Engi 9867 Advanced Computing Concepts for Engineering

R. Shahidi (c) 2021

April 16, 2021

Total marks: 100
Time: 3 hrs
Name:
Student #:

You have 180 minutes to write this final examination. After you have completed your solutions, please upload them to the appropriate folder in D2L after first scanning your solutions using a scanner or your favourite phone scanning app (e.g., CamScanner).

You are not to communicate with any other human about the exam during the exam period. You are permitted to consult any of the course material available on D2L, but nothing beyond that.

The following definitions may be of use.

- An *undirected graph* G is a pair of sets (V, E) such that each edge (i.e., each member of E) is a set consisting of 1 or 2 vertices (elements of V).
- A *finite path* in a graph $G = (V, E)$ is an alternating sequence of edges and vertices

$$[v_0, e_0, v_1, e_1, ..., e_{n-1}, v_n] \ ,$$

  starting and ending with a vertex, such that, for each i, $v_i, v_{i+1} \in e_i$. The length of a path is the number of edges it has.
- A path is *acyclic* iff no vertex appears twice.
- A *Hamiltonian cycle* in $G = (V, E)$ is a path in that starts and ends at the the same vertex, but otherwise contains each vertex exactly once.
- The *Hamiltonian Cycle Problem* is the NP-Complete problem: Given an undirected graph G, does it have a Hamiltonian cycle?
- A *Hamiltonian path* in $G = (V, E)$ is a path in the graph G which contains each vertex exactly once.
- The *Hamiltonian Path Problem* is the NP-Complete problem: Given an undirected graph G, does it have a Hamiltonian path?

Student #: _____

Q0 Upper Bounds [14]
Consider the following problem:

- **Input**: An array a of length N of numbers and an array of numbers b of length N.
- **Output**: An array p of length N of integers such that for each $i \in \{0, ..N\}$, $p'(i)$ is the number of numbers in array a that are greater than $b(i)$.
- Assume that the only operations allowed on arrays a and b are comparing items using $<$ and possibly swapping items of the arrays.

Show that $\Omega(N \log N)$ is a lower bound for this problem.

Student #: _____

Q1 Complexity [**8**] Consider the following functions.

$$f(n) = n \log_2 n + n$$
$$g(n) = 10^9 n^2$$
$$h(n) = e^n$$

True or false

- $f \in O(g)$

- $f \in \Omega(g)$

- $h \in O(g)$

- $h \in \Omega(g)$

Student #: _____

Q2 Regular languages [22]

Consider the language L over the alphabet $\Sigma = \{a, b, c\}$, such that a word is in L if and only if the sum of the numbers of a's and b's in the word is even *or* the number of c's is even.

(a) [**8**] Construct an NDFR that will recognize L.

(b) [**8**] Convert the NDFR in part (a) to a DFR.

(c) [**6**] Find a regular expression for L.

Student #: _____

Q3 Reducibility [12]

(a) [**4**] Define what it means for a problem Q to be polynomially reducible to problem P.

(b) [**8**] Show that the Hamiltonian path problem is polynomially reducible to the Hamiltonian circuit problem

Student #: _____

Q4 Formal Specification [22]

Consider the following specification where a, b and c are arrays of integers of length n

$$f = \langle \forall i \in \{0, ..n\} \cdot c\prime(i) = (\max j \in \{0, ..i\} \cdot a(j) + b(j)) \rangle$$

Read the whole question before beginning to answer. The idea is to develop a command of the form

$$f \sqsubseteq m_0; \textbf{while } \mathcal{A} \textbf{ do } h_0$$

(a) [**5**] Propose a loop invariant $\mathcal{I}$

$\mathcal{I}$ is

(b) [**5**] Let $g = (\langle \mathcal{I} \rangle \Rightarrow f)$. Let $m = \langle \mathcal{I}' \wedge a' = a \rangle$ so that $f \sqsubseteq m; g$.

Give a command that refines m.

$m \sqsubseteq$

(c) [**5**] Give an expression $\mathcal{A}$ and such that

$$\mathcal{I} \wedge \neg \mathcal{A} \Rightarrow \forall i \in \{0, ..n\} \cdot c(i) = (\max j \in \{0, ..i\} \cdot a(j) + b(j))$$

is valid, i.e., so that $\langle \neg \mathcal{A} \rangle \Rightarrow g \sqsubseteq \textbf{skip}$.

$\mathcal{A}$ is

(d) [**2**] Propose a bound expression for the loop.

(e) [**5**] Let $h = \langle \mathcal{A} \wedge \mathcal{I} \Rightarrow \mathcal{I}' \wedge a' = a \rangle$ so that

$$g \sqsubseteq \textbf{if } \mathcal{A} \textbf{ then } (h; g) \textbf{ else skip}$$

Give a command that refines h and that makes the bound expression smaller.

Student #: _____

Q5 Context-Free Grammars [16] Let G be the grammar

$$\begin{aligned} S' &\rightarrow S\$ \\ S &\rightarrow aB|bA \\ A &\rightarrow a|aS|bAA \\ B &\rightarrow b|bS|aBB \end{aligned}$$

For the string $aaabbabbba\$$ find a

(a) **[6]** leftmost derivation

(b) **[6]** parse tree

(c) **[4]** Is G in $LL(1)$? Why or why not?

Student #: _____

Q6 Neural Networks [6]

Recall from lectures that if a function can be implemented in time $T(n)$, then there is a neural network of size $cT(n)^2 + b$ for some positive real numbers b and c such that there is an assignment of weights implementing the same function. Comment on the complexity of finding the optimal assignment of weights in the neural network implementing the same function.

Student #: _____

Please sign the following declaration:

I did the exam alone without extra help outside of the course material posted on D2L and did the exam without consulting any other human being. I also did not help anybody else with the exam.

_____

(Your signature)