L2 - Resumo

Sintaxe

$$
\begin{array}{rcl}
e & ::= & n \mid b \mid e_1 \ op \ e_2 \mid \texttt{if} \ e_1 \ \texttt{then} \ e_2 \ \texttt{else} \ e_3 \\
 & \mid & l \texttt{:=} e \mid \texttt{!} \ l \\
 & \mid & \texttt{skip} \mid e_1 \texttt{;} \ e_2 \\
 & \mid & \texttt{while} \ e_1 \ \texttt{do} \ e_2 \\
 & \mid & fn \ x{:}T \Rightarrow e \mid e_1 \ e_2 \mid x \\
 & \mid & \texttt{let} \ x{:}T = e_1 \ \texttt{in} \ e_2 \ \texttt{end} \\
 & \mid & \texttt{let rec} \ f{:}T_1 \to T_2 = (fn \ y{:}T_1 \Rightarrow e_1) \ \texttt{in} \ e_2 \ \texttt{end}
\end{array}
$$

$$
v \ ::= \ n \mid b \mid \texttt{skip} \mid fn \ x{:}T \Rightarrow e
$$

onde

$$
\begin{array}{rcl}
b & \in & \{\texttt{true}, \texttt{false}\} \\
n & \in & \textit{conjunto de numerais inteiros} \\
l & \in & \textit{conjunto de endereços} \\
op & \in & \{+, \geq\}
\end{array}
$$

$$
\begin{array}{rcl}
T & ::= & \mathsf{int} \mid \mathsf{bool} \mid \mathsf{unit} \mid T_1 \to T_2 \\
T_{loc} & ::= & \mathsf{int\ ref}
\end{array}
$$

Semântica Operacional

$$
\frac{[\![n]\!] = [\![n_1 + n_2]\!]}{\langle n_1 + n_2, \ \sigma\rangle \longrightarrow \langle n, \ \sigma\rangle} \qquad (\text{OP}+)
$$

$$
\frac{[\![b]\!] \models [\![n_1 \geq n_2]\!]|}{\langle n_1 \geq n_2, \ \sigma\rangle \longrightarrow \langle b, \ \sigma\rangle} \qquad (\text{OP}\geq)
$$

$$
\frac{\langle e_1, \ \sigma\rangle \longrightarrow \langle e_1', \ \sigma'\rangle}{\langle e_1 \ op \ e_2, \ \sigma\rangle \longrightarrow \langle e_1' \ op \ e_2, \ \sigma'\rangle} \qquad (\text{OP1})
$$

$$
\frac{\langle e_2, \ \sigma\rangle \longrightarrow \langle e_2', \ \sigma'\rangle}{\langle v \ op \ e_2, \ \sigma\rangle \longrightarrow \langle v \ op \ e_2', \ \sigma'\rangle} \qquad (\text{OP2})
$$

$$
\langle \texttt{if true then} \ e_2 \ \texttt{else} \ e_3, \ \sigma\rangle \longrightarrow \langle e_2, \ \sigma\rangle \qquad (\text{IF1})
$$

$$
\langle \texttt{if false then} \ e_2 \ \texttt{else} \ e_3, \ \sigma\rangle \longrightarrow \langle e_3, \ \sigma\rangle \qquad (\text{IF2})
$$

$$
\frac{\langle e_1, \ \sigma\rangle \longrightarrow \langle e_1', \ \sigma'\rangle}{\langle \texttt{if} \ e_1 \ \texttt{then} \ e_2 \ \texttt{else} \ e_3, \ \sigma\rangle \longrightarrow \langle \texttt{if} \ e_1' \ \texttt{then} \ e_2 \ \texttt{else} \ e_3, \ \sigma'\rangle} \qquad (\text{IF3})
$$

$$
\langle \texttt{skip;} e_2, \ \sigma\rangle \longrightarrow \langle e_2, \ \sigma\rangle \qquad (\text{SEQ1})
$$

$$\frac{\langle e_1,\ \sigma\rangle \longrightarrow \langle e_1',\ \sigma'\rangle}{\langle e_1\texttt{;}e_2,\ \sigma\rangle \longrightarrow \langle e_1'\texttt{;}e_2,\ \sigma'\rangle} \quad (\textsc{seq2})$$

$$\frac{l \in Dom(\sigma)}{\langle l\texttt{:=}\ n,\ \sigma\rangle \longrightarrow \langle \texttt{skip},\ \sigma[l \mapsto n]\rangle} \quad (\textsc{atr1})$$

$$\frac{\langle e,\ \sigma\rangle \longrightarrow \langle e',\ \sigma'\rangle}{\langle l\texttt{:=}\ e,\ \sigma\rangle \longrightarrow \langle l\texttt{:=}\ e',\ \sigma'\rangle} \quad (\textsc{atr2})$$

$$\frac{l \in Dom(\sigma) \quad \sigma(l) = n}{\langle \texttt{!}\ l,\ \sigma\rangle \longrightarrow \langle n,\ \sigma\rangle} \quad (\textsc{deref})$$

$$\langle \texttt{while}\ e_1\ \texttt{do}\ e_2,\ \sigma\rangle \longrightarrow \langle \texttt{if}\ e_1\ \texttt{then}\ (e_2\texttt{;while}\ e_1\ \texttt{do}\ e_2)\ \texttt{else}\ \texttt{skip},\ \sigma\rangle \quad (\textsc{while})$$

$$\langle (fn\ x\texttt{:}T \Rightarrow e)\ v,\ \sigma\rangle \longrightarrow \langle \{v/x\}e,\ \sigma\rangle \quad (\beta)$$

$$\frac{\langle e_2,\ \sigma\rangle \longrightarrow \langle e_2',\ \sigma'\rangle}{\langle v\ e_2,\ \sigma\rangle \longrightarrow \langle v\ e_2',\ \sigma'\rangle} \quad (\textsc{app1})$$

$$\frac{\langle e_1,\ \sigma\rangle \longrightarrow \langle e_1',\ \sigma'\rangle}{\langle e_1\ e_2,\ \sigma\rangle \longrightarrow \langle e_1'\ e_2,\ \sigma'\rangle} \quad (\textsc{app2})$$

$$\langle \texttt{let}\ x\texttt{:}T = v\ \texttt{in}\ e_2\ \texttt{end},\ \sigma\rangle \longrightarrow \langle \{v/x\}e_2,\ \sigma\rangle \quad (\textsc{let1})$$

$$\frac{\langle e_1,\ \sigma\rangle \longrightarrow \langle e_1',\ \sigma'\rangle}{\langle \texttt{let}\ x\texttt{:}T = e_1\ \texttt{in}\ e_2\ \texttt{end},\ \sigma\rangle \longrightarrow \langle \texttt{let}\ x\texttt{:}T = e_1'\ \texttt{in}\ e_2\ \texttt{end},\ \sigma'\rangle} \quad (\textsc{let2})$$

$$\begin{array}{c} \langle \texttt{let rec}\ f\texttt{:}T_1 \to T_2 = (fn\ y\texttt{:}T_1 \Rightarrow e_1)\ \texttt{in}\ e_2\ \texttt{end},\ \sigma\rangle \\ \longrightarrow \\ \langle \{(fn\ y\texttt{:}T_1 \Rightarrow \texttt{let rec}\ f\texttt{:}T_1 \to T_2 = (fn\ y\texttt{:}T_1 \Rightarrow e_1)\ \texttt{in}\ e_1\ \texttt{end})/f\}e_2,\ \sigma\rangle \end{array} \quad (\textsc{letrec})$$

Sistema de Tipos

$$\Gamma;\Delta \vdash n : \mathsf{int} \quad (\textsc{Tint})$$

$$\Gamma;\Delta \vdash b : \mathsf{bool} \quad (\textsc{Tbool})$$

$$\frac{\Gamma;\Delta \vdash e_1 : \mathsf{int} \qquad \Gamma;\Delta \vdash e_2 : \mathsf{int}}{\Gamma;\Delta \vdash e_1 + e_2 : \mathsf{int}} \quad (\textsc{T}+)$$

$$\frac{\Gamma;\Delta \vdash e_1 : \mathsf{int} \qquad \Gamma;\Delta \vdash e_2 : \mathsf{int}}{\Gamma;\Delta \vdash e_1 \geq e_2 : \mathsf{bool}} \quad (\textsc{T}\geq)$$

$$\frac{\Gamma;\Delta \vdash e_1 : \mathsf{bool} \qquad \Gamma;\Delta \vdash e_2 : T \qquad \Gamma;\Delta \vdash e_3 : T}{\Gamma;\Delta \vdash \texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ e_3 : T} \quad (\textsc{Tif})$$

$$\frac{\Gamma;\Delta \vdash e : \mathsf{int} \qquad \Delta(l) = \mathsf{int\ ref}}{\Gamma;\Delta \vdash l \mathtt{:=} e : \mathsf{unit}} \tag{TATR}$$

$$\frac{\Delta(l) = \mathsf{int\ ref}}{\Gamma;\Delta \vdash \mathtt{!}\ l : \mathsf{int}} \tag{TDEREF}$$

$$\Gamma;\Delta \vdash \mathtt{skip} : \mathsf{unit} \tag{TSKIP}$$

$$\frac{\Gamma;\Delta \vdash e_1 : \mathsf{unit} \qquad \Gamma;\Delta \vdash e_2 : T}{\Gamma;\Delta \vdash e_1 \mathtt{;} e_2 : T} \tag{SEQ}$$

$$\frac{\Gamma;\Delta \vdash e_1 : \mathsf{bool} \qquad \Gamma;\Delta \vdash e_2 : \mathsf{unit}}{\Gamma;\Delta \vdash \mathtt{while}\ e_1\ \mathtt{do}\ e_2 : \mathsf{unit}} \tag{TWHILE}$$

$$\frac{\Gamma(x) = T}{\Gamma;\Delta \vdash x : T} \tag{TVAR}$$

$$\frac{\Gamma, x : T;\Delta \vdash e : T'}{\Gamma;\Delta \vdash fn\ x : T \Rightarrow e : T \to T'} \tag{TFN}$$

$$\frac{\Gamma;\Delta \vdash e_1 : T \to T' \qquad \Gamma;\Delta \vdash e_2 : T}{\Gamma;\Delta \vdash e_1\ e_2 : T'} \tag{TAPP}$$

$$\frac{\Gamma;\Delta \vdash e_1 : T \qquad \Gamma, x : T;\Delta \vdash e_2 : T'}{\Gamma;\Delta \vdash \mathtt{let}\ x \mathtt{:} T = e_1\ \mathtt{in}\ e_2\ \mathtt{end} : T'} \tag{TLET}$$

$$\frac{\Gamma, f : T_1 \to T_2, y : T_1;\Delta \vdash e_1 : T_2 \qquad \Gamma, f : T_1 \to T_2;\Delta \vdash e_2 : T}{\Gamma;\Delta \vdash \mathtt{let\ rec}\ f \mathtt{:} T_1 \to T_2 = (fn\ y \mathtt{:} T_1 \Rightarrow e_1)\ \mathtt{in}\ e_2\ \mathtt{end} : T} \tag{TLETREC}$$

Propriedades

Teorema 1 (Determinismo) Se $\langle e,\ \sigma\rangle \longrightarrow \langle e',\ \sigma'\rangle$ e se $\langle e,\ \sigma\rangle \longrightarrow \langle e'',\ \sigma''\rangle$ então $\langle e',\ \sigma'\rangle = \langle e'',\ \sigma''\rangle$.

Teorema 2 (Progresso) Se e é fechado e $\Gamma;\Delta \vdash e : T$ e $Dom(\Delta) \subseteq Dom(\sigma)$ então e é um valor ou existe $e';\sigma'$ tal que $\langle e,\ \sigma\rangle \longrightarrow \langle e',\ \sigma'\rangle$ e e' é fechado.

Teorema 3 (Preservação) Se e é fechado e $\Gamma;\Delta \vdash e : T$, $Dom(\Delta) \subseteq Dom(\sigma)$ e $\langle e,\ \sigma\rangle \longrightarrow \langle e',\ \sigma'\rangle$ então $\Gamma;\Delta \vdash e' : T$ e $Dom(\Delta) \subseteq Dom(\sigma')$.

Teorema 4 (Decidibilidade da Tipabilidade) Dados ambiente Γ e expressão e, existe algoritmo que decide se existe tipo T tal que $\Gamma;\Delta \vdash e : T$ é verdadeiro ou não.