

0 is not neutral in a serie
when
S in a Grandi's series
is
a matrix

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1 Introduction

In this text, we leave the notation of 0 intentionally ambiguous. I will fix this in a following text.

As everybody knows, the Grandi's series is demonstrated by the equality.

$$1 - S = S$$

Some mathematicians clame that it converges to $-1/12$.

2 Computing the wrong way

$$1 - S = S$$

where $S = 1 - 1 + 1 - 1 + \dots$

Wrong step is following !

$$1 - S = 1 - (1 - 1 + 1 - 1 + 1 + \dots)$$

Note that the wrong step is used in the Grandi series demonstration in order to prove $S = -1/12$.

3 Computing the right way

Let

$$1 = (1, 0, 0, 0, \dots, 0) \text{ where } \dots \text{ is all } 0\text{'s}$$

Then make the correct computation for $1 - S$ by subtracting 2 matrices.

Then notice that $S \neq 1 - S$

4 Another wrong way ...

We could then think we just need to subtract 2 matrices of the same size.

$$S + 0 = (1, -1, 1, -1, \dots, -1, 0)$$

or

$$0 + S = (0, 1, -1, 1, \dots)$$

Note that if we add those two matrices we get.

$$0 + S - S + 0 \neq 0$$

which would prove adding 0 is not a commutative operation when dealing with infinite series.

It would also prove that 0 is not the a neutral when dealing with infinite series.

4.1 What it truly is !

4.1.1 $1 - S$ produces the complement vector

A matrix is also a vector in geometry. When using multiple dimensions in a geographical space, the Natural number 1 is

$$1 = (1, 0, 0, 0, \dots, 0) \text{ where } \dots \text{ is all zero's}$$

then, if we say that $1 - S$ is subtracting S to the Natural number 1,

$$(1 - S) = (0, -1, 1, -1, \dots) \text{ where } \dots \text{ is all } 1 \text{ or } -1 \text{ one after another}$$

4.1.2 Vector arithmetics resolves correctly

You can then project this vector on the same origin than the vector (S) and find out the vector $+(1 - S)$ is $-(S)$.

$+(1 - S) + -(S) = 0$ becomes $-(S) + (S) = 0$ which resolves to $(0) = 0$.

4.1.3 Algebra also resolves correctly

Let

$$(0) + +(S) + -(S) = 0$$

becomes

$$(0, \dots, 0) + (1, -1, 1, -1, \dots) - (-1, 1, -1, 1, \dots) = 0$$

You will be able to make yourself this formal proof after reading the text where I prove that $(+\infty) + (-\infty) = 0$.

Let's remind that $(-S) = (1 - S)$.

5 Conclusion

$S = S$, $1 - S = -S$, $S - S = 0$ and $0 = 0$ when resolving $1 - S = S$ properly.

I will provide another more formal proof in a following text but I first need to introduce notations. This will be done in the $(+\infty) + (-\infty) = 0$ article.