# 0 is not neutral in a serie when S in a Grandi's series is a matrix

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## 1 Introduction

In this text, we leave the notation of 0 intentionally ambiguous. I will fix this in a following text.

As everybody knows, the Grandi's series is demonstrated by the equality.

$$1 - S = S$$

Some mathematicians clame that it converges to 1/2.

## 2 Computing the wrong way

1 - S = S

where S = 1 - 1 + 1 - 1 + ...

Wrong step is following !
1 - S = 1 - (1 - 1 + 1 - 1 + 1 + ...)

Note that the wrong step is used in the Grandi series demonstration in order to prove $S = 1/2$.

# 3 Computing the right way

Let

$$1 = (1, 0, 0, 0, ..., 0) \text{ where ... is all 0's}$$

Then make the correct computation for 1 - S by subtracting 2 matrices.

Then notice that S $\neq$ 1 - S

# 4 Another wrong way ...

We could then think we just need to subtract 2 matrices of the same size.

$$S + 0 = (1, -1, 1, -1, ... , -1 , 0)$$

or

$$0 + S = (0, 1, - 1, 1, ...)$$

Note that if we add those two matrices we get.

$$0 + S - S + 0 \neq 0$$

which would prove adding 0 is not a commutative operation when dealing with infinite series.

It would also prove that 0 is not the a neutral when dealing with infinite series.

## 4.1 What it truly is !

### 4.1.1 1 - S produces the complement vector

A matrix is also a vector in geometry. When using multiple dimensions in a geographical space, the Natural number 1 is

$$1 = (1, 0, 0, 0, ..., 0) \text{ where ... is all zero's}$$

then, if we say that 1 - S is subtracting S to the Natural number 1,

$$(1 - S) = (0, -1, 1, -1, ... ) \text{ where ... is all 1 or -1 one after another}$$

### 4.1.2 Vector arithmethics resolves correctly

You can then project this vector on the same origin than the vector (S) and find out the vector +(1 - S) is -(S).

+(1 - S) + -(S) = 0 becomes -(S) + (S) = 0 which resolves to (0) = 0.

#### 4.1.3 Algebra also resolves correctly

Let

$$(0) + +(S) + -(S) = 0$$

becomes

$$(0, ..., 0) + (1, -1, 1, -1, ...) - (-1, 1, -1, 1, ...) = 0$$

You will be able to make yourself this formal proof after reading the texte where I prove thaty $(+\infty) + (-\infty) = 0$.

Let's remind that (-S) = (1 - S).

## 5 Conclusion

S = S, 1 - S = -S, S - S = 0 and 0 = 0 when resolving 1 - S = S properly.

I will provide another more formal proof in a following text but I first need to introduce notations. This will be done in the $(+\infty) + (-\infty) = 0$ article.