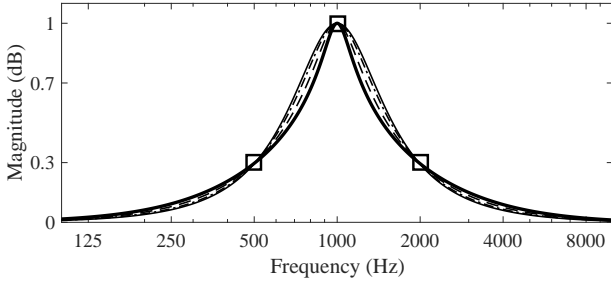
Fig. 2. Normalized amplitude response of the 1-kHz band filter, which has a gain of 0.3 times its dB-peak gain at 500 Hz and 2 kHz, with different peak gain values: 3 dB (thin solid), 10 dB (dash-dot), 17 dB (dashed), and 24 dB (thick solid line). The squares indicate the points where the responses meet.

and $B_{10} = 0.76\omega_{c,10}$ make these filters proportional at their lower neighboring center frequency. The bandwidths used in this work are 46.88, 93.75, 187.5, 375.0, 750.0, 1500, 3000, 5580, 9360, and 12160 Hz.

The above specification of bandwidth gain minimizes variations in the filter amplitude response shape at the center frequency and at one or two neighboring frequencies. To demonstrate this, Fig. 2 shows the normalized amplitude response of one of the parametric EQs for different peak gain values when $c = 0.3$. The responses have been normalized on the dB scale by dividing them by the dB gain $20\log(G_m)$, as suggested in [19], to reveal the differences in the response shapes. Fig. 2 shows that all responses at different gains indeed meet at three points. At these frequencies, the magnitude response of the filter is proportional to the gain coefficient. This is advantageous because now three elements on each row of the interaction matrix remain the same for different prototype gain values.

Another design parameter, in addition to c, is the prototype gain g_p. We have found that, when the command gains vary between 12 and -12 dB, the best approximations are obtained with the prototype gains between 10 and 23 dB. We have chosen to use $g_p = 17$ dB. Fig. 3 shows the amplitude response of the octave EQ when $c = 0.3$ and the interaction matrix $\mathbf{A}$ has been set up using the 17-dB prototype gain for all filters (thin solid line). An improvement from Fig. 1 is observed. This is the consequence of having controlled interaction at all gain values, which can be accounted for by the interaction matrix. Nevertheless, the design is still unsatisfactory for high-fidelity audio, as the maximum errors in Fig. 3 exceed the 1-dB limit between command points. Next we show how to improve this design to reach an acceptable accuracy.

B. Extra Frequency Points and Updated Interaction Matrix

The accuracy of the proposed design can be increased further by expanding the interaction matrix to include one extra design point between each of the command frequencies and by updating the interaction matrix with more realistic prototype gains. Both ideas were also tested separately, but only their combined use leads to a sufficient improvement. The new frequency points are chosen at the geometric mean of the command frequencies to maintain a logarithmic spacing. The octave EQ design has 19 frequency points instead of 10.
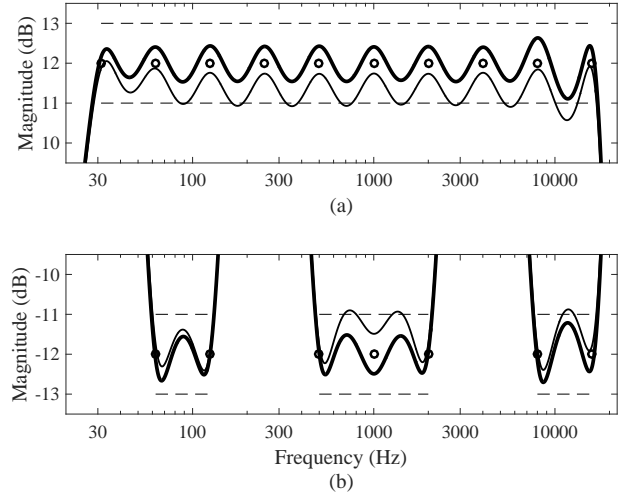

Fig. 3. Amplitude responses obtained using the non-iterative design at command frequencies (thin solid line) and the iterative design with extra points (thick solid line) for (a) the constant +12-dB and (b) the special zigzag settings, cf. Fig. 1. The dashed horizontal lines indicate the ± 1-dB tolerances.

The new, expanded interaction matrix with $2M - 1$ by M elements is defined as

$$\mathbf{B}_{k,m} = A_m(e^{j\omega_k T_s})/g_p, \tag{8}$$

where $k = 1, 2, 3, ... 2M - 1$ and $m = 1, 2, 3, ... M$. Since the interaction matrix $\mathbf{B}$ is non-square, the solution of the least-squares optimal coefficients now requires the use of its pseudoinverse [20], which we denote as $\mathbf{B}^+$. The filter gains $\mathbf{g}$ are obtained as

$$\mathbf{g} = \mathbf{B}^+ \mathbf{t}_1 = (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \mathbf{t}_1, \tag{9}$$

where $\mathbf{t}_1$ is a vector with $2M - 1$ elements containing the original target dB-gain values in odd rows and their linearly interpolated intermediate values in even rows. Matrix $\mathbf{B}^+$ can be computed conveniently in advance (off-line). The design (9) reduces the error, but it still exceeds 1 dB in some cases.

Accurate filter gains are then obtained by defining a second interaction matrix $\mathbf{B}_1$ using gains $\mathbf{g}$ obtained from (9) instead of the prototype gain g_p. The M columns of matrix $\mathbf{B}_1$ are constructed by sampling the amplitude responses of the M band filters (6) in dB, each computed with its own near-optimum peak gain $G(m)$ and normalized with the corresponding dB gain $g(m)$, at the $2M - 1$ frequencies. The gain vector $\mathbf{g}_1$ after this iteration step is

$$\mathbf{g}_1 = \mathbf{B}_1^+ \mathbf{t}_1 = (\mathbf{B}_1^T \mathbf{B}_1)^{-1} \mathbf{B}_1^T \mathbf{t}_1. \tag{10}$$

These first-iteration filter gains must be converted to linear gain factors to be used in (6) and (7).

The responses of the improved design (10) using the iterated gain vector $\mathbf{g}_1$ are included in Fig. 3 (thick solid lines). Now the amplitude response oscillates nearly symmetrically around the command gains, staying within the 1-dB tolerances. The maximum error at the command points is 0.63 dB in Fig. 3(a) and 0.49 dB in Fig. 3(b), which is satisfactory. The amplitude response does not exceed the tolerances between the command

TABLE I
MAXIMUM ERRORS OF THE OCTAVE BAND EQUALIZERS IN DECIBELS.
THE BEST RESULT IN EACH CASE IS HIGHLIGHTED

Case	OJ	EQ4	PGE	Proposed
Zigzag	1.2 dB	1.8 dB	0.91 dB	**0.25 dB**
Every third down	0.67 dB	1.2 dB	0.76 dB	**0.52 dB**
OJ worst case	2.3 dB	1.5 dB	0.69 dB	**0.49 dB**

TABLE II
NUMBER OF OPERATIONS PER OUTPUT SAMPLE

Operation	OJ	EQ4	PGE	Proposed
ADD	40	140	80	40
MUL	50	150	81	50
TOTAL	90	290	161	90

TABLE III
AVERAGE COMMAND GAIN UPDATE TIMES IN MATLAB

OJ	EQ4	PGE	Proposed
0.045 ms	0.0082 ms	19 ms	2.2 ms

points either. This method has been tested extensively, and its error has not exceeded 1 dB at the command or extra points or between them when neighboring command gains are the same. We thus assume that the proposed design method is sufficiently accurate for high-fidelity equalization in audio.

IV. COMPARISON WITH PREVIOUS METHODS

In this section, the proposed design method in the case of an octave (10-band) EQ is compared with three previous graphic EQ designs: the Oliver-Jot (OJ) [17], the cascaded fourth-order (EQ4) [12], and the high-precision parallel (PGE) [10], [1] graphic EQ designs. Here, the octave PGE design follows the description given in [1], but to improve its performance, the lowest pole is set to 12 Hz and the target gains below the first and above the last command gain is set to half of their target gains in dB. The sample rate used is 44.1 kHz.

Table I compares the accuracy of the different EQs with three different command gain settings by showing the maximum errors of the amplitude responses at the command frequencies. The first case is a zigzag setting in which the command gains alternate between ±12 dB. Here, the behavior of the PGE and the proposed design is acceptable, as the error is less than 1 dB. The OJ and EQ4 methods are unacceptable.

In the second case in Table I, every third command gain is set to −12 dB (first, fourth etc.) and the rest to 0 dB. This example demonstrates the overlapping of the neighboring filters. Only the error of EQ4 exceeds 1 dB whereas the other designs can reach the 0 dB-level between the negative peaks.

Finally, the third case depicts the most difficult situation reported by Oliver and Jot [17], which combines the zigzag form with plateaus [see Fig. 1(b)]. Even in this case, the PGE and proposed methods stay within the 1-dB error tolerance. As can be seen, the performance of the proposed EQ design is the best in terms of maximum error in all cases in Table I.

Table II lists for each method the number of operations necessary during real-time filtering. As can be seen, the OJ and the proposed method use the smallest number of operations due the second-order design with direct form I implementation [20]. PGE needs more operations mainly because it contains twice as many filters. Finally, EQ4 requires the largest number of operations due to its special, high-order structure [10].

Table III compares another aspect of graphic EQ design by showing the parameter computing time required when

the command gains are changed. The update times were calculated in MATLAB as an average of 1000 updates, when all command gains were set to random values. The Internet connection and all other programs were shut down, so that they would not affect the execution times.

In Table III, the update time for EQ4 is the fastest, since only three parameters are recalculated, even though square root and tangent functions, divisions, and additions are needed. Next come OJ and the proposed design, which use the same design formulas (6) and (7). However, the proposed method requires additional linear interpolation between the command gains, a matrix inversion in (10), and larger matrix multiplications, leading to a much longer execution time. Finally, the gain update time for PGE is larger than for the other methods due to the computation of the high-resolution target magnitude and phase response and the pseudoinverse of a large matrix [10]. Thus, the command gain update for the proposed method is an order of magnitude faster than for the PGE, which is the only other EQ design achieving the ±1-dB accuracy.

V. CONCLUSION

This letter proposes a new design method for a graphic equalizer, which consists of cascaded second-order filters. The method assumes one such filter per command band, as in the simplest graphic EQs. Extensive testing with command gains in the range of ±12 dB over command frequencies spaced an octave apart has empirically shown that the proposed design does not deviate more than ±1 dB from the gain specification. The design requires linear interpolation of the dB gains, an inversion of a square matrix, a few matrix multiplications, and the computation of IIR filter coefficients using closed-form formulas. The proposed design is considerably simpler, and its filter implementation is 44% more efficient than those of an accurate parallel graphic EQ.

A key idea in the proposed design is to use second-order filters with an adjustable gain at the bandwidth so that the magnitude response variations at the neighboring center frequencies with varying peak gain are minimized. To reduce the approximation error further, the proposed method evaluates the interaction between the filters at the command frequencies and at one extra point between each of them. An interaction matrix is determined using one iteration. The only adjustable parameters remaining are the coefficient c, which determines the gain at the neighboring command frequencies, and the prototype gain g_p used for determining the first-iteration interaction matrix. The relevant MATLAB code is available online [25].

A digital filter can be converted to its parallel form using the partial fraction expansion [20]. As the parallel structure has certain advantages, future work will examine how to conveniently apply this technique to turn the proposed cascade graphic EQ into a parallel one.

REFERENCES

[1] V. Välimäki and J. D. Reiss, "All about audio equalization: Solutions and frontiers," *Appl. Sci.*, vol. 6, no. 5, p. 129, May 2016.

[2] S. Stasis, R. Stables, and J. Hockman, "Semantically controlled adaptive equalisation in reduced dimensionality parameter space," *Appl. Sci.*, vol. 6, no. 4, p. 116, Apr. 2016.

[3] J. Rämö, V. Välimäki, and M. Tikander, "Perceptual headphone equalization for mitigation of ambient noise," in *Proc. IEEE Int. Conf. Acoust. Speech Signal Process. (ICASSP)*, Vancouver, BC, Canada, May 2013, pp. 724–728.

[4] F. Vidal Wagner and V. Välimäki, "Automatic calibration and equalization of a line array system," in *Proc. Int. Conf. Digital Audio Effects (DAFx-15)*, Trondheim, Norway, Nov. 2015, pp. 123–130.

[5] D. Griesinger, "Accurate timbre and frontal localization without head tracking through individual eardrum equalization of headphones," in *Proc. Audio Eng. Soc. 141st Conv.*, Los Angeles, CA, USA, Sept. 2016.

[6] R. A. Greiner and M. Schoessow, "Design aspects of graphic equalizers," *J. Audio Eng. Soc.*, vol. 31, no. 6, pp. 394–407, 1983.

[7] Motorola Inc., "Digital stereo 10-band graphic equalizer using the DSP56001," 1988, Application note.

[8] S. Tassart, "Graphical equalization using interpolated filter banks," *J. Audio Eng. Soc.*, vol. 61, no. 5, pp. 263–279, May 2013.

[9] Z. Chen, G. S. Geng, F. L. Yin, and J. Hao, "A pre-distortion based design method for digital audio graphic equalizer," *Digital Signal Process.*, vol. 25, pp. 296–302, Feb. 2014.

[10] J. Rämö, V. Välimäki, and B. Bank, "High-precision parallel graphic equalizer," *IEEE/ACM Trans. Audio Speech Lang. Process.*, vol. 22, no. 12, pp. 1894–1904, Dec. 2014.

[11] P. A. Regalia and S. K. Mitra, "Tunable digital frequency response equalization filters," *IEEE Trans. Acoust. Speech Signal Process.*, vol. 35, no. 1, pp. 118–120, Jan. 1987.

[12] M. Holters and U. Zölzer, "Graphic equalizer design using higher-order recursive filters," in *Proc. Int. Conf. Digital Audio Effects*, Montreal, Canada, Sept. 2006, pp. 37–40.

[13] J. Rämö and V. Välimäki, "Optimizing a high-order graphic equalizer for audio processing," *IEEE Signal Process. Lett.*, vol. 21, no. 3, pp. 301–305, Mar. 2014.

[14] J. A. Belloch and V. Välimäki, "Efficient target response interpolation for a graphic equalizer," in *Proc. IEEE Int. Conf. Acoust. Speech Signal Process. (ICASSP)*, Shanghai, China, Mar. 2016, pp. 564–568.

[15] S.-A. Azizi, "A new concept of interference compensation for parametric and graphic equalizer banks," in *Proc. Audio Eng. Soc. 111th Conv.*, New York, USA, Nov. 2001.

[16] R. Miller, "Equalization methods with true response using discrete filters," in *Proc. Audio Eng. Soc. 116th Conv.*, Berlin, Germany, May 2004.

[17] R. J. Oliver and J.-M. Jot, "Efficient multi-band digital audio graphic equalizer with accurate frequency response control," in *Proc. Audio Eng. Soc. 139th Conv.*, New York, USA, Oct. 2015.

[18] Y. Lee, R. Kim, G. Cho, and S. J. Choi, "An adjusted-Q digital graphic equalizer employing opposite filters," in *Lect. Notes Comput. Sci.* Berlin, Germany: Springer-Verlag, 2005, vol. 3768, pp. 981–992.

[19] J. S. Abel and D. P. Berners, "Filter design using second-order peaking and shelving sections," in *Proc. Int. Computer Music Conf.*, Miami, FL, USA, Nov. 2004.

[20] L. B. Jackson, *Digital Filters and Signal Processing*, 2nd ed. Norwell, MA: Kluwer, 1989.

[21] R. Bristow-Johnson, "The equivalence of various methods of computing biquad coefficients for audio parametric equalizers," in *Proc. Audio Eng. Soc. 97th Conv.*, San Francisco, CA, USA, Nov. 1994.

[22] J. D. Reiss, "Design of audio parametric equalizer filters directly in the digital domain," *IEEE Trans. Audio Speech Lang. Process.*, vol. 19, no. 6, pp. 1843–1848, Aug. 2011.

[23] J.-M. Jot, "Proportional parametric equalizers—Application to digital reverberation and environmental audio processing," in *Proc. Audio Eng. Soc. 139th Conv.*, New York, USA, Oct. 2015.

[24] S. J. Orfanidis, *Introduction to Signal Processing*. Piscataway, NJ: Rutgers Univ., 2010.

[25] J. Liski and V. Välimäki, "Companion page to Accurate Cascade Graphic Equalizer [Online]," http://research.spa.aalto.fi/publications/papers/spl-acge/, 2016, [accessed Dec. 23, 2016].