Analysis of the Battleship Game

Damien R. Bose

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1 Introduction

This article will try and analyse the game of battleship. Our goal is to come up with a strategy for optimal (or close to optimal) play, inspired by an article by Anthony Rochford's analysis[2]. However, I try to formalise these ideas in my own way. Furthermore, I try pay more attention to about the behaviour of the opponent.

2 Rules of the Game[1]

3 Mathematical Setup

3.1 Modelling the Game

We first come to the question of modelling the opponents board. First, however, notice that the game lies on a 10×10 grid. Hence, there are 100 tiles. Therefore, we can model them as a series of random variables.

Definition 1 *X^i denotes wether the i^{th} tile is a ship or not. $X^i = 1$ if the i^{th} tile is a ship and $X^i = 0$ if it doesn't.*

$X^i \sim Ber(\theta^i)$. Hence $P(X^i = 1) = \theta^i$ is the probability that tile i has a boat.

Notice that we make no i.i.d assumptions for X^i. Since, for example, if we know tile i has a ship, then we can conclude that adjacent tiles are more likely to be ships as well. In fact, conditioning on previous knowledge will guide our strategy.

3.2 Modelling Performance

Now lets not forget out goal: be the first to hit all 17 boat tiles of the opponent. One way to this is by minimising the time it takes to hit all ships.

Definition 2 *T_s is the number of moves/time it takes for us to do so with strategy s.*

We can therefore quantify the performance of our strategy using $E[T_s]$.

Note, however, $E[T_s]$ does not capture the entire picture regarding the success of a strategy: winning is dependent on the performance of the other player as well. For example, let's say player 1 has strategy s_1 and player 2 has strategy s_2, and $E[T_{s_1}] = E[T_{s_2}]$. Then, s_1 might be a better strategy than s_2 if $var[T_{s_1}] > var[T_{s_2}]$. However, this is simply an intuition and has to be proven and/or tested. Lets simply focus on finding s which minimises $E[T_s]$ for now.

4 Strategy

Now that we have the mathematical setup, we can start exploring our strategy.

During turn 1, we have no information about the opponents board. So we simply pick the tile i which maximises $P(X_i = 1)$. Lets say tile 55 gets picked. We find out that $x_i = 1$. Hit! Now, we pick the tile $j \neq i$ which maximises $P(X_j = 1|X_i = 1)$, but now $x_j = 0$. We missed. We now pick tile k with max $P(X_k = 1|X_j = 0, X_i = 1)$, and so on iteratively.

A possible improvement to this strategy is instead of picking i which maximises the probability of that tile having a ship, we should pick the our next target j randomly with corresponding probability $P(X_j = 1|...)$. This likely has an equal $E[T_s]$ to the initial strategy but will likely have a higher $var[T_s]$.

References

[1] https://www.hasbro.com/common/instruct/battleship.pdf

[2] https://austinrochford.com/posts/2021-09-02-battleship-bayes.html

[3] http://thevirtuosi.blogspot.com/2011/10/linear-theory-of-battleship.html