Weekly Report

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1 Summary

For this update I was tasked with looking at my implementation of the stopping conditions (18) and (19) of Chen et al. [1] to determine whether they are appropriate or not. I was also asked to read Hu et al. [2] and [3] so that I can apply the code I currently have to the Boltzmann equation.

2 Progress

2.1 Analyzing the Stopping Conditions

The stopping conditions for Example 5.1.1 in Chen et al. [1] are satisfied when

$$\frac{\|u^{n+1} - u^n\| + h^p}{\|u^n - u^{n-1}\|} > 1 \tag{1}$$

and

$$\|u^{n+1} - u^n\| - h^{p+1} < 0 \tag{2}$$

hold true. Here p is the truncation error of the method. I set $p = 3$ as it says in the paper. Call the condition in Equation 1 c_1 and the condition in Equation 2 c_2. In my code I have a while loop that runs while $\neg(c_1 \wedge c_2)$ which is logically equivalent to $\neg c_1 \vee \neg c_2$. This is the condition I originally had in my code. I just forgot the reasoning behind it during the meeting.

	N	L_1 Error	L_1 Order	L_inf Error	L_inf Order	Iterations
0	40	1.009e-03	NaN	3.171e-03	NaN	81
1	80	1.503e-04	2.746754	6.168e-04	2.362200	209
2	160	1.961e-05	2.938453	1.201e-04	2.360192	505
3	320	2.222e-06	3.141978	2.451e-05	2.293147	1154
4	640	2.419e-07	3.199434	4.894e-06	2.324276	2582
5	1280	2.510e-08	3.268100	9.391e-07	2.381678	5718
6	2560	2.410e-09	3.380740	1.616e-07	2.538457	12622

Figure 1: This plot is an attempt to reproduce the results from Table 1 in Chen et al. This is the condition that I used in my code.

I went on to plot the evolution of the L_1 error of the method. I got the following results.

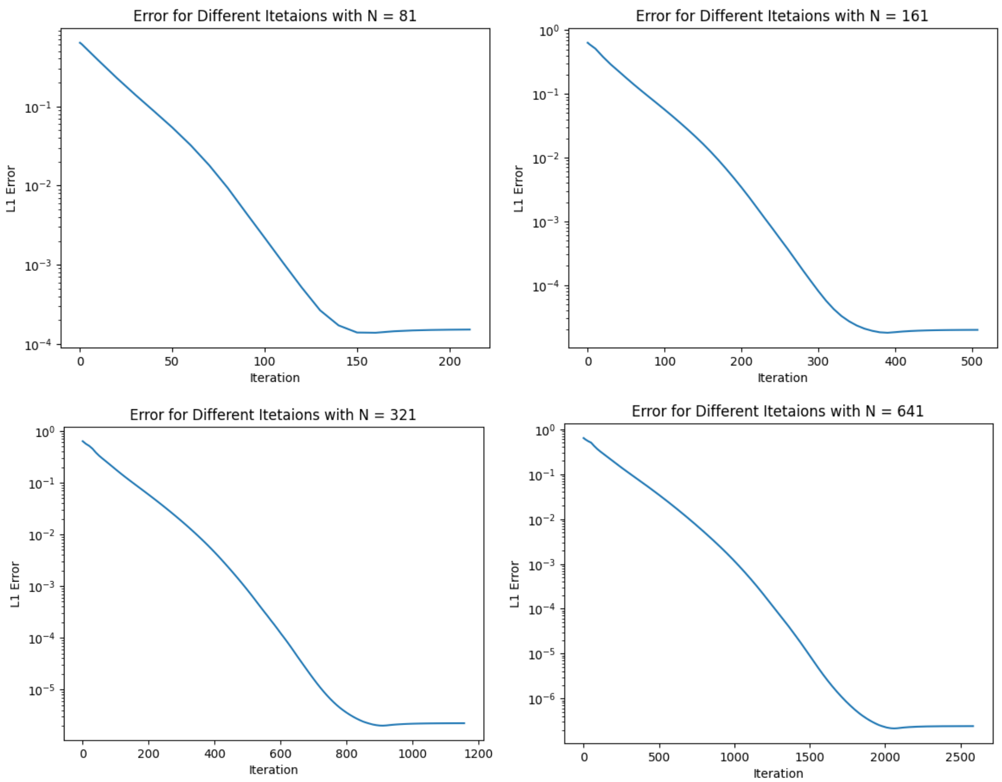

Figure 2: The shape of the error curve for the case of $N = 80$ is similar to the right plot in Figure 2 of [1]. The major differences are that the error here converges faster and my error curve has a long flat tail. This flat tail indicates that the method converged a good while before the stopping conditions were met. For this case at least the stopping conditions seem a bit conservative.

2.2 Reading

There is a great deal of material in these two papers. It would help me to talk about precisely which sections I need to understand in order to achieve the next milestone. That being said, I came up with a list of questions about different parts of the reading.

1. What sections of these papers should I thoroughly understand before continuing?
2. What collision kernels should I test the method on? What should the boundary conditions, initial conditions, and solution domain be for the tests on these sample problems?
3. Do I use the low-rank representation from Section 2 of Hu et al. [2] in my current implementation? If so, what basis should I use?
4. I suppose the rank changing feature of Section 3 in Hu et al. [2] is irrelevant since we are only going for the steady state. Is this correct?
5. I would like to see some references on the loss and gain term splitting in the collision operator in Su et al. [3].

3 To Do

I would like to meet and determine specific test problems so that I can get started on the implementation.

References

[1] Weitao Chen, Ching-Shan Chou, and Chiu-Yen Kao. Lax–friedrichs fast sweeping methods for steady state problems for hyperbolic conservation laws. *Journal of Computational Physics*, 234:452–471, 2013.

[2] Jingwei Hu and Yubo Wang. An adaptive dynamical low rank method for the nonlinear boltzmann equation, 2021.

[3] Wei Su, Peng Wang, Yonghao Zhang, and Lei Wu. Implicit discontinuous galerkin method for the boltzmann equation, 2019.