Math 170S Midterm

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Contents

Problem 1

```
Class <- c('0-9', '10-19', '20-29', '30-39', '40-49', '50-59')
Frequency <- c(13, 15, 9, 14, 16, 8)
MidPoint <- c(4.5, 14.5, 24.5, 34.5, 44.5, 54.5)
xf <- Frequency * MidPoint
cf <- c(13, 28, 37, 51, 67, 75)
df <- data.frame(Class, Frequency, MidPoint, xf, cf)
df
```

```
##   Class Frequency MidPoint    xf  cf
## 1   0-9        13      4.5  58.5 13
## 2 10-19        15     14.5 217.5 28
## 3 20-29         9     24.5 220.5 37
## 4 30-39        14     34.5 483.0 51
## 5 40-49        16     44.5 712.0 67
## 6 50-59         8     54.5 436.0 75
```

```
mean1 <- sum(xf)/75
print(paste0("the mean is ", mean1))
```

```
## [1] "the mean is 28.3666666666667"
```

```
median1 <- 30 + (1/14)*9
print(paste0("the median is ", median1))
```

```
## [1] "the median is 30.6428571428571"
```

```
variance1 <- (sum(Frequency * ((MidPoint - mean1)^2))) / 75
sd1 <- sqrt(variance1)
print(paste0("the standard deviation is ", sd1))
```

```
## [1] "the standard deviation is 16.5644867781112"
```

```
percentile10 <- ((10*75)/100)/13*9
print(paste0("the 10th percentile is ", percentile10))
```

```
## [1] "the 10th percentile is 5.19230769230769"
```

```
q1 <- 14.5 + (6.5/15)*9
q3 <- 44.5 +(6.5/16)*9
iqr1 <- q3 - q1
print(paste0("the first quartile is ", q1))
```

```
## [1] "the first quartile is 18.4"
```

```
print(paste0("the IQR is ", iqr1))
```

```
## [1] "the IQR is 29.75625"
```

Problem 2

Part A

```
data("USArrests")
summary(USArrests$Murder)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   0.800   4.075   7.250   7.788  11.250  17.400
```

```
summary(USArrests$Assault)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    45.0   109.0   159.0   170.8   249.0   337.0
```

```
summary(USArrests$UrbanPop)
```

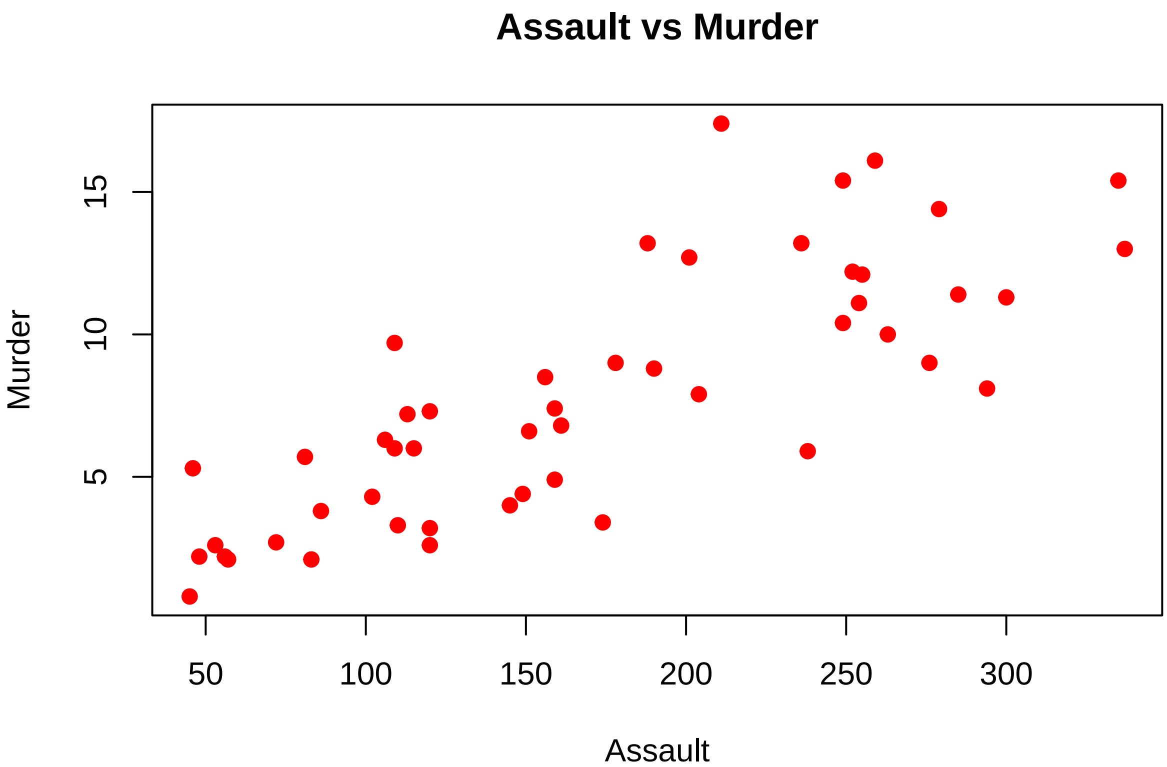
```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   32.00   54.50   66.00   65.54   77.75   91.00
```

```r
summary(USArrests$Rape)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    7.30   15.07   20.10   21.23   26.18   46.00
```

Part B

```r
plot(USArrests$Assault, USArrests$Murder,
     xlab = "Assault", ylab = "Murder", main = "Assault vs Murder",
     col = "red", pch = 19)
```



There's a slightly positive trend in the scatterplot. On average, an increase in assaults implies an increase in murders

Part C

```r
summary(lm(USArrests$Murder ~ USArrests$UrbanPop))
```

```
##
## Call:
## lm(formula = USArrests$Murder ~ USArrests$UrbanPop)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -6.537 -3.736 -0.779  3.332  9.728
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)         6.41594    2.90669   2.207   0.0321 *
```

```
## USArrests$UrbanPop  0.02093    0.04333   0.483   0.6312
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.39 on 48 degrees of freedom
## Multiple R-squared:  0.00484,    Adjusted R-squared:  -0.01589
## F-statistic: 0.2335 on 1 and 48 DF,  p-value: 0.6312
```

The intercept is 6.416 suggesting that when we have population 0, we can expect 6.416 murders. We also see the slope is 0.0209 meaning with a 1 unit increase in population, we expect the average murder increase to be 0.0209. The R squared value, however, is relatively low meaning that the model explains very little of the variability in murder, and the p-value of 0.6312 is rather high, suggesting our results are not very statistically significant

Problem 3

Part A

MLE for $Poisson(\lambda)$

Poisson distribution: $P(X = x) = \frac{\lambda^x e^{-x}}{x!}$

$L(\lambda) = \Pi_{i=1}^{n} \frac{\lambda^x e^{-x}}{x!}$

$ln(L(\lambda)) = ln(\Pi_{i=1}^{n} \frac{\lambda^x e^{-x}}{x!})$

$= \Sigma_{i=1}^{n} ln(\frac{\lambda^x e^{-x}}{x!})$

$= \Sigma_{i=1}^{n} (ln(\lambda^{x_i}) + ln(e^{-\lambda}) - ln(x_i!))$

$= \Sigma_{i=1}^{n} (ln(\lambda^{x_i}) - \lambda ln(e) - ln(x_i!))$

$= \Sigma_{i=1}^{n} (ln(\lambda^{x_i}) - \lambda - ln(x_i!))$

$= ln(\lambda)\Sigma_{i=1}^{n} x_i - n\lambda - \Sigma_{i=1}^{n}(x_i!)$

$\frac{d}{d\lambda} ln(L(\lambda)) = \frac{1}{\lambda}\Sigma_{i=1}^{n} x_i - n$

$0 = \frac{1}{\lambda}\Sigma_{i=1}^{n} x_i - n$

$n = \frac{1}{\lambda}\Sigma_{i=1}^{n} x_i$

$n\lambda = \Sigma_{i=1}^{n} x_i$

$\lambda = \frac{1}{n}\Sigma_{i=1}^{n} x_i$

Part B

MLE for $Normal(\mu, \sigma^2)$

Normal distribution: $f(x \mid \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$

$L(\mu, \sigma^2) = \Pi_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$

$= \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}\Sigma_{i=1}^{n}(x_i-\mu)^2}$

$ln(L(\mu, \sigma^2)) = ln(\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}\Sigma_{i=1}^{n}(x_i-\mu)^2})$

$= ln(\frac{1}{\sigma\sqrt{2\pi}}) + ln(e^{-\frac{1}{2\sigma^2}\Sigma_{i=1}^{n}(x_i-\mu)^2})$

$= ln((2\pi\sigma^2)^{-\frac{n}{2}}) + ln(e^{-\frac{1}{2\sigma^2}\Sigma_{i=1}^n(x_i-\mu)^2})$

$= -\frac{n}{2}ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\Sigma_{i=1}^n(x_i-\mu)^2$

$= -\frac{n}{2}ln(2\pi) - \frac{n}{2}ln(\sigma^2) - \frac{1}{2\sigma^2}\Sigma_{i=1}^n(x_i-\mu)^2$

$\frac{\partial}{\partial\mu}ln(L(\mu,\sigma^2)) = \frac{\partial}{\partial\mu}(-\frac{n}{2}ln(2\pi) - \frac{n}{2}ln(\sigma^2) - \frac{1}{2\sigma^2}\Sigma_{i=1}^n(x_i-\mu)^2)$

$= \frac{1}{\sigma^2}\Sigma_{i=1}^n(x_i-\mu)$

$= \frac{1}{\sigma^2}(\Sigma_{i=1}^n x_i - n\mu)$

$0 = \frac{1}{\sigma^2}(\Sigma_{i=1}^n x_i - n\mu)$

$0 = \Sigma_{i=1}^n x_i - n\mu$

$n\mu = \Sigma_{i=1}^n x_i$

$\boxed{\mu = \frac{1}{n}\Sigma_{i=1}^n x_i}$

$\frac{\partial}{\partial\sigma^2}ln(L(\mu,\sigma^2)) = \frac{\partial}{\partial\sigma^2}(-\frac{n}{2}ln(2\pi) - \frac{n}{2}ln(\sigma^2) - \frac{1}{2\sigma^2}\Sigma_{i=1}^n(x_i-\mu)^2)$

$= -\frac{n}{2\sigma^2} - (\frac{1}{2}\Sigma_{i=1}^n(x_i-\mu)^2)\frac{d}{d\sigma^2}(\frac{1}{\sigma^2})$

$= -\frac{n}{2\sigma^2} - (\frac{1}{2}\Sigma_{i=1}^n(x_i-\mu)^2)(-\frac{1}{(\sigma^2)^2})$

$= -\frac{n}{2\sigma^2} + (\frac{1}{2}\Sigma_{i=1}^n(x_i-\mu)^2)\frac{1}{(\sigma^2)^2}$

$= \frac{1}{2\sigma^2}(\frac{1}{\sigma^2}\Sigma_{i=1}^n(x_i-\mu)^2 - n)$

Let's suppose $\sigma^2 \neq 0$. Then we have

$0 = \frac{1}{2\sigma^2}(\frac{1}{\sigma^2}\Sigma_{i=1}^n(x_i-\mu)^2 - n)$

$0 = \frac{1}{\sigma^2}\Sigma_{i=1}^n(x_i-\mu)^2 - n$

$n = \frac{1}{\sigma^2}\Sigma_{i=1}^n(x_i-\mu)^2$

$\sigma^2 n = \Sigma_{i=1}^n(x_i-\mu)^2$

$\boxed{\sigma^2 = \frac{1}{n}\Sigma_{i=1}^n(x_i-\mu)^2}$

Part C

MLE for $Gamma(\alpha)$

Gamma distribution: $f(x \mid \alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}$

$L(\alpha,\beta) = \Pi_{i=1}^n\frac{\beta^\alpha}{\Gamma(\alpha)}x_i^{\alpha-1}e^{-\beta x_i}$

$ln(L(\alpha,\beta)) = \Pi_{i=1}^n(\alpha ln(\beta) - ln(\Gamma(\alpha)) + (\alpha-1)ln(x_i) - \beta x_i)$

$\frac{\partial}{\partial\alpha}(ln(L(\alpha,\beta))) = n\alpha ln(\beta) - nln(\Gamma(\alpha)) + (\alpha-1)\Sigma_{i=1}^n ln(x_i) - \beta\Sigma_{i=1}^n x_i$

$0 = n\alpha ln(\beta) - nln(\Gamma(\alpha)) + (\alpha-1)\Sigma_{i=1}^n ln(x_i) - \beta\Sigma_{i=1}^n x_i$

$0 = nln(\beta) - n\frac{1}{\Gamma(\alpha)}\Gamma'(\alpha) + \Sigma_{i=1}^n ln(x_i)$

Let $\Psi(\alpha) = \frac{\Gamma'(\alpha)}{\Gamma(\alpha)}$. We have

$\boxed{\Psi(\alpha) = ln(\beta) + \frac{1}{n}\Sigma_{i=1}^n ln(x_i)}$

Part D

MLE for $Binomial(p)$

Binomial distribution: $P(X=x) = \binom{n}{x} p^x (1-p)^{n-x} = \frac{n!}{(n-x)!x!} p^x (1-p)^{n-x}$

$L(p) = \Pi_{i=1}^n \frac{n!}{(n-x_i)!x_i!} p^{x_i} (1-p)^{n-x_i}$

$= (\Pi_{i=1}^n \frac{n!}{(n-x_i)!x_i!}) \Pi_{i=1}^n p^{x_i} (1-p)^{n-x_i}$

$= (\Pi_{i=1}^n \frac{n!}{(n-x_i)!x_i!}) p^{x_i} (1-p)^{n-\Sigma_{i=1}^n x_i}$

$ln(L(P)) = \Sigma_{i=1}^n x_i ln(p) + (n - \Sigma_{i=1}^n x_i) ln(1-p)$

$\frac{d}{dp} ln(L(p)) = \frac{1}{p} \Sigma_{i=1}^n x_i - (n - \Sigma_{i=1}^n x_i) \frac{1}{1-p}$

$0 = \frac{1}{p} \Sigma_{i=1}^n x_i - (n - \Sigma_{i=1}^n x_i) \frac{1}{1-p}$

$0 = \frac{(1-p)\Sigma_{i=1}^n x_i - (n - \Sigma_{i=1}^n x_i)p}{p(1-p)}$

$0 = (1-p)\Sigma_{i=1}^n x_i - (n - \Sigma_{i=1}^n x_i)p$

$0 = \Sigma_{i=1}^n x_i - p\Sigma_{i=1}^n x_i - np + (\Sigma_{i=1}^n x_i)p$

$0 = \Sigma_{i=1}^n x_i - np$

$np = \Sigma_{i=1}^n x_i$

$\boxed{p = \frac{\Sigma_{i=1}^n x_i}{n}}$

Part E

MLE for $Geometric(p)$

Geometric distribution: $f(x \mid p) = (1-p)^{x-1} p$

$L(p) = \Pi_{i=1}^n (1-p)^{x-1} p$

$= p^n (1-p)^{\Sigma_{i=1}^n x_i - n}$

$ln(L(p)) = ln(p^n (1-p)^{\Sigma_{i=1}^n x_i - n})$

$= nln(p) + (\Sigma_{i=1}^n x_i - n) ln(1-p)$

$\frac{d}{dp} ln(L(p)) = \frac{d}{dp}(nln(p) + (\Sigma_{i=1}^n x_i - n) ln(1-p))$

$= \frac{n}{p} - \frac{\Sigma_{i=1}^n (x_i - n)}{1-p}$

$0 = \frac{n}{p} - \frac{\Sigma_{i=1}^n (x_i - n)}{1-p}$

$\frac{n}{p} = \frac{\Sigma_{i=1}^n (x_i - n)}{1-p}$

$(1-p)\frac{n}{p} = \Sigma_{i=1}^n (x_i - n)$

$\frac{n}{p} - n = \Sigma_{i=1}^n (x_i - n)$

$\frac{n}{p} = \Sigma_{i=1}^n x_i$

$n = p\Sigma_{i=1}^n x_i$

$\boxed{p = \frac{n}{\Sigma_{i=1}^n x_i}}$

Part F

MLE for $Exponential(\mu)$

Exponential distribution: $f(x \mid \mu) = \frac{1}{\mu}e^{\frac{-x}{\mu}}$

$L(\mu) = \Pi_{i=1}^{n}\frac{1}{\mu}e^{\frac{-x}{\mu}}$

$= \frac{1}{\mu^n}e^{\frac{\Sigma_{i=1}^{n}x_i}{\mu}}$

$ln(L(\mu)) = -nln(\mu) - \frac{1}{\mu}\Sigma_{i=1}^{n}x_i$

$\frac{d}{d\mu}ln(L(\mu)) = \frac{-n}{\mu} + \frac{1}{\mu^2}\Sigma_{i=1}^{n}x_i$

$0 = \frac{-n}{\mu} + \frac{1}{\mu^2}\Sigma_{i=1}^{n}x_i$

$\frac{n}{\mu} = \frac{1}{\mu^2}\Sigma_{i=1}^{n}x_i$

$\mu n = \Sigma_{i=1}^{n}x_i$

$\mu = \frac{1}{n}\Sigma_{i=1}^{n}x_i$

Problem 4

Part A

```
df2 <- data.frame(Y = c(15.4, 18, 19, 22, 27, 30, 38.5, 46, 55, 58, 60, 62, 64.6, 68, 71),
                  X = c(9.89, 11, 15.3, 15, 17, 19, 20.9, 22, 28, 30.3, 31, 35, 36.6, 38, 39))
summary(lm(df2$Y ~ df2$X))
```

```
##
## Call:
## lm(formula = df2$Y ~ df2$X)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -6.314 -2.529 -1.339  2.499  7.392
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -5.04348    2.62132  -1.924   0.0765 .
## df2$X        1.98416    0.09935  19.971 3.87e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.735 on 13 degrees of freedom
## Multiple R-squared:  0.9684, Adjusted R-squared:  0.966
## F-statistic: 398.8 on 1 and 13 DF,  p-value: 3.869e-11
```

Part B

```
# R-squared
summary(lm(df2$Y ~ df2$X))[[9]]
```

```
## [1] 0.9660049
```

```r
# intepretation
print(paste0("the model explains ", summary(lm(df2$Y ~ df2$X))[[9]]*100,
             "% of variability in Y"))
```

```
## [1] "the model explains 96.6004944898044% of variability in Y"
```

Part C

```r
# Y value when X = 16
summary(lm(df2$Y ~ df2$X))[[4]][[1]] + summary(lm(df2$Y ~ df2$X))[[4]][[2]]*16
```

```
## [1] 26.70313
```

```r
# Y value when X = 25
summary(lm(df2$Y ~ df2$X))[[4]][[1]] + summary(lm(df2$Y ~ df2$X))[[4]][[2]]*25
```

```
## [1] 44.5606
```

Problem 5

Part A

```r
library("MASS")
```

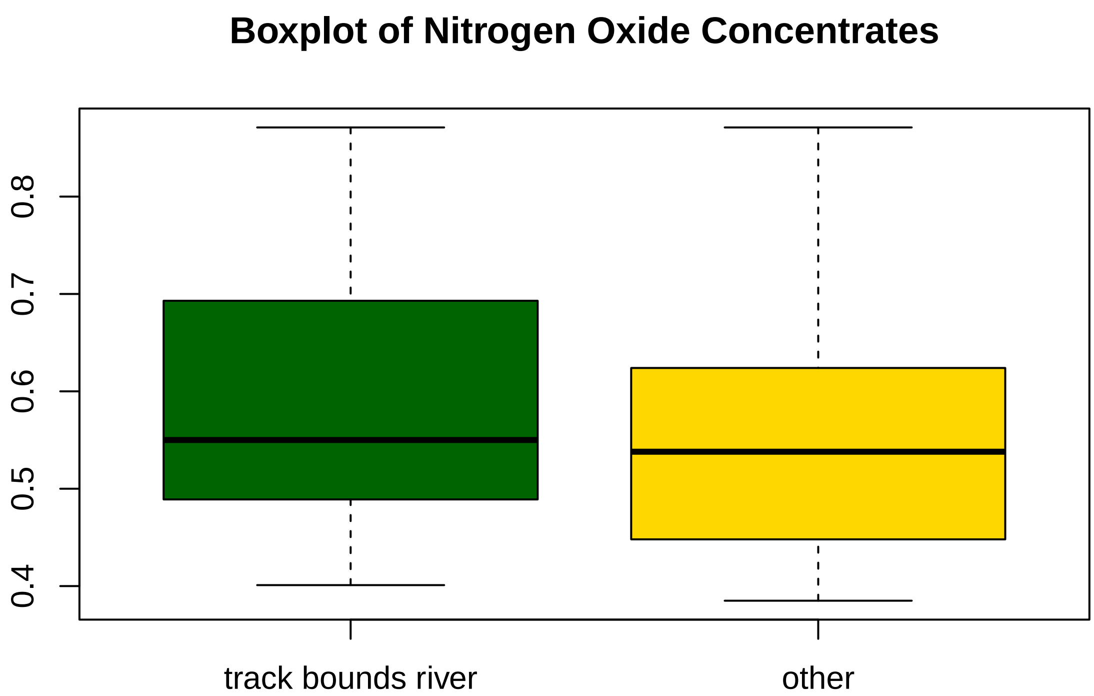
```
## Warning: package 'MASS' was built under R version 4.1.2
```

```r
data("Boston")
l <- list(crim = Boston$crim, rad = Boston$rad)
meanMedianSDiqr <- function(x){
  c(mean = mean(x), median = median(x), standard_deviation = sd(x), iqr = IQR(x))
}
print(sapply(l, meanMedianSDiqr))
```

```
##                        crim       rad
## mean               3.613524  9.549407
## median             0.256510  5.000000
## standard_deviation 8.601545  8.707259
## iqr                3.595038 20.000000
```

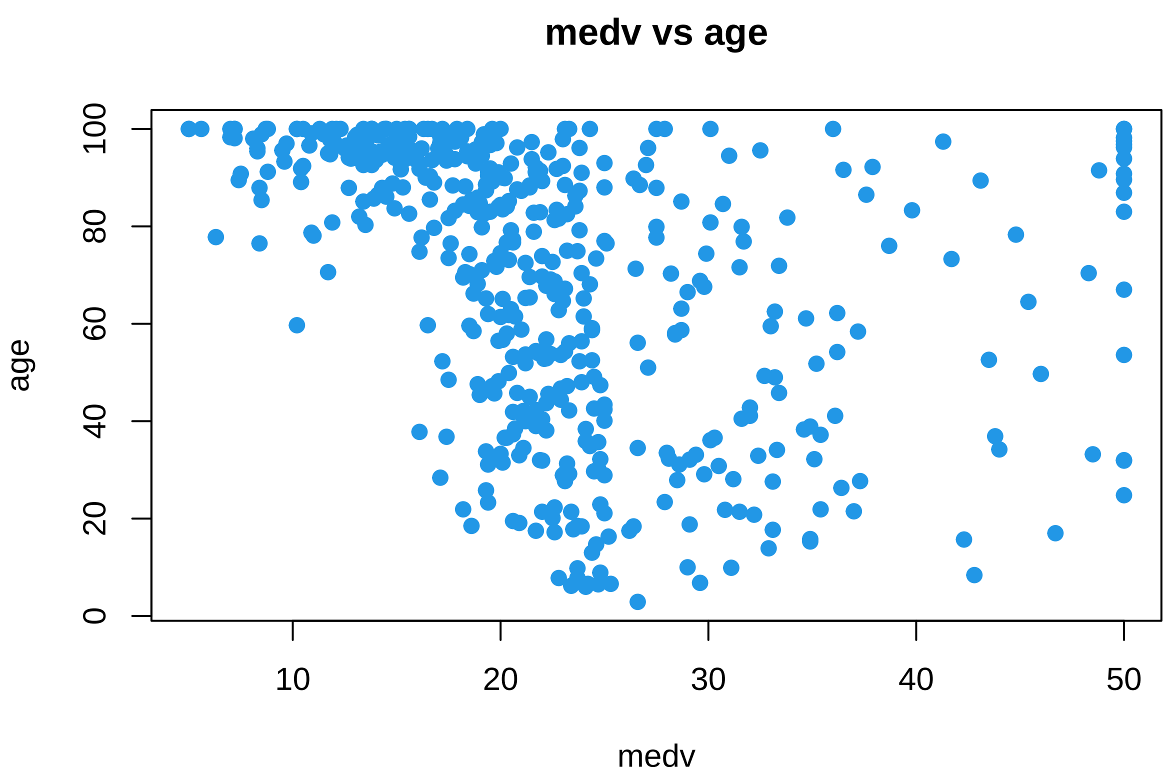
Part B

```r
boxplot(Boston$nox[which(Boston$chas == 1)], Boston$nox[which(Boston$chas == 0)],
        main = "Boxplot of Nitrogen Oxide Concentrates",
        col = c("darkgreen", "gold"), names = c("track bounds river", "other"))
```



Part C

```
plot(Boston$medv, Boston$age, xlab = "medv", ylab = "age", main = "medv vs age",
     col = 4, pch = 19)
```



There doesn't seem to be any particular trend in the scatterplot. Rather, the points seem to be scattered randomly, so medv and age may not be strongly linearly correlated. Some notable features are that there are fewer points in the lower left at the portion of the graph representing low age and low medv so there is no data for young people with medv, and that there are more points clustered together in the middle of the plot compared to the far right perhaps suggesting that higher levels of medv aren't too common.

Part D

```r
# crim 95% confidence interval
c(mean(Boston$crim) - (pt(unname(t.test(Boston$crim)[[1]]),
                          length(Boston$crim-1)))*(sd(Boston$crim)/length(Boston$crim)),
  mean(Boston$crim) + (pt(unname(t.test(Boston$crim)[[1]]),
                          length(Boston$crim-1)))*(sd(Boston$crim)/length(Boston$crim)))
```

```
## [1] 3.596524 3.630523
```

```r
# tax 95% confidence interval
c(mean(Boston$tax) - (pt(unname(t.test(Boston$tax)[[1]]),
                         length(Boston$tax-1)))*(sd(Boston$tax)/length(Boston$tax)),
  mean(Boston$tax) + (pt(unname(t.test(Boston$tax)[[1]]),
                         length(Boston$tax-1)))*(sd(Boston$tax)/length(Boston$tax)))
```

```
## [1] 407.9041 408.5702
```

Part E

```r
summary(lm(Boston$medv ~ Boston$crim))
```

```
##
## Call:
## lm(formula = Boston$medv ~ Boston$crim)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -16.957  -5.449  -2.007   2.512  29.800
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 24.03311    0.40914   58.74   <2e-16 ***
## Boston$crim -0.41519    0.04389   -9.46   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 8.484 on 504 degrees of freedom
## Multiple R-squared:  0.1508, Adjusted R-squared:  0.1491
## F-statistic: 89.49 on 1 and 504 DF,  p-value: < 2.2e-16
```

```r
# R-squared
summary(lm(Boston$medv ~ Boston$crim))[[9]]
```

```
## [1] 0.1490955
```

```r
# Interpretation
print(paste0("the model explains ", summary(lm(Boston$medv ~ Boston$crim))[[9]]*100,
             "% of the variability in medv"))
```

```
## [1] "the model explains 14.9095509662951% of the variability in medv"
```

The very low p-value suggests our results are statistically significant. The intercept of 24.03311 suggests that at 0 crim, we can expect 24.03311 crim on average. Our slope of -0.41519 suggest that with a 1 unit increase in crim, we see an 0.41519 descrease in medv

Problem 6

$X \sim Normal(\mu, \sigma_0^2)$

PDF: $\frac{1}{\sigma_0\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{x-\mu}{\sigma_0})^2}$

$L(\mu, \sigma_0^2) = \Pi_{i=1}^n(\frac{1}{\sigma_0\sqrt{2\pi}})e^{-\frac{1}{2}(\frac{x-\mu}{\sigma_0})^2}$

$= (\frac{1}{\sigma_0\sqrt{2\pi}})^n e^{-\frac{1}{2}\Sigma_{i=1}^n(\frac{x_i-\mu}{\sigma_0})^2}$

Prior distribution: $M \sim Normal(\mu_0, \rho_0)$

PDF: $n(\mu) = \frac{1}{\rho_0\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{\mu-\mu_0}{\rho_0})^2}$

WTS posterior density is $P_{\gamma_M}(\mu \mid x_1, ..., x_n) = \kappa e^{\frac{-(\mu-c)^2}{2\tau^2}}$

$P_\mu(\mu \mid x_1, ..., x_n) = (\frac{1}{\sigma_0\sqrt{2\pi}})^n e^{-\frac{1}{2}\Sigma_{i=1}^n(\frac{x_i-\mu}{\sigma_0})^2} \cdot \frac{1}{\rho_0\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{\mu-\mu_0}{\rho_0})^2}$

$= (\frac{1}{\sigma_0\sqrt{2\pi}})^n \frac{1}{\sigma_0\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{\mu-\mu_0}{\rho_0})^2+\frac{1}{\sigma_0^2}\Sigma_{i=1}^n(x_i-\mu)^2}$

$= e^{-\frac{1}{2}(\frac{1}{\rho_0}(\mu^2-2\mu\mu_0+mu_0^2)+\frac{1}{\sigma^2}(\Sigma_{i=1}^n x_i^2-2\mu\Sigma_{i=1}^n+n\mu^2))}$

$= e^{-\frac{1}{2}\frac{\Sigma_{i=1}^n x_i^2}{\sigma_0^2}+\frac{\mu_0^2}{\rho_0^2}} \cdot e^{-\frac{1}{2}(\frac{1}{\rho^3}(\mu_2-2\mu\mu_0)-\frac{2\mu}{\sigma_0^2}\Sigma_{i=1}^n x_i=\frac{n-\mu^2}{\sigma_0^2})}$

$= e^{-\frac{1}{2}\frac{\mu^2}{\rho_0^2}-\frac{2\mu\mu_0}{\rho_0^2}-\frac{2\mu}{\sigma_0^2}\Sigma_{i=1}^n x_i+\frac{n\mu^2}{\sigma_0^2}}$

$= e^{-\frac{1}{2}\frac{\sigma_0^2\mu^2-\sigma_0^2 2\mu\mu_0-2\rho_0^2\mu\Sigma_{i=1}^n x_i+n\mu^2\rho_0^2}{\sigma_0^2\rho_0^2}}$

$= e^{-\frac{1}{2}\frac{\mu^2(\sigma_0^2+n\rho_0^2)-2\mu(\sigma_0^2\mu_0=\rho_0^2\Sigma_{i=1}^n x_i)}{\sigma_0^2\rho_0^2}}$

$= e^{-\frac{1}{2}\frac{\mu^2-2\mu\frac{(\sigma_0^2\mu_0=\rho_0^2\Sigma_{i=1}^n x_i)}{\sigma_0^2+n\rho_0^2}}{\frac{\sigma_0^2\rho_0^2}{\sigma_0^2+n\rho_0^2}}}$

$= e^{-\frac{1}{2}\frac{\mu-(\frac{(\sigma_0^2\mu_0=\rho_0^2\Sigma_{i=1}^n x_i)}{\sigma_0^2+n\rho_0^2})^2}{\frac{\sigma_0^2\rho_0^2}{\sigma_0^2+n\rho_0^2}}}$

$P_\mu(\mu \mid x_1, ..., x_n) = \kappa e^{\frac{-(\mu-c)^2}{2\tau^2}}$ where $c = \frac{\sigma_0^2\mu_0+\rho_0^2\Sigma_{i=1}^n x_i}{\sigma_0^2+n\rho_0^2}$ and $\tau = \sqrt{\frac{\sigma^2\rho_0^2}{\sigma^2+n\rho_0^2}} = \frac{\sigma\rho_0}{\sqrt{\sigma^2+n\rho_0^2}}$

Problem 7

```
sigma_w <- sqrt(((5.2^2)/40)+((3.98^2)/49))
# confidence interval
c(14.5-13.75-(qnorm(p=.01/2, lower.tail=F)*sigma_w),
  14.5-13.75+(qnorm(p=.01/2, lower.tail=F)*sigma_w))
```

```
## [1] -1.824893  3.324893
```

```r
# interpretation/conclusion
print(paste0("we're 99% confident the true difference in means lies between ",
             c(14.5-13.75-(qnorm(p=.01/2, lower.tail=F)*sigma_w),
               14.5-13.75+(qnorm(p=.01/2, lower.tail=F)*sigma_w))[[1]],
             " and ", c(14.5-13.75-(qnorm(p=.01/2, lower.tail=F)*sigma_w),
                        14.5-13.75+(qnorm(p=.01/2, lower.tail=F)*sigma_w))[[2]]))
```

```
## [1] "we're 99% confident the true difference in means lies between -1.82489342411128 and 3.324893424
```

Since the confidence interval includes 0, there is a good chance that there is not a significant difference between the two means