$$S = 1 - S$$

doesn't resolve to $-1/12$

Damiens ROBERT

2019-12-16

1 Grandi's series

As every body knows, the Grandi's series is demonstrated by subtracting 2 said identical series.

- S
- 1 − S

Let's show why those 2 series are actually different.

2 S is a 1 dimension matrix

2.1 Transform a serie in a 1 dimension matrix

$$S = 1 - 1 + 1 - 1 + \ldots + 1$$

or

$$S = 1 - 1 + 1 - 1 + \dots - 1$$

We can transform S into a 1 dimension matrix using this method :

- Using only addition in the series by replacing the substraction of the number n_i by $+(-n_i)$ where i is the position in the serie.
- ullet Put each term of the serie of T terms in a 1 dimension matrix of size T

For the serie S, we then get :

$$S = (1, -1, ..., 1)$$

or

$$S = (1, -1, ..., -1)$$

2.2 0 as a 1 dimension matrix?

0 depends on the the serie it is being added.

2.3 justification

In order to be able to add 2 matrices, the matrices needs to have the same size.

2.4 Example with S

$$S = (1, -1, ..., 1)$$

$$S = (0, 0, ..., 0)$$

or

$$S = (1, -1, ..., -1)$$

$$S = (0, 0, ..., 0)$$

3 Conclustion

We cannot substract S from 1 because 1 is a natural number and S is a matrix.

What is 1 as a matrix? All 1's? Only one 1? Multiple 1's?

The 1 from natural number encoded as a matrix of size N is this matrix :

$$1=(1,\,1,\,\ldots\,\,,\,1)$$

or this one :

$$1 = (1, 0, \dots, 0)$$

or yet another one?

What is sure, is that 1 - S won't be the same dependending on your definition of 1 represented as a matrix and this ambiguity is what leads to a mistake in the Grandi's series demonstration.